Hindawi
Scientific Programming
Volume 2022, Article ID 2272672, 27 pages
https://doi.org/10.1155/2022/2272672

Research Article

An Improved Adaptive Human Learning Optimization Algorithm with Reasoning Learning

Pinggai Zhang[1,2] **Baoling Hu**[3] **Dengji Li**[2] **Quanle Wang**[1] **and Yi Zhou**[1]

[1]*Industrial Process Control Optimization and Automation Engineering Research Center, School of Electronic Engineering, Chaohu University, Chaohu, Anhui 238024, China*
[2]*Shanghai Key Laboratory of Power Station Automation Technology, School of Mechatronics Engineering and Automation, Shanghai University, Shanghai 200072, China*
[3]*School of Information Engineering, Chaohu University, Chaohu Anhui 238024, China*

Correspondence should be addressed to Pinggai Zhang; zhangpinggai@shu.edu.cn and Baoling Hu; hubaoling_chu@yeah.net

Received 24 December 2021; Revised 7 March 2022; Accepted 18 March 2022; Published 4 May 2022

Academic Editor: Qianchuan Zhao

Human Learning Optimization (HLO) is a simple yet highly efficient metaheuristic developed based on a simplified human learning model. To further extend the research of HLO, the social reasoning learning operator (SRLO) is introduced. However, the learning ability of social imitating learning operator (SILO) and SRLO is constant in the process of iterations, which is not true in a real human population as humans often adopt dynamic learning strategies to solve the problem. Inspired by this fact, an improved adaptive human learning optimization algorithm with reasoning learning (AHLORL) is proposed to enhance the global search ability, in which an adaptive *ps* strategy is carefully designed to sufficiently motivate the roles of SILO and SRLO and dynamically adjust the learning efficiency of the algorithm at different stages of iterations. Then, a comprehensive parameter study is performed to explain why the proposed adaptive strategy can exploit the optimization ability of SILO and SRLO effectively. Finally, the AHLORL is applied to solve the CEC 15 benchmark functions as well as multidimensional knapsack problems (MKPs), and its performance is compared with the previous HLO variants as well as the other recent metaheuristics. The experimental results show that the proposed AHLORL outperforms the other algorithms in terms of search accuracy and scalability.

1. Introduction

With the increasing complexity of industrial production, traditional optimization algorithms cannot effectively solve the optimal solution of the problem. To break through the limitations of traditional optimization algorithms, some new metaheuristics, i.e., Particle Swarm Optimization (PSO) [1], Ant Colony Optimization (ACO) [2], Artificial Bee Colony (ABC) [3], Monarch Butterfly Optimization (MBO) [4], Moth Search Algorithm (MSA) [5], Harris Hawk Optimization (HHO) [6], and Slime Mould Algorithm (SMA) [7], have emerged in the field of intelligent optimization to solve the complex optimization problems, such as medical analysis [8], fault diagnosis [9], and model design [10]. Compared with other mechanisms in nature, humans have a higher level of intelligence and strong learning ability, which can solve complex problems that other creatures cannot. Inspired by this fact, Wang et al. proposed a new Human Learning Optimization Algorithm (HLO) [11] by employing the learning ability of human beings. The HLO [11] algorithm is a highly effective metaheuristic algorithm transforming the accumulated knowledge into computational intelligence, in which three learning operators, i.e., the random learning operator (RLO), the individual learning operator (ILO), and the social learning operator (SLO), are developed to yield new candidates to search for the optimal solution.

To further improve the performance of HLO, a few enhanced variants have been subsequently developed. In 2015, an adaptive simplified human learning optimization algorithm (ASHLO) [12] is proposed to achieve a better trade-off between exploration and exploitation, in which the *pr* and *pi*, which are two control parameters determining the probabilities of performing RLO, ILO, and SLO, adopt the linearly decreasing adaptive strategy and linearly increasing adaptive strategy during the whole search process to strengthen the search efficiency of the algorithm and relieve the effort of parameter setting. Later, to dynamically switch the ability between the global search and local search, a sine-cosine adaptive human learning optimization algorithm (SCHLO) [13] is developed in which the *pr* and *pi* are adjusted by the sine and cosine functions to help SCHLO escape from the local optimal and get better results. Recently, an improved adaptive human learning optimization algorithm (IAHLO) [14] is presented to further take an insight into the role of RLO, in which the control parameter *pr* is precisely tuned so that IAHLO can efficiently explore the interesting solution areas at the early stages of iterations and perform the local search at the later stages of the search process. Inspired by the IQ scores of humans, a diverse human learning optimization algorithm (DHLO) [15] is proposed in which the values of control parameter *pi* are randomly initiated by a Gaussian distribution and dynamically updated based on the *pi* value of the best individual over the course of the search process.

The adaptive strategies of *pr* and *pi* provide a significant improvement in HLO. However, the problem of trapping in the local optimum remains. So, the re-learning operation [16] is introduced to help HLO escape from the local optima and acquire better performance if its fitness is not renewed in a certain number of iterations. To further extend HLO, a new hybrid-coded human learning optimization algorithm (HcHLO) [17] is presented to efficiently tackle mixed-variable optimization problems, in which a continuous human learning optimization (CHLO) is proposed to solve real-coded parameters while other variables are optimized by the standard HLO. Besides, a novel discrete human learning optimization algorithm is presented to tackle the scheduling problem [18]. Until now, the HLO algorithms have been successfully applied to tackle the different types of problems, such as financial markets forecasting [19], engineering optimization problems [14], knapsack problems [16], optimal power flow calculation [20, 21], extractive text summarization [22], furnace flame recognition [23], scheduling problems [24], intelligent control [25, 26], and image segmentation [27]. Especially, HLO achieved the best-so-far results on two well-studied sets of multidimensional knapsack problems, i.e., 5.100 and 10.100 [16], as well as the set of mixed-variable optimization problems [17] compared to the other publicly reported metaheuristics, which proves that the HLO algorithm is a promising metaheuristic optimization algorithm and has important research significance.

To further improve the global search ability of HLO, a novel social reasoning learning operator (SRLO) is developed and human learning optimization with reasoning learning (HLORL) [28] is presented. In HLORL, the SILO (original SLO) and SRLO are inspired by social imitating learning strategy and social reasoning learning strategy, respectively. Among them, social imitating learning is an efficient learning strategy [29], which can efficiently accumulate knowledge by copying optimal individuals in a certain environment. For example, students generally imitate the thinking of their teachers to build a new knowledge framework efficiently [30], and preschool children usually imitate the behavior of their parents to solve problems effectively [31]. Compared with the social imitating learning strategy, social reasoning learning is a powerful learning strategy with logical thinking [32], which can excavate some deeper common characteristics by using surface-related information in an uncertain environment. For instance, economists usually use reasoning ability to predict economic changes [33], and police often adopt the found evidence to reason the truth effectively when they are analyzing the criminal case [34]. Based on the different characteristics between social imitating learning and social reasoning learning, these two learning strategies can effectively play different roles at different stages of human learning.

Although the learning efficiency of SILO and SRLO will change with the environment of human learning, it is not considered in the standard HLORL, which uses two fixed learning probabilities to perform SILO and SRLO over the course of the search process. Now cultural evolution researchers [35, 36] believe that humans often reasonably choose the corresponding learning strategy and perform the optimal behaviors in different learning environments, which is better for human beings to further improve the learning efficiency and obtain better learning results. Therefore, an improved adaptive human learning optimization algorithm with reasoning learning (AHLORL) is proposed in this paper, in which the adaptive learning probabilities of SILO and SRLO are introduced to dynamically adjust the learning efficiency. And a thorough analysis and comparison are performed to explain why the proposed adaptive strategy can effectively exploit the optimization ability of SILO and SRLO and further enhance the global search ability of the algorithm. This paper makes the following contributions:

(1) This paper proposes an improved adaptive human learning optimization algorithm with reasoning learning (AHLORL)
(2) An adaptive learning probability of SILO and SRLO is introduced to dynamically adjust the learning efficiency of the algorithm during the iterative search
(3) A thorough analysis and comparison are performed to explain why the proposed adaptive strategy can effectively exploit the optimization ability of SILO and SRLO and further enhance the global search ability of the algorithm
(4) The results demonstrate that the proposed AHLORL has significant advantages over the previous HLO variants

The rest of this paper is organized as follows: Section 2 proposes the proposed AHLORL algorithm in detail. A comprehensive parameter learning is performed in Section 3 to analyze and explain the superiority of the proposed AHLORL algorithm. After that, we describe the experiment with settings, CEC 15 benchmark functions, and MKPs and analyze the results in Section 4. Finally, the conclusion is given with the overall result of this proposed AHLORL algorithm.

2. Adaptive Human Learning Optimization with Reasoning Learning

2.1. Initialization. Like the HLORL, all computations in AHLORL are also a discrete process and the binary-coding framework is adopted to denote the population. At the initialization stage, the individual x_i is initialized as a binary string of length M with "0" or "1" stochastically as the following equation:

$$x_i = \left[x_{i1} x_{i2} \ldots x_{ij} \ldots x_{iM}\right], x_{ij} \in \{0,1\}, 1 \le i \le N, 1 \le j \le M, \tag{1}$$

where x_i denotes the i-th individual, x_{ij} indicates the j-th bit of the i-th individual, and N and M are the number of individuals and the dimension of solutions, respectively.

2.2. Learning Operators

2.2.1. Random Learning Operator. Random learning [37] always occurs at the early stages of human learning because of the lack of prior knowledge of new problems. With the progress of the search, this random learning strategy also remains, which can help AHLORL keep the exploration ability to obtain new strategies. And therefore, AHLORL adopts the random learning operator as equation (2) to imitate these phenomena of human random learning:

$$x_{ij} = RE(0,1) = \begin{cases} 0, & 0 \le r_1 \le 0.5, \\ 1, & \text{else}, \end{cases} \tag{2}$$

where r_1 is a stochastic number between 0 and 1.

2.2.2. Individual Learning Operator. Individual learning [38] is an important learning ability in the process of human evolution, which can build an individual's knowledge database to help humans avoid the same mistakes and improve learning efficiency more effectively. Inspired by this learning mechanism, the best individual solutions are saved in the individual knowledge database (IKD) as

$$\mathrm{IKD} = \begin{bmatrix} ikd_1 \\ ikd_2 \\ \vdots \\ ikd_i \\ \vdots \\ ikd_N \end{bmatrix} 1 \le i \le N, \tag{3}$$

$$ikd_i = \begin{bmatrix} ikd_{i1} \\ ikd_{i2} \\ \vdots \\ ikd_{ip} \\ \vdots \\ ikd_{iL} \end{bmatrix} = \begin{bmatrix} ik_{i1,1} & ik_{i1,2} & \cdots & ik_{i1,j} & \cdots & ik_{i1,M} \\ ik_{i2,1} & ik_{i2,2} & \cdots & ik_{i2,j} & \cdots & ik_{i2,M} \\ \vdots & \vdots & & \vdots & & \vdots \\ ik_{ip,1} & ik_{ip,2} & \cdots & ik_{ip,j} & \cdots & ik_{ip,M} \\ \vdots & \vdots & & \vdots & & \vdots \\ ik_{iL,1} & ik_{iL,2} & \cdots & ik_{iL,j} & \cdots & ik_{iL,M} \end{bmatrix} 1 \le p \le L, \tag{4}$$

where ikd_i stands for the IKD of individual i; N and L are the size of IKD and ikd_i, respectively; and ikd_{ip} denotes the p-th best solution of the individual i. When AHLORL performs the individual learning operator (ILO), a new candidate solution is generated from the IKD as the following equation:

$$x_{ij} = ik_{ip,j}. \tag{5}$$

2.2.3. Social Imitating Learning Operator. Social imitating learning [39] is a potentially cheap way of acquiring valuable information and plays a fundamental role in development, communication, interaction, learning, and culture, which can greatly hasten the process of independent learning by enabling the subject to perform the correct response sooner than others. And human beings usually use the social imitating learning strategy to learn from others' better experiences and improve learning efficiency. To simulate this advanced learning strategy, the social knowledge data (SKD) is adopted to reserve the best social solution as follows:

$$\mathrm{SKD} = \begin{bmatrix} skd_1 \\ skd_2 \\ \vdots \\ skd_q \\ \vdots \\ skd_H \end{bmatrix} = \begin{bmatrix} sk_{11} & sk_{12} & \cdots & sk_{1j} & \cdots & sk_{1M} \\ sk_{21} & sk_{22} & \cdots & sk_{2j} & \cdots & sk_{2M} \\ \vdots & \vdots & \vdots & \vdots & & \\ sk_{q1} & sk_{q2} & \cdots & sk_{qj} & \cdots & sk_{qM} \\ \vdots & \vdots & \vdots & \vdots & & \\ sk_{H1} & sk_{H2} & \cdots & sk_{Hj} & \cdots & sk_{HM} \end{bmatrix} 1 \le q \le H, \tag{6}$$

where skd_q means the q-th solution in the SKD, H is the size of SKD, and skd_q stands for the j-th dimension of the q-th solution in the SKD. The social imitating learning operator (SILO) is performed to generate a new candidate solution as follows:

$$x_{ij} = sk_{qj}. \tag{7}$$

2.2.4. Social Reasoning Learning Operator. Social reasoning learning is a hallmark of human intelligence [40], which allows humans to attempt powerful generalizations from sparse data when learning about unobserved properties and causal relationships. Demetriou and Kazi [41] point out that logic in the mind is the culmination of a long developmental process, extending into adolescence, and Cesana-Arlotti et al. [42] discover that infants can also use elementary logical representations to frame and prune hypotheses. Modern psychologists are in reasonable agreement [43] that humans usually adopt the predictable methods of social reasoning learning to get deeper characteristic information. Inspired by the social reasoning learning strategy, a social reasoning learning operator (SRLO) is designed to generate a new candidate solution as the following equations:

$$x_{ij} = \mathrm{SRLO}(0,1), \tag{8}$$

$$\mathrm{SRLO}(0,1) = \begin{cases} 1, & \text{if } r_2 \le f\left(srl_{ij}\right), \\ 0, & \text{otherwise}, \end{cases} \tag{9}$$

$$f\left(srl_{ij}\right) = \begin{cases} 0.0005, & \text{if } c = 0, \\ 0.6633, & \text{if } c = 1, \\ 0.3367, & \text{if } c = 2, \\ 0.9995, & \text{if } c = 3, \end{cases} \tag{10}$$

$$c = ik_{i1,j} + ik_{i2,j} + ik_{i3,j}, \tag{11}$$

where $\mathrm{SRLO}(0,1)$ stands for the social reasoning learning operation; r_2 is a stochastic number between 0 and 1; $f(srl_{ij})$ means the social reasoning learning probability model; $ik_{i1,j}$, $ik_{i2,j}$, and $ik_{i3,j}$ are the j-th dimension the best knowledge saved in the IKD of three randomly chosen individuals, i.e., individuals i_1, i_2, and i_3, and $i_1 \neq i_2 \neq i_3 \neq i$.

In summary, AHLORL adopts the random learning operator, individual learning operator, social imitating learning operator, and social reasoning learning operator to yield new candidate solutions and searches for the optimal, which are presented as follows:

$$x_{ij} = \begin{cases} \mathrm{RE}(0,1), & 0 \le r \le pr, \\ ik_{ip,j}, & pr < r \le pi, \\ sk_{qj}, & pi < r \le ps, \\ \mathrm{SRLO}(0,1), & \text{else}, \end{cases} \tag{12}$$

where r means a stochastic number between 0 and 1 and pr, $(pi - pr)$, $(ps - pi)$, and $(1 - ps)$ are the probabilities of performing RLO, ILO, SILO, and SRLO, respectively.

2.3. Adaptive Strategy of SILO and SRLO. In HLORL, the SILO and SRLO are simultaneously adopted to accumulate useful information in social groups more effectively. Among them, the SILO can quickly accumulate knowledge by imitating the current global optimal [44, 45]. However, copying comes with pitfalls in SILO that the acquired knowledge may be outdated [46], misleading or inappropriate if the knowledge of the learned individual is inaccurate. The imitated individual is important in the progress of the search, which can directly militate the learning result. Therefore, the SILO can play an accurate and efficient learning role in a certainty environment. On the other hand, the SRLO is also an efficient learning strategy for the accumulation of human culture, especially in an uncertain environment [47], which can stimulate the learning ability of human beings to dig out some deeper common characteristics by using surface-related knowledge [48, 49]. And reasoning learning [31] is generally used to avoid the inferiority of imitating learning and further exploit the unlimited potential ability learning of human beings. Due to the different learning mechanisms between the SILO and the SRLO, they can play different learning roles at different stages of iterations.

Based on the insight of the roles of SILO and SRLO, we argue that an adaptive strategy for the SILO and SRLO to enhance the optimization ability of AHLORL needs to meet the requirements as follows:

(1) As the initial population is randomly generated, the global optimal solution has the largest uncertainty at the beginning. Therefore, the value of ps should be small so that the individuals can efficiently use the SRLO to find the optimal solutions.

(2) With the progress of the search, more and more optimal solutions are found by SRLO and the uncertainty of population gradually decreases. Therefore, the value of ps should be increased so that the SILO can accumulate the found optimal solutions effectively and further guide the whole population to learn the global optimal information better.

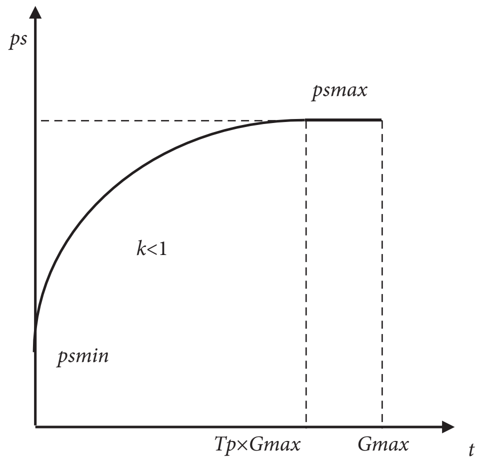


Figure 1: Example of the proposed adaptive strategy.

