Research Article
Forecast of Water Structure Based on GM (1, 1) of the Gray System

Yanan Dong, Zheng Ren, and Lian Hui Li

1Institute of Water Conservation and Hydro-Electric Power, Hebei University of Engineering, Handan, Hebei 056000, China
2Northwestern Polytechnical University, Xi'an, Shaanxi 710129, China

Correspondence should be addressed to Lian Hui Li; 2010100407@mail.nwpu.edu.cn

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A forecast approach of water structure based on GM (1, 1) of the gray system is proposed. Based on economic and water information of Hebei Province from 2000 to 2018, the water use structure of Hebei's industrial sector from 2019 to 2030 is forecasted according to the composition data and gray system GM (1, 1) model. The forecasting results by the proposed approach shows that the water structure of the tertiary industry has changed from 62.8 : 10.3 : 26.9 in 2018 to 60.5 : 10.2 : 29.3 in 2030. The proportion of water used in the primary and secondary industries has decreased slightly, the proportion of water used in the tertiary industry has increased, and the proportion of water used in the tertiary industry has not changed significantly.

1. Introduction

Regional water resources are both driving economic growth and improving the living standards of the people. The improvement of the quality of material life and the enhancement of the ability to survive are also key conditions for achieving sustainable human development. Faced with the contradiction between the shortage of large-scale water resources and the demand for water resources in economic development, new industrial restructuring will be inevitable to achieve greater economic efficiency in the face of shortages. Under the constraints of limited available water resources, the current economic growth model and water supply and consumption structure need to be optimized and adjusted to ensure the sustainable utilization of regional water resources and the healthy and sustainable development of economy and society [1–7].

How to make rational development and utilization of limited water resources and make it exert the maximum utility under the premise of sustainable development has become an important problem faced by human society. This is the motivation of this study.

Scholars have studied the relationship between industrial structure and water use structure, and achieved a lot of results. Jia et al. [1] proposed to first improve the efficiency of industrial water use, upgrade the secondary industry, and optimize the industrial structure to reduce water consumption. Few authors [2, 3] advocate the transfer of high-water-consumption industries to small-water-consuming industries to increase efficiency in water resources utilization. Su et al. [4] studied the impact of water resources on industrial structure adjustment in Henan Province by means of cointegration, vector error correction estimation, impulse response, and variance decomposition. Jiao et al. [5] analyzed the temporal and spatial variation characteristics of the coupling coordination degree of water resources and industrial structure in Henan Province by establishing a coupling evaluation model. Fan and Wen [6] made quantitative analysis and comparison between industrial structure and water use system in Gansu Province based on the correlation analysis method and gray system theory, and put forward the direction of industrial structure adjustment. Jia et al. [7] used the system dynamics method to construct the collaborative evolution model of regional water use structure and national economy, and predicted and analyzed the collaborative evolution path and evolution law of water use structure and national economy industrial structure in Shandong Province.

The development of information technology and artificial intelligence has brought new opportunities to the
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and the proportion of water used in the three industries will be equal to one. By analyzing the time trend of a single value of the three secondary industries and the sum of the study of Beijing.

The existing studies have more single variable analysis of regional industrial structure and water use structure, but there are the following deficiencies. The method is relatively simple and lacks the analysis of industrial structure development under the constraints of water resources. There are many studies on the relationship between water resources and industrial structure, but there is a lack of in-depth exploration on the coordinated development relationship between water resources and industry.

This paper mainly analyzes the evolution of industrial structure and water structure through the collection and collation of economic data and water information in Hebei Province for 2000–2018. The relationship between the two countries is to reveal the coordination between them, and to forecast the water structure in Hebei province in 2030, and to propose the direction for research on industrial restructuring.

2. Research Methodology

There are certain conditions between the three secondary industries and water use, that is, the proportion of the output value of the three secondary industries and the sum of the proportion of water used in the three secondary industries are equal to one. By analyzing the time trend of a single indicator, the result will inevitably appear in each forecast year. The proportion of output value of the three industries and the proportion of water used in the three industries will no longer equal 1. Therefore, a single indicator prediction method cannot be used for a set of constrained variable indicators [19, 20].

