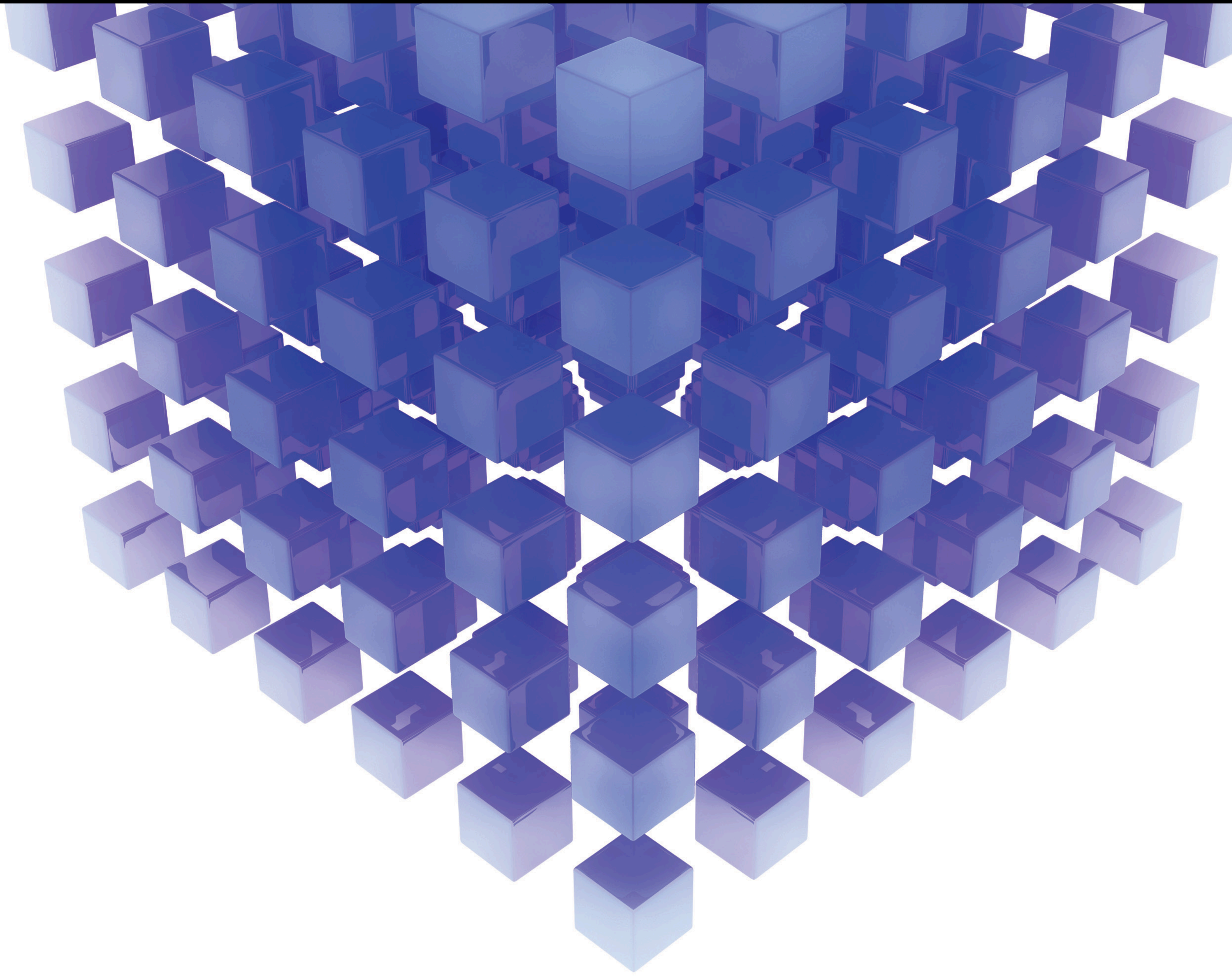
Mathematical Problems in Engineering

Building Mathematical Models for Multicriteria and Multiobjective Applications 2019

Lead Guest Editor: Adiel T. de Almeida

Guest Editors: Love Ekenberg, Juan C. Leyva-Lopez, and Danielle Morais



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Editorial

Building Mathematical Models for Multicriteria and Multiobjective Applications 2019

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This is the third special issue (SI) dealing with "Building Mathematical Models for Multicriteria and Multiobjective Applications 2019." The first one was published in 2016, and based on its success, another one was published in 2018, and now this one in 2019. The ambition is to henceforth publish an annual special issue. This series has been attracting the multicriteria decision-making/aid (MCDM/A) and multiobjective community, researchers, and practitioners.

The focus of this special issue is to demonstrate how MCDM/A and multiobjective methods can be highly useful for decision-makers (DMs) in solving decision problems involving multiple criteria. Many researchers submitted a large number of high-quality papers for consideration in this SI, which went through a rigorous peer-review process, with an acceptance rate of 25%.

This special issue offers 18 original research papers covering a variety of applications for real-world problems while combining theoretical methodology and mathematical analysis. The authors of these papers are from different countries around the world, namely, Brazil, Bulgaria, China, India, Mexico, Taiwan, and Turkey. Six papers bring new methods or methodology to deal with multicriteria problems and also contain different applications in several contexts. Eight papers discuss larger applications of multicriteria decision-making thoroughly, and four papers are focused on multiobjective applications.

Two papers discuss novel aspects of the TOPSIS method. One model is based on distance, similarity, and correlation, and another is based on the improved grey relational analysis. Other innovative approaches explore, respectively, elicitation processes based on pairwise comparisons, an outranking model for a nominal classification problem, a model for deriving the priority weights from hesitant triangular fuzzy preference relations, and a group decision model with uncertain weights using interval-valued intuitionistic fuzzy reasoning.

Three papers utilize varieties of the analytic hierarchy process (AHP) for the topology of diesel engine cylinder block, suitability analysis for the matching area in underwater geomagnetism-aided inertial navigation, and as a group decision tool combined with Dempster–Shafer Theory (DST). Other multicriteria methods were also applied to various problems. The PROMETHEE method was used for analyzing and suggesting reasonable transport plans for high-speed rails. A multilevel Borda model (MLBM) was the base of a quantitative risk evaluation model for a multilevel complex structure hierarchical system in the petrochemical industry. Furthermore, hybrid multicriteria methods (Fuzzy Delphi, DEMATEL, ANP, and TOPSIS) were used for selecting the location of women's fitness centers, and intuitionistic fuzzy set theory is applied to risks of mergers and acquisitions by grey relational analysis and evaluation of enterprise learning performance in the process of cooperation innovation.

In relation to the multiobjective applications, this special issue brings four different contexts of applications. One paper presents new product development projects while applying the evolutionary approach to portfolio optimization. Another paper presents a modern machine learning techniques for univariate tunnel settlement forecasting using particle swarm optimization (PSO) and support vector regression (SVR). In addition, one paper suggests a model for postdisruption recoveries of aircraft and passengers using a loop-based multiobjective genetic algorithm (GA). Finally, we have included a model for multitarget strike path planning based on the aircraft's threat tolerance and the battlefield threat by constraining the balance between mission execution and the combat survival using a multiobjective evolutionary algorithm based on decomposition (MOEA/D).

We hope that the papers presented in this special issue will be useful and stimulating for further developments and applications of multicriteria and multiobjective models and that we again have been able to highlight the extensive range of contexts over which these methods can be used.

Conflicts of Interest

The editors declare that they have no conflicts of interest.

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Research Article

Considering Passenger Preferences in Integrated Postdisruption Recoveries of Aircraft and Passengers

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After a disruption to its operations, an airline needs to dispatch its aircraft and accommodate affected passengers in order to resume normal operations. When facing a flight delay or cancelation, passengers usually have two options—switching to a different itinerary or receiving ticket refunds. This paper focuses on the integrated recovery of both aircraft and passengers by taking into consideration passengers' preferences regarding the two options. Objectives include minimizing the financial loss of airlines and minimizing the utility loss of passengers. To solve the optimization problem, we present a loop-based multiobjective genetic algorithm (GA) by leveraging a special characteristic of flight networks. Experiments based on real-world data demonstrate the effectiveness and stability of the algorithm in various situations. The outcome has theoretical and practical implications for disruption management and airline operations.

1. Introduction

According to statistics from the International Air Transport Association (IATA), the global civil aviation industry transported 4.1 billion passengers in 2017, and its growth rate was 7.4%, which is the greatest one in history. However, the total punctuality rate in 2017 was 76.35% and this rate varies significantly among different airlines, which claims that different operational strategies may have great impacts on the punctuality rate. In China, with development of the Chinese economy, civil aviation transportation has unique advantages in convenience and time. With the support of various policies, the number of airline passengers of different countries as well as cities in China has rapidly grown. At the same time, however, complaints from civil aviation passengers have also risen sharply. According to statistics from the China Civil Aviation Administration (CAAC), the total number of complaints received from passengers reached more than twenty thousand in 2017, nearly twice as many as 2016, and flight delay problems accounted for 53% of the complaints. Why did this happen?

One of the most important reasons for the low customer satisfaction with airlines' handling of disruptions is that airlines often prioritize their own operational convenience and economic impact over passengers' preferences. Such prioritization of their own margins during flight rescheduling is very common for profit-seeking business like airlines. However, it is extremely unfair and intolerable for their disrupted passengers. The incident of a passenger dragged off a United flight, posted by CNN wire on Apr. 11th 2017, highlighted such ignorance of passengers' preferences. Meanwhile, the core competitiveness of an airline is its ability to satisfy passengers' needs. Chang et al. [1] evaluated airline competitiveness and determined that customer service quality is one of the primary considerations of passengers. Thus, during the postdisruption recovery process, airlines should pay more attention to passengers' preferences when deciding how to accommodate them.

Common methods for airlines flight scheduling recovery from disruptions include flight delays and cancellation, aircraft swapping and ferrying, crew repair, and passenger reassigning; these have been considered in various prior studies, such as Clause et al. [2], Le et al. [3], Castro et al. [4], and Artigues et al. [5]. A topic of concern to an increasing number of researchers is integrated recovery of multiple resources, such as aircraft and crews. In contrast, recovery of passengers' itineraries, especially their preferences regarding alternate itineraries, has gone unheeded by most studies. However, during the practical recovery process, there is a close connection between passenger recovery and aircraft recovery, and passengers' satisfaction is paid significant attention by most airlines because of its potential impact on their own reputation and competition level in the aviation market. Therefore, it is critical to integrate interests of an airline and its passengers into a single recovery model, especially with consideration of passengers' preferences regarding alternate itineraries.

In practice, the cost of recovery for airlines is still an important aspect for the operation of an airline so that a multicriteria model should be built to reflect these two considerations. In the field of integrated of flight, multicriteria model is a common modeling strategy, such as Lee et al. [6], Liu et al. [7], and Khaled [8]. The consideration of different kind of objects for the recovery and using functions separately to describe these objects is the key of this modeling style. After that, well-designed mechanisms are introduced to balance these objects. Although the most common way to handle multiobjective problems is to set weights of different objects so that they can be transferred into one single object, which is relatively easy to solve, we still try to keep two independent objective functions so that both objects can be fully considered. Moreover, the separate consideration of two objects avoid the complexity of how the weight of each object is set during the solving process. Thence, aiming to model the integrated postdisruption recoveries of aircraft and passengers, a multicriteria model is necessary.

The goal of this paper is to define, formulate, and efficiently solve the problem of satisfying passenger preferences in the integrated recovery of both aircraft and passengers. To achieve this goal, it was not only necessary to develop and extend mathematical formulation to consider the disrupted passengers' preferences in the integrated recovery process but also to design an efficient solution method for the integrated recovery problem with consideration of passengers' willingness. This paper provides an efficient and computationally manageable method for integrating the aircraft and passenger recovery problems considering passengers' preferences by establishing a multiobjective optimization model formulation. The objectives of the formulation include clearly describing the airline recovery cost and passengers' interest loss during disruption recovery. Additionally, a loop-based multiple objective genetic algorithm (GA) that leverages the characteristics of flight networks is also designed to obtain Pareto-optimal solutions for this problem extremely efficiently.

The remainder of the paper is organized as follows: after a review of relevant studies of airlines' disruption management in Section 1, the problem and model are formally described in Section 2. Section 3 presents a heuristic method for the integrated recovery problem with passenger preferences. Computational experiments that evaluate our approach are presented in Section 4. Section 5 summarizes our work and discusses directions for future work.

2. Literature Review and Contributions

2.1. Postdisruption Recoveries of Aircraft and Passengers. Airline disruption recovery, which is an important guarantee of airline daily operation, includes total aircraft, crew recovery, and passenger recovery such as Clause et al. [2]. Moreover, Hu et al. [9] think passenger itinerary recovery is the most important measure to determine the quality of airline service. The initial research topic focuses on aircraft recovery. Teodorovic et al. [37 performed the first aircraft recovery study using an optimization method. Subsequently, many scholars have primarily focused on the problem of rearranging aircraft after disruptions using minimum–cost network flow theory and corresponding algorithms such as Gershkoff [11], Jarrah et al. [12], Yan et al. [13], and Thengvall et al. [14]. A time-band network was constructed by Bard et al. [15] for the aircraft rerouting optimization problem; it is often combined with the Column Generation algorithm and Bender Decomposition algorithm, to generate feasible aircraft routes in the airline disruption recovery problem by Eggenberg et al. [16]. For the large-scale flight delay recovery problem, metaheuristic algorithms are preferred to obtain near-optimal solutions efficiently, such as Argüello et al. [17] and Løve et al. [18]. However, most studies only focus on single objectives concerning the cost of deviations from original flight schedules. Only a few attempts, such as Liu et al. [19], have employed a hybrid multiobjective GA to find solutions for a daily short-haul aircraft schedule recovery problem.

Besides recovering aircraft, it is also important to recover several resources simultaneously in airline disruption management. Since integrating recovery of the total resources is a difficult task due to the size of the resultant problem, only a few attempts, such as Lettovsky [20], Peterse et al. [21], and Arıkan et al. (2013), have appeared in the literature.

Most studies try to describe the integrated recovery problem of aircraft and passengers in a single model formulation. Bratu et al. [22] were the first to model passenger recovery using a passenger delay metric model (PDM) and a disrupted passenger metric model (DPM). Both models consider the passengers' disrupted cost and airlines' operation cost in a single objective. The latter one is validated in three different levels of scenarios and three different degrees of disruption. Jafari et al. [23] presented a single-objective model that simultaneously recovered aircraft and passengers. Their dataset contained 13 aircraft of 2 fleets, and they were able to address disruptions in a small-scale airline.

Bisaillon et al. [24] developed a large neighborhood search heuristic for an airline recovery problem combining aircraft routing and passenger reassignment with one objective of minimizing operating costs and impacts on passengers. Sinclair et al. [25] improved the heuristic by adding some additional steps in each phase for the same problem. Then, Sinclair et al. [26] solved the similar problem with a column generation postoptimization heuristic, which slightly increased the allotted computing time. The heuristic is based on a mixed-integer programming mode and forms various hierarchies of passengers, flights, and other elements to recursively solve the problem. Zhang et al. [27] proposed a three-stage sequential math-heuristic framework to solve the integrated airline service recovery problem. Time-space network flow representations, mixed-integer programming formulations, and algorithms that take advantage of the underlying problem structures were proposed for each of the three stages. However, the computational results of the above studies were employed for the 2009 ROADEF Challenge that was organized by Artigues et al. [5] instead of for real-life situations.

Hu et al. [28] and Hu et al. [9] aimed to find the optimal trade-off between passenger delay costs, passenger reassignment costs, and the cost of refunding tickets in a single-objective model for the integrated recovery problem of both aircraft and passengers. They both consider the airline's point of view instead of those of the passengers. The former solved the model using CPLEX Solver, and the latter design a meta-heuristic based on GRASP to obtain the near-optimal solutions. In the former research, the average time for solving a real example, including 319 flights and 59 airports, is 49 seconds and the average optimization efficiency is about 10%. Therefore, the computational time and optimization efficiency still have chances to improve.

From the literature review analysis, we observe that most of these papers combine utility losses of passengers with airlines' financial losses to form a total loss as the single objective of the optimization model. This combined object simplifies the problem-solving process with a certain degree of relevance. However, unlike aircraft or crew members, passengers do not belong to the airline. They are two juxtaposed participants during flight production. Faced with the unforeseen disruption and the following recovery process, an airline cannot cover and compensate for the total loss of disrupted passengers, especially the utility cost and dissatisfaction of the passengers with the airline's service quality. Therefore, the disruption and recovery costs of the airline and passengers cannot be simply combined. They should be subdivided in order to obtain a recovery solution that is more suitable for both participants in long-term plans.

2.2. Passengers Preferences. In recent years, research on passengers' travel preferences has gained popularity. Most studies use a logit model to analyze the choice of passengers between flights or railways by Moeckel et al. [29], revealing factors influencing passengers' flight choices by Algers et al. [30], Yan et al. [31], Hagmann et al. [32], and Fleischer et al. [33], predicting the total number of air travelers [34], or optimizing passenger flows in a flight network, Dou et al. [35] and Yang et al.(2017). The above models are only suitable for normal transportation networks, not analysis of passengers' preferences in the case of an accident or disruptions. Only a few attempts have discussed the possible behavior of railway passengers during emergency evacuation, and the analysis concluded that gender may result in significant difference in evacuation behavior in the ordered logit model.

However, although an airline's handling of disruptions is a major source of its passengers' dissatisfaction, Bratu et al. [22], no study on airline postdisruption recovery has attempted to incorporate passenger preferences. This paper tries to address this gap by quantifying passenger preference via utility losses and considering such losses along with airlines' financial losses in a multiobjective optimization model. To solve such a multiobjective optimization problem, we choose to use GAs.

2.3. Multiobjective Model and Optimization. In most studies, as stated above, traditional airline recovery problems are only described as single-objective models. Even when faced with multiobjective problems, they often propagate a linear sum on objectives to change them into a single one in the mathematical formulations. However, it is stated that the single-objective solution cannot overall reflect the true interests of both participants, airlines and passengers, Fieldsend et al. [36]. Therefore, a multiobjective model is naturally in-need to describe these two considerations. On one hand, the introduction of multiobjective model is used on the modeling, which means different kinds of object functions are built. Liu et al. [7] used a five-object model to manage a disruption and different weights were set to continue the solving process. On the other hand, the multiobjective idea can be used in the process of solving, such as Burke et al. [37], Chou et al. [38], and Lee et al. [6]. However, there are very few studies that consider the multiobjective optimization problems of integrated recovery of aircraft and passengers. Liu et al. [19] constructed a multiobjective combinational optimization model formulation for daily short-haul recovery problems and developed a hybrid evolutionary algorithm composed of an adaptive evaluated vector and the inequality-based multiobjective GA. A simulated disturbance experiment involving daily domestic airline plans in Taiwan with only 7 aircraft and 39 flights was performed. The efficiency was relatively low. Moreover, the objectives lack formal descriptions of flight cancellation and passenger reassignment, which are common in most airline recovery situations. The short-haul recovery approach is not suitable for general-scale disruptions, especially in larger airlines.

2.4. Genetic Algorithm. GAs represent a widely used method for intricate multiobjective optimization, especially for airlines' integrated recovery problems. This method, based on a Pareto noninferior solution, is a powerful multiobjective tool, Goldberg [39], that exhibits the superiority and inferiority of each solution. These solutions are then classified to choose the parent individuals during the solution process, leading the whole population to the forefront of Pareto solutions. GAs are especially suitable for problems with large feasible regions and complex constraints.

Especially in the research field of flight rescheduling, the aircraft flow being continuous in time and space increases the difficulty of the integrated recovery problem with multiple objectives, Hu et al. [40]. It is also the reason why few studies have investigated multiobjective optimization for airline recovery. However, the characters of flight loops, especially in the Chinese flight network, can package and protect the aircraft flow as continuous in time and space. Therefore, a loop-based multiple objective GA can be designed by combining an efficient coding strategy and the loop characteristics of the flight networks, in order to obtain near-optimal solutions of the integrated recovery problem.

2.5. Contributions. The main contributions of our research are as follows.

(1) An integer programming model with two objectives constructed in this study reflects the practical interests of two major participants: airline and the disrupted passengers during the disruption recovery process. One objective refers to reducing the financial costs of airlines, and the other one focuses on reducing passengers' utility losses.

(2) Different from the previous research topic, passengers' preferences between refunding or endorsing are first considered in the integrated recovery model formulation of aircraft and passengers in order to improve passengers' satisfaction. It is the first attempt in which passengers' psychological behaviors are quantified in a probabilistic form in the integrated recovery optimization model.

(3) It is the first time that a recovery solution approach for resolving multiobjective airline rescheduling problems that combines a characteristic flight loop network and a GA is developed. The combination results in distinctly improved efficiency and better solution performance for the integrated airline recovery problem.

3. Problem Description

3.1. Passenger Preferences Analysis. An airline must arrange its aircraft to cover its flights each day. In the case of inclement weather, emergency repairs, and aviation control, the availability of aircraft could change, causing disruptions to the airline's operations. The airport operation control center (AOCC) will collect all information and start the recovery period. A typical recovery period involves rearranging the assignments of aircraft to flights, mainly with the goal to reduce the financial loss for airlines. However, such a recovery process often ignores the preferences of passengers whose itineraries are affected by the disruption. In other words, it is assumed that passengers will accept the new itinerary provided by the airline.

Figure 1 shows the decision process of a passenger who belongs to a disrupted flight. If the flight is not canceled, which means the flight will land off later, we assume this passenger will still wait for the delayed flight. However, if the flight is canceled, the problem for the passenger of this flight is whether to apply for refund. If so, this passenger will receive a refund from the airline and his/her journey will end. If not, this passenger insists his/her journey and wants to move to another flight. In this paper, a proportion factor is used to illustrate how many passengers want to choose endorsement when the cancellation of flight happened. This factor is external and based on the operation information of the airline. In this situation, the problem waves back to the airline: are there enough available seats for the passenger who wants to endorse? The answer relies on the recovery decision the airline made. If the passenger's demand can be satisfied, he/she can continue the journey. If not, the passenger will be annoyed about the recovering process so that the utility loss of passenger increases. In conclusion, in the whole recovering process, passengers only need to show their preference to which extend they choose refunding and this is the information they need to give to the airline when adjusted flight schedule is announced.

According to the statement above, in the recovery process, although passengers do not participate in creating a new flight schedule, they can pick between two options. One is endorsing, which means the passenger chooses to transfer to another flight with the same airline. The other one is ticket refunding, which means passenger cancels their current itinerary with this airline, and the airline does not have to provide any extra transformation service for them but rather helps them apply for a refund. In this paper, passengers do not passively accept the adjustment but actively choose to endorse or refund when flight cancellations occur. The airline reassigns the aircraft based on the passengers' willingness during the recovery period. For the endorsing choice, some passengers' willingness cannot be satisfied due to the limited available aircraft capabilities and other reasons, which causes a decline in satisfaction. Thus, the utility loss is used to describe passenger dissatisfaction with the airlines. For ticket refunding choice of passengers, the service task seems easy for the airline; however, due to the failure of the service, the airline has to absorb both the economic cost and reputation cost due to the disruption.

Considering that the utility loss in the delay situation has a high degree of consistency with the economic loss, which may produce multiple collinearities, and that a significant amount of computation is needed to calculate the endorsed passengers' utility loss, this research considers only the utility loss of the passengers' willingness to endorse but cannot be satisfied when a cancellation occurs.

3.2. Recovery Operation Hypothesis. The options of integrated recovery in this paper are as follows: cancel—no aircraft will be arranged for this flight, and the passengers will be reassigned according to their willingness and aircraft capacity; naturally delay the same aircraft—a new timetable of flights will be provided according to the available times of the delayed aircraft; swapping aircraft—the matches between aircraft and flights will be rearranged.

During recovery operations, it is assumed that the crew requirements are not considered, so modifications to aircraft routes are not limited by crew availability. When an aircraft route is selected, it is acceptable as long as appropriate crew members can be found to fly the aircraft. Because the time window of the recovery is only a single day, the arrival and departure times of the involved flight must be the same day to prevent interference with the flight schedule of the following day. The time length of a single day does not exceed the maximal duty time of most crews according to the Airline Operation Manual.

In reality, a fleet is widely used in civil aviation to describe a type of aircraft, such as the Boeing 787 fleet. The main reason for this use is that within the same fleet, the crew members, maintenance team, and other resources can be shared. Some flights can be flown only by a specific type of aircraft due to distance limitations, arrival airport situations, and other reasons. For example, remote aircraft are usually

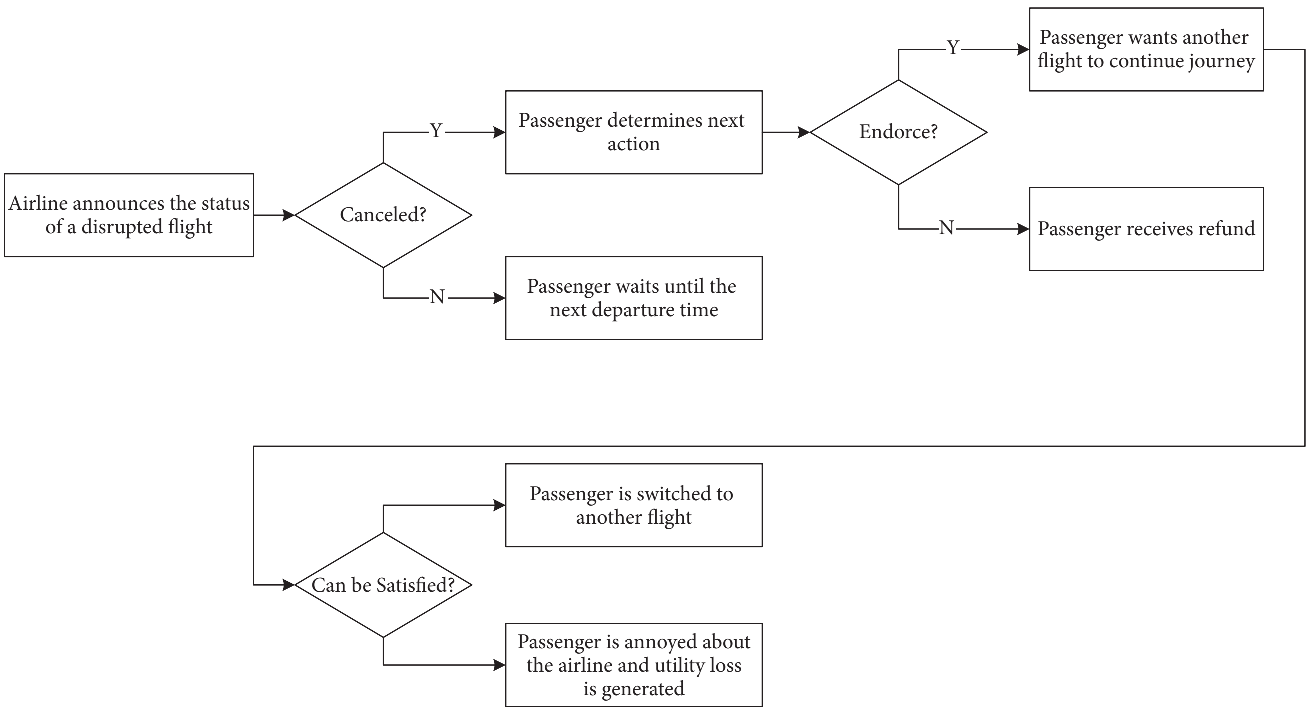

Figure 1: Passenger decision process.

used for intercontinental flights. Therefore, in this paper, we do consider multiple fleet types and the fact that each fleet has different characteristics that limit cross-assignments. Substitution for flights is allowed between some fleets in accordance with airline policies and aircraft configurations.