(3) As the learning efficiency of SILO and SRLO is closely related to the reliability of the population, the adaptive strategy of control parameter *ps* should be nonlinear so that SILO quickly accumulates the found optimal solutions more, which can further help AHLORL maximize the learning ability of SILO and SRLO more effectively.

(4) At the later stage of iterations, the risk of trapping in the local optimum remains because the greedy strategy is adopted in the SILO. Therefore, the value of *ps* should remain constant so that AHLORL can keep the exploration ability to explore the interesting solution areas more efficiently.

As analyzed above, a novel adaptive strategy is proposed to dynamically adjust the control parameter *ps* between SILO and SRLO as follows:

$$ps = \begin{cases} ps_{\min} + (ps_{\max} - ps_{\min}) \times \left(\dfrac{t}{Tp \times \text{iter}_{\max}} \right)^k, & 0 < t \leq Tp \times \text{iter}_{\max}, \\ ps_{\max}, & x > Tp \times \text{iter}_{\max}, \end{cases} \quad (13)$$

where t and $\text{iter}_{\max}$ denote the current iteration and the maximum number of iterations, respectively. Tp is a predefined turning point, and $ps_{\min}$ and $ps_{\max}$ are the minimum value and the maximum value in the whole process, respectively.

For the proposed adaptive strategy, $ps_{\min}$ should be small to meet the requirements in Point (1). Tp should be set greater than 0.5 so that AHLORL effectively maximizes the learning ability of SILO and SRLO, which satisfies the demands in Points (2), (3), and (4). Finally, k should be less than 1 to meet the requirements in Point (3). With the introduction of adaptive strategy, AHLORL can achieve a better trade-off between exploration and exploitation. Figure 1 draws an example of the proposed adaptive strategy.

2.4. Updating the IKD and the SKD. For AHLORL, the IKD and the SKD are updated like the HLORL. After the new candidate solutions of all individuals are generated, the fitness of all individuals is calculated by the predefined fitness function. If the new fitness value is superior to the old one, the new candidate solutions will be adopted to update the old one in the IKDs. Otherwise, the individual's solution in the IKDs will not be updated. And the SKD is updated according to the same way. Since AHLORL is not a Pareto algorithm, the sizes of the IKD and the SKD are both set to 1.

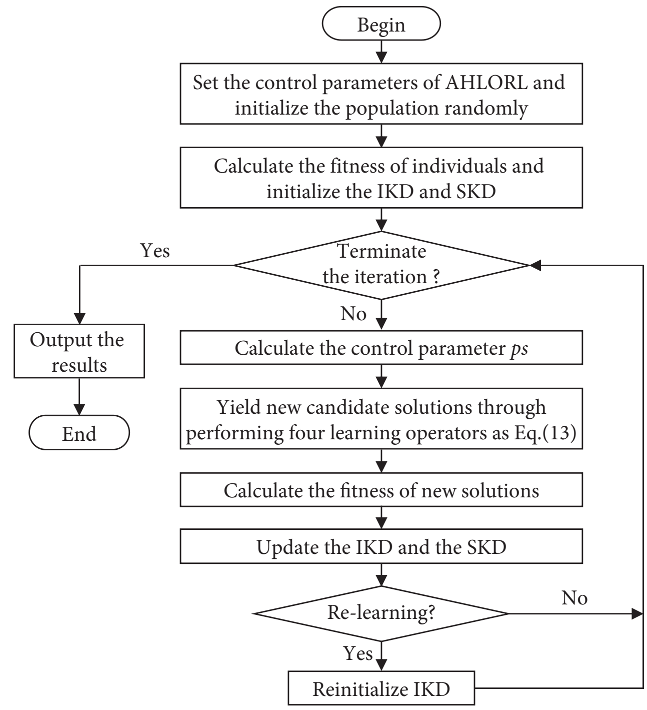


Figure 2: The flowchart of AHLORL.

2.5. Algorithm Complexity. Like the standard HLO, the AHLORL also has two phases, i.e., the population initialization and the iterative search. The running time of generating the initial population X, individual knowledge database (IKD), and social knowledge database (SKD) are $N \times M$, $N \times M$ and $(M + \log N)$, respectively, where M and N are the dimension of solutions and number of individuals, respectively. So, the total running time of the population initialization is $((2N + 1) \times M + \log N)$. During the iterative search of AHLORL, generating new individuals costs time $N \times M$, and updating the IKD and SKD costs time $N \times (M + \log L)$ and $(\log N + \log H + M)$, respectively, where L is the size of ikd_i and H denotes the size of SKD. Therefore, the running time of each iterative is $((2N + 1) \times M + \log(N \times H \times L^N))$. Assume that the maximum generation of AHLORL is G, so the iterative search phase takes time $G \times ((2N + 1) \times M + \log(N \times H \times L^N))$. In general, the maximum generation G is much greater than N, L, and H, and therefore the time complexity of AHLORL is $O((2N + 1) \times G \times M)$.

A flowchart illustrating the implementation of AHLORL is presented in Figure 2.

Table 1: Factor levels in AHLORL.

Parameters	Factor level								
	1	2	3	4	5	6	7	8	9
$ps_{\min}$	0.82	0.83	0.84	0.85	0.86	0.87	0.88	0.89	0.90
$ps_{\max}$	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00
Tp	0.4	0.5	0.6	0.65	0.7	0.75	0.8	0.9	1.0
k	1/5	1/3	1/2	2/3	1	1.5	2.0	3.0	5.0

Table 2: The CEC 15 benchmark functions.

Type	No.	Functions name	Dimension	$F_i^* = F_i(x^*)$
Unimodal functions	F1	Rotated high conditioned elliptic function	10/30	100
	F2	Rotated cigar function	10/30	200
Simple multimodal functions	F3	Shifted and rotated Ackley's function	10/30	300
	F4	Shifted and rotated Rastrigin's function	10/30	400
	F5	Shifted and rotated Schwefel's function	10/30	500
Hybrid functions	F6	Hybrid function 1 ($N = 3$)	10/30	600
	F7	Hybrid function 2 ($N = 4$)	10/30	700
	F8	Hybrid function 3 ($N = 5$)	10/30	800
Composition functions	F9	Composition function 1 ($N = 3$)	10/30	900
	F10	Composition function 2 ($N = 3$)	10/30	1000
	F11	Composition function 3 ($N = 5$)	10/30	1100
	F12	Composition function 4 ($N = 5$)	10/30	1200
	F13	Composition function 5 ($N = 5$)	10/30	1300
	F14	Composition function 6 ($N = 7$)	10/30	1400
	F15	Composition function 7 ($N = 10$)	10/30	1500

3. Parameter Study of AHLORL

3.1. Analysis of the Control Parameters. To evaluate the proposed adaptive strategy, a parameter study was performed in this section, which mainly considers $ps_{\min}$, $ps_{\max}$, Tp, and k. For simplicity, the values of pr and pi remain unchanged in AHLORL, i.e., $pr = 5/M$ and $pi = 0.82$, because the role of the RLO and the ILO in AHLORL is as same as that in HLORL. The orthogonal experimental design method was used, and all the combinations of control parameters are given in Table 1. It should be noted that k is recommended to be less than 1 in the last section. However, for a throughout parameter study, the cases that k is equal to or more than 1 are also considered. The two functions, i.e., F1 and F5 chosen from the CEC 15 benchmark functions [50], were adopted to test the influence of those four parameters on the performance of AHLORL. And the characteristics of selected functions as well as the other 13 functions were used as benchmarks to verify the superiority of the AHLORL in the next section, which is listed in Table 2. The number of populations and the maximum number of iterations for the 10-dimensional/30-dimensional were set as 50/100 and 3000/5000, respectively. Each decision variable was coded by 30 bits and the times of each testing were set to 100 independently. The result of the mean value (mean) was used to evaluate the optimization ability, which is given in Table 3 where the best values have been highlighted in bold.

As mentioned above, $ps_{\min}$, $ps_{\max}$, Tp, and k jointly decide the probabilities of SILO and SRLO over the course of the search process, which are dependent on problems and these also interact with each other, and therefore AHLORL needs a set of suitable values to obtain a better the optimization search ability. Table 3 shows that AHLORL obtains the best comprehensive results on the F1 and F5 of CEC 15 benchmark functions when $ps_{\min}$, $ps_{\max}$, Tp, and k are set to 0.85, 0.98, 0.7, and 2/3, which are chosen as the default values in this work. And the influences of the four control parameters can be concluded as follows:

(1) The value of $ps_{\min}$ is of great importance for the AHLORL, and it should be small so that AHLORL can efficiently utilize the reasoning ability of SRLO to find the optimal solutions at the beginning of the search, which boost the effectiveness and confidence of the following learning operation and consequently enhance the exploitation ability of AHLORL. However, too small $ps_{\min}$ still causes lower convergence speed and spoils the performance of the algorithm. According to Table 3, the value of $ps_{\min}$ should be no less than 0.84.

(2) The larger the $ps_{\max}$ is, the more accurate search the AHLORL performs at the later stage of the search. However, the results show that $ps_{\max}$ should be more than level 7, i.e., 0.98, which indicates that a too big $ps_{\max}$ would greatly reduce the efficiency of search, and consequently the performance of the algorithm is worsened.

(3) It is suggested that Tp should be big enough and do not exceed 0.8 so that enough generations can be guaranteed for the AHLORL to switch between SILO and SRLO search abilities more efficiently.

Table 3: The results of the parameter study.

Trail	Parameter factors $ps_{\min}$	$ps_{\max}$	Tp	k	**10D-F1** Mean	Rank	**10D-F5** Mean	Rank	**30D-F1** Mean	Rank	**30D-F5** Mean	Rank	Ranks
1	0.85	0.92	0.65	0.67	$2.266E+05$	65	$7.836E+01$	55	$3.907E+06$	58	$2.203E+03$	70	67
2	0.87	0.97	1	0.2	$1.775E+05$	26	$6.801E+01$	30	$4.050E+06$	65	$1.469E+03$	17	34
3	0.87	0.93	0.9	1	$2.185E+05$	59	$8.375E+01$	62	$4.062E+06$	67	$2.326E+03$	71	70
4	0.86	0.98	1	3	$2.362E+05$	73	$8.600E+01$	65	$4.510E+06$	72	$2.482E+03$	73	73
5	0.85	1	0.75	3	$2.036E+05$	51	$6.730E+01$	26	$3.612E+06$	42	$1.714E+03$	55	48
6	0.85	0.97	0.8	0.33	$1.741E+05$	23	$5.990E+01$	8	$3.370E+06$	20	$1.462E+03$	13	7
7	0.89	0.94	0.75	5	$1.855E+05$	34	$7.275E+01$	41	$3.887E+06$	56	$1.864E+03$	64	53
8	0.88	0.98	0.5	0.2	$2.345E+05$	72	$7.492E+01$	47	$4.301E+06$	69	$1.563E+03$	37	61
9	0.83	0.93	0.65	0.2	$1.829E+05$	32	$7.081E+01$	37	$3.086E+06$	4	$1.540E+03$	34	21
10	0.89	0.93	0.4	2	$1.693E+05$	20	$6.481E+01$	18	$4.068E+06$	68	$1.503E+03$	27	30
11	0.9	0.98	0.4	0.5	$1.877E+05$	39	$6.475E+01$	17	$4.547E+06$	73	$1.520E+03$	31	44
12	0.9	0.94	0.65	2	$1.653E+05$	15	$7.067E+01$	35	$3.837E+06$	54	$1.444E+03$	10	24
13	0.82	0.95	1	2	$4.025E+05$	81	$1.813E+02$	81	$8.272E+06$	81	$4.377E+03$	81	81
14	0.82	0.98	0.7	0.67	$1.618E+05$	13	$6.346E+01$	13	$3.455E+06$	27	$1.495E+03$	26	15
15	0.87	1	0.7	2	$1.772E+05$	24	$7.243E+01$	39	$3.279E+06$	10	$1.516E+03$	30	20
16	0.85	0.95	0.6	0.2	$1.719E+05$	21	$6.663E+01$	22	$3.432E+06$	24	$1.458E+03$	12	16
17	0.84	0.96	0.4	3	$1.619E+05$	14	$6.686E+01$	23	$3.063E+06$	3	$1.595E+03$	47	18
18	0.84	0.94	0.8	0.2	$1.568E+05$	8	$7.245E+01$	40	$3.531E+06$	34	$1.414E+03$	5	19
19	0.83	0.99	0.9	0.67	$1.655E+05$	16	$5.968E+01$	7	$3.478E+06$	30	$1.491E+03$	24	13
20	0.84	0.93	0.7	0.33	$1.774E+05$	25	$7.699E+01$	50	$3.622E+06$	43	$1.572E+03$	40	43
21	0.87	0.94	0.4	0.67	$1.852E+05$	33	$8.852E+01$	67	$3.047E+06$	2	$1.430E+03$	8	23
22	0.87	0.95	0.5	0.5	$1.403E+05$	1	$6.015E+01$	9	$3.518E+06$	33	$1.414E+03$	6	5
23	0.85	0.93	1	1.5	$3.310E+05$	79	$1.336E+02$	79	$6.816E+06$	79	$4.050E+03$	79	79
24	0.9	0.92	1	5	$2.207E+05$	61	$9.464E+01$	73	$4.896E+06$	78	$2.664E+03$	75	75
25	0.82	0.99	0.8	1.5	$2.336E+05$	71	$8.953E+01$	68	$3.487E+06$	31	$1.681E+03$	53	59
26	0.83	1	0.4	1.5	$1.913E+05$	44	$7.572E+01$	48	$3.565E+06$	38	$1.594E+03$	46	50
27	0.82	0.97	0.65	3	$2.249E+05$	63	$8.010E+01$	58	$3.990E+06$	61	$1.744E+03$	58	65
28	0.9	0.97	0.6	0.67	$1.525E+05$	6	$8.295E+01$	60	$3.544E+06$	36	$1.472E+03$	19	26
29	0.84	0.99	1	1	$1.823E+05$	31	$7.399E+01$	44	$3.344E+06$	17	$1.570E+03$	39	28
30	0.85	0.96	0.7	0.5	$1.454E+05$	2	$5.766E+01$	4	$2.989E+06$	1	$1.395E+03$	3	1
31	0.88	0.99	0.65	0.5	$2.109E+05$	57	$5.862E+01$	5	$3.844E+06$	55	$1.477E+03$	20	33
32	0.89	0.95	0.65	1	$1.516E+05$	4	$5.895E+01$	6	$3.747E+06$	48	$1.438E+03$	9	8
33	0.85	0.98	0.9	2	$2.315E+05$	68	$8.426E+01$	63	$3.594E+06$	41	$1.753E+03$	59	63
34	0.82	0.94	0.9	0.33	$1.882E+05$	40	$7.457E+01$	45	$3.507E+06$	32	$1.583E+03$	43	45
35	0.84	0.92	0.6	0.5	$2.090E+05$	54	$9.579E+01$	75	$3.902E+06$	57	$2.046E+03$	68	69
36	0.86	0.97	0.9	0.5	$1.490E+05$	3	$7.306E+01$	42	$3.338E+06$	15	$1.463E+03$	14	11
37	0.87	0.98	0.8	5	$2.083E+05$	53	$7.755E+01$	53	$3.588E+06$	40	$1.714E+03$	56	56
38	0.89	0.99	0.7	0.2	$2.228E+05$	62	$8.200E+01$	59	$4.670E+06$	75	$1.580E+03$	41	64
39	0.89	0.96	1	0.67	$1.576E+05$	10	$6.463E+01$	16	$3.191E+06$	6	$1.456E+03$	11	4
40	0.9	0.93	0.5	3	$2.383E+05$	74	$6.726E+01$	25	$3.757E+06$	49	$1.598E+03$	48	54
41	0.9	1	0.9	0.2	$2.102E+05$	56	$7.959E+01$	57	$4.895E+06$	77	$1.524E+03$	32	58
42	0.86	0.93	0.8	0.67	$2.202E+05$	60	$6.847E+01$	31	$3.307E+06$	12	$1.877E+03$	66	46
43	0.84	0.95	0.9	5	$3.889E+05$	80	$1.716E+02$	80	$7.184E+06$	80	$4.052E+03$	80	80
44	0.88	0.96	0.9	1.5	$2.100E+05$	55	$7.471E+01$	46	$3.340E+06$	16	$1.568E+03$	38	41
45	0.87	0.96	0.65	0.33	$1.524E+05$	5	$6.373E+01$	14	$3.383E+06$	21	$1.465E+03$	15	6
46	0.85	0.94	0.5	1	$1.795E+05$	29	$6.021E+01$	10	$3.324E+06$	13	$1.472E+03$	18	9
47	0.88	1	1	0.33	$1.693E+05$	19	$9.225E+01$	69	$4.051E+06$	66	$1.508E+03$	28	51
48	0.89	0.92	0.9	3	$2.460E+05$	76	$1.033E+02$	77	$4.631E+06$	74	$2.977E+03$	78	77
49	0.88	0.97	0.4	1	$1.788E+05$	28	$5.469E+01$	2	$3.463E+06$	29	$1.487E+03$	23	17
50	0.82	0.96	0.5	5	$2.559E+05$	77	$9.382E+01$	72	$3.454E+06$	26	$1.667E+03$	52	62
51	0.88	0.93	0.6	5	$1.905E+05$	41	$7.847E+01$	56	$3.534E+06$	35	$1.865E+03$	65	55
52	0.83	0.96	0.6	2	$2.014E+05$	49	$6.739E+01$	28	$3.276E+06$	9	$1.588E+03$	44	27
53	0.88	0.92	0.8	2	$2.285E+05$	66	$9.350E+01$	71	$4.508E+06$	71	$2.868E+03$	77	74
54	0.82	0.92	0.4	0.2	$1.997E+05$	47	$7.724E+01$	52	$3.925E+06$	59	$1.713E+03$	54	57
55	0.83	0.94	1	0.5	$2.331E+05$	69	$9.867E+01$	76	$4.017E+06$	64	$1.997E+03$	67	72
56	0.85	0.99	0.4	5	$1.905E+05$	42	$7.070E+01$	36	$3.653E+06$	44	$1.616E+03$	49	47
57	0.83	0.95	0.8	3	$2.752E+05$	78	$1.232E+02$	78	$4.752E+06$	76	$2.615E+03$	74	78
58	0.87	0.99	0.6	3	$1.858E+05$	36	$6.903E+01$	33	$3.361E+06$	19	$1.635E+03$	51	38
59	0.84	0.97	0.75	2	$1.858E+05$	35	$7.807E+01$	54	$3.275E+06$	8	$1.776E+03$	60	42
60	0.84	1	0.5	0.67	$1.905E+05$	43	$6.625E+01$	20	$3.462E+06$	28	$1.630E+03$	50	39

Table 3: Continued.

