Hindawi
Scientific Programming
Volume 2023, Article ID 7235996, 18 pages
https://doi.org/10.1155/2023/7235996

Research Article

Evaluating the Quality of Medical Services Using Intuitionistic Hesitant Fuzzy Aczel–Alsina Aggregation Information

Shahzaib Ashraf,[1] Aziz Khan,[2] Muhammad Kamran,[1,4] and M. K. Pandit[3]

[1]*Institute of Mathematics, Khwaja Fareed University of Engineering & Information Technology, Rahim Yar Khan 64200, Punjab, Pakistan*
[2]*Department of Mathematics, Abdul Wali Khan University, Mardan 23200, Khyber Pakhtunkhwa, Pakistan*
[3]*Department of Mathematics, Jahangirnagar University, Savar, Dhaka, Bangladesh*
[4]*Department of Mathematics, Thal University Bhakkar, 30000, Punjab, Pakistan*

Correspondence should be addressed to M. K. Pandit; mkpandit@juniv.edu

Received 8 July 2022; Revised 6 March 2023; Accepted 17 April 2023; Published 15 May 2023

Academic Editor: Debo Cheng

In modern civilization, individuals are increasingly concerned with evaluating the quality of medical services. Evaluation of the quality of medical services enables medical care providers to monitor and improve their service quality. The evaluation of medical service quality is efficiently addressed by the novel concept of Aczel–Alsina operators in an intuitionistic hesitant fuzzy (IHF) environment as multicriteria decision-making (MCDM) problem. Thus, this paper presents the IHF Aczel–Alsina weighted geometric operators for IHF information. We first apply the Aczel–Alsina norms to IHF scenarios and present novel operations of intuitionistic hesitant fuzzy sets. This article develops a unique strategy for evaluating the quality of medical services based on the specified operators, including a quantitative framework for evaluating medical service quality and a novel MCDM technique. Finally, this article presents a numerical example of the novel approach used to evaluate medical services for hospitals and compares it to conventional MCDM methods to highlight the suggested superiority method. According to the comparative results, our strategy outperforms the insufficiency of lacking decision flexibility in the existing MAGDM method.

1. Introduction

In light of the significant development of medical services and the healthcare industry, the society is becoming more concerned about the quality of medical treatment and urging companies to enhance quality [1]. Additionally, the World Health Organization declared in 2000 that assessing a medical service provider's quality has been a worldwide problem [2]. Healthcare practitioners need an appropriate and effective approach for evaluating the quality of healthcare services in order to monitor and enhance the quality of medical services they provide. If patients want to receive better healthcare, they should evaluate various hospitals using a variety of factors in order to choose the best option. Numerous researchers have devoted to studies on the assessment of medical service quality on the basis of this framework. For example, Shieh [3] utilised the DEMATEL approach to identify the essential aspects in the quality of medical services, such as the atmosphere, registration system, and appearance. Mccarthy [4] classified medical service quality like respect and care, effectiveness and continuity, suitability, and communication, efficiency, meals, initial impression, and staff versatility. By combining the work of the previous researchers, Fei et al. [5] presented a set of medical quality assessment.

The method of employing expert assessment data to analyse, rank, and select the best solutions using a specific decision-making (DM) approach is known as multiattribute group decision making (MAGDM) [6–8]. Giving pertinent information is a necessary step in the decision-making process, as it is selecting one DM technique over others. As it is impossible to anticipate the future and because specialists' knowledge is limited, improving the MADM technique has become a hot topic in today's DM field [9–11]. While analysing data, the uncertainty and incompleteness are always an issue. For instance, according to the idea of hard sets, an item either belongs to or does not belong to a particular class. However, many things in the real world could not be described in such detail [12, 13].

Since the assessment problem of medical service quality is related to many factors and the evaluation finding is the foundation of decision making (DM) on the selection of medical service provider, it may be regarded as a multi-attribute decision-making (MADM) problem [14, 15]. MADM refers to the process of assessing, classifying, and choosing the best options from a set of decision support data and a set of decision support models. The most crucial parts of making a decision are specialists' knowledge and the choice of applicable decision assistance approaches. Expertise and the complexity of society have necessitated that the expansion and refinement of the MADM technique are better decisions in decision supporting difficulties. In the DM process, there is inconsistency and uncertainty, and Zadeh's [16] idea of fuzzy sets (FSs) provides a highly effective technique to deal with these challenges. Following that, intuitionistic FSs (IFSs) [17] were developed, which used +ve and −ve membership grades to express uncertainty in decision-making processes. When presented with a DM issue, decision makers began to employ IF numbers to indicate their ranking for various choices. As a result, an increasing number of researchers are getting interested in IF data [18–20].

To compile the information gathered from the specialists, we will need to use some aggregation operators (Agop). Some Agops, such as the IF averaging operator, were created by Xu [21]. Some Einstein Agops, such as IFE averaging/geometric operators, were introduced by Wang and Liu [22]. Yu and Xu [23] created a prioritised list of Agops and explored how they could be used to solve DM issues. Liu and Wang [24] used a linguistic IF technique to create several innovative Agops and an algorithm to solve complex uncertain DM situations. Xu and Yager [25] established the decision-making technique using Bonferroni means Agop under IF data.

On the basis of prioritised IF Agop under linguistic data set, Arora and Garg [26] constructed the group DMA. Zhao et al. [27] examined the uses of generalised IF Agops, such as the generalised IF averaing/geometric operators, to deal with uncertainty in Dcmp. Yu [28] demonstrated certain IF Agops depending on confidence levels and solved challenging real-world Dcmps. Yu [29] created the IF Agop and discussed its usefulness in decision making using Heronian mean. Jiang et al. [30] devised a decision-making strategy based on the IF power Agop and the entropy measure. Senapati et al. [31] developed an Aczel–Alsina (Acz-Aln) norm based on some IF Acz-Aln Agops and applied it to the IF MADM process [32]. Khan et al. [33] created new generalised IF soft details Agops and investigated their use in DM.

T-norms and t-conorms (e.g., Algebaric t-norm and t-conorm, Einstein t-norm and t-conorm, Hamacher t-norm and t-conorm) are well-known as essential operations in FSs and other fuzzy systems [34]. Aczel and Alsina established the Acz-Aln t-norm and Acz-Aln t-conorm operations, which offer the benefit of parameter adaptability [35]. The goal of this research is to propose the Acz-Aln t-norm and t-conorm operations, as well as a list of new Agop, in an IHF context, and to develop a MAGDM strategy to solve the favoured ranking of alternatives in MADM utilising these operators. Assessing the quality of medical services as an MCDM challenge is an effective way to equip medical practitioners with a reasonable appraisal. We provide a novel Acz-Aln Agop-based approach for evaluating medical services. Finally, we demonstrate the supremacy of our technique by providing a numerical example and a comparison to the existing approaches. Depending on the outcomes, we can assist governments on how to improve the quality of medical care providers.

1.1. Motivation and Gap of Study. The literature had a large number of AOs based on t-norm (TN) and t-conorm (TCN). Uninorms were first introduced to the FS theory by Deschrijver and Kerre [36]. IF AO was created by Xia et al. [37] using Archimedean TN and TCN.

IF AOs were created by Wang and Liu [22] utilising Einstein TN and TCN. Based on Einstein TN and TCN, Wei and Zhao [38] presented induced hesitant interval-valued AOs. Liu [39] created interval-valued IF (IVIFS) AOs using Hamacher TN and TCN. By utilising Dombi TN and TCN, Ullah et al. [40] established interval-valued TSF AOs. IF power AOs were created by Zhang et al. [41] utilising Frank TN and TCN. This TN and TCN had a significant impact on the use of FSs in decision making. A novel pair of TN and TCN that is more flexible than the previous TN and TCN listed above was initially introduced by Aczel and Alsina [35]. Senapati et al. [42] developed the Aczel–Alsina AOs for the IFS framework as well as for interval-valued IFSs (IVIFSs) and discussed the application to MADM, and both highlight the importance of the Aczel–Alsina TN and TCN due to the variableness of the involved parameters. We can infer from the investigation abovemenioned that the AOs used in MADM are complicated by actual phenomena. Naeem et al. [43] presented the decision methodology based on the picture fuzzy Aczel–Alsina geometric aggregation operators to tackle the uncertainty in decision making problems.

The information should be handled more readily in order to get the optimal alternative in MADM. Moreover, IHFSs operate in a more ambiguity-tolerant environment than IFSs, IVIFSs, Pythagorean FSs (PyFSs), q-rung orthopair FSs (qROFSs), and spherical FSs (SFSs). The usage of Aczel–Alsina TN and TCN in the framework of IHFSs has not yet been discovered. These elements inspire us to formalise the idea of Aczel–Alsina AOs in the design of IHFSs and then investigate how they apply to MADM. Using this structure, the problem is additionally categorised by changing the physical importance of the reference parameters. For the following reasons, many novel aggregation operators (AOs), including the IHF Acz-Aln operations operators, are recommended as follows:

(a) IFS and HFS are related ideas that provide IHFS decision makers more latitude.
(b) IHFS uses approximate spaces for hesitation as opposed to IFS.
(c) Weighted averaging and geometric aggregation operators are unable to account for experts' levels of knowledge with the examined items when performing initial evaluation; however, IHF Acz-Aln AOs can.
(d) The IHF Acz-Aln AOs are simple and cover the decision-making process; therefore, this article aims to address more complex and advanced data.
(e) In the suggested work, all flaws are fixed.

The following are the objectives of the paper:

(1) In order to make up for the lack of algebraic, Einstein, and Hamacher operations and to depict the relationship between IHFNs, we develop a series of Acz-Aln operations for IHFNs.
(2) In order to support IHF data, we develop Acz-Aln Agop to IHF Acz-Aln Agop: IHF Acz-Aln weighted geometric (IHFAWG) operator, IHF Acz-Aln order weighted geometric (IHFAOWG) operator, and IHF Acz-Aln hybrid weighted geometric (IHFAHWG) operator.
(3) We developed an algorithm to handle MAGDM problem using IHF information.
(4) We used a MAGDM problem regarding the quality of medical care providers to describe the suitability and reliability of the proposed IHF Acz-Aln Agop.
(5) The results demonstrated that the developed approach is incrementally more potent and produces more authentic findings than existing approaches.

The following is the order in which the rest of this article is organised. Section 2 discusses the basics of t-norms, t-conorms, Acz-Aln t-norms, IHFSs, and a number of operational principles in the context of IHFNs. Section 3 discusses the Acz-Aln operating rules as well as the characteristics of IHFNs. In Section 4, we interpret several IHF Acz-Aln Agops and discuss their distinguishing characteristics. In the next section, we will look at the MADM problem with the help of IHF Acz-Aln Agop. In Section 6, we present a novel method for assessing the quality of medical services by seeing it as an MCDM problem. The new technique aids in the efficiency and accuracy of assessing the quality of medical services, allowing medical service providers to enhance and amaze their customers. In Section 7, we examine how a parameter influences DM results. Section 8 contrasts the results of the suggested Agop to those obtained through existing Agops, and it summarises the article and makes recommendations for further research.

2. Fundamental Concepts

We will look at some key topics in this section that will be significant in the creation of this article.

2.1. Aczel–Alsina Norm

Definition 1. Assume $\Upsilon, \beth, \eth \in [0,1]$, a relation $\mathscr{R}$: $[0,1] \times [0,1] \longrightarrow [0,1]$ is a t-norm, if it is fulfilled.

(1) $\mathscr{R}(\Upsilon, \beth) = \mathscr{R}(\beth, \Upsilon)$;
(2) $\mathscr{R}(\Upsilon, \beth) \leq \mathscr{R}(\Upsilon, \eth)$ if $\beth \leq \eth$;
(3) $\mathscr{R}(\Upsilon, \mathscr{R}(\beth, \eth)) = \mathscr{R}(\mathscr{R}(\Upsilon, \beth), \eth)$;
(4) $\mathscr{R}(\Upsilon, 1) = \Upsilon$.

Definition 2. Assume $\Upsilon, \beth, \eth \in [0,1]$, a relation $\mathscr{S}$: $[0,1] \times [0,1] \longrightarrow [0,1]$ is an s-norm, if it is fulfilled.

(1) $\mathscr{S}(\Upsilon, \beth) = \mathscr{S}(\beth, \Upsilon)$;
(2) $\mathscr{S}(\Upsilon, \beth) \leq \mathscr{S}(\Upsilon, \eth)$ if $\beth \leq \eth$;
(3) $\mathscr{S}(\Upsilon, \mathscr{S}(\beth, \eth)) = \mathscr{S}(\mathscr{S}(\Upsilon, \beth), \eth)$;
(4) $\mathscr{S}(\Upsilon, 0) = \Upsilon$.

Acz-Aln norms are two useful processes that have the benefit of being changeable with the parametric activity [40, 41].

Definition 3. A relation $(\mathscr{R}_\alpha^\rho)_{\rho \in [0,\infty]}$ is a (Aczel–Alsina) Acz-Aln t-norm, if it is fulfilled.

$$\mathscr{R}_\alpha^\rho(\Upsilon, \beth) = \begin{cases} \mathscr{R}_{\mathscr{D}}(\Upsilon, \beth), & \text{if } \rho = 0, \\ \min(\Upsilon, \beth), & \text{if } \rho = \infty, \\ e^{-((-\ell n \Upsilon)^\rho + (-\ell n \beth)^\rho)^{1/\rho}}, & \text{otherwise,} \end{cases} \tag{1}$$

where $\Upsilon, \beth \in [0,1]$, ρ is +ve constant and $\mathscr{R}_{\mathscr{D}}$ is severe t-norm, and defined as follows:

$$\mathscr{R}_{\mathscr{D}}(\Upsilon, \beth) = \begin{cases} \Upsilon, & \text{if } \beth = 1, \\ \beth, & \text{if } \Upsilon = 1, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

Definition 4. A relation $(\mathscr{S}_\alpha^\rho)_{\rho \in [0,\infty]}$ is Acz-Aln s-norm, if it is fulfilled.

$$\mathscr{S}_\alpha^\rho(\Upsilon, \beth) = \begin{cases} \mathscr{S}_{\mathscr{D}}(\Upsilon, \beth), & \text{if } \rho = 0, \\ \max(\Upsilon, \beth), & \text{if } \rho = \infty, \\ 1 - e^{-((-\ell n(1-\Upsilon))^\rho + (-\ell n(1-\beth))^\rho)^{1/\rho}}, & \text{otherwise,} \end{cases} \tag{3}$$

where $\Upsilon, \beth \in [0,1]$, ρ is +ve constant and $\mathscr{S}_{\mathscr{D}}$ is severe s-norm, and defined as follows:

$$\mathcal{S}_{\mathcal{D}}(\Upsilon, \daleth) = \begin{cases} \Upsilon, & \text{if } \daleth = 0, \\ \daleth, & \text{if } \Upsilon = 0, \\ 0, & \text{otherwise.} \end{cases} \quad (4)$$

For every $\rho \in [0, \infty]$, the t-norm $\mathcal{R}_{\alpha}^{\rho}$ and s-norm $\mathcal{S}_{\alpha}^{\rho}$ are dual to one another.