When flight interference occurs, passengers' itineraries are forced to be adjusted. We claim that only the flights are cancelled, and the corresponding passengers can choose new itineraries. Otherwise, all passengers should travel on their original flights. The passengers of the cancelled flight make their decisions based on their own characteristics. As described above, faced with flight disruption, passengers have two options: endorsing or ticket refunding. Endorsing is the first choice of most passengers except when there are not enough seats to accommodate them. So, some assumption about passengers' endorsing should be presented clearly. Since different fleets have different passenger carrying capacities, this paper considers only aircraft that belong to the same fleet for passenger exchanges. In this circumstance, the class and the capacity are the same, which provides more feasibility for airlines and passengers. Additionally, it is assumed that a passenger has only one flight in an itinerary due to the low connecting flight rates and that most domestic flights will not change flight numbers during a voyage, which has substantial equivalence to a single flight. This paper focuses on Chinese domestic flights, for which the majority of the passengers can complete their trip in one flight.

3.3. Problem Objectives. The problem raised in this paper reflects the practical operation of an airline when accidents force its operating plan to be altered. When some aircraft are delayed or grounded due to unforeseen events, the flights covered by these aircraft and passengers' itineraries will be disrupted. To recover the schedule and passengers' itineraries as soon as possible, available aircraft need to be rerouted and passengers reassigned according to their willingness. Several objectives have been established to guide the recovery and evaluate the total losses of the airlines. In this paper, the objectives include the following two aspects.

(1) Economic loss. The objective is to minimize the airline financial cost, including compensation for passenger delays, the transiting cost of transferring them to other flights, and ticket refunding cost when the passengers cannot be reassigned. The flight delay cost refers to the direct cost due to flight delays, such as additional fuel cost, overtime payment, and additional meal cost for each passenger. It is apparently associated not only with delay time but also with the number of passengers. The transiting cost is the service cost for each passenger transiting from a cancelled flight to the other flight. It is related to the difference between the actual departure time of the new flight and the scheduled departure time of the cancelled flight. The refunding cost mainly refers to airline compensation for the passenger original ticket payment due to the airline service discontinuing.

(2) Utility loss. The utility cost is mainly to describe the dissatisfaction of passengers on airline recovery operations and their potential economic loss. Faced with the itinerary disruption, most passengers prefer endorsing. However, whether endorsement is feasible depends on the capacity of the aircraft, which covers the passengers' new itinerary. If the new arrangement does not have sufficient capacity to accommodate all passengers who endorse, some of the

passengers will need to have their tickets refunded, which will generate a utility cost for these passengers. Compared with passengers who prefer refunding, the passengers who want to be endorsed but cannot be satisfied need to be emphasized because they suffer more from the cancellation; thus, the utility loss of this type of passenger is relatively high. The more dissatisfied the passengers become with the airline, the less likely they are to choose airline's flights in the future, which can generate potential economic loss and loss of market share for the airline.

3.4. Problem Constraints. As the integrated recovery problem in this paper must be solved in practice, the flight rescheduling, aircraft rerouting, and passenger recovery must also obey the following constraints.

First, each flight can only be adjusted once or cancelled in the feasible aircraft route, its adjusted departure time must not be earlier than the original time, and its delay time must be less than the maximum delayed time. Moreover, for every couple of neighborhood flights in one flight sequence, the former flight's arrival airport must be the same as the latter flight's departure airport, each neighborhood flight's interval time must be smaller than the minimum turnaround time (the minimum time for the aircraft to rearrange for the next flight after landing), and the arrival time of the last flight must be earlier than the curfew time.

Each aircraft can cover at most one route to form one feasible aircraft route that remains continuous in the time and space connection network. Additionally, no aircraft scheduled for maintenance service during the recovery period will have its original route altered. Finally, at the end of the recovery period, there should be a sufficient number of appropriate aircraft types positioned at each airport to ensure that the published schedule beyond the recovery period can be executed to avoid disruption continuation into the following day.

For every endorsement willingness, the available flight's departure time must be later than the departure time of the cancelled flight, and the aircraft type between two switched flights must be the same. Moreover, the routes must be the same between two switched flights, i.e., passengers who choose to endorse should be promised to arrive at their original destination. Finally, the capacity of the aircraft must allow for the extra passengers; otherwise, they will have to refund their tickets.

3.5. Mathematical Formulation. To accurately describe the problem, an integer programming formulation is constructed. This model aims to allocate limited aircraft to satisfy the needs of all types of flights and the constraints of aircraft flow and airport curfew and to reallocate the passenger into available flights with sufficient capacity according to their desire.

Index

c: aircraft fleet index;

p: aircraft route index;

a: aircraft index;

f: flight index;

s: airport index.

Sets

C: set of aircraft fleet;

P: set of aircraft routes;

A: set of aircraft;

F: set of flights;

S: set of airports;

$A(c)$: set of aircraft that belong to fleet c; $c \in C$;

$PF(f)$: set of aircraft routes that include flight f; $f \in F$;

$PS(s)$: set of aircraft routes that end in airport s; $s \in S$;

$PA(a)$: set of aircraft routes that can be covered by aircraft a; $a \in A$;

$PC(c)$: set of aircraft routes that can be covered by aircraft fleet c; $c \in C$;

$FR(f)$: set of flights that can receive passengers from flight f; $f \in F$;

$FS(f)$: set of flights that can send passengers to flight f; $f \in F$.

Parameters

CD_{fpa}: cost of each passenger who belongs to the delay flight f covered by route p and aircraft a; $f \in F$, $p \in P$, $a \in A$;

CC_f: cost of each passenger who belongs to the cancelled flight f; $f \in F$;

CT_f^{gpa}: cost of each passenger who belongs to the cancelled flight f and transfers to a flight covered by the route and aircraft;

TD_{fpa}: delay time of flight f covered by route p and aircraft a;

M: the maximum amount of time that a flight can be delayed;

NT_{cs}: number of aircraft that belong to fleet c and should terminate at airport s; $c \in C$, $s \in S$;

NI_f: number of passengers who are initially in flight f; $f \in F$;

NS_c: number of seats in aircraft fleet c; $c \in C$;

α_f: the rate of passengers who want to endorse and belong to the cancelled flight f; $f \in F$;

θ_f: the coefficient of utility loss of each passenger who belongs to the cancelled flight f and want to transfer but cannot be accommodated; $f \in F$.

Decision Variables

x_p: if route p is covered by one aircraft; if not, $x_p = 0$; $p \in P$;

t_f^{gpa}: the actual number of passengers who transfer from flight f to a flight covered by route and aircraft; $f \in F$, $g \in FR(f)$, $p \in PF(g)$;

r_f: the actual number of passengers who refund tickets and belong to flight f.

Main Formula

$$\min(object1) = \sum_{f\in F, a\in A, p\in PE(f)} CD_{fpa} NI_f x_p + \sum_{f\in F, g\in FR(f), p\in P, r\in RF(k)} CT_f^{gpa} t_f^{gpa} + \sum_{f\in F} CC_f r_f \quad (1)$$

$$\min(object2) = \sum_{f\in F} \theta_f \left(\alpha_f NI_f - \sum_{p\in PF(f), g\in FR(f), a\in A} t_f^{gpa} \right) \quad (2)$$

s.t.

$$\sum_{p\in PF} x_p \le 1, \quad f \in F \quad (3)$$

$$\sum_{p\in PS(s)\cap PC(c)} x_p = NT_{cs}, \quad s \in S,\ c \in C \quad (4)$$

$$\sum_{p\in PA(a)} x_r \le 1, \quad a \in A \quad (5)$$

$$\sum_{a\in A, p\in PF(f)} TD_{fpa} x_r \le M, \quad f \in F \quad (6)$$

$$\sum_{p\in PF(f), g\in FR(f), a\in A} t_f^{gpa} + r_f \le NI_f \left(1 - \sum_{p\in PF(f)} x_p \right), \quad f \in F \quad (7)$$

$$\sum_{g\in FS(f), a\in A(c), p\in PF(f)} t_f^{gpa} \le \left(NS_c - NI_f \right) \sum_{p\in PF(f)\cap PA(a)} x_p, \quad f \in F,\ c \in C \quad (8)$$

$$\sum_{g\in FR(f), p\in PF(g), a\in A} t_f^{gpa} \le a_f NI_f, \quad f \in F \quad (9)$$

$$x_p = \{0, 1\}, \quad p \in P \quad (10)$$

$$r_f = \{0, 1, 2, \ldots\}, \quad f \in F \quad (11)$$

$$t_f^{gpa} = \{0, 1, 2, \ldots\}, \quad g \in FR(f),\ p \in PF(g),\ a \in A \quad (12)$$

The objective of function (1) is to minimize the sum of the delayed cost, the transferring cost, and the refunding cost. The objective of function (2) is to minimize the sum of utility losses. Constraint (3) ensures that every flight is either cancelled or covered by a flight. Constraint (4) ensures that a sufficient number of aircraft are arranged for the needed airport prior to the curfew time. Constraint (5) ensures that every route should be covered by at most one aircraft. Constraint (6) ensures that the maximum delay time satisfies the demand. Constraint (7) ensures the balance of the transferring stream. Constraint (8) ensures that the adjustment satisfies the volume of aircraft. Constraint (9) emphasizes that some desires of passengers cannot be satisfied. Constraints (10) - (12) define the decision variables as integer or binary.

The proposed model (1)-(12) is an integer programming model based on a set covering model formulation with side constraints about flights and passenger. In some studies, an integrated recovery model of aircraft and passengers has been directly solved using business software, such as CPLEX or LINGO [28] Arıkan et al. (2013) and Arıkan et al. (2016). However, considering the exponential relationship between the aircraft route and aircraft and the extreme complexity of passenger transiting process with their willingness, plenty of time and storage must be used for business software to generate all of possible transiting relationship and all possible aircraft routes. Some other studies have introduced the column-generation algorithm and bender composition algorithm to solve this problem. However, the biggest resistance for the above algorithm is that there are two objectives in the proposed model (1)-(12), and the complicated passenger transiting relationship between flights may negatively affect the efficiency of the above algorithm.

In contrast, faced with this complicated biobjective integrated recovery problem, it is critical to design an efficient heuristic algorithm to obtain a series of nondominant recovery solutions that meet the selection requirements of the AOCC staff according to the real-time environment. In this paper, we provide a recovery solution method with more competitive performance and more efficiency by designing a heuristic based on a multiple objective GA and flight loop network characteristics. Detailed information about the proposed solution method is described in Section 4.

4. Solution Method

Under the consideration of the proposed biobjective model (1)-(12), referring to the characteristics of flights in China and the rules for actual flight operations, a novel recovery solution approach (denoted as LBMGA) is designed, based on a multiple objective GA and flight loop network characteristics, to solve the problem efficiently. The significant efficiency is mainly derived from excellent parallel searching ability of GAs in complex conditions and excellent structural compatibility with a multiobjective mathematical model of the proposed approach. Additionally, since the clash between the mechanical application of a general GA processing strategy and the space continuity characteristics of the flight sequence may result in a negative effect on the approach efficiency, a clever encoding method is designed according to loop features of the flight network, which can perfectly avoid the

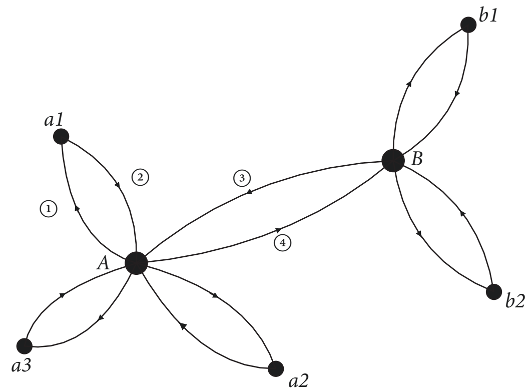

Figure 2: A flight network example.

Table 1: Information about the air hubs and the spokes in the flight network.

Air Hub	Air Spoke	Air Hub	Air Spoke
A	a1	B	b1
	a2		b2
	a3		

Table 2: Example based on Figure 2 about explaining the gene sequence.

Flight loop for A	Aircraft	Flight loop for B	Aircraft
(A⟶a1,a1⟶A)	1	(B⟶b1,b1⟶B)	3
(A⟶a2,a2⟶A)	1	(B⟶b2, b2⟶B)	4
(A⟶a3,a3⟶A)	2	(B⟶A, A⟶B)	4

negative effect without blemishes. Moreover, the path of the searching for better solutions between two objects is also significant for the total performance of the algorithm. Thus, we also chiefly introduce the multiple objective choosing strategy in the following subsection.

Overall, the subsections are organized as follows. Based on graphical analysis of the flight loop features in Section 4.1, a novel coding strategy of the GA is presented in Section 4.2, and then the description of the fitness calculation method follows immediately. Finally, the choosing strategy of multiple objectives of the proposed model in the recovery solution method is introduced in detail.

4.1. Feature Analysis of Flight Schedules. Flight schedules indicate how flights are organized by an airline. In this paper, we analyze current practical flight schedules and find a loop-based flight network character. In the flight network, one flight sequence connects each air spoke via air hubs; no direct connections exist among air spokes. Simultaneously, all available airports are separated into groups; each group contains one air hub and some air spokes. Thus, each air hub only services the same group of air spokes. In most cases, an aircraft that flies from an air hub to an air spoke must return to the same air hub to create a flight loop (FL). In other cases, the aircraft flies from an air hub to another air hub and then immediately returns to the original hub; this situation also makes an FL.

An FL consists of two adjacent flights performed by the same aircraft, including a contrast origin, a terminal point, and at least one air hub.

Figure 2 and Table 1 show a small example of this special system. A and B are air hubs, and the remaining letters represent air spokes. Flight ① and flight ② constitute a flight loop, and flight ③ and flight ④ constitute a flight loop. If a flight is b1⟶b2, the passengers of the flight need to fly from b1 to B and then from B to b2. However, if a flight is b1⟶a1, the passengers need to transit by B and A to reach the destination.

Referring to some existing flight schedules in China, numerous Chinese airways choose this system to arrange their routine. The most significant characteristic of this special system is that the percentage of FL is at an extremely and remarkably high level.

4.2. Gene Coding. The traditional theory of a chromosome is to encode a partial schedule of all flights. If a sample consists of n flights and m aircraft, the length of the chromosome is n, which changes from 0 to m for each gene point. For example, the gene sequence (2, 0, 3) indicates that the first flight is performed by aircraft 2, the second flight is cancelled, and the third flight is performed by aircraft 3. This strategy is intuitionistic, and the algorithm can be easily realized. Concerning massive restrictions that need to be obeyed during the recovery problem, especially the continuity constraint, the traditional choice will be very inefficient and slow.

In this paper, a segmented encoding theory based on FLs is proposed to improve efficiency and accelerate the problem-solving processes. According to the features of an FL, if a recovery case contains n air hubs, the chromosome will be divided into n sections, and the length of each section is determined by the number of specific sets of FLs of the air hub. Thus, each gene point represents a specific FL. Using the information in Figure 2 as an example, supposed the flights belonging to air hub A is A⟶a1,a2,a3; a1,a2,a3 ⟶A so there are three flight loops for air hub A and the corresponding aircraft is shown in Table 2. It means that aircraft 1 flights the loops (A⟶a1,a1⟶A) and (A⟶a2,a2⟶A). The information about air hub B is also shown in Table 2. Therefore, the gene sequence of this example is (1,1,2) (3,4,4). Particularly, if the number of aircraft is 0, it means there is no aircraft is arranged to the flight loop so that both flights of the loop will be canceled

The major concern of this coding theory is to cope with the continuity constraint. Using an FL, every crossover, mutation, and other operations for each gene section will not break the continuity constraint to reduce the number of calculations. In this special strategy, only the FL can be involved in the recovery process. Because the percentage of FLs is very high in the Chinese flight network, the optimization efficiency is guaranteed.

4.3. Fitness Calculation. Fitness one aims to calculate the total economic cost for each flight schedule. Using the previous

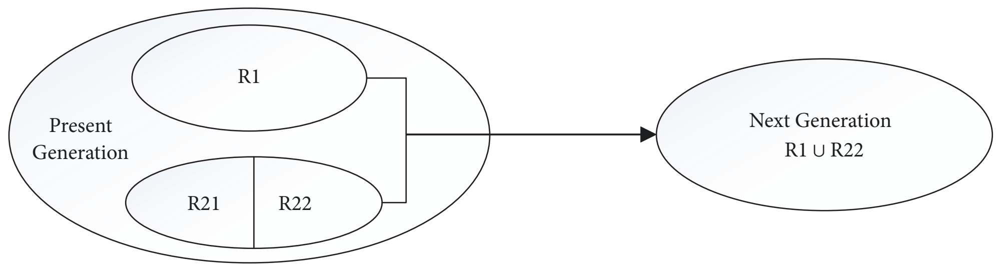

Figure 3: A simple case of choosing multiple target operations.

method, when every flight is confirmed to be performed by an aircraft, the flight schedule and the cancel table is confirmable. By matching every original departure time with the present departure time and matching every original arrival time with the present arrival time of each flight, the delay time of every noncancelled flight can be calculated to obtain the delay cost. By adding the cancellation cost for the cancelled fights, the total economic cost can be calculated.

Fitness two aims to calculate the utility losses. Every flight schedule corresponds to a cancellation table. For every cancelled flight f, there exists a set of flights ($FR(f)$) that can receive passengers from flight f. $FR(f)$ is a dynamic set. For every flight $f_e \in FR(f)$, the number of available seats (AS_{fe}) has been given as background information. AS_f represents the number of empty seats after an airline stops selling tickets for flight f_e. Thus, for every cancelled flight f, the total number of available endorsing seats can be calculated based on a specific flight schedule.

Flight f_e needs to obey the available constraint, the airport constraint, the time constraint, and the aircraft type constraint. The airport and the aircraft type of flight f_e is relatively fixed, whereas the available situation, the present departure time, and the present arrival time of flight f_e is dynamic based on every flight schedule.

Based on the actual situation, some passengers of the cancelled flight choose to endorse. Thus, the percentage α_i is set to reflect the degree to which passengers will choose endorsing when a cancellation occurs for flight i. Based on the cancellation table, the number of passengers endorsing willingness can be calculated, and the number of dissatisfied passengers who want to endorse can be calculated based on the real situation.

4.4. Multiple Objective Choosing Strategy. Two types of fitness exist for every member of the population. Thus, the method for weighting two objects to optimize the population is a critical process for the LBMGA. The main idea of coping with multiobjects in this paper is to rank every solution to layer the population; within each rank, solutions need to be ordered.

The population p, which includes the present population and the filial population that is generated during the crossover operation, is assumed. In this paper, the degree of adaptation is used to distinguish rank, and the relative density is used to order the solutions in every rank. The degree of adaptation is described by the domination: if solution p dominates solution q, both objects of p are better than both objects of solution q. The solutions within the same rank cannot dominate each other.

Assume $p_i \in RankR$, the relative density of p_i is d_{p_i}, and the definition of is given in formula (13):

$$d_{p_i} = \begin{cases} M_1, & i = 1 \\ \sqrt{\left(O_{1p_{i-1}} - O_{1p_i}\right)^2 + \left(O_{2p_{i+1}} - O_{2p_i}\right)^2}, & 1 < i < |R| \\ M_2, & i = |R| \end{cases} \quad (13)$$

In formula (13), M_1 and M_2 are both sufficiently large numbers, and $M_1 > M_2$, where O_{ap_i} represents the value of object a of p_i and $|R|$ represents the number of solutions in R.

The multiple target choosing operation is divided into two parts. The first part is to layer the population into different ranks to confirm the dominating relationships among solutions. The larger is the number of solutions dominated by the rank, and the smaller is the ordinal of this rank. The second part is to calculate the relative density of each solution in every rank. The solution is filled with the rank ordinal ascending and the relative density ascending in every rank until the next population is filled.

In Figure 3, a small case is shown to reveal the multiple target choosing operation. The present generation can be divided into four parts: R1, R21, R22, and others. R1 dominates R21 & R22, and R21 & R22 dominates others. Regarding the constraint of volume, only some of the solutions can be inherited by the next generation. For R21 & R22, although they are in the same rank, R22 is better than the other part of R21 based on the relative density; thus, R22 will be given priority during choosing. According to the volume, R1 and R22 are determined in order to form the next generation.

4.5. Total Algorithm Flow. A flow chart of the entire process of the loop-based GA is shown in Figure 4. At the beginning point, the original population is formed based on the natural delayed flight schedule. Then, a crossover operation is performed to form the filial population. The object values of these two populations are calculated and ranked to form the next population.

After the next population is formed, mutation and cancellation operations are performed. Then, the population undergoes a stop inspection. If the number of generations has

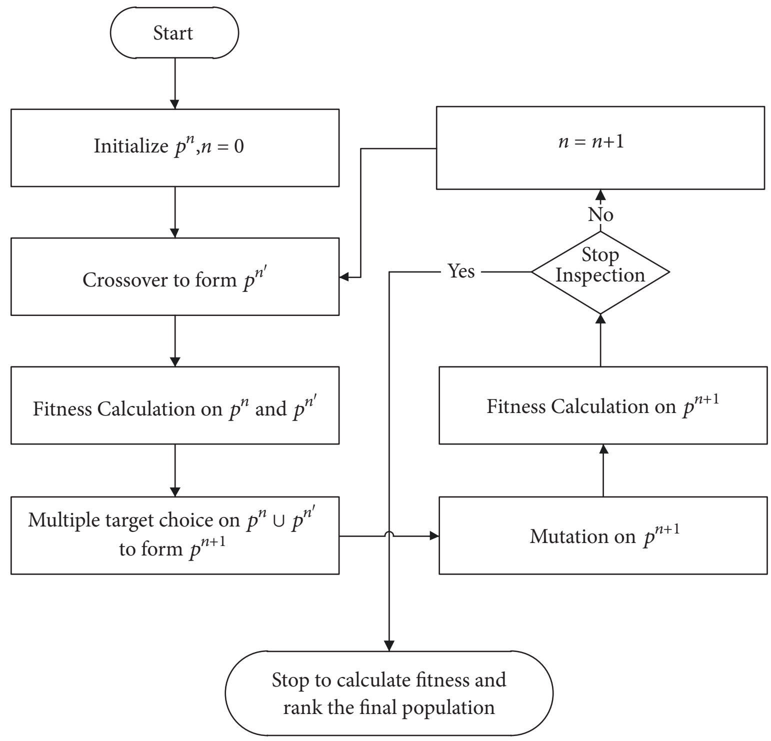

Figure 4: Complete algorithm flow.

reached the limit, circling will stop, and the final population will be calculated. If not, circling will continue.

In total, the special structural design of the algorithm ensures the improvement of the efficiency, and the settings of two types of objects make airlines try to find a balance between the economic profit and the passengers' willingness. In other words, in this process, the passengers' willingness is quantized and deeply considered such that the operator of airlines can clearly receive the changes of passengers' willingness among different recovery plans and choose a preferred one among them. Thus, compared with the traditional recovery process, the passengers have more chances to receive the information of the flight adjustment early and accept it when then new recovery process mentioned in this paper is performed. The reasons can be separated into two aspects: first, the earlier passengers are informed by the airline about the recovery process means more time for the passengers to adjust their journey, and second, the preconsideration of the passenger's willingness means more passengers can be satisfied. Hence, the overall service level of the airline can be significantly improved, as can the market competitiveness.

5. Computational Experiments

To explain and evaluate the effectiveness of the proposed recovery solution method, two types of examples are generated. First, a small example with 5 aircraft is shown to explain how our method works. Then, real-world empirical data regarding the Boeing 737 fleet of a major Chinese airline are used to evaluate the effectiveness of the recovery solution method. In this section, we will discuss the construction of problem instances and the experimental results. All computations were performed on a HASEE HYTIK5AIDIC0SE0 with i7-4270HQ CPU and 7.89 gigabits of RAM. The proposed algorithm was coded in C++.

5.1. Experiments with a Simple Case.

A simple case in Table 3 for 5 aircraft and 24 flights is first introduced to analyze the performance of the proposed solution method compared with the benchmark solution method (traditional GA). In Figures 7–10, FN, DA, AA, DT, AT, AM and OA refer to the flight number, the departure airport, the arrival airport, the scheduled departure time, the scheduled arrival time, the aircraft fleet type, and the original aircraft ID that performs the flight, respectively.

The disruption information is set as follows: Flight 4 and Flight 5 cannot be available until 15:00 due to emergency repairs. The remaining aircraft have been available since 09:00. During the recovery process, the minimum turnaround time is assumed to be 40 min, the maximum delay time is 5 h, and the curfew time is 24:00 on the current day. For reflecting the comparison results fairly, the proposed recovery solution method and the traditional GA share the same crossover rate, mutation rate, cancellation rate, and generation limitation (40). The values of the coefficients refer to the present operating plans of the airlines and the relevant laws and regulations in China.

Based on the solving process mentioned before, LBMGA and GA both run several times and the optimal example of both algorithms is given to be analyzed in depth. In the

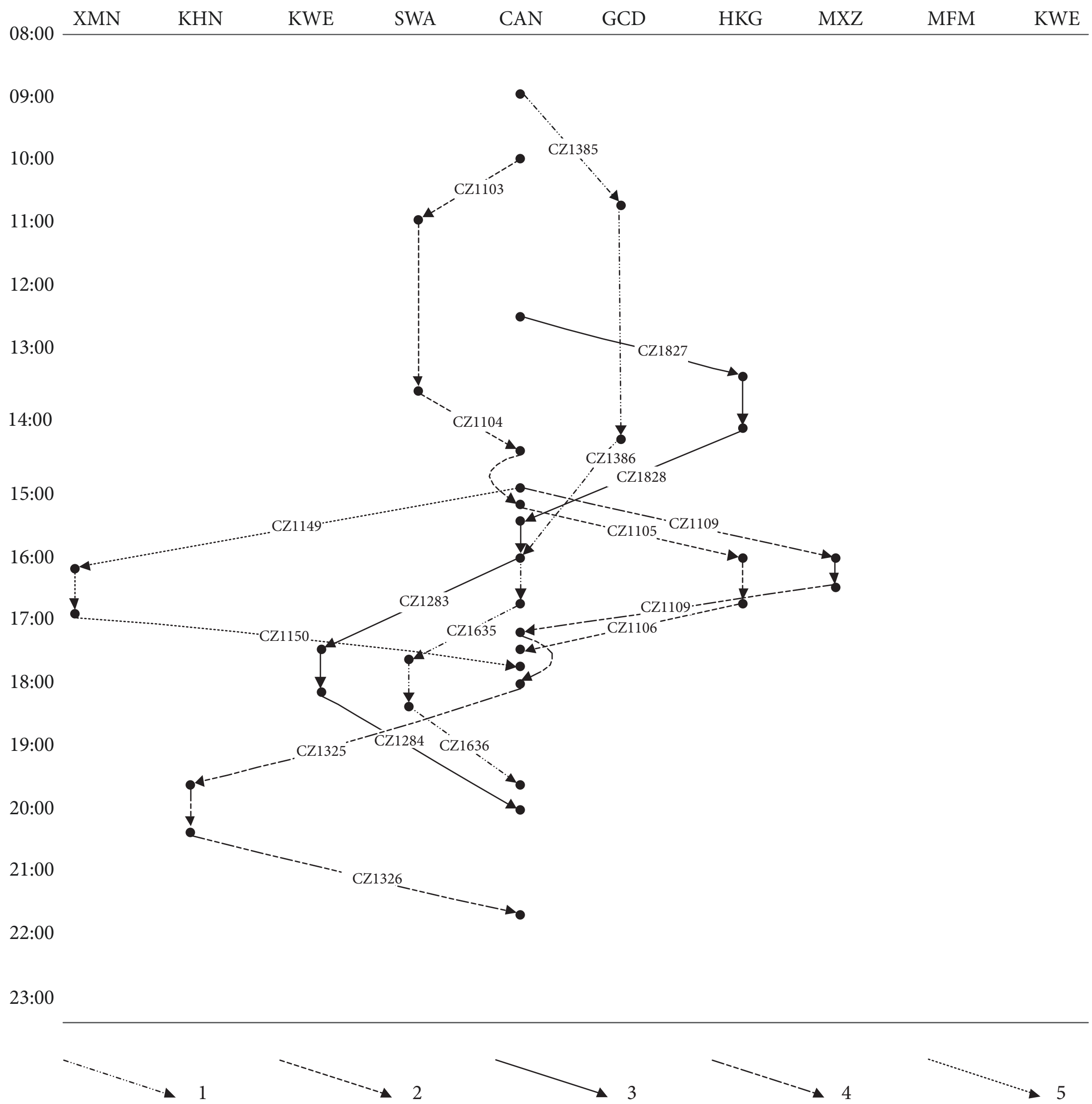

Figure 5: Naturally delayed aircraft arrangement of the simple case.

example of GA, there is only one solution ranking first in the final generation. In the example of LBMGA, two solutions rank first together in the final generation. In order to make an intuitive comparison, the circumstance of natural delay is also given. Figures 5–8 show the different performances of the proposed method and the benchmark method and their contradistinctions are displayed in Table 3. Figure 5 shows the naturally delayed arrangement at the beginning of the recovery process, which indicates that the chromosome is formed based on Table 3. The time route auto-generation strategy is employed to determine which flight is cancelled, and it obeys the rule that no aircraft is rearranged.