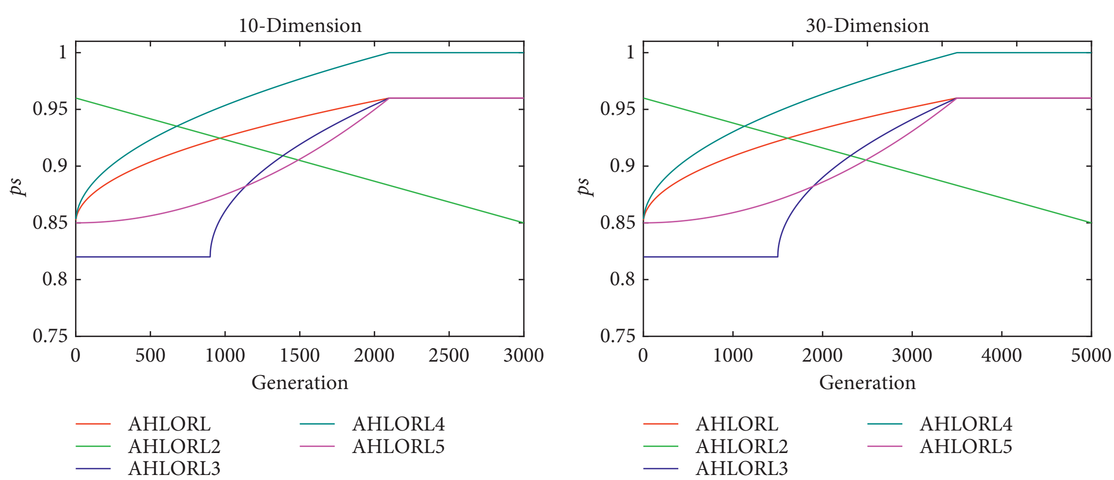
61	0.86	0.94	0.6	1.5	$1.866E+05$	38	$6.999E+01$	34	$3.349E+06$	18	$1.589E+03$	45	31
62	0.88	0.95	0.75	0.67	$2.014E+05$	50	$6.709E+01$	24	$3.581E+06$	39	$1.492E+03$	25	35
63	0.86	0.99	0.5	2	$1.957E+05$	45	$7.365E+01$	43	$3.201E+06$	7	$1.553E+03$	36	29
64	0.86	0.92	0.7	1	$2.254E+05$	64	$8.737E+01$	66	$4.009E+06$	63	$2.434E+03$	72	71
65	0.86	1	0.65	5	$1.861E+05$	37	$7.087E+01$	38	$3.776E+06$	51	$1.810E+03$	62	52
66	0.86	0.95	0.4	0.33	$1.528E+05$	7	$5.601E+01$	3	$3.442E+06$	25	$1.367E+03$	2	2
67	0.89	0.98	0.6	0.33	$2.010E+05$	48	$5.444E+01$	1	$3.945E+06$	60	$1.508E+03$	29	36
68	0.82	0.93	0.75	0.5	$2.179E+05$	58	$9.267E+01$	70	$3.828E+06$	53	$2.067E+03$	69	68
69	0.89	0.97	0.5	1.5	$1.600E+05$	11	$6.644E+01$	21	$3.393E+06$	22	$1.483E+03$	21	12
70	0.9	0.99	0.75	0.33	$1.573E+05$	9	$6.870E+01$	32	$3.994E+06$	62	$1.527E+03$	33	32
71	0.84	0.98	0.65	1.5	$1.679E+05$	18	$7.616E+01$	49	$3.159E+06$	5	$1.547E+03$	35	22
72	0.89	1	0.8	0.5	$2.293E+05$	67	$6.501E+01$	19	$3.764E+06$	50	$1.466E+03$	16	40
73	0.9	0.96	0.8	1	$1.785E+05$	27	$6.797E+01$	29	$3.334E+06$	14	$1.336E+03$	1	10
74	0.88	0.94	0.7	3	$2.060E+05$	52	$8.327E+01$	61	$3.718E+06$	47	$1.822E+03$	63	60
75	0.83	0.97	0.7	5	$2.401E+05$	75	$8.498E+01$	64	$3.705E+06$	45	$1.781E+03$	61	66
76	0.86	0.96	0.75	0.2	$1.973E+05$	46	$6.462E+01$	15	$3.706E+06$	46	$1.420E+03$	7	25
77	0.83	0.98	0.75	1	$1.737E+05$	22	$6.108E+01$	11	$3.426E+06$	23	$1.485E+03$	22	14
78	0.83	0.92	0.5	0.33	$1.823E+05$	30	$7.700E+01$	51	$3.544E+06$	37	$1.732E+03$	57	49
79	0.9	0.95	0.7	1.5	$1.603E+05$	12	$6.336E+01$	12	$3.282E+06$	11	$1.408E+03$	4	3
80	0.82	1	0.6	1	$1.673E+05$	17	$6.736E+01$	27	$3.820E+06$	52	$1.582E+03$	42	37
81	0.87	0.92	0.75	1.5	$2.335E+05$	70	$9.523E+01$	74	$4.330E+06$	70	$2.747E+03$	76	76

Note. 10D-F1 is the 10-dimension of F1 function.

Table 4: Parameter settings of five AHLORLs.

Algorithm	Parameters
AHLORL	$pr=5/M,\ pi=0.82,\ ps_{\min}=0.85,\ ps_{\max}=0.96,\ Tp=0.7,\ k=1/2$
AHLORL2	$pr=5/M,\ pi=0.82,\ ps_{\min}=0.85,\ ps_{\max}=0.96$
AHLORL3	$pr=5/M,\ pi=0.82,\ ps_{\min}=0.82,\ ps_{\max}=0.96,\ Tp_1=0.3,\ Tp_2=0.7,\ k=1/2$
AHLORL4	$pr=5/M,\ pi=0.82,\ ps_{\min}=0.85,\ ps_{\max}=1.00,\ Tp=0.7,\ k=1/2$
AHLORL5	$pr=5/M,\ pi=0.82,\ ps_{\min}=0.85,\ ps_{\max}=0.96,\ Tp=0.7,\ k=2.0$

Note. M is the dimension of solutions.



(a) (b)

Figure 3: The ps curves of AHLORL, AHLORL2, AHLORL3, AHLORL4, and AHLORL5.

(4) The value of k is also important for the AHLORL, and it should be smaller than 1 so that AHLORL can efficiently utilize the imitating ability of SILO to quickly accumulate the found optimal solutions, which boost the effectiveness and confidence of the following learning operation and consequently enhance the exploitation ability of AHLORL. However, too small k still weakens the reasoning ability of

Table 5: The results of five AHLORLs on the 10-dimensional CEC 15 benchmarks.

Fun	Metric	AHLORL	AHLORL2	AHLORL3	AHLORL4	AHLORL5
F1	Best	7.0185E+03	1.6356E+04	1.6287E+04	7.4843E+03	**5.0977E+03**
	Mean	**1.4321E+05**	3.1414E+05	1.7902E+05	1.7662E+05	1.9387E+05
	Std	**1.3145E+05**	2.7232E+05	1.3685E+05	1.7698E+05	1.5487E+05
	t-Test	—	1	1	0	1
	W-test	—	1	1	1	1
F2	Best	1.4326E+03	6.8963E+03	1.3610E+03	2.2374E+03	**1.3546E+03**
	Mean	8.4642E+05	5.9482E+06	1.0999E+06	1.6162E+06	**7.0918E+05**
	Std	1.9368E+06	7.3543E+06	2.5008E+06	3.3453E+06	**8.7764E+05**
	t-Test	—	1	0	1	0
	W-test	—	1	0	0	0
F3	Best	2.5798E+00	2.9440E+00	**2.0660E+00**	8.2877E+00	3.4195E+00
	Mean	1.8580E+01	1.8803E+01	**1.8114E+01**	1.9799E+01	1.8904E+01
	Std	4.3836E+00	4.1082E+00	**5.0023E+00**	1.4746E+00	3.7793E+00
	t-Test	—	0	0	1	0
	W-test	—	1	0	1	1
F4	Best	**5.5808E−01**	3.0217E+00	2.0195E+00	1.1184E+00	1.3070E+00
	Mean	**4.4647E+00**	5.8591E+00	4.8625E+00	4.7293E+00	5.1812E+00
	Std	**1.4911E+00**	1.5632E+00	1.6063E+00	1.3735E+00	1.7328E+00
	t-Test	—	1	0	0	1
	W-test	—	1	0	0	1
F5	Best	2.5876E+00	5.5531E+00	**6.7090E−01**	1.7313E+00	8.0449E+00
	Mean	**5.7225E+01**	8.2763E+01	6.4581E+01	6.5224E+01	7.6558E+01
	Std	**5.0841E+01**	6.9845E+01	6.2850E+01	6.1357E+01	7.1917E+01
	t-Test	—	1	0	0	1
	W-test	—	1	0	0	1
F6	Best	4.5969E+01	5.9686E+01	6.2284E+01	**3.2633E+01**	6.7267E+01
	Mean	1.3756E+03	2.5942E+03	**1.2486E+03**	1.9808E+03	1.6455E+03
	Std	1.3486E+03	6.5814E+03	**1.2807E+03**	1.9696E+03	1.4654E+03
	t-Test	—	0	0	1	0
	W-test	—	1	0	1	0
F7	Best	1.5272E−01	2.9158E−01	1.7449E−01	1.5402E−01	**1.0669E−01**
	Mean	**9.1146E−01**	1.0890E+00	9.2265E−01	1.0449E+00	9.2102E−01
	Std	**3.6188E−01**	4.0394E−01	3.2331E−01	3.7025E−01	3.5455E−01
	t-Test	—	1	0	1	0
	W-test	—	1	0	1	0
F8	Best	2.8786E+00	5.5281E+00	**2.1503E+00**	8.0870E+00	1.4100E+01
	Mean	3.3990E+02	5.2568E+02	**3.1279E+02**	4.6150E+02	3.2763E+02
	Std	3.3356E+02	7.6387E+02	**3.4204E+02**	5.1608E+02	2.5034E+02
	t-Test	—	1	0	0	0
	W-test	—	0	0	0	0
F9	Best	1.0009E+02	1.0014E+02	1.0007E+02	**1.0006E+02**	1.0008E+02
	Mean	1.0017E+02	1.0025E+02	1.0019E+02	**1.0017E+02**	1.0019E+02
	Std	3.2463E−02	5.7059E−02	3.4712E−02	**3.7246E−02**	3.7393E−02
	t-Test	—	1	1	0	1
	W-test	—	1	1	0	1
F10	Best	2.2384E+02	2.5789E+02	**2.2198E+02**	2.2334E+02	2.3119E+02
	Mean	3.5107E+02	4.2840E+02	**3.5086E+02**	3.6722E+02	3.6328E+02
	Std	6.6269E+01	1.3156E+02	**7.1596E+01**	6.4233E+01	6.1590E+01
	t-Test	—	1	0	0	0
	W-test	—	1	0	1	0
F11	Best	2.7164E+00	3.2384E+00	**2.0988E+00**	2.1370E+00	3.0977E+00
	Mean	2.1417E+01	1.2580E+02	2.2096E+01	2.7560E+01	**9.9780E+00**
	Std	6.4015E+01	1.4263E+02	6.3877E+01	7.4865E+01	**2.9252E+01**
	t-Test	—	1	0	0	0
	W-test	—	1	0	0	0

Table 5: Continued.

Fun	Metric	AHLORL	AHLORL2	AHLORL3	AHLORL4	AHLORL5
F12	Best	**1.0052E + 02**	1.0140E + 02	1.0102E + 02	1.0076E + 02	1.0091E + 02
	Mean	1.0155E + 02	1.0249E + 02	1.0166E + 02	**1.0155E + 02**	1.0168E + 02
	Std	4.3034E − 01	5.9355E − 01	3.6841E − 01	**4.3902E − 01**	3.5919E − 01
	t-Test	—	1	0	0	1
	W-test	—	1	0	0	1
F13	Best	**2.1308E + 01**	2.2542E + 01	2.2485E + 01	2.1947E + 01	2.2726E + 01
	Mean	**2.5728E + 01**	2.7713E + 01	2.6419E + 01	2.6109E + 01	2.6735E + 01
	Std	**1.8822E + 00**	2.0439E + 00	1.7183E + 00	1.9366E + 00	1.7349E + 00
	t-Test	—	1	1	0	1
	W-test	—	1	1	0	1
F14	Best	3.3572E + 02	5.1767E + 02	**1.7705E + 02**	3.4012E + 02	3.3437E + 02
	Mean	2.7176E + 03	4.1555E + 03	2.6734E + 03	2.9169E + 03	**2.5102E + 03**
	Std	9.2981E + 02	1.9663 E + 03	7.3991E + 02	8.8523E + 02	**8.2462E + 02**
	t-Test	—	1	0	0	0
	W-test	—	1	0	0	0
F15	Best	1.0002E + 02	1.0002E + 02	1.0010E + 02	1.0015E + 02	**1.0002E + 02**
	Mean	**1.0234E + 02**	1.0622E + 02	1.0268E + 02	1.0293E + 02	1.0253E + 02
	Std	**1.9637E + 00**	4.3625E + 00	2.1304E + 00	2.6496E + 00	1.3014E + 00
	t-Test	—	1	0	0	0
	W-test	—	1	0	0	0

SRLO and spoils the performance of the algorithm. According to Table 3, the value of k should be no less than 1/3.

3.2. Influences of the Adaptive Strategies on SILO and SRLO. With the deep analysis of the control parameters of AHLORL, the influences of the adaptive strategies on SILO and SRLO are further investigated. The AHLORL with the default values and the other four versions with modified parameters, i.e., AHLORL2, AHLORL3, AHLORL4, and AHLORL5, are compared with each other to testify the characteristics of different adaptive strategies. The parameter settings of AHLORL, AHLORL2, AHLORL3, AHLORL4, and AHLORL5 are given in Table 4, and the corresponding ps curves are drawn in Figure 3. All the algorithms were used to solve the 10-dimensional and 30-dimensional CEC 15 functions, and the numerical results, including mean, the best value (Best), and the standard deviation (Std), are listed in Table 5, where the best results are marked in bold. And the Student's t-test (t-test) and Wilcoxon signed-rank test (W-test) [51] are performed and the corresponding results are also shown in Table 6, in which "1/0/−1" indicates that the numerical result of AHLORL is obviously better than, similar to, or worse than the compared algorithm in the 95% confidence, respectively. The t-test is a parameter test that needs to fulfill the normality and homogeneity of variance, while the W-test is a nonparametric test that does not need to satisfy the above characteristics [12]. For convenience, the results of the t-test and W-test are summarized in Tables 7 and 8. Besides, to deeply inspect the influence of the adaptive strategies on SILO and SRLO, two evaluating indicators were used as follows:

Definition 1: Index 1 is the percent of the same bit values obtained by SILO and SRLO in a generation.

Definition 2: average distance (AD) is the average Hamming distance *between* the global optimal solution and the other individual optimal solutions in a generation.

Tables 5–8 clearly show that AHLORL outperforms AHLORL2, AHLORL3, AHLORL4, and AHLORL5. Especially 30-dimensional functions achieve a better optimization performance. Specifically, the proposed AHLORL obtains the best numerical results on 12 out of fifteen 30-dimensional functions. Besides, the summary of t-test and W-test results on the 30-dimensional CEC 15 functions in Table 8 indicates that the proposed AHLORL surpasses AHLORL2, AHLORL3, AHLORL4, and AHLORL5 on 15, 5, 8, and 7 out of 15 functions. And the W-test results support that AHLORL outperforms these variants on 15, 6, 7, and 8 out of 15 functions. By systematically analyzing the differences between AHLORL and the other four versions, it will be easier to understand the influences of various adaptive strategies on the performance and learn about how to meet the requirements of the ideal balance between exploration and exploitation.

To analyze the superiority of the proposed AHLORL algorithm more clearly, the Index 1 curves of AHLORL and AHLORL2 on F1 and F5 over 100 independent runs are drawn in Figure 4, and the AD curves and corresponding fitness values of AHLORL, AHLORL2, AHLORL3, AHLORL4, and AHLORL5 are drawn in Figure 5, respectively. Figures 3–6 clearly show that the relationship between AHLORL, AHLORL2, AHLORL3, AHLORL4, and AHLORL5 and the evaluating indicators result was caused by the change of ps.

Table 6: The results of five AHLORLs on the 10-dimensional CEC 15 benchmarks.