2.2. Intuitionistic Hesitant Fuzzy Sets

Definition 5. A IFS ζ in $\eth$ is defined as

$$\zeta = \left\{\mathcal{O}_{\zeta}(\flat), \mathcal{G}_{\zeta}(\flat) \in [0,1] \mid \flat \in \eth\right\}, \quad (5)$$

where +ve grade $\mathcal{O}_{\zeta}$ and −ve grade $\mathcal{G}_{\zeta}$ of the element $\flat$ to intuitionistic fuzzy set ζ fulfilled that $0 \le \mathcal{O}_{\zeta} + \mathcal{G}_{\zeta} \le 1$, for each $\flat \in \eth$.

Definition 6. A IHFS ζ in $\eth$ is defined as

$$\zeta = \left\{\mathcal{O}_{\zeta}(\flat), \mathcal{G}_{\zeta}(\flat) \in [0,1] \mid \flat \in \eth\right\}, \quad (6)$$

where +ve grade $\mathcal{O}_{\zeta}$ and −ve grade $\mathcal{G}_{\zeta}$ of the element $\flat$ to intuitionistic fuzzy set ζ fulfilled that $(\max(\mathcal{O}_{\zeta}(\flat)))^2 + (\min(\mathcal{G}_{\zeta}(\flat)))^2 \le 1$ and $(\min(\mathcal{O}_{\zeta}(\flat)))^2 + (\max(\mathcal{G}_{\zeta}(\flat)))^2 \le 1$.

Definition 7. Let $\zeta_{\sqsupset} = \left\{\mathcal{O}_{\zeta_{\sqsupset}}, \mathcal{G}_{\zeta_{\sqsupset}}\right\}$ be two intuitionistic hesitant fuzzy numbers (IHFNs), where $(\sqsupset = 1, 2)$.

(1) $\zeta_1 \subseteq \zeta_2$ if $\mathcal{O}_{\zeta_1} \le \mathcal{O}_{\zeta_2}$ and $\mathcal{G}_{\zeta_1} \ge \mathcal{G}_{\zeta_2}$ for all $\flat \in \eth$;

(2) $\zeta_1 = \zeta_2$ if $\zeta_1 \subseteq \zeta_2$ and $\zeta_2 \subseteq \zeta_1$;

(3) $\zeta_1 \cap \zeta_2 = \left\{\bigcup_{\substack{\beta_1 \in \mathcal{G}_{\zeta_1} \\ \beta_2 \in \mathcal{G}_{\zeta_2}}} (\min(\beta_1, \beta_2)), \bigcup_{\substack{\alpha_1 \in \mathcal{O}_{\zeta_1} \\ \alpha_2 \in \mathcal{O}_{\zeta_2}}} (\max(\alpha_1, \alpha_2))\right\}$;

(4) $\zeta_1 \cup \zeta_2 = \left\{\bigcup_{\substack{\alpha_1 \in \mathcal{O}_{\zeta_1} \\ \alpha_2 \in \mathcal{O}_{\zeta_2}}} (\max(\alpha_1, \alpha_2)), \bigcup_{\substack{\beta_1 \in \mathcal{G}_{\zeta_1} \\ \beta_2 \in \mathcal{G}_{\zeta_2}}} (\min(\beta_1, \beta_2))\right\}$;

(5) $(\zeta_1)^c = \left\{\mathcal{G}_{\zeta_1}, \mathcal{O}_{\zeta_1}\right\}$.

Definition 8. Let $\zeta_{\sqsupset} = \left\{\mathcal{O}_{\zeta_{\sqsupset}},, \mathcal{G}_{\zeta_{\sqsupset}}\right\}$ be two IHFNs, where $(\sqsupset = 1, 2)$. The operations about any two IHFNs are introduced as follows:

(1) $\zeta_1 \oplus \zeta_2 = \left\{\mathcal{O}_{\zeta_1} + \mathcal{O}_{\zeta_2} - \mathcal{O}_{\zeta_1}\mathcal{O}_{\zeta_2}, \mathcal{G}_{\zeta_1}\mathcal{G}_{\zeta_2}\right\}$;

(2) $\zeta_1 \otimes \zeta_2 = \left\{\mathcal{O}_{\zeta_1}\mathcal{O}_{\zeta_2}, \mathcal{G}_{\zeta_1} + \mathcal{G}_{\zeta_2} - \mathcal{G}_{\zeta_1}.\mathcal{G}_{\zeta_2}\right\}$;

(3) $\eta \cdot \zeta_1 = \left\{1 - (1 - \mathcal{O}_{\zeta_1})^{\eta}, (\mathcal{G}_{\zeta_1})^{\eta}\right\}, \eta > 0$;

(4) $(\zeta_1)^{\eta} = \left\{(\mathcal{O}_{\zeta_1})^{\eta}, 1 - (1 - \mathcal{G}_{\zeta_1})^{\eta}\right\}, \eta > 0$.

Wei [44] derived the following operations using the Definition 8 as follows:

Definition 9. Let $\zeta_{\sqsupset} = \left\{\mathcal{O}_{\zeta_{\sqsupset}}, \mathcal{G}_{\zeta_{\sqsupset}}\right\}$ be collection of IHFNs, where $(\sqsupset = 1, 2, \ldots, n)$ and $\eta_1, \eta_1 > 0$, then

(1) $\zeta_1 \oplus \zeta_2 = \zeta_2 \oplus \zeta_1$;

(2) $\zeta_1 \otimes \zeta_2 = \zeta_2 \otimes \zeta_1$;

(3) $\eta_1(\zeta_1 \oplus \zeta_2) = \eta_1\zeta_1 \oplus \eta_1\zeta_2$;

(4) $(\zeta_1 \otimes \zeta_2)^{\eta_1} = \zeta_1^{\eta_1} \otimes \zeta_2^{\eta_1}$;

(5) $\eta_1\zeta_1 \oplus \eta_2\zeta_1 = (\eta_1 + \eta_2)\zeta_1$;

(6) $\zeta_1^{\eta_1} \otimes \zeta_1^{\eta_2} = \zeta_1^{(\eta_1 + \eta_2)}$;

(7) $(\zeta_1^{\eta_1})^{\eta_2} = \zeta_1^{\eta_1\eta_2}$.

Definition 10. Let $\zeta = \left\{\mathcal{O}_{\zeta}, \mathcal{G}_{\zeta}\right\}$ be IHFN. The score $\Theta(\zeta)$ and accuracy $\alpha(\zeta)$ are given as follows:

(1) $\Theta(\zeta) = (1/l(\mathcal{O}_{\zeta})\oplus_{\sqsupset=1}^{\ell}\mathcal{O}_{\zeta} - 1/l(\mathcal{G}_{\zeta})\oplus_{\sqsupset=1}^{\ell}\mathcal{G}_{\zeta}) \in [0, 1]$;

(2) $\alpha(\zeta) = (\mathcal{O}_{\zeta} + \mathcal{G}_{\zeta}) \in [-1, 1]$.

Definition 11. Let $\zeta_{\sqsupset} = \left\{\mathcal{O}_{\zeta_{\sqsupset}}, \mathcal{G}_{\zeta_{\sqsupset}}\right\}$ be two IHFNs, where $(\sqsupset = 1, 2)$. Then, the comparison technique of IHFNs can be defined as follows:

(1) $\Theta(\zeta_1) > \Theta(\zeta_2)$ implies that $\zeta_1 > \zeta_2$;

(2) $\Theta(\zeta_1) = \Theta(\zeta_2)$ and $\alpha(\zeta_1) > \alpha(\zeta_2)$ implies that $\zeta_1 > \zeta_2$;

(3) $\Theta(\zeta_1) = \Theta(\zeta_2)$ and $\alpha(\zeta_1) = \alpha(\zeta_2)$ implies that $\zeta_1 = \zeta_2$.

Ashraf et al. [45] prepared the algebraic Agop under IHFSs portrayed in the succeeding definition.

Definition 12. Let $\zeta_{\sqsupset} = \left\{\mathcal{O}_{\zeta_{\sqsupset}}, \mathcal{G}_{\zeta_{\sqsupset}}\right\}$ be the collection of IHFNs, where $(\sqsupset = 1, 2, \ldots, \ell)$. An IHF weighted geometric (IFWG) Agop of dimention ℓ is a relation $\mathcal{P}^{\ell} \longrightarrow \mathcal{P}$ with the weight vector $\sigma = (\sigma_1, \sigma_1, \ldots, \sigma_{\ell})^T$ such that $\sigma_{\sqsupset} > 0$ and $\oplus_{\sqsupset=1}^{\ell}\sigma_{\sqsupset} = 1$ as follows:

$$\mathrm{IHFWG}(\zeta_1, \zeta_2, \ldots, \zeta_{\ell}) = \prod_{\sqsupset=1}^{\ell}(\zeta_{\sqsupset})^{\sigma_{\sqsupset}} = \bigcup_{\substack{\alpha \in \mathcal{O}_{\zeta_{\sqsupset}} \\ \beta \in \mathcal{G}_{\zeta_{\sqsupset}}}} \left\{\prod_{\sqsupset=1}^{\ell}(\alpha)^{\sigma_{\sqsupset}}, 1 - \prod_{\sqsupset=1}^{\ell}(1 - \beta)^{\sigma_{\sqsupset}}\right\}. \quad (7)$$

Definition 13. Let $\zeta_{\sqsupset} = \left\{\mathcal{O}_{\zeta_{\sqsupset}}, \mathcal{G}_{\zeta_{\sqsupset}}\right\}$ be two IHFNs, where $(\sqsupset = 1, 2)$ and ρ is the positive constant. Then, Acz-Aln norms-based operations for IHFNs are introduced as follows:

3. Aczel–Alsina Operation for IHFNs

We discussed Acz-Aln operations in relation to IHFNs, taking into account Acz-Aln t-norm and Acz-Aln t-conorm.

(1) $\zeta_1 \oplus \zeta_2 = \bigcup_{(\alpha_{\zeta_\sqsupset}, \beta_{\zeta_\sqsupset}) \in (\mathcal{O}_{\zeta_\sqsupset}, \mathcal{G}_{\zeta_\sqsupset})(\sqsupset=1,2)} \left\{ \begin{array}{c} 1 - e^{-((-\ell n(1-\alpha_{\zeta_1}))^\rho + (-\ell n(1-\alpha_{\zeta_2}))^\rho)^{1/\rho}}, \\ e^{-((-\ell n \beta_{\zeta_1})^\rho + (-\ell n \beta_{\zeta_2})^\rho)^{1/\rho}} \end{array} \right\};$

(2) $\zeta_1 \otimes \zeta_2 = \bigcup_{(\alpha_{\zeta_\sqsupset}, \beta_{\zeta_\sqsupset}) \in (\mathcal{O}_{\zeta_\sqsupset}, \mathcal{G}_{\zeta_\sqsupset})(\sqsupset=1,2)} \left\{ \begin{array}{c} e^{-((-\ell n \alpha_{\zeta_1})^\rho + (-\ell n \alpha_{\zeta_2})^\rho)^{1/\rho}}, \\ 1 - e^{-((-\ell n(1-\beta_{\zeta_1}))^\rho + (-\ell n(1-\beta_{\zeta_2}))^\rho)^{1/\rho}} \end{array} \right\};$

(3) $\eta \cdot \zeta_1 = \bigcup_{(\alpha_{\zeta_1}, \beta_{\zeta_1}) \in (\mathcal{O}_{\zeta_1}, \mathcal{G}_{\zeta_1})} \{1 - e^{-(\eta(-\ell n(1-\alpha_{\zeta_1}))^\rho)^{1/\rho}}, e^{-(\eta(-\ell n \beta_{\zeta_1})^\rho)^{1/\rho}}\}, \eta > 0;$

(4) $(\zeta_1)^\eta = \bigcup_{(\alpha_{\zeta_1}, \beta_{\zeta_1}) \in (\mathcal{O}_{\zeta_1}, \mathcal{G}_{\zeta_1})} \{e^{-(\eta(-\ell n \alpha_{\zeta_1})^\rho)^{1/\rho}}, 1 - e^{-(\eta(-\ell n(1-\beta_{\zeta_1}))^\rho)^{1/\rho}}\}, \eta > 0.$

Theorem 1. *Let $\zeta_\sqsupset = \{\mathcal{O}_{\zeta_\sqsupset}, \mathcal{G}_{\zeta_\sqsupset}\}$ be the collection of IHFNs, where $(\sqsupset = 1, 2, \ldots, n)$ and $\eta_1, \eta_1 > 0$, then*

(1) $\zeta_1 \oplus \zeta_2 = \zeta_2 \oplus \zeta_1$;

(2) $\zeta_1 \otimes \zeta_2 = \zeta_2 \otimes \zeta_1$;

(3) $\eta_1(\zeta_1 \oplus \zeta_2) = \eta_1 \zeta_1 \oplus \eta_1 \zeta_2$;

(4) $(\zeta_1 \otimes \zeta_2)^{\eta_1} = \zeta_1^{\eta_1} \otimes \zeta_2^{\eta_1}$;

(5) $\eta_1 \zeta_1 \oplus \eta_2 \zeta_1 = (\eta_1 + \eta_2)\zeta_1$;

(6) $\zeta_1^{\eta_1} \otimes \zeta_1^{\eta_2} = \zeta_1^{(\eta_1 + \eta_2)}$;

(7) $(\zeta_1^{\eta_1})^{\eta_2} = \zeta_1^{\eta_1 \eta_2}$.