Statistically, a combination of the individual share data for a set of constrained variables is called the component data (generally assuming that the sum of the shares of each variable is equal to 1), in under this constraint, economic models are established, and trends in variable composition are analyzed and predicted in a comprehensive manner. The gray system theory considers the socioeconomic system to be a native gray system [21]. It treats random processes of varying amounts of gray over a range of time as gray processes. The metric amount corresponding to a time series is the gray amount that is changing and the process of change can be considered gray. For this reason, the method of dedimensional [22] of component data was used. Combined with the GM (1, 1) model of the gray system, the basic steps for modeling the prediction of component data are given, and the three industrial structures and water structures of Beijing, Tianjin, and Hebei have been analyzed in a prediction and are of satisfactory results.

\textbf{Definition 1.} \( X = \{ (x_1, x_2, \ldots, x_m) \in R^m | \sum_{i=1}^{m} x_i = 1.0 \leq x_i \leq 1 \}. \) \( X \) is called the component data series, and \( x_i \) is the component data for the component \( i \).

\textbf{Definition 2.} \( X^t = \{ (x^t_1, x^t_2, \ldots, x^t_m) \in R^m | \sum_{i=1}^{m} x^t_i = 1.0 \leq x^t_i \leq 1 \} t = 1, 2, \ldots, T. \) \( t \) indicates the time.

The basic question of modeling forecasts for time component data: given the time component data sequence \( x_t \), how to build a mathematical model, and predict \( T + l \) the component data for the moment \( x^t_{T+l} \).

Using the component data dedimensional approach and the gray GM (1, 1) prediction method, the basic ideas for solving problems are presented, with the following steps:

1. Make a nonlinear transformation of the original data

\[
\begin{align*}
y^t_i &= \sqrt{x^t_i}, \quad i = 1, 2, \ldots, m; \\
&= 1, 2, \ldots, T.
\end{align*}
\]

Remember \( Y^t = (y^t_1, y^t_2, \ldots, y^t_m), \quad t = 1, 2, \ldots, T. \) Then, there are

\[
\begin{align*}
Y^{t+2} &= \sum_{i=1}^{m} (y^t_i)^2 \\
&= 1. \tag{2}
\end{align*}
\]

2. For any, the \( t = 1, 2, \ldots, T \) \( Y^t = (y^t_1, y^t_2, \ldots, y^t_m) \in R^m \) is distributed over \( m \) a spherical face with a radius of 1, as can be seen by the formula (2). Change \( Y^t = (y^t_1, y^t_2, \ldots, y^t_m), t = 1, 2, \ldots, T \) from the right-angle coordinate system to the spherical coordinate system \( (r^t, \theta_2, \ldots, \theta_m) \in \Theta^m \), because there is a mapping relationship (where \( (r^t)^2 = Y^{t+2} = 1R^m \rightarrow \Theta^m \))

\[
0 < \theta_i \leq \Pi/2 i = 2, 3, \ldots, m, \]

\[
\begin{align*}
y^t_1 &= \sin\theta^t_1 \sin\theta^t_2 \cdots \sin\theta^t_m, \\
y^t_2 &= \cos\theta^t_1 \sin\theta^t_2 \sin\theta^t_3 \cdots \sin\theta^t_m, \\
y^t_3 &= \cos\theta^t_1 \cos\theta^t_2 \sin\theta^t_3 \cdots \sin\theta^t_m, \\
&\vdots \\
y^t_{m-1} &= \cos\theta^t_{m-1} \sin\theta^t_m, \\
y^t_m &= \cos\theta^t_m. \tag{3}
\end{align*}
\]

3. The component data are reduced from the original dimension space to \( m(m-1) \) the dimension space \( m(m-1) \) during the conversion from the right-angle coordinate system to the spherical coordinate system, so that the original linear-related variables are converted into independent variables (corners), and the angle variables are obtained by recursion according to the formula (3).
\[ \theta_m' = \arccos y_m', \]
\[ \theta_{m-1}' = \arccos \left( \frac{y_{m-1}'}{\sin \theta_m' \sin \theta_{m-1}'} \right), \]
\[ \theta_{m-2}' = \arccos \left( \frac{y_{m-2}'}{\sin \theta_m' \sin \theta_{m-1}'} \right), \]
\[ \vdots \]
\[ \theta_1' = \arccos \left( \frac{y_1'}{\sin \theta_2' \sin \theta_1'} \right) \]
\[ t = 1, 2, \ldots, T. \]  
\[ (4) \]