Fun	Metric	AHLORL	AHLORL2	AHLORL3	AHLORL4	AHLORL5
F1	Best	**3.5098E + 05**	1.4133*E* + 06	1.0411*E* + 06	8.4655*E* + 05	7.4393*E* + 05
	Mean	**2.9313E + 06**	6.1536*E* + 06	3.4012*E* + 06	3.4200*E* + 06	3.0466*E* + 06
	Std	**1.5240E + 06**	4.0321*E* + 06	1.7932*E* + 06	2.5401*E* + 06	1.2297*E* + 06
	t-Test	—	1	0	0	0
	W-test	—	1	0	0	0
F2	Best	1.0894*E* + 06	1.2545*E* + 07	7.6671*E* + 05	**3.3620E + 05**	2.1615*E* + 06
	Mean	2.5609*E* + 07	2.7935*E* + 08	**2.0945E + 07**	3.7516*E* + 07	3.0009*E* + 07
	Std	2.0518*E* + 07	3.2278*E* + 08	**1.6985E + 07**	2.8472*E* + 07	2.1323*E* + 07
	t-Test	—	1	0	1	0
	W-test	—	1	0	1	0
F3	Best	**2.0015E + 01**	2.0480*E* + 01	2.0051*E* + 01	2.0032*E* + 01	2.0095*E* + 01
	Mean	**2.0088E + 01**	2.0736*E* + 01	2.0194*E* + 01	2.0099*E* + 01	2.0248*E* + 01
	Std	**4.4153E − 02**	8.1513*E* − 02	8.3327*E* − 02	4.1915*E* − 02	9.7447*E* − 02
	t-Test	—	1	1	0	1
	W-test	—	1	1	1	1
F4	Best	**1.6636E + 01**	2.5367*E* + 01	1.7398*E* + 01	2.0935*E* + 01	1.9020*E* + 01
	Mean	**3.5857E + 01**	4.6908*E* + 01	3.7162*E* + 01	3.8234*E* + 01	3.7059*E* + 01
	Std	**7.3883E + 00**	1.0394*E* + 01	7.4725*E* + 00	7.8884*E* + 00	8.2516*E* + 00
	t-Test	—	1	0	1	0
	W-test	—	1	0	0	0
F5	Best	**3.1978E + 02**	7.7955*E* + 02	5.7000*E* + 02	4.4575*E* + 02	6.0766*E* + 02
	Mean	**1.3751E + 03**	1.8290*E* + 03	1.6460*E* + 03	1.5358*E* + 03	1.6217*E* + 03
	Std	**4.0458E + 02**	4.7633*E* + 02	3.7057*E* + 02	4.0991*E* + 02	3.7949*E* + 02
	t-Test	—	1	1	1	1
	W-test	—	1	1	1	1
F6	Best	5.9882*E* + 04	7.2691*E* + 04	6.3261*E* + 04	1.1807*E* + 05	**3.9817E + 04**
	Mean	**4.0067E + 05**	5.3018*E* + 05	4.5017*E* + 05	5.2750*E* + 05	4.9655*E* + 05
	Std	**2.2508E + 05**	4.4915*E* + 05	2.5779*E* + 05	3.9484*E* + 05	3.0791*E* + 05
	t-Test	—	1	0	1	1
	W-test	—	1	0	1	1
F7	Best	**6.4758E + 00**	6.9204*E* + 00	7.4183*E* + 00	7.5758*E* + 00	7.8512*E* + 00
	Mean	1.0124*E* + 01	1.2739*E* + 01	1.0199*E* + 01	1.0704*E* + 01	**1.0105E + 01**
	Std	1.3229*E* + 00	2.0361*E* + 00	1.1682*E* + 00	1.2033*E* + 00	**1.1061E + 00**
	t-Test	—	1	0	1	0
	W-test	—	1	0	1	0
F8	Best	9.5273*E* + 03	1.5489*E* + 04	1.7583*E* + 04	1.7795*E* + 04	**6.1712E + 03**
	Mean	**1.0991E + 05**	1.6202*E* + 05	1.1810*E* + 05	1.5987*E* + 05	1.1532*E* + 05
	Std	**6.0527E + 04**	9.5848*E* + 04	6.8404*E* + 04	8.8975*E* + 04	6.4274*E* + 04
	t-Test	—	1	0	1	0
	W-test	—	1	0	1	0
F9	Best	1.0244*E* + 02	1.0297*E* + 02	1.0250*E* + 02	1.0249*E* + 02	**1.0237E + 02**
	Mean	**1.0288E + 02**	1.0432*E* + 02	1.0295*E* + 02	1.0293*E* + 02	1.0294*E* + 02
	Std	**2.1536E − 01**	1.4430*E* + 00	1.9787*E* − 01	2.4404*E* − 01	2.0690*E* − 01
	t-Test	—	1	1	0	1
	W-test	—	1	1	0	1
F10	Best	**8.0989E + 03**	3.1026*E* + 04	2.3093*E* + 04	1.4837*E* + 04	1.6642*E* + 04
	Mean	**1.5791E + 05**	2.2310*E* + 05	1.8441*E* + 05	2.3865*E* + 05	1.6185*E* + 05
	Std	**9.7338E + 04**	1.5798*E* + 05	9.7905*E* + 04	1.4587*E* + 05	9.0725*E* + 04
	t-Test	—	1	0	1	0
	W-test	—	1	0	1	0
F11	Best	3.0624*E* + 02	3.0737*E* + 02	3.1190*E* + 02	**3.0621E + 02**	3.0970*E* + 02
	Mean	**3.2617E + 02**	6.0056*E* + 02	3.3005*E* + 02	3.5009*E* + 02	3.3688*E* + 02
	Std	**3.8006E + 01**	1.6202*E* + 02	2.5714*E* + 01	8.5204*E* + 01	3.6888*E* + 01
	t-Test	—	1	0	1	0
	W-test	—	1	1	0	1

Table 6: Continued.

Fun	Metric	AHLORL	AHLORL2	AHLORL3	AHLORL4	AHLORL5
F12	Best	$1.0417E+02$	$1.0500E+02$	**1.0407E + 02**	$1.0426E+02$	$1.0442E+02$
	Mean	**1.0555E + 02**	$1.0679E+02$	$1.0578E+02$	$1.0556E+02$	$1.0578E+02$
	Std	**5.3918E − 01**	$7.4995E-01$	$5.4516E-01$	$5.2658E-01$	$5.5395E-01$
	t-Test	—	1	1	0	1
	W-test	—	1	1	0	1
F13	Best	**7.5673E + 01**	$8.4114E+01$	$7.8988E+01$	$7.9979E+01$	$8.0951E+01$
	Mean	**9.1145E + 01**	$9.9983E+01$	$9.3561E+01$	$9.1643E+01$	$9.5014E+01$
	Std	**6.8232E + 00**	$5.5460E+00$	$5.6323E+00$	$5.3644E+00$	$4.8753E+00$
	t-Test	—	1	1	0	1
	W-test	—	1	1	0	1
F14	Best	$3.1272E+04$	$3.1326E+04$	**3.1176E + 04**	$3.1297E+04$	$3.1460E+04$
	Mean	**3.2927E + 04**	$3.3531E+04$	$3.2934E+04$	$3.2993E+04$	$3.2947E+04$
	Std	**8.3761E + 02**	$1.1279E+03$	$8.0199E+02$	$7.9269E+02$	$7.1843E+02$
	t-Test	—	1	0	0	0
	W-test	—	1	0	0	0
F15	Best	$1.0182E+02$	$1.0434E+02$	$1.0234E+02$	**1.0121E + 02**	$1.0222E+02$
	Mean	**1.0562E + 02**	$1.1068E+02$	$1.0591E+02$	$1.0579E+02$	$1.0615E+02$
	Std	**1.3104E + 00**	$4.2595E+00$	$1.3376E+00$	$1.6978E+00$	$1.4430E+00$
	t-Test	—	1	0	0	1
	W-test	—	1	0	0	1

Table 7: Summary of the t-test and W-test results on the 10-dimensional CEC 15 functions.

Metric	AHLORL	AHLORL2	AHLORL3	AHLORL4	AHLORL5
t-Test	1	13	3	4	6
	0	2	12	11	9
	−1	0	0	0	0
W-test	1	14	3	5	7
	0	1	12	10	8
	−1	0	0	0	0

Table 8: Summary of the t-test and W-test results on the 30-dimensional CEC 15 functions.

Metric	AHLORL	AHLORL2	AHLORL3	AHLORL4	AHLORL5
t-Test	1	15	5	8	7
	0	0	10	7	8
	−1	0	0	0	0
W-test	1	15	6	7	8
	0	0	9	8	7
	−1	0	0	0	0

Based on the above experiments, the characteristics of AHLORL and the influences of the adaptive strategies can be concluded as follows:

(1) The Index 1 curves of AHLORL in Figure 4 increase gradually. This character indicates that AHLORL needs to use a large probability of SRLO to find the optimal bit values at the beginning of the search, which can effectively reduce the uncertainty impact of the randomly initialized population. With the progress of the search, the learning probability of SILO is increased to accumulate the found optimal bit values, which further boosts the effectiveness and confidence of the following learning operation. Besides, the Index 1 curves of AHLORL2 have a significant drop at the later stage of iteration, which does not meet the characteristics of knowledge changes of the population. And therefore, the increasing strategy of control parameter *ps* can utilize the learning ability between SILO and SRLO more effectively and efficiently.

(2) Figure 5 shows that the AD curves of AHLORL2 represent converging fast and then performing the

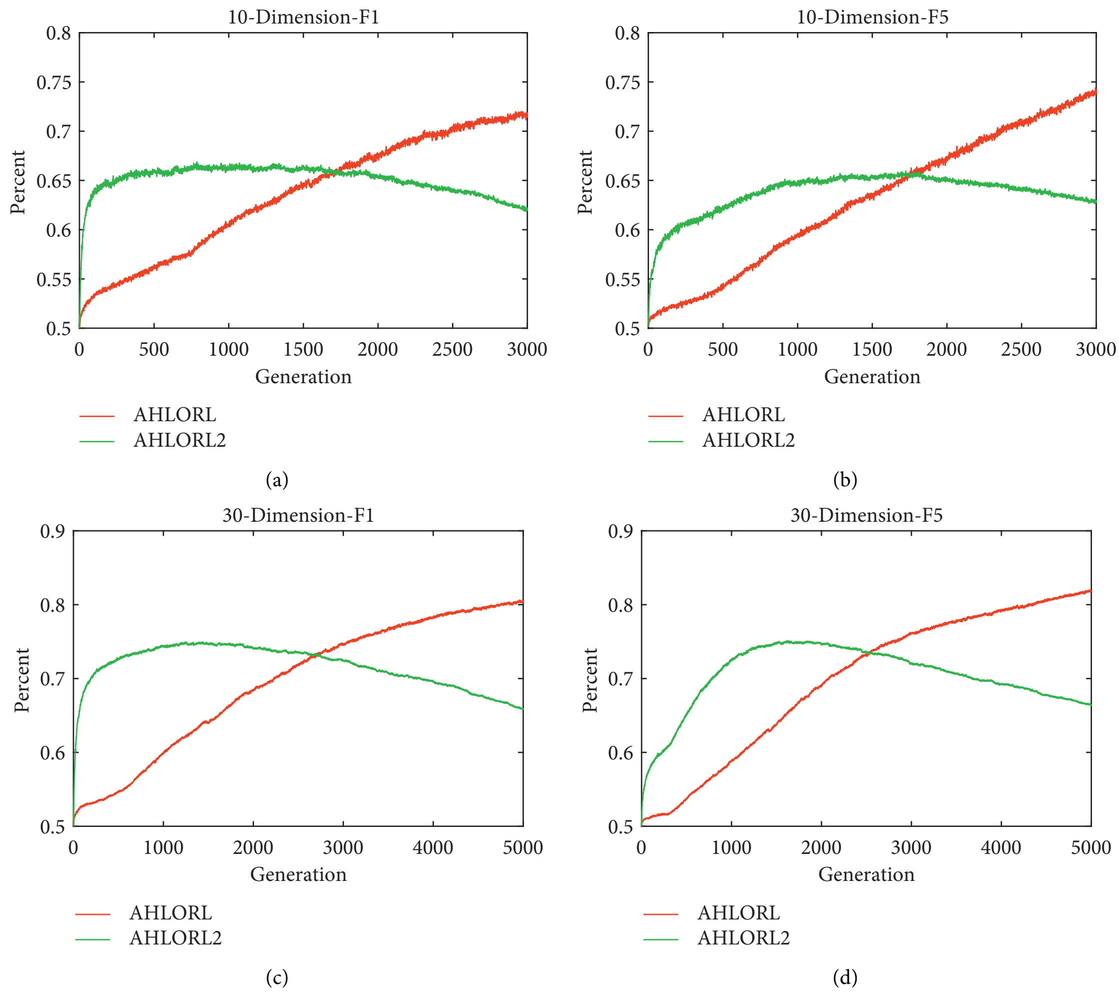


Figure 4: The Index 1 curves of AHLORL and AHLORL2.

accurate search at the beginning. Although AHLORL2 converges fastest, Figure 6 displays that AHLORL2 likely stucks in the local optima because it cannot widely explore the interesting solution areas and sufficiently perform the accurate search at the later stages of iterations, and consequently its results are the worst among all the algorithms.

(3) Figure 3 shows that the *ps* curves of AHLORL3 can maintain diversity during a long period to sufficiently explore the interesting solution areas and quickly enhance the local search ability in the middle of the searching process. From Figures 5 and 6, it reveals that the values of AD are almost unchanged in the first $Gmax \times Tp_1$ generations because the found useful knowledge cannot be effectively accumulated by SILO, and the functions fitness is not improved. Besides, the *ps* value of AHLORL3 quickly rises in the middle of the searching process, and AHLORL3 performs efficient accurate search promptly. Correspondingly, the fitness of AHLORL3 is greatly improved; meanwhile, the value of *AD* curves obviously changes. However, the final results of AHLORL3 are not good enough due to the limited resources.

(4) AHLORL4 has a similar problem with AHLORL3, and it also cannot effectively accumulate the found useful knowledge. Compared with AHLORL3, AHLORL4 obtains a better result before the $Gmax \times Tp_1$ generations because it can perform SILO with a certain probability. However, the final results of AHLORL4 are worse than AHLORL3 because the AHLORL4 is relatively slow to enhance the learning ability of SILO.

(5) Figures 5 and 6 clearly show that the AD curves of AHLORL5 drop quickly, and it obtains a better result before the $Gmax \times Tp$ generations. Although the AD curve of AHLORL5 continues to drop after the $Gmax \times Tp$ generations, the fitness values are almost unchanged, which displays that AHLORL5 is likely stuck in the local optima. As the greedy strategy is adopted for updating IKDs and SKD, the ILO and SILO perform copy strategy to get the same bit value from the IKDs and the SKD. Therefore, if the

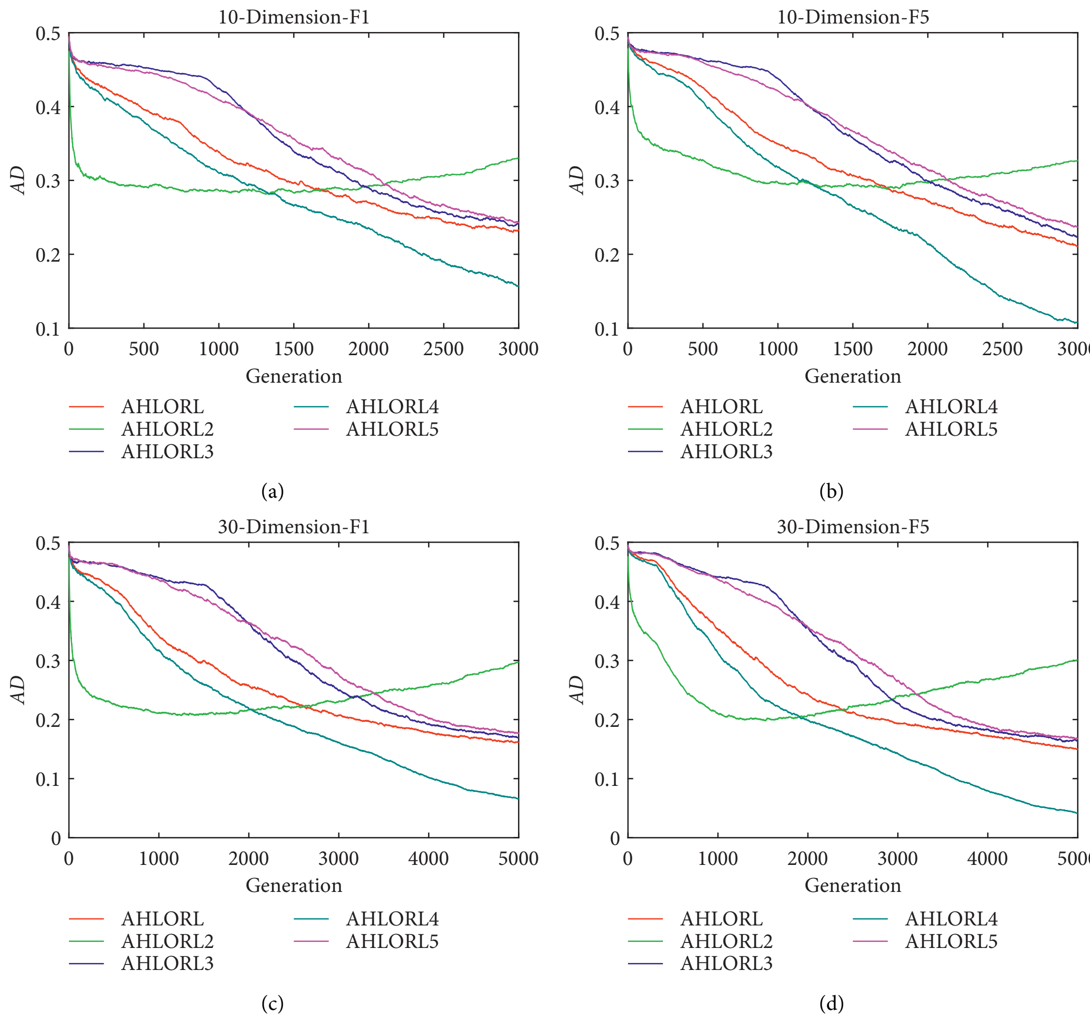


Figure 5: The *AD* curves of AHLORL, AHLORL2, AHLORL3, AHLORL4, and AHLORL5.

corresponding bit value from the IKDs and SKD is the same, for example, being "1" but the optimal value being "0," AHLORL5 cannot efficiently regain "0" by the ILO and SILO, and the only chance of the algorithm to obtain "0" depends on the RLO. However, the rate of performing the RLO is 5/M; that is, the probability of generating "0" for a certain bit on the 30-dimensional functions is 0.0028, which is quite inefficient.