Proof

(1) Since $\zeta_\sqsupset = \{\mathcal{O}_{\zeta_\sqsupset}, \mathcal{G}_{\zeta_\sqsupset}\}$ be the collection of IHFNs, where $(\sqsupset = 1, 2, \ldots, n)$ and $\eta_1, \eta_1 > 0$, then by Definition 13, we have

$$\begin{aligned} \zeta_1 \oplus \zeta_2 &= \bigcup_{\left(\alpha_{\zeta_\sqsupset}, \beta_{\zeta_\sqsupset}\right) \in \left(\mathcal{O}_{\zeta_\sqsupset}, \mathcal{G}_{\zeta_\sqsupset}\right)(\sqsupset=1,2)} \left\{ \begin{array}{c} 1 - e^{-\left(\left(-\ell n\left(1-\alpha_{\zeta_1}\right)\right)^\rho + \left(-\ell n\left(1-\alpha_{\zeta_2}\right)\right)^\rho\right)^{1/\rho}}, \\ e^{-\left(\left(-\ell n \beta_{\zeta_1}\right)^\rho + \left(-\ell n \beta_{\zeta_2}\right)^\rho\right)^{1/\rho}} \end{array} \right\} \\ &= \bigcup_{\left(\alpha_{\zeta_\sqsupset}, \beta_{\zeta_\sqsupset}\right) \in \left(\mathcal{O}_{\zeta_\sqsupset}, \mathcal{G}_{\zeta_\sqsupset}\right)(\sqsupset=1,2)} \left\{ \begin{array}{c} 1 - e^{-\left(\left(-\ell n\left(1-\alpha_{\zeta_2}\right)\right)^\rho + \left(-\ell n\left(1-\alpha_{\zeta_1}\right)\right)^\rho\right)^{1/\rho}}, \\ e^{-\left(\left(-\ell n \beta_{\zeta_2}\right)^\rho + \left(-\ell n \beta_{\zeta_1}\right)^\rho\right)^{1/\rho}} \end{array} \right\} \\ &= \zeta_2 \oplus \zeta_1. \end{aligned} \tag{8}$$

(2) By Definition 13, we have

$$\begin{aligned} \zeta_1 \otimes \zeta &= \bigcup_{\left(\alpha_{\zeta_\sqsupset}, \beta_{\zeta_\sqsupset}\right) \in \left(\mathcal{O}_{\zeta_\sqsupset}, \mathcal{G}_{\zeta_\sqsupset}\right)(\sqsupset=1,2)} \left\{ \begin{array}{c} e^{-\left(\left(-\ell n \alpha_{\zeta_1}\right)^\rho + \left(-\ell n \alpha_{\zeta_2}\right)^\rho\right)^{1/\rho}}, \\ 1 - e^{-\left(\left(-\ell n\left(1-\beta_{\zeta_1}\right)\right)^\rho + \left(-\ell n\left(1-\beta_{\zeta_2}\right)\right)^\rho\right)^{1/\rho}} \end{array} \right\} \\ &= \bigcup_{\left(\alpha_{\zeta_\sqsupset}, \beta_{\zeta_\sqsupset}\right) \in \left(\mathcal{O}_{\zeta_\sqsupset}, \mathcal{G}_{\zeta_\sqsupset}\right)(\sqsupset=1,2)} \left\{ \begin{array}{c} e^{-\left(\left(-\ell n \alpha_{\zeta_2}\right)^\rho + \left(-\ell n \alpha_{\zeta_1}\right)^\rho\right)^{1/\rho}}, \\ 1 - e^{-\left(\left(-\ell n\left(1-\beta_{\zeta_2}\right)\right)^\rho + \left(-\ell n\left(1-\beta_{\zeta_1}\right)\right)^\rho\right)^{1/\rho}} \end{array} \right\} \\ &= \zeta_2 \otimes \zeta_1. \end{aligned} \tag{9}$$

(3) By Definition 13, we have

$$\begin{aligned}
\eta_1(\zeta_1\oplus\zeta_2) &== \bigcup_{\left(\alpha_{\zeta_\sqsupset},\beta_{\zeta_\sqsupset}\right)\in\left(\mathcal{O}_{\zeta_\sqsupset},\mathcal{G}_{\zeta_\sqsupset}\right)(\sqsupset=1,2)} \eta_1\left\{\begin{array}{c} 1-e^{-\left(\left(-\ell n\left(1-\alpha_{\zeta_1}\right)\right)^{\rho}+\left(-\ell n\left(1-\alpha_{\zeta_2}\right)\right)^{\rho}\right)^{1/\rho}}, \\ e^{-\left(\left(-\ell n\beta_{\zeta_1}\right)^{\rho}+\left(-\ell n\beta_{\zeta_2}\right)^{\rho}\right)^{1/\rho}} \end{array}\right\} \\
&= \bigcup_{\left(\alpha_{\zeta_\sqsupset},\beta_{\zeta_\sqsupset}\right)\in\left(\mathcal{O}_{\zeta_\sqsupset},\mathcal{G}_{\zeta_\sqsupset}\right)(\sqsupset=1,2)} \left\{\begin{array}{c} 1-e^{-\left(\eta_1\left(-\ell n\left(1-\alpha_{\zeta_1}\right)\right)^{\rho}+\eta_1\left(-\ell n\left(1-\alpha_{\zeta_2}\right)\right)^{\rho}\right)^{1/\rho}}, \\ e^{-\left(\eta_1\left(-\ell n\beta_{\zeta_1}\right)^{\rho}+\eta_1\left(-\ell n\beta_{\zeta_2}\right)^{\rho}\right)^{1/\rho}} \end{array}\right\} \\
&= \bigcup_{\left(\alpha_{\zeta_\sqsupset},\beta_{\zeta_\sqsupset}\right)\in\left(\mathcal{O}_{\zeta_\sqsupset},\mathcal{G}_{\zeta_\sqsupset}\right)(\sqsupset=1,2)} \left\{\begin{array}{c} \left(\begin{array}{c} 1-e^{-\left(\eta_1\left(-\ell n\left(1-\alpha_{\zeta_1}\right)\right)^{\rho}\right)^{1/\rho}}, \\ e^{-\left(\eta_1\left(-\ell n\beta_{\zeta_1}\right)^{\rho}\right)^{1/\rho}} \end{array}\right)\oplus \\ \left(\begin{array}{c} 1-e^{-\left(\eta_1\left(-\ell n\left(1-\alpha_{\zeta_1}\right)\right)^{\rho}\right)^{1/\rho}}, \\ e^{-\left(\eta_1\left(-\ell n\beta_{\zeta_1}\right)^{\rho}\right)^{1/\rho}} \end{array}\right) \end{array}\right\} \\
&= \eta_1\zeta_1\oplus\eta_1\zeta_2.
\end{aligned} \tag{10}$$

Proof (4) is similar to proof (3).

(5) By Definition 13, we have

$$\begin{aligned}
\eta_1\zeta_1\oplus\eta_2\zeta_1 &= \bigcup_{\left(\alpha_{\zeta_1},\beta_{\zeta_1}\right)\in\left(\mathcal{O}_{\zeta_1},\mathcal{G}_{\zeta_1}\right)} \left\{\begin{array}{c} \left(\begin{array}{c} 1-e^{-\left(\eta_1\left(-\ell n\left(1-\alpha_{\zeta_1}\right)\right)^{\rho}\right)^{1/\rho}}, \\ e^{-\left(\eta_1\left(-\ell n\beta_{\zeta_1}\right)^{\rho}\right)^{1/\rho}} \end{array}\right)\oplus \\ \left(\begin{array}{c} 1-e^{-\left(\eta_2\left(-\ell n\left(1-\alpha_{\zeta_1}\right)\right)^{\rho}\right)^{1/\rho}}, \\ e^{-\left(\eta_2\left(-\ell n\beta_{\zeta_1}\right)^{\rho}\right)^{1/\rho}} \end{array}\right) \end{array}\right\} \\
&= \bigcup_{\left(\alpha_{\zeta_1},\beta_{\zeta_1}\right)\in\left(\mathcal{O}_{\zeta_1},\mathcal{G}_{\zeta_1}\right)} \left\{\begin{array}{c} 1-e^{-\left(\eta_1\left(-\ell n\left(1-\alpha_{\zeta_1}\right)\right)^{\rho}+\eta_2\left(-\ell n\left(1-\alpha_{\zeta_1}\right)\right)^{\rho}\right)^{1/\rho}}, \\ e^{-\left(\eta_1\left(-\ell n\beta_{\zeta_1}\right)^{\rho}+\eta_2\left(-\ell n\beta_{\zeta_1}\right)^{\rho}\right)^{1/\rho}} \end{array}\right\} \\
&= \bigcup_{\left(\alpha_{\zeta_1},\beta_{\zeta_1}\right)\in\left(\mathcal{O}_{\zeta_1},\mathcal{G}_{\zeta_1}\right)} \left\{\begin{array}{c} 1-e^{-\left(\eta_1+\eta_2\left(-\ell n\left(1-\alpha_{\zeta_1}\right)\right)^{\rho}\right)^{1/\rho}}, \\ e^{-\left(\eta_1+\eta_2\left(-\ell n\beta_{\zeta_1}\right)^{\rho}\right)^{1/\rho}} \end{array}\right\} \\
&= (\eta_1+\eta_2)\zeta_1.
\end{aligned} \tag{11}$$

The proof of (6) and (7) are similar as the proof (5). □

4. Aczel–Alsina Geometric Aggregation Operators for IHFNs

Acz-Aln norms-based novel Agop under IHF information are proposed in this section.

Definition 14. Let $\zeta_\daleth = \left\{\mathscr{O}_{\zeta_\daleth}, \mathscr{G}_{\zeta_\daleth}\right\}$ be the collection of IHFNs, where $(\daleth = 1, 2, \ldots, \ell)$. An IHF Acz-Aln weighted geometric (IHFAWG) Agop of dimention ℓ is a relation $\mathscr{P}^\ell \longrightarrow \mathscr{P}$ with weight vector $\sigma = (\sigma_1, \sigma_1, \ldots, \sigma_\ell)^T$ such that $\sigma_\daleth > 0$ and $\oplus_{\daleth=1}^{\ell}\sigma_\daleth = 1$ as follows:

$$\mathrm{IHFAWG}(\zeta_1, \zeta_2, \ldots, \zeta_\ell) = \prod_{\daleth=1}^{\ell} (\zeta_\daleth)^{\sigma_\daleth}. \tag{12}$$

Theorem 2. *Suppose $\zeta_\daleth = \left\{\mathscr{O}_{\zeta_\daleth}, \mathscr{G}_{\zeta_\daleth}\right\}$ be the collection of IHFNs, where $(\daleth = 1, 2, \ldots, \ell)$. An IHF Acz-Aln weighted geometric (IHFAWG) Agop of dimention ℓ is a relation $\mathscr{P}^\ell \longrightarrow \mathscr{P}$ with weight vector $\sigma = (\sigma_1, \sigma_1, \ldots, \sigma_\ell)^T$ such that $\sigma_\daleth > 0$ and $\oplus_{\daleth=1}^{\ell}\sigma_\daleth = 1$ is defined as follows:*

$$\mathrm{IHFAWG}(\zeta_1, \zeta_2, \ldots, \zeta_\ell) = \prod_{\daleth=1}^{\ell} (\zeta_\daleth)^{\sigma_\daleth} = \bigcup_{\left(\alpha_{\zeta_\daleth}, \beta_{\zeta_\daleth}\right) \in \left(\mathscr{O}_{\zeta_\daleth}, \mathscr{G}_{\zeta_\daleth}\right)} \left\{ \begin{array}{c} e^{-\left(\oplus_{\daleth=1}^{\ell}\sigma_\daleth\left(-\ell n\alpha_{\zeta_\daleth}\right)^\rho\right)^{1/\rho}}, \\ 1 - e^{-\left(\oplus_{\daleth=1}^{\ell}\sigma_\daleth\left(-\ell n\left(1-\beta_{\zeta_\daleth}\right)\right)^\rho\right)^{1/\rho}} \end{array} \right\}. \tag{13}$$

Proof. Using mathematical induction, the proof of Theorem 2 is derived as follows:

Step-1: for $\ell = 2$, we get

$$\mathrm{IHFAWG}(\zeta_1, \zeta_2) = (\zeta_1)^{\sigma_1} \otimes (\zeta_2)^{\sigma_2}. \tag{14}$$

By Definition 13, we have

$$\begin{aligned} (\zeta_1)^{\sigma_1} &= \bigcup_{\left(\alpha_{\zeta_1}, \beta_{\zeta_1}\right) \in \left(\mathscr{O}_{\zeta_1}, \mathscr{G}_{\zeta_1}\right)} \left\{ e^{-\left(\sigma_1\left(-\ell n\alpha_{\zeta_1}\right)^\rho\right)^{1/\rho}}, 1 - e^{-\left(\sigma_1\left(-\ell n\left(1-\beta_{\zeta_1}\right)\right)^\rho\right)^{1/\rho}} \right\}, \\ (\zeta_2)^{\sigma_2} &= \bigcup_{\left(\alpha_{\zeta_1}, \beta_{\zeta_2}\right) \in \left(\mathscr{O}_{\zeta_1}, \mathscr{G}_{\zeta_2}\right)} \left\{ e^{-\left(\sigma_2\left(-\ell n\alpha_{\zeta_2}\right)^\rho\right)^{1/\rho}}, 1 - e^{-\left(\sigma_2\left(-\ell n\left(1-\beta_{\zeta_2}\right)\right)^\rho\right)^{1/\rho}} \right\}. \end{aligned} \tag{15}$$