(4) Using the calculated angle data from the formula (4), \{ \theta_i', t = 1, 2, 3, \ldots, T \}, \[ i = 2, 3, \ldots, m, \] create a gray G \( m - 1 \) M (1, 1) forecast model, which predicts the angle of the moment \( T + l \theta_i'^{l+1}, i = 2, 3, \ldots, m. \)

(5) Use the formula (3) to calculate the forecast for the time of day \( T + l \theta_i'^{l+1} = (y_1^{l+1}, \ldots, y_m^{l+1}) \). Obviously,
\[ \sum_{i=1}^{m} (y_i^{l+1})^2 = 1. \]  
\[ (5) \]

(6) Use formula (1) to get the forecast value of the component data at the time of day: \( T + l \)
\[ x_i^{l+1} = \left( y_i^{l+1} \right)^2, i = 1, 2, \ldots, m. \]  
\[ (6) \]

(7) Create a GM (1, 1) model to predict the value of the angle in 2030.

GM (1, 1) Modeling process:

2.1. Additive Generation. Set original number column: \( x^{(0)} = \{ x_1^{(0)}, x_2^{(0)}, x_3^{(0)}, \ldots, x_t^{(0)} \} \) number of columns total. To reduce uncertainty, add up to get a new set of numbers listed:
\[ x^{(1)} = (x_1^{(1)}, x_2^{(1)}, x_3^{(1)}, \ldots, x_t^{(1)}), \]
\[ x_t^{(1)} = \sum_{i=1}^{t} x_i^{(0)}. \]  
\[ (7) \]

In the formula, \( x_t^{(0)} \) is the data of the original number column; \( x_t^{(1)} \) is the data of the cumulative number column.

2.2. Mean Calculations.
\[ \varepsilon_t^{(1)} = \frac{1}{2} \left[ x_t^{(1)} + x_{t-1}^{(1)} \right]. \]  
\[ (8) \]

In the formula, the initial value: \( t = 2 \).

2.3. Curve Fit. To establish a differential equation,
\[ \frac{dx^{(1)}}{dt} + ax^{(1)} = u. \]  
\[ (9) \]

In the formula, \( a \) is a factor and is taken in the range [2]; \( a, u \) is a gray contribution \( \alpha = \begin{pmatrix} a \\ u \end{pmatrix} \); the matrix is formed, using the smallest two-multiplication solution.

To create a matrix and constant item vector for a new array created by the additive: BY
\[ \begin{bmatrix} -z_2^{(1)} & 1 \\ -z_3^{(1)} & 1 \\ \vdots & \vdots \\ -z_i^{(1)} & 1 \end{bmatrix}, \]
\[ (10) \]
\[ \bar{\alpha} = \left( B^T B \right)^{-1} B^T Y. \]  
\[ (11) \]

It will be substituted and evaluated for solution
dx^{(1)}/dt + ax^{(1)} = u.
\[ \bar{x}_t^{(1)} = \left( x_1^{(0)} - \frac{u}{a} \right) e^{-a(t-1)} + \frac{u}{a}. \]  
\[ (12) \]

Or
\[ \bar{x}_{t+1}^{(1)} = \left( x_1^{(0)} - \frac{u}{a} \right) e^{-at} + \frac{u}{a} \]  
\[ (13) \]
\( \bar{x}_t^{(1)} \) is the ratio of accumulated output value in the forecast year (%), \( x_1^{(1)} \): the specific weight of the new series (%) for the forecast value of the starting year, and \( a, u \): the parameter.

2.4. Residual Analysis. By predicting the forecast value of the original series and doing a residual analysis with the original value, the residual difference is smaller and can be predicted directly with the model, which is much needed. Make corrections to the residual series data by creating a gray model.

The difference between the new agricultural series and the year of adjacent water consumption is the predicted annual demand:
\[ V_t = \bar{x}_t^{(1)} - \bar{x}_{t-1}^{(1)}. \]  
\[ (14) \]

The residual differential is calculated as
\[ \varepsilon_t^{(0)} = V_t^{(0)} - V_t. \]  
\[ (15) \]

The relative error is calculated as...
\[ \delta_t^{(0)} = \frac{\xi_t^{(0)}}{x_t^{(0)}} \times 100\%. \quad (16) \]

2.5. Postdifferential Inspection. The accuracy level for the postdifferential inspection is shown in Table 1 [23].

Residual mean value calculation formula is as follows:

\[ \bar{\varepsilon} = \frac{1}{n} \sum_{t=1}^{n} \varepsilon_t. \quad (17) \]

