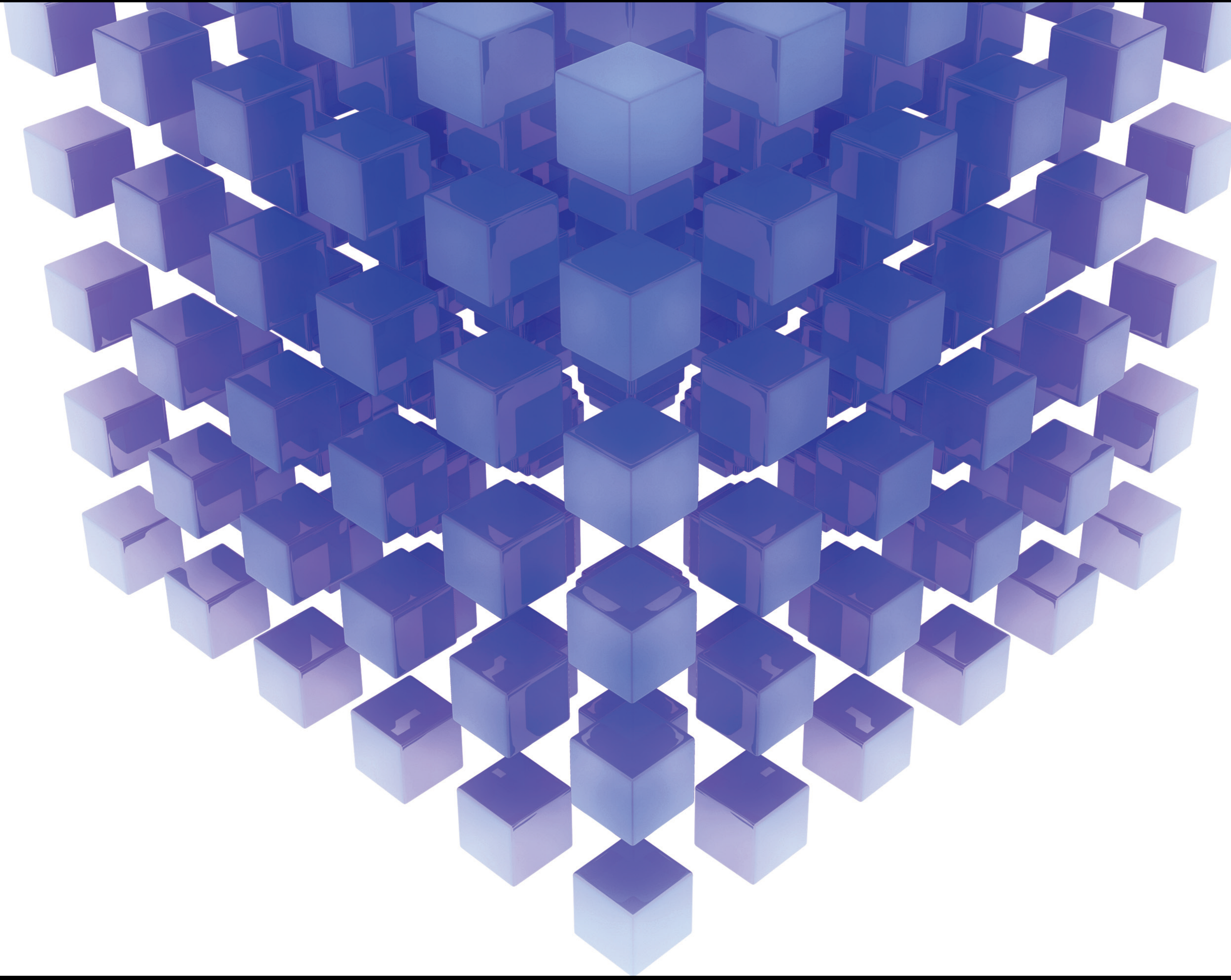
Mathematical Problems in Engineering

Different Approaches in Treating Uncertainty in the Application of Multicriteria Decision Modeling

Lead Guest Editor: Darko Božanić
Guest Editors: Cristiano Fragassa and Radu-Emil Precup



Different Approaches in Treating Uncertainty in the Application of Multicriteria Decision Modeling

Mathematical Problems in Engineering

Different Approaches in Treating Uncertainty in the Application of Multicriteria Decision Modeling

Lead Guest Editor: Darko Božanić
Guest Editors: Cristiano Fragassa and Radu-Emil Precup

Paolo Boscariol, Italy
Daniela Boso, Italy
Guillermo Botella-Juan, Spain
Abdesselem Boulkroune, Algeria
Boulaïd Boulkroune, Belgium
Fabio Bovenga, Italy
Francesco Braghin, Italy
Ricardo Branco, Portugal
Julien Bruchon, France
Matteo Bruggi, Italy
Michele Brun, Italy
Maria Elena Bruni, Italy
Maria Angela Butturi, Italy
Bartłomiej Błachowski, Poland
Dhanamjayulu C, India
Raquel Caballero-Águila, Spain
Filippo Cacace, Italy
Salvatore Caddemi, Italy
Zuowei Cai, China
Roberto Caldelli, Italy
Francesco Cannizzaro, Italy
Maosen Cao, China
Ana Carpio, Spain
Rodrigo Carvajal, Chile
Caterina Casavola, Italy
Sara Casciati, Italy
Federica Caselli, Italy
Carmen Castillo, Spain
Inmaculada T. Castro, Spain
Miguel Castro, Portugal
Giuseppe Catalanotti, United Kingdom
Alberto Cavallo, Italy
Gabriele Cazzulani, Italy
Fatih Vehbi Celebi, Turkey
Miguel Cerrolaza, Venezuela
Gregory Chagnon, France
Ching-Ter Chang, Taiwan
Kuei-Lun Chang, Taiwan
Qing Chang, USA
Xiaoheng Chang, China
Prasenjit Chatterjee, Lithuania
Kacem Chehdi, France
Peter N. Cheimets, USA
Chih-Chiang Chen, Taiwan
He Chen, China
Kebing Chen, China
Mengxin Chen, China
Shyi-Ming Chen, Taiwan
Xizhong Chen, Ireland
Xue-Bo Chen, China
Zhiwen Chen, China
Qiang Cheng, USA
Zeyang Cheng, China
Luca Chiapponi, Italy
Francisco Chicano, Spain
Tirivanhu Chinyoka, South Africa
Adrian Chmielewski, Poland
Seongim Choi, USA
Gautam Choubey, India
Hung-Yuan Chung, Taiwan
Yusheng Ci, China
Simone Cinquemani, Italy
Roberto G. Citarella, Italy
Joaquim Ciurana, Spain
John D. Clayton, USA
Piero Colajanni, Italy
Giuseppina Colicchio, Italy
Vassilios Constantoudis, Greece
Enrico Conte, Italy
Alessandro Contento, USA
Mario Cools, Belgium
Gino Cortellessa, Italy
Carlo Cosentino, Italy
Paolo Crippa, Italy
Erik Cuevas, Mexico
Guozeng Cui, China
Mehmet Cunkas, Turkey
Giuseppe D'Aniello, Italy
Peter Dabnichki, Australia
Weizhong Dai, USA
Zhifeng Dai, China
Purushothaman Damodaran, USA
Sergey Dashkovskiy, Germany
Adiel T. De Almeida-Filho, Brazil
Fabio De Angelis, Italy
Samuele De Bartolo, Italy
Stefano De Miranda, Italy
Filippo De Monte, Italy

José António Fonseca De Oliveira Correia, Portugal
Jose Renato De Sousa, Brazil
Michael Defoort, France
Alessandro Della Corte, Italy
Laurent Dewasme, Belgium
Sanku Dey, India
Gianpaolo Di Bona, Italy
Roberta Di Pace, Italy
Francesca Di Puccio, Italy
Ramón I. Diego, Spain
Yannis Dimakopoulos, Greece
Hasan Dinçer, Turkey
José M. Domínguez, Spain
Georgios Dounias, Greece
Bo Du, China
Emil Dumic, Croatia
Madalina Dumitriu, United Kingdom
Premraj Durairaj, India
Saeed Eftekhar Azam, USA
Said El Kafhali, Morocco
Antonio Elipe, Spain
R. Emre Erkmen, Canada
John Escobar, Colombia
Leandro F. F. Miguel, Brazil
FRANCESCO FOTI, Italy
Andrea L. Facci, Italy
Shahla Faisal, Pakistan
Giovanni Falsone, Italy
Hua Fan, China
Jianguang Fang, Australia
Nicholas Fantuzzi, Italy
Muhammad Shahid Farid, Pakistan
Hamed Faroqi, Iran
Yann Favennec, France
Fiorenzo A. Fazzolari, United Kingdom
Giuseppe Fedele, Italy
Roberto Fedele, Italy
Baowei Feng, China
Mohammad Ferdows, Bangladesh
Arturo J. Fernández, Spain
Jesus M. Fernandez Oro, Spain
Francesco Ferrise, Italy
Eric Feulvarch, France
Thierry Floquet, France
Eric Florentin, France
Gerardo Flores, Mexico
Antonio Forcina, Italy
Alessandro Formisano, Italy
Francesco Franco, Italy
Elisa Francomano, Italy
Juan Frausto-Solis, Mexico
Shujun Fu, China
Juan C. G. Prada, Spain
HECTOR GOMEZ, Chile
Matteo Gaeta, Italy
Mauro Gaggero, Italy
Zoran Gajic, USA
Jaime Gallardo-Alvarado, Mexico
Mosè Gallo, Italy
Akemi Gálvez, Spain
Maria L. Gandarias, Spain
Hao Gao, Hong Kong
Xingbao Gao, China
Yan Gao, China
Zhiwei Gao, United Kingdom
Giovanni Garcea, Italy
José García, Chile
Harish Garg, India
Alessandro Gasparetto, Italy
Stylianos Georgantzinos, Greece
Fotios Georgiades, India
Parviz Ghadimi, Iran
Ștefan Cristian Gherghina, Romania
Georgios I. Giannopoulos, Greece
Agathoklis Giaralis, United Kingdom
Anna M. Gil-Lafuente, Spain
Ivan Giorgio, Italy
Gaetano Giunta, Luxembourg
Jefferson L.M.A. Gomes, United Kingdom
Emilio Gómez-Déniz, Spain
Antonio M. Gonçalves de Lima, Brazil
Qunxi Gong, China
Chris Goodrich, USA
Rama S. R. Gorla, USA
Veena Goswami, India
Xunjie Gou, Spain
Jakub Grabski, Poland

Bekir Sahin, Turkey
Laxminarayan Sahoo, India
John S. Sakellariou, Greece
Michael Sakellariou, Greece
Salvatore Salamone, USA
Jose Vicente Salcedo, Spain
Alejandro Salcido, Mexico
Alejandro Salcido, Mexico
Nunzio Salerno, Italy
Rohit Salgotra, India
Miguel A. Salido, Spain
Sinan Salih, Iraq
Alessandro Salvini, Italy
Abdus Samad, India
Sovan Samanta, India
Nikolaos Samaras, Greece
Ramon Sancibrian, Spain
Giuseppe Sanfilippo, Italy
Omar-Jacobo Santos, Mexico
J Santos-Reyes, Mexico
José A. Sanz-Herrera, Spain
Musavarah Sarwar, Pakistan
Shahzad Sarwar, Saudi Arabia
Marcelo A. Savi, Brazil
Andrey V. Savkin, Australia
Tadeusz Sawik, Poland
Roberta Sburlati, Italy
Gustavo Scaglia, Argentina
Thomas Schuster, Germany
Hamid M. Sedighi, Iran
Mijanur Rahaman Seikh, India
Tapan Senapati, China
Lotfi Senhadji, France
Junwon Seo, USA
Michele Serpilli, Italy
Silvestar Šesnić, Croatia
Gerardo Severino, Italy
Ruben Sevilla, United Kingdom
Stefano Sfarra, Italy
Dr. Ismail Shah, Pakistan
Leonid Shaikhet, Israel
Vimal Shanmuganathan, India
Prayas Sharma, India
Bo Shen, Germany
Hang Shen, China
Xin Pu Shen, China
Dimitri O. Shepelsky, Ukraine
Jian Shi, China
Amin Shokrollahi, Australia
Suzanne M. Shontz, USA
Babak Shotorban, USA
Zhan Shu, Canada
Angelo Sifaleras, Greece
Nuno Simões, Portugal
Mehakpreet Singh, Ireland
Piyush Pratap Singh, India
Rajiv Singh, India
Seralathan Sivamani, India
S. Sivasankaran, Malaysia
Christos H. Skiadas, Greece
Konstantina Skouri, Greece
Neale R. Smith, Mexico
Bogdan Smolka, Poland
Delfim Soares Jr., Brazil
Alba Sofi, Italy
Francesco Soldovieri, Italy
Raffaele Solimene, Italy
Yang Song, Norway
Jussi Sopanen, Finland
Marco Spadini, Italy
Paolo Spagnolo, Italy
Ruben Specogna, Italy
Vasilios Spitas, Greece
Ivanka Stamova, USA
Rafał Stanisławski, Poland
Miladin Stefanović, Serbia
Salvatore Strano, Italy
Yakov Strelniker, Israel
Kangkang Sun, China
Qiuqin Sun, China
Shuaishuai Sun, Australia
Yanchao Sun, China
Zong-Yao Sun, China
Kumarasamy Suresh, India
Sergey A. Suslov, Australia
D.L. Suthar, Ethiopia
D.L. Suthar, Ethiopia
Andrzej Swierniak, Poland
Andras Szekrenyes, Hungary
Kumar K. Tamma, USA

Zaoli Yang, China
Jun Ye, China
Min Ye, China
Luis J. Yebra, Spain
Peng-Yeng Yin, Taiwan
Muhammad Haroon Yousaf, Pakistan
Yuan Yuan, United Kingdom
Qin Yuming, China
Elena Zaitseva, Slovakia
Arkadiusz Zak, Poland
Mohammad Zakwan, India
Ernesto Zambrano-Serrano, Mexico
Francesco Zammori, Italy
Jessica Zangari, Italy
Rafal Zdunek, Poland
Ibrahim Zeid, USA
Nianyin Zeng, China
Junyong Zhai, China
Hao Zhang, China
Haopeng Zhang, USA
Jian Zhang, China
Kai Zhang, China
Lingfan Zhang, China
Mingjie Zhang, Norway
Qian Zhang, China
Tianwei Zhang, China
Tongqian Zhang, China
Wenyu Zhang, China
Xianming Zhang, Australia
Xuping Zhang, Denmark
Yinyan Zhang, China
Yifan Zhao, United Kingdom
Debao Zhou, USA
Heng Zhou, China
Jian G. Zhou, United Kingdom
Junyong Zhou, China
Xueqian Zhou, United Kingdom
Zhe Zhou, China
Wu-Le Zhu, China
Gaetano Zizzo, Italy
Mingcheng Zuo, China

Contents

Hindawi
Mathematical Problems in Engineering
Volume 2023, Article ID 9843106, 1 page
https://doi.org/10.1155/2023/9843106

Retraction

Retracted: Research on the English Classroom Teaching Effect Evaluation with Interval-Valued Intuitionistic Fuzzy Grey Relational Analysis Method

Mathematical Problems in Engineering

Received 11 July 2023; Accepted 11 July 2023; Published 12 July 2023

This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope
(2) Discrepancies in the description of the research reported
(3) Discrepancies between the availability of data and the research described
(4) Inappropriate citations
(5) Incoherent, meaningless and/or irrelevant content included in the article
(6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] Z. Zhang and P. Su, "Research on the English Classroom Teaching Effect Evaluation with Interval-Valued Intuitionistic Fuzzy Grey Relational Analysis Method," *Mathematical Problems in Engineering*, vol. 2022, Article ID 7445250, 11 pages, 2022.

Hindawi
Mathematical Problems in Engineering
Volume 2023, Article ID 9767024, 1 page
https://doi.org/10.1155/2023/9767024

Retraction

Retracted: Grey Relational Analysis Method for Probabilistic Double Hierarchy Linguistic Multiple Attribute Group Decision Making and Its Application to College Tennis Classroom Teaching Effect Evaluation

Mathematical Problems in Engineering

Received 11 July 2023; Accepted 11 July 2023; Published 12 July 2023

This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope
(2) Discrepancies in the description of the research reported
(3) Discrepancies between the availability of data and the research described
(4) Inappropriate citations
(5) Incoherent, meaningless and/or irrelevant content included in the article
(6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] L. Wang, H. Li, J. Zhang, and J. Yang, "Grey Relational Analysis Method for Probabilistic Double Hierarchy Linguistic Multiple Attribute Group Decision Making and Its Application to College Tennis Classroom Teaching Effect Evaluation," *Mathematical Problems in Engineering*, vol. 2022, Article ID 7419496, 17 pages, 2022.

Hindawi
Mathematical Problems in Engineering
Volume 2022, Article ID 8442123, 9 pages
https://doi.org/10.1155/2022/8442123

Research Article

An Approach of Decision-Making under the Framework of Fermatean Fuzzy Sets

Muhammad Sarwar Sindhu,[1] Imran Siddique ,[2] Muhammad Ahsan,[1] Fahd Jarad ,[3,4,5] and Taner Altunok[6]

[1]*Virtual University of Pakistan, Lahore 54770, Pakistan*
[2]*Department of Mathematics, University of Management and Technology, Lahore 54770, Pakistan*
[3]*Department of Mathematics, Cankaya University, Etimesgut, Ankara, Turkey*
[4]*Department of Mathematics, King Abdulaziz University, Jeddah, Saudi Arabia*
[5]*Department of Medical Research, China Medical University Hospital, China Medical University, Taichung, Taiwan*
[6]*Department of Industrial Engineering, Konya Food and Agriculture University, Meram, Konya, Turkey*

Correspondence should be addressed to Imran Siddique; imransmsrazi@gmail.com and Fahd Jarad; fahd@cankaya.edu.tr

Received 21 January 2022; Revised 3 March 2022; Accepted 21 June 2022; Published 8 July 2022

Academic Editor: Gianpaolo Di Bona

Because of its influence on various elements of human life experiences and conditions, the building industry is a significant business. In the recent past, environmental considerations have been incorporated in the design and planning stages of building supply chains. The process of evaluating and selecting suppliers is one of the most important issues in supply chain management. A multicriteria decision-making (MCDM) problem can be utilized to handle such issues. The goal of this research is to present a new and efficient technique for selecting suppliers with ambiguous data. The suggested methodology's structure is based on technology for order of preference by similarity to ideal solution (TOPSIS), with Fermatean fuzzy sets (F_r FSs) employed to cope with information uncertainty. In this article, authors modified the distance between F_r FSs to propose the similarity measure and implemented it to form the MCDM model to resolve the vague and uncertain data. Moreover, we used this similarity measure to choose the optimal alternative. A practical example for alternative selection is provided, along with a comparison of the acquired findings to existing approach. Finally, to strengthen the outcome obtained through the proposed model, sensitivity analysis and time complexity analysis are performed.

1. Introduction

In real-world situations, we frequently encounter tasks and activities that necessitate the usage of decision-making (DM_g) procedures. DM_g may be viewed as a problem-solving process that yields an ideal, or at the very least reasonable, solution. In general, DM_g is a mental and reasoning process that leads to choose an ideal option from a collection of possible alternatives in a DM_g circumstance. TOPSIS is a valuable method for MCDM issues in the real world. Hwang and Yoon [1] first proposed this strategy in 1981, with Yoon continuing the process in 1987. TOPSIS rates options and determines the best compromise between them and the ideal solution. TOPSIS is an effective approach for ranking and picking a number of generally recognized alternatives using distance metrics that is both practical and helpful. TOPSIS is the best compromise choice, having the lowest distance from the positive-ideal solution and the greatest distance from the negative-ideal solution [2–4]. So far, TOPSIS has been thoroughly investigated by explorers and experts, and it has been successfully applied to a wide range of DM_g situations [5–8].

The DM_g procedure demands the analysis of a small number of possibilities stated in terms of evaluative criteria for the most part. Instead, when analyzing all of the criteria at once, the issue may be to rank these possibilities in terms of how desirable they are to the decision-maker. If all parameters are assessed at the same time, another goal would be to discover the best choice or to estimate the relative over all preferences of each alternative. The basic goal of MCDM is to solve challenges like these: (1) PROMETHEE, (2) ELECTRE, (3) AHP, (4) VIKOR, (5) Fuzzy AHP, (6) TOPSIS, and (7) Fuzzy TOPSIS are the seven most significant MCDM approaches. Hundreds of experts have implemented TOPSIS in many domains, updated or modified the TOPSIS approach to meet unique issues.

One of the inevitabilities of dealing with DM_g challenges is the ambiguity of information. The opinions and expressions of decision-makers are frequently the source of this ambiguity. We may describe and capture information uncertainty in a variety of ways. Fuzzy sets (FSs) theory has been a popular method for dealing with uncertainty in DM_g situations in recent years. Furthermore, the linear programming (LP) presented in [9] was used to calculate the weights of criteria [10–12] based on decision-makers' evaluations. The F_r F-TOPSIS approach was created in a variety of fuzzy situations. The study's key contribution is the use of F_r FSs to expand the F_r F-TOPSIS approach and apply the enlarged methodology to evaluating green building suppliers.

2. Literature Review

In the middle of the 1960s, Zadeh [13] proposed the concept of FSs, which ushered in a new era for scholars. In real-world situations, FSs typically reflect uncertainty and ambiguity. The majority of the experts have concentrated on FS expansions and applications. In 1986, Atanassov [14] proposed the notion of intuitionistic fuzzy sets (IFSs), which is one of the most important extensions of FSs and have two number of degrees named, membership degree (MD), and nonmembership degree (NMD) such that $0 \leq MD + NMD \leq 1$.

Recently, Pythagorean fuzzy sets (P_g FS) [15] have gotten more concentration from the experts and implemented in different fields of DM_g procedures. When comparing two items based on their unequal content, distance measures are quite useful. Zeng et al. [16] demonstrated the use of various P_g F distance and similarity measurements in MCDM. Hussain and Yang [17] provided various Hausdorff metric-based P_g F distance and similarity measures with P_g F-TOPSIS applicability. Li and Lu [18] presented some generalized distance measurements and their continuous versions for P_g FSs. Ejegwa [19] provided several distance and similarity measurements for P_g FSs based on membership grades. Wei and Wei [20] proposed some cosine function-based P_g F similarity measurements. Peng et al. [21] presented 12 P_g F distance and similarity measurements, along with their applicability (2017). Although P_g FSs have a wide spectrum of uses, they are unable to handle circumstances, where $MD^2 + NMD^2 > 1$, for instance, if $MD = 0.8$ and $NMD = 0.7$, then $0.8^2 + 0.7^2 = 0.64 + 0.49 = 1.13 > 1$. To overcome such situations, Senapati and Yager [22] introduced as a new sort of FSs recently, named F_r FSs. F_r FSs make up of both MD and NMD which satisfies the condition $MD^3 + NMD^3 < 1$, so it handles the abovementioned circumstances accurately. F_r FSs are derived from the ideas of IFS and P_g FS. F_r FSs, on the other hand, use novel concepts to manage uncertain data that make them more flexible and efficient than IFSs and P_g FS [23, 24]. Because they are all confined within the space of F_r FSs, F_r FSs are more powerful than FSs, IFSs, and P_g FSs. Senapati and Yager [24] presented certain F_r FS aggregation operators and their application in decision-making. Mishra and Rani [25] proposed the weighted aggregated sum product assessment (WASPAS) method in the Fermatean fuzzy (F_r F) environment. Garg et al. [26] demonstrated the use of FF aggregating functions in the COVID-19 testing facility. The continuities and derivatives of FF functions were investigated by Yang et al. [27]. Sergi and Sari [28] proposed some FF capital budgeting approaches. Sahoo [29] suggested some FFS scoring functions and their application to transportation issues and decision-making.

The major reason we used F_r FSs in designing the current study's strategy is because of its flexibility in dealing with unclear information. The goal of this research is to develop a new and efficient system for evaluating and selecting green suppliers in a building supply chain where there is uncertainty. In the evaluation process, the technique described in this study takes into account the ambiguity of information given by decision-makers. To deal with information uncertainty, we employed F_r FSs. The suggested technique is based on the extended TOPSIS (E-TOPSIS) and LP methods, which is both efficient and helpful.

Failure mode effect analysis (FMEA) is a common and effective technique that may be used to assess risk and improve the safety of a repairable engineering system, according to Kushwaha et al. [30]. Yorulmaz et al. [31] proposed TOPSIS based on modified Mahalanobis distance measure to rank the 81 Turkish provinces by considering distinct levels of development. One of the most important activities in the purchasing department is supplier selection. By assisting in the selection of the most suitable supplier, choosing the correct supplier makes a strategic difference in an organization's capacity to decrease costs and improve product quality. Cakar and Cavus [32] implemented fuzzy TOPSIS to select the best supplier. The criteria for choosing an air traffic control (ATC) radar station that effectively fulfills the job of radar in air traffic management are developed and assessed in [33]. Picture fuzzy set and rough setbased approaches are proposed in this study to consider the unclear concerns linked with students' job decision since they are shown to be appropriate due to their inherent qualities to cope with incomplete and imprecise information [34]. To select the construction machinery, Bozanic et al. [35] offered the Neuro-Fuzzy System as a decision-making aid.

There has been no previous study employing the F_r F-TOPSIS approach with F_r FSs to deal with MCDM, to the best of the authors' knowledge. The primary contributions of this study can be summarized as follows:

(1) To tackle MCDM situations with ambiguous knowledge that may be stated by a number of decision-makers, a novel D M_g technique based on F_r F-TOPSIS and F_r FSs is proposed.

(2) An example demonstrates the effectiveness of the proposed technique for evaluating green building providers.

The remainder of the paper is arranged as follows: Section 2 contains some fundamental and relevant knowledge. In Section 3, the features of novel F_r FSs are thoroughly examined. To address the ambiguous information, an MCDM model based on F_r F-TOPSIS is created. An MCDM issues relevant to select the supplier is provided in Section 5. The validity of the suggested model is explored in Section 6. Subsection 7.1 examines a complete comparison based on TC. Figure 1 represents the research process of this article.

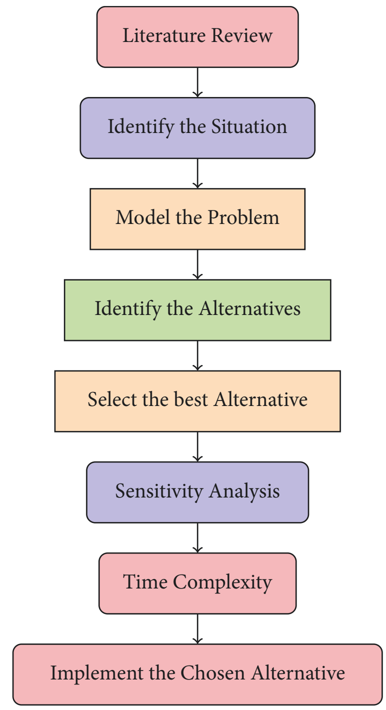


Figure 1: Research process.

3. Basic Concepts

Some basic ideas connected to the present work such as FSs, IFSs, F_r FSs, and LP are briefly penned in this section.

Definition 1. [13] A FS $\mathscr{F}$ over $Y = \{y_1, y_2, \ldots, y_n\}$ can be illustrated as follows:

$$\mathscr{F} = \{(y, \mu_{\mathscr{F}}(y)) | y \in Y\}. \tag{1}$$

where $\mu_{\mathscr{F}}(y))$: $X \longrightarrow [0, 1]$ is a MD so that $y \in Y$ to $\mathscr{F}$.

Definition 2. [14] Let Y be a fixed set, an IFS $\mathscr{I}$ on Y is characterized as follows:

$$\mathscr{I} = \{\langle y, \alpha_{\mathscr{I}}(y), \beta_{\mathscr{I}}(y)\rangle | y \in Y\}, \tag{2}$$

where $\alpha_{\mathscr{I}}(y), \beta_{\mathscr{I}}(y) \in [0, 1]$ are called the MD and NMD of $y \in Y$ to set $\mathscr{I}$ with the following condition: $0 \leq \alpha_{\mathscr{I}}(y) + \beta_{\mathscr{I}}(y) \leq 1$, for all $y \in Y$.

For all $y \in Y$, $\omega_{\mathscr{I}}(y)$ is known as hesitancy degree of $y \in \mathscr{I}$, where $\omega_{\mathscr{I}}(y) = 1 - \alpha_{\mathscr{I}}(y) - \beta_{\mathscr{I}}(y)$.

Definition 3. [36] A P_g FS $\mathscr{P}$ over Y is given by

$$\mathscr{P} = \{(y, <\alpha_{\mathscr{P}}(y), \beta_{\mathscr{P}}(y)>) | y \in Y\}, \tag{3}$$

where $\alpha_{\mathscr{P}}(y), \beta_{\mathscr{P}}(y) \in [0.1]$ are the MD and NMD of y to $\mathscr{P}$ such that $0 \leq \alpha^2_{\mathscr{P}}(y) + \beta^2_{\mathscr{P}}(y) \leq 1$. The degree of hesitancy or indeterminacy represented by $\eta_{\mathscr{P}}(y)$ is written as $\eta_{\mathscr{P}}(y) = \sqrt{1 - \alpha^2_{\mathscr{P}}(y) - \beta^2_{\mathscr{P}}(y)}$.

Definition 4. [22] A Fermatean fuzzy set over the set $Y = \{y_1, y_2, \ldots, y_n\}$ is defined as follows:

$$F = \{\langle y, \alpha_F(y), \eta_F(y)\rangle | y \in Y\}, \tag{4}$$

where $\alpha_F(y), \eta_F(y) \in [0, 1]$ and are called the MD, NMD of $y \in Y$ to the set F, respectively and $\alpha_F(y), \eta_F(y)$ fulfil the condition: $0 \leq \alpha^3_F(y) + \eta^3_F(y) \leq 1$, for all $y \in Y$. Also $\zeta_F(y) = \sqrt[3]{1 - \alpha^3_F(y) - \eta^3_F(y)}$, then $\zeta_F(y)$ is supposed to be an indeterminacy membership degree (IMD) of $y \in Y$ in F. For simplicity, F_r FSs over Y is read as F_r FSs(Y).

Definition 5. Reference [9]. The following is the formula of an LP model:

$$\begin{aligned} \text{Maximize}: S &= c_1t_1 + c_2t_2 + c_3t_3 + \cdots + c_nt_n, \\ \text{Subject to}: &\ a_{11}t_1 + a_{12}t_2 + a_{13}t_3 + \cdots + a_{1n}t_n \leq b_1, \\ &\ a_{21}t_1 + a_{22}t_2 + a_{23}t_3 + \cdots + a_{2n}t_n \leq b_2, \\ &\ \vdots \\ &\ a_{m1}t_1 + a_{m2}t_2 + a_{m3}t_3 + \cdots + a_{mn}t_n \leq b_m, \\ &\ t_1, t_2, \ldots, t_n \geq 0. \end{aligned} \tag{5}$$

In LP model, m indicates the cardinality of constraints and n shows the number of decision variables.

4. A Modified Distance Measure between F_r FSs

A modified Hamming distance measure between two F_r FSs is presented to tackle the vague data in this section.

Definition 6. Suppose that F_1 and F_2 be two F_r FSs defined on a fixed set $Y = \{y_1, y_2, y_3\}$, then the distance $D_F(F_1, F_2)$ is defined as follows:

$$D_F(F_1, F_2) = \frac{1}{3n}\sum_{i=1}^{n}\left(\begin{array}{c}\left[\left|\alpha^3_{F_1}(y_i) - \alpha^3_{F_2}(y_i)\right| + \left|\eta^3_{F_1}(y_i) - \eta^3_{F_2}(y_i)\right|\right] + \\ \max\left[\left|\alpha^3_{F_1}(y_i) - \alpha^3_{F_2}(y_i)\right|, \left|\eta^3_{F_1}(y_i) - \eta^3_{F_2}(y_i)\right|\right]\end{array}\right). \tag{6}$$

Example 1. Let F_1 and F_2 be two F_r FSs over $Y = \{y_1, y_2, y_3\}$ given by $F_1 = \{(y_1, (0.8, 0.7)), (y_2, (0.9, 0.8), (y_3, (0.5, 0.9))\}$ and $F_2 = \{(y_1, (0.8, 0.62)), (y_2, (0.7, 0.6)), (y_3, (0.9, 0.6))\}$, based on Definition 6, we get, $D_F(F_1, F_2) = 0.2974$.

Theorem 1. *Let D be a mapping such that D: $F_rFSs(X) \times F_rFSs(X) \longrightarrow [0,1]$. If the requirements below are achieved, then $D_F(F_1, F_2)$ is a distance measure.*

(1) $0 \leq D_F(F_1, F_2) \leq 1$;

(2) $D_F(F_1, F_2) = 0\sqrt{b^2 - 4ac}$ iff $F_1 = F_2$;

(3) $D_F(F_1, F_2) = D_F(F_2, F_1)$;

(4) $D_F(F_1, F_3) \geq D_F(F_1, F_2)$ and $D_F(F_1, F_3) \geq D_F(F_2, F_3)$, for any $F_1, F_2, F_3 \in F_r$ FSs(X).

Proof. As, (6) is easy to prove, however, the last condition (4) is proved as follows: For any $F_1, F_2, F_3 \in F_r$ FSs(X), and $F_1 \subseteq F_2 \subseteq F_3$, then on the basis of Definition 5, we get

$$\begin{aligned} &\left|\alpha^3_{F_1}(x_i) - \alpha^3_{F_3}(x_i)\right| \geq \left|\alpha^3_{F_1}(x_i) - \alpha^3_{F_2}(x_i)\right|, \\ &\left|\alpha^3_{F_1}(x_i) - \eta^3_{F_3}(x_i)\right| \geq \left|\alpha_{F_1}(x_i) - \eta^3_{F_2}(x_i)\right|. \end{aligned} \tag{7}$$

By adding equation (7), we get

$$\begin{aligned} &\left|\alpha^3_{F_1}(x_i) - \alpha^3_{F_3}(x_i)\right| + \left|\alpha^3_{F_1}(x_i) - \eta^3_{F_3}(x_i)\right| \\ &\geq \left|\alpha^3_{F_1}(x_i) - \alpha^3_{F_2}(x_i)\right| + \left|\alpha^3_{F_1}(x_i) - \eta^3_{F_2}(x_i)\right|, \\ &\Rightarrow \\ &\left|\alpha^3_{F_1}(x_i) - \alpha^3_{F_3}(x_i)\right| + \left|\alpha^3_{F_1}(x_i) - \eta^3_{F_3}(x_i)\right| \\ &+ \max\left\{\left|\alpha^3_{F_1}(x_i) - \alpha^3_{F_3}(x_i)\right|, \left|\alpha^3_{F_1}(x_i) - \eta^3_{F_3}(x_i)\right|\right\} \\ &\geq \left|\alpha^3_{F_1}(x_i) - \alpha^3_{F_2}(x_i)\right| + \left|\alpha^3_{F_1}(x_i) - \eta^3_{F_2}(x_i)\right| \\ &+ \max\left\{\left|\alpha^3_{F_1}(x_i) - \alpha^3_{F_2}(x_i)\right|, \left|\alpha^3_{F_1}(x_i) - \eta^3_{F_2}(x_i)\right|\right\}, \end{aligned} \tag{8}$$

$\Rightarrow D_F(F_1, F_3) \geq D_F(F_1, F_3)$, similarly, we can show, $D_F(F_1, F_3) \geq D_F(F_2, F_3)$. □

Since, criteria's weights have great impact in DM_g, we transform the Definition 2.6 into a weighted distance measure (WDM) between two F_r FSs as follows:where $w_j(1 \leq j \leq m)$ denotes the m criteria weights such that $\sum_{j=1}^m w_j = 1$.

Definition 7. Suppose that F_1 and F_2 are two F_r FSs over $Y = \{y_1, y_2, \ldots, y_n\}$ and w_j are the m criteria's weights satisfying the condition $\sum_{j=1}^m w_j = 1$. Then the WDM $D^w_F(F_1, F_2)$ is penned as below:

$$\begin{aligned} &D^w_F(F_1, \mathscr{B}) \\ &= \sum_{i=1}^n w_j \begin{pmatrix} \left[\left|\alpha^3_{F_1}(y_i) - \alpha^3_{F_2}(y_i)\right| + \left|\eta^3_{F_1}(y_i) - \eta^3_{F_2}(y_i)\right|\right] + \\ \max\left[\left|\alpha^3_{F_1}(y_i) - \alpha^3_{F_2}(y_i)\right|, \left|\eta^3_{F_1}(y_i) - \eta^3_{F_2}(y_i)\right|\right] \end{pmatrix}. \end{aligned} \tag{9}$$

Example 2. Let. F_1. and F_1 be two F_r FSs on a set $Y = \{y_1, y_2, y_3\}$. Example 1 takes the result by using the weights of y_1, y_2 and y_3 as $w_1 = 0.25, w_2 = 0.35$ and $w_3 = 0.4$, respectively. based on Definition 2.7, $D^w_F(F_1, F_2) = 0.7539$.

Theorem 2. *The WDM $D^w_F(F_1, F_2)$ between two F_r FSs F_1 and F_2 satisfy the following four conditions:*

(1) $0 \leq D^w_F(F_1, F_2) \leq 1$;

(2) $D^w_F(F_1, F_2) = 0$ iff $F_1 = F_2$;

(3) $D^w_F(F_1, F_2) = D_F(F_2, F_1)$;

(4) $D^w_F(F_1, F_3) \geq D^w_F(F_1, F_2)$ and $D^w_F(F_1, F_3) \geq D^w_F(F_2, F_3)$, for any $F_1, F_2, F_3 \in F_r$ FSs(X).

Proof. In order to prove Theorem 2, follow the same strategy as Theorem 1. □

Definition 8. Suppose that F_1 and F_2 are two F_r FSs over $Y = \{y_1, y_2, \ldots, y_n\}$. Then measure of similarity $S_p(F_1, F_2)$ on the basis of Definition 7 is penned as follows:

$$\begin{aligned} &S_F(F_1, F_2) \\ &= 1 - \sum_{i=1}^n w_j \begin{pmatrix} \left[\left|\alpha^3_{F_1}(y_i) - \alpha^3_{F_2}(y_i)\right| + \left|\eta^3_{F_1}(y_i) - \eta^3_{F_2}(y_i)\right|\right] + \\ \max\left[\left|\alpha^3_{F_1}(y_i) - \alpha^3_{F_2}(y_i)\right|, \left|\eta^3_{F_1}(y_i) - \eta^3_{F_2}(y_i)\right|\right] \end{pmatrix}. \end{aligned} \tag{10}$$

Definition 9. A mapping S: $F_rFSs(X) \times F_rFSs(X) \longrightarrow [0,1]$. $S_F(F_1, F_2)$ is supposed to be a measure of similarity if $S_F(F_1, F_2)$ fulfills the following four axioms:

(1) $0 \leq S_F(F_1, F_2) \leq 1$;

(2) $S_F(F_1, F_2) = 1$ iff $F_1 = F_2$;

(3) $S_F(F_1, F_2) = S_F(F_2, F_1)$;

(4) $S_F(F_1, F_3) \leq S_F(F_1, F_2)$ and $S_F(F_1, F_3) \leq S_F(F_2, F_3)$, for any $F_1, F_2, F_3 \in F_r$ FSs(X) and $F_1 \subseteq F_2 \subseteq F_3$.

5. MCDM Model Based on Fermatean Fuzzy TOPSIS (F_r F-Topsis)

We suggested an MCDM using F_r F information based on TOPSIS employing LP methodology in this part. The LP model is used to assess the weights of criteria under various restrictions. Suppose that $H = \{H_1, H_2, \ldots, H_n\}$ be a collection of alternatives, and $G = \{G_1, G_2, \ldots, G_m\}$ be the collection of criteria with $\mu = \{\mu_1, \mu_2, \ldots, \mu_m\}$, where $\sum_{j=1}^m \mu_j = 1$ as the weight vector of the criteria G_j, where $j = 1, 2, 3, \ldots, m$. A F_r F decision matrix denoted by $\mathscr{F} = [\Omega_{ij}]_{n\times m} = [(\alpha_{ij}, \eta_{ij})]_{n\times m}$ with α_{ij} as MD and η_{ij} NMD that the alternatives $A_i(i = 1, 2, \ldots, n)$ fulfills, respectively. To reach the optimal solution, follow the steps of proposed MCDM model.

Step 1. Developed a F_r F decision matrix denoted by $\mathscr{F} = [\Omega_{ij}]_{n\times m}$ according to the given information presented by the DM.

Step 2. Figure out the F_r F positive-ideal solution (F_r FPIS), Ω^+_p and F_r F negative-ideal solution (F_r FNIS), Ω^-_p as follows:

Table 1: F_r F decision matrix.

Alternatives	
Q_1	$\{(y_1, 0.7, 0.3), (y_2, 0.4, 0.6), (y_3, 0.5, 0.5), (y_4, 0.8, 0.2), (y_5, 0.8, 0.4)\}$
Q_2	$\{(y_1, 0.5, 0.8), (y_2, 0.8, 0.6), (y_3, 0.4, 0.5), (y_4, 0.7, 0.4), (y_5, 0.6, 0.5)\}$
Q_3	$\{(y_1, 0.9, 0.6), (y_2, 0.8, 0.1), (y_3, 0.6, 0.4), (y_4, 0.7, 0.5), (y_5, 0.9, 0.3)\}$
Q_4	$\{(y_1, 0.6, 0.7), (y_2, 0.8, 0.3), (y_3, 0.7, 0.2), (y_4, 0.5, 0.3), (y_5, 0.7, 0.3)\}$

$$\Omega_p^+ = \left\{\left(\alpha_{ij}^+, \eta_{ij}^+\right)\right\} = \begin{pmatrix} \left\{\left(\max_j\left(\alpha_{ij}\right), \max_j\left(\eta_{ij}\right)\right)\right\}: U_j \in \xi_1 \\ \left\{\left(\min_j\left(\alpha_{ij}\right), \min_j\left(\eta_{ij}\right)\right)\right\}: U_j \in \xi_2 \end{pmatrix}. \tag{11}$$

$$\Omega_p^- = \left\{\left(\alpha_{ij}^-, \eta_{ij}^-\right)\right\} = \begin{pmatrix} \left\{\left(\min_j\left(\alpha_{ij}\right), \min_j\left(\eta_{ij}\right)\right)\right\}: U_j \in \xi_1 \\ \left\{\left(\min_j\left(\alpha_{ij}\right), \min_j\left(\eta_{ij}\right)\right)\right\}: U_j \in \xi_2 \end{pmatrix}, \tag{12}$$

where ξ_1 and ξ_2 are subcollections of beneficial and cost criteria, respectively, so that $\xi_1 \cap \xi_2 = \varnothing$.

Step 3. Compute the weighted similarity degree (WSD) $S_{F_i}^{+w}$ between F_r FPIS Ω_F^+ and each alternative likewise the WSD $S_{F_i}^{-w}$ between F_r FNIS Ω_F^- by using equation (12), respectively:

$$S_{Fi}^{+w}\left(H_i, \Omega_F^+\right) = 1 - \sum_{j=1}^{m} w_j \begin{pmatrix} \left[\left|\alpha_{F_1}(x_i) - \alpha_{ij}^+\right| + \left|\eta_{F_1}(x_i) - \eta_{ij}^+\right|\right] + \\ \max\left[\left|\alpha_{F_1}(x_i) - \alpha_{ij}^+\right|, \left|\eta_{F_1}(x_i) - \eta_{ij}^+\right|\right] \end{pmatrix}. \tag{13}$$

$$S_{Fi}^{-w}\left(H_i, \Omega_F^-\right) = 1 - \sum_{j=1}^{m} w_j \begin{pmatrix} \left[\left|\alpha_{F_1}(x_i) - \alpha_{ij}^-\right| + \left|\eta_{F_1}(x_i) - \eta_{ij}^-\right|\right] + \\ \max\left[\left|\alpha_{F_1}(x_i) - \alpha_{ij}^-\right|, \left|\eta_{F_1}(x_i) - \eta_{ij}^-\right|\right] \end{pmatrix}, \tag{14}$$

where, $1 \le i \le n$.

Step 4. Based on equations (15) and (16), construct the model to find the objective function Z for the weights of criteria as follows:

$$Z = \left(S_{Fi}^{+w}\left(H_i, \Omega_F^+\right) - S_{Fi}^{-w}\left(H_i, \Omega_F^-\right)\right). \tag{15}$$

Step 5. We derive the weights μ_j of the criterion $G_j\,(j = 1, 2, 3, \ldots, m)$ by solving the LP model described in [30], so that the objective function Z produced in Step 4 is maximized.

Step 6. Based on equations (15) and (16), calculate the degree of similarity and evaluate $S_{F_i}^{+w}$ and $S_{F_i}^{-w}$ on the basis of equations (9) and (10) between each option and the components achieved in F_r FPIS Ω_p^+ and F_r FNIS Ω_p^-, respectively.

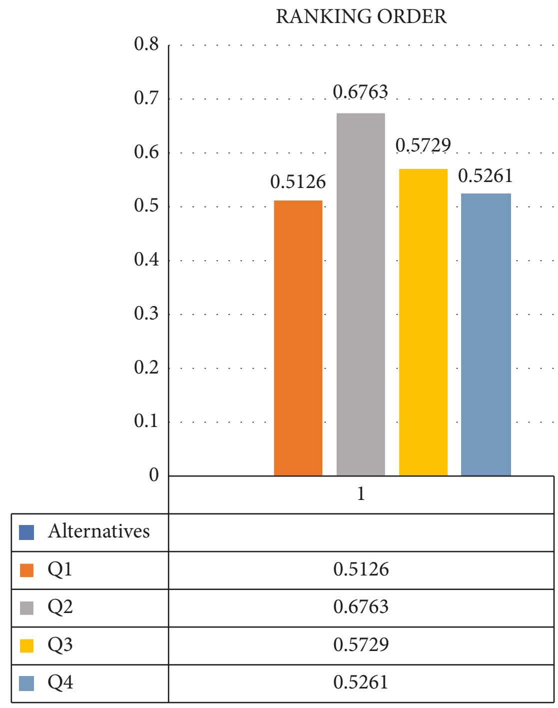


Figure 2: Ranking of alternatives.

Step 7. Determine the coefficient of relative closeness $\mathscr{R}^C{}_i$ of each alternative H_i with respect to the F_r FPIS Ω_F^+ as follows:

$$\mathscr{R}_i^C = \frac{S_{F_i}^{+w}}{S_{F_i}^{+w} + S_{F_i}^{-w}}. \tag{16}$$

The greater the value $\mathscr{R}^C{}_i$ of the alternatives to F_r FPIS (Ω_p^{+w}), the more likely we are to find the greatest choice from a set of alternatives H_i, where $1 \le i \le n$.

6. Solution of Problems Based on F_r F-Topsis

The authors used the proposed MCDM model to recognize the pattern and breakout of dengue disease in this section.

Step 1. F_r F decision matrix $P_c = [\Omega_{ij}]_{4\times5}$ denoted in Table 1.

Step 2. The ideal solution $\Omega_F^+ = \{(y_1,0.9000, 0.8000), (y_2,0.8000,0.6000), (y_3,0.7000, 0.5000), (y_4,0.8000, 0.5000), (y_5,0.9000, 0.5000)\}$ $\Omega_F^- = \{(y_1,0.5000, 0.3000), (y_2,0.4000, 0.1000), (y_3,0.4000, 0.2000), (y_4, 0.5000, 0.2000), (y_5,0.6000, 0.3000)\}$

Step 3. The WSD $S_{F_i}^{+w}$ between F_r FPIS Ω_F^+ and each alternative as well as the WSD $S_{F_i}^{-w}$ between F_r FNIS Ω_F^-

Table 2: Results obtained for altering the weights of criteria.

Alternatives	Original	Increment in w_1	Increment in w_2	Increment in w_3	Increment in w_4	Increment in w_5
Q_1	0.5126	0.5013	0.5214	0.5180	0.5127	0.5210
Q_2	0.6763	0.6600	0.6508	0.6691	0.6707	0.6700
Q_3	0.5729	0.5630	0.5621	0.5592	0.5730	0.5745
Q_4	0.5261	0.5187	0.5201	0.5150	0.5271	0.5268

by using equations (13) and (14), respectively, in terms of weights.

Step 4. Based on equations (13) and (14), evaluate $Z = -0.0250w_1 - 0.3650w_2 - 0.1450w_3 - 0.0550w_4$ which is written in equation (17).

Step 5. Based on LP model penned in [9], the weights w_j of the criteria P_j, where $j = 1, 2, 3, 4, 5$ are obtained as follows:

$$w_1 = 0.2, w_2 = 0.3, w_3 = 0.25, w_4 = 0.1 \text{ and } w_5 = 0.15. \tag{17}$$

Step 6. Degree of positive and negative weighted similarities $S_{+}^{f_{ri}}$ and $S_{-}^{f_{ri}}$ are obtained by using equations (7) and (8) as follows:

$$\begin{aligned}
&S_{+}^{f_{r1}}(Q_1, \Omega_F^{+}) = 0.5100, S_{+}^{f_{r2}}(Q_2, \Omega_F^{+}) = 0.7000,\\
&S_{+}^{f_{r3}}(Q_3, \Omega_F^{+}) = 0.5700, S_{+}^{f_{r4}}(Q_4, \Omega_F^{+}) = 0.5550, \text{ and}\\
&S_{-}^{f_{r1}}(Q_1, \Omega_F^{-}) = 0.4850, S_{-}^{f_{r2}}(Q_2, \Omega_F^{-}) = 0.3350,\\
&S_{-}^{f_{r3}}(Q_3, \Omega_F^{-}) = 0.4250, S_{-}^{f_{r4}}(Q_4, \Omega_F^{-}) = 0.5000.
\end{aligned} \tag{18}$$

Step 7. Values of $\mathscr{R}^C{}_i$ of each alternative is the following:

$$\begin{aligned}
\mathscr{R}_1^C &= 0.5126,\\
\mathscr{R}_2^C &= 0.6763,\\
\mathscr{R}_3^C &= 0.5729,\\
\mathscr{R}_4^C &= 0.5261.
\end{aligned} \tag{19}$$

Step 8. Arrange the alternatives according to the values of R_i^C as obtained in Step 4. We get, $Q_2 \prec Q_3 \prec Q_4 \prec Q_1$. Hence, the optimal alternative attained is Q_2 which is illustrated in Figure 2.

Example 3. A construction company wanted to select four suppliers, Q_1, Q_2, Q_3, and Q_5 according to certain criteria. Suppliers are evaluated against five parameters, P_1, P_2, P_3, and p_5. Weights of criteria have great impact in decisions, authors have used LP model to compute the weights. Assume that the evaluation values of the alternatives in relation to each criterion provided by the committee are represented by F_r FNs, as shown in the F_r F decision matrix given in Table 1.

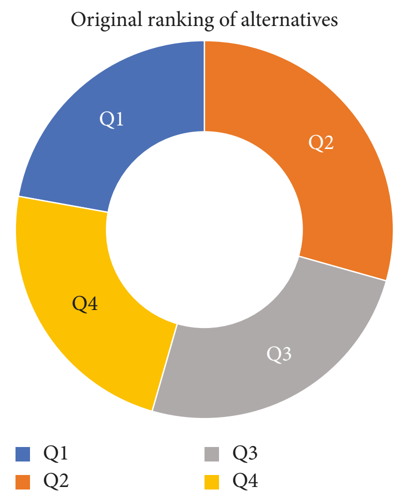


Figure 3: Original result.

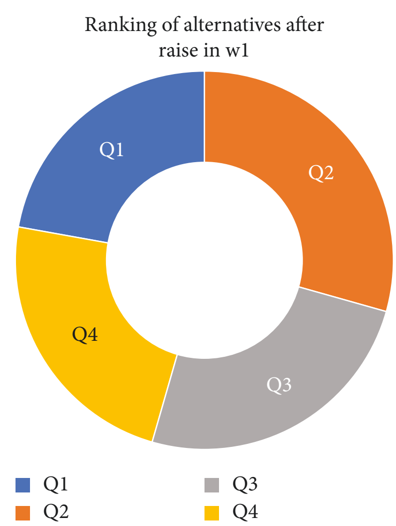


Figure 4: 0.05 to 0.1 raise in w_1.

7. Sensitivity Analysis

Because the information for MCDM problems is frequently uncertain and ambiguous, there is a need for a tool that can assist us make more correct decisions. Sensitivity analysis (SA) can help in this regard. In this part, weighted SA is used to evaluate the impact of changing the weights of criteria on the results provided by the proposed model. A formula described in [37] is used to generate a new vector for criterion weights, and the behavior of the findings obtained by the suggested model is then examined. We changed the

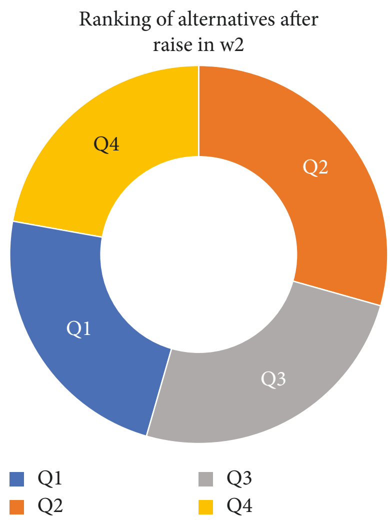


Figure 5: 0.05 to 0.1 raise in w_2.

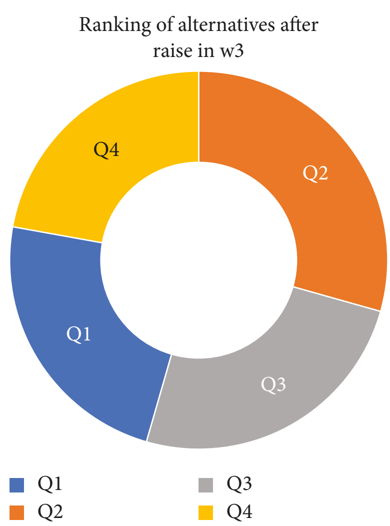


Figure 6: 0.05 to 0.1 raise in w_3.

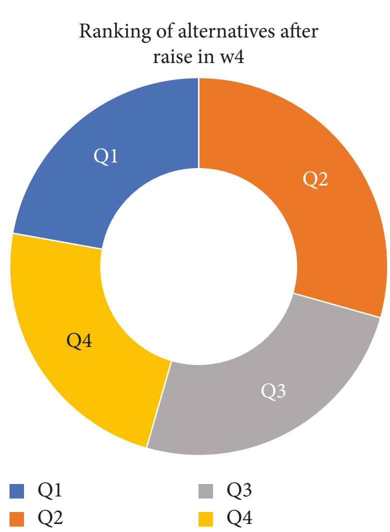


Figure 7: 0.05 to 0.1 raise in w_4.

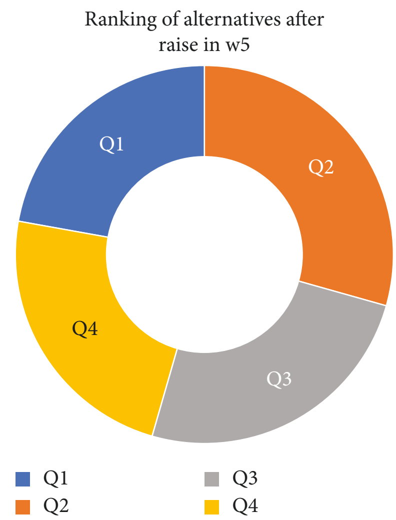


Figure 8: 0.05 to 0.1 raise in w_5.

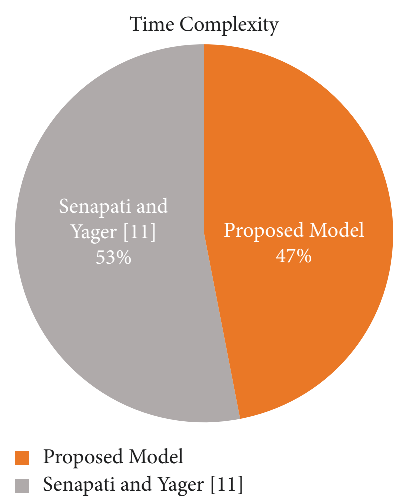


Figure 9: Graphical view of TC analysis.

Table 3: TC among the proposed and existing technique.

Techniques	Executing time
Proposed model	0.4513 seconds
Senapati and Yager [23]	0.510 seconds

weights of individual criteria by raising different ratios and looked at the effect on the final findings. Table 2 shows the outcomes achieved by varying the weights of criterion. Figures 3 to 8 show that raising 0.05 to 0.1 in each weight results in a little change in the numeric values, but the ranking orders remain same, demonstrating the usefulness and strength of our suggested model.

7.1. Comparison Based on Time Complexity (TC). In order to strengthen the results obtained from the proposed MCDM model, TC analysis is performed in the present subsection. TC is the time required to execute an algorithm to reach the final result. TC is measured among the proposed and the existing techniques presented by Senapati and Yager [23]. The executing time of each technique is evaluated with the help of MATLAB which is presented in Table 3 and its graphical view is illustrated in Figure 9. From Table 3, it can be seen that our approach takes less time as compared to others; hence, the proposed MCDM model is more effective and resolves the issues rapidly.

8. Conclusions

TOPSIS is one of the most well-known MCDM approaches. The focus of this research was on TOPSIS extensions named F_r. F-TOPSIS is used in complicated decision scenarios with uncertainty. The total of squares of MD and NMD to which an item meeting a criteria supplied by expert is subjected in some real-world situations may be greater than one, but their cube sum may be less than or equal to one. As a result, P_g FS is unable to handle such a situation. From this perspective, the F_r FS might be used to mimic some D M_g scenarios that P_g FS cannot handle. In this study, we offer an MCDM technique based on TOPSIS in a F_r FS environment. Finally, we provided an example to demonstrate how this method might be utilized efficiently.

In light of the foregoing, future research could concentrate on:

(1) Using other traditional objective and subjective multicriteria decision-making methods in conjunction with F_r FS to determine and evaluate criteria for the selection of the alternative.

(2) Aside from that, the benefits of the current strategy can be enhanced by considering the objective weight of risk factors, which are not taken into account in this study.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] C. Hwang and K. Yoon, *Multiple Attributes Decision Making Methods and Applications*, Springer, Berlin, Heidelberg, 1981.

[2] G. Dwivedi, R. K. Srivastava, and S. K. Srivastava, "A generalised fuzzy topsis with improved closeness coefficient," *Expert Systems with Applications*, vol. 96, pp. 185–195, 2018.

[3] S. Sadic, J. P. de Sousa, and J. A. Crispim, "A two-phase milp approach to integrate order, customer and manufacturer characteristics into dynamic manufacturing network formation and operational planning," *Expert Systems with Applications*, vol. 96, pp. 462–478, 2018.

[4] F. Shen, X. Ma, Z. Li, Z. Xu, and D. Cai, "An extended intuitionistic fuzzy topsis method based on a new distance measure with an application to credit risk evaluation," *Information Sciences*, vol. 428, pp. 105–119, 2018.

[5] S. M. Chen and C. H. Chang, "Fuzzy multiattribute decision making based on transformation techniques of intuitionistic fuzzy values and intuitionistic fuzzy geometric averaging operators," *Information Sciences*, vol. 352-353, pp. 133–149, 2016.

[6] J. R. S. Cristobal, "Multi - criteria decision-making in the selection of a renewable energy project in Spain: the Vikor method," *Renewable Energy*, vol. 36, no. 2, pp. 498–502, 2011.

[7] T. T. T. Duong and N. X. Thao, "A novel dissimilarity measure on picture fuzzy sets and its application in multi-criteria decision making," *Soft Computing*, vol. 25, no. 1, pp. 15–25, 2021.

[8] D. Kannan, A. B. L. d. S. Jabbour, and C. J. C. Jabbour, "Selecting green suppliers based on GSCM practices: using fuzzy TOPSIS applied to a Brazilian electronics company," *European Journal of Operational Research*, vol. 233, no. 2, pp. 432–447, 2014.

[9] R. J. Vanderbei, *Linear Programming: Foundations and Extensions*, Springer-Verlag, Berlin, Heidelberg, 2014.

[10] M. S. Sindhu, T. Rashid, and A. Kashif, "Modeling of linear programming and extended TOPSIS in decision making problem under the framework of picture fuzzy sets," *PLoS One*, vol. 14, no. 8, Article ID e0220957, 2019.

[11] M. S. Sindhu, T. Rashid, and A. Kashif, "An approach to select the investment based on bipolar picture fuzzy sets," *International Journal of Fuzzy Systems*, vol. 23, pp. 1–13, 2021.

[12] M. S. Sindhu, M. Ahsan, A. Rafiq, and I. A. Khan, "Multiple criteria decision making based on bipolar picture fuzzy sets and extended TOPSIS," *The Journal of Mathematics and Computer Science*, vol. 23, no. 1, pp. 49–57, 2021.

[13] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.

[14] K. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, 1986.

[15] R. R. Yager, "Pythagorean Fuzzy Subsets," in *Proceedings of the 2013 Joint IFSA World congress and NAFIPS Annual Meeting (IFSA/NAFIPS)*, pp. 57–61, IEEE, Edmonton, Canada, June 2013.

[16] W. Zeng, D. Li, and Q. Yin, "Distance and similarity measures of Pythagorean fuzzy sets and their applications to multiple criteria group decision making," *International Journal of Intelligent Systems*, vol. 33, no. 11, pp. 2236–2254, 2018.

[17] Z. Hussian and M. S. Yang, "Distance and similarity measures of Pythagorean fuzzy sets based on the Hausdorff metric with application to fuzzy TOPSIS," *International Journal of Intelligent Systems*, vol. 34, no. 10, pp. 2633–2654, 2019.

[18] Z. Li and M. Lu, "Some novel similarity and distance measures of Pythagorean fuzzy sets and their applications," *Journal of Intelligent and Fuzzy Systems*, vol. 37, no. 2, pp. 1781–1799, 2019.

[19] P. A. Ejegwa, "Improved composite relation for pythagorean fuzzy sets and its application to medical diagnosis," *Granular Computing*, vol. 5, no. 2, pp. 277–286, 2020.

[20] G. Wei and Y. Wei, "Similarity measures of Pythagorean fuzzy sets based on the cosine function and their applications," *International Journal of Intelligent Systems*, vol. 33, no. 3, pp. 634–652, 2018.

[21] X. Peng, H. Yuan, and Y. Yang, "Pythagorean fuzzy information measures and their applications," *International Journal of Intelligent Systems*, vol. 32, no. 10, pp. 991–1029, 2017.

[22] T. Senapati and R. R. Yager, "Some new operations over Fermatean fuzzy numbers and application of Fermatean fuzzy

WPM in multiple criteria decision making," *Informatica*, vol. 30, no. 2, pp. 391–412, 2019.
[23] T. Senapati and R. R. Yager, "Fermatean fuzzy sets," *Journal of Ambient Intelligence and Humanized Computing*, vol. 11, no. 2, pp. 663–674, 2020.
[24] T. Senapati and R. R. Yager, "Fermatean fuzzy weighted averaging/geometric operators and its application in multi-criteria decision-making methods," *Engineering Applications of Artificial Intelligence*, vol. 85, pp. 112–121, 2019.
[25] A. R. Mishra and P. Rani, "Multi-criteria healthcare waste disposal location selection based on Fermatean fuzzy WASPAS method," *Complex & Intelligent Systems*, vol. 7, no. 5, pp. 2469–2484, 2021.
[26] H. Garg, G. Shahzadi, and M. Akram, "Decision-making analysis based on Fermatean fuzzy Yager aggregation operators with application in COVID-19 testing facility," *Mathematical Problems in Engineering*, p. 2020, 2020.
[27] Z. Yang, H. Garg, and X. Li, "Differential calculus of fermatean fuzzy functions: continuities, derivatives, and differentials," *International Journal of Computational Intelligence Systems*, vol. 14, no. 1, pp. 282–294, 2021.
[28] D. Sergi and I. U. Sari, "Fuzzy capital budgeting using fermatean fuzzy sets," in *Proceedings of the International Conference on Intelligent and Fuzzy Systems*, pp. 448–456, Springer, Istanbul, Turkey, July 2020.
[29] L. Sahoo, "Some score functions on Fermatean fuzzy sets and its application to bride selection based on TOPSIS method," *International Journal of Fuzzy System Applications*, vol. 10, no. 3, pp. 18–29, 2021.
[30] D. K. Kushwaha, D. Panchal, A. Panchal, and A. Sachdeva, "Risk analysis of cutting system under intuitionistic fuzzy environment," *Reports in Mechanical Engineering*, vol. 1, no. 1, pp. 162–173, 2020.
[31] O. Yorulmaz, S. K. Yildirim, and B. F. Yildirim, "Robust Mahalanobis distance based TOPSIS to evaluate the economic development of provinces," *Operational Research in Engineering Sciences: Theory and Applications*, vol. 4, no. 2, pp. 102–123, 2021.
[32] B. Cavus and T. Cakar, "Supplier selection process in dairy industry using fuzzy-topsis method," *Operational Research in Engineering Sciences: Theory and Applications*, vol. 4, no. 1, pp. 82–98, 2021.
[33] I. Petrovic and M. Kankaras, "A hybridized IT2FS-DEMATEL-AHP-TOPSIS multicriteria decision making approach: case study of selection and evaluation of criteria for determination of air traffic control radar position," *Decision Making: Applications in Management and Engineering*, vol. 3, no. 1, pp. 146–164, 2020.
[34] R. Sahu, S. R. Dash, and S. Das, "Career selection of students using hybridized distance measure based on picture fuzzy set and rough set theory," *Decision Making: Applications in Management and Engineering*, vol. 4, no. 1, pp. 104–126, 2021.
[35] D. Bozanic, D. Tesic, D. Tesic, A. Marinkovic, and A. Milic, "Modeling of neuro-fuzzy system as a support in decision-making processes," *Reports in Mechanical Engineering*, vol. 2, no. 1, pp. 222–234, 2021.
[36] R. Yager, "Pythagorean membership grades in multicriteria decision making," *IEEE Transactions on Fuzzy Systems*, vol. 22, no. 4, pp. 958–965, 2014.
[37] A. Alinezhad and A. Amini, "Sensitivity analysis of TOPSIS technique: the results of change in the weight of one attribute on the final ranking of alternatives," *Journal of Optimization in Industerial Engeering*, vol. 7, pp. 23–28, 2011.

Hindawi
Mathematical Problems in Engineering
Volume 2022, Article ID 5480848, 17 pages
https://doi.org/10.1155/2022/5480848

Research Article

A New Spherical Fuzzy LBWA-MULTIMOOSRAL Framework: Application in Evaluation of Leanness of MSMEs in India

Sanjib Biswas,[1] **Dragan Pamučar**,[2] **Darko Božanić**,[2] **and Bikash Halder**[1]

[1]*Decision Sciences and Operations Management Area, Calcutta Business School, South 24 Parganas, Bishnupur, West Bengal 743503, India*
[2]*Military Academy, University of Defence in Belgrade, Belgrade 11000, Serbia*

Correspondence should be addressed to Darko Božanić; dbozanic@yahoo.com

Received 18 February 2022; Revised 10 May 2022; Accepted 10 June 2022; Published 8 July 2022

Academic Editor: Ardashir Mohammadzadeh

The present paper aims to propose a new hybrid multi-criteria decision-making (MCDM) framework with spherical fuzzy numbers (SFNs). We extend two recently developed algorithms such as level-based weight assessment (LBWA) and MULTIMOOSRAL in spherical fuzzy (SF) domain. We present a case study on six MSME units belonging to engineering cluster for examining their leanness. MSMEs form the backbone of the socioeconomic growth and therefore garner attention of the policy makers. Lean manufacturing (LM) has been a key enabler for the last three decades which help the organizations to achieve business growth. We consider the criteria like leadership, supplier focus, customer focus, process management, waste, culture, human resource focus, technology use and communication, and awareness to compare leanness of the MSMEs using expert opinions. We find that committed leadership, waste reduction, and customer value are given more weightage by the experts for achieving leanness in SMEs. Furthermore, the results show that medium and small units with focused product line score high in terms of leanness. We validate the results obtained by our proposed method by comparing with the same derived by using another widely used approach such as Technique for Order Preference by Similarity to Ideal Solution (TOPSIS). We carry out sensitivity analysis for examining the stability in the solution with the changes in the given condition such as variations in the criteria weights. Our results using SF-LBWA-MULTIMOOSRAL show reasonable accuracy and stability.

1. Introduction

The last two decades are characterized by revolutionary progress in technology, extreme volatility and disruption, cut-throat competition, rise in knowledgeable customer base, and rapid speed of innovation. The organizations are challenged by increasing demand from the market in terms of superior quality, variety, quick response, convenience, and affordability with respect to cost [1]. As a result, the organizations have no other choice but to optimize their processes and put all efforts to deliver maximum possible value with optimum utilization of resources [2]. In other words, organizations strive to become lean. The concept of lean management and/or lean manufacturing (LM) was first defined by Krafcik [3] and later got popularized in 1990 with explanations given by Womack and his colleagues [4]. Gupta et al. [5] defined LM as "an integrated multi-dimensional approach encompassing a wide variety of management practices based on the philosophy of eliminating waste through continuous improvement." The work in [6–8] portrays the benefits of practicing the principles and tools of LM as reduction in defect rates and waste, human efforts, process hours, space requirement, and operational cost while increasing value, customer satisfaction, demand, flow of the process, and morale of the employees among others. LM paves the way to global excellence through continuous introspection and improvement for the organizations by imbibing the philosophy and implementing the concepts and tools [9]. In this regard, leanness is the extent to which the concepts and practices of LM are adapted and implemented vis-à-vis organizational goals and customers' requirement. In simple term, leanness indicates how lean is an organization [10].

In India as per the provisions of Micro, Small, and Medium Enterprises Development (MSMED) Act, 2006, the micro, small, and medium-sized enterprises (MSMEs) are defined as the organizations having investment up to 10 crores with an annual turnover ranging from 5 to 250 crores. The market size of MSMEs in India is around 6.3 crores with an increase in number by CAGR 18.5 percent in 2020 [11]. MSMEs provide foundation to the socioeconomic growth of a nation, contributing to employment generation, empower the youth and improving livelihood (especially self-help groups and women), income distribution, provide support to large-scale industries, resource mobilization, reduce regional disparity, export balancing, and accelerate social reform [12–18]. Needless to mention that for the inclusive development of the country like India, empowerment and growth of MSMEs are very important. The estimated growth of India's manufacturing sector is USD 1 trillion by 2025 wherein MSMEs play the role as one of the key enablers for fostering the promise of "Make-in-India" initiative taken by the Govt. of India (GOI). LM helps to improve the dynamic capabilities and competitiveness of the MSMEs by combating the constraints like fund, space, skill, waste, imbalanced process, manpower, maintenance, and facilities, among others [19, 20].

Therefore, from the facts and figures, it is evident that MSME sector has huge potential for the growth of India. Further, LM plays an important role in accelerating the growth of MSMEs and improving their competitiveness. Having understood the benefits and relevance of LM for MSMEs, it is quite imperative to assess the leanness of MSMEs. In this context, the present study attempts to find answers of the following research questions. (RQ1) How to measure the leanness of MSMEs? (RQ2) How to compare the competitiveness of the MSMEs from multiple perspectives? However, in this regard, we observe that the extant literature does not show adequate evidence in favour of competitive assessment of leanness of MSMEs. Our paper fills the gap to the literature by providing a framework for comparing the achievement of leanness of a group of MSMEs.

It is evident that for a holistic comparison of MSMEs in terms of their leanness, a complex multi-criteria analysis is involved. The problem can be formally expressed as $X = \begin{pmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{pmatrix}_{m\times n}$.

Here, m is the number of MSME units under comparison with respect to n criteria that are manifesting leanness of the organization, and x_{ij} is the leanness achievement of i^{th} MSME unit with respect to j^{th} criterion. In this paper, we present a sample case study of six MSMEs located in the eastern part of India wherein the sample units are compared on the basis of the on-field diagnostic study and opinions by a group of three experts. The dimensions or criteria for comparison are derived from literature supported by the opinions of the experts vis-à-vis RQ1. Hence, the present study calls for a complex and subjective opinion-based group decision-making approach to answer RQ2. We address this problem by carrying out our analysis in SF domain.

The concept of fuzzy sets (FSs) and fuzzy numbers (FNs) were introduced to handle impreciseness of information under uncertain and ambiguous environment by Zadeh [21]. Unlike the crisp sets, FS considers the varying degree of membership of the elements ranging from 0 to 1. However, there are real-life situations wherein the degree of non-membership and neutrality add significant complexity. In view of the requirement to provide additional flexibility to the analyst for decision making in uncertain environment, Atanassov [22] introduced intuitionistic fuzzy sets (IFSs) as a generalization of FS. IFS considers both degree of membership (μ) and non-membership (υ) with the condition that $0 \le \mu + \vartheta \le 1$. Moving further, Atanassov and Gargov [23] extended the concept of IFS with interval values, aka interval-valued IFS (IVIFS). The strand of literature that worked on FS, IFS, and IVIFS later observed the conditions where $\mu + \vartheta > 1$. To solve this problem, Yager [24] propounded a new variant, called Pythagorean fuzzy set (PyFS) as a generalization of IFS. PyFS follows the norms $0 \le \mu^2 + \vartheta^2 \le 1$. As a further generalization of PyFS, q rung orthopair fuzzy set (qROFS) was proposed (with a relation $0 \le \mu^q + \vartheta^q \le 1$) [25]. In effect, $q = 2$ converts qROFS into PyFS and for $q = 1$, it becomes IFS. However, researchers [26, 27] felt the importance of considering the degree of membership, non-membership, neutrality (Υ), and refusal too. As a result, a new branch of the broad domain of FS, known as picture fuzzy sets (PFSs), was introduced. PFS is more capable of countering the issue of vagueness and imprecise information and satisfies the condition $0 \le \mu + \vartheta + \gamma \le 1$. The rationale behind use of the recently developed wings of extended fuzzy sets such as spherical fuzzy set (SFS) stems from the disadvantage of PFS in some cases wherein $\mu + \vartheta + \gamma > 1$ [28]. SFS is grounded on the concept of three-dimensional spherical geometry and the membership degrees follow the condition $0 < \mu^2 + \vartheta^2 + \gamma^2 < 1$. SFS is an advanced extension of neutrosophic fuzzy sets [29] and type 2 IFS [30]. SFS provides a number of advantages [31, 32] such as

(i) Unlike IFS, it considers the sum of membership and non-membership degrees greater than 1.

(ii) In contrast to PyFS, it considers the degree of hesitancy.

(iii) Compared to PFS, SFS works well in a typical situation wherein, for instance, $\mu = 0.7$; $\vartheta = 0.3$; $\gamma = 0.5$ that does not satisfy $\mu + \vartheta + \gamma < 1$ (PFS) but does not violate the assumption of SFS, i.e., $\mu^2 + \vartheta^2 + \gamma^2 < 1$. Therefore, SFS provides the decision makers more flexibility and larger space.

Furthermore, if compared with qROFS, SFS provides the benefits like consideration of hesitancy and lesser complexity in computation and visualization. SFS considers three-dimensional space or volume which is more easy to conceptualize, visualize, and handle as compared with qROFS. SFS is also less complex that Fermatean fuzzy sets [33]. In this paper, we apply SFS to solve the issue of performance evaluation of SMEs using a combined novel framework of LBWA and MULTIMOOSRAL approach. LBWA is a recently developed algorithm that works on level-based partitioning of the criteria as per their relative significance to decide the criteria weights [34]. The MULTIMOOSRAL approach combines the weighted sum, weighted product, and logarithmic approximations to rank the alternatives subject to the influence of the criteria [35]. In effect, the MULTIMOOSRAL algorithm is an upgraded synthesis of the popular approaches like multi-objective optimization on the basis of ratio analysis (MOORA), multi-objective optimization on the basis of simple ratio analysis (MOOSRA), multi-objective analysis by ratio analysis plus the full multiplicative form (MULTIMOORA), weighted aggregated sum product assessment (WASPAS), and combined compromise solution (CoCoSo).

The proposed SF-LBWA-MULTIMOOSRAL framework (as used in this paper) provides the following advantages.

(i) Greater flexibility to the decision makers (DM) in rating as SFS allows more space in selecting the values of membership, non-membership, and hesitancy.
(ii) Lesser computational complexity as the framework uses simple arithmetic operators.
(iii) The model uses less number of pairwise comparisons for determining criteria weights and subsequently reduces the possibility of the subjective bias unlike its counterparts like analytic hierarchy process (AHP).
(iv) Ability to work with a large criteria and alternative set with subjective and objective information.
(v) Ability to withstand large variations in the criteria values.
(vi) Combination of addition, subtraction, multiplication, division, and logarithmic approach in evaluation.
(vii) Reasonably accurate and stable results.

The motivations behind the present paper are as follows.

(a) We find that several authors have worked on establishing the importance of practicing LM for achieving competitive advantage for the organizations. Furthermore, the authors have also advocated in favour of maintaining leanness in the processes for optimization and mobilization of the resources and adding value to the customers. The extant literature has provided definitions and measurement of leanness. In the context of MSMEs, past works have shown the utility of practicing LM. However, there is a scantiness of work that measures and compares leanness of MSMEs in Indian context.
(b) From the methodological point of view, SFS has been recently introduced to overcome the drawbacks of IFS and PyFS. SFS as compared with IFS, PyFS, and PFS provides more flexibility in selection of membership, non-membership, and hesitancy grades which allow the researchers to apply in real-life situations. We observe that SFS has not been used for subjective opinion-based group decision-making cases using LBWA and MULTIMOOSRAL.
(c) As we have mentioned the benefits of our proposed model above, we do not find any literature that has used an integrated framework of LBWA and MULTIMOOSRAL.

The major contributions of the present paper are as follows.

(i) The present paper provides a comprehensive multi-criteria-based evaluation framework for comparing leanness of the MSMEs in Indian context. The extant literature shows a scantiness of research in considering multiple dimensions of assessment of leanness through a comparative study.
(ii) A novel hybrid framework of LBWA-MULTIMOOSRAL for multi-criteria group decision making is proposed.
(iii) In the present study, we provide a new extension of LBWA-MULTIMOOSRAL using SFS. The use of SFS in solving various research problems is growing but has not been explored exhaustively for applications in MCDM problems.

The reminder of this paper is organized as follows. In Section 2, we include some of the recent related work. Section 3 presents some preliminary concepts of SFS and SFN. Section 4 navigates the research methodology. The summary of results is included in Section 5. Section 6 provides the discussion on the results and sheds light on some of the research implications. In Section 7, we make the concluding remarks and mention some of the future scopes.

2. Related Work

In this section, we present some of the relative work on assessment of leanness, SFS, and applications of LBWA and MULTIMOOSRAL methods.

In the last decade, a number of studies have been conducted towards developing measure of leanness. For instance, Seyedhosseini et al. [36] utilized the balanced scorecard framework to define leanness measures. Azevedo et al. [37] put emphasis on agility for ensuring leanness in the context of supply chain management. The use of an integrated AHP-DEMATEL model is noticed in [38] in exploring the priority of the factors responsible for implementation of LM. The work of Patil et al. [39] focused on the new product development process and contributed five measures such as knowledge management, customer value, design cost, and schedule. Maasouman and Demirli [40] stressed on leadership, people management, facility management, process, working condition, quality, and just-in-time operations to assess the leanness of cellular manufacturing. In this regard, Basu et al. [41] advocated for ensuring employee welfare for supporting the successful implementation of LM. In the study [42], the authors presented a leanness assessment framework considering leadership, supplier focus, customer value, process management, and employee development and further developed a value stream map in the context of a large-scale organization belonging to Indian plywood industry. Some authors relied on probabilistic and statistical approaches to examine the root of lean elements and their interactions [43, 44]. Kroes et al. [45] investigated the causal effect of lean practices on the performance of the retail firms. The study in [46] enquired the preparedness of pharmaceutical organization for implementing LM tools. Tekez and Taşdeviren [47] extended the strand of literature with their advocacy for innovation as a measure of leanness. Therefore, we have noticed that a good number of studies have been done to find out the measures of leanness. However, there is a lack of evidence of using all these dimensions to carry out comparative evaluation of the organizations in terms of leanness.

In the context of MSMEs, there has been a notable number of attempts made towards constructing leanness measurement framework. For example, Ravikumar et al. [48] followed a two-stage approach. The authors considered the attributes like leadership, organizational culture, financial support, communication, performance management system, skill set, training, planning, critical thinking, and customer focus. In the first stage, the authors applied structural equation modelling to ascertain the causal relationship of the attributes with leanness, while in the second stage, a TOPSIS-based MCDM framework was utilized to prioritize the attributes and carry out a comparative assessment of selected MSME units. Prabhakar et al. [49] endeavoured to identify the enablers of LM and prioritize using fuzzy AHP-ISM method. Singh et al. [50] took the discussion to a different level by incorporating the environmental aspect and considered product quality, environmental impact, green product development, and optimization of cost for successfully ensuring leanness in MSMEs. The authors applied the best-worst method (BWM). The work of [51] used the fuzzy AHP-DEMATEL framework to prioritize enablers and barriers and reported that management support, training and knowledge, and technology are some of the top influencing factors. In [52], the researchers attempted to measure leanness in terms of the financial outcomes. In the same line, the study of [53] applied an integrated AHP-ISM model to identify and rank the enablers for leanness in MSMEs. It is evident from the review of the extant literature that the authors have established importance of LM for MSMEs. The authors have applied various algorithms for investigating critical success factors (CSFs) and challenges of implementation of LM for MSMEs. However, there is a lack of confluence of CSF and measurable attributes for LM and subsequently application of MCDM-based approaches for holistically comparing leanness of MSMEs. It is an established fact that LM enables the MSMEs to achieve competitive advantage. But, in a country like India, there is a lack of governance and awareness about leanness particularly for MSMEs. Of late, National Productivity Council (NPC) of India initiated a nationwide drive for assessing leanness and formulating intervention measures for supporting the MSMEs to embrace LM under the Government of India agenda of Make in India. We notice that in Indian context, a comparative multi-criteria-based analysis of extent of implementation of leanness has not been explored in the extant literature. Table 1 provides a comparative study of the present paper with some of the past work.

SFS has garnered attention from the researchers for extensive use in solving various real-life issues like medical diagnosis problem using trigonometric similarity measures and Choquet integral-based SF operation [54]; 3D printer selection using interval valued SF-additive ratio assessment (ARAS) method [55]; waste disposal location selection using SF-REGIME approach [56]; insurance policy selection using SF bi-objective linear decision-making model [57]; SF-analytic hierarchy process for supplier selection [58]; process mining application with SF-AHP [59]; energy management using SF linear Diophantine fuzzy soft rough sets [60]; cosine similarity-based medical diagnosis [61]; advertisement strategy formulation with SF-TOPSIS method [62]; Earth science application with SF [63]; SF-based failure mode and effect analysis in marble manufacturing [64]; assessment of efficacies of Facebook ads using SF-VIKOR in a group decision-making setup [65]; present value analysis in wealth management [66]; agricultural management in Industry 4.0 using interval-valued SF [67]; hospital performance evaluation using interval-valued SF-AHP [68]; and fraud detection with interval-valued AHP-MULTIMOORA method [69], among others. We have found that though there is an increasing number of applications of SFS in variety of areas, there is scope for further extensions of existing MCDM algorithms.

LBWA has been a popular method of late as is evident from the literature. The extant literature shows application of LBWA in various real-life situations, for example, social entrepreneurship [70], facility location planning [71, 72], talent acquisition [73], technology management [74], risk management in merger and acquisition [75], supplier selection [76], military applications [77–80], fleet management [81], healthcare operation [82, 83], and energy management and preservation [84, 85], among others. On the other hand, the applications of the MULTIMOOSRAL method have not yet reached the level of proliferation. Some of the recent applications of MULTIMOOSRAL method include supplier selection [35] and sustainable energy source selection [86], among others.

3. Preliminaries

In this section, we discuss about definitions and some fundamental properties and operations of SFS and SFN based on the past work [28, 31, 32, 87–89].

Let U be the universe of discourse.

Definition 1. A spherical fuzzy set (SFS) is defined as

$$\widetilde{S} = \left\{x, \left(\mu_{\widetilde{S}}(x), \vartheta_{\widetilde{S}}(x), \gamma_{\widetilde{S}}(x)\right) | x \in U\right\}, \tag{1}$$

where $\mu_{\widetilde{S}}(x), \vartheta_{\widetilde{S}}(x), \gamma_{\widetilde{S}}(x): U \longrightarrow [0,1]$; $0 \leq \mu_{\widetilde{S}}(x)^2 + \vartheta_{\widetilde{S}}(x)^2 + \gamma_{\widetilde{S}}(x)^2 \leq 1 \quad \forall x \in U$.

$\mu_{\widetilde{S}}(x), \vartheta_{\widetilde{S}}(x), \gamma_{\widetilde{S}}(x)$, respectively, represent the degree of positive, negative, and hesitancy.

Table 1: Comparison of the present study with some of the related work.

Author(s) (paper reference)	Objective(s) of the study: Establishing the importance of leanness for business growth	Measures of leanness	Critical success factor for implementation of LM	Multi-criteria-based comparison of leanness achievement of organizations	Application for MSME	Analysis with imprecise information
[36]	✓	✓				
[37]	✓		✓			
[38]			✓			
[39]		✓				
[40]		✓				
[41]			✓			
[42]		✓				
[45]	✓					
[46]		✓	✓			
[47]	✓	✓				
[48]		✓	✓	✓	✓	
[49]			✓		✓	
[50]		✓	✓		✓	
[51]			✓		✓	
[52]		✓			✓	
[53]			✓		✓	
Present study		✓		✓	✓	✓

Figure 1 gives a pictorial representation of the difference among IFS, type 2 IFS (IFS2), neutrosophic fuzzy set (NS), and SFS.

Definition 2. Basic operations.

Let us represent the SFS in terms of the spherical fuzzy number (SFN) as $\widetilde{S} = \{\mu, \vartheta, \gamma\}$ without losing the meaning of usual terms. Let $\widetilde{S}_1 = \{\mu_1, \vartheta_1, \gamma_1\}$ and $\widetilde{S}_2 = \{\mu_2, \vartheta_2, \gamma_2\}$ be two SFNs. Some of the basic operations are defined as follows.

Addition:

$$\widetilde{S}_1 \oplus \widetilde{S}_2 = \left\{\left(\mu_1^2 + \mu_2^2 - \mu_1^2\mu_2^2\right)^{1/2}, \vartheta_1\vartheta_2, \left(\left(1-\mu_2^2\right)\gamma_1^2 + \left(1-\mu_1^2\right)\gamma_2^2 - \gamma_1^2\gamma_2^2\right)^{1/2}\right\}. \quad (2)$$

Multiplication:

$$\widetilde{S}_1 \otimes \widetilde{S}_2 = \left\{\mu_1\mu_2, \left(\vartheta_1^2 + \vartheta_2^2 - \vartheta_1^2\vartheta_2^2\right)^{1/2}, \left(\left(1-\vartheta_2^2\right)\gamma_1^2 + \left(1-\vartheta_1^2\right)\gamma_2^2 - \gamma_1^2\gamma_2^2\right)^{1/2}\right\}. \quad (3)$$

Multiplication by a scalar: $w > 0$.

$$w.\widetilde{S} = \left(1-\left(1-\mu^2\right)^w\right)^{1/2}, \vartheta^w, \left(\left(1-\mu^2\right)^w - \left(1-\mu^2-\gamma^2\right)^w\right)^{1/2}. \quad (4)$$

Power of $\widetilde{S}$: $w > 0$.

$$\widetilde{S}^w = \left\{\mu^w, \left(1-\left(1-\vartheta^2\right)^w\right)^{1/2}, \left(\left(1-\vartheta^2\right)^w - \left(1-\vartheta^2-\gamma^2\right)^w\right)^{1/2}\right\}. \quad (5)$$

Complement of $\widetilde{S}$:

$$\widetilde{S}^c = \{\vartheta, \mu, \gamma\}. \quad (6)$$

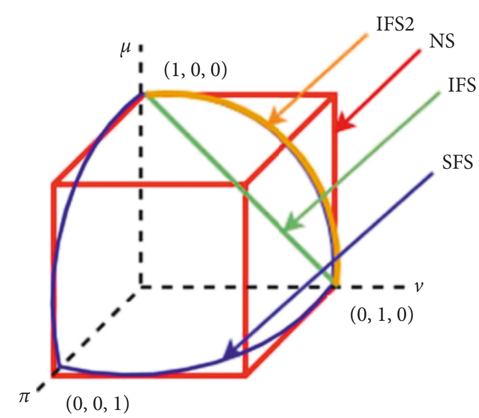


Figure 1: Difference of different types of fuzzy sets (adopted from [28]).

Definition 3. Spherical weighted average.

Let $w = (w_1, w_2, w_3, \ldots, w_n)$ be the weights of the SFNs $\widetilde{S}_1, \widetilde{S}_2, \widetilde{S}_3, \ldots, \widetilde{S}_n$ where n is finite; $w_j \in [0, 1]$; $\sum_{j=1}^{n} w_j = 1$.