The comparison of Table 3 and Figure 5 concludes that six flights are delayed, and six flights are cancelled. Flights CZ1849 and CZ1850 are cancelled because the AT of CZ1850 will break the curfew time. CZ1849 is in the same flight loop with CZ1850 and thus is also cancelled. The flight loops CZ1457-CZ1458 and CZ1143-CZ1144 are cancelled due to the curfew constraint. Consequently, 72 passengers who want to endorse cannot be satisfied, which results in utility losses of the disrupted passengers. Due to the high coefficient of utility loss of flight loops CZ1849-CZ1850 and CZ1143-CZ1144, which are both the end flight loops of the routes, few substance flights are available, and the total utility losses of this arrangement are relatively high.

The remaining flights of aircraft 4 are delayed by 3.5 hours, whereas the remaining flights of aircraft 5 are delayed by 4 hours. Six flights are cancelled. As a result, the economic loss of this arrangement is very high.

Figures 6 and 7 list the Rank-1 Pareto solutions of the final generation by the proposed recovery solution method. The total solution time is 1.1379 s.

The arrangement shown in Figure 6 enables CZ1109-CZ1110 to be cancelled and ten flights to be delayed, which only causes 20 endorsing passengers to be dissatisfied. The arrangement shown in Figure 6 enables CZ1109-CZ1110 and

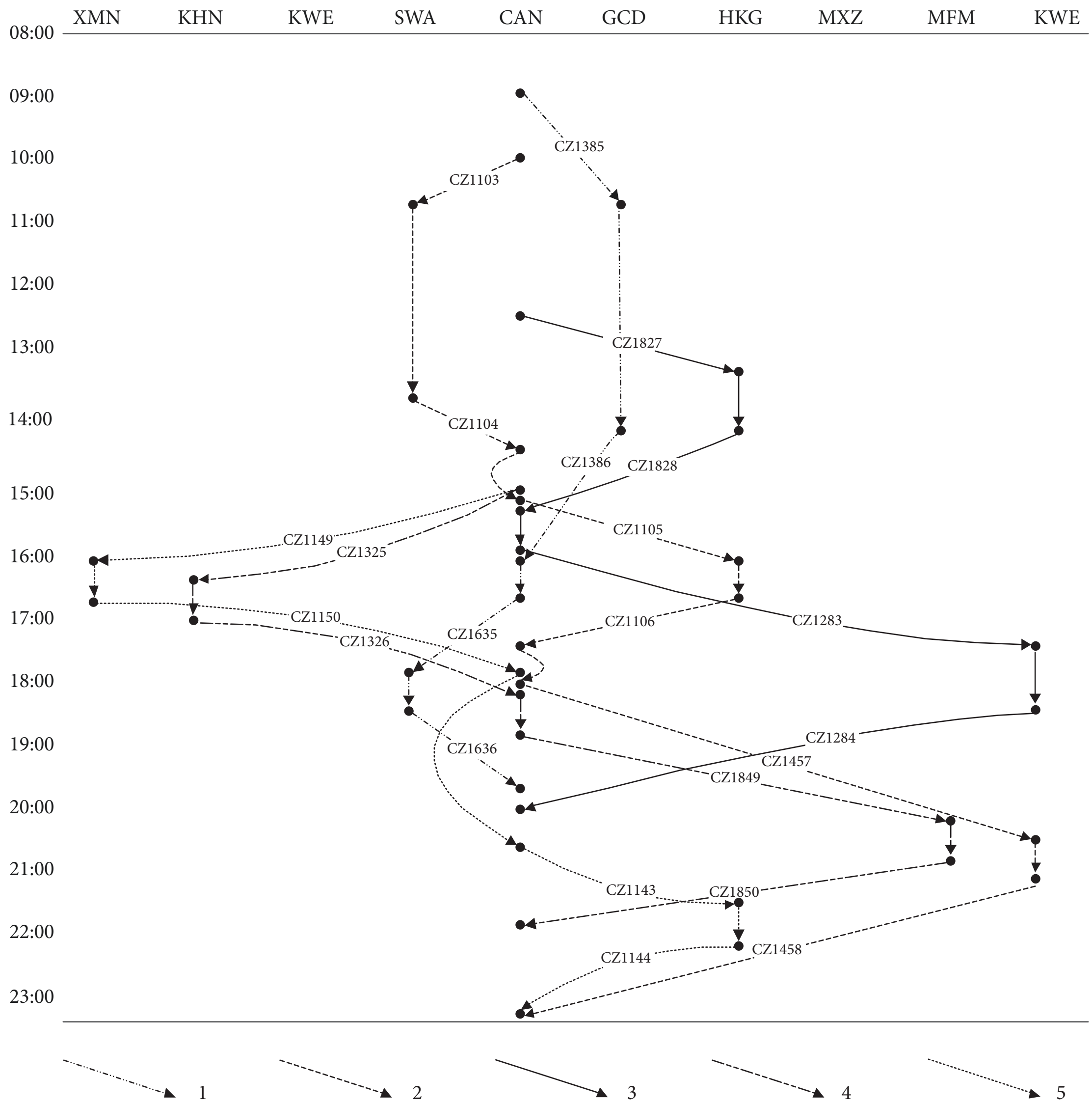

Figure 6: Aircraft in one solution of the proposed method.

CZ1149-1150 to be cancelled and eight flights to be delayed. Compared with the arrangement in Figure 5, the total delayed time in this timetable is significantly smaller, and the cancellation fee of CZ1149-CZ1150 is relatively lower, which produces a relatively low economic loss. However, two more flights are cancelled so that 48 passengers endorsing willingness cannot be satisfied. Due to the increase in cancelled flights, the utility cost of the arrangement in Figure 7 is higher than the utility cost of the arrangement in Figure 6.

Figure 8 shows the Rank-1 Pareto solutions of the final generation by the traditional GA. First, a coding strategy is directly applied. The length of the chromosome is 24, and for each point, the number changes from zero to five. Second, a flight sequence generation strategy is employed to obtain a suitable solution. Last, all chromosome operations are randomly performed, and a punishment strategy is employed to constrain the solution.

The total solution time is 4.5532 s. The arrangement in the table enables CZ1325-CZ1326 and CZ1149-CZ1150 to be cancelled and eight flights to be delayed. Consequently, 40 passengers who want to endorse cannot be satisfied. Although the total delayed time is smaller than that in Figure 6, the economic loss is larger because the case is not such as that in Figure 6; the cancellation fee fills the gap and overflow. Compared with the arrangement in Figure 6, even though the number of cancelled flights is the same, the coefficient of utility loss of CZ1325-CZ1326 is smaller than that of CZ1109-CZ1110, which produces a smaller utility loss in Figure 8.

Table 4 provides a comparison between traditional GA and the proposed method LBMGA. Although the optimization efficiency of the proposed method is slightly better than that of traditional GA, the speed of the proposed method is substantially faster. More importantly, via the proposed method, a relatively even result can be generated, which

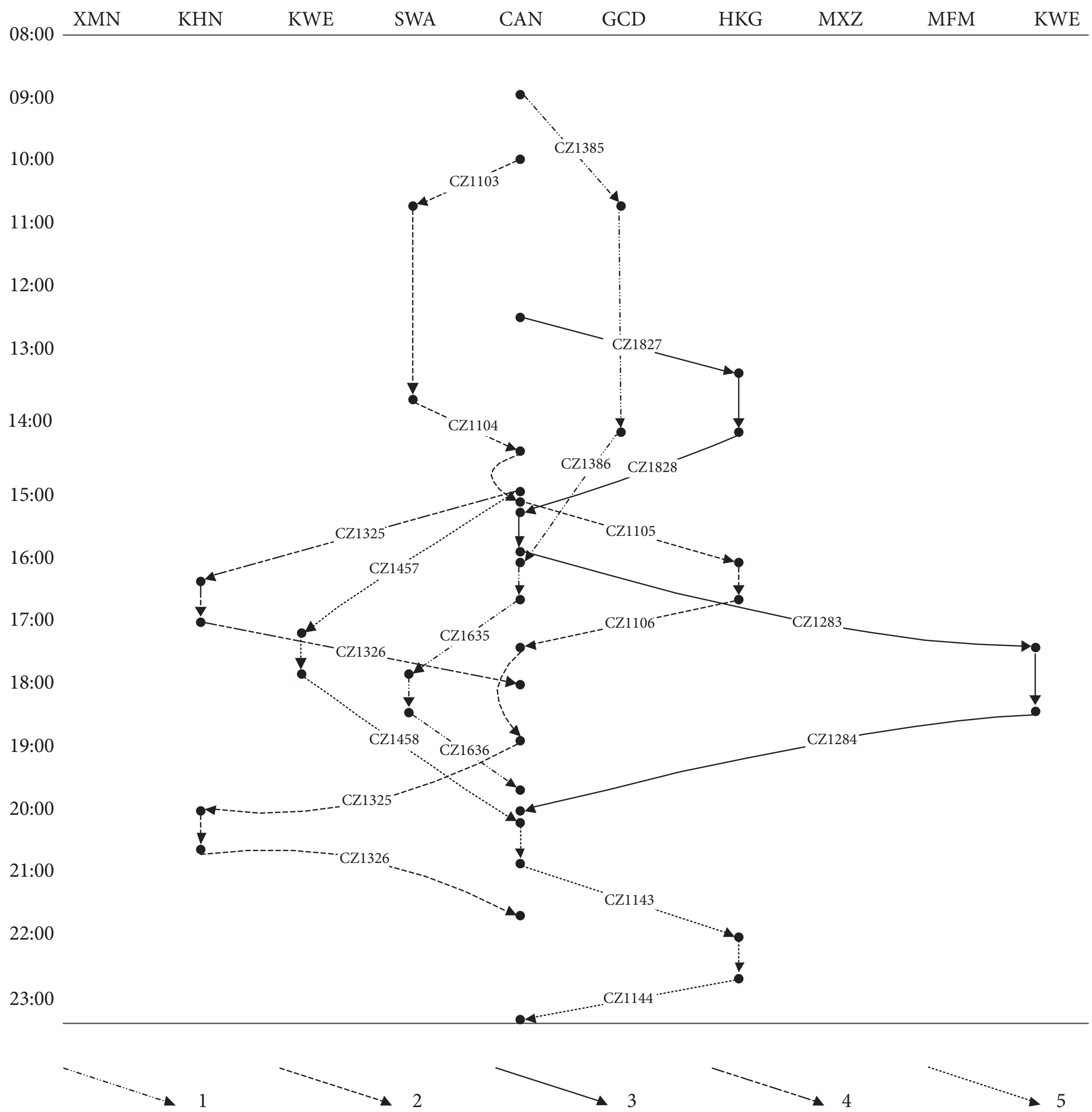

Figure 7: Aircraft of another solution of the proposed method.

means the willingness from the passengers can be fully considered so that the rearrangement is more likely able to satisfy the passengers

Additional analysis of the solution process indicates that approximately 65% of the time is allocated to forming a timetable in the case of the proposed method. Conversely, approximately 80% of the time is allocated to generating suitable flight sequences and calculating the punishment value in the case of the traditional GA.

According to the previous analysis, we know that the characteristics of the flight, especially the relationship between the arrival time and the curfew time, and the coefficient of utility loss have a great impact on the total benefit, on either the economy side or the utility side. In detail, the later the initial arrival time of the canceled flight is, the less the possibility that the passenger can move to another flight. Moreover, it is obvious that a high coefficient of utility loss of a flight results in high total utility loss if this flight is canceled. Therefore, the airline should pay more attention to these two factors when rearranging flights, which means that it should try to not cancel the latest flight of a day or high utility-costing flights.

Moving to the perspective of coding, this simple case shows the importance of data structure. Based on a well-designed data structure fitting the structural features of the mathematical model, plenty of time and space can be saved when solving these problems, and the additional analysis indicates that the feasibility and the punishment value process are two of the key points to improve the efficiency.

5.2. Experiments Based on Real-World Data. In this study, a large case based on a present flight schedule in China is employed to verify the effectiveness and efficiency of the proposed algorithm. Table 5 lists the scheduled information about the real data case.

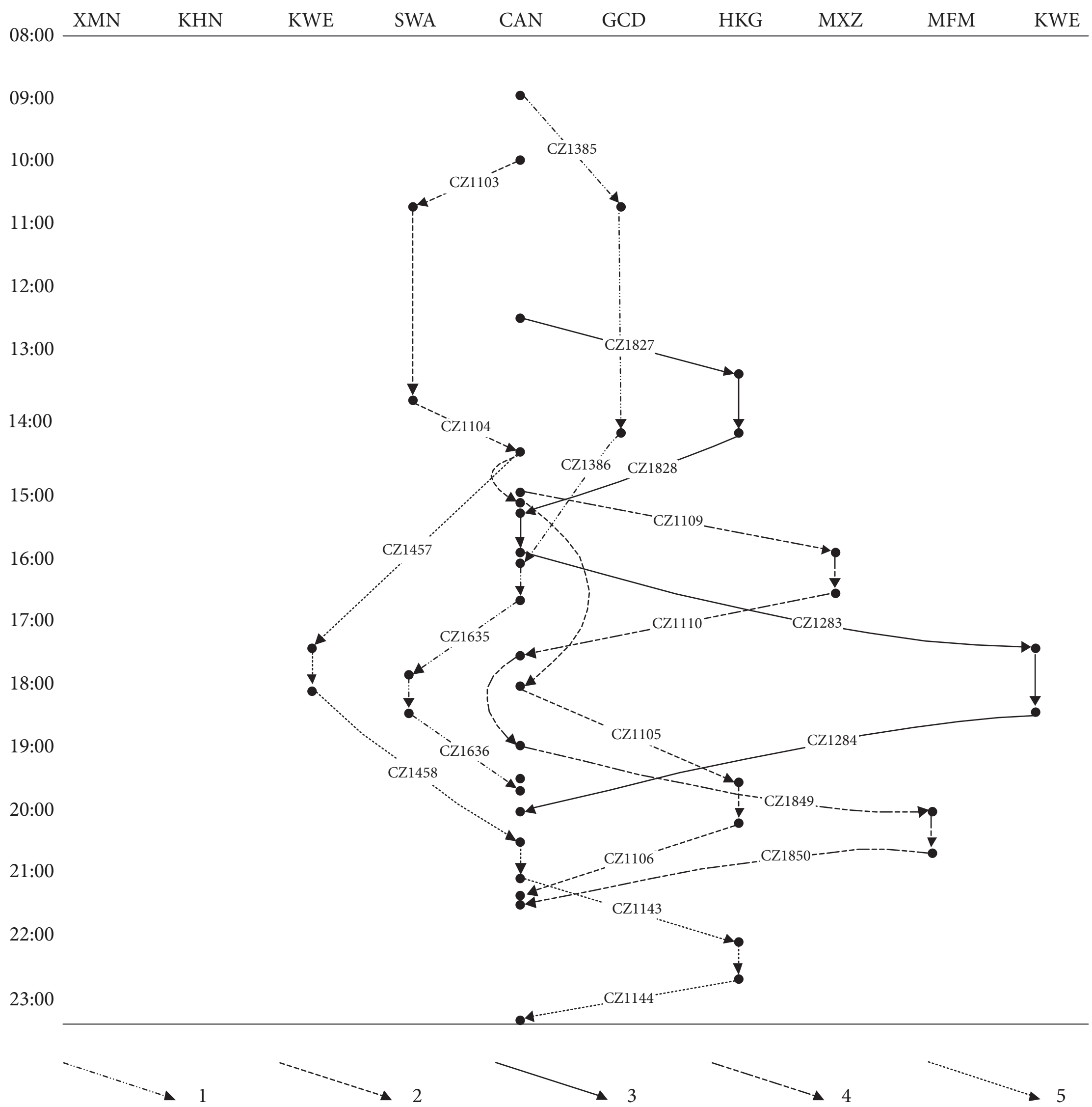

Figure 8: Aircraft for one solution of traditional GA.

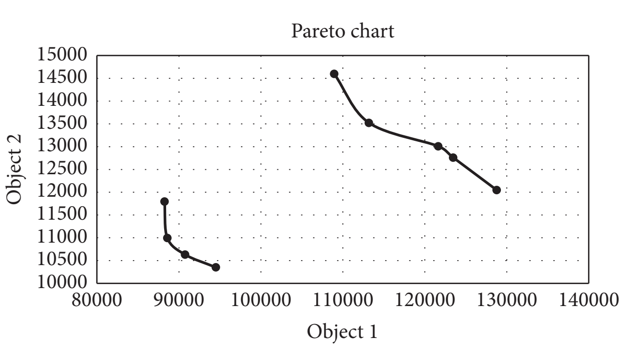

Figure 9: Pareto chart.

The delayed aircraft are randomly chosen. For each specific delayed table, the program runs 50 times to retrieve an average. The optimization efficiency is defined in formula (14):

$$\begin{aligned} &\textit{Optimization efficiency} \\ &\quad = 50\% * \frac{\textit{final object}1}{\textit{original object}1} + 50\% \\ &\quad * \frac{\textit{final object}2}{\textit{original object}2} \end{aligned} \tag{14}$$

In formula (14), original object1 means the value of object 1 in the circumstance natural delay recovery process happens, so as the original2. The final object1 means the value of object1 under the recovery process given by the LBMGA, so as the original2.

The functionality parameters are chosen in prior experiments for excellent performance. The operating environment is an Intel Core i7-4720HQ 2.60 GHz CPU, 8 G memory, Win 10 OS and Dev-C++ 5.11 compiler. This program uses only a single core of the CPU.

Table 3: Information about the simple case.

FN	DA	AA	DT	AT	AM	OA
CZ1385	CAN	CGD	09:00:00	10:44:00	A320	1
CZ1386	CGD	CAN	14:25:00	16:02:00	A320	1
CZ1635	CAN	SWA	16:43:00	17:43:00	A320	1
CZ1636	SWA	CAN	18:30:00	19:41:00	A320	1
CZ1103	CAN	SWA	09:58:00	10:46:00	A320	2
CZ1104	SWA	CAN	13:40:00	14:33:00	A320	2
CZ1105	CAN	HKG	15:15:00	16:04:00	A320	2
CZ1106	HKG	CAN	16:45:00	17:34:00	A320	2
CZ1827	CAN	HKG	12:30:00	13:31:00	A320	3
CZ1828	HKG	CAN	14:15:00	15:24:00	A320	3
CZ1283	CAN	KWE	16:05:00	17:37:00	A320	3
CZ1284	KWE	CAN	18:20:00	19:58:00	A320	3
CZ1109	CAN	MXZ	11:30:00	12:27:00	A320	4
CZ1110	MXZ	CAN	13:10:00	14:11:00	A320	4
CZ1325	CAN	KHN	14:52:00	16:13:00	A320	4
CZ1326	KHN	CAN	16:55:00	18:20:00	A320	4
CZ1849	CAN	MFM	19:00:00	20:06:00	A320	4
CZ1850	MFM	CAN	20:55:00	21:51:00	A320	4
CZ1149	CAN	XMN	11:00:00	12:11:00	A320	5
CZ1150	XMN	CAN	12:52:00	13:59:00	A320	5
CZ1457	CAN	KWE	14:40:00	17:02:00	A320	5
CZ1458	KWE	CAN	17:42:00	20:08:00	A320	5
CZ1143	CAN	HKG	20:50:00	21:52:00	A320	5
CZ1144	HKG	CAN	22:42:00	23:42:00	A320	5

Table 4: Comparison between two types of GAs.

	Naturally delayed aircraft arrangement (Figure 5)	Aircraft for one solution of LBMGA (Figure 6)	Aircraft of another solution of LBMGA (Figure 7)	Aircraft for one solution of the traditional GA (Figure 8)
Fitness 1	11292	4878	4184	5656
Fitness 2	8766	3962	5852	5061
Operation time	\	1.1379s	1.1379s	4.5532s
Number of delayed flights	6	10	8	8
Number of cancelled flights	6	2	4	4
Number of dissatisfaction passengers	72	20	48	40

In a real situation, some flights belong to flight loops. To accelerate the recovery process, only flight loops become involved with the aircraft switch, which indicates that these non-flight-loop flights will be naturally delayed if accidents occur. For the case in this section, the chromosome will be divided into three sections, which correspond to three air hubs. Only 144 flights (72 flight loops) will participate in the aircraft switch. The remaining flights will remain in their original aircraft and naturally perform a delay.

In the practical application of GA, dynamic mutation is commonly introduced to expand the search areas and avoid the solution premature. In LBMGA, the mutation rate is the same for every section of a solution, but the rate among solutions may differ.

Mutation rate for solution i:

$$PM_i = \begin{cases} 0.03 \times \dfrac{\max[n_i] - n_i}{\max[n_i] - \overline{n_i}}, & n_i \geq \overline{n_i} \\ 0.03, & otherwise \end{cases} \quad (15)$$

In formula (15), n_i means the optimization efficiency of solution i. For every generation, the denominator is solid but for every solution, the molecule is different. Moreover, the molecule, $\max[n_i] - n_i$, means that if the solution is more different than the best one, this solution may have more chance to mutate so that the search areas expend. More importantly, this action effectively avoids the solutions concentrating on the average so that the premature less

Table 5: Scheduled information about the real data case.

Number of aircraft	59
Number of flights	209
Number of flight loops	72
Number of airports	42
Number of air hubs	3
Number of passengers	24860

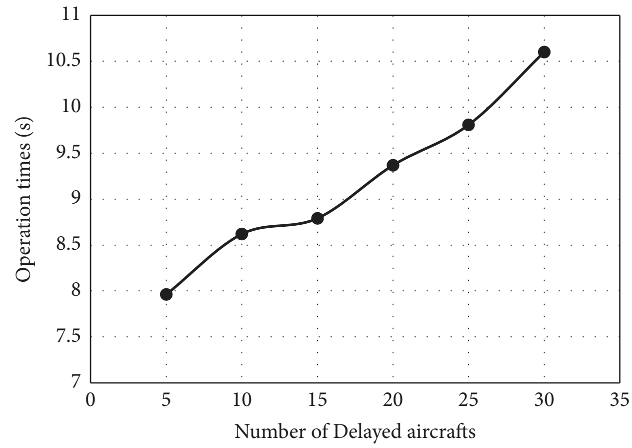

Figure 10: Operation speed analysis.

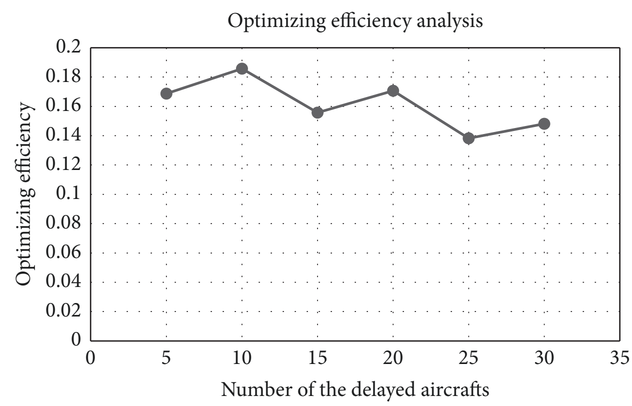

Figure 11: Optimization efficiency analysis.

likely appearing. Meanwhile, the dynamic mutation rate only affects the solutions above the average level and the wave of mutation is controlled to a reasonable extent so that the searching process can be more effective.

In Figure 9, the number of delayed aircraft is 15; the data are obtained from a specific running result. The outside line represents the Rank-1 Pareto solutions of generation 5, and the inside line represents the situation of final generation (100). To vividly display the data, the population size in Figure 9 is 32. From the two groups of solutions, it can be concluded that the quality of the solution has been significantly improved. Moreover, the distribution of the Rank-1 Pareto solutions has also been noticeably changed from sparse to dense. This result shows that the proposed algorithm has good convergence to determine an area of good solutions

Figure 10 shows the relationship between operation time and the number of delayed aircraft. An increase in the number of delayed aircraft causes a steady increase in operation times. As shown in Figure 10, as the number of delayed aircraft increases, the optimization efficiency gradually fluctuates and shows a rising trend. This result was observed because for every additional delayed aircraft, the disturbed flights are limited, and the number of the total disturbed flights is increased; thus, extra computation is needed to handle the new flights involved in the recovery process, which results in an increase in the time to obtain a solution. Moreover, from the analysis of the proposed algorithm, the generation of the new route timetable for every aircraft occupies most of the operations. Therefore, each newly added delayed aircraft means a new route timetable must be calculated, which increases the computational costs and time needed to obtain a solution significantly. Moreover, in the situation in which the economic loss is the only object, the relationship between the two factors above shows a trend similar to that in Figure 10, which means that the consideration of utility loss does not have significant impact on the operation speed when the scale of delayed aircraft becomes larger. Hence, considering the utility loss is reasonable in terms of in-time reaction for airlines.

It can be concluded from Figures 10 and 11 that the stability of optimization efficiency decreases as the number of delayed aircraft increases, and the complexity of the situation increases, which significantly guarantees the degree of effectiveness of the algorithm. This phenomenon can be explained by two aspects. First, as more aircraft are delayed, the feasible solutions become fewer; thus, the possible adjustment operations also become fewer and the optimization efficiency decreases. Second, a gene algorithm is based on stochastic processes; thus, the optimization efficiency shows some volatility.