Residual differential calculation formula is as follows:

\[ S_2^2 = \frac{1}{n} \sum_{t=1}^{n} (\xi_t - \bar{\varepsilon})^2. \quad (18) \]

The original mean value calculation formula is as follows:

\[ \bar{x} = \frac{1}{n} \sum_{t=1}^{n} x_t. \quad (19) \]

The formula for calculating the original value variance is as follows:

\[ S_1^2 = \frac{1}{n} \sum_{t=1}^{n} [x_t^{(0)} - \bar{x}]^2. \quad (20) \]

The formula for calculating the postcheck ratio is as follows:

\[ C = \frac{S_1}{S_2}. \quad (21) \]

Small probability of error calculation formula is as follows:

\[ P = P \left[ |\varepsilon_t - \bar{\varepsilon}| < 0.6745 \times S_2 \right]. \quad (22) \]

### 3. Water Structure Forecast

Using the composition data and the gray system GM (1, 1), it is predicted that the water structure in Hebei Province will be used in 2030 to calculate the water bias between Hebei industry and industry. The difference factor compared with 2018 will be used to analyze water structure in Hebei Province for the next decade [24–27].

According to water data of the three secondary industries in Hebei Province 2000–2018, we establish a model of the dynamic distribution of water usage patterns in Hebei’s industrial water structure and forecast water structure for Hebei industry in 2030 [28–31]. The angle values for the nonlinear mapping of the three-way industrial water structure are shown in Table 2.

The forecast for the angle of water structure of the three industrial sectors in Hebei Province is performed, \( \theta_1^{(0)} \) \( \theta_2^{(0)} \) are calculated, and the average fit error is shown in Table 3.

The accuracy of the prediction of the angle of water structure is shown in Table 4.

According to Table 1, the angle forecast model for the industrial water structure of Hebei Province meets the level 2 accuracy requirement (qualified), which is qualified. \( t = 2019, 2020, 2021, \ldots, 2030 \). The angle forecast is shown in Table 5.

The forecast of water usage structure for the three subindustries in Hebei Province is shown in Table 6.

Based on the actual value of the three-way industrial water structure, 2000–2018, and the forecast value of the three-way industrial water structure, 2019–2030, the evolution of industrial water use structure in 2019–2030 is shown in Figure 1.

Figure 1 shows the evolution of water structure in three industrial sectors of Hebei Province from 2000 to 2030. The coarse point part is the data from 2000 to 2018 and the fine point part is the prediction from 2019 to 2030. As can be seen from the diagram, (1) Component data modeling combined with the gray system GM(1,1) model predicts that Hebei Province will have a good fit for its future industrial water structure. (2) The proportion of water used in the first industrial sector in Hebei Province has been slowly decreasing, the proportion of water used in the second industry has been gradually decreasing, the proportion of water used in the third industry has been increasing slowly, and the gap between the proportion of water used in the first industry has gradually narrowed. According to the results of the calculation of the model and according to the forecast
development trend, by 2030, the water structure of the three subindustries in Hebei province will be adjusted to 60.4% of the water use in the first industry, 10.2% of the water use in the second industry, and 29.4% of the water use in the third industry.

Based on the analysis of the forecasting results, the growth of domestic water and industrial water may be due to the improvement of economic level, the acceleration of urbanization and the rapid development of industrial enterprises. The increase in water use for ecological environment is mainly due to the improvement of awareness of greening and environmental protection. On the one hand, the slow growth of agricultural water consumption is due to the vigorous development of tourism in many places and the transformation of many farmlands into landscape land, resulting in the reduction of the effective area of farmland irrigation. On the other hand, the continuous upgrading and transformation of agricultural water-saving measures and the construction of high standards of farmland have increased the coefficient of farmland irrigation water, thus reducing the consumption of agricultural water.