Therefore,

$$\begin{aligned} \mathrm{IHFAWG}(\zeta_1, \zeta_2) &= \bigcup_{\left(\alpha_{\zeta_\daleth}, \beta_{\zeta_\daleth}\right) \in \left(\mathscr{O}_{\zeta_\daleth}, \mathscr{G}_{\zeta_\daleth}\right)(\daleth=1,2)} \left\{ \begin{array}{c} \left(\begin{array}{c} e^{-\left(\sigma_1\left(-\ell n\alpha_{\zeta_1}\right)^\rho\right)^{1/\rho}}, \\ 1 - e^{-\left(\sigma_1\left(-\ell n\left(1-\beta_{\zeta_1}\right)\right)^\rho\right)^{1/\rho}} \end{array} \right) \otimes \\ \left(\begin{array}{c} e^{-\left(\sigma_2\left(-\ell n\alpha_{\zeta_2}\right)^\rho\right)^{1/\rho}}, \\ 1 - e^{-\left(\sigma_2\left(-\ell n\left(1-\beta_{\zeta_2}\right)\right)^\rho\right)^{1/\rho}} \end{array} \right) \end{array} \right\} \\ &= \bigcup_{\left(\alpha_{\zeta_\daleth}, \beta_{\zeta_\daleth}\right) \in \left(\mathscr{O}_{\zeta_\daleth}, \mathscr{G}_{\zeta_\daleth}\right)(\daleth=1,2)} \left\{ \begin{array}{c} e^{-\left(\sigma_1\left(-\ell n\alpha_{\zeta_1}\right)^\rho + \sigma_2\left(-\ell n\alpha_{\zeta_2}\right)^\rho\right)^{1/\rho}} \\ 1 - e^{-\left(\sigma_1\left(-\ell n\left(1-\beta_{\zeta_1}\right)\right)^\rho + \sigma_2\left(-\ell n\left(1-\beta_{\zeta_2}\right)\right)^\rho\right)^{1/\rho}} \end{array} \right\} \\ &= \bigcup_{\left(\alpha_{\zeta_\daleth}, \beta_{\zeta_\daleth}\right) \in \left(\mathscr{O}_{\zeta_\daleth}, \mathscr{G}_{\zeta_\daleth}\right)(\daleth=1,2)} \left\{ \begin{array}{c} e^{-\left(\sum_{\daleth=1}^{2}\sigma_\daleth\left(-\ell n\alpha_{\zeta_\daleth}\right)^\rho\right)^{1/\rho}}, \\ 1 - e^{-\left(\sum_{\daleth=1}^{2}\sigma_\daleth\left(-\ell n\left(1-\beta_{\zeta_\daleth}\right)\right)^\rho\right)^{1/\rho}} \end{array} \right\}. \end{aligned} \tag{16}$$

Thus, Theorem 2 is valid if $\ell = 2$.

Now, we assume, Theorem 2 is valid if $\ell = d$, we have

$$\mathrm{IHFAWG}(\zeta_1,\zeta_2,\ldots,\zeta_d) = \bigcup_{\left(\alpha_{\zeta_\beth},\beta_{\zeta_\beth}\right)\in\left(\mathcal{O}_{\zeta_\beth},\mathcal{G}_{\zeta_\beth}\right)(\beth=1\ldots d)} \left\{ \begin{array}{c} e^{-\left(\sum_{\beth=1}^{d}\sigma_\beth\left(-\ell n\alpha_{\zeta_\beth}\right)^\rho\right)^{1/\rho}}, \\ 1 - e^{-\left(\sum_{\beth=1}^{d}\sigma_\beth\left(-\ell n\left(1-\beta_{\zeta_\beth}\right)\right)^\rho\right)^{1/\rho}} \end{array} \right\}. \tag{17}$$

We have to show that Theorem 2 is true for $\ell = d+1$.

$$\begin{aligned}
\mathrm{HFAWG}(\zeta_1,\zeta_2,\ldots,\zeta_d,\zeta_{d+1}) &= \prod_{\beth=1}^{\ell}(\zeta_\beth)^{\sigma_\beth}\otimes(\zeta_{d+1})^{\sigma_{d+1}} \\
\prod_{\beth=1}^{\ell}(\zeta_\beth)^{\sigma_\beth}\otimes(\zeta_{d+1})^{\sigma_{d+1}} &= \bigcup_{\left(\alpha_{\zeta_\beth},\beta_{\zeta_\beth}\right)\in\left(\mathcal{O}_{\zeta_\beth},\mathcal{G}_{\zeta_\beth}\right)(\beth=1\ldots d)} \left\{ \begin{array}{c} e^{-\left(\sum_{\beth=1}^{d}\sigma_\beth\left(-\ell n\alpha_{\zeta_\beth}\right)^\rho\right)^{1/\rho}}, \\ 1 - e^{-\left(\sum_{\beth=1}^{d}\sigma_\beth\left(-\ell n\left(1-\beta_{\zeta_\beth}\right)\right)^\rho\right)^{1/\rho}} \end{array} \right\} \\
&\otimes \bigcup_{\left(\alpha_{\zeta_\beth},\beta_{\zeta_\beth}\right)\in\left(\mathcal{O}_{\zeta_\beth},\mathcal{G}_{\zeta_\beth}\right)} \left\{ \begin{array}{c} e^{-\left(\sigma_{d+1}\left(-\ell n\alpha_{\zeta_{d+1}}\right)^\rho\right)^{1/\rho}}, \\ 1 - e^{-\left(\sigma_{d+1}\left(-\ell n\left(1-\beta_{\zeta_{d+1}}\right)\right)^\rho\right)^{1/\rho}} \end{array} \right\} \\
&= \bigcup_{\left(\alpha_{\zeta_\beth},\beta_{\zeta_\beth}\right)\in\left(\mathcal{O}_{\zeta_\beth},\mathcal{G}_{\zeta_\beth}\right)(\beth=1\ldots d+1)} \left\{ \begin{array}{c} e^{-\left(\sum_{\beth=1}^{d}\sigma_\beth\left(-\ell n\alpha_{\zeta_\beth}\right)^\rho+\sigma_{d+1}\left(-\ell n\alpha_{\zeta d+1}\right)^\rho\right)^{1/\rho}}, \\ 1 - e^{-\left(\begin{array}{c}\sum_{\beth=1}^{d}\sigma_\beth\left(-\ell n\left(1-\beta_{\zeta_\beth}\right)\right)^\rho + \\ \sigma_{d+1}\left(-\ell n\left(1-\beta_{\zeta d+1}\right)\right)^\rho\end{array}\right)^{1/\rho}} \end{array} \right\} \\
&= \bigcup_{\left(\alpha_{\zeta_\beth},\beta_{\zeta_\beth}\right)\in\left(\mathcal{O}_{\zeta_\beth},\mathcal{G}_{\zeta_\beth}\right)(\beth=1\ldots d+1)} \left\{ \begin{array}{c} e^{-\left(\sum_{\beth=1}^{d+1}\sigma_\beth\left(-\ell n\alpha_{\zeta_\beth}\right)^\rho\right)^{1/\rho}} \\ 1 - e^{-\left(\sum_{\beth=1}^{d+1}\sigma_\beth\left(-\ell n\left(1-\beta_{\zeta_\beth}\right)\right)^\rho\right)^{1/\rho}} \end{array} \right\}.
\end{aligned} \tag{18}$$

Hence, Theorem 2 is valid for all ℓ.

We show the following characteristics properly by employing the IHFAWG operator. □

Theorem 3. *(Idempotency) Let $\zeta_\beth = \left\{\mathcal{O}_{\zeta_\beth}, \mathcal{G}_{\zeta_\beth}\right\}(\beth = 1,2,\ldots,\ell)$ be the collection of equivalent IHFNs, i.e., $\zeta_\beth = \zeta$ for each $(\beth = 1,2,\ldots,\ell)$. Then,*

$$\mathrm{IHFAWG}(\zeta_1,\zeta_2,\ldots,\zeta_\ell) = \zeta. \tag{19}$$

Proof. Since

$$\mathrm{IHFAWG}(\zeta_1,\zeta_2,\ldots,\zeta_\ell) = \bigcup_{\left(\alpha_{\zeta_\beth},\beta_{\zeta_\beth}\right)\in\left(\mathcal{O}_{\zeta_\beth},\mathcal{G}_{\zeta_\beth}\right)(\beth=1\ldots\ell)} \left\{ \begin{array}{c} e^{-\left(\oplus_{\beth=1}^{\ell}\sigma_\beth\left(-\ell n\alpha_{\zeta_\beth}\right)^\rho\right)^{1/\rho}}, \\ 1 - e^{-\left(\oplus_{\beth=1}^{\ell}\sigma_\beth\left(-\ell n\left(1-\beta_{\zeta_\beth}\right)\right)^\rho\right)^{1/\rho}} \end{array} \right\}. \tag{20}$$

Put $\zeta_\daleth = \left\{\mathcal{O}_{\zeta_\daleth}, \zeta_{\zeta_\daleth}, \mathcal{G}_{\zeta_\daleth}\right\} = \zeta$ ($\daleth = 1, 2, \ldots, \ell$), we have

$$\begin{aligned} \text{IHFAWG}(\zeta_1, \zeta_2, \ldots, \zeta_\ell) &= \bigcup_{\left(\alpha_{\zeta_\daleth}, \beta_{\zeta_\daleth}\right) \in \left(\mathcal{O}_{\zeta_\daleth}, \mathcal{G}_{\zeta_\daleth}\right)(\daleth=1\ldots\ell)} \left\{ \begin{array}{c} e^{-\left(\oplus_{\daleth=1}^{\ell} \sigma_\daleth \left(-\ell n\alpha_\zeta\right)^\rho\right)^{1/\rho}} \\ 1 - e^{-\left(\oplus_{\daleth=1}^{\ell} \sigma_\daleth \left(-\ell n\left(1-\beta_\zeta\right)\right)^\rho\right)^{1/\rho}} \end{array} \right\} \\ &= \bigcup_{\left(\alpha_{\zeta_\daleth}, \beta_{\zeta_\daleth}\right) \in \left(\mathcal{O}_{\zeta_\daleth}, \mathcal{G}_{\zeta_\daleth}\right)(\daleth=1\ldots\ell)} \left\{ e^{-\left(\left(-\ell n\alpha_\zeta\right)^\rho\right)^{1/\rho}}, 1 - e^{-\left(\left(-\ell n\left(1-\beta_\zeta\right)\right)^\rho\right)^{1/\rho}} \right\} \\ &= \bigcup_{\left(\alpha_{\zeta_\daleth}, \beta_{\zeta_\daleth}\right) \in \left(\mathcal{O}_{\zeta_\daleth}, \mathcal{G}_{\zeta_\daleth}\right)(\daleth=1\ldots\ell)} \left(\alpha_\zeta, \mathcal{G}_\zeta\right) = \zeta. \end{aligned} \tag{21}$$

Thus, $\text{IHFAWG}(\zeta_1, \zeta_2, \ldots, \zeta_\ell) = \zeta$ holds. □

Theorem 4. *(Boundedness) Let* $\zeta_\daleth = \left\{\mathcal{O}_{\zeta_\daleth}, \mathcal{G}_{\zeta_\daleth}\right\}$ *($\daleth = 1, 2, \ldots, \ell$) be the collection of IHFNs. Let* $\zeta_\daleth^- = (\min_\daleth\left\{\mathcal{O}_{\zeta_\daleth}\right\}, \max_\daleth\left\{\mathcal{G}_{\zeta_\daleth}\right\})$ *and* $\zeta_\daleth^+ = (\max_\daleth\left\{\mathcal{O}_{\zeta_\daleth}\right\}, \min_\daleth\left\{\mathcal{G}_{\zeta_\daleth}\right\})$ *($\daleth = 1, 2, \ldots, \ell$). Then,*

$$\zeta_\daleth^- \leq \text{IHFAWG}(\zeta_1, \zeta_2, \ldots, \zeta_\ell) \leq \zeta_\daleth^+. \tag{22}$$

Proof. We have, $\min_\daleth\left\{\mathcal{O}_{\zeta_\daleth}\right\} \leq \mathcal{O}_{\zeta_\daleth} \leq \max_\daleth\left\{\mathcal{O}_{\zeta_\daleth}\right\}$, i.e.,

$$\bigcup_{\left(\alpha_{\zeta_\daleth}\right) \in \left(\mathcal{O}_{\zeta_\daleth}\right)} \left\{ e^{-\left(\oplus_{\daleth=1}^{\ell} \sigma_\daleth \left(-\ell n\left(\min \alpha_{\zeta_\daleth}\right)\right)^\rho\right)^{1/\rho}} \leq e^{-\left(\oplus_{\daleth=1}^{\ell} \sigma_\daleth \left(-\ell n\alpha_{\zeta_\daleth}\right)^\rho\right)^{1/\rho}} \leq e^{-\left(\oplus_{\daleth=1}^{\ell} \sigma_\daleth \left(-\ell n\left(\max \alpha_{\zeta_\daleth}\right)\right)^\rho\right)^{1/\rho}} \right\}. \tag{23}$$

Similarly, we have

$$\bigcup_{\left(\beta_{\zeta_\daleth}\right) \in \left(\mathcal{G}_{\zeta_\daleth}\right)} \left\{ \begin{array}{c} 1 - e^{-\left(\sum_{\daleth=1}^{d} \sigma_\daleth \left(-\ell n\left(\max\left(1-\beta_{\zeta_\daleth}\right)\right)\right)^\rho\right)^{1/\rho}} \leq 1 - e^{-\left(\sum_{\daleth=1}^{d} \sigma_\daleth \left(-\ell n\left(1-\beta_{\zeta_\daleth}\right)\right)^\rho\right)^{1/\rho}} \\ \leq 1 - e^{-\left(\sum_{\daleth=1}^{d} \sigma_\daleth \left(-\ell n\left(\min\left(1-\beta_{\zeta_\daleth}\right)\right)\right)^\rho\right)^{1/\rho}} \end{array} \right\}. \tag{24}$$

Therefore,

$$\zeta_\daleth^- \leq \text{IHFAWG}(\zeta_1, \zeta_2, \ldots, \zeta_\ell) \leq \zeta_\daleth^+. \tag{25}$$

□

Theorem 5. *Let* $\zeta_\daleth = \left\{\mathcal{O}_{\zeta_\daleth}, \mathcal{G}_{\zeta_\daleth}\right\}$ *and* $\zeta_\daleth^* = \left\{\mathcal{O}_{\zeta_\daleth}^*, \mathcal{G}_{\zeta_\daleth}^*\right\}$ *($\daleth = 1, 2, \ldots, \ell$) be the two collection of IHFNs, if* $\zeta_\daleth \leq \zeta_\daleth^*$ *for ($\daleth = 1, 2, \ldots, \ell$). Then,*

$$\text{IHFAWG}(\zeta_1, \zeta_2, \ldots, \zeta_\ell) \leq \text{IHFAWG}(\zeta_1^*, \zeta_2^*, \ldots, \zeta_\ell^*). \tag{26}$$

Proof. Obviously, □

Definition 15. Let $\zeta_\daleth = \left\{\mathcal{O}_{\zeta_\daleth}, \mathcal{G}_{\zeta_\daleth}\right\}$ be the collection of IHFNs, where ($\daleth = 1, 2, \ldots, \ell$). An IHF Acz-Aln ordered weighted geometric (IHFAOWG) Agop of dimention ℓ is a relation $\mathcal{P}^\ell \longrightarrow \mathcal{P}$ with a weight vector $\sigma = (\sigma_1, \sigma_1, \ldots, \sigma_\ell)^T$ such that $\sigma_\daleth > 0$ and $\oplus_{\daleth=1}^{\ell} \sigma_\daleth = 1$ as follows:

$$\text{IHFAOWG}(\zeta_1, \zeta_2, \ldots, \zeta_\ell) = \prod_{\daleth=1}^{\ell} \left(\zeta_{\tau(\daleth)}\right)^{\sigma_\daleth}, \tag{27}$$

where $(\tau(1), \tau(2), \ldots, \tau(\ell))$ are the permutation in such a way as $\zeta_{\tau(\daleth)} \leq \zeta_{\tau(\daleth-1)}$.

