Research Article

Construction of Mathematical Modeling for Teaching Evaluation Index System Based on the Delphi AHP Method

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China plays an important role in the global basic education of mathematics. Moreover, China has a lot of educational experience and methods and trains many excellent talents in the field of mathematics. However, there are still various disadvantages in the process of mathematics teaching. The traditional exam-oriented education makes the talent thought of mathematics education lack openness. Teachers also focus on difficult problems, partial questions, and exam questions during teaching. When teaching, teachers cannot contact students’ daily life, which has low value. Mathematics teaching methods are too rigid and lack attraction. Most teachers focus on contents in textbooks while students cannot understand the value of mathematics in learning. This results in the decline of most students’ interest in mathematics courses. To solve this problem, mathematical modeling teaching is put forward. China has a short application time in this field, and mathematical modeling is complex and systematic. When dealing with mathematical modeling problems, we must be able to skillfully use the basic knowledge learned and also involve the knowledge of other disciplines, so we must realize the integration of the knowledge of various disciplines. In addition, mathematical modeling teaching attaches great importance to students’ communication ability, mathematical interpretation ability, and teamwork ability. Teachers can improve students’ ability; however, our teachers’ abilities in mathematical modeling teaching are uneven and there are no relevant standards. Therefore, it is necessary to establish an evaluation index system to accurately judge teachers’ abilities. Based on this, this study uses the Delphi AHP method to construct the evaluation index system and selects four criteria level indexes and 12 index level indexes to judge teachers’ mathematical modeling teaching ability.

1. Introduction

At present, China’s education reform has entered a new period, with a variety of new education and teaching modes, and the emergence of Delphi and analytic hierarchy process (AHP) speeds up the pace of education reform. This method can more accurately evaluate the weight, importance, and role of various indicators in the process of mathematical modeling teaching. Through mathematical modeling teaching, students’ mathematical ability and thinking ability can be strengthened. However, at present, most of the research is on mathematical modeling teaching for middle school students, college students, and graduate students, and there is less content on evaluating mathematical modeling teaching [1]. However, our teachers’ abilities in mathematical modeling teaching are uneven, and there are no relevant standards. It is urgent to use the most advanced methods to evaluate the quality of teaching modeling. This is used to judge the teaching quality and future development direction of the mathematical modeling in schools and colleges.

The ability of students and individuals during mathematical modeling is not only the ability of mathematical modeling but also the necessary ability to successfully perform modeling tasks. When analyzing the teaching ability of mathematical modeling in colleges and universities, this study starts from four aspects, namely, modeling options, teaching monitoring, modeling process, and cooperative learning. This ideological index is used as the criterion layer of the evaluation index system of mathematical modeling teaching established in this study and further defines the 12 indexes of the index layer. By
calculating the weight of each index, the importance of each index in the teaching modeling evaluation index system is judged according to the weight of the index.

The innovations and contributions of this study in the process of studying the evaluation index system of mathematical modeling teaching are

(1) Describe in detail the two methods used in this study, namely, the Delphi method and analytic hierarchy process (AHP), list the setting and specific operation process of expert group personnel in the Delphi method, and explain the way to modify expert opinions.

(2) This study focuses on the analysis of the mathematical modeling process and explains in detail the three mathematical modeling models that are widely used.

(3) We use the Delphi AHP method to construct the evaluation index system and select four criteria level indexes and 12 index level indexes to judge teachers' mathematical modeling teaching ability.

(4) The 1–5 scale method of analytic hierarchy process (AHP) is selected to construct the judgment matrix and establish the evaluation index system and model of mathematical modeling teaching. The selected indexes can fully cover all aspects.

The rest of the paper is organized as follows. In Section 2, we offer an overview of the related work. Section 3 is about the Delphi method. Mathematical modeling is presented in Section 4. In Section 5, the proposed system is discussed. Moreover, results are also illustrated in this section. Finally, Section 6 concludes this study and offers several directions for further research and investigation.