Spherical weighted arithmetic average (SWAA) is defined as

$$SWAA_w\left(\widetilde{S}_1, \widetilde{S}_2, \widetilde{S}_3, \ldots, \widetilde{S}_n\right) = \left\{\left[1 - \prod_{i=1}^{n}\left(1-\mu_i^2\right)^{w_i}\right]^{1/2}, \prod_{i=1}^{n}\vartheta_i^{w_i}, \left[\prod_{i=1}^{n}\left(1-\mu_i^2\right)^{w_i} - \prod_{i=1}^{n}\left(1-\mu_i^2-\gamma_i^2\right)^{w_i}\right]^{1/2}\right\}. \quad (7)$$

Spherical weighted geometric average (SWGA) is defined as

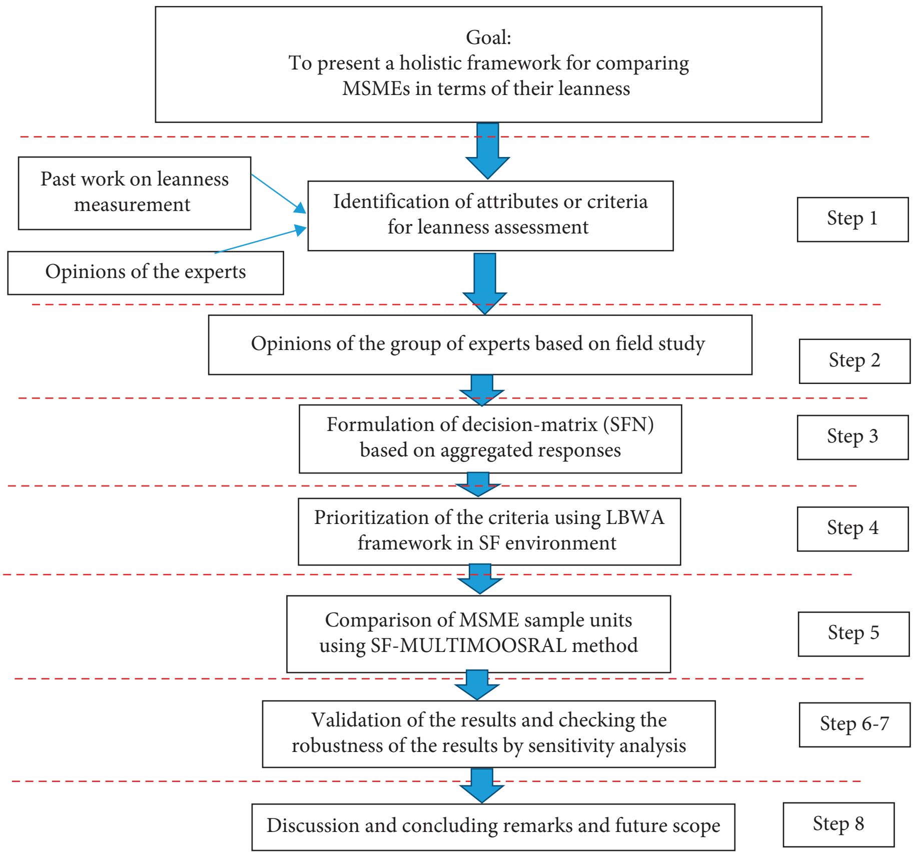


Figure 2: Research framework of the present paper.

$$\begin{aligned} &SWGA_w\left(\tilde{S}_1, \tilde{S}_2, \tilde{S}_3, \ldots, \tilde{S}_n\right) \\ &= \left\{\prod_{i=1}^{n}\mu_i^{w_i}, \left[1 - \prod_{i=1}^{n}\left(1-\vartheta_i^2\right)^{w_i}\right]^{1/2}, \right. \\ &\left. \left[\prod_{i=1}^{n}\left(1-\mu_i^2\right)^{w_i} - \prod_{i=1}^{n}\left(1-\vartheta_i^2-\gamma_i^2\right)^{w_i}\right]^{1/2}\right\}. \end{aligned} \tag{8}$$

Definition 4. Score and accuracy function.

The score function is defined as [31]

$$Sc\left(\tilde{S}\right) = \frac{1}{3}\left(2 + \mu - \gamma - \vartheta\right). \tag{9}$$

The accuracy function is given as [31]

$$Ac\left(\tilde{S}\right) = \left(\mu - \gamma\right). \tag{10}$$

In this context, the certainty function is defined as [31]

$$Cr\left(\tilde{S}\right) = \mu. \tag{11}$$

Rule:

(i) If $Sc(\tilde{S}_1) > Sc(\tilde{S}_2)$, then $\tilde{S}_1 > \tilde{S}_2$.

(ii) If $Sc(\tilde{S}_1) < Sc(\tilde{S}_2)$, then $\tilde{S}_1 < \tilde{S}_2$.

(iii) If $Sc(\tilde{S}_1) = Sc(\tilde{S}_2)$, then

If $Ac(\tilde{S}_1) > Ac(\tilde{S}_2)$, then $\tilde{S}_1 > \tilde{S}_2$.

If $Ac(\tilde{S}_1) < Ac(\tilde{S}_2)$, then $\tilde{S}_1 < \tilde{S}_2$.

(iv) If $Sc(\tilde{S}_1) = Sc(\tilde{S}_2)$ and $Ac(\tilde{S}_1) = Ac(\tilde{S}_2)$, then

If $Cr(\tilde{S}_1) > Cr(\tilde{S}_2)$, then $\tilde{S}_1 > \tilde{S}_2$.

Definition 5. Defuzzification.

The defuzzified value of $\tilde{S}$ is given as

$$S = \left(\left|100 \times \left[\left(3\mu - \frac{\gamma}{2}\right)^2 - \left(\frac{\vartheta}{2} - \gamma\right)^2\right]\right|\right)^{1/2}. \tag{12}$$

4. Materials and Methods

In this section, we present the overall steps of the research methodology followed in this paper and the case study on six MSME sample units under study. The flow of the steps is shown in Figure 2.

4.1. Case Study. In this paper, we consider six MSME sample units belonging to engineering cluster and located in the eastern part of the country. Table 2 provides brief information about the sample units. For confidentiality purpose, we do not disclose their real names in this paper. Therefore, the units are mentioned as $A1, A2, \ldots A6$ in our paper. These units act as alternatives in the multi-criteria decision-making framework presented in this paper. A group of three experts took part in the field study and opinion making. The experts ($E1$, $E2$ and $E3$) have significant experience in

Table 2: Descriptions of the SMEs (sample units or alternatives).

Unit's no.	A1
Category of unit (micro/small/ medium)	Medium
Year of establishment	2006
Turnover	Rs. 200 crores p.a.
Business activity	Manufacturing (main product: welding consumables)
Unit's no.	A2
Category of unit (micro/small/ medium)	Micro
Year of establishment	1994
Turnover	Rs. 2 crores p.a.
Business activity	Designing and manufacturing Manufacturer of machine tools and inspection instruments, die, and spares
Unit's no.	A3
Category of unit (micro/small/ medium)	Micro
Year of establishment	2006
Turnover	Rs. 3 crores p.a.
Business activity	Manufacturing of battery charger, transformer, L.T. control panel
Unit's no.	A4
Category of unit (micro/small/ medium)	Small
Year of establishment	1986
Turnover	Rs. 9.90 cr. p.a. (Unit I), rs. 5.70 cr. (Unit II)
Business activity	UNIT-I: conveyor components, idlers for coal handling plants, steel plants, cement plants, etc. UNIT-II: heavy structural fabrication
Unit's no.	A5
Category of unit (micro/small/ medium)	Small
Year of establishment	2008
Turnover	Rs. 12 crore p.a.
Business activity	Manufacturer of PVC pipe and related products
Unit's no.	A6
Category of unit (micro/small/ medium)	Small
Year of establishment	1992
Turnover	28 crore p.a.
Business activity	Fabrication, forging, heat treatment, machining, assembly of bogie and wagon components, and other engineering goods

implementation of LM techniques in large organizations with industrial experience of 15, 18, and 22 years, respectively. We consider the criteria as derived from the literature (see Table 3).

4.2. LBWA Method. The algorithmic steps of LBWA [34] are briefly mentioned below.

Step 1: determination of the most important criterion.

Let C_j (where, $j = 1, 2, 3, \ldots, n$) be the criteria from the criteria set indicated by $C = \{C_1, C_2, C_3, \ldots, C_n\}$. Let the i^{th} criterion ($C_i \in C$) be the most important criterion according to the decision maker.

Step 2: formation of subsets of criteria by grouping based on level of significance.

The grouping process is demonstrated below.

Table 3: List of criteria.

S/L	Criteria	Effect direction
C1	Top management support and leadership	Max
C2	Lean culture	Max
C3	Communication and awareness	Max
C4	Customer focus	Max
C5	Human resource focus	Max
C6	Process management	Max
C7	Waste	Min
C8	Supplier partnership	Max
C9	Technology usage	Max

Level L_1: group the criteria and form the subset with the criteria having equal to or up to twice as less as the significance of the criterion C_i.

Level L_2: group the criteria and form the subset with the criteria having exactly twice as less as the significance of the criterion C_i or up to three times as less as the significance of the criterion C_i

Level L_3: group the criteria and form the subset with the criteria having exactly three times as less as the significance of the criterion C_i or up to four times as less as the significance of the criterion C_i

Level L_k: group the criteria and form the subset with the criteria having exactly "k" times as less as the significance of the criterion C_i or up to "$k+1$" times as less as the significance of the criterion C_i. Hence,

$$L = L_1 \cup L_2 \cup L_3 \cup L_k. \tag{13}$$

If $l(C_j)$ is the significance of the j^{th} criterion, it can be stated that

$$L_k = \{C_j \in L\colon k \le l(C_j) \le k+1\}. \tag{14}$$

Also, the following condition holds good to appropriately define the grouping:

$$L_p \cap L_q = \varnothing; \text{where} p, q \in \{1, 2, \ldots, k\}, \quad p \neq q. \tag{15}$$

Step 3: find out comparative significance of the criteria within the subsets.

Based on the comparison, each criterion $C_j \in L_k$ is assigned with an integer value $I_{C_j} \in \{0, 1, 2, \ldots, r\}$ where r is the maximum value on the scale for comparison and is given by

$$r = \max\{|L_1|, |L_2|, |L_3| \ldots\ldots |L_k|\}. \tag{16}$$

Conditions followed in this context are

(i) The integer value of the most important criterion, i.e.,

$$I_{C_i} = 0. \tag{17}$$

(ii) If C_p is more significant than C_q, then

$$I_{C_p} < I_{C_q}. \tag{18}$$

(iii) If C_p is equally significant with C_q, then

$$I_{C_p} = I_{C_q}. \tag{19}$$

Step 4: defining the elasticity coefficient.

The elasticity coefficient r_0 is defined as any number belonging the set of real numbers which meets the condition $r_0 > r$ and $r_0 \in \mathbb{R}$ where $\mathbb{R}$ represents a set of real numbers.

Step 5: deriving the influence function of the criteria.

For a particular criterion $C_j \in L_k$, the influence function can be defined as $f\colon L \longrightarrow R$.

It is calculated as

$$f(C_j) = \frac{r_0}{kr_0 + I_{C_j}}, \tag{20}$$

where k is the number of level or subset to which C_j belongs and $I_{C_j} \in \{0, 1, 2, \ldots, r\}$ is the value assigned to the criterion C_j within that level.

Step 6: calculation of the optimum values of the criteria weights for most significant criterion:

$$w_i = \frac{1}{1 + f(C_1) + f(C_2) + \cdots + f(C_n)}, \tag{21}$$

where $i \in j$; $j = 1, 2, \ldots, n$.

For other criteria: $w_{j \neq i} = f(C_j) w_i$.

4.3. MULTIMOOSRAL Method. The computational steps are given below [35].

Step 1. Formation of the evaluation matrix (EM) for decision making.

Let $A = [a_{ij}]_{m \times n}$ be the EM where m is the number of alternatives and n is the number of criteria.

Step 2. Normalize EM.

The normalized EM (NEM) is obtained by

$$b_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{m} (a_{ij})^2}}. \tag{22}$$

Step 3. Calculation of the overall utility of the alternatives using ratio scale (RS) approach.

The following steps are followed.

First, the overall importance of the alternatives is calculated as

$$y_i = \sum_{j \in j^+} w_j b_{ij} - \sum_{j \in j^-} w_j b_{ij}. \tag{23}$$

The overall utility is given by

$$m_i = \begin{cases} y_i; & \text{if } \max_i (y_i) > 0, \\ y_i + 1 & \text{if } \max_i (y_i) = 0, \\ -\dfrac{1}{y_i} & \text{if } \max_i (y_i) < 0. \end{cases} \tag{24}$$

The normalized overall utility is obtained as

$$m_i' = \frac{m_i - \min(m_i)}{\max(m_i) - \min(m_i)}. \tag{25}$$

Step 4. Calculate the utility of alternatives using reference point (RP) approach.

First, the reference point is determined as

$$b^* = (b_1^*, b_2^*, \ldots, b_m^*) = \left\{\max_i b_{ij} \text{if} \quad j \in j^+; \min_i b_{ij} \text{if} \quad j \in j^-;\right\}. \tag{26}$$

The maximal distance of each alternative with respect to the RP is given as

$$t_i = \max_j\left(w_j \left| b_j^* - b_{ij} \right| \right). \tag{27}$$

The normalized maximal distance is obtained as

$$t_i' = \frac{\max(t_i) - t_i}{\max(t_i) - \min(t_i)}. \tag{28}$$

The normalized maximal distance is the overall normalized utility of the alternatives based on RP approach.

Step 5. Obtain the utility of the alternatives using full multiplicative form (FMF).

The overall utility using FMF is given as

$$u_i = \frac{\prod_{j\in j^+} w_j b_{ij}}{\prod_{j\in j^-} w_j b_{ij}}. \tag{29}$$

The normalized overall utility of the alternatives is given as

$$u_i' = \frac{u_i - \min(u_i)}{\max(u_i) - \min(u_i)}. \tag{30}$$

Step 6. Obtain the utility of the alternatives using addition form (AF).

The overall utility using AF is given as

$$v_i = \frac{\sum_{j\in j^+} w_j b_{ij}}{\sum_{j\in j^-} w_j b_{ij}}. \tag{31}$$

The normalized overall utility of the alternatives is given as

$$v_i' = \frac{v_i - \min(v_i)}{\max(v_i) - \min(v_i)}. \tag{32}$$

Step 7. Obtain the utility of the alternatives using logarithmic approach (LA).

The overall utility is given by

$$k_i = \sum_{j\in j^+} \ln\left(1 + w_j b_{ij}\right) + \frac{1}{\sum_{j\in j^-} \ln\left(1 + w_j b_{ij}\right)}. \tag{33}$$

The normalized overall utility of the alternatives is given as

$$k_i' = \frac{k_i - \min(k_i)}{\max(k_i) - \min(k_i)}, \tag{34}$$

Step 8. Ranking of the alternatives based on total utility value.

The total utility value of an alternative is obtained as

$$UV_i = m_i' + u_i' + v_i' + k_i'. \tag{35}$$

The higher the total utility, the better the alternative.

4.4. Proposed SF-LBWA-MULTIMOOSRAL Method. The procedural steps are in line with the descriptions of LBWA and MULTIMOOSRAL method given in Sections 4.2 and 4.3. The steps are given below.

Step 1. Formulate the SF linguistic rating matrix for the criteria for each expert. At this step, SFS-based analysis helps the analyst to select a wide range of values of membership and non-membership.

Step 2. Aggregate the expert opinions using SWGA operator (see expression (8)) to obtain the SF criteria rating matrix. SWGA operator helps to offset variations in the selection of membership and non-membership values.

Step 3. Obtain the score of the SF criteria rating matrix by using expression (9). The score function includes all membership values including degree of hesitancy and therefore is an improved measure of uncertainty.

Step 4. Follow the steps of the LBWA method (see Section 4.2 and expressions (13)–(21)) to derive the criteria weights.

Step 5. Formulate the SF linguistic rating matrix for the alternatives with respect to the criteria for each expert.

Step 6. Aggregate the expert opinions using SWGA operator (see expression (8)) to obtain the SF evaluation matrix (SFEM).

Step 7. Normalize the SFEM (NSFEM), Here,

$$\begin{aligned} \widetilde{S} &= \widetilde{S} \quad \text{for } j \in j^+, \\ \widetilde{S} &= \widetilde{S}^c \quad \text{for } j \in j^-. \end{aligned} \tag{36}$$

Use expression (6).

Step 8. Obtain the score values of NSFEM.

Step 9. Follow steps 3 to 8 (expressions (23) to (35)) of the MULTIMOOSRAL approach (see Section 4.3) to rank the alternatives.

5. Results and Discussion

We use the linguistic rating scale for criteria rating as given in Table 4.

The experts expressed their rating to prioritize the criteria as per their relative importance as given in Table 5. In our problem, we have 9 criteria. Use of LBWA helps to reduce the number of pairwise comparisons substantially than AHP. In addition, for a large criteria set, AHP finds it difficult to reach the consistency. Therefore, LBWA provides the advantages like reduction in computational complexity and subjective bias.

We apply the SWGA operator (see expression (8)) to aggregate the individual responses for obtaining the SF criteria rating matrix whose elements are SFNs and apply expression (9) to derive corresponding weights. Table 6 provides the SF criteria rating matrix and corresponding score values.

We now proceed to find out the criteria weights using the LBWA method. We follow the procedural steps as given in

Table 4: Linguistic scale and SFN values for criteria rating.

Linguistic term	μ	ν	Υ
Very high (VH)	0.9	0.1	0.1
High (H)	0.7	0.3	0.3
Moderate (M)	0.5	0.5	0.5
Low (L)	0.3	0.7	0.3
Very low (VL)	0.1	0.9	0.1

Table 5: Experts' rating of the criteria.

Criteria	Expert		
	E1	E2	E3
C1	VH	H	VH
C2	H	VH	H
C3	L	L	M
C4	VH	VH	H
C5	H	M	H
C6	M	H	H
C7	H	VH	VH
C8	L	VL	L
C9	L	H	M

Table 6: SF criteria rating values and scores.

Criteria	μ	ν	Υ	Score
C1	0.57	0.329	0.323	0.638
C2	0.44	0.424	0.401	0.538
C3	0.05	0.897	0.327	0.274
C4	0.57	0.329	0.323	0.638
C5	0.25	0.616	0.534	0.365
C6	0.25	0.616	0.534	0.365
C7	0.57	0.329	0.323	0.638
C8	0.01	0.975	0.133	0.300
C9	0.11	0.807	0.419	0.293

Section 4.2. As we see, C1 has the highest score value of 0.638. Therefore, we compare all other criteria with respect to C1. The integer value assigned to C1 is zero. Following the steps of LBWA, we partition the criteria as C1, C7, C4, C2, C5, and C6 in level 1 and C8, C9, and C3 in the level 2. The final criteria weights are given in Table 7 along with their respective functional values.

Now we move to rank the alternatives using experts' opinions. The experts carried out field visits to investigate the leanness of the organizations and rate the sample units with respect to the criteria considered using the rating scale as given in Table 8.

Accordingly, the sample units (alternatives) are rated by the individual experts (see Tables 9–11).

We then aggregate the opinions using SWGA operator and derive the SFEM (see Table 12). We normalize the SFEM using expression (36) and apply expression (9) to get the score values of the SFEM and NSFEM (see Tables 13 and 14).

To find the weighted NSFEM, we use expression (4), and thereafter, we find the score values of the weighted NSFEM (see Table 15). This is required for the usual steps of MULTIMOOSRAL starting from step 3 (see Section 4.3).

Table 7: Criteria weights (LBWA method).

Criteria	Function	Weight
C1	1.000	0.170
C2	0.700	0.119
C3	0.412	0.070
C4	0.778	0.132
C5	0.636	0.108
C6	0.583	0.099
C7	0.875	0.149
C8	0.467	0.079
C9	0.438	0.074
Σ		1.0000

Table 8: Rating scale for ranking alternatives.

Linguistic term	μ	ν	Υ
Completely lean (CL)	0.9	0.1	0.1
Largely lean (LL)	0.7	0.3	0.3
Moderately lean (ML)	0.5	0.5	0.5
Largely traditional (LT)	0.3	0.7	0.3
Completely traditional (CT)	0.1	0.9	0.1

Table 9: Rating of alternatives by first expert.

Criteria	C1	C2	C3	C4	C5	C6	C7	C8	C9
Alternatives	(+)	(+)	(+)	(+)	(+)	(+)	(−)	(+)	(+)
A1	CL	CL	ML	LL	LL	ML	ML	CL	ML
A2	LL	CL	ML	LL	ML	LT	LT	ML	ML
A3	CL	CL	LL	ML	LL	LL	CT	LL	LT
A4	ML	LL	LT	CL	CT	ML	ML	LT	CT
A5	LL	ML	CL	ML	LL	ML	LL	ML	ML
A6	LT	LL	ML	LL	LT	CT	ML	ML	CT

Table 10: Rating of alternatives by second expert.

Criteria	C1	C2	C3	C4	C5	C6	C7	C8	C9
Alternatives	(+)	(+)	(+)	(+)	(+)	(+)	(−)	(+)	(+)
A1	LL	CL	ML	LL	LL	LL	LL	CL	LT
A2	LL	ML	ML	ML	ML	LT	ML	LL	ML
A3	ML	LL	LL	LT	LT	CL	LT	ML	CT
A4	LL	ML	CT	CL	CT	ML	CT	LT	CT
A5	ML	CT	CL	ML	LL	LT	LL	ML	ML
A6	LT	ML	ML	LL	CT	CT	ML	ML	CT

Table 11: Rating of alternatives by third expert.

Criteria	C1	C2	C3	C4	C5	C6	C7	C8	C9
Alternatives	(+)	(+)	(+)	(+)	(+)	(+)	(−)	(+)	(+)
A1	CL	CL	LL	CL	ML	LL	ML	LL	ML
A2	CL	LL	LL	ML	LL	CT	LT	ML	LT
A3	LL	LL	ML	LT	ML	LL	CT	ML	ML
A4	LT	ML	LT	LL	CT	ML	LT	CT	LT
A5	LL	LT	LL	ML	LL	LT	ML	LT	ML
A6	CT	ML	ML	LL	LT	CT	ML	ML	CT

We now follow the usual steps of MULTIMOOSRAL (see Section 4.3) to find out the normalized overall utility values of the alternatives using RS, RP, AF, FMF, and LA and

Table 12: SF-evaluation matrix.

Weight		0.1698			0.1189			0.0699	
Criteria		C1			C2			C3	
Alternatives		(+)			(+)			(+)	
A1	0.567	0.329	0.323	0.729	0.172	0.171	0.175	0.699	0.554
A2	0.441	0.424	0.401	0.315	0.569	0.523	0.175	0.699	0.554
A3	0.315	0.569	0.523	0.441	0.424	0.401	0.245	0.616	0.534
A4	0.105	0.807	0.419	0.175	0.699	0.554	0.009	0.975	0.133
A5	0.245	0.616	0.534	0.015	0.963	0.187	0.567	0.329	0.323
A6	0.009	0.975	0.133	0.175	0.699	0.554	0.125	0.760	0.545
Weight		0.1321			0.1081			0.0991	
Criteria		C4			C5			C6	
Alternatives		(+)			(+)			(+)	
A1	0.441	0.424	0.401	0.245	0.616	0.534	0.245	0.616	0.534
A2	0.175	0.699	0.554	0.175	0.699	0.554	0.009	0.975	0.133
A3	0.045	0.897	0.327	0.105	0.807	0.419	0.441	0.424	0.401
A4	0.567	0.329	0.323	0.001	0.997	0.032	0.125	0.760	0.545
A5	0.125	0.760	0.545	0.343	0.496	0.450	0.045	0.897	0.327
A6	0.343	0.496	0.450	0.009	0.975	0.133	0.001	0.997	0.032
Weight		0.1486			0.0793			0.0743	
Criteria		C7			C8			C9	
Alternatives		(−)			(+)			(+)	
A1	0.175	0.699	0.554	0.567	0.329	0.323	0.075	0.844	0.426
A2	0.045	0.897	0.327	0.175	0.699	0.554	0.075	0.844	0.426
A3	0.003	0.991	0.069	0.175	0.699	0.554	0.015	0.963	0.187
A4	0.015	0.963	0.187	0.009	0.975	0.133	0.003	0.991	0.069
A5	0.245	0.616	0.534	0.075	0.844	0.426	0.125	0.760	0.545
A6	0.125	0.760	0.545	0.125	0.760	0.545	0.001	0.997	0.032

Table 13: Score values of SFEM.

Weight	0.1698	0.1189	0.0699	0.1321	0.1081	0.0991	0.1486	0.0793	0.0743
Criteria	C1	C2	C3	C4	C5	C6	C7	C8	C9
Alternatives									
A1	0.638	0.795	0.307	0.538	0.365	0.365	0.307	0.638	0.268
A2	0.538	0.407	0.307	0.307	0.307	0.300	0.274	0.307	0.268
A3	0.407	0.538	0.365	0.274	0.293	0.538	0.314	0.307	0.288
A4	0.293	0.307	0.300	0.638	0.324	0.273	0.288	0.300	0.314
A5	0.365	0.288	0.638	0.273	0.466	0.274	0.365	0.268	0.273
A6	0.300	0.307	0.273	0.466	0.300	0.324	0.273	0.273	0.324

Table 14: Score values of NSFEM.

Weight	0.170	0.119	0.070	0.132	0.108	0.099	0.149	0.079	0.074
Criteria	C1	C2	C3	C4	C5	C6	C7	C8	C9
Alternatives									
A1	0.6384	0.7954	0.3075	0.5385	0.3652	0.3652	0.6566	0.6384	0.2680
A2	0.5385	0.4074	0.3075	0.3075	0.3075	0.3004	0.8418	0.3075	0.2680
A3	0.4074	0.5385	0.3652	0.2736	0.2927	0.5385	0.9728	0.3075	0.2884
A4	0.2927	0.3075	0.3004	0.6384	0.3241	0.2733	0.9204	0.3004	0.3143
A5	0.3652	0.2884	0.6384	0.2733	0.4656	0.2736	0.6123	0.2680	0.2733
A6	0.3004	0.3075	0.2733	0.4656	0.3004	0.3241	0.6968	0.2733	0.3241

Table 15: Score values of the weighted NSFEM.

Criteria Alternatives	C1 (+)	C2 (+)	C3 (+)	C4 (+)	C5 (+)	C6 (+)	C7 (−)	C8 (+)	C9 (+)
A1	0.421	0.466	0.303	0.369	0.313	0.313	0.399	0.382	0.304
A2	0.379	0.324	0.303	0.296	0.299	0.321	0.513	0.302	0.304
A3	0.327	0.366	0.314	0.303	0.304	0.360	0.697	0.302	0.319
A4	0.299	0.298	0.323	0.405	0.330	0.293	0.594	0.322	0.328
A5	0.313	0.315	0.377	0.289	0.341	0.306	0.388	0.303	0.297
A6	0.318	0.298	0.298	0.345	0.321	0.330	0.410	0.296	0.331

Table 16: Utility values and final ranking of the alternatives (MULTIMOOSRAL method).

Alternatives	mi'	ti'	Ui'	Vi'	Ki,	Si	Rank
A1	1.000	1.000	1.000	1.000	1.000	5.000	1
A2	0.204	0.381	0.038	0.348	0.368	1.339	4
A3	0.000	0.180	0.000	0.000	0.000	0.180	6
A4	0.187	0.043	0.064	0.188	0.199	0.680	5
A5	0.446	0.000	0.195	0.813	0.867	2.321	2
A6	0.398	0.061	0.163	0.707	0.756	2.085	3

Table 17: Ranking comparison.

Alternatives	Ranking order	
	SF-MULTIMOOSRAL	SF-TOPSIS
A1	1	1
A2	4	3
A3	6	5
A4	5	6
A5	2	2
A6	3	4

Table 18: Spearman's rank correlation test.

Spearman's rho	SF_TOPSIS
SF_MULTIMOOSRAL	0.886*

*Correlation is significant at the 0.05 level (2-tailed).

to calculate the total overall utility values. Table 16 provides the final ranking of the alternatives.

5.1. Validation and Sensitivity Analysis. The results obtained by using multi-criteria decision-making (MCDM) methods, especially in a group decision-making setup, are vulnerable to the changes in the given conditions such as changes in the criteria values, alternative and criteria set, exclusion or inclusion of the criteria and alternatives, and changes in the weights, among others [90–92]. Therefore, it is essential to examine the validity testing and checking of stability in the results.

In this paper, for validation purpose, we use the methodology followed in [93–95]. We utilize the score values of the SFEM to carry out the usual steps of the TOPSIS method [96]. TOPSIS allows the researchers to compare the alternatives in terms of Euclidean distance with respect to two extreme points, i.e., positive and negative ideal solutions. It considers the alternatives having farthest distance from negative ideal as the best one. PROBID considers all possible positive ideal solutions and also takes into account the distance from the average point like EDAS. Table 17 shows that the ranking results obtained from our proposed methodology and SF score based TOPSIS are consistent to each other. Table 18 statistically confirms the statement by Spearman's rank correlation test.

To examine the stability in the result, we carry out the sensitivity analysis as conducted in [97–99]. Table 19 exhibits the scheme for sensitivity analysis. We exchange the weights of the top priority criterion, C1, with all others and carry out eight experiments. Figure 3 provides the result of sensitivity analysis and pictorially confirms that our method provides absolutely stable result. However, to statistically confirm the fact, we conduct Friedman test (Table 20) and Kendall's test (Table 21) using the final overall utility values of the alternatives under different experiments. We observe that statistically no significant change is incurred. Figure 4 reflects the findings of Tables 20 and 21.

5.2. Comparative Analysis of the Present Framework with Some of the Existing Models. The MULTIMOOSRAL method considers logarithmic approximations in addition to weighted sum and weighted product (as used in WASPAS and CoCoSo methods). Like COPRAS method, MULTIMOOSRAL algorithm also considers ratio system and reference point. Therefore, MULTIMOOSRAL can be considered as an improved version of MCDM method that works with a wide range of performance values of the alternatives. According to Brauers and Zavadskas [100], a combination of three different types of operators provides more reliable and robust solution. Therefore, MULTIMOOSRAL provides reasonably robust and reliable solutions by combining three different types of operators, namely, weighted sum, weighted product, and logarithmic approximations. Further, unlike MOORA and MULTIMOORA approaches (that uses dominance theory), the present method ranks the alternative units using a combined performance scores of four types of utility values. On the other hand, LBWA works efficiently with a large criteria set, reduces the computational complexity, and reasonably offsets the subjective bias. Furthermore, it provides a greater flexibility to the decision makers by varying the values of elasticity coefficient. With these added methodological benefits and use of SFN, our model allows the decision

Table 19: Sensitivity analysis scheme.

Criteria	C1	C2	C3	C4	C5	C6	C7	C8	C9
Original	0.1698	0.1189	0.0699	0.1321	0.1081	0.0991	0.1486	0.0793	0.0743
Exp1	**0.1189**	**0.1698**	0.0699	0.1321	0.1081	0.0991	0.1486	0.0793	0.0743
Exp2	**0.0699**	0.1189	**0.1698**	0.1321	0.1081	0.0991	0.1486	0.0793	0.0743
Exp3	**0.1321**	0.1189	0.0699	**0.1698**	0.1081	0.0991	0.1486	0.0793	0.0743
Exp4	**0.1081**	0.1189	0.0699	0.1321	**0.1698**	0.0991	0.1486	0.0793	0.0743
Exp5	**0.0991**	0.1189	0.0699	0.1321	0.1081	**0.1698**	0.1486	0.0793	0.0743
Exp6	**0.1486**	0.1189	0.0699	0.1321	0.1081	0.0991	**0.1698**	0.0793	0.0743
Exp7	**0.0793**	0.1189	0.0699	0.1321	0.1081	0.0991	0.1486	**0.1698**	0.0743
Exp8	**0.0743**	0.1189	0.0699	0.1321	0.1081	0.0991	0.1486	0.0793	**0.1698**

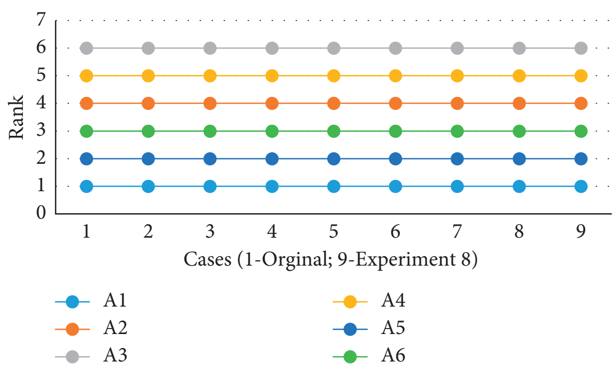


Figure 3: Result of sensitivity analysis.

Table 20: Friedman test result.

Chi-square	10.384
df	8
Asymp. Sig.	0.239

maker to derive more accurate and reliable decisions while working with imprecise information.

6. Discussion

We observe that experts put more emphasis on committed leadership, waste reduction, and customer value which are the cornerstone principles of lean. The primary focus is on achieving more with less, and hence, technology usage does not fetch more weight. We find that committed leadership, waste reduction, and customer value are given more weightage by the experts for achieving leanness in SMEs. Furthermore, the results show that medium and small units with focused product line (A1 and A5) score high in terms of leanness. The findings imply that there is a need to focus on microunits and incorporate policies to revive them through effective implementation of LM.

The present paper provides a SFS-based analysis that provides more flexibility with reasonably less complexity to the analysts as compared with IFS, PyFS, PFS, and qROFS. Therefore, our framework has extended the growing strand of literature with a new MCDM framework with uncertain information that can work with

Table 21: Result of Kendall test (Kendall's coefficient of concordance).

Kendall's W	0.216
Chi-square	10.384
df	8
Asymp. Sig.	0.239

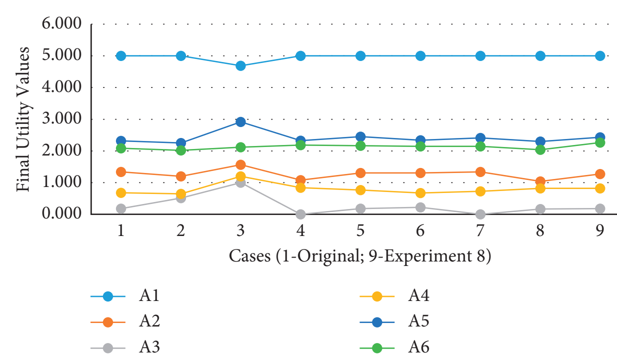


Figure 4: Final overall utility values of the alternatives under different experiments.

larger criteria and alternative set with reasonable accuracy and stability. However, the model proposed in this paper may be fine-tuned with using type 3 fuzzy logic which is an improved version of generalized type 2 fuzzy system for handling susceptibility of MCDM models in handling uncertainties. In recent times, several researchers (for example, [101–103]) have used type 3 fuzzy based analysis in solving complex real-life problems. These models may be used in solving our problem and a comparative analysis may be carried out.

Nevertheless, the findings of the present paper provide an important direction to the strategic decision makers as it is revealed that concentrated effort in product offerings lowers the possibility of waste which might help in achieving leanness. After achieving leanness, the organization may take the practice forward for reaching to the level of maturity and move forward to customization. An organization wide approach supported by top management is the necessity. However, we contend that in Indian context, still microorganizations need policy support and fund mobilization with better governance for achieving leanness.

7. Conclusion

In this paper, we have conducted a case study on six MSME sample units producing engineering products. We have presented a new hybrid SF-LBWA-MULTIMOOSRAL framework to carry out a comparative assessment of the leanness of the sample units. We consider 9 criteria from the perspectives of leadership, supplier focus, customer focus, process management, waste, culture, human resource focus, technology use, and communication and awareness. In this aspect, the present study provides a more holistic approach than the existing research papers which multi-criteria-based comparison of MSMEs vis-à-vis leanness is quite rare. The criteria are obtained through literature survey and opinions of the three experts who took part in our study. The experts carried out a field study and rated the sample units. We observe that medium and small units having focused product portfolio score high in terms of leanness. The present paper provides a holistic multi-criteria-based assessment of leanness which is not seen in plenty in Indian context. Further, we propose a novel extension of LBWA and MULTIMOOSRAL with SFN. SFNs have been proven as superior than IFS, PyFS, and PFS as evident from the discussion in the extant literature. Therefore, the present work provides a more flexible and effective framework for group decision making. Further, in the previous work, we have not noticed any attempt to integrate LBWA and MULTIMOOSRAL approach although these methods possess substantial benefits. The result of validation test and sensitivity analysis suggests that our method provides stable and accurate result.

However, one of the limitations of our model is that given the close rating of the criteria, our model may not give distinct partitioning of the criteria. In a further study, one may attempt to examine the causal relationship of the criteria with the soft and hard outcomes of practicing LM. Further, our model may be tested in other complex scenarios. In addition, in the present study, we did not calculate the time complexity which may be another limitation. Nevertheless, we are hopeful that our model may solve other complex real-life problems and the framework of measuring leanness shall provide a holistic and easy way to assess the performance of the MSME units.

Data Availability

The responses used to support the findings of this study are provided in the paper. For the sake of confidentiality, the names of the respondents are not disclosed.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All authors contributed equally and significantly in conducting this research work and writing this paper.

References

[1] J. G. Sarhan, B. Xia, S. Fawzia, A. Karim, A. O. Olanipekun, and V. Coffey, "Framework for the implementation of lean construction strategies using the interpretive structural modelling (ISM) technique: a case of the Saudi construction industry," *Engineering Construction and Architectural Management*, vol. 27, no. 1, pp. 1–23, 2019.

[2] S. Biswas, "Implications of industry 4.0 vis-à-vis lean Six-Sigma: a multi-criteria group decision approach," in *Proceedings of the JD Birla International Management Conference on "Strategic Management in Industry*, Technology Driven Operations Management, Kolkata, India, September 2019.

[3] J. F. Krafcik, "Triumph of the lean production system," *Sloan Management Review*, vol. 30, no. 1, pp. 41–52, 1988.

[4] J. P. Womack, D. T. Jones, and D. Roos, *The Machine that Changed the World: The story of Lean Production--Toyota's Secret Weapon in the Global Car Wars that Is Now Revolutionizing World Industry*, Simon & Schuster, New York, NY, USA, 2007.

[5] S. Gupta, M. Sharma, and M. V. Sunder, "Lean services: a systematic review," *International Journal of Productivity and Performance Management*, vol. 65, no. 8, pp. 1025–1056, 2016.

[6] W. J. Hopp and M. L. Spearman, "To pull or not to pull: what is the question?" *Manufacturing & Service Operations Management*, vol. 6, no. 2, pp. 133–148, 2004.

[7] A. Anvari, N. Zulkifli, S. Sorooshian, and O. Boyerhassani, "An integrated design methodology based on the use of group AHP-DEA approach for measuring lean tools efficiency with undesirable output," *International Journal of Advanced Manufacturing Technology*, vol. 70, no. 9-12, pp. 2169–2186, 2014.

[8] T. T. Pullan, M. Bhasi, and G. Madhu, "Decision support tool for lean product and process development," *Production Planning & Control*, vol. 24, no. 6, pp. 449–464, 2013.

[9] A. Chiarini and F. Brunetti, "What really matters for a successful implementation of Lean production? A multiple linear regression model based on European manufacturing companies," *Production Planning & Control*, vol. 30, no. 13, pp. 1091–1101, 2019.

[10] G. Pal and S. Biswas, "A review of the critical factors affecting the success of lean implementation in Indian manufacturing organizations – challenges & way forward," *CBS Journal of Management Practices*, vol. 2, pp. 103–112, 2015.

[11] IBEF report, "MSME Industry in India," 2021, https://www.ibef.org/industry/msme.aspx.

[12] I. R. Andaningsih and A. H. Susanto, "Empowering MSMEs in the creative economy of the agrobusiness industrial sector in the baranangsiang area, BogorCity, west java," *Asian Journal of Management, Entrepreneurship and Social Science*, vol. 2, no. 01, pp. 48–59, 2022.

[13] S. K. Das and M. Chakraborty, "A Forecasting Model for MSMEs of Manufacturing and Engineering Sector of India," *SEDME (Small Enterprises Development, Management & Extension Journal)*, vol. 48, no. 1, pp. 66–73, Article ID 09708464211054882, 2022.

[14] S. Gupta and V. Singh, "Determinants influencing the growth of MSMEs women entrepreneurs in India," *Gorteria Journal*, vol. 34, no. 5, pp. 198–205, 2021.

[15] N. K. M. Tri Utari, "Marketing strategy of small and medium Enterprises (MSMES) through instagram in the marketing

era 4.0," *Jurnal Ekonomi & Bisnis JAGADITHA*, vol. 8, no. 1, pp. 15–22, 2021.
[16] R. Basak and M. D. Gupta, "Literature review on importance of MSMEs in empowerment of women participants," *Journal of Entrepreneurship and Management*, vol. 7, no. 3, p. 22, 2018.
[17] O. S. Ajuwon, S. Ikhide, and J. O. Akotey, "MSMEs and employment generation in Nigeria," *The Journal of Developing Areas*, vol. 51, no. 3, pp. 229–249, 2017.
[18] S. K. Dey, "MSMEs in India: it's growth and prospects," *Abhinav National Monthly Refereed Journal of Research in Commerce & Management*, vol. 3, no. 8, pp. 26–33, 2014.
[19] I. Bhattacharya and A. Ramachandran, "Lean manufacturing techniques–Implementation in Indian MSMEs and benefits realized thereof," *Indian Journal of Engineering and Materials Sciences*, vol. 28, pp. 89–101, 2021.
[20] M. N. Mishra, A. Mohan, and A. Sarkar, "Role of lean six sigma in the Indian MSMEs during COVID-19," *International Journal of Lean Six Sigma*, vol. 12, no. 4, pp. 697–717, 2021.
[21] L. A. Zadeh, "Fuzzy sets," *Fuzzy sets, Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.
[22] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, 1986.
[23] K. T. Atanassov and G. Gargov, "Interval valued intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 31, no. 3, pp. 343–349, 1989.
[24] R. R. Yager, "Pythagorean fuzzy subsets," in *Proceedings of the 2013 Joint IFSA World congress and NAFIPS Annual Meeting (IFSA/NAFIPS)*, pp. 57–61, IEEE, Edmonton, Canada, June 2013.
[25] R. R. Yager, "Generalized orthopair fuzzy sets," *IEEE Transactions on Fuzzy Systems*, vol. 25, no. 5, pp. 1222–1230, 2017.
[26] B. C. Cuong and V. Kreinovich, "Picture fuzzy sets," *Journal of Computer Science and Cybernetics*, vol. 30, no. 4, pp. 409–420, 2014.
[27] B. C. Cuong and V. Kreinovich, "Picture Fuzzy Sets-A New Concept for Computational Intelligence Problems," in *Proceedings of the 2013 Third World congress on Information and Communication Technologies (WICT 2013)*, pp. 1–6, IEEE, Hanoi, Vietnam, December 2013.
[28] F. K. Gündoǧdu and C. Kahraman, "Spherical fuzzy analytic hierarchy process (ahp) and its application to industrial robot selection," in *Proceedings of the International Conference on Intelligent and Fuzzy Systems*, pp. 988–996, Springer, New York, NY, USA, January 2019.
[29] F. Smarandache, "A unifying field in logics: neutrosophic logic," in *Philosophy*, pp. 1–141, American Research Press, Mexico, MX, USA, 1999.
[30] K. Atanassov, *Geometrical Interpretation of the Elements of the Intuitionistic Fuzzy Objects*, pp. 1–89, Preprint IM-MFAIS, Sofia, Bulgaria, 1989.
[31] S. Ashraf, S. Abdullah, T. Mahmood, F. Ghani, and T. Mahmood, "Spherical fuzzy sets and their applications in multi-attribute decision making problems," *Journal of Intelligent and Fuzzy Systems*, vol. 36, no. 3, pp. 2829–2844, 2019.
[32] A. Menekşe and H. Camgöz Akdağ, "Distance education tool selection using novel spherical fuzzy AHP EDAS," *Soft Computing*, vol. 26, no. 4, pp. 1617–1635, 2022.
[33] T. Senapati and R. R. Yager, "Fermatean fuzzy sets," *Journal of Ambient Intelligence and Humanized Computing*, vol. 11, no. 2, pp. 663–674, 2020.
[34] M. Žižović and D. Pamucar, "New model for determining criteria weights: level Based Weight Assessment (LBWA) model," *Decision Making: Applications in Management and Engineering*, vol. 2, no. 2, pp. 126–137, 2019.
[35] A. Ulutaş, D. Stanujkic, D. Karabasevic et al., "Developing of a novel integrated MCDM MULTIMOOSRAL approach for supplier selection," *Informatica*, vol. 32, no. 1, pp. 145–161, 2021.
[36] S. M. Seyedhosseini, A. E. Taleghani, A. Bakhsha, and S. Partovi, "Extracting leanness criteria by employing the concept of Balanced Scorecard," *Expert Systems with Applications*, vol. 38, no. 8, Article ID 10454, 2011.
[37] S. G. Azevedo, K. Govindan, H. Carvalho, and V. Cruz-Machado, "An integrated model to assess the leanness and agility of the automotive industry," *Resources, Conservation and Recycling*, vol. 66, pp. 85–94, 2012.
[38] M. A. Almomani, A. Abdelhadi, A. Mumani, A. Momani, and M. Aladeemy, "A proposed integrated model of lean assessment and analytical hierarchy process for a dynamic road map of lean implementation," *International Journal of Advanced Manufacturing Technology*, vol. 72, no. 1-4, pp. 161–172, 2014.
[39] B. A. Patil, M. S. Kulkarni, and P. V. M. Rao, "A methodology for assessing leanness in NPD process," in *ICoRD*, vol. 13, pp. 735–744, Springer, New Delhi, India, 2013.
[40] M. A. Maasouman and K. Demirli, "Development of a lean maturity model for operational level planning," *International Journal of Advanced Manufacturing Technology*, vol. 83, no. 5-8, pp. 1171–1188, 2016.
[41] P. Basu, S. Chowdhury, and P. A. Alam, "A model-based approach of flexibility and its impact on organization and employee welfare in lean environment," *Decision*, vol. 42, no. 3, pp. 269–277, 2015.
[42] K. Shrivastava, A. Bose, and S. Biswas, "Assessment of implementation of lean manufacturing practices in Indian plywood industries–A case study," *CBS Journal of Management Practices*, vol. 3, pp. 35–46, 2016.
[43] G. Narayanamurthy and A. Gurumurthy, "Systemic leanness: an index for facilitating continuous improvement of lean implementation," *Journal of Manufacturing Technology Management*, vol. 27, no. 8, pp. 1014–1053, 2016.
[44] M. Soliman, T. A. Saurin, and M. J. Anzanello, "The impacts of lean production on the complexity of socio-technical systems," *International Journal of Production Economics*, vol. 197, pp. 342–357, 2018.
[45] J. R. Kroes, A. S. Manikas, and T. F. Gattiker, "Operational leanness and retail firm performance since 1980," *International Journal of Production Economics*, vol. 197, pp. 262–274, 2018.
[46] J. A. Garza-Reyes, I. E. Betsis, V. Kumar, and M. A. Radwan Al-Shboul, "Lean readiness–the case of the European pharmaceutical manufacturing industry," *International Journal of Productivity and Performance Management*, vol. 67, no. 1, pp. 20–44, 2018.
[47] E. K. Tekez and G. Taşdeviren, "Measuring the influence values of lean criteria on leanness," *Journal of Manufacturing Technology Management*, vol. 31, no. 7, pp. 1391–1416, 2020.
[48] M. M. Ravikumar, K. Marimuthu, P. Parthiban, and H. A. Zubar, "Evaluating lean execution performance in Indian MSMEs using SEM and TOPSIS models," *International Journal of Operational Research*, vol. 26, no. 1, p. 104, 2016.
[49] V. Prabhakar, A. Sagar, and R. Singh, "A comprehensive method for modelling leanness enablers and measuring

leanness index in MSMEs using an integrated AHP-ISM-MICMAC and multi-grade fuzzy approach," *International Journal of Six Sigma and Competitive Advantage*, vol. 13, no. 1/2/3, p. 377, 2021.

[50] M. Singh, R. Rathi, and J. A. Garza-Reyes, "Analysis and prioritization of Lean Six Sigma enablers with environmental facets using best worst method: a case of Indian MSMEs," *Journal of Cleaner Production*, vol. 279, Article ID 123592, 2021.

[51] M. Singh and R. Rathi, "Empirical investigation of lean six sigma enablers and barriers in Indian MSMEs by using multi-criteria decision making approach," *Engineering Management Journal*, vol. 1-22, pp. 1–22, 2021.

[52] M. A. A. Triono, S. T. Ahmad, C. Ermiati, and H. Sihombing, "Can the financial performance of the micro, small, and medium-sized enterprise production sector in Medan be a signal in the use of a leanness strategy?" *F1000Research*, vol. 10, no. 1183, p. 1183, 2021.

[53] V. Prabhakar and A. Sagar, "Using hybrid AHP-ISM technique for modelling of lean management enablers in MSMEs," in *Advances in Industrial and Production Engineering*, pp. 613–619, Springer, Springer, Singapore, 2021.

[54] M. Unver, M. Olgun, and E. Türkarslan, "Cosine and cotangent similarity measures based on Choquet integral for Spherical fuzzy sets and applications to pattern recognition," *Journal of Computational and Cognitive Engineering*, vol. 1, no. 1, pp. 1–11, 2022.

[55] A. Aydoğdu and S. Gül, "New entropy propositions for interval-valued spherical fuzzy sets and their usage in an extension of ARAS (ARAS-IVSFS)," *Expert Systems*, vol. 39, no. 4, pp. 1–19, Article ID e12898, 2021.

[56] B. Oztaysi, C. Kahraman, and S. C. Onar, "Spherical fuzzy REGIME method waste disposal location selection," in *Intelligent and Fuzzy Techniques for Emerging Conditions and Digital Transformation. INFUS 2021. Lecture Notes in Networks and Systems*, C. Kahraman, S. Cebi, S. Cevik Onar, B. Oztaysi, A. C. Tolga, and I. U. Sari, Eds., vol. 308, Cham, Switzerland, Springer, 2022.

[57] S. A. Seyfi-Shishavan, F. Kutlu Gündoğdu, Y. Donyatalab, E. Farrokhizadeh, and C. Kahraman, "A novel spherical fuzzy bi-objective linear assignment method and its application to insurance options selection," *International Journal of Information Technology and Decision Making*, vol. 20, no. 02, pp. 521–551, 2021.

[58] I. M. Sharaf, "Global supplier selection with spherical fuzzy analytic hierarchy process," in *Decision Making with Spherical Fuzzy Sets Studies in Fuzziness and Soft Computing*, C. Kahraman and F. Kutlu Gündoğdu, Eds., vol. 392, Cham, Switzerland, Springer, 2021.

[59] O. Dogan, "Process mining technology selection with spherical fuzzy AHP and sensitivity analysis," *Expert Systems with Applications*, vol. 178, Article ID 114999, 2021.

[60] M. R. Hashmi, S. T. Tehrim, M. Riaz, D. Pamucar, and G. Cirovic, "Spherical linear diophantine fuzzy soft rough sets with multi-criteria decision making," *Axioms*, vol. 10, no. 3, p. 185, 2021.

[61] T. Mahmood, M. Ilyas, Z. Ali, and A. Gumaei, "Spherical fuzzy sets-based cosine similarity and information measures for pattern recognition and medical diagnosis," *IEEE Access*, vol. 9, Article ID 25835, 2021.

[62] E. Farrokhizadeh, S. A. Seyfi-Shishavan, F. Kutlu Gündoğdu, Y. Donyatalab, C. Kahraman, and S. H. Seifi, "A spherical fuzzy methodology integrating maximizing deviation and TOPSIS methods," *Engineering Applications of Artificial Intelligence*, vol. 101, Article ID 104212, 2021.

[63] M. Fernández-Martínez and J. M. Sánchez-Lozano, "Assessment of near-earth asteroid deflection techniques via spherical fuzzy sets," *Advances in Astronomy*, vol. 2021, Article ID 6678056, 12 pages, 2021.

[64] M. Gul and M. F. Ak, "A modified failure modes and effects analysis using interval-valued spherical fuzzy extension of TOPSIS method: case study in a marble manufacturing facility," *Soft Computing*, vol. 25, no. 8, pp. 6157–6178, 2021.

[65] M. Akram, C. Kahraman, and K. Zahid, "Group decision-making based on complex spherical fuzzy VIKOR approach," *Knowledge-Based Systems*, vol. 216, Article ID 106793, 2021.

[66] E. Boltürk and S. Seker, "Present worth analysis using spherical fuzzy sets," in *Intelligent and Fuzzy Techniques for Emerging Conditions and Digital Transformation. INFUS 2021 Lecture Notes in Networks and Systems*, C. Kahraman, S. Cebi, S. Cevik Onar, B. Oztaysi, A. C. Tolga, and I. U. Sari, Eds., vol. vol 308, Cham, Switzerland, Springer, 2022.

[67] M. Erdoğan, "Assessing farmers' perception to Agriculture 4.0 technologies: a new interval-valued spherical fuzzy sets based approach," *International Journal of Intelligent Systems*, vol. 37, no. 2, pp. 1751–1801, 2022.

[68] F. Kutlu Gündoğdu and C. Kahraman, "Hospital performance assessment using interval-valued spherical fuzzy analytic hierarchy process," in *Decision Making with Spherical Fuzzy Sets*, pp. 349–373, Springer, Cham, Switzerland, 2021.

[69] S. Hamal and O. Senvar, "A novel integrated AHP and MULTIMOORA method with interval-valued spherical fuzzy sets and single-valued spherical fuzzy sets to prioritize financial ratios for financial accounting fraud detection," *Journal of Intelligent and Fuzzy Systems*, vol. 42, no. 1, pp. 337–364, 2021.

[70] S. Biswas, S. Majumder, D. Pamucar, and S. K. Dawn, "An extended LBWA framework in picture fuzzy environment using actual score measures application in social enterprise systems," *International Journal of Enterprise Information Systems*, vol. 17, no. 4, pp. 37–68, 2021.

[71] S. Biswas and D. Pamucar, "Facility location selection for b-schools in Indian context: a multi-criteria group decision based analysis," *Axioms*, vol. 9, no. 3, p. 77, 2020.

[72] M. Deveci, E. Özcan, R. John, C. F. Covrig, and D. Pamucar, "A study on offshore wind farm siting criteria using a novel interval-valued fuzzy-rough based Delphi method," *Journal of Environmental Management*, vol. 270, Article ID 110916, 2020.

[73] V. Jakovljevic, M. Zizovic, D. Pamucar, Ž. Stević, and M. Albijanic, "Evaluation of human resources in transportation companies using multi-criteria model for ranking alternatives by defining relations between ideal and anti-ideal alternative (RADERIA)," *Mathematics*, vol. 9, no. 9, p. 976, 2021.

[74] A. E. Torkayesh and S. E. Torkayesh, "Evaluation of information and communication technology development in G7 countries: an integrated MCDM approach," *Technology in Society*, vol. 66, Article ID 101670, 2021.

[75] L. M. Chien and K. J. Tu, "Establishing merger feasibility simulation model based on multiple-criteria decision-making method: case study of taiwan's property management industry," *Sustainability*, vol. 13, no. 5, p. 2448, 2021.

[76] M. Yazdani, D. Pamucar, P. Chatterjee, and A. E. Torkayesh, "A multi-tier sustainable food supplier selection model

under uncertainty," *Operations Management Research*, pp. 1–30, 2021.
[77] D. Božanić, D. Jurišić, and D. Erkić, "LBWA–Z-MAIRCA model supporting decision making in the army," *Operational Research in Engineering Sciences: Theory and Applications*, vol. 3, no. 2, pp. 87–110, 2020a.
[78] D. Božanić, A. Ranđelović, M. Radovanović, and D. Tešić, "A hybrid lbwa-ir-mairca multi-criteria decision-making model for determination of constructive elements of weapons," *Facta Universitatis – Series: Mechanical Engineering*, vol. 18, no. 3, p. 399, 2020b.
[79] N. Hristov, D. Pamucar, and M. E Amine, "Application of a D number based LBWA model and an interval MABAC model in selection of an automatic cannon for integration into combat vehicles," *Defence Science Journal*, vol. 71, no. 1, pp. 34–45, 2021.
[80] Ž. Jokić, D. Božanić, and D. Pamučar, "Selection of fire position of mortar units using LBWA and Fuzzy MABAC model," *Operational Research in Engineering Sciences: Theory and Applications*, vol. 4, no. 1, pp. 115–135, 2021.
[81] D. Pamucar, M. Deveci, F. Canıtez, and V. Lukovac, "Selecting an airport ground access mode using novel fuzzy LBWA-WASPAS-H decision making model," *Engineering Applications of Artificial Intelligence*, vol. 93, Article ID 103703, 2020.
[82] D. Pamučar, M. Žižović, D. Marinković, D. Doljanica, S. V. Jovanović, and P. Brzaković, "Development of a multicriteria model for sustainable reorganization of a healthcare system in an emergency situation caused by the COVID-19 pandemic," *Sustainability*, vol. 12, no. 18, p. 7504, 2020b.
[83] A. E. Torkayesh, D. Pamucar, F. Ecer, and P. Chatterjee, "An integrated BWM-LBWA-CoCoSo framework for evaluation of healthcare sectors in Eastern Europe," *Socio-Economic Planning Sciences*, vol. 70, 2021.
[84] D. Pamučar, M. Behzad, D. Božanić, and M. Behzad, "Decision making to support sustainable energy policies corresponding to agriculture sector: case study in Iran's Caspian Sea coastline," *Journal of Cleaner Production*, vol. 292, Article ID 125302, 2021.
[85] F. Ecer, D. Pamucar, A. Mardani, and M. Alrasheedi, "Assessment of renewable energy resources using new interval rough number extension of the level based weight assessment and combinative distance-based assessment," *Renewable Energy*, vol. 170, pp. 1156–1177, 2021.
[86] S. A. Mousavi, A. Hafezalkotob, V. Ghezavati, and F. Abdi, "An integrated framework for new sustainable waste-to-energy technology selection and risk assessment: an R-TODIM-R-MULTIMOOSRAL approach," *Journal of Cleaner Production*, vol. 335, Article ID 130146, 2022.
[87] S. Ashraf, S. Abdullah, and L. Abdullah, "Child development influence environmental factors determined using spherical fuzzy distance measures," *Mathematics*, vol. 7, no. 8, p. 661, 2019.
[88] M. Kovač, S. Tadić, M. Krstić, and M. B. Bouraima, "Novel Spherical Fuzzy MARCOS Method for Assessment of Drone-Based City Logistics Concepts," *Complexity*, Article ID 2374955, 2021.
[89] J. Ali, "A novel score function based CRITIC-MARCOS method with spherical fuzzy information," *Computational and Applied Mathematics*, vol. 40, no. 8, p. 280, 2021.
[90] P. K. D. Pramanik, S. Biswas, S. Pal, D. Marinković, and P. Choudhury, "A comparative analysis of multi-criteria decision-making methods for resource selection in mobile crowd computing," *Symmetry*, vol. 13, no. 9, p. 1713, 2021.
[91] S. Biswas and D. S. Pamučar, "Combinative distance based assessment (CODAS) framework using logarithmic normalization for multi-criteria decision making," *Serbian Journal of Management*, vol. 16, no. 2, pp. 321–340, 2021.
[92] S. Biswas, D. Pamucar, S. Kar, and S. S. Sana, "A new integrated FUCOM–CODAS framework with fermatean fuzzy information for multi-criteria group decision-making," *Symmetry*, vol. 13, no. 12, p. 2430, 2021.
[93] D. Pamucar, A. E. Torkayesh, and S. Biswas, "Supplier selection in healthcare supply chain management during the COVID-19 pandemic: a novel fuzzy rough decision-making approach," *Annals of Operations Research*, pp. 1–43, 2022.
[94] S. Biswas, D. Pamucar, P. Chowdhury, and S. Kar, "A new decision support framework with picture fuzzy information: comparison of video conferencing platforms for higher education in India," *Discrete Dynamics in Nature and Society*, vol. 10, no. 2, pp. 286–303, 2021.
[95] S. Biswas, S. Majumder, and S. K. Dawn, "Comparing the Socioeconomic Development of G7 and BRICS Countries and Resilience to COVID-19: An Entropy–MARCOS Framework," *Business Perspectives and Research*, vol. 10, no. 2, pp. 286–303, Article ID 22785337211015406, 2021.
[96] P. Karmakar, P. Dutta, and S. Biswas, "Assessment of mutual fund performance using distance based multi-criteria decision making techniques-An Indian perspective," *Research Bulletin*, vol. 44, no. 1, pp. 17–38, 2018.
[97] D. Pamucar, M. Žižović, S. Biswas, and D. Božanić, "A new logarithm methodology of additive weights (LMAW) for multi-criteria decision-making: application in logistics," *Facta Universitatis – Series: Mechanical Engineering*, vol. 19, no. 3, p. 361, 2021.
[98] S. Biswas, "Measuring performance of healthcare supply chains in India: a comparative analysis of multi-criteria decision making methods," *Decision Making: Applications in Management and Engineering*, vol. 3, no. 2, pp. 162–189, 2020.
[99] S. Biswas and O. P. Anand, "Logistics competitiveness index-based comparison of BRICS and G7 countries: an integrated PSI-PIV approach," *IUP Journal of Supply Chain Management*, vol. 17, no. 2, pp. 32–57, 2020.
[100] W. K. M. Brauers and E. K. Zavadskas, "Robustness of MULTIMOORA: a method for multi-objective optimization," *Informatica*, vol. 23, no. 1, pp. 1–25, 2012.
[101] M. W. Tian, S. R. Yan, A. Mohammadzadeh et al., "Stability of interval type-3 fuzzy controllers for autonomous vehicles," *Mathematics*, vol. 9, no. 21, p. 2742, 2021.
[102] A. Mohammadzadeh, M. H. Sabzalian, and W. Zhang, "An interval type-3 fuzzy system and a new online fractional-order learning algorithm: theory and practice," *IEEE Transactions on Fuzzy Systems*, vol. 28, no. 9, pp. 1940–1950, 2020.
[103] J. H. Wang, J. Tavoosi, A. Mohammadzadeh et al., "Non-singleton type-3 fuzzy approach for flowmeter fault detection: experimental study in a gas industry," *Sensors*, vol. 21, no. 21, p. 7419, 2021.

Hindawi
Mathematical Problems in Engineering
Volume 2023, Article ID 9767024, 1 page
https://doi.org/10.1155/2023/9767024

Retraction

Retracted: Grey Relational Analysis Method for Probabilistic Double Hierarchy Linguistic Multiple Attribute Group Decision Making and Its Application to College Tennis Classroom Teaching Effect Evaluation

Mathematical Problems in Engineering

Received 11 July 2023; Accepted 11 July 2023; Published 12 July 2023

This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope
(2) Discrepancies in the description of the research reported
(3) Discrepancies between the availability of data and the research described
(4) Inappropriate citations
(5) Incoherent, meaningless and/or irrelevant content included in the article
(6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] L. Wang, H. Li, J. Zhang, and J. Yang, "Grey Relational Analysis Method for Probabilistic Double Hierarchy Linguistic Multiple Attribute Group Decision Making and Its Application to College Tennis Classroom Teaching Effect Evaluation," *Mathematical Problems in Engineering*, vol. 2022, Article ID 7419496, 17 pages, 2022.

Hindawi
Mathematical Problems in Engineering
Volume 2022, Article ID 7419496, 17 pages
https://doi.org/10.1155/2022/7419496

Research Article

Grey Relational Analysis Method for Probabilistic Double Hierarchy Linguistic Multiple Attribute Group Decision Making and Its Application to College Tennis Classroom Teaching Effect Evaluation

Lihua Wang, Huiming Li , Jianpeng Zhang , and Jin Yang

School of Physical Education, Yunnan Agricultural University, Kunming 650201, Yunnan, China

Correspondence should be addressed to Huiming Li; 2000021@ynau.edu.cn

Received 25 February 2022; Revised 19 April 2022; Accepted 20 April 2022; Published 10 June 2022

Academic Editor: Darko Božanić

The college tennis classroom teaching effect evaluation is viewed as the multiattribute group decision making (MAGDM). The probabilistic double hierarchy linguistic term set (PDHLTS) not only conforms to people's language expression habit of "adverb + adjective" but also can accurately depict its importance in real MAGDM. Therefore, this paper comes up with the probabilistic double hierarchy linguistic grey relational analysis (PDHL-GRA) method based on the grey relational analysis (GRA) process for MAGDM based on PDHLTS environment and applies it to the college tennis classroom teaching effect evaluation. Finally, a practical case for college tennis classroom teaching effect evaluation is presented to demonstrate the steps of our method, and a comparison analysis illustrates its feasibility and effectiveness.

1. Introduction

To better fuse decision information, MAGDM technology came into being [1–5]. After MAGDM theory came into being, it has been widely used in finance, engineering, corporate decision making, and many other aspects [6–10]. In view of the intricateness and fuzzification of decision circumstances [11–15], in many MAGDM issues, expert opinions are often stated as fuzzy data [16–18]. For this reason, Zadeh [19] raised concept of a linguistic variable for approximate reasoning. In many environments, the linguistic variable cannot exactly formulate proficient's perspective. Hence, hesitant fuzzy LTS (HFLTS) was proposed by Rodriguez, Martinez and Herrera [20]. An idea about probabilistic linguistic term sets (PLTSs) was proposed by Pang et al. [21]. Soon afterwards, critical malfunction matters were finished off by the PLAMM and PLWAMM formulas derived by Liu and Teng [22]. The performance estimation system of college teachers was finished off by the PLPA and PLPWG formulas derived by Kobina et al. [23]. Wei et al. [24] built the EDAS method for PL-MAGDM. The extensive similarity measure based on probabilistic language circumstances was derived by Wei et al. [25]. Su et al. [26] defined the PT-TODIM method for PL-MAGDM. Lin et al. [27] defined the probabilistic uncertain linguistic term sets (PULTs). Wang et al. [28] developed the GRP and CRITIC methods for PUL-MAGDM. Wei et al. [29] built the generalized Dice similarity measures for PUL-MAGDM. Zhao et al. [30] built the PUL-TODIM method based on prospect theory. He et al. [31] built the taxonomy-based MAGDM method with probabilistic uncertain linguistic assessment information. He et al. [32] built the bidirectional projection method for PUL-MAGDM. Nevertheless, a few sophisticated proficient estimation perspectives cannot be remarked in existing language terms such as "only a tiny bit poor" or "only a tiny bit good." Hence, Gou et al. [33] made a conceptual layout about double hierarchy linguistic term set (DHLTS) and double hierarchy hesitant fuzzy linguistic term set (DHHFLTS). Many research results have emerged one after another [34–41]. Soon afterwards, Gou et al. [42]

made a project about probabilistic double hierarchy linguistic term set (PDHLTS). Lei et al. [43] built the PDHL-CODAS model to rank online shopping platform. Lei et al. [44] defined a sequence of probabilistic double hierarchy linguistic polymerization formulas. Lei et al. [45] defined the PDHL-EDAS method for MAGDM.

GRA was initially defined by Deng [46] to cope with real MAGDM. Compared with other real MAGDM methods [47–51], the GRA method could consider the shape similarity of every given alternative from PIS as well as NIS. Javanmardi et al. [52] explored grey system theory-based methods and applications in sustainability studies. Javanmardi and Liu [53] explored the human cognitive capacity in understanding systems: a grey system theory perspective. Zhang et al. [54] used the GRA method based on cumulative prospect theory for IF-MAGDM. Javanmardi et al. [55] explored the philosophical paradigm of grey system theory as a postmodern theory. With the purpose of discerning the carbon market, Zhu et al. [56] took advantage of the GRA process as well as EMD. Malek et al. [57] built a revised hybrid GRA for green supply. Kung and Wen [58] used the GRA process to solve grey MADM. Javanmardi and Liu [59] explored grey system theory-based methods and applications in analyzing socioeconomic systems. Javanmardi et al. [60] explored the philosophical foundations of grey system theory. Alptekin et al. [61] solved the low carbon development based on the GRA process. Zhang et al. [62] defined the SF-GRA method based on cumulative prospect theory for MAGDM.

The main contributions of this paper are to utilize the GRA algorithm to build the MAGDM matters on the strength of PDHLTSs. The main research work of this paper is arranged as follows: (1) the GRA is constructed on account of PDHLTSs; (2) the PDHL-GRA method is applied to finish off the MAGDM issue under PDHLTSs; (3) a practical case for college tennis classroom teaching effect evaluation is presented to demonstrate the steps of our method; and (4) a comparison analysis illustrates its feasibility and effectiveness. The framework of this article is as follows. Section 2 reviews some concepts of PDHLTSs. Section 3 designs a PDHL-GRA method for MAGDM with entropy weight. Section 4 provides a practical example to illustrate the method and a comparison analysis illustrates its effectiveness. Finally, Section 5 summarizes this study.

2. Preliminaries

First, let us learn some basics about PDHLTS.

Definition 1. (see [33]). Let us say $DHL = \{\Gamma_{\vartheta\langle I_\Omega\rangle} | \vartheta = -A, \cdots, -1, 0, 1, \cdots A; \Omega = -B, \cdots, -1, 0, 1, \cdots B;\}$ is a DHLTS, and the definition of the DHLTS is

$$DHLTS = \left\{\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle} | \vartheta = -A, \cdots, -1, 0, 1, \cdots A; \Omega = -B, \cdots, -1, 0, 1, \cdots B;\right\}, \tag{1}$$

where $\Delta = 1, 2, \ldots, \Xi DHL$, the $\Delta - \mathrm{th}$ double hierarchy linguistic element (DHLE) is narrated as $\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}$, the quantity of all DHLEs is ΞDHL, and all DHLEs are sorted in ascending sequence.

Definition 2 (see [42]). Let us say $DHL = \{\Gamma_{\vartheta\langle I_\Omega\rangle} | \vartheta = -A, \cdots, -1, 0, 1, \cdots A; \Omega = -B, \cdots, -1, 0, 1, \cdots B;\}$ is a DHLTS, and the PDHLTS is created as

$$PDHL(\lambda) = \left\{\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}(\lambda^{\Delta}) | \Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle} \in DHL, \lambda^{\Delta} \geq 0, \sum_{\Delta=1}^{\Xi P\ DH\ L(\lambda)} \lambda^{\Delta} \leq 1\right\}, \tag{2}$$

where $\Delta = 1, 2, \ldots, \Xi PDHL(\lambda)$, the $\Delta - \mathrm{th}$ probabilistic double hierarchy linguistic element (PDHLE) is narrated as $\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}(\lambda^{\Delta})$, the quantities of all PDHLEs are denoted as $\Xi PDHL(\lambda)$, and according to $\Upsilon(\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}(\lambda^{\Delta}))$, PDHLE is sorted in ascending order; the function is determined by formula (3).

Definition 3 (see [42]). Let $DHL = \{\Gamma_{\vartheta\langle I_\Omega\rangle} | \vartheta = -A, \cdots, -1, 0, 1, \cdots A; \Omega = -B, \cdots, -1, 0, 1, \cdots B;\}$ be a DHLTS, and $PDHL(\lambda) = \{\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}(\lambda^{\Delta}) | \Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle} \in DHL, \lambda^{\Delta} \geq 0, \sum_{\Delta=1}^{\Xi P\ DH\ L(\lambda)} \lambda^{\Delta} \leq 1\}$ be a PDHLTS. The above conversion function Υ for PDHLE $\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}(\lambda^{\Delta})$ is designed as follows:

$$\begin{aligned}
&\Upsilon\colon [-A, A] \times [-B, B] \longrightarrow [0, 1], \Upsilon(\vartheta, \Omega),\\
&= \frac{\Omega + (A + \vartheta)B}{2AB} = \varpi,\\
&\Upsilon^{-1}\colon [0, 1] \longrightarrow [-A, A] \times [-B, B],\\
&\Upsilon^{-1}(\varpi) = [2A\varpi - A]_{\langle I_{B((2A\varpi-A)-[2A\varpi-A])}\rangle or}[2A\varpi - A]\\
&+ 1_{\langle I_{B((2A\varpi-A)-[2A\varpi-A])-B}\rangle or}.
\end{aligned} \tag{3}$$

Because the probability sum of all PDHLEs in PDHLTS may be less than 1, we had to standardize PDHLTS, and the specific measures are as follows:

$$PDHL(\lambda) = \left\{\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}(\lambda^{\Delta}) | \Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle} \in DHL, \lambda^{\Delta} \geq 0, \sum_{\Delta=1}^{\Xi PDHL(\lambda)} \lambda^{\Delta} \leq 1\right\}, \tag{4}$$

where $\tilde{\lambda}^{\Delta} = \lambda^{\Delta} / \sum_{\Delta=1}^{\Xi PDHL(\lambda)} \lambda^{\Delta}$; $\vartheta \in [-A, A]$; $\Omega \in [-B, B]$; A, B are all integers.

Definition 4. (see [43]). Let $DHL = \{\Gamma_{\vartheta\langle I_\Omega\rangle} | \vartheta = -A, \cdots, -1, 0, 1, \cdots A; \Omega = -B, \cdots, -1, 0, 1, \cdots B;\}$ be a DHLTS and $PDH\tilde{L}_1(\tilde{\lambda}) = \{\Gamma^{\Delta}_{1\vartheta\langle I_\Omega\rangle}(\tilde{\lambda}^{\Delta}_1) | \Gamma^{\Delta}_{1\vartheta\langle I_\Omega\rangle} \in DHL; \Delta = 1, 2, \ldots, \Xi PDH\tilde{L}_1(\tilde{\lambda})\}$ and $PDH\tilde{L}_2(\tilde{\lambda}) = \{\Gamma^{\Delta}_{2\vartheta\langle I_\Omega\rangle}(\tilde{\lambda}^{\Delta}_2) | \Gamma^{\Delta}_{2\vartheta\langle I_\Omega\rangle} \in DHL; \Delta = 1, 2, \ldots, \Xi PDH\tilde{L}_2(\tilde{\lambda})\}$ be two different PDHLTSs, where $\#P\ DH\ \tilde{L}_1(\tilde{\lambda})$, $\#PDH\tilde{L}_2(\tilde{\lambda})$ are the lengths of all PDHLEs in $PDH\tilde{L}_1(\tilde{\lambda})$ and $PDH\tilde{L}_2(\tilde{\lambda})$, respectively. Especially, if $\Xi PDH\tilde{L}_1(\tilde{\lambda}) > \Xi PDH\tilde{L}_2(\tilde{\lambda})$, then the lengths of $\Xi PDH\tilde{L}_1(\tilde{\lambda}) - \Xi PDH\tilde{L}_2(\tilde{\lambda})$ DHLEs are raised to $PDH\tilde{L}_2(\tilde{\lambda})$. The added PDHLEs should not be greater than any of the elements in the $\mathrm{PDHL}_2(\tilde{\lambda})$, and the probability should be set to 0.

Definition 5. (see [42]). Let $\mathrm{PDHL}(\tilde{\lambda}) = \left\{\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}(\tilde{\lambda}^{\Delta}) | \Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle} \in \mathrm{DHL}; \Delta = 1, 2, \ldots, \Xi\mathrm{PDHL}(\tilde{\lambda})\right\}$ be a PDHLTS, and the expected values $\chi(\mathrm{PDHL}(\tilde{\lambda}))$ and deviation degree $\gamma(\mathrm{PDHL}(\tilde{\lambda}))$ of $\mathrm{PDHL}(\tilde{\lambda})$ are built as $\chi(\mathrm{PDHL}_1(\tilde{\lambda})) = \chi(\mathrm{PDHL}_2(\tilde{\lambda}))$:

$$\chi(\mathrm{PDHL}(\tilde{\lambda})) = \frac{\sum_{\Delta=1}^{\Xi\mathrm{PDHL}(\tilde{\lambda})} \Upsilon(\mathrm{PDHL}(\tilde{\lambda}))\tilde{\lambda}^{\Delta}}{\sum_{\Delta=1}^{\mathrm{PDHL}(\tilde{\lambda})} \tilde{\lambda}^{\Delta}},$$

$$\gamma(\mathrm{PDHL}(\tilde{\lambda})) = \frac{\sqrt{\sum_{\Delta=1}^{\Xi\mathrm{PDHL}(\tilde{\lambda})} \left(\Upsilon(\mathrm{PDHL}(\tilde{\lambda}))\tilde{\lambda}^{\Delta} - \chi(\mathrm{PDHL}(\tilde{\lambda}))\right)^2}}{\sum_{\Delta=1}^{\Xi\mathrm{PDHL}(\tilde{\lambda})} \tilde{\lambda}^{\Delta}}. \quad (5)$$

Definition 6. (see [43]). Let $\mathrm{DHL} = \left\{\Gamma_{\vartheta\langle I_\Omega\rangle} | \vartheta = -A, \cdots, -1, 0, 1, \cdots A; \quad \Omega = -B, \cdots, -1, 0, 1, \cdots B;\right\}$ be a DHLTS, and $\mathrm{PDHL}_1(\tilde{\lambda}) = \left\{\Gamma^{\Delta}_{1\vartheta\langle I_\Omega\rangle}(\lambda_1^{\Delta}) | \Gamma^{\Delta}_{1\vartheta\langle I_\Omega\rangle} \in \mathrm{DHL}; \Delta = 1, 2, \ldots, \Xi\mathrm{PDHL}_1(\tilde{\lambda})\right\}$ and $\mathrm{PDHL}_2(\tilde{\lambda}) = \left\{\Gamma^{\Delta}_{2\vartheta\langle I_\Omega\rangle}(\lambda_2^{\Delta}) | \Gamma^{\Delta}_{2\vartheta\langle I_\Omega\rangle} \in \mathrm{DHL}; \Delta = 1, 2, \ldots, \Xi\mathrm{PDHL}_2(\tilde{\lambda})\right\}$ are two PDHLTSs, where $\Xi\mathrm{PDHL}_1(\tilde{\lambda}) = \Xi\mathrm{PDHL}_2(\tilde{\lambda}) = \Xi\mathrm{PDHL}(\tilde{\lambda})$; then, Hamming distance $\mathrm{HD}(\mathrm{PDHL}_1(\tilde{\lambda}), \mathrm{PDHL}_2(\tilde{\lambda}))$ is determined.

$$\mathrm{HD}\left(\mathrm{PDHL}_1(\tilde{\lambda}), \mathrm{PDHL}_2(\tilde{\lambda})\right) = \frac{\sum_{\Delta=1}^{\Xi\mathrm{PDHL}(\tilde{\lambda})} \Upsilon\left(\Gamma^{\Delta}_{1\vartheta\langle I_\Omega\rangle}\right)\lambda_1^{\Delta} - \Gamma^{\Delta}_{2\vartheta\langle I_\Omega\rangle}\lambda_2^{\Delta}}{\Xi\mathrm{PDHL}(\tilde{\lambda})}. \quad (6)$$

3. PDHL-GRA Method for MAGDM with Entropy Weight

Now, GRA mean in the context of PDHLTSs is proposed to deal with MAGDM matters. Also, a complete MAGDM issue is narrated as follows. Whole alternatives is shown as $C = \{C_1, C_2, \cdots, C_a\}$, $D = \{D_1, D_2, \cdots, D_b\}$ is denoted a sequence of attributes, and the weight vector is $\mathfrak{I} = (\mathfrak{I}_1, \mathfrak{I}_2, \cdots, \mathfrak{I}_b)$, where $\mathfrak{I}_\sigma \in [0, 1]$, $\sigma = 1, 2, \cdots, b$, $\sum_{\sigma=1}^{b} \mathfrak{I}_b = 1$, and $\mathrm{JK} = \{\mathrm{JK}_1, \mathrm{JK}_2, \cdots, \mathrm{JK}_T\}$ are T experts, and $\mathfrak{R} = (\mathfrak{R}_1, \mathfrak{R}_2, \cdots, \mathfrak{R}_T)$ is weight vector of all experts. Suppose that q-th expert JK_q is evaluated $\tau -$ th alternative C_τ under $\sigma -$ th attribute D_σ as $\mathrm{PDHL}^{(q)}_{\tau\sigma}(\lambda) = \left\{\Gamma^{\Delta(q)}_{\tau\sigma\vartheta\langle I_\Omega\rangle}(\lambda^{\Delta(q)}_{\tau\sigma}) | \Gamma^{\Delta(q)}_{\tau\sigma\vartheta\langle I_\Omega\rangle} \in \mathrm{DHL}, \lambda^{\Delta(q)}_{\tau\sigma} \geq 0, \sum_{\Delta=1}^{\Xi\mathrm{PDHL}(\lambda)} \lambda^{\Delta(q)}_{\tau\sigma} \leq 1\right\}$ $(\tau = 1, 2, \cdots, a, \sigma = 1, 2, \cdots, b, q = 1, 2, \cdots, \mathrm{T})$.

Furthermore, PDHL-GRA mean is created to dispose of MAGDM issue with entropy weight.

Step 1. Establish all decision makers' decision matrixes PDHLTS $Q^{(q)} = (\mathrm{PDHL}^{(q)}(\lambda))_{a\times b}$.

$$\mathrm{PDHLTS} = \mathop{\oplus}_{q=1}^{\mathrm{T}} w_q \mathrm{PDHL}_q(\tilde{\lambda}), = \left\{\Upsilon^{-1}\left(\cup\left(1 - \prod_{q=1}^{\mathrm{T}}\left(1 - \Upsilon\left(\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}\right)\right)^{\mathfrak{R}_q}\right)\right)\frac{\sum_{q=1}^{T}\lambda_q^{\Delta}}{q}\right\}. \quad (7)$$

Step 2. Convert cost index into benefit index. Let $\mathrm{PDHL}(\tilde{\lambda}) = \left\{\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}(\lambda^{\Delta}) | \Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle} \in \mathrm{DHL}; \Delta = 1, 2, \ldots, \Xi\mathrm{PDHL}(\tilde{\lambda})\right\}$ be a PDHLTS; if $\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}(\tilde{\lambda}^{\Delta})$ is an evaluation on cost, we need to translate it into the benefit evaluation $\Gamma^{\Delta}_{-\vartheta\langle I_\Omega\rangle}(\lambda^{\Delta})$.

Step 3. Compute the normalized decision matrix $\tilde{Q}^{(q)} = (\mathrm{PDHL}^{(q)}_{\tau\sigma}(\tilde{\lambda}))_{a\times b}$.

$$\mathrm{PDHL}^{(q)}_{\tau\sigma}(\tilde{\lambda}) = \frac{\sum_{\Delta=1}^{\Xi\mathrm{PDHL}(\tilde{\lambda})} \Upsilon\left(\Gamma^{\Delta(q)}_{\tau\sigma\vartheta\langle I_\Omega\rangle}\right)\left(\lambda^{\Delta(q)}_{\tau\sigma}\right)}{\sum_{\tau=1}^{a}\sum_{\Delta=1}^{\Xi\mathrm{PDHL}(\tilde{\lambda})} \Upsilon\left(\Gamma^{\Delta(q)}_{\tau\sigma\vartheta\langle I_\Omega\rangle}\right)\left(\lambda^{\Delta(q)}_{\tau\sigma}\right)}, \sigma = 1, 2, \cdots, b. \quad (8)$$

Step 4. The proportion of each attribute is calculated depending on the entropy formula.

Entropy [63] is one of the important tools to ascertain the proportion of each attribute.

The first thing to do is ascertaining the normalized decision matrix $\mathrm{NL}_{ij}(p)$:

Secondly, the Shannon entropy $E = (E_1, E_2, \cdots, E_b)$ is obtained by the following formula:

$$E_\sigma = -\frac{1}{\ln a}\sum_{\tau=1}^{a} \mathrm{PDHL}^{(q)}_{\tau\sigma}(\tilde{\lambda})\ln \mathrm{PDHL}^{(q)}_{\tau\sigma}(\tilde{\lambda}), \quad (9)$$

and $\mathrm{PDHL}^{(q)}_{\tau\sigma}(\tilde{\lambda})\ln \mathrm{PDHL}^{(q)}_{\tau\sigma}(\tilde{\lambda})$ is defined as 0, if $\mathrm{PDHL}^{(q)}_{\tau\sigma}(\tilde{\lambda}) = 0$.

Finally, the attribute weights $\mathfrak{I} = (\mathfrak{I}_1, \mathfrak{I}_2, \cdots, \mathfrak{I}_b)$ are computed:

$$\mathfrak{I}_\sigma = \frac{1 - E_\sigma}{\sum_{\sigma=1}^{b} (1 - E_\sigma)}, \sigma = 1, 2, \cdots, b. \tag{10}$$

Step 5. Confirm the probabilistic double hierarchy linguistic positive ideal scheme more than zero (PDHLPIS) and probabilistic double hierarchy linguistic negative ideal scheme less than zero (PDHLNIS):

$$\begin{aligned}\mathrm{PDHLPIS} &= (\mathrm{PDHLPIS}_1, \mathrm{PDHLPIS}_2, \cdots, \mathrm{PDHLPIS}_b),\\ \mathrm{PDHLPIS} &= (\mathrm{PDHLPIS}_1, \mathrm{PDHLPIS}_2, \cdots, \mathrm{PDHLPIS}_b),\\ \mathrm{PDHLNIS} &= (\mathrm{PDHLNIS}_1, \mathrm{PDHLNIS}_2, \cdots, \mathrm{PDHLNIS}_b),\end{aligned} \tag{11}$$

where

$$\begin{aligned}\mathrm{PDHLPIS}_\sigma &= \left\{\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}\left(\tilde{\lambda}^{\Delta}\right) \mid \Delta = 1, 2, \ldots, \Xi\mathrm{PDHL}(\tilde{\lambda})\right\},\\ &= \left\{\max_{\tau} \Upsilon\left(\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}\left(\tilde{\lambda}^{\Delta}\right)\right)\right\},\\ \mathrm{PDHLNIS}_\sigma &= \left\{\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}\left(\tilde{\lambda}^{\Delta}\right) \mid \Delta = 1, 2, \ldots, \Xi\mathrm{PDHL}(\tilde{\lambda})\right\},\\ &= \left\{\min_{\tau} \Upsilon\left(\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}\left(\tilde{\lambda}^{\Delta}\right)\right)\right\}.\end{aligned} \tag{12}$$

Step 6. Compute the grey rational coefficients of every given attribute of every given alternative from the PDHLPIS and PDHLNIS.

$$\begin{aligned}\mathrm{PDHLPIS}(\xi_{\tau\sigma}) &= \frac{\min_{1\le i\le m}\min_{1\le j\le n} d(\mathrm{PDHLA}_{\tau\sigma}, \mathrm{PDHLPIS}_\sigma) + \rho\max_{1\le i\le m}\max_{1\le j\le n} d(P\ DH\ LA_{\tau\sigma}, \mathrm{PDHLPIS}_\sigma)}{d(\mathrm{PDHLA}_{\tau\sigma}, \mathrm{PDHLPIS}_\sigma) + \rho\max_{1\le i\le m}\max_{1\le j\le n} d(\mathrm{PDHLA}_{\tau\sigma}, \mathrm{PDHLPIS}_\sigma)},\\ \mathrm{PDHLPIS}(\xi_{\tau\sigma}) &= \frac{\min_{1\le i\le m}\min_{1\le j\le n} d(\mathrm{PDHLA}_{\tau\sigma}, \mathrm{PDHLNIS}_\sigma) + \rho\max_{1\le i\le m}\max_{1\le j\le n} d(P\ DH\ LA_{\tau\sigma}, \mathrm{PDHLNIS}_\sigma)}{d(\mathrm{PDHLA}_{\tau\sigma}, \mathrm{PDHLNIS}_\sigma) + \rho\max_{1\le i\le m}\max_{1\le j\le n} d(\mathrm{PDHLA}_{\tau\sigma}, \mathrm{PDHLNIS}_\sigma)},\\ &\tau = 1, 2, \cdots, a, \sigma = 1, 2, \cdots, b.\end{aligned} \tag{13}$$

Step 7. Figure out the degree of GRC of all given alternatives from PDHLPIS as well as PDHLNIS:

$$\begin{aligned}\mathrm{PDHLPIS}(\xi_\tau) &= \sum_{\tau=1}^{a} \mathfrak{I}_\tau \mathrm{PDHLPIS}(\xi_{\tau\sigma}), \tau = 1, 2, \cdots, a,\\ \mathrm{PDHLPIS}(\xi_\tau) &= \sum_{\tau=1}^{a} \mathfrak{I}_\tau \mathrm{PDHLNIS}(\xi_{\tau\sigma}), \tau = 1, 2, \cdots, a.\end{aligned} \tag{14}$$

Step 8. Compute each alternative's PDHL relative relational degree (PDHLRRD) of all given alternatives from PDHLPIS:

$$\mathrm{PDHLRRD}_\tau = \frac{\mathrm{PDHLPIS}(\xi_\tau)}{\mathrm{PDHLNIS}(\xi_\tau) + \mathrm{PDHLPIS}(\xi_\tau)}, \tau = 1, 2, \cdots, a. \tag{15}$$

Step 9. According to $\mathrm{PDHLRRD}_\tau (\tau = 1, 2, \cdots, a)$. The highest value of $\mathrm{PDHLRRD}_\tau (\tau = 1, 2, \cdots, a)$, the optimal choice is.