Theorem 6. *Suppose* $\zeta_\daleth = \left\{\mathcal{O}_{\zeta_\daleth}, \mathcal{G}_{\zeta_\daleth}\right\}$ *be the collection of IHFNs, where ($\daleth = 1, 2, \ldots, \ell$). An IHF Acz-Aln ordered weighted geometric (IHFAOWG) Agop of dimension ℓ is a relation* $\mathcal{P}^\ell \longrightarrow \mathcal{P}$ *with a weight vector* $\sigma = (\sigma_1, \sigma_1, \ldots, \sigma_\ell)^T$ *such that* $\sigma_\daleth > 0$ *and* $\oplus_{\daleth=1}^{\ell} \sigma_\daleth = 1$ *is defined as follows:*

$$\mathrm{IHFAOWG}(\zeta_1,\zeta_2,\ldots,\zeta_\ell)=\prod_{\beth=1}^{\ell}\left(\zeta_{\tau(\beth)}\right)^{\sigma_\beth}=\bigcup_{\left(\alpha_{\zeta_\beth},\beta_{\zeta_\beth}\right)\in\left(\mathcal{O}_{\zeta_\beth},\mathcal{G}_{\zeta_\beth}\right)(\beth=1\ldots\ell)}\left\{\begin{array}{c} e^{-\left(\oplus_{\beth=1}^{\ell}\sigma_\beth\left(-\ell n\alpha_{\zeta_{\tau(\beth)}}\right)^{\rho}\right)^{1/\rho}},\\ 1-e^{-\left(\oplus_{\beth=1}^{\ell}\sigma_\beth\left(-\ell n\left(1-\beta_{\zeta_{\tau(\beth)}}\right)\right)^{\rho}\right)^{1/\rho}}\end{array}\right\}, \tag{28}$$

where $(\tau(1),\tau(2),\ldots,\tau(\ell))$ *are the permutation in such a way as* $\zeta_{\tau(\beth)}\leq\zeta_{\tau(\beth-1)}$.

We may show the following characteristics properly by employing the IHFAOWG operator.

Theorem 7

(1) *(Idempotency) Let* $\zeta_\beth=\{\mathcal{O}_{\zeta_\beth},\mathcal{G}_{\zeta_\beth}\}(\beth=1,2,\ldots,\ell)$ *be the collection of equivalent IHFNs, i.e.,* $\zeta_\beth=\zeta$ *for each* $(\beth=1,2,\ldots,\ell)$. *Then,*

$$IHFAOWG(\zeta_1,\zeta_2,\ldots,\zeta_\ell)=\zeta. \tag{29}$$

(2) *(Boundedness) Let* $\zeta_\beth=\{\mathcal{O}_{\zeta_\beth},\mathcal{G}_{\zeta_\beth}\}(\beth=1,2,\ldots,\ell)$ *be the collection of IHFNs. Let* $\zeta_\beth^-=(\min_\beth\{\mathcal{O}_{\zeta_\beth}\},\max_\beth\{\mathcal{G}_{\zeta_\beth}\})$ *and*

$$\zeta_\beth^+=\left(\max_\beth\{\mathcal{O}_{\zeta_\beth}\},\min_\beth\{\mathcal{G}_{\zeta_\beth}\}\right). \tag{30}$$

$(\beth=1,2,\ldots,\ell)$. *Then,*

$$\zeta_\beth^-\leq\mathrm{IHFAOWG}(\zeta_1,\zeta_2,\ldots,\zeta_\ell)\leq\zeta_\beth^+. \tag{31}$$

(3) *Let* $\zeta_\beth=\{\mathcal{O}_{\zeta_\beth},\zeta_{\zeta_\beth},\mathcal{G}_{\zeta_\beth}\}$ *and* $\zeta_\beth^*=\{\mathcal{O}_{\zeta_\beth}^*,\zeta_{\zeta_\beth}^*,\mathcal{G}_{\zeta_\beth}^*\}$ $(\beth=1,2,\ldots,\ell)$ *be the two collection of IHFNs, if* $\zeta_\beth\leq\zeta_\beth^*$ *for* $(\beth=1,2,\ldots,\ell)$. *Then,*

$$\mathrm{IHFAOWG}(\zeta_1,\zeta_2,\ldots,\zeta_\ell)\leq\mathrm{IHFAOWG}(\zeta_1^*,\zeta_2^*,\ldots,\zeta_\ell^*). \tag{32}$$

Proof. Prove of this theorem is similarly done by using Theorem 3, 4, and 5. □

Definition 16. Let $\zeta_\beth=\{\mathcal{O}_{\zeta_\beth},\mathcal{G}_{\zeta_\beth}\}$ be the collection of IHFNs, where $(\beth=1,2,\ldots,\ell)$. An IHF Acz-Aln hybrid weighted geometric (IHFAHWG) Agop of dimention ℓ is a relation $\mathcal{P}^\ell\longrightarrow\mathcal{P}$ with a weight vector $\sigma=(\sigma_1,\sigma_2,\ldots,\sigma_\ell)^T$ such that $\sigma_\beth>0$ and $\oplus_{\beth=1}^{\ell}\sigma_\beth=1$ as follows:

$$\mathrm{IHFA}\daleth WG(\zeta_1,\zeta_2,\ldots,\zeta_\ell)=\prod_{\beth=1}^{\ell}\left(\zeta_{\tau(\beth)}^*\right)^{\Psi_\beth}, \tag{33}$$

where $\Psi=(\Psi_1,\Psi_2,\ldots,\Psi_\ell)^T$ are the associated weights such that $\Psi_\beth>0$ and $\oplus_{\beth=1}^{\ell}\Psi_\beth=1$; also, $\zeta_{\tau(\beth)}^*=(\zeta_{\tau(\beth)}^*=n\sigma_\beth\zeta_{\tau(\beth)})$ $(\beth=1,2,\ldots,\ell)$ and $(\tau(1),\tau(2),\ldots,\tau(\ell))$ are the permutation in such a way as $\zeta_{\tau(\beth)}^*\leq\zeta_{\tau(\beth-1)}^*$.

Theorem 8. *Suppose* $\zeta_\beth=\{\mathcal{O}_{\zeta_\beth},\mathcal{G}_{\zeta_\beth}\}$ *be the collection of IHFNs, where* $(\beth=1,2,\ldots,\ell)$. *An IHF Acz-Aln hybrid weighted geometric (IHFAHWG) Agop of dimension* ℓ *is a relation* $\mathcal{P}^\ell\longrightarrow\mathcal{P}$ *with a weight vector* $\sigma=(\sigma_1,\sigma_1,\ldots,\sigma_\ell)^T$ *such that* $\sigma_\beth>0$ *and* $\oplus_{\beth=1}^{\ell}\sigma_\beth=1$ *is defined as follows:*

$$\mathrm{IHFA}\daleth WG(\zeta_1,\zeta_2,\ldots,\zeta_\ell)=\prod_{\beth=1}^{\ell}\left(\zeta_{\tau(\beth)}^*\right)^{\Psi_\beth}=\bigcup_{\left(\alpha_{\zeta_\beth},\beta_{\zeta_\beth}\right)\in\left(\mathcal{O}_{\zeta_\beth},\mathcal{G}_{\zeta_\beth}\right)(\beth=1\ldots\ell)}\left\{\begin{array}{c} e^{-\left(\oplus_{\beth=1}^{\ell}\Psi_\beth\left(-\ell n\alpha_{\zeta_{\tau(\beth)}^*}\right)^{\rho}\right)^{1/\rho}},\\ 1-e^{-\left(\oplus_{\beth=1}^{\ell}\Psi_\beth\left(-\ell n\left(1-\beta_{\zeta_{\tau(\beth)}^*}\right)\right)^{\rho}\right)^{1/\rho}}\end{array}\right\}, \tag{34}$$

where $\Psi=(\Psi_1,\Psi_2,\ldots,\Psi_\ell)^T$ *are the associated weights such that* $\Psi_\beth>0$ *and* $\bigoplus_{\beth=1}^{\ell}\Psi_\beth=1$; *also,* $\zeta_{\tau(\beth)}^*=(\zeta_{\tau(\beth)}^*=n\sigma_\beth\zeta_{\tau(\beth)})$ $(\beth=1,2,\ldots,\ell)$ *and* $(\tau(1),\tau(2),\ldots,\tau(\ell))$ *are the permutation in such a way as* $\zeta_{\tau(\beth)}^*\leq\zeta_{\tau(\beth-1)}^*$.

We may show the following characteristics properly by employing the IHFAHWG operator.

Theorem 9

(1) *(Idempotency) Let* $\zeta_\beth=\{\mathcal{O}_{\zeta_\beth},\mathcal{G}_{\zeta_\beth}\}(\beth=1,2,\ldots,\ell)$ *be the collection of equivalent IHFNs, i.e.,* $\zeta_\beth=\zeta$ *for each* $(\beth=1,2,\ldots,\ell)$. *Then,*

$$\mathrm{IHFA}\daleth WG(\zeta_1,\zeta_2,\ldots,\zeta_\ell)=\zeta. \tag{35}$$

(2) *(Boundedness)* Let $\zeta_{\beth} = \{\mathcal{O}_{\zeta_{\beth}}, \mathcal{G}_{\zeta_{\beth}}\}\,(\beth = 1, 2, \ldots, \ell)$ be the collection of IHFNs. Let $\zeta_{\beth}^{-} = (\min_{\beth}\{\mathcal{O}_{\zeta_{\beth}}\}, \max_{\beth}\{\mathcal{G}_{\zeta_{\beth}}\})$ and

$$\zeta_{\beth}^{+} = \left(\max_{\beth}\{\mathcal{O}_{\zeta_{\beth}}\}, \min_{\beth}\{\mathcal{G}_{\zeta_{\beth}}\}\right). \tag{36}$$

$(\beth = 1, 2, \ldots, \ell)$. Then,

$$\zeta_{\beth}^{-} \leq \text{IHFA}\daleth WG(\zeta_1, \zeta_2, \ldots, \zeta_{\ell}) \leq \zeta_{\beth}^{+}. \tag{37}$$

(3) Let $\zeta_{\beth} = \{\mathcal{O}_{\zeta_{\beth}}, \mathcal{G}_{\zeta_{\beth}}\}$ and $\zeta_{\beth}^{*} = \{\mathcal{O}_{\zeta_{\beth}}^{*}, \mathcal{G}_{\zeta_{\beth}}^{*}\}$ $(\beth = 1, 2, \ldots, \ell)$ be the two collection of IHFNs, if $\zeta_{\beth} \leq \zeta_{\beth}^{*}$ for $(\beth = 1, 2, \ldots, \ell)$. Then,

$$\text{IHFA}\daleth WG(\zeta_1, \zeta_2, \ldots, \zeta_{\ell}) \leq \text{IHFA}\daleth WG(\zeta_1^{*}, \zeta_2^{*}, \ldots, \zeta_{\ell}^{*}). \tag{38}$$

Proof. By using Theorem 3, 4, and 5, prove of this theorem is similarly done. □

5. Decision Support Algorithm

In order to validate the utility of the IHF Acz-Aln geometric Agops in this study, a new MCGDM approach is developed to deal with complex ambiguous data in real-life decision support difficulties. The following are the particular measures to take.

Assume that "set of ℓ alternatives $\{\mathfrak{I}_1, \mathfrak{I}_2, \ldots, \mathfrak{I}_{\ell}\}$, and adequately evaluate by a set of attributes $\{\mathcal{R}_1, \mathcal{R}_2, \ldots, \mathcal{R}_m\}$. Then, the impotence of attributes $\mathcal{R}_{\imath}\,(\imath = 1, 2, \ldots, m)$ is specified by a weight vector $\sigma = (\sigma_1, \sigma_1, \ldots, \sigma_m)^T$ such that $\sigma_{\imath} > 0$ and $\sum_{\imath=1}^{m} \sigma_{\imath} = 1$.

Let $\zeta_{\beth\imath} = \{\mathcal{O}_{\zeta_{\beth\imath}}, \mathcal{G}_{\zeta_{\beth\imath}}\}$ for $\mathcal{O}_{\zeta_{\beth\imath}}, \mathcal{G}_{\zeta_{\beth\imath}} \in [0, 1]$ be the satisfactory assessment of each attribute for each alternative, where $\mathcal{O}_{\zeta_{\ell m}}$ indicates the +ve grade function that the alternative $\mathfrak{I}_{\beth}$ $(\beth = 1, 2, \ldots, \ell)$ satisfies $\mathcal{R}_{\imath}\,(\imath = 1, 2, \ldots, m)$ and $\mathcal{G}_{\zeta_{\ell m}}$ indicates the neutral grade function and the −ve grade function sequentially. The decision matrix of IHFNs can be calculated using all evaluation values as follows: $\zeta = (\zeta_{\beth\imath})_{\ell m}$.