2. Related Work

Mathematical modeling was first formed from practical applied mathematics [2]. As early as the 1970s, the mathematical modeling course was carried out in the UK, and the mathematical modeling course is mainly used by doctoral and master students. After years of development, it has been extended to undergraduates and primary and secondary school students. In 1985, the United States launched a global undergraduate mathematical modeling competition, that is, MCM competition. The increasing scale of the competition in the world strengthened more college students’ mathematical awareness and improved their mathematical practice ability [3]. By 1989, the United States took mathematical modeling as one of the main reform projects in the mathematical reform [4]. China’s mathematical modeling has gone through many processes. It was the first to carry out mathematical modeling competition nationwide and developed into setting up mathematical modeling courses in colleges and universities [1]. At present, mathematical modeling courses are added to the national curriculum system. In 1992, China launched the largest mathematical modeling competition for college students and took the competition content as an important project to cultivate college students’ competition ability. Since then, most engineering, science, and even some liberal arts colleges in China have also set up mathematical experiment courses and mathematical modeling teaching courses. The education department adds the mathematical experiment course and mathematical modeling course into the university teaching curriculum system as elective courses or compulsory courses [5].

Wang et al. believe that mathematical modeling ability refers to students’ creativity, design ability, and ability to build mathematical models [6]. Using teaching activities can fully reflect students’ personal mathematical ability. ICMI study (14) defines mathematical modeling ability as the ability to translate various questions, variables, assumptions, relationships, and other assumptions into mathematical language, evaluate and explain the mathematical ability to answer them, and test the scope and nature of the model by investigating assumptions, so as to judge the ability suitable for the model [7]. In the 21st century, the field of mathematics education began to study the mathematical modeling ability. During the research period, scholars such as Jun F et al. reasonably distinguished six different mathematical modeling abilities, such as clarifying objectives, simplifying assumptions, describing problems, clarifying parameter constants, and variables, selecting models and returning to practical problems [8]. Wang et al. (2020) analyzed the five different stages of mathematical modeling and pointed out that “the five subabilities of mathematical modeling ability mainly include logical reasoning ability, reading comprehension ability, calculation ability, mathematical ability, and self-learning ability [9].” Mathematical modeling ability is a very complex mathematical ability, which usually needs a mixture of multiple mathematical activities, including mathematical ideas, mathematical methods and knowledge, and the ability to deal with practical problems [10]. Therefore, while improving personal mathematical modeling ability, we should also strengthen our own mathematical ability [11].

3. Index System Based on the Delphi AHP Method

3.1. Delphi Method. Delphi method was first used in the expert group of problem definition and screening. It usually needs to complete three rounds of consultation [12]. Figure 1 shows the research process of the Delphi method [10, 13].

At present, the ideal number of consultants of the expert group has not been determined. Usually, the number of expert groups is in the range of 10 to 30. Generally, the first round of questionnaire consultation is highly open, and some primary information can be provided to the expert group personnel. After receiving the information, the expert group personnel should expand from their personal professional knowledge. However, in some studies, experts will redefine and classify questions, so there is no need to carry out the first round of consultation and start directly from the second round, which is called the modified Delphi method. After entering the second round, the expert group shall rearrange and score specific items. The standards followed
by the feedback results are median, mean, and coefficient of variation (CoV). Experts can directly express their opinions on evaluation indicators. Complete or carry out the third round of inquiry to get an accurate project ranking and score.

Based on the constructed evaluation index system, the weight opinion consultation form and expert opinion consultation form are prepared, and the expert opinions are revised in two rounds through the modified Delphi method. On the basis of expert feedback results, the index system is modified, added, and deleted again to make the content of the index system more perfect.

3.2. Analytic Hierarchy Process. In the 1970s, T.L. Saaty proposed a multicriteria decision-making method combining quantitative and qualitative analysis, namely, analytic hierarchy process (AHP) [14]. In this method, experienced and knowledgeable experts in the field qualitatively compare the importance of decision-making objectives at all levels, and then, researchers use the scaling method to calculate the value. Based on various index systems, this study compiles the weight questionnaire of mathematical modeling teaching evaluation index system, and experts use the 1–5 scale method to evaluate the importance of different levels of indicators. Finally, Yaahp software is used to analyze the expert evaluation data to calculate the weight of different levels of indicators [15].

4. Mathematical Modeling

4.1. Mathematical Modeling Process. Figure 2 is the mathematical modeling flowchart of China’s senior high school mathematics curriculum. The figure describes in detail all links of mathematical modeling, mainly including asking questions according to the actual situation, how to build a mathematical model according to the questions raised, how to answer the mathematical model questions, and how to test the results. If the results are difficult to explain the actual situation problems, the constructed model should be improved and modified. Mathematical modeling also includes the content of the improved model [16].