In Figure 9, the data describe how the optimization efficiency changes for different minimum turnaround times and delayed aircraft; the delayed aircraft are randomly chosen for each group. When the number of delayed aircraft is 15, 20, and 30, the optimization efficiency shows a tendency to fluctuate, and the fluctuation is mainly caused by the nature of the GA, which is related to the randomness of the solution. Further analysis in the process data unveils the reason why the optimization efficiency does not change significantly for different minimum turnaround times; when the time increases, the total delayed time of naturally delayed flights in the initial solution also increases. Therefore, although the absolute quantity of the final good solutions increases with increasing maximum delay time, the optimization fluctuates. However, there is a dramatic fluctuation in the optimization when the number of delayed aircraft is 25, especially when the minimum turnaround time is 60. Analysis of the delayed aircraft shows that there are 3 aircraft only being delayed in this circumstance but not in others, and these aircraft are in the same fleet. Thus, the main extra cost is from the flights performed by this fleet. However, the number of aircraft in this fleet is very small, and only a few flights can

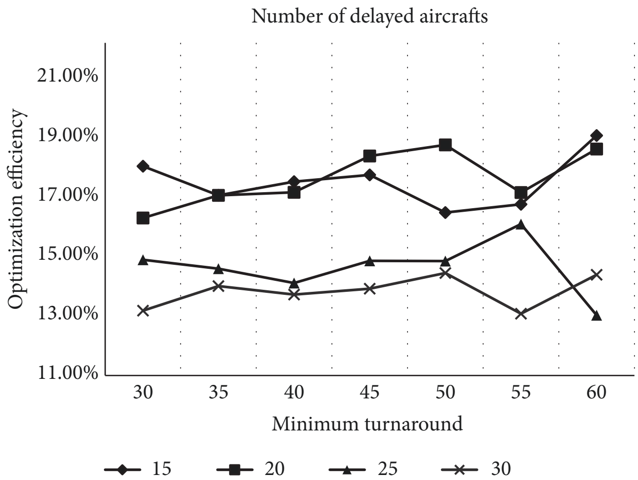

Figure 12: Minimum turnaround time analysis.

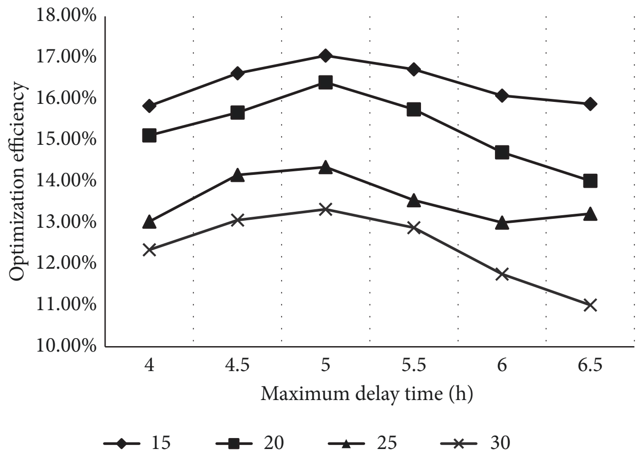

Figure 13: Maximum delay time analysis.

be performed by this fleet. When the minimum turnaround time is less than 60, these flights are not severely affected. However, when the turnaround time reaches 60, they are severely disturbed. Therefore, the importance of each aircraft in the flight recovery system is not equal; the available time of some aircraft can have a large influence on the recovery process. According to the lines in Figure 9, it is obvious that the smaller turnaround time always brings the higher optimization efficiency. However, cutting down the turnaround time is a tough challenge for airlines because it highly depends on the ability level of the ground staff and the situation of the airport, which means that it is quite a challenge for the airline to behave much better than its rivals. Therefore, trying to find a different idea for improvement, such as considering the utility cost mentioned in this paper, is more feasible for the airline to improve the competitiveness. An interesting point is that if the airline only chases the minimize turnaround time, it may have a negative effect on the optimization efficiency. The line 25 mins in Figure 12 is a good example. Wiser advice is that the turnaround time should be well-designed with the time sensitivity of the total

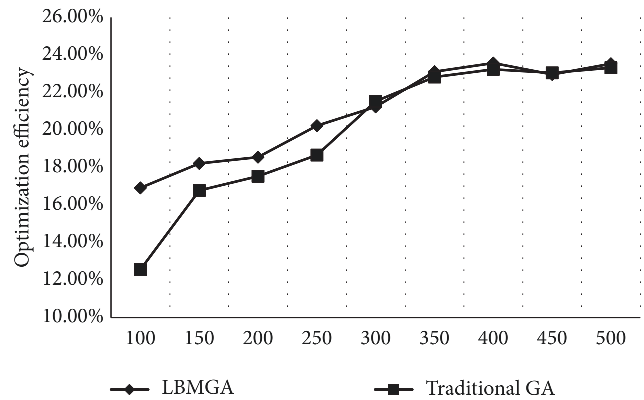

Figure 14: Maximum generation analysis.

fight arrangement. This paper suggests that the airline can set their turnaround time based on the average gap-time between the flights.

In Figure 13, all of the curves reach their peaks when the maximum delay time is 5 and then show overall declines. The rising section can be interpreted as that with a relatively small maximum delay time; many flights are forced to be canceled due to the maximum delay time constraint. Thus, the higher cancellation costs and the dissatisfaction of the passengers on these canceled flights are inevitable. However, with more relaxed maximum delay time restrictions, the recovery arrangement has more flexibility to determine whether a flight is canceled or not. However, the process data show that in this circumstance, the total delayed time is likely to increase and more flights may surpass the curfew time. More importantly, the optimization process consists of the optimization of two objects. When the maximum delay time increases, object 2 shows a more pronounced downward because the coefficient in object 2 is more sensitive than object 1 when cancellation happens; moreover, as the arrival time of the flight approaches the curfew time, the utility loss will also increase. Data analysis also supports the sharp decline in the optimization of object 2. Therefore, the maximum delay time should be set wisely. Moreover, the setting process should depend on the flights in the system, especially the later flights.

Figure 14 compares the results of the traditional GA and the proposed algorithm for different maximum generations. The operation time of the proposed algorithm is significantly smaller than that of the traditional GA for all maximum generation levels. Theoretically, a larger search area increases the likelihood of finding a good solution, which is obviously the advantage of the normal gene algorithm because its searching process includes many infeasible solutions. However, the curves in Figure 13 show that when the maximum generation is relatively low, the algorithm in this paper clearly performs better, although the two algorithms convergence to a roughly equal level. Further research into the solution of the iterative process reveals that the distribution of feasible solutions is sparse but locally concentrated due to the massive constraints and the 0-1 encoding strategy. In this condition, the number of infeasible solutions is much larger than that of feasible solutions, and the searching process jumps from one area to another. Therefore, although a given sufficient maximum generation level, the normal gene algorithm can convergence to a satisfactory result, an algorithm that is based on the feature of the solutions and running within the feasible solutions can improve the search efficiency and the convergence speed significantly. Moreover, the global optimality can be ensured at the same time.

6. Conclusions

This study focused on the problem of integrated recovery of aircraft and passengers with passengers' choice willingness between itinerary changes and the ticket refunding to put the concept of passenger-oriented service into practice and make the airlines better serve the construction of the civil aviation industry in China. The following objectives were proposed: minimizing the total economic cost of recovering flights, aircraft, and passengers and minimizing the total utility loss of passengers. Based on these objectives, an integer programming model formulation was built for describing and analyzing the problem.

The LBMGA was introduced to lead the solution process and ensure that the willingness of passengers was on the same level as the airlines' profits. Then, a special time route auto-generation operation was designed to establish a new flight schedule to fit various constraints. A rank-based multiobject strategy was used to balance the two objectives in order to find better solutions.

The results of a small-scale example used to present the changes in a flight schedule imply that the LBMGA is more available and efficient than the normal GA. Then, a large-scale example based on a real-world situation was performed to prove the robustness and efficiency of LBMGA.

Considering the importance of passengers' willingness, future research needs to focus on how to describe the utility more accurately and consider more circumstances, such as using statistical methods to determine the utility parameters of the passengers and expanding one-stroke to multiple-stroke. Additionally, future research will generalize this algorithm to solve the general flight recovery problem, for example, by expanding the LBMGA to embrace more types of flight networks. The pertinent improvement of the GA can increase the efficiency to a new level, such as the application of a new data structure and a new crossover or mutation operation.

Data Availability

The data, used for validating the efficiency and effectiveness of loop-based multiple objective genetic algorithm (LBMGA), can be divided into two parts. One part is a sample case. The basic data of the sample case is displayed in Table 2 in the article. The other part is real data case. The flight schedules are derived from Air China. The aircraft delay information is randomly generated according to analysis on real situation. As the scheduled database is not publicly available because of the commercial sensitivity, we could not provide the basic data. In terms of verifying the results of an article, we think that the sample case is enough for verifying the results of an article. We also have given detailed analysis for validating the results of the sample case and the real data case in the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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Research Article

An Interval-Based Evolutionary Approach to Portfolio Optimization of New Product Development Projects

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The growth of large enterprises in the manufacturing market commonly depends on good New Product Development (NPD) projects; these projects represent a strategy to overcome competitors inside a competitive environment. The management of such projects is usually complex and involves risk due to the changing and conflicting environment. The approaches that tackle the problem lack an explicit consideration of the DM's attitude facing uncertainty and imprecision related to the risk and particularly in the presence of time-interdependencies. This paper proposes a model of the time-related effects, under imperfect knowledge, and their influence in choosing optimal NPD portfolios. The proposed approach is an interval-based method to solve NPD portfolio optimization problems under different forms of imperfect knowledge. This approach has the advantage of a unified and simple way to model the different sources of imprecision, vagueness, uncertainty, and arbitrariness. The attitude of the DM facing the imperfect knowledge is adjusted by using some meaningful parameters. The research focuses particularly in creating a method useful for risk-averse DMs. The proposal was tested through an experimental design that compared the results achieved by the new method against the expected value in portfolios. The results revealed that high levels of conservatism might prevent wasting resources in failed projects.

1. Introduction

It is commonly accepted that good research and development practices are essential for growing and well positioned large enterprises in the manufacturing market, commonly having dozens or hundreds of projects. The development of competitive new products is likely the most important task that allows a manufacturing enterprise surveillance within a competitive environment [1, 2]; such NPD projects can positively impact in the company competitive position allowing a better efficiency in the product management, and/or the generation of social benefits [3]. Hence, for the growth of enterprises under stiff competition, good New Product Development (NPD) projects are required. However, the competence can affect the projects of an enterprise in different ways; e.g., strong competence is related to reduction in real prizes (cf. [4, 5]), or technical inefficiency can be reduced improving external and/or internal competence [6]. According to [7], planning the interdependency among projects in a portfolio is a dynamic hard and complex task that must optimize resources considering the competitors movements in the market. Given the time is one of the wider resources spread in the development of a project, studying its effects must be considered in NPD projects.

To a great extent, a successful NPD can produce large benefit (profit, prestigious, market share, etc.), but needs complex management and involves high risk, mainly due to the fast changing and conflicting environment, as well as technological innovations. According to some authors most NPD projects have low probability of success (e.g., [1]). Many authors argue the importance of good practices to handle the innovation risk, thus increasing chances of having successful new products (e.g., [8–11]).

Since often there are numerous good projects but there are not sufficient resources to develop all of them, the decision makers should select the most appropriate NPD project portfolios, expecting that these portfolios allow developing several, even many, attractive, and successful products that generate growing benefits [12]. To balance risk and potential benefits is a crucial aspect in selecting appropriate NPD portfolios (e.g., [13]).

NPD portfolio selection is a particular case of research and development projects portfolio selection. But NPD projects are distinguished from other innovation and research projects by some relevant features:

(i) Uncertain market payoffs that change over time.

(ii) Strong dependence of benefits on imperfectly known project completion times, technological innovations, potential competitor products, and their interactions.

(iii) Sometimes, NPD projects can be preceded by applied research projects. Then, effects related to time-interdependence among different projects should be considered as an additional feature in portfolio selection problems.

Chan and Ip [14] proposed a framework for a decision support system that aids in NPD through the assessment of product values based on three types of influence factors, which are the product, the customers, and the market. The system uses two models to estimate the Net Customer Lifetime Value (NCLV) from predicted customer purchasing behaviour; the NCLV can be used to rank a set of potential products. The selection process is performed through a ranking process based on such value.

Loch and Kavadias [13] proposed a model based on marginal analysis to solve a dynamic version of the Portfolio Selection of NPD programs. Using a probabilistic approach, this paper handles multiple periods, synergy, uncertainty, managerial risk, and obsolescence, but not different levels of conservatism from decision makers (DMs).

Wei and Chang [1] and Lin et al. [15] proposed approaches that integrate fuzzy set theory and multicriteria group decision-making method into a NPD Project Portfolio Selection Model (PPSM) which allows the management of risk. These proposals use multiattribute fuzzy group decision techniques to deal with fuzziness, uncertainty and inaccuracy. The PPSM proposes the use of a Project Fuzzy Decision Index in combination with Fuzzy Gates (FG) to eliminate not so good projects. The FG should be calculated from information provided by the decision makers; the FG are used by a Go-Kill method together with a threshold (which is defined by the DM regarding enterprise resources, risk tolerance, and so on); finally, the total dominance method is used for the Go-Kill decision. The definition of the FG might consider information related to evaluation of risks and other aspects. These works do not directly involve the influence of time-interdependencies such as the obsolescence of a project or the apparition of new competing products from other companies.

Badizadeh and Khanmohammadi [2] proposed a Fuzzy Multicriteria Decision-Making model for evaluation and prioritizing NPD under uncertainty. Project selection is performed through a ranking process. This paper considered three types of uncertainties that influence in the value of products; these are market uncertainty, technology uncertainty, and process uncertainty (related to internal issues of the organization). This work can involve a large number of criteria to be handled by a DM, a situation that can lead to an exhaustive cognitive effort from her/him to provide the required information.

Reich [16] proposes to map the Portfolio Selection Problem of NPD to the domain of the Constraint Satisfaction Problems. The proposal consists in a decision support system that aids in portfolio selection of NPDs, taking as guiding feature the desired reliability of products, i.e., the capacity of a product to perform the required function. The strategy relies on a fuzzy system and historical data to estimate the NPDs costs; these costs are involved in the constraint satisfaction optimization model to make the appropriate selection of projects. This approach successfully includes the reliability feature in the selection process for NPDs.

Wei et al. [17] present a PPSM for NPD based on fuzzy sets theory to rank product lines, dealing with synergy, risk, and uncertainties. The approach is an improved PPSM for fuzzy multicriteria group decision-making; it proposes an NPD project Go-Kill decision index named project fuzzy synthetic rating (PFSR) to aggregate fuzzy weights and fuzzy evaluations of the NPD projects. Once the PFSR is calculated, the method uses the Go-Kill threshold provided by the DMs based on risk and performance to compare the crisp PFSR and determine which projects survive for the next stage. The method ends with a ranking of the projects.

Reich and Pawlewski [18] use neural networks and fuzzy sets theory to provide a solution for PPSM for NPD. The approach offers a rational structure of NPD project evaluation reflecting vagueness or ambiguity that appears in the business environment. The combined use of neural networks and fuzzy systems offers an evaluation of product lines that is used to rank the set of projects; such rank aids in the selection process and with it, in the construction of an NPD portfolio.

Recognizing that launching a new product is highly risky due to uncertainties of the market and competitors, Tolga [8] proposed a model based on compound options with type-2 fuzzy numbers. He stated that those uncertainties cannot be represented by crisp numbers and traditional techniques are inappropriate to solve real life problems.

Tiwari et al. [10] proposed a method to become a NPD process more effective, evaluating design concepts in uncertain environments of early design stages. In order to improve the overall effectiveness of the process, design concepts are characterized by soft sets and customer's preferences by Shannon entropy. The approach to incorporate costumers in early NPD stages is validated through examples.

Most of the related works in the particular framework of NPD are subject to at least one of the following criticisms: (i) no consideration of complex time-interdependencies related to competing products and technologic obsolescence; (ii) no explicit consideration of the DM's attitude facing uncertainty and imprecision; (iii) no solution of an optimization problem in the portfolio space; the best portfolio is not necessarily composed by the best projects, because the complex interdependencies among projects and their influence on the DM's preferences (c.f. [19]); (iv) lack of knowledge about market, competitors, technological changes, etc. is mainly modelled by fuzzy sets. Fuzzy Sets theory is a powerful tool to model vagueness, but this is not the unique (probably not the main) source of imperfect knowledge in NPD decision-making.

The difference between uncertainty and vagueness in the framework of R&D project selection has been approached by several authors (e.g., [20]). The uncertainty of projects is related to the degree of precision with which the variations in outcome, resources, and work processes of projects can be forecast [21]. From the point of view of multicriteria decisions, other authors prefer to use the more general term "imperfect knowledge" (e.g., [22–26]); the sources of this imperfect knowledge are arbitrariness, imprecision, ill-determination, and uncertainty in data and model parameters [22]. In this sense, the term "uncertainty" is related to the values of certain data in a more or less distant future [22].

There is a vast literature modelling imperfect knowledge in the field of Research & Development project portfolio selection (e.g., [20, 27–29]), using statistical information or fuzzy sets. Interval analysis is another approach that has been recently applied to model imperfect knowledge in project portfolio selection and stock portfolio optimization (c.f. [30–35]). Interval analysis combined with outranking methods was recently applied to portfolio optimization in [24, 26]. The interval approach is a natural and simple way in which to express imperfect information, it does not matter its source.

NPD portfolio selection can be modelled as a multiobjective optimization problem under imperfect knowledge. Unlike most research projects, the benefits provided by NPD projects are strongly depending on time; but the time-related features of NPD projects are often poorly known. In this paper, we are not mainly interested in addressing the multicriteria aspects concerning with NPD portfolio selection; these aspects are not essentially different from those of research and development project selection, which have received a great attention for many years. Our aim is to propose a way to face the risk provoked by imperfect knowledge in time-related interdependencies which are typical of NPD.

Three different moments are generally present in any NPD project: (1) the estimated completion time; (2) the moment in which the competence become significant; and (3) the moment in which the developed product becomes old. Those moments have strong dependences with the final benefit produced by a project, e.g., the longer periods of completion of a NPD project or the apparition of competence in the market might provoke smaller benefits, even null benefits if the project becomes old (i.e., it has no longer relevance in the market). These specific forms of dependences should be established by the management teams of each project; nevertheless, long lead times of R&D projects combined with a complex market and technology dynamics make it very difficult to collect reliable data [36], originating the presence of risk.

This paper is primarily oriented to the modelling of the time-related effects, under imperfect knowledge, and their influence in choosing optimal NPD-oriented project portfolios. This work proposes an interval-based method to concern with NPD portfolio optimization problems under the above forms of imperfect knowledge. This approach has the advantage of a unified and simple way to model the different sources of imprecision, vagueness, uncertainty, and arbitrariness. The attitude of the DM facing the imperfect knowledge is adjusted by using some meaningful parameters. We are particularly interested in creating a method useful for risk-averse decision makers. Although interested in benefits from NPDs, risk-averse decision makers are also interested in controlling and minimizing his/her regret as a consequence of spending resources in risky projects with low probability of success.

The remaining of this paper is organized as follows: Section 2 presents the general problem formulation for NPD-oriented project portfolio optimization with time-interdependencies under imperfect knowledge. Section 3 gives a brief description of the algorithmic solution, approach based on an evolutionary algorithm. Section 4 presents an illustrative example, which allowed us to evaluate the performance of the proposed approach. Finally, Section 5 contains the conclusions.

2. Problem Formulation

Beyond the expected market payoffs from the new products or revenues from innovation projects, there are usually other objectives to be taken into account by the decision makers. These criteria may concern with social responsibility, image, alignment with long term objectives of the enterprise, portfolio balance, etc. To consider several criteria is more general than the simple single-objective formulation based on expected revenues. In NPD-oriented portfolios, at least one of the criteria aggregates the expected revenues from the products that will be introduced in the market. Most of the candidate projects should be oriented to the market, although there could be some projects with nonprofit goals (e.g., applied research projects).

The project portfolio selection problem is formulated below as a multiobjective optimization problem. This model follows the well-known binary formulation for the stationary problem of project selection, which has been broadly studied in the literature (e.g., [37–42]). The basic features of the model that describe any instance of the project portfolio problem are (a) the number of projects M competing for financing; (b) the number of criteria (or objectives) N considered by the DM for measuring the quality of portfolios and projects; and (c) the requirements R for resources.

A portfolio is a subset of projects and is represented by a binary vector $a = \langle a_1, a_2, a_3, \dots, a_M \rangle$ where $a_i = 0$ means that project i will not be financed, and $a_i = 1$ means that the project receives support. The vector of the impacts of portfolio a is associated with N objectives $\overrightarrow{f}(a) =$

$\langle f_1(a), f_2(a), \dots, f_N(a)\rangle$. Assuming that the DM's preference increases with the value of the objectives, the decision-making problem can be expressed as

$$\max\{\langle f_1(a), f_2(a), \dots, f_N(a)\rangle\} \quad a \in R_F, \tag{1}$$

where R_F is the space of feasible portfolios and is usually determined by the available resources and timing relations among projects.

Considering the imperfect knowledge in project objectives, requirements, and budget availability, problem (1) can be expressed as in (2).

$$\begin{aligned} \max_a \quad & \{f_1(a) + \Delta f_1(a) \dots, f_M(a) + \Delta f_M(a)\} \\ \text{s.t.} \quad & R_{k,0}(a) \le P_{k.0} \quad k = 1, \dots n_r \\ & R_{k,0}(a) + \Delta R_k(a) <\approx P_{k.0} + \Delta P_k \\ & \qquad k = 1, \dots n_r \\ & T_n(a) = 0 \quad n = 1, \dots n_p \end{aligned} \tag{2}$$

where

(i) $f_i(a)$, i=1,…M represents the estimated value of the i-th objective in portfolio a;

(ii) $\Delta f_i(a)$ i=1,…M represents the imperfect knowledge associated with the i-th portfolio objective;

(iii) n_r is the number of different classes of resources;

(iv) $R_{k,0}(a)$ is the estimated consumption of the k-th resource for the portfolio a;

(v) $P_{k.0}$ is the estimated availability of the k-th resource;

(vi) $\Delta R_k(a)$ represents the imperfect knowledge related to the consumption of the k-th resource for the portfolio a;

(vii) ΔP_k represents the imperfect knowledge related to the availability of the k-th resource;

(viii) $<\approx$ denotes "less with sufficient likelihood";

(ix) T_n(a)=0 denotes the fulfillment of the precedence relationships between projects in portfolio a;

(x) n_p denotes the number of precedence relationships among projects.

Let us also denote by

(i) $f_{i,j}$: the estimated impact produced by the jth project on the ith objective;

(ii) $\Delta f_{i,j}$: a representation of the imperfect knowledge related to the ith objective related to the jth project;

(iii) $r_{k,j}$: the estimated consumption of the kth resource for the jth Project;

(iv) $\Delta r_{k,j}$: the imperfect knowledge associated with $r_{k,j}$.

Remark 1. $f_{i,j}$, $\Delta f_{i,j}$, $r_{k,j}$, and $\Delta r_{k,j}$, j=1,…N, i=1,…M, are aggregated at portfolio level using certain aggregation functions which can model different synergies. Let us denote these aggregation functions as follows:

(i) $f_i(a) = f_i(a_1 f_{i,1}, \dots a_N f_{i,N})$, $i = 1, \dots, M$

(ii) $\Delta f_i(a) = \Delta f_i(a_1 \Delta f_{i,1}, \dots a_N \Delta f_{i,N})$, $i = 1, \dots, M$

(iii) $R_{k,0}(a) = R_{k,0}(a_1 r_{k,1}, \dots a_N r_{k,N})$, $k = 1, \dots, n_r$

(iv) $\Delta R_k(a) = \Delta R_k(a_1 \Delta r_{k,1}, \dots a_N \Delta r_{k,N})$, $k = 1, \dots, n_r$

Formulation (2) could correspond to a portfolio selection problem of basic and applied research project, in which the influence of time-related effects on objective values can be neglected. Our proposal pretends to underline the differences between research and NPD project portfolio optimization. In manufacturing enterprises, during its last phase, an applied research project becomes NPD project, oriented to market and whose results are strongly influenced by time-related effects, mainly concerning competitors and technological changes.

Since in this paper we are primarily interested in modelling the timing effects and their influence in solving problem (2), let us introduce the following notation:

(i) t_{cj}: the estimated completion time of the jth project;

(ii) Δt_{cj}: the imperfect knowledge associated with the completion time of the jth project;

(iii) t_{sj}: the estimated start of the jth project;

(iv) Δt_{sj}: it represents the imperfect knowledge associated with the start of the jth project;

(v) $P_{a,j}$: the set of projects which are predecessor of the jth project.

Two precedence conditions should be fulfilled for each project in a feasible portfolio according to (3) and (4):

$$t_{sj} = \max\{t_{si} + t_{ci}\}, \quad i \in P_{a,j} \tag{3}$$

$$t_{sj} + \Delta t_{sj} = \max\{t_{si} + \Delta t_{si} + t_{ci} + \Delta t_{ci}\}, \quad i \in P_{a,j} \tag{4}$$

Remark 2.

(i) $r_{k,j}$ depends on t_{cj}; this dependence is usually an increasing function, because more time for completion likely implies more resources spending;

(ii) Δr_{kj} depends on Δt_{cj} for the above reason.

Since a subset of projects is oriented to the development of new products, one of the objectives in problem (2) is the revenue obtained by the projects that achieve certain portion of the market. This portion depends on the level of the competency for the same market.

Remark 3.

(i) f_{ij} can depend on the estimated moment of completion, that is $(t_{sj} + t_{cj})$ and on its relationship with the moment when the competence for the same market become significant; when the impact of the project can be degraded by competitors, later completions makes it more likely stronger presence of competitors; so, the function that model such a dependence is often a decreasing one;

(ii) Δf_{ij} can depend on $(\Delta t_{sj} + \Delta t_{cj})$ for the above argument.

Let us introduce two new concepts:

(i) $t_{comp,j}$: the moment in which the competence of the product developed by the jth project becomes significant;

(ii) $t_{old,j}$: the moment in which the product developed by the jth project can be considered old.

Assumption 4. $f_{i,j}+\Delta f_{i,j}$, $r_{k,j}+\Delta r_{k,j}$, j=1,...,N, i=1,...,M, $t_{cj}+\Delta t_{cj}$, $P_{k.0}+\Delta P_k$, $t_{sj}+\Delta t_{sj}$, $t_{comp,j}$, $t_{old,j}$, j=1,...N, i=1,...M, k=1,...n_r, can be represented as intervals. In the following, interval numbers will be denoted by boldface italic letters.

Assumption 5. The aggregation functions in Remark 1 can be expressed by using the basic operations of the interval arithmetic (see Appendix).

Under the above assumptions, problem (2) can be expressed as

$$\begin{aligned}
\text{Maximize}_a \quad & [\boldsymbol{f}_1(a),\ldots \boldsymbol{f}_N(a)] \\
\text{s.t.} \quad & R_{k,0}(a) \le P_{k.0}, \quad k=1,\ldots,n_r \\
& \boldsymbol{R}_k(a) <\approx \boldsymbol{P}_k \quad k=1,\ldots,n_r \\
& \boldsymbol{t}_{sj} = 0, \\
& \text{for all } j \text{ such that } a_j = 1, \text{ and } P_{a,j} = \varnothing \\
& \boldsymbol{t}_{sj} >\approx \max\{\boldsymbol{t}_{si} + \boldsymbol{t}_{ci} \mid i \in P_{a,j}\}, \\
& \qquad \text{for all } j \text{ such that } a_j = 1 \\
& \boldsymbol{t}_{sj} + \boldsymbol{t}_{cj} <\approx \boldsymbol{t}_{old,j} \\
& \qquad \text{for all } j \text{ such that } a_j = 1
\end{aligned} \tag{5}$$

where the symbol "≈" means "with sufficient likelihood".