The created IHF Acz-Aln geometric operators were used to resolve the MCDM problem in this research, and the process for sorting the best raking from alternatives is as follows:

Step 1: identify a set of characteristics that are relevant for the evaluation problem being solved.

A literature review is conducted to acquire potential evaluation qualities, which are then filtered by a technical committee to produce a suitable list of evaluation attributes.

$$\mathcal{R}_{\imath}\,(\imath = 1, 2, \ldots, m),$$

$$D_{\beth\times\imath} = \begin{matrix} \\ \mathfrak{I}_1 \\ \mathfrak{I}_2 \\ \vdots \\ \mathfrak{I}_{\ell} \end{matrix} \begin{pmatrix} \mathcal{R}_1 & \mathcal{R}_2 & & \mathcal{R}_m \\ (\mathcal{O}_{\zeta_{11}}, \mathcal{G}_{\zeta_{11}}) & (\mathcal{O}_{\zeta_{12}}, \mathcal{G}_{\zeta_{12}}) & \ldots & (\mathcal{O}_{\zeta_{1m}}, \mathcal{G}_{\zeta_{1m}}) \\ (\mathcal{O}_{\zeta_{21}}, \mathcal{G}_{\zeta_{21}}) & (\mathcal{O}_{\zeta_{22}}, \mathcal{G}_{\zeta_{22}}) & \ldots & (\mathcal{O}_{\zeta_{2m}}, \mathcal{G}_{\zeta_{2m}}) \\ \vdots & \vdots & \ddots & \vdots \\ (\mathcal{O}_{\zeta_{\ell 1}}, \mathcal{G}_{\zeta_{\ell 1}}) & (\mathcal{O}_{\zeta_{\ell 2}}, \mathcal{G}_{\zeta_{\ell 2}}) & \ldots & (\mathcal{O}_{\zeta_{\ell m}}, \mathcal{G}_{\zeta_{\ell m}}) \end{pmatrix}. \tag{39}$$

Step 2: normalization yields the normalized decision matrix as follows:

$$\mathcal{N}_{\beth\times\imath} = \begin{cases} (\mathcal{O}_{\zeta_{\beth\imath}}, \mathcal{G}_{\zeta_{\beth\imath}}) & if \quad C_I, \\ (\mathcal{G}_{\zeta_{\beth\imath}}, \mathcal{O}_{\zeta_{\beth\imath}}) & if \quad C_{II}, \end{cases} \tag{40}$$

where C_I refers to "if $\mathcal{R}_{\imath}\,(\imath = 1, 2, \ldots, m)$ is a benefit criterion" and C_{II} refers to "if $\mathcal{R}_{\imath}\,(\imath = 1, 2, \ldots, m)$ is a cost criterion".

Step 3: collected specialist uncertain data: IHFWA/IHFWG to combine the specialist uncertain data in decision support issues, aggregation operators are used.

Step 4: illustrate the significance (weighting) of the attributes to be considered $\{\mathcal{R}_1, \mathcal{R}_2, \ldots, \mathcal{R}_m\}$ using IHF entropy measure as follows:

$$\kappa_{\imath} = \bigcup_{(\alpha_{\zeta_{\beth}}, \beta_{\zeta_{\beth}}) \in (\mathcal{O}_{\zeta_{\beth}}, \mathcal{G}_{\zeta_{\beth}})} \left\{ \frac{1 + (1/\ell)\sum_{\beth=1}^{\ell}\left(\alpha_{\zeta_{\beth\imath}}\log(\alpha_{\zeta_{\beth\imath}}) + \beta_{\zeta_{\beth\imath}}\log(\beta_{\zeta_{\beth\imath}})\right)}{\sum_{\imath=1}^{m}\left(1 + (1/\ell)\sum_{\beth=1}^{\ell}\alpha_{\zeta_{\beth\imath}}\log(\alpha_{\zeta_{\beth\imath}}) + \beta_{\zeta_{\beth\imath}}\log(\beta_{\zeta_{\beth\imath}})\right)} \right\}. \tag{41}$$

Step 5: aggregated data: developed IHF Acz-Aln geometric operators exploited to aggregate the specialist uncertain data of decision support problems.

Step 5(a): exploited IHFAWG operator to amalgamate aggregated data.

Step 5(b): exploited IHFAOWG operator to amalgamate aggregated data.

Step 5(c): exploited IHFAHWG operator to amalgamate aggregated data.

Step 6: according to the score function in Definition 10, the score values of $\mathfrak{I}_{\beth}$ ($\beth = 1, 2, \ldots, \ell$) are derived."

Step 7: all options are sorted in highest to the lowest depending on the scores, and the best one with the highest score value is chosen.

6. Numerical Representation of the Suggested Method

We approach an uncertain challenge of assessing the quality of healthcare care in verifying the usefulness and adaptability of the created strategy to the MADM challenge.

6.1. Case Study: Evaluating the Quality of Medical Services. The assessment framework for evaluating the quality of medical services developed in this article contains four parameters. The following provides an explanation of the alternatives for evaluating the quality of medical services:

The focus of this research is nearly entirely on evaluating the medical care process at the level of physician-patient interaction. As a result, it excludes processes that are primarily concerned with the successful delivery of medical care in the community. Furthermore, the administrative components of quality control are not addressed in this study. Many of the studies discussed here arose from the urgent requirement to assess and control the quality of care in organised medical care systems. Nonetheless, these studies will be discussed solely in terms of their contribution to evaluation procedures rather than their broader social objectives. Specific construction standards have a significant impact on health maintenance and enhancement, as well as the safety and comfort of people in health care facilities. Health-care institutions are unusual structures in which several habitats coexist. A wide variety of people, activities in each setting, and risk factors are all implicated in the pathogenesis of a wide range of diseases. Nursing units, operating theatres, diagnostic facilities (radiology units, laboratory units, and so on), outpatient departments, administration areas (offices), dietary facilities, linen services, engineering services and equipment areas, corridors, and passages are all classified using functional organisation criteria. Health professionals, staff members, patients (long-stay inpatients, acute inpatients, and outpatients), and visitors make up the hospital population. The processes include activities particular to health care—diagnostic, therapeutic, and nursing tasks—as well as activities common to many public facilities, such as office work, technological maintenance, and food preparation. Physical agents (ionising and nonionising radiation, noise, lighting, and microclimatic factors), chemicals (e.g., organic solvents and disinfectants), biological agents (viruses, bacteria, and fungi), ergonomics (postures, lifting, and so on), and psychological and organisational factors are among the risk factors (e.g., environmental perceptions and work hours). Four general topics on quality assessment were quite useful in the preparation of this study.

(1) Hospital environment ($\mathscr{R}_1$): the patients and the hospital are inextricably linked. Patients visit hospital for treatment, and they are happy with the clean environment produced by the providers. The hospital environment is influenced by a variety of elements, including travel accessibility, ward space, ward's quietness, patient and medication care, and environmental characteristics that restrict or improve communications. When a patient's surrounding is exceedingly loud, he or she may have difficulty sleeping or being comfortable, impairing their ability to heal. The patients like the clean and secure atmosphere provided by the medical service provider. Environmental services (ESs), often known as housekeeping or cleaning services, is a phrase used specifically in healthcare to guide the act of cleaning and disinfecting medical devices, patient rooms, and other public area inside healthcare facilities by highly trained support service workers. ES assist reduce HAIs and prevent diseases from spreading.

(2) Medical procedures ($\mathscr{R}_2$): a medical procedure is a sequence of actions performed in the provision of healthcare to accomplish a purpose. A medical assessment is conducted to ascertain, quantify, or diagnose a patient's condition or parameter. The medical procedure is impacted by a number of factors, including the availability of the medical care, the information collected during the assessment, the diagnostic process, and continued health guidance. Due to the sheer risk of becoming infected spreading and/or resource constraints (both apparatus and manpower), the acceptability and capability of conducting abortion interventions and surveillance throughout a pandemic influenza outbreak are likely to be altered. To improve outcomes in critically sick patients treated in and outside intensive care units (ICUs, careful planning for operation execution and monitoring will be required. Although there are little data to support conclusive recommendations, the following standard operating procedure (SOP) is based on the understanding of influenza features, experience from prior respiratory epidemics (such as SARS), specialist consensus, and a safety-first attitude. In order to properly undertake monitoring and interventions in patients with and without influenza sickness during an epidemic, ensure that suitable resources are accessible and appropriate protocols are implemented.

(3) Service attitude ($\mathscr{R}_3$): a hospital is a platform where families go to get their health problems fixed and receive treatment. Due to their trauma, the patients are already in a condition of anguish and worry. The patients want to be treated with respect, cared for, and reassured. They require a promise of hope that their condition will improve as a result of treatment at the aforementioned facility. They must receive a favourable attitude from the hospital's healthcare specialists in order to find consolation. Any profession benefits from having a positive attitude since it provides it an edge. However, in the healthcare field, attitude is extremely important, with a positive attitude being a requirement. Yes, when in the hospital, all healthcare staff, from the most senior surgeons and physicians to the most junior ward boy, must maintain a happy demeanour and a caring demeanour. This is due to the fact that the attitude of healthcare personnel in a hospital has an impact on the hospital's relationship with its patients. Patient relationship management is essential for any hospital's performance, as delighted patients result in increased referrals and repeat business. Patient happiness, on the other hand, is more dependent on the attitude of healthcare workers, which is what healthcare HR management focuses on in order to achieve hospital efficiency. Professional healthcare consulting organisations provide positive attitude training to hospital workers in order to assist them maintain a friendly and caring attitude toward patients. This boosts income and improves the hospital's reputation by increasing patient happiness and loyalty. Having a superb customer service attitude entails being aware of client expectations, going above and beyond, and acting as a customer supporter. Expressing helpfulness, sincere passion, and respect has an effect on customer behaviour, converting uninterested customers to loyal ones. The administration's attitude plays a vital role in the process of delivering medical services. A multitude of factors impact service attitude, including patience in answering queries, prevarication in responding, prejudice in service, and efficiency in achieving findings.

(4) Medical expenses ($\mathscr{R}_4$): medical expenditures incurred during the provision of medical services are crucial and must be within the patients' acceptable financial reach. Medication costs, medical examination charges, fee transparency, prescription prices, hospital bed costs, expenditure correspondence, charge precision, and food costs are all encompassed in the context of healthcare expenses. The participants in our full medical insurance plan (FMIP) and the supplementary medical insurance plan (SMIP) have lately noticed that their donations have increased in order to avoid a predicted increase in insurance premiums beginning in January 1998. Apart from paying the higher payments, the participants can take steps to help keep future premium increases to a minimum. In today's competitive medical-care industry, medical product and service consumers have a variety of options when it comes to medical spending. A group insurance plan's participants create a risk community, and in such a community, each individual's consumption patterns influence the overall plan's performance and thus directly affect the amount of premiums paid to the insurer. As a result, we would want to encourage all members of our medical insurance plan to use the same caution and discipline while incurring medical expenses as they would when making a large purchase or repairing a car. In such instances, one would undoubtedly take the time to shop around for the best deal before spending the money. The same is true for medical care, where a customer can learn about his or her options, get cost estimates, and/or "shop around" for the best value for money before making a decision. While it is true that medical treatment, more than other products/services, incorporates nonquantifiable factors such as trust, it is nevertheless crucial to be aware that we have rights and options as patients. It is also critical to send a message to medical providers that their charges and rates will be scrutinised. The mindset that "the insurer pays regardless" is a certain way to boost premiums. As a result, it is critical that we take responsibility for our collective medical insurance plan and exercise extreme caution when it comes to medical spending.

Assume that the set of hospitals is $\mathfrak{I} = \{\mathfrak{I}_1, \mathfrak{I}_2, \mathfrak{I}_3, \mathfrak{I}_4\}$ and $\{\mathscr{R}_1, \mathscr{R}_2, \mathscr{R}_3, \mathscr{R}_4\}$ be a set of criteria for assessing the quality of medical services specified by a weight vector $\sigma = (0.1, 0.3, 0.5, 0.1)^T$. The following are the various steps involved in developing a novel scheme for evaluating the quality of medical services on the basis of suggested operators:

Step 1: the specialist matrices of IHFNs is enclosed in Table 1

Step 2: Table 2 evaluates the normalized decision matrices after normalisation

Step 5(a): employed IHFAWG operator to integrate aggregated data enclosed in Table 3 as follows:

$$\mathrm{IHFAWG}(\zeta_1, \zeta_2, \ldots, \zeta_\ell) = \bigcup_{\left(\alpha_{\zeta_\beth}, \beta_{\zeta_\beth}\right) \in \left(\mathscr{O}_{\zeta_\beth}, \mathscr{G}_{\zeta_\beth}\right)} \left\{ \begin{array}{c} e^{-\left(\oplus_{\beth=1}^{\ell} \sigma_\beth \left(-\ell n \alpha_{\zeta_\beth}\right)^{\rho}\right)^{1/\rho}} \\ 1 - e^{-\left(\oplus_{\beth=1}^{\ell} \sigma_\beth \left(-\ell n \left(1-\beta_{\zeta_\beth}\right)\right)^{\rho}\right)^{1/\rho}} \end{array} \right\}. \quad (42)$$

Table 1: Specialist matrix 1 of IHFNs.

	$\mathscr{R}_1$	$\mathscr{R}_2$	$\mathscr{R}_3$	$\mathscr{R}_4$
$\mathfrak{T}_1$	$\left\{\begin{array}{c}(0.2,0.3),\\(0.4,0.5)\end{array}\right\}$	{(0.1, 0.4)}	{(0.2, 0.7)}	{(0.3, 0.5)}
$\mathfrak{T}_2$	{(0.3, 0.4)}	$\left\{\begin{array}{c}(0.3,0.6),\\(0.1,0.2)\end{array}\right\}$	{(0.2, 0.4)}	{(0.3, 0.6)}
$\mathfrak{T}_3$	{(0.2, 0.5)}	{(0.1, 0.5)}	$\left\{\begin{array}{c}(0.2,0.5),\\(0.2,0.3)\end{array}\right\}$	{(0.1, 0.7)}
$\mathfrak{T}_4$	{(0.1, 0.6)}	{(0.2, 0.4)}	{(0.2, 0.3)}	$\left\{\begin{array}{c}(0.1,0.8),\\(0.3,0.4)\end{array}\right\}$

Table 2: Normalized specialist matrix of IHFNs.