The industrial structure restricts the proportion of water used by different users and affects the overall water use efficiency. The improvement of water use efficiency can produce greater economic value and promote the optimization and adjustment of industrial structure. Through the correlation analysis between industrial structure and water use, combined with the analysis of the coordinated

### Table 3: The average fit error table is projected at the corner of the water structure.

<table>
<thead>
<tr>
<th>Forecast function</th>
<th>Average fit error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_2^{(1)} )</td>
<td>0.0042</td>
</tr>
<tr>
<td>( \theta_3^{(1)} )</td>
<td>0.0658</td>
</tr>
</tbody>
</table>

### Table 4: The results of the inspection of the accuracy of the projection of the industrial water structure.

<table>
<thead>
<tr>
<th>Corner</th>
<th>Residual spread mean (( \tilde{r} ))</th>
<th>Residual variance (( S_1^2 ))</th>
<th>Original mean (( \tilde{x} ))</th>
<th>Original value variance (( S_2^2 ))</th>
<th>Postcheck ratio (C)</th>
<th>Small probability of error (P value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_2^{(1)} )</td>
<td>0</td>
<td>0.0018</td>
<td>1.1833</td>
<td>0.0078</td>
<td>0.4848</td>
<td>0.90</td>
</tr>
<tr>
<td>( \theta_3^{(1)} )</td>
<td>0</td>
<td>0.0007</td>
<td>1944</td>
<td>0.0041</td>
<td>0.4213</td>
<td>0.90</td>
</tr>
</tbody>
</table>

### Table 5: Industrial water structure angle forecast.

<table>
<thead>
<tr>
<th>( \theta_2 )</th>
<th>2019</th>
<th>2020</th>
<th>2021</th>
<th>2022</th>
<th>2023</th>
<th>2024</th>
<th>2025</th>
<th>2026</th>
<th>2027</th>
<th>2028</th>
<th>2029</th>
<th>2030</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1826</td>
<td>1.1825</td>
<td>1.1825</td>
<td>1.1824</td>
<td>1.1823</td>
<td>1.1823</td>
<td>1.1822</td>
<td>1.1822</td>
<td>1.1821</td>
<td>1.1821</td>
<td>1.1820</td>
<td>1.1820</td>
<td></td>
</tr>
</tbody>
</table>

### Table 6: The forecast of the water structure of the tertiary industry.

<table>
<thead>
<tr>
<th></th>
<th>2019</th>
<th>2020</th>
<th>2021</th>
<th>2022</th>
<th>2023</th>
<th>2024</th>
<th>2025</th>
<th>2026</th>
<th>2027</th>
<th>2028</th>
<th>2029</th>
<th>2030</th>
</tr>
</thead>
<tbody>
<tr>
<td>First industry</td>
<td>0.6855</td>
<td>0.6732</td>
<td>0.6664</td>
<td>0.6596</td>
<td>0.6527</td>
<td>0.6459</td>
<td>0.6390</td>
<td>0.6321</td>
<td>0.6252</td>
<td>0.6183</td>
<td>0.6114</td>
<td>0.6045</td>
</tr>
<tr>
<td>Secondary industry</td>
<td>0.1132</td>
<td>0.1126</td>
<td>0.1115</td>
<td>0.1104</td>
<td>0.1093</td>
<td>0.1082</td>
<td>0.1071</td>
<td>0.1060</td>
<td>0.1048</td>
<td>0.1037</td>
<td>0.1026</td>
<td>0.1015</td>
</tr>
<tr>
<td>Tertiary industry</td>
<td>0.2103</td>
<td>0.2142</td>
<td>0.2221</td>
<td>0.2300</td>
<td>0.2380</td>
<td>0.2459</td>
<td>0.2539</td>
<td>0.2619</td>
<td>0.2699</td>
<td>0.2780</td>
<td>0.2860</td>
<td>0.2940</td>
</tr>
</tbody>
</table>

**Figure 1:** Trends in the evolution of the water structure of the three secondary industries, 2000–2030.
relationship between water resources and industrial development, the scale of economic and social development and the range of water use efficiency under the constraint of water resources can be obtained.

4. Conclusions

As we can see from the above results, with the development of comprehensive water conservation activities in Hebei Province and the readjustment of the industrial structure, the water quality of the three industries has been continuously improved and the water efficiency has been improved. It is consistent with the law of economic and social development. The water structure of the tertiary industry has changed from 62.8:10.3:26.9 in 2018 to 60.5:10.2:29.3 in 2030. The proportion of water used in the primary and secondary industries has decreased slightly, the proportion of water used in the tertiary industry has increased, and the proportion of water used in the tertiary industry has not changed significantly. From this, it is evident that the water use structure of Hebei’s industrial sector is in line with the future industrial development plan.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References