Remark 6.

(a) The presence of competitors in a given project j provokes a reduction of $\boldsymbol{t}_{old,j}$ and a strong dependence of some impact functions $\boldsymbol{f}_{i,j}$ on $(\boldsymbol{t}_{sj} + \boldsymbol{t}_{cj})$. In such a case, $\boldsymbol{f}_{i,j}$ is a function of $(\boldsymbol{t}_{sj} + \boldsymbol{t}_{cj})$, $\boldsymbol{t}_{comp,j}$, and $\boldsymbol{t}_{old,j}$. $\boldsymbol{f}_{i,j}$ is a monotonically decreasing function on $(\boldsymbol{t}_{sj} + \boldsymbol{t}_{cj})$, and increasing on $\boldsymbol{t}_{comp,j}$, and $\boldsymbol{t}_{old,j}$. As greater $\boldsymbol{t}_{comp,j}$ is, greater $\boldsymbol{t}_{old,j}$ should be. The specific forms of those dependences should be established by the management team of each project supported by market studies and expert opinions. If a project j is related to results that do not become old, its $t_{old,j}$ is set to infinite.

(b) The strong dependences on the impact of the criteria due to time effects are included in the formulation of problem (5); such dependences are distinctive of NPD projects and mark differences from the more general portfolio optimization problem provided by problem (2). Basically, problem (5) contains the time-related effects addressed by our proposal.

(c) The nature and volume of the information required for solving problem (5) can be seen as an important criticism. However, many recent papers emphasize the importance of involving numerous stakeholders like shareholders, financial institutions, suppliers, buyers, customers, dealers, and different sources of design expertise in the early stages of the development process to get information about customer's preferences, technological changes, and competitors (e.g., [9, 10, 43]). In this sense, the importance of new information and communication technologies involving many stakeholders to get the necessary information has been recognized by several recent papers (e.g., [44–46]). According to Zhong and Lou [44], rich information about competitors can be obtained through collaboration and communication with buyers and suppliers. In such a collaborative environment, with the use of expert consensus methods, it should be possible to make a reasonable estimation of the input data required by problem (5).

With the use of the possibility measure for interval numbers (see (A.2) in Appendix), problem (5) can be transformed into

$$\begin{aligned}
\text{Maximize}_a \quad & [\boldsymbol{f}_1(a),\ldots \boldsymbol{f}_N(a)] \\
\text{s.t.} \quad & R_{k,0}(a) \le P_{k.0}, \quad k=1,\ldots n_r \\
& Poss\left(\boldsymbol{R}_k(a) \le \boldsymbol{P}_k\right) \ge \gamma, \quad k=1,\ldots n_r \\
& \boldsymbol{t}_{sj} = 0, \\
& \text{for all } j \text{ such that } a_j = 1, \text{and } P_{a,j} = \varnothing \\
& Poss\left(\boldsymbol{t}_{sj} \ge \max\{\boldsymbol{t}_{si} + \boldsymbol{t}_{ci} \mid i \in P_{a,j}\}\right) \\
& \ge \delta, \quad \text{for all } j \text{ such that } a_j = 1 \\
& Poss\left(\boldsymbol{t}_{sj} + \boldsymbol{t}_{cj} \le \boldsymbol{t}_{old,j}\right) \ge \delta, \\
& \qquad \text{for all } j \text{ such that } a_j = 1
\end{aligned} \tag{6}$$

where γ and δ are thresholds related to the phrase "with sufficient likelihood".

Definition 7. We say that a portfolio a is (γ,δ)-feasible if and only if a fulfills the constraints in problem (6).

Definition 8. For γ and δ from Definition 7, let a_i and a_j be two (γ,δ)-feasible portfolios. We say that a_i β-dominates a_j (denoted $a_i\text{D}(\beta)a_j$) if and only if $Poss(\boldsymbol{f}_k(a_i) \ge \boldsymbol{f}_k(a_j)) \ge \beta \ge 0.5$), k=1,...N, and $Poss(\boldsymbol{f}_k(a_i) \ge \boldsymbol{f}_k(a_j)) > \beta$ for some k.

Remark 9. The level of conservatism of the DM increases with γ, δ, and β. Although such parameters are related to the DM's conservatism, these refer to different portfolio features, and therefore, they have not necessarily equal values.

Definition 10. A (γ,δ,β)- Pareto portfolio is defined as a (γ,δ)-feasible portfolio a_i in problem (6) such that there is no (γ,δ)-feasible portfolio a_j that fulfills $a_j \mathrm{D}(\beta) a_i$.

Remark 11. The set of (γ,δ,β)-Pareto portfolios form the (γ,δ, β)-Pareto frontier. The threshold parameters can be modified, thus exploring different degrees of conservatism. It is obvious that the "best" solution to problem (6) is an element of the (γ,δ, β)-Pareto frontier for certain values of γ, δ, and β. Once these values have been set by the DM and the decision analyst, the best compromise solution to problem (6) depends on the DM's multicriteria preferences. In this paper, we are not interested in modelling the DM's preferences, but the DM's degree of conservatism facing the risk provoked by imperfect knowledge. Hence, our proposal is limited to generate the (γ,δ,β)-Pareto frontier. From this set, through a posteriori articulation of preferences, the DM will choose his/her best compromise.

Setting precise values of the conservatism-related thresholds is certainly demanding for the DM. Parameters γ and β were introduced by Balderas et al. [26] in the frame of interval-based project portfolio optimization. The present paper has introduced δ to model the DM' attitude facing the risk related to time effects. As stated by Balderas et al. [26], those conservatism parameters should be set in a coconstructive process involving a DM-decision analyst pair. In terms of the conservatism thresholds, an interaction between the DM and the decision analyst is mandatory in which the DM should understand the meaning of γ-feasibility and β-dominance. In [26] the reader can find illustrative examples of this interaction. The concept of a γ-feasible portfolio is easier than non-β-dominance; hence, the value of γ should be set first; this setting can be achieved by comparing the interval number associated with the available resources with different interval numbers representing potential levels of resource consumption (see [26]). As a result of these comparisons, acceptable values of γ can be identified.

Now, from the concept of non-β-dominance, the DM sets a starting value of β. Given γ and the starting β, solving problem (6) with different values of δ allows the DM to identify a value compatible with his/her level of conservatism facing the risk related to time effects. The setting of δ will be illustrated in Section 4. Once γ and δ have been determined, solving problem (6) with different values of β helps the DM-analyst pair to identify good solutions that are non β-dominated with appropriate levels of conservatism. As a consequence of this process, a good compromise can be determined that will be a non-β-dominated solution for some β in the interval [0.5, 1).

When N=2,3 problem (6) can be efficiently solved by using the I-NSGA-II method proposed in [34]. In this range of evaluation criteria, the human cognitive limitations are not an obstacle to compare multiobjective solutions, and the DM can identify a final compromise. Higher dimensions problems can be addressed by the I-NOSGA method [26], which has the capacity to handle many objective functions incorporating DM preferences through the interval-based outranking approach by Fernández et al. [23].

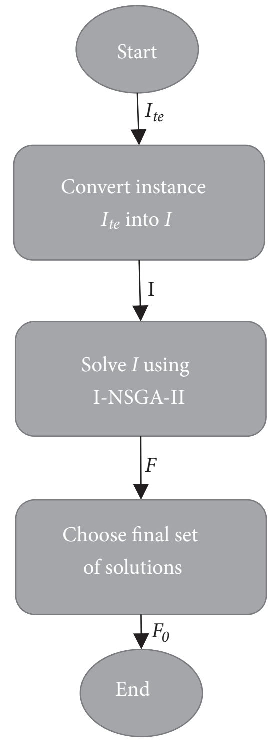

Figure 1: Illustration of the main steps for solving problem (6) with N = {2, 3}.

3. Brief Description of the Algorithm

This section presents the methodology based, shown in Figure 1, on the algorithm I-NSGA-II (cf. [34]). The I-NSGA-II deals with interval multiobjective optimization problems and, in the proposed methodology, it is used to solve problem (5), i.e., the optimization of NPD project portfolios under time effects. The methodology works in three phases. In the first phase, it transforms the time effects of the instance I_{te} of problem (5) into an instance involving just intervals in objectives and constraints (denoted I). During the second phase, the instance I is solved using the I-NSGA-II algorithm in order to obtain an approximation of the RoI (denoted as F). Finally, the best portfolios are chosen among the solution of F (i.e., the portfolios that belong to the front zero F_0).

The I-NSGA-II algorithm, shown in Algorithm 1, is an adaptation of the most relevant strategies involved in its predecessor NSGA-II [47] (which are *fast-nondominated-sort* and *crowding-distance-sort*) using interval mathematics (see Algorithms 2 and 3). The I-NSGA-II algorithm focuses on the creation of nondominated fronts using interval mathematics. Each generation of I-NSGA-II the fronts are ordered, under the interval domain, based on the individual's nondominance and crowding distance (see Lines 4 and 11, respectively). The I-NSGA-II is supported by the *crowded-comparison-operator* (or *crowding operator*, denoted by $\prec_n$); this operator guides the selection process to achieve diversity on the optimal interval Pareto front. The *crowding operator* employs as interval comparison threshold the value 0.5. Finally, the I-NSGA-II uses the population generated through the

Input: Iter_{MAX}, N
Output: F_0 of the last iteration of the algorithm
Initialize: $P \longleftarrow$ **RandomPopulation**(N), $Q \longleftarrow \emptyset$
for $T = 1$ **to** Iter_{MAX} **do**
  $R_T \longleftarrow P_T \cup Q_T$
  $F \longleftarrow$ **interval-fast-non-dominated-sort**(R_T)
  $i \longleftarrow 0$, $P_{T+1} \longleftarrow \emptyset$
  while $|P_{T+1}| + |F_i| \leq N$
    interval-crowding-distance-assignment(F_i)
    $P_{T+1} \longleftarrow P_{T+1} \cup F_i$
    $i \longleftarrow i + 1$
  end while
  interval-crowding-distance-sort$(F_i, \prec_n)$ //ascending sorting by crowding distance
  $P_{T+1} \longleftarrow P_{T+1} \cup F_i[1 : N - |P_{T+1}|]$
  $Q_{T+1} \longleftarrow$ **make-new-population**(P_{T+1})
end for
return F_0

Algorithm 1: I-NSGA-II.

Input: A population P in which each individual is an interval vector representing a portfolio
Output: Individuals of P sorted in dominance fronts $\{F_0, F_1, \ldots\}$ according to their level of dominance
Inicialize: $S_P \longleftarrow \emptyset$, $n(p) = 0$, $H \longleftarrow \emptyset$, $i = 1$
for each $p \in P$ **do**
  $S_p \longleftarrow \emptyset$
  $n_p \longleftarrow 0$
  for each $q \in P$ **do**
    if $p \succ_\beta q$ **then**
      $S_p \longleftarrow S_p \cup \{q\}$
    else if $q \succ_\beta p$ **then**
      $n_p \longleftarrow n_p + 1$
  if $n_p = 0$ **then**
      $F_0 \longleftarrow F_0 \cup \{p\}$
$i = 0$
while $F_i \neq \emptyset$ **do**
  $F_{i+1} \longleftarrow \emptyset$
  for each $p \in F_i$ **do**
      for each $q \in S_P$ **do**
    $n_q \longleftarrow n_q - 1$
    if $n_q = 0$ **then**
      $F_{i+1} \longleftarrow F_{i+1} \cup \{q\}$
$i \longleftarrow i + 1$
return $\{F_0, F_1, \ldots\}$

Algorithm 2: Interval-fast-nondominated-sort(P).

Input: Non-dominated set
Output: None. The crowding distance value is assigned to each member in I in this method
Inicializar: $l \longleftarrow |I|$
for each i, **set** $I[i]_{distancia}$=0
for each objective m **do**
  $I \longleftarrow$ **sort**(I, m)
  $I[1] = I[l] = \infty$
  for $i = 2$ **to** $l - 1$ **do**
    $I[i]_{distance} = I[i]_{distance} + (\text{I[i + 1].m–I[i–1].m})/(f_m^{max} - f_m^{min})$

Algorithm 3: Interval-crowding-distance-assignment(I).

previously described orderings (see Line 12) to create the set of individuals that will form part of the new population for the next generation (see Line 13). The I-NSGA-II returns the best front obtained in the last generation (see Line 15).

Algorithm 2 is the pseudocode of the method *interval-fast-nondominated-sort*; this algorithm was former proposed in [48], where it was explained in detail. The I-NSGA-II algorithms uses this method to rank the population P_T each iteration T and to create the interval nondominated fronts.

Algorithm 3 is the pseudocode of the method *interval-crowding-distance-assignment*. The I-NSGA-II algorithm uses an interval extension of the method presented in [47]. The operations over the objective values and the ordering are made through interval mathematics described in Appendix.

4. Study Case: Portfolio Selection of New Product Development (PSNPD)

In order to validate the proposed model defined in problem (6) and its solution using the algorithm I-NSGA-II, we address here the NPD case for a company involving the features described in Section 4.1. The experimental design developed to evaluate the case of study is described in Section 4.2, and the results and their analysis are presented in Section 4.3.

4.1. Optimization Model. Let us consider a problem as a particular case of project portfolio selection of new product development that is characterized by the following circumstances:

(1) The impact of the portfolio is described by N=2 objectives, which involve imperfect knowledge and are denoted by $\{\boldsymbol{f_1}, \boldsymbol{f_2}\}$.

(2) There are M candidate projects of New Product Development.

(3) The estimated impact of each project $j \in M$ over each objective $i \in N$ involves imperfect knowledge, and it is denoted by the interval function $\boldsymbol{f_{i,j}}$. The impact of a given portfolio a over the each objective i is obtained by a simple aggregation of the values of $\boldsymbol{f_{i,j}}$ for each $j \in a$.

(4) The *budget* is the single class of resource involved ($n_r = 1$) in this problem. The available budget, denoted by $P_{1.0}$, represents the estimated availability of such resource. Now, its estimated consumption for a given portfolio a, denoted by $R_{1,0}(a)$, is computed by adding the individual costs of each project, denoted $r_{1.j}$.

(5) There is imperfect knowledge related to the availability and the consumption of budget; such imperfections are represented by $\boldsymbol{P_{1.0}}$ and $\boldsymbol{R_1}(a)$, respectively. The resource consumption of the j-th project is imperfectly known and denoted by $\boldsymbol{r_{1,j}}$. The consumed budget $\boldsymbol{R_1}(a)$ is the total consumption of portfolio a (obtained by simple aggregation of the individual project costs $\boldsymbol{r_{1,j}}$ for each $j \in a$), and it must not exceed the available budget $\boldsymbol{P_{1.0}}$.

(6) All the projects has no precedence relations, i.e., $P_{a,j} = \varnothing$ for any project j of any portfolio a. Hence, the starting time of any project can be considered as $\boldsymbol{t_{sj}} = 0$.

(7) The j-th project is associated with the following time components:

 (a) $\boldsymbol{t_{comp,j}}$: the moment in which the competence of the product developed by the jth project becomes significant;

 (b) $\boldsymbol{t_{old,j}}$: the moment in which the product developed by the jth project can be considered old;

 (c) $\boldsymbol{t_{cj}}$: its estimated completion time.

(8) There are certain functions $\boldsymbol{f_{i,j}}$ that has a strong dependence on the time conditions $\{\boldsymbol{t_{comp,j}}, \boldsymbol{t_{old,j}}, \boldsymbol{t_{cj}}\}$, and their impact is defined according to (7), where ω and π are degradation coefficients defined by the management team of each project in collaboration with the marketing department of the enterprise.

$$\boldsymbol{f_{i,j}} = \begin{cases} \boldsymbol{f_{i,j}} & \text{si } Poss\left(\boldsymbol{t_{comp,j}} \geq \boldsymbol{t_{cj}}\right) \geq \delta \\ \pi_{i,j} \bullet \boldsymbol{f_{i,j}} & \text{si } Poss\left(\boldsymbol{t_{cj}} \geq \boldsymbol{t_{old,j}}\right) \geq 1 - \delta \\ \omega_{i,j} \bullet \boldsymbol{f_{i,j}} & \text{otherwise} \end{cases} \quad (7)$$

Equation (7) is a simplified model that reflects the impact on the i-th objective from the j-th project in dependence on its completion time. Particularly, those effects can be traduced as follows. The conclusion of a project when its competence has not yet appeared implies that it provides all the benefit for the objectives where it impacts. However, if a project is concluded after it has become obsolete, then all the benefits on objectives affected by this situation are reduced by a factor π. Finally, the apparition of the competence before the projects is ended affects its benefits by a factor $\omega > \pi$.

This case of study of the problem PPSP can be formally described by (8), which is an instance of problem (6). Note that a portfolio a is a binary vector $\langle a_1, a_2, \ldots, a_M \rangle$ where a_j=1 indicates the inclusion of the j-th project in the portfolio.

$$\begin{aligned} \text{Maximize}_a \quad & \left[\boldsymbol{f_1}(a), \boldsymbol{f_2}(a)\right] \\ \text{s.t.} \quad & R_{1,0}(a) \leq P_{1.0}, \\ & Poss\left(\boldsymbol{R}_1(a) \leq \boldsymbol{P}_1\right) \geq \gamma, \\ & Poss\left(\boldsymbol{t_{c\mathrm{j}}} \leq \boldsymbol{t_{old,\mathrm{j}}}\right) \geq \delta, \\ & \qquad \text{for all } j \text{ such that } a_j = 1 \end{aligned} \quad (8)$$

where γ and δ are thresholds related to the phrase "with sufficient likelihood".

The present work proposes the development of a method that solves problem (8) based on the definition of a (γ,δ,β)-feasible portfolio a, i.e., a portfolio that satisfies the time-related and resource constraints as defined previously (according to the conservatism levels γ, δ defined by the DM), and for a given dominance parameter β.

4.2. Experimental Design. This section describes the experiment conducted to evaluate the performance of the proposed I-NSGA-II as solver of problem (6) in the special case PSNPD described in Section 4.1. The organization is as follows: Section 4.2.1 presents the general information of the instances used for the experiments; Section 4.2.2 describes the different conservatism levels that were tested, and Section 4.2.3 details the experiments.

4.2.1. Instance Definition. According to the description in Section 4.1, Table 1 shows the particular instance with M=100, N=2, and budget $\boldsymbol{P}_{10} = [240000, 260000]$ used in the experiment. Let us recall that all the bold variables involve information with uncertainty expressed by intervals. For simplicity, the strong degradation factor π was set to zero; the weak degradation factor ω was set to 0.5.

4.2.2. Definition of Parameters γ, δ, and β. The level of conservatism is a crucial aspect concerning the way in which a DM faces imperfect knowledge on his/her addressed problem. Parameters γ, δ, and β are used here to model this aspect of the DM's subjectivity. As stated in Remark 9, these parameters refer to different portfolio features, and therefore, they have not necessarily equal values. The decision maker-decision analyst pair is in charge to set appropriate values of these conservatism levels. The concept of γ,δ-feasible portfolio is easier than non-β-dominance; hence, the value of γ and δ should be set firstly. Both parameters allude to certain constraints (on resources and time, respectively), so one could expect that their values may be not very different. In this experiment γ was set as 0.66, and δ as 0.66, 0.75, and 0.90. β is set as 0.5, which reflects a nonconservative attitude with respect to dominance.

4.2.3. Experimental Design Based on the Expected Value Method (EVM). This section describes the experiment conducted to evaluate the performance of the proposed I-NSGA-II as solver of problem (6) in the special case PSNPD described in Section 4.1. The experiment analyses the proposed model against the situation when the time-related imperfect knowledge are modelled by uniform probability distributions and the project performances are represented by their expected values.

Under uncertain conditions, risk-neutral DMs and other nonconservative DMs can make decisions based on average values. Risk-neutral DMs have linear utility functions; in risky events, their certainty equivalent equals the average value of the lottery. Also, the use of average values represents a simple way of handling uncertainty and imprecision; so, this strategy can be used by DMs that are unfamiliar with (or reject) an in-depth modelling of imperfect knowledge.

The use of expected values eliminates the uncertain time effects (initially provided as intervals) through the computation of the expected impact $\boldsymbol{f}_{\mathbf{i},\boldsymbol{j}}^*$ of each objective i in all the projects j. The new impact $\boldsymbol{f}_{\mathbf{i},\boldsymbol{j}}^*$ is calculated as follows:

(1) Generate a realization t_{cj}, $t_{comp,j}$ for each pair of intervals ($\boldsymbol{t}_{\boldsymbol{c},\boldsymbol{j}}$, $\boldsymbol{t}_{\boldsymbol{comp},\boldsymbol{j}}$), respectively, satisfying $Poss(\boldsymbol{t}_{\boldsymbol{comp},j} \geq \boldsymbol{t}_{\boldsymbol{c}j}) \geq 0.5$.

(2) Assuming a strong correlation, compute the value of a realization $t_{old,j}$ in $\boldsymbol{t}_{\boldsymbol{old},\boldsymbol{j}}$ as $t_{old,j} = \min\{lower(\boldsymbol{t}_{\boldsymbol{old},\boldsymbol{j}}) + (t_{comp,j} - lower(\boldsymbol{t}_{\boldsymbol{comp},\boldsymbol{j}})), \text{upper}(\boldsymbol{t}_{\boldsymbol{old},\boldsymbol{j}})\}$.

(3) Compute the new impact $\boldsymbol{f}_{\mathbf{i},\boldsymbol{j}\ \boldsymbol{new}}$ of each objective $\boldsymbol{f}_{\mathbf{i},\boldsymbol{j}}$ subject of degradation due to the current values of t_{cj}, $t_{\text{comp,j}}$, $t_{\text{old,j}}$.

(4) Accumulate in $\boldsymbol{f}_{\mathbf{i},\boldsymbol{j}\ \boldsymbol{new\text{-}acum}}$ the value of $\boldsymbol{f}_{\mathbf{i},\boldsymbol{j}\ \boldsymbol{new}}$.

(5) Repeat steps (1) to (4) from 1 until 1000.

(6) Compute $\boldsymbol{f}_{\mathbf{i},\boldsymbol{j}}^* = \mathbf{f}_{\mathbf{i},\boldsymbol{j}\ \boldsymbol{new\text{-}acum}}/1000$.

The average contribution of projects, for the instance shown in Table 1, is presented in Columns 2-3 in Table 2. The average contributions of the j-th project are considered as the contributions of the j-th project of a new instance, but in which the time-related effects have been already considered in the average process.

A risk-neutral decision maker may focus his/her attention on the average values provided by Table 2 and make his/her decisions based on those values. On this background, the next step consisted in using I-NSGA-II to solve the problem, derived from the definition of objectives impact due to time effects as shown in Table 2:

$$\begin{aligned} \text{Maximize}_a \quad & [\boldsymbol{f}_{\mathbf{1}}^*(a), \boldsymbol{f}_{\mathbf{2}}^*(a)] \\ \text{s.t.} \quad & R_{1,0}(a) \leq P_{1.0}, \\ & Poss(\boldsymbol{R}_1(a) \leq \boldsymbol{P}_1) \geq \gamma \end{aligned} \tag{9}$$

where $*$ denotes the information of the average projects in Table 3; the requirements are the same given in Table 1, second column.

Solving problem (9), after 30 I-NSGA-II runs, 88 nondominated solutions were obtained. The ideal solution is $f_1^{*\max} = [1410460, 1418960]$ and $f_2^{*\max} = [332195, 341496]$ and is formed by identifying the maximum impact value for each objective in the nondominated set of solutions using interval mathematics.

In order to identify the closest nondominated solution to the ideal, we used the mean point of the intervals as representative of the intervals that describe solutions in the objective space. Having computed the mean point for each solution provided by I-NSGA-II, the Euclidean distance was used to identify the closest one to the ideal (which was also defined by mean points). The closest solution in the objective space for EVM is shown in Table 3 and in the portfolio space (denoted as C_{EVM}) is shown in Table 8 along with the others corresponding to the different level of conservatism considered.

The portfolio built using EVM is justified in the sense that a DM that is neutral to the risk will choose it, while someone with more conservative attitude would feel more comfortable with a portfolio composed of less risky project. A risk-averse DM would feel a great regret for projects whose

Table 1: Description of projects and their time-related properties.

P_j	$r_{1,j}$	$f_{1,j}$	$f_{2,j}$	t_{cj}	$t_{comp,j}$	$t_{old,j}$
1	[6025, 6225]	[12405, 12705]	[9095, 9195]	[6, 10]	[6, 10]	[7, 10]
2	[4975, 5375]	[45390, 45590]	[5825, 6225]	[3, 6]	[5, 6]	[6, 10]
3	[5180, 5280]	[48910, 49110]	[7085, 7385]	[2, 4]	[2, 5]	[4, 6]
4	[8575, 8875]	[23480, 23680]	[8790, 9090]	[3, 6]	[4, 5]	[5, 9]
5	[7245, 7445]	[33225, 33325]	[8275, 8375]	[4, 8]	[4, 8]	[5, 9]
6	[6765, 7065]	[45190, 45290]	[9805, 9905]	[6, 9]	[8, 10]	[10, 11]
7	[6440, 6740]	[32895, 32995]	[8330, 8430]	[3, 5]	[1, 5]	[3, 7]
8	[5885, 6285]	[48770, 49070]	[9415, 9815]	[1, 4]	[1, 5]	[3, 4]
9	[5900, 6100]	[29290, 29690]	[8890, 9090]	[3, 5]	[3, 4]	[4, 5]
10	[6835, 7035]	[40235, 40435]	[9400, 9500]	[2, 3]	[1, 2]	[3, 4]
11	[7485, 7885]	[28800, 29000]	[9360, 9660]	[6, 9]	[6, 7]	[8, 10]
12	[5060, 5260]	[43415, 43615]	[8060, 8260]	[1, 4]	[1, 3]	[1, 4]
13	[7770, 8070]	[33875, 34075]	[8075, 8175]	[4, 8]	[5, 8]	[6, 10]
14	[5265, 5565]	[27110, 27310]	[5705, 5905]	[7, 8]	[8, 10]	[10, 14]
15	[8805, 8905]	[42030, 42230]	[5915, 6315]	[5, 8]	[5, 6]	[7, 8]
16	[5700, 5800]	[8830, 8930]	[6230, 6430]	[4, 7]	[3, 6]	[4, 8]
17	[6925, 7225]	[22730, 23030]	[9755, 9855]	[7, 8]	[7, 8]	[9, 13]
18	[8150, 8250]	[20295, 20395]	[9790, 9890]	[3, 7]	[1, 5]	[3, 6]
19	[5910, 6010]	[43380, 43480]	[8695, 8995]	[6, 9]	[5, 7]	[6, 7]
20	[9580, 9780]	[32950, 33250]	[6380, 6780]	[6, 10]	[7, 8]	[8, 9]
21	[5290, 5690]	[23555, 23855]	[9280, 9580]	[5, 9]	[7, 9]	[7, 10]
22	[8580, 8680]	[32800, 32900]	[8785, 8985]	[1, 3]	[1, 5]	[2, 5]
23	[8755, 8855]	[26410, 26610]	[6990, 7090]	[8, 10]	[9, 10]	[11, 15]
24	[9295, 9495]	[38920, 39120]	[5755, 5855]	[9, 10]	[7, 8]	[7, 10]
25	[9200, 9500]	[33730, 34030]	[8480, 8580]	[4, 8]	[6, 8]	[8, 10]
26	[8285, 8485]	[40585, 40985]	[6215, 6315]	[8, 9]	[9, 10]	[9, 12]
27	[8035, 8435]	[15850, 16250]	[9025, 9225]	[2, 5]	[3, 5]	[5, 8]
28	[8085, 8285]	[20310, 20510]	[9145, 9345]	[4, 6]	[2, 6]	[3, 6]
29	[7015, 7215]	[8360, 8760]	[8025, 8125]	[8, 9]	[7, 10]	[8, 10]
30	[7810, 8210]	[29440, 29840]	[8130, 8430]	[8, 10]	[8, 10]	[10, 13]
31	[6465, 6665]	[31890, 31990]	[6325, 6625]	[7, 10]	[6, 10]	[8, 9]
32	[5965, 6165]	[25105, 25505]	[6215, 6415]	[7, 9]	[9, 10]	[10, 12]
33	[7175, 7375]	[16220, 16420]	[6665, 6865]	[9, 10]	[9, 10]	[11, 13]
34	[5125, 5325]	[8970, 9170]	[8120, 8520]	[8, 10]	[9, 10]	[11, 14]
35	[8795, 9195]	[5955, 6355]	[9790, 9990]	[9, 10]	[9, 10]	[11, 15]
36	[5030, 5430]	[8480, 8580]	[9300, 9600]	[9, 10]	[9, 10]	[9, 13]
37	[8595, 8695]	[38560, 38760]	[7785, 8085]	[8, 9]	[9, 10]	[10, 13]
38	[7575, 7675]	[17840, 17940]	[9785, 10085]	[7, 8]	[6, 10]	[8, 9]
39	[5005, 5205]	[8820, 8920]	[8595, 8895]	[2, 6]	[2, 3]	[3, 6]
40	[9505, 9605]	[6450, 6750]	[8560, 8960]	[3, 5]	[3, 5]	[3, 7]
41	[9325, 9725]	[10395, 10695]	[9150, 9450]	[3, 7]	[3, 7]	[5, 8]
42	[8225, 8525]	[6785, 7185]	[6790, 6890]	[8, 9]	[6, 10]	[8, 9]
43	[6390, 6690]	[30905, 31005]	[6165, 6465]	[8, 10]	[9, 10]	[10, 15]
44	[6515, 6915]	[32585, 32785]	[7970, 8170]	[9, 10]	[9, 10]	[10, 13]
45	[9690, 9990]	[41190, 41590]	[9150, 9450]	[6, 9]	[7, 8]	[8, 9]
46	[9105, 9505]	[40875, 41075]	[5345, 5745]	[3, 7]	[4, 5]	[6, 7]
47	[8140, 8240]	[19905, 20105]	[9125, 9225]	[3, 5]	[3, 5]	[5, 6]
48	[7260, 7460]	[38110, 38210]	[9325, 9625]	[7, 9]	[8, 10]	[9, 12]

Table 1: Continued.