(6) Compared with AHLORL2, AHLORL3, AHLORL4, and AHLORL5, it is fair to declare that AHLORL can effectively utilize the learning ability of SILO and SRLO and significantly improve the search results because the proposed adaptive strategy is carefully designed according to the different requirements at different search stages. Figures 5 and 6 indicate that the proposed adaptive strategy brings about noticeable improvements in search performance. The reason is that AHLORL achieves a practically perfect trade-off between exploration and exploitation through the proposed adaptive strategy. Specifically, at the beginning of the search, the efficiency and reliability of SILO are low due to the random initialization of the population. At this time, the SRLO can efficiently find the optimal bit values by reasoning. With the progress of the search, the learning probability of SILO is quickly increased to efficiently accumulate found the optimal bit values, which further enhance the effectiveness of the following learning operation and consequently boost the exploitation ability of AHLORL. At the later stage of iterations, the risk of trapping in the local optimum remains because the greedy strategy is adopted in the SILO and ILO, and the SRLO efficiently retrieves the optimal bit values lost by the SILO and ILO. Therefore, the global search ability is significantly enhanced.

4. Inherent Search Mechanisms of AHLORL

To further understand the inherent search mechanisms of AHLORL, the structural similarities and differences between

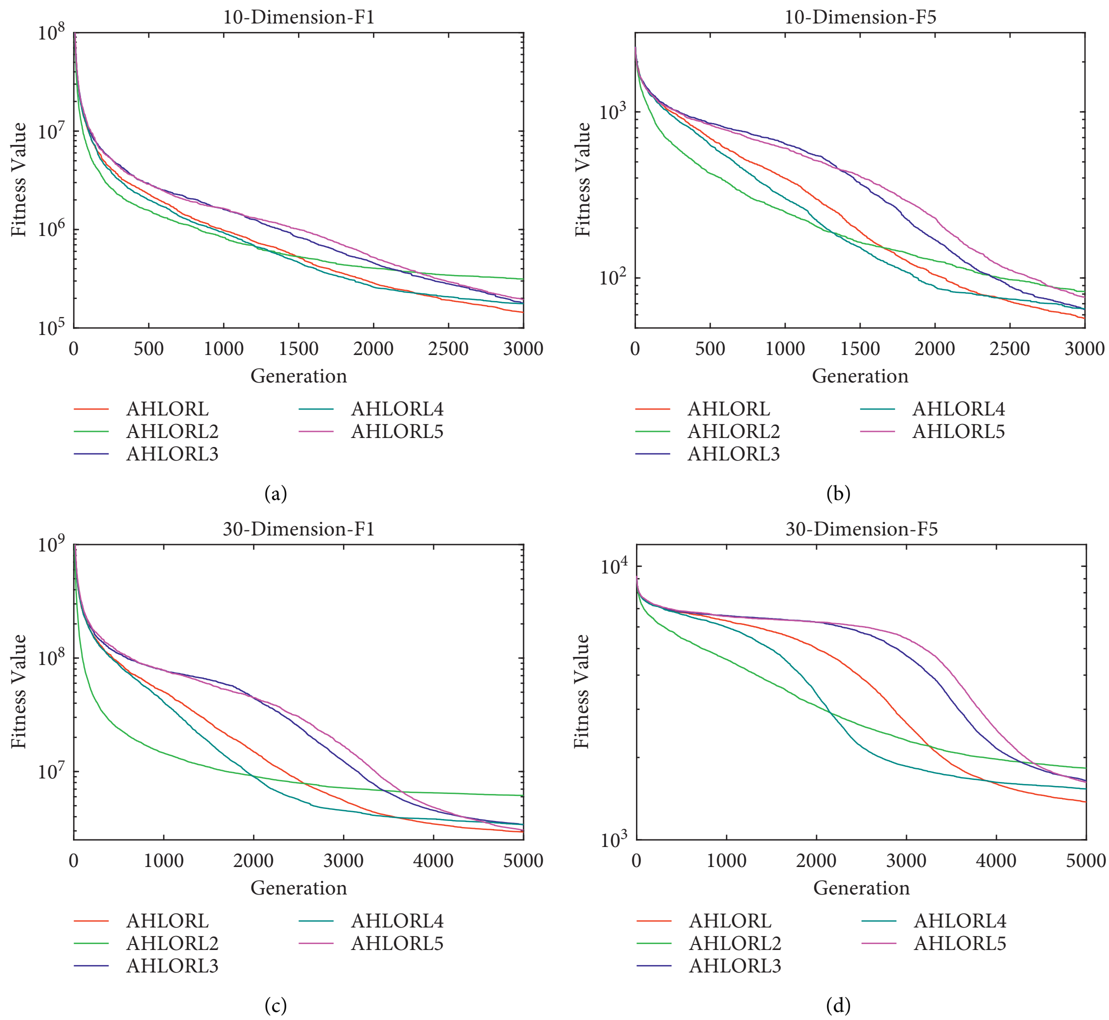


Figure 6: The fitness curves of AHLORL, AHLORL2, AHLORL3, AHLORL4, and AHLORL5.

Table 9: Parameter settings of AHLORL, HLORL, IAHLO, SCHLO, SFPSO, BGWO, BAAA, and IBDE.

Algorithms	Parameter settings
AHLORL	$pr = 5/M$, $pi = 0.82$, $ps_{\min} = 0.85$, $ps_{\max} = 0.96$, $Tp = 0.7$, $k = 0.5$
HLORL [18]	$pr = 5/M$, $pi = 0.82$, $ps = 0.92$
IAHLO [4]	$pr_{\min1} = 0.02$, $pr_{\min2} = 0.05$, $pr_{\max} = 0.15$, $pi = 0.85 + 2/M$, $Sp = 0.2 \times G_{\max}$
SCHLO [3]	$pr_{\text{mid}} = 0.1$, $pi_{\text{mid}} = 0.9$
SFPSO [43]	$\omega\text{min} = 0.95$, $\omega\text{max} = 0.99$, $\text{c1} = \text{c2} = 2.05$, $\text{vmax} = 6$, $\text{vmin} = -6$
BGWO [44]	$a = 2 - t \times 2/T$, $A = 2a \times \text{r1} - a$, $C = 2 \times \text{r2}$
BAAA [45]	$\beta = 0.5$, $UMSP = 0.5$, $Ap = 0.5$, $DSP = 0.66$
IBDE [46]	$\delta = 0.05$, $a = 1.0$, $p_{\text{sm}} = 0.008$, $b = 5$

Note. M is the dimension of solutions.

AHLORL and two mainstream metaheuristic algorithms, i.e., Genetic Algorithms (GAs) and Particle Swarm Optimization (PSO), are compared and discussed in this section.

In Genetic Algorithms (GAs), the mutation operator has a similar effect on the RLO of AHLORL. However, there are still differences in the execution strategy of the operator. In AHLORL, each bit of an individual has an independent mutation probability $pr/2$. In binary GAs, the mutation of all bits of an individual will not occur for the simple mutation operator or the boundary mutation operator. Besides, the combination of the ILO, SILO, and SRLO of AHLORL can be seen as a complicated variable multipoint crossover operator in GAs. However, there are still significant differences between the combination of ILO, SILO, and SRLO

Table 10: The results of all the algorithms on the 10-dimensional benchmark functions.

Fun	Metric	AHLORL	HLORL	IAHLO	SCHLO	SFPSO	BGWO	BAAA	IBDE
F1	Best	**7.0185E+03**	1.3446E+04	9.3633E+03	4.7152E+04	3.3678E+04	1.2398E+05	6.8415E+04	5.1655E+05
	Mean	**1.4321E+05**	1.9159E+05	2.1120E+05	4.0181E+05	2.4912E+06	1.4807E+07	6.7169E+06	8.8931E+06
	Std	**1.3145E+05**	1.9685E+05	1.9791E+05	3.6145E+05	4.2897E+06	1.7463E+07	1.2579E+07	6.0931E+06
	t-test	—	1	1	1	1	1	1	1
	W-test	—	1	1	1	1	1	1	1
F2	Best	**1.4326E+03**	1.4944E+03	4.2125E+03	1.0268E+04	2.7518E+05	4.2573E+05	2.1975E+05	4.3900E+05
	Mean	**8.4642E+05**	3.2602E+06	2.9041E+06	3.0073E+06	1.9300E+07	1.2978E+08	2.5241E+07	1.2781E+08
	Std	**1.9368E+06**	5.2737E+06	3.3760E+06	3.3770E+06	1.6533E+07	1.9796E+08	2.6981E+07	1.8336E+08
	t-test	—	1	1	1	1	1	1	1
	W-test	—	1	1	1	1	1	1	1
F3	Best	2.5798E+00	3.7595E+00	**1.3525E+00**	1.8761E+01	6.0587E+00	2.0000E+01	1.0695E+01	1.9732E+01
	Mean	**1.8580E+01**	1.9164E+01	1.8836E+01	2.0013E+01	1.9777E+01	2.0019E+01	1.9919E+01	2.0010E+01
	Std	**4.3836E+00**	3.3914E+00	4.2628E+00	1.3203E−01	1.7255E+00	1.9284E−02	9.2719E−01	2.9474E−02
	t-test	—	0	0	1	1	1	1	1
	W-test	—	1	1	1	0	0	1	0
F4	Best	5.5808E−01	**1.2431E−01**	1.4518E+00	3.6097E+00	3.2684E+00	7.1134E+00	6.3039E+00	8.9835E+00
	Mean	**4.4647E+00**	4.6057E+00	6.0314E+00	9.1071E+00	1.1282E+01	2.3981E+01	1.5214E+01	2.5032E+01
	Std	**1.4911E+00**	1.8133E+00	2.2456E+00	2.2050E+00	3.9832E+00	1.0248E+01	6.0930E+00	7.2519E+00
	t-test	—	0	1	1	1	1	1	1
	W-test	—	0	1	1	1	1	1	1
F5	Best	2.5876E+00	**1.7577E+00**	3.1217E+00	2.1931E+01	1.4759E+01	1.4030E+02	1.6839E+01	1.1878E+02
	Mean	**5.7225E+01**	6.3487E+01	1.0786E+02	2.3633E+02	3.1082E+02	5.5590E+02	3.6302E+02	6.2041E+02
	Std	**5.0841E+01**	6.8584E+01	8.0639E+01	1.0189E+02	1.5763E+02	2.1552E+02	1.6622E+02	1.8303E+02
	t-test	—	0	1	1	1	1	1	1
	W-test	—	0	1	1	1	1	1	1
F6	Best	4.5969E+01	4.7732E+01	**4.2252E+01**	8.0843E+01	9.5246E+01	1.4213E+03	2.2540E+02	1.2421E+03
	Mean	1.3756E+03	1.1842E+03	**1.0852E+03**	2.0278E+03	5.3354E+04	4.7971E+05	7.2668E+04	1.4828E+05
	Std	1.3486E+03	1.3757E+03	**8.8106E+02**	1.8091E+03	6.7434E+04	7.8036E+05	7.0640E+04	1.6664E+05
	t-test	—	0	0	1	1	1	1	1
	W-test	—	0	0	1	1	1	1	1
F7	Best	1.5272E−01	**6.6477E−02**	2.2471E−01	4.7651E−01	4.2832E−01	6.1152E−01	8.7150E−01	2.9339E−01
	Mean	**9.1146E−01**	9.4744E−01	1.0279E+00	1.2819E+00	1.8793E+00	3.1382E+00	2.2034E+00	3.1227E+00
	Std	**3.6188E−01**	4.3133E−01	3.7009E−01	3.1360E−01	5.9668E−01	1.4746E+00	7.2780E−01	1.2871E+00
	t-test	—	0	1	1	1	1	1	1
	W-test	—	0	1	1	1	1	1	1
F8	Best	**2.8786E+00**	7.9561E+00	3.1862E+00	5.9463E+00	1.1982E+02	3.2531E+02	4.0006E+01	4.7011E+02
	Mean	**3.3990E+02**	3.8820E+02	4.0453E+02	4.5745E+02	1.0047E+04	3.3140E+05	8.5433E+03	6.4974E+04
	Std	**3.3356E+02**	4.6690E+02	5.2586E+02	4.5645E+02	1.9573E+04	6.6362E+05	1.0979E+04	1.7631E+05
	t-test	—	1	0	1	1	1	1	1
	W-test	—	1	0	1	1	1	1	1
F9	Best	**1.0009E+02**	1.0010E+02	1.0012E+02	1.0014E+02	1.0020E+02	1.0033E+02	1.0018E+02	1.0021E+02
	Mean	**1.0017E+02**	1.0020E+02	1.0026E+02	1.0028E+02	1.0043E+02	1.0082E+02	1.0057E+02	1.0073E+02
	Std	**3.2463E−02**	5.0998E−02	5.9280E−02	6.4821E−02	1.5868E−01	4.5483E−01	2.1405E−01	7.4180E−01
	t-test	—	1	1	1	1	1	1	1
	W-test	—	1	1	1	1	1	1	1
F10	Best	**2.2384E+02**	2.5058E+02	2.3869E+02	2.5461E+02	2.7677E+02	3.3769E+02	3.3496E+02	5.0671E+02
	Mean	**3.5107E+02**	3.5465E+02	4.1261E+02	4.1754E+02	2.5943E+03	3.8343E+04	4.0855E+03	1.7553E+04
	Std	**6.6269E+01**	6.8455E+01	1.9901E+02	1.2079E+02	2.8501E+03	8.4870E+04	5.4270E+03	3.2145E+04
	t-test	—	0	1	1	1	1	1	1
	W-test	—	0	1	1	1	1	1	1
F11	Best	2.7164E+00	**2.3079E+00**	2.6666E+00	4.0966E+00	5.2569E+00	1.2480E+01	7.4946E+00	1.3264E+01
	Mean	**2.1417E+01**	7.1509E+01	6.5715E+01	3.9942E+01	2.4030E+02	3.3103E+02	2.7509E+02	2.5003E+02
	Std	**6.4015E+01**	1.2156E+02	1.1735E+02	8.7214E+01	1.1758E+02	1.0919E+02	8.2864E+01	1.0680E+02
	t-test	—	1	1	0	1	1	1	1
	W-test	—	1	1	1	1	1	1	1

Table 10: Continued.

Fun	Metric	AHLORL	HLORL	IAHLO	SCHLO	SFPSO	BGWO	BAAA	IBDE
F12	Best	**1.0052E + 02**	1.0115E + 02	1.0133E + 02	1.0118E + 02	1.0166E + 02	1.0218E + 02	1.0227E + 02	1.0165E + 02
	Mean	**1.0155E + 02**	1.0195E + 02	1.0270E + 02	1.0230E + 02	1.0384E + 02	1.0670E + 02	1.0565E + 02	1.0353E + 02
	Std	**4.3034E − 01**	4.0969E − 01	5.8725E − 01	5.9069E − 01	1.2615E + 00	3.0044E + 00	2.0406E + 00	1.0769E + 00
	t-test	—	1	1	1	1	1	1	1
	W-test	—	1	1	1	1	1	1	1
F13	Best	**2.1308E + 01**	2.1427E + 01	2.4364E + 01	2.3509E + 01	2.2514E + 01	2.4153E + 01	2.4025E + 01	3.0650E + 01
	Mean	**2.5728E + 01**	2.6305E + 01	2.8891E + 01	2.9169E + 01	3.0810E + 01	3.5695E + 01	3.3104E + 01	3.7691E + 01
	Std	**1.8822E + 00**	2.2454E + 00	2.0852E + 00	1.7060E + 00	3.0419E + 00	3.8189E + 00	3.8470E + 00	3.0153E + 00
	t-test	—	1	1	1	1	1	1	1
	W-test	—	1	1	1	1	1	1	1
F14	Best	3.3572E + 02	2.8040E + 02	**1.4995E + 02**	3.9862E + 02	4.2596E + 02	1.7775E + 03	4.3885E + 02	9.9638E + 02
	Mean	2.7176E + 03	4.0533E + 03	3.4954E + 03	**2.5245E + 03**	5.2164E + 03	6.6428E + 03	6.6212E + 03	4.1075E + 03
	Std	9.2981E + 02	1.9211E + 03	2.0107E + 03	**9.5392E + 02**	2.3174E + 03	2.9204E + 03	2.7114E + 03	1.2864E + 03
	t-test	—	1	1	0	1	1	1	1
	W-test	—	1	1	0	1	1	1	1
F15	Best	**1.0002E + 02**	1.0035E + 02	1.0002E + 02	1.0009E + 02	1.0056E + 02	1.0335E + 02	1.0240E + 02	1.0659E + 02
	Mean	**1.0234E + 02**	1.0610E + 02	1.0306E + 02	1.0387E + 02	1.1060E + 02	1.1615E + 02	1.1081E + 02	1.1903E + 02
	Std	**1.9637E + 00**	4.3975E + 00	2.4058E + 00	3.0619E + 00	3.8583E + 00	7.8143E + 00	3.8961E + 00	6.4351E + 00
	t-test	—	1	1	1	1	1	1	1
	W-test	—	1	1	1	1	1	1	1

Table 11: The summary results of the t-test and W-test on the 10-dimensional benchmark functions.