There is no direct relationship between mathematical modeling activities and the development of mathematical knowledge system, so the teaching of mathematical modeling activities shall not focus on specific knowledge and content [17]. During mathematics teaching, the time of mathematical modeling activities can be reasonably arranged according to the actual needs to ensure that the number of class hours exceeds 4. The original intention of compiling the textbook is to improve students’ mathematical level. The basic structure selected is “teaching modeling activity Case + mathematical modeling activity a + mathematical modeling activity B.” The purpose of “mathematical modeling activity case” is to use the way of classroom teaching to let students really feel the mathematical modeling process and complete the preparation of mathematical teaching activity report, so as to better understand this activity and master a lot of experience in mathematical
modeling activities. Based on students’ personal experience, teachers can expand the contents outside the teaching materials and formulate mathematical modeling tasks consistent with students [18].

4.2. Mathematical Modeling. At present, the well-known mathematical modeling model is “four stage modeling process,” which is described in detail below:

(i) Analyze the problem: analyze the background materials related to the problem and find the factors leading to the problem.

(ii) Simplified hypothesis: clarify the factors that interfere with the research object, focus on the analysis of the main factors, and simplify the secondary factors, which can reduce the difficulty of problem analysis and grasp the essence of the problem more deeply.

(iii) Modeling and solving: build a mathematical model based on analysis and solve the model by computer program or mathematical method.

(iv) Modification and verification: check the model, judge whether it is consistent with the reality, and then further explain it, and apply it in real life and production to form economic and social benefits.

Mathematical modeling five-step model is also widely used in practice, and the result is relatively simple, which is favored by a large number of scholars at home and abroad [19]. Especially, in the process of mathematical modeling in middle school, mechanism analysis is a common modeling method. The following describes the five-step model in detail and studies the process of building the model based on mechanism analysis. Figure 4 shows the five step model of mathematical modeling.

Blum and other experts proposed the modeling process, that is, the initial form of the seven-step mathematical modeling model [20]. The detailed process is listed in Figure 5.

5. Construction of the Evaluation Index System of Mathematical Modeling Teaching Based on the Delphi AHP Method

5.1. Establishment of Mathematical Modeling Teaching Evaluation System. By sorting out and analyzing the literature on mathematical modeling teaching, mathematical modeling, and teaching evaluation indicators, combined with teaching materials and expert opinions, this study determines the basic teaching concept of mathematical modeling and extracts the teaching characteristics and mathematical modeling. Based on the developmental teaching evaluation, according to the developmental teaching evaluation index and mathematics teaching quality evaluation system (MQI), take it as the main evaluation tool and establish the mathematical modeling teaching evaluation index system from the three points of student subject, teacher leadership, and teaching effect, which is shown in Figure 6 [21].

5.2. Construction of Judgment Matrix. Based on the 1–5 scale method of analytic hierarchy process (AHP), the feedback opinions of all experts are assigned, and the corresponding judgment matrix of each expert is constructed [22]. There are 13 expert feedback results obtained in this study. The criteria level index judgment matrix is listed in Table 1.

After constructing the judgment matrix, check the consistency of the judgment matrix, and check whether there is a logical inconsistency between the feedback data provided by experts. In the analytic hierarchy process, the consistency of the judgment matrix is tested according to the random consistency proportion C scale. The results show that the CR result is less than 0.1, which cannot ensure that the judgment matrices given by all experts meet the consistency requirements. If there is inconsistency in some judgment matrix data, the judgment matrix data can be automatically corrected and stopped until the consistency requirements are met. The consistency correction algorithm of automatic selection judgment matrix is deeply explored to test whether the judgment results given by various experts are consistent or not.

After analysis and judgment, 21 judgment matrix data inconsistencies appear in the judgment matrix established in this study. Among these judgment matrices, 12 adopt the “minimum change” algorithm to correct them, which meets the consistency requirements after correction. The other five matrices also meet the consistency requirements by using the “optimal direction” algorithm. There is no need to further study and process the above judgment matrix data. The remaining three judgment matrices need to modify the contents of two data items, accounting for 20% of the total data items. Only a small part of the data needs to be corrected without in-depth processing of the judgment matrix data. There is also a judgment matrix to correct three of the data, accounting for 30% of all the data, so the judgment matrix cannot be used. Here, only the consistency test results of the judgment matrix given by experts are listed, which are shown in Table 2.