P_j	$r_{1,j}$	$f_{1,j}$	$f_{2,j}$	t_{cj}	$t_{comp,j}$	$t_{old,j}$
49	[5395, 5595]	[11585, 11785]	[8615, 8815]	[5, 7]	[5, 6]	[6, 10]
50	[4965, 5165]	[19200, 19300]	[5510, 5610]	[2, 5]	[2, 6]	[4, 7]
51	[9415, 9715]	[17995, 18095]	[8810, 9210]	[4, 7]	[4, 5]	[4, 7]
52	[6765, 6965]	[32745, 32845]	[8000, 8400]	[7, 9]	[5, 7]	[6, 10]
53	[6280, 6480]	[44090, 44290]	[7185, 7485]	[6, 9]	[4, 5]	[6, 10]
54	[9315, 9515]	[7160, 7260]	[7720, 8120]	[4, 7]	[3, 6]	[5, 9]
55	[6470, 6670]	[9480, 9780]	[8725, 9125]	[9, 10]	[9, 10]	[10, 13]
56	[9255, 9455]	[29750, 30150]	[9495, 9595]	[2, 3]	[3, 4]	[5, 9]
57	[8460, 8660]	[33665, 33765]	[9885, 9985]	[1, 4]	[1, 4]	[2, 4]
58	[5180, 5480]	[20825, 21125]	[9750, 10050]	[9, 10]	[9, 10]	[10, 14]
59	[6125, 6525]	[45230, 45330]	[6300, 6700]	[3, 7]	[3, 7]	[5, 6]
60	[6750, 7150]	[11890, 12190]	[8735, 8835]	[6, 8]	[8, 9]	[10, 11]
61	[9100, 9300]	[46750, 46850]	[5970, 6270]	[3, 4]	[4, 7]	[6, 8]
62	[4980, 5180]	[39395, 39795]	[9055, 9355]	[5, 9]	[5, 7]	[6, 7]
63	[6890, 7090]	[10690, 10990]	[5455, 5655]	[4, 8]	[3, 6]	[4, 8]
64	[8575, 8675]	[21910, 22010]	[7355, 7555]	[5, 9]	[3, 4]	[5, 7]
65	[4805, 5205]	[5795, 6095]	[5655, 5955]	[6, 10]	[5, 8]	[7, 10]
66	[7355, 7655]	[38940, 39240]	[5095, 5495]	[9, 10]	[8, 9]	[10, 14]
67	[8595, 8895]	[29980, 30180]	[6585, 6985]	[7, 10]	[5, 6]	[5, 9]
68	[5550, 5750]	[22725, 22925]	[9390, 9590]	[5, 7]	[3, 6]	[3, 7]
69	[6755, 6955]	[38570, 38870]	[8700, 8800]	[7, 8]	[6, 8]	[7, 9]
70	[7925, 8025]	[40810, 40910]	[5455, 5655]	[4, 8]	[5, 6]	[6, 8]
71	[4905, 5205]	[41835, 42235]	[8115, 8515]	[6, 9]	[5, 8]	[7, 8]
72	[8980, 9380]	[7000, 7300]	[7585, 7985]	[4, 8]	[5, 6]	[5, 8]
73	[6915, 7215]	[47955, 48355]	[9140, 9240]	[1, 3]	[1, 2]	[2, 4]
74	[5250, 5450]	[37840, 38140]	[5555, 5855]	[6, 9]	[6, 7]	[7, 10]
75	[5515, 5815]	[2040, 2440]	[6245, 6645]	[6, 9]	[6, 7]	[7, 9]
76	[8350, 8450]	[33115, 33315]	[9005, 9405]	[8, 10]	[7, 9]	[8, 10]
77	[5170, 5570]	[14525, 14725]	[7440, 7740]	[2, 5]	[2, 4]	[4, 5]
78	[7385, 7685]	[38735, 38835]	[7870, 8270]	[3, 4]	[1, 3]	[3, 7]
79	[7265, 7665]	[15595, 15695]	[9610, 9810]	[1, 3]	[1, 2]	[1, 5]
80	[8810, 9110]	[46160, 46560]	[9495, 9795]	[3, 4]	[1, 4]	[3, 6]
81	[5100, 5300]	[45055, 45255]	[8040, 8140]	[7, 9]	[6, 8]	[7, 8]
82	[9200, 9400]	[42020, 42220]	[6200, 6500]	[7, 10]	[5, 8]	[6, 10]
83	[8565, 8965]	[33690, 33890]	[7275, 7475]	[8, 10]	[8, 9]	[9, 12]
84	[9135, 9435]	[49390, 49690]	[8845, 9145]	[5, 9]	[5, 7]	[6, 10]
85	[5325, 5725]	[18500, 18900]	[7750, 7850]	[7, 10]	[7, 8]	[9, 12]
86	[8185, 8585]	[21385, 21685]	[8550, 8750]	[2, 6]	[1, 3]	[2, 3]
87	[4805, 5205]	[33835, 34135]	[8355, 8455]	[9, 10]	[8, 9]	[8, 10]
88	[9450, 9750]	[34160, 34560]	[6790, 6990]	[9, 10]	[7, 8]	[7, 9]
89	[7835, 8135]	[41500, 41800]	[8050, 8450]	[2, 6]	[1, 4]	[2, 6]
90	[9420, 9520]	[36365, 36665]	[7925, 8325]	[1, 4]	[1, 2]	[3, 7]
91	[6795, 7095]	[30200, 30600]	[7200, 7500]	[6, 10]	[7, 8]	[7, 10]
92	[7125, 7425]	[16720, 16820]	[7140, 7540]	[9, 10]	[8, 9]	[8, 10]
93	[5960, 6260]	[25645, 26045]	[6565, 6665]	[3, 7]	[1, 3]	[1, 5]
94	[9780, 9980]	[44425, 44725]	[5655, 5755]	[3, 6]	[1, 4]	[3, 5]
95	[8015, 8115]	[27105, 27505]	[7220, 7620]	[1, 5]	[1, 2]	[1, 5]
96	[8085, 8285]	[19625, 19825]	[8025, 8225]	[3, 7]	[1, 4]	[2, 6]
97	[8980, 9380]	[42100, 42500]	[8950, 9350]	[1, 3]	[1, 2]	[3, 5]
98	[6710, 6810]	[6430, 6830]	[9125, 9525]	[6, 7]	[5, 6]	[6, 7]
99	[7930, 8130]	[24165, 24265]	[8065, 8165]	[6, 7]	[4, 6]	[5, 8]
100	[7300, 7500]	[31670, 31770]	[6940, 7340]	[3, 7]	[1, 3]	[3, 6]

Table 2: Average project contributions.

Project Id	$f_{1,j}^*$	$f_{2,j}^*$
1	[6574, 6733]	[6639, 6712]
2	[35404, 35560]	[5825, 6225]
3	[35704, 35850]	[7085, 7385]
4	[11270, 11366]	[7911, 8181]
5	[15948, 15996]	[5792, 5862]
6	[40671, 40761]	[9805, 9905]
7	[6907, 6928]	[5997, 6069]
8	[27798, 27969]	[9226, 9618]
9	[7322, 7422]	[6400, 6544]
10	[0, 0]	[9400, 9500]
11	[5472, 5510]	[7488, 7728]
12	[12590, 12648]	[2337, 2395]
13	[19986, 20104]	[7025, 7112]
14	[27110, 27310]	[5705, 5905]
15	[7565, 7601]	[4732, 5052]
16	[1324, 1339]	[2554, 2636]
17	[12274, 12436]	[9755, 9855]
18	[2841, 2855]	[4699, 4747]
19	[3036, 3043]	[1739, 1799]
20	[11532, 11637]	[3955, 4203]
21	[17666, 17891]	[6960, 7185]
22	[24600, 24675]	[8345, 8535]
23	[20863, 21021]	[6990, 7090]
24	[0, 0]	[0, 0]
25	[26646, 26883]	[8480, 8580]
26	[40585, 40985]	[6215, 6315]
27	[10461, 10725]	[9025, 9225]
28	[5686, 5742]	[4663, 4765]
29	[4681, 4905]	[6741, 6825]
30	[11776, 11936]	[8130, 8430]
31	[11161, 11196]	[3605, 3776]
32	[25105, 25505]	[6215, 6415]
33	[9245, 9359]	[6665, 6865]
34	[6906, 7060]	[8120, 8520]
35	[2441, 2605]	[9790, 9990]
36	[5172, 5233]	[5673, 5856]
37	[38560, 38760]	[7785, 8085]
38	[12131, 12199]	[9785, 10085]
39	[1058, 1070]	[3609, 3735]
40	[3289, 3442]	[4365, 4569]
41	[5405, 5561]	[7777, 8032]
42	[2103, 2227]	[6043, 6132]
43	[25651, 25734]	[6165, 6465]
44	[15314, 15408]	[7970, 8170]
45	[21830, 22042]	[7503, 7749]
46	[12671, 12733]	[4436, 4768]
47	[10151, 10253]	[9125, 9225]
48	[35442, 35535]	[9325, 9625]
49	[3012, 3064]	[6633, 6787]
50	[11712, 11773]	[5344, 5441]

Table 2: Continued.

Project Id	$f_{1,j}^*$	$f_{2,j}^*$
51	[3778, 3799]	[1850, 1934]
52	[0, 0]	[960, 1008]
53	[0, 0]	[1437, 1497]
54	[1503, 1524]	[6098, 6414]
55	[4076, 4205]	[8725, 9125]
56	[29750, 30150]	[9495, 9595]
57	[15485, 15531]	[7117, 7189]
58	[10620, 10773]	[9750, 10050]
59	[21258, 21305]	[4977, 5293]
60	[11890, 12190]	[8735, 8835]
61	[46750, 46850]	[5970, 6270]
62	[10242, 10346]	[3803, 3929]
63	[1389, 1428]	[1581, 1639]
64	[0, 0]	[1103, 1133]
65	[753, 792]	[3279, 3453]
66	[0, 0]	[5095, 5495]
67	[0, 0]	[0, 0]
68	[2045, 2063]	[845, 863]
69	[8871, 8940]	[7047, 7128]
70	[11834, 11863]	[3273, 3393]
71	[10877, 10981]	[5274, 5534]
72	[2660, 2774]	[2882, 3034]
73	[12468, 12572]	[7037, 7114]
74	[5676, 5721]	[2721, 2868]
75	[367, 439]	[3622, 3854]
76	[6291, 6329]	[3962, 4138]
77	[5229, 5301]	[6993, 7275]
78	[0, 0]	[5666, 5954]
79	[3742, 3766]	[2306, 2354]
80	[5077, 5121]	[7501, 7738]
81	[3153, 3167]	[2733, 2767]
82	[3781, 3799]	[1302, 1365]
83	[7411, 7455]	[5892, 6054]
84	[9384, 9441]	[3980, 4115]
85	[2775, 2835]	[6432, 6515]
86	[1496, 1517]	[1453, 1487]
87	[0, 0]	[0, 0]
88	[0, 0]	[0, 0]
89	[8300, 8360]	[3783, 3971]
90	[5818, 5866]	[6498, 6826]
91	[12382, 12546]	[2952, 3075]
92	[0, 0]	[0, 0]
93	[0, 0]	[0, 0]
94	[2221, 2236]	[2657, 2704]
95	[3794, 3850]	[1010, 1066]
96	[196, 198]	[882, 904]
97	[10946, 11050]	[8950, 9350]
98	[0, 0]	[4380, 4572]
99	[0, 0]	[1935, 1959]
100	[0, 0]	[2220, 2348]

Table 3: The closest solution C_{EVM} to the ideal one (in the portfolio space).

Resource consumption		$f_{1,j}^*$		$f_{2,j}^*$		Cardinality
240265	250165	1384860	1393360	298740	308040	37

Table 4: Sets of solutions with different degree of conservatism.

Poss(C≤B) ≥ γ	Poss($t_{old,j} \geq t_{c,j}$) ≥ $\boldsymbol{\delta}$	Cardinality of the Non-Dominated Solution Set	Average number of projects
0.66	0.66	61	35.1
	0.75	30	33.8
	0.90	1	28.0

Table 5: Solution corresponding to δ =0.9 in the objective space.

Resource consumption		$f_{1,j}$		$f_{2,j}$		Cardinality
202655	209855	809470	816570	224080	230580	28

Table 6: Closest solution to the ideal one in the objective space (δ =0.66).

Cost		$f_{1,j}$		$f_{2,j}$		Cardinality
239880	249680	988032	995627	256481	263721	36

Table 7: Closest solution to the ideal one in the objective space (δ =0.75).

Cost		$f_{1,j}$		$f_{2,j}$		Cardinality
240770	249870	866389	872789	224375	231115	33

results were obsolete. Since most decision makers are risk-averse, a method to control regret for delayed products should be welcome.

4.3. Results Obtained by Our Proposal. After 30 runs of I-NSGA-II solving problem (8), with different levels of conservatism related to the time-constraints, we obtained the results given in Table 4.

The portfolios obtained with δ = 0.66 contain more projects, but these are riskier. A significant part of the supported projects will not give benefits or only will give a portion of them.

The single solution (in the objective space) corresponding to δ = 0.9 is provided by Table 5. Tables 6 and 7 show the closest solution (also in the objective space) to the ideal one in Euclidean sense for δ = 0.66 and δ = 0.75, respectively. In the portfolio space, these solutions, denoted by $C_{0.9}$, $C_{0.66}$, $C_{0.75}$, respectively, are shown in Table 8.

In order to compare the solutions obtained by I-NSGA-II and the one obtained using the EVM, the best portfolios were compared in terms of the number of failure projects and wasted budget. For this purpose, the time effects derived from $\boldsymbol{t_{c,j}}$, $\boldsymbol{t_{old,j}}$, and $\boldsymbol{t_{comp,j}}$ were simulated using a sample of 1000 possible outcomes of their impact. Each outcome is calculated from realizations $t_{c,j}$, $t_{comp,j}$, and $t_{old,j}$. The first ones are randomly chosen from the intervals $\boldsymbol{t_{c,j}}$ and $\boldsymbol{t_{comp,j}}$ (the values in the range are considered uniformly distributed); the realization $t_{old,j}$ is calculated assuming a strong correlation with $t_{comp,j}$, taking $t_{old,j}$=min{*lower*($\boldsymbol{t_{old,j}}$) + ($t_{comp,j}$ - *lower*($\boldsymbol{t_{comp,j}}$)), upper($\boldsymbol{t_{old,j}}$)}.

Now, the performance of a particular portfolio p is measured in terms of the number of *successful projects*, *partial failure projects*, *complete failure projects* and the *wasted budget* derived from it under the following rules: (1) if $t_{c,j} < t_{old,j}$ and $t_{c,j} \geq t_{comp,j}$ then a project in p is considered a *partial failure* because the project was finished after the competence appear in the market and part of its impact is degraded; (2) if $t_{c,j} \geq t_{old,j}$ then a project in p is considered a *complete failure* because its relevance disappeared even before it could be finished; and (3) when $t_{c,j} < t_{old,j}$, the j-th project is considered successful and its budget is count as well-invested. Table 9 presents a summary of the values for *successful*, *partial failure*, and *complete failure projects*, derived from the portfolios created by I-NSGA-II and EVM; Table 10 presents the information related to the budget that is *well-invested*, *partial wasted*, or *complete wasted*.

Based on the results presented in Tables 9 and 10, we can conclude that the results are in agreement with an increment of the level of conservatism. This is mainly because the probability of having failed projects is reduced 60%, and more importantly, to 0 when it refers to complete failures. Wasted budget can be strongly reduced. A risk-averse decision maker should prefer $C_{0.90}$. Note that portfolio $C_{0.90}$ does not use the whole available budget because the optimization process did not find sufficient nonrisky projects; hence, the portfolio is very robust with respect to the imperfect information of resources availability and consumption. This portfolio has no budget invested in projects that end in a complete fail.

Concerning the question about the appropriate value of δ, we note that the solution with δ = 0.75 is not more

TABLE 8: Closest solutions (in the portfolio space) to the ideal ones.

Project Id	C_{EVM}	$C_{0.66}$	$C_{0.75}$	$C_{0.90}$
1	0	1	0	0
2	1	1	1	1
3	1	0	0	1
4	0	0	0	0
5	1	1	0	0
6	1	0	0	1
7	1	1	1	0
8	1	0	0	0
9	1	0	0	0
10	1	0	0	1
11	1	0	0	0
12	1	0	0	0
13	1	1	1	0
14	1	1	1	1
15	0	1	0	0
16	0	1	1	0
17	0	1	0	1
18	0	1	0	0
19	1	0	0	0
20	0	0	0	0
21	1	0	0	0
22	0	0	0	0
23	0	0	1	1
24	0	1	1	0
25	0	1	1	1
26	0	0	1	1
27	0	0	0	1
28	0	0	0	0
29	0	0	0	0
30	0	0	0	1
31	1	1	1	0
32	0	1	1	1
33	0	1	1	1
34	0	0	0	1
35	0	0	0	1
36	0	1	1	0
37	0	0	0	1
38	0	0	0	1
39	0	1	0	0
40	0	0	0	0
41	0	0	0	0
42	0	1	1	0
43	1	1	1	1
44	1	1	0	1
45	0	0	0	0
46	0	1	1	0
47	0	0	0	1
48	1	0	0	1
49	0	0	0	0
50	0	1	1	0
51	0	0	0	0
52	1	0	0	0
53	1	1	0	0
54	0	0	0	0
55	0	0	0	1
56	0	0	0	1
57	1	0	0	0
58	1	0	0	1
59	1	1	0	0
60	0	0	0	1
61	0	0	1	1
62	1	0	0	0
63	0	1	1	0
64	0	0	0	0
65	0	1	1	0
66	1	0	1	1
67	0	0	1	0
68	1	0	0	0
69	1	0	0	0
70	1	1	1	0
71	1	0	0	0
72	0	0	1	0
73	1	0	0	0
74	1	1	0	0
75	0	0	0	0
76	0	0	0	0
77	0	1	1	0
78	0	0	0	0
79	0	1	1	0
80	1	0	0	0
81	1	1	0	0
82	0	0	1	0
83	0	1	1	0
84	1	0	0	0
85	1	1	1	0
86	0	0	0	0
87	0	1	0	0
88	0	0	1	0
89	1	0	0	0
90	0	0	0	0
91	0	0	0	0
92	0	0	1	0
93	0	1	0	0
94	0	1	0	0
95	0	0	0	0
96	0	0	1	0
97	1	0	0	1
98	0	0	0	0
99	0	0	1	0
100	0	0	0	0

TABLE 8: Continued.

Note: "1" means that the related project belongs to the portfolio.

Table 9: Average performance of the portfolios generated by I-NSGA II and EVM.

Portfolio	Cardinality	Successful projects	Partial Failure projects	Complete Failure projects
C_{EVM}	37	13.50	12.76	10.74
$C_{0.66}$	36	13.02	10.67	12.31
$C_{0.75}$	33	13.05	8.31	11.65
$C_{0.9}$	28	18.82	9.18	0

Table 10: Budget-related performance of the portfolios generated by I-NSGA II and EVM.

Portfolio	Partial Wasted Budget	Complete Wasted Budget	Budget Well Invested	Total Budget
C_{EVM}	[86264, 89869]	[68106, 70547]	[85896, 89749]	[240265, 250165]
$C_{0.66}$	[73323, 76245]	[81474, 84630]	[85083, 88805]	[239880, 249680]
$C_{0.75}$	[57921, 60356]	[90701, 93869]	[92148, 95645]	[240770, 249870]
$C_{0.9}$	[66374, 68875]	[0, 0]	[136281, 140980]	[202655, 209855]

conservative than the risk-neutrality defined by EVM. It seems that risk-averse decision makers should set values close to 0.9. This should be confirmed by more experimentation.

5. Conclusions

In manufacturing enterprises, during its last phase, an applied research project becomes NPD project, oriented to market and whose results are strongly influenced by time-related effects, mainly concerning competitors and technological changes. These time-dependencies are usually neglected by the project portfolio optimization literature, but they must be taken into account for NPD project evaluation.

The research presented in this work involves a solution method for the portfolio selection problem of new product developments (NPD) under uncertainty and imprecision, considering effects related to time-interdependence among different projects and imperfect knowledge on completion times. The three important characteristics of NPD projects included in the developed model are (1) uncertain market payoffs that change over time; (2) strong dependence of benefits on imperfectly known project completion times, technological innovations, potential competitor products, and their interactions; and (3) probable precedence of applied research projects. Particularly, time effects are the completion times of projects, the moment when the competence becomes relevant, and the moment in which a project is outdated and possibly no longer of interest, which are the considered time-related effects; all of them have impact on one or more objectives that characterize the project performances, degrading their values, and hence, making it important to study the handling of risk, particularly in case of risk-averse decision makers.

The proposed optimization model integrates in its design the following novel characteristics: (1) management of imperfect knowledge through interval mathematic; (2) incorporation of the conservatism level of a decision maker in the optimization process; and (3) a set of parameters that allows the adjustment of the conservatism level, at least in the management of the budget and the constraints derived from time-related effects. The multiobjective optimization problem is solved by the interval-based evolutionary algorithm I-NSGAII, which could approximate the Pareto frontier in the interval domain. The use of this method inherits the limitations of NSGA-II; i.e., it works well only with 2 or 3 objectives; in cases where the portfolio is described by N > 3 objectives a different algorithm of solution would be required, e.g., as the one published in [26].

To be operational, the proposal requires a significant volume of information, although not necessarily precise: (a) mathematical models of the way in which the impact of the projects depend on time; (b) time-related interdependencies among projects; (c) other interdependencies among projects; (d) three parameters describing different aspects concerning the DM's conservatism facing uncertainty and imprecision. It is important to underline that the DM is not along in this task. The DM-decision analyst pair should be supported by experts, project manager teams, and marketing departments. Fortunately, in large manufacturing enterprise each NPD project is managed and supported by an expert team. This team, working closely to the marketing department, is in charge of collecting the necessary information. Several recent papers insist in involving many stakeholders in the early stages of NPD processes to get information about customer's preferences, technological changes, and competitors (e.g., [43, 44]). The DM is required only to reflect his/her attitude facing the risk produced by imperfect knowledge and to make decisions about the final portfolio.

The adoption of our proposal would imply challenges and new tasks to marketing departments and supporting teams of NPD projects. In this sense, our approach requires some organizational changes to be fully operational.

Handling the imprecision and uncertainty unavoidable in such volume of information is a crucial point. As stated by Fernández et al. [23], to express the values of parameters and estimations as intervals is a much easier and a more manageable task than setting precise values. Such interval-based models can also better satisfying group of experts with conflicting opinions, because they can agree easier on specifying a range of values from their opinions rather than a single one.

The proposal was tested through an experimental design that had the purpose of comparing the results achieved by the new method against the expected value based portfolio.

This strategy computed the average of a sample of possible outcomes about how a portfolio might end after considering the time-related effects. The results revealed that high levels of conservatism might prevent wasting resources in failed projects, i.e., projects that if were chosen as part of a portfolio, their impact will reduce drastically due to the time-related effects. The results show that, adjusting the parameter that characterizes the time-related conservatism, the decision maker can control, reduce, and even eliminate his/her regret produced by the budget wasted in failed projects. The example seems to suggest that time-related conservative portfolios are associated with resource-related conservative portfolios; this should be confirmed by extensive tests.

Appendix

The concept of interval number was originated in the so-called interval analysis theory (Moore, 1962). Such a number represents a numerical quantity whose exact value is unknown. Moore (1962) describes an interval number in terms of a range, $\boldsymbol{E} = [\underline{E}, \overline{E}]$, where $\underline{E}$ represents the lower limit and $\overline{E}$ the upper limit of the interval number. A real number is a particular case of interval numbers when $\underline{E} = \overline{E}$.

Let $\boldsymbol{D} = [\underline{D}, \overline{D}]$ and $\boldsymbol{E} = [\underline{E}, \overline{E}]$ be interval numbers. Basic arithmetic operations can be defined for interval numbers as follows:

$$\begin{aligned} \boldsymbol{D} + \boldsymbol{E} &= \left[\underline{D} + \underline{E}, \overline{D} + \overline{E}\right], \\ \boldsymbol{D} - \boldsymbol{E} &= \left[\underline{D} - \overline{E}, \overline{D} - \underline{E}\right], \\ \boldsymbol{D} * \boldsymbol{E} &= \left[\min\left\{\underline{D}\,\underline{E}, \underline{D}\overline{E}, \overline{D}\underline{E}, \overline{D}\,\overline{E}\right\},\right. \\ &\quad \left.\max\left\{\underline{D}\,\underline{E}, \underline{D}\overline{E}, \overline{D}\underline{E}, \overline{D}\,\overline{E}\right\}\right], \\ \frac{\boldsymbol{D}}{\boldsymbol{E}} &= \left[\underline{D}, \overline{D}\right] * \left[\frac{1}{\overline{E}}, \frac{1}{\underline{E}}\right]. \end{aligned} \tag{A.1}$$

Yao et al. (2011) introduced certain order relation rules over interval numbers. This order relation rests on a possibility measure of $\boldsymbol{E} \geq \boldsymbol{D}$. It is easy to prove that this possibility measure is equivalent to the following equation:

$$Poss\left(\boldsymbol{E} \geq \boldsymbol{D}\right) = \begin{cases} 1 & if\ p_{ED} > 1, \\ p_{ED} & if\ 0 \leq p_{ED} \leq 1, \\ 0 & if\ p_{ED} \leq 0, \end{cases} \tag{A.2}$$

where $p_{ED} = (\overline{E} - \underline{D})/((\overline{E} - \underline{E}) + (\overline{D} - \underline{D}))$.