Metric	AHLORL	HLORL	IAHLO	SCHLO	SFPSO	BGWO	BAAA	IBDE
t-test	1	9	12	13	15	15	15	15
	0	6	3	2	0	0	0	0
	−1	0	0	0	0	0	0	0
W-test	1	10	13	14	14	14	15	14
	0	5	2	1	1	1	0	1
	−1	0	0	0	0	0	0	0

and the crossover operator. In AHLORL, the ILO and SILO yield a new candidate by copying the values of the solutions in the IKD and SKD, and the SRLO yields a new candidate based on the best knowledge saved in the IKD of three randomly chosen individuals. In binary GAs, the crossover operator chooses two solutions in the current population to generate a new offspring. Note that the best solution of an individual, i.e., the knowledge stored in the IKD, may not survive in the selection of GAs since there is no certain mechanism to save it for the next generation. Therefore, the inherent search mechanisms of GAs and AHLORL are different.

Compared with binary GAs, Particle Swarm Optimization may be more similar to AHLORL in the structure of the algorithm as the information of the individual best solutions and the global best solution is also adopted in PSO. However, the underlying search mechanisms of PSO and AHLORL are also different. For the standard PSO, it is a real-coded algorithm inspired by the foraging of birds. But the proposed AHLORL is a binary-coded algorithm that mimics the learning mechanism of humans. In the updating of the population, PSO and its binary variants generate solutions based on the "velocity," and the information of the "velocity" is updated based on its inertia information and the individual/global best information. But, there is no corresponding definition of the "velocity" or inertia information in AHLORL. Besides, PSO performs the new search based on its current position while the following search in AHLORL does not depend on its current solution. Therefore, the forms of the operators of AHLORL and binary PSO are different, and the candidates generated by AHLORL and binary PSO with the same population are also distinct.

5. Experimental Results and Discussion

In this section, the proposed AHLORL, as well as seven recent algorithms, i.e., HLORL [28], IAHLO [14], SCHLO [13], Scale-Free Binary Particle Swarm Optimization (SFPSO) [52], Binary Grey Wolf Optimizer (BGWO) [53], Binary Artificial Algae Algorithm (BAAA) [54], and Improved Binary Differential Evolution (IBDE) [55], were applied to solve the 10/30-dimensional CEC 15 benchmark functions [50] and multidimensional knapsack problems (MKPs) [56]. For a fair comparison, the parameter settings for all the algorithms adopted the recommended values, which are listed in Table 9. Besides, the simulation

Table 12: The results of all the algorithms on the 30-dimensional benchmark functions.

Fun	Metric	AHLORL	HLORL	IAHLO	SCHLO	SFPSO	BGWO	BAAA	IBDE
F1	Best	**3.5098E + 05**	9.0963E + 05	4.2587E + 06	4.9958E + 06	2.7440E + 06	3.7163E + 06	6.1783E + 06	1.2036E + 07
	Mean	**2.9313E + 06**	5.0158E + 06	1.3071E + 07	1.1129E + 07	1.8469E + 07	3.5357E + 07	3.4367E + 07	3.9808E + 07
	Std	**1.5240E + 06**	4.5879E + 06	5.1957E + 06	4.2216E + 06	1.1233E + 07	1.8509E + 07	2.1090E + 07	1.4055E + 07
	t-test	—	1	1	1	1	1	1	1
	W-test	—	1	1	1	1	1	1	1
F2	Best	**1.0894E + 06**	4.2357E + 06	2.5375E + 08	8.0420E + 06	8.3441E + 07	1.4856E + 08	1.8754E + 08	6.8389E + 08
	Mean	**2.5609E + 07**	1.1108E + 08	8.6618E + 08	1.6169E + 08	1.0878E + 09	1.3677E + 09	1.2748E + 09	2.6559E + 09
	Std	**2.0518E + 07**	1.1473E + 08	3.6072E + 08	1.2576E + 08	6.5628E + 08	7.9023E + 08	7.8449E + 08	9.4198E + 08
	t-test	—	1	1	1	1	1	1	1
	W-test	—	1	1	1	1	1	1	1
F3	Best	**2.0015E + 01**	2.0027E + 01	2.0673E + 01	2.0129E + 01	2.0611E + 01	2.0024E + 01	2.0023E + 01	2.0017E + 01
	Mean	**2.0088E + 01**	2.0199E + 01	2.0874E + 01	2.0375E + 01	2.0835E + 01	2.0133E + 01	2.0133E + 01	2.0119E + 01
	Std	**4.4153E − 02**	8.6295E − 02	5.6902E − 02	8.2217E − 02	7.0497E − 02	7.9673E − 02	8.2617E − 02	7.3520E − 02
	t-test	—	1	1	1	1	1	1	1
	W-test	—	1	1	1	1	1	1	1
F4	Best	**1.6636E + 01**	2.9605E + 01	1.0905E + 02	5.5509E + 01	6.1622E + 01	7.1585E + 01	6.6964E + 01	8.1246E + 01
	Mean	**3.5857E + 01**	5.9574E + 01	1.6191E + 02	9.2484E + 01	9.4954E + 01	1.1528E + 02	1.1890E + 02	1.3745E + 02
	Std	**7.3883E + 00**	1.3293E + 01	1.5608E + 01	1.6034E + 01	1.8212E + 01	2.3329E + 01	2.4313E + 01	1.8938E + 01
	t-test	—	1	1	1	1	1	1	1
	W-test	—	1	1	1	1	1	1	1
F5	Best	**3.1978E + 02**	1.0839E + 03	3.6009E + 03	2.3346E + 03	1.7347E + 03	1.6199E + 03	1.8477E + 03	2.0324E + 03
	Mean	**1.3751E + 03**	1.9456E + 03	5.0128E + 03	3.3259E + 03	3.2133E + 03	2.8057E + 03	2.8749E + 03	3.0347E + 03
	Std	**4.0458E + 02**	4.6736E + 02	4.3888E + 02	3.9043E + 02	5.8685E + 02	5.4174E + 02	5.0916E + 02	3.6628E + 02
	t-test	—	1	1	1	1	1	1	1
	W-test	—	1	1	1	1	1	1	1
F6	Best	5.9882E + 04	**1.3325E + 04**	4.0064E + 04	2.3135E + 05	1.9820E + 05	6.2731E + 05	1.0287E + 06	6.4738E + 05
	Mean	**4.0067E + 05**	4.4431E + 05	5.4702E + 05	1.0361E + 06	2.8680E + 06	1.2642E + 07	8.7032E + 06	7.3282E + 06
	Std	**2.2508E + 05**	3.4413E + 05	4.2097E + 05	4.8414E + 05	2.0881E + 06	8.5527E + 06	5.3332E + 06	4.1596E + 06
	t-test	—	0	1	1	1	1	1	1
	W-test	—	0	1	1	1	1	1	1
F7	Best	6.4758E + 00	6.8847E + 00	1.0866E + 01	9.8167E + 00	8.6934E + 00	1.1447E + 01	**6.0209E + 00**	1.3647E + 01
	Mean	**1.0124E + 01**	1.1213E + 01	1.5438E + 01	1.3752E + 01	2.2356E + 01	3.7480E + 01	3.6721E + 01	2.3279E + 01
	Std	**1.3229E + 00**	1.7691E + 00	1.5947E + 00	1.4168E + 00	1.7295E + 01	2.9827E + 01	3.3357E + 01	7.9519E + 00
	t-test	—	1	1	1	1	1	1	1
	W-test	—	1	1	1	1	1	1	1
F8	Best	**9.5273E + 03**	9.8358E + 03	1.8766E + 04	3.8604E + 04	7.4961E + 04	2.4240E + 05	7.3224E + 04	2.7181E + 05
	Mean	**1.0991E + 05**	1.3257E + 05	1.3993E + 05	2.0626E + 05	7.6158E + 05	3.4480E + 06	2.3953E + 06	1.9233E + 06
	Std	**6.0527E + 04**	8.4546E + 04	1.0217E + 05	1.0570E + 05	6.0954E + 05	2.6455E + 06	2.1905E + 06	1.2221E + 06
	t-test	—	1	1	1	1	1	1	1
	W-test	—	1	1	1	1	1	1	1
F9	Best	**1.0244E + 02**	1.0261E + 02	1.0470E + 02	1.0368E + 02	1.0417E + 02	1.0394E + 02	1.0420E + 02	1.0539E + 02
	Mean	**1.0288E + 02**	1.0333E + 02	1.0680E + 02	1.0462E + 02	1.0761E + 02	1.1690E + 02	1.2238E + 02	1.1573E + 02
	Std	**2.1536E − 01**	1.1594E + 00	1.8305E + 00	5.5225E − 01	3.4149E + 00	4.2117E + 01	5.5823E + 01	6.7971E + 00
	t-test	—	1	1	1	1	1	1	1
	W-test	—	1	1	1	1	1	1	1
F10	Best	**8.0989E + 03**	4.1676E + 04	2.2123E + 04	3.5227E + 04	5.8620E + 04	1.1142E + 05	3.5327E + 05	2.4152E + 05
	Mean	**1.5791E + 05**	1.8300E + 05	2.7241E + 05	2.9698E + 05	1.4694E + 06	4.0229E + 06	4.7046E + 06	3.2672E + 06
	Std	**9.7338E + 04**	9.2415E + 04	2.3919E + 05	1.9105E + 05	9.9794E + 05	3.2280E + 06	6.3750E + 06	2.4546E + 06
	t-test	—	0	1	1	1	1	1	1
	W-test	—	1	1	1	1	1	1	1
F11	Best	**3.0624E + 02**	3.0852E + 02	3.1583E + 02	3.1800E + 02	3.1903E + 02	3.1587E + 02	3.1734E + 02	3.3006E + 02
	Mean	**3.2617E + 02**	4.8949E + 02	6.1431E + 02	3.8016E + 02	8.5938E + 02	9.3240E + 02	8.3604E + 02	4.6217E + 02
	Std	**3.8006E + 01**	1.4960E + 02	2.5687E + 02	1.0738E + 02	1.0920E + 02	1.3880E + 02	2.6787E + 02	1.7508E + 02
	t-test	—	1	1	1	1	1	1	1
	W-test	—	1	1	1	1	1	1	1

Table 12: Continued.

Fun	Metric	AHLORL	HLORL	IAHLO	SCHLO	SFPSO	BGWO	BAAA	IBDE
F12	Best	**1.0417E + 02**	1.0482E + 02	1.0777E + 02	1.0545E + 02	1.0797E + 02	1.0681E + 02	1.0739E + 02	1.0603E + 02
	Mean	**1.0555E + 02**	1.0592E + 02	1.0985E + 02	1.0719E + 02	1.0982E + 02	1.1025E + 02	1.1067E + 02	1.0822E + 02
	Std	**5.3918E − 01**	6.2218E − 01	1.1221E + 00	7.3087E − 01	1.1802 E + 00	1.9445E + 00	1.8683E + 00	9.9629E − 01
	t-test	—	1	1	1	1	1	1	1
	W-test	—	1	1	1	1	1	1	1
F13	Best	7.5673E + 01	**7.3943E + 01**	1.1390E + 02	9.7683E + 01	9.7090E + 01	9.8463E + 01	8.4401E + 01	1.0127E + 02
	Mean	**9.1145E + 01**	9.1641E + 01	1.2233E + 02	1.0993E + 02	1.1661E + 02	1.1086E + 02	1.0969E + 02	1.1720E + 02
	Std	**6.8232E + 00**	5.9686E + 00	3.1306E + 00	3.8369E + 00	5.1212E + 00	5.9995E + 00	7.5160E + 00	4.4689E + 00
	t-test	—	0	1	1	1	1	1	1
	W-test	—	0	1	1	1	1	1	1
F14	Best	**3.1272E + 04**	3.1353E + 04	3.1820E + 04	3.1425E + 04	3.1611E + 04	3.1730E + 04	3.1699E + 04	3.2629E + 04
	Mean	**3.2927E + 04**	3.3319E + 04	3.4153E + 04	3.3370E + 04	3.4475E + 04	3.4700E + 04	3.4791E + 04	3.4534E + 04
	Std	**8.3761E + 02**	8.8714E + 02	1.0854E + 03	9.9792E + 02	1.0511E + 03	1.1733E + 03	1.5108E + 03	9.1681E + 02
	t-test	—	1	1	1	1	1	1	1
	W-test	—	1	1	1	1	1	1	1
F15	Best	**1.0182E + 02**	1.0285E + 02	1.0937E + 02	1.0383E + 02	1.0888E + 02	1.0589E + 02	1.0689E + 02	1.2353E + 02
	Mean	**1.0562E + 02**	1.0782E + 02	1.1710E + 02	1.0952E + 02	1.1862E + 02	2.1332E + 02	1.6383E + 02	1.5057E + 02
	Std	**1.3104E + 00**	2.4117E + 00	3.2758E + 00	2.8086E + 00	4.9390E + 00	3.3686E + 02	2.1802E + 02	3.2049E + 01
	t-test	—	1	1	1	1	1	1	1
	W-test	—	1	1	1	1	1	1	1

Table 13: The summary results of the t-test and W-test on the high-dimensional benchmark functions.

Metric	AHLORL	HLORL	IAHLO	SCHLO	SFPSO	BGWO	BAAA	IBDE
	1	12	15	15	15	15	15	15
t-test	0	3	0	0	0	0	0	0
	−1	0	0	0	0	0	0	0
	1	13	15	15	15	15	15	15
W-test	0	2	0	0	0	0	0	0
	−1	0	0	0	0	0	0	0

environment was Eclipse platform and Java encoding on Windows 7, 64-bit operating system, the configuration of computer was Intel Xeon E3-1230 v3 @3.30 GHz 16G RAMs.

5.1. Results of the CEC 15 Benchmark Functions

5.1.1. Low-Dimensional Benchmark Functions. The optimization results of all algorithms on the 10-dimensional benchmark functions are presented in Table 10, where the best numerical results are highlighted in bold. Besides, the Student's t-test (t-test) and the Wilcoxon signed-rank test (W-test) are also summarized in Table 11. As can be seen from Tables 10 and 11, the AHLORL is significantly superior to these compared algorithms, which obtains 13 best numerical results out of 15 functions. Besides, the results of the t-test clearly indicate that the proposed AHLORL is substantially better than HLORL, IAHLO, SCHLO, SFPSO, BGWO, BAAA, and IBDE on 9, 12, 13, 15, 15, 15, and 15 out of 15 functions. And the results of the W-test also unfold that the proposed AHLORL is obviously superior to HLORL, IAHLO, SCHLO, SFPSO, BGWO, BAAA, and IBDE on 10, 13, 14, 14, 14, 15, and 14 out of 15 functions, respectively.

5.1.2. High-Dimensional Benchmark Functions. The numerical results of all algorithms on the 30-dimensional CEC 15 benchmark functions are listed in Table 12, where the best optimization results are also marked in bold. For convenience, the summary results of the t-test and W-test are counted in Table 13. From Tables 12 and 13, AHLORL obtains the best numerical results on all the functions. Besides, the t-test results explicitly show that the proposed AHLORL significantly surpasses HLORL, IAHLO, SCHLO, SFPSO, BGWO, BAAA, and IBDE on 12, 15, 15, 15, 15, 15, and 15 out of 15 functions, respectively. And the W-test results also show that the proposed AHLORL is significantly better than HLORL, IAHLO, SCHLO, SFPSO, BGWO, BAAA, and IBDE on 13, 15, 15, 15, 15, 15, and 15 out of 15 functions, respectively.

5.2. Results of the Multidimensional Knapsack Problems (MKPs).

To further verify the optimization ability of AHLORL, a total of 30 multidimensional knapsack problems (MKPs) [56], i.e., the instances 10.500.00-29, was adopted as the test function to evaluate the performance of AHLORL. The times of simulation test for all problems were 100 independently, and the population size and the maximal

Table 14: The results of all the algorithms on the multidimensional knapsack problems (MKPs).