	$\mathscr{R}_1$	$\mathscr{R}_2$	$\mathscr{R}_3$	$\mathscr{R}_4$
$\mathfrak{T}_1$	$\left\{\begin{array}{c}(0.3,0.2),\\(0.5,0.4)\end{array}\right\}$	{(0.4, 0.1)}	{(0.7, 0.2)}	{(0.5, 0.3)}
$\mathfrak{T}_2$	{(0.4, 0.3) }	$\left\{\begin{array}{c}(0.6,0.3),\\(0.2,0.1)\end{array}\right\}$	{(0.4, 0.2)}	(0.6, 0.3)
$\mathfrak{T}_3$	{(0.5, 0.2)}	(0.5, 0.1)	$\left\{\begin{array}{c}(0.5,0.2),\\(0.3,0.2)\end{array}\right\}$	(0.7, 0.1)
$\mathfrak{T}_4$	{(0.6, 0.1)}	(0.4, 0.2)	(0.3, 0.2)	$\left\{\begin{array}{c}(0.8,0.1),\\(0.4,0.3)\end{array}\right\}$

Table 3: Intuitionistic hesitant fuzzy aggregated data (IHFAWG).

$\mathfrak{T}_1$	{(0.4618, 0.2023), (0.5045, 0.2499)}
$\mathfrak{T}_2$	{(0.4487, 0.2625), (0.3015, 0.2185)}
$\mathfrak{T}_3$	{(0.5104, 0.1751), (0.3664, 0.1751)}
$\mathfrak{T}_4$	{(0.3542, 0.1485), (0.2653, 0.1850)}

Table 4: Scores of the normalized data.

	$\mathscr{R}_1$	$\mathscr{R}_2$	$\mathscr{R}_3$	$\mathscr{R}_4$
$\mathfrak{T}_1$	0.1	0.3	0.5	0.2
$\mathfrak{T}_2$	0.1	0.7	0.6	0.3
$\mathfrak{T}_3$	0.3	0.4	0.2	0.6
$\mathfrak{T}_4$	0.5	0.2	0.1	0.4

Step 5(b): employed IHFAOWG operator to integrate aggregated data enclosed in Tables 4–6

Arranged row wise normalized matrix

Step 5(c): employed IHFAHWG operator to integrate aggregated data enclosed in Tables 7–10

Step 6: observing the score function in Definition 10, the score values of $\mathfrak{T}_\daleth$ ($\daleth = 1, 2, 3, 4$) are enclosed in Table 11:

Step 7: under all the suggested Acz-Aln operators, $\mathfrak{T}_3$ has the largest score value; therefore, $\mathfrak{T}_3$ is our finest choice in terms of giving attributes.

Table 5: Updated normalized specialist matrix of IHFNs.

	$\mathscr{R}_1$	$\mathscr{R}_2$	$\mathscr{R}_3$	$\mathscr{R}_4$
$\mathfrak{T}_1$	{(0.7, 0.2)}	{(0.4, 0.1)}	{(0.5, 0.3)}	{((0.3, 0.2), (0.5, 0.4))}
$\mathfrak{T}_2$	{((0.6, 0.3), (0.2, 0.1))}	{(0.4, 0.2)}	{(0.6, 0.3)}	{(0.4, 0.3)}
$\mathfrak{T}_3$	{(0.7, 0.1)}	{(0.5, 0.1)}	{(0.5, 0.2)}	{((0.5, 0.2), (0.3, 0.2))}
$\mathfrak{T}_4$	{(0.6, 0.1)}	{((0.8, 0.1), (0.4, 0.3))}	{(0.4, 0.2)}	{(0.3, 0.2)}

Table 6: Intuitionistic hesitant fuzzy aggregated (IHFAOWG) data.

$\mathfrak{T}_1$	{(0.4351, 0.2543), (0.4687, 0.2844)}
$\mathfrak{T}_2$	{(0.4823, 0.2792), (0.3966, 0.2688)}
$\mathfrak{T}_3$	{(0.5104, 0.1751), (0.4659, 0.1751)}
$\mathfrak{T}_4$	{(0.4352, 0.1751), (0.3947, 0.2391)}

7. Comparison Analysis

To show the applicability and appropriateness of the established operators-based methodology, we compared the approach based on the suggested operator in accordance with existing decision assistance approaches to validate the advantages and stability of the intuitionistic hesitant Acz-Aln Agops.

Table 7: Hybrid data sets.

	$\mathscr{R}_1$	$\mathscr{R}_2$	$\mathscr{R}_3$	$\mathscr{R}_4$
$\mathfrak{I}_1$	$\left\{\begin{array}{c}(0.5719, 0.0983),\\ (0.7249, 0.2111)\end{array}\right\}$	$\{0.5415, 0.0681\}$	$\{0.7534, 0.1623\}$	$\{0.7249, 0.1526\}$
$\mathfrak{I}_2$	$\{0.6536, 0.1526\}$	$\left\{\begin{array}{c}(0.7103, 0.2124),\\ (0.3406, 0.0681)\end{array}\right\}$	$\{0.4833, 0.1623\}$	$\{0.7889, 0.1526\}$
$\mathfrak{I}_3$	$\{0.7249, 0.0983\}$	$\{0.6288, 0.0681\}$	$\left\{\begin{array}{c}(0.5769, 0.1623),\\ (0.3846, 0.1623)\end{array}\right\}$	$\{0.8474, 0.0477\}$
$\mathfrak{I}_4$	$\{0.7889, 0.0477\}$	$\{0.5415, 0.2897\}$	$\{0.3846, 0.1623\}$	$\left\{\begin{array}{c}(0.9017, 0.0477),\\ (0.6536, 0.1526)\end{array}\right\}$

Table 8: Scores of the hybrid data set.

	$\mathscr{R}_1$	$\mathscr{R}_2$	$\mathscr{R}_3$	$\mathscr{R}_4$
$\mathfrak{I}_1$	0.4937	0.4734	0.5911	0.5723
$\mathfrak{I}_2$	0.501	0.3852	0.321	0.6363
$\mathfrak{I}_3$	0.6266	0.5607	0.3184	0.7997
$\mathfrak{I}_4$	0.7412	0.2518	0.2223	0.6775

Table 9: Updated normalized specialist matrix of IHAHFNs.

	$\mathscr{R}_1$	$\mathscr{R}_2$	$\mathscr{R}_3$	$\mathscr{R}_4$
$\mathfrak{I}_1$	{(0.7, 0.2)}	{(0.5, 0.3)}	{((0.3, 0.2), (0.5, 0.4))}	{(0.4, 0.1)}
$\mathfrak{I}_2$	{(0.6, 0.3)}	{(0.4, 0.3)}	{((0.6, 0.3), (0.2, 0.1))}	{(0.4, 0.2)}
$\mathfrak{I}_3$	{(0.7, 0.1)}	{(0.5, 0.2)}	{(0.5, 0.1)}	{((0.5, 0.2), (0.3, 0.2))}
$\mathfrak{I}_4$	{(0.6, 0.1)}	{((0.8, 0.1), (0.4, 0.3))}	{(0.4, 0.2)}	{(0.3, 0.2)}

Table 10: Intuitionistic hesitant fuzzy aggregated (IHFAHWG) data.

$\mathfrak{I}_1$	{(0.3613, 0.2391), (0.4949, 0.3518)}
$\mathfrak{I}_2$	{(0.4823, 0.2935), (0.2618, 0.2372)}
$\mathfrak{I}_3$	{(0.5104, 0.1585), (0.4659, 0.1585)}
$\mathfrak{I}_4$	{(0.4352, 0.1751), (0.3947, 0.2391)}

Table 11: Score and ranking of IHFNs.

Operators	Scores				Rankings
	$\Theta(\mathfrak{I}_1)$	$\Theta(\mathfrak{I}_2)$	$\Theta(\mathfrak{I}_3)$	$\Theta(\mathfrak{I}_4)$	
IHFAWG	0.2570	0.1346	0.2633	0.143	$\mathfrak{I}_3 > \mathfrak{I}_1 > \mathfrak{I}_4 > \mathfrak{I}_2$
IHFAOWG	0.1825	0.1654	0.3130	0.2078	$\mathfrak{I}_3 > \mathfrak{I}_4 > \mathfrak{I}_1 > \mathfrak{I}_2$
IHFA$\gimel WG$	0.1326	0.1067	0.3296	0.2078	$\mathfrak{I}_3 > \mathfrak{I}_4 > \mathfrak{I}_1 > \mathfrak{I}_2$

Peng et al. [46] evolved novel intuitionistic hesitant fuzzy weighted Agops to sort out the best alternative. Table 12 gives the comparison findings and shows that the IHFWG operator is a special instance of our developed IHFAWG operator, and it acquires when $\rho = 3$ and $w = (0.15, 0.15, 0.375, 0.325)$.

As a nutshell, our suggested solutions are expected to become more extensive and flexible than a few existing strategies for addressing IHF MADM problems. The collected specialist data [46] are listed in Table 13:

Comparative studies with collected specialist data by [22] is enclosed in Table 14 as follows:

7.1. Advantages. The benefits are summarised as follows:

(i) A multicriteria framework with a different approach to generating IF numbers is the IHF Acz-Aln method.

(ii) By using the inverse sorting algorithm, the IHF Acz-Aln methodology standardises the home matrix elements, which permits I retention of the placement of normalized values on the measuring scale and the absence of domain shift or a distortion of the source data.

(iii) The display of expert preferences is substantially facilitated by the IHF Acz-Aln algorithm, which is

Table 12: Collected specialist data under PHFNs.

$\mathfrak{I}_1$	$\left\{\begin{pmatrix} 0.3948, 0.4099, 0.4118, 0.3999, 0.4157, 0.4177, \\ 0.3976, 0.4130, 0.4150, 0.4028, 0.4190, 0.4211, \\ 0.5656, 0.6372, 0.6495, 0.5864, 0.6781, 0.6963, \\ 0.5767, 0.6582, 0.6731, 0.5997, 0.7099, 0.7353 \end{pmatrix}, \\ (0.1915, 0.2459, 0.2000, 0.2505)\right\}$
$\mathfrak{I}_2$	$\left\{\begin{pmatrix} 0.3332, 0.3355, 0.3724, 0.3761, 0.3721, 0.3758, \\ 0.4302, 0.4360, 0.3922, 0.3965, 0.4637, 0.4714, \\ 0.3335, 0.3362, 0.3731, 0.3768, 0.3727, 0.3764, \\ 0.4312, 0.4371, 0.3930, 0.3974, 0.4650, 0.4728 \end{pmatrix}, \\ (0.4287, 0.4371, 0.5021, 0.5069, 0.4296, 0.4379, 0.5026, 0.5073)\right\}$
$\mathfrak{I}_3$	$\left\{\begin{pmatrix} 0.4793, 0.4802, 0.4831, 0.4841, 0.5002, 0.5013, \\ 0.5047, 0.5059, 0.5111, 0.5123, 0.5160, 0.5173, \\ 0.5651, 0.5669, 0.5727, 0.5746, 0.6102, 0.6129, \\ 0.6214, 0.6244, 0.6382, 0.6417, 0.6526, 0.6565 \end{pmatrix}, \\ (0.2609, 0.2704)\right\}$
$\mathfrak{I}_4$	$\left\{\begin{pmatrix} 0.5521, 0.5581, 0.5990, 0.6077, 0.6270, 0.6382, \\ 0.5688, 0.5756, 0.6242, 0.6351, 0.6599, 0.6752 \end{pmatrix}, \\ (0.2560, 0.2647, 0.2877, 0.2942, 0.2696, 0.2773, 0.2978, 0.3037)\right\}$

Table 13: Specialist evaluation information.

	$\mathscr{R}_1$	$\mathscr{R}_2$	$\mathscr{R}_3$	$\mathscr{R}_4$
$\mathfrak{I}_1$	$\{\{0.2, 0.7\}, \{0.2\}\}$	$\{\{0.6, 0.8\}, \{0.1, 0.2\}\}$	$\{\{0.6, 0.7\}, \{0.2, 0.3\}\}$	$\{\{0.5, 0.7, 0.8\}, \{0.2\}\}$
$\mathfrak{I}_2$	$\{\{0.7, 0.8\}, \{0.1, 0.2\}\}$	$\{\{0.2, 0.3, 0.4\}, \{0.5\}\}$	$\{\{0.3, 0.4\}, \{0.5, 0.6\}\}$	$\{\{0.6, 0.7\}, \{0.1, 0.3\}\}$
$\mathfrak{I}_3$	$\{\{0.3, 0.5\}, \{0.4\}\}$	$\{\{0.4, 0.5, 0.6\}, \{0.3\}\}$	$\{\{0.7, 0.8\}, \{0.1, 0.2\}\}$	$\{\{0.8, 0.9\}, \{0.1\}\}$
$\mathfrak{I}_4$	$\{\{0.5, 0.6\}, \{0.2, 0.3\}\}$	$\{\{0.6\}, \{0.3, 0.4\}\}$	$\{\{0.5, 0.6, 0.7\}, \{0.3\}\}$	$\{\{0.7, 0.8\}, \{0.1, 0.2\},\}$

Table 14: Score and ranking of IHFNs for comparative studies.

	Scores				Rankings
	$\Theta(\mathfrak{I}_1)$	$\Theta(\mathfrak{I}_2)$	$\Theta(\mathfrak{I}_3)$	$\Theta(\mathfrak{I}_4)$	
Wang and Liu [22]	0.7907	0.5803	0.8155	0.7354	$\mathfrak{I}_3 > \mathfrak{I}_1 > \mathfrak{I}_4 > \mathfrak{I}_2$
IHFAWG (proposed)	0.2795	0.08805	0.2898	0.278	$\mathfrak{I}_3 > \mathfrak{I}_1 > \mathfrak{I}_4 > \mathfrak{I}_2$

based on the ranking of criteria based on IHF values. Also, this fixes the issue with some multicriteria techniques' limited range for predefined scales for comparing criteria.