4. Numerical Example and Comparative Analysis

4.1. Numerical Example. Based on the research on the development of tennis teachers in colleges and universities and the evaluation requirements of the new round of basic education curriculum reform, it is of great significance to measure whether tennis teaching meets the expected goals. The core courses in the curriculum reform were implemented, and the fundamental way is to implement curriculum classroom. Curriculum reform embodies an important issue that every school and teacher is thinking about. Classroom evaluation reform to carry out scientific and effective evaluation of classroom teaching and establish an effective evaluation system mechanism should be the core of the curriculum reform. According to the current and future period of teaching reform and development, classroom evaluation should be "developmental classroom evaluation." Classroom evaluation helps to overcome the limitations and deficiencies of current evaluation. Classroom evaluation reflects the latest trend of current teacher evaluation, evaluation of advanced ideas, and evaluation functions. Classroom evaluation conducts reflection and analysis on teachers, evaluates teachers' development potential, teachers' classroom status and the process of value judgment. However, the evaluation of teaching in the field of teaching is a worldwide problem, but also the key to promoting quality education process. There is a clear gap between the current evaluation theories, methods and systems, and quality education. Similar problems exist in teacher teaching evaluation. These serious constraints restricted the promotion of quality education. Therefore, the establishment of the quality

Table 1: The PDHLTS evaluation of all alternatives is provided by JK_1.

	D_1	D_2	D_3	D_4
C_1	$\{\Gamma_{-2\langle I_{-1}\rangle}(0.1), \Gamma_{-1\langle I_1\rangle}(0.3), \Gamma_{0\langle I_1\rangle}(0.6)\}$	$\{\Gamma_{-3\langle I_1\rangle}(0.2), \Gamma_{0\langle I_{-2}\rangle}(0.1), \Gamma_{1\langle I_1\rangle}(0.7)\}$	$\{\Gamma_{-3\langle I_0\rangle}(1.0)\}$	$\{\Gamma_{2\langle I_{-1}\rangle}(0.5), \Gamma_{-1\langle I_{-2}\rangle}(0.3), \Gamma_{-1\langle I_{-1}\rangle}(0.2)\}$
C_2	$\{\Gamma_{-3\langle I_1\rangle}(1)\}$	$\{\Gamma_{-3\langle I_0\rangle}(0.3), \Gamma_{0\langle I_2\rangle}(0.7)\}$	$\{\Gamma_{-3\langle I_{-2}\rangle}(0.6), \Gamma_{-3\langle I_{-1}\rangle}(0.2), \Gamma_{2\langle I_{-1}\rangle}(0.2)\}$	$\{\Gamma_{1\langle I_2\rangle}(0.3), \Gamma_{-2\langle I_0\rangle}(0.4), \Gamma_{-1\langle I_1\rangle}(0.3)\}$
C_3	$\{\Gamma_{0\langle I_{-1}\rangle}(0.1), \Gamma_{1\langle I_{-1}\rangle}(0.1), \Gamma_{2\langle I_2\rangle}(0.8)\}$	$\{\Gamma_{3\langle I_0\rangle}(0.2), \Gamma_{2\langle I_0\rangle}(0.4), \Gamma_{-1\langle I_1\rangle}(0.4)\}$	$\{\Gamma_{-3\langle I_0\rangle}(0.8), \Gamma_{1\langle I_{-2}\rangle}(0.1), \Gamma_{3\langle I_0\rangle}(0.1)\}$	$\{\Gamma_{-3\langle I_1\rangle}(0.1), \Gamma_{1\langle I_0\rangle}(0.8), \Gamma_{2\langle I_1\rangle}(0.1)\}$
C_4	$\{\Gamma_{2\langle I_{-1}\rangle}(0.3), \Gamma_{1\langle I_2\rangle}(0.3), \Gamma_{2\langle I_2\rangle}(0.4)\}$	$\{\Gamma_{-2\langle I_1\rangle}(0.4), \Gamma_{0\langle I_{-3}\rangle}(0.4), \Gamma_{3\langle I_{-2}\rangle}(0.2)\}$	$\{\Gamma_{-3\langle I_2\rangle}(0.4), \Gamma_{-1\langle I_2\rangle}(0.5), \Gamma_{1\langle I_3\rangle}(0.1)\}$	$\{\Gamma_{-1\langle I_2\rangle}(0.5), \Gamma_{-1\langle I_3\rangle}(0.5),\}$
C_5	$\{\Gamma_{-2\langle I_0\rangle}(0.5), \Gamma_{-1\langle I_2\rangle}(0.2), \Gamma_{0\langle I_3\rangle}(0.3)\}$	$\{\Gamma_{-1\langle I_{-1}\rangle}(0.4), \Gamma_{2\langle I_{-1}\rangle}(0.4), \Gamma_{1\langle I_0\rangle}(0.2)\}$	$\{\Gamma_{0\langle I_0\rangle}(0.4), \Gamma_{2\langle I_1\rangle}(0.6)\}$	$\{\Gamma_{2\langle I_1\rangle}(1.0)\}$

Table 2: The PDHLTS evaluation of all alternatives is provided by JK_2.

	D_1	D_2	D_3	D_4
C_1	$\{\Gamma_{-3\langle I_2\rangle}(0.7), \Gamma_{-1\langle I_1\rangle}(0.3)\}$	$\{\Gamma_{1\langle I_{-1}\rangle}(1.0)\}$	$\{\Gamma_{1\langle I_{-3}\rangle}(0.2), \Gamma_{1\langle I_{-2}\rangle}(0.6), \Gamma_{1\langle I_0\rangle}(0.2)\}$	$\{\Gamma_{-1\langle I_{-2}\rangle}(0.5), \Gamma_{-1\langle I_1\rangle}(0.1), \Gamma_{2\langle I_2\rangle}(0.4)\}$
C_2	$\{\Gamma_{2\langle I_{-1}\rangle}(0.6), \Gamma_{2\langle I_0\rangle}(0.1), \Gamma_{2\langle I_1\rangle}(0.3)\}$	$\{\Gamma_{-3\langle I_1\rangle}(0.7), \Gamma_{-3\langle I_2\rangle}(0.1), \Gamma_{-3\langle I_3\rangle}(0.2),\}$	$\{\Gamma_{0\langle I_0\rangle}(1.0)\}$	$\{\Gamma_{-2\langle I_{-3}\rangle}(0.2), \Gamma_{1\langle I_1\rangle}(0.5), \Gamma_{2\langle I_1\rangle}(0.3)\}$
C_3	$\{\Gamma_{1\langle I_{-1}\rangle}(0.1), \Gamma_{1\langle I_0\rangle}(0.3), \Gamma_{2\langle I_1\rangle}(0.6)\}$	$\{\Gamma_{-3\langle I_2\rangle}(0.8), \Gamma_{2\langle I_1\rangle}(0.2)\}$	$\{\Gamma_{-2\langle I_1\rangle}(0.1), \Gamma_{-2\langle I_2\rangle}(0.9)\}$	$\{\Gamma_{1\langle I_{-1}\rangle}(0.5), \Gamma_{2\langle I_{-1}\rangle}(0.5)\}$
C_4	$\{\Gamma_{1\langle I_0\rangle}(0.6), \Gamma_{2\langle I_0\rangle}(0.2), \Gamma_{1\langle I_3\rangle}(0.2)\}$	$\{\Gamma_{-2\langle I_0\rangle}(0.3), \Gamma_{-2\langle I_1\rangle}(0.6), \Gamma_{-1\langle I_1\rangle}(0.1)\}$	$\{\Gamma_{-2\langle I_1\rangle}(0.8), \Gamma_{-2\langle I_3\rangle}(0.2)\}$	$\{\Gamma_{-2\langle I_1\rangle}(0.3), \Gamma_{-1\langle I_1\rangle}(0.1), \Gamma_{-1\langle I_2\rangle}(0.6)\}$
C_5	$\{\Gamma_{1\langle I_1\rangle}(0.2), \Gamma_{1\langle I_2\rangle}(0.5), \Gamma_{2\langle I_2\rangle}(0.3)\}$	$\{\Gamma_{-2\langle I_{-2}\rangle}(0.5), \Gamma_{-1\langle I_{-1}\rangle}(0.4), \Gamma_{0\langle I_2\rangle}(0.1)\}$	$\{\Gamma_{3\langle I_{-2}\rangle}(1.0)\}$	$\{\Gamma_{1\langle I_{-3}\rangle}(0.1), \Gamma_{1\langle I_{-1}\rangle}(0.2), \Gamma_{2\langle I_1\rangle}(0.7)\}$

Table 3: The PDHLTS evaluation of all alternatives is provided by JK_3.

	D_1	D_2	D_3	D_4
C_1	$\{\Gamma_{-1\langle I_1\rangle}(0.4), \Gamma_{1\langle I_1\rangle}(0.2), \Gamma_{0\langle I_3\rangle}(0.4)\}$	$\{\Gamma_{-3\langle I_{-3}\rangle}(0.3), \Gamma_{-3\langle I_{-2}\rangle}(0.3), \Gamma_{1\langle I_{-2}\rangle}(0.4)\}$	$\{\Gamma_{1\langle I_0\rangle}(1.0)\}$	$\{\Gamma_{-2\langle I_{-3}\rangle}(0.6), \Gamma_{1\langle I_1\rangle}(0.4)\}$
C_2	$\{\Gamma_{0\langle I_3\rangle}(1.0)\}$	$\{\Gamma_{-1\langle I_0\rangle}(0.3), \Gamma_{1\langle I_1\rangle}(0.7)\}$	$\{\Gamma_{2\langle I_{-2}\rangle}(0.4), \Gamma_{2\langle I_{-1}\rangle}(0.4), \Gamma_{2\langle I_0\rangle}(0.2)\}$	$\{\Gamma_{0\langle I_{-3}\rangle}(0.1), \Gamma_{2\langle I_{-1}\rangle}(0.2), \Gamma_{2\langle I_0\rangle}(0.7)\}$
C_3	$\{\Gamma_{1\langle I_{-2}\rangle}(0.7), \Gamma_{2\langle I_{-1}\rangle}(0.2), \Gamma_{2\langle I_0\rangle}(0.1)\}$	$\{\Gamma_{-1\langle I_3\rangle}(0.2), \Gamma_{1\langle I_1\rangle}(0.5), \Gamma_{2\langle I_1\rangle}(0.3)\}$	$\{\Gamma_{2\langle I_{-1}\rangle}(0.4), \Gamma_{-1\langle I_0\rangle}(0.6)\}$	$\{\Gamma_{0\langle I_{-3}\rangle}(0.3), \Gamma_{1\langle I_0\rangle}(0.1), \Gamma_{2\langle I_1\rangle}(0.6)\}$
C_4	$\{\Gamma_{-1\langle I_{-3}\rangle}(0.3), \Gamma_{-2\langle I_1\rangle}(0.6), \Gamma_{1\langle I_1\rangle}(0.1)\}$	$\{\Gamma_{-2\langle I_{-2}\rangle}(0.4), \Gamma_{0\langle I_0\rangle}(0.4), \Gamma_{1\langle I_0\rangle}(0.5)\}$	$\{\Gamma_{-3\langle I_{-2}\rangle}(0.4), \Gamma_{-1\langle I_{-3}\rangle}(0.1), \Gamma_{1\langle I_{-3}\rangle}(0.5)\}$	$\{\Gamma_{1\langle I_{-1}\rangle}(0.3), \Gamma_{1\langle I_1\rangle}(0.4), \Gamma_{2\langle I_2\rangle}(0.3)\}$
C_5	$\{\Gamma_{-3\langle I_{-1}\rangle}(0.5), \Gamma_{-2\langle I_0\rangle}(0.4), \Gamma_{2\langle I_1\rangle}(0.1)\}$	$\{\Gamma_{1\langle I_{-3}\rangle}(0.1), \Gamma_{1\langle I_{-1}\rangle}(0.3), \Gamma_{1\langle I_3\rangle}(0.6)\}$	$\{\Gamma_{-3\langle I_{-2}\rangle}(0.6), \Gamma_{-1\langle I_1\rangle}(0.2), \Gamma_{1\langle I_3\rangle}(0.2)\}$	$\{\Gamma_{-1\langle I_{-2}\rangle}(0.5), \Gamma_{2\langle I_2\rangle}(0.1), \Gamma_{3\langle I_0\rangle}(0.4)\}$

Table 4: The standardized decision matrix is provided by JK_1.

	D_1	D_2	D_3	D_4
C_1	$\{\Gamma_{-2\langle I_{-1}\rangle}(0.1), \Gamma_{-1\langle I_{1}\rangle}(0.3), \Gamma_{0\langle I_{1}\rangle}(0.6)\}$	$\{\Gamma_{-3\langle I_{1}\rangle}(0.2), \Gamma_{0\langle I_{-2}\rangle}(0.1), \Gamma_{1\langle I_{1}\rangle}(0.7)\}$	$\{\Gamma_{-3\langle I_{0}\rangle}(0), \Gamma_{-3\langle I_{0}\rangle}(0), \Gamma_{-3\langle I_{0}\rangle}(1.0)\}$	$\{\Gamma_{2\langle I_{-1}\rangle}(0.5), \Gamma_{-1\langle I_{-2}\rangle}(0.3), \Gamma_{-1\langle I_{-1}\rangle}(0.2)\}$
C_2	$\{\Gamma_{-3\langle I_{1}\rangle}(0), \Gamma_{-3\langle I_{1}\rangle}(0), \Gamma_{-3\langle I_{1}\rangle}(1)\}$	$\{\Gamma_{-3\langle I_{0}\rangle}(0), \Gamma_{-3\langle I_{0}\rangle}(0.3), \Gamma_{0\langle I_{2}\rangle}(0.7)\}$	$\{\Gamma_{-3\langle I_{-2}\rangle}(0.6), \Gamma_{-3\langle I_{-1}\rangle}(0.2), \Gamma_{2\langle I_{-1}\rangle}(0.2)\}$	$\{\Gamma_{1\langle I_{2}\rangle}(0.3), \Gamma_{-2\langle I_{0}\rangle}(0.4), \Gamma_{-1\langle I_{1}\rangle}(0.3)\}$
C_3	$\{\Gamma_{0\langle I_{-1}\rangle}(0.1), \Gamma_{1\langle I_{-1}\rangle}(0.1), \Gamma_{2\langle I_{2}\rangle}(0.8)\}$	$\{\Gamma_{3\langle I_{0}\rangle}(0.2), \Gamma_{2\langle I_{0}\rangle}(0.4), \Gamma_{-1\langle I_{1}\rangle}(0.4)\}$	$\{\Gamma_{-3\langle I_{0}\rangle}(0.8), \Gamma_{1\langle I_{-2}\rangle}(0.1), \Gamma_{3\langle I_{0}\rangle}(0.1)\}$	$\{\Gamma_{-3\langle I_{1}\rangle}(0.1), \Gamma_{1\langle I_{0}\rangle}(0.8), \Gamma_{2\langle I_{1}\rangle}(0.1)\}$
C_4	$\{\Gamma_{2\langle I_{-1}\rangle}(0.3), \Gamma_{1\langle I_{2}\rangle}(0.3), \Gamma_{2\langle I_{2}\rangle}(0.4)\}$	$\{\Gamma_{-2\langle I_{1}\rangle}(0.4), \Gamma_{0\langle I_{-3}\rangle}(0.4), \Gamma_{3\langle I_{-2}\rangle}(0.2)\}$	$\{\Gamma_{-3\langle I_{2}\rangle}(0.4), \Gamma_{-1\langle I_{2}\rangle}(0.5), \Gamma_{1\langle I_{3}\rangle}(0.1)\}$	$\{\Gamma_{-1\langle I_{2}\rangle}(0), \Gamma_{-1\langle I_{2}\rangle}(0.5), \Gamma_{-1\langle I_{3}\rangle}(0.5),\}$
C_5	$\{\Gamma_{-2\langle I_{0}\rangle}(0.5), \Gamma_{-1\langle I_{2}\rangle}(0.2), \Gamma_{0\langle I_{3}\rangle}(0.3)\}$	$\{\Gamma_{-1\langle I_{-1}\rangle}(0.4), \Gamma_{2\langle I_{-1}\rangle}(0.4), \Gamma_{1\langle I_{0}\rangle}(0.2)\}$	$\{\Gamma_{0\langle I_{0}\rangle}(0), \Gamma_{0\langle I_{0}\rangle}(0.4), \Gamma_{2\langle I_{1}\rangle}(0.6)\}$	$\{\Gamma_{2\langle I_{1}\rangle}(0), \Gamma_{2\langle I_{1}\rangle}(0), \Gamma_{2\langle I_{1}\rangle}(1.0)\}$

Table 5: The standardized decision matrix is provided by JK_2.

	D_1	D_2	D_3	D_4
C_1	$\{\Gamma_{-3\langle I_2\rangle}(0), \Gamma_{-3\langle I_2\rangle}(0.7), \Gamma_{-1\langle I_1\rangle}(0.3)\}$	$\{\Gamma_{1\langle I_{-1}\rangle}(0), \Gamma_{1\langle I_{-1}\rangle}(0), \Gamma_{1\langle I_{-1}\rangle}(1.0)\}$	$\{\Gamma_{1\langle I_{-3}\rangle}(0.2), \Gamma_{1\langle I_{-2}\rangle}(0.6), \Gamma_{1\langle I_0\rangle}(0.2)\}$	$\{\Gamma_{-1\langle I_{-2}\rangle}(0.5), \Gamma_{-1\langle I_1\rangle}(0.1), \Gamma_{2\langle I_2\rangle}(0.4)\}$
C_2	$\{\Gamma_{2\langle I_{-1}\rangle}(0.6), \Gamma_{2\langle I_0\rangle}(0.1), \Gamma_{2\langle I_1\rangle}(0.3)\}$	$\{\Gamma_{-3\langle I_1\rangle}(0.7), \Gamma_{-3\langle I_2\rangle}(0.1), \Gamma_{-3\langle I_3\rangle}(0.2),\}$	$\{\Gamma_{0\langle I_0\rangle}(0), \Gamma_{0\langle I_0\rangle}(0), \Gamma_{0\langle I_0\rangle}(1.0)\}$	$\{\Gamma_{-2\langle I_{-3}\rangle}(0.2), \Gamma_{1\langle I_1\rangle}(0.5), \Gamma_{2\langle I_1\rangle}(0.3)\}$
C_3	$\{\Gamma_{1\langle I_{-1}\rangle}(0.1), \Gamma_{1\langle I_0\rangle}(0.3), \Gamma_{2\langle I_1\rangle}(0.6)\}$	$\{\Gamma_{-3\langle I_2\rangle}(0), \Gamma_{-3\langle I_2\rangle}(0.8), \Gamma_{2\langle I_1\rangle}(0.2)\}$	$\{\Gamma_{-2\langle I_1\rangle}(0), \Gamma_{-2\langle I_1\rangle}(0.1), \Gamma_{-2\langle I_2\rangle}(0.9)\}$	$\{\Gamma_{1\langle I_{-1}\rangle}(0), \Gamma_{1\langle I_{-1}\rangle}(0.5), \Gamma_{2\langle I_{-1}\rangle}(0.5)\}$
C_4	$\{\Gamma_{1\langle I_0\rangle}(0.6), \Gamma_{2\langle I_0\rangle}(0.2), \Gamma_{1\langle I_3\rangle}(0.2)\}$	$\{\Gamma_{-2\langle I_0\rangle}(0.3), \Gamma_{-2\langle I_1\rangle}(0.6), \Gamma_{-1\langle I_1\rangle}(0.1)\}$	$\{\Gamma_{-2\langle I_1\rangle}(0), \Gamma_{-2\langle I_1\rangle}(0.8), \Gamma_{-2\langle I_3\rangle}(0.2)\}$	$\{\Gamma_{-2\langle I_1\rangle}(0.3), \Gamma_{-1\langle I_1\rangle}(0.1), \Gamma_{-1\langle I_2\rangle}(0.6)\}$
C_5	$\{\Gamma_{1\langle I_1\rangle}(0.2), \Gamma_{1\langle I_2\rangle}(0.5), \Gamma_{2\langle I_2\rangle}(0.3)\}$	$\{\Gamma_{-2\langle I_{-2}\rangle}(0.5), \Gamma_{-1\langle I_{-1}\rangle}(0.4), \Gamma_{0\langle I_2\rangle}(0.1)\}$	$\{\Gamma_{3\langle I_{-2}\rangle}(0), \Gamma_{3\langle I_{-2}\rangle}(0), \Gamma_{3\langle I_{-2}\rangle}(1.0)\}$	$\{\Gamma_{1\langle I_{-3}\rangle}(0.1), \Gamma_{1\langle I_{-1}\rangle}(0.2), \Gamma_{2\langle I_1\rangle}(0.7)\}$

Table 6: The standardized decision matrix is provided by JK_3.

	D_1	D_2	D_3	D_4
C_1	$\{\Gamma_{-1\langle I_1\rangle}(0.4), \Gamma_{1\langle I_1\rangle}(0.2), \Gamma_{0\langle I_3\rangle}(0.4)\}$	$\{\Gamma_{-3\langle I_{-3}\rangle}(0.3), \Gamma_{-3\langle I_{-2}\rangle}(0.3), \Gamma_{1\langle I_{-2}\rangle}(0.4)\}$	$\{\Gamma_{1\langle I_0\rangle}(0), \Gamma_{1\langle I_0\rangle}(0), \Gamma_{1\langle I_0\rangle}(1.0)\}$	$\{\Gamma_{-2\langle I_{-3}\rangle}(0), \Gamma_{-2\langle I_{-3}\rangle}(0.6), \Gamma_{1\langle I_1\rangle}(0.4)\}$
C_2	$\{\Gamma_{0\langle I_3\rangle}(0), \Gamma_{0\langle I_3\rangle}(0), \Gamma_{0\langle I_3\rangle}(1.0)\}$	$\{\Gamma_{-1\langle I_0\rangle}(0), \Gamma_{-1\langle I_0\rangle}(0.3), \Gamma_{1\langle I_1\rangle}(0.7)\}$	$\{\Gamma_{2\langle I_{-2}\rangle}(0.4), \Gamma_{2\langle I_{-1}\rangle}(0.4), \Gamma_{2\langle I_0\rangle}(0.2)\}$	$\{\Gamma_{0\langle I_{-3}\rangle}(0.1), \Gamma_{2\langle I_{-1}\rangle}(0.2), \Gamma_{2\langle I_0\rangle}(0.7)\}$
C_3	$\{\Gamma_{1\langle I_{-2}\rangle}(0.7), \Gamma_{2<\langle I_{-1}\rangle}(0.2), \Gamma_{2\langle I_0\rangle}(0.1)\}$	$\{\Gamma_{-1\langle I_3\rangle}(0.2), \Gamma_{1\langle I_1\rangle}(0.5), \Gamma_{2\langle I_1\rangle}(0.3)\}$	$\{\Gamma_{2\langle I_{-1}\rangle}(0), \Gamma_{2\langle I_{-1}\rangle}(0.4), \Gamma_{-1\langle I_0\rangle}(0.6)\}$	$\{\Gamma_{0\langle I_{-3}\rangle}(0.3), \Gamma_{1\langle I_0\rangle}(0.1), \Gamma_{2\langle I_1\rangle}(0.6)\}$
C_4	$\{\Gamma_{-1\langle I_{-3}\rangle}(0.3), \Gamma_{-2\langle I_1\rangle}(0.6), \Gamma_{1\langle I_1\rangle}(0.1)\}$	$\{\Gamma_{-2\langle I_{-2}\rangle}(0.4), \Gamma_{0\langle I_0\rangle}(0.4), \Gamma_{1\langle I_0\rangle}(0.5)\}$	$\{\Gamma_{-3\langle I_{-2}\rangle}(0.4), \Gamma_{-1\langle I_{-3}\rangle}(0.1), \Gamma_{1\langle I_{-3}\rangle}(0.5)\}$	$\{\Gamma_{1\langle I_{-1}\rangle}(0.3), \Gamma_{1\langle I_1\rangle}(0.4), \Gamma_{2\langle I_2\rangle}(0.3)\}$
C_5	$\{\Gamma_{-3\langle I_{-1}\rangle}(0.5), \Gamma_{-2\langle I_0\rangle}(0.4), \Gamma_{2\langle I_1\rangle}(0.1)\}$	$\{\Gamma_{1\langle I_{-3}\rangle}(0.1), \Gamma_{1\langle I_{-1}\rangle}(0.3), \Gamma_{1\langle I_3\rangle}(0.6)\}$	$\{\Gamma_{-3\langle I_{-2}\rangle}(0.6), \Gamma_{-1\langle I_1\rangle}(0.2), \Gamma_{1\langle I_3\rangle}(0.2)\}$	$\{\Gamma_{-1\langle I_{-2}\rangle}(0.5), \Gamma_{2\langle I_2\rangle}(0.1), \Gamma_{3\langle I_0\rangle}(0.4)\}$

TABLE 7: The overall evaluation matrix.

	D_1	D_2	D_3	D_4
C_1	$\{\Gamma_{3\langle I_{0.5899}\rangle}(0.1667), \Gamma_{-2\langle I_{0.1756}\rangle}(0.4000), \Gamma_{-2\langle I_{1.2370}\rangle}(0.4333)\}$	$\{\Gamma_{-3\langle I_{0.7617}\rangle}(0.1667), \Gamma_{-3\langle I_{0.9152}\rangle}(0.1333), \Gamma_{-1\langle I_{2.1598}\rangle}(0.7000)\}$	$\{\Gamma_{-3\langle I_{0.4904}\rangle}(0.0667), \Gamma_{-3\langle I_{1.5523}\rangle}(0.2000), \Gamma_{-1\langle I_{0.3719}\rangle}(0.7333)\}$	$\{\Gamma_{-3\langle I_{2.9741}\rangle}(0.3333), \Gamma_{-3\langle I_{1.3877}\rangle}(0.3333), \Gamma_{-2\langle I_{1.5639}\rangle}(0.3333)\}$
C_2	$\{\Gamma_{-3\langle I_{2.0078}\rangle}(0.2000), \Gamma_{-3\langle I_{0.3578}\rangle}(0.0333), \Gamma_{-1\langle I_{2.9446}\rangle}(0.7667)\}$	$\{\Gamma_{-3\langle I_{0.5894}\rangle}(0.2333), \Gamma_{-3\langle I_{0.6609}\rangle}(0.2333), \Gamma_{-2\langle I_{2.2160}\rangle}(0.5333)\}$	$\{\Gamma_{-3\langle I_{2.4770}\rangle}(0.3333), \Gamma_{-3\langle I_{1.6502}\rangle}(0.2000), \Gamma_{-1\langle I_{0.1429}\rangle}(0.4667)\}$	$\{\Gamma_{-3\langle I_{1.8320}\rangle}(0.2000), \Gamma_{-2\langle I_{0.9213}\rangle}(0.3667), \Gamma_{-2\langle I_{2.8878}\rangle}(0.4333)\}$
C_3	$\{\Gamma_{-3\langle I_{2.8891}\rangle}(0.3000), \Gamma_{-3\langle I_{2.4560}\rangle}(0.2000), \Gamma_{-1\langle I_{2.1545}\rangle}(0.5000)\}$	$\{\Gamma_{-3\langle I_{2.4000}\rangle}(0.1333), \Gamma_{-1\langle I_{0.8095}\rangle}(0.5667), \Gamma_{-2\langle I_{1.2134}\rangle}(0.3000)\}$	$\{\Gamma_{-3\langle I_{0.3171}\rangle}(0.2667), \Gamma_{-3\langle I_{2.0040}\rangle}(0.2000), \Gamma_{-0\langle I_{0.6000}\rangle}(0.5333)\}$	$\{\Gamma_{-3\langle I_{0.8605}\rangle}(0.1333), \Gamma_{-2\langle I_{2.4539}\rangle}(0.4667), \Gamma_{-1\langle I_{0.1944}\rangle}(0.4000)\}$
C_4	$\{\Gamma_{-2\langle I_{1.5865}\rangle}(0.4000), \Gamma_{-2\langle I_{1.7293}\rangle}(0.3667), \Gamma_{-2\langle I_{0.6822}\rangle}(0.2333)\}$	$\{\Gamma_{-3\langle I_{1.0658}\rangle}(0.3667), \Gamma_{-3\langle I_{2.9481}\rangle}(0.4667), \Gamma_{-3\langle I_{2.2129}\rangle}(0.1667)\}$	$\{\Gamma_{-3\langle I_{0.4637}\rangle}(0.2667), \Gamma_{-3\langle I_{2.5820}\rangle}(0.4667), \Gamma_{-2\langle I_{0.0994}\rangle}(0.2667)\}$	$\{\Gamma_{-3\langle I_{1.5703}\rangle}(0.2000), \Gamma_{-2\langle I_{0.1473}\rangle}(0.3333), \Gamma_{-2\langle I_{2.9973}\rangle}(0.4667)\}$
C_5	$\{\Gamma_{-3\langle I_{2.7494}\rangle}(0.4000), \Gamma_{-2\langle I_{0.5766}\rangle}(0.3667), \Gamma_{-2\langle I_{0.6239}\rangle}(0.2333)\}$	$\{\Gamma_{-3\langle I_{1.7130}\rangle}(0.3333), \Gamma_{-2\langle I_{1.0831}\rangle}(0.3667), \Gamma_{-2\langle I_{0.8293}\rangle}(0.3000)\}$	$\{\Gamma_{-3\langle I_{2.2406}\rangle}(0.2000), \Gamma_{-3\langle I_{2.4432}\rangle}(0.2000), \Gamma_{0\langle I_{0.4612}\rangle}(0.6000)\}$	$\{\Gamma_{-3\langle I_{2.4888}\rangle}(0.2000), \Gamma_{-3\langle I_{1.5492}\rangle}(0.1000), \Gamma_{1\langle I_{0.6000}\rangle}(0.7000)\}$

Table 8: The PDHLPIS.

D_1	D_2
$\left\{\begin{matrix}\Gamma_{-3\langle I_{2.8891}\rangle}(0.3000), \Gamma_{-3\langle I_{2.4560}\rangle}(0.2000), \\ \Gamma_{-1\langle I_{2.1545}\rangle}(0.5000)\end{matrix}\right\}$	$\left\{\begin{matrix}\Gamma_{-3\langle I_{2.4000}\rangle}(0.1333), \Gamma_{-1\langle I_{0.8095}\rangle}(0.5667), \\ \Gamma_{-2\langle I_{1.2134}\rangle}(0.3000)\end{matrix}\right\}$
D_3	D_4
$\left\{\begin{matrix}\Gamma_{-3\langle I_{2.2406}\rangle}(0.2000), \Gamma_{-3\langle I_{2.4432}\rangle}(0.2000), \\ \Gamma_{0\langle I_{0.4612}\rangle}(0.6000)\end{matrix}\right\}$	$\left\{\begin{matrix}\Gamma_{-3\langle I_{2.4888}\rangle}(0.2000), \Gamma_{-3\langle I_{1.5492}\rangle}(0.1000), \\ \Gamma_{1\langle I_{0.6000}\rangle}(0.7000)\end{matrix}\right\}$

Table 9: The PDHLNIS.

D_1	D_2
$\left\{\begin{matrix}\Gamma_{3\langle I_{0.5899}\rangle}(0.1667), \Gamma_{-2\langle I_{0.1756}\rangle}(0.4000), \\ \Gamma_{-2\langle I_{1.2370}\rangle}(0.4333)\end{matrix}\right\}$	$\left\{\begin{matrix}\Gamma_{-3\langle I_{1.0658}\rangle}(0.3667), \Gamma_{-3\langle I_{2.9481}\rangle}(0.4667), \\ \Gamma_{-3\langle I_{2.2129}\rangle}(0.1667)\end{matrix}\right\}$
D_3	D_4
$\left\{\begin{matrix}\Gamma_{-3\langle I_{0.4637}\rangle}(0.2667), \Gamma_{-3\langle I_{2.5820}\rangle}(0.4667), \\ \Gamma_{-2\langle I_{0.0994}\rangle}(0.2667)\end{matrix}\right\}$	$\left\{\begin{matrix}\Gamma_{-3\langle I_{2.9741}\rangle}(0.3333), \Gamma_{-3\langle I_{1.3877}\rangle}(0.3333), \\ \Gamma_{-2\langle I_{1.5639}\rangle}(0.3333)\end{matrix}\right\}$

Table 10: GRC of each alternative from PDHLPIS.

Alternatives	D_1	D_2	D_3	D_4
C_1	0.5725	1.0000	0.2233	0.5195
C_2	0.4343	0.5443	0.3075	0.6043
C_3	0.5949	0.7925	1.0000	1.0000
C_4	0.4081	0.6043	0.3243	0.5931
C_5	1.0000	0.5443	0.3075	0.6281

Table 11: GRC of each alternative from PDHLNIS.

Alternatives	D_1	D_2	D_3	D_4
C_1	0.4072	1.0000	0.3067	0.5645
C_2	0.6795	1.1100	0.8900	0.9433
C_3	0.3312	0.4759	1.0000	0.5047
C_4	0.8900	0.6272	0.6043	1.0000
C_5	1.0000	0.5645	0.3739	0.5869

Table 12: PDHLPIS(ξ_τ) and PDHLNIS(ξ_τ) of every alternative.

Alternatives	IVIFPIS(ξ_i)	IVIFNIS(ξ_i)
C_1	0.6953	0.5446
C_2	0.6089	1.0698
C_3	0.9824	0.5046
C_4	0.6156	0.8749
C_5	0.7575	0.6166

Table 13: PDHLRRD of each alternative from PDHLPIS.

Alternatives	C_1	C_2	C_3	C_4	C_5
PDHLRRD$_\tau$	0.1748	0.4049	0.2386	0.4233	0.5373

of classroom education development of the concept of evaluation system is the full implementation of the objective of quality education, and at the same time, it also pushes the design and implementation of teaching activities to a new stage. However, these problems can be attributed to the MAGDM problem. This paper analyzes college tennis classroom teaching effect evaluation problems based on the proposed PDHL-GRA method. There are five given latent college tennis teachers $C = \{C_1, C_2, C_3, C_4, C_5\}$, who may be the best. For the sake of assessing the college tennis classroom teaching effect fairly, three experts $JK = \{JK_1, JK_2, JK_3\}$ (expert's weight $\mathfrak{R} = [0.40, 0.33, 0.27]$) are invited. All experts depict their assessment information through four subsequent attributes: ① D_1 is teaching attitude; ② D_2 represents the teaching methods; ③ D_3 is student feedback; and ④ D_4 is teaching quality. Obviously, all attributes are benefit, and $\mathfrak{I} = (\mathfrak{I}_1, \mathfrak{I}_2, \mathfrak{I}_3, \mathfrak{I}_4)$ is the weight of four attributes where $\mathfrak{I}_\sigma \in [0,1]$, $\mathfrak{I} = 1,2,3,4$, $\sum_{\sigma=1}^{4} \mathfrak{I}_\sigma = 1$. Suppose that $q-$th expert JK_q evaluated $\tau-$th alternative C_τ under $\sigma-$th attribute D_σ as $\mathrm{PDHL}_{\tau\sigma}^{(q)}(\lambda) = \left\{\Gamma_{\tau\sigma\vartheta\langle I_\Omega\rangle}^{\Delta(q)}(\lambda_{\tau\sigma}^{\Delta(q)}) \;|\Gamma_{\tau\sigma\vartheta\langle I_\Omega\rangle}^{\Delta(q)} \in \mathrm{DHL}, \lambda_{\tau\sigma}^{\Delta(q)} \geq 0, \sum_{\Delta=1}^{\Xi\mathrm{PDHL}(\lambda)} \lambda_{\tau\sigma}^{\Delta(q)} \leq 1\right\}(\tau = 1,2,\cdots,5, \sigma = 1,2,\cdots,4, q = 1,2,3.)$ where the double linguistic hierarchy evaluation information tables are given as follows:

Table 14: The numerical results and rank derived by the PDHL-CODAS.

	PDHL-TOPSIS	Rank	PDHL-CODAS	Rank
C_1	0.8804	5	−0.5040	5
C_2	0.5782	3	0.0735	3
C_3	0.6145	4	0.1672	2
C_4	0.4358	2	0.0734	4
C_5	0.3846	1	0.2847	1
	The expected values of PDHLWA operator	Rank	The expected values of PDHLWA operator	Rank
C_1	0.4409	5	0.4433	5
C_2	0.4444	4	0.6599	3
C_3	0.6488	3	0.7611	2
C_4	0.7333	2	0.5609	4
C_5	0.8841	1	0.8823	1
	The expected values of PDHLPWA operator	Rank	The expected values of PDHLPWG operator	Rank
C_1	0.4431	5	0.4455	5
C_2	0.6466	3	0.6621	3
C_3	0.5510	4	0.7633	2
C_4	0.7355	2	0.5631	4
C_5	0.8863	1	0.8845	1

$$\begin{aligned}\Gamma = \{&\Gamma_{-3} = \text{extremely poor}, \Gamma_{-2} = \text{very poor}, \Gamma_{-1} \\ &= \text{poor}, \Gamma_0 = \text{medium}, \\ &\Gamma_1 = \text{good}, \Gamma_2 = \text{very good}, \Gamma_3 = \text{extremely good}\}, \\ \mathrm{I} = \{&\mathrm{I}_{-3} = \text{far form} \mathrm{I}_{-2} = \text{only } a \text{ little}, \mathrm{I}_{-1} \\ &= a \text{ little}, \mathrm{I}_0 = \text{just right}, \\ &\mathrm{I}_1 = \text{much}, \mathrm{I}_2 = \text{very much}, \mathrm{I}_3 = \text{extirely much}\}.\end{aligned} \tag{16}$$

Then, the decision matrixes of each invited expert are expressed in Tables 1–3.

Now, the built PDHL-GRA method is used to select the optimal latent college tennis teacher.

Step 1. Standardize the evaluation matrix of the three experts (Tables 4–6).

Step 2. According to the weighted average operator, the evaluation of three experts is aggregated into a total decision matrix, which has been converted to the PDHLTSs (see Table 7).

Step 3. Calculate the weight of the decision attribute.

$$\mathfrak{I}_1 = 0.1432\mathfrak{I}_2 = 0.3496\mathfrak{I}_3 = 0.3217\mathfrak{I}_4 = 0.1855. \tag{17}$$

Step 4. The PDHLPIS and the PDHLNIS are determined according to the global decision matrix, which has been converted to the PDHLTSs (see Tables 8 and 9).

Step 5. Figure out the GRC of every alternative from PDHLPIS as well as PDHLNIS (Tables 10 and 11).

Step 6. Figure out the degree of GRC of all alternatives from PDHLPIS as well as PDHLNIS (Table 12).

Step 7. Calculate the PDHLRRD$_\tau$ of each given alternative from PDHLPIS (Table 13).

Step 8. According to the PDHLRRD$_\tau$, all given alternatives are ranked, the higher the PDHLRRD$_\tau$, the better the alternative selected. Evidently, the order is $C_5 > C_4 > C_2 > C_3 > C_1$ and C_5 is the best one.

4.2. Comparative Analysis. Finally, we compared it with the PDHL-VIKOR method [64], PDHL-CODAS method [43], PDHLWA operator, PDHLWG operator, PDHLPWA operator, and PDHLPWG operator. The results and analysis are as follows (see Table 14). It can be seen from Table 14 that although the six methods are different, the optimal scheme obtained is the same. Only schemes 3 and 4 have slight differences between the PDHLWA operator and other methods. Therefore, the PDHL-GRA method proposed by us can scientifically and effectively solve the investment decision problem.

5. Conclusion

Life changes and people's ideas and educational expectation have brought great challenges to contemporary school education, especially to college tennis education. With the gradual development of social needs, schools seem hard to meet the more and more advanced and complex education needs of the society. In order to promote whole-person education to students, family-school cooperation has become one of the effective ways to collect common effort and establish collaboration for education. Family and school cooperation not only provides an opportunity for in-depth development by prioritizing education environment and exploring potentiality of education resources but also is a booster for the development of students' physical and mental health. However, while there are achievements in family-school cooperative management, there are still difficulties and problems. Also, the theoretical basis and teaching practices need further exploration. Affordance theory proposed by Gibson [65] claims that there is an interaction between humans (individuals) and the environment (the nature). There is potentiality of potential act in the affordance environment. Its existence is closely related to actors' capability and understanding of the environment. That is to say, affordance is characterized not only by the environment but also by the individuals and emerges only when the two factors interact. Generally, we may put our focus on the affordance of language, the affordance of social culture, and the affordance of situations. Although focal difference exists between these types of affordances, there are similarities. Classroom management can be considered as an environment created together by the child, the teacher, and the parents, as compared with the traditional classroom management, which put emphasis on the interactive rule of the teacher and the student and the environment managed by the teacher. However, parents' participation in college tennis class management provides a possible route for affordable learning environment. This paper defines an useful method for this kind of issue, since it builds the PDHL-GRA method for college tennis classroom teaching effect evaluation. And then a numerical example is used to evaluate the College tennis classroom teaching effect. Furthermore, to check on the feasibility as well as availability of the new proposed method, useful comparative analysis is also designed. In the near future, we shall pay attention to the consensus reaching process [66–71], influence of DMs' psychological factors [72–77], and how to deal with the situations when criteria weights are incompletely known [78–83].

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] J. Ye and S. Du, "Some distances, similarity and entropy measures for interval-valued neutrosophic sets and their relationship," *International Journal of Machine Learning and Cybernetics*, vol. 10, no. 2, pp. 347–355, 2019.

[2] J. Ye, "Multiple attribute group decision-making method with single-valued neutrosophic interval number information," *International Journal of Systems Science*, vol. 50, no. 1, pp. 152–162, 2019.

[3] J. Lu, S. Zhang, J. Wu, and Y. Wei, "COPRAS method for multiple attribute group decision making under picture fuzzy environment and their application to green supplier selection," *Technological and Economic Development of Economy*, vol. 27, no. 2, pp. 369–385, 2021.

[4] F. Lei, G. Wei, W. Shen, and Y. Guo, "PDHL-EDAS method for multiple attribute group decision making and its application to 3D printer selection," *Technological and Economic Development of Economy*, vol. 28, no. 1, pp. 179–200, 2021.

[5] D. Zhang, Y. Su, M. Zhao, and X. Chen, "CPT-TODIM method for interval neutrosophic MAGDM and its application to third-party logistics service providers selection," *Technological and Economic Development of Economy*, vol. 28, no. 1, pp. 201–219, 2021.

[6] H. Garg, M. Munir, K. Ullah, T. Mahmood, and N. Jan, "Algorithm for T-spherical fuzzy multi-attribute decision making based on improved interactive aggregation operators," *Symmetry*, vol. 10, no. 12, p. 670, 2018.

[7] M. Akram, A. Khan, and A. B. Saeid, "Complex Pythagorean Dombi fuzzy operators using aggregation operators and theirdecision-making," *Expert Systems*, vol. 38, 2021.

[8] H. Zhang, G. Wei, and X. Chen, "Spherical fuzzy Dombi power Heronian mean aggregation operators for multiple attribute group decision-making," *Computational and Applied Mathematics*, vol. 41, no. 3, p. 98, 2022.

[9] H. Zhang, G. Wei, and C. Wei, "TOPSIS method for spherical fuzzy MAGDM based on cumulative prospect theory and combined weights and its application to residential location," *Journal of Intelligent and Fuzzy Systems*, vol. 42, no. 3, pp. 1367–1380, 2022.

[10] Z. Ali, T. Mahmood, T. Mahmood, K. Ullah, and Q. Khan, "Einstein geometric aggregation operators using a novel complex interval-valued pythagorean fuzzy setting with application in green supplier chain management," *Reports in Mechanical Engineering*, vol. 2, no. 1, pp. 105–134, 2021.

[11] W. Yang and Y. Pang, "Hesitant interval-valued Pythagorean fuzzy VIKOR method," *International Journal of Intelligent Systems*, vol. 34, no. 5, pp. 754–789, 2019.

[12] J. Xui, J. Y. Dong, S. P. Wan, and J. Gao, "Multiple attribute decision making with triangular intuitionistic fuzzy numbers based on zero-sum game approach," *Iranian Journal of Fuzzy Systems*, vol. 16, pp. 97–112, 2019.

[13] Z. Xu and S. Zhang, "An overview on the applications of the hesitant fuzzy sets in group decision-making: theory, support and methods," *Frontiers of Engineering Management*, vol. 6, no. 2, pp. 163–182, 2019.

[14] R. Rostamzadeh, A. Esmaeili, A. S. Nia, J. Saparauskas, and M. Keshavarz Ghorabaee, "A fuzzy ARAS method for supply chain management performance measurement in smes under uncertainty," *Transformations in Business and Economics*, vol. 16, pp. 319–348, 2017.

[15] E. K. Zavadskas, J. Antucheviciene, J. Saparauskas, and Z. Turskis, "MCDM methods WASPAS and MULTIMOORA: verification of robustness of methods when assessing alternative solutions," *Economic Computation & Economic Cybernetics Studies & Research*, vol. 47, pp. 5–20, 2013.

[16] M. Yazdani, P. Zarate, A. Coulibaly, and E. K. Zavadskas, "A group decision making support system in logistics and supply chain management," *Expert Systems with Applications*, vol. 88, pp. 376–392, 2017.

[17] H. Hashemi, S. Mousavi, E. Zavadskas, A. Chalekaee, and Z. Turskis, "A new group decision model based on grey-intuitionistic fuzzy-ELECTRE and VIKOR for contractor assessment problem," *Sustainability*, vol. 10, no. 5, p. 1635, 2018.

[18] E. K. Zavadskas, R. Bausys, and I. Mazonaviciute, "Safety evaluation methodology of urban public parks by multi-criteria decision making," *Landscape and Urban Planning*, vol. 189, pp. 372–381, 2019.

[19] L. A. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning," *Learning Systems and Intelligent Robots*, vol. 8, pp. 1–10, 1974.

[20] R. M. Rodriguez, L. Martinez, and F. Herrera, "Hesitant fuzzy linguistic term sets for decision making," *IEEE Transactions on Fuzzy Systems*, vol. 20, no. 1, pp. 109–119, 2012.

[21] Q. Pang, H. Wang, and Z. Xu, "Probabilistic linguistic term sets in multi-attribute group decision making," *Information Sciences*, vol. 369, pp. 128–143, 2016.

[22] P. Liu and F. Teng, "Some Muirhead mean operators for probabilistic linguistic term sets and their applications to multiple attribute decision-making," *Applied Soft Computing*, vol. 68, pp. 396–431, 2018.

[23] A. Kobina, D. Liang, and X. He, "Probabilistic linguistic power aggregation operators for multi-criteria group decision making," *Symmetry*, vol. 9, no. 12, p. 320, 2017.

[24] G. Wei, C. Wei, and Y. Guo, "EDAS method for probabilistic linguistic multiple attribute group decision making and their application to green supplier selection," *Soft Computing*, vol. 25, no. 14, pp. 9045–9053, 2021.

[25] G. Wei, C. Wei, J. Wu, and Y. Guo, "Probabilistic linguistic multiple attribute group decision making for location planning of electric vehicle charging stations based on the generalized Dice similarity measures," *Artificial Intelligence Review*, vol. 54, no. 6, pp. 4137–4167, 2021.

[26] Y. Su, M. Zhao, C. Wei, and X. Chen, "PT-TODIM method for probabilistic linguistic MAGDM and application to industrial control system security supplier selection," *International Journal of Fuzzy Systems*, vol. 24, no. 1, pp. 202–215, 2022.

[27] M. Lin, Z. Xu, Y. Zhai, and Z. Yao, "Multi-attribute group decision-making under probabilistic uncertain linguistic environment," *Journal of the Operational Research Society*, vol. 69, no. 2, pp. 157–170, 2018.

[28] S. Wang, G. Wei, J. Lu, J. Wu, C. Wei, and X. Chen, "GRP and CRITIC method for probabilistic uncertain linguistic MAGDM and its application to site selection of hospital constructions," *Soft Computing*, vol. 26, no. 1, pp. 237–251, 2022.

[29] G. Wei, R. Lin, J. Lu, J. Wu, and C. Wei, "The generalized Dice similarity measures for probabilistic uncertain linguistic MAGDM and its application to location planning of electric vehicle charging stations," *International Journal of Fuzzy Systems*, vol. 24, no. 2, pp. 933–948, 2022.

[30] M. Zhao, H. Gao, G. Wei, C. Wei, and Y. Guo, "Model for network security service provider selection with probabilistic uncertain linguistic TODIM method based on prospect theory," *Technological and Economic Development of Economy*, vol. 28, 2022.

[31] Y. He, G. Wei, and X. Chen, "Taxonomy-based multiple attribute group decision making method with probabilistic uncertain linguistic information and its application in supplier selection," *Journal of Intelligent and Fuzzy Systems*, vol. 41, no. 2, pp. 3237–3250, 2021.

[32] Y. He, G. Wei, X. Chen, and Y. Wei, "Bidirectional projection method for multi-attribute group decision making under probabilistic uncertain linguistic environment," *Journal of Intelligent and Fuzzy Systems*, vol. 41, no. 1, pp. 1429–1443, 2021.

[33] X. Gou, H. Liao, Z. Xu, and F. Herrera, "Double hierarchy hesitant fuzzy linguistic term set and MULTIMOORA method: a case of study to evaluate the implementation status of haze controlling measures," *Information Fusion*, vol. 38, pp. 22–34, 2017.

[34] X. Wang, X. Gou, and Z. Xu, "Assessment of traffic congestion with ORESTE method under double hierarchy hesitant fuzzy linguistic environment," *Applied Soft Computing*, vol. 86, Article ID 105864, 2020.

[35] R. Krishankumar, K. S. Ravichandran, V. Shyam, S. V. Sneha, S. Kar, and H. Garg, "Multi-attribute group decision-making using double hierarchy hesitant fuzzy linguistic preference information," *Neural Computing & Applications*, vol. 32, no. 17, pp. 14031–14045, 2020.

[36] R. Krishankumar, K. S. Ravichandran, S. Kar, P. Gupta, and M. K. Mehlawat, "Double-hierarchy hesitant fuzzy linguistic term set-based decision framework for multi-attribute group decision-making," *Soft Computing*, vol. 25, no. 7, 2020.

[37] R. Krishankumar, K. S. Ravichandran, H. Liao, and S. Kar, "An integrated decision framework for group decision-making with double hierarchy hesitant fuzzy linguistic information and unknown weights," *International Journal of Computational Intelligence Systems*, vol. 13, no. 1, p. 624, 2020.

[38] J. Montserrat-Adell, Z. Xu, X. Gou, and N. Agell, "Free double hierarchy hesitant fuzzy linguistic term sets: an application on ranking alternatives in GDM," *Information Fusion*, vol. 47, pp. 45–59, 2019.

[39] X. Gou, H. Liao, Z. Xu, R. Min, and F. Herrera, "Group decision making with double hierarchy hesitant fuzzy linguistic preference relations: consistency based measures, index and repairing algorithms and decision model," *Information Sciences*, vol. 489, pp. 93–112, 2019.

[40] X. Gou, Z. Xu, H. Liao, and F. Herrera, "Multiple criteria decision making based on distance and similarity measures under double hierarchy hesitant fuzzy linguistic environment," *Computers & Industrial Engineering*, vol. 126, pp. 516–530, 2018.

[41] X. Gou, Z. Xu, and F. Herrera, "Consensus reaching process for large-scale group decision making with double hierarchy hesitant fuzzy linguistic preference relations," *Knowledge-Based Systems*, vol. 157, pp. 20–33, 2018.

[42] X. Gou, Z. Xu, H. Liao, and F. Herrera, "Probabilistic double hierarchy linguistic term set and its use in designing an improved VIKOR method: the application in smart healthcare," *Journal of the Operational Research Society*, vol. 72, no. 12, pp. 2611–2630, 2021.

[43] F. Lei, G. Wei, and X. Chen, "Model-based evaluation for online shopping platform with probabilistic double hierarchy linguistic CODAS method," *International Journal of Intelligent Systems*, vol. 36, no. 9, pp. 5339–5358, 2021.

[44] F. Lei, G. Wei, and X. Chen, "Some self-evaluation models of enterprise's credit based on some probabilistic double hierarchy linguistic aggregation operators," *Journal of Intelligent and Fuzzy Systems*, vol. 40, no. 6, pp. 11809–11828, 2021.

[45] F. Lei, G. Wei, W. Shen, and Y. Guo, "PDHL-EDAS method for multiple attribute group decision making and its application to 3D printer selection," *Technological and Economic Development of Economy*, vol. 28, 2021.

[46] J. L. Deng, "Introduction to grey system," *Journal of Grey System*, vol. 1, pp. 1–24, 1989.

[47] H. Garg, "New exponential operation laws and operators for interval-valued q-rung orthopair fuzzy sets in group decision making process," *Neural Computing & Applications*, vol. 33, no. 20, pp. 13937–13963, 2021.

[48] P. Liu, Y. Li, and F. Teng, "Bidirectional projection method for probabilistic linguistic multi-criteria group decision-making based on power average operator," *International Journal of Fuzzy Systems*, vol. 21, no. 8, pp. 2340–2353, 2019.

[49] W. Xue, Z. Xu, X. Zhang, and X. Tian, "Pythagorean fuzzy LINMAP method based on the entropy theory for railway project investment decision making," *International Journal of Intelligent Systems*, vol. 33, no. 1, pp. 93–125, 2018.

[50] Z. Wu, J. Xu, and Z. Xu, "A multiple attribute group decision making framework for the evaluation of lean practices at

logistics distribution centers," *Annals of Operations Research*, vol. 247, no. 2, pp. 735–757, 2016.
[51] N. Liao, G. Wei, and X. Chen, "TODIM method based on cumulative prospect theory for multiple attributes group decision making under probabilistic hesitant fuzzy setting," *International Journal of Fuzzy Systems*, vol. 24, no. 1, pp. 322–339, 2022.
[52] E. Javanmardi, S. Liu, and N. Xie, "Exploring grey systems theory-based methods and applications in sustainability studies: a systematic review approach," *Sustainability*, vol. 12, no. 11, p. 4437, 2020.
[53] E. Javanmardi and S. Liu, "Exploring the human cognitive capacity in understanding systems: a grey systems theory perspective," *Foundations of Science*, vol. 25, no. 3, pp. 803–825, 2019.
[54] S. Zhang, H. Gao, G. Wei, and X. Chen, "Grey relational analysis method based on cumulative prospect theory for intuitionistic fuzzy multi-attribute group decision making," *Journal of Intelligent and Fuzzy Systems*, vol. 41, no. 2, pp. 3783–3795, 2021.
[55] E. Javanmardi, S. Liu, and N. Xie, "Exploring the philosophical paradigm of grey systems theory as a postmodern theory," *Foundations of Science*, vol. 25, no. 4, pp. 905–925, 2019.
[56] B. Zhu, L. Yuan, and S. Ye, "Examining the multi-timescales of European carbon market with grey relational analysis and empirical mode decomposition," *Physica A: Statistical Mechanics and Its Applications*, vol. 517, pp. 392–399, 2019.
[57] A. Malek, S. Ebrahimnejad, and R. Tavakkoli-Moghaddam, "An improved hybrid grey relational analysis approach for green resilient supply chain network assessment," *Sustainability*, vol. 9, no. 8, p. 1433, 2017.
[58] C.-Y. Kung and K.-L. Wen, "Applying Grey Relational Analysis and Grey Decision-Making to evaluate the relationship between company attributes and its financial performance-A case study of venture capital enterprises in Taiwan," *Decision Support Systems*, vol. 43, no. 3, pp. 842–852, 2007.
[59] L. Javanmardi and Liu, "Exploring grey systems theory-based methods and applications in analyzing socio-economic systems," *Sustainability*, vol. 11, no. 15, p. 4192, 2019.
[60] E. Javanmardi, S. Liu, and N. Xie, "Exploring the philosophical foundations of grey systems theory: subjective processes, information extraction and knowledge formation," *Foundations of Science*, vol. 26, no. 2, pp. 371–404, 2020.
[61] O. Alptekin, N. Alptekin, and B. Sarac, "Evaluation of low carbon development of European union countries and Turkey using grey relational analysis," *Tehnicki Vjesnik-Technical Gazette*, vol. 25, pp. 1497–1505, 2018.
[62] H. Zhang, G. Wei, and X. Chen, "SF-GRA method based on cumulative prospect theory for multiple attribute group decision making and its application to emergency supplies supplier selection," *Engineering Applications of Artificial Intelligence*, vol. 110, Article ID 104679, 2022.
[63] C. E. Shannon, "A mathematical theory of communication," *Bell System Technical Journal*, vol. 27, no. 3, pp. 379–423, 1948.
[64] X. Gou, Z. Xu, H. Liao, and F. Herrera, "Probabilistic double hierarchy linguistic term set and its use in designing an improved VIKOR method: the application in smart healthcare," *Journal of the Operational Research Society*, vol. 72, no. 12, pp. 2611–2630, 2020.
[65] J. J. Gibson, The theory of affordances. In R. E. Shaw & J. Bransford (Eds.), Perceiving, Acting, and Knowing, pp. 67–82, Lawrence Erlbaum, Hillsdale, MI, USA, 1977.
[66] Y. Dong, C.-C. Li, Y. Xu, and X. Gu, "Consensus-based group decision making under multi-granular unbalanced 2-tuple linguistic preference relations," *Group Decision and Negotiation*, vol. 24, no. 2, pp. 217–242, 2015.
[67] Z. Zhang, Y. Gao, and Z. Li, "Consensus reaching for social network group decision making by considering leadership and bounded confidence," *Knowledge-Based Systems*, vol. 204, Article ID 106240, 2020.
[68] Y. Dong, Y. Wu, H. Zhang, and G. Zhang, "Multi-granular unbalanced linguistic distribution assessments with interval symbolic proportions," *Knowledge-Based Systems*, vol. 82, pp. 139–151, 2015.
[69] Z. Zhang, J. Gao, Y. Gao, and W. Yu, "Two-sided matching decision making with multi-granular hesitant fuzzy linguistic term sets and incomplete criteria weight information," *Expert Systems with Applications*, vol. 168, Article ID 114311, 2021.
[70] A. Punkka and A. Salo, "Preference Programming with incomplete ordinal information," *European Journal of Operational Research*, vol. 231, no. 1, pp. 141–150, 2013.
[71] Y. Wu, C.-C. Li, X. Chen, and Y. Dong, "Group decision making based on linguistic distributions and hesitant assessments: maximizing the support degree with an accuracy constraint," *Information Fusion*, vol. 41, pp. 151–160, 2018.
[72] L. F. Autran Monteiro Gomes and L. A. Duncan Rangel, "An application of the TODIM method to the multicriteria rental evaluation of residential properties," *European Journal of Operational Research*, vol. 193, no. 1, pp. 204–211, 2009.
[73] L. F. A. M. Gomes, L. A. D. Rangel, and F. J. C. Maranhão, "Multicriteria analysis of natural gas destination in Brazil: an application of the TODIM method," *Mathematical and Computer Modelling*, vol. 50, no. 1-2, pp. 92–100, 2009.
[74] Z. Jiang, G. Wei, and X. Chen, "EDAS method based on cumulative prospect theory for multiple attribute group decision-making under picture fuzzy environment," *Journal of Intelligent and Fuzzy Systems*, vol. 42, no. 3, pp. 1723–1735, 2022.
[75] Z. Jiang, G. Wei, and Y. Guo, "Picture fuzzy MABAC method based on prospect theory for multiple attribute group decision making and its application to suppliers selection," *Journal of Intelligent and Fuzzy Systems*, vol. 42, no. 4, pp. 3405–3415, 2022.
[76] Y. Huang, R. Lin, and X. Chen, "An enhancement EDAS method based on prospect theory," *Technological and Economic Development of Economy*, vol. 27, no. 5, pp. 1019–1038, 2021.
[77] N. Liao, H. Gao, G. Wei, and X. Chen, "CPT-MABAC-based multiple attribute group decision making method with probabilistic hesitant fuzzy information," *Journal of Intelligent and Fuzzy Systems*, vol. 41, no. 6, pp. 6999–7014, 2021.
[78] P. Tanadtang, D. Park, and S. Hanaoka, "Incorporating uncertain and incomplete subjective judgments into the evaluation procedure of transportation demand management alternatives," *Transportation*, vol. 32, no. 6, pp. 603–626, 2005.

Hindawi
Mathematical Problems in Engineering
Volume 2022, Article ID 8118952, 28 pages
https://doi.org/10.1155/2022/8118952

Research Article

A Novel Integrated Model under Fuzzy Environments as Support for Determining the Behavior of Pedestrians at Unsignalized Pedestrian Crossings

Biljana Ivanović,[1] Jelena Mitrović Simić ,[2] Vuk Bogdanović ,[2] Valentina Mirović ,[2] Dillip Kumar Das ,[3] and Vladimir Jakovljevic[4]

[1]*University of Montenegro, Faculty of Civil Engineering, Džordža Vašingtona Bb, 81000 Podgorica, Montenegro*
[2]*University of Novi Sad, Faculty of Technical Sciences, Trg Dositeja Obradovića 6, Novi Sad 21000, Serbia*
[3]*Civil Engineering, School of Engineering, University of Kwazulu Natal, Sustainable Transportation Research Group, Durban 4041, South Africa*
[4]*School of Mathematics and Statistics, Faculty of Science, University of Sydney, Carslaw Building, Eastern Ave, Camperdown, NSW 2006, Australia*

Correspondence should be addressed to Jelena Mitrović Simić; mjelena@uns.ac.rs

Received 11 October 2021; Revised 1 November 2021; Accepted 21 March 2022; Published 11 May 2022

Academic Editor: Darko Božanić

Pedestrians as a vulnerable category of traffic participants demand a special attention, particularly regarding their behavior at unsignalized pedestrian crossings. Unquestionably, when crossing a road at these types of pedestrian crossings, there is a potential risk, for both the pedestrians and other traffic participants, as well. Accordingly, this article shows the research on pedestrians' behavior at unsignalized intersections, conducted at four locations in the urban environment of Novi Sad. The main goals of this study are reflected in developing a multiphase model by integrating different approaches into one original unique model. First, the efficiency of the observed locations of pedestrian crossings was determined by applying a model consisting of DEA (Data Envelopment Analysis), fuzzy DEA, entropy, CRITIC (CRiteria Importance Through Intercriteria Correlation), fuzzy FUCOM (Full Consistency Method), fuzzy PIPRECIA (PIvot Pairwise RElative Import Criteria Assessment), and fuzzy MARCOS (Measurement of alternatives and ranking according to COmpromise solution). Then, the following aim of this study is to determine the values of the critical interval and then to compare these values with the accepted interval, which can be considered one of the criteria of safe pedestrians' crossing the roadway. Apart from this, the aim is related to determining the characteristics of pedestrians' behavior at unsignalized crossings, with a special reference to gender differences, as well to the fact whether the pedestrian crosses the roadway as an individual or within a group. After the empirical research and data classification, efficiency calculation, an extensive statistical and verification analysis was conducted to determine the set goals. The results imply that the relationship of the values of the accepted and critical intervals indicates the occurrence of the risky behavior of a certain number of pedestrians, which is reflected in accepting the intervals that are not completely safe for crossing the roadway and which can negatively affect the sustainable functioning of the traffic system.

1. Introduction

Behavior of pedestrians and drivers at pedestrian crossings directly affects the level of service of pedestrian flows, since the pedestrian waiting for an appropriate gap causes delays, which are the basic parameter for determining the level of service of pedestrian flows [1, 2]. Simultaneously, pedestrian flows can also affect vehicles' delays at unsignalized intersections [3]. Pedestrians crossing the roadway depend on numerous factors that affect their decision and the way of crossing the roadway (age and gender of pedestrians, drivers' behavior, vehicles' characteristics, road geometry, built environment of streets, construction measures, etc.) [4]. Researchers found evidence that women were more inclined than men to use the crossing [5]. Taking into account the different needs of users, the goal is to provide infrastructural facilities and elements that are planned and designed according to the security principles and that correspond to the projected speed and road function as well as safe infrastructure for different groups of pedestrians, such as children, the elderly, and persons with disabilities [6]. From the traffic safety aspect, for example, it was concluded that the construction of a raised pedestrian crosswalk had a positive effect on the pedestrian traffic conditions. This improvement is reflected in the reduction of pedestrian delays and in an increase in the level of service offered to pedestrians [7]. At the pedestrian crossings with the refuge island, it was proved that pedestrians accept shorter time intervals between vehicles for road crossing when they have previously gone across the road part to the refuge island. Factors such as road width, number of traffic lanes, and allowed speed affect pedestrian crossing behavior and have an impact on pedestrian-vehicle conflicts [8].

For every pedestrian crossing, the value of the critical interval can be determined, that is, the minimum necessary time for pedestrians to safely cross the roadway at a certain speed of movement. When pedestrians are in front of a pedestrian crossing, they estimate by assessing the traffic situation whether the available time interval to the vehicle's arrival at the pedestrian crossing is sufficient for them to cross the roadway safely and they make a decision "yes" or "no"; that is, they decide whether to accept or refuse the offered interval. Thus, a pedestrian assesses each interval for the specific traffic situation and accepts those intervals for which they assess to be longer than the critical ones; that is, that they are sufficient for safe crossing the road. The accepted and the refused intervals by the pedestrians form a unique set of conditions that can be used in the statistical analysis, which will be shown in this article.

In the region of Southeastern Europe, there have not been any significant research studies on the behavior of pedestrians when crossing the roadway; therefore, there have not been any analyses of the acceptable intervals. Generally, the research studies in this field both in Europe and in the world are sparse, in relation to some other parameters of the traffic flow, which have been more analysed (flow, velocity and the density of traffic flow, critical gaps and headway, travel time, etc.). Since pedestrians represent an integral part of the sustainable traffic system of a city, it is extremely important to know the patterns in which these categories of participants behave in local traffic conditions so as to enable city's traffic and urban development towards a sustainable direction.

The aim of the research conducted for the needs of this article was to determine the value of the acceptable intervals at several locations of the unsignalized pedestrian crossings, different in their geometrical characteristics and traffic conditions. Comparative analysis of the acceptable and critical interval was used for creating the model of pedestrians' behavior depending on the characteristics of the location of the unsignalized pedestrian crossing. In addition, the aim was to determine the influence of different factors on the behavior of pedestrians when crossing the roadway in the conditions of local traffic, as well as to conduct the comparative analysis of the obtained results of the research studies conducted in the world at unsignalized pedestrian crossings. Namely, the results of the research studies conducted in the world imply that the factors such as gender and the number of pedestrians in a group when crossing the roadway affect the values of the acceptable pedestrian intervals. Considering the fact that traffic conditions, regulations, and habits, as well as traffic culture, are usually different around the world, the results of the research conducted on the territory of Europe [4, 9, 10] are completely different from those conducted, for example, in Asia [11–13]. The research studies conducted in Europe show that women choose shorter intervals in comparison with men and that pedestrians circulating in groups choose longer intervals for crossing. Additionally, the aim and contribution of this article are reflected in forming an original integrated MDCM model for determining criterion weights, which involves a combination of two objective methods in a crisp form and two subjective methods in a fuzzy form. Integration of subjective-objective methods was made in order to achieve more accurate and approximately optimal results from criterion weights aspect. Such integration should ensure precise answers to various questions and give potential approximately optimal solutions in various fields taking into account different constraints. After defuzzification, their values were averaged using the Bonferroni aggregator, which gives additional significance to this model. Previously, the DEA and fuzzy DEA methods were applied to determine the efficiency of the observed pedestrian crossings, and the final efficiency was determined using the fuzzy MARCOS method. The model that takes into account the combination of objectivity, subjectivity, and fuzzy theory can be applied in other fields as well.

In addition to the introductory notes on the cause and the aims of the research, there is also an overview of the basic terms related to the characteristics of pedestrians' behavior at the pedestrian crossing, as well as a short retrospective of previous research studies conducted in this field. The method and procedure of the conducted research at four locations of the pedestrian crossings were described, after which the most important results were shown. The comparative analysis of pedestrian intervals (gaps) was performed and the comparison with the values of the critical interval for each location. Then, the intervals were analysed depending on the fact whether the pedestrian crosses the roadway alone or in a group. After the discussion of the achieved results and the comparison with similar research studies conducted in the world, the conclusion as well as the directions on further research studies in this field was given. The obtained results can be used for a detailed analysis of the microlocation of the pedestrian crossing and the formation of a plan of possible infrastructural and regulatory interventions at the location of the pedestrian crossing, in order to raise the level of pedestrian safety and increase the level of service.

2. Short Overview—Characteristics of the Behaviour of Pedestrians at the Pedestrian Crossing

The process of pedestrians' crossing the roadway is defined on the basis of subjective, that is, individual characteristics of the pedestrian, which interact with objective factors (location, traffic density, vehicles velocity, and vehicular follow-up gap). When analysing the behavior of the pedestrian and the drivers approaching the pedestrian crossing in the vehicle, a special attention is directed to the following characteristics:

(i) Demographic characteristics of the participants (gender, age)
(ii) Pedestrian delay gap—the time interval for waiting/making decision for crossing
(iii) Velocity of the pedestrian and the vehicle
(iv) Category and position of the vehicle in relation to the pedestrian
(v) Risky behavior of pedestrians when crossing the roadway.

The process of pedestrians crossing the roadway in case the interval is accepted consists of the following procedures:

(i) Arrival of the passengers at the place where they want to cross the roadway
(ii) Process of waiting for the adequate interval for crossing
(iii) Process of crossing the roadway
(iv) Stepping on the opposite edge of the roadway.

In order to present the basic characteristics of the accepted intervals, it is necessary to understand the terms and make distinction among several types of intervals found in the literature [14, 15]. There are intervals that are defined in relation to the location characteristics (adequate and critical intervals), as well as the intervals that depend on the conditions that are relevant at the moment when a pedestrian is trying to cross the roadway at the pedestrian crossing (available, accepted, and rejected interval).

The available interval is the time interval that is available to the pedestrian and represents the current time distance between the pedestrian stepping onto the roadway and the approaching vehicle. This time interval is used as a comparative criterion for pedestrian's decision whether to accept the interval or not. If the pedestrian accepts the available interval, that is, if they cross the roadway within that interval, then it becomes the accepted interval. Otherwise, the available interval becomes the rejected interval. Adequate interval or critical interval for every location is determined when the distance the pedestrian has to cover is divided by the pedestrian's velocity, and the adequate starting time is added to that value. However, it should be emphasized that in this calculation, the approximate velocity of pedestrian circulation is used, while the real velocity of each pedestrian differs, which actually depends mostly on age and physical abilities, alongside other conditions occurring at the observed location. Comparison of the values of the accepted and critical interval is used as one of the criteria for determining the term of safe pedestrian crossing the roadway [16].

Pedestrian delay, as one of the parameters occurring in research studies, implies that with the increase in the delay, the pedestrians become impatient and they accept shorter intervals for crossing the roadway [17]. The same authors reached a conclusion that the probability of accepting the smaller interval increases with the number of missed opportunities for crossing. Similar results have been found in other research studies [18, 19]. Observing the individual characteristics of pedestrians, such as gender, it was established in the research studies that women have greater delays than men; that is, they wait longer for the adequate crossing interval [20, 21]. Accordingly, the research studies have shown that women spend 27% of time longer waiting at the pedestrian crossing [11], while the crossing velocity is higher with men than with women [22, 23]. The research studies conducted on the territory of Asia show that pedestrians circulating in groups choose shorter gaps, consequently their behaviour is more aggressive, and the process of roadway crossing is more risky. The authors explain this result with the fact that pedestrians feel more protected within a group, and for that reason, they act more aggressively [24, 25]. Considering the fact that traffic conditions, regulations, and habits, as well as traffic culture, are completely different, the results of the research conducted on the territory of Europe are completely different from those conducted in Asia. Namely, the authors Yanis et al. [9] reached a conclusion that pedestrians within a group choose longer intervals for crossing the roadway in relation to those who do that individually. The pedestrian age is one of the most influential variables in the risk-taking behaviour at crosswalks [26, 27]. The findings of the generic model concluded that with the increase in the pedestrian age, there is a significant decrease in the probability of road crossing and it further decreases with the increase in the number of vehicle lanes [28].

The pedestrian behaviour, as well as an analysis of the dynamics between pedestrians and vehicles at unsignalized intersections, is usually a great source of data for mathematical modelling. Statistical analysis of the parameters, which affect the process of accepting a certain interval for crossing, enables the formation of mathematical models used for assessing the probability of the accepted pedestrian intervals. Logistic curve (logit) is usually used for the assessment of the accepted and rejected intervals, and it actually represents the probability of accepting the interval of a certain length. In this way, the acceptable pedestrian interval can be determined for a certain percentage of the population [9, 10, 29]. In accordance with modern technology and the development of traffic systems, there is a need to explore the relation between personal characteristics of pedestrians and their crossing behaviour in front of an automated vehicle (AV). The results of generalized linear mixed models showed that besides the distance from the approaching vehicle and existence of a zebra crossing, pedestrians' crossing decisions are significantly affected by the participants' age, familiarity with AVs, the communication between the AV and the pedestrian, and whether the approaching vehicle is an AV [30]. In another study, the game theory is used to analyse the interactions between pedestrians and autonomous vehicles, with a focus on yielding at crosswalks. Because autonomous vehicles will be risk-averse, the model suggests that pedestrians will be able to behave with impunity, and autonomous vehicles may facilitate a shift toward pedestrian-oriented urban neighborhoods [31]. The review of used literature and their contributions are summarized in Table 1 [9–11, 14–31].

As previously mentioned, local traffic conditions, different law enforcement and traffic culture, can lead to different patterns of pedestrian behaviour. Therefore, it is important to investigate the behaviour of pedestrians in local traffic conditions, because the obtained parameters enable the formation of a model that is based on variables that are the result of local measurements. In that way, the influences and specific qualities of the local environment would be valued, which was not the case on research locations of the study presented in this article. That would contribute to a more precise determination of the level of service at pedestrian crossings and future infrastructural and regulatory interventions on the street network.

MCDM methodology as part of operation research is very often applied for solving different problems in various fields. It is a very important and powerful tool for decision-making in traffic end transport engineering. Regardless of the fact that these are young methods that have been exploited for only a few years (fuzzy FUCOM, fuzzy PIPRECIA, fuzzy MARCOS), their applicability is at an enviable level. Apart from them, using entropy, CRITIC, and DEA methods mentioned previously represents a very comprehensive methodology for solving questions of efficiency. Table 2 shows a short review of the application of the MCDM method used in this study [32–43].

3. The Research Methods and Procedure

The flow chart of the conducted research study is shown in Figure 1 presented in the appendix. The overall flow of the research and the proposed methodology consists of 4 extensive phases and 14 steps with a larger number of activities at the lowest hierarchical level.

3.1. The First Phase. The first phase of research includes defining influential factors and data collection. It consists of four steps. The first step refers to recognizing the needs for research through a literature review and previous experience of the authors and knowledge of gap in the field that can be fulfilled by this research study. The second step of the first phase involves defining the influential factors related to the locations where the research was conducted. In the third step, the parameters of the model were defined: five inputs and two outputs in order to determine the efficiency of the observed locations. Inputs are the number of traffic lanes, vehicles' movement direction, length of pedestrian crossing, crossing time, and waiting time (Tables 3 and 4), while outputs are pedestrian flow and vehicle flow (Table 3). The vehicle flow is expressed in passenger car unit (PCU). It is common practice to consider the passenger car as the standard vehicle unit to convert the other vehicle classes. In the last, fourth step, the typical characteristics of pedestrians were defined in order to be able to form an adequate model of their behaviour. In order to collect relevant data, which would be used for forming a certain database, the research was conducted at four typical unsignalized pedestrian crossings in Novi Sad (Figure 2). The criterion for the selection was the number of traffic lanes and vehicles movement direction; thus, four types of locations were analysed: one traffic lane, one-way vehicles movement; two traffic lanes, two-way vehicles movement; two traffic lanes, one-way vehicles movement; and more than two traffic lanes, two-way vehicles movement.

The basic parameters necessary for the analysis are pedestrian delay, crossing velocity, and the lengths of the accepted and rejected intervals. All these mentioned parameters were obtained by local measurements with considering all specific features related to the behaviour of participants in typical situations. Data regarding all analysed parameters were collected by means of the method of the analysis of the video recordings made at the chosen locations. Measuring traffic flow parameters by processing videos is one of the oldest but also the safest methods that has been proven to be an efficient way of gathering data needed for analysis in a large number of researches so far. For that purpose, traffic flow of vehicles and pedestrians at the locations of the chosen unsignalized pedestrian crossings was taped. The recording was made in 18 March 2015 (Wednesday) during the period of morning peak hour (10 : 00–11 : 00). According to previous traffic research conducted on the territory of the city of Novi Sad, it has been determined that the morning peak hour is in the specified period, and it is recommended that all measurements be made in this interval, which is relevant for determining traffic flow parameters. For capturing traffic conditions for a typical weekday, it is recommended to collect field data on weekdays, such as Tuesday, Wednesday, and Thursday; and during months, such as September through November and/or February to April since these time periods represent more typical commute patterns. At this stage of the research, interviews were not conducted, because these kinds of data were not necessary for the model. The research was carried out in the real traffic conditions and can be repeated in the relevant periods. The recordings were then analysed in a certain software package used for video recording processing. The analysis of the video recording also enabled data collection regarding pedestrian delays. For the needs of the analysis, the following time sections were recorded:

t1: Pedestrian's arrival time to the pedestrian crossing

t2: The moment the pedestrian started the roadway crossing

Table 1: Review of the used literature and their contribution.

Reference	Objective of study	Contribution/Findings
HCM (2010)	Concepts, guidelines, and computational procedures for computing the capacity and quality of service of various highway facilities	Methodology of the level of service (LOS) for pedestrian flows at pedestrian crossings
MUTCD (2009)	Standards, guidance, options, and supporting information relating to the traffic control devices	Standardization of traffic control devices for pedestrian
Fitzpatrick et al. (2006)	Improving pedestrian safety at unsignalized crossings	Analysis of pedestrian intervals and determination of influential factors
Lobjois et al. (2013)	The effects of age and traffic density on street-crossing behaviour	With the increase in the delay, the pedestrians become impatient and they accept shorter intervals for crossing the roadway
Herrero-Fernández et al. (2016) and Nor et al. (2017)	Risky behaviour in young adult pedestrians/analysis of pedestrian gap acceptance and crossing decision	The probability of accepting the smaller interval increases with the number of missed opportunities for crossing
DiPietro and King (1970) and Hamed (2001)	Analysis of pedestrian gap acceptance/analysis of pedestrians' behaviour at pedestrian crossings	Women have greater delays than men at pedestrian crossings
Tiwari et al. (2007)	Pedestrian risk exposure at signalized intersections	Women spend 27% of time longer waiting at the pedestrian crossing
Rastogi et al. (2011) and Tarawneh (2001)	Study of pedestrian speeds at mid-block crossings/evaluation of pedestrian speed with the investigation of some contributing factors	The crossing velocity is higher with men than with women
Pawar and Patil (2015)/Wang et al. (2010)	Pedestrian temporal and spatial gap acceptance at mid-block street crossing/study of pedestrians' gap acceptance behaviour	Pedestrians circulating in groups choose shorter gaps, their behaviour is more aggressive, and the process of roadway crossing is more risky
Yanis et al. (2010)	Pedestrian gap acceptance for mid-block street crossing	Pedestrians within a group choose longer intervals for crossing the roadway
Lord et al. (2018) and Shaaban et al. (2018)	Perceptions of risk and crossing behaviours among the elderly/analysis of illegal pedestrian crossing behaviour	The pedestrian age is one of the most influential variables in the risk-taking behaviour at crosswalks
Kadali and Vedagiri (2020)	Role of number of traffic lanes on pedestrian gap acceptance and risk-taking behaviour at uncontrolled crosswalk locations	With the increase in the pedestrian age, there is a significant decrease in the probability of road crossing and it further decreases with the increase in the number of vehicle lanes
Papadimitriou et al. (2009)/Zhao et al. (2019)	Pedestrian behaviour models/gap acceptance probability model for pedestrians at unsignalized mid-block crosswalks based on logistic regression	Using the logistic curve (logit) for the assessment of the accepted and rejected intervals
Rad et al. (2020)	Pedestrians' road crossing behaviour in front of automated vehicles (AV)	Pedestrians' crossing decisions are significantly affected by the participants' age, familiarity with AVs, and the communication between the AV and the pedestrian
Millard-Ball (2018)	Analysis of the interactions between pedestrians and autonomous vehicles	Because autonomous vehicles will be risk-averse, the model suggests that pedestrians will be able to behave with impunity, and autonomous vehicles may facilitate a shift toward pedestrian-oriented urban neighbourhoods

t3: The moment the pedestrian finished the roadway crossing

t4: The headway of the approaching vehicle to the pedestrian crossing.

On the basis of the collected data, pedestrian delays occurred due to waiting at pedestrian crossings (t2-t1) and the time necessary for a pedestrian to cross the roadway (t3-t2) have been calculated , whereby the average pedestrian velocity was calculated, since the length of the pedestrian crossing was known for the given location. Critical interval for the location was determined by dividing the distance the pedestrian had to cover by the velocity of the pedestrian and then a certain starting time (3 s) is added to the value. The accepted intervals are obtained as the time difference between the moment when the passenger started crossing and the time headway of the vehicle approaching the pedestrian crossing. The rejected intervals are calculated as the time difference between two follow-up vehicles through the pedestrian crossing, in cases while the pedestrian was standing at the edge of the roadway and waited for the adequate interval for crossing.

The rejected intervals lower than 1 s are by previous research study recommendations excluded from the analysis, due to the assumption that these intervals are not acceptable for a single pedestrian because they occurred in the situations of vehicles approaching and the pedestrians stepping onto the pedestrian crossing at

Table 2: Short review of used MCDM methods in different fields.

Reference	Applied methods	Field of application
Deveci and Torkayesh, (2021)	Interval-valued neutrosophic set, which uses Shannon's entropy and mixed aggregation by comprehensive normalization technique	Selection of the most appropriate charging type for urban electric buses
Blagojević et al. (2020)	Fuzzy AHP and DEA	Measurement of the efficiency of freight transport railway undertakings
Torkayesh and Deveci (2021)	mulTi-noRmalization mUlti-distance aSsessmenT (TRUST)	Selection of the optimal battery swapping station for electric scooters
Krishankumar et al. (2021)	Attitudinal evidence-based Bayesian approach, variance approach, and (EDAS) approach	Prioritization of zero-carbon measures for sustainable urban mobility
Vesković et al. (2020)	Fuzzy PIPRECIA	Determining criteria significance in selecting reach stackers
Deveci et al. (2021)	CoCoSo with the logarithmic method and the power Heronian function	Prioritization of autonomous vehicles in real-time traffic management
Gokasar et. Al. (2021)	T2NN-based fuzzy WASPAS and TOPSIS	Rank the bridge maintenance projects
Memis et al. (2020)	Fuzzy PIPRECIA	Prioritization of road transportation risks
Simić et al. (2021)	Fermatean fuzzy set and CODAS method	Taxation of public transit investments
Nenadić (2019)	FUCOM and WASPAS	Ranking dangerous sections of the road
Simić et al. (2021)	CRITIC- and MABAC-based type-2 neutrosophic model	Public transportation pricing system selection
Pamučar et al. (2021)	Fuzzy Hamacher WASPAS decision-making model	For prioritization of sustainable supply chain of electric ferry implementation in public transportation

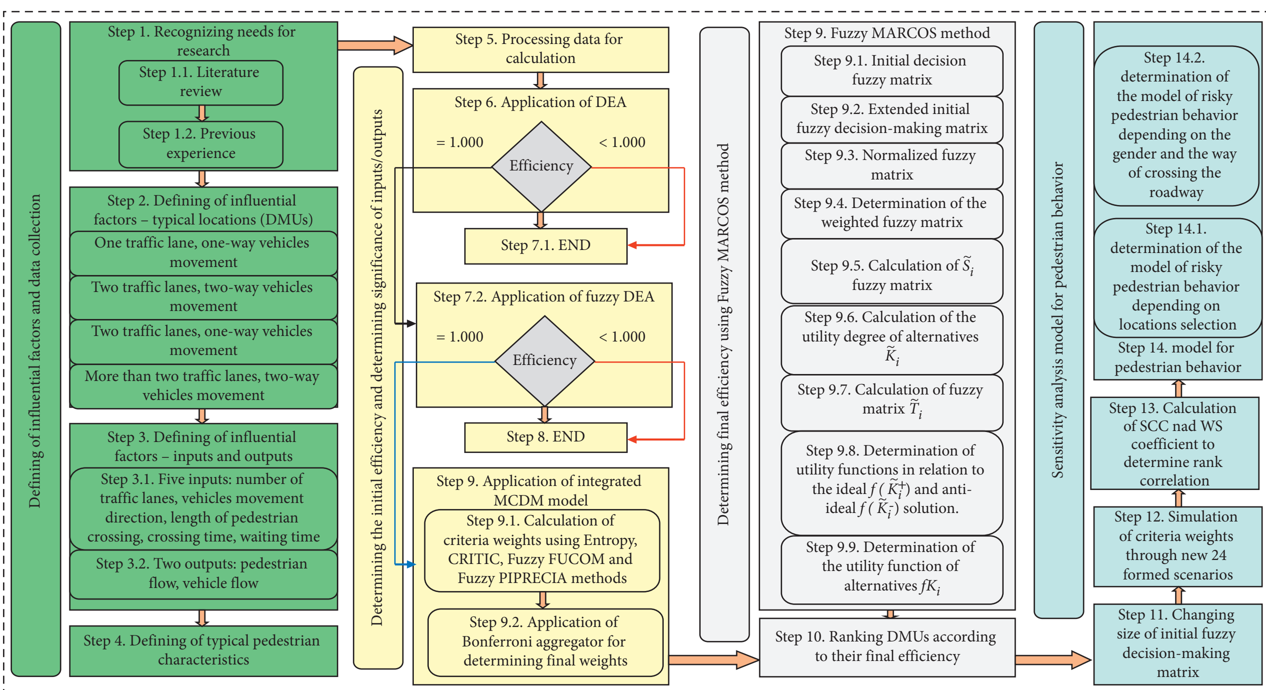


Figure 1: Research flow diagram with proposed integrated methodology.

approximately the same moment. Analogously, all accepted intervals higher than 12 s are also rejected due to the assumption that these gaps are acceptable for every pedestrian.

For every pedestrian crossing the roadway, the fact whether they did it individually or in a group was recorded, as well as whether they were male or female. When pedestrians were going across the roadway in a group, the data were established for the leading pedestrian, that is, the one who started the procedure of road crossing the first in front of the group, and the previously mentioned parameters were analysed and calculated only for them.

The analysis of the video recording and data collecting in the field resulted in the basis of about 450 intervals of pedestrians going across the roadway. Based on crossing time and the length of the pedestrian crossing, the average pedestrian velocity was calculated, as well as the pedestrian delay.

Table 3: Characteristics of the location for the analysis of pedestrian intervals.

Marking	Location name	Number of traffic lanes	Vehicles movement direction	Length of pedestrian crossing (m)	Pedestrian flow (ped/h)	Vehicle flow (PCU/h)	Level of service
K1	Fruškogorska street (1)	1	One-way	4	418	342	A
K2	Fruškogorska street (2)	2	Two-way	7	199	1092	E
K3	Braće ribnikar street	2	One-way	6	370	644	B
K4	Bulevar Kralja petra I	5	Two-way	16.5	157	1754	F

Table 4: Data obtained after recording the crossing of pedestrians at locations K1, K2, K3, and K4.

	Male	Female	One pedestrian	Group of pedestrians	Average
K1					
Waiting time (s)	1.49	0.83	1.2	1.02	1.14
Crossing time (s)	3.61	3.24	3.32	3.58	3.41
Crossing velocity (m/s)	1.13	1.29	1.25	1.16	1.22
85% Accepted (s)	7.35	6.79	7.03	7.1	7.05
85% Rejected (s)	3.16	4.31	3.6	3.5	3.338
$t_{c\ (s)}$					6.28
K2					
Waiting time (s)	4.06	3.9	3.24	6.41	3.98
Crossing time (s)	5.02	5.37	5.12	5.39	5.2
Crossing velocity (m/s)	1.5	1.4	1.48	1.34	1.45
85% Accepted (s)	8.11	6.88	7.41	8.01	7.56
85% Rejected (s)	3.797	4.66	3.79	4.67	4.115
t_c (s)					7.83
K3					
Waiting time (s)	1.74	1.04	1.26	1.81	1.42
Crossing time (s)	4.62	4.65	4.54	4.87	6.54
Crossing velocity (m/s)	1.34	1.33	1.36	1.27	1.33
85% Accepted (s)	8.11	6.88	5.881	5.803	5.862
85% Rejected (s)	3.173	2.54	2.826	3.025	2.776
$t_{c\ (s)}$					7.51
K4					
Waiting time (s)	3.21	6.5	5.13	3.53	4.56
Crossing time (s)	9.68	10	9.65	10.1	9.81
Crossing velocity (m/s)	1.74	1.71	1.77	1.65	1.73
85% Accepted (s)	7.517	6.68	7.124	7.579	7.277
85% Rejected (s)	4.192	3.608	3.773	3.925	3.802
t_c (s)					7.77

At the pedestrian crossing K1 (Fruškogorska street) during the morning peak hour (10 : 00–11 : 00), 418 pedestrians and 342 PCU/h were recorded. From the recording lasting for 1 h, altogether 95 crossings were recorded, out of which 62 crossings were by individual pedestrians, while the other crossings (33) were the crossings of groups of pedestrians. During the crossings, 108 gaps were recorded, out of which 86 accepted and 22 rejected gaps. The value of the critical interval obtained with the measured average velocity of pedestrian circulation at the location was 6.28 s.