Problem	Algorithm	Best	Mean	Worst	Std	t-Test	W-test
10.500.0	AHLORL	**113770.0**	**112515.5**	**111112.0**	**520.6**	—	—
	HLORL	113392.0	111882.3	110844.0	508.9	1	1
	IAHLO	103981.0	101229.4	99509.0	817.3	1	1
	SCHLO	110985.0	109034.1	106690.0	819.4	1	1
	SFPSO	105522.0	103397.3	99615.0	1083.6	1	1
	BGWO	98719.0	94972.4	91396.0	1452.8	1	1
	BAAA	109324.0	107065.6	102318.0	1361.2	1	1
	IBDE	98658.0	95399.1	93779.0	932.2	1	1
10.500.1	AHLORL	**115343.0**	**114185.3**	**112536.0**	**536.1**	—	—
	HLORL	114829.0	113352.3	112249.0	510.1	1	1
	IAHLO	104791.0	102300.2	99872.0	883.3	1	1
	SCHLO	112654.0	110545.4	108797.0	880.0	1	1
	SFPSO	107092.0	104492.1	101672.0	1032.7	1	1
	BGWO	98755.0	95498.8	91153.0	1340.1	1	1
	BAAA	111014.0	108443.1	104757.0	1118.7	1	1
	IBDE	98355.0	95931.3	93363.0	1019.0	1	1
10.500.2	AHLORL	**115115.0**	**114113.1**	**113027.0**	**482.8**	—	—
	HLORL	114597.0	113382.7	112109.0	470.0	1	1
	IAHLO	104090.0	102071.7	99972.0	799.1	1	1
	SCHLO	112137.0	110417.4	108348.0	756.7	1	1
	SFPSO	107126.0	104349.0	102060.0	1037.5	1	1
	BGWO	98531.0	95154.7	91693.0	1459.0	1	1
	BAAA	110563.0	108404.6	105710.0	1060.5	1	1
	IBDE	98484.0	95936.1	93732.0	925.0	1	1
10.500.3	AHLORL	**115177.0**	**113690.2**	**112607.0**	**501.9**	—	—
	HLORL	114500.0	113000.5	111978.0	490.9	1	1
	IAHLO	103401.0	101526.7	99638.0	783.1	1	1
	SCHLO	111905.0	109826.5	107634.0	855.7	1	1
	SFPSO	105730.0	103726.9	101340.0	910.6	1	1
	BGWO	97955.0	95011.9	91703.0	1370.7	1	1
	BAAA	110349.0	107690.8	104607.0	1147.7	1	1
	IBDE	98176.0	95512.3	93028.0	986.6	1	1
10.500.4	AHLORL	**112734.0**	**111477.5**	**110234.0**	**438.7**	—	—
	HLORL	112142.0	110843.5	109754.0	488.2	1	1
	IAHLO	102241.0	100001.5	97861.0	935.1	1	1
	SCHLO	109843.0	108015.9	105552.0	859.6	1	1
	SFPSO	105108.0	102428.1	99452.0	1134.1	1	1
	BGWO	97090.0	93680.8	90287.0	1443.4	1	1
	BAAA	108672.0	106086.9	103229.0	1054.5	1	1
	IBDE	96521.0	93964.0	92109.0	981.0	1	1
10.500.5	AHLORL	**115816.0**	**114534.9**	**113008.0**	**545.9**	—	—
	HLORL	115172.0	113916.6	112556.0	466.1	1	1
	IAHLO	103176.0	101530.9	100124.0	721.4	1	1
	SCHLO	112958.0	110823.1	108643.0	943.8	1	1
	SFPSO	107709.0	104292.7	102048.0	1154.5	1	1
	BGWO	98901.0	94952.6	90938.0	1504.2	1	1
	BAAA	110665.0	108352.3	105058.0	1003.0	1	1
	IBDE	97354.0	94899.5	92461.0	1007.2	1	1
10.500.6	AHLORL	**115971.0**	**114783.5**	**113426.0**	**486.1**	—	—
	HLORL	115261.0	114186.3	112740.0	497.9	1	1
	IAHLO	104943.0	102615.8	100549.0	828.7	1	1
	SCHLO	113446.0	111451.9	109636.0	802.9	1	1
	SFPSO	106527.0	105021.0	101896.0	994.4	1	1
	BGWO	99483.0	95590.0	91570.0	1533.4	1	1
	BAAA	111964.0	109092.6	106503.0	1107.5	1	1
	IBDE	98693.0	95908.3	93870.0	1067.9	1	1

Table 14: Continued.

Problem	Algorithm	Best	Mean	Worst	Std	t-Test	W-test
10.500.7	AHLORL	**114142.0**	**113243.2**	**112113.0**	**459.7**	—	—
	HLORL	113305.0	112510.0	111271.0	420.6	1	1
	IAHLO	103489.0	101443.4	99754.0	864.6	1	1
	SCHLO	111576.0	109702.2	107436.0	883.5	1	1
	SFPSO	105719.0	103562.6	100190.0	1078.6	1	1
	BGWO	98657.0	94523.5	91059.0	1421.6	1	1
	BAAA	109896.0	107575.2	104160.0	1096.9	1	1
	IBDE	98332.0	95295.1	92864.0	959.8	1	1
10.500.8	AHLORL	**113833.0**	**112635.4**	**111321.0**	**504.6**	—	—
	HLORL	112964.0	112040.4	110359.0	508.3	1	1
	IAHLO	103400.0	101125.5	99572.0	823.2	1	1
	SCHLO	111031.0	109272.2	105571.0	924.4	1	1
	SFPSO	105540.0	103384.4	100600.0	1019.1	1	1
	BGWO	97854.0	94697.9	91237.0	1402.8	1	1
	BAAA	109954.0	107151.7	103881.0	1249.5	1	1
	IBDE	98882.0	95393.9	93069.0	1043.7	1	1
10.500.9	AHLORL	**114983.0**	**114184.6**	**113012.0**	**434.7**	—	—
	HLORL	114788.0	113580.5	112484.0	474.3	1	1
	IAHLO	104168.0	102295.6	100085.0	786.6	1	1
	SCHLO	112202.0	110479.1	108249.0	839.4	1	1
	SFPSO	107208.0	104519.9	101627.0	1204.7	1	1
	BGWO	98752.0	95404.3	91729.0	1425.1	1	1
	BAAA	110823.0	108415.5	102170.0	1339.6	1	1
	IBDE	98965.0	96162.1	94342.0	952.5	1	1
10.500.10	AHLORL	**212879.0**	**211667.0**	**210108.0**	**515.8**	—	—
	HLORL	212239.0	210406.5	208933.0	557.1	1	1
	IAHLO	203992.0	201407.8	199591.0	784.8	1	1
	SCHLO	209760.0	208011.5	205448.0	976.9	1	1
	SFPSO	207024.0	203787.1	200739.0	923.2	1	1
	BGWO	195031.0	189837.5	186272.0	1707.4	1	1
	BAAA	209238.0	207181.9	204201.0	1040.1	1	1
	IBDE	193836.0	190033.6	187542.0	1138.6	1	1
10.500.11	AHLORL	**214822.0**	**213715.3**	**211772.0**	**516.5**	—	—
	HLORL	214022.0	212461.0	211032.0	578.3	1	1
	IAHLO	204449.0	202539.6	200745.0	720.4	1	1
	SCHLO	212097.0	209782.1	208038.0	762.9	1	1
	SFPSO	207488.0	205121.6	202628.0	933.2	1	1
	BGWO	193401.0	190283.1	186498.0	1569.5	1	1
	BAAA	211544.0	208978.8	206337.0	994.1	1	1
	IBDE	192953.0	190320.0	188121.0	1074.3	1	1
10.500.12	AHLORL	**213703.0**	**212376.1**	**211168.0**	**514.3**	—	—
	HLORL	212398.0	211141.2	209222.0	580.3	1	1
	IAHLO	204266.0	201329.4	199793.0	794.9	1	1
	SCHLO	210798.0	208467.0	206135.0	978.1	1	1
	SFPSO	206413.0	203802.0	201967.0	927.5	1	1
	BGWO	192662.0	189101.0	186318.0	1504.8	1	1
	BAAA	209650.0	207360.8	203869.0	1273.6	1	1
	IBDE	191857.0	189242.2	187582.0	912.2	1	1
10.500.13	AHLORL	**212625.0**	**211482.2**	**209900.0**	**489.5**	—	—
	HLORL	211326.0	210310.1	209271.0	498.5	1	1
	IAHLO	202212.0	200167.8	197838.0	820.2	1	1
	SCHLO	209298.0	207633.1	205217.0	908.6	1	1
	SFPSO	205407.0	202710.3	199971.0	1025.4	1	1
	BGWO	191428.0	187601.5	183122.0	1731.4	1	1
	BAAA	209058.0	206739.7	203571.0	1217.1	1	1
	IBDE	191141.0	187978.1	185301.0	1135.0	1	1

Table 14: Continued.

Problem	Algorithm	Best	Mean	Worst	Std	t-Test	W-test
10.500.14	AHLORL	**209743.0**	**208596.5**	**207667.0**	**460.2**	—	—
	HLORL	208863.0	207382.8	206203.0	562.9	1	1
	IAHLO	200131.0	198124.2	196529.0	849.1	1	1
	SCHLO	206980.0	204761.5	202417.0	854.0	1	1
	SFPSO	202602.0	200608.0	198290.0	996.2	1	1
	BGWO	189680.0	186146.3	182509.0	1479.1	1	1
	BAAA	206637.0	204316.4	200682.0	1035.2	1	1
	IBDE	189184.0	186891.2	184202.0	1053.8	1	1
10.500.15	AHLORL	**210871.0**	**209569.9**	**207919.0**	**564.3**	—	—
	HLORL	209850.0	208598.7	206823.0	553.6	1	1
	IAHLO	200124.0	198504.7	196616.0	782.2	1	1
	SCHLO	208065.0	205853.3	203907.0	949.9	1	1
	SFPSO	203250.0	200889.3	198190.0	920.7	1	1
	BGWO	190424.0	186044.0	180941.0	1762.4	1	1
	BAAA	207532.0	204522.0	201071.0	1296.2	1	1
	IBDE	189469.0	186533.5	184525.0	1009.2	1	1
10.500.16	AHLORL	**213979.0**	**212733.5**	**211607.0**	**447.9**	—	—
	HLORL	212788.0	211584.1	210003.0	551.2	1	1
	IAHLO	204257.0	201416.1	199614.0	884.6	1	1
	SCHLO	210549.0	208953.1	206989.0	824.8	1	1
	SFPSO	206749.0	204229.6	201470.0	989.1	1	1
	BGWO	193898.0	188963.6	183804.0	1906.9	1	1
	BAAA	210623.0	207895.1	204029.0	1334.9	1	1
	IBDE	192404.0	189239.5	187040.0	1112.4	1	1
10.500.17	AHLORL	**216086.0**	**214444.2**	**213022.0**	**584.7**	—	—
	HLORL	214649.0	213125.9	212126.0	532.0	1	1
	IAHLO	205296.0	203324.2	201504.0	818.6	1	1
	SCHLO	212509.0	210242.7	208412.0	877.8	1	1
	SFPSO	207927.0	205715.2	203416.0	837.5	1	1
	BGWO	196171.0	191007.8	187060.0	1875.3	1	1
	BAAA	212163.0	209648.1	206546.0	1139.5	1	1
	IBDE	196214.0	191684.0	189545.0	975.8	1	1
10.500.18	AHLORL	**210263.0**	**208878.4**	**207740.0**	**508.1**	—	—
	HLORL	209673.0	207895.3	206792.0	488.9	1	1
	IAHLO	200589.0	198119.2	195967.0	812.7	1	1
	SCHLO	207557.0	205288.4	202595.0	866.1	1	1
	SFPSO	203146.0	200614.0	198019.0	976.6	1	1
	BGWO	190317.0	185829.2	181380.0	1766.6	1	1
	BAAA	206481.0	204376.6	199841.0	1147.8	1	1
	IBDE	189435.0	186275.1	184217.0	1009.3	1	1
10.500.19	AHLORL	**216256.0**	**214914.9**	**213692.0**	**506.7**	—	—
	HLORL	215023.0	213617.5	212300.0	590.9	1	1
	IAHLO	206333.0	204386.1	201917.0	833.5	1	1
	SCHLO	212876.0	210941.3	208463.0	978.6	1	1
	SFPSO	209270.0	206606.9	203959.0	949.3	1	1
	BGWO	195908.0	192282.2	188736.0	1445.6	1	1
	BAAA	213062.0	210214.8	207258.0	1105.2	1	1
	IBDE	195994.0	192959.8	191050.0	1064.3	1	1
10.500.20	AHLORL	**302215.0**	**301298.5**	**300236.0**	**425.6**	—	—
	HLORL	301585.0	300838.4	300115.0	366.4	1	1
	IAHLO	292406.0	290153.6	288741.0	719.5	1	1
	SCHLO	300292.0	298668.5	296817.0	656.6	1	1
	SFPSO	294739.0	292406.7	290576.0	951.7	1	1
	BGWO	287155.0	284406.9	281145.0	1408.7	1	1
	BAAA	298427.0	296235.6	293445.0	882.2	1	1
	IBDE	285664.0	283137.3	280969.0	1048.1	1	1

Table 14: Continued.

Problem	Algorithm	Best	Mean	Worst	Std	t-Test	W-test
10.500.21	AHLORL	**300047.0**	**299211.0**	**298122.0**	**405.6**	—	—
	HLORL	299638.0	298799.1	297873.0	327.1	1	1
	IAHLO	289863.0	288335.8	286770.0	677.9	1	1
	SCHLO	298287.0	296598.2	294000.0	740.7	1	1
	SFPSO	292800.0	290481.4	287934.0	863.6	1	1
	BGWO	286431.0	282791.7	279058.0	1648.1	1	1
	BAAA	296665.0	294396.9	292360.0	880.0	1	1
	IBDE	284002.0	281474.2	279179.0	928.3	1	1
10.500.22	AHLORL	**300385.0**	**299216.8**	**298454.0**	**349.2**	—	—
	HLORL	299515.0	298766.9	297999.0	330.4	1	1
	IAHLO	291114.0	288994.8	286745.0	710.2	1	1
	SCHLO	298391.0	296598.6	295026.0	688.5	1	1
	SFPSO	292972.0	290987.8	288860.0	807.3	1	1
	BGWO	287567.0	283495.6	280420.0	1557.7	1	1
	BAAA	296630.0	294581.2	291335.0	951.9	1	1
	IBDE	284770.0	282112.8	280297.0	954.0	1	1
10.500.23	AHLORL	**298371.0**	**297631.9**	**296972.0**	**351.0**	—	—
	HLORL	298305.0	297154.6	296254.0	402.6	1	1
	IAHLO	288461.0	286591.8	284931.0	735.8	1	1
	SCHLO	296509.0	295004.6	292914.0	646.8	1	1
	SFPSO	291285.0	288761.1	285978.0	893.4	1	1
	BGWO	284344.0	280812.0	277107.0	1410.5	1	1
	BAAA	294646.0	292441.2	289923.0	930.5	1	1
	IBDE	282517.0	279893.4	277913.0	991.9	1	1
10.500.24	AHLORL	**301958.0**	**301178.2**	**300267.0**	**364.3**	—	—
	HLORL	301937.0	300726.7	299962.0	397.6	1	1
	IAHLO	291715.0	289829.9	287890.0	755.7	1	1
	SCHLO	300078.0	298565.6	296701.0	757.8	1	1
	SFPSO	293581.0	292161.5	289638.0	809.6	1	1
	BGWO	287947.0	283995.3	280454.0	1566.0	1	1
	BAAA	298585.0	295992.7	294506.0	837.1	1	1
	IBDE	285550.0	283100.8	280529.0	1065.3	1	1
10.500.25	AHLORL	**299260.0**	**298446.4**	**297617.0**	**335.9**	—	—
	HLORL	298901.0	297980.0	297156.0	372.2	1	1
	IAHLO	290256.0	288533.4	286662.0	759.2	1	1
	SCHLO	297688.0	295979.4	294106.0	644.7	1	1
	SFPSO	292616.0	290445.8	288062.0	867.3	1	1
	BGWO	286462.0	283057.1	279369.0	1295.0	1	1
	BAAA	295892.0	293902.7	290951.0	961.4	1	1
	IBDE	285118.0	282043.4	279656.0	1006.0	1	1
10.500.26	AHLORL	**303031.0**	**301745.0**	**301001.0**	**371.9**	—	—
	HLORL	302049.0	301300.9	300374.0	361.9	1	1
	IAHLO	293550.0	290964.8	289203.0	748.5	1	1
	SCHLO	300923.0	299336.0	297649.0	609.4	1	1
	SFPSO	295285.0	293236.9	290848.0	846.7	1	1
	BGWO	288028.0	285192.2	280841.0	1441.5	1	1
	BAAA	298930.0	296835.7	294529.0	877.2	1	1
	IBDE	287349.0	283893.9	281788.0	1073.6	1	1
10.500.27	AHLORL	**294409.0**	**293232.0**	**292501.0**	**378.3**	—	—
	HLORL	293538.0	292822.7	291939.0	334.3	1	1
	IAHLO	285759.0	283169.4	281179.0	759.5	1	1
	SCHLO	292222.0	290747.6	289393.0	644.2	1	1
	SFPSO	286925.0	285025.9	282668.0	858.5	1	1
	BGWO	280920.0	277702.1	273044.0	1518.9	1	1
	BAAA	290489.0	288805.8	287020.0	818.9	1	1
	IBDE	279027.0	276672.2	273862.0	959.2	1	1

Table 14: Continued.

Problem	Algorithm	Best	Mean	Worst	Std	t-Test	W-test
10.500.28	AHLORL	**299442.0**	**298382.7**	**297751.0**	**347.8**	—	—
	HLORL	298678.0	297848.4	296912.0	364.1	1	1
	IAHLO	290118.0	287148.3	285486.0	739.9	1	1
	SCHLO	297333.0	295810.7	293097.0	730.5	1	1
	SFPSO	291563.0	289273.7	286573.0	932.5	1	1
	BGWO	285815.0	281373.3	277453.0	1529.9	1	1
	BAAA	295388.0	293188.0	290847.0	1029.9	1	1
	IBDE	283464.0	280183.5	277694.0	1048.9	1	1
10.500.29	AHLORL	**304836.0**	**303754.2**	**303009.0**	**351.8**	—	—
	HLORL	304383.0	303361.8	302190.0	387.9	1	1
	IAHLO	295204.0	292865.5	290276.0	872.5	1	1
	SCHLO	303018.0	301395.3	299597.0	712.2	1	1
	SFPSO	297197.0	294974.5	292606.0	958.2	1	1
	BGWO	290420.0	286599.3	283284.0	1725.2	1	1
	BAAA	300634.0	298845.4	296797.0	929.4	1	1
	IBDE	289072.0	285729.0	283654.0	968.1	1	1

Table 15: The summary results of the t-test and W-test on the multidimensional knapsack problems (MKPs).

Metric	AHLORL	HLORL	IAHLO	SCHLO	SFPSO	BGWO	BAAA	IBDE
t-Test	1	30	30	30	30	30	30	30
	0	0	0	0	0	0	0	0
	−1	0	0	0	0	0	0	0
W-test	1	30	30	30	30	30	30	30
	0	0	0	0	0	0	0	0
	−1	0	0	0	0	0	0	0

generation number were set to 100 and 5000, respectively. The MKPs is a multiconstrained problem and the objective of MKPs is to find out an optimal subset for the maximum total profit and with multiple constraints, which is presented as follows:

$$\max f(x_1, x_2, \ldots, x_N) = \sum_{j=1}^{N} p_j x_j s.t. \begin{pmatrix} \sum_{j=1}^{N} r_{ij} x_j \le c_i, i \in \{1, 2, \ldots, M\}, \\ x_j \in \{0, 1\}, j \in \{1, 2, \ldots, N\}, \end{pmatrix}, \tag{14}$$

where N and M are the number of items and constraints, respectively. p_j stands for the profit of the j-th item, c_i means the capacity of the i-th knapsack, and r_{ij} represents the weight of the j-th item in the i-th knapsack with capacity constraint c_i.