If $\boldsymbol{E}$ and $\boldsymbol{D}$ are real numbers $[e, e]$ and $[d, d]$, respectively, then

$$Poss\left(\boldsymbol{E} \geq \boldsymbol{D}\right) = \begin{cases} 1 & if\ e \geq d \\ 0 & otherwise \end{cases} \tag{A.3}$$

The order relation between two interval numbers is defined as follows:

(i) If $\underline{D} = \underline{E}$ and $\overline{D} = \overline{E}$, this means that $\boldsymbol{D}$ is equal to $\boldsymbol{E}$ ($\boldsymbol{D} = \boldsymbol{E}$). Then, $P(\boldsymbol{D} \leq \boldsymbol{E}) = 0.5$.

(ii) If $\underline{E} > \overline{D}$, this means that $\boldsymbol{E}$ is greater than $\boldsymbol{D}$ ($\boldsymbol{E} > \boldsymbol{D}$). Then, $P(\boldsymbol{D} \leq \boldsymbol{E}) = 1$.

(iii) If $\overline{E} < \underline{D}$, this means that $\boldsymbol{E}$ is smaller than $\boldsymbol{D}$ ($\boldsymbol{E} < \boldsymbol{D}$). Then, $P(\boldsymbol{D} \leq \boldsymbol{E}) = 0$.

(iv) If $\underline{D} \leq \underline{E} \leq \overline{D} \leq \overline{E}$ or $\underline{D} \leq \underline{E} \leq \overline{E} \leq \overline{D}$, when $P(\boldsymbol{D} \leq \boldsymbol{E}) > 0.5$, this means that $\boldsymbol{E}$ is greater than $\boldsymbol{D}$($\boldsymbol{E} > \boldsymbol{D}$). When $P(\boldsymbol{D} \leq \boldsymbol{E}) < 0.5$, this means that $\boldsymbol{E}$ is smaller than $\boldsymbol{D}$ ($\boldsymbol{E} < \boldsymbol{D}$).

A real number e within the interval $[\underline{E}, \overline{E}]$ is said to be a *realization* of the interval number $\boldsymbol{E}$. $P(\boldsymbol{D} \leq \boldsymbol{E}) = \alpha$ is interpreted as a likelihood degree of the statement "once both realizations are determined, e will be greater than or equal to d".

Data Availability

The experimental design data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Research Article

A Group Decision-Making Approach Based on DST and AHP for New Product Selection under Epistemic Uncertainty

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Selecting the most appropriate new product(s) is regarded as a critical decision which greatly influences the development of manufacturing enterprises. In order to improve the accuracy of selection, more experts are required to be invited to predict key indicators for new products selection. Due to limited knowledge, experts use fuzzy numbers more confidently than using numerical values in the prediction. Therefore, new product selection is a multiattribute group decision-making process under epistemic uncertainty. The purpose of this paper is to introduce a new hybrid decision-making approach based on Analytic Hierarchy Process (AHP) and Dempster-Shafer Theory (DST) to evaluate and select a new product. AHP and DST are used in weight determination to improve the accuracy and objectivity. In addition, this paper proposes that DST is a proper mathematical framework to deal with the epistemic uncertainty on the indicators of new product scheme selection. In particular, the initial assessments from experts are disassembled and then combined into the evidence information. By setting confidence degree, reliability function and likelihood function are used to evaluate and rank new products. A case study in a home appliance manufacturer is provided to illustrate the proposed hybrid approach and demonstrate its applicability.

1. Introduction

As the market is increasingly competitive, it is more important for enterprises to choose an appropriate new product. Good new products can bring hope to dying enterprises. On the contrary, bad schemes make prosperous enterprises fail. There are eight phases typically in new product development (NPD), namely, idea generation, idea screening, concept development and testing, marketing strategy development, business analysis, product development, market testing, and commercialization. Idea screening, also known as new product selection, is choosing the best one from a number of schemes, which is the most important task for NPD. Therefore, the research on new product selection is of vital significance.

It is difficult to choose an optimal scheme from many designs. The main reasons are as follows. First of all, there are many factors to be considered while making decision, such as product performance, market potential, project risk, and customer demand. Secondly, because decision making is for the future, the impact factors are full of uncertainty. That is why accurate number is difficult to use to represent factors. Last but not least, these factors are estimated by experts on the basis of their past experience and personal preferences, which leads to different evaluations by different experts. If only one expert is invited for evaluation, the results would be influenced by his own experience and preferences. If more than one expert is invited, different estimations would be obtained. How to deal with these differences is an important problem to be solved. Therefore, new product selection is a problem worthy for studying.

Based on the first point, it is very necessary to select several indicators to help the researchers make decisions. Therefore, the study is a multiple criteria decision-making (MCDM) process. Based on the second point, fuzzy number is advised to express indicators which are difficult to express by accurate number. And a fuzzy set should be used in the study of new product selection. In order to eliminate the impact of individual preferences and experience, a number of experts would be invited to estimate indicators. Thus, a conclusion can be drawn that it is a group decision-making problem. That is to say, new product selection is a fuzzy multiattribute group decision-making model.

There are three key problems to deal with in this issue. The first one is how to judge the relationship between those indicators. That is to say, which indicators are more important and which can be slightly sacrificed. Therefore, weight determination is the first key point in the model. The second question is how to calculate and judge the attributes expressed by fuzzy numbers, so as to carry out comprehensive evaluation. The last one is how to deal with these differences on indicators estimated by various experts and to get attribute values more accurately. In a word, it is uneasy to select an optimal scheme from multiple designs.

The purpose of this paper is to present a group decision-making approach based on Dempster-Shafer Theory (DST) and Analytic Hierarchy Process (AHP) for new product selection under epistemic uncertainty. Firstly, many experts are asked to grade indexes according to scale. And AHP is applied to calculate attribute weights. Due to different experiences and preferences, weights given by different experts are various. And DST is used for the final weight to eliminate the difference. In addition, DST is used as a proper mathematical framework to deal with the epistemic uncertainty. By building reliability function and likelihood function, comprehensive evaluation of new scheme is carried out. Finally, the validity of this method is verified by a case study. The structure of this paper is as follows: the second part is related work; the third part is an introduction to Dempster-Shafer Theory; the fourth section is the establishment of the evaluation model, and the fifth section is case study.

2. Related Work

NPD is the foundation for enterprises to survive and develop. Therefore, a key task for top management is to choose appropriate new products. When evaluating product plans, top management should take many factors into consideration and prediction, such as market trend, product market competitiveness, product cost, and technical requirements. The work of new product evaluation is very important and complex for enterprises. It is also an important topic for researchers. Therefore, many researchers have proposed different tools or methods to evaluate new product schemes.

(i) Risk assessment: Katie et al. [1] proposed a risk identification framework from system perspectives, which can be used to assess risks quantitatively of new products. Patil et al. [2] predicted business risks at the beginning of new product design and established a risk framework. Fang and Marle [3] designed a way to identify inherent risks and cross risks of projects according to the direction of risk communication. Mohit [4] designed a risk network corresponding to different functional organizations based on Bayesian network method to measure risks of all new projects.

(ii) Key factors: Kim et al. [5] summarized key factors in NPD by studying cases of developing new products in virtual teams of SME. Cooper et al. [6] identified the critical factors that set the most successful firms apart from their competitors by benchmarking at the company or macro level, to ensure that resources are allocated appropriately. Lam et al. [7] used AHP to study the most important 13 factors in the process of new product development from the perspective of conflict management. Cunha [8] studied the key success factors by summarizing the empirical and conceptual studies of NPD in the past 30 years.

(iii) Quality function deployment (QFD): Lowe et al. [9] developed a new product evaluation tool based on QFD to evaluate potential products for an innovative metal forming process. Hanumaiah et al. [10] built a rapid hard tooling process selection though QFD and AHP methodology. Lee [11] selected the critical factors though Fuzzy Delphi method and constructed houses of quality for QFD in NPD. Zhang Xuefeng [12] proposed an integrated approach based on QFD and DEA to ensure user collaboration which has been recognized as a critical factor in successful product development. Yu L, Wang L, and Bao Y [13] proposed a new product evaluation approach based on QFD integrated IVIF and CI.

(iv) Artificial neural network: Thieme et al. [14] evaluated and chose the new plan by constructing the neural network model. HO and Tsai [15] proposed a new method based on structural equation model (SEM) and adaptive neural fuzzy inference system (ANFIS) to predict the effect of value process quality on the performance of NPD.

(v) AHP/ANP and TOPSIS: Lin et al. [16] constructed a method based on AHP and TOPSIS to identify customer requirements and design features, so as to evaluate design schemes. Shyur et al. [17] used ANP to determine evaluation criteria and weights and used the improved TOPSIS to arrange alternative designs. Chiuh et al. [18] presented a fuzzy analytic network process (FANP) for solving the product selection. Akkaya et al. [19] constructed a new method based on fuzzy AHP and fuzzy MOORA for industrial engineering sector choosing problem. Joshi and Kumar [20] held that, in the process of group decision-making, the interaction of decision criteria and concept uncertainty has a great influence on the membership function.

(vi) Fuzzy set: When hen solving the optimal new product plan, some attributes are hard to be expressed numerically. Many scholars have adopted fuzzy sets, for example, Carrera Diego A et al. [21] and Chen H. H et al. [22]. On the basis of considering the favorable and unfavorable factors, Lin and Yang [23] calculated the fuzzy attractiveness and determined portfolio selection problems. Kit et al. [24] proposed a forward selection based on fuzzy regression (FS-FR) to correlate engineering characteristics with consumer preferences regarding a new product. Büyüközkan G et al. [25] constructed a multicriteria group decision-making approach based on intuitionist fuzzy TOPSIS for smart phone selection. Kumru et al. [26] proposed a new hybrid approach for multicriteria decision-making problems through combining intuitionist fuzzy analytic hierarchy process and intuitionist fuzzy multiobjective optimization by ratio analysis.

(vii) Group decision making: Ren et al. [27] used fuzzy information and group decision analysis method to construct an extension method of fuzzy measurement, so as to solve the problem of evaluation and selection. Ebrahimnejad S. et al. [28] introduced a new hierarchical multi criteria group decision-making method to solve the optimal selection of new product design alternatives. What is more, it proposed sets to evaluate and analyze new products. Lo C. C. et al. [29] and Chung et al. [30] used both group decision-making and fuzzy number applied an axiomatic design method using fuzzy linguistic to make multiattribute group decision. Tuzkaya [31] suggested that IFCI operators should adopt MCDM method, take advantage of the interaction between fuzziness of decision environment and decision criteria, and combine supplier evaluation processes. Gülçin Büyüközkan et al. [32] presented a creative approach to evaluate the smart medical device selection process in a group decision-making with intuitionistic fuzzy set based on fuzzy choquet integral. Mousavi et al. [33] adopted a hierarchical group decision-making approach to new product selection based on VIKOR.

(viii) Other methods: Sarı et al. [34] used fuzzy Monte Carlo simulation to evaluate new product investment plans, so as to select the best. Huanghong et al. [35] adopted computational intelligence techniques for New Product Design. Wang wenpei [36] presented a 2-tuple fuzzy linguistic computing approach to deal with heterogeneous information and information loss problems during the processes of subjective evaluation integration in NDP. Gülçin et al. [37] identified the decision points in the NPD process and the uncertainty factors affecting those points and proposed an integrated approach based on fuzzy logic to shape the decisions.

Although many scholars put forward different methods for new product scheme selection from different perspectives, there are some external conditions which are not taken into consideration in these methods. Firstly, the majority of scholars hold the view that new product scheme selection is a multiattribute decision-making (MADM) problem. And many MADM methods are used for new product selection. The MAMD methods differ in different fields. Some MADM methods are used to solve one specific problem and they are not suitable for other problems. These MADM methods can be divided into two categories. One is compensatory methods, such as AHP, TOPSIS, and VIKOR. The other is noncompensatory methods, such as Dominance, Max-min, Conjunctive-Satisfying, and Elimination methods. When choosing a new product, some attributes considered are cross-cutting. So, it is more appropriate to use compensatory MADM approach. Each compensatory method has its advantages and disadvantages. Compared with TOPSIS and VIKOR, AHP is more suitable for determining index weight in NPD. This is found in many literatures. Attribute weights is very important for the final scheme selection. It may be inaccurate to judge the importance of each indicator only by personal knowledge and experience. Inviting multiple experts to evaluate indicators weights and dealing with different weights are seldom mentioned in previous literatures. So, in this paper, an approach based on DST and AHP is proposed to solve this problem.

In addition, the attribute values of new products are estimated for future, which makes it difficult for experts to express the attributes of the new scheme numerically relying on limited knowledge and experience. Experts use fuzzy number more confidently compared to using specific value to represent the attributes of new schemes. In order to improve the accuracy of prediction, some scholars use hesitant fuzzy sets. However, new products from R&D to production and sales involve many links, and it is difficult for an expert to grasp all the information. More experts should be invited to evaluate the future performance of new products in order to improve the accuracy of the estimations. However, the research on this aspect has hardly been mentioned in previous papers. The fact that various experts would have different knowledge leads to different results given by different experts to the same attribute of the same design. In general, the best choice from new product plans is a fuzzy multicriteria group decision-making problem and an evaluation approach based on DST is proposed.

3. Dempster-Shafer Theory

DST was put forward by Professor P. Dempster of Harvard University. Glenn Shafer, a student of Professor P. Dempster, has further developed evidence theory and constructed a mathematical method for calculating uncertainty. This method is mainly used in information fusion, expert system, intelligence analysis, legal case analysis, and multiattribute decision-making. This method (Certa et al. [38]; Shafer [39]; Li dawei et al. [40]) is based on three different measures, namely, Basic Probability Assignment (BPA), Belief Measure (BEL), and Plausibility Measure (PL).

Definition 1 (basic probability assignment (BPA)). Suppose U is the frame of discernment. BPA on frame of discernment is a function m of $2^{U} \longrightarrow [0,1]$. On BPAs the following assumptions hold:

$$m(\emptyset) = 1 \quad (1)$$

$$\sum_{A \subseteq U} m(A) = 1 \quad (2)$$

$$m(A) > 0 \quad (3)$$

Definition 2 (belief function). Belief Function is the sum of BPAs for all subsets of A. On the frame of discernment U, reliability function based on BPA is expressed by Bel (A), and its calculation formula is as follows:

$$\mathrm{Bel}(A) = \sum_{B \subseteq A} m(B) \tag{4}$$

Definition 3 (plausibility function). Plausibility Function is the sum of BPAs of all subsets intersecting with A. On the frame of discernment U, plausibility function based on BPA is expressed by Pl (A), and its calculation formula is as follows:

$$\mathrm{Pl}(A) = \sum_{B \cap A \neq \emptyset} m(B) \tag{5}$$

Definition 4 (trust intervals). The Trust Interval of A is denoted as [Bel(A), Pl(A)]. Bel(A) is the lower bound of the Trust Interval, and Pl(A) is the upper bound. For example, the trust interval of event A is (0.25, 0.85). The probability that A is true is 0.25, the probability that A is false is 0.15, and the probability that A is uncertain is 0.6

Definition 5 (Dempster aggregation rule). DST can be used to integrate evidences from multiple independent sources. There are a finite number of mass functions on the recognition framework U, namely, $m_1, m_2, m_3, m_4, \ldots m_n$. The aggregation rules are as follows:

$$(m_1 \oplus m_2 \oplus \cdots \oplus m_n)(A) = \frac{1}{K} \sum_{A_1 \bigcap A_2 \bigcap \cdots \bigcap A_n = A} m_1(A_1) \cdot m_2(A_2) \cdots m_n(A_n) \tag{6}$$

$$K = \sum_{A_1 \bigcap A_2 \bigcap \cdots \bigcap A_n \neq \emptyset} m_1(A_1) \cdot m_2(A_2) \cdots m_n(A_n) \tag{7}$$

$$= 1 - \sum_{A_1 \bigcap A_2 \bigcap \cdots \bigcap A_n = \emptyset} m_1(A_1) \cdot m_2(A_2) \cdots m_n(A_n) \tag{8}$$

K is called normalization factor, and the value of 1-K reflects the degree of evidence conflict.

4. A New FMCGDM Approach Model Based on AHP and DST

In order to study the complex problem in NPD, this paper focuses on the FMCGDM (fuzzy multiple criteria group decision-making). In this paper, a new FMCGDM approach based on DST and AHP is proposed. This method consists of four important parts. Firstly, an index system for new product selection is constructed, and attribute weight can be analyzed by using AHP and DST. Then, multiple experts are asked to predict attributes performance of all new schemes and to express it by fuzzy number. Finally, DST is applied to group decision-making, and the optimal solution is obtained. The process is shown in Figure 1.

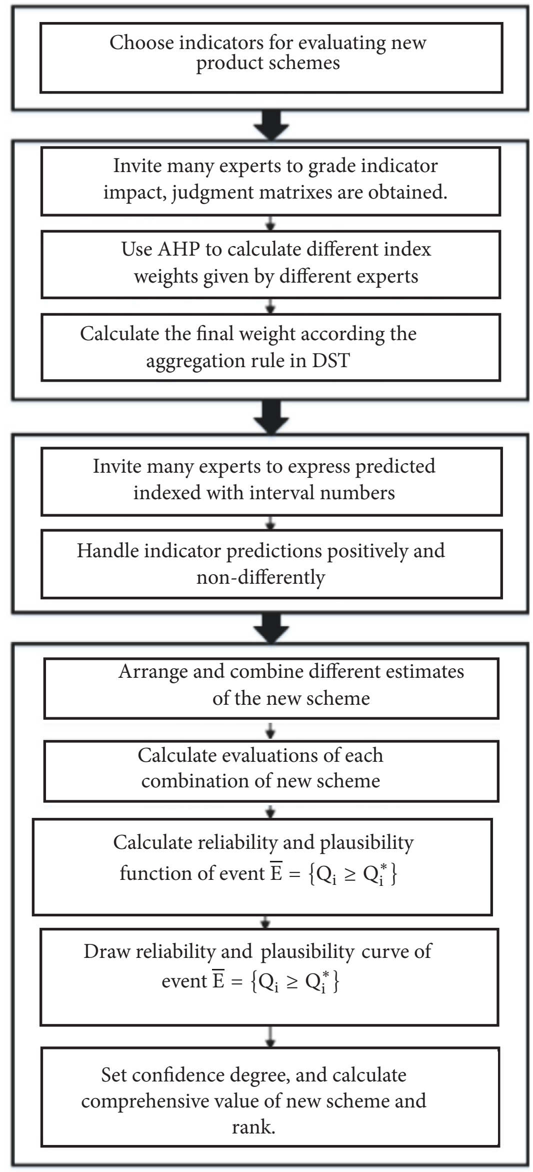

Figure 1: Conceptual framework of the proposed FMCGDM approach based on AHP and DST for new product selection.

4.1. The Construction of Evaluation Index System. A new design plan can be evaluated by scoring. The principle of scoring is that schemes which are conducive to achieve our goals will get higher scores, and vice versa. To evaluate new product plans, goals of developing new product should be made clear firstly. Several criteria can be selected according to program purposes, and evaluation index system for new product selection is set up. Different enterprises have different goals in designing and producing new products. Enterprises design new products in order to open a new market, gain a higher market share, contribute to the environment, or obtain higher profits.

When evaluating specific product designs, indicators may vary according to previous research. For example, when a mobile phone manufacture enterprise develops a new product, indexes may be battery life, storage capacity, core processor efficiency, screen size, camera pixel, development

Table 1: Scaling Method of Judgment Matrix.

Scaling value	Implication
1	The two indicators are equally important
3	The former indicator is slightly more important than the later.
5	The former indicator is obviously more important than the later.
7	The former indicator is much more important than the later.
9	The former indicator is extremely more important than the later.
2, 4, 6, 8	The median value is between two adjacent discriminates
reciprocal	If the matrix score is a_{ij} when the i-th indicator is compared with the j-th, the matrix score is $1/a_{ij}$ when the j-th indicator is compared with the i-th.

cost, and product weight. Therefore, when selecting indicators, decision makers should modify indexes according to the goals.

4.2. The Weights Determination Method Based on AHP and DST. Through Section 4.1 content, indicators for new product selection have been chosen. Suppose that indexes are $(U_1 \ \ U_2 \ \ \cdots \ \ U_N)$. Next, we need to clarify relationship between indicators, that is, to calculate indicator weights. Weights obtained by objective methods often do not conform to the reality, and these obtained by subjective methods are influenced by expert preferences. To avoid these shortcomings, a combination of subjective and objective methods is used to calculate index weights. This method is to invite many experts to evaluate indicators importance and use AHP to calculate indicator weights. Because weights given by various experts are different, so in this paper, DST is used to calculate the final weights.

4.2.1. The Judgment Matrix Given by Experts. Firstly, many experts are invited to grade the impact of all indicators on the target. Experts can give evaluating information on the importance of the indicators according to their own professional knowledge and experience. This evaluating information can be converted into scores by comparing two indicators, as shown in Table 1.

According to this method, judgment matrix on the importance of indexes given by all experts can be obtained. Then judgment matrix given by the m-th expert is represented by A^m:

$$A^m = \left[\begin{pmatrix} a_{11}^m & \cdots & a_{1n}^m \\ \vdots & \ddots & \vdots \\ a_{n1}^m & \cdots & a_{nn}^m \end{pmatrix} \right] \tag{9}$$

4.2.2. Weight-Making by AHP. In order to estimate the impact of all indexes on the design selection, AHP [41] is used to calculate index weights. The calculation process is as follows.

Normalization. In order to calculate eigenvalues, judgment matrix given by every expert is normalized firstly. According to formula (10), judgment matrix is normalized and the normalized matrix $A_{ij}^{m'}$ is obtained. According to formula (11), the normalized matrix is summed by rows:

$$a_{ij}^{m'} = \frac{a_{ij}^m}{\sum_{i=1}^n a_{ij}^m} \tag{10}$$

$$\begin{aligned} \overline{W^m} &= \left(\overline{\omega_1^m} \ \ \overline{\omega_2^m} \ \ \cdots \ \ \overline{\omega_n^m}\right) \\ &= \left(\sum_{j=1}^n a_{1j}^{m'} \ \ \sum_{j=1}^n a_{2j}^{m'} \ \ \cdots \ \ \sum_{j=1}^n a_{nj}^{m'}\right) \end{aligned} \tag{11}$$

Eigenvalues and Maximum Eigenvectors of Judgment Matrix. According to formula (12), eigenvectors are calculated, and indicator weights given by the m-th expert are obtained. According to formula (13), the maximum eigenvalue is calculated:

$$\begin{aligned} W^m &= \left(\omega_1^m \ \ \omega_2^m \ \ \cdots \ \ \omega_n^m\right) \\ &= \left(\frac{\overline{\omega_1^m}}{\sum_{i=1}^n \overline{\omega_i^m}} \ \ \frac{\overline{\omega_2^m}}{\sum_{i=1}^n \overline{\omega_i^m}} \ \ \cdots \ \ \frac{\overline{\omega_n^m}}{\sum_{i=1}^n \overline{\omega_i^m}}\right) \end{aligned} \tag{12}$$

$$\lambda_{max}^m = \sum_{i=1}^n \frac{(AW^m)_i}{n\omega_i^m} \tag{13}$$

Consistency Test. Consistency test is to judge the credibility of experts' scores:

$$CR = \frac{CI}{RI} \tag{14}$$

where CR is the consistency test ratio, RI is a random consistency index, as shown in Table 2, and CI is consistency coefficient of judgment matrix, and it can be calculated as

$$CI = \frac{\lambda_{max} - n}{n - 1} \tag{15}$$

According to formula (14), the value of CR can be calculated. If $CR < 0.1$ is true, the inconsistency degree is within the acceptable range. The weights given by this expert are considered to be valid. Conversely, the inconsistency degree is unacceptable.

Table 2: The table of Random Consistency Indicators RI.

n	1	2	3	4	5	6	7	8	9	10	11
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49	1.51

Table 3: Index weight given by different expert.

Expert	Index A	Index B	Index C
Expert 1	0.4	0.3	0.3
Expert 2	0.3	0.35	0.35

4.2.3. The Final Weight Determination Based on DST. In Section 4.2.2, index weights can be calculated by using judgment matrixes given by all experts. If the weights given by different experts are the same with each other, index weights can be obtained. However, judgment matrix given by different experts is often various due to their respective knowledge and experiences. So these weights calculated by judgment matrixes given by different experts are different. DST is used to synthetically analyze different weights of the same index given by different experts and calculate the final index weights. The aggregation rule of multiple independent information sources in DST can be used to solve this problem.

For example, three indicator weights given by two experts are calculated through AHP method. The calculation results are shown in Table 3.

Using aggregation rule in DST, the calculation process of the final index weights is as follows:

$$\begin{aligned} K &= 1 - m_1(A) * m_2(B) + m_1(A) * m_2(C) + m_1(B) \\ &\quad * m_2(A) + m_1(B) * m_2(C) + m_1(C) * m_2(A) \\ &\quad + m_1(C) * m_2(B) = 1 - (0.4 * 0.35 + 0.4 * 0.35 \\ &\quad + 0.3 * 0.3 + 0.3 * 0.35 + 0.3 * 0.3 + 0.3 * 0.35) \\ &\quad = 0.33 \\ \text{Bel}(A) &= m_1(A) * \frac{m_2(A)}{K} = 0.4 * \frac{0.3}{0.33} = 0.3636 \\ \text{Bel}(B) &= m_1(B) * \frac{m_2(B)}{K} = 0.3 * \frac{0.35}{0.33} = 0.3182 \\ \text{Bel}(C) &= m_1(C) * \frac{m_2(C)}{K} = 0.3 * \frac{0.35}{0.33} = 0.3182 \end{aligned} \quad (16)$$

4.3. The Indicator Prediction Represented by Interval Number. Multiple experts are invited to estimate the future performance of all indicators of new product schemes. Because it is a prediction for future, experts use fuzzy number more confidently compared to using numerical values to assess the possible future performance of every scheme.

For different indicators, date dimension is different. So it needs to be put in the same dimension so as to avoid mistake in the final results. Index system to evaluate new schemes includes positive index and negative index. In order to facilitate calculation, the negative data needs to be put forward. So there are two main steps in data processing, namely, nondimensional process and positive management.

4.3.1. Nondimensional Process. The indicators are represented by values or interval numbers. In this paper, min-max is mainly used to nondimensionalize.

For numerical data, the nondimensional formula (17) is as follows:

$$x'_{ij} = \frac{x_{ij} - x_{min,j}}{x_{max,j} - x_{min,j}} \quad (17)$$

where x_{ij} represents the value of the jth index of the ith scheme and x'_{ij} represents the nondimensional data of the jth index of the ith scheme. $x_{max,j}$ and $x_{min,j}$ are used to represent the maximum and minimum values of the jth index data, respectively. $i = 1, 2, \cdots, M$ and $j = 1, 2, \cdots, N$. M is the total number of programs, and N is the total number of indicators.