The second typical pedestrian crossing, K2, is also in the same street, Fruškogorska street, but in the section where the two-way movement of vehicles is allowed. During the morning peak hour (10 : 00–11 : 00), 199 pedestrians and 1,092 PCU/h were recorded. The analysis of the video recording for one hour shows that there are 56 roadway crossings, out of which 43 crossings were by individual pedestrians, while the rest (13) were group crossings. A total of 107 intervals were recorded, out of which 52 were the accepted ones, and 55 were the rejected ones. The value of the critical interval obtained by the measuring the average velocity of pedestrian circulation at the given location was 7.83 s.

The third typical pedestrian crossing, K3, is in the street Braće Ribnikar. The profile of the street is such that there are two carriageway lanes separated by the divisional island with two pedestrian crossings, so that pedestrians cross the roadway in two phases. For every phase traffic flow, parameters are determined separately, such as pedestrian flow [44], vehicular flow, delays, and

Location K1 – Fruškogorska Street (1)

Location K2 – Fruškogorska Street (2)

Location K3 – Braće Ribnikar Street

Location K4 – Bulevar kralja Petra I

Figure 2: Display of the researched locations.

level of service; therefore, the interval analysis was conducted only for one phase. In that case, pedestrians cross a one-way carriageway lane with two traffic lanes. During the morning peak hour (10 : 00–11 : 00), 370 pedestrians and 644 PCU/h were recorded. The analysis of the video recording in the abovementioned period shows that there were 87 road crossings, out of which 62 were by individual pedestrians, while the rest of the crossings (25) were by group of pedestrians. During the crossings, 116 intervals were recorded, out of which 77 accepted ones and 39 rejected ones. The value of the critical interval measured by the average velocity of the pedestrian's circulation at the location was 7.51 s.

The fourth typical pedestrian crossing, K4, is in the boulevard called Bulevar Kralja Petra I. Pedestrians cross more than five traffic lanes, and vehicles go in both directions. This pedestrian crossing is typical by the fact that pedestrians use the so-called "rolling-gap" crossing method for going across the roadway. This way of crossing is typical of multilane arterials. Namely, the pedestrian starts the crossing, steps on the roadway, and pays all the attention to only one, the closest, traffic lane. With this kind of attention, the pedestrian gets to the second lane, waiting for the new acceptable interval for the crossing from the same or the opposite direction. During the morning peak hour (10 : 00–11 : 00), 157 pedestrians and 1,754 PCU/h were recorded. From the recording, which lasted for one hour, 51 crossings were recorded, out of which 33 were by the individual pedestrians, while the rest of the crossings (18) were by groups of pedestrians. During the crossings, 108 intervals were recorded, out of which 40 were the accepted ones and 68 the rejected ones. Since pedestrians in the first phase of roadway crossing pay attention only to the vehicles approaching from one direction, that is, from the left, in the analysis, it was taken into account that the value of the critical interval is calculated only for one half of the trajectory that a pedestrian is to cover. The value of the critical interval obtained by measuring the average velocity of pedestrian circulation at the location was 7.77 s. Table 4 shows all the data for all four typical locations, which are necessary for further analysis.

3.2. The Second Phase. The second phase is determining the initial efficiency and determining the significance of inputs/outputs. This phase represents the integration of several approaches into a single model to determine the efficiency of the observed locations where the research was conducted. The first step of this phase, that is, the fifth step of the overall methodology, involves the preparation and processing of data for further calculation. In the sixth step, the conventional DEA was applied (steps presented in 3.2.1) in order to determine the efficiency of the locations where the research regarding pedestrians was conducted. The algorithm is set up to react causally, which means that depending on the results of the DEA method, further steps are taken. If the results of the DEA method show that efficiency for all locations is less than 1.000, then the procedure is completed. If after the application of the DEA method, there are more than one location with efficiency = 1.000, then it proceeds to Step 7.2 in which the fuzzy DEA method is applied (steps presented in 3.2.2). After that, the procedure is the same as in the sixth step. Since the final efficiencies of all observed locations have not been obtained even when applying the fuzzy DEA method, it further implements the ninth step in which four MCDM methods for obtaining input and output weight values are integrated. There are two subjective methods in a crisp form: entropy (steps presented in 3.2.3) and CRITIC (3.2.4) and two subjective methods in a fuzzy form: fuzzy FUCOM (3.2.5) and fuzzy PIPRECIA (3.2.6). In order to obtain the final significance of the model parameters, the Bonferroni aggregator (3.2.7) was used to average the values of the criteria obtained by applying the above four methods.

3.2.1. DEA Method. This method is one of the most common methods when it comes to determining the efficiency of variant solutions [33]. It was developed by Charnes et al. [45]. this section of the study only presents the output-oriented model, which was applied to determine the efficiency of locations, that is, DMUs (decision-making units). The DEA CCR output-oriented model (max) is

$$\begin{aligned} &\mathrm{DEA}_{\mathrm{output}} = \max \sum_{i=1}^{s} w_i y_{i-\mathrm{output}} \\ &st\colon \sum_{i=1}^{m} w_i x_{ij} - \sum_{i=m+1}^{m+s} w_i y_{ij} \geq 0, \quad j = 1, \ldots, n, \sum_{i=1}^{m} w_i x_{i-\mathrm{input}} = 1 \quad w_i \geq 0, \quad i = 1, \ldots, m+s. \end{aligned} \tag{1}$$

DMU consists of m input parameters for each alternative x_{ij}, while s represents output parameters for each alternative y_{ij}, taking into account the weights of the parameters denoted by w_i. In addition, n represents the total number of DMUs.

3.2.2. Fuzzy DEA Method. This section presents an algorithm of fuzzy DEA CCR output-oriented model (max) based on linguistic variables transformed into triangular fuzzy numbers (TFNs) shown in Figure 3:

$$\begin{aligned} &\mathrm{DEA}_{\mathrm{output}} = \max \sum_{i=1}^{s} \overline{w}_i \overline{y}_{i-\mathrm{output}} \\ &st\colon \sum_{i=1}^{m} \overline{w}_i \overline{x}_{ij} - \sum_{i=m+1}^{m+s} \overline{w}_i \overline{y}_{ij} \leq 0, \quad j = 1, \ldots, n \sum_{i=1}^{m} \overline{w}_i \overline{x}_{i-\mathrm{input}} = 1 \overline{w}_i \geq 0, \quad i = 1, \ldots, m+s, \end{aligned} \tag{2}$$

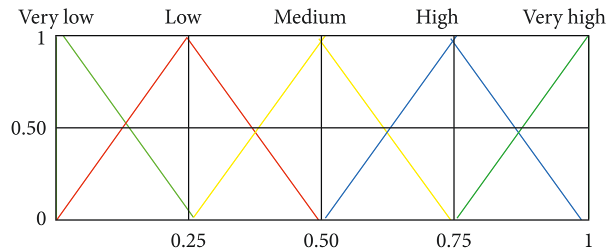


Figure 3: Fuzzy scale for the evaluation of DMUs in fuzzy DEA and fuzzy MARCOS.

where the parameters are the same as for crisp DEA, except that they are expressed in TFNs.

3.2.3. Entropy Method. The entropy method consists of the steps shown as follows [46]:

Step 1. It is necessary to normalize the initial matrix given as

$$n_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}}. \tag{3}$$

Step 2. In this step, the computation of the entropy measure is performed as

$$e_j = -\frac{1}{\ln(m)} \sum_{i=1}^{m} r_{ij} \ln(n_{ij}). \tag{4}$$

Step 3. By applying this step, the values of the objective calculation of criterion weight are obtained:

$$w_j = \frac{1 - e_j}{\sum_{j=1}^{n} (1 - e_j)}. \tag{5}$$

3.2.4. CRITIC Method. This method consists of the following steps [47]:

Step 1: Forming an initial matrix

$$x_{ij} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} i = 1, 2, \ldots, m;\; j = 1, 2, \ldots, n, \tag{6}$$

where (x_{ij}) represents the characteristics of i alternative in relation to the j criterion.

Step 2: Normalization of the initial matrix depending on the type of criteria:

$$r_{ij} = \frac{x_{ij} - \min_i x_{ij}}{\max_i x_{ij} - \min_i x_{ij}} \quad \text{if } j \in B \longrightarrow \max, \tag{7}$$

$$r_{ij} = \frac{x_{ij} - \max_i x_{ij}}{\min_i x_{ij} - \max_i x_{ij}} \quad \text{if } j \in C \longrightarrow \min. \tag{8}$$

Step 3. Determining a symmetric linear correlation matrix is as

$$r_{ij} = \frac{n \sum x_i y_i - \sum x_i \sum y_i}{\sqrt{n \sum x_i^2 - \left(\sum x_i\right)^2} \cdot \sqrt{n \sum y_i^2 - \left(\sum y_i\right)^2}}. \tag{9}$$

Step 4. Calculation of the standard deviation (σ) is given as

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \overline{x})^2}, \tag{10}$$

where n represents the total number of data in a sample and $\overline{x}$ is the mean value of the data in a sample. And the calculation of the sum of the matrix 1-rij is given as

$$\sum_{j=1}^{n} (1 - r_{ij}). \tag{11}$$

Step 5. Determining the amount of information in relation to each criterion by

$$C_j = \sigma \sum_{j'=1}^{n} 1 - r_{ij}. \tag{12}$$

Step 6. Calculation of criterion weights is given by

$$W_j = \frac{C_j}{\sum_{j=1}^{n} C_j}. \tag{13}$$

3.2.5. Fuzzy FUCOM Method. This section presents the methodology of the fuzzy FUCOM method [48]:

Step 1. Creating a set of criteria.

Step 2. Ranking the criteria based on experts' preferences by criterion importance:

$$C_{j(1)} > C_{j(2)} > \ldots > C_{j(k)}. \tag{14}$$

k denotes the ranking of the last-ranked criterion.

Step 3. Comparing the criteria using TFNs and a fuzzy linguistic scale. Referring to the criterion importance, fuzzy comparative importance $\widetilde{\varphi}_{k/(k+1)}$ is obtained using

$$\widetilde{\varphi}_{k/(k+1)} = \frac{\widetilde{\varpi}_{C_{j(k)}}}{\widetilde{\varpi}_{C_{j(k+1)}}} = \frac{\left(\varpi^l_{C_{j(k)}}, \varpi^m_{C_{j(k)}}, \varpi^u_{C_{j(k)}}\right)}{\left(\varpi^l_{C_{j(k+1)}}, \varpi^m_{C_{j(k+1)}}, \varpi^u_{C_{j(k+1)}}\right)}. \quad (15)$$

Hence, A fuzzy vector of comparative importance of evaluation criteria is obtained as follows:

$$\widetilde{\Phi} = \left(\widetilde{\varphi}_{1/2}, \widetilde{\varphi}_{2/3}, \ldots, \widetilde{\varphi}_{k/(k+1)}\right). \quad (16)$$

where $\widetilde{\varphi}_{k/(k+1)}$ is the importance of the criterion of $C_{j(k)}$ rank in comparison with the criterion of $C_{j(k+1)}$ rank.

Step 4. Calculating the optimal fuzzy weights. The final values of the fuzzy weight coefficients of the criteria $(\widetilde{w}_1, \widetilde{w}_2, \ldots, \widetilde{w}_n)^T$ are obtained. The final values of the weight coefficients should meet the conditions given by the following equations:

$$\frac{\widetilde{w}_k}{\widetilde{w}_{k+1}} = \widetilde{\varphi}_{k/(k+1)}, \quad (17)$$

$$\frac{\widetilde{w}_k}{\widetilde{w}_{k+2}} = \widetilde{\varphi}_{k/(k+1)} \otimes \widetilde{\varphi}_{(k+1)/(k+2)}. \quad (18)$$

$\varphi_{k/(k+1)}$ is the comparative importance of $C_{j(k)}$ and $C_{j(k+1)}$ criteria.

Then, it is required to calculate the values of the weight coefficients of the criteria $(\widetilde{w}_1, \widetilde{w}_2, \ldots, \widetilde{w}_n)^T$ meeting the condition that $|\widetilde{w}_k/\widetilde{w}_{k+1} - \widetilde{\varphi}_{k/(k+1)}| \leq \chi$ and $|\widetilde{w}_k/\widetilde{w}_{k+2} - \widetilde{\varphi}_{k/(k+1)} \otimes \widetilde{\varphi}_{k+1/(k+2)}| \leq \chi$, with the minimization of χ. Considering the above, the final nonlinear model $(\widetilde{w}_1, \widetilde{w}_2, \ldots, \widetilde{w}_n)^T$ is defined as

$$\min \chi,$$

$$\text{s.t.} \begin{cases} \left|\dfrac{\widetilde{w}_k}{\widetilde{w}_{k+1}} - \widetilde{\varphi}_{k/(k+1)}\right| \leq \chi, \quad \forall j, \\ \left|\dfrac{\widetilde{w}_k}{\widetilde{w}_{k+2}} - \widetilde{\varphi}_{k/(k+1)} \otimes \widetilde{\varphi}_{(k+1)/(k+2)}\right| \leq \chi, \quad \forall j, \\ \displaystyle\sum_{j=1}^{n} \widetilde{w}_j = 1, \\ w^l_j \leq w^m_j \leq w^u_j, \\ w^l_j \geq 0, \ \forall j, \\ j = 1, 2, \ldots, n. \end{cases} \quad (19)$$

$\widetilde{w}_j = (w^l_j, w^m_j, w^u_j)$, and $\widetilde{\varphi}_{k/(k+1)} = (\varphi^l_{k/(k+1)}, \varphi^m_{k/(k+1)}, \varphi^u_{k/(k+1)})$.

3.2.6. Fuzzy PIPRECIA Method. The fuzzy PIPRECIA method was created in the study [49] and consists of the steps presented as follows [50]:

Step 1. Forming a set of criteria and sorting the criteria according to marks from the first to the last, and this means that they need to be sorted unclassified.

Step 2. Each decision-maker individually evaluates presorted criteria by starting from the second criterion:

$$\overline{s^r_j} = \begin{cases} > \overline{1} & \text{if } C_j > C_{j-1}, \\ = \overline{1} & \text{if } C_j = C_{j-1}, \\ < \overline{1} & \text{if } C_j < C_{j-1}. \end{cases} \quad (20)$$

$\overline{s^r_j}$ denotes the assessment of criteria by a decision-maker r.

Step 3. Determining the coefficient $\overline{k}_j$ by

$$\overline{k}_j = \begin{cases} = \overline{1} & \text{if } j = 1, \\ 2 - \overline{s}_j & \text{if } j > 1. \end{cases} \quad (21)$$

Step 4. Determining the fuzzy weight $\overline{q}_j$ by

$$\overline{q}_j = \begin{cases} = \overline{1} & \text{if } j = 1, \\ \dfrac{\overline{q_{j+1}}}{\overline{k}_j} & \text{if } j > 1. \end{cases} \quad (22)$$

Step 5. Determining the relative weight of the criterion $\overline{w}_j$ by

$$\overline{w}_j = \frac{\overline{q}_j}{\sum_{j=1}^{n} \overline{q}_j}. \quad (23)$$

In the following steps, the inverse methodology of the fuzzy PIPRECIA method needs to be applied.

Step 6. Performing the assessment, but this time starting from a penultimate criterion:

$$\overline{s^{r'}_j} = \begin{cases} > \overline{1} & if \quad C_j > C_{j+1}, \\ = \overline{1} & if \quad C_j = C_{j+1}, \\ < \overline{1} & if \quad C_j < C_{j+1}. \end{cases} \quad (24)$$

Step 7. Determining the coefficient $\overline{k}'_j$ by

$$\overline{k}'_j = \begin{cases} = \overline{1} & \text{if } j = n, \\ 2 - \overline{s}'_j & \text{if } j > n. \end{cases} \quad (25)$$

Step 8. Determining the fuzzy weight $\overline{q}'_j$ by

$$\overline{q}'_j = \begin{cases} = \overline{1} & \text{if } j = n, \\ \dfrac{\overline{q'_{j+1}}}{\overline{k}'_j} & \text{if } j > n. \end{cases} \quad (26)$$

Step 9. Determining the relative weight of the criterion $\overline{w}'_j$ by

$$\overline{w}'_j = \frac{\overline{q}'_j}{\sum_{j=1}^{n} \overline{q}'_j}. \quad (27)$$

Step 10. In order to determine the final weights of criteria, it is first necessary to perform the defuzzification of the fuzzy values $\overline{w}_j$ and $\overline{w}'_j$:

$$\overline{w}''_j = \frac{1}{2}\left(w_j + w'_j\right). \quad (28)$$

Step 11. Checking the results obtained by applying Spearman and Pearson correlation coefficients.

3.2.7. Bonferroni Aggregator. In order to determine the final values of inputs and outputs that will be implemented further in the MCDM model, the Bonferroni aggregator is applied [51]:

$$a_{ij} = \left(\frac{1}{e(e-1)} \sum_{\substack{i,j=1 \\ i \neq j}}^{e} a_i^p \otimes a_j^q \right)^{1/p+q}. \quad (29)$$

In this research, e represents the number of methods used to determine the significance of the criteria, while p, $q \geq 0$ are a set of non-negative numbers.

3.3. The Third Phase. Following the previously applied methodology, explained in detail in the previous section, the final efficiency of the observed locations was determined using the fuzzy MARCOS method through the ninth step in the research diagram. After that, in the tenth step, DMUs were ranked according to their finally determined efficiency.

The fuzzy MARCOS method [52] consists of the following steps:

Step 1. Creating an initial fuzzy decision matrix.

Step 2. Expanding the previous matrix with the anti-ideal solution (AAI) as

$$\begin{aligned} \widetilde{A}(\mathrm{AI}) = \min_i \widetilde{x}_{ij} \quad & \text{if } j \in B \\ \max_i \widetilde{x}_{ij} \quad & \text{if } j \in C, \end{aligned} \quad (30)$$

and the ideal solution (AI) as

$$\begin{aligned} \widetilde{A}(\mathrm{ID}) = \max_i \widetilde{x}_{ij} \quad & \text{if } j \in B \\ \min_i \widetilde{x}_{ij} \quad & \text{if } j \in C. \end{aligned} \quad (31)$$

Step 3. Normalizing the initial fuzzy decision matrix as

$$\begin{aligned} \widetilde{n}_{ij} &= \left(n_{ij}^l, n_{ij}^m, n_{ij}^u\right) \\ &= \left(\frac{x_{id}^l}{x_{ij}^u}, \frac{x_{id}^l}{x_{ij}^m}, \frac{x_{id}^l}{x_{ij}^l} \right) \quad \text{if } j \in C, \end{aligned} \quad (32)$$

$$\begin{aligned} \widetilde{n}_{ij} &= \left(n_{ij}^l, n_{ij}^m, n_{ij}^u\right) \\ &= \left(\frac{x_{ij}^l}{x_{id}^u}, \frac{x_{ij}^m}{x_{id}^u}, \frac{x_{ij}^u}{x_{id}^u} \right) \quad \text{if } j \in B. \end{aligned} \quad (33)$$

Step 4. Weighting the normalized decision matrix as

$$\begin{aligned} \widetilde{v}_{ij} &= \left(v_{ij}^l, v_{ij}^m, v_{ij}^u\right) \\ &= \widetilde{n}_{ij} \otimes \widetilde{w}_j \\ &= \left(n_{ij}^l \times w_j^l, n_{ij}^m \times w_j^m, n_{ij}^u \times w_j^u\right). \end{aligned} \quad (34)$$

Step 5. Calculation of the S_i matrix is given as

$$\widetilde{S}_i = \sum_{i=1}^{n} \widetilde{v}_{ij}. \quad (35)$$

Step 6. Calculation of the degree of usefulness K_i is given as

$$\begin{aligned} \widetilde{K}_i^- &= \frac{\widetilde{S}_i}{\widetilde{S}_{ai}} \\ &= \left(\frac{s_i^l}{s_{ai}^u}, \frac{s_i^m}{s_{ai}^m}, \frac{s_i^u}{s_{ai}^l} \right), \end{aligned} \quad (36)$$

$$\begin{aligned} \widetilde{K}_i^+ &= \frac{\widetilde{S}_i}{\widetilde{S}_{id}} \\ &= \left(\frac{s_i^l}{s_{id}^u}, \frac{s_i^m}{s_{id}^m}, \frac{s_i^u}{s_{id}^l} \right). \end{aligned} \quad (37)$$

Step 7. Calculation of the fuzzy matrix $\widetilde{T}_i$ is given as

$$\begin{aligned}\tilde{T}_i &= \tilde{t}_i \\ &= \left(t_i^l, t_i^m, t_i^u\right) \\ &= \tilde{K}_i^- \oplus \tilde{K}_i^+ \\ &= \left(k_i^{-l} + k_i^{+l}, k_i^{-m} + k_i^{+m}, k_i^{-u} + k_i^{+u}\right).\end{aligned} \tag{38}$$

Determining the fuzzy number $\tilde{D}$ is given as

$$\tilde{D} = \left(d^l, d^m, d^u\right) = \max_i \tilde{t}_{ij}. \tag{39}$$

Step 8. Defuzzification of fuzzy numbers is given as

$$df_{\text{crisp}} = \frac{l + 4m + u}{6}. \tag{40}$$

Step 9. Determining the utility functions $f(\tilde{K}_i)$ is given as

$$f\left(\tilde{K}_i^+\right) = \frac{\tilde{K}_i^-}{df_{\text{crisp}}} = \left(\frac{k_i^{-l}}{df_{\text{crisp}}}, \frac{k_i^{-m}}{df_{\text{crisp}}}, \frac{k_i^{-u}}{df_{\text{crisp}}}\right), \tag{41}$$

$$f\left(\tilde{K}_i^-\right) = \frac{\tilde{K}_i^+}{df_{\text{crisp}}} = \left(\frac{k_i^{+l}}{df_{\text{crisp}}}, \frac{k_i^{+m}}{df_{\text{crisp}}}, \frac{k_i^{+u}}{df_{\text{crisp}}}\right). \tag{42}$$

Step 10. Calculation of the final utility function is given as

$$f(K_i) = \frac{K_i^+ + K_i^-}{1 + 1 - f(K_i^+)/f(K_i^+) + 1 - f(K_i^-)/f(K_i^-)}. \tag{43}$$

Step 11. Ranking alternatives.

3.4. The Fourth Phase. In the last phase of the research, a sensitivity analysis and verification of previously obtained results were performed, as well as the creation of a model of pedestrian behaviour. In the eleventh step of the applied methodology, the sensitivity of the model to changing the initial matrix size was determined, while in the twelfth step, 24 new scenarios were formed in which the weight values of the criteria were simulated and the sensitivity of the model to changing the criterion significance was determined. Subsequently, in the thirteen step, rank correlations were calculated for all 24 scenarios using the Spearman correlation coefficient (SCC) and the WS coefficient. In the last fourteenth step, a model of pedestrian behaviour was created: determination of the model of risky pedestrian behaviour depending on location selection, and determination of the model of risky pedestrian behaviour depending on the gender and the way of crossing the roadway.

4. The Research Results

4.1. Application of DEA and Fuzzy DEA Methods for Determining Efficiency. As previously mentioned, the conventional DEA method was first applied to determine the efficiency of the observed locations. The model parameters that include inputs and outputs, and their measured values are presented in Table 5.

The results obtained by (1) showed that all locations, DMU1 = DMU2 = DMU3 = DMU4, have a value of 1.000, which can be observed from two aspects: that all locations are fully efficient or that conventional DEA in this case is not applicable to determine efficiency. The reason is in fact that in our example relation about required number inputs, outputs and DMUs are not satisfied. The second aspect was taken, and then, the fuzzy DEA method was applied by (2), the parameters of which were determined based on Figure 3 and Table 5, and are shown in Table 6.

The results of the applied fuzzy DEA method showed that the second location, that is, DMU2, is not efficient and then is eliminated further from the model. The results are as follows: DMU1 = 1.000, DMU2 = 0.889, $DM\ U3$ = 1.000, $DMU4$ = 1.000. Furthermore, the model that is solved by applying the integrated MCDM model includes three DMUs with a value of 1.000.

4.2. Application of Entropy, CRITIC, Fuzzy FUCOM, and Fuzzy PIPRECIA Methods for Determining the Significance of Inputs and Outputs. Using the entropy method, that is, Equations (3)–(5), the weight values of inputs and outputs were obtained. The complete calculation and results are shown in Table 7.

Using the CRITIC method, that is, Equations (6)–(13), the weight values of inputs and outputs were obtained. The complete calculation and results are shown in Table 8.

After applying the two methods that belong to objective methods for determining the weight values of criteria, two subjective methods in a fuzzy form were also applied. When Equations (14)–(18) are applied in the fuzzy FUCOM method, the model setting expressed by (19) is obtained:

Table 5: Measured values of inputs and outputs at four locations.

	Inputs					Outputs	
	Number of traffic lanes	Vehicles movement direction	Length of pedestrian crossing (m)	Crossing time (s)	Waiting time (s)	Pedestrian flow (ped/h)	Vehicle flow (PCU/h)
DMU1	1	1	4	3.41	1.14	418	342
DMU2	2	2	7	5.20	3.98	199	1092
DMU3	2	1	6	6.54	1.42	370	644
DMU4	5	2	16.5	9.81	4.56	157	1754

Table 6: Parameters for calculation by applying the fuzzy DEA model.

	I1	I2	I3	I4	I5	O1	O2
DMU1	(0.75, 1, 1)	(0.5, 0.75, 1)	(0.75, 1, 1)	(0.75, 1, 1)	(0.75, 1, 1)	(0.75, 1, 1)	(0, 0.25, 0.5)
DMU2	(0.5, 0.75, 1)	(0.25, 0.5, 0.75)	(0.25, 0.5, 0.75)	(0.5, 0.75, 1)	(0, 0.25, 0.5)	(0, 0.25, 0.5)	(0.5, 0.75, 1)
DMU3	(0.5, 0.75, 1)	(0.5, 0.75, 1)	(0.5, 0.75, 1)	(0.5, 0.75, 1)	(0.75, 1, 1)	(0.75, 1, 1)	(0.25, 0.5, 0.75)
DMU4	(0, 0.25, 0.5)	(0.25, 0.5, 0.75)	(0, 0.25, 0.5)	(0, 0.25, 0.5)	(0, 0.25, 0.5)	(0, 0.25, 0.5)	(0.75, 1, 1)

Table 7: Calculation and results obtained by applying the entropy method.

	I1	I2	I3	I4	I5	O1	O2
DMU1	1	1	4	3.41	1.14	418	342
DMU3	2	1	6	6.54	1.42	370	644
DMU4	5	2	16.5	9.81	4.56	157	1754
n_{ij}							
DMU1	0.125	0.250	0.151	0.173	0.160	0.442	0.125
DMU3	0.250	0.250	0.226	0.331	0.199	0.392	0.235
DMU4	0.625	0.500	0.623	0.496	0.640	0.166	0.640
ln (n_{ij})							
DMU1	−2.079	−1.386	−1.891	−1.757	−1.832	−0.816	−2.081
DMU3	−1.386	−1.386	−1.485	−1.106	−1.612	−0.938	−1.448
DMU4	−0.470	−0.693	−0.474	−0.700	−0.446	−1.795	−0.446
$\sum_{i=1}^{m} r_{ij} \ln (n_{ij})$	−0.900	−1.040	−0.917	−1.017	−0.900	−1.026	−0.886
e_j	0.819	0.946	0.834	0.926	0.819	0.934	0.806
$1\text{-}e_j$	0.181	0.054	0.166	0.074	0.181	0.066	0.194
$\sum_{j=1}^{n} (1-e_j)$				0.915			
w_j	**0.197**	**0.059**	**0.181**	**0.081**	**0.197**	**0.072**	**0.212**

w_j are the weight values of inputs and outputs.

Table 8: Calculation and results obtained by applying the CRITIC method.

	I1	I2	I3	I4	I5	O1	O2
DMU1	1	1	4	3.41	1.14	418	342
DMU3	2	1	6	6.54	1.42	370	644
DMU4	5	2	16.5	9.81	4.56	157	1754
Max	5.00	2.00	16.50	9.81	4.56	418.00	1754.00
min	1.00	1.00	4.00	3.41	1.14	157.00	342.00
Normalization							
	I1	I2	I3	I4	I5	O1	O2
DMU1	1.000	1.000	1.000	1.000	1.000	1.000	0.000
DMU3	0.750	1.000	0.840	0.511	0.918	0.816	0.214
DMU4	0.000	0.000	0.000	0.000	0.000	0.000	1.000
STdev	0.520	0.577	0.537	0.500	0.555	0.532	0.527
Correlation (r_{ij})							
	I1	I2	I3	I4	I5	O1	O2
I1	1.000	0.971	0.996	0.964	0.986	0.998	−0.999
I2	0.971	1.000	0.989	0.872	0.997	0.985	−0.979
I3	0.996	0.989	1.000	0.935	0.997	1.000	−0.998
I4	0.964	0.872	0.935	1.000	0.906	0.944	−0.953
I5	0.986	0.997	0.997	0.906	1.000	0.995	−0.991

Table 8: Continued.

	I1	I2	I3	I4	I5	O1	O2
O1	0.998	0.985	1.000	0.944	0.995	1.000	−1.000
O2	−0.999	−0.979	−0.998	−0.953	−0.991	−1.000	1.000
1- r_{ij}							
I1	0.000	0.029	0.004	0.036	0.014	0.002	1.999
I2	0.029	0.000	0.011	0.128	0.003	0.015	1.979
I3	0.004	0.011	0.000	0.065	0.003	0.000	1.998
I4	0.036	0.128	0.065	0.000	0.094	0.056	1.953
I5	0.014	0.003	0.003	0.094	0.000	0.005	1.991
O1	0.002	0.015	0.000	0.056	0.005	0.000	2.000
O2	1.999	1.979	1.998	1.953	1.991	2.000	0.000
SUM	2.085	2.165	2.082	2.332	2.110	2.079	11.921
C_j	1.085	1.250	1.118	1.166	1.172	1.106	6.278
w_j	**0.082**	**0.095**	**0.085**	**0.089**	**0.089**	**0.084**	**0.476**

$$
\begin{aligned}
&\min \chi,\\
&\text{s.t.}\begin{cases}
\left(\frac{w_6^l}{w_7^u}-1.30\right)\le\chi;\left(\frac{w_6^m}{w_7^m}-1.40\right)\le\chi;\left(\frac{w_6^u}{w_7^l}-1.50\right)\le\chi;\left(\frac{w_7^l}{w_4^u}-0.93\right)\le\chi;\left(\frac{w7}{w_4^m}-1.07\right)\le\chi;\left(\frac{w_7^u}{w_4^l}-1.23\right)\le\chi;\\
\left(\frac{w_4^l}{w_5^u}-1.06\right)\le\chi;\left(\frac{w_4^m}{w_5^m}-1.09\right)\le\chi;\left(\frac{w_4^u}{w_5^l}-1.18\right)\le\chi;\left(\frac{w_5^l}{w_3^u}-1.00\right)\le\chi;\left(\frac{w_5^m}{w_3^m}-1.11\right)\le\chi;\left(\frac{w_5^u}{w_3^l}-1.23\right)\le\chi;\\
\left(\frac{w_3^l}{w_1^u}-1.05\right)\le\chi;\left(\frac{w_3^m}{w_1^m}-1.15\right)\le\chi;\left(\frac{w_3^u}{w_1^l}-1.26\right)\le\chi;\left(\frac{w_1^l}{w_2^u}-1.00\right)\le\chi;\left(\frac{w_1^m}{w_2^m}-1.09\right)\le\chi;\left(\frac{w_1^u}{w_2^l}-1.18\right)\le\chi;\\
\left(\frac{w_6^l}{w_4^u}-1.21\right)\le\chi;\left(\frac{w_6^m}{w_4^m}-1.50\right)\le\chi;\left(\frac{w_6^u}{w_4^l}-1.85\right)\le\chi;\left(\frac{w_7^l}{w_5^u}-0.99\right)\le\chi;\left(\frac{w_7^m}{w_5^m}-1.28\right)\le\chi;\left(\frac{w_7^u}{w_5^l}-1.66\right)\le\chi;\\
\left(\frac{w_4^l}{w_3^u}-1.06\right)\le\chi;\left(\frac{w_4^m}{w_3^m}-1.33\right)\le\chi;\left(\frac{w_4^u}{w_3^l}-1.66\right)\le\chi;\left(\frac{w_5^l}{w_1^u}-1.05\right)\le\chi;\left(\frac{w_5^m}{w_1^m}-1.28\right)\le\chi;\left(\frac{w_5^u}{w_1^l}-1.55\right)\le\chi;\\
\left(\frac{w_3^l}{w_2^u}-1.05\right)\le\chi;\left(\frac{w_3^m}{w_2^m}-1.25\right)\le\chi;\left(\frac{w_3^u}{w_2^l}-1.49\right)\le\chi;\\
\frac{\left(w_1^l+4\cdot w_1^m+w_1^u\right)}{6}+\frac{\left(w_2^l+4\cdot w_2^m+w_2^u\right)}{6}+\frac{\left(w_3^l+4\cdot w_3^m+w_3^u\right)}{6}+\frac{\left(w_4^l+4\cdot w_4^m+w_4^u\right)}{6}+\frac{\left(w_5^l+4\cdot w_5^m+w_5^u\right)}{6}+\frac{\left(w_6^l+4\cdot w_6^m+w_6^u\right)}{6}+\frac{\left(w_7^l+4\cdot w_7^m+w_7^u\right)}{6}=1;\\
w_1^l\le w_1^m\le w_1^u;w_2^l\le w_2^m\le w_2^u;w_3^l\le w_3^m\le w_3^u;w_4^l\le w_4^m\le w_4^u;w_5^l\le w_5^m\le w_5^u;w_6^l\le w_6^m\le w_6^u;w_7^l\le w_7^m\le w_7^u;\\
w_1^l,w_2^l,w_3^l,w_4^l,w_5^l,w_6^l,w_7^l\ge 0.
\end{cases}
\end{aligned}
\tag{44}
$$

By solving the set problem, it is obtained the fuzzy values of criteria, which are

$$
\begin{aligned}
w_1 &= (0.100, 0.107, 0.107),\\
w_2 &= (0.085, 0.090, 0.111),\\
w_3 &= (0.111, 0.119, 0.134),\\
w_4 &= (0.143, 0.148, 0.175),\\
w_5 &= (0.121, 0.134, 0.146),\\
w_6 &= (0.228, 0.228, 0.253),\\
w_7 &= (0.158, 0.158, 0.163).
\end{aligned}
\tag{45}
$$

After that, (40) is applied for defuzzification, so the following values are obtained:

$$
\begin{aligned}
w_1 &= 0.106,\\
w_2 &= 0.093,\\
w_3 &= 0.120,\\
w_4 &= 0.152,\\
w_5 &= 0.134,\\
w_6 &= 0.232,\\
w_7 &= 0.163.
\end{aligned}
\tag{46}
$$

The fuzzy PIPRECIA method was used as another subjective method for determining the weight values of the criteria. Using Equations (20)–(23), the calculation shown in Table 9 was performed.

The inverse fuzzy PIPRECIA methodology, that is, Equations (24)–(27), was then applied. The results are shown in Table 10.

Table 9: Results by steps applying fuzzy PIPRECIA.

	$\bar{s}_j$	$\bar{k}_j$	$\bar{q}_j$	$\bar{w}_j$	DFwj
I1		(1, 1, 1)	(1, 1, 1)	(0.05, 0.1, 0.16)	0.103
I2	(0.5, 0.67, 1)	(1, 1.33, 1.5)	(0.67, 0.75, 1)	(0.03, 0.08, 0.16)	0.084
I3	(1.2, 1.3, 1.35)	(0.65, 0.7, 0.8)	(0.83, 1.07, 1.54)	(0.04, 0.11, 0.25)	0.121
I4	(1.3, 1.45, 1.5)	(0.5, 0.55, 0.7)	(1.19, 1.95, 3.08)	(0.06, 0.2, 0.5)	0.225
I5	(0.5, 0.67, 1)	(1, 1.33, 1.5)	(0.79, 1.46, 3.08)	(0.04, 0.15, 0.5)	0.189
O1	(1.2, 1.3, 1.35)	(0.65, 0.7, 0.8)	(0.99, 2.09, 4.73)	(0.05, 0.21, 0.77)	0.278
O2	(0.5, 0.67, 1)	(1, 1.33, 1.5)	(0.66, 1.57, 4.73)	(0.03, 0.16, 0.77)	0.240
SUM			(6.14, 9.88, 19.16)	(0.05, 0.1, 0.16)	

Table 10: Results by steps applying fuzzy PIPRECIA-I.

	$\bar{s}'_j$	$\bar{k}'_j$	$\bar{q}'_j$	$\bar{w}'_j$	Df wj
I1	(1.1, 1.15, 1.2)	(0.8, 0.85, 0.9)	(0.47, 0.38, 0.62)	(0.08, 0.08, 0.11)	0.087
I2	(0.4, 0.5, 0.67)	(1.33, 1.5, 1.6)	(0.42, 0.33, 0.49)	(0.08, 0.07, 0.09)	0.074
I3	(0.33, 0.4, 0.5)	(1.5, 1.6, 1.67)	(0.67, 0.49, 0.66)	(0.12, 0.11, 0.12)	0.110
I4	(1.1, 1.15, 1.2)	(0.8, 0.85, 0.9)	(1.12, 0.78, 0.99)	(0.2, 0.17, 0.17)	0.175
I5	(1.01, 0.5, 0.67)	(1.33, 1.5, 0.99)	(1.01, 0.67, 0.79)	(0.18, 0.14, 0.14)	0.149
O1	(1, 1, 1.05)	(0.95, 1, 1)	(1, 1, 1.05)	(0.18, 0.21, 0.19)	0.204
O2		(1, 1, 1)	(1, 1, 1)	(0.18, 0.21, 0.18)	0.202
SUM			(5.69, 4.65, 5.6)	(0.08, 0.08, 0.11)	

In order to calculate the final weight values of the criteria using the fuzzy and inverse fuzzy PIPRECIA methods, (29) was applied, and the following values were obtained in a crisp form because the defuzzification was previously performed using (40):

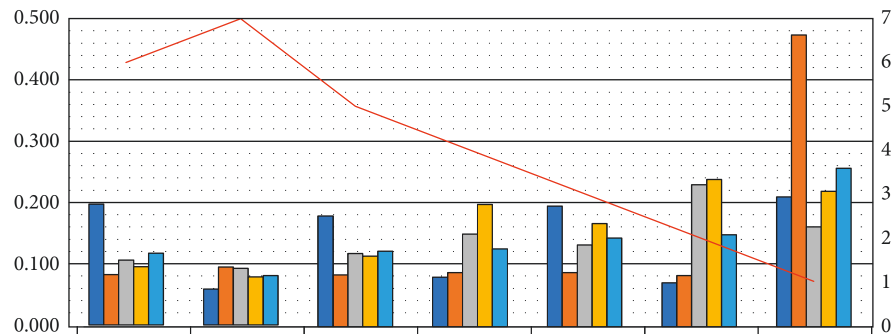
$$\begin{aligned} w_1 &= 0.095,\\ w_2 &= 0.079,\\ w_3 &= 0.115,\\ w_4 &= 0.200,\\ w_5 &= 0.169,\\ w_6 &= 0.241,\\ w_7 &= 0.211. \end{aligned} \tag{47}$$

4.3. Application of Bonferroni Aggregator for Determining the Final Values of Inputs and Outputs.

Using the Bonferroni aggregator, the final values of all criteria were obtained, which is shown in Figure 4. The values are obtained as follows:

$$\begin{aligned} &\mathrm{BM}^{p=1,q=1} = (0.197, 0.082, 0.106, 0.095)\\ &\varpi_{C_1} = \left(\frac{1}{4(4-1)}\sum_{\substack{i,j=1\\ i\neq j}}^{4}\varpi_{C_1 i}^{p}\varpi_{C_1 j}^{q}\right)^{1/1+1} = \left(0.083\left(\begin{array}{c} 0.197^1\cdot 0.082^1 + 0.197^1\cdot 0.106^1 + 0.197^1\cdot 0.095^1\\ +0.082^1\cdot 0.197^1 + 0.082^1\cdot 0.106^1 + 0.082^1\cdot 0.095^1\\ +0.106^1\cdot 0.197^1 + 0.106^1\cdot 0.082^1 + 0.106^1\cdot 0.095^1\\ +0.095^1\cdot 0.197^1 + 0.095^1\cdot 0.082^1 + 0.095^1\cdot 0.106^1 \end{array}\right)\right)^{1/1+1}\\ &= 0.117. \end{aligned} \tag{48}$$

According to the results shown in Figure 4, which were obtained by applying the integrated objective-subjective model (entropy-CRITIC-fuzzy FUCOM-fuzzy PIPRECIA, and Bonferroni aggregator), the output O2 has the highest value, that is, the seventh criterion with a value of 0.259. The second most significant parameter is the sixth criterion with



	C1	C2	C3	C4	C5	C6	C7
Entropy	0.197	0.059	0.181	0.081	0.197	0.072	0.212
CRITIC	0.082	0.095	0.085	0.089	0.089	0.084	0.476
Fuzzy FUCOM	0.106	0.093	0.120	0.152	0.134	0.232	0.163
Fuzzy PIPRECIA	0.095	0.079	0.115	0.200	0.169	0.241	0.221
Bonferroni	0.117	0.081	0.124	0.127	0.145	0.150	0.259
Rank	6	7	5	4	3	2	1

Entropy
CRITIC
Fuzzy FUCOM
Fuzzy PIPRECIA
Bonferroni
Rank

Figure 4: Final values of the criteria after the application of the subjective-objective model and the Bonferroni aggregator.

a value of 0.150. The most significant input is the waiting time at the pedestrian crossing with a value of 0.145. The least significant input is vehicles' movement direction with a value of 0.081.

4.4. Application of the Fuzzy MARCOS Method for Determining the Final Efficiency of Pedestrian Crossings. This section presents the results obtained by applying the fuzzy MARCOS method for determining the final ranking according to the efficiency of pedestrian crossings, DMU1, DMU3, and DMU4. It is important to note that the linguistic scale from the original fuzzy MARCOS method was not used for the initial matrix, but the scale in Figure 4. Based on this scale and the data from Table 5, the extended fuzzy initial decision matrix shown in Table 11 was formed.

Since the orientation of the criteria was taken into account when evaluating DMUs by all parameters using the linguistic scale, it means that all criteria were marked as benefit further in applying the fuzzy MARCOS method and (30) and (31) were applied to extend the initial fuzzy matrix.

Equation (33) was then used to perform the normalization of the initial fuzzy matrix and (34) to calculate the weighted normalized matrix shown in Table 12.

The applying Equations (35)–(43), the results presented in Table 13 were obtained.

Based on the final efficiencies of the observed locations of pedestrian crossings obtained using the entropy-CRITIC-fuzzy FUCOM-fuzzy PIPRECIA model based on the Bonferroni aggregator and the fuzzy MARCOS method, it can be seen that the second location showed the highest efficiency in relation to the measured input-output parameters of the model. Implications of this model can be manifested through monitoring these locations in future in order to increase their efficiency, especially the worst ranked.

4.5. Testing and Verification of Results. In this section of the study, the effect of changing the size of the initial fuzzy matrix was first tested by forming two sets in which the last-ranked DMU was eliminated from the calculation. Figure 5 shows the results obtained for this part of the model robustness testing.

From Figure 5, it can be seen that the size of the initial fuzzy matrix has no effect on changing the results in terms of the final ranking of alternatives, while their values change, but slightly.

Furthermore, the results were tested in relation to a change in the significance of the criteria; that is, a sensitivity analysis was performed. A total of 24 scenarios was formed in which new criterion values were simulated based on

$$\widetilde{W}_{n\beta} = \left(1 - \widetilde{W}_{n\alpha}\right) \frac{\widetilde{W}_{\beta}}{\left(1 - \widetilde{W}_{n}\right)}. \quad (49)$$

The 24 scenarios were formed by reducing the values of four most significant criteria by 15–90% of their own value. In scenarios S1–S6, the values of the most significant criterion, O2, were reduced. In scenarios S7–S12, S13–S18, and S19–S24, the values of criteria O1, I5, and I4 were reduced, respectively.

The results given in Figure 6 show that a change in the most significant criterion has an impact on a change in the rank of DMUs. In scenarios S2–S6, the final rank of DMUs changes because the value of the most significant criterion, O2, decreases by a range of 30–90%, which shows that the traffic flow of vehicles has an impact on the efficiency of the

Table 11: Extended fuzzy initial decision matrix.

	I1	I2	I3	I4	I5	O1	O2
AAI	(0, 0.25, 0.5)	(0.25, 0.5, 0.75)	(0, 1, 1)	(0.75, 1, 1)	(0.75, 1, 1)	(0.75, 1, 1)	(0.75, 1, 1)
DMU1	(0.75, 1, 1)	(0.5, 0.75, 1)	(0.75, 1, 1)	(0.75, 1, 1)	(0.75, 1, 1)	(0.75, 1, 1)	(0, 0.25, 0.5)
DMU3	(0.5, 0.75, 1)	(0.5, 0.75, 1)	(0.5, 0.75, 1)	(0.5, 0.75, 1)	(0.75, 1, 1)	(0.75, 1, 1)	(0.25, 0.5, 0.75)
DMU4	(0, 0.25, 0.5)	(0.25, 0.5, 0.75)	(0, 0.25, 0.5)	(0, 0.25, 0.5)	(0, 0.25, 0.5)	(0, 0.25, 0.5)	(0.75, 1, 1)
AI	(0.75, 1, 1)	(0.5, 0.75, 1)	(0.75, 0.25, 0.5)	(0, 0.25, 0.5)	(0, 0.25, 0.5)	(0, 0.25, 0.5)	(0, 0.25, 0.5)

Table 12: Weighted normalized fuzzy initial decision matrix.

	I1	I2	I3	I4	I5	O1	O2
AAI	(0, 0.03, 0.06)	(0.02, 0.04, 0.06)	(0, 0.03, 0.06)	(0, 0.03, 0.06)	(0, 0.04, 0.07)	(0, 0.04, 0.08)	(0, 0.06, 0.13)
DMU1	(0.09, 0.12, 0.12)	(0.04, 0.06, 0.08)	(0.09, 0.12, 0.12)	(0.1, 0.13, 0.13)	(0.11, 0.15, 0.15)	(0.11, 0.15, 0.15)	(0, 0.06, 0.13)
DMU3	(0.06, 0.09, 0.12)	(0.04, 0.06, 0.08)	(0.06, 0.09, 0.12)	(0.06, 0.1, 0.13)	(0.11, 0.15, 0.15)	(0.11, 0.15, 0.15)	(0.06, 0.13, 0.19)
DMU4	(0, 0.03, 0.06)	(0.02, 0.04, 0.06)	(0, 0.03, 0.06)	(0, 0.03, 0.06)	(0, 0.04, 0.07)	(0, 0.04, 0.08)	(0.19, 0.26, 0.26)
AI	(0.09, 0.12, 0.12)	(0.04, 0.06, 0.08)	(0.09, 0.12, 0.12)	(0.1, 0.13, 0.13)	(0.11, 0.15, 0.15)	(0.11, 0.15, 0.15)	(0.19, 0.26, 0.26)

Table 13: Final results obtained by applying the integrated model.

	$f(\tilde{K}_i^-)$	$f(\tilde{K}_i^+)$	K-	$K+$	fK-	fK+	Ki	Rank
DMU1	(0.05, 0.08, 0.11)	(0.1, 0.27, 4.05)	9.330	0.823	0.077	0.872	0.772	**2**
DMU3	(0.05, 0.07, 0.12)	(0.09, 0.26, 4.35)	9.788	0.815	0.076	0.915	0.802	**1**
DMU4	(0.02, 0.04, 0.08)	(0.04, 0.16, 3.01)	6.589	0.499	0.047	0.616	0.321	**3**

The bold values are final ranking of DMUs (decision making units), which are in fact the locations of pedestrian crossings.

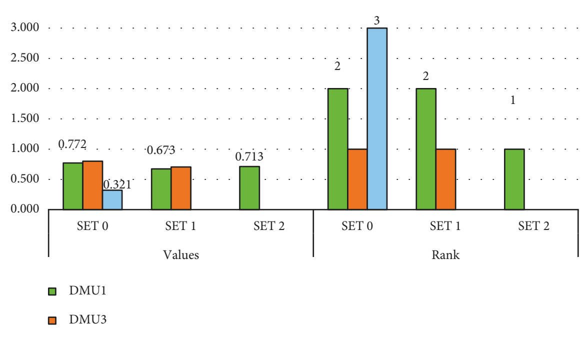


Figure 5: Testing the results depending on the size of the initial fuzzy decision matrix.

observed locations of pedestrian crossings. In other scenarios, when the values of other criteria are reduced, there is no change in the final ranks. Due to the occurrence of the change in ranks, rank correlations were then calculated for all 24 scenarios using the Spearman correlation coefficient (SCC) and the WS coefficient [53].

Figure 7 shows the rank correlation calculated by changing the SCC and WS coefficients. For all rank changes in scenarios S2–S6, the correlation coefficient is 0.500, while in other scenarios, there is a total rank correlation. Observing the average total value of rank correlation with a value of 0.896, it can be concluded that there is a large correlation.

4.6. Comparative Analysis of Pedestrian Intervals. The obtained results served as a basis for the comparative analysis of pedestrian intervals, whereby it was assumed that the values of pedestrian intervals are different types of pedestrian crossings, which were divided into four typical ones. Also, it is assumed that there are different relations in values between the accepted, the rejected, and the critical intervals depending on the type of the pedestrian crossing. Typical pedestrian crossings were chosen on the basis of the number of lanes the pedestrian has to cross, as well as of the direction of the vehicle approaching to the pedestrian crossing. In accordance with similar research studies [16, 17], the values of 85% of the accepted intervals are taken as the representative values. Also, it was established that the accepted intervals behave by normal distribution and the rejected ones by log-normal distribution.

Figure 8 shows the cumulative distribution of the accepted intervals, for each typical location separately.

Figure 9 shows the cumulative distribution of the rejected intervals, for each typical location separately.

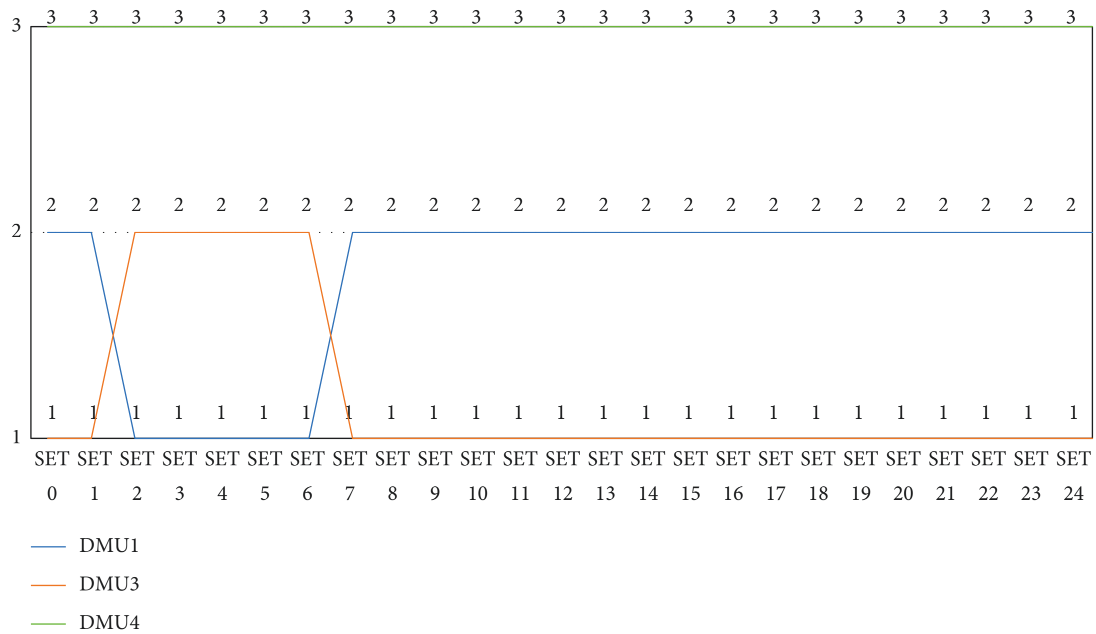
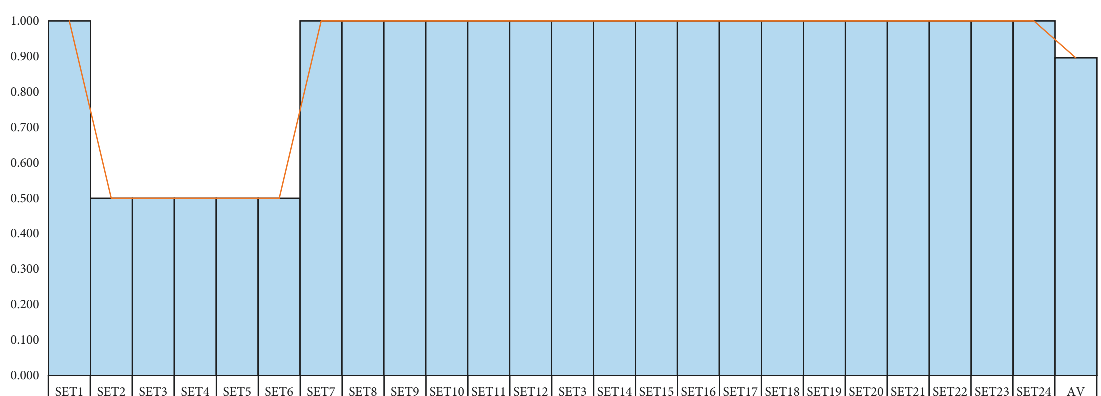


Figure 6: Ranks of alternatives in relation to 24 newly formed scenarios.



	SET1	SET2	SET3	SET4	SET5	SET6	SET7	SET8	SET9	SET10	SET11	SET12	SET3	SET14	SET15	SET16	SET17	SET18	SET19	SET20	SET21	SET22	SET23	SET24	AV
SCC	1.000	0.500	0.500	0.500	0.500	0.500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.896
WS	1.000	0.500	0.500	0.500	0.500	0.500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.896

Figure 7: Calculation of SCC and WS coefficients for determining rank correlation.

Figure 10 shows the comparative display of cumulative distribution of the accepted intervals at the chosen typical locations K1, K2, K3, and K4.

As it can be seen in Figure 10, the shortest accepted intervals were recorded the location K3 (5.86 s), where the pedestrians were crossing two traffic lanes, and the vehicles were approaching only from one direction. The explanation for this occurrence is in the fact that the pedestrian crossing K3 represents the so-called boulevard type of pedestrian crossings, where pedestrians cross the roadway in two phases, whereby between the traffic lanes of the opposite vehicles' movement directions, there is a divisional island. During the accepted intervals recording, not only the crossings of the pedestrians who start the first phase but the crossings that were a part of the second phase of crossing were considered. Namely, the analysis of the behaviour of pedestrians during roadway crossing showed that pedestrians in the first phase already pay attention to the vehicles, which are conflicting for their crossing in the second phase. In that way, pedestrians have more time for the assessment, they have more confidence, they are more visible to the drivers since they have already started moving, and for these reasons, they choose shorter intervals for crossing the roadway. At other locations, approximately the same values of the accepted intervals were recorded. At crossing K1, pedestrians were crossing only one traffic lane, whereby the vehicles were approaching only from one direction. The value of the accepted intervals at this location was 7.05 s. At the crossing K4 (more than two traffic lanes, two-way direction of vehicles movement), the value of the accepted intervals was 7.28 s. The longest accepted pedestrian intervals were noticed at crossing K2 (7.56 s). At that location,

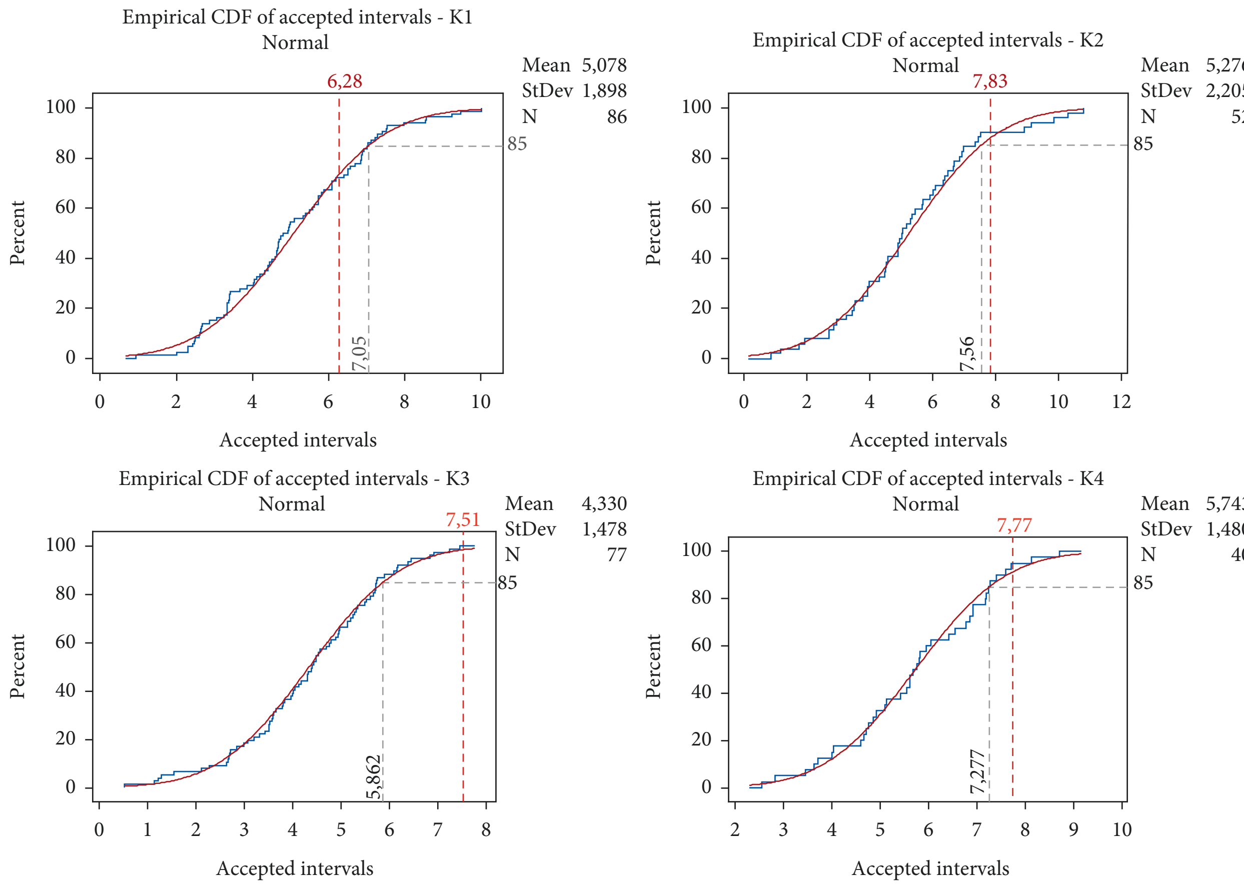


Figure 8: Cumulative distribution of the accepted intervals by locations.

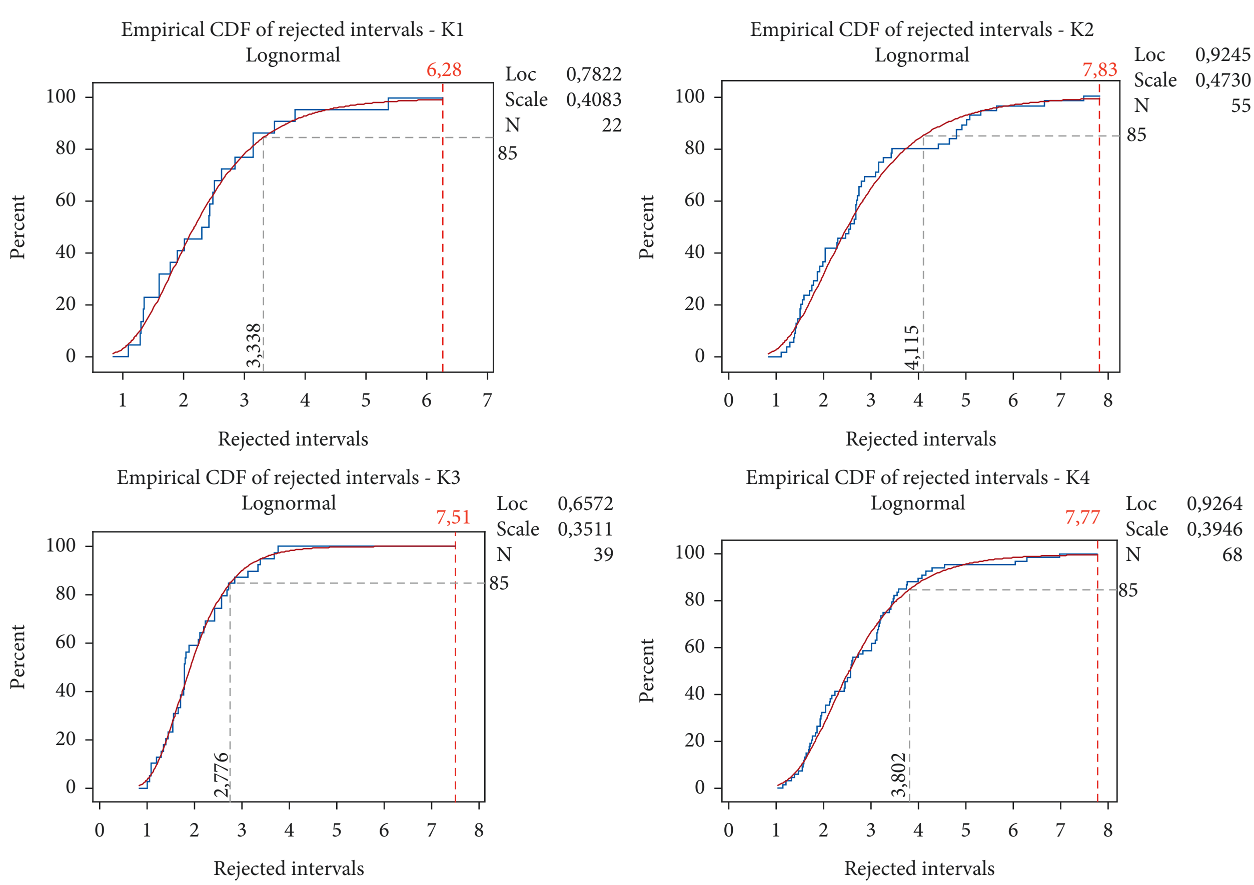


Figure 9: Cumulative distribution of the rejected intervals by locations.

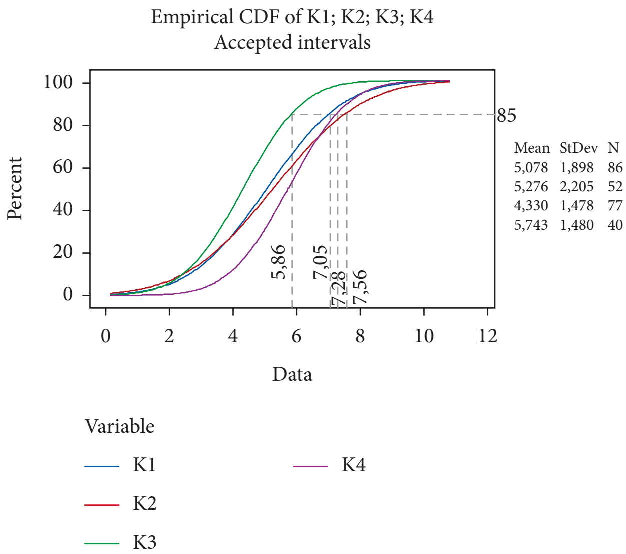


Figure 10: Comparative display of cumulative distribution of the accepted intervals for four typical pedestrian crossings.

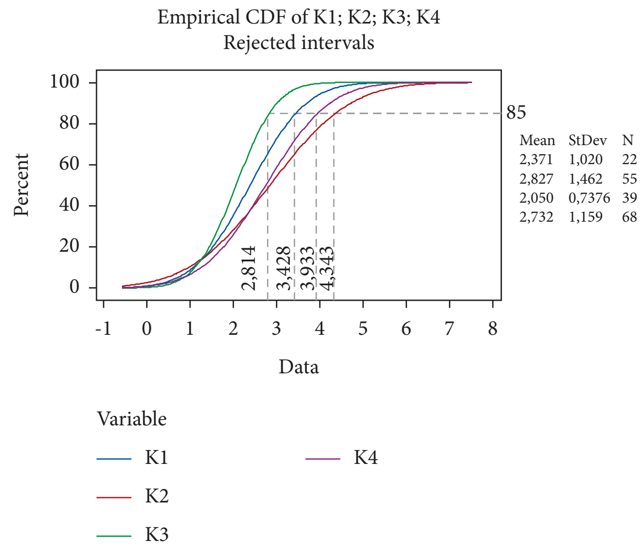


Figure 11: Comparative display of cumulative distribution of the rejected intervals for four typical pedestrian crossings.

pedestrians go across two traffic lanes, and vehicles approach from both directions. The explanation for the longest intervals is in the fact that when pedestrians go across the roadway, they have to pay attention to the vehicles approaching from both directions, and for these reasons, they are more careful and indecisive when choosing the interval. When they cross the roadway at this type of pedestrian crossing, pedestrians have to choose an interval of sufficient length in order to avoid the conflict with vehicles approaching from both directions, unlike at previous types of pedestrian crossings, where vehicles were approaching only from one direction. As previously mentioned, at location K4, although there is two-way direction movement of vehicles, due to "rolling-gap" crossing the roadway by pedestrians, intervals were chosen by the assessment of the movement of vehicles approaching from only one direction, that is, during the first phase of the roadway crossing.

Based on the research results, a comparative analysis of the rejected intervals at typical locations was conducted, whereby the obtained results are similar to those in the case of the accepted intervals (Figure 11). The longest rejected intervals were noticed at location K2 (3.343 s), while the shortest rejected intervals were at location K3 (2.814 s), which confirmed the assumptions about the influence of previously described traffic conditions at pedestrian crossings on the value of the interval when crossing the roadway.

In order to compare the values of the critical, rejected, and accepted intervals, critical intervals were calculated for each of the locations, as well as 85% of the values of the accepted and rejected intervals (Figure 12). Highway Capacity Manual (HCM) defines the critical interval as the time expressed in seconds within which the pedestrian will not start going across the pedestrian crossing. Thus, critical time interval represents the minimum necessary time during which the pedestrian can cross the roadway. Of all analysed locations, only at location K1, 85% of the value of the accepted intervals was higher than the value of the critical

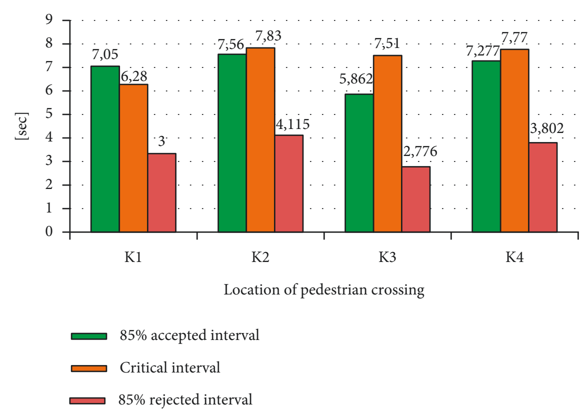


Figure 12: Value of the accepted, rejected, and critical intervals at typical locations of pedestrian crossings.

interval, which practically means that a certain number of pedestrians choose the intervals for crossing that are longer than the critical one, therefore safer for crossing. At all other locations, 85% of the value of the accepted intervals is lower than the critical intervals determined by the HCM method.

The biggest difference between the accepted intervals and critical intervals is noticed at location K3, where 85% of the value of the accepted interval is 5.86 s, and the critical interval is 7.51 s. The explanation for this occurrence is similar to the case when at the same location, the lowest value of the accepted intervals out of all observed locations was recorded. Namely, parts of pedestrians who start the second phase of crossing the roadway, in the first phase assess the distance and the velocity of the approaching vehicles. Pedestrians at the same time also have their own velocity of movement, which makes them more noticeable to drivers than in the case when they stand and assume that in that case, drivers will react to reduce the velocity so as to avoid the conflict. The lowest difference between the accepted and critical intervals is at location K2 (difference of 0.27 s), whereby the difference of the accepted intervals is approximately the same as the time, which is, by calculations, necessary for the pedestrians to reach the other side of the roadway.

After the analysis, it can be concluded that the values of the accepted and critical intervals imply that the behaviour of some pedestrians is risky, which is reflected in accepting the intervals, which are not completely safe for crossing the roadway. Figure 13 shows the percentage of pedestrians who chose an interval that is smaller or larger than the critical interval.

At location K1, in relation to the critical interval ($t_c = 6.28$ s), it was established that 73.7% of pedestrians choose the intervals that are shorter than the time necessary for crossing to the other side of the roadway. It means that only 26.3% of pedestrians choose the intervals that are longer than the critical ones, whereby safer for going across the pedestrian crossing. This result is a possible consequence of the fact that, at this pedestrian crossing, pedestrians go across only one traffic lane and they pay attention and assess the approaching of the vehicles only from one direction.

The lowest percentage of the pedestrians who chose the intervals higher than the critical one, only 1.6%, was recorded at location K3. At this pedestrian crossing, pedestrians cross two traffic lanes, while vehicles approach the pedestrian crossing from both sides. Considering the research results, this location has the highest percentage of unsafe crossings of pedestrians.

Figure 13: Comparative display of the accepted intervals of pedestrians and critical intervals at typical locations.

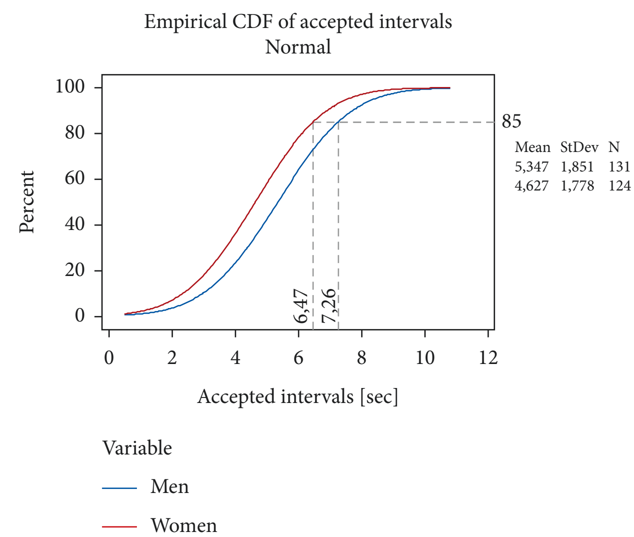


Figure 14: Cumulative distribution of the accepted intervals in relation to the gender of pedestrians at all locations altogether.

4.7. The Accepted Intervals Depending on the Pedestrian's Gender. During the analysis of the accepted intervals depending on the gender of pedestrians, 255 crossing of pedestrians were considered, at four locations, out of which 131 pedestrians were male (51%) and 124 pedestrians were female (49%).

Figure 14 shows united data of cumulative distribution of the accepted intervals for all four locations; 85% of the value of the accepted intervals for women is 6.47 s and for men 7.26 s. Figure 14 shows the cumulative distributions of the accepted intervals depending on the gender of the individual pedestrian by locations. Apart from 85% of the value of the accepted intervals for men and women, the figure also shows the values of critical intervals calculated for each typical location.

What is common for all the locations is the fact that female pedestrians choose shorter intervals for crossing the roadway, which confirms the assumption that the gender of pedestrians has an influence on the choice of crossing interval.

Based on the data collected at four locations, the average waiting time was calculated as well as the average velocity of pedestrian's going across the roadway in relation to the gender. The analysis results showed that the waiting time of the male pedestrians is 2.4 s/pedestrian, while for female ones, the obtained value was slightly lower and it is 2.35 s/pedestrian. The pedestrian velocities are identical, regardless of the gender: in average, they are both for men and women 1.39 m/s, which represents the velocity, which is higher than the recommended value by HCM (1.2 m/s). It is higher than the value that is adopted when pedestrian crossing signals are designed in the Republic of Serbia (from 0.8 m/s to 1.2 m/s) and that depends on the character and the size of the pedestrian flows, as well as on the way of regulation of pedestrian traffic, which is applied [54].

4.8. The Accepted Intervals Depending on the Type of Crossing the Roadway. During the analysis of the accepted intervals depending on the type of crossing the roadway, 255 crossings of pedestrians were taken into account, both individual and group ones, at 4 typical locations, out of which 177 were the crossings by individuals (69%) and 78 were group crossings (31%). Figure 15 shows the data consolidation regarding cumulative distribution of the accepted intervals for all four locations; 85% of the value of the accepted intervals for pedestrians who individually cross the roadway is 6.85 s, and for the pedestrians who cross the roadway in groups, it is 7.04 s (Figure 16). Figure 17 shows cumulative distributions of the accepted intervals depending

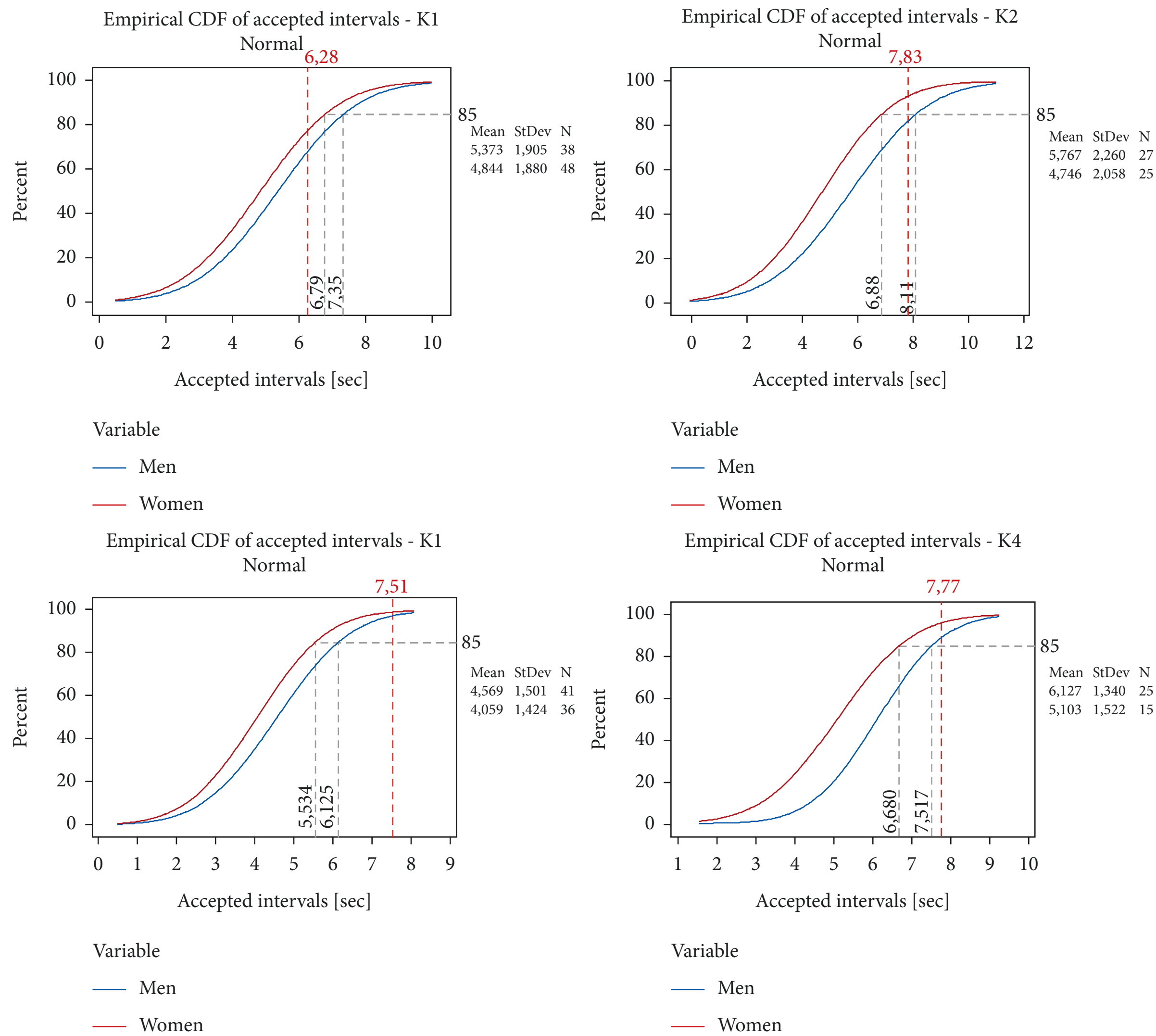


FIGURE 15: Cumulative distribution of the accepted intervals in relation to the pedestrian's gender at the locations K1–K4.

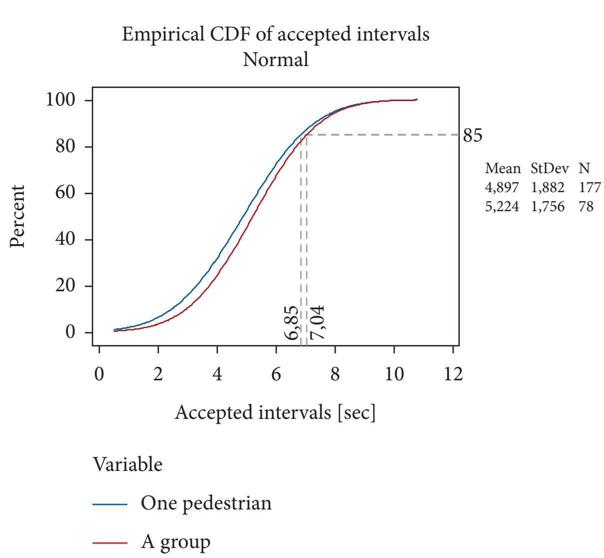


FIGURE 16: Cumulative distribution of the accepted intervals in relation to the type of crossing for all locations altogether.

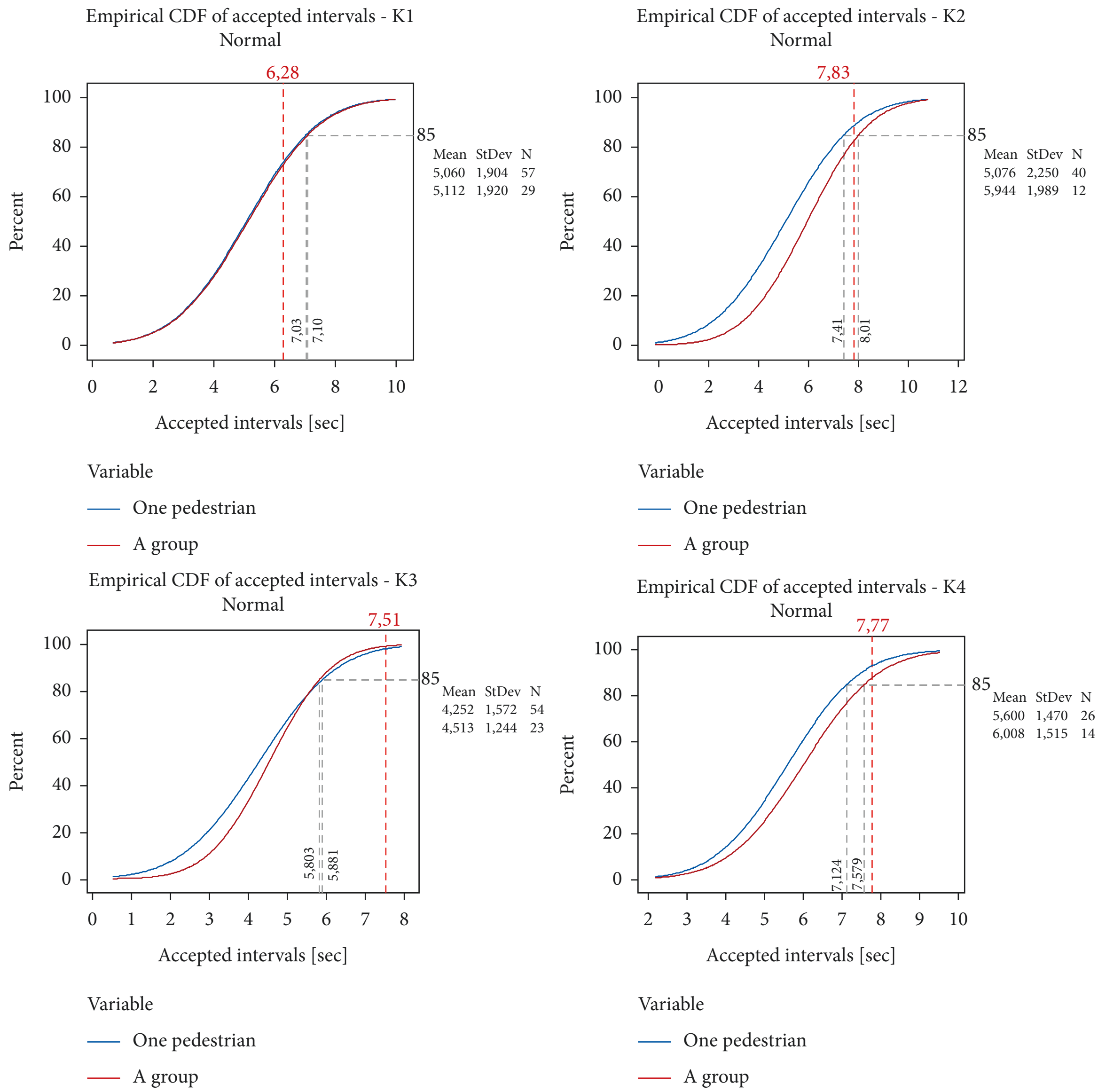


Figure 17: Cumulative distribution of the accepted intervals in relation to the type of crossing at typical locations K1, K2, K3, and K4.

on the type of the crossing individually by locations. Apart from 85% of the value of the accepted intervals for crossing the roadway by an individual pedestrian and groups of pedestrians, the flow charts show the values of critical intervals, which are calculated for each typical location.

What is common for all locations is when pedestrians cross the roadway alone, they choose shorter intervals, in comparison with the crossing in a group, when they choose longer intervals. The analysis results confirm the assumption that the type of crossing affects the length of the accepted interval for crossing the roadway.

Based on the data collected at four locations, the average waiting time was calculated, as well as the average speed of pedestrians' going across the roadway in relation to the type of crossing. The analysis results show that the waiting time of the pedestrian standing alone at the edge of the roadway is 2.3 s, while for a group of pedestrians, the average waiting is longer and it is 2.54 s/pedestrian. The velocities of pedestrians depending on the type of crossing differ, and for individual pedestrians, it is 1.42 m/s, while the average velocity for the group is 1.31 m/s.

5. Discussion

Starting from initial assumptions, the analysis of pedestrian intervals during crossing the roadway was conducted. Around 450 intervals were analysed (accepted and rejected) at four typical locations. Statistical analysis showed that the accepted intervals behave by the normal distribution and the rejected ones by log-normal distribution, which is in accordance with the previous research studies conducted in this area [9, 12, 16, 55]. The results showed that the accepted intervals differ in relation to the characteristics of the location (number of lanes, which a pedestrian has to cross, and the direction of the approaching vehicles towards the pedestrian crossing). The analysis established that in many cases, pedestrians choose the interval, which is shorter than the critical one, and they create different risky traffic situations. According to the analysis results, the type of the location, which has the highest percentage of unsafe crossings of pedestrians, is the pedestrian crossing at the two-way road with two lanes, when pedestrians cross two traffic lanes, and vehicles approach the pedestrian crossing from both sides.

Apart from the characteristics of the location, pedestrian intervals were analysed from the aspect of gender characteristics of pedestrians (men and women) and the type of crossing the roadway (crossing of the individual and of a group of pedestrians). What is common for all the locations is that pedestrians of the female gender choose shorter intervals for crossing the roadway. It means that pedestrians of the male gender are less prone to risk than women and they choose longer intervals. This result is in accordance with the research studies carried out on the territory of Europe [9], while the research carried out on the territory of Asia showed that men choose shorter intervals for crossing in comparison with women [11–13].

Analysis of the average velocities of pedestrians when crossing the roadway showed that there are no significant differences in average values of velocities regarding the gender (1.39 m/s), which was proved in research studies [16, 46]. However, most authors came to conclusion that the velocity of male pedestrians is slightly higher than that of female pedestrians [22, 23, 56]. Similar results were obtained with the waiting time: in research studies carried out mostly on the territory of Asia, the waiting time for male pedestrians is shorter in comparison with women [21, 56, 57]. The research results for the needs of this article showed that the average waiting time of pedestrians is approximately the same in relation to the gender: for men, it is 2.4 s, while for women, it is 2.35 s.

The analysis results show that the type of crossing (individual crossing or a group of pedestrians) affects the length of the accepted interval for crossing the roadway also by the fact that pedestrians who are in a group choose longer intervals (7.04 s) in comparison with individual roadway crossings (6.85 s). The same conclusions were reached by the authors of one of the rare research studies from this field conducted in Europe [9], while the research conducted on the territory of Asia showed the opposite results [24, 25]. However, during the analysis of the velocities of pedestrians' circulation, the results showed that the velocity of the pedestrians who cross the roadway individually is higher (1.42 m/s) in comparison with the velocity of the group of pedestrians (1.31 m/s). This is in accordance with most research studies conducted in the world regarding velocity of the pedestrians when crossing the roadway [20, 23, 58]. In accordance with the stated, it was established that the average waiting time of a group of pedestrians is longer than the waiting time of an individual pedestrian (2.54 s in relation to 2.3 s).

Proximity to facilities such as schools, preschools, and eldercare facilities significantly affects the structure of pedestrians at the pedestrian crossing. Different categories of traffic participants have different speeds, but also different psychophysical abilities on which their behavior in traffic depends. The ability of different groups of pedestrians to select appropriate intervals depends on their ability to estimate the speed of an oncoming vehicle and the time it takes them to cross the pedestrian crossing. In addition, the location of the pedestrian crossing can be observed from the point of view of geometry, that is, the type of road construction. In that case, there are two basic types of location: crossing at intersections and crossing at a mid-block crossing. The geometric characteristics of the road affect the crossing from the aspect of the spatial distance that the pedestrian has to overcome. The number of traffic lanes is a very important factor due to the distance that pedestrians cross, because with the increase in the number of traffic lanes, the need for the introduction of refuge islands increases. Knowing the structure of pedestrians by some of the aforementioned categories, as well as the geometry of the intersection on a larger sample of locations, would certainly give a more precise picture and more detailed analysis that could form models for a specific category of participants and location of pedestrian crossing depending on geometry.

6. Conclusion

With the assumption that factors like traffic conditions at the pedestrian crossing, the characteristics of pedestrians and the number of pedestrians who in a group cross the roadway, affect the length of the accepted intervals, the analysis of four typical locations of pedestrian crossings was conducted. An original integrated multiphase model for determining the efficiency of pedestrian crossings was created. First, the DEA method was applied in a crisp form, which showed that all locations were efficient. Due to the drawback of the classical DEA method manifested in this article too, the fuzzy DEA method was applied, the results of which show that the second location is not efficient in terms of the observed parameters. In order to determine the final efficiency, the fuzzy MARCOS method was applied. Before that, it was integrated an objective-subjective model for determining the weights of the criteria based on the Bonferroni aggregator for averaging and obtaining final values. Four methods were applied: entropy, CRITIC, fuzzy FUCOM, and fuzzy PIPRECIA. The created multiphase model that treats objectivity and subjectivity can be applied in future for different studies.

The analysis results showed that the accepted intervals differ in relation to the characteristics of the locations (number of lanes, which a pedestrian has to cross, and the direction of the approaching vehicles in relation to the pedestrian crossing). The shortest accepted intervals were recorded at the pedestrian crossing where pedestrians cross two traffic lanes with vehicles approaching from one direction. At this location, the percentage of the accepted intervals, which are shorter than the critical interval, was the highest, which implies that pedestrians at this type of pedestrian crossing create different risky situations when crossing the roadway.

The analysis of the accepted intervals at all locations showed that women choose shorter intervals in relation to men, but there were no significant differences in the average values of the crossing velocities and waiting time at the pedestrian crossing. Observing the number of pedestrians in a group who cross the roadway, it was noticed that pedestrians who cross the roadway alone choose shorter intervals for crossing, they move faster when crossing the roadway, and they wait shorter for the adequate crossing interval. The obtained results are in accordance with the research studies conducted on the territory of Europe [9, 10], while the research studies conducted on the territory of Asia showed opposite results [12, 22, 23, 56].