5.3. Calculation of Index System Weight. Yaahp software, as auxiliary software of analytic hierarchy process, is developed based on Yaahp software. By comparing Yaahp software, Yaahp software has the advantage of supporting various scale types, including 1–5 scale, 1–9 scale, 1–2 scale, and 1–3 scale. This study uses the 1–5 scale method on Yaahp software to study the expert scoring data and calculate the weight of the evaluation index system of mathematical modeling teaching. The following is the detailed process:

(1) First, establish the hierarchical structure model. Based on mathematical modeling and mathematical evaluation index system, clarify the control criteria and control objectives and complete the drawing of hierarchical model based on this.

(2) Secondly, construct judgment matrix. This study selects the 1–5 scale method to count the weight score results given by experts and constructs the index judgment matrix of experts for different levels.
(3) Again, check the consistency of the judgment matrix. Evaluate the judgment matrix data given by experts to judge whether it meets the consistency requirements. Yaahp software comes with two automatic repair consistency algorithms, namely, the optimal direction algorithm and the minimum selection algorithm. This method is used to automatically repair the judgment matrix of inconsistent data. Based on the judgment matrix described by Yaahp software and the given processing decision, it is judged whether to further process the judgment matrix.

(4) Finally, analyze the weight data. The common ways to calculate the judgment matrix are (i) sum method, (ii) power method, and (iii) root method. Combined with the group decision-making expert data and element weight, the calculation results are obtained, and the results are deeply analyzed to obtain the single ranking and final total ranking of different levels of indicators.

Based on the hierarchical single sorting, the combined weights of different indicators can be obtained by multiplying the aggregated secondary indicator weights and the corresponding primary indicator weights. The detailed calculation results are listed in Table 3.

5.4. Construction of the Evaluation Model. After all the weights of the evaluation index system of mathematical modeling teaching based on the Delphi AHP method have been determined, the following mathematical modeling teaching evaluation models are established by using the weighted average method [23]:

\[
S = \sum_{i=1}^{12} W_i T_{jk},
\]

where \(W_i\) represents the combined weight of the \(i\) secondary index, \(S\) represents the total score of mathematical modeling teaching, and \(T_{ik} T_{jk}\) represents the score of secondary index, evaluates the overall level of mathematical modeling teaching based on the total score, and makes up for the problems and deficiencies existing during mathematical modeling teaching according to the score of each index.

According to the feedback of experts, this study establishes the evaluation index system of mathematical modeling teaching.
Practical problems

Reality model
Mathematical model

Solution mathematical model

Answer real world questions

To simplify the mathematical model

To solve

Inspection

Real world world of mathematics

Figure 5: Seven-step model of Blum mathematical modeling.

Figure 6: Evaluation index system of mathematical modeling teaching based on the Delphi AHP method.

Table 1: Criteria layer index judgment matrix.

<table>
<thead>
<tr>
<th></th>
<th>Modeling topics $T_1$</th>
<th>Teaching monitoring $T_2$</th>
<th>The modeling process $T_3$</th>
<th>Cooperative learning $T_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modeling topics $T_1$</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Teaching monitoring $T_2$</td>
<td>1/2</td>
<td>1</td>
<td>1/3</td>
<td>1</td>
</tr>
<tr>
<td>The modeling process $T_3$</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Cooperative learning $T_4$</td>
<td>1/3</td>
<td>1</td>
<td>1/3</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 2: Consistency test results of index judgment matrix of any expert at all levels.

<table>
<thead>
<tr>
<th>Judgment matrix</th>
<th>The maximum characteristic root $\lambda_{\text{max}}$</th>
<th>Consistency ratio CR</th>
<th>Consistency test</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>Before correction</td>
<td>The revised</td>
<td>Before correction</td>
</tr>
<tr>
<td>$T_1$</td>
<td>3.24</td>
<td>3.14</td>
<td>0.21</td>
</tr>
<tr>
<td>$T_2$</td>
<td>---</td>
<td>5.36</td>
<td>---</td>
</tr>
<tr>
<td>$T_3$</td>
<td>---</td>
<td>5.06</td>
<td>---</td>
</tr>
<tr>
<td>$T_4$</td>
<td>---</td>
<td>3.07</td>
<td>---</td>
</tr>
</tbody>
</table>
teaching based on the Delphi AHP method, calculates the weight of each index by using the above method, and obtains the total score by solving formula (1). The calculation formula is as follows:

\[
S = 0.12T_{11} + 0.08T_{12} + 0.06T_{13} + 0.05T_{21} + 0.03T_{22} + 0.03T_{23} + 0.08T_{31} + 0.05T_{32} + 0.03T_{33} + 0.05T_{41} + 0.05T_{42} + 0.05T_{43}.
\]