(iv) The suggested mathematical model serves two purposes. It can be used to define the weighting factors for attributes and alternatives as well as to prioritise.

(v) Both individual and group decision making can be done using the proposed multicriteria framework.

8. Conclusion

Every day, we work with complex and cutting-edge data. We created approaches and tools for this kind of data in order to operate more efficiently and compute thorough information. The cost of condensing a large volume of data into a single value is a fundamental cost of aggregation. The IHFS was developed as a very effective combination of an IFS and HFS for scenarios where each item has a range of possible values determined by MD and non-MD.

We go into further detail about these strategies' benefits as follows:

(i) First, it discusses the implementation of the IHFAWG, IHFAOWG, and IHFAHWG operators, which are capable of capturing the interrelationships between all attributes and highlighting possible factors.

(ii) Second, it discusses various characteristics of the IHFAWG, IHFAOWG, and IHFAHWG operators and also investigated some particular cases of the abovementioned operators and discussed their relationships between those operators in more detail.

(iii) Third, it develops a unique method for evaluating the quality of medical services via the use of a criterion framework and a scheme. We use this novel methodology to assess four hospitals in order to show the method's availability and conduct a comparison study to establish the method's efficacy and superiority.

(iv) The finding is that our strategy has a lot of benefits and trustworthy in IHF data DM. In other words, we may have easily described fuzzy information and make the information aggregation method more transparent than certain existing systems by supplying a parameter.

(v) The existing Agops [46] do not make data aggregate more versatile. As a consequence, our suggested aggregation operations in IHF data DM are more comprehensive and reliable.

(vi) The concept of Acz-Aln Agops may be extended to other fuzzy environments for assessing the quality of medical services in the future. Additionally, group decision making [47, 48] might be explored to strengthen the assessment result's robustness.

(vii) In our next work, we will investigate the theoretical foundations of IHFS' Acz-Aln operations utilising cutting-edge decision-making techniques including TOPSIS, VIKOR, TODAM, GRA, and EDAS. We will also discuss how these methods are applied in a variety of fields, such as soft computing, robotics, horticulture, intelligent systems, social sciences, finance, and human resource management.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All authors have contributed equally to this article.

References

[1] R. X. Qin, L. Velin, E. F. Yates et al., "Building sustainable and resilient surgical systems: a narrative review of opportunities to integrate climate change into national surgical planning in the Western Pacific region," *The Lancet Regional Health-Western Pacific*, vol. 22, Article ID 100407, 2022.

[2] A. Erjaee, S. Hendiani, S. Moradi, and M. Bagherpour, "Healthcare sustainability evaluation using a hybrid fuzzy multi-criteria decision-making model," *International Journal of Fuzzy Systems*, vol. 24, no. 2, pp. 1182–1202, 2022.

[3] R. C. Barclay, E. A. Cudney, S. Shetty, and J. Antony, "Determining critical success factors for lean implementation," *Total Quality Management and Business Excellence*, vol. 33, no. 7-8, pp. 818–832, 2022.

[4] D. J. McDonough, M. A. Helgeson, W. Liu, and Z. Gao, "Effects of a remote, YouTube-delivered exercise intervention on young adults' physical activity, sedentary behavior, and sleep during the COVID-19 pandemic: randomized controlled trial," *Journal of Sport and Health Science*, vol. 11, no. 2, pp. 145–156, 2022.

[5] L. Fei, J. Lu, and Y. Feng, "An extended best-worst multi-criteria decision-making method by belief functions and its applications in hospital service evaluation," *Computers & Industrial Engineering*, vol. 142, Article ID 106355, 2020.

[6] M. Deveci, P. R. Brito-Parada, D. Pamucar, and E. A. Varouchakis, "Rough sets based Ordinal Priority Approach to evaluate sustainable development goals (SDGs) for sustainable mining," *Resources Policy*, vol. 79, Article ID 103049, 2022.

[7] M. Deveci, I. Gokasar, O. Castillo, and T. Daim, "Evaluation of Metaverse integration of freight fluidity measurement alternatives using fuzzy Dombi EDAS model," *Computers & Industrial Engineering*, vol. 174, Article ID 108773, 2022.

[8] S. Ashraf, S. Abdullah, T. Mahmood, and M. Aslam, "Cleaner production evaluation in gold mines using novel distance measure method with cubic picture fuzzy numbers," *International Journal of Fuzzy Systems*, vol. 21, no. 8, pp. 2448–2461, 2019.

[9] M. J. Khan, P. Kumam, S. Ashraf, and W. Kumam, "Generalized picture fuzzy soft sets and their application in decision support systems," *Symmetry*, vol. 11, no. 3, p. 415, 2019.

[10] S. Khan, S. Abdullah, and S. Ashraf, "Picture fuzzy aggregation information based on Einstein operations and their application in decision making," *Mathematical Sciences*, vol. 13, no. 3, pp. 213–229, 2019.

[11] S. Khan, S. Abdullah, L. Abdullah, and S. Ashraf, "Logarithmic aggregation operators of picture fuzzy numbers for multi-attribute decision making problems," *Mathematics*, vol. 7, no. 7, p. 608, 2019.

[12] M. Kamran, S. Nadeem, S. Ashraf, A. Alam, and I. N. Cangul, "Novel decision modeling for manufacturing sustainability under single-valued neutrosophic hesitant fuzzy rough aggregation information," *Computational Intelligence and Neuroscience*, vol. 2022, 36 pages, 2022.

[13] D. Pamucar, I. Gokasar, A. E. Torkayesh, M. Deveci, L. Mart ínez, and Q. Wu, "Prioritization of unmanned aerial vehicles in transportation systems using the integrated stratified fuzzy rough decision-making approach with the hamacher operator," *Information Sciences*, vol. 622, pp. 374–404, 2023.

[14] S. Al-Humairi, A. Hizami, A. A. Zaidan et al., "Towards sustainable transportation: a pavement strategy selection based on the extension of dual-hesitant fuzzy multi-criteria decision-making methods," *IEEE Transactions on Fuzzy Systems*, vol. 31, 2022.

[15] M. Kamran, R. Ismail, E. H. A. Al-Sabri, N. Salamat, M. Farman, and S. Ashraf, "An optimization strategy for MADM framework with confidence level aggregation operators under probabilistic neutrosophic hesitant fuzzy rough environment," *Symmetry*, vol. 15, no. 3, p. 578, 2023.

[16] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.

[17] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, 1986.

[18] F. Shen, X. Ma, Z. Li, Z. Xu, and D. Cai, "An extended intuitionistic fuzzy TOPSIS method based on a new distance measure with an application to credit risk evaluation," *Information Sciences*, vol. 428, pp. 105–119, 2018.
[19] M. C. Wu and T. Y. Chen, "The ELECTRE multicriteria analysis approach based on Atanassov's intuitionistic fuzzy sets," *Expert Systems with Applications*, vol. 38, no. 10, pp. 12318–12327, 2011.
[20] Y. X. Xue, J. X. You, X. D. Lai, and H. C. Liu, "An interval-valued intuitionistic fuzzy MABAC approach for material selection with incomplete weight information," *Applied Soft Computing*, vol. 38, pp. 703–713, 2016.
[21] Z. Xu, "Intuitionistic fuzzy aggregation operators," *IEEE Transactions on Fuzzy Systems*, vol. 15, no. 6, pp. 1179–1187, 2007.
[22] W. Wang and X. Liu, "Intuitionistic fuzzy information aggregation using Einstein operations," *IEEE Transactions on Fuzzy Systems*, vol. 20, no. 5, pp. 923–938, 2012.
[23] X. Yu and Z. Xu, "Prioritized intuitionistic fuzzy aggregation operators," *Information Fusion*, vol. 14, no. 1, pp. 108–116, 2013.
[24] P. Liu and P. Wang, "Some improved linguistic intuitionistic fuzzy aggregation operators and their applications to multiple-attribute decision making," *International Journal of Information Technology and Decision Making*, vol. 16, no. 3, pp. 817–850, 2017.
[25] Z. Xu and R. R. Yager, "Intuitionistic fuzzy Bonferroni means," *IEEE Transactions on Systems, Man, and Cybernetics - Part B: Cybernetics: A Publication of the IEEE Systems, Man, and Cybernetics Society*, vol. 41, no. 2, pp. 568–578, 2011.
[26] R. Arora and H. Garg, "Group decision-making method based on prioritized linguistic intuitionistic fuzzy aggregation operators and its fundamental properties," *Computational and Applied Mathematics*, vol. 38, no. 2, pp. 36–32, 2019.
[27] H. Zhao, Z. Xu, M. Ni, and S. Liu, "Generalized aggregation operators for intuitionistic fuzzy sets," *International Journal of Intelligent Systems*, vol. 25, no. 1, pp. 1–30, 2010.
[28] D. Yu, "Intuitionistic fuzzy information aggregation under confidence levels," *Applied Soft Computing*, vol. 19, pp. 147–160, 2014.
[29] D. Yu, "Intuitionistic fuzzy geometric Heronian mean aggregation operators," *Applied Soft Computing*, vol. 13, no. 2, pp. 1235–1246, 2013.
[30] W. Jiang, B. Wei, X. Liu, X. Li, and H. Zheng, "Intuitionistic fuzzy power aggregation operator based on entropy and its application in decision making," *International Journal of Intelligent Systems*, vol. 33, no. 1, pp. 49–67, 2018.
[31] T. Senapati, G. Chen, and R. R. Yager, "Aczel–Alsina aggregation operators and their application to intuitionistic fuzzy multiple attribute decision making," *International Journal of Intelligent Systems*, vol. 37, no. 2, pp. 1529–1551, 2021.
[32] S. Qahtan, H. A. Alsattar, A. A. Zaidan, M. Deveci, D. Pamucar, and W. Ding, "A novel fuel supply system modelling approach for electric vehicles under Pythagorean probabilistic hesitant fuzzy sets," *Information Sciences*, vol. 622, pp. 1014–1032, 2023.
[33] M. J. Khan, P. Kumam, P. Liu, W. Kumam, and S. Ashraf, "A novel approach to generalized intuitionistic fuzzy soft sets and its application in decision support system," *Mathematics*, vol. 7, no. 8, p. 742, 2019.
[34] D. Pamucar, M. Deveci, I. Gokasar, M. Tavana, and M. Kö ppen, "A metaverse assessment model for sustainable transportation using ordinal priority approach and Aczel-Alsina norms," *Technological Forecasting and Social Change*, vol. 182, Article ID 121778, 2022.
[35] J. Aczél and C. Alsina, "Characterizations of some classes of quasilinear functions with applications to triangular norms and to synthesizing judgements," *Aequationes Mathematicae*, vol. 25, no. 1, pp. 313–315, 1982.
[36] G. Deschrijver and E. E. Kerre, "Uninorms in L^*-fuzzy set theory," *Fuzzy Sets and Systems*, vol. 148, no. 2, pp. 243–262, 2004.
[37] M. Xia, Z. Xu, and B. Zhu, "Some issues on intuitionistic fuzzy aggregation operators based on Archimedean t-conorm and t-norm," *Knowledge-Based Systems*, vol. 31, pp. 78–88, 2012.
[38] G. Wei and X. Zhao, "Induced hesitant interval-valued fuzzy Einstein aggregation operators and their application to multiple attribute decision making," *Journal of Intelligent and Fuzzy Systems*, vol. 24, no. 4, pp. 789–803, 2013.
[39] P. Liu, "Some Hamacher aggregation operators based on the interval-valued intuitionistic fuzzy numbers and their application to group decision making," *IEEE Transactions on Fuzzy Systems*, vol. 22, no. 1, pp. 83–97, 2014.
[40] K. Ullah, H. Garg, Z. Gul, T. Mahmood, Q. Khan, and Z. Ali, "Interval valued T-spherical fuzzy information aggregation based on Dombi t-norm and Dombi t-conorm for multi-attribute decision making problems," *Symmetry*, vol. 13, no. 6, p. 1053, 2021.
[41] X. Zhang, P. Liu, and Y. Wang, "Multiple attribute group decision making methods based on intuitionistic fuzzy frank power aggregation operators," *Journal of Intelligent and Fuzzy Systems*, vol. 29, no. 5, pp. 2235–2246, 2015.
[42] T. Senapati, G. Chen, and R. R. Yager, "Aczel–Alsina aggregation operators and their application to intuitionistic fuzzy multiple attribute decision making," *International Journal of Intelligent Systems*, vol. 37, no. 2, pp. 1529–1551, 2022.
[43] M. Naeem, Y. Khan, S. Ashraf, W. Weera, and B. Batool, "A novel picture fuzzy Aczel-Alsina geometric aggregation information: application to determining the factors affecting mango crops," *AIMS Mathematics*, vol. 7, no. 7, pp. 12264–12288, 2022.
[44] G. Wei, "Picture fuzzy aggregation operators and their application to multiple attribute decision making," *Journal of Intelligent and Fuzzy Systems*, vol. 33, no. 2, pp. 713–724, 2017.
[45] S. Ashraf, T. Mahmood, S. Abdullah, and Q. Khan, "Different approaches to multi-criteria group decision making problems for picture fuzzy environment," *Bulletin of the Brazilian Mathematical Society, New Series*, vol. 50, no. 2, pp. 373–397, 2019.
[46] J. J. Peng, J. Q. Wang, X. H. Wu, H. Y. Zhang, and X. H. Chen, "The fuzzy cross-entropy for intuitionistic hesitant fuzzy sets and their application in multi-criteria decision-making," *International Journal of Systems Science*, vol. 46, no. 13, pp. 2335–2350, 2015.
[47] L. Li, R. Zhang, J. Wang, and X. Shang, "Some q-rung orthopair linguistic Heronian mean operators with their application to multi-attribute group decision making," *Archives of Control Sciences*, vol. 28, no. 4, 2018.
[48] A. Biswas and A. Sarkar, "Development of dual hesitant fuzzy prioritized operators based on Einstein operations with their application to multi-criteria group decision making," *Archives of Control Sciences*, vol. 28, pp. 527–549, 2018.