Analysis results regarding pedestrians' behaviour during roadway crossing showed in the article imply the basic characteristics of pedestrians' behaviour noticed in local traffic conditions. Some of the limitations in the research are the small number of considered pedestrian crossing locations, as well as the limitation in the initial phase of the model when the DEA model is applied, and the ratio of the number of inputs, outputs, and DMUs. With the application of the defined model, the future research could be performed by the evaluation of the influence and specific conditions of the local environment (school zones, zones with greater attraction, slow traffic zones), as well as traffic flow characteristics (speed, flow, density) and different categories of pedestrians as traffic participants (children, the elderly, people with disabilities, mothers with children), which has not been the case so far. That would contribute to a more precise determination of the level of service at different types of pedestrian crossings as well as to defining special measures in the field of pedestrian traffic, in order to achieve a sustainable and safe traffic system in cities. The recommendations for future works from this field should be determining the influence of other factors (drivers' behaviour, vehicles' characteristics, road geometry, built street environment, etc.) on the behaviour of pedestrians during roadway crossing. Special attention should be paid to vehicle category, vehicle position in the traffic lane, number of traffic lanes, presence of illegal parked cars, motorist yield rate, and pedestrian crossing designs and equipment. The pedestrian's accepted gaps have a unique set of conditions, which can be used in statistical analysis. In such a way, certain models can be modelled, and they can be used for evaluating the probability of the accepted crossing gap, which has not been carried out so far at pedestrian crossings in the city of Novi Sad. The application of the models and recommendations that are the results of the research will enable experts in this field to obtain the results that correspond to the actual traffic conditions in the process of analysing the level of service at pedestrian crossings. Accordingly, it will be easier to choose appropriate measures in the field of traffic engineering in order to improve traffic conditions and safety of all participants in the traffic system. Implications of this model can be manifested through monitoring these locations in future in order to increase their efficiency, especially the worst ranked.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare no conflicts of interest.

References

[1] J. Mitrović Simić and V. Bogdanović, "Motorist yield rate at unsignalized crossings in Novi Sad," in *Proceedings of the 5th International Conference "Towards a humane city"*, Faculty of Technical Sciences, Novi Sad, Republic of Serbia, November 2015.

[2] J. Mitrović Simić, V. Bogdanović, and V. Basarić, "Analysis of motorist yield rate at unsignalized pedestrian crossing in urban areas," *Suvremeni promet – Modern Traffic*, vol. 34, no. 3-4, pp. 221–225, 2014.

[3] N. Saulić, N. Šarac, and N. Garunović, "Investigation of the impact of pedestrian flows on the time losses of first-class vehicles at three-lane unsignalized intersections," *Put i saobraćaj*, vol. 62, no. 1, pp. 47–51, 2016.

[4] J. Mitrović Simić, V. Basarić, and V. Bogdanović, "Pedestrian crossing behaviour at unsignalized crossings," in *Proceedings of the 1st International Conference "Transport for Today's Society"*, pp. 166–172, Bitola, The Republic of North Macedonia, May 2016.

[5] E. O'Dowd and T. V. Pollet, "Gender differences in use of a pedestrian crossing: an observational study in newcastle upon tyne," *Letters of Evolutionary Behaviorial Science*, vol. 9, no. 1, pp. 1–4, 2018.

[6] J. Bunevska Talevska, M. Ristov, M. Ristov, and M. Malenkovska Todorova, "Development of the methodology for selecting the optimal type of pedestrian crossing," *Decision Making: Applications in Management and Engineering*, vol. 2, no. 1, pp. 105–114, 2019.

[7] N. Garunović, V. Bogdanović, J. M. Simić, G. Kalamanda, and B. Ivanović, "The influence of the construction of raised pedestrian crossing on traffic conditions on urban segments," *Građevinar*, vol. 72, no. 8, pp. 681–691, 2020.

[8] C. Zhang, F. Chen, and Y. Wei, "Evaluation of pedestrian crossing behavior and safety at uncontrolled mid-block crosswalks with different numbers of lanes in China," *Accident Analysis & Prevention*, vol. 123, pp. 263–273, 2019.

[9] G. Yannis, E. Papadimitriou, and A. Theofilatos, "Pedestrian gap acceptance for mid-block street crossing," in *Proceedings of the 12th World Conference on Transport Research Society*, Lisbon, Portugal, July 2010.

[10] E. Papadimitriou, G. Yannis, and J. Golias, "A critical assessment of pedestrian behaviour models," *Transportation Research Part F: Traffic Psychology and Behaviour*, vol. 12, no. 3, pp. 242–255, 2009.

[11] G. Tiwari, S. Bangdiwala, A. Saraswat, and S. Gaurav, "Survival analysis: pedestrian risk exposure at signalized intersections," *Transportation Research Part F: Traffic Psychology and Behaviour*, vol. 10, no. 2, pp. 77–89, 2007.

[12] B. Raghuram Kadali and V. Perumal, "Pedestrians' gap acceptance behavior at mid block location," *International Journal of Engineering & Technology*, vol. 4, no. 2, pp. 158–161, 2012.

[13] F. M. Khan, M. Jawaid, H. Chotani, and S. Luby, "Pedestrian environment and behavior in Karachi, Pakistan," *Accident Analysis & Prevention*, vol. 31, no. 4, pp. 335–339, 1999.

[14] "Highway Capacity Manual - H. C. M, ," Transportation Research Board of The National Research Council, Washington D.C., USA, 2010.

[15] "Manual on uniform traffic control devices for streets and highways - mutcd, ," U.S. Department of Transportation, Washington D.C., USA, 2009.

[16] K. Fitzpatrick, S. Turner, M. Brewer et al., "Improving pedestrian safety at unsignalized crossings," TCRP Report 112/ NCHRP Report 562, Transportation Research Board, Washington, D.C., USA, 2006.

[17] R. Lobjois, N. Benguigui, and V. Cavallo, "The effects of age and traffic density on street-crossing behavior," *Accident Analysis & Prevention*, vol. 53, pp. 166–175, 2013.

[18] D. Herrero-Fernández, P. Macía-Guerrero, L. Silvano-Chaparro, L. Merino, and E. C. Jenchura, "Risky behavior in young adult pedestrians: personality determinants, correlates with risk perception, and gender differences," *Transporation Research: Part F*, vol. 36, pp. 14–24, 2016.

[19] S. N. M. Nor, D. D. Basil, R. Hamidun et al., "Analysis of pedestrian gap acceptance and crossing decision in Kuala Lumpur," *MATEC Web of Conferences*, vol. 103, Article ID 08014, 2017.

[20] C. M. DiPietro and L. E. King, "Pedestrian gap-acceptance," *Highway Research Record*, vol. 308, pp. 80–91, 1970.

[21] M. M. Hamed, "Analysis of pedestrians' behavior at pedestrian crossings," *Safety Science*, vol. 38, no. 1, pp. 63–82, 2001.

[22] R. Rastogi, S. Chandra, J. Vamsheedhar, and V. R. Das, "Parametric study of pedestrian speeds at midblock crossings," *Journal of Urban Planning and Development*, vol. 137, no. 4, pp. 381–389, 2011.

[23] M. S. Tarawneh, "Evaluation of pedestrian speed in Jordan with investigation of some contributing factors," *Journal of Safety Research*, vol. 32, no. 2, pp. 229–236, 2001.

[24] D. S. Pawar and G. R. Patil, "Pedestrian temporal and spatial gap acceptance at mid-block street crossing in developing world," *Journal of Safety Research*, vol. 52, pp. 39–46, 2015.

[25] T. Wang, J. Wu, P. Zheng, and M. McDonald, "Study of pedestrians' gap acceptance behavior when they jaywalk outside crossing facilities," in *Proceedings of the 13th International IEEE Annual Conference on Intelligent Transportation Systems*, pp. 1295–1300, University of Porto, Porto, Portugal, September 2010.

[26] S. Lord, M.-S. Cloutier, B. Garnier, and Z. Christoforou, "Crossing road intersections in old age-With or without risks? Perceptions of risk and crossing behaviours among the elderly," *Transportation Research Part F: Traffic Psychology and Behaviour*, vol. 55, pp. 282–296, 2018.

[27] K. Shaaban, D. Muley, and A. Mohammed, "Analysis of illegal pedestrian crossing behavior on a major divided arterial road," *Transportation Research Part F: Traffic Psychology and Behaviour*, vol. 54, pp. 124–137, 2018.

[28] R. Kadali and P. Vedagiri, "Role of number of traffic lanes on pedestrian gap acceptance and risk taking behaviour at uncontrolled crosswalk locations," *Journal of Transport & Health*, vol. 19, Article ID 100950, 2020.

[29] J. Zhao, J. O. Malenje, Y. Tang, and Y. Han, "Gap acceptance probability model for pedestrians at unsignalized mid-block crosswalks based on logistic regression," *Accident Analysis & Prevention*, vol. 129, pp. 76–83, 2019.

[30] S. Razmi Rad, G. Homem de Almeida Correia, and M. Hagenzieker, "Pedestrians' road crossing behaviour in front of automated vehicles: results from a pedestrian simulation experiment using agent-based modelling," *Transportation Research Part F: Traffic Psychology and Behaviour*, vol. 69, pp. 101–119, 2020.

[31] A. Millard-Ball, "Pedestrians, autonomous vehicles, and cities," *Journal of Planning Education and Research*, vol. 38, no. 1, pp. 6–12, 2018.

[32] M. Deveci and A. E. Torkayesh, "Charging type selection for electric buses using interval-valued neutrosophic decision support model," *IEEE Transactions on Engineering Management*, 2021.

[33] A. Blagojević, S. Vesković, S. Kasalica, A. Gojić, and A. Allamani, "The application of the fuzzy AHP and DEA for measuring the efficiency of freight transport railway," *Operational Research in Engineering Sciences: Theory and Applications*, vol. 3, no. 2, 2020.

[34] A. E. Torkayesh and M. Deveci, "A mulTi-noRmalization mUlti-distance aSsessmenT (TRUST) approach for locating a battery swapping station for electric scooters," *Sustainable Cities and Society*, vol. 74, Article ID 103243, 2021.

[35] R. Krishankumar, D. Pamucar, M. Deveci, and K. S. Ravichandran, "Prioritization of zero-carbon measures for sustainable urban mobility using integrated double hierarchy decision framework and EDAS approach," *The Science of the Total Environment*, vol. 797, Article ID 149068, 2021.

[36] S. Vesković, S. Milinković, B. Abramović, and I. Ljubaj, "Determining criteria significance in selecting reach stackers by applying the fuzzy PIPRECIA method," *Operational Research in Engineering Sciences: Theory and Applications*, vol. 3, no. 1, pp. 72–88, 2020.

[37] M. Deveci, D. Pamucar, and I. Gokasar, "Fuzzy Power Heronian function based CoCoSo method for the advantage prioritization of autonomous vehicles in real-time traffic management," *Sustainable Cities and Society*, vol. 69, Article ID 102846, 2021.

[38] I. Gokasar, M. Deveci, and O. Kalan, "CO2 Emission based prioritization of bridge maintenance projects using neutrosophic fuzzy sets based decision making approach," *Research in Transportation Economics*, vol. 91, Article ID 101029, 2021.

[39] S. Memiş, E. Demir, Ç. Karamaşa, and S. Korucuk, "Prioritization of road transportation risks: an application in Giresun province," *Operational Research in Engineering Sciences: Theory and Applications*, vol. 3, no. 2, pp. 111–126, 2020.

[40] V. Simic, I. Gokasar, M. Deveci, and M. Isik, "Fermatean fuzzy group decision-making based CODAS approach for taxation of public transit investments," *IEEE Transactions on Engineering Management*, 2021.

[41] D. Nenadić, "Ranking dangerous sections of the road using MCDM model," *Decision Making: Applications in Management and Engineering*, vol. 2, no. 1, pp. 115–131, 2019.

[42] V. Simic, I. Gokasar, M. Deveci, and A. Karakurt, "An integrated CRITIC and MABAC based Type-2 neutrosophic model for public transportation pricing system selection," *Socio-Economic Planning Sciences*, vol. 80, Article ID 101157, 2021.

[43] D. Pamucar, M. Deveci, I. Gokasar, and M. Popovic, "Fuzzy Hamacher WASPAS decision-making model for advantage prioritization of sustainable supply chain of electric ferry implementation in public transportation," *Environment, Development and Sustainability*, vol. 24, pp. 1–40, 2021.

[44] M. Subotić, B. Stević, B. Ristić, and S. Simić, "The selection of a location for potential roundabout construction–a case study of Doboj," *Operational Research in Engineering Sciences: Theory and Applications*, vol. 3, no. 1, pp. 41–56, 2020.

[45] A. Charnes, W. W. Cooper, and E. Rhodes, "Measuring the efficiency of decision making units," *European Journal of Operational Research*, vol. 2, no. 6, pp. 429–444, 1978.

[46] A. Blagojević, Ž. Stevi, D. Marinković, S. Kasalica, and S. Rajilić, "A novel entropy-fuzzy PIPRECIA-DEA model for safety evaluation of railway traffic," *Symmetry*, vol. 12, no. 9, p. 1479, 2020.

[47] J. Mitrović Simić, Ž. Stević, E. K. Zavadskas, V. Bogdanović, M. Subotić, and A. Mardani, "A novel CRITIC-fuzzy FUCOM-DEA-fuzzy MARCOS model for safety evaluation of road sections based on geometric parameters of road," *Symmetry*, vol. 12, no. 12, p. 2006, 2020.

[48] D. Pamucar and F. Ecer, "Prioritizing the weights of the evaluation criteria under fuzziness: the fuzzy full consistency method - fucom-f," *Facta Universitatis – Series: Mechanical Engineering*, vol. 18, no. 3, pp. 419–437, 2020.

[49] Ž. Stević, Ž. Stjepanović, Z. Božičković, D. K. Das, and D. Stanujkić, "Assessment of conditions for implementing information technology in a warehouse system: a novel fuzzy piprecia method," *Symmetry*, vol. 10, no. 11, p. 586, 2018.

[50] I. Đalić, J. Ateljević, Ž. Stević, and S. Terzić, "An integrated swot–fuzzy piprecia model for analysis of competitiveness in order to improve logistics performances," *Facta Universitatis – Series: Mechanical Engineering*, vol. 18, no. 3, pp. 439–451, 2020.

[51] D. Pamucar, "Normalized weighted Geometric Dombi Bonferoni Mean Operator with interval grey numbers: application in multicriteria decision making," *Reports in Mechanical Engineering*, vol. 1, no. 1, pp. 44–52, 2020.

[52] M. Stanković, Ž. Stević, D. K. Das, M. Subotić, and D. Pamučar, "A new fuzzy MARCOS method for road traffic risk analysis," *Mathematics*, vol. 8, no. 3, p. 457, 2020.

[53] W. Sałabun and K. Urbaniak, "A new coefficient of rankings similarity in decision-making problems," in *Proceedings of the International Conference on Computational Science*, pp. 632–645, Springer, Cagliari, Italy, June 2020.

[54] T. Đorđević, *Regulisanje Saobraćajnih Tokova Svetlosnom Signalizacijom - Regulation of Traffic Flows by Light Signaling*, Road institute, Belgrade, Republic of Serbia, 1997, (in serbian).

[55] R. Kadali, B. Raghuram, and P. Vedagiri, "Modelling pedestrian road crossing behaviour under mixed traffic condition," *European Transport\Trasporti Europei*, vol. 55, no. 3, pp. 1–17, 2013.

[56] A. Jain, A. Gupta, and R. Rastogi, "Pedestrian crossing behaviour analysis at intersections," *International Journal for Traffic and Transport Engineering*, vol. 4, no. 1, pp. 103–116, 2014.

[57] P. B. Shahi and B. P. Devkota, "Modeling pedestrians' behavior at road crossings: a case study in kathmandu," in *Proceedings of the IOE Graduate Conference*, Tribhuvan University, Institute of Engineering, Lalitpur, Bagmati Pradesh, Nepal, November 2013.

[58] T. J. Gates, D. A. Noyce, A. R. Bill, and E. N. Van, "Recommended walking speeds for pedestrian clearance timing based on pedestrian characteristics," in *Proceedings of the 85th Annual Meeting of the Transportation Research Board*, Transportation Research Board of The National Research Council, Washington D.C., USA, January 2006.

Hindawi
Mathematical Problems in Engineering
Volume 2022, Article ID 9299246, 13 pages
https://doi.org/10.1155/2022/9299246

Research Article

A New Interactive Decision Approach with Probabilistic Linguistic Data: An Application in the Academic Sector

K.S. Ravichandran,[1] **R. Krishankumar**,[2] **Sibnath Deb**,[1] **Dragan Pamucar**,[3] **David Paul**,[1] **and Darko Božanić**[4]

[1]*Rajiv Gandhi National Institute of Youth Development, Sriperumbudur, TN, India*
[2]*Department of Computer Science and Engineering, Amrita School of Engineering, Amrita Vishwa Vidyapeetham, Coimbatore, TN, India*
[3]*Department of Logistics, Military Academy, University of Defence in Belgrade, Belgrade, Serbia*
[4]*Military Academy, University of Defence in Belgrade, Belgrade, Serbia*

Correspondence should be addressed to Dragan Pamucar; dragan.pamucar@va.mod.gov.rs

Received 8 December 2021; Revised 14 January 2022; Accepted 22 March 2022; Published 21 April 2022

Academic Editor: Ali Ahmadian

As an innovative generalization to a linguistic term set, the probabilistic variant is gaining abundant attraction in the decision process. However, earlier studies with this variant for decision-making have not adequately explored hesitation in data articulation and interactive ranking. Driven by the claim, in this paper, a new integrated approach is put forward under the probabilistic linguistic context, which attempts to address the claims by presenting a regret/rejoice technique and an interactive WASPAS algorithm for determining the significance of factors and personalized ranking of alternatives. To test the usefulness of the approach, the online course prioritization problem based on empirical data is exemplified, and a comparison demonstrates the benefits and limitations of the proposed work.

1. Introduction

Decision-making under uncertainty is an interesting and complex problem in day-to-day life [1]. Zadeh [2] introduced the concept of linguistic decision-making that was further ameliorated by the work of Herrera et al. [3]. Rodriguez identified that the linguistic term set (LTS) could not accept more than one instance at a given point in time. To resolve the issue, hesitant fuzzy linguistic information (HFLI) [4] was put forward, which embedded the idea of hesitation and allowed more than one instance as preference information. Driven by this feature, many researchers used HFLI for decision-making [5]. Pang et al. [6] identified that though HFLI allowed multiple information, the confidence associated with each element is either ignored or assumed to be the same. A probabilistic linguistic term set (PLTS) was introduced to handle the issue.

PLTS can accept more than one term and associate occurrence probability to each term. This setting gained a lot of attention in the decision-making context in both in theoretical (such as operational laws/aggregation functions [7–13]; ranking [14–19]; measures [20–22]) and the application perspectives [23–29]. Two popular reviews dealing with the applicability of PLTS in decision-making [30] and aggregation operators in the PLTS context [31] give a holistic view of the core importance of the structure for the decision process.

Based on these two reviews, specific challenges can be identified, as follows:

(i) consideration of views from a large population that is heterogeneous is challenging to manage,

(ii) hesitation during opinion sharing is not adequately captured during weight estimation,

(iii) personalized prioritization based on personal choices on alternatives is lacking, along with the idea of capturing interaction among factors.

These challenges motivated the present study, and the following contributions are henceforth made:

(i) PLTS is used to transform the Likert-scale rating from multiple participants to a holistic decision matrix for prioritization of alternatives,

(ii) the regret/rejoice approach is put forward to effectively capture hesitation under the PLTS context; weights of the rating personnel are considered during factor significance calculation,

(iii) a new interactive algorithm with the WASPAS (weighted aggregated sum product assessment) technique as the base formulation is developed to consider personal choices on alternatives and the nature of factors.

These contributions add value to the PLTS-based decision-making and aid in rational selection. Further organization of the paper is as follows. First, literature relating to regret theory and WASPAS are presented in Section 2, tracked by the methodology in Section 3, which covers the core aspect of the paper by detailing step-wise the methods for significance calculation and prioritization. An empirical case example is exemplified in Section 4 to clarify the usefulness of the developed interactive approach. Comparison with other models is also presented to showcase the efficacy and shortcomings of the work. Finally, concluding remarks with future research scope are provided in Section 5.

2. Literature Review

2.1. Regret Theory. Regret theory (RT) is a concept adapted from psychology into decision-making that deals with a clear understanding of the mindset of an expert upon choosing a candidate over another [32]. This idea resembles the human decision process and is a viable approach for determining the hesitation of experts during the decision process [33]. Some review articles [34–37] on RT are prepared by scholars that infer that the concept (i) is elegant and effective in decision-making; (ii) allows efficient capturing of experts' hesitation; and (iii) promotes methodical determination of weights of factors. Chen et al. [38] used the fuzzy context with axiomatic design and RT for logistic provider evaluation under an omnichannel environment. Wang et al. [39] presented a stock grading model with RT and compared the result with prospect theory in the context of fuzzy interval-neutrosophic probability. Qu et al. [40] gave an extended RT with group satisfaction measure under dual hesitant context for grading shared-bikes for an investment. Xia [41] developed the multiobjective model by adopting the RT idea under the hesitant fuzzy linguistic domain to solve firms' decision problems. Gong et al. [42] gave a cloud model with linguistic structure by extending the RT concept over dual expectation of stock evaluation. Wang et al. [43] developed RT-based TOPSIS (a technique for order of preference by similarity to ideal solution) with interval type-2 data in a three-way decision context. Liu et al. [44] assessed projects for venture capitalists under probabilistic hesitant setting by adopting RT and mathematical model formulated using entropy and water-filling strategy. Ren et al. [45] developed an extension to RT under an intuitionistic fuzzy environment by adopting Canberra distance for solving decision problems of supplier selection for assembly components. Gong et al. [46] performed a portfolio assessment with multiobjective programming by considering the RT formulation and the DEA (data envelopment analysis) approach. Liang et al. [47] solved decision problems in probabilistic interval hesitant fuzzy data by developing an integrated RT-gain/lost dominance approach.

2.2. WASPAS Method. The method's inception is from [48], which linearly combines the weighted sum and product measures. Driven by the simplicity of the method, many researchers used the technique for decision-making. Mardani et al. [49] prepared a detailed review on WASPAS showcasing the method's usefulness in the decision process. Mishra et al. [50] evaluated green suppliers in an industry by extending WASPAS with exponential divergence concept with hesitant fuzzy data. Tus and Adali [51] prepared the CRITIC-WASPAS approach as an integrated framework for software evaluation in firms. Pamucar et al. [52] gave a new extension to WASPAS under neutrosophic context for assessing advisors in the process of transporting hazardous goods. Krishankumar et al. [53] came up with a variance-WASPAS integrated method for green supplier selection in a linguistic environment. Bouchraki et al. [54] provided an integrated AHP-WASPAS with fuzzy numbers for assessing claims of customers concerning drinking water service in a firm. Ilbahar et al. developed a Pythagorean fuzzy WASPAS model for renewable energy selection by considering sustainable factors. Krishankumar et al. [55] ranked risk management strategies in the construction sector by proposing a combined framework with variance and WASPAS technique under a double hierarchy setting. Pamucar et al. [56] selected suitable transport modes for reaching airports in Istanbul by preparing a model with fuzzy numbers, a level-based weight assessment technique, and WASPAS. Simic et al. [57] ranked last-mile travel modes of goods by presenting WASPAS under picture fuzzy context. Ali et al. [58] came up with a new framework under an uncertain linguistic setting with arithmetic operations, fusion functions, entropy, and the WASPAS technique for selecting suppliers in a firm. Osintsev et al. [59] assessed compression methods for aerial images by gathering linguistic ratings and adopting neutrosophic WASPAS algorithm. Bozanic et al. [60] prepared a new extension of WASPAS and AHP approaches to ordered fuzzy values for rationally ranking improvement projects.

2.3. Insights from the Review. Based on the previously prepared previously review, it is clear that PLTS is a sophisticated preference structure that can associate confidence levels to terms and aid in heterogeneous data transformation into holistic data forms. Further, determining significance values by properly considering hesitation is crucial in rational decision-making. Finally, the idea of personalized ordering with appropriate consideration to the nature of criteria is key for effective decision-making. The insights are in line with the challenges discussed in the study, which are circumvented by the contributions made in this work.

Figure 1 presents the working model of the proposed interactive approach by utilizing PLD. As claimed in [6], PLD is a flexible preference style that can effectively model diverse opinions from heterogeneous candidates/participants. The procedure adopted for converting the rating information into PLD is explained in Section 3. Data on each online course is given by a diverse set of participants with a different count, background, expertise, demography, and so on. PLTS is a flexible structure adopted to transform the various data into a holistic data matrix to prioritize online professional courses. The data is collected empirically from participants of the short-term online certification course hosted by RGNIYD, an academic institution during the pandemic time. Officials who hosted the certification program acted as experts and offered their opinion on each factor considered for rating the courses. By using the regret/rejoice technique, the significance values of elements are calculated. Later, the data matrix and the significance vector are used by the interactive WASPAS algorithm for the prioritization of online courses given job opportunities for Indian youths. The officials collect personal choices on each course as a choice vector. The nature of factors is also being considered in this interactive WASPAS algorithm for rational prioritization of courses.

3. Methodology

3.1. Preliminaries. The authors provide some basic concepts related to LTS, HFLTS, and PLTS.

Definition 1. Reference [3]. Let $TS = \{s_z | z = 0, 1, \ldots, q\}$ be an LTS with s_0 and s_q as the initial and final objects with v being a positive integer. The features of S are as follows:

If

$$\begin{aligned} &za > zb \text{ then } s_{za} > s_{zb}, \\ &neg(s_{za}) = s_{zb}, \end{aligned} \tag{1}$$

where

$$za + zb = q. \tag{2}$$

Definition 2 [4]. Let TS be as before. Then, an HFLTS is given by

$$D_F = \{a, h_{D_F}(a) | a \in A\}, \tag{3}$$

where

$$h_{D_F}(a) = h(a) = \{s_z^k | k = 1, 2, \ldots, \#h(a)\}. \tag{4}$$

Definition 3 [6]. Let TS be as before. Then, PLTS is given by

$$D(p) = \left\{ D^k\left(p^k\right) | D^k \in TS,\ 0 \leq p^k \leq 1,\ \sum_k p^k \leq 1,\ k = 1, 2, \ldots, \#D(p) \right\}, \tag{5}$$

where $D^k(p^k)$ is the k^{th} instance with p^k being the occurrence probability that is associated with the term D^k and $\#D(p)$ refers to the number of instances.

Note 1. $d_i = \{(s_z^k)_i (p_i^k)\}$ is the probabilistic linguistic element (or) probabilistic linguistic data (PLE/PLD) and many such elements constitute the PLTS. Terms have the following semantics:

$$\left\{ \begin{array}{c} s_o = \text{none}, \\ s_1 = \text{extremely low}, \\ s_2 = \text{very low}, \\ s_2 = \text{low}, \\ s_4 = \text{moderate}, \\ s_5 = \text{high}, \\ s_6 = \text{very high}, \\ s_7 = \text{extremely high}. \end{array} \right\}. \tag{6}$$

Definition 4. Reference [21]. Two PLEs d_1 and d_2 considered. Then, the operations are given by

$$\begin{aligned} d_1 \oplus d_2 &= f^{-1}(f(d_1) + f(d_2)), \\ d_1 \oplus d_2 &= f^{-1}(f(d_1) \times f(d_2)), \end{aligned} \tag{7}$$

where f and f^{-1} are obtained from [21]

$$f\colon \tau = \frac{z}{4q} + 0.5, \tag{8}$$

and

$$f^{-1}\colon s_z = s_{(2\tau - 1)\times 2q}. \tag{9}$$

3.2. Data Transformation. This section focuses on converting Likert scale ratings from a heterogeneous set of participants into holistic data for decision-making. PLTS is a suitable structure for supporting this conversion process. Occurrence probability values are associated as confidence values to the different rating terms that give an overview of all the participants and their rating for a particular instance.

The present study considered four professional courses for short-term certification programs conducted online during 2020 (pandemic time). The participant count was

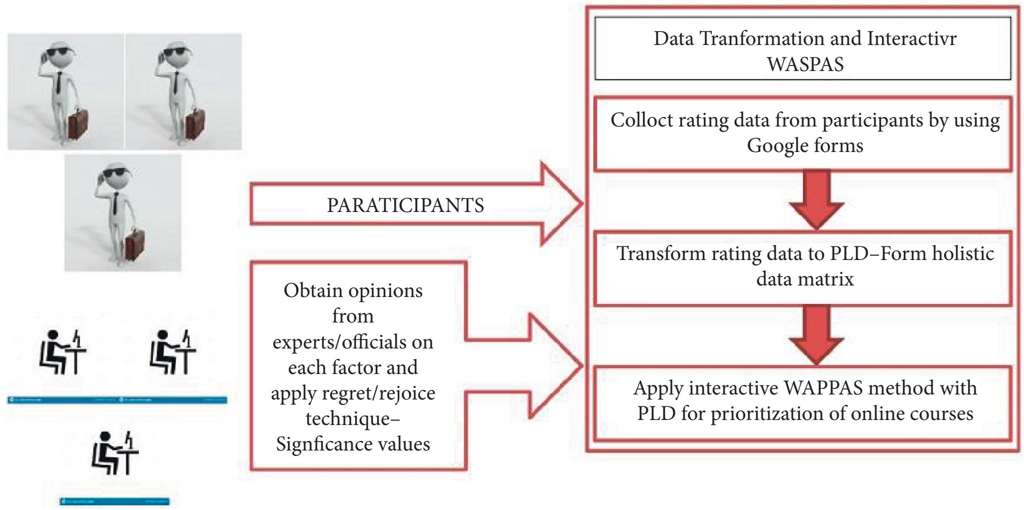


Figure 1: Working model for online professional course selection with PLD.

diverse for each course, and a normal rating was insufficient to grasp the data correctly. Thus, the rating information is transformed to PLD. Proper consideration is given to the diverse participants pertaining to each course by doing the transformation. As a simple walkthrough example, suppose there are two courses, namely, course A and course B, with participants as three and five, respectively, for each course. Rating information is obtained as A(1) = s_4, A(2) = s_3, and A(3) = s_4; B(1) = s_3, B(2) = s_3, B(3) = s_3, B(4) = s_2, and B(5) = s_5. It must be noted that A(1) to A(3) denotes Likert scale rating from three students on course A. Similarly, B(1) to B(5) denotes Likert scale rating from five students on course B. PLEs with respect to course A based on the rating information from three students are calculated as follows:

$$\begin{aligned} A &= \left\{ \begin{matrix} s_4\left(\frac{2}{3}\right) \\ s_3\left(\frac{1}{3}\right) \end{matrix} \right\} = \left\{ \begin{matrix} s_4(0.67) \\ s_3(0.33) \end{matrix} \right\}, \\ B &= \left\{ \begin{matrix} s_3\left(\frac{3}{5}\right) \\ s_2\left(\frac{1}{5}\right) \\ s_5\left(\frac{1}{5}\right) \end{matrix} \right\} = \left\{ \begin{matrix} s_3(0.6) \\ s_2(0.2) \\ s_5(0.2) \end{matrix} \right\}. \end{aligned} \tag{10}$$

For course A, the factor 2/3 for s_4 is obtained since two students out of three rated course A as s_4. On the other hand, for course B, a factor of 1/5 is obtained for s_2 since one student rated course B as s_2 out of five students. Similarly, other values can be determined. This mechanism is adopted to transform the input data matrix for ranking online professional courses within the perspective of job opportunities for Indian youths. It must be noted that the instances chosen for the study depend on the expert. For the present study, two cases in high probability are chosen.

3.3. Significance Calculation: Regret/Rejoice Technique. The factors considered for evaluating online courses are competing with one another and pose unbiased or heterogeneous importance. Kao [56] stated that this needs to be methodically determined to avoid subjectivity and inaccuracies from direct elicitation. Popularly, weights are determined either with partial information or unknown information. Programming models are used for partial context, and methods such as entropy [20], analytical methods [62–64], variance [53], and so on are used for unknown context. Though these methods estimate weights, hesitation during opinion sharing is not adequately captured, and the regret factor incurred by the personnel during the decision process is not properly realized. The regret/rejoice technique is put forward under the PLTS context to mitigate the issue. Regret theory [32] is an exciting concept that deals with the psychological aspect and demonstrates the behavior based on a particular choice made by the expert. The technique that evolves from the theory (i) is simple and elegant, (ii) effectively represents the hesitation of experts, and (iii) captures the regret factor incurred by an expert during the decision process. From the review, it is clear that the technique is suitable for the significance determination of factors.

Inspired by the claims, steps for calculation of significance of factors are presented as follows:

Step 1. Experts share their opinions on each factor as a PLE. g vectors of $1 \times y$ order are obtained.

Step 2. Neumann utility function is applied to the score measures of PLEs by using (5) and (6). Neumann values are determined for each opinion value from the expert leading to $g \times y$ values.

$$Sd_{lj} = \sum_k \left(z_{lj}^k \times p_{lj}^k\right), \tag{11}$$

$$UF_j = \sum_{l=1}^{g} v\left(Sd_{lj}\right) + R\left(v\left(Sd_{lj}\right)\right) - v\left(\widetilde{Sd_{lj}}\right), \tag{12}$$

where $v(Sd_{lj}) = (Sd_{lj})^{\eta}$ is the von-Neumann function, $R(v(Sd_{lj})) = 1 - e^{-\beta \times v(Sd_{lj})}$ is the regret function and

$$v\left(\widetilde{Sd_{lj}}\right) = \max_{j\in \text{Benefit}}\left(v\left(Sd_{lj}\right)\right) \text{or} \min_{j\in \text{Cost}}\left(v\left(Sd_{lj}\right)\right). \tag{13}$$

Here, η is the power value, and β is the risk aversion value. Both are considered as 0.50 in this study.

Step 3. Normalize the values from the utility function by using (7) to get the significance vector for the factors. Values from Step 2 are considered as input to determine the significance values.

$$Sf_j = \frac{UF_j}{\sum_j UF_j}, \tag{14}$$

where Sf_j is in the range 0 to 1, with the sum being unity, referred to as the significance value.

3.4. Ranking Alternatives: WASPAS Method. The focus of the section is to present an approach for ranking alternatives based on the set of criteria. As discussed earlier, criteria are heterogeneous and pose biased significance. Therefore, the calculated significance vector from the previous section is utilized here, along with the preference data.

Based on the review previously made, it is clear that WASPAS [48] (i) is simple and elegant; (ii) popularly used in decision-making, and (iii) works using sum/product functions as base formulation. It can be noted that the WASPAS method does not consider the nature of criteria and cannot accept the personal choices of experts during the ranking process. Motivated by the issue, in this section, an interactive extension of WASPAS is put forward, and the stepwise formulation is given as follows.

Step 1. Based on the participants rating, a holistic PLTS data matrix is formed of $x \times y$ order by adopting the procedure given in Section 3.2. Here x denotes the number of professional courses (alternatives), and y represents the number of factors.

Step 2. Apply transformation functions given in (15) to effectively accommodate the nature of factors (criteria) in the rank estimation.

$$d_{ij} = \begin{cases} d_{ij} = d_{ij} \text{ for } j \in \text{ benefit}, \\ d_{ij} = d_{ij}^c \text{ for } j \in \text{ cost}, \end{cases} \tag{15}$$

where $d_{ij}^c = \left\{s_{q-z_{ij}}^k (1 - p_{ij}^k)\right\}$ is the complement of d_{ij}. Ideally, the probability values are normalized to retain the property of a PLE. Here q is as denoted in Definition 1.

Step 3. Determine each alternative's weighted sum and product values based on the significance vector and holistic data matrix. Equations (9) and (10) are applied for this purpose.

$$AF_i = \sum_{j=1}^{y} Sf_j \times \left(\sum_k \left(po_i \times \left(\frac{z_{ij}^k \times p_{ij}^k}{\sum_j \left(z_{ij}^k \times p_{ij}^k\right)}\right)\right)\right), \tag{16}$$

$$PF_i = \prod_{j=1}^{y} \left(\left(\sum_k \left(po_i \times \left(\frac{z_{ij}^k \times p_{ij}^k}{\sum_j \left(z_{ij}^k \times p_{ij}^k\right)}\right)\right)\right)\right)^{Sf_j}, \tag{17}$$

where po_i is the personal opinion on alternative i in the unit interval with $\sum_i po_i$ as unity, po_i is a vector of $1 \times x$ order, Sf_j is the significance of factor j, z_{ij}^k is the subscript of the linguistic term in the k^{th} instance for i^{th} professional course rated based on j^{th} factor, p_{ij}^k is the occurrence probability value in the k^{th} instance for i^{th} professional course rated based on j^{th} factor, and AF_i and PF_i are the weighted sum and weighted product values associated with each alternative.

Equations (9) and (10) yield two vectors each of order $1 \times x$, and Sf_j is the significance value of the factor j calculated by applying (14). po_i is another parameter that denotes the personal opinion on an alternative i given by the experts.

It must be noted that when instances of the PLEs are not equal, the procedure mentioned in Definition 6 of [6] is adopted, which makes the instances in the PLEs equal.

Step 4. Calculate the net rank TR_i of each alternative i by adopting the idea of a linear combination of weighted sum and weighted product determined from (9) and (10). Equation (18) is used for the calculation that yields a rank vector of order $1 \times x$.

$$TR_i = \theta \times AF_i + (1 - \theta) \times PF_i, \tag{18}$$

where θ is the strategy measure between 0 and 1.

It must be noted that (18) yields a vector of $1 \times x$ order that contains the rank values of each professional certification course that is considered in the case example. By increasing the strategy values stepwise from 0.1 to 0.9, nine vectors of $1 \times x$ can be obtained from (18). Arrange the values obtained from (18) in the descending order for forming the ordering of alternatives.

4. Case Example: Online Professional Course Selection

This section attempts to exemplify the applicability of the research model. For this, a case example with empirical data from participants is adopted to select suitable online courses for Indian youths focused on job creation for the youth population with an IT background. Four online courses, namely, data science, machine learning, cyber security, and cloud computing were conducted as short-term certifications in RGNIYD, an academic institution. Youths with fair IT background attended the program, and it accounted for 69, 164, 64, and 103, respectively. Out of these, participants who volunteered for data collection were 47, 110, 44, and 83, respectively. It can be seen that the population size is heterogeneous for each online course. To better formulate the data, PLTS is adopted. During the pandemic, these courses were hosted online for the betterment of youths. Resource personnel from the institute of national importance were invited to deliver lectures and hands-on training to the participants.

To further understand the efficacy of these courses in terms of job creation for youths, we invited volunteers to participate in a semistructured questionnaire created using Google forms and circulated online for data collection. The study aimed to rank online courses as per the perception of youth. Factors utilized in the study for rating online courses are the usefulness of the course, job creation from the course, resource personnel content knowledge, expected prerequisite/preparation of the course, stress due to pandemic, and connectivity issues. Based on the literature [65–67] and intuition, these factors are finalized for the study. The last three factors are cost type, and the other factors are benefit type.

For the sake of implementation, authors refer to online courses (cyber security, machine learning, data science, and cloud computing) as CC_1, CC_2, CC_3, and CC_4; factors as FC_1, FC_2, FC_3, FC_4, FC_5, and FC_6; course organizers as CO_1, CO_2, and CO_3. The last three factors are of cost type, while the rest are benefit type. Steps for ranking the online courses are given as follows.

Step 1. Consider rating data from each participant on the four online courses based on the six factors. Likert-scale rating is adopted by participants transformed to PLE by adopting the procedure described in Section 3.2.

Table 1 gives the PLEs as a data matrix for the participants' four courses rated on six linguistically (Likert-scale). The transformation procedure presented in Section 3.2 is used for constructing PLEs from the linguistic data. This is crucial because participants for each course are heterogeneous in terms of count, demography, and so on. The authors considered the top two linguistic terms based on the associated occurrence probability values to build the decision matrix.

Step 2. Officials/organizers of the course (online) provide their rating on the factors that helps in determining the significance of the elements (Table 2). The procedure developed in Section 3.3 is used for this purpose.

The procedure put forward in Section 3.3 is applied to determine the utility values of factors by considering regret/rejoice factors and von-Neumann values (as shown in Figure 2). $v(\widetilde{Sd_{lj}})$ is determined as 2.025, 2.023, 2.098, 1.643, 1.265, and 1.732. Equations (6) and (7) yield the significance values of factors as 0.097, 0.230, 0.090, 0.203.0.170, and 0.210, respectively.

Step 3. With the help of data from Step 1 and vector from Step 2, online courses are ranked by adopting the algorithm proposed in Section 3.4. From the data transformation procedure, the heterogeneous participants' data of each course is holistically transformed into a data matrix of order 4×6. Significance value is a vector of order 1×6.

Personal Opinion. po_i is considered for each course as 0.25, 0.20, 0.35, and 0.20, respectively. From Table 3, the parameter values associated with the improved WASPAS for each online professional course are obtained. The TR_i values indicate the ordering of courses as $CC_3 \succ CC_1 \succ CC_4 \succcurlyeq CC_2$, which infers that the data science course is considered most suited for job opportunities for Indian youths, followed by cyber security, machine learning, and cloud computing. In particular, machine learning and cloud computing are equally preferred by Indian youths in terms of job opportunities in organizations.

Step 4. Conduct sensitivity analysis with significance and strategy values of factors and experts by altering values systematically by shift operations.

Sensitivity measure is investigated in both inter/intra-context by varying the significance of factors through shift operations and periodically increasing step size of strategy values. In the intercontext, the effect of new sets of significance vectors on rank values is determined, and in the intracontext, the impact of strategy values on rank values is determined. Figures 3(a) to 3(f) show the effect of both the values (alteration of weights (inter) and alteration of strategy values (intra) on the ordering of the online professional courses. Six bar graphs are depicted for six weight sets (obtained by shift operation of significance values), and within each graph, strategy values are altered from 0.1 to 0.9. Rank values of each course is plotted as the bar. It can be seen that the Indian youths highly prefer data science in terms of job opportunities in organizations. Courses such as machine learning and cloud computing are equally considered in their respective ranking regarding job opportunities for the youth population. The empirical case study conducted by RGNIYD, an academic institute, serves as a pilot study in effectively understanding the importance of online training (teaching/learning) during pandemic situations and the courses that fetch job opportunities to Indian youths based on their data. The inter/intrasensitivity analysis shows that the proposed framework is robust even after adequate alterations are incorporated.

4.1. Comparison Study. The authors attempt to showcase the efficacy of the proposed work by comparing the model with a close counterpart method [48]. From the sensitivity graph shown in Figure 3, it is clear that the proposed work is highly robust even after alterations to factor significance, and

Table 1: Linguistic data transformed to PLE for decision-making.

Factors	Professional online courses			
	CC_1	CC_2	CC_3	CC_4
FC_1	$\{s_5(0.5), s_4(0.44)\}$	$\{s_4(0.46), s_6(0.36)\}$	$\{s_5(0.5), s_2(0.5)\}$	$\{s_4(0.44), s_3(0.56)\}$
FC_2	$\{s_5(0.45), s_3(0.4)\}$	$\{s_4(0.66), s_5(0.3)\}$	$\{s_6(0.52), s_3(0.4)\}$	$\{s_2(0.35), s_3(0.47)\}$
FC_3	$\{s_3(0.6), s_5(0.4)\}$	$\{s_6(0.55), s_4(0.4)\}$	$\{s_6(0.46), s_4(0.52)\}$	$\{s_5(0.54), s_6(0.43)\}$
FC_4	$\{s_5(0.6), s_2(0.33)\}$	$\{s_4(0.44), s_3(0.33)\}$	$\{s_3(0.63), s_4(0.33)\}$	$\{s_5(0.44), s_3(0.33)\}$
FC_5	$\{s_2(0.25), s_3(0.27)\}$	$\{s_3(0.6), s_4(0.4)\}$	$\{s_5(0.55), s_6(0.33)\}$	$\{s_4(0.62), s_2(0.28)\}$
FC_6	$\{s_6(0.5), s_4(0.5)\}$	$\{s_6(0.35), s_5(0.6)\}$	$\{s_3(0.42), s_2(0.25)\}$	$\{s_3(0.52), s_4(0.44)\}$

Table 2: Opinions for determining factors' significance.

Factors	Officials/organizers		
	CO_1	CO_2	CO_3
FC_1	$\{s_3(0.45), s_4(0.44)\}$	$\{s_4(0.4), s_5(0.5)\}$	$\{s_3(0.6), s_5(0.3)\}$
FC_2	$\{s_5(0.5), s_4(0.4)\}$	$\{s_5(0.6), s_3(0.25)\}$	$\{s_4(0.45), s_5(0.4)\}$
FC_3	$\{s_4(0.4), s_3(0.5)\}$	$\{s_4(0.35), s_6(0.5)\}$	$\{s_4(0.4), s_3(0.6)\}$
FC_4	$\{s_2(0.55), s_4(0.4)\}$	$\{s_5(0.35), s_6(0.4)\}$	$\{s_4(0.45), s_2(0.55)\}$
FC_5	$\{s_3(0.3), s_2(0.35)\}$	$\{s_5(0.4), s_4(0.45)\}$	$\{s_4(0.5), s_5(0.5)\}$
FC_6	$\{s_3(0.6), s_4(0.3)\}$	$\{s_5(0.5), s_4(0.45)\}$	$\{s_5(0.6), s_2(0.4)\}$

Table 3: Rank values associated with each online professional course.

Online courses	Parameters of interactive WASPAS algorithm		
	AF_i	PF_i	TR_i
CC_1	0.066523	0.062433	0.064478
CC_2	0.048126	0.04509	0.046608
CC_3	0.090113	0.083139	0.086626
CC_4	0.047729	0.045634	0.046891

strategy values are done adequately. To further realize the efficacy of the work, the model in [53] is compared with the proposed work. Table 4 gives the summarized view of the theoretical benefits of the proposed work over [53]. Besides, authors also compare the proposed work with extant models, namely, [13, 17] and [23], which actively use PLTS in their framework for attaining rational decisions. Finally, data from the case example are provided to these models, and the courses are ranked, and they are shown in Table 5.

Table 5 and Figure 4 show that the proposed model produces a unique ranking order of professional courses. It can be intuitively inferred owing to the proposed model's ability to consider the nature of factors during the ranking of courses (alternatives) and capture experts' personal opinions during ranking, which provides a sense of personalization and is lacking in existing models. Apart from this, specific theoretical merits of the framework are listed below. Based on the briefing in Table 4, it is inferred that the proposed work is novel and innovative. To further detail the claim, specific points are presented as follows:

(i) PLD is a sophisticated structure that allows for the elegant transformation of rating data from multiple heterogeneous candidates/users. Furthermore, the structure ensures data without loss of generality.

(ii) Driven by the inference from Kao [61], weights are rationally determined by considering risk attitudes and the nature of factors, which is lacking in the close counterpart approach.

(iii) Unlike the framework [53], the ranking algorithm in the proposed work considers the nature of factors and personal choices to provide an interactive, personalized variant of WASPAS with PLD.

O The nature of factors, which is a potential parameter in the decision process, is considered both in weight assessment and ranking, lacking in [53].

O Also, the new formulation allows experts/agents to share their personal opinion on each alternative option (as a vector) that acts as potential information in influencing the ordering of options.

Statistically, the comparison is further extended to realize the superiority of the proposed work. For this purpose, 300 matrices are generated that are used in the simulation experiment. These matrices are of the same dimension as the data in the case example. They are given as input to both the proposed models [53]. Rank vectors are estimated via the algorithm provided in each model. It can be seen that each algorithm obtains 300 vectors of 1×4. Statistical variance is calculated for each vector, so 300 values are obtained, which are plotted in Figure 5. The graph shows that the proposed model can better discriminate alternatives (online courses here) by producing broader vectors than its close counterpart. The graph shows that the proposed model has about six times better discrimination than the counterpart approach. Besides the test for uniqueness of the proposed model put forward in Figure 4, the uniqueness measure is also determined for the 300 orders yielded by the proposed model from the simulation experiment. Spearman rank correlation is applied for the rank values produced by the proposed and counterpart approach. It can be inferred that due to consideration of personal choices, the proposed model produces an order that is unique compared to its close counterpart with an average uniqueness score of 0.7807 for the 300 simulated matrices (Figure 6).

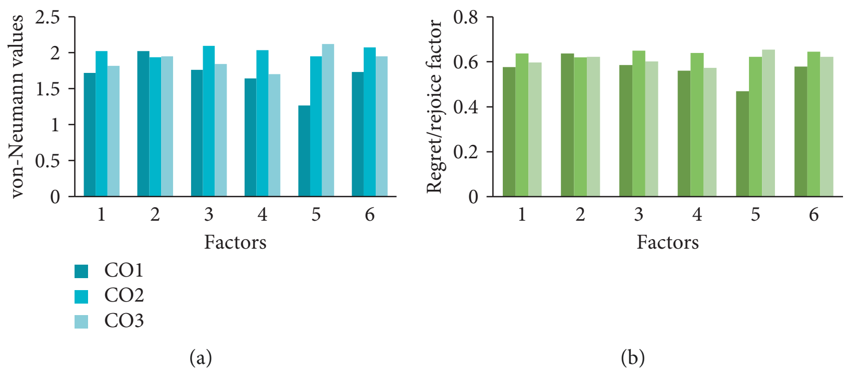


Figure 2: (a) von-Neumann values and (b) regret/rejoice values.

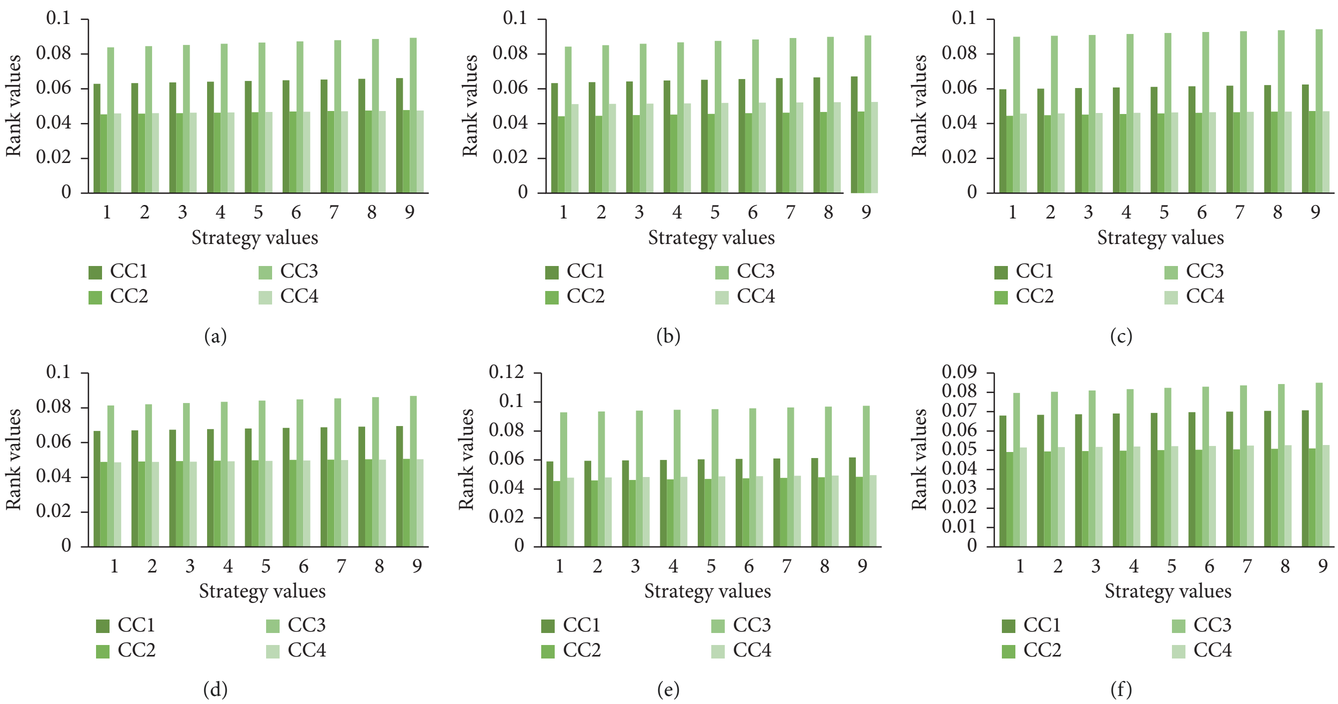


Figure 3: Sensitivity measure of factors' significance: (a) to (f) is set 1 to set 6 (X axis 1 to 9 indicate strategy values from 0.1 to 0.9, resp., with step size 0.1).

In a nutshell, the proposed model is analyzed from both the theoretical and statistical perspective to effectively understand the superiority of the model that is put forth in the present study. It is clear that (i) the model adds value to the PLTS-based decision-making by putting forward a novel framework, whose usefulness is demonstrated by using real case example of professional online course evaluation by collecting empirical data from RGNIYD, an academic institution in India; (ii) later, the test for uniqueness (Figures 4 and 6) shows that the proposed model produces an unique order of alternatives (online courses), which is intuitively backed by the novelty in the formulation of the model that allows consideration of personal choices; and (iii) finally, the test for discriminative power (Figure 5) also reveals that the proposed model produces broader and sensible rank values that aid in better discrimination of alternatives (online courses) for rational decision-making.

Table 4: Summary of characteristics: proposed and other models.

Context	Proposed work	[53]
Data	PLD	PLD
Weights of factors	Calculated methodically	Calculated methodically
Regret attitude characterization	Done, regret/rejoice factor	Not done
Nature of factors	Considered, both during weight assessment and ranking	Not considered
Personal choices of experts	Considered during ranking	Not considered

Table 5: Order of professional courses obtained from PLTS models: proposed versus others.

Course	Proposed	Reference [48] (0.2)	[13]	[17]	[23]
CC_1	2	3	2	3	3
CC_2	4	1	1	1	1
CC_3	1	2	3	2	2
CC_4	3	4	4	4	4

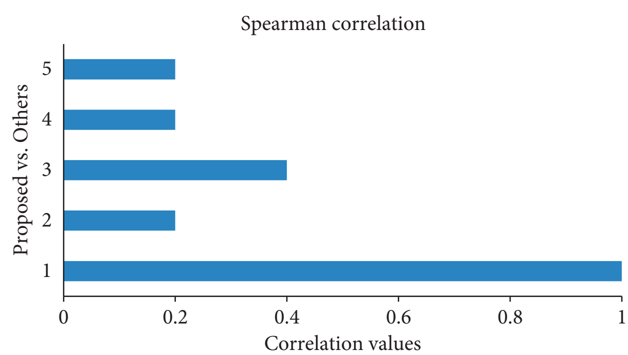


Figure 4: Test of uniqueness in ordering.

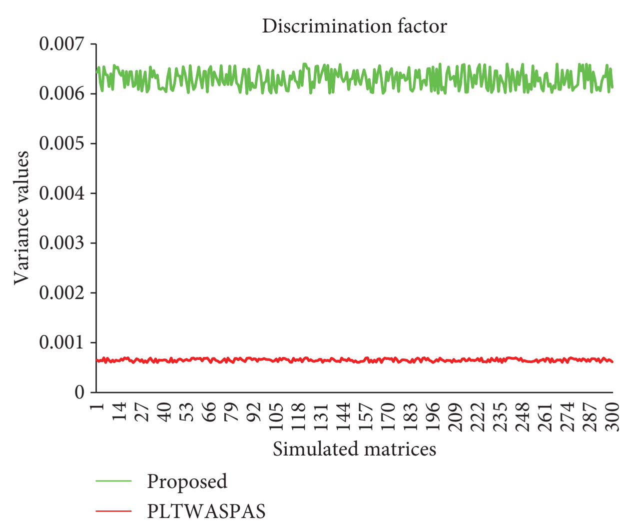


Figure 5: Broadness measure for realizing discrimination factor.

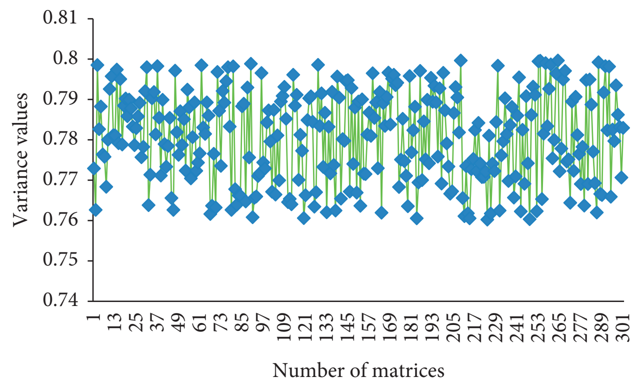


FIGURE 6: Consistency measure through Spearman correlation.

TABLE 6: Symbols, semantics, and the respective values.

Symbol	Meaning and value
s_z	Linguistic term with subscript z that can have values as $0, 1, \ldots, q$. q is 6
$d_i = \{(s_z^k)_i (p_i^k)\}$	Probabilistic linguistic element/data with s_z^k as the k^{th} linguistic term and p_z^k as the probability value associated with the k^{th} linguistic term
k	Index for the instance that can take values as $1, 2, \ldots, \#D(p)$
Sd_{lj}	Score measure associated with l^{th} expert and j^{th} factor
Sf_j	Weight/importance associated with factor j. Values are in the range 0 to 1
j	Index associated with the factors
i	Index associated with the alternatives (online courses)
l	Index associated with the experts
g	Number of experts. g is 3
y	Number of factors. y is 6
x	Number of alternatives. x is 4
z_{ij}^k	Subscript of the k^{th} linguistic term given as rating to rate the alternative i based on factor j
p_{ij}^k	Probability associated with the k^{th} linguistic term that is used for rating the alternative i based on factor j
po_i	Personal opinion associated with alternative i. Values are in the range 0 to1. Here, we considered values as 0.25, 0.20, 0.35, and 0.20, respectively for each online course.
AF_i	Weighted sum of alternative i
PF_i	Weighted product of alternative i
TR_i	Net rank value of alternative i
θ	Strategy value that can have values in the range 0 to 1
η	Power value. Considered as 0.50
β	Risk aversion. Considered as 0.50

5. Conclusion

The present study offers an integrated technique put into a framework for decision-making with PLD. The framework adds value to the research in the field of PLTS. The method in the framework calculates weights/significance of factors rationally by considering the hesitation attitudes of experts/agents. Furthermore, an extension to the WASPAS approach is provided with PLD that enables the method to consider the nature of factors during the ordering and personal choices of alternative options from experts/agents. This personal choice as a vector offers a sense of the personalized ordering of options. The framework's benefits can be theoretically and statistically verified by an empirical case study of professional online course selection during the pandemic time. Comprehensive inter-/intrasensitivity analysis and comparison (with close counterpart) reveal the framework's superiority in robustness and acceptable discrimination level.

Some shortcomings of the study are (i) the importance of experts/agents is not methodically derived, and (ii) consideration of top two linguistic instances with its associated single confidence value to the terms (Likert scales) may cause some information loss in the practical sense. On the other hand, a few implications from the managerial perspectives are (i) the framework can be readily adapted for other decision problems in academics and other fields; (ii) transforming rating data from heterogeneous participants into a holistic data matrix by using the PLTS concept is a flexible way for data representation; (iii) though some loss exists in the data, it may be addressed by extending the framework to complete data zone; (iv) the framework gives educational policymakers to effectively plan courses for youth population so that they gain the state-of-the-art skill and knowledge to become ready of industry; (v) finally, some training with the model is expected to aid policymakers in the decision process.

In the future, authors plan to resolve the previously mentioned shortcomings by presenting algorithms for expert weight assessment and considering complete data zone for decision-making. Further, plans are made to propose an integrated approach in the fuzzy variants, such as orthopair sets [68–70] and interval variants of linguistic forms, such as PLTS [6] and double hierarchy variants [71]. Finally, machine learning concepts can be embedded with decision approaches for solving large-scale decision problems in academic and other contexts.

Appendix

The symbols, their notations, and respective values are provided in Table 6 for clarity to readers.

Data Availability

The data used to support the findings of this study are included within this article. However, the reader may contact the corresponding author for more details on the data.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

Each author has participated and contributed sufficiently to take public responsibility for appropriate portions of the content.

References

[1] Z. Ali, T. Mahmood, T. Mahmood, K. Ullah, and Q. Khan, "Einstein geometric aggregation operators using a novel complex interval-valued pythagorean fuzzy setting with application in green supplier chain management," *Reports in Mechanical Engineering*, vol. 2, no. 1, pp. 105–134, 2021.

[2] L. A. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning-I," *Information Sciences*, vol. 8, no. 3, pp. 199–249, 1975.

[3] F. Herrera, E. Herrera-Viedma, and J. L. Verdegay, "A sequential selection process in group decision making with a linguistic assessment approach," *Information Sciences*, vol. 85, no. 4, pp. 223–239, 1995.

[4] Y. Ali, M. Ahmad, M. Ahmad, M. Sabir, and S. A. Shah, "Regional development through energy infrastructure: a comparison and optimization of Iran-Pakistan-India (IPI) & Turkmenistan- Afghanistan-Pakistan-India (TAPI) gas pipelines," *Operational Research in Engineering Sciences: Theory and Applications*, vol. 4, no. 3, pp. 82–106, 2021, https://doi.org/10.31181/oresta091221082a.

[5] T. K. Biswas and M. C. Das, "Selection of the barriers of supply chain management in Indian manufacturing sectors due to COVID-19 impacts," *Operational Research in Engineering Sciences: Theory and Applications*, vol. 3, no. 3, pp. 1–12, 2020.

[6] Q. Pang, H. Wang, and Z. Xu, "Probabilistic linguistic term sets in multi-attribute group decision making," *Information Sciences*, vol. 369, pp. 128–143, 2016.

[7] A. Kobina, D. Liang, and X. He, "Probabilistic linguistic power aggregation operators for multi-criteria group decision making," *Symmetry*, vol. 9, no. 12, pp. 320-321, 2017.

[8] P. Liu and F. Teng, "Some Muirhead mean operators for probabilistic linguistic term sets and their applications to multiple attribute decision-making," *Applied Soft Computing*, vol. 68, pp. 396–431, 2018.

[9] S.-P. Wan, W.-B. Huang Cheng, and J.-Y. Dong, "Interactive multi-criteria group decision-making with probabilistic linguistic information for emergency assistance of COVID-19," *Applied Soft Computing*, vol. 107, no. 169, Article ID 107383, 2021.

[10] S.-P. Wan, W.-C. Zou, J.-Y. Dong, and L. Martínez, "A probabilistic linguistic dominance score method considering individual semantics and psychological behavior of decision makers," *Expert Systems with Applications*, vol. 184, Article ID 115372, 2021.

[11] W. Yu, H. Zhang, and B. Li, "Operators and comparisons of probabilistic linguistic term sets," *International Journal of Intelligent Systems*, vol. 34, no. 7, pp. 1476–1504, 2019.

[12] P. Liu and Y. Li, "Multi-attribute decision making method based on generalized maclaurin symmetric mean aggregation operators for probabilistic linguistic information," *Computers & Industrial Engineering*, vol. 131, pp. 282–294, 2019.

[13] D. S. Pamucar and M. Savin, "Multiple-criteria model for optimal off-road vehicle selection for passenger transportation: BWM-COPRAS model," *Military Technical Courier*, vol. 68, no. 1, pp. 28–64, 2020.

[14] X. Gou, Z. Xu, H. Liao, and F. Herrera, "Probabilistic double hierarchy linguistic term set and its use in designing an improved VIKOR method: the application in smart healthcare," *Journal of the Operational Research Society*, vol. 72, no. 12, pp. 2611–2630, 2020.

[15] P. Liu and F. Teng, "Probabilistic linguistic TODIM method for selecting products through online product reviews," *Information Sciences*, vol. 485, pp. 441–455, 2019.

[16] H. Liao, L. Jiang, Z. Xu, J. Xu, and F. Herrera, "A linear programming method for multiple criteria decision making with probabilistic linguistic information," *Information Sciences*, vol. 415-416, pp. 341–355, 2017.

[17] D. Liang, A. Kobina, and W. Quan, "Grey relational analysis method for probabilistic linguistic multi-criteria group decision-making based on geometric bonferroni mean," *International Journal of Fuzzy Systems*, vol. 20, no. 7, pp. 2234–2244, 2018.

[18] X. Wu, H. Liao, Z. Xu, A. Hafezalkotob, and F. Herrera, "Probabilistic linguistic multimoora: a multicriteria decision making method based on the probabilistic linguistic expectation function and the improved borda rule," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 6, pp. 3688–3702, 2018.

[19] P. Liu and Y. Li, "The PROMTHEE II method based on probabilistic linguistic information and their application to decision making," *Informatica*, vol. 29, no. 2, pp. 303–320, 2018.

[20] H. Liu, L. Jiang, and Z. Xu, "Entropy measures of probabilistic linguistic term sets," *International Journal of Computational Intelligence Systems*, vol. 11, no. 1, pp. 45–57, 2018.

[21] X. Gou and Z. Xu, "Novel basic operational laws for linguistic terms, hesitant fuzzy linguistic term sets and probabilistic linguistic term sets," *Information Sciences*, vol. 372, pp. 407–427, 2016.

[22] G.-l. Xu, S.-P. Wan, and J.-Y. Dong, "An entropy-based method for probabilistic linguistic group decision making and

its application of selecting car sharing platforms," *Informatica*, vol. 31, no. 3, pp. 621–658, 2020.

[23] R. Sivagami, R. Krishankumar, V. Sangeetha, K. S. Ravichandran, S. Kar, and A. H. Gandomi, "Assessment of cloud vendors using interval-valued probabilistic linguistic information and unknown weights," *International Journal of Intelligent Systems*, vol. 36, no. 8, pp. 3813–3851, 2021.

[24] X. Cheng, J. Gu, and Z. Xu, "Venture capital group decision-making with interaction under probabilistic linguistic environment," *Knowledge-Based Systems*, vol. 140, pp. 82–91, 2018.

[25] X. Wu and H. Liao, "An approach to quality function deployment based on probabilistic linguistic term sets and ORESTE method for multi-expert multi-criteria decision making," *Information Fusion*, vol. 43, pp. 13–26, 2018.

[26] R. Sivagami, K. S. Ravichandran, R. Krishankumar et al., "A scientific decision framework for cloud vendor prioritization under probabilistic linguistic term set context with unknown/partial weight information," *Symmetry*, vol. 11, no. 5, p. 682, 2019.

[27] S. Ramadass, R. Krishankumar, K. S. Ravichandran, H. Liao, S. Kar, and E. Herrera-Viedma, "Evaluation of cloud vendors from probabilistic linguistic information with unknown/partial weight values," *Applied Soft Computing*, vol. 97, Article ID 106801, 2020.

[28] X.-B. Mao, M. Wu, J.-Y. Dong, S.-P. Wan, and Z. Jin, "A new method for probabilistic linguistic multi-attribute group decision making: application to the selection of financial technologies," *Applied Soft Computing*, vol. 77, pp. 155–175, 2019.

[29] S.-P. Wan, J. Yan, and J.-Y. Dong, "Personalized individual semantics based consensus reaching process for large-scale group decision making with probabilistic linguistic preference relations and application to COVID-19 surveillance," *Expert Systems with Applications*, vol. 191, Article ID 116328, 2022.

[30] H. Liao, X. Mi, and Z. Xu, "A survey of decision-making methods with probabilistic linguistic information: bibliometrics, preliminaries, methodologies, applications and future directions," *Fuzzy Optim. Decis. Mak.*, 2019.

[31] X. Mi, H. Liao, X. Wu, and Z. Xu, "Probabilistic linguistic information fusion: a survey on aggregation operators in terms of principles, definitions, classifications, applications, and challenges," *International Journal of Intelligent Systems*, vol. 35, no. 3, pp. 529–556, 2020.

[32] D. E. Bell, "Regret in decision making under uncertainty," *Operations Research*, vol. 30, no. 5, pp. 961–981, 1982.

[33] G. Loomes and R. Sugden, "Regret theory: an alternative theory of rational choice under uncertainty," *The Economic Journal*, vol. 92, no. 368, pp. 805–824, 1982.

[34] D. S. Pamucar and S. R. Dimitrijević, "Multiple-criteria model for optimal anti-tank ground missile weapon system procurement," *Military Technical Courier*, vol. 69, no. 4, pp. 792–827, 2021.

[35] I. Mukhametzyanov, "Specific character of objective methods for determining weights of criteria in MCDM problems: entropy, CRITIC and SD," *Decision Making: Applications in Management and Engineering*, vol. 4, no. 2, pp. 76–105, 2021.

[36] D. K. Kushwaha, D. Panchal, D. Panchal, and A. Sachdeva, "Risk analysis of cutting system under intuitionistic fuzzy environment," *Reports in Mechanical Engineering*, vol. 1, no. 1, pp. 162–173, 2020.

[37] V. Romanuke, "Refinement of acyclic-and-asymmetric payoff aggregates of pure strategy efficient Nash equilibria in finite noncooperative games by maximultimin and superoptimality," *Decision Making: Applications in Management and Engineering*, vol. 4, no. 2, pp. 178–199, 2021.

[38] W. Chen, M. Goh, and Y. Zou, "Logistics provider selection for omni-channel environment with fuzzy axiomatic design and extended regret theory," *Applied Soft Computing*, vol. 71, pp. 353–363, 2018.

[39] Y. Wang, J.-Q. Wang, and T.-L. Wang, "Fuzzy stochastic multi-criteria decision-making methods with interval neutrosophic probability based on regret theory," *Journal of Intelligent and Fuzzy Systems*, vol. 35, no. 2, pp. 2309–2322, 2018.

[40] G. Qu, T. Li, X. Zhao, W. Qu, Q. An, and J. Yan, "Dual hesitant fuzzy stochastic multiple attribute decision making method based on regret theory and group satisfaction degree," *Journal of Intelligent and Fuzzy Systems*, vol. 35, no. 6, pp. 6479–6488, 2018.

[41] M. Xia, "A hesitant fuzzy linguistic multi-criteria decision-making approach based on regret theory," *International Journal of Fuzzy Systems*, vol. 20, no. 7, pp. 2135–2143, 2018.

[42] X. Gong, C. Yu, and Z. Wu, "An extension of regret theory based on probabilistic linguistic cloud sets considering dual expectations: an application for the stock market," *IEEE Access*, vol. 7, pp. 171046–171060, 2019.

[43] T. Wang, H. Li, Y. Qian, B. Huang, and X. Zhou, "A regret-based three-way decision model under interval type-2 fuzzy environment," *IEEE Transactions on Fuzzy Systems*, vol. 30, no. 1, pp. 175–189, 2022.

[44] X. Liu, Z. Wang, S. Zhang, and J. Liu, "Probabilistic hesitant fuzzy multiple attribute decision-making based on regret theory for the evaluation of venture capital projects," *Economic Research-Ekonomska Istraživanja*, vol. 33, no. 1, pp. 672–697, 2020.

[45] H. Ren, Y. Gao, and T. Yang, "A novel regret theory-based decision-making method combined with the intuitionistic fuzzy Canberra distance," *Discrete Dynamics in Nature and Society*, vol. 2020, pp. 1–9, Article ID 8848031, 2020.

[46] X. Gong, C. Yu, L. Min, and Z. Ge, "Regret theory-based fuzzy multi-objective portfolio selection model involving DEA cross-efficiency and higher moments," *Applied Soft Computing*, vol. 100, Article ID 106958, 2021.

[47] W. Liang and Y.-M. Wang, "A probabilistic interval-valued hesitant fuzzy gained and lost dominance score method based on regret theory," *Computers & Industrial Engineering*, vol. 159, Article ID 107532, 2021.

[48] S. Chakraborty and E. K. Zavadskas, "Applications of WASPAS method in manufacturing decision making," *Informatica*, vol. 25, no. 1, pp. 1–20, 2014.

[49] A. Mardani, M. Nilashi, N. Zakuan et al., "A systematic review and meta-Analysis of SWARA and WASPAS methods: theory and applications with recent fuzzy developments," *Applied Soft Computing*, vol. 57, pp. 265–292, 2017.

[50] A. R. Mishra, P. Rani, K. R. Pardasani, and A. Mardani, "A novel hesitant fuzzy WASPAS method for assessment of green supplier problem based on exponential information measures," *Journal of Cleaner Production*, vol. 238, Article ID 117901, 2019.

[51] A. Tuş and E. Aytaç Adalı, "The new combination with CRITIC and WASPAS methods for the time and attendance software selection problem," *Opsearch*, vol. 56, no. 2, pp. 528–538, 2019.

[52] D. Pamučar, S. Sremac, Ž. Stević, G. Ćirović, and D. Tomić, "New multi-criteria LNN WASPAS model for evaluating the work of advisors in the transport of hazardous goods," *Neural Computing & Applications*, vol. 31, no. 9, pp. 5045–5068, 2019.

[53] R. Krishankumar, R. Saranya, R. P. Nethra, K. S. Ravichandran, and S. Kar, "A decision-making framework under probabilistic linguistic term set for multi-criteria group decision-making problem," *Journal of Intelligent and Fuzzy Systems*, vol. 36, no. 6, pp. 5783–5795, 2019.

[54] F. Bouchraki, A. Berreksi, and S. Hamchaoui, "Evaluating the policy of listening to customer claims in a drinking water utility using fuzzy-AHP approach and WASPAS method," *Water Policy*, vol. 23, no. 1, pp. 167–186, 2021.

[55] R. Krishankumar, L. S. Subrajaa, K. S. Ravichandran, S. Kar, and A. B. Saeid, "A framework for multi-attribute group decision-making using double hierarchy hesitant fuzzy linguistic term set," *International Journal of Fuzzy Systems*, vol. 21, pp. 1–14, 2019.

[56] D. Pamucar, M. Deveci, F. Canıtez, and V. Lukovac, "Selecting an airport ground access mode using novel fuzzy LBWA-WASPAS-H decision making model," *Engineering Applications of Artificial Intelligence*, vol. 93, Article ID 103703, 2020.

[57] V. Simi and D. Lazarevi, "Picture fuzzy WASPAS method for selecting last-mile delivery mode : a case study of Belgrade," vol. 6, 2021.

[58] J. Ali, Z. Bashir, and T. Rashid, "WASPAS-based decision making methodology with unknown weight information under uncertain evaluations," *Expert Systems with Applications*, vol. 168, Article ID 114143, 2021.

[59] N. Osintsev, A. Rakhmangulov, and V. Baginova, "Evaluation of logistic flows in green supply chains based on the combined DEMATEL-ANP method," *Facta Universitatis – Series: Mechanical Engineering*, vol. 19, no. 3, pp. 473–498, 2021.

[60] D. Bozanic, A. Milic, D. Tešic, W. Salabun, and D. Pamucar, "D numbers – FUCOM – fuzzy RAFSI model for selecting the group of construction machines for enabling mobility," *Facta Universitatis – Series: Mechanical Engineering*, vol. 19, no. 3, pp. 447–471, 2021.

[61] S.-H. Lin, X. Zhao, J. Wu et al., "An evaluation framework for developing green infrastructure by using a new hybrid multiple attribute decision-making model for promoting environmental sustainability," *Socio-Economic Planning Sciences*, vol. 75, Article ID 100909, 2021.

[62] S. Seker and C. Kahraman, "Socio-economic evaluation model for sustainable solar PV panels using a novel integrated MCDM methodology: a case in Turkey," *Socio-Economic Planning Sciences*, vol. 77, Article ID 100998, 2021.

[63] G. Büyüközkan, E. Mukul, and E. Kongar, "Health tourism strategy selection via SWOT analysis and integrated hesitant fuzzy linguistic AHP-MABAC approach," *Socio-Economic Planning Sciences*, vol. 74, Article ID 100929, 2021.

[64] D. M. d. Oliveira Dias, D. R. d. O. Albergarias Lopes, and A. C. Teles, "Will virtual replace classroom teaching? Lessons from virtual classes via zoom in the times of COVID-19," *Journal of Advances in Education and Philosophy*, vol. 4, no. 5, pp. 208–213, 2020.

[65] A. Rahman, "Using students' experience to derive effectiveness of COVID-19-lockdown-induced emergency online learning at undergraduate level: evidence from Assam, India," *Higher Education for the Future*, vol. 8, no. 1, pp. 71–89, 2021.

[66] M. Aggarwal, "Attitudinal Choquet integrals and applications in decision making," *International Journal of Intelligent Systems*, vol. 33, no. 4, pp. 879–898, 2018.

[67] C. Kao, "Weight determination for consistently ranking alternatives in multiple criteria decision analysis," *Applied Mathematical Modelling*, vol. 34, no. 7, pp. 1779–1787, 2010.

[68] S. Zeng, M. Shoaib, S. Ali, F. Smarandache, H. Rashmanlou, and F. Mofidnakhaei, "Certain properties of single-valued neutrosophic graph with application in food and agriculture organization," *International Journal of Computational Intelligence Systems*, vol. 14, no. 1, pp. 1516–1540, 2021.

[69] S. Zeng, N. Zhang, C. Zhang, W. Su, and L.-A. Carlos, "Social network multiple-criteria decision-making approach for evaluating unmanned ground delivery vehicles under the Pythagorean fuzzy environment," *Technological Forecasting and Social Change*, vol. 175, Article ID 121414, 2022.

[70] S. Zeng, S. Ali, M. K. Mahmood, F. Smarandache, and D. Ahmad, "Decision-making problems under the environment of m-polar diophantine neutrosophic N-soft set," *C. - Comput. Model. Eng. Sci.*, vol. 129, no. 1, 2021.

[71] X. Gou, P. Xiao, D. Huang, and F. Deng, "Probabilistic double hierarchy linguistic alternative queuing method for real economy development evaluation under the perspective of economic financialization," *Economic Research-Ekonomska Istraživanja*, pp. 1–60, 2021.

Hindawi
Mathematical Problems in Engineering
Volume 2023, Article ID 9843106, 1 page
https://doi.org/10.1155/2023/9843106

Retraction

Retracted: Research on the English Classroom Teaching Effect Evaluation with Interval-Valued Intuitionistic Fuzzy Grey Relational Analysis Method

Mathematical Problems in Engineering

Received 11 July 2023; Accepted 11 July 2023; Published 12 July 2023

This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope
(2) Discrepancies in the description of the research reported
(3) Discrepancies between the availability of data and the research described
(4) Inappropriate citations
(5) Incoherent, meaningless and/or irrelevant content included in the article
(6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] Z. Zhang and P. Su, "Research on the English Classroom Teaching Effect Evaluation with Interval-Valued Intuitionistic Fuzzy Grey Relational Analysis Method," *Mathematical Problems in Engineering*, vol. 2022, Article ID 7445250, 11 pages, 2022.

Hindawi
Mathematical Problems in Engineering
Volume 2022, Article ID 7445250, 11 pages
https://doi.org/10.1155/2022/7445250

Research Article

Research on the English Classroom Teaching Effect Evaluation with Interval-Valued Intuitionistic Fuzzy Grey Relational Analysis Method

Zhen Zhang [1] and Pengfei Su [2]

[1]School of Foreign Language Studies, Xi'an Aeronautical Institute, Xi' an 710077, Shaanxi, China
[2]Modern Chemistry Research Institute, Xi'an 710065, Shaanxi, China

Correspondence should be addressed to Zhen Zhang; janezhen@xaau.edu.cn

Received 18 January 2022; Revised 22 February 2022; Accepted 22 March 2022; Published 15 April 2022

Academic Editor: Darko Božanić

The English classroom teaching effect evaluation is looked as the multiattribute group decision-making (MAGDM). Thus, a useful MAGDM algorithm is needed to cope with it. Depending on the classical GRA process and interval-valued IFSs (IVIFSs), this study builds the IVIF-GRA process to assess the English classroom teaching effect. First of all, the concepts of IVIFSs are reviewed. In addition, the weights of criteria are derived through the CRITIC method. Afterwards, the GRA model is extended to IVIFSs to get the final result of the alternative. Therefore, all alternatives could be ranked and the optimal one with English classroom teaching effect can be identified. At last, a given numerical example and some given comparative studies are obtained. The analysis results show that the defined algorithms are effective for solving the English classroom teaching effect evaluation.

1. Introduction

Since the decision-making process is full of uncertainty and fuzziness [1–6], Zadeh [7] designed the fuzzy sets (FSs) to deal with the accuracy issues of decision-making. Atanassov [8] built the intuitionistic fuzzy sets (IFSs) to depict uncertain issues. Milovanović et al. [9] built uncertainty modeling using IFSs. Gupta et al. [10] designed the fuzzy mathematical entropy under IFSs. Xiao et al. [11] built the intuitionistic fuzzy Taxonomy decision method. Bao et al. [12] built the prospect theory as well as novel evidential reasoning with IFSs. Phochanikorn and Tan [13] merged DEMATEL through ANP to obtain interdependencies as well as uncertainties under IFSs. Rouyendegh [14]used the ELECTRE process with IFSs. Hao et al. [15] devised the novel decision fields' method for IF-MADM. Li et al. [16] defined the grey target based on real decision-making under IFSs. Zhao et al. [17] built the IF-MABAC model based on cumulative prospect theory for MAGDM. Liang et al. [18] built the IF-MABAC method with distance measures. Khan and Lohani [19] devised the similarity mathematical measure about IFSs. Chen et al. [20] developed IF-TOPSIS under similarity measures. Zhang and He [21] built the geometric interaction information-fused methods under IFSs. Joshi and Kumar [22] defined an extended VIKOR method under IFSs. Kumar and Garg [23] defined the TOPSIS method under IVIFSs. Xiao et al. [24] used the taxonomy method for MAGDM based on IVIFSs with given entropy.

Similar to the TOPSIS method [25–27], EDAS method [28–31], and TODIM method [32–37], the GRA model was initially defined by Deng [38] to cope with real MAGDM. Compared with other MAGDM [39–44], the GRA model could consider the given shape similarity of every given alternative from given PIS as well as NIS. Javanmardi et al. [45] explored the philosophical foundations of grey theory. Javanmardi et al. [46] explored grey systems theory-based methods and applications in sustainability studies. Javanmardi et al. [47] explored the philosophical paradigm of grey theory. Javanmardi and Liu [48] explored the grey theory perspective. Javanmardi [49] explored grey theory-based methods and applications in real socio-economic systems. Zhang et al. [50] used GRA algorithm based on cumulative prospect theory for IF-MAGDM. With the purpose of discerning the carbon market, Zhu et al. [51] took advantage of GRA algorithm as well as EMD. Alptekin et al. [52] solved the low carbon issues based on GRA process. Malek et al. [53] built a revised hybrid GRA algorithm for green supply. Kung and Wen [54] used GRA algorithm to solve the given grey MADM. Xie et al. [55] extended the grey relational analysis with the comparable degree for dual probabilistic multiplicative linguistic term set.

English writing is very important to college school English learning, but the current teaching situation of English writing is not satisfactory. Students lack interest in English writing, and teachers have made a lot of effort but have achieved little effect [56]. Many studies attempt to adopt the teaching mode of flipped classroom from the perspective of ecological teaching to increase students' interest in English writing and improve their English writing ability effectively [57]. The mode of flipped classroom subverts the teaching mode "teaching before practicing" in traditional class and replaces teachers' teaching during the whole class by students' learning independently before class. In class, the interaction between teacher and students can attribute to completing the internalization of knowledge and enabling students to become true masters in the class [58]. The theoretical foundations of the flipped classroom are ecological teaching. Ecological teaching is the application of the concept of ecology to education. It regard teachers, students, teaching environment, and educational resources as ecological factors, aiming to create a dynamic, interactive, and balanced teaching environment [59]. Xu [60] evaluated the teaching effect with intuitionistic trapezoidal fuzzy numbers. Geng [61] assessed the multimedia teaching effect based on deep neural networks. Liu and Qi [62] assessed the flipped classroom in oral English teaching. Wang [63] built the TOPSIS for teaching effect evaluation of college English with IVIFSs.

Unfortunately, in the given existing literature, we fail to find out the corresponding works of the IVIF-GRA method with CRITIC. Thus, it is imperative to extend the GRA method in IVIFSs. The elementary aim of such study is to solve the MAGDM efficiently with GRA and IVIFSs. Most especially, extend GRA algorithm to the IVIFSs. On the other side, the CRITIC method is used to get the attribute's weight. Then, a given application is used to certify such defined model, and several given comparative studies are used to certify the advantages of the built model. The motivations of the paper can be given as follows: (1) the GRA algorithm is used to deal with MAGDM under IVIFSs, (2) the weights of attribute is obtained objectively through the given CRITIC method, (3) a numerical example for English classroom teaching effect evaluation is given to show the built approach, and (4) some given comparative studies are given.

The reminder of this study is as follows. Some necessary ideas of IVIFSs are given in Section 2. The GRA process for real MAGDM is revised with IVIFSs; then, the calculating procedures are designed in Section 3. A numerical example for English classroom teaching effect evaluation is given; also, some comparative analysis is given in Section 4. In the end, we give the conclusion in Section 5.

2. Preliminaries

2.1. IVIFSs

Definition 1 (see [64]). The IVIFSs based on X is defined:

$$P = \{\langle x, \tilde{\mu}_P(x), \tilde{\nu}_P(x)\rangle | x \in X\}, \tag{1}$$

which $\tilde{\mu}_P(x) \subset [0,1]$ is used to show as "membership of P," $\tilde{\nu}_P(x) \subset [0,1]$ is used to show as "nonmembership of P," and $\tilde{\mu}_P(x)$ and $\tilde{\nu}_P(x)$ meet condition, $0 \le \sup\tilde{\mu}_P(x) + \sup\tilde{\nu}_P(x) \le 1$, $\forall x \in X$.

Definition 2 (see [65]). Let $P_1 = ([\mu_1^L, \mu_1^R], [\nu_1^L, \nu_1^R])$ and $P_2 = ([\mu_2^L, \mu_2^R], [\nu_2^L, \nu_2^R])$ be two IVIFNs; then, the operation rules are defined:

$$\begin{aligned}
P_1 \oplus P_2 &= \left(\left[\mu_1^L + \mu_2^L - \mu_1^L\mu_2^L, \mu_1^R + \mu_2^R - \mu_1^R\mu_2^R\right], \left[\nu_1^L\nu_2^L, \nu_1^R\nu_2^R\right]\right),\\
P_1 \otimes P_2 &= \left(\left[\mu_1^L\mu_2^L, \mu_1^R\mu_2^R\right], \left[\nu_1^L + \nu_2^L - \nu_1^L\nu_2^L, \nu_1^R + \nu_2^R - \nu_1^R\nu_2^R\right]\right),\\
\lambda P_1 &= \left(\left[1-\left(1-\mu_1^L\right)^{\lambda}, 1-\left(1-\mu_1^R\right)^{\lambda}\right], \left[\left(\nu_1^L\right)^{\lambda}, \left(\nu_1^R\right)^{\lambda}\right]\right), \quad \lambda > 0,\\
P_1^{\lambda} &= \left(\left[\left(\mu_1^L\right)^{\lambda}, \left(\mu_1^R\right)^{\lambda}\right], \left[1-\left(1-\lambda_1^L\right)^{\lambda}, 1-\left(1-\lambda_1^R\right)^{\lambda}\right]\right), \quad \lambda > 0.
\end{aligned} \tag{2}$$

Definition 3 (see [66]). Let $P_1 = ([\mu_1^L, \mu_1^R], [\nu_1^L, \nu_1^R])$ and $P_2 = ([\mu_2^L, \mu_2^R], [\nu_2^L, \nu_2^R])$ be two IVIFNs; the score and accuracy functions are defined:

$$\begin{aligned}
S(P_1) &= \frac{\left(1 + \mu_1^L - \nu_1^L\right) + \left(1 + \mu_1^R - \nu_1^R\right)}{4}\\
S(P_2) &= \frac{\left(1 + \mu_2^L - \nu_2^L\right) + \left(1 + \mu_2^R - \nu_2^R\right)}{4},\\
H(P_1) &= \frac{\mu_1^L + \nu_1^L + \mu_1^R + \nu_1^R}{4},\\
H(P_2) &= \frac{\mu_2^L + \nu_2^L + \mu_2^R + \nu_2^R}{4}.
\end{aligned} \tag{3}$$

Definition 4 (see [67]). Let $P_1 = ([\mu_1^L, \mu_1^R], [\nu_1^L, \nu_1^R])$ and $P_2 = ([\mu_2^L, \mu_2^R], [\nu_2^L, \nu_2^R])$ be two IVIFNs; the Hamming distance between two IVIFNs is

$$\mathrm{HD}(P_1, P_2) = \frac{\left|\mu_1^L - \mu_2^L\right| + \left|\mu_1^R - \mu_2^R\right| + \left|\nu_1^L - \nu_2^L\right| + \left|\nu_1^R - \nu_2^R\right|}{4}. \tag{4}$$

2.2. Two Operators under IVIFSs. The IVIFWA and IVIFWG operator is given [68].

Definition 5 (see [68]). Let $P_j = ([\mu^L_{P_j}, \mu^R_{P_j}], [\nu^L_{P_j}, \nu^R_{P_j}])$ $(j = 1, 2, \ldots, n)$ be a group of IVIFNs; the IVIFWA is

$$\text{IVIFWA}_\omega(P_1, P_2, \ldots, P_n) = \oplus_{j=1}^{n}(\omega_j P_j) = \left(\left[1 - \prod_{j=1}^{n}\left(1 - \mu^L_{P_j}\right)^{\omega_j}, 1 - \prod_{j=1}^{n}\left(1 - \mu^R_{P_j}\right)^{\omega_j}\right], \left[\prod_{j=1}^{n}\left(\nu^L_{P_j}\right)^{\omega_j}, \prod_{j=1}^{n}\left(\nu^R_{P_j}\right)^{\omega_j}\right]\right), \tag{5}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is weight information of P_j $(j = 1, 2, \ldots, n)$ and $\omega_j > 0, \sum_{j=1}^{n} \omega_j = 1$.