At the same time, previous work [47] demonstrates that the penalty function method, called *pCOR*, has the best results on solving MKPs, and therefore *pCOR* is used in this paper which can be presented as follows:

$$p\mathrm{COR}(x) = \frac{p_{\max} + 1}{r_{\min}} \times \max\{\mathrm{CV}(x, i)\}, \tag{15}$$

$$\mathrm{CV}(x, i) = \max\left(0, \sum r_{ij} x_j - c_j\right), \tag{16}$$

where $p\mathrm{COR}(x)$ is the penalty coefficient used in the penalty function for infeasible solutions, $p_{\max}$ is the maximum profit coefficient, $r_{\min}$ is the minimum resource consumption, and $\mathrm{CV}(x, i)$ is the amount of constraint violation for constraint i.

The results of all algorithms on the multidimensional knapsack problems (MKPs) are given in Table 14, where the best solutions have been highlighted in bold. To analyze the superiority of the AHLORL, the summary results of the t-test and W-test are summarized in Table 15. From Table 14, the proposed AHLORL has the best performance on the multidimensional knapsack problems (MKPs). Specifically, AHLORL obtains the best numerical results on all the problems, and the superiority of AHLORL is also reflected in Table 15 where no algorithm is competitive to it. Therefore, it can be concluded that the AHLORL algorithm is a promising binary metaheuristic algorithm.

Based on the numerical simulation results on the multidimensional knapsack problems (MKPs) and CEC 15 benchmarks functions as well as the results of the parameter study, it is fair to claim that AHLORL has overwhelming advantages over previous HLO variants, as well as SFPSO, BGWO, BAAA, and IBDE, because the proposed adaptive strategy can exploit the optimization ability of SILO and SRLO more effectively. And therefore, the optimization search ability of AHLORL is significantly enhanced.

6. Conclusions and Future Work

The SRLO and SILO are both important learning operators for HLORL, which can play different roles and functions at different stages during the search process. The reasonable execution probability of SRLO and SILO can effectively enhance the learning efficiency of the algorithm, and therefore the learning performance is significantly improved. Inspired by this, an improved adaptive human learning optimization algorithm with reasoning learning is proposed, and a new adaptive strategy is presented based on the search requirements to utilize the optimization ability of SILO and SRLO more efficiently and effectively.

A comprehensive parameter study is performed to evaluate the influences of the proposed adaptive strategy. On that basis, the analysis on each parameter is given and the deep insights of the roles and functions of SRLO and SILO are taken. Then, the necessity for the adaptive *ps* strategy is concluded. The comparison results of different adaptive strategies demonstrate the efficiency and superiority of the proposed AHLORL and reveal why the proposed adaptive strategy can achieve the practically perfect trade-off between exploration and exploitation at different search stages of the algorithm. Finally, the experimental results show that the proposed AHLORL outperforms the other algorithms in terms of search accuracy and scalability.

It is well known that humans can adaptively choose and adjust their strategies to solve problems more efficiently and effectively, and the performance of AHLORL is also influenced by the parameter *pi*, which determines the learning probabilities of operating ILO and SILO. Therefore, our future work will focus on the relationship between *pi* and *ps* and try to develop a cooperatively adaptive strategy for both *pi* and *ps* to further balance the exploration-exploitation ability and enhance the performance of the algorithm.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by Key Project of Science and Technology Commission of Shanghai Municipality (Nos. 16010500300, 19510750300, 19500712300, and 21190780300), Natural Science Research Programme of Colleges and Universities of Anhui Province (Nos. KJ2020ZD39 and KJ2021A1025), Open Research Fund of AnHui Key Laboratory of Detection Technology and Energy Saving Devices, AnHui Polytechnic University (Nos. DTESD2020A02 and JCKJ2021A05), School-level Scientific Research Project of Chaohu University (No. XLY-202101), 2021 Discipline Construction Quality Improvement Project of Chaohu University (no. kj21gczx02), and 111 Project under Grant no. D18003.

References

[1] X. Xia, L. Gui, Y. Fei, H. Wu, B. Wei, and Y. Zhang, "Triple archives particle swarm optimization," *IEEE Transactions on Cybernetics*, vol. 50, no. 12, pp. 4862–4875, 2019.

[2] Q. Luo, H. Wang, Y. Zheng, and J. He, "Research on path planning of mobile robot based on improved ant colony algorithm," *Neural Computing & Applications*, vol. 32, no. 6, pp. 1555–1566, 2020.

[3] H. Gao, Y. Shi, C.-M. Pun, and S. Kwong, "An improved artificial bee colony algorithm with its application," *IEEE Transactions on Industrial Informatics*, vol. 15, no. 4, pp. 1853–1865, 2018.

[4] Y. Feng, S. Deb, G.-G. Wang, and A. H. Alavi, "Monarch butterfly optimization: a comprehensive review," *Expert Systems with Applications*, vol. 168, Article ID 114418, 2021.

[5] G.-G. Wang, "Moth search algorithm: a bio-inspired metaheuristic algorithm for global optimization problems," *Memetic Computing*, vol. 10, no. 2, pp. 151–164, 2018.

[6] A. H. Ali, S. Mirjalili, H. Faris, I. Aljarah, M. Mafarja, and H. Chenf, "Harris hawks optimization: algorithm and applications," *Future Generation Computer Systems*, vol. 97, pp. 849–872, 2019.

[7] S. Li, H. Chen, M. Wang, A. A. Heidari, and S. Mirjalili, "Slime mould algorithm: a new method for stochastic optimization," *Future Generation Computer Systems*, vol. 111, pp. 300–323, 2020.

[8] X. Zhang, H. Wang, C. Du et al., "Custom-molded offloading footwear effectively prevents recurrence and amputation, and lowers mortality rates in high-risk diabetic foot patients: a multicenter, prospective observational study diabetes," *Metabolic Syndrome and Obesity: Targets and Therapy*, vol. 15, pp. 103–109, 2022.

[9] H. Cui, Y. Guan, and H. Chen, "Rolling element fault diagnosis based on VMD and sensitivity MCKD," *IEEE Access*, vol. 9, pp. 120297–120308, 2021.

[10] E. Q. Wu, M. C. Zhou, D. Hu, and L. Zhu, "Self-paced dynamic infinite mixture model for fatigue evaluation of pilots' brains," *IEEE Transactions on Cybernetics*, 2020.

[11] L. Wang, H. Ni, R. Yang, M. Fei, and W. Ye, "A simple human learning optimization algorithm," *Computational Intelligence, Networked Systems and Their Applications*, pp. 56–65, Springer, Berlin, Heidelberg, 2014.

[12] L. Wang, H. Ni, R. Yang, P. M. Pardalos, X. Du, and M. Fei, "An adaptive simplified human learning optimization algorithm," *Information Sciences*, vol. 320, pp. 126–139, 2015.

[13] R. Yang, M. Xu, J. He, S. Ranshous, and N. F. Samatova, "An intelligent weighted fuzzy time series model based on a sine-cosine adaptive human learning optimization algorithm and its application to financial markets forecasting," in *Proceedings of the International Conference on Advanced Data Mining and Applications*, pp. 595–607, Springer, Singapore, November 2017.

[14] L. Wang, J. Pei, Y. Wen, J. Pi, M. Fei, and P. M. Pardalos, "An improved adaptive human learning algorithm for engineering optimization," *Applied Soft Computing*, vol. 71, pp. 894–904, 2018.

[15] L. Wang, A. Lu, J. Pi, M. Fei, and M. Panos, "A diverse human learning optimization algorithm[J]," *Journal of Global Optimization*, vol. 67, no. 1-2, pp. 283–323, 2017.

[16] L. Wang, R. Yang, H. Ni, W. Ye, M. Fei, and P. M. Pardalos, "A human learning optimization algorithm and its application to multi-dimensional knapsack problems," *Applied Soft Computing*, vol. 34, pp. 736–743, 2015.

[17] L. Wang, J. Pei, M. I. Menhas, J. Pi, M. Fei, and P. M. Pardalos, "A hybrid-coded human learning optimization for mixed-variable optimization problems," *Knowledge-Based Systems*, vol. 127, pp. 114–125, 2017.

[18] X. Li, J. Yao, L. Wang, and M. I. Menhas, "Application of human learning optimization algorithm for production scheduling optimization," in *Proceedings of the International Conference on Life System Modeling and Simulation International Conference on Intelligent Computing for Sustainable Energy and Environment*, pp. 242–252, Springer, Nanjing, China, September 2017.

[19] R. Yang, J. He, M. Xu, H. Ni, P. Jones, and N. Samatova, "An intelligent and hybrid weighted fuzzy time series model based on empirical mode decomposition for financial markets forecasting," *Advances in Data Mining. Applications and Theoretical Aspects*, pp. 104–118, Springer, Salmon, NY, USA, 2018.

[20] C. Jia, Y. Zheng, X. Xu, G. He, and S. Huang, "Optimal power flow calculation in AC/DC hybrid power system based on adaptive simplified human learning optimization algorithm," *Journal of Modern Power Systems and Clean Energy*, vol. 4, no. 4, pp. 690–701, 2016.

[21] C. Jia, Y. Zheng, and G. He, "Application of multi-objective human learning optimization method to solve AC/DC multi-objective optimal power flow problem," *International Journal of Emerging Electric Power Systems*, vol. 17, no. 3, pp. 327–337, 2016.

[22] R. Alguliyev, R. Aliguliyev, and N. Isazade, "A sentence selection model and HLO algorithm for extractive text summarization," in *Proceedings of the 2016 IEEE 10th International Conference on Application of Information and Communication Technologies (AICT)*, pp. 1–4, IEEE, Baku, Azerbaijan, October 2016.

[23] P. Zhang, M. Fei, L. Wang, C. Peng, and W. Zhou, "Identification method for furnace flame based on adaptive color model," *Scientia Sinica Informationis*, vol. 48, no. 7, pp. 856–870, 2018.

[24] W. Fan, J. Pei, X. Liu, P. M. Pardalos, and M. Kong, "Serial-batching group scheduling with release times and the combined effects of deterioration and truncated job-dependent learning," *Journal of Global Optimization*, vol. 71, no. 1, pp. 147–163, 2018.

[25] Z. Han, H. Qi, L. Wang, M. I. Menhas, and M. Fei, "Water level control of nuclear power plant steam generator based on intelligent virtual reference feedback tuning," *Advances in Green Energy Systems and Smart Grid*, pp. 14–23, Springer, Salmon, NY, USA, 2018.

[26] Y. Wen, L. Wang, W. Peng, M. I. Menhas, and L. Qian, "Application of intelligent virtual reference feedback tuning to temperature control in a heat exchanger," *Intelligent Computing and Internet of Things*, pp. 311–320, Springer, Salmon, NY, USA, 2018.

[27] A. K. Bhandari and I. V. Kumar, "A context sensitive energy thresholding based 3D Otsu function for image segmentation using human learning optimization," *Applied Soft Computing*, vol. 82, Article ID 105570, 2019.

[28] P. Zhang, J. Du, L. Wang, M. Fei, T. Yang, and M. Panos, "A human learning optimization with reasoning learning," *Applied Soft Computing*, vol. 122, Article ID 108816, 2022.

[29] T. Stenlund, F. U. Jönsson, and B. Jonsson, "Group discussions and test-enhanced learning: individual learning outcomes and personality characteristics," *Educational Psychology*, vol. 37, no. 2, pp. 145–156, 2017.

[30] A. Jimoyiannis, "Designing and implementing an integrated technological pedagogical science knowledge framework for science teachers professional development," *Computers & Education*, vol. 55, no. 3, pp. 1259–1269, 2010.

[31] M. Paulus, "How and why do infants imitate? An ideomotor approach to social and imitative learning in infancy (and beyond)," *Psychonomic Bulletin & Review*, vol. 21, no. 5, pp. 1139–1156, 2014.

[32] N. D. Goodman, C. L. Baker, and B. Joshua, "Tenenbaum. Tenenbaum. Cause and intent: social reasoning in causal learning," in *Proceedings of the 31st Annual Conference of the Cognitive Science Society*, Amsterdam, The Netherlands, August 2009.

[33] M. Almlund, A. L. Duckworth, J. Heckman, and T. Kautz, "Personality psychology and economics," *Handbook of the Economics of Education*, Elsevier, vol. 4, pp. 1–181, 2011.

[34] M. Oaksford and N. Chater, "Précis ofBayesian rationality: the probabilistic approach to human reasoning," *Behavioral and Brain Sciences*, vol. 32, no. 1, pp. 69–84, 2009.

[35] E. Levin, R. Pieraccini, and W. Eckert, "A stochastic model of human-machine interaction for learning dialog strategies," *IEEE Transactions on Speech and Audio Processing*, vol. 8, no. 1, pp. 11–23, 2000.

[36] F. Khan, B. Mutlu, and J. Zhu, "How do humans teach: on curriculum learning and teaching dimension," *Advances in Neural Information Processing Systems*, 2011.

[37] C. Gary, *Without Miracles: Universal Selection Theory and the Second Darwinian revolution*, pp. 49–70, MIT press, Cambridge, MA, USA, 1997.

[38] E. Heled, A. Somech, and L. Waters, "Psychological capital as a team phenomenon: mediating the relationship between learning climate and outcomes at the individual and team levels," *The Journal of Positive Psychology*, vol. 11, no. 3, pp. 303–314, 2016.

[39] L. Rendell, L. Fogarty, W. J. E. Hoppitt et al., "Cognitive culture: theoretical and empirical insights into social learning strategies," *Trends in Cognitive Sciences*, vol. 15, no. 2, pp. 68–76, 2011.

[40] A. Collins and E. Koechlin, "Reasoning, learning, and creativity: frontal lobe function and human decision-making," *PLoS Biology*, vol. 10, no. 3, Article ID e1001293, 2012.

[41] A. Demetriou and S. Kazi, *Unity and Modularity in the Mind and Self: Studies on the Relationships between Self-Awareness, Personality, and Intellectual Development from Childhood to adolescence*, Routledge, England, UK, 2013.

[42] N. Cesana-Arlotti, A. Martín, E. Téglás, L. Vorobyova, R. Cetnarski, and L. L. Bonatti, "Precursors of logical reasoning in preverbal human infants," *Science*, vol. 359, no. 6381, pp. 1263–1266, 2018.

[43] A. Mesoudi, L. Chang, K. Murray, and H. J. Lu, "Higher frequency of social learning in China than in the West shows cultural variation in the dynamics of cultural evolution,"

Proceedings of the Royal Society B: Biological Sciences, vol. 282, no. 1798, Article ID 20142209, 2015.

[44] N. Andrew, "Meltzoff. Infants' brains are wired to learn from culture: implications for social robots," in *Proceedings of the 1st Workshop on Modeling Interpersonal Synchrony and influence*, pp. 3-4, ACM, Washington, DC, USA, November 2015.

[45] C. Heyes, "Who knows? Metacognitive social learning strategies," *Trends in Cognitive Sciences*, vol. 20, no. 3, pp. 204–213, 2016.

[46] L. Alain Giraldeau, T. J. Valone, and J. J. Templeton, "Potential disadvantages of using socially acquired information," *Philosophical Transactions of the Royal Society of London Series B Biological Sciences*, vol. 357, no. 1427, pp. 1559–1566, 2002.

[47] K. G. Volz and G. Gigerenzer, "Cognitive processes in decisions under risk are not the same as in decisions under uncertainty," *Frontiers in Neuroscience*, vol. 6, p. 105, 2012.

[48] J. B. Tenenbaum, T. L. Griffiths, and C. Kemp, "Theory-based Bayesian models of inductive learning and reasoning," *Trends in Cognitive Sciences*, vol. 10, no. 7, pp. 309–318, 2006.

[49] P. Julia Beth, D. Coley John, and L. Medin Douglas, "Expertise and category-based induction," *Journal of Experimental Psychology: Learning, Memory, and Cognition*, vol. 26, no. 4, p. 811, 2000.

[50] J. J. Liang, B. Y. Qu, P. N. Suganthan, and Q. Chen, "Problem definitions and evaluation criteria for the CEC 2015 competition on learning-based real-parameter single objective optimization," Technical Report201411A, Computational Intelligence Laboratory, vol. 29, pp. 625–640, Zhengzhou University,Zhengzhou, China,, 2014.

[51] I. Cyprian Anaene Oyeka and G. Uwawunkonye Ebuh, "Modified Wilcoxon signed-rank test," *Open Journal of Statistics*, vol. 2, no. 2, p. 172, 2012.

[52] S. L. Gupta, A. S. Baghel, and A. Iqbal, "Big data classification using scale-free binary particle swarm optimization," *Harmony Search and Nature Inspired Optimization Algorithms*, pp. 1177–1187, Springer, Salmon, NY, USA, 2019.

[53] L. K. Panwar, S. Reddy K, A. Verma, B. K. Panigrahi, and R. Kumar, "Binary Grey Wolf Optimizer for large scale unit commitment problem," *Swarm and Evolutionary Computation*, vol. 38, pp. 251–266, 2018.

[54] S. Korkmaz, M. S. Kiran, and K. Servet, "An artificial algae algorithm with stigmergic behavior for binary optimization," *Applied Soft Computing*, vol. 64, pp. 627–640, 2018.

[55] S. Qian, Y. Ye, Y. Liu, and G. Xu, "An improved binary differential evolution algorithm for optimizing PWM control laws of power inverters," *Optimization and Engineering*, vol. 19, no. 2, pp. 271–296, 2018.

[56] K. Deep and J. Chand Bansal, "A socio-cognitive particle swarm optimization for multi-dimensional knapsack problem," in *Proceedings of the 2008 1st International Conference on Emerging Trends in Engineering and Technology*, pp. 355–360, Nagpur, Washington, DC, USA, July 2008.