5.5. Discussion on Evaluation Results. In this study, the current mainstream statistical algorithm Delphi and analytic hierarchy process (AHP) are used to establish the evaluation index system of mathematical modeling teaching, which enriches the field of teaching modeling evaluation, promotes the sustainable development of this field, and formulates the corresponding judgment standards to measure the evaluation of mathematical modeling teaching. Starting from the research on teaching modeling evaluation, the carrier of most of the current research is the division of mathematical modeling level and modeling process. A variety of practical evaluation tools are designed from the perspective of students, focusing on students’ modeling ability and cultivating students’ personal quality. This study focuses on mathematical modeling teaching and selects criteria level indicators for modeling topic selection, teaching monitoring, modeling process, and cooperative learning. The indicators on the indicator level mainly include appropriateness, development, attraction, strategy guidance, quality supervision, comprehensive evaluation, model establishment, model solving, model improvement, component team, clear division of labor, and effective cooperation, with a total of 12 indicators. Based on this index, the evaluation index system and evaluation model of mathematical modeling teaching are established. The basic idea of this study is the teaching process of mathematical modeling. According to the achievements of scholars, the teaching of mathematical modeling is divided into three stages: learning and skillfully using mathematical models, understanding the modeling process, and the practical application of mathematical modeling.

In this study, the evaluation index system of mathematical modeling teaching based on the Delphi AHP method is established. The calculated primary index weights are 0.276, 0.192, 0.376, and 0.156, respectively. The importance of each index is judged according to the weight value. The most important index is the modeling process, followed by modeling options, then modeling monitoring, and finally cooperative learning. Among the secondary indicators, the most important is appropriateness, with a weight of 0.12, followed by clear division of labor, effective cooperation, strategic guidance, team formation, and development. The weights of the above six indicators exceed 0.05. After accumulating each indicator, the proportion in the total indicator weight is 1/2 left and right.

In the evaluation index system, the value range of Cronbach coefficient of different dimensions is 0.81 to 0.95, the first-order Cronbach coefficient is 0.92, and the second-order Cronbach coefficient is 0.98, indicating that the reliability test level and reliability of the evaluation index system of mathematical modeling teaching based on the Delphi AHP method are high, and the corresponding half reliability coefficients are 0.91 and 0.97. The i-cvi [22] of each index is taken in the range of 0.8 to 1. The internal consistency reliability is calculated by SPSS software, and the results are more than 0.9. Therefore, the evaluation index system has high reliability.

6. Conclusions and Future Work

With the development of science and technology and the promotion of the rapid reform of education, the traditional mathematics teaching model has been unable to meet people’s needs. This promotes the comprehensive application of mathematical modeling teaching methods in various countries. However, there is a large gap in each teacher’s mathematical modeling teaching ability, so it is impossible to accurately evaluate the teacher’s ability. Aiming at this
problem, this paper studies the evaluation of mathematical modeling teaching by combining Delphi and analytic hierarchy process (AHP), selects modeling options, teaching monitoring, modeling process, and cooperative learning as the criterion level indicators, establishes the evaluation index system of mathematical modeling teaching based on Delphi AHP, and describes Delphi and analytic hierarchy process (AHP) in detail. This study introduces the teaching process of mathematical modeling, which lays the foundation for the establishment of mathematical modeling. After calculation, the weights of the first-class indicators of this model are 0.276, 0.192, 0.376, and 0.156, respectively. The importance of the indicators is arranged according to the weight. Among them, the modeling process is the most important, followed by modeling options and modeling monitoring, and finally cooperative learning. In the future, we aim to clarify the importance of each indicator in evaluating mathematical modeling teaching, so as to judge teachers’ mathematical modeling teaching ability.

Data Availability

The data can be obtained from the corresponding author upon request.

Conflicts of Interest

The author declares no conflicts of interest.

References