Definition 6 (see [68]). Let $P_j = ([\mu^L_{P_j}, \mu^R_{P_j}], [\nu^L_{P_j}, \nu^R_{P_j}])$ $(j = 1, 2, \ldots, n)$ be a group of IVIFNs; the IVIFWG is

$$\text{IVIFWG}_\omega(P_1, P_2, \ldots, P_n) = \otimes_{j=1}^{n}(P_j)^{\omega_j} = \left(\left[\prod_{j=1}^{n}\left(\mu^L_{P_j}\right)^{\omega_j}, \prod_{j=1}^{n}\left(\mu^R_{P_j}\right)^{\omega_j}\right], \left[1 - \prod_{j=1}^{n}\left(1 - \nu^L_{P_j}\right)^{\omega_j}, 1 - \prod_{j=1}^{n}\left(1 - \nu^R_{P_j}\right)^{\omega_j}\right]\right), \tag{6}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight of $P_j (j = 1, 2, \ldots, n)$ and $\omega_j > 0, \sum_{j=1}^{n} \omega_j = 1$.

3. GRA Algorithm for MAGDM with IVIFNs

This section proposes the IVIF-GRA process for MAGDM. Let $A = \{A_1, A_2, \ldots, A_n\}$ be the set of attributes and $w = \{w_1, w_2, \ldots, w_n\}$ be the attribute weight A_j, where $w_j \in [0, 1], j = 1, 2, \ldots, n, \sum_{j=1}^{n} w_j = 1$. Assume $H = \{H_1, H_2, \ldots, H_l\}$ be a group of DMs that have weight information of $h = \{h_1, h_2, \ldots, h_l\}$, where $h_k \in [0, 1]$, $k = 1, 2, \ldots, l$ and $\sum_{k=1}^{l} h_k = 1$. Let $P = \{P_1, P_2, \ldots, P_m\}$ be a group of alternatives. And $Q = (\overset{ij}{q})_{m\times n}$ is the decision matrix, where $\overset{ij}{q}$ depicts the given value of P_i for A_j. Afterwards, the calculating steps are listed.

Step 1: build the matrix $Q^{(k)} = (q^k_{ij})_{m\times n}$; then, the overall matrix is depicted as $Q = (q_{ij})_{m\times n}$:

$$Q^{(k)} = \left[q^k_{ij}\right]_{m\times n} = \begin{bmatrix} q^k_{11} & q^k_{12} & \cdots & q^k_{1n} \\ q^k_{21} & q^k_{22} & \cdots & q^k_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ q^k_{m1} & q^k_{m2} & \cdots & q^k_{mn} \end{bmatrix},$$
$$Q = \left[q_{ij}\right]_{m\times n} = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1n} \\ q_{21} & q_{22} & \cdots & q_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ q_{m1} & q_{m2} & \cdots & q_{mn} \end{bmatrix}, \tag{7}$$
$$q_{ij} = \left(\left[\prod_{k=1}^{l}\left(\mu^L_{q^k_{ij}}\right)^{h_k}, \prod_{k=1}^{l}\left(\mu^R_{q^k_{ij}}\right)^{h_k}\right], \left[1 - \prod_{k=1}^{l}\left(1 - \nu^L_{q^k_{ij}}\right)^{h_k}, 1 - \prod_{k=1}^{l}\left(1 - \nu^R_{q^k_{ij}}\right)^{h_k}\right]\right),$$

where q^k_{ij} is the IVIFNs of $P_i (i = 1, 2, \ldots, m)$ for $A_j (j = 1, 2, \ldots, n)$ and $H_k (k = 1, 2, \ldots, l)$.

Step 2: normalize the matrix $Q = (q_{ij})_{m\times n}$ to $Q^N = [q^N_{ij}]_{m\times n}$ with IVIFNs:

$$q_{ij}^{N} = \begin{cases} \left(\left[\mu_{ij}^{L}, \mu_{ij}^{R}\right], \left[\nu_{ij}^{L}, \nu_{ij}^{R}\right]\right), & A_j \text{ is a benefit criterion,} \\ \left(\left[\nu_{ij}^{L}, \nu_{ij}^{R}\right], \left[\mu_{ij}^{L}, \mu_{ij}^{R}\right]\right), & A_j \text{ is a cost criterion.} \end{cases} \tag{8}$$

Step 3: use the CRITIC to get the attributes' weight.

The CRITIC was defined by Diakoulaki et al. [69]. The CRITIC has been used in different setting and connected with methods [70–74]. Whereafter, the compute procedures of such method are designed.

(1) Depending on the normalized matrix, the given correlation coefficient between attributes could be obtained:

$$CC_{jr} = \frac{\sum_{i=1}^{m}\left(H\left(q_{ij}^{N}\right) - H\left(q_{j}^{N}\right)\right)\left(H\left(q_{ir}^{N}\right) - H\left(q_{r}^{N}\right)\right)}{\sqrt{\sum_{i=1}^{m}\left(H\left(q_{ij}^{N}\right) - H\left(q_{j}^{N}\right)\right)^{2}}\sqrt{\sum_{i=1}^{m}\left(H\left(q_{ir}^{N}\right) - H\left(q_{r}^{N}\right)\right)^{2}}}, \quad j, r = 1, 2, \ldots, n, \tag{9}$$

where $H(q_j^N) = (1/m)\sum_{i=1}^{m} H(q_{ij}^N)$ and $H(q_r^N) = (1/m)\sum_{i=1}^{m} H(q_{ir}^N)$.

(2) Calculate attributes' standard deviation:

$$\mathrm{SD}_j = \sqrt{\frac{1}{m-1}\sum_{i=1}^{m}\left(H\left(q_{ij}^{N}\right) - H\left(q_{j}^{N}\right)\right)^{2}}, \quad j = 1, 2, \ldots, n, \tag{10}$$

where $H(q_j^N) = (1/m)\sum_{i=1}^{m} H(q_{ij}^N)$.

(3) Calculate the attributes' weights:

$$ta_j = \frac{\mathrm{SD}_j \sum_{t=1}^{n}\left(1 - CC_{jr}\right)}{\sum_{j=1}^{n}\left(\mathrm{SD}_j \sum_{t=1}^{n}\left(1 - CC_{jr}\right)\right)}, \quad j = 1, 2, \ldots, n, \tag{11}$$

where $ta_j \in [0, 1]$ and $\sum_{j=1}^{n} ta_j = 1$.

Step 4: build positive ideal solution (PIS) $\mathrm{IVIFPIS}_j$ and the corresponding negative ideal solution (NIS) $\mathrm{IVIFNIS}_j$ through equations (12) and (13):

$$\mathrm{IVIFPIS}_j = \left(\left[\mu_j^{L+}, \mu_j^{R+}\right], \left[\nu_j^{L+}, \nu_j^{R+}\right]\right), \tag{12}$$

$$\mathrm{IVIFNIS}_j = \left(\left[\mu_j^{L+}, \mu_j^{R+}\right], \left[\nu_j^{L+}, \nu_j^{R+}\right]\right), \tag{13}$$

where $\mathrm{IVIFPIS}_j = \begin{pmatrix} [\max_j(\mu_{ij}^L), \max_j(\mu_{ij}^R)], \\ [\min_j(\nu_{ij}^L), \min_j(\nu_{ij}^R)] \end{pmatrix}$ and $\mathrm{IVIFNIS}_j = \begin{pmatrix} [\min_j(\mu_{ij}^L), \min_j(\mu_{ij}^R)], \\ [\max_j(\nu_{ij}^L), \max_j(\nu_{ij}^R)] \end{pmatrix}$.

Step 5: the grey relational coefficient (GRC) of each alternative between each alternative and IVIFIVPIS and IVIFPIS is given

$$\begin{aligned} \mathrm{IVIFPIS}\left(\xi_{ij}\right) &= \frac{\min_{1\le i\le m}\mathrm{HD}\left(q_{ij}^{N}, \mathrm{IVIFPIS}_j\right) + \rho\max_{1\le i\le m}\mathrm{HD}\left(q_{ij}^{N}, \mathrm{IVIFPIS}_j\right)}{\mathrm{HD}\left(q_{ij}^{N}, \mathrm{IVIFPIS}_j\right) + \rho\max_{1\le i\le m}\mathrm{HD}\left(q_{ij}^{N}, \mathrm{IVIFPIS}_j\right)}, \\ \mathrm{IVIFNIS}\left(\xi_{ij}\right) &= \frac{\min_{1\le i\le m}\mathrm{HD}\left(q_{ij}^{N}, \mathrm{IVIFNIS}_j\right) + \rho\max_{1\le i\le m}\mathrm{HD}\left(q_{ij}^{N}, \mathrm{IVIFNIS}_j\right)}{\mathrm{HD}\left(q_{ij}^{N}, \mathrm{IVIFNIS}_j\right) + \rho\max_{1\le i\le m}\mathrm{HD}\left(q_{ij}^{N}, \mathrm{IVIFNIS}_j\right)}, \\ & i = 1, 2, \ldots, m, j = 1, 2, \ldots, n. \end{aligned} \tag{14}$$

Step 6: figuring out the degree of GRC of all alternatives from IVIFPIS and IVIFNIS,

$$\begin{aligned} \mathrm{IVIFPIS}\left(\xi_i\right) &= \sum_{j=1}^{n} w_j \mathrm{IVIFPIS}\left(\xi_{ij}\right), \quad i = 1, 2, \ldots, m, \\ \mathrm{IVIFNIS}\left(\xi_i\right) &= \sum_{j=1}^{n} w_j \mathrm{IVIFNIS}\left(\xi_{ij}\right), \quad i = 1, 2, \ldots, m. \end{aligned} \tag{15}$$

Step 7: compute each alternative's IVIF relative relational degree (IVIFRRD) from IVIFPIS:

$$\mathrm{IVIFRRD}_i = \frac{\mathrm{IVIFNIS}\left(\xi_i\right)}{\mathrm{IVIFNIS}\left(\xi_i\right) + \mathrm{IVIFPIS}\left(\xi_i\right)}, \quad i = 1, 2, \ldots, m. \tag{16}$$

Step 8: according to $\mathrm{IVIFRRD}_i\,(i = 1, 2, \ldots, m)$, the highest value of $\mathrm{IVIFRRD}_i\,(i = 1, 2, \ldots, m)$, the optimal choice is.

Table 1: IVIF matrix by H_1.

	A_1	A_2	A_3	A_4
P_1	([0.59,0.62],[0.26,0.38])	([0.63,0.70],[0.25,0.30])	([0.37,0.45],[0.55,0.60])	([0.62,0.70],[0.25,0.30])
P_2	([0.65,0.75],[0.20,0.25])	([0.35,0.40],[0.55,0.60])	([0.55,0.62],[0.28,0.38])	([0.36,0.40],[0.55,0.60])
P_3	([0.37,0.40],[0.53,0.60])	([0.42,0.48],[0.50,0.52])	([0.55,0.60],[0.32,0.40])	([0.37,0.45],[0.50,0.55])
P_4	([0.61,0.65],[0.30,0.35])	([0.38,0.42],[0.52,0.58])	([0.52,0.62],[0.30,0.38])	([0.70,0.80],[0.10,0.20])
P5	([0.35,0.45],[0.50,0.55])	([0.61,0.65],[0.30,0.35])	([0.59,0.65],[0.30,0.35])	([0.19,0.25],[0.70,0.75])

Table 2: IVIF matrix by H_2.

	A_1	A_2	A_3	A_4
P_1	([0.41,0.45],[0.50,0.55])	([0.36,0.41],[0.56,0.59])	([0.73,0.81],[0.14,0.22])	([0.60,0.70],[0.25,0.30])
P_2	([0.36,0.40],[0.57,0.60])	([0.70,0.80],[0.15,0.20])	([0.57,0.62],[0.30,0.38])	([0.29,0.36],[0.58,0.64])
P_3	([0.29,0.35],[0.60,0.65])	([0.55,0.62],[0.27,0.38])	([0.35,0.40],[0.51,0.60])	([0.32,0.40],[0.55,0.60])
P_4	([0.53,0.60],[0.35,0.40])	([0.28,0.46],[0.50,0.54])	([0.62,0.70],[0.25,0.30])	([0.60,0.65],[0.30,0.35])
P_5	([0.46,0.52],[0.40,0.48])	([0.52,0.60],[0.35,0.40])	([0.25,0.31],[0.66,0.71])	([0.47,0.55],[0.40,0.45])

Table 3: IVIF matrix by H_3.

	A_1	A_2	A_3	A_4
P_1	([0.65,0.72],[0.21,0.28])	([0.63,0.75],[0.15,0.25])	([0.50,0.50],[0.50,0.50])	([0.55,0.60],[0.30,0.40])
P_2	([0.65,0.70],[0.25,0.30])	([0.55,0.60],[0.30,0.40])	([0.43,0.46],[0.51,0.54])	([0.24,0.35],[0.58,0.65])
P_3	([0.56,0.62],[0.30,0.38])	([0.36,0.42],[0.52,0.59])	([0.71,0.78],[0.17,0.22])	([0.31,0.42],[0.50,0.58])
P_4	([0.52,0.60],[0.35,0.41])	([0.59,0.65],[0.30,0.35])	([0.23,0.34],[0.58,0.66])	([0.19,0.30],[0.65,0.70])
P_5	([0.72,0.80],[0.15,0.20])	([0.38,0.45],[0.50,0.55])	([0.60,0.66],[0.30,0.34])	([0.73,0.80],[0.10,0.20])

Table 4: IVIF matrix by H_4.

	A_1	A_2	A_3	A_4
P_1	([0.17,0.22],[0.65,0.78])	([0.34,0.42],[0.50,0.58])	([0.59,0.66],[0.30,0.35])	([0.66,0.75],[0.20,0.25])
P_2	([0.32,0.40],[0.55,0.60])	([0.18,0.25],[0.70,0.75])	([0.57,0.62],[0.32,0.38])	([0.60,0.65],[0.30,0.35])
P_3	([0.43,0.47],[0.50,0.53])	([0.32,0.40],[0.55,0.60])	([0.68,0.75],[0.20,0.25])	([0.35,0.40],[0.55,0.60])
P_4	([0.32,0.39],[0.41,0.61])	([0.28,0.36],[0.57,0.64])	([0.41,0.52],[0.40,0.48])	([0.58,0.63],[0.30,0.37])
P_5	([0.25,0.30],[0.55,0.70])	([0.44,0.48],[0.50,0.52])	([0.74,0.80],[0.15,0.21])	([0.52,0.62],[0.32,0.38])

Table 5: IVIF matrix by H_5.

	A_1	A_2	A_3	A_4
P_1	([0.28,0.32],[0.60,0.68])	([0.34,0.41],[0.53,0.59])	([0.41,0.55],[0.37,0.45])	([0.69,0.75],[0.18,0.25])
P_2	([0.49,0.55],[0.40,0.45])	([0.27,0.34],[0.60,0.66])	([0.62,0.72],[0.20,0.28])	([0.39,0.45],[0.48,0.55])
P_3	([0.33,0.43],[0.51,0.57])	([0.59,0.65],[0.27,0.35])	([0.19,0.25],[0.19,0.25])	([0.26,0.32],[0.60,0.68])
P_4	([0.41,0.45],[0.50,0.55])	([0.40,0.45],[0.50,0.55])	([0.33,0.42],[0.50,0.58])	([0.34,0.41],[0.52,0.59])
P_5	([0.35,0.40],[0.50,0.60])	([0.46,0.55],[0.30,0.45])	([0.28,0.35],[0.58,0.65])	([0.57,0.67],[0.26,0.33])

4. Numerical Example and Some Comparative Analysis

4.1. Numerical Example. Some existing studies are based on the development of English teachers, on the basis of a new round of reform of the basic education curriculum evaluation requirements to measure compliance with foreign language teaching and learning to achieve the intended objectives, the analysis is presented in the experiment and some of the basic theory and practice. The core courses in the curriculum reform were implemented, and the fundamental way is to implement curriculum classroom. How to play the main channel function of the quality of education, curriculum reform which embodied the idea that each school and teachers are thinking about an important issue, reform of classroom assessment to classroom teaching scientific and effective evaluation, and the establishment of an effective evaluation mechanism should be the core of the curriculum reform issues or one important aspect because evaluation of reform often plays a guiding role, promoting or constraining role. According to current and future period for the reform and development of teaching, classroom assessment should be “developmental classroom evaluation.” Class A is conducive to overcoming the limitations and deficiencies of current evaluation. It could reflect the latest trend of current teacher evaluation, advanced ideas and evaluation function, teacher reflection analysis, teacher future

Table 6: Overall IVIF evaluation matrix.

	A_1	A_2	A_3	A_4
P_1	([0.4473,0.5186],[0.3847,0.4725])	([0.5266,0.4874],[0.2908,0.4054])	([0.4872,0.4706],[0.4123,0.4152])	([0.5354,0.4146],[0.4533,0.3644])
P_2	([0.3032,0.3588],[0.4872,0.6413])	([0.5627,0.3402],[0.3011,0.3549])	([0.5874,0.6826],[0.2512,0.3188])	([0.6571,0.7492],[0.2662,0.4506])
P_3	([0.3632,0.4297],[0.4241,0.5704])	([0.4122,0.4682],[0.3625,0.5312])	([0.5142,0.5862],[0.3421,0.4056])	([0.5233,0.5802],[0.3444,0.4176])
P_4	([0.4332,0.5057],[0.4272,0.4946])	([0.4769,0.5633],[0.3492,0.4303])	([0.3017,0.3643],[0.5678,0.4322])	([0.5315,0.3014],[0.3232,0.3967])
P_5	([0.4702,0.4165],[0.4312,0.4833])	([0.3062,0.4132],[0.5152,0.3890])	([0.4920,0.5554],[0.3651,0.4422])	([0.4082,0.3861],[0.3562,0.4165])

Table 7: The normalized IVIF matrix.

	A_1	A_2	A_3	A_4
P_1	([0.4473,0.5186],[0.3847,0.4725])	([0.2908,0.4054], [0.5266,0.4874])	([0.4872,0.4706],[0.4123,0.4152])	([0.5354,0.4146],[0.4533,0.3644])
P_2	([0.3032,0.3588],[0.4872,0.6413])	([0.3011,0.3549], [0.5627,0.3402])	([0.5874,0.6826],[0.2500,0.3188])	([0.6571,0.7492],[0.2662,0.4506])
P_3	([0.3632,0.4297],[0.4241,0.5704])	([0.3625,0.5312, [0.4122,0.4682]])	([0.5142,0.5862],[0.3421,0.4056])	([0.5233,0.5802],[0.3444,0.4176])
P_4	([0.4332,0.5057],[0.4272,0.4946])	([0.3492,0.4303], [0.4769,0.5633])	([0.3017,0.3643],[0.5678,0.4322])	([0.5315,0.3014],[0.3232,0.3967])
P_5	([0.4702,0.4165],[0.4312,0.4833])	([0.5152,0.3890], [0.3062,0.4132])	([0.4920,0.5554],[0.3651,0.4422])	([0.4082,0.3861],[0.3562,0.4165])

Table 8: The weights ta_j.

	A_1	A_2	A_3	A_4
w_j	0.1245	0.3564	0.3365	0.1736

Table 9: The GRC from IVIFPIS.

Alternatives	A_1	A_2	A_3	A_4
P_1	0.5737	1.0000	0.2286	0.5157
P_2	0.4326	0.5465	0.3095	0.6094
P_3	0.5984	0.7928	1.0000	1.0000
P_4	0.4037	0.6036	0.3294	0.5932
P_5	1.0000	0.5499	0.3083	0.6279

development potentiality, teacher classroom status, and value judgment process. However, the evaluation of teaching in the field of teaching is not only a worldwide problem but also the key to promoting quality education process. My current evaluation theory, methods and systems, and quality education are for the obvious gap. Evaluation of teachers teaching there are similar problems. These serious constraints restricted the promotion of quality education. Therefore, the establishment of the quality of classroom education development of the concept of evaluation system is the full implementation of the objective of quality education, while also teaching the design and implementation of our activities into a new stage. In this section, an empirical application is given evaluation to the English classroom teaching effect through IVIF-GRA. Since the school hopes to choose the best English teachers, there are five latent English teachers $P_i\,(i = 1, 2, 3, 4, 5)$. For evaluating the English classroom teaching effect fairly, five experts $H = \{H_1, H_2, H_3, H_4, H_5\}$ (expert's weight values w = (0.20, 0.20, 0.20, 0.20, 0.20) are asked. All invited experts express their evaluation through four given attributes: ① A1 is the teaching contents; ② A2 is the teaching cost; ③ A3 is the teaching atmosphere; ④ A4 is the teacher quality. Only A_2 is cost attribute.

Step 1: build the IVIF matrix $Q^{(k)} = (q_{ij}^k)_{m\times n}$ as in Tables 1–5. Then, the overall matrix is derived in Table 6.

Step 2: normalize the IVIF matrix $Q = [{}^{ij}_{q}]_{m\times n}$ to $Q^N = [q_{ij}^N]_{m\times n}$ (see Table 7).

Step 3: obtain the given weights through CRITIC (see Table 8).

Step 4: determine IVIFPIS_j and IVIFNIS_j by using equations (12) and (13):

$$\begin{aligned}
\text{IVIFPIS}_1 &= \langle [0.4702, 0.5186], [0.3847, 0.4725]\rangle,\\
\text{IVIFPIS}_2 &= \langle [0.5152, 0.5312], [0.3062, 0.3402]\rangle,\\
\text{IVIFPIS}_3 &= \langle [0.5874, 0.6826], [0.2500, 0.3188]\rangle,\\
\text{IVIFPIS}_4 &= \langle [0.6571, 0.7492], [0.2662, 0.3644]\rangle,\\
\text{IVIFNI}_1 &= \langle [0.3032, 0.3588], [0.4872, 0.6413]\rangle,\\
\text{IVIFNIS}_2 &= \langle [0.2908, 0.3890], [0.5627, 0.5633]\rangle,\\
\text{IVIFNIS}_3 &= \langle [0.3017, 0.3643], [0.5678, 0.4422]\rangle,\\
\text{IVIFNIS}_4 &= \langle [0.4082, 0.3014], [0.4533, 0.4506]\rangle.
\end{aligned} \tag{17}$$

Step 5: figure out the GRC of every alternative from IVIFPIS and IVIFNIS (Tables 9 and 10).

Step 6: figure out the degree of GRC from IVIFPIS and IVIFNIS (Table 11).

Step 7: calculate $\text{IVIFRRD}(\xi_i)$ from IVIFPIS (Table 12).

Step 8: According to $\text{IVIFRRD}(\xi_i)$, the higher $\text{IVIFRRD}(\xi_i)$, the better the alternative. Thus, the order is $P_3 > P_1 > P_5 > P_4 > P_2$ and P_3 is the best one.

4.2. Compare Analysis. The designed method is always compared with four methods to show the superiority. Firstly, make a comparison between our designed method with IVIFWA and IVIFWG operator. For IVIFWA operator, the calculating value is $S(P_1) = 0.5771$, $S(P_2) = 0.4623$, $S(P_3) = 0.6146$, $S(P_4) = 0.4988$, and $S(P_5) = 0.5409$. Thus, the

Table 10: The GRC from IVIFNIS.

Alternatives	A_1	A_2	A_3	A_4
P_1	0.4070	1.0000	0.3062	0.5654
P_2	0.6796	0.1100	0.8900	0.9476
P_3	0.3421	0.4755	1.0000	0.5063
P_4	0.8900	0.6298	0.6027	1.0000
P_5	1.0000	0.5639	0.3764	0.5839

Table 11: IVIFPIS (ξ_i) and IVIFNIS (ξ_i).

Alternatives	IVIFPIS (ξ_i)	IVIFNIS (ξ_i)
P_1	0.6952	0.5441
P_2	0.6087	1.0676
P_3	0.9825	0.5037
P_4	0.6154	0.8752
P_5	0.7577	0.6138

Table 12: The IFRRD from IFPIS.

Alternatives	P_1	P_2	P_3	P_4	P_5
IVIFRRD (ξ_i)	0.4236	0.1742	0.5371	0.2384	0.4042

Table 13: The compared results of other methods.

Models	Order	The best choice	The worst choice
IVIFWA[68]	$P_3 > P_1 > P_5 > P_4 > P_2$	P_3	P_2
IVIFWG [68]	$P_3 > P_1 > P_5 > P_4 > P_2$	P_3	P_2
IVIF-CODAS [75]	$P_3 > P_1 > P_5 > P_4 > P_2$	P_3	P_2
The built method	$P_3 > P_1 > P_5 > P_4 > P_2$	P_3	P_2

ranking order is $P_3 > P_1 > P_5 > P_4 > P_2$. For IVIFWG operator, the order is $S(P_1) = 0.5525$, $S(P_2) = 0.4411$, $S(P_3) = 0.6156$, $S(P_4) = 0.4988$, and $S(P_5) = 0.5322$. The order is $P_3 > P_1 > P_5 > P_4 > P_2$.

Finally, our method is compared with IVIF-CODAS [75]. The total assessment score is $AS_1 = 0.9041$, $AS_2 = -1.4527$, $AS_3 = 1.5382$, $AS_4 = -1.018$, and $AS_5 = 0.6395$. Therefore, the order is $P_3 > P_1 > P_5 > P_4 > P_2$.

The results of four methods are shown in Table 13.

5. Conclusion

Life changes and people's ideas and educational expectation have brought great challenges to contemporary school education, especially to English education. With the gradual development of social needs, schools seem hard to meet the more and more advanced and complex education needs of the society. In order to promote whole-person education to students, family-school cooperation has become one of the effective ways to collect common effort and establish collaboration for education. Family and school cooperation not only provides an opportunity for in-depth development by prioritizing education environment and exploring potentiality of education resources but also is a booster for the development of students' physical and mental health. However, while there are achievements in family-school cooperative management, there are still difficulties and problems. And the theoretical basis and teaching practices need further exploration. Affordance theory proposed by Gibson [76] claims that there is an interaction between humans (individuals)and the environment (the nature). There is potentiality of potential act in the affordance environment. Its existence is closely related to actors' capability and understanding of the environment. That is to say, affordance is characterized not only by the environment but also by the individuals and emerges only when the two factors interact. Generally, we may put our focus on the affordance of language, the affordance of social culture, and the affordance of situations. Although focal difference exists between these types of affordances, there are similarities. Classroom management can be considered as an environment created together by the child, the teacher, and the parents, as compared with the traditional classroom management, which put emphasis on the interactive rule of the teacher and the student and the environment managed by the teacher. However, parents' participation in English class management provides a possible route for affordable learning environment. This study builds the useful method for this kind of given issue since it builds the IVIF-GRA method for English classroom teaching effect evaluation. And then, a numerical example gives evaluation to English classroom teaching effect. Furthermore, some useful comparative analysis is also given. The main contributions of such study are outlined: (1) the GRA algorithm is used to deal with MAGDM issue under IVIFSs, (2) the weights are derived through CRITIC model, (3) a numerical example for English classroom teaching effect evaluation is given, and (4) some comparative studies are given. At the same time, the main limitations of such study are outlined: (1) the built GRA method does not consider influence of DMs' psychological factors on the given decision result under IVIFSs; (2) the weights of attribute do not consider subjective weight information.

In the near future, we shall pay attention to the consensus reaching process and how to deal with the situations when criteria weights are incompletely known [77–82].

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] A. Alosta, O. Elmansuri, and I. Badi, "Resolving a location selection problem by means of an integrated AHP-RAFSI approach," *Reports in Mechanical Engineering*, vol. 2, pp. 135–142, 2021.

[2] J. Lu, S. Zhang, J. Wu, and Y. Wei, "COPRAS method for multiple attribute group decision making under picture fuzzy environment and their application to green supplier selection," *Technological and Economic Development of Economy*, vol. 27, no. 2, pp. 369–385, 2021.

[3] B. Kizielewicz, J. Więckowski, A. Shekhovtsov, J. Wątróbski, R. Depczyński, and W. Sałabun, "Study towards the time-based m ranking analysis-a supplier selection case study," *Facta Universitatis – Series: Mechanical Engineering*, vol. 19, no. 3, pp. 381–399, 2021.

[4] Z. Jiang, G. Wei, and Y. Guo, "Picture fuzzy MABAC method based on prospect theory for multiple attribute group decision making and its application to suppliers selection," *Journal of Intelligent and Fuzzy Systems*, vol. 42, no. 4, pp. 3405–3415, 2022.

[5] Ç. Karamaşa, D. Karabasevic, D. Stanujkic, A. R. Kookhdan, M. Arunodaya Raj, and M. Ertürk, "An extended single-valued neutrosophic AHP and MULTIMOORA method to evaluate the optimal training aircraft for flight training organizations," *Facta Universitatis – Series: Mechanical Engineering*, vol. 19, pp. 555–578, 2021.

[6] H. Zhang, G. Wei, and X. Chen, "Spherical fuzzy Dombi power Heronian mean aggregation operators for multiple attribute group decision-making," *Computational and Applied Mathematics*, vol. 41, no. 3, p. 98, 2022.

[7] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.

[8] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, 1986.

[9] V. R. Milovanović, A. V. Aleksić, V. S. Sokolović, and M. A. Milenkov, "Uncertainty modeling using intuitionistic fuzzy numbers," *Military Technical Courier*, vol. 69, pp. 905–929, 2021.

[10] P. Gupta, H. D. Arora, and P. Tiwari, "Generalized entropy for intuitionistic fuzzy sets," *Malaysian Journal of Mathematical Sciences*, vol. 10, pp. 209–220, 2016.

[11] L. Xiao, S. Zhang, G. Wei et al., "Green supplier selection in steel industry with intuitionistic fuzzy Taxonomy method," *Journal of Intelligent and Fuzzy Systems*, vol. 39, no. 5, pp. 7247–7258, 2020.

[12] T. Bao, X. Xie, P. Long, and Z. Wei, "MADM method based on prospect theory and evidential reasoning approach with unknown attribute weights under intuitionistic fuzzy environment," *Expert Systems with Applications*, vol. 88, pp. 305–317, 2017.

[13] P. Phochanikorn and C. Q. Tan, "A new extension to a multi-criteria decision-making model for sustainable supplier selection under an intuitionistic fuzzy environment," *Sustainability*, vol. 11, p. 24, 2019.

[14] B. D. Rouyendegh, "The intuitionistic fuzzy ELECTRE model," *International Journal of Management Science and Engineering Management*, vol. 13, no. 2, pp. 139–145, 2018.

[15] Z. Hao, Z. Xu, H. Zhao, and R. Zhang, "Novel intuitionistic fuzzy decision making models in the framework of decision field theory," *Information Fusion*, vol. 33, pp. 57–70, 2017.

[16] P. Li, J. Liu, S. F. Liu, X. Su, and J. Wu, "Grey target method for intuitionistic fuzzy decision making based on grey incidence analysis," *Journal of Grey System*, vol. 28, pp. 96–109, 2016.

[17] M. Zhao, G. Wei, X. Chen, and Y. Wei, "Intuitionistic fuzzy MABAC method based on cumulative prospect theory for multiple attribute group decision making," *International Journal of Intelligent Systems*, vol. 36, no. 11, pp. 6337–6359, 2021.

[18] R. X. Liang, S. S. He, J. Q. Wang, K. Chen, and L. Li, "An extended MABAC method for multi-criteria group decision-making problems based on correlative inputs of intuitionistic fuzzy information," *Computational and Applied Mathematics*, vol. 38, p. 28, 2019.

[19] M. S. Khan and Q. M. D. Lohani, "A Similarity Measure for Atanassov Intuitionistic Fuzzy Sets and its Application to Clustering," in *Proceedings of the 2016 International Workshop on Computational Intelligence (IWCI)*, Dhaka, Bangladesh, December, 2016.

[20] S.-M. Chen, S.-H. Cheng, and T.-C. Lan, "Multicriteria decision making based on the TOPSIS method and similarity measures between intuitionistic fuzzy values," *Information Sciences*, vol. 367-368, pp. 279–295, 2016.

[21] L. Zhang and Y. He, "Extensions of intuitionistic fuzzy geometric interaction operators and their application to cognitive microcredit origination," *Cognitive Computation*, vol. 11, no. 5, pp. 748–760, 2019.

[22] R. Joshi and S. Kumar, "An intuitionistic fuzzy information measure of order-$$(\alpha , \beta)$$ (α, β) with a new approach in supplier selection problems using an extended VIKOR method," *Journal of Applied Mathematics and Computing*, vol. 60, no. 1-2, pp. 27–50, 2019.

[23] K. Kumar and H. Garg, "TOPSIS method based on the connection number of set pair analysis under interval-valued intuitionistic fuzzy set environment," *Computational and Applied Mathematics*, vol. 37, no. 2, pp. 1319–1329, 2018.

[24] L. Xiao, G. Wei, Y. Guo, and X. Chen, "Taxonomy method for multiple attribute group decision making based on interval-valued intuitionistic fuzzy with entropy," *Journal of Intelligent and Fuzzy Systems*, vol. 41, no. 6, pp. 7031–7045, 2021.

[25] Y.-J. Lai, T.-Y. Liu, C.-L. Hwang, and M. O. D. M. Topsis For, "TOPSIS for MODM," *European Journal of Operational Research*, vol. 76, no. 3, pp. 486–500, 1994.

[26] B. F. Yıldırım, Ö. Yorulmaz, and S. Kuzu Yıldırım, "Robust mahalanobis distance based TOPSIS to evaluate the economic development of provinces," *Operational Research in Engineering Sciences: Theory and Applications*, vol. 4, pp. 102–123, 2021.

[27] H. Zhang, G. Wei, and C. Wei, "TOPSIS method for spherical fuzzy MAGDM based on cumulative prospect theory and combined weights and its application to residential location," *Journal of Intelligent and Fuzzy Systems*, vol. 42, no. 3, pp. 1367–1380, 2022.

[28] M. Keshavarz Ghorabaee, E. K. Zavadskas, L. Olfat, and Z. Turskis, "Multi-criteria inventory classification using a new method of evaluation based on distance from average solution (EDAS)," *Informatica*, vol. 26, no. 3, pp. 435–451, 2015.

[29] Y. Su, M. Zhao, G. Wei, C. Wei, and X. Chen, "Probabilistic uncertain linguistic EDAS method based on prospect theory for multiple attribute group decision-making and its application to green finance," *International Journal of Fuzzy Systems*, 2022.

[30] Z. Jiang, G. Wei, and X. Chen, "EDAS method based on cumulative prospect theory for multiple attribute group decision-making under picture fuzzy environment," *Journal of Intelligent and Fuzzy Systems*, vol. 42, no. 3, pp. 1723–1735, 2022.

[31] Y. Huang, R. Lin, and X. Chen, "An enhancement EDAS method based on prospect theory," *Technological and Economic Development of Economy*, vol. 27, no. 5, pp. 1019–1038, 2021.

[32] L. F. Autran Monteiro Gomes and L. s. A. Duncan Rangel, "An application of the TODIM method to the multicriteria rental evaluation of residential properties," *European Journal of Operational Research*, vol. 193, no. 1, pp. 204–211, 2009.

[33] N. Liao, G. Wei, and X. Chen, "TODIM method based on cumulative prospect theory for multiple attributes group decision making under probabilistic hesitant fuzzy setting," *International Journal of Fuzzy Systems*, vol. 24, no. 1, pp. 322–339, 2022.

[34] Y. Su, M. Zhao, C. Wei, and X. Chen, "PT-TODIM method for probabilistic linguistic MAGDM and application to industrial control system security supplier selection," *International Journal of Fuzzy Systems*, vol. 24, no. 1, pp. 202–215, 2022.

[35] D. Zhang, Y. Su, M. Zhao, and X. Chen, "CPT-TODIM method for interval neutrosophic MAGDM and its application to third-party logistics service providers selection," *Technological and Economic Development of Economy*, vol. 28, pp. 201–219, 2022.

[36] M. Zhao, G. Wei, Y. Guo, and X. Chen, "CPT-TODIM method for interval-valued bipolar fuzzy multiple attribute group decision making and application to industrial control security service provider selection," *Technological and Economic Development of Economy*, vol. 27, no. 5, pp. 1186–1206, 2021.

[37] M. Zhao, G. Wei, J. Wu, Y. Guo, and C. Wei, "TODIM method for multiple attribute group decision making based on cumulative prospect theory with 2-tuple linguistic neutrosophic sets," *International Journal of Intelligent Systems*, vol. 36, no. 3, pp. 1199–1222, 2021.

[38] J. L. Deng, "Introduction to grey system," *Journal of Grey System*, vol. 1, pp. 1–24, 1989.

[39] H. Garg, "New exponential operation laws and operators for interval-valued q-rung orthopair fuzzy sets in group decision making process," *Neural Computing & Applications*, vol. 33, no. 20, pp. 13937–13963, 2021.

[40] P. Liu, Y. Li, and F. Teng, "Bidirectional projection method for probabilistic linguistic multi-criteria group decision-making based on power average operator," *International Journal of Fuzzy Systems*, vol. 21, no. 8, pp. 2340–2353, 2019.

[41] C. Jin, H. Wang, and Z. Xu, "Uncertain probabilistic linguistic term sets in group decision making," *International Journal of Fuzzy Systems*, vol. 21, no. 4, pp. 1241–1258, 2019.

[42] G. Wei, C. Wei, and Y. Guo, "EDAS method for probabilistic linguistic multiple attribute group decision making and their application to green supplier selection," *Soft Computing*, vol. 25, no. 14, pp. 9045–9053, 2021.

[43] W. Xue, Z. Xu, X. Zhang, and X. Tian, "Pythagorean fuzzy LINMAP method based on the entropy theory for railway project investment decision making," *International Journal of Intelligent Systems*, vol. 33, no. 1, pp. 93–125, 2018.

[44] Z. Wu, J. Xu, and Z. Xu, "A multiple attribute group decision making framework for the evaluation of lean practices at logistics distribution centers," *Annals of Operations Research*, vol. 247, no. 2, pp. 735–757, 2016.

[45] E. Javanmardi, S. Liu, and N. Xie, "Exploring the philosophical foundations of grey systems theory: subjective processes, information extraction and knowledge formation," *Foundations of Science*, vol. 26, no. 2, pp. 371–404, 2020.

[46] E. Javanmardi, S. Liu, and N. Xie, "Exploring grey systems theory-based methods and applications in sustainability studies: a systematic review approach," *Sustainability*, vol. 12, no. 11, p. 4437, 2020.

[47] E. Javanmardi, S. Liu, and N. Xie, "Exploring the philosophical paradigm of grey systems theory as a postmodern theory," *Foundations of Science*, vol. 25, no. 4, pp. 905–925, 2019.

[48] E. Javanmardi and S. Liu, "Exploring the human cognitive capacity in understanding systems: a grey systems theory perspective," *Foundations of Science*, vol. 25, no. 3, pp. 803–825, 2019.

[49] L. Javanmardi, "Exploring grey systems theory-based methods and applications in analyzing socio-economic systems," *Sustainability*, vol. 11, no. 15, p. 4192, 2019.

[50] S. Zhang, H. Gao, G. Wei, and X. Chen, "Grey relational analysis method based on cumulative prospect theory for intuitionistic fuzzy multi-attribute group decision making," *Journal of Intelligent and Fuzzy Systems*, vol. 41, no. 2, pp. 3783–3795, 2021.

[51] B. Zhu, L. Yuan, and S. Ye, "Examining the multi-timescales of European carbon market with grey relational analysis and empirical mode decomposition," *Physica A: Statistical Mechanics and Its Applications*, vol. 517, pp. 392–399, 2019.

[52] O. Alptekin, N. Alptekin, and B. Sarac, "Evaluation of low carbon development of European union countries and Turkey using grey relational analysis," *Tehnicki Vjesnik-Technical Gazette*, vol. 25, pp. 1497–1505, 2018.

[53] A. Malek, S. Ebrahimnejad, and R. Tavakkoli-Moghaddam, "An improved hybrid grey relational analysis approach for green resilient supply chain network assessment," *Sustainability*, vol. 9, 2017.

[54] C.-Y. Kung and K.-L. Wen, "Applying Grey Relational Analysis and Grey Decision-Making to evaluate the relationship between company attributes and its financial performance-A case study of venture capital enterprises in Taiwan," *Decision Support Systems*, vol. 43, no. 3, pp. 842–852, 2007.

[55] W. Xie, Z. Xu, Z. Ren, and E. Herrera-Viedma, "Expanding grey relational analysis with the comparable degree for dual probabilistic multiplicative linguistic term sets and its

Hindawi
Mathematical Problems in Engineering
Volume 2022, Article ID 2793538, 10 pages
https://doi.org/10.1155/2022/2793538

Research Article

Decision-Making with Risk under Interval Uncertainty Based on Area Metrics

Ying Yan[1] and Bin Suo[2]

[1]*School of Economics and Management, Southwest University of Science and Technology, Mianyang 621010, China*
[2]*School of Information Engineering, Southwest University of Science and Technology, Mianyang 621010, China*

Correspondence should be addressed to Bin Suo; suo.y.y@163.com

Received 9 February 2022; Revised 11 March 2022; Accepted 17 March 2022; Published 14 April 2022

Academic Editor: Darko Božanić

From the perspective of D-S evidence theory and area measurement, a risk-based comprehensive decision-making method that considers both the expected utility and the uncertainty of the scheme is proposed under the interval uncertainty environment of attribute values. The upper and lower bounds of the synthetic probability distribution of attributes values in different natural states are constructed based on the belief measure and plausibility measure. Based on the area measurement, a method for calculating the expected utility of each scheme is proposed. To reflect the influence of the uncertainty in the evaluation value of each scheme attribute on the final decision result, two indexes are defined: the evaluation uncertainty of attributes (EUA) and the uncertainty of the expected utility of scheme (UEU). Finally, considering the expected value of the expected utility and its uncertainty, three decision methods, namely, risk-neutral, risk-averse, and risk-preference, are constructed. An example is considered to show that the proposed method is effective and practical, and the uncertainty of the expected utility has a significant impact on the result of risky decisions. The new method can solve the problems of existing methods that overlook the impact of epistemic uncertainty on the decision-making process.

1. Introduction

As a special form of multiattribute decision-making, risky decision-making is characterized by the presence of different natural states in the decision-making process, each of which has a certain occurrence probability, and the attribution of values as the natural state changes. Risky decision-making is common in investment decision-making [1, 2], emergency decision-making [3, 4], ecological risk assessment [5, 6], and other fields and has attracted extensive attention in recent years.

Due to the complexity, uncertainty, and unpredictability of risky decision-making problems, it is often difficult to accurately predict information such as attribute values and natural state occurrence probabilities during the decision-making process, leading to epistemic uncertainty, which has been described in various ways, such as fuzzy numbers/intuitive fuzzy sets [7, 8], interval numbers [9, 10], and linguistic variables [11, 12]. To obtain the final decision-making conclusion in different uncertain environments, various methods of converting risky decision-making into deterministic decision-making have been proposed.

Generally, two approaches are used to solve the risk decision-making problem. In the first approach, the interval probability is transformed into a point probability. Reference [13] used the continuous ordered weighted average (C-OWA) operator to convert the interval probability into a point probability. Reference [14] proposed an interval probability conversion method based on the Monte Carlo simulation method. Reference [10] proposed another interval probability conversion method based on belief and plausibility measures to transform interval risky decision-making into deterministic decision-making. These methods rank the decisions based on the expected utility theory without considering the psychological factors of decision-makers. In the second approach, the psychological and behavioral factors of decision-makers are accounted for. Representative methods mainly include prospect theory-based methods and regret theory-based methods. Reference [15] calculated the weighted prospect value (interval number) of each scheme and used the expected value of the interval number as the basis for deterministic decision-making. Reference [16] calculated the value of the potential response result related to each criterion based on cumulative prospect theory and determined the prospect value of each alternative by aggregating the values and weights of the response results, based on which the alternatives were sorted. Considering that it is difficult for prospect theory-based methods to determine reference point information, some researchers have investigated risky decision-making methods based on regret theory. Reference [17] proposed a decision analysis method that considers the regret-aversion psychological behaviors of decision-makers. In this method, the alternatives are sorted based on the calculated overall regret value and overall gratification value of each alternative relative to other alternatives. Reference [18] proposed the VIKOR method based on regret theory. A decision-making mechanism coefficient was introduced to measure the impact of the maximum group utility value and the minimum individual regret value on the decision-making result, and an optimization model was constructed and then solved to obtain the final decision-making result.

The aforementioned methods can be used to address risky decision-making problems from different perspectives. However, previous studies have focused on transforming risky decision-making problems into deterministic decision-making problems while overlooking the influence of uncertainty information in the decision-making process on the decision-making result. Because the attribute values and natural state occurrence probabilities of different schemes often contain massive amounts of uncertainty information, uncertainty is always present regardless of the description method used (e.g., intuitionistic fuzzy sets, interval numbers, and linguistic variables). As the uncertainty of a scheme increases, the uncertainties contained in the expected utility value or prospect value increase, so ignoring the influence of these uncertainties and only sorting the schemes based on the mathematical expectation of the expected utility or prospect value may lead to irrational decisions. For example, the expected values of the expected utility of schemes A and B are 1 million yuan and 0.9 million yuan, respectively, and scheme A is superior to scheme B if the schemes are sorted according to the expected value; however, if the uncertainties of the expected utility of schemes A and B are 300,000 yuan and 30,000 yuan, respectively, then, for risk-averse decision-makers, scheme B is superior to scheme A.

Current methods to deal with uncertainty include probability theory [19, 20], fuzzy theory [21–23], and Dempster–Shafer (D-S) evidence theory [24, 25]. D-S evidence theory has a strong ability to deal with epistemic uncertainty. Compared with probability theory, fuzzy theory, and other approaches, it can be used to evaluate and quantify the existing uncertainty only by using the obtained information without any additional assumptions, for example, by assuming a random distribution and a membership function. Based on the above analysis, in this paper, from the perspective of D-S evidence theory, we consider the case in which the attribute value is an interval number and construct the upper and lower bounds of the comprehensive probability distribution of the attribute evaluation values in various natural states based on the plausibility measure and belief measure. We propose an expected utility value calculation method based on area metrics. In addition, we consider the influence of the uncertainty in the final decision evaluation information by defining two indicators of the scheme: the evaluation uncertainty of attributes (EUA) and the uncertainty of the expected utility of schemes (UEU). Finally, we make a comprehensive decision by simultaneously considering the expected utility and the UEU based on the different risk preferences of decision-makers (risk-preferred, risk-averse, and risk-neutral). The new evaluation framework considers the preferences of decision-makers and their aversion to risk and can thus provide a more comprehensive basis for decision-makers with different risk preference types when making decisions in the real world.

2. D-S Evidence Theory

D-S evidence theory is an uncertainty reasoning method proposed by A. P. Dempster and further expanded by his student G. Shafer. It is based on the frame of discernment, which represents a nonempty set containing all possible results that are generally expressed as a nonempty set Θ.

Definition 1. [26]: Basic probability assignment (BPA) is a mapping from a power set to interval numbers [0, 1], i.e., m: $2\Theta \longrightarrow [0, 1]$. The reliability of a set A is denoted as $m(A)$, which represents the degree of confidence in A but not any subset of A. Reliability has the following basic attributes:

$$\begin{cases} m(\varnothing) = 0, \\ 0 \leq m(A) \leq 1,\ \forall A \subseteq \Theta, \\ \sum_{A\subseteq\Theta} m(A) = 1 \end{cases}. \tag{1}$$

If $m(A) > 0$, then A is called a focal element.

Definition 2. [27]: For a proposition A, the degree of confidence in this proposition can be represented by interval numbers $[Bel(A), Pl(A)]$, and $Bel(A)$ and $Pl(A)$ are both numbers between 0 and 1, as shown in Figure 1. $Bel(\cdot)$ and $Pl(\cdot)$ are called the belief function and the plausibility function, respectively, and are defined as follows:

$$\begin{aligned} Bel(A) &= \sum_{B\subseteq A} m(B), \\ Pl(A) &= \sum_{B\cap A\neq\varnothing} m(B). \end{aligned} \tag{2}$$

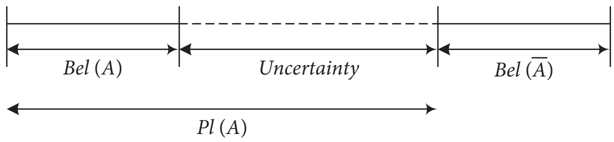


Figure 1: Belief function and plausibility function.

3. Risky Decision-Making Method Based on an Area Measure

3.1. Problem Description. In a risky multiattribute decision-making problem, there are n schemes, denoted as $\mathbf{a} = \{a_1, a_2, \cdots, a_n\}$, $a_i\,(i = 1, 2, \cdots, n) \in \ell$; 1 is the decision space with N natural states, denoted as $W = \{W_1, W_2, \cdots, W_N\}$; the probability of the occurrence of the j^{th} natural state $W_j\,(j = 1, 2, \cdots, N)$ is $p_j\,(j = 1, 2, \cdots, N)$; and there are m decision attributes, denoted as $\mathbf{C} = \{C_1, C_2, \cdots, C_m\}$, with attribute weights of $\omega = \{\omega_1, \omega_2, \cdots, \omega_m\}$ that satisfy $\omega_k > 0\,(k = 1, 2, \cdots, m)$ and $\sum \omega_k = 1$.

In general, attributes $C_k\,(k = 1, 2 \cdots, m)$ are evaluated with two types of indicators: benefit and cost. For benefit-type indicators, a greater value is better, while for cost-type indicators, a smaller value is better.

For the j^{th} natural state, the decision-maker's evaluation value of attribute $C_k\,(k = 1, 2 \cdots, m)$ is an interval number $[x_{jk}^L, x_{jk}^U]$, and the expected utility of each scheme according to the expected monetary value criterion is as follows:

$$E_i = \sum_{j=1}^{N} p_j u_{ij}, \tag{3}$$

where u_{ij} is the utility value of scheme a_i in natural state W_j.

3.2. Area Metrics Definition of Attribute Evaluation Value. For attribute $C_k\,(k = 1, 2 \cdots, m)$ under scheme $a_i\,(i = 1, 2, \cdots, n)$, the decision information for different natural states is a set of data, as shown in Table 1.

For N natural states, the evaluation values can be expressed as a set of D-S evidence theory focal elements:

$$\begin{cases} h_{i,1k} = \left[x_{i,1k}^L, x_{i,1k}^U\right] \\ h_{i,2k} = \left[x_{i,2k}^L, x_{i,2k}^U\right] \\ \cdots \\ h_{i,Nk} = \left[x_{i,1k}^L, x_{i,1k}^U\right] \end{cases}. \tag{4}$$

The BPA corresponding to each focal element is as follows:

$$\begin{cases} m\left(h_{i,1k}\right) = p_1 \\ m\left(h_{i,2k}\right) = p_2 \\ \cdots \\ m\left(h_{i,Nk}\right) = p_N \end{cases}. \tag{5}$$

Based on (4), the upper and lower bounds of attribute C_k can be obtained as follows:

Table 1: Decision information of C_k under scheme a_i.

W_1	W_2	$\cdots$	W_N
$[x_{i,1k}^L, x_{i,1k}^U]$	$[x_{i,2k}^L, x_{i,2k}^U]$	$\cdots$	$[x_{i,Nk}^L, x_{i,Nk}^U]$

$$\begin{cases} X_{i,k}^L = \min\left(x_{i,1k}^L, x_{i,2k}^L, \cdots, x_{i,Nk}^L\right) \\ X_{i,k}^U = \max\left(x_{i,1k}^U, x_{i,2k}^U, \cdots, x_{i,Nk}^U\right) \end{cases}. \tag{6}$$

Based on the above information, the belief function and plausibility function of the attribute evaluation value of attribute C_k can be calculated as follows:

$$Bel_{i,k}\left(x < x^*\right) = \begin{cases} \sum\limits_{\sup\left(h_{i,jk}\right) < x^*} m\left(h_{i,jk}\right) x^* \in \left[X_{i,k}^L, X_{i,k}^U\right] \\ 1\, x^* > X_{i,k}^U \\ 0\, x^* < X_{i,k}^L \end{cases}, \tag{7}$$

$$Pl_{i,k}\left(x < x^*\right) = \begin{cases} \sum\limits_{\inf\left(h_{i,jk}\right) < x^*} m\left(h_{i,jk}\right) \left[X_{i,k}^L, X_{i,k}^U\right] \\ 1\, x^* > X_{i,k}^U \\ 0\, x^* < X_{i,k}^L \end{cases}. \tag{8}$$

In this manner, the upper and lower bounds of the comprehensive probability distribution of attribute C_k are constructed; $Bel_{i,k}\,(x < x^*)$ is the lower bound, and $Pl_{i,k}\,(x < x^*)$ is the upper bound, as shown in Figure 2.

In Figure 2, $Bel_{i,k}\,(x < x^*)$ represents the lower bound of the comprehensive probability distribution of evaluation values in various natural states, and $Pl_{i,k}\,(x < x^*)$ represents the upper bound of the comprehensive probability distribution, while the actual probability distribution $P_{i,k}\,(x < x^*) \in [Bel_{i,k}\,(x < x^*), Pl_{i,k}\,(x < x^*)]$ is shown as the double-dotted line in Figure 2.

Definition 3. Area metric of the attribute evaluation value (AMA). For $Pl_{i,k}\,(x < x^*)$, the area metric is defined as follows:

$$A_{i,k}^L = \int_0^1 {Pl_{i,k}}^{-1}\,(x < x^*)\mathrm{d}x. \tag{9}$$

Clearly, a greater evaluation value of attribute C_k indicates that $Pl_{i,k}\,(x < x^*)$ is closer to the right side of the coordinate axis and greater values of $A_{i,k}^L$; this function can reflect the size of the evaluation value of attribute C_k. If C_k is a benefit-type index, then a value of $A_{i,k}^L$ is better; if C_k is a cost-type index, a smaller value of $A_{i,k}^L$ is better. As indicated by (9), the area metric index $A_{i,k}^L$ is a point value that realizes the transformation from a random probability distribution to a deterministic index and is thus beneficial to subsequent decision-making.

Similarly, the area measure for the lower bound of the probability of attribute C_k can be obtained as follows:

$$A_{i,k}^U = \int_0^1 {Bel_{i,k}}^{-1}\,(x < x^*)\mathrm{d}x. \tag{10}$$

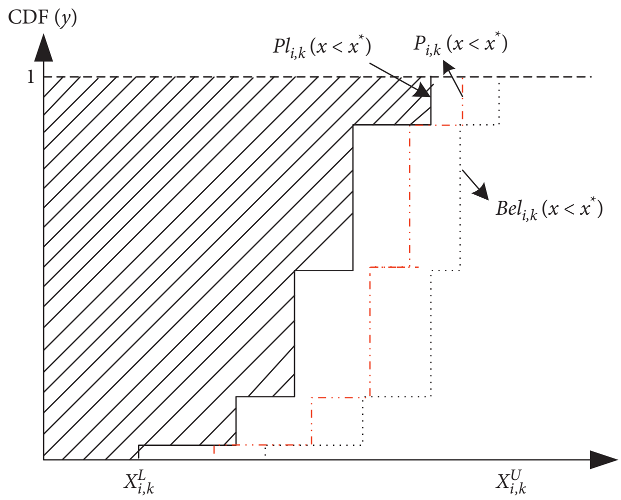


FIGURE 2: Comprehensive probability distribution of attribute C_k.

Then, the AMA indicator of attribute C_k is $A_{i,k} = [A_{i,k}^{L}, A_{i,k}^{U}]$, and its expected value is the median of the interval:

$$\overset{\%}{A}_{i,k} = \frac{\left(A_{i,k}^{L} + A_{i,k}^{U}\right)}{2}. \tag{11}$$

For all attributes, the AMA expectation vector can be calculated as follows:

$$\overset{\%}{A}_{i} = \left(\overset{\%}{A}_{i,1}, \overset{\%}{A}_{i,2}, \cdots, \overset{\%}{A}_{i,m}\right). \tag{12}$$

Definition 4. Area metric of the expected utility (AME) of the scheme. Based on the AMA indicator of each attribute, the AME value of scheme a_i is as follows:

$$\widehat{E}_i = \sum_{k=1}^{m} \omega_k \widetilde{A}_{i,k}. \tag{13}$$

Because the evaluation value of attribute C_k is an interval number, it describes the epistemic uncertainty of the decision-maker on the value of the attribute; a greater epistemic uncertainty indicates a greater uncertainty of the expected utility value $\widehat{E}_i$ reflected in the final scheme. The greater the uncertainty is, the greater the expected volatility of the scheme is, and the worse the worst-case scenario of its expected utility is. This information is also an important indicator in decision-making. Therefore, in this study, we define the EUA and UEU of an attribute to reflect the information.

Definition 5. EUA of an attribute. The evaluation uncertainty of C_k is the area enclosed between $Bel_{i,k}(x<x^*)$ and $Pl_{i,k}(x<x^*)$:

$$EUA_{i,k} = \int_{X_{i,k}^{L}}^{X_{i,k}^{U}} \left(Pl_{i,k}^{-1}(x<x^*) - Bel_{i,k}^{-1}(x<x^*)\right)\mathrm{d}x. \tag{14}$$

Based on (14), the greater the $EUA_{i,k}$ value is, the greater the EUA of attribute C_k is, and vice versa; if $Bel_{i,k}(x<x^*) = Pl_{i,k}(x<x^*)$, i.e., if the epistemic uncertainty disappears and only random uncertainty remains, then the probability envelope is transformed into a deterministic probability distribution $P_{i,k}(x<x^*)$, where the EUA of attribute C_k is zero.

Definition 6. UEU of a scheme. For all attributes, the EUA indicator vector is given as follows:

$$EUA_i = \left(EUA_{i,1}, EUA_{i,2}, \cdots, EUA_{i,m}\right). \tag{15}$$

The UEU indicator of scheme a_i is defined as follows:

$$UEU_i = \sum_{k=1}^{m} \omega_k EUA_{i,k}. \tag{16}$$

In summary, $\widehat{E}_i$ reflects the expected value of the expected utility of scheme a_i, and UEU_i reflects the uncertainty of the expected utility of scheme a_i; a greater $\widehat{E}_i$ value is better, while a smaller UEU_i value is better. These two indicators need to be considered when making decisions.

3.3. Decision-Making Algorithm. The diagram of the proposed decision-making algorithm is shown in Figure 3.

Step 1. If the dimensions and scales of the attribute evaluation values of $C_1, C_2, \cdots, C_m$ are identical, then go to Step 2 directly; otherwise, first perform nondimensionalization as follows:

If the evaluation value of attribute a_i of scheme C_k in the j^{th} natural state is the interval number $h_{i,jk} = [x_{i,jk}^{L}, x_{i,jk}^{U}]$, then for benefit-type attributes, the upper and lower bounds of the interval after nondimensionalization are as follows:

$$\begin{cases} h_{i,jk}^{U} = \dfrac{x_{i,jk}^{U}}{\sum_i \left(x_{i,jk}^{L} + x_{i,jk}^{U}\right)/2n} \\ h_{i,jk}^{L} = \dfrac{x_{i,jk}^{L}}{\sum_i \left(x_{i,jk}^{L} + x_{i,jk}^{U}\right)/2n} \end{cases}. \tag{17}$$

For cost-type attributes, the upper and lower bounds of the interval after nondimensionalization are as follows:

$$\begin{cases} h_{i,jk}^{U} = \dfrac{1/x_{i,jk}^{L}}{\sum_i \left(1/x_{i,jk}^{L} + 1/x_{i,jk}^{U}\right)/2n} \\ h_{i,jk}^{L} = \dfrac{1/x_{i,jk}^{U}}{\sum_i \left(1/x_{i,jk}^{L} + 1/x_{i,jk}^{U}\right)/2n} \end{cases}. \tag{18}$$

Step 2. Construct the upper and lower bounds ($Pl_{i,k}(x<x^*)$ and $Bel_{i,k}(x<x^*)$) of the probability distribution of the evaluation values of attribute C_k using equations (8) and (9).

Step 3. Calculate the area metric index $\widetilde{A}_{i,k}$ and the $EUA_{i,k}$ of attribute C_k using equations (10) and (15).

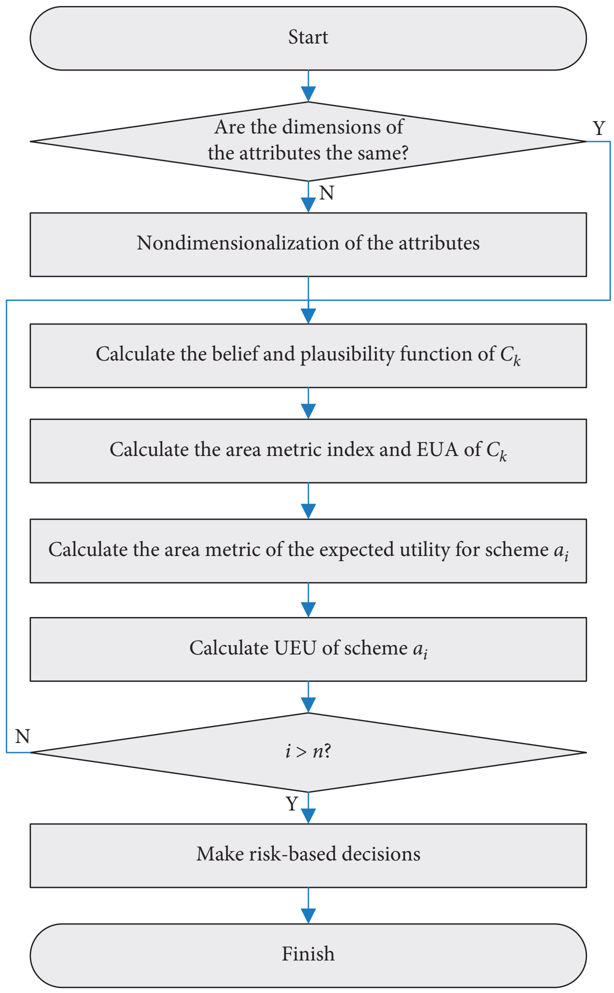


Figure 3: Diagram of the proposed decision-making algorithm.

Step 4. Calculate the $\widehat{E}_i$ of scheme a_i using equation (14).

Step 5. Calculate the UEU_i of scheme a_i using equations (16) and (17).

Step 6. Repeat Steps 2 to 5 to calculate the $\widehat{E}_i$ and UEU_i values of all n schemes.

Step 7. The decision-maker makes risk-based decisions on n schemes according to the following principles:

(1) Risk-neutral decision-makers: Decisions are made directly according to the order of $\widehat{E}_i$. If the $\widehat{E}_i$ values of the two schemes are identical, the scheme with a smaller UEU_i value is preferred.

(2) Risk-averse decision-makers: Set the risk aversion coefficient to $\alpha\,(0 \le \alpha \le 1)$ and sort the schemes using the following equation:

$$\widehat{E}_i^L = \widehat{E}_i - \alpha \cdot UEU_i. \quad (19)$$

(3) Risk-preferred decision-makers: Set the risk preference coefficient to $\beta\,(0 \le \beta \le 1)$ and sort the schemes using the following equation:

$$\widehat{E}_i^L = \widehat{E}_i + \beta \cdot UEU_i. \quad (20)$$

4. Case Study

A new energy vehicle is to be selected to support the company plans to invest in a power battery project. There are four investment schemes for selection: ternary lithium batteries, lithium iron phosphate batteries, nickel-metal hydride batteries, and hydrogen fuel cells, denoted as $\mathbf{a} = \{a_1, a_2, a_3, a_4\}$. The attributes of the schemes include sales volume C1 (unit: 10,000 units/year), rate of return C2 (unit: %/year), R&D cost C3 (unit: 10,000 yuan/unit), and payback period C4 (unit: year). Of these attributes, C1 and C2 are benefit-type indicators, and C3 and C4 are cost-type indicators. The decision-maker assigns weights to the four attributes as $\omega = (0.35, 0.2, 0.2, 0.25)$. In addition, after the product is put on the market, there are three natural states, $\mathbf{W} = \{W_1, W_2, W_3\}$, corresponding to fast-selling, fair, and slow-selling, respectively. The probabilities of occurrence of the three natural states are determined by experts to be $\mathbf{p} = (0.5, 0.3, 0.2)$. The risk decision information of each scheme is shown in Tables 2-4.

First, the data in Tables 2-4 are nondimensionalized, and the results are shown in Tables 5-7.

Next, the upper and lower bounds ($Pl_k(x < x^*)$ and $Bel_k(x < x^*)$) of the comprehensive evaluation probability distribution of attribute C_k are constructed. Taking the attribute C_1 of scheme a_1 as an example, the probability distribution of the evaluation values of C_1 can be obtained through (7) and (8), as shown in Figure 4.

Using (9) and (10), $A_{1,1}^L = 0.8910$ and $A_{1,1}^U = 1.1160$ can be obtained. Thus, (11) yields the expected value of the evaluation value of $C_1 \widetilde{A}_{i,k} = 1.0035$, and the evaluation uncertainty is $EUA_{1,1} = 0.2250$. Similarly, the expected values and EUA values of attributes C2–C4 can be calculated, as listed in Table 8.

Similarly, the comprehensive evaluation results of each attribute of scheme a_2-a_4 can be obtained, as shown in Tables 9-11.

Assuming the coefficient of risk aversion and the coefficient of risk preference are $\alpha = 1$ and $\beta = 1$, respectively, and using Tables 8-11 and (13) and (16), the expected values ($\widehat{E}_i$) and uncertainty values (UEU_i) of the expected utility of the four alternatives can be calculated. The results are listed in Table 12.

Based on the calculation results in Table 12, the comprehensive evaluation results for the risk-preferred, risk-averse, and risk-neutral cases are obtained using the decision-making method described in Step 7 of Section 2.3, as shown in Table 13.

Table 2: Risk decision information table of each scheme (natural state W_1).

Attribute		C_1	C_2	C_3	C_4
Scheme	a_1	[45, 60]	[15, 20]	[3.2, 3.6]	[4.5, 6.0]
	a_2	[42, 54]	[18, 22]	[3.1, 3.4]	[5.5, 6.5]
	a_3	[38, 46]	[12, 18]	[2.5, 2.8]	[4.0, 5.0]
	a_4	[40, 70]	[13, 17]	[3.8, 4.3]	[5.0, 7.0]

Table 3: Risk decision information table of each scheme (natural state W_2).

Attribute		C_1	C_2	C_3	C_4
Scheme	a_1	[31, 35]	[12, 16]	[3.5, 4.1]	[5.5, 7.0]
	a_2	[22, 34]	[13, 17]	[3.4, 3.9]	[6.5, 7.5]
	a_3	[27, 30]	[10, 11]	[3.0, 3.2]	[4.8, 6.4]
	a_4	[24, 39]	[11, 13]	[4.2, 4.5]	[6.5, 8.5]

Table 4: Risk decision information table of each scheme (natural state W_3).

Attribute		C_1	C_2	C_3	C_4
Scheme	a_1	[12, 15]	[8, 12]	[3.7, 4.4]	[8.5, 10.0]
	a_2	[10, 13]	[7, 10]	[3.8, 4.2]	[10.0, 12.0]
	a_3	[11, 14]	[6, 9]	[3.3, 3.5]	[9.0, 10.5]
	a_4	[10, 18]	[5, 10]	[4.4, 4.7]	[12.0, 13.0]

Table 5: Risk decision information table of each scheme (natural state W_1).

Attribute		C_1	C_2	C_3	C_4
Scheme	a_1	[0.91, 1.21]	[0.88, 1.18]	[0.90, 1.01]	[0.87, 1.17]
	a_2	[0.85, 1.09]	[1.06, 1.30]	[0.95, 1.04]	[0.81, 0.95]
	a_3	[0.76, 0.93]	[0.71, 1.06]	[1.16, 1.30]	[1.05, 1.31]
	a_4	[0.81, 1.41]	[0.77, 1.00]	[0.75, 0.85]	[0.75, 1.05]

Table 6: Risk decision information table of each scheme (natural state W_2).

Attribute		C_1	C_2	C_3	C_4
Scheme	a_1	[31, 35]	[12, 16]	[3.5, 4.1]	[5.5, 7.0]
	a_2	[22, 34]	[13, 17]	[3.4, 3.9]	[6.5, 7.5]
	a_3	[27, 30]	[10, 11]	[3.0, 3.2]	[4.8, 6.4]
	a_4	[24, 39]	[11, 13]	[4.2, 4.5]	[6.5, 8.5]

Table 7: Risk decision information table of each scheme (natural state W_3).

Attribute		C_1	C_2	C_3	C_4
Scheme	a_1	[31, 35]	[12, 16]	[3.5, 4.1]	[5.5, 7.0]
	a_2	[22, 34]	[13, 17]	[3.4, 3.9]	[6.5, 7.5]
	a_3	[27, 30]	[10, 11]	[3.0, 3.2]	[4.8, 6.4]
	a_4	[24, 39]	[11, 13]	[4.2, 4.5]	[6.5, 8.5]

As shown in Table 13, when deciding about the four alternatives, the risk-neutral, risk-averse, and risk-preferred decision-makers show completely different decision-making results.

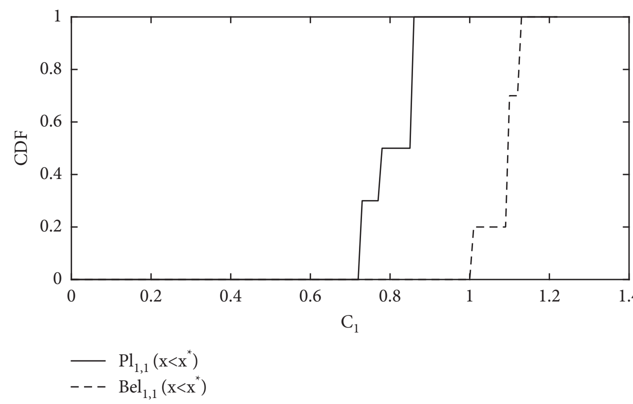


Figure 4: Comprehensive probability distribution of attribute a_1.

Table 8: Comprehensive evaluation results of each attribute under scheme a_1.

Attribute	C_1	C_2	C_3	C_4
$A^L_{1,k}$	0.8910	0.8970	0.8950	0.8440
$A^U_{1,k}$	1.1160	1.1700	1.0290	1.0950
$\tilde{A}_{1,k}$	1.0035	1.0335	0.9620	0.9695
$EUA_{1,k}$	0.2250	0.2730	0.1340	0.2510

Table 9: Comprehensive evaluation results of each attribute under scheme a_2.

Attribute	C_1	C_2	C_3	C_4
$A^L_{1,k}$	0.7950	0.9940	0.9350	0.7720
$A^U_{1,k}$	1.0810	1.2840	1.0470	0.9000
$\tilde{A}_{1,k}$	0.9380	1.1390	0.9910	0.8360
$EUA_{1,k}$	0.2860	0.2900	0.1120	0.1280

Table 10: Comprehensive evaluation results of each attribute under scheme a_3.

Attribute	C_1	C_2	C_3	C_4
$A^L_{1,k}$	0.7650	0.7280	1.1430	0.9570
$A^U_{1,k}$	0.9220	0.9990	1.2510	1.2080
$\tilde{A}_{1,k}$	0.8435	0.8635	1.1970	1.0825
$EUA_{1,k}$	0.1570	0.2710	0.1080	0.2510

Table 11: Comprehensive evaluation results of each attribute under scheme a_4.

Attribute	C_1	C_2	C_3	C_4
$A^L_{1,k}$	0.7450	0.7580	0.7820	0.7060
$A^U_{1,k}$	1.2890	1.0380	0.8640	0.9310
$\tilde{A}_{1,k}$	1.0170	0.8980	0.8230	0.8185
$EUA_{1,k}$	0.5440	0.2800	0.0820	0.2250

Table 12: Expected utility evaluation results of four alternatives.

Attribute		$\widehat{E}_i$	UEU_i
Scheme	a_1	0.9927	0.2229
	a_2	0.9633	0.2125
	a_3	0.9780	0.1935
	a_4	0.9048	0.3191

Table 13: Comprehensive evaluation results of alternative schemes under different risk preferences.

Attribute		Risk-neutral		Risk-averse		Risk-preferred	
		Value	Rating	Value	Rating	Value	Rating
Alternatives	a_1	0.9927	1	0.7698	2	1.2156	2
	a_2	0.9633	3	0.7508	3	1.1758	3
	a_3	0.9780	2	0.7845	1	1.1715	4
	a_4	0.9048	4	0.5857	4	1.2239	1

Table 14: Comparison of the proposed method and other methods for different α and β.

Method	Condition	a_1	a_2	a_3	a_4
Proposed method	$\alpha = 0.1$	1	3	2	4
	$\alpha = 0.3$	1	3	2	4
	$\alpha = 0.5$	1	3	2	4
	$\alpha = 0.7$	1	4	3	2
	$\alpha = 0.9$	1	3	4	2
	$\beta = 0.1$	1	3	2	4
	$\beta = 0.3$	1	3	2	4
	$\beta = 0.5$	1 (tied)	3	1 (tied)	4
	$\beta = 0.7$	2	3	1	4
	$\beta = 0.9$	2	3	1	4
[9]	—	1	3	2	4

Table 15: Risk decision information table of each scheme (natural state W_1, uncertainty increased by 20%).

Attribute		C_1	C_2	C_3	C_4
Scheme	a_1	[0.94, 1.18]	[0.91, 1.15]	[0.911, 0.999]	[0.9, 1.14]
	a_2	[0.874, 1.066]	[1.084, 1.276]	[0.959, 1.031]	[0.824, 0.936]
	a_3	[0.777, 0.913]	[0.745, 1.025]	[1.174, 1.286]	[1.076, 1.284]
	a_4	[0.87, 1.35]	[0.793, 0.977]	[0.76, 0.84]	[0.78, 1.02]

Table 16: Risk decision information table of each scheme (natural state W_2, uncertainty increased by 20%).

Attribute		C_1	C_2	C_3	C_4
Scheme	a_1	[1.033, 1.137]	[0.961, 1.209]	[0.905, 1.025]	[0.935, 1.135]
	a_2	[0.76, 1.08]	[1.032, 1.288]	[0.944, 1.056]	[0.863, 0.967]
	a_3	[0.9, 0.98]	[0.778, 0.842]	[1.147, 1.203]	[1.033, 1.297]
	a_4	[0.839, 1.231]	[0.865, 0.985]	[0.816, 0.864]	[0.773, 0.957]

Table 17: Risk decision information table of each scheme (natural state W_3, uncertainty increased by 20%).

Attribute		C_1	C_2	C_3	C_4
Scheme	a_1	[0.953, 1.137]	[0.998, 1.382]	[0.907, 1.043]	[0.702, 0.798]
	a_2	[0.793, 0.977]	[0.866, 1.154]	[0.94, 1.02]	[0.582, 0.678]
	a_3	[0.873, 1.057]	[0.746, 1.034]	[1.127, 1.183]	[0.671, 0.759]
	a_4	[0.832, 1.328]	[0.65, 1.13]	[0.836, 0.884]	[0.534, 0.566]

Table 18: Risk decision information table of each scheme (natural state W_1, uncertainty decreased by 20%).

Attribute		C_1	C_2	C_3	C_4
Scheme	a_1	[0.88, 1.24]	[0.85, 1.21]	[0.889, 1.021]	[0.84, 1.2]
	a_2	[0.826, 1.114]	[1.036, 1.324]	[0.941, 1.049]	[0.796, 0.964]
	a_3	[0.743, 0.947]	[0.675, 1.095]	[1.146, 1.314]	[1.024, 1.336]
	a_4	[0.75, 1.47]	[0.747, 1.023]	[0.74, 0.86]	[0.72, 1.08]

Risk-neutral decision-makers conclude that scheme a_1 is the best, and they sort the schemes as follows: $a_1 > a_3 > a_2 > a_4$.

Risk-averse decision-makers conclude that scheme a_3 is the best, and they sort the schemes as follows: $a_3 > a_1 > a_2 > a_4$.

Table 19: Risk decision information table of each scheme (natural state W_2, uncertainty decreased by 20%).

Attribute		C_1	C_2	C_3	C_4
Scheme	a_1	[1.007, 1.163]	[0.899, 1.271]	[0.875, 1.055]	[0.885, 1.185]
	a_2	[0.68, 1.16]	[0.968, 1.352]	[0.916, 1.084]	[0.837, 0.993]
	a_3	[0.88, 1]	[0.762, 0.858]	[1.133, 1.217]	[0.967, 1.363]
	a_4	[0.741, 1.329]	[0.835, 1.015]	[0.804, 0.876]	[0.727, 1.003]

Table 20: Risk decision information table of each scheme (natural state W_3, uncertainty decreased by 20%).

Attribute		C_1	C_2	C_3	C_4
Scheme	a_1	[0.907, 1.183]	[0.902, 1.478]	[0.873, 1.077]	[0.678, 0.822]
	a_2	[0.747, 1.023]	[0.794, 1.226]	[0.92, 1.04]	[0.558, 0.702]
	a_3	[0.827, 1.103]	[0.674, 1.106]	[1.113, 1.197]	[0.649, 0.781]
	a_4	[0.708, 1.452]	[0.53, 1.25]	[0.824, 0.896]	[0.526, 0.574]

Table 21: Comparison of the proposed method and other methods under different uncertainties.

Condition	Method	a_1	a_2	a_3	a_4
Uncertainty decreased by 20%	Proposed method $\alpha = 0.5$	1	3	2	4
	Proposed method $\beta = 0.5$	1	3	2	4
	[9]	1	3	2	4
Uncertainty increased by 20%	Proposed method $\alpha = 0.5$	1	4	3	2
	Proposed method $\beta = 0.5$	2	3	1	4
	[9]	1	3	2	4

Risk-preferred decision-makers conclude that scheme a_4 is the best, and they sort the schemes as follows: $a_4 > a_1 > a_2 > a_3$.

By carefully analyzing the results in Table 13, although the expected value of the expected utility of scheme a_1 is the greatest, its uncertainty is also higher (ranks second), so it ranks first when its uncertainty is ignored; however, when considering the risk of uncertainty during decision-making, scheme a_1 is no longer the best choice. Scheme a_4 has the greatest uncertainty and the greatest risk, but from the perspective of risk-preferred decision-makers, it also has the greatest opportunity and enables the highest return in the best case, so it is the best choice for risk-preferred decision-makers.

5. Validation of Results

To further verify the proposed method, the risk preference coefficient α and the risk aversion coefficient β are set to different values, and the schemes are sorted using the proposed method. The results are then compared with the ranking results in [9], as shown in Table 14.

As shown in Table 14, when the risk preference coefficient α and the risk aversion coefficient β are set to low values, the ranking results of the four schemes are identical and consistent with the ranking results of [9]: $a_1 > a_3 > a_2 > a_4$. When α and β are set to high values, the ranking results begin to change; for example, when $\alpha = 0.7$, $a_1 > a_4 > a_3 > a_2$, and when $\beta = 0.7$, $a_2 > a_3 > a_1 > a_4$. The ranking result is associated with the values of α and β and the values of $\widehat{E}_i$ and UEU_i.

To assess the influence of attribute uncertainty on the decision-making results, the uncertainty of the estimated values of the various attributes in Tables 5–7 under different natural states is reduced by 20% and expanded by 20%, respectively. The results are shown in Tables 15–20.

For $\alpha = 0.5$ and $\beta = 0.5$, the schemes are sorted, and the results are compared with the results from [9], as shown in Table 21.

As shown in Table 21, when the uncertainties in the attributes are reduced by 20%, the ranking results of the four schemes are identical and consistent with the ranking results of [9], i.e., $a_1 > a_3 > a_2 > a_4$. However, when the uncertainties are increased by 20%, the ranking results begin to change. For example, for $\alpha = 0.5$, $a_1 > a_4 > a_3 > a_2$, while for $\beta = 0.5$ and the method in [9], the results are $a_2 > a_3 > a_1 > a_4$ and $a_1 > a_3 > a_2 > a_4$, respectively.

This case study demonstrates that uncertainty in decision-making information can have a great impact on the final decision-making result and is thus an important factor that must be considered in risky decision-making. In view of previous studies, regardless of the method used, uncertain decision-making information is converted into accurate information to make final decisions. Clearly, these decision-making methods overlook uncertainty, which may lead decision-makers to overlook risks and make incorrect choices.

6. Conclusion

In multiattribute risky decision-making processes, the attribute evaluation information of a scheme often contains interval epistemic uncertainty, which has a significant impact on the decision outcome. From the perspective of D-S evidence theory, in this paper, we construct the area metric indicator AME for the expected utility of the scheme to measure the expected value of the expected utility of the scheme; we also construct the uncertainty index UEU of the expected utility of the scheme to measure the risks and opportunities of the expected utility of alternative schemes so that quantitative risk and opportunity measures for decision-makers with different risk preferences can be provided. When comparing and selecting schemes, decision-makers must comprehensively consider the area metric index AME and the uncertainty index UEU of the expected value of the expected utility to make decisions that are more aligned with reality.

The main contributions of the risk-based decision-making method proposed in this paper are as follows:

(1) The area metric of the attribute evaluation value is proposed. The calculation process of the index does not require any artificial assumptions, and the results are more objective.

(2) Different from the existing methods that only consider the expected utility index, the method proposed in this paper establishes the expected utility uncertainty index at the same time. Decision-makers can comprehensively evaluate alternatives according to the two indexes and draw more objective and consistent conclusions.

(3) The proposed evaluation framework considers the preferences of decision-makers and their aversion to risk, so it provides a more comprehensive basis for decision-makers with different risk preference types when making decisions in the real world.

In future work, more complex application scenarios will be explored. For example, the uncertainty of attribute weights and the uncertainty of natural state probability will be considered [28].

Data Availability

The data were curated by the authors and are available upon request.

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (Grant nos. 71702156 and U183010080) and the Research Foundation of Southwest University of Science and Technology (Grant no. 21zx7125).

References

[1] Y. Chang, C. Liu, and M. Liu, “Differentiation degree combination weighting method for investment decision-making risk assessment in power grid construction projects,” *Global Energy Interconnection*, vol. 2, no. 5, pp. 465–477, 2019.

[2] H. Seiti, A. Hafezalkotobb, and E. Herrera-Viedma, “A novel linguistic approach for multi-granular information fusion and decision-making using risk-based linguistic D numbers,” *Information Sciences*, vol. 530, no. 8, pp. 43–65, 2020.

[3] M. Y. Li and P. P. Cao, “Extended TODIM method for multi-attribute risk decision making problems in emergency response,” *Computers & Industrial Engineering*, vol. 135, no. 9, pp. 1286–1293, 2018.

[4] X. P. Yin, X. H. Xu, and B. Pan, “Selection of strategy for large group emergency decision-making based on risk measurement,” *Reliability Engineering & System Safety*, vol. 208107325 pages, 2021.

[5] N. Manap and N. Voulvo, “Risk-based decision-making framework for the selection of sediment dredging option,” *The Science of the Total Environment*, vol. 496, no. 15, pp. 607–623, 2014.

[6] E. Ahmadisharaf and B. L. Benham, “Risk-based decision making to evaluate pollutant reduction scenarios,” *The Science of the Total Environment*, vol. 702, no. 1, 135022 pages, 2020.

[7] A. Karaşan, İ. Kaya, M. Erdoğan, and M. Çolak, “A multi-criteria decision making methodology based on two-dimensional uncertainty by hesitant Z-fuzzy linguistic terms with an application for blockchain risk evaluation,” *Applied Soft Computing*, vol. 113108014 pages, 2021.

[8] H. Y. Zhang, H. G. Peng, J. Wang, and J. Wang, “An extended outranking approach for multi-criteria decision-making problems with linguistic intuitionistic fuzzy numbers,” *The Journal*, vol. 59, no. 5, pp. 462–474, 2017.

[9] J. J. Zhu, Z. Z. Ma, H. H. Wang, and Y. Chen, “Risk decision-making method using interval numbers and its application based on the prospect value with multiple reference points,” *Information Sciences*, vol. 385-386, pp. 415–437, 2017.

[10] Y. Yan, Z. L. Zhou, and M. Yuan, “A novel method for risky interval probability decision making in view of evidence theory,” *Statistics & Decisions*, vol. 34, no. 3, pp. 62–64, 2018.

[11] P. D. Liu, F. Jin, X. Zhang, Y. Su, and M. Wang, “Research on the multi-attribute decision-making under risk with interval probability based on prospect theory and the uncertain linguistic variables,” *Knowledge-Based Systems*, vol. 24, no. 4, pp. 554–561, 2011.

[12] W. Zhou and Z. S. Xu, “Generalized asymmetric linguistic term set and its application to qualitative decision making involving risk appetites,” *European Journal of Operational Research*, vol. 254, no. 2, pp. 610–621, 2016.

[13] M. Rezvani, F. Nickravesh, A. D. Astaneh, and N. Kazemi, “A risk-based decision-making approach for identifying natural-based tourism potential areas,” *Journal of Outdoor Recreation and Tourism*, vol. 37, Article ID 100485, 2022.

[14] D. Y. He and R. X. Zhou, “Study on methods of decision-making under interval probability,” *Journal of Systems Management*, vol. 19, no. 2, pp. 210–214, 2010.

[15] D. P. Liu, “Method for multi-attribute decision-making under risk with the uncertain linguistic variables based on prospect theory,” *Control and Decision*, vol. 26, no. 6, pp. 893–897, 2011.

[16] Y. Liu, Z. P. Fan, and Y. Zhang, “Risk decision analysis in emergency response: a method based on cumulative prospect theory,” *Computers & Operations Research*, vol. 42, pp. 75–82, 2014.

[17] X. Zhang, Z. P. Fan, and F. D. Chen, “Risky multiple attribute decision making with regret aversion,” *Journal of Systems Management*, vol. 23, no. 1, pp. 111–117, 2016.

[18] C. Q. Tan and X. D. Zhang, "VIKOR method for uncertain risky multi-attribute decision making based on regret theory," *Statistics & Decisions*, vol. 35, no. 1, pp. 47–51, 2019.
[19] M. G. R. Faes, M. A. Valdebenito, D. Moens, and M. Beer, "Operator norm theory as an efficient tool to propagate hybrid uncertainties and calculate imprecise probabilities," *Mechanical Systems and Signal Processing*, vol. 152, no. 1, p. 107482, 2021.
[20] Z. R. Wang, Y. J. Li, X. Tong, and J. H. Gong, "Risk probability evaluation for the effect of obstacle on CO2 leakage and dispersion indoors based on uncertainty theory," *Journal of Loss Prevention in the Process Industries*, vol. 74, no. 1, p. 104652, 2022.
[21] M. A. Alao, O. M. Popoola, and T. R. Ayodele, "A novel fuzzy integrated MCDM model for optimal selection of waste-to-energy-based-distributed generation under uncertainty: a case of the City of Cape Town, South Africa," *Journal of Cleaner Production*, vol. 343, no. 1, p. 130824, 2022.
[22] N. Foroozesh, B. Karimi, and S. M. Mousavi, "Green-resilient supply chain network design for perishable products considering route risk and horizontal collaboration under robust interval-valued type-2 fuzzy uncertainty: a case study in food industry," *Journal of Environmental Management*, vol. 307, no. 1, p. 114470, 2022.
[23] N. Gopal and D. Panchal, "A structured framework for reliability and risk evaluation in the milk process industry under fuzzy environment," *Facta Universitatis – Series: Mechanical Engineering*, vol. 19, no. 2, pp. 307–333, 2021.
[24] L. X. Cao, J. Liu, X. H. Meng, Y. Zhao, and Z. B. Yu, "Inverse uncertainty quantification for imprecise structure based on evidence theory and similar system analysis," *Structural and Multidisciplinary Optimization*, vol. 64, pp. 2183–2198, 2021.
[25] R. Li, Z. Chen, H. Li, and Y. C. Tang, "A new distance-based total uncertainty measure in Dempster-Shafer evidence theory," *Applied Intelligence*, vol. 52, pp. 1209–1237, 2022.
[26] A. P. Dempster, "Upper and lower probabilities induced by a multi valued mapping," *The Annals of Mathematical Statistics*, vol. 38, no. 2, pp. 325–339, 1967.
[27] G. A. Shafer, *Mathematical Theory of Evidence*, Princeton University Press, New Jersey, 1976.
[28] X. H. Xua, L. L. Wang, X. H. Chen et al., "Large group emergency decision-making method with linguistic risk appetites based on criteria mining," *Knowledge-Based Systems*, vol. 182, no. 15, pp. 1–13, 2019.

Hindawi
Mathematical Problems in Engineering
Volume 2022, Article ID 2958519, 12 pages
https://doi.org/10.1155/2022/2958519

Research Article

The Allocation of Business Model Components under Presence of Uncertainties by the Branch-and-Bound Method

Snežana Nestić ,[1] Aleksandar Aleksić ,[1] Jaime Gil Lafuente,[2] Tijana Cvetić ,[1] and Goran Đurić[3]

[1]*University of Kragujevac, Faculty of Engineering, Kragujevac 34000, Serbia*
[2]*University of Barcelona, Faculty of Economics and Business Science, Barcelona 08001–08042, Spain*
[3]*Faculty of Mechanical Engineering, University of Belgrade, Belgrade 11120, Serbia*

Correspondence should be addressed to Aleksandar Aleksić; aaleksic@kg.ac.rs

Received 9 December 2021; Revised 31 December 2021; Accepted 20 January 2022; Published 16 February 2022

Academic Editor: Darko Božanić

As the only constant in business is change, business transformation is essential for adopting new perspectives and business trends. One of the keys to performing successful business transformation is to be fully aware of the current components of the business model. This research aims to allocate the business model components (BMCs) to defined business model components groups (BMCGs) by developing a new approach that integrates fuzzy sets and heuristic algorithms. The allocation results enable a comprehensive analysis of business model frameworks and give a good connection to research in the domain of strategic management and business process modeling. For allocation, the decision-makers (DMs) are employing the linguistic terms modeled by the fuzzy sets theory. The considered problem is stated as an integer programming model where the optimal solution is given by a B&B algorithm. The model is tested on a sample of forty experts from four different economic sectors.

1. Introduction

During the last decades, defining business models has become a very important issue in the domain of business and management since a lot of scholars believe that a company's business can be presented through a business model. The term business model (BM) has been described by many authors appointing that it covers the architecture of product flows, services, and information, including the description of different business entities and their roles, as well as a description of potential benefits for different business entities and a description of the source of income [1]. BMs are used for determining the structure, relations, and success factors of an organization. They can serve as generators of competencies, especially in terms of rapid changes in the market. BMs describe how marketable information, products, and/or services are generated utilizing a company's value-added component. In addition to value creation, different components are taken into consideration to achieve generating and securing the competitive advantage. Those are related to strategic, customer, and market components. In literature, five different perspectives of this term can be found: business model activities, business model logics, business model archetypes, business model alignment, and business model components (BMCs) [2].

BMCs perspective is taken by authors who propose structuring BM based on its essential components to capture the important parts of the business and to create the operational framework. Several studies have investigated various definitions and lists of BMCs [3–5].

Stating the fact that BM should interpret the most significant segments of the business and the basic features of the enterprise, the most significant issue in this research area is defining the BMCs. For more efficient and effective management of the enterprise, a certain number of scholars denote the idea of allocating BMCs to BMCGs with the same purpose or some other attributes.

Awareness of BMCs and their organization in a business model is crucial for the business transformation and achieving the long-term sustainability of the company. This is highly applicable for the companies that are shifting their production to the requirements of industry 4.0. The motivation for this research comes from the fact that there is no genuine set of BMCs in literature, nor is the allocation of BMCs to BMCGs performed in an exact way. Such allocation is highly dependent on the experience of decision-makers (DMs). At the same time, the wider audience is questioning the need for the business model development or BMs improvement, considering the legacy of established companies [6]. As a known management tool, an affinity diagram could be used for this purpose, although there is a certain level of ambiguity related to BMCs classification. The authors believe that the mentioned classification tool should be enhanced. This complies with the ongoing research trend that existing methodologies should be modified to address more complex situations [7].

By using the words of natural language, DMs can better express their assessment compared to the situation when they use real numbers. Linguistic expressions can be quantitatively described by using the fuzzy sets theory [8]. If fuzzy sets are employed as a tool for describing different variables, many literature sources stand for the application of type-1 fuzzy numbers [9–11] in many research fields.

Allocation of BMCs to BMCGs can be denoted as a medium-sized instances Integer Linear Programming (ILP) problem with a linear fitness objective function and a set of linear constraints. In literature, many exact methods address the problems of combinatorial optimization [12]. The Branch-and-Bound (B&B) method has been significantly employed for solving a variety of combinatorial optimization problems, for example, production planning problems [13, 14] and energy system problems [15, 16]. It is worth mentioning that most software solutions dedicated to combinatorial optimization problems are based on the B&B method such as Gurobi 9.1 that is employed in the scope of this research.

In compliance with the stated, the objective of this research is to (1) define the appropriate BMCs that may be used for the constitution of the enterprise BM at the level of the considered industry, (2) model the existing uncertainties by using fuzzy sets theory, and (3) allocate BMCs to defined BMCGs by using an exact method, to help the corporate managers understand BMs and help them make strategic choices.

The paper is organized in the following way: Section 2 provides a detailed analysis of papers that can be found in literature covering different research domains, for example, a business model, modeling of uncertainty, and allocation domain. Section 3 describes the used methodology. In Section 4, the proposed model is tested based on real-life data. The discussion of the given results and conclusion are presented in Section 5.

2. Literature Review

This section introduces the wide analysis of relevant literature sources divided into two parts: (i) analysis of business model components and (ii) allocation problem under uncertainties by using the B&B method.

2.1. Business Model Components. For the BCMs' allocation in this study, authors have analyzed many literature sources in which the BMCs have been stated. BMCs may be analyzed by considering mutual interaction between business subjects, creating values and income sources [17]. The significance of the economic component in BM is emphasized in the conducted research [18], as well as firms' economic dimension with components that correspond to the determinants of firms' profitability [19]. Some scholars [6] look at BMs at a general level and provide the ontology of BM by emphasizing the significance of the component, differentiation, and strategic control, which represent the economic need to differentiate and protect revenue streams.

To utilize the technology to increase the effectiveness of the company, the components that emphasize informational technologies are introduced into the BM [20, 21].

In studies [22, 23], the inclusion of value in the model as its component was used for the first time. The value proposition dimension might be enhanced with three components: competitors, key business components, and structure [24]. For an easier understanding of how the BM fulfilled a potent value proposition profitably, the business model framework was defined [25]. The authors defined their framework by placing components in four groups: Customer Value Proposition, Profit Formula, Key Resources, and Key Processes. The Key Resources group put focus on the key components that create value for the customer and the company and, unlike other authors, further introduced components of equipment and brand. The Key Processes group also included rules, metrics, and norms. A consolidated view of the components [23] might be based on the value proposition (the offering, the target customer, and the basic strategy), the value creation and delivery system (resources and capabilities, organization, and position in the value network), and the value capture (revenue sources and the economics of the business).

As a component of BMs, trading mechanisms, trading protocols was introduced with increased employment of dynamics of electronic commerce [26]. The more comprehensive research in the domain of e-commerce models has resulted in the introduction of a new component entitled product innovation [27]. The authors then identified 9 of the most common BMCs [28]. They included all the components related to competition and implementation of BM. Considering that these components, although they are connected with the BM, are not their internal part, the authors introduced a component delivery channel.

The component entitled goods and services production and exchanges was introduced through the presentation of an analytical framework for comparing different BMs for producing information goods and digital services [29].

The review of literature on BMs in the contexts of technological, organizational, and social innovation brought more BMs components [30]. They proposed components including value proposition, supply chain, customer interface, and financial model that BMs should meet to be sustainable. The business model framework was analyzed from the sustainability perspective [1], so the following components were introduced for the first time: governance, process measure, and value configuration in the BM.

A new set of BM components had been developed based on BM innovations, and a new component entitled core competences was introduced [31].

One of the conducted research projects emphasized the heterogeneousness of contents of BM in literature, which manifested as T [32]. The authors had systematized all the components from literature and introduced the component procurement. Based on the detailed analysis of these studies, it can be said that the allocation was performed based on the assessment of the DMs and, to a great degree, depending on their knowledge and experience [3, 5, 22]. An affinity diagram can be used to categorize BM components found in literature [4].

2.2. Allocation of BMCs Problem under Uncertainties. This section is supplied with the literature review emphasizing the importance of linguistic variables modeling with FST and solving a variety of optimization problems in different research domains by the B&B method.

2.2.1. Modeling by Using FST. A significant number of scholars stand beside the fact that it is suitable for DM to employ approximate information and uncertainty to generate decisions. The development of mathematics, especially probability theory and FST [8], has enabled the quantitative description of linguistic expressions. The application of a stochastic approach in the processing of uncertainty requires the existence of any relevant data records and a large complexity of computation. On the other hand, FST is a valuable tool that copes with two major problematic areas of the treated problem such as imprecision and ambiguity.

A fuzzy set is represented by its membership function, and the shape of the membership functions can be based on one's experience, the subjective belief of DMs, intuition, and contextual knowledge about the concept modeled [8]. The selection of membership function shapes can be treated as a problem itself. Many authors use trapezoidal fuzzy numbers (TrFNs) and triangular fuzzy numbers (TFNs). The range of maximum triangular membership must be around the crisp point of the triangle. The range of the maximum trapezoidal membership function is wider. Using TFNs decreases the complexity of calculations, and at the same time, the calculated results are accurate enough. In this research, modeling of judgments of the DMs is performed by TFNs.

The granularity depends on the treated problem size. Based on literature, the seven categories can be used at most. In this research, the five have been employed TFNs by analogy [11, 33].

The domain of fuzzy sets can be defined on different measurement scales, for instance, common measurement scale, [1–5], or [0–1] as in this research.

If the problem is presented as a fuzzy group decision-making problem, an aggregation procedure should be conducted [8, 34].

2.2.2. A Branch-and-Bound Method Analysis. The B&B method is based on dividing the total set of feasible solutions into smaller subsets of solutions. These smaller subsets can be evaluated until the best solution is obtained. This exact method requires large computer resources to solve very large problems, and therefore a heuristic is required for most real problems. According to the B&B method, the node with the smallest lower bound is extended at each iteration. If the number of decision variables is low, the B&B method is very useful and easily implemented to obtain the integer solution.

The problem of the assembly line design with parallel stations could be stated as ILP [35]. In this case, the objective function is defined as minimizing the number of stations respecting different conditions. A similar problem has been treated by [16]. Also, the energy-efficient management problem has been analyzed [36]. This problem is defined as a Mixed-Integer Linear Programming model. The objective function is defined as the maximization of the total capacity. The deviation of the variables to the operating point presents a constraint. The scheduling problem related to the jobs that may be realized at several machines is considered by [14]. The new B&B method is proposed, and it is tested on problems with 100 instances.

Another kind of problem that may be solved by B&B is carpooling, which consists of defining the subsets of passengers that will share each vehicle and the routes that the drivers should follow. Several authors introduce the presumption that the vehicles and drivers are not known beforehand. Carpooling problem is treated by [15]. The objective function is formulated as minimizing three different costs. Constrains are given by using 14 linear equations. It has been shown that, by applying the B&B method, the optimal solution for the problem is efficiently obtained. The optimal design of energy supply systems in consideration of multiperiod operation is formulated as a Mixed-Integer Linear Programming task by [37]. The objective function is defined as the sum of the annual capital cost of equipment, the annual demand charge of utilities, and the annual energy charge of utilities per hour at each period. The optimal solution is found from the condition when the goal function reaches a minimum while satisfying all constraints.

One important application of the B&B method is presented in the domain of inventory management [13]. In the presented model, the objective function is defined as integrated profit. The optimal solution is found when the goal function reaches the maximum value while satisfying the set cost limits.

3. The Methodology

In this section, the hybrid model, which integrates panel discussion, fuzzy sets theory, ILP, and Branch-and-Bound algorithm, is presented. The proposed methodology is presented in Figure 1 for one of four BMCGs defined by [4], which is explained in Section 3.2. The methodology is repeated for each BMCG.

During the development of business model theory, many scholars have appointed different BMCs. The finite number and definition of BMCs, in the present research,

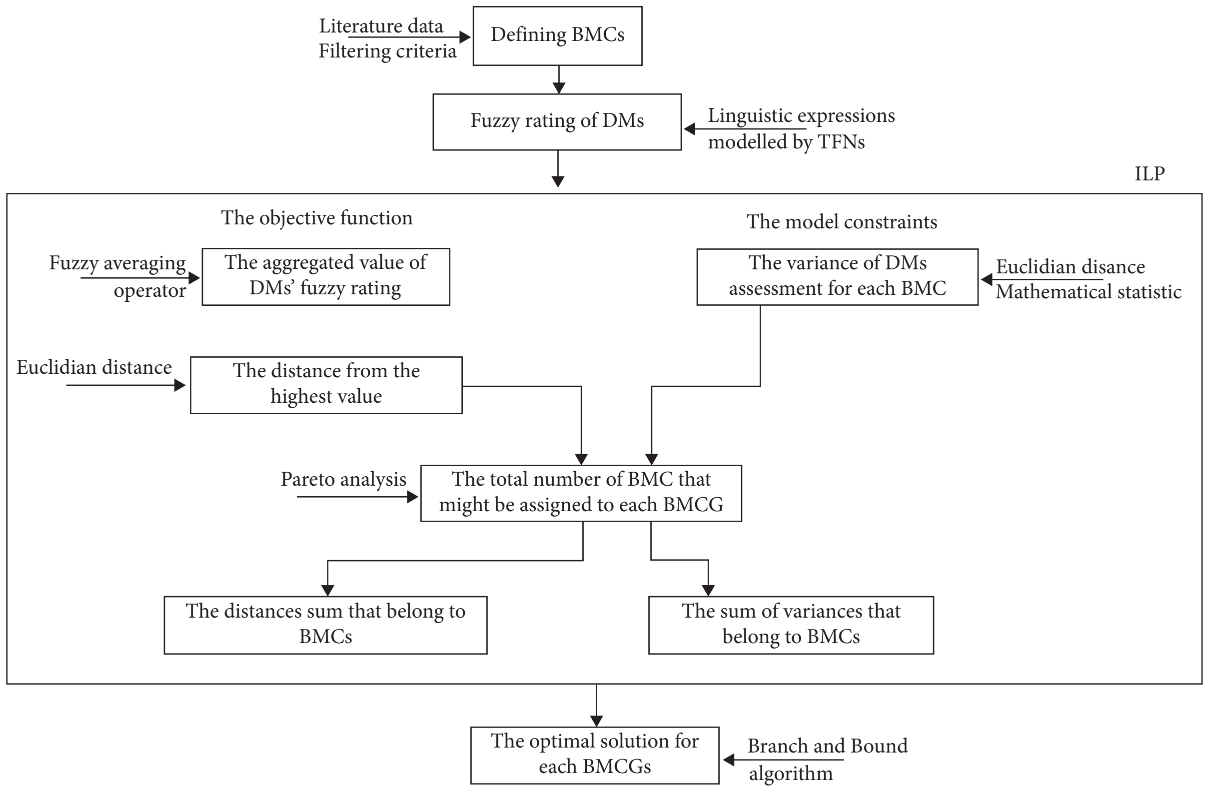


Figure 1: The model for BMCs' allocation to BMCGs.

start with the activities of literature data search and filtering criteria. At the same time, that finite number of identified BMCs should be adjoined to the four BMCGs defined by [4]. In the scope of the proposed research, the four economic sectors are considered (extraction, production, services, and research and development sector). The finite number of DMs is presented in Section 3.3. The DMs should assess the belonging of each identified BMC to the proposed BMCG by using predefined linguistic expressions. Since four groups of DMs corresponds to four different economic sectors, an aggregation procedure should be conducted.

At the level of each BMCG, the DMs are assessing the belonging of treated BMC. The first problem of the proposed methodology is to determine the total number of BMCs that should be assigned to each BMCG. This problem has been treated in literature in an almost negligible manner. Our research presumes that the total number of BMCs that might be assigned to each BMCG is defined by respecting the Pareto analysis. The second problem of the research is how to allocate the identified BMCs to BMCGs in an exact manner. For the course to a solution of this, the ILP model is defined to allocate each BMC to BMCG at the level of each BMCG. The ILP model consists of the objective function and the constraints.

The objective function is defined as a minimum of distances sum that belongs to BMCs derived as the output from the Pareto analysis. The distance from the aggregated value of DMs' fuzzy rating and the highest value that implies belief of certain belonging of BMC to BMCG is calculated.

In the scope of the proposed research, the errors of the DMs' assessment are presented by the variances. The variance of DMs assessment for each BMC at the level of each BMCG is determined. The model constraints are subject to the mean value of the variance of DMs assessment, which should be less than a predefined threshold value. The variance of BMCs that are considered for this calculation corresponds to the output from the Pareto analysis. As the consensus is reached when DMs use 3 consecutive expressions at most, those values are used for the calculation of variance, so the obtained value represents the threshold value.

The optimal solution for each BMCG content is obtained by the ILP model and Branch-and-Bound algorithm.

3.1. Definition of a Finite Set of BMCs. BMCs are formally represented as a set of indexes $\{1, \ldots, i, \ldots, I\}$. The total number of BMCs is designated I and i, $i = 1, .., I$ is an index of BMC. Many studies are dealing with the problem of defining the BMs, which have been published in the last 20 years. By analyzing these studies, 317 BMCs have been identified. Considering the already mentioned issues, it could be noticed that some elements are very similar or very much the same, or many components which have been defined in similar ways have different titles. During the research, the technique of criteria filtering has been applied to consider the mentioned facts. The most important criteria stand if the proposed BMC is an integral part of other BMCs. In this way, the total number of 317 identified BMCs is decreased to 59 unique BMCs that are further considered in the research. To support the genuine of the identified BMCs, their explanation in the existing literature is presented in Table 1.

3.2. Definition of a Finite Set of BMCGs. BMCGs can be formally represented as a set of indexes $j = \{1, \ldots, j, \ldots, J\}$. A total number of BMCGs is designated J and j, $j = 1, .., J$ is an index of BMCG. In this study, BMCGs are determined according to the author's suggestion, the reason being that it gives a very large distribution of terms that are used to define BM. These BMCGs are strategic choices $(j = 1)$, value network $(j = 2)$, creating value $(j = 3)$, and capturing value $(j = 4)$.

3.3. Definition of a Finite Set of DMs. Assessment of BMC i, $i = 1.., I$ belonging to BMCG j, $j = 1, .., J$ on the scale of companies that belong to different economic sectors has been performed by the DMs. In this research, the DM is defined as the representative of the company from four different economic sectors in the Republic of Serbia. The competence of DMs is based on formal education and position in the company. The criterion of formal education is fulfilled if a person holds a master's degree or a higher-level degree. The criterion of position in a company, no matter if it is a private or public company, is fulfilled if a person is ranked at a senior management position or higher. Each of the four economics sectors has 10 representatives from different companies that participate in the panel discussion, which means 40 in total. Each BMC is assessed by different economic sector DMs, so 10 DMs are bringing assessment during the panel discussion by using consensus. The DMs are formally presented by a set of indices $\varepsilon = \{1, \ldots, e, \ldots, E\}$. The total number of DMs is denoted as E, and e, $e = 1, .., E$ is an index of DM.

3.4. Choice of Appropriate Linguistic Variables for Describing the Values of BMCs. In this paper, it is assumed that the DMs expressed their assessments using one of the five predefined linguistic expressions. These linguistic expressions are modeled by TFNs:

It almost does not belong $(S1) = (0, 0, 0.25)$.

Very small degree of belonging $(S2) = (0.05, 0.3, 0.55)$.

Belongs spatially $(S3) = (0.25, 0.5, 0.75)$.

Belongs significantly $(S4) = (0.45, 0.7, 0.95)$.

Almost certainly belongs $(S5) = (0.75, 1, 1)$.

Domains of these TFNs are defined in the real numbers set in the interval $[0, 1]$. The value 0 and value 1 mark that the element i, $i = 1, .., I$ does not belong or that it fully belongs to the group j, $j = 1, .., J$, respectively.

The motivation for employment TFNs is supported by the fact that their usage does not demand complex mathematical operations. Simultaneously, the obtained solutions are accurate in a very sufficient manner taking into account the existing uncertainty in the treated problem.

4. The Proposed Algorithm

The algorithm is executed through the defined steps.

Step 1. A fuzzy rating of DMs can be presented:

$$\tilde{v}_{ij}^{e} = \left(l_{ij}^{e}, m_{ij}^{e}, u_{ij}^{e}\right). \quad (1)$$

Step 2. The aggregated value of DMs' fuzzy rating for each BMC i, $i = 1, .., I$ at the level of each BMCG j, $j = 1, .., J$ is given by applying the fuzzy averaging method:

$$\tilde{v}_{ij} = \frac{1}{E} \cdot \sum_{e=1,..,E} \tilde{v}_{ij}^{e}. \quad (2)$$

According to the rules of fuzzy algebra, $\tilde{v}_{\text{ij}}$ is TFN, too.

Step 3. Let us calculate the variance of fuzzy rating of DMs for each BMC i, $i = 1, .., I$ at the level of each BMCG j, $j = 1, .., J$:

$$s_{ij}^{2} = \frac{1}{E-1} d^{2}\left(\tilde{v}_{ij}^{e}, \tilde{v}_{ij}\right). \quad (3)$$

Step 4. Let us set the ILP problem:

The objective function is as follows:

$$\min_{i'} \sum d\left(\tilde{v}_{ij}, (1, 1, 1)\right), \quad (4)$$

for each j, $j = 1, .., J$,

where $d\ (\tilde{v}_{ij}, (1, 1, 1))$ is calculated as the Euclidean distance between two TFNs [38].

The constraints are as follows:

$$\frac{1}{I'} \cdot \sum_{i=1,\ldots,I'} s_{ij}^{2} \leq \left(\sigma^{2}\right)^{*}. \quad (5)$$

I' is the total number of BMCs that are allocated to each BMCG by respecting the Pareto analysis.

The value of the right side of constraints $(\sigma^2)^*$ is defined as the variance threshold value of the fuzzy rating of DMs. It is a value of the variance where DMs reach consensus. In this research, an assumption is introduced that DMs are reaching consensus if three linguistic expressions in a row are used for fuzzy rating of DMs.

Step 5. By using the Gurobi solver, which is enhanced with the B&B method, the arranged I' set of BMCs is sequentially introduced into BMCGs, the allocation of which is random.

5. Case Study

The input data for the proposed methodology is obtained through the online panel discussion. The period of conducting the research was 2020. As DMs are brought from four economic sectors, the ten DMs from each sector were put together to participate in the panel discussion. At the level of each group of ten panelists, the analysis of the finite list BMCs was performed. In compliance with the principles

Table 1: The review of BMCs according to literature data.

I	Business model components (BMCs)	[17]	[39]	[18]	[6]	[20]	[21]	[40]	[22]	[26]	[28]	[24]	[19]	[27]	[29]	[25]	[23]	[30]	[31]	[1]	[32]
$i=1$	Alliances															X					
$i=2$	Business architecture	X										X									
$i=3$	Brand															X					
$i=4$	Capabilities												X							X	
$i=5$	Capital (capital model)						X													X	
$i=6$	Commerce process model			X																	
$i=7$	Competitors											X									
$i=8$	Connected activities												X								
$i=9$	Core competences																		X		
$i=10$	Cost structure and revenue stream, profit model								X							X				X	
$i=11$	Customer interface						X											X			
$i=12$	Customer relations model						X				X								X		
$i=13$	Customers (customer segments)				X							X									X
$i=14$	Customized (or personalized) services		X																		
$i=15$	Governance																			X	
$i=16$	Delivery channel													X							
$i=17$	Differentiation and strategic control				X																
$i=18$	Distribution, distribution channel			X												X					
$i=19$	Equipment															X					
$i=20$	Finances										X							X			X
$i=21$	Goods and services production and exchanges														X						
$i=22$	Implementation												X								
$i=23$	IS architecture, IT infrastructure		X							X											
$i=24$	Key business components											X									
$i=25$	Legal issues, legalities					X															
$i=26$	Market segment							X													
$i=27$	Marketing strategy	X					X		X												
$i=28$	Mission, mission structure					X															
$i=29$	Norms															X					
$i=30$	Offering		X													X					

Table 1: Continued.

I	Business model components (BMCs)	[17]	[39]	[18]	[6]	[20]	[21]	[40]	[22]	[26]	[28]	[24]	[19]	[27]	[29]	[25]	[23]	[30]	[31]	[1]	[32]
$i = 31$	Organization (form and characteristics)			X																	
$i = 32$	Partner network															X			X	X	X
$i = 33$	People															X					
$i = 34$	Price (scope price, pricing model, and strategies)			X									X		X						
$i = 35$	Process measure (nonfinancial) activity																			X	
$i = 36$	Processes					X										X					
$i = 37$	Procurement																				X
$i = 38$	Profit																		X	X	
$i = 39$	Product innovation										X										
$i = 40$	Product/service	X					X														
$i = 41$	Service provision																				X
$i = 42$	Relationship														X					X	
$i = 43$	Resources (system, pooling, model)		X				X					X									X
$i = 44$	Revenue (model, sources, stream)	X				X	X					X	X	X	X	X				X	X
$i = 45$	Rules and metrics															X					
$i = 46$	Scope				X								X								
$i = 47$	Stakeholder (benefits and network)	X										X		X						X	
$i = 48$	Structure											X									
$i = 49$	Supply chain																	X			
$i = 50$	Sustainability												X		X						
$i = 51$	Sales (target customer, target market)															X			X	X	
$i = 52$	Technology (core investments)		X			X															
$i = 53$	Trading mechanisms, trading protocols									X											
$i = 54$	Value capture				X												X				
$i = 55$	Value chain								X										X		
$i = 56$	Value configuration																			X	
$i = 57$	Value creation design																X				
$i = 58$	Value network, value network configuration								X												
$i = 59$	Value proposition			X			X		X			X	X	X			X	X	X	X	X

of the Affinity Diagram technique, each DMs' group had the goal to affiliate each of 59 BMCs to the 4 BMCGs, considering that each BMC may be affiliated to each BMCG. The decision was brought by using consensus. After the performed panel discussion, the collected data was used as input data for testing the proposed methodology.

Each group of ten DMs has performed independently as is defined in the proposed algorithm. Also, each group of

DMs is supplied with the five predefined linguistic expressions to perform this assessment.

5.1. An Application of the Proposed Model. The algorithm is executed by following the procedure defined in Section 4. The proposed procedure (Step 1 to Step 4 of the proposed algorithm) is illustrated by an example.

Let us assess the degree of belonging of BMC core competences $(i = 9)$ to BMCG strategic choices $(j = 1)$:

$$\begin{aligned} \tilde{v}_{91}^{1} &= S3, \\ \tilde{v}_{91}^{2} &= S3, \\ \tilde{v}_{91}^{3} &= S5, \\ \tilde{v}_{91}^{4} &= S4. \end{aligned} \tag{6}$$

The aggregated value of fuzzy rating of DMs presented for BMC $(i = 9)$ and BMCG $(j = 1)$ is

$$\tilde{v}_{91} = \frac{1}{4} \cdot \{(0.25, 0.5, 0.75) + (0.25, 0.5, 0.75) + (0.75, 1, 1) + (0.45, 0.7, 0.95)\} = (0.425, 0.675, 0.862). \tag{7}$$

The variance of fuzzy rating of DMs is stated as (4.3) so that

$$s_{91}^{2} = \frac{1}{4-1} \cdot \left\{ \begin{array}{l} \frac{1}{3} \cdot \left[(0.25-0.425)^2 + (0.5-0.675)^2 + (0.75-0.862)^2 \right] + \\ \frac{1}{3} \cdot \left[(0.25-0.425)^2 + (0.5-0.675)^2 + (0.75-0.862)^2 \right] + \\ \frac{1}{3} \cdot \left[(0.75-0.425)^2 + (1-0.675)^2 + (1-0.862)^2 \right] + \\ \frac{1}{3} \cdot \left[(0.45-0.425)^2 + (0.7-0.675)^2 + (0.95-0.862)^2 \right] \end{array} \right\} = 0.129. \tag{8}$$

The distance between $\tilde{v}_{91}$ and point $(1, 1, 1)$ is denoted as d_{91}. It is calculated:

$$d_{91} = \sqrt{\frac{\left[(1-0.425)^2 + (1-0.675)^2 + (1-0.862)^2 \right]}{3}} = 0.39. \tag{9}$$

The aggregated values are calculated in a similar way, as well as variance and distances of the rest of BMCs at the level of considered BMCGs Table 2.

Based on the known data from the Pareto analysis, it is known that 20% of the considered items have the greatest importance for the considered problem. Respecting this fact, in this case, 12 BMCs best describe each BMCG. For this purpose, a model was developed whose application makes it possible to allocate BMCs to 4 BMCGs in the exact way as shown in Figure 2. By using Gurobi 9.1 (Step 5 of the proposed algorithm), the optimal solutions are found and presented.

In the scope of the proposed research, the Gurobi solver has employed a gap between the best and possible solutions. The threshold of 0,0% has been introduced, which provides the status of the optimal solution. The analysis of Figure 2 indicates that all four BMCGs have the same importance. In this way, different business models can be described by using the proposed BMCs with creating their descriptions and interactions.

5.2. The Discussion of the Results. As is well known, the use of BMs allows management to understand how it creates value for the customer and how it makes a profit. To understand it better, it is necessary to analyze each BMCG in more detail by using the appropriate BMCs (Figure 2).

The conducted analysis reveals that most of the treated BMCs are defined in Business Model Canvas except the component denoted as resources. This component is broken down into capital (including human capital and intellectual capital), finances, and equipment. In this way, scholars and companies might propose their own view of the business model framework and customize it for their own needs.

Strategic choices

- Business architecture
- Brand
- Capital (capital model)
- Commerce process model
- Governance
- Differentiation and strategic control
- Market segment
- Marketing strategy
- Mission, mission structure
- Organization (form and characteristics)
- Scope
- Sustainability

Values network

- Customer relations model
- Customers (customer segments)
- Delivery channel
- Distribution, distribution channel
- Goods and services production and exchanges
- Partner network
- Procurement
- Service provision
- Relationship
- Stakeholder (benefits and network)
- Supply chain
- Value network, value network configuration

Creating value

- Capabilities
- Connected activities
- Equipment
- Implementation
- Process measure (nonfinancial) Activity
- Processes
- Product/service
- Rules and metrics
- Technology (core investments)
- Value configuration
- Value creation design
- Value proposition

Capturing value

- Core competences
- Cost structure and Revenue stream, profit model
- Customized (or personalized) services
- Finances
- Offering
- Price (scope price, pricing model, and strategies)
- Profit
- Revenue (model, sources, stream)
- Sales (Target customer, target market)
- Trading mechanisms, Trading protocols
- Value capture
- Value chain

FIGURE 2: Allocation of BMCs to each BMCG.

TABLE 2: The variance and distances of the BMCs at the level of considered BMCGs.

Strategic choice			Value network			Creating value			Capturing value		
	d_{i1}	s_{i1}^2		d_{i2}	s_{i2}^2		d_{i3}	s_{i3}^2		d_{i4}	s_{i4}^2
$i = 1$	0.247	0.061	$i = 1$	0.288	0.134	$i = 1$	0.678	0.129	$i = 1$	0.824	0.141
$i = 2$	0.144	0.000	$i = 2$	0.774	0.134	$i = 2$	0.824	0.141	$i = 2$	0.677	0.301
$i = 3$	0.144	0.000	$i = 3$	0.678	0.129	$i = 3$	0.582	0.264	$i = 3$	0.535	0.264
$i = 4$	0.681	0.030	$i = 4$	0.475	0.395	$i = 4$	0.193	0.046	$i = 4$	0.582	0.292
$i = 5$	0.193	0.046	$i = 5$	0.725	0.268	$i = 5$	0.587	0.110	$i = 5$	0.824	0.141
$i = 6$	0.144	0.000	$i = 6$	0.725	0.268	$i = 6$	0.494	0.110	$i = 6$	0.725	0.268
$i = 7$	0.304	0.046	$i = 7$	0.464	0.566	$i = 7$	0.873	0.046	$i = 7$	0.569	0.566
$i = 8$	0.774	0.134	$i = 8$	0.582	0.292	$i = 8$	0.144	0.000	$i = 8$	0.678	0.209
$i = 9$	0.390	0.129	$i = 9$	0.475	0.395	$i = 9$	0.435	0.221	$i = 9$	0.678	0.129
$i = 10$	0.363	0.000	$i = 10$	0.774	0.134	$i = 10$	0.678	0.209	$i = 10$	0.144	0.000
$i = 11$	0.523	0.445	$i = 11$	0.823	0.061	$i = 11$	0.346	0.097	$i = 11$	0.416	0.495
$i = 12$	0.304	0.046	$i = 12$	0.193	0.046	$i = 12$	0.774	0.134	$i = 12$	0.774	0.134
$i = 13$	0.304	0.046	$i = 13$	0.193	0.046	$i = 13$	0.678	0.129	$i = 13$	0.678	0.129
$i = 14$	0.775	0.046	$i = 14$	0.523	0.445	$i = 14$	0.390	0.209	$i = 14$	0.575	0.296
$i = 15$	0.144	0.000	$i = 15$	0.774	0.296	$i = 15$	0.677	0.301	$i = 15$	0.823	0.061
$i = 16$	0.592	0.221	$i = 16$	0.193	0.046	$i = 16$	0.428	0.405	$i = 16$	0.824	0.141

Table 2: Continued.

Strategic choice			Value network			Creating value			Capturing value		
$i=17$	0.144	0.000	$i=17$	0.725	0.268	$i=17$	0.729	0.000	$i=17$	0.725	0.268
$i=18$	0.587	0.110	$i=18$	0.193	0.046	$i=18$	0.428	0.405	$i=18$	0.774	0.134
$i=19$	0.435	0.221	$i=19$	0.667	0.483	$i=19$	0.288	0.134	$i=19$	0.873	0.046
$i=20$	0.346	0.097	$i=20$	0.873	0.046	$i=20$	0.824	0.141	$i=20$	0.193	0.046
$i=21$	0.727	0.152	$i=21$	0.247	0.061	$i=21$	0.247	0.061	$i=21$	0.823	0.061
$i=22$	0.582	0.212	$i=22$	0.725	0.268	$i=22$	0.144	0.000	$i=22$	0.823	0.061
$i=23$	0.390	0.209	$i=23$	0.376	0.301	$i=23$	0.540	0.160	$i=23$	0.774	0.134
$i=24$	0.435	0.221	$i=24$	0.677	0.301	$i=24$	0.376	0.301	$i=24$	0.582	0.292
$i=25$	0.247	0.061	$i=25$	0.667	0.483	$i=25$	0.824	0.141	$i=25$	0.824	0.141
$i=26$	0.247	0.061	$i=26$	0.630	0.221	$i=26$	0.481	0.212	$i=26$	0.535	0.264
$i=27$	0.231	0.141	$i=27$	0.727	0.097	$i=27$	0.449	0.120	$i=27$	0.435	0.221
$i=28$	0.144	0.000	$i=28$	0.824	0.141	$i=28$	0.824	0.141	$i=28$	0.774	0.134
$i=29$	0.273	0.296	$i=29$	0.823	0.061	$i=29$	0.571	0.395	$i=29$	0.774	0.296
$i=30$	0.494	0.110	$i=30$	0.587	0.030	$i=30$	0.369	0.483	$i=30$	0.288	0.134
$i=31$	0.193	0.046	$i=31$	0.587	0.110	$i=31$	0.727	0.097	$i=31$	0.774	0.134
$i=32$	0.449	0.040	$i=32$	0.144	0.000	$i=32$	0.774	0.134	$i=32$	0.774	0.296
$i=33$	0.331	0.188	$i=33$	0.390	0.129	$i=33$	0.428	0.405	$i=33$	0.725	0.268
$i=34$	0.405	0.030	$i=34$	0.873	0.046	$i=34$	0.774	0.296	$i=34$	0.144	0.000
$i=35$	0.634	0.120	$i=35$	0.681	0.030	$i=35$	0.193	0.046	$i=35$	0.725	0.268
$i=36$	0.587	0.110	$i=36$	0.727	0.097	$i=36$	0.144	0.000	$i=36$	0.678	0.209
$i=37$	0.727	0.097	$i=37$	0.247	0.061	$i=37$	0.416	0.495	$i=37$	0.628	0.394
$i=38$	0.774	0.134	$i=38$	0.725	0.188	$i=38$	0.628	0.394	$i=38$	0.144	0.000
$i=39$	0.390	0.129	$i=39$	0.630	0.221	$i=39$	0.369	0.483	$i=39$	0.725	0.268
$i=40$	0.571	0.395	$i=40$	0.405	0.030	$i=40$	0.231	0.141	$i=40$	0.540	0.160
$i=41$	0.774	0.134	$i=41$	0.247	0.061	$i=41$	0.346	0.129	$i=41$	0.304	0.046
$i=42$	0.727	0.097	$i=42$	0.144	0.000	$i=42$	0.727	0.097	$i=42$	0.582	0.292
$i=43$	0.288	0.134	$i=43$	0.510	0.708	$i=43$	0.449	0.040	$i=43$	0.824	0.141
$i=44$	0.494	0.110	$i=44$	0.571	0.395	$i=44$	0.774	0.134	$i=44$	0.310	0.531
$i=45$	0.435	0.141	$i=45$	0.823	0.061	$i=45$	0.247	0.061	$i=45$	0.774	0.296
$i=46$	0.144	0.000	$i=46$	0.475	0.395	$i=46$	0.677	0.301	$i=46$	0.677	0.301
$i=47$	0.405	0.030	$i=47$	0.231	0.141	$i=47$	0.678	0.129	$i=47$	0.527	0.264
$i=48$	0.247	0.061	$i=48$	0.582	0.212	$i=48$	0.571	0.395	$i=48$	0.873	0.046
$i=49$	0.727	0.097	$i=49$	0.144	0.000	$i=49$	0.449	0.120	$i=49$	0.774	0.296
$i=50$	0.144	0.000	$i=50$	0.678	0.129	$i=50$	0.725	0.268	$i=50$	0.873	0.046
$i=51$	0.346	0.129	$i=51$	0.390	0.209	$i=51$	0.630	0.221	$i=51$	0.523	0.445
$i=52$	0.304	0.046	$i=52$	0.774	0.134	$i=52$	0.193	0.046	$i=52$	0.540	0.080
$i=53$	0.523	0.365	$i=53$	0.435	0.221	$i=53$	0.678	0.209	$i=53$	0.331	0.268
$i=54$	0.677	0.301	$i=54$	0.824	0.141	$i=54$	0.774	0.296	$i=54$	0.144	0.000
$i=55$	0.523	0.445	$i=55$	0.390	0.129	$i=55$	0.369	0.483	$i=55$	0.630	0.141
$i=56$	0.540	0.160	$i=56$	0.678	0.209	$i=56$	0.144	0.000	$i=56$	0.824	0.141
$i=57$	0.582	0.292	$i=57$	0.582	0.292	$i=57$	0.144	0.000	$i=57$	0.774	0.134
$i=58$	0.628	0.394	$i=58$	0.144	0.000	$i=58$	0.535	0.264	$i=58$	0.824	0.141
$i=59$	0.569	0.566	$i=59$	0.582	0.292	$i=59$	0.193	0.046	$i=59$	0.582	0.292

From the managerial implications' perspective, it should be noticed that business product/service added value is crucial. Creating value is described by enabling conditions, value proposition and configuration, implementation, and establishing processes, so different strategic concepts may be further employed to derive the rest of business activities. Capturing value is described through core competences, revenue model, value chain, and other value and finance components. Above all, managers, while creating value, should create and maintain a value network.

6. Conclusion

In the scope of the research, extensive literature regarding the BM domain is considered. By applying the criteria filtering technique, 59 BMCs that are mostly used in literature have been identified. In the research, DMs from various industry companies have been assessing the significance of identified BMCs at the level of each BMCG. DMs used one of the five predefined linguistic expressions, which were modeled by TFNs. The assessment problem is stated as a fuzzy group decision-making problem. Since it is considered that all DMs have equal importance, the aggregated value of the fuzzy rating of DMs is given by using the fuzzy averaging operator.

Allocation problem is stated as ILP and appropriate BMCs at the level of each BMCG by using B&B. In this way, the obtained solutions are less burdened with DMs prejudices than in papers that can be found in literature.

The contributions of this research could be denoted as theoretical. The contributions in the theoretical domain are as follows: (1) the analysis of BMCs that have been defined in the last 20 years and have been systematized and (2) methodological enhancement of affinity diagram. The enhancement of affinity diagram embraces (i) assessment of the belonging to each proposed group, which is based on usage of linguistic expressions by DMs, so it implies accurate assessment, (ii) modeling linguistic terms, which is based on fuzzy sets theory, and (iii) allocation of BMCs to BMCGs, which is determined in an exact way.

The proposed procedure can be used to analyze business enterprises that exist in different economic domains. This can be marked as the practical contribution of the study. The proposed methodology could be used for solving different management problems where the same relative importance of DMs is employed.

The main constraint of the proposed method is that DMs must have significant knowledge and experience in different areas to correctly conduct an assessment. As the model is large-sized, DMs need to spend significant time to complete the survey. Also, it is worth mentioning that each BMC is not uniquely defined as different scholars suggest the diverse scope of each BMC.

Future research should be focused on the determination of the relationship between BMCs under each BMCG. In this way, the improvement of BMs may be achieved by enhancing business processes derived from BMCs, or through the reengineering of the BMs by applying different frameworks. The interaction of the business processes could be performed through different analyses, such as as-is process analysis.

Data Availability

All data generated or analyzed during this study are included in this paper.

Conflicts of Interest

The authors declare no conflicts of interest.

References

[1] A. Upward and P. Jones, "An ontology for strongly sustainable business models: defining an enterprise framework compatible with natural and social science," *Organization & Environment*, vol. 29, no. 1, pp. 97–123, 2015.

[2] T. Ritter and C. Lettl, "The wider implications of business-model research," *Long Range Planning*, vol. 51, no. 1, pp. 1–8, 2018.

[3] M. M. Al-Debei and D. Avison, "Developing a unified framework of the business model concept," *European Journal of Information Systems*, vol. 19, no. 3, pp. 359–376, 2010.

[4] S. M. Shafer, H. J. Smith, and J. C. Linder, "The power of business models," *Business Horizons*, vol. 48, no. 3, pp. 199–207, 2005.

[5] Y. Zhang, S. Zhao, and X. Xu, "Business model innovation: an integrated approach based on elements and functions," *Information Technology and Management*, vol. 17, no. 3, pp. 303–310, 2015.

[6] D. W. Stewart and Q. Zhao, "Internet marketing, business models, and public policy," *Journal of Public Policy and Marketing*, vol. 19, no. 2, pp. 287–296, 2018.

[7] S. Widjajanto and E. Rimawan, "Modified failure mode and effect analysis approaching to improve organization performance based on baldrige criteria - a case study of an electromedic industry," *Operational Research in Engineering Sciences: Theory and Applications*, vol. 4, no. 3, pp. 39–58, 2021.

[8] H.-J. Zimmermann, "Fuzzy set theory," *Wiley Interdisciplinary Reviews: Computational Statistics*, vol. 2, no. 3, pp. 317–332, 2010.

[9] W. He, R. M. Rodríguez, B. Dutta, and L. Martínez, "A type-1 OWA operator for extended comparative linguistic expressions with symbolic translation," *Fuzzy Sets and Systems*, 2021, in Press.

[10] S. Nestic, J. F. Lampón, A. Aleksic, P. Cabanelas, and D. Tadic, "Ranking manufacturing processes from the quality management perspective in the automotive industry," *Expert Systems*, vol. 36, no. 6, p. e12451, 2019.

[11] D. Tadic, A. Aleksic, P. Mimovic, H. Puskaric, and M. Misita, "A model for evaluation of customer satisfaction with banking service quality in an uncertain environment," *Total Quality Management and Business Excellence*, vol. 29, no. 11–12, pp. 1342–1361, 2016.

[12] E.-G. Talbi, *Metaheuristics: From Design to Implementation*, John Wiley & Sons, Hoboken, NJ, USA, 2009.

[13] B. Khara, J. K. Dey, and S. K. Mondal, "An integrated imperfect production system with advertisement dependent demand using branch and bound technique," *Flexible Services and Manufacturing Journal*, vol. 33, no. 2, pp. 508–546, 2021.

[14] O. Ozturk, M. A. Begen, and G. S. Zaric, "A branch and bound algorithm for scheduling unit size jobs on parallel batching machines to minimize makespan," *International Journal of Production Research*, vol. 55, no. 6, pp. 1815–1831, 2016.

[15] M. Tamannaei and I. Irandoost, "Carpooling problem: a new mathematical model, branch-and-bound, and heuristic beam search algorithm," *Journal of Intelligent Transportation Systems*, vol. 23, no. 3, pp. 203–215, 2018.

[16] R. Walter and P. Schulze, "On the performance of task-oriented branch-and-bound algorithms for workload smoothing in simple assembly line balancing," *International Journal of Production Research*, pp. 1–14, 2021.

[17] P. Timmers, "Business models for electronic markets," *Electronic Markets*, vol. 8, no. 2, pp. 3–8, 1998.

[18] J. Linder and S. Cantrell, *Changing Business Models: Surveying the Landscape*Accenture Institute for Strategic Change, Cambridge, UK, 2000.

[19] A.. Afuah, *Business Models: A Strategic Management Approach*, McGraw-Hill/Irwin, New York, NY, USA, 2004.

[20] R. Alt and H.-D. Zimmermann, "Preface: introduction to special section - business models," *Electronic Markets*, vol. 11, no. 1, pp. 3–9, 2001.

[21] O. Petrovic, C. Kittl, and R. D. Teksten, "Developing business models for ebusiness," *SSRN Electronic Journal*, 2001.

[22] H. Chesbrough, R. S. Rosenbloom, and J. S. Brown, "The role of the business model in capturing value from innovation: evidence from xerox corporation's technology spinoff companies," *Industrial and Corporate Change*, vol. 11, pp. 529–555, 2002.

[23] J. Richardson, "The business model: an integrative framework for strategy execution," *Strategic Change*, vol. 17, no. 5–6, pp. 133–144, 2008.

[24] J. Hedman and T. Kalling, "The business model concept: theoretical underpinnings and empirical illustrations," *European Journal of Information Systems*, vol. 12, no. 1, pp. 49–59, 2003.

[25] M. W. Johnson, C. M. Christensen, and H. Kagermann, "Reinventing your business model," *Harvard Business Review*, vol. 86, pp. 50–59, 2008.

[26] K. Lyytinen and S. McGann, "Capturing the dynamics of eBusiness models: the eBusiness analysis framework and the electronic trading infrastructure," in *Proceedings of the 15th Annual Bled Electronic Commerce Conference*, pp. 36–54, Bled, Slovenia, 2002.

[27] A. Osterwalder and Y. Pigneur, "15th bled electronic commerce conference e-reality: constructing the e-economy an e-business model ontology for modeling e-business," in *Proceedings of the Electronic Commerce Conference – EReality: Constructing the EEconomy*, pp. 75–91, Bled, Slovenia, 2002.

[28] A. Osterwalder and Y. Pigneur, "An ontology for e-business models," in *Value Creation from E-Business Models*, W. Currie, Ed., pp. 65–97, Butterworth-Heinemann, Lausanne, Switzerland, 2003.

[29] E. Brousseau and T. Penard, "The economics of digital business models: a framework for analyzing the economics of platforms," *Review of Network Economics*, vol. 6, no. 2, pp. 81–114, 2007.

[30] F. Boons and F. Lüdeke-Freund, "Business models for sustainable innovation: state-of-the-art and steps towards a research agenda," *Journal of Cleaner Production*, vol. 45, pp. 9–19, 2013.

[31] Y. Taran, H. Boer, and P. Lindgren, "A business model innovation typology," *Decision Sciences*, vol. 46, no. 2, pp. 301–331, 2015.

[32] B. W. Wirtz, A. Pistoia, S. Ullrich, and V. Göttel, "Business models: origin, development and future research perspectives," *Long Range Planning*, vol. 49, no. 1, pp. 36–54, 2016.

[33] D. Tadić, S. Arsovski, A. Aleksić, M. Stefanović, and S. Nestić, "A fuzzy evaluation of projects for business processes' quality improvement," *Intelligent Systems Reference Library*, vol. 87, pp. 559–579, 2015.

[34] Z. Ali, T. Mahmood, T. Mahmood, K. Ullah, and Q. Khan, "Einstein geometric aggregation operators using a novel complex interval-valued pythagorean fuzzy setting with application in green supplier chain management," *Reports in Mechanical Engineering*, vol. 2, no. 1, pp. 105–134, 2021.

[35] J. Bukchin and J. Rubinovitz, "A weighted approach for assembly line design with station paralleling and equipment selection," *IIE Transactions*, vol. 35, pp. 73–85, 2003.

[36] B. Mayer, M. Killian, and M. Kozek, "A branch and bound approach for building cooling supply control with hybrid model predictive control," *Energy and Buildings*, vol. C, no. 128, pp. 553–566, 2016.

[37] R. Yokoyama, Y. Shinano, Y. Wakayama, and T. Wakui, "Model reduction by time aggregation for optimal design of energy supply systems by an MILP hierarchical branch and bound method," *Energy*, vol. 181, pp. 782–792, 2019.

[38] P. Grzegorzewski, "Distances between intuitionistic fuzzy sets and/or interval-valued fuzzy sets based on the Hausdorff metric," *Fuzzy Sets and Systems*, vol. 148, no. 2, pp. 319–328, 2004.

[39] S. Bagchi and B. Tulskie, "E-Business models: integrating learning from strategy development experiences and empirical research," in *Proceedings of the 20th Annual International Conference of the Strategic Management Society*, pp. 15–18, Vancouver, Canada, 2000.

[40] J. F. Rayport and B. J. Jaworski, *Introduction to E-Commerce*, McGraw-Hill Irwin MarketspaceU, Boston, MA, USA, 2004.

Hindawi
Mathematical Problems in Engineering
Volume 2022, Article ID 2659826, 15 pages
https://doi.org/10.1155/2022/2659826

Research Article

Energy Source Allocation Decision-Making in Textile Industry: A Novel Symmetric and Asymmetric Spherical Fuzzy Linear Optimization Approach

Sajida Kousar,[1] **Urooj Shafqat**,[1] **Nasreen Kausar**,[2] **Dragan Pamucar**,[3] **and Yaé Ulrich Gaba**[4,5,6]

[1]*Department of Mathematics and Statistics, International Islamic University Islamabad, Pakistan*
[2]*Department of Mathematics, Faculty of Arts and Sciences, Yildiz Technical University, Esenler, 34210, Istanbul, Turkey*
[3]*Department of Logistics, University of Defence in Belgrade, Belgrade, Serbia*
[4]*Quantum Leap Africa (QLA), AIMS Rwanda Center, Remera Sector KN 3, Kigali, Rwanda*
[5]*Institut de Mathématiques et de Sciences Physiques (IMSP/UAC), Laboratoire de Topologie Fondamentale, Computationnelle et leurs Applications (Lab-ToFoCApp), BP 613, Porto-Novo, Benin*
[6]*African Center for Advanced Studies, P.O. Box 4477, Yaounde, Cameroon*

Correspondence should be addressed to Yaé Ulrich Gaba; yaeulrich.gaba@gmail.com

Received 3 November 2021; Revised 20 December 2021; Accepted 17 January 2022; Published 16 February 2022

Academic Editor: Akif Akgul

In this study, the authors extended the concept of spherical fuzzy optimization models by considering different parameters of spherical fuzzy linear programming problem as symmetric and asymmetric spherical numbers. Eight spherical fuzzy linear programming models are discussed by converting decision variables, parameters, and coefficients of objective function and constraints into symmetric and asymmetric spherical fuzzy numbers. To verify the validity and efficiency of this study in contrast with a linear programming numerical and a physical energy optimization model for the textile industry is considered. The application of these symmetric and asymmetric spherical fuzzy optimization models is discussed along with the postoptimal analysis of the best optimization models that provide the feasible and most optimal solution.

1. Introduction

Growing urbanization is directly related to the increase in energy demands, usage, and cost. Energy optimization is globally targeted by every sector. Mostly, an industrial sector is the one who consumed most of the produced or natural energy. The key factor of rising production cost in the industrial sector is abrupt energy usage. Since industrial sectors cannot ignore this factor, getting help through mathematical modeling such as optimizing cost, profits, loss, and energy for such matters is sane act. A lot of work is performed for the optimal utilization of energy in different areas as Wang *et al.* presented their general guidelines regarding energy optimization in iron and steel industry by using mass-thermal network optimization [1]. Ullah *et al.* presented bio-inspired energy optimization techniques with the purpose of power scheduling in an office [2]. According to Ozturk *et al.*, energy consumption could be decreased by using the waste-heat recovery systems for the industrial sector so they presented eighty-five techniques for the reduction of energy consumption in their study where thirteen of them were prioritized and applied as energy-efficient techniques [3]. Kimutai [4] proposed the physical energy optimization model for the textile industry and optimized the energy cost by using linear programming (LP). In the manufacturing sector, textile industries are considered major energy consumption units globally due to their several production stages. In the textile industry, mostly electricity and fuel, such as charcoal and petroleum, are used to create all the required kinds of energy. In Pakistan, this particular industrial sector is having a great share up to 8.5% towards GDP (gross domestic product) and considered Asia's 8th largest textile exporter [5]. Pakistan is having a huge textile industrial sector and now facing many uncertainties due to the unpredicted policy shift 2020–25 and COVID. According to National Electric Power Regulatory Authority's (NEPRA) report, one energy unit fluctuation cost causes almost 4 to 5 hours closure in the production of textile's products [6]. To overcome this loss, it is best to optimize the usage and wastage of energy as much as possible. The most extensively adopted procedure for the optimal solution of modeled problem was linear programming (LP) due to its easy applicable nature that was first introduced by Kantorovich [7]. Advancement in this traditional LP generated several extensions such as bi-level LP, multilevel LP, and multiobjective. These LP extensions are highly applicable in real life such as it gave optimized solutions for transportation, supply chain, energy, profit, loss, and cost optimization problems.

Huge modification happened in linear programming after the introduction of fuzzy sets by Zadeh [8]. Fuzzy linear programming was introduced by Zimmerman [9], who originated the technique to solve the multiobjective linear programming in a fuzzy environment. This method was defined according to the natural environmental uncertainties as all the optimization conditions can be considered in fuzzy. Improvements are continuously occurring till now; firstly, membership degree was considered well enough to understand decision-makers choices, but Atanssove [10] created an intuitionistic fuzzy set dealing with the degree of membership and nonmembership clearly, recognizing the choice of an element from the decision set. This definition of the intuitionistic fuzzy set became another reason for improvement in optimization techniques, and firstly, intuitionistic fuzzy (IF) optimization got revealed by Angelov [11]. A lot of work has been carried out in intuitionistic fuzzy linear programming (IFLP). Afterwards, Yager presented the concept of another generalization of fuzzy sets and named it Pythagorean fuzzy set by refining the condition that membership and nonmembership can be independent of each other and their sum of squares must be less than 1 [12]. In 1999, Smarandache [13] introduced a neutrosophic set, which covers the third predictable choice of decision-makers that might be neutral or indeterminacy. In the neutrosophic environment, many optimization models were considered and solved by Ahmad et al. [14, 15].

Recently, a spherical fuzzy set has been introduced by Gundogdu and Kahraman [16]. A spherical fuzzy set is defined with the compliance of positive, neutral, and negative membership functions under the condition that the sum of their squares must be less than 1 providing more general way to cope with uncertainty. It is considered that a spherical fuzzy set is a superset of fuzzy, Pythagorean fuzzy, and intuitionistic fuzzy sets. Ahmad and Adhami [17] presented their work on spherical fuzzy linear programming problem (SFLPP). They presented different types of optimization models under the spherical fuzzy (SF) environment. In this study, we are presenting symmetric and asymmetric energy optimization models inspired by the work of Ahmad and Adhami [17]. For this purpose, the LP model for the textile industry is considered in the spherical fuzzy environment as a numeric example to validate the working of generated energy optimization models in the SF environment. For the conversion of LP into SFLP, parameters were considered spherical fuzzy numbers (SFNs). By targeting each parameter one by one, different SF optimization models are constructed. Every model further contains two submodels in it on the basis of symmetric spherical fuzzy number (SSFN) and asymmetric spherical fuzzy numbers (ASFN) parameters. The deterministic version corresponding to SFNs is based on the spherical fuzzy set theory. Conclusions are based on the application of these spherical fuzzy models on the energy optimization model. The postoptimal analysis of the best feasible optimized SF model is also discussed.

2. Preliminaries

A spherical fuzzy set (SFS) is defined by Rafiq et al. [18] as the following set:

$$\left\{\langle u, p_{\widetilde{SF}}(u), n_{\widetilde{SF}}(u), f_{\widetilde{SF}}(u)\rangle : u \in U\right\}. \tag{1}$$

Considering U the universal discourse and $\widetilde{SF}$ representing spherical fuzzy set such that

$$\begin{aligned} &p_{\widetilde{SF}}: U \longrightarrow [0,1],\\ &n_{\widetilde{SF}}: U \longrightarrow [0,1],\\ &f_{\widetilde{SF}}: U \longrightarrow [0,1] \text{ with } 0 \leq p^2_{\widetilde{SF}}(u) + n^2_{\widetilde{SF}}(u) + f^2_{\widetilde{SF}}(u) \leq 1 \forall u \in U, \end{aligned} \tag{2}$$

where $p_{\widetilde{SF}}(u)$ is positive membership degree, and $n_{\widetilde{SF}}(u)$ and $f_{\widetilde{SF}}(u)$ is representing neutral and negative membership degree of each $u \in U$, respectively, to $\widetilde{SF}$. A spherical fuzzy number is a fuzzy number $\widetilde{r} = \langle p_{\widetilde{r}}, n_{\widetilde{r}}, f_{\widetilde{r}}\rangle$ with positive, neutral, and negative membership functions defined as

$$\left(p_{\widetilde{SF}}(u), n_{\widetilde{SF}}(u), f_{\widetilde{SF}}(u)\right) = \begin{cases} \left(\dfrac{(x-p_{\tilde{r}})\alpha}{n_{\tilde{r}}-p_{\tilde{r}}}, \dfrac{(x-p_{\tilde{r}})\beta}{n_{\tilde{r}}-p_{\tilde{r}}}, \dfrac{(x-p_{\tilde{r}})\gamma}{n_{\tilde{r}}-p_{\tilde{r}}}\right), & \text{if; } p_{\tilde{r}} \le u < n_{\tilde{r}}, \\ \left(\dfrac{(f_{\tilde{r}}-x)\alpha}{f_{\tilde{r}}-n_{\tilde{r}}}, \dfrac{(f_{\tilde{r}}-x)\beta}{f_{\tilde{r}}-n_{\tilde{r}}}, \dfrac{(f_{\tilde{r}}-x)\gamma}{f_{\tilde{r}}-n_{\tilde{r}}}\right), & \text{if; } n_{\tilde{r}} \le u \le f_{\tilde{r}}, \\ (0,0,0), & \text{otherwise.} \end{cases} \tag{3}$$

Here, $\alpha, \beta, \gamma \in [0,1]$ such that $0 \le \alpha^2 + \beta^2 + \gamma^2 \le 1$. Let $\tilde{r} = \langle p_{\tilde{r}}, n_{\tilde{r}}, f_{\tilde{r}} \rangle$ and $\tilde{m} = \langle p_{\tilde{r}}, n_{\tilde{r}}, f_{\tilde{r}} \rangle$ are two spherical fuzzy numbers, and then, the algebraic operations [18] between them are defined as follows:

(1) $\tilde{r} \oplus \tilde{m} = \langle \sqrt{p_{\tilde{r}}^2 + p_{\tilde{m}}^2 - p_{\tilde{r}}^2 . p_{\tilde{m}}^2}, n_{\tilde{r}} . n_{\tilde{m}}, f_{\tilde{r}} . f_{\tilde{m}} \rangle$

(2) $\tilde{r} \circledast \tilde{m} = \langle p_{\tilde{r}} . p_{\tilde{m}}, n_{\tilde{r}} . n_{\tilde{m}}, \sqrt{f_{\tilde{r}}^2 + f_{\tilde{m}}^2 - f_{\tilde{r}}^2 . f_{\tilde{m}}^2} \rangle$

(3) $\kappa \tilde{r} = \langle \sqrt{1 - (1 - p_{\tilde{r}}^2)^\kappa}, (n_{\tilde{r}})^\kappa, (f_{\tilde{r}})^\kappa \rangle; \quad \text{for } \kappa \ge 0$

A SFN will be considered symmetric spherical fuzzy number (SSFN) if there exists a relation between positive and neutral, positive, and negative membership. For example,

$$\begin{aligned} \tilde{r} &= \langle p_{\tilde{r}}, n_{\tilde{r}}, f_{\tilde{r}} \rangle \\ &= \langle p_{\tilde{r}}, p_{\tilde{r}} + \lambda, p_{\tilde{r}} + k\lambda \rangle, \end{aligned} \tag{4}$$

where $\lambda > 0$, $k > 1$, and $\lambda, k \in \mathbb{R}$; otherwise, it is considered asymmetric spherical fuzzy number (ASFN).

3. Spherical Fuzzy Linear Programming Problem

Ideally for optimal solution of mathematically modeled problem, linear programming (LP) is considered the most convenient way [7]. Since this LP does not accommodate the fuzziness of nature, the best real-life modeled problem solution requires a method of fuzzy optimization. A lot of work is already carried out for fuzzy optimization modeling by utilizing different techniques such as intuitionistic fuzzy linear programming [11, 12] and neutrosophic LP [14–19]. Ahmad and Adhami presented different models for the solution of spherical fuzzy LP [17]. By continuing their idea for SF modeling, different spherical fuzzy models are constructed in this study. In the first model, only constraint coefficients were considered spherical fuzzy numbers, whereas all the other decision variables and parameters are real quantities. In the second model, two factors demand and constraint coefficients are taken as SFNs, while cost is taken as a real number. In the third model, other than decision variables, all the other factors are considered in the spherical fuzzy number, whereas in the fourth model, the cost and demand are in SF numbers. Table 1 is designed to illustrate all these cases.

In Table 1, $\widetilde{S^{ssf}}$, $\widetilde{D^{ssf}}$, and $\widetilde{C^{ssf}}$ are symmetric spherical fuzzy, $\widetilde{S^{asf}}$, $\widetilde{D^{asf}}$, and $\widetilde{C^{asf}}$ asymmetric spherical fuzzy parameters, and s, d, c real-valued parameters.

4. Numerical Example

Consider the following linear programming problem:

$$\begin{aligned} &\text{Max } z = 5x + 3y \\ &\text{subjected to,} \end{aligned} \tag{5}$$

$$3x + 5y \le 15, \tag{6}$$

$$5x + 2y \le 10. \tag{7}$$

The SSF and ASF for the above LP are presented in Table 2.

From Table 3, it is clear that SSF model-I results in the highest optimal solution value. Since the solution of LP is 12.3684 and we are looking for more better feasible solution, all those models whose values are greater; that is, higher than LP output is considered better. Here, in Table 3, we obtain that

$$\begin{aligned} &\text{LP} < \text{ASF Model} - II < \text{SSF Model} - II < \text{FLP} < \text{IFLP} \\ &\quad < \text{ASF Model} - I < \text{SSF Model} - I, \end{aligned} \tag{8}$$

where the remaining models result in a value less than LP solution so these are not considered better than LP. All the models are providing feasible solution, and the best one is provided by SSF model-I as it results in the highest objective output.

5. Application

To elaborate the working efficiency of the above-defined SSF and ASF optimization models of our study, we construct an energy optimization model for the textile industry with five stages shown in Figure 1.

Suppose X_i is the number of units of product that processed at stage i. In the objective function, the cost coefficients are according to the type of energy used for the preparation of per unit product of stage i. D_i is the monthly demand of each product X_i and availability of working hours that helped to form the following demand constraint equations according to the stages presented in Figure 1:

$$\begin{aligned} x_1 - x_2 &\ge 400, \\ 0.03x_2 - x_3 + 0.07x_4 &= 0, \\ 0.97x_2 - x_4 &= 0, \\ 0.93x_4 - x_5 &\ge 600, \\ 0..96x_5 &\ge 20000, \\ 0.007x_1 - 0.07x_2 + 0.013x_4 + 0.0062x_5 &\le 720. \end{aligned} \tag{9}$$

Each stage in Figure 1 is also presenting the production cost per unit of each stage's product along with information about how much quantity is going to be processed further in

Table 1: Spherical fuzzy linear programming models.

Model	Symmetric spherical fuzzy model (a)	Asymmetric spherical fuzzy model (b)
I	Optimize $\alpha = \sum_{i=1}^{K} c_i x_i$ subjected to $\Sigma_{i=1}^{K} \widetilde{S_{ij}^{ssf}} x_i \leq \ = \ \geq d_j, \forall j = 1, 2, 3 \ldots l$	Optimize $\alpha = \sum_{i=1}^{K} c_i x_i$ subjected to $\Sigma_{i=1}^{K} \widetilde{S_{ij}^{asf}} x_i \leq \ = \ \geq d_j, \forall j = 1, 2, 3 \ldots l$
II	Optimize $\alpha = \sum_{i=1}^{K} c_i x_i$ subjected to $\Sigma_{i=1}^{K} \widetilde{S_{ij}^{ssf}} x_i \leq \ = \ \geq \widetilde{D_j^{ssf}}$, $\forall j = 1, 2, 3 \ldots l$	Optimize $\alpha = \sum_{i=1}^{K} c_i x_i$ subjected to $\Sigma_{i=1}^{K} \widetilde{S_{ij}^{asf}} x_i \leq \ = \ \geq \widetilde{D_j^{asf}}, \forall j = 1, 2, 3 \ldots l$
III	Optimize $\alpha = \Sigma_{i=1}^{K} \widetilde{C_i^{ssf}} x_i$ subjected to $\Sigma_{i=1}^{K} \widetilde{S_{ij}^{ssf}} x_i \leq \ = \ \geq \widetilde{D_j^{ssf}}, \forall j = 1, 2, 3 \ldots l$	Optimize $\alpha = \Sigma_{i=1}^{K} \widetilde{C_i^{asf}} x_i$ subjected to $\Sigma_{i=1}^{K} \widetilde{S_{ij}^{asf}} x_i \leq \ = \ \geq \widetilde{D_j^{asf}}, \forall j = 1, 2, 3 \ldots l$
IV	Optimize $\alpha = \Sigma_{i=1}^{K} \widetilde{C_i^{ssf}} x_i$ subjected to $\sum_{i=1}^{K} s_{ij} x_i \leq \ = \ \geq \widetilde{D_j^{ssf}}$, $\forall j = 1, 2, 3 \ldots l$	Optimize $\alpha = \Sigma_{i=1}^{K} \widetilde{C_i^{asf}} x_i$ subjected to $\sum_{i=1}^{K} s_{ij} x_i \leq \ = \ \geq \widetilde{D_j^{asf}}$, $\forall j = 1, 2, 3 \ldots l$

Table 2: Spherical fuzzy linear programming models.

Model	Symmetric spherical fuzzy model (a)	Asymmetric spherical fuzzy model (b)
I	$\text{Max } Z = 5x + 3y$ subjected to $\widetilde{3^{ssf}} x + \widetilde{5^{ssf}} y \leq 15$,	$\text{Max } Z = 5x + 3y$ subjected to $\widetilde{3^{asf}} x + \widetilde{5^{asf}} y \leq 15$
II	$\text{Max } Z = 5x + 3y$ subjected to $\widetilde{3^{ssf}} x + \widetilde{5^{ssf}} y \leq \widetilde{15^{ssf}}$,	$\text{Max } Z = 5x + 3y$ subjected to $\widetilde{3^{asf}} x + \widetilde{5^{asf}} y \leq \widetilde{15^{asf}}$
III	$\text{Max } Z = \widetilde{5^{ssf}} x + \widetilde{3^{ssf}} y$ subjected to $\widetilde{3^{ssf}} x + \widetilde{5^{ssf}} y \leq \widetilde{15^{ssf}}$,	$\text{Max } Z = \widetilde{5^{asf}} x + \widetilde{3^{asf}} y$ subjected to $\widetilde{3^{asf}} x + \widetilde{5^{asf}} y \leq \widetilde{15^{asf}}$
IV	$\text{Max } Z = \widetilde{5^{ssf}} x + \widetilde{3^{ssf}} y$ subjected to $3x + 5y \leq \widetilde{15^{ssf}}$,	$\text{Max } Z = \widetilde{5^{asf}} x + \widetilde{3^{asf}} y$ subjected to $3x + 5y \leq \widetilde{15^{asf}}$

In Table 2, $\widetilde{2^{ssf}} = (2, 1, 0)$, $\widetilde{3^{ssf}} = (3, 2, 1)$, $\widetilde{5^{ssf}} = (5, 3, 1)$, $\widetilde{10^{ssf}} = (10, 6, 2)$, $\widetilde{15^{ssf}} = (15, 10, 5)$, and $\widetilde{2^{asf}} = (2, 1, 0.5)$, $\widetilde{3^{asf}} = (3, 1.5, 0)$, $\widetilde{5^{asf}} = (5, 4, 1)$, $\widetilde{10^{asf}} = (10, 8, 1)$, $\widetilde{15^{asf}} = (15, 12, 3)$. Since objective function is needed to be maximize, preference will be given to the model with a higher optimal value. By solving 0.3 in different fuzzy and crisp environment as in fuzzy, intuitionistic fuzzy, and spherical fuzzy, we obtain the results shown in Table 3).

Table 3: Comparison of LP and different fuzzy optimization models for the numerical example.

Optimization model	Objective function	x	y
LP	12.3684	1.05263	2.36842
FLP	15.13159	1.44737	2.63158
IFLP	16.31579	1.052632	2.368421
SSF Model-I	22.1171	3.119093	2.173913
ASF Model-I	16.58654	1.442308	3.125
SSF Model-II	12.97593	1.100301	2.491472
ASF Model-II	12.71875	1.0625	2.46875
SSF Model-III	9.691474	1.100301	2.491472
ASF Model-III	8.957813	1.0625	2.46875
SSF Model-IV	6.808947	0.684211	1.8894744
ASF Model-IV	6.660789	0.768421	1.878947

the next stage by considering electricity cost 20.62 PKR/kWh, fuel (furnace oil) price PKR85.68/litre, and LPG cost at the rate of PKR19.4103/litre [20, 21]. The last constraint is regarding availability of total working time and production rate, that is, how much hours are needed for the production of 1 kg product of spinning, weaving, and final stages. Objective function constructed through Figure 1 is

$$\min z = 51.55x_1 + 24.25653x_2 + 41.24x_3 + 15.465x_4 + 177.8921x_5. \tag{10}$$

To conduct optimization in the spherical fuzzy environment, the uncertainty in demand, supply parameters, and energy cost per unit fluctuation is kept in mind and considered symmetric and asymmetric SFNs. The considered positive, neutral, and negative membership degree of acceptance is throughout $(0.5, 0.3, 0.2)$.

6. Model-I(a)

Optimize $\alpha = \sum_{i=1}^{5} c_i x_i$ subjected to $\Sigma_{i=1}^{5} \widetilde{S_{ij}^{ssf}} x_i \leq \ = \ \geq d_j$, $\forall j = 1, 2, \ldots, 5$, where spherical fuzzy coefficients $\widetilde{S_{ij}^{ssf}}$ are considered symmetric with x_i real decision variables where d_j is real-valued demands. After the conversion of an in-consideration energy optimization model for the textile industry into the symmetric spherical fuzzy model-I(a), it is represented as follows:

Cost set is $C = \{c_i, i = 1, 2, 3, 4, 5\}$ $C = \{51.55, 24.5653, 41.24, 15.46, 177.8921\}$, where symmetric spherical fuzzy constraint coefficients are

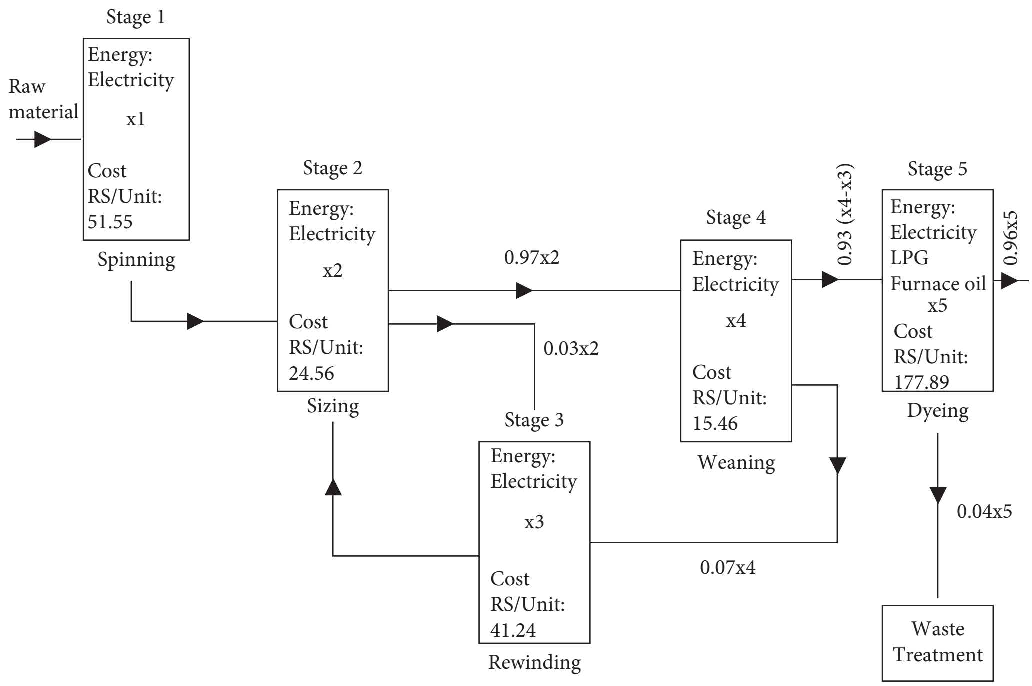


Figure 1: Production stages.

$$\begin{aligned}
\widetilde{S_{21}^{ssf}} &= (0.03)^{ssf} = (0.03, 0.08, 0.13),\\
\widetilde{S_{23}^{ssf}} &= (0.007)^{ssf} = (0.007, 0.012, 0.017),\\
\widetilde{S_{31}^{ssf}} &= (0.97)^{ssf} = (0.97, 1.02, 1.07),\\
\widetilde{S_{41}^{ssf}} &= (0.93)^{ssf} = (0.93, 0.98, 1.03),\\
\widetilde{S_{51}^{ssf}} &= (0.96)^{ssf} = (0.96, 1.01, 1.06),\\
\widetilde{S_{63}^{ssf}} &= (0.013)^{ssf} = (0.013, 0.018, 0.023),\\
\widetilde{S_{64}^{ssf}} &= (0.0062)^{ssf} = (0.0062, 0.0067, 0.0072),\\
\widetilde{S_{ij}^{ssf}} &= (1)^{ssf}\\
&= (1, 0, 0) \cdot \text{for all the values of \$ij\$ other than above.}
\end{aligned} \tag{11}$$

with the monthly production demand in kg for three products and total availability of working hours in a month as $D = \{d_1 = 400, d_2 = 600, d_3 = 20000, d_4 = 720\}$. Mathematically, symmetric spherical fuzzy energy optimization is framed as follows:

$$\begin{aligned}
\min \widetilde{\alpha^{\Upsilon}} &= \sum_{j=1}^{m} c_j x_j\\
&= 51.55x_1 + 24.25653x_2 + 41.24x_3 + 15.465x_4\\
&\quad + 177.8921x_5.
\end{aligned} \tag{12}$$

Subjected to

$$\begin{aligned}
x_1 - x_2 &\geq 400,\\
(0.03)^{ssf} x_2 - x_3 + (0.07)^{ssf} x_4 &= 0,\\
(0.97)^{ssf} x_2 - x_4 &= 0,\\
(0.93)^{ssf} x_4 - x_5 &\geq 600,\\
(0.96)^{ssf} x_5 &\geq 20000,\\
(0.007)^{ssf} x_1 - (0.007)^{ssf} x_2 + (0.013)^{ssf} x_4 + (0.0062)^{ssf} x_5 &\leq 720,\\
x_i \geq 0, \quad i &= 1, 2, 3, 4, 5.
\end{aligned} \tag{13}$$

The above symmetric spherical fuzzy liner programming model is then converted into LP, and the following defuzzified constraints are obtained:

$$\begin{aligned} x_1 - x_2 &\geq 400, \\ 0.065x_2 - x_3 + 0.0105x_4 &= 0, \\ 1.005x_2 - x_4 &= 0, \\ 0.965x_4 &\geq 600, \\ 0.995x_5 &\geq 20000, \\ 0.0105x_1 - 0.0105x_2 + 0.0165x_4 + 0.00655x_5 &\leq 720, \\ x_i \geq 0, \quad i &= 1, 2, 3, 4, 5. \end{aligned} \tag{14}$$

7. Model-I(b)

Now in this model, the spherical fuzzy coefficients $\widetilde{S_{ij}^{asf}}$ are considered asymmetric with x_i real-valued decision variables with d_j, c_i, which are real demands and costs.

For this purpose, by changing the values of spherical fuzzy coefficients by asymmetric spherical fuzzy in the above model-I(a) in (9) for the textile industry, the following changes occurred:

$$\begin{aligned} \widetilde{S_{21}^{asf}} &= (0.03)^{asf} = (0.03, 0.08, 1.08), \\ \widetilde{S_{23}^{asf}} &= (0.007)^{asf} = (0.007, 0.012, 1.012), \\ \widetilde{S_{31}^{asf}} &= (0.97)^{asf} = (0.97, 1.02, 2.02), \\ \widetilde{S_{41}^{asf}} &= (0.93)^{asf} = (0.93, 0.98, 1.98), \\ \widetilde{S_{41}^{asf}} &= (0.96)^{asf} = (0.96, 1.01, 2.06), \\ \widetilde{S_{53}^{asf}} &= (0.013)^{asf} = (0.013, 0.018, 1.023), \\ \widetilde{S_{54}^{asf}} &= (0.0062)^{asf} = (0.0062, 0.0067, 1.0072), \\ \widetilde{S_{ij}^{asf}} &= (1)^{asf} = (1, 0, 0) \cdot \text{for all the values of \$ij\$ other than above.} \end{aligned} \tag{15}$$

The monthly production demand in kg for three products and total availability of working hours in a month are $D = \{d_1 = 400, d_2 = 600, d_3 = 20000, d_4 = 720\}$. The asymmetric spherical fuzzy energy optimization model is

$$\begin{aligned} \min \widetilde{\alpha^{\gamma}} &= \sum_{j=1}^{m} c_j x_j \\ &= 51.55x_1 + 24.25653x_2 + 41.24x_3 \\ &\quad + 15.465x_4 + 177.8921x_5, \end{aligned} \tag{16}$$

subjected to constraints

$$\begin{aligned} x_1 - x_2 &\geq 400, \\ (0.03)^{asf} x_2 - x_3 + (0.07)^{asf} x_4 &= 0, \\ (0.97)^{asf} x_2 - x_4 &= 0, \\ (0.93)^{asf} x_4 - x_5 &\geq 600, \\ (0.96)^{asf} x_5 &\geq 20000, \\ (0.007)^{asf} x_1 - (0.007)^{asf} x_2 + (0.013)^{asf} x_4 + (0.0062)^{asf} x_5 &\leq 720, \\ x_i \geq 0, \quad i &= 1, 2, 3, 4, 5. \end{aligned} \tag{17}$$

The defuzzyfied form of the above ASF linear programming model is given as

$$\begin{aligned} x_1 - x_2 &\geq 400, \\ 0.255x_2 - x_3 + 0.2095x_4 &= 0, \\ 1.195x_2 - x_4 &= 0, \\ 1.155x_4 &\geq 600, \\ 1.185x_5 &\geq 20000, \\ 0.2095x_1 - 0.2095x_2 + 0.2155x_4 + 0.20645x_5 &\leq 720, \\ x_i \geq 0, \quad i &= 1, 2, 3, 4, 5. \end{aligned} \tag{18}$$

8. Model-II(a)

In this model, constraint coefficients and demand is considered in the symmetric spherical fuzzy number and cost remains unchanged as follows:

$$\begin{aligned} \widetilde{D_1^{ssf}} &= (400)^{ssf} = (400, 450, 500), \\ \widetilde{D_2^{ssf}} &= (0)^{ssf} = (0, 0, 0) = \widetilde{D_3^{ssf}}, \\ \widetilde{D_4^{ssf}} &= (600)^{ssf} = (600, 650, 700), \\ \widetilde{D_5^{ssf}} &= (20000)^{ssf} = (20000, 20050, 20100), \\ \widetilde{D_6^{ssf}} &= (720)^{ssf} = (720, 770, 820), \\ \widetilde{S_{21}^{ssf}} &= (0.03)^{ssf} = (400, 450, 500), \\ \widetilde{S_{23}^{ssf}} &= (0.007)^{ssf} = (400, 450, 500), \\ \widetilde{S_{31}^{ssf}} &= (0.97)^{ssf} = (400, 450, 500), \\ \widetilde{S_{41}^{ssf}} &= (0.96)^{ssf} = (400, 450, 500), \\ \widetilde{S_{53}^{ssf}} &= (0.013)^{ssf} = (400, 450, 500), \\ \widetilde{S_{54}^{ssf}} &= (0.0062)^{ssf} = (400, 450, 500), \\ \widetilde{S_{ij}^{ssf}} &= (1)^{ssf} = (1, 0, 0) \cdot \text{for all the values of \$ij\$ other than above.} \end{aligned} \tag{19}$$

The symmetric SF energy optimization model-II(a) for the textile industry becomes

$$\begin{aligned} \min \widetilde{\alpha}^{\Upsilon} = 51.55x_1 + 24.25653x_2 + 41.24x_3 \\ + 15.465x_4 + 177.8921x_5. \end{aligned} \tag{20}$$

Under the constraints

$$\begin{aligned} x_1 - x_2 &\geq (400)^{ssf}, \\ (0.03)^{ssf} x_2 - x_3 + (0.07)^{ssf} x_4 &= (0)^{ssf}, \\ (0.97)^{ssf} x_2 - x_4 &= (0)^{ssf}, \\ (0.93)^{ssf} x_4 - x_5 &\geq (600)^{ssf}, \\ (0.96)^{ssf} x_5 &\geq (20000)^{ssf}, \\ (0.007)^{ssf} x_1 - (0.007)^{ssf} x_{,2} + (0.013)^{ssf} x_4 + (0.0062)^{ssf} x_5 &\leq (720)^{ssf} \\ x_i \geq 0, \quad i &= 1, 2, 3, 4, 5, \end{aligned} \tag{21}$$

converting the above constraint into LP:

$$\begin{aligned}
x_1 - x_2 &\geq 435,\\
0.065x_2 - x_3 + 0.0105x_4 &= 0,\\
1.005x_2 - x_4 &= 0,\\
0.965x_4 &\geq 635,\\
0.995x_5 &\geq 20035,\\
0.0105x_1 - 0.0105x_2 + 0.0165x_4 + 0.00655x_5 &\leq 755,\\
x_i \geq 0, \quad i &= 1,2,3,4,5.
\end{aligned} \tag{22}$$

Solve the objective function 0.13 subjected to set of constraints 0.15 to obtain the solution.

9. Model-II(b)

In this model, constraint coefficients and demand is considered asymmetric spherical fuzzy number:

$$\begin{aligned}
\widetilde{D_1^{asf}} &= (400)^{asf} = (400, 450, 511),\\
\widetilde{D_2^{asf}} &= (0)^{asf} = (0, 0, 0) = \widetilde{D_3^{asf}},\\
\widetilde{D_4^{asf}} &= (600)^{asf} = (600, 650, 711),\\
\widetilde{D_5^{asf}} &= (20000)^{asf} = (20000, 20050, 20111),\\
\widetilde{D_6^{asf}} &= (720)^{asf} = (720, 770, 831),\\
\widetilde{S_{21}^{asf}} &= (0.03)^{asf} = (400, 450, 500),\\
\widetilde{S_{23}^{ssf}} &= (0.007)^{ssf} = (400, 450, 500),\\
\widetilde{S_{31}^{ssf}} &= (0.97)^{ssf} = (400, 450, 500),\\
\widetilde{S_{41}^{ssf}} &= (0.96)^{ssf} = (400, 450, 500),\\
\widetilde{S_{53}^{ssf}} &= (0.013)^{ssf} = (400, 450, 500),\\
\widetilde{S_{54}^{ssf}} &= (0.0062)^{ssf} = (400, 450, 500),\\
\widetilde{S_{ij}^{ssf}} &= (1)^{ssf} = (1, 0, 0) \text{ for all the values of ij other than above.}
\end{aligned} \tag{23}$$

The asymmetric SF energy optimization model-II(b) with objective function 0.13 for the textile industry under the constraints:

$$\begin{aligned}
x_1 - x_2 &\geq (400)^{asf},\\
(0.03)^{asf}x_2 - x_3 + (0.07)^{asf}x_4 &= (0)^{asf},\\
(0.97)^{asf}x_2 - x_4 &= (0)^{asf},\\
(0.93)^{asf}x_4 - x_5 &\geq (600)^{asf},\\
(0.96)^{asf}x_5 &\geq (20000)^{asf},\\
(0.007)^{asf}x_1 - (0.007)^{asf}x_2 + (0.013)^{asf}x_4 + (0.0062)^{asf}x_5 &\leq (720)^{asf},\\
x_i \geq 0, i &= 1,2,3,4,5,
\end{aligned} \tag{24}$$

converting the above constraint into real-valued:

$$\begin{aligned} x_1 - x_2 &\geq 437.2, \\ 0.255x_2 - x_3 + 0.2095x_4 &= 0, \\ 1.195x_2 - x_4 &= 0, \\ 1.155x_4 &\geq 637.2, \\ 1.185x_5 &\geq 20037.2, \\ 0.2095x_1 - 0.2095x_2 + 0.2155x_4 + 0.20645x_5 &\leq 727.2 \\ x_i \geq 0, i &= 1, 2, 3, 4, 5. \end{aligned} \tag{25}$$

10. Model-III(a)

In this model, all the cost and demand coefficients are in SSFNs, so here the objective function is expressed as follows:

$$\begin{aligned} Optimize\ \alpha &= \sum_{i=1}^{K} \widetilde{C_i^{ssf}} x_i = \min \widetilde{\alpha^{\curlyvee}} = \sum_{i=1}^{5} \widetilde{C_i^{asf}} \circledast x_j, \\ &= ((51.55, 56.55, 61.55) \circledast x_1 \oplus (24.5653, 29.5653, 34.5653) \circledast x_2, \\ &\quad \oplus (41.24 \\ &\quad \oplus ((177.892, 182.892, 187.892) \circledast x_5, \end{aligned} \tag{26}$$

under the constraints

$$\begin{aligned} x_1 - x_2 &\geq (400)^{ssf}, \\ (0.03)^{ssf} x_2 - x_3 + (0.07)^{ssf} x_4 &= 0, \\ (0.97)^{ssf} x_2 - x_4 &= (0)^{ssf}, \\ (0.93)^{ssf} x_4 - x_5 &\geq (600)^{ssf}, \\ (0.96)^{ssf} x_5 &\geq (20000)^{ssf}, \\ (0.007)^{ssf} x_1 - (0.007)^{ssf} x_2 + (0.013)^{ssf} x_4 + (0.0062)^{ssf} x_5 &\leq (720)^{ssf}, \\ x_i \geq 0, i &= 1, 2, 3, 4, 5. \end{aligned} \tag{27}$$

after converting the above model into general LP, 0.15 becomes required constraints for the objective function:

$$\begin{aligned} \min \widetilde{\alpha^{\curlyvee}} &= 55.05x_1 + 28.0653x_2 + 44.74x_3 + 18.965x_4 \\ &\quad + 181.3921x_5. \end{aligned} \tag{28}$$

11. Model-III(b)

By solving objective function,

$$\begin{aligned} Optimize\ \alpha &= \sum_{i=1}^{K} \widetilde{C_i^{asf}} x_i, \\ &= \min \widetilde{\alpha^{\curlyvee}} = \sum_{i=1}^{5} \widetilde{C_i^{asf}} \circledast x_j, \\ &= ((51.55, 56.55, 67.55) \circledast x_1 \oplus (24.5653, 29.5653, 40.5653) \circledast x_2, \\ &\quad \oplus (41.24 \\ &\quad \oplus ((177.892, 182.892, 193.892) \circledast x_5 \end{aligned} \tag{29}$$

Under the subjected constraint 0.16, we obtain an infeasible solution after converting into LP objective function by using the weight formula, $\min \widetilde{\alpha^{\curlyvee}} = 56.25x_1 + 29.2653x_2 + 45.94x_3 + 20.165x_4 + 182.5921x_5$.

12. Model-IV(a)

In this model, we considered cost and demand (right-hand side of constraint equations) a symmetric spherical fuzzy number. The spherical fuzzy energy optimization model for the textile industry according to this model is as follows:

$$\begin{aligned} \min \widetilde{\alpha^{\curlyvee}} &= \sum_{j=1}^{m} \widetilde{C_j^{ssf}} \circledast x_j, \\ &= \widetilde{(51.55)}^{ssf} \circledast x_1, \\ &\oplus \widetilde{(24.5653)}^{ssf} \circledast x_2, \\ &\oplus \widetilde{(41.24)}^{ssf} \circledast x_3, \\ &\oplus \widetilde{(15.465)}^{ssf} \circledast x_4, \\ &\oplus \widetilde{(177.8921)}^{ssf} \circledast x_5, \end{aligned} \tag{30}$$

subjected to

$$\begin{aligned} x_1 - x_2 &\geq (400)^{ssf}, \\ 0.03x_2 - x_3 + 0.07x_4 &= 0^{ssf}, \\ 0.97x_2 - x_4 &= 0^{ssf}, \\ 0.93x_4 - x_5 &\geq (600)^{ssf}, \\ 0.96x_5 &\geq (20000)^{ssf}, \\ 0.007x_1 - 0.007x_2 + 0.013x_4 + 0.0062x_5 &\leq (720)^{ssf}, \\ x_i \geq 0, \quad i &= 1, 2, 3, 4, 5, \end{aligned} \tag{31}$$

converting the above model into LP, we obtain

$$\min \widetilde{\alpha^{\curlyvee}} = 55.05x_1 + 28.0653x_2 + 44.74x_3 + 18.965x_4 + 181.392x_5, \tag{32}$$

under the real constraints

$$\begin{aligned} x_1 - x_2 &\geq 435, \\ 0.03x_2 - x_3 + 0.07x_4 &= 0, \\ 0.97x_2 - x_4 &= 0, \\ 0.93x_4 - x_5 &\geq 635, \\ 0.96x_5 &\geq 20035, \\ 0.007x_1 - 0.007x_2 + 0.013x_4 + 0.0062x_5 &\leq 755, \\ x_i \geq 0, i &= 1, 2, 3, 4, 5. \end{aligned} \tag{33}$$

13. Model-IV(b)

$$\begin{aligned} \min \widetilde{\alpha^{\curlyvee}} &= \sum_{j=1}^{m} \widetilde{C_j^{asf}} \circledast x_j, \\ &= \widetilde{(51.55)}^{asf} \circledast x_1, \\ &\oplus \widetilde{(24.5653)}^{asf} \circledast x_2, \\ &\oplus \widetilde{(41.24)}^{asf} \circledast x_3, \\ &\oplus \widetilde{(15.465)}^{asf} \circledast x_4, \\ &\oplus \widetilde{(177.8921)}^{asf} \circledast x_5, \end{aligned} \tag{34}$$

Subjected to

$$\begin{aligned} x_1 - x_2 &\geq (400)^{asf}, \\ 0.03x_2 - x_3 + 0.07x_4 &= 0^{asf}, \end{aligned} \tag{35}$$

subjected to

$$\begin{aligned} 0.97x_2 - x_4 &= 0^{asf}, \\ 0.93x_4 - x_5 &\geq (600)^{asf}, \\ 0.96x_5 &\geq (20000)^{asf}, \\ 0.007x_1 - 0.007x_2 + 0.013x_4 + 0.0062x_5 &\leq (720)^{asf}, \\ x_i \geq 0, \quad i &= 1, 2, 3, 4, 5, \end{aligned} \tag{36}$$

converting the above model into LP, we obtain

$$\begin{aligned} \min \widetilde{\alpha^{\curlyvee}} &= 56.25x_1 + 29.2653x_2 + 45.94x_3 \\ &+ 20.165x_4 + 182.5921x_5, \end{aligned} \tag{37}$$

under defuzzified constraints

$$\begin{aligned} x_1 - x_2 &\geq 437.2, \\ 0.03x_2 - x_3 + 0.07x_4 &= 0, \\ 0.97x_2 - x_4 &= 0, \\ 0.93x_4 - x_5 &\geq 637.2, \\ 0.96x_5 &\geq 20037.2, \\ 0.007x_1 - 0.007x_2 + 0.013x_4 + 0.0062x_5 &\leq 757.2, \\ x_i \geq 0, \quad i &= 1, 2, 3, 4, 5. \end{aligned} \tag{38}$$

Table 4 shows the optimal output we obtained through different models.

14. Postoptimal Analysis

In present study eight models with different symmetric and asymmetric spherical fuzzy changes were tried. Out of eight models only five models provided us a feasible solution for spherical fuzzy energy optimization model 0.5 and one of

Table 4: Optimal solutions.

Optimization model	Objective value	x_1	x_2	x_3	x_4	x_5
LP	5987499	24195.38	23759.38	2326.043	23046.59	20833.33
IFLP	5927615.1	24159.4	23759.4	874.104	23046.6	20833.33
Model-I(a)	5702683	21744.58	21344.58	3639.78	21451.3	20100.50
Model-I(b)	Infeasible solution	0	0	0	0	0
Model-II(a)	5717777.8	21851.93	21416.93	3652.12	21524.02	20135.68
Model-II(b)	Infeasible solution	0	0	0	0	0
Model-III(a)	6027917.33	21851.93	21416.93	3652.12	21524.02	20135.68
Model-III(b)	Infeasible solution	0	0	0	0	0
Model-IV(a)	6333697.02	24273.59	23838.59	2333.8	23123.43	20869.79
Model-IV(b)	6448227.44	24280.77	23843.57	2334.28	23128.26	20872.08

Table 5: Model-I(a) sensitivity analysis report.

Variables	Final value	Reduced cost	Objective coefficient	Allowable increase	Allowable decrease
x_1	21744.57	0	51.55	1E+30	51.55
x_2	21344.57	0	24.56	1E+30	98.68
x_3	3639.78	0	41.24	1E+30	578.68
x_4	21451.3	0	15.46	1E+30	98.19
x_5	20100.50	0	177.89	1E+30	279.64
Constraint	Final value	Shadow price	RHS	Allowable increase	Allowable decrease
1	400	51.55	400	21923.36	21744.57
2	4.55E−13	−41.24	0	3639.78	1E+30
3	0	78.4	0	1E+30	21451.3
4	600	101.75	600	13462.94	20700.50
5	20000	281.05	20000	9685.38	20000
6	489.80	0	720	1E+30	230.19

them provided the best optimal result as compared to all SF models and LP as shown in Table 4 while all the other spherical fuzzy models resulted those objective values, which are much higher than real LP. Overall, if we combine the objective value in the form of inequality keeping LP greatest we get:

$$\text{SSFModel} - I(a) < \text{SSFModel} - II(a) < \text{IFLP} < \text{LP}, \quad (39)$$

the objective values obtained from these methods also satisfy the relation as

$$5702683 < 5717777.8 < 5927615.1 < 5933824 < 5987499. \quad (40)$$

Except one ASF model, all the other ASF (models-I, II, III) energy optimization models gave "Infeasible solution" in 0.5 and one ASF model-IV, which provided a solution that have the most highest optimal value (greater than LP) where objective function was needed to minimize, whereas in 0.3 where the objective function was needed to maximize the similar models, ASF (models-I, II, III) provided feasible solution, but other than SSF and AS (models- I, II), all the other remaining models provided outputs less than LP; that is, the outputs of these models are much lesser than LP's outputs. Here, in the following tables, the sensitivity report and limits of all those SF models in 0.5 are discussed whose optimal solutions were not greater than the LP textile energy model.

In Table 5 the flexibility regarding allowable change in optimal and feasible conditions is mentioned. Since solution is effected by both and impacts the efficiency of model, how much cost and demand fluctuation can be handled by these models in 0.5 is discussed, whereas in Tables 6, 7 and 8 the feasibility range of the decision variable is discussed along with the optimal solution between that range. In Table 9 the validity range of all the parameters and decision variables of the best optimal model for 0.5 is mentioned. According to the above postoptimal analysis, we obtain the following information about the range of each factor of the energy optimization model in a spherical environment. SSF model-I(a) is providing a minimal optimal value of the objective function for a longer range of coefficients that insure the feasibility for a huge change in 0.5. In both situations, where objective function is needed to maximize 0.3 or to minimize 0.5, SSF model-I(a) provided the best results.

From Figure 2 it is clearly seen that all the decision variables in SSF model-I for 0.5 is having least value as compared to other models except x_3. This x_3 is providing its minimal value in the intuitionistic fuzzy environment.

In Figure 3 an objective value of 0.5 in different fuzzy and LP environments is graphically presented. Three bars in the right-hand side denote ASF models and are clearly higher than even LP's bar. Therefore, they are not considered the best models for 0.5. SSF model-I is providing the best results as compared to others.

Table 6: Model-I(a) limits report.

Variable	Values	Lower limit	Objective result	Upper limit	Objective result
x_1	21744.57	21744.57	5702683.001	43667.94	6832832.271
x_2	21344.57	21344.57	5702683.001	21344.57	5702683.001
x_3	3639.78	3639.78	5702683.001	3639.78	5702683.001
x_4	21451.3	21451.3	5702683.001	21451.30	5702683.001
x_5	20100.50	20100.50	5702683.001	20100.50	5702683.001

Table 7: Model-II(a) sensitivity analysis report.

Variables	Final value	Reduced cost	Objective coefficient	Allowable increase	Allowable decrease
x_1	21851.93	0	51.55	1E+30	51.55
x_2	21416.93	0	24.56	1E+30	98.68
x_3	3652.12	0	41.24	1E+30	578.68
x_4	21524.02	0	15.46	1E+30	98.19
x_5	20135.68	0	177.89	1E+30	279.64
Constraint	Final value	Shadow price	RHS	Allowable increase	Allowable decrease
1	435	51.55	435	25085.47	21851.9
2	4.55E−13	−41.24	0	3652.12	1E+30
3	0	78.4	0	1E+30	21524.02
4	635	101.75	635	15404.76	20770.68
5	20035	281.045	20035	11082.36	20035
6	491.60	0	755	1E+30	263.4

Table 8: Model-II(a) limits report.

Variable	Values	Lower limit	Objective result	Upper limit	Objective result
x_1	21851.93	21851.93	5717777.832	46937.41	7010934.09
x_2	21416.93	21416.93	5717777.832	21416.93	5717777.832
x_3	3652.12	3652.12	5717777.832	3652.12	5717777.832
x_4	21524.02	21524.02	5717777.832	21524.02	5717777.832
x_5	20135.68	20135.68	5717777.832	20135.69	5717777.832

Table 9: Validity range of SSF-I(a).

Factors	SSF model-I(a)	Range
x_1	21744.56 units	21923.38 unit
x_2	21344.56 units	0
x_3	3539.73 units	0
x_4	21451.3 units	0 unit
x_5	20100 units	0 unit
c_1	51.55 Rs./unit	51.55 Rs/unit
c_2	24.56 Rs./units	98.68 Rs/unit
c_3	41.24 Rs./units	578.71 Rs/unit
c_4	15.46 Rs./units	98.2 Rs/unit
c_5	177.89 Rs./units	279.64 Rs/unit
d_1	400 kg/month	43667 kg/month
d_2	0	3639.78 kg/month
d_3	0	21451.3 kg/month
d_4	20000 kg/month	29685.39 kg/month
d_5	600 kg/month	43163.44 kg/month
d_6	720 working hours/month	489.81 hours/month

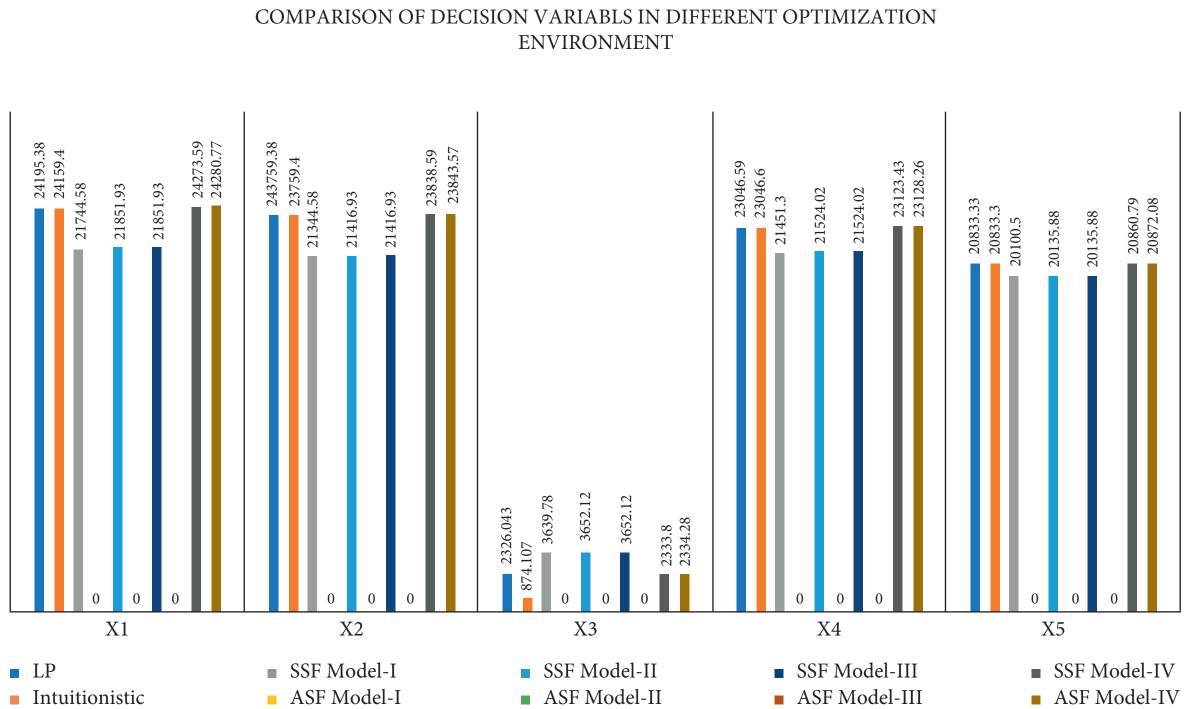


Figure 2: Comparison of decision variables.

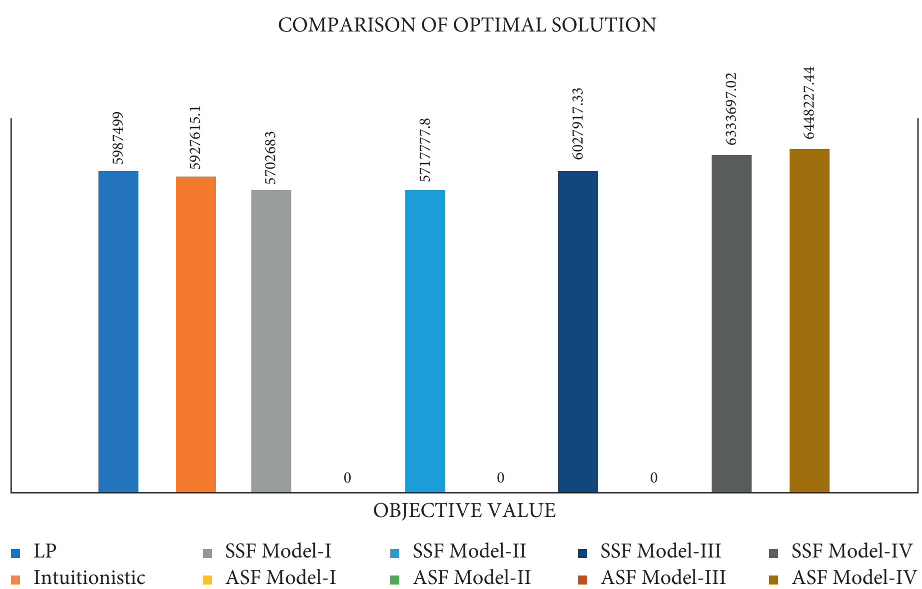


Figure 3: Comparison of objective function values.

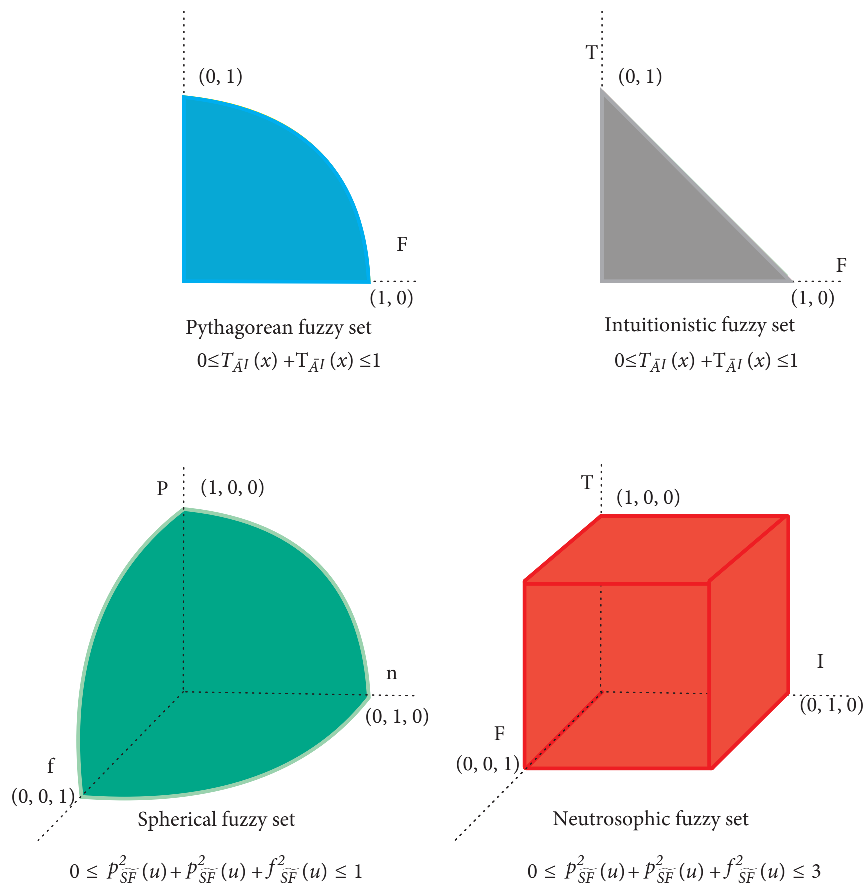


FIGURE 4: Comparison of SF with other fuzzy sets.

15. Conclusion

Spherical fuzzy set (SFS) is a well-known generalization of widely studied fuzzy sets, and significant research has been carried out to investigate SFS set theocratic properties and applications in other fields. Due to the involvement of positive, neutral, and negative grades, SFS can handle uncertainties better than fuzzy sets. In this article, linear optimization is carried out by utilizing spherical fuzzy numbers. The method is superior to previously defined techniques as shown in.

(1) Due to the spherical region, parameters are highly flexible and can provide optimal solution between a long range

(2) Spherical fuzzy set is a super set of the intuitionistic and Pythagorean fuzzy set, so it will cover more area graphically (see Figure 4) and can easily target those points for solution that are far away to those points, which are obtained through LP or IF technique

(3) No need to construct or change a model for a huge change due to long-range flexibility of parameters

The method can be used in any decision-making problem simply by identifying the objectives, parameters, and the constraints imposed to maximize or minimize the objective. In recent years, the fuzzy set and its generalizations are used widely in decision-making related to real-life problems [22, 23]. Any advancement in basic fuzzy set theocratic concepts will ultimately improve the accuracy of its implementation by incorporating the imprecision and vagueness in the data. Spherical fuzzy optimization techniques can be used to find the suitable weights for the best criteria over others in the decision-making process. [24–26].

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

Authors' Contributions

All authors contributed equally to the preparation of this manuscript.

Acknowledgments

The fifth author (YUG) would like to acknowledge that this publication was made possible by a grant from the Carnegie Corporation of New York.

References

[1] R. Q. Wang, L. Jiang, Y. D. Wang, and A. P. Roskilly, "Energy saving technologies and mass-thermal network optimization for decarbonized iron and steel industry: a review," *Journal of Cleaner Production*, vol. 274, Article ID 122997, 2020.

[2] I. Ullah, Z. Khitab, M. Khan, and S. Hussain, "An efficient energy management in office using bio-inspired energy optimization algorithms," *Processes*, vol. 7, no. 3, Article ID 142, 2019.

[3] E. Ozturk, N. C. Cinperi, and M. Kitis, "Improving energy efficiency using the most appropriate techniques in an integrated woolen textile facility," *Journal of Cleaner Production*, vol. 254, Article ID 120145, 2020.

[4] I. Kimutai, P. Maina, and A. Makokha, "Energy optimization model using linear programming for process industry: a case study of textile manufacturing plant in Kenya," *International Journal of Energy Engineering*, vol. 9, no. 2, pp. 45–52, 2019.

[5] https://invest.gov.pk/textilepakistan, 2021.

[6] K. Mustafa, "Despite go-ahead by PM twice: textile Policy 2020-25 in the doldrums," Available at: https://Thenews.com.pk, Jan 24 2021.

[7] L. V. Kantorovich, "Mathematical methods of organizing and planning production," *Management Science*, vol. 6, no. 4, pp. 366–422, 1960.

[8] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.

[9] H.-J. Zimmermann, "Fuzzy programming and linear programming with several objective functions," *Fuzzy Sets and Systems*, vol. 1, no. 1, pp. 45–55, 1978.

[10] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, 1986.

[11] P. P. Angelov, "Optimization in an intuitionistic fuzzy environment," *Fuzzy Sets and Systems*, vol. 86, no. 3, pp. 299–306, 1997.

[12] R. R. Yager, "Pythagorean fuzzy subsets," in *2013 joint IFSA world congress and NAFIPS annual meeting (IFSA/NAFIPS)*, pp. 57–61, Edmonton, AB, Canada, 24-28 June 2013.

[13] F. Smarandche, *A Unifying Field in Logics: Neutrosophic Logic*, American Press, Reston, Virginia, Philosophy, 1999.

[14] F. Ahmad, A. Y. Adhami, and F. Smarandache, "Neutrosophic optimization model and computational algorithm for optimal shale gas water management under uncertainty," *Symmetry*, vol. 11, no. 4, p. 544, 2019.

[15] F. Ahmad, A. Y. Adhami, and F. Smarandache, "Modified neutrosophic fuzzy optimization model for ooptimal closed-loop supply chain management under uncertainity," *Optimization Theory Based on Neutrosophic and Plithogenic Sets*, vol. 11, no. 4, pp. 343–403, 2020.

[16] G. F. Kutlu and C. Kahraman, "Spherical fuzzy sets and spherical fuzzy TOPSIS method," *Journal of Intelligent and Fuzzy Systems*, vol. 36, no. 4, pp. 337–352, 2019.

[17] F. Ahmad and A. Y. Adhami, "Spherical fuzzy linear programming problem," *Decision Making with Spherical Fuzzy Sets*, vol. 392, pp. 455–472, 2021.

[18] A. Rafiq, S. Ashraf, S. Abdullah, T. Mahmood, and S. Muhammad, "The cosine similarity measure of spherical fuzzy sets and their applications in decision making," *Journal of Intelligent and Fuzzy Systems*, vol. 36, no. 8, pp. 1–15, 2019.

[19] F. Ahmad and A. Y. Adhami, "Neutrosophic programming approach to multiobjective nonlinear transportation problem with fuzzy parameters," *International Journal of Management Science and Engineering Management*, vol. 14, no. 3, pp. 218–229, 2019.

[20] https://www.nepra.org, 2021.

[21] http://www.ogra.org, 2021.

[22] M. A. Alsalem, R. Mohammed, O. S. Albahri et al., "Rise of multiattribute decision making in combating COVID-19: a systematic review of the state of the art literature," *International Journal of Intelligent Systems*, 2021.

[23] A. S. Albahri, A. A. Zaidan, O. S. Albahri et al., "Development of IoT-based mhealth framework for various cases of heart disease patients," *Health Technology*, vol. 11, no. 5, pp. 1013–1033, 2021.

[24] A. S. Albahri, O. S. Albahri, A. A. Zaidan et al., "Integration of fuzzy-weighted zero-inconsistency and fuzzy decision by opinion score methods under a q-rung orthopair environment: a distribution case study of COVID-19 vaccine doses," *Computer Standards & Interfaces*, vol. 80, Article ID 103572, 2022.

[25] O. S. Albahri, A. A. Zaidan, M. M. Salih et al., "Multidimensional benchmarking of the active queue management methods of network congestion control based on extension of fuzzy decision by opinion score method," *International Journal of Intelligent Systems*, vol. 36, no. 2, pp. 796–831, 2021.

[26] M. A. Alsalem, H. A. Alsattar, A. S. Albahri et al., "Based on T-spherical fuzzy environment: a combination of FWZIC and FDOSM for prioritising COVID-19 vaccine dose recipients," *Journal of Infection and Public Health*, vol. 14, no. 10, pp. 1513–1559, 2021.

Hindawi
Mathematical Problems in Engineering
Volume 2022, Article ID 3705244, 13 pages
https://doi.org/10.1155/2022/3705244

Research Article

Fruit Production Planning in Semiarid Zones: A Novel Triangular Intuitionistic Fuzzy Linear Programming Approach

Sajida Kousar,[1] **Afia Zafar**,[1] **Nasreen Kausar**,[2] **Dragan Pamucar**,[3] **and Parameshwari Kattel**[4]

[1]*Department of Mathematics and Statistics, International Islamic University, Islamabad, Pakistan*
[2]*Department of Mathematics, Faculty of Arts and Sciences, Yildiz Technical University, Esenler, Istanbul 34210, Turkey*
[3]*Department of Logistics, University of Defence, Belgrade, Serbia*
[4]*Department of Mathematics, Tri-Chandra Multiple Campus, Tribhuvan University, Kathmandu, Nepal*

Correspondence should be addressed to Parameshwari Kattel; parameshwari.kattel@trc.tu.edu.np

Received 13 November 2021; Accepted 11 January 2022; Published 12 February 2022

Academic Editor: Cristiano Fragassa

A triangular intuitionistic fuzzy linear programming (TIFLP) model is formulated for the planning of sustainable fruit production system for hyperarid regions while assuming the availability of resources and existing knowledge. A remarkable advancement is achieved through the composition of intuitionistic fuzzy concept with the linear programming by considering all parameters and variables in the form of triangular intuitionistic fuzzy numbers, which provides a planning or strategic tool for handling uncertain situations with more control and in a realistic way. This fuzzy optimization model is redesigning the feasible region obtained by linear programming which is presented in graphical form. Moreover, the practical application and implementation of this fruit production system for planning in real-life scenarios are accomplished considering the case study of fruit orchards of Baluchistan, Pakistan.

1. Introduction

Have you ever imagined experiencing the world without agriculture? In that case, most of the world's population could not outlive hunger, and the remaining ones would be hunting for food. In fact, you would no longer be here to read this paper because the path of modern civilization would be lost forever with the absence of agriculture. Agriculture is art, science, and business of all types of crop production which flourished into seven major branches named as agronomy, horticulture, forestry, animal husbandry, agricultural engineering, fishery, and home science [1]. The beginning of human civilization started with agricultural development referred to as first agricultural revolution. Later on, agriculture and farming spread into different regions around the world and broadened with livestock, industrial agriculture, agronomy, and much more. The history of human civilization is reflected by the inventions, methods, and techniques used to enhance the agriculture and its different branches in a productive manner. Throughout modification in agricultural field, it has been improved and transformed into much more ultramodern form known as "sustainable agriculture" which equally impacts the environment, society, and economy [2].

The ultimate motive of sustainable agriculture is the satisfaction of all human needs and necessities with the major contribution to economy in healthy environmental conditions. The improvement of our food security system is the mostly targeted goal for the betterment of present and future generation. The sustainable development goal is the eradication of hunger by accomplishing food security and improving the nutrition intake by 2030 [3]. A thorough analysis was carried out about the achievement of "zero hunger" goal by studying all the existing scientific literature to assess their contribution to the achievement of the sustainable development goal [4]. According to a latest study, the fourth agricultural revolution demands the balance between the agricultural production and world's population together with the environment [5]. To eradicate the undernourishment of the world, fruit consumption rate of the world per capita should be according to diverging health conditions. The low intake of fruit and vegetable increases the worldwide burden of disease, which can be controlled through the ample amount of fruit consumption and production [6]. Analytical study reveals that approximately 22% of difference exists between the demand and supply of fruit production, whereas this percentage increases to 58% for the underdeveloped countries, which is increasing with the passage of time [7].

Pakistan, being a middle-income developing country, produces five major crops, wheat, rice, sugarcane, maize, and cotton, along with the most importantly fruits and vegetables with pulses and oilseeds [8]. The production of fruits and vegetables is approximately 12 million tons per year. More precisely, fruits contribute 2.48% to agricultural gross domestic product of Pakistan, producing apples, mangoes, grapes, dates, citrus, peaches, cherries, plums, loquat, pears, and guava. According to a rough analysis, Pakistan earned $730 million by exporting 1.165 million tons of fruits and vegetables in a year [9]. The study of Pakistan recommends investing in research and development to find innovative strategies to enhance production and quality and reduce postharvest losses in order to boost fruit and vegetable export competitiveness [10]. The global horticultural products trade for the past two decades was maximized by four times by making earnings of USD 51 billion in 2001 to USD 200 billion in 2018 [11]. The international trade competitiveness of Pakistan is evaluated through the analysis of competitive and comparative demand and supply of vegetables and fruits [12]. The overwhelming pressure on the demand of food security caused by population increase and global development results in the destruction of natural resources and food crises [13]. Additionally, COVID-19 and intense climate changes severely escalate the demand of food by decreasing the average agricultural production [14].

Real-life situations can be assessed mathematically. For modeling and management of certain scenarios, mathematical analysis of real-life occurrences utilized quantitative and qualitative methodologies. Linear programming is a generalized and renowned technique presented by Kantorovich [15] to optimize agricultural aims and objectives by allocation and restriction of certain demand and availability constraints [16]. In light of our current agricultural requirements, our objective is not only food supply but also the ample amount and quality of food provision around the world. Therefore, agricultural planning is carried out for this goal using operational mathematical approaches in the most efficient way in order to eliminate food security issues [17, 18]. It is used as a single objective as well as multiple objectives to minimize and maximize the cost and profit by the utilization and management of natural resources, labor, techniques, research, capital regarding land allocation, cropping patterns, optimization of water resources, raising livestock, and production maximization with cost minimization [19].

Food production system must be thoroughly modified and armed with resilience and adaptivity and have high diversity against different situations and factors (climate change, pest attacks and diseases, governmental policies at national and international level, social and cultural stability factors) [20]. For perfection in the precision of goals regarding planning, this area still needs much more modifications in terms of changing environmental, ecological, and social factors [21]. Globally, agricultural output continuously confronts drastic fluctuations due to which sustainable agriculture is constantly evolving with the passage of time and demand of the world is changing continuously regarding various aspects. These factors generate uncertainly and vagueness in environment, which is assessed by using the concept of fuzzy sets introduced by Zadeh [22]. Indeed, fuzzy set and its generalizations such as intuitionistic fuzzy sets [23] are utilized to present data that is fuzzy in nature. Eventually, fuzzy optimization theory was initiated by Zimmermann for effective decision making in fuzzy environment [24].

Fuzzy linear programming approach was further investigated through meticulous application to decision making and management problems considered in uncertain environment, and it obtained much more precise and feasible output [25]. Under unpredictable circumstances in energy-water nexus, an integrated fuzzy optimization approach was proposed for agricultural water and land resource management [26]. Multiobjective fuzzy methodology having three goals was considered as maximization of net benefits, agricultural output, and labor employment for Pune city of Maharashtra State, India [27]. Another study was conducted by applying intuitionistic fuzzy optimization technique in agricultural production planning, with a focus on smallholder farmers in north Bihar, India [28].

Specifically, fruit production planning by using linear programming is done, which is generalized for production maximization in hyperarid regions with available resources, labor, capital, etc. Further, in order to evaluate a targeted objective function that stays valid and optimal under the influence of climatic, social, and economic conditions, triangular intuitionistic fuzzy linear programming has been constructed more accurately and meticulously. The article is divided into five sections, where all the basic and essential information is provided in Preliminaries section. The objective function and constraints for optimal fruit production in crisp and intuitionistic fuzzy environment are defined in Methodology section. The model is then applied to a real-life example by considering fruit production data from Baluchistan province of Pakistan. The superiority of the proposed methodology is supported by comparative and postoptimal analysis.

2. Preliminaries

2.1. Fuzzy Set. Let X be the universal set. A fuzzy set $\widetilde{A}$ [22] consists of a pair defined as $\widetilde{A} = \left\{(x, \mu_{\widetilde{A}}(x)), x \in X\right\}$, in which the first element x of $(x, \mu_{\widetilde{A}}(x))$ belongs to classical set and the second element defined as $\mu_{\widetilde{A}}(x)\colon X \longrightarrow [0, 1]$ refers to the membership degree of x in $\widetilde{A}$, called the membership function of $\widetilde{A}$.

2.2. Fuzzy Intuitionistic Sets. Let X be denoted as a universal set. An intuitionistic fuzzy set (IFS) $\widetilde{A}^I$ [23] is defined as set of ordered triplets $\widetilde{A}^I = \left\{(x, \mu^I_{\widetilde{A}}(x), \nu^I_{\widetilde{A}}(x)); x \in X\right\}$, in which the functions $\mu^I_{\widetilde{A}}(x)\colon X \longrightarrow [0,1]$ and $\nu^I_{\widetilde{A}}(x)\colon X \longrightarrow [0,1]$ represent membership and nonmembership degree of x in $\widetilde{A}$, respectively, for each element $x \in X$ satisfying $0 \le \mu^I_{\widetilde{A}}(x) + \nu^I_{\widetilde{A}}(x) \le 1$.

2.3. Triangular Intuitionistic Fuzzy Number. A triangular intuitionistic fuzzy number (TIFN) [29] $\breve{S}^I$ is an especial IFN with the membership function and nonmembership function defined as follows:

$$\mu_{\breve{S}^I}(x) = \begin{cases} 0, & \text{if } x < a, \\ \dfrac{x-a}{b-a}, & \text{if } a \le x \le b, \\ 1, & \text{if } x = b, \\ \dfrac{c-x}{c-b}, & \text{if } b \le x \le c, \\ 0, & \text{if } x > c, \end{cases}$$
$$\nu_{\breve{S}^I}(x) = \begin{cases} 1, & \text{if } x < d; \\ \dfrac{b-x}{b-d}, & \text{if } d \le x \le b; \\ 0, & \text{if } x = s_2, \\ \dfrac{x-b}{e-b}, & \text{if } b \le x \le e, \\ 1, & \text{if } x > e, \end{cases} \tag{1}$$

where $d \le a \le b \le c \le e$, denoted by $\breve{S}^I = (a,b,c;d,b,e)$ or TIFN. Membership and nonmembership functions of TIFN are presented in Figure 1.

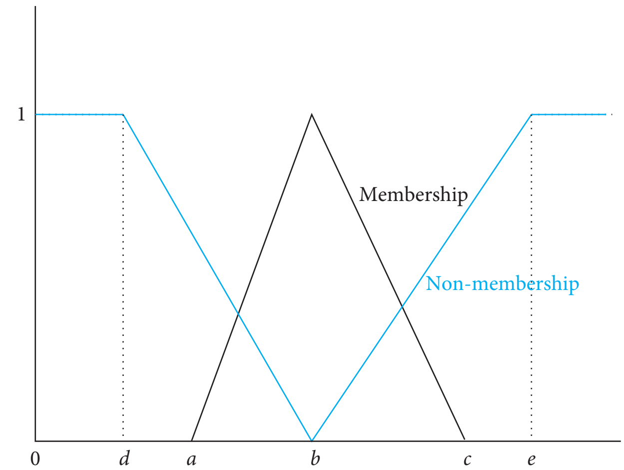


Figure 1: Triangular intuitionistic fuzzy number.

2.4. Accuracy Function. The accuracy function [30] for triangular intuitionistic fuzzy numbers $\breve{A} = (a_1, a_2, a_3; a_1', a_2, a_3')$ is defined as

$$H\left(\breve{A}^I\right) = \frac{(a_1 + 2a_2 + a_3) + (a_1' + 2a_2 + a_3')}{8}. \tag{2}$$

3. Operations on Triangular Intuitionistic Fuzzy Number

A triangular intuitionistic fuzzy number $\breve{S}^I = (s_1, s_2, s_3; s_1', s_2, s_3')$ is said to be nonnegative if and only if $s_l' \ge 0$.

The arithmetic operations of triangular intuitionistic fuzzy number [29], i.e., addition, subtraction, multiplications, and division, are defined by considering two nonnegative triangular intuitionistic fuzzy numbers $\breve{S}^I = (s_1, s_2, s_3; s_1', s_2, s_3')$ and $\breve{R}^I = (r_1, r_2, r_3; r_1', r_2, r_3')$. Two triangular intuitionistic fuzzy numbers are equal, $\breve{S}^I = \breve{R}^I$, if and only if $s_1 = r_1$, $s_2 = r_2$, $s_3 = r_3$, $s_1' = r_1'$, and $s_3' = r_3'$.

3.1. Addition

$$\begin{aligned} \breve{S}^I \oplus \breve{R}^I &= (s_1, s_2, s_3; s_1', s_2, s_3') \oplus (r_1, r_2, r_3; r_1', r_2, r_3') \\ &= (s_1 + r_1, s_2 + r_2, s_3 + r_3; s_1' + r_1', s_2 + r_2, s_3' + r_3'). \end{aligned} \tag{3}$$

3.2. Subtraction

$$\begin{aligned} \breve{S}^I \ominus \breve{R}^I &= (s_1, s_2, s_3; s_1', s_2, s_3') \ominus (r_1, r_2, r_3; r_1', r_2, r_3'), \\ &= (s_1, s_2, s_3; s_1', s_2, s_3') \ominus (r_1, r_2, r_3; r_1', r_2, r_3'). \end{aligned} \tag{4}$$

3.3. Symmetric Property

$$-\left(\breve{S}^I\right) = (-s_1, -s_2, -s_3; -s_1', -s_2, -s_3'). \tag{5}$$

3.4. Scalar Multiplication. Let α be any scalar; then,

$$\begin{aligned} \alpha\left(\breve{S}^I\right) &== (\alpha s_1, \alpha s_2, \alpha s_3; \alpha s_1', \alpha s_2, \alpha s_3'), \alpha \ge 0, \\ \alpha\left(\breve{S}^I\right) &= (\alpha s_3, \alpha s_2, \alpha s_1; \alpha s_3', \alpha s_2, \alpha s s_1'), \alpha < 0. \end{aligned} \tag{6}$$

3.5. Multiplication

$$\begin{aligned} \breve{S}^I \otimes \breve{R}^I &= (s_1, s_2, s_3; s_1', s_2, s_3') \otimes (r_1, r_2, z_3; r_1', r_2, r_3') \\ &\cong (s_1 r_1, s_2 r_2, s_3 r_3; s_1' r_1', s_2 r_2, s_3' r_3'). \end{aligned} \tag{7}$$

Remark 1. If $\breve{S}$ and $\breve{R}$ are not nonnegative triangular fuzzy numbers, then their multiplication will be performed as

$$\breve{S}^I \otimes \breve{R}^I = (a, b, c; a', b', c'), \quad (8)$$

where

$$\begin{aligned} a &= \min(s_1r_1, s_1r_3, s_3r_1, s_3r_3), \\ a' &= \min(s'_1r'_1, s'_1r'_3, s'_3r'_1, s'_3r'_3), \\ b &= s_2r_2, \\ b' &= s_2r_2, \\ c &= \max(s_1r_1, s_1r_3, s_3r_1, s_3r_3), \\ c' &= \max(s'_1r'_1, s'_1r'_3, s'_3r'_1, s'_3r'_3). \end{aligned} \quad (9)$$

4. Linear Programming Model

General linear programming [16] is defined as

$$(\text{Max})Z(x) = \sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij}x_{ij}, \quad (10)$$

subject to the following constraints:

$$\sum_{i=1}^{p}\sum_{j=1}^{q} a_{ij}x_{ij} = u_i. \quad (11)$$

Condition of nonnegativity is as follows:

$$x_{\text{ij}} \geq 0 \text{ for all } i = 1, 2, \ldots, p;\ j = 1, 2, \ldots, q, \quad (12)$$

where $x_{\text{ij}}, c_{\text{ij}}, a_{\text{ij}}$, and u_i are the decision variables, coefficients of quantity which we have to maximize or minimize, constraints coefficients, and constants, respectively. This represents the crisp modeling of the problem, but for the most beneficial implementation of this model in our daily life problems, we used its modified form "triangular intuitionistic fuzzy linear programming" which is endowed with the generalized techniques for the absorbtion of fuzziness due to unpredictable and unfortunate scenario.

5. Triangular Intuitionistic Fuzzy Linear Programming Model

Triangular intuitionistic fuzzy linear programming enhances the targeted requirements by evaluating the problem specifications meticulously using the generalization of fuzzy logics intuitionistic fuzzy sets. A triangular intuitionistic fuzzy linear programming [25] can be formulated as follows:

$$(\text{Max})\breve{Z}^I = \sum_{i=1}^{p}\sum_{j=1}^{q} \breve{c}_{ij}^I \otimes \breve{x}_{ij}^I, \quad (13)$$

subject to the following constraints:

$$\sum_{i=1}^{p}\sum_{j=1}^{q} \breve{a}_{ij}^I \otimes \breve{x}_{ij}^I = \breve{u}_i^I. \quad (14)$$

Condition of nonnegativity is as follows:

$$\breve{x}_{ij}^I \geq 0 \text{ for all } i = 1, 2, \ldots, p;\ j = 1, 2, \ldots, q, \quad (15)$$

where the model contains all coefficients, variables, and constants in the form of triangular intuitionistic fuzzy numbers; for example, $\breve{c}_{ij}^I = (c_{ij,1}, c_{ij,2}, c_{ij,3}; c'_{ij,1}, c_{ij,2}, c'_{ij,3})$, $\breve{a}_{ij}^I = (a_{ij,1}, a_{ij,2}, a_{ij,3}; a'_{ij,1}, a_{ij,2}, a'_{ij,3})$, and $\breve{u}_i^I = (u_{i,1}, u_{i,2}, u_{i,3}; u'_{i,1}, u_{i,2}, u'_{i,3})$ are triangular intuitionistic fuzzy cost coefficients, triangular intuitionistic fuzzy constraints coefficients, and constants, respectively, with $\breve{x}_{ij}^I = (x_{ij,1}, x_{ij,2}, x_{ij,3}; x'_{ij,1}, x_{ij,2}, x'_{ij,3})$ being triangular intuitionistic fuzzy decision variables. Ultimately, $\breve{Z}$ is the maximum triangular intuitionistic fuzzy objective value.

6. Methodology

The linear programming for fruit production maximization is developed as

$$(\text{Max})\text{FP}_M(x) = \sum_{i=1}^{p}\sum_{j=1}^{q} P_{ij}x_{ij}, \quad (16)$$

subject to the following constraints:

$$\sum_{i=1}^{p}\sum_{j=1}^{q} A_{ij}x_{ij} = v_i. \quad (17)$$

Condition of nonnegativity is as follows:

$$x_{ij} \geq 0 \text{ for all } i = 1, 2, \ldots, p;\ j = 1, 2, \ldots, q, \quad (18)$$

where FP_M is maximized fruit production; x_{ij} refers to activities (cutting, pruning, harvesting, thinning, leveling, sales, etc.); P_{ij} indicates objective coefficients (market prices of variables, product profit, etc.); A_{ij} denotes constraints coefficients (utilized resources and capital per unit of fruit production); and v_i is the total available amount/units/volume of supplies per hector.

Generally defined constraints for major fruit production are further written as follows:

$$\begin{aligned} &\text{total land availability constraints: } \textstyle\sum_{i=1}^{h} G_i^l \leq \text{TL}, \\ &\text{maximum sowing area constraints: } G_1^l \leq \text{TL}^G, \\ &: G_2^l \leq \text{TL}^A \mathbb{Z}, \\ &: G_3^l \leq \text{TL}^C, \\ &: G_4^l \leq \text{TL}^{\text{AL}}, \\ &: G_5^l \leq \text{TL}^{\text{PL}}, \\ &\text{availability of labor units constraints: } \textstyle\sum H_iR_i \leq \text{TH}_i, \\ &\text{balanced fertilizers input constraints: } \textstyle\sum F_iR_i = 0, \\ &\text{pesticide input constraints: } \textstyle\sum S_iR_i = 0, \\ &\text{cost constraints: } \textstyle\sum B_iR_i = 0, \\ &\text{average yield constraints: } \textstyle\sum Y_iR_i - M_i = 0, \end{aligned} \quad (19)$$

where h is the total number of fruit crops, TL is the total cultivated land, G_i^l is the total available area for each fruit,

TL^G is the total area for grapes, TL^A is the total area for apples, TL^C is the total area for cherry, TL^{AL} is the total area for almond, TL^{PL} is the total area for plum, TH_i is the total available hours or man-days for labor, R_i is the area for each fruit crop, H_i is the required working hours or man-days for each ith crop, F_i represents the required amount of fertilizer per hector, S_i represents the required amount of pesticide per hector, B_i is the total cost per hector, Y_i is the amount of yields in kg per hector, and M_i is the market selling price of yield per kg.

Then, we need much more precision regarding data and situation analysis because of changing factors and circumstances in our universe. The world we are living in is not like before; it is constantly changing, which makes it more challenging for us to change ourselves and our methods according to that change. The simple linear programming is not enough for our environment changes like climate changes, economic downfall, fluctuation of prices and demand, unsuitability of resources, pest and diseases, governmental policies, international trade agreements, topography, and political and social factors. We made a conscious effort regarding this issue especially for the hyperarid zones of Pakistan to improve our food security and GDP. Here, a triangular fuzzy linear programming is formulated according to the present situation analysis of fruit production of Pakistan for improvement.

The triangular intuitionistic fuzzy linear programming for fruit production maximization is developed as

$$(\text{Max})\breve{\text{FP}}_M^I = \sum_{i=1}^{p}\sum_{j=1}^{q}\breve{P}_{\text{ij}}^I \otimes \breve{x}_{\text{ij}}^I, \tag{20}$$

subject to the following constraints:

$$\sum_{i=1}^{p}\sum_{j=1}^{q}\breve{K}_{\text{ij}}^I \otimes \breve{x}_{\text{ij}}^I = \breve{v}_i^I. \tag{21}$$

Condition of nonnegativity is as follows:

$$\breve{x}_{\text{ij}}^I \geq 0^I \text{ for all } i = 1, 2, \ldots, p;\ j = 1, 2, \ldots, q, \tag{22}$$

where $\breve{\text{FP}}_M^I$ is the triangular intuitionistic fuzzy maximized fruit production; $\breve{x}_{\text{ij}}^I$ refers to the triangular intuitionistic fuzzy activities (cutting, pruning, harvesting, thinning, leveling, sales, etc.); $\breve{P}_{\text{ij}}^I$ indicates the objective triangular intuitionistic fuzzy coefficients (market prices of variables, product profit, etc.); $\breve{K}_{\text{ij}}^I$ represents the triangular intuitionistic fuzzy constraints coefficients (utilized resources and capital per unit of fruit production); and $\breve{v}_i^I$ is the total available triangular intuitionistic fuzzy amount/units/volume of supplies per hector.

The objective function and constraints equations will be written as

$$\begin{aligned}(\text{Max})\breve{\text{FP}}_M^I &= \sum_{i=1}^{p}\sum_{j=1}^{q}\left(P_{\text{ij},1}, P_{\text{ij},2}, P_{\text{ij},3}; P_{\text{ij},1}', P_{\text{ij},2}, P_{\text{ij},3}'\right) \otimes \left(x_{\text{ij},1}, x_{\text{ij},2}, x_{\text{ij},3}; x_{\text{ij},1}', x_{\text{ij},2}, x_{\text{ij},3}'\right),\\ &\cdot \sum_{i=1}^{p}\sum_{j=1}^{q}\left(K_{\text{ij},1}, K_{\text{ij},2}, K_{\text{ij},3}; K_{\text{ij},1}', K_{\text{ij},2}, K_{\text{ij},3}'\right) \otimes \left(x_{\text{ij},1}, x_{\text{ij},2}, x_{\text{ij},3}; x_{\text{ij},1}', x_{\text{ij},2}, x_{\text{ij},3}'\right) = \left(u_{i,1}, u_{i,2}, u_{i,3}; u_{i,1}', u_{i,2}, u_{i,3}'\right),\\ &\cdot \left(x_{\text{ij},1}, x_{\text{ij},2}, x_{\text{ij},3}; x_{\text{ij},1}', x_{\text{ij},2}, x_{\text{ij},3}'\right) \succcurlyeq 0^I.\end{aligned} \tag{23}$$

By using the operations of triangular fuzzy numbers,

$$\begin{aligned}(\text{max})\breve{\text{FP}}_M^I &= \sum_{i=1}^{p}\sum_{j=1}^{q}\left(P_{ij,1}x_{ij,1}, P_{ij,2}x_{ij,2}, P_{ij,3}x_{ij,3}; P_{ij,1}'x_{ij,1}', P_{ij,2}x_{ij,2}, P_{ij,3}'x_{ij,3}'\right),\\ &\cdot \sum_{i=1}^{p}\sum_{j=1}^{q}\left(K_{ij,1}x_{ij,1}, K_{ij,2}x_{ij,2}, K_{ij,3}x_{ij,3}; K_{ij,1}'x_{ij,1}', K_{ij,2}x_{ij,2}, K_{ij,3}'x_{ij,3}'\right) = \left(u_{i,1}, u_{i,2}, u_{i,3}; u_{i,1}', u_{i,2}, u_{i,3}'\right),\\ &x_{ij,3}' \geq 0, x_{ij,3} - x_{ij,3}' \geq 0, x_{ij,2} - x_{ij,3} \geq 0, x_{ij,1} - x_{ij,2} \geq 0, x_{ij,1}' - x_{ij,1} \geq 0.\end{aligned} \tag{24}$$

Further simplification was carried out using accuracy function on the triangular intuitionistic fuzzy objective function.

$$\begin{aligned}
&\mathbf{W}\max\left(\sum_{i=1}^{p}\sum_{j=1}^{q}P_{\mathrm{ij},1}x_{ij,1},\sum_{i=1}^{p}\sum_{j=1}^{q}P_{\mathrm{ij},2}x_{\mathrm{ij},2},\sum_{i=1}^{p}\sum_{j=1}^{q}P_{\mathrm{ij},3}x_{\mathrm{ij},3};\sum_{i=1}^{p}\sum_{j=1}^{q}P_{\mathrm{ij},1}'x_{\mathrm{ij},1}',\right.\\
&\left.\sum_{i=1}^{p}\sum_{j=1}^{q}P_{\mathrm{ij},2}x_{\mathrm{ij},2},\sum_{i=1}^{p}\sum_{j=1}^{q}P_{\mathrm{ij},3}'x_{\mathrm{ij},3}'\right)\\
&=\frac{1}{8}\left(\sum_{i=1}^{p}\sum_{j=1}^{q}P_{\mathrm{ij},1}x_{\mathrm{ij},1}+2\sum_{i=1}^{p}\sum_{j=1}^{q}P_{\mathrm{ij},2}x_{\mathrm{ij},2}+\sum_{i=1}^{p}\sum_{j=1}^{q}P_{\mathrm{ij},3}x_{\mathrm{ij},3}\right)\\
&+\frac{1}{8}\left(\sum_{i=1}^{p}\sum_{j=1}^{q}P_{\mathrm{ij},1}'x_{\mathrm{ij},1}'+2\sum_{i=1}^{p}\sum_{j=1}^{q}P_{\mathrm{ij},2}x_{\mathrm{ij},2}+\sum_{i=1}^{p}\sum_{j=1}^{q}P_{\mathrm{ij},3}'x_{\mathrm{ij},3}'\right).
\end{aligned}\tag{25}$$

Ultimately, triangular intuitionistic fuzzy objective function is transmuted into linear objective function by accuracy function, and regarding that reference, the constraints are thoroughly modified into

$$\begin{aligned}
&\left(\sum_{i=1}^{p}\sum_{j=1}^{q}K_{\mathrm{ij},1}x_{\mathrm{ij},1},\sum_{i=1}^{p}\sum_{j=1}^{q}K_{\mathrm{ij},2}x_{\mathrm{ij},2},\sum_{i=1}^{p}\sum_{j=1}^{q}K_{\mathrm{ij},3}x_{\mathrm{ij},3};\sum_{i=1}^{p}\sum_{j=1}^{q}K_{\mathrm{ij},1}'x_{\mathrm{ij},1}',\sum_{i=1}^{p}\sum_{j=1}^{q}K_{\mathrm{ij},2}x_{\mathrm{ij},2},\sum_{i=1}^{p}\sum_{j=1}^{q}K_{\mathrm{ij},3}'x_{\mathrm{ij},3}'\right)\\
&=\left(u_{i,1},u_{i,2},u_{i,3};u_{i,1}',u_{i,2},u_{i,3}'\right).
\end{aligned}\tag{26}$$

Using the equality condition of triangular intuitionistic fuzzy number, we have

$$\begin{aligned}
\sum_{i=1}^{p}\sum_{j=1}^{q}K_{\mathrm{ij},1}x_{\mathrm{ij},1}&=u_{i,1},\\
\sum_{i=1}^{p}\sum_{j=1}^{q}K_{\mathrm{ij},2}x_{\mathrm{ij},2}&=u_{i,2},\\
\sum_{i=1}^{p}\sum_{j=1}^{q}K_{\mathrm{ij},3}x_{\mathrm{ij},3}&=u_{i,3},\\
\sum_{i=1}^{p}\sum_{j=1}^{q}K_{\mathrm{ij},1}'x_{\mathrm{ij},1}'&=u_{i,1}',\\
\sum_{i=1}^{p}\sum_{j=1}^{q}K_{\mathrm{ij},2}x_{\mathrm{ij},2}&=u_{i,2},\\
\sum_{i=1}^{p}\sum_{j=1}^{q}K_{\mathrm{ij},3}'x_{\mathrm{ij},3}'&=u_{i,3}'.
\end{aligned}\tag{27}$$

Now, the model is converted into simple linear problem which can be easily solved through LP algorithm or Excel Solver. Then, we get the values of unknowns (decision variables) that are substituted into the triangular intuitionistic fuzzy objective function to get the maximized result in the form of triangular intuitionistic fuzzy number.

7. Application

The provinces of Punjab and Baluchistan produce abundant amount of fruit where Baluchistan lies in the arid regions of Pakistan. Baluchistan is the largest province on the basis of area occupying 347,190 square kilometres and located in southwest direction. The climatic conditions of Baluchistan region are characterized by very cold winter and very hot summer with maximum of 50°C to 53°C [31]. Moreover, strong windstorms and temperature make the area very hot arid zone, which is referred to as hyperarid zone. Baluchistan contributes nearly 4.9% to GDP which is far less than other provinces. Recently, water availability for the expansion of sustainable agricultural land is achieved by making Mirani Dam on the Dasht River which irrigates 35,000 km^2 of area [32]. For practical application of our formulated models, data for fruit production is collected from Baluchistan and is arranged in tabular form for easy further use.

8. Mathematical Model Formulation

The practical formulation of the model is carried out through the application of the above statistics that are specifically gathered from the Baluchistan province based on the data given in Tables 1–3.

Objective function is as follows:

$$\text{Max} \quad Z_{\text{FP}} = -110x_5 + X_7 + 160x_9 + 120x_{10} + 150x_{11} + 150x_{12} + 200x_{13}, \tag{28}$$

subject to the following constraints:

$$\begin{aligned}
x_1 + x_2 + x_3 + x_4 + x_5 &= 120,\\
x_1 &\le 30,\\
x_2 &\le 30,\\
x_3 + x_4 + x_5 &\le 60,\\
-300x_1 - 250x_2 - 200x_3 - 210x_4 - 230x_5 + x_6 &= 0,\\
3000x_1 + 2850x_2 + 2900x_3 + 3100x_4 + 3050x_5 - x_7 &= 0,\\
-6.1x_1 - 3.5x_2 - 4.5x_3 - 3.2x_4 - 3.5x_5 + x_8 &\le 20.5,\\
13700x_1 - x_9 &= 0,\\
17100x_1 - x_{10} &= 0,\\
25000x_1 - x_{11} &= 0,\\
81600x_1 - x_{12} &= 0,\\
52800x_1 - x_{13} &= 0,\\
52800x_1 - x_{13} &= 0.
\end{aligned} \tag{29}$$

In this model, we used fertilizers, all types of cost, available labor hours, and average fruit yield as constraints to find the optimal fruit production. After the above developments, we used Excel Solver for the maximum yield which gives objective value $Z_{\text{FP}} = 858880500$ kg. Afterwards, fuzzy modification of model is carried out to figure out more optimal way of modeling the existing methodology. The triangular fuzzy intuitionistic linear programming is given as follows.

Intuitionistic fuzzy objective function is as follows:

$$\begin{aligned}
(Max)\breve{Z}^{I}_{\text{FP}} = &\left(-(120, 110, 100; 130, 110, 90) \otimes \breve{x}^{I}_{6}\right) + \left((1.3, 1, 0.7; 1.6, 1, 0.4) \otimes \breve{x}^{I}_{7}\right) + \left((180, 160, 140; 200, 160, 120) \otimes \breve{x}^{I}_{9}\right)\\
&+ \left((140, 120, 100; 160, 120, 180) \otimes \breve{x}^{I}_{10}\right) + \left((175, 150, 125; 200, 150, 100) \otimes \breve{x}^{I}_{11}\right)\\
&+ \left((160, 150, 140; 170, 150, 130) \otimes \breve{x}^{I}_{12}\right) + \left((220, 200, 180; 240, 200, 160) \otimes \breve{x}^{I}_{13}\right),
\end{aligned} \tag{30}$$

Table 1: Orchard area statistics.

Specifications	Occupied area (ha)	Percentage of average cultivated land (%)	Number of trees (\ha)
Apple	30	25	900
Grapes	30	25	1000
Apricot	20	16.66	455
Peach	20	16.66	450
Plum	20	16.66	430
Total	120	100	3235

Table 2: Orchard production statistics.

Specifications	Yield (kg\ha)	Price (Rs\kg)
Apple	13700	160
Grapes	17100	120
Apricot	25000	150
Peach	81600	150
Plum	52800	200

Table 3: Material consumption statistics.

	Available units (kg, hrs, Rs\ha)		
Specifications	Fertilizers (kg\ha)	Cost (Rs\ha)	Labor (hrs\ha)
Apple	300	3000	6.1
Grapes	250	2850	3.5
Apricot	200	2900	4.5
Peach	210	3100	3.2
Plum	230	3050	3.5

Price of the fertilizer is Rs 110/kg.

subject to the following intuitionistic fuzzy constraints:

$$
\begin{aligned}
&(1.4, 1, 0.6; 1.8, 1, 0.2)\breve{x}_1^I + (1.3, 1, 0.7; 1.6, 1, 0.4)\breve{x}_2^I + (1.2, 1, 0.8; 1.4, 1, 0.6)\breve{x}_3^I + (1.1, 1, 0.9; 1.2, 1, 0.8)\\
&\cdot \breve{x}_4^I + (1.4, 1, 0.6; 1.8, 1, 0.2)\breve{x}_5^I = (125, 120, 115; 130, 120, 110),\\
&\cdot\ (1.3, 1, 0.7; 1.6, 1, 0.4)\breve{x}_1^I \le (35, 30, 25; 40, 30, 20),\\
&\cdot\ (1.3, 1, 0.7; 1.6, 1, 0.4)\breve{x}_2^I \le (35, 30, 25; 40, 30, 20),\\
&\cdot\ (1.4, 1, 0.6; 1.8, 1, 0.2)\breve{x}_3^I + (1.2, 1, 0.8; 1.4, 1, 0.6)\breve{x}_4^I + (1.1, 1, 0.9; 1.2, 1, 0.8)\breve{x}_5^I \le (65, 60, 55; 70, 60, 50),\\
&-(310, 300, 290; 320, 300, 280)\breve{x}_1^I - (260, 250240; 270, 250, 230)\breve{x}_2^I - (205, 200, 195; 210, 200, 190)\breve{x}_3^I -\\
&\cdot (220, 210, 200; 230, 210, 190)\breve{x}_4^I - (240, 230, 220; 250, 230, 210)\breve{x}_5^I + (1.4, 1, 0.6; 1.8, 1, 0.2)\breve{x}_6^I = \breve{0}^I,\\
&\cdot\ (3050, 3000, 2950; 3100, 3000, 2900)\breve{x}_1^I + (2900, 2850, 2800; 2950, 2850, 2750)\breve{x}_2^I +\\
&\cdot (2925, 2900, 2875; 2950, 2900, 2850)\breve{x}_3^I + (3200, 3100, 3000; 3300, 3100, 2900)\breve{x}_4^I +\\
&\cdot (3100, 3050, 3000; 3150, 3050, 2950)\breve{x}_5^I - (1.4, 1, 0.6; 1.8, 1, 0.2)\breve{x}_7^I = \breve{0}^I,\\
&-(6.4, 6.1, 5.8; 6.7, 6.1, 5.5)\breve{x}_1^I - (4, 3.5, 3; 4.4, 3.5, 2.5)\breve{x}_2^I - (4.8, 4.5, 4.2; 5.1, 4.5, 3.9)\breve{x}_3^I - (3.4, 3.2, 3;\\
&\cdot 3.6, 3.2, 2.8)\breve{x}_4^I - (3.75, 3.5, 3.25; 4, 3.5, 3)\breve{x}_5^I + (1.4, 1, 0.6; 1.8, 1, 0.2)\breve{x}_8^I \le (21, 20.5, 20; 21.5, 20.5, 19.5),\\
&\cdot\ (13800, 13700, 13600; 13900, 13700, 13500)\breve{x}_1^I - (1.4, 1, 0.6; 1.8, 1, 0.2)\breve{x}_9^I = \breve{0}^I,\\
&\cdot\ (17200, 17100, 17000; 17300, 17100, 16900)\breve{x}_2^I - (1.4, 1, 0.6; 1.8, 1, 0.2)\breve{x}_{10}^I = \breve{0}^I,\\
&\cdot\ (25100, 25000, 24900; 25200, 2500, 24800)\breve{x}_3^I - (1.4, 1, 0.6; 1.8, 1, 0.2)\breve{x}_{11}^1 = \breve{0}^I,\\
&\cdot\ (81700, 81600, 81500; 81800, 81600, 81400)\breve{x}_4^I - (1.4, 1, 0.6; 1.8, 1, 0.2)\breve{x}_{12}^I = \breve{0}^I,\\
&\cdot\ (52850, 52800, 52750; 52900, 52800, 52700)\breve{x}_5^I - (1.4, 1, 0.6; 1.8, 1, 0.2)\breve{x}_{13}^I = \breve{0}^I.
\end{aligned}
\tag{31}
$$

This is the mathematical formulation of triangular intuitionistic fuzzy linear programming in which all the decision variables and the regarding coefficients are triangular intuitionistic fuzzy numbers. As stated above, we cannot directly solve this model. Ultimately, we convert this model into crisp linear programming by using the accuracy function and arithmetic operations of triangular intuitionistic fuzzy numbers accordingly.

$$\begin{aligned}
&\Big(\Big(-(120, 110, 100; 130, 110, 90) \otimes \breve{x}_6^I\Big) + \Big((1.3, 1, 0.7; 1.6, 1, 0.4) \otimes \breve{x}_7^I\Big) + ((180, 160, 140; 200, 160, 120) \\
&\otimes \breve{x}_9^I\Big) + \Big((140, 120, 100; 160, 120, 180) \otimes \breve{x}_{10}^I\Big) + \Big((175, 150, 125; 200, 150, 100) \otimes \breve{x}_{11}^I\Big) + (\,(160, 150, 140; \\
&170, 150, 130) \otimes \breve{x}_{12}^I\Big) + \Big((220, 200, 180; 240, 200, 160) \otimes \breve{x}_{13}^I\Big)\Big), \\
&(\text{Max})\breve{Z}_{\text{FP}}^I = \frac{1}{8}\left\{-120x_{6,1} + 1.3x_{7,1} + 180x_{9,1} - 140x_{10,1} - 175x_{11,1} + 160x_{12,1} + 220x_{13,1}\right\} + \frac{4}{8}\left\{-110x_{6,2}\right. \\
&\quad \left. + 1x_{7,2} + 160x_{9,2} - 120x_{10,2} - 150x_{11,2} + 150x_{12,2} + 200x_{13,2}\right\} + \frac{1}{8}\left\{-100x_{6,3} + 0.7x_{7,3}\right. \\
&\quad \left. + 140x_{9,3} - 100x_{10,3} - 125x_{11,3} + 140x_{12,3} + 180x_{13,3}\right\} + \frac{1}{8}\left\{-130x'_{6,1} + 1.6x'_{7,4} + 200x'_{9,4}\right. \\
&\quad \left. - 160x'_{10,1} - 200x'_{11,1} + 170x'_{12,1} + 240x'_{13,1}\right\} + \frac{1}{8}\left\{-90x'_{6,3} + 0.4x'_{7,3} + 120x'_{9,3} - 180x'_{10,3}\right. \\
&\quad \left. - 100x'_{11,3} + 130x'_{12,3} + 160x'_{13,3}\right\}.
\end{aligned} \tag{32}$$

Along with the linear constraints simplification, which is carried out using the arithmetic operations of multiplication and equality of triangular intuitionistic fuzzy numbers according to the methodology, we have the crisp LP model which is simply solved through Excel Solver to find the values of decision variables. The values of decision variables obtained are

$$\begin{aligned}
\breve{x}_1^I &= (21.72619048, 30, 35.7142871; 16.66666667, 30, 50), \\
\breve{x}_2^I &= (26.92307692, 30, 35.71428571; 25, 30, 50), \\
\breve{x}_3^I &= (0, 0, 53.57142857; 0, 0, 40), \\
\breve{x}_4^I &= (54.16666667, 60, 28.57142857; 50, 60, 70), \\
\breve{x}_5^I &= (0, 0, 0; 0, 0, 0), \\
\breve{x}_6^I &= (18322.70408, 29100, 58482.14286; 13101.85185, 29100, 232000), \\
\breve{x}_7^I &= (226910.8124, 361500, 741815.4762; 161342.5926, 361500, 2997500), \\
\breve{x}_8^I &= (0, 0, 0; 0, 0, 0), \\
\breve{x}_9^I &= (214158.1633, 411000, 809523.8095; 128703.7037, 411000, 3375000), \\
\breve{x}_{10}^I &= (330769.2308, 513000, 1011904.762; 240277.7778, 513000, 4225000), \\
\breve{x}_{11}^I &= (0, 0, 2223214.286; 0, 0, 4960000), \\
\breve{x}_{12}^I &= (3161011.905, 4896000, 3880952.381; 2272222.222, 4896000, 28490000), \\
\breve{x}_{13}^I &= (0, 0, 0; 0, 0, 0).
\end{aligned} \tag{33}$$

The triangular intuitionistic fuzzy objective value is obtained by putting the values of decision variables $\breve{x}_1^I, \breve{x}_2^I, \breve{x}_3^I, \breve{x}_4^I,\ \breve{x}_5^I \ldots \breve{x}_{13}^I$ into the triangular intuitionistic fuzzy objective function as follows:

$$\breve{Z}_{\text{FP}}^I = (588714344, 859026000, 1042552710; 449017835.6, 859026000, 5345519000), \tag{34}$$

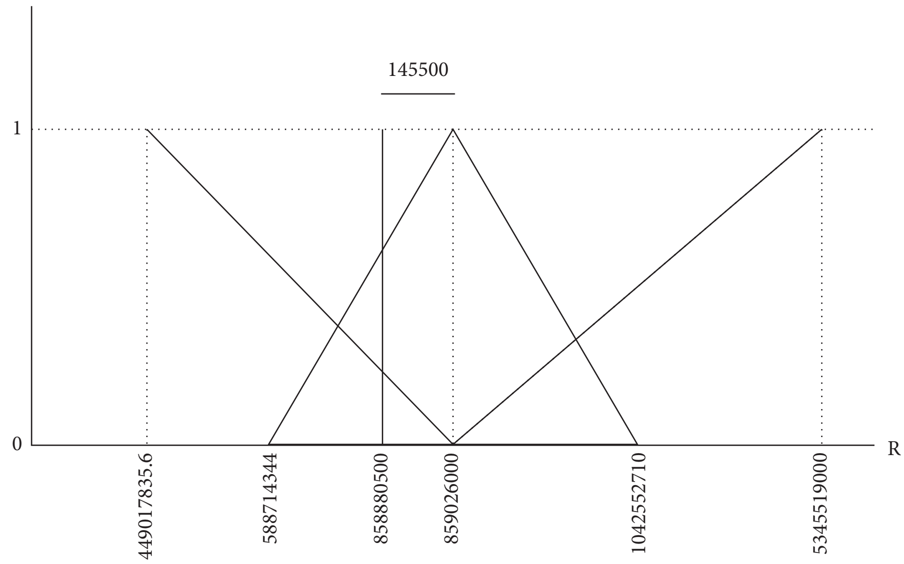


Figure 2: Graphical comparison of optimal solution.

Table 4: Sensitivity report (variables).

Variable name	Final value	Objective coefficient	Allowable increase	Allowable decrease	Variable name	Final value	Objective coefficient	Allowable increase	Allowable decrease
$x_{1,1}$	21.72619048	0	10082.65797	333727.8781	$x'_{7,1}$	161342.5926	0.2	$1E+30$	50.32943262
$x_{1,2}$	30	0	$1.00E+30$	67325	$x'_{7,3}$	2997500	0.05	264.5900754	157.5387006
$x_{1,3}$	35.71428571	0	$1E+30$	5840117.708	$x_{8,1}$	0	0	0	$1E+30$
$x'_{1,1}$	16.66666667	0	23365.45139	65707.87037	$x_{8,2}$	0	0	0	$1E+30$
$x'_{1,3}$	50	0	$1E+30$	1394217.5	$x_{8,3}$	0	0	0	$1E+30$
$x_{2,1}$	26.92307692	0	$1E+30$	9362468112	$x'_{8,1}$	0	0	0	$1E+30$
$x_{2,2}$	30	0	67325	$1E+30$	$x'_{8,3}$	0	0	0	$1E+30$
$x_{2,3}$	35.71428571	0	$1E+30$	6706814.583	$x_{9,1}$	214158.1633	22.5	1.022878344	33.8564514
$x'_{2,1}$	25	0	$1E+30$	20769.29012	$x_{9,2}$	411000	80	$1E+30$	4.914233577
$x'_{2,3}$	50	0	$1E+30$	1626235	$x_{9,3}$	809523.8095	17.5	$1E+30$	257.6522518
$x_{3,1}$	0	0	955994.5206	$7.45517E+21$	$x'_{9,1}$	128703.7037	25	3.025741906	8.508932854
$x_{3,2}$	0	0	4244550	$2.29518E+22$	$x'_{9,3}$	3375000	15	$1E+30$	20.65507407
$x_{3,3}$	53.57142857	0	1197645.461	$1E+30$	$x_{10,1}$	330769.2308	17.5	$1E+30$	0.762061358
$x'_{3,1}$	$7.105E-15$	0	875857.0547	$1.16921E+20$	$x_{10,2}$	513000	60	3.937134503	$1E+30$
$x'_{3,3}$	40	0	1355195.833	$1E+30$	$x_{10,3}$	1011904.762	12.5	$1E+30$	236.7111029
$x_{4,1}$	54.16666667	0	$6.08437E+19$	101852.0147	$x'_{10,1}$	240277.7778	20	$1E+30$	2.160966602
$x_{4,2}$	60	0	$1E+30$	841125	$x'_{10,3}$	4225000	10	$1E+30$	19.245 384 62
$x_{4,3}$	28.57142857	0	$1E+30$	1233965.205	$x_{11,1}$	0	21.875	53.32240354	$1E+30$
$x'_{4,1}$	50	0	$1E+30$	32853.93519	$x_{11,2}$	0	75	169.782	$1E+30$
$x'_{4,3}$	70	0	$1E+30$	2393337.5	$x_{11,3}$	2223214.286	15.625	28.85892678	$1E+30$
$x_{5,1}$	0	0	9364.34684	$1E+30$	$x'_{11,1}$	0	25	62.56121819	$1E+30$
$x_{5,2}$	0	0	841125	$1E+30$	$x'_{11,3}$	4960000	12.5	10.92899866	$1E+30$
$x_{5,3}$	0	0	4442274.739	$1E+30$	$x_{12,1}$	3161011.905	20	$1.04261E+15$	1.745322162
$x'_{5,1}$	0	0	28160.51587	$1E+30$	$x_{12,2}$	4896000	75	$1E+30$	10.30790441
$x'_{5,3}$	$3.552E-15$	0	5265342.5	$1E+30$	$x_{12,3}$	3880952.381	17.5	$1E+30$	9.08440642
$x_{6,1}$	18322.70408	−15	470.5240384	2701.020218	$x'_{12,1}$	2272222.222	21.25	$1E+30$	0.722947229
$x_{6,2}$	29100	−55	42056.25	1346.5	$x'_{12,3}$	28490000	16.25	$1E+30$	5.880436118
$x_{6,3}$	58482.1486	−12.5	18573.97104	47352.30574	$.x_{13,1}.$	0	27.5	2.473227731	$1E+30$
$x'_{6,1}$	13101.85185	−16.25	2588.173077	601.3940678	$x_{13,2}$	0	100	15.93039773	$1E+30$
$x'_{6,3}$	232000	−11.25	3682.057692	1366.879902	$x_{13,3}$	0	22.5	50.52824348	$1E+30$
$x_{7,1}$	226910.8124	0.1625	$1.00E+30$	190.3709959	$x'_{13,1}$	0	30	0.958202809	$1E+30$
$x_{7,2}$	361500	0.5	$1E+30$	448.8333333	$x'_{13,3}$	0	20	19.98232448	$1E+30$
$x_{7,3}$	741815.4762	0.0875	1579.475463	$1E+30$					

Table 5: Sensitivity report (constraints).

Constraints (LHS)	Final value	Shadow price	Constraints (RHS)	Allowable increase	Allowable decrease
Total land 1 LHS	125	156298.7883	125	7.275641026	30.41666667
Total land 2 LHS	120	1013675	120	0	30
Total land 3 LHS	115	−9081770.833	115	4.761904762	6.696428571
Total land 4 LHS	130	105839.5062	130	15	30
Total land 5 LHS	110	−1983712.5	110	70	13.33333333
Land for apple 1 LHS	28.24404762	0	35	$1E+30$	6.755952381
Land for apple 2 LHS	30	67325	30	30	0
Land for apple 3 LHS	25	8343025.297	25	7.8125	5.555555556
Land for apple 4 LHS	26.66666667	0	40	$1E+30$	13.33333333
Land for apple 5 LHS	20	3485543.75	20	26.66666667	20
Land for grapes 1 LHS	35	7201.898548	35	30.41666667	7.275641026
Land for grapes 2 LHS	30	0	30	$1E+30$	$-7.10543E-15$
Land for grapes 3 LHS	25	9581163.69	25	0	4.761904762
Land for grapes 4 LHS	40	12980.80633	40	30	15
Land for grapes 5 LHS	20	4065587.5	20	13.33333333	20
Land for drupes 1 LHS	65	827690.3965	65	33.18181818	0
Land for drupes 2 LHS	60	5096325	60	30	0
Land for drupes 3 LHS	55	13183684.9	55	5.952380952	3.571428571
Land for drupes 4 LHS	70	597840.9392	70	35	17.5
Land for drupes 5 LHS	50	13651262.5	50	10	23.33333333
Fertilizers 1 LHS	$-1.66619E-09$	−10.71428571	0	$1E+30$	25651.78571
Fertilizers 2 LHS	$-3.63798E-12$	−55	0	$1E+30$	29100
Fertilizers 3 LHS	$5.31873E-09$	−20.83333333	0	$1E+30$	35089.28571
Fertilizers 4 LHS	$5.96629E-10$	−9.027777778	0	$1E+30$	23583.33333
Fertilizers 5 LHS	$-1.05501E-08$	−56.25	0	$1E+30$	46400
Costs 1 LHS	$-8.24803E-08$	−0.116071429	0	317675.1374	$1E+30$
Costs 2 LHS	0	−0.5	0	361500	$1E+30$
Costs 3 LHS	$2.69793E-07$	−0.145833333	0	445089.2857	$1E+30$
Costs 4 LHS	$-1.17405E-07$	−0.111111111	0	290416.6667	$1E+30$
Costs 5 LHS	$2.18092E-06$	−0.25	0	599500	$1E+30$
Labor 1 LHS	−430.9065934	0	21	$1E+30$	451.9065934
Labor 2 LHS	−480	0	20.5	$1E+30$	500.5
Labor 3 LHS	−625	0	20	$1E+30$	645
Labor 4 LHS	−404.1666667	0	21.5	$1E+30$	425.6666667
Labor 5 LHS	−752	0	19.5	$1E+30$	771.5
Apple yield 1 LHS	$-7.78819E-08$	−16.07142857	0	299821.4286	$1E+30$
Apple yield 2 LHS	$5.82077E-11$	−80	0	411000	$1E+30$
Apple yield 3 LHS	$2.94473E-07$	−29.16666667	0	485714.2857	$1E+30$
Apple yield 4 LHS	$-9.3627E-08$	−13.88888889	0	231666.6667	$1E+30$
Apple yield 5 LHS	$2.45555E-06$	−75	0	675000	$1E+30$
Grapes yield 1 LHS	$4.81319E-07$	−12.5	0	463076.9231	$1E+30$
Grapes yield 2 LHS	0	−60	0	513000	$1E+30$
Grapes yield 3 LHS	$-1.47265E-06$	−20.83333333	0	607142.8571	$1E+30$
Grapes yield 4 LHS	$-1.74856E-07$	−11.11111111	0	432500	$1E+30$
Grapes yield 5 LHS	$3.07418E-06$	−50	0	845000	$1E+30$
Apricot yield 1 LHS	0	−53.7124311	0	1165357.143	0
Apricot yield 2 LHS	0	−244.782	0	1500000	0
Apricot yield 3 LHS	$-3.23541E-06$	−26.04166667	0	1333928.571	$1E+30$
Apricot yield 4 LHS	$1.79057E-10$	−48.64512122	0	980000	$1.79057E-10$
Apricot yield 5 LHS	$3.60981E-06$	−62.5	0	992000	$1E+30$
Peach yield 1 LHS	$-1.83983E-05$	−14.28571429	0	4425416.667	$1E+30$
Peach yield 2 LHS	0	−75	0	4896000	$1E+30$
Peach yield 3 LHS	$2.25897E-05$	−29.16666667	0	2328571.429	$1E+30$
Peach yield 4 LHS	$6.61286E-06$	−11.80555556	0	4090000	$1E+30$
Peach yield 5 LHS	$-8.29175E-05$	−81.25	0	5698000	$1E+30$
Plum yield 1 LHS	0	−21.40944838	0	3122954.545	0
Plum yield 2 LHS	0	−115.9303977	0	3168000	0
Plum yield 3 LHS	0	−121.7137391	0	418650.7937	0
Plum yield 4 LHS	0	−17.19900156	0	2057222.222	0
Plum yield 5 LHS	$1.87228E-10$	−199.9116224	0	1676818.182	$1.87228E-10$

with membership and nonmembership degree as follows:

$$\mu_{\breve{Z}_{FP}^{I}}(x)=\begin{cases}0, & x<588714344,\\ \dfrac{x-588714344}{270311656}, & 588714344\leq x\leq 859026000,\\ 1, & x=859026000,\\ \dfrac{1042552710-x}{183526710}, & 859026000\leq x\leq 1042552710,\\ 0, & x>1042552710,\end{cases}$$

$$\nu_{\breve{Z}_{FP}^{I}}(x)=\begin{cases}1, & x<449017835.6,\\ \dfrac{859026000-x}{410008165}, & 449017835.6\leq x\leq 859026000,\\ 0, & x=859026000,\\ \dfrac{x-859026000}{4486493000}, & 859026000\leq x\leq 5345519000,\\ 1, & x>5345519000.\end{cases} \quad (35)$$

8.1. Interpretation and Comparison of Results. For comparison, the results obtained by optimization model considered in fuzzy environment should be compared with the linear programming in crisp environment. The general linear programming specifically designed for fruit production gives the output of 858880500 kg which is maximum fruit yield by consuming the available resources and inputs. The modified triangular intuitionistic fuzzy linear programming yields the result of

$$\breve{Z}_{FP}^{I}=(588714344, 859026000, 1042552710; 449017835.6, 859026000, 5345519000), \quad (36)$$

which is clearly maximum fruit production output in the form of triangular intuitionistic fuzzy number. These results are further explained and demonstrated trough detailed analysis in the form of graphical representation in Figure 2 which shows the output of both techniques. The level of satisfaction increases with the production increase from 588 714 344 to 859 026 000, reaches the maximum over 859 026 000 with membership degree 1, and then decreases afterwards to 1042 552 710. It is obvious that degree of nonmembership decreases with the increase in membership degree simultaneously. The vertical line in the graph at 858 880 500 represents the results of linear programming. In comparison, the graph already shows that 145 500 kg of yield increased by triangular intuitionistic fuzzy linear programming and the optimal region obtained from this technique is much more acceptable due to the feasibility levels at certain situations.

8.2. Postoptimality (Sensitivity) Analysis. Sensitivity analysis (postoptimality analysis) is the process of determining how changes in the optimal solution influence it, within certain limits. The sensitivity analysis is carried out by changing the coefficients of objective function and the right-hand side (RHS) values of constraints. Here, postoptimality (sensitivity) analysis of triangular intuitionistic fuzzy linear programming is assessed using the Tables 4 and 5. The solution remains optimal and feasible within the specified limits of variables and parameters. Range of optimality is dependent on the coefficients of objective function, which means that change in the coefficients of objective function affects the optimality of solution, which is represented by Table 4. This table contains the limits for the coefficients of each variable in the form of allowable increase and decrease. For example, the limit of coefficient of $x_{1,1}$ having original value 0 is between 10082.65797 and 333727.8781, and the solution remains optimal for this range. The cell containing value $1E+30$ in the form of allowable increase or decrees means that there is no limit for the increase or decrease of that specific variable.

In Table 5, the range of each constraint is presented with the shadow increase in objective value, which is only valid for given ranges. A change in the right-hand side of a constraint directly changes the feasible region which perhaps influences the optimal solution. From Table 5, it is clear that our feasibility region remains feasible and the same if the constraints change within the allowable range. As observed from Table 5, the total land constraint 1 has a range between 7.275641026 and 30.41666667 in which feasibility region of the model remains unchanged. Moreover, shadow price is also given per unit increase in the right-hand side of the constraint providing improvement in the value of the optimal solution. The above analysis indicated that this technique is providing flexible optimal solution with the original data.

9. Conclusion

The comparison of methodologies, postoptimality (sensitivity) analysis, and compiled statistics stated that the triangular intuitionistic fuzzy linear programming is providing best results for management of real-life problems. The feasible region for optimal production in fuzzy environment remains feasible and optimal within sufficient range. In future, we can consider this model in different fuzzy environments to optimize production and observe the optimality and feasibility levels more accurately. To maintain the level of food security nationally or internationally, we can design a multilevel model in fuzzy environment for the achievement of best optimal agricultural production with least cost by consuming available resources.

Data Availability

Fruit production data were collected from local farmers to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

Authors' Contributions

All authors contributed equally to the preparation of this manuscript.

References

[1] https://en.wikipedia.org/wiki/Agriculture.

[2] M. D. Anderson and W. Lockeretz, "Sustainable agriculture research in the ideal and in the field," *Journal of Soil and Water Conservation*, vol. 47, no. 1, pp. 100–111, 1992.

[3] C. Vogliano, L. Murray, J. Coad et al., "Progress towards SDG 2: zero hunger in melanesia–a state of data scoping review," *Global Food Security*, vol. 29, Article ID 100519, 2021.

[4] H. Calderon, P. Oscar, Y. Ricardongel et al., "A bibliometric analysis of the scientific production related to "zero hunger" as a sustainable development goal: trends of the pacific alliance towards 2030," *Agriculture & Food Security*, vol. 10, no. 1, pp. 1–15, 2021.

[5] R. Christian, D. Wheeler, W. Rebecca, L. Michael, C. Matt, and A. Charlotte, "Agriculture 4.0: making it work for people, production, and the planet," *Land Use Policy*, vol. 100, Article ID 104933, 2021.

[6] J. Pomerleau, K. Lock, M. McKee, and D. R. Altmann, "The challenge of measuring global fruit and vegetable intake," *Journal of Nutrition*, vol. 134, no. 5, pp. 1175–1180, 2004.

[7] K. Siegel, M. Ali, K. Srinivasiah, N. Adithi, N. Rachel, and K. M. Venkat, "Do we produce enough fruits and vegetables to meet global health need?" *PLoS One*, vol. 9, no. 8, Article ID e104059, 2014.

[8] https://en.wikipedia.org/wiki/Agriculture.

[9] https://viewsweek.com/pakistan-fruit-production/.

[10] B. Ahmad, M. Anwar, B. Hammad, M. Mehdi, and T. Farooq, "Analyzing export competitiveness of major fruits and vegetables of Pakistan: an application of revealed comparative advantage indices," *Pakistan Journal of Agricultural Sciences*, vol. 58, no. 2, pp. 719–730, 2021.

[11] S. A. Husiani and J. Rehman, "A study on horticultural sector of Pakistan. Understanding the bottlenecks and opportunities in value-added exports of fruits and vegetables," pp. 1–64, 2020, https://invest.gov.pk/sites/default/files//PBC-Horticulture-Sector-StudyReport-compressed-compressed.pdf.

[12] H. Manzoor, M. Safyan, and F. Manzoor, "Trade competitiveness of Pakistan's fruits and vegetables in world market," 2020.

[13] R. Moeini and M. Soltani-nezhad, "Extension of the constrained gravitational search algorithm for solving multi-reservoir operation optimization problem," *Journal of Environmental Informatics*, vol. 36, no. 2, 2020.

[14] P. Udmale, I. Pal, S. Szabo, M. Pramanik, and A. Large, "Global food security in the context of COVID-19: a scenario-based exploratory analysis," *Progress in Disaster Science*, vol. 7, Article ID 100120, 2020.

[15] L. V. Kantorovich, "Mathematical methods of organizing and planning production," *Management Science*, vol. 6, no. 4, pp. 366–422, 1960.

[16] H. A. Eiselt and C. L. Sandblom, *Linear Programming and its Applications*, Springer Science & Business Media, Berlin, Germany, 2007.

[17] O. A. Solaja, J. A. Abiodun, M. A. Abioro, J. E. Ekpudu, and O. M. Olasubulumi, "Application of linear programming in production planning," *International Journal of Applied Operational Research*, vol. 9, no. 3, pp. 11–19, 2019.

[18] G. Vico and R. Bodiroga, "Tools for planning in agriculture–linear programming approach," *Agri-Base*, 2017.

[19] M. Li, V. P. Singh, Q. Fu, D. Liu, T. Li, and Y. Zhou, "Optimization of agricultural water-food-energy nexus in a random environment: an integrated modelling approach," *Stochastic Environmental Research and Risk Assessment*, vol. 35, no. 1, pp. 3–19, 2021.

[20] M. G. Muluneh, "Impact of climate change on biodiversity and food security: a global perspective-a review article," *Agriculture & Food Security*, vol. 10, no. 1, pp. 1–25, 2021.

[21] A. Gupta, C. P. Sawant, K. V. R. Rao, and A. Sarangi, "Results of century analysis of rainfall and temperature trends and its impact on agriculture production in Bundelkhand region of Central India," *Mausam*, vol. 72, no. 2, pp. 473–488, 2021.

[22] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.

[23] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, 1986.

[24] H.-J. Zimmermann, "Description and optimization of fuzzy systems†," *International Journal of General Systems*, vol. 2, no. 1, pp. 209–215, 1975.

[25] R. Ghanbari, K. Ghorbani-Moghadam, N. Mahdavi-Amiri, and B. De Baets, "Fuzzy linear programming problems: models and solutions," *Soft Computing*, vol. 24, pp. 1–31, 2019.

[26] Z. Luo, Y. Xie, L. Ji, Y. Cai, Z. Yang, and G. Huang, "Regional agricultural water resources management with respect to fuzzy return and energy constraint under uncertainty: an integrated optimization approach," *Journal of Contaminant Hydrology*, vol. 242, Article ID 103863, 2021.

[27] D. V. Morankar, K. Srinivasa Raju, and D. Nagesh Kumar, "Integrated sustainable irrigation planning with multi-objective fuzzy optimization approach," *Water Resources Management*, vol. 27, no. 11, pp. 3981–4004, 2013.

[28] S. K. Bharati and S. R. Singh, "Intuitionistic fuzzy optimization technique in agricultural production planning: a small farm holder perspective," *International Journal of Computer Applications*, vol. 89, no. 6, pp. 17–23, 2014.

[29] M. C. J. Anand and J. Bharatraj, "Theory of triangular fuzzy number," *Proceedings of NCATM*, vol. 80, 2017.

[30] https://www.sciencedirect.com/topics/computer-science/weighted-sum-method.

[31] S. Khan, S. Shahab, M. I. Fani, A. Wahid, M. Hasan, and A. Khan, "Climate and weather condition of Balochistan province, Pakistan," *Internation Journal of Economic and Enivronmental Geology*, vol. 12, no. 2, pp. 65–71, 2021.

[32] https://en.wikipedia.org/wiki/Balochistan.