For interval numbers, the nondimensional formula is as follows:

$$a_{ij}^{-\prime} = \frac{a_{ij}^{-} - \min_{0\le k\le m}\left\{a_{kj}^{-}\right\}}{\max_{0\le k\le m}\left\{a_{kj}^{+}\right\} - \min_{0\le k\le m}\left\{a_{kj}^{-}\right\}} \quad (18)$$

$$a_{ij}^{+\prime} = \frac{a_{ij}^{+} - \min_{0\le k\le m}\left\{a_{kj}^{-}\right\}}{\max_{0\le k\le m}\left\{a_{kj}^{+}\right\} - \min_{0\le k\le m}\left\{a_{kj}^{-}\right\}} \quad (19)$$

where $[a_{ij}^{-}, a_{ij}^{+}]$ represents the value of the jth index of the ith scheme, and $[a_{ij}^{-\prime}, a_{ij}^{+\prime}]$ is used to represent the nondimensional data of the jth index of the ith scheme. $\min_{0\le k\le m}\{a_{kj}^{-}\}$ is the minimum lower limit value of the jth index. $\max_{0\le k\le m}\{a_{kj}^{+}\}$ is the maximum upper limit value of the jth index. $i = 1, 2, \cdots, M$, $j = 1, 2, \cdots, N$, where M represents the total number of programs and N represents the total number of indicators.

4.3.2. Positive Management. Indicators can be divided into positive and negative indicators. Positive indicators are the indicators that mean that the higher the values are, the better the schemes are. The negative indicators are the indicators that mean that the smaller the indexes are, the better the product is. In order to make calculation more convenient, negative indicators need to be transformed into positive indicators.

For numerical data, forward processing formula is as follows:

$$x''_{ij} = 1 - x'_{ij} \tag{20}$$

where x'_{ij} stands for the nondimensional data of the jth index of the ith scheme, and x''_{ij} stands for the forward data of the jth index of the ith scheme

For interval number, forward processing formula is as follows:

$$a^{+\prime\prime}_{ij} = 1 - a^{-\prime}_{ij} \tag{21}$$

$$a^{-\prime\prime}_{ij} = 1 - a^{+\prime}_{ij} \tag{22}$$

where $[a^{-\prime}_{ij}, a^{+\prime}_{ij}]$ stands for the nondimensional data of the jth index of the ith scheme, and $[a^{-\prime\prime}_{ij}, a^{+\prime\prime}_{ij}]$ stands for the forward data of the jth index of the ith scheme.

4.4. Evaluation of New Product Scheme Based on DST. It is difficult for experts to predict future data accurately, especially on accurate number. Therefore, experts use interval numbers more confidently to predict future data based on their experience and knowledge. What is more, in order to avoid the preference of individual expert and improve the accuracy of prediction, several experts were invited to predict the indicators. The assumption that these experts are equally familiar with new products is made. This means that every expert has the same credibility. In other words, indicator values proposed by every expert appear with the same probability. For example, n experts are asked to estimate the j-th index of the i-th scheme and n estimates are obtained. Then, the probability of every estimate is 1/n.

All experts should estimate all indicators of all new schemes. Let interval number a_{ijm} characterize the estimated value given by the m-th expert to the j-th index attribute of the i-th scheme. In order to take into account all possible future values of new scheme, different estimates would be arranged and combined. If there are P experts and Q indicators, there will be P^Q possible combinations for every scheme. Only through analyzing comprehensively these combinations can evaluation results of new schemes be obtained.

For clarity, suppose there are three indicators and two experts. The two experts predicted all indicators of a new scheme, and the results are shown in Table 4. All possible combinations of the new scheme are as shown in Table 5. The number of combinations is $2^3 = 8$. The probability of occurrence of every combination is equal; all are 1/8.

Next, comprehensive evaluation of all combination can be calculated, which is equal to multiplying the attributes and the corresponding weights and then accumulating them. The calculation process of the comprehensive value of the r-th combination of the i-th new scheme is as follows:

$$Q_{ir} = AW = \left[a_{ir1} \;\; a_{ir2} \;\; \cdots \;\; a_{irn}\right] * \left[w_1 \;\; w_2 \;\; \cdots \;\; w_n\right]^T = \sum_{j=1}^{n} a_{irj} * w_j = \left[\sum_{j=1}^{n} a^{-}_{irj} * w_j, \sum_{j=1}^{n} a^{+}_{irj} * w_j\right] \tag{23}$$

A project would have P^Q comprehensive evaluations, and they are interval numbers. This makes sorting all new schemes difficult. A numerical value Q_i^* which represented the final evaluation of the i-th scheme is needed. In order to synthesize the available information and make them useful for new product selection, let us consider event $\overline{E} = \{Q_i \geq Q_i^*\}$, where the evaluation of the i-th new product, i.e. Q_i, is compared with a generic threshold value Q_i^*. To reckon Q_i^* by this method is calculated from a conservative point of view. If event $\overline{E} = \{Q_i \geq Q_i^*\}$ occurs, the actual evaluation of new scheme is more than Q_i^*. If the evaluation of new scheme Q_i^* is acceptable, the probability that the actual evaluation of the new scheme is greater than Q_i^* is very large. So the new scheme must be acceptable. The probability of occurrence of event $\overline{E} = \{Q_i \geq Q_i^*\}$ should be calculated though Dempster aggregation rules. The greater the probability of evidence support is, the more likely the event is to occur. Because it is difficult to calculate the probability of event $\overline{E} = \{Q_i \geq Q_i^*\}$ directly, the confidence interval of inverse event E can be calculated firstly. By definition, $[Bel(E), Pl(E)]$ stands for the trust interval of event E. Bel (E) denotes the lower limit, and Pl (E) denotes the upper limit.

(i) The reliability function of event E can be calculated according to

$$Bel(E) = Bel\left(Q_i \leq Q_i^*\right) = \sum_{Q_i \in [0, Q_i^*]} m\left(Q_{ir}\right) \tag{24}$$

(ii) The plausibility function of event E can be calculated according to

$$Pl(E) = Pl\left(Q_i \leq Q_i^*\right) = \sum_{Q_i \cap [0, Q_i^*] = \emptyset} m\left(Q_{ir}\right) \tag{25}$$

(iii) The reliability function and plausibility function of inverse event $\overline{E}$ can be obtained by calculating the reliability function and plausibility function of event E according to

$$Bel\left(\overline{E}\right) = Bel\left(Q_i \geq Q_i^*\right) = 1 - Pl\left(Q_i \leq Q_i^*\right) \tag{26}$$

$$Pl\left(\overline{E}\right) = Pl\left(Q_i \geq Q_i^*\right) = 1 - Bel\left(Q_i \leq Q_i^*\right) \tag{27}$$

It is more reasonable to determine the Q_i^* from the curve depicted by the last equation, since the greater the comprehensive evaluation is, the better the scheme is. Set a confidence degree g. As long as plausibility function of inverse event $\overline{E} = \{Q_i \geq Q_i^*\}$ is greater than the confidence degree g, the value of Q_i^* is reasonable. The value of Q_i^* is the abscissa of the intersection point of line y = g (parallel to x-axis) and plausibility function curve. In this way, Q_i^* can be used to rank the new schemes to get the optimal one.

Here, there is still a problem; that is, the Q_i^* of more than one schemes obtained by plausibility function may be equal. In order to rank the schemes with same Q_i^*, reliability value r of event $\overline{E} = \{Q_i \geq Q_i^*\}$ is solved for sorting. The r is the longitudinal coordinate of the intersection point of line x=

Table 4: Experts' Predictions to Indicators.

Expert	Indicator 1	Indicator 2	Indicator 3
Expert A	[0.5, 0.6]	[0.3, 0.4]	[0.9, 1]
Expert B	[0.3, 0.4]	[0.2, 0.3]	[0.8, 0.9]

Table 5: All possible combinations.

combination	Indicator 1	Indicator 2	Indicator 3
1	[0.5, 0.6]	[0.3, 0.4]	[0.9, 1]
2	[0.5, 0.6]	[0.3, 0.4]	[0.8, 0.9]
3	[0.5, 0.6]	[0.2, 0.3]	[0.9, 1]
4	[0.5, 0.6]	[0.2, 0.3]	[0.8, 0.9]
5	[0.3, 0.4]	[0.3, 0.4]	[0.9, 1]
6	[0.3, 0.4]	[0.3, 0.4]	[0.8, 0.9]
7	[0.3, 0.4]	[0.2, 0.3]	[0.9, 1]
8	[0.3, 0.4]	[0.2, 0.3]	[0.8, 0.9]

Q_i^* and reliability function curve. If the r is equal, change confidence degree g and then solve Q_i^*. Repeat the above steps until a comparison can be made.

The calculation process is as follows.

(i) Set a confidence level g. Draw a line y = g parallel to the X axis, and the intersection point of this line and plausibility function curve is $(Q_i^*\ \ g)$, so as to obtain Q_i^*.

(ii) By comparing Q_i^*, new schemes are sorted.

(iii) If there are no two or more schemes with equal Q_i^*, the calculation process is over.

(iv) If there is more than one scheme with equalQ_i^*, the reliability value r of event $\overline{E} = \{Q_i \geq Q_i^*\}$ should be calculated. Draw a line $x = Q_i^*$ parallel to Y axis, and the intersection point of this line and reliability function curve is (Q_i^*, r). By this way, the r can be obtained.

(v) By comparing r, scheme with equal Q_i^* are sorted.

(vi) If the reliability value r of new schemes with equal Q_i^* are not equal, the calculation process is over.

(vii) If there are more than one scheme with equal Q_i^* and r, a new confidence level g' is set to rank schemes with equal r and Q_i^*, and $g' \neq g$. Repeat steps (i)-(v) until all schemes are sorted completely. The calculation process can be shown in Figure 2.

5. Case Study

In order to prove the application of this method in new product selection, a case study of a household appliances manufacturing enterprise would be introduced in this section. This household appliances manufacturer is located in Qingdao, China. Regarding product innovation and user's needs as key factors for its development, this company has invested a lot of money in product development and market research. This also enables this enterprise to obtain many new product schemes. How to choose the right products is an important problem. The names and technical characteristics of new schemes are not provided in this study, since these are confidential contents to this corporation. The name of new schemes is replaced by Arabic numerals, and estimations are given by experts directly without discussing the feature information.

5.1. Decision Attributes for NPD. The goal of choosing new products to this enterprise is for the better development. How to select appropriate attributes from many factors which affect enterprise growth is the first problem to be solved. The correctness of enterprise decision-making is influenced by the quality and quantity of attributes. In order to construct appropriate indicator system, the company leaders invited several departmental representatives to set up a new product committee. These departmental representatives are from marketing, finance, R&D, and product department. Through analyzing and screening all factors, the five indicators are selected in the final, namely, Technical Difficulty (C_1), Product Performance (C_2), Market Potential (C_3), Project Risk (C_4), and Project Cost (C_5). Technical Difficulty (C_1) is product complexities which highlight physical possibility of product realization. Product Performance (C_2) is superior quality and new product features. Market Potential (C_3) reflects the market demand. Project Risk (C_4) is about all things that might prevent successful completion. Project Cost (C_5) depicts the consumption of resources. The specific contents of the indicators are shown in Table 6.

5.2. A Weight Determination Method Based on AHP and DST for NPD. The indicators for evaluating new product schemes are selected though Section 5.1, and the next important thing is to determine these indicator weights. In this paper, four experts involved from different departments in the company

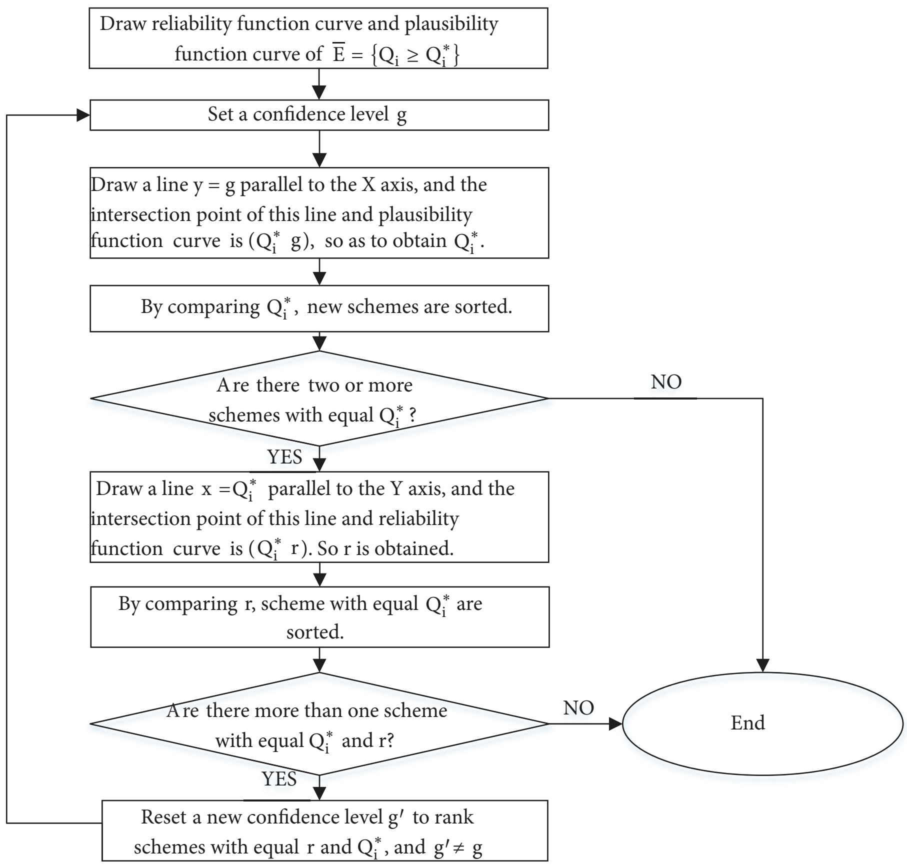

Figure 2: Flow diagram of the prioritization procedure for new product selection.

were invited to score the importance of the indicators. They are from marketing, finance, R&D, and product department and have been working in this company for more than five years. The four managers scored the indicator importance according to the scoring scale of Table 1. Four judgment matrices are obtained, as follows:

$$A^1 = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{2} & 1 \\ 2 & 1 & \frac{1}{2} & 1 & 2 \\ 3 & 2 & 1 & 1 & 3 \\ 2 & 1 & 1 & 1 & 2 \\ 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{2} & 1 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 1 & 3 & 1 & 3 & 2 \\ \frac{1}{3} & 1 & \frac{1}{2} & 1 & \frac{1}{2} \\ 1 & 2 & 1 & 2 & 1 \\ \frac{1}{3} & 1 & \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & 2 & 1 & 2 & 1 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 1 & \frac{1}{2} & 2 & \frac{1}{2} & 1 \\ 2 & 1 & 3 & 1 & 2 \\ \frac{1}{2} & \frac{1}{3} & 1 & \frac{1}{2} & \frac{1}{2} \\ 2 & 1 & 2 & 1 & 2 \\ 1 & \frac{1}{2} & 2 & \frac{1}{2} & 1 \end{pmatrix}$$

$$A^4 = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{3} & \frac{1}{2} \\ 2 & 1 & 1 & \frac{1}{2} & \frac{1}{2} \\ 2 & 1 & 1 & \frac{1}{2} & \frac{1}{2} \\ 3 & 2 & 2 & 1 & 1 \\ 3 & 2 & 2 & 1 & 1 \end{pmatrix} \quad (28)$$

According to each expert's score on each index, the weight of each index is calculated through AHP as shown in Table 7. Because of various work experience and knowledge, multiple experts assigned different weights to the same indicators. In order to obtain more reliable weights, the aggregation rules in DST are used to calculate the final weight. It is mainly based on that the greater the evidence is, the greater the probability of events is. According to formula (6) and (8) in Section 3, the final index weights are obtained. The result of empowerment is as follows:

$$w_1 = 0.09$$
$$w_2 = 0.17$$
$$w_3 = 0.20 \quad (29)$$
$$w_4 = 0.37$$
$$w_5 = 0.17$$

5.3. Data Collection and Preprocessing for NPD. In Sections 5.1 and 5.2, the indicators and their weights for evaluating new product are determined. The next thing to do is to predict the future performance of all indicators of new schemes. Through the active efforts of all staff, ten new products are collected. Two experts who are very familiar with these new products took the initiative to participate in the estimation of all indicators. One expert (A) is the manager of R&D and the other (B) is the manager of marketing department. They have been working for more than five years and are very familiar with market and enterprise. They gave, respectively, a suitable score which is between 1 and 10 points for all indicators. Because it is a prediction, experts use interval numbers more confidently than using numerical values to assess the possible future performance of each indicator. The estimated values are shown in Table 8.

According to the data processing formula in Section 4.3, the estimated values are handled positively and nondifferently. And the results are shown in Table 9.

5.4. Evaluation and Ranking of New Product Schemes Based on DST. In Section 5.3, two experts estimated the future performance of all indicators of the ten new schemes. Because they are very familiar with these new products, the probability of occurrence of the indicator value given by the two experts is equal, namely, 1/2. Following the decomposition and merging approach given in Section 4.4, the five indicators given by the two experts are combined. The number of combinations is 2^5, namely, 32.

Every combination would get a comprehensive evaluation according to formula (23), and the occurrence probability of every comprehensive value is 1/32. According to the calculation process in Section 4.4, plausibility curve and belief curve of event $\overline{E} = \{Q_i \geq Q_i^*\}$ are drawn. Set confidence level g as 0.9, and draw a line y = g parallel to the X axis. The intersection point of this line and plausibility function curve is $(Q_i^* \ \ g)$, so as to obtain Q_i^*. Due to the limitation of space, it is impossible to list all the schemes. However, in order to illustrate this method, the new product scheme 2 is illustrated as an example. Table 10 shows 32 combinations of scheme 2 and their comprehensive values. Figure 3 shows the Belief and Plausibility curves of event $\overline{E} = \{Q_2 \geq Q_2^*\}$, and Q_2^* is 0.5.

According to the same method, the evaluation results Q_i^* of these ten new products are obtained. The new products are

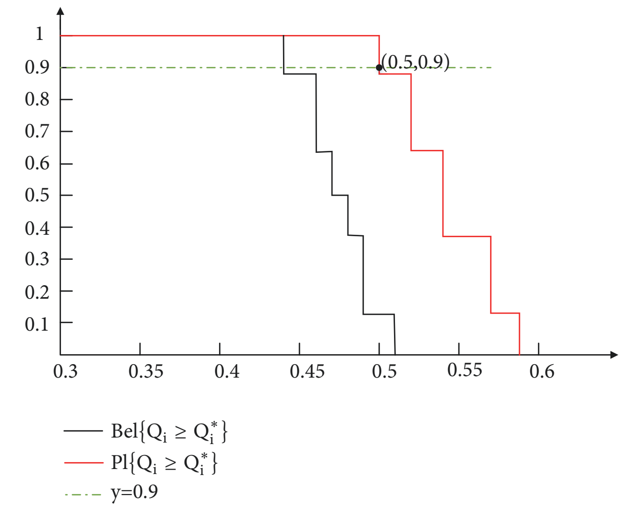

Figure 3: Belief and plausibility curves of new product Scheme 2.

sorted according to Q_i^*, and the ranking results are shown in Table 11.

Among them, the evaluation results Q_i^* of new product schemes 4 and 5 are both 0.59. They are equally ranked in the third position of Table 11. For the sake of clarity, the reliability value of event $\overline{E} = \{Q_i \geq 0.59\}$ is solved for sorting. Draw a line x = 0.59 parallel to the Y axis, and the intersection point of this line and reliability function curve leads to $\text{Bel}(Q_4 \geq 0.59) = 0$, and $\text{Bel}(Q_5 \geq 0.59) = 0$. Therefore, it is still impossible to rank the two schemes. Aiming at better discriminating their actual criticality, we reset confidence level g as 0.8. And draw a line y = 0.8 parallel to the X axis. The intersection point of this line and plausibility function curve leads to $Q_4^{**} = 0.61$, whereas $Q_5^{**} = 0.59$, as shown in Figures 4 and 5. The new product scheme 4 results are more critical than scheme 5. As a consequence, the new product scheme 4 still remains in the third position of the final ranking, whereas scheme 5 shifts to the forth. The final ranking of new product schemes is shown in Table 12. In this scheme selection, scheme 6 is the best choice.

In real life, the top manager in this enterprise finally adopted scheme 8 and scheme 3 through many seminars and repeated evaluations. After successful R&D and putting scheme 8 and scheme 3 into the market, it is found that the actual situation is the same as the result of the evaluation in this paper. New product 8 has achieved good economic returns and new scheme 3 has achieved lower economic returns, which is the same as our final evaluation.

6. Conclusions

The evaluation and selection of new product schemes is a prediction and decision-making process for future problems, which has many risks and uncertainties. It is one of the key events which influences the future development of enterprises. Therefore, it is imperative to take a reliable approach to assess new products. In this paper, a new method based on AHP and DST is proposed to make the best decisions in

Table 6: The specific contents of the indicators.

Indicator	The specific contents
Technical Difficulty	Technological difficulty in Research, Development and Production
Product Performance	The new properties, costs and physical characteristics
Market Potential	Market capacity, demand trend, profit potential, competitive ability, market competition
Project Risk	Financial risks, managerial risks, envisioning risks, design risks, and execution risks
Project Cost	Financial consumption, human consumption and material consumption

Table 7: Weights assigned by different expert.

Indictor	Expert A	Expert B	Expert C	Expert D
Technical Difficulty	0.1103	0.3276	0.1568	0.0975
Product Performance	0.2097	0.1103	0.2945	0.1568
Market Potential	0.3276	0.2422	0.0975	0.1568
Project Risk	0.2422	0.1103	0.2945	0.2945
Project Cost	0.1103	0.2097	0.1568	0.2945

Table 8: Table of Experts' Prediction for New Products.

Indictor	C_1		C_2		C_3		C_4		C_5	
Expert	A	B	A	B	A	B	A	B	A	B
Scheme 1	[7, 8]	[6, 7]	[8, 9]	9	[7, 8]	8	[89]89	8	9	9
Scheme 2	[7, 8]	[5, 6]	[4, 5]	4	[3, 4]	4	[5, 6]	5	[8, 9]	8
Scheme 3	[4, 5]	4	[1, 2]	2	[2, 3]	2	[3, 4]	3	[4, 5]	4
Scheme 4	[8, 9]	[7, 8]	[4, 5]	[5, 6]	[4, 5]	5	[7, 8]	[6, 7]	8	[7, 8]
Scheme 5	[8, 9]	[8, 9]	7	[7, 8]	8	[7, 8]	[7, 8]	[7, 8]	4	5
Scheme 6	[2, 3]	3	[2, 3]	[2, 3]	[9, 10]	9	[8, 9]	[7, 8]	9	[8, 9]
Scheme 7	[4, 5]	[5, 6]	[2, 3]	3	[4, 5]	[3, 4]	[5, 6]	[4, 5]	[4, 5]	[3, 4]
Scheme 8	[7, 8]	[8, 9]	[4, 5]	[3, 4]	2	[2, 3]	[4, 5]	[5, 6]	[2, 3]	[1, 2]
Scheme 9	[2, 3]	[3, 4]	[4, 5]	[5, 6]	[4, 5]	[4, 5]	[2, 3]	2	[4, 5]	[5, 6]
Scheme 10	[4, 5]	[3, 4]	2	[2, 3]	[1, 2]	[2, 3]	[1, 2]	1	[5, 6]	[5, 6]

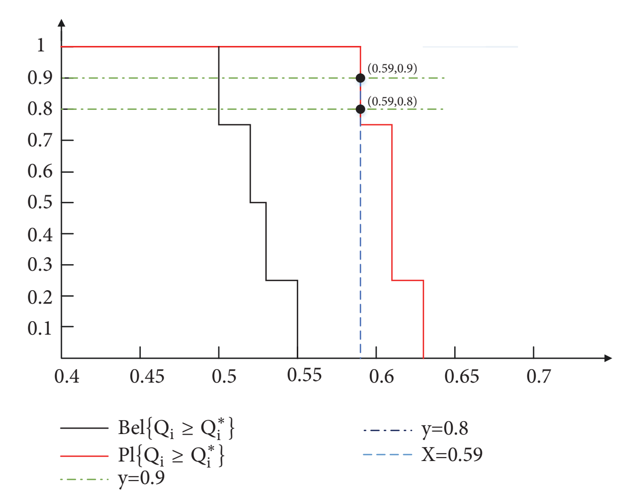

Figure 4: Belief and plausibility curves of new product Scheme 4.

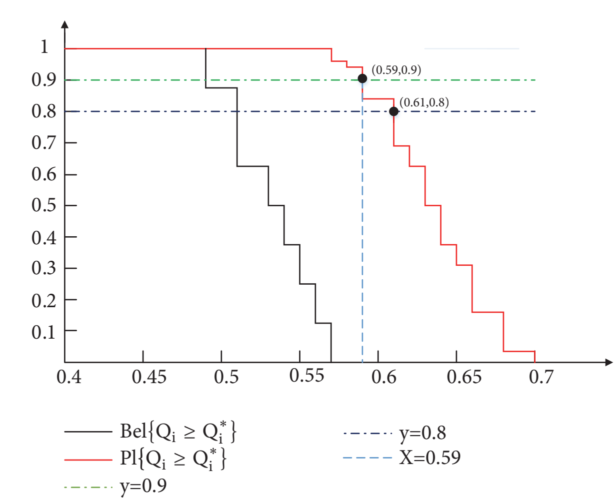

Figure 5: Belief and plausibility curves of new product Scheme 5.

a fuzzy environment. There are four stages in this method. The first stage is attributes selection for evaluation, which is the first stage for all decision-making problems. At this stage, we need to make clear what indicators influence new

Table 9: Pre-processing result table.

Indictor	C_1		C_2		C_3		C_4		C_5	
Expert	A	B	A	B	A	B	A	B	A	B
Scheme 1	[0.71, 0.86]	[0.57, 0.71]	[0, 0.12]	[0, 0]	[0.22, 0.33]	0.22	[0.88, 1]	0.88	0	0
Scheme 2	[0.71, 0.86]	[0.43, 0.57]	[0.5, 0.62]	0.62	[0.67, 0.78]	0.67	[0.5, 0.63]	0.5	[0, 0.12]	0.12
Scheme 3	[0.29, 0.43]	[0.29, 0.29]	[0.87, 0.1]	[0.87, 0.87]	[0.78, 0.89]	0.89	[0.25, 0.38]	0.25	[0.5, 0.62]	0.62
Scheme 4	[0.86, 1]	[0.71, 0.86]	[0.5, 0.62]	[0.37, 0.5]	[0.56, 0.67]	[0.67, 0.78]	[0.75, 0.88]	[0.63, 0.75]	0.12	[0.12, 0.25]
Scheme 5	[0.86, 1]	[0.86, 1]	0.25	[0.12, 0.25]	0.22	[0.22, 0.33]	[0.75, 0.88]	[0.75, 0.88]	0.62	0.5
Scheme 6	[0, 0.14]	[0.14, 0.14]	[0.75, 0.87]	[0.75, 0.87]	[0, 0.11]	0.11	[0.88, 1]	[0.75, 0.88]	0	[0, 0.12]
Scheme 7	[0.29, 0.43]	[0.43, 0.57]	[0.75, 0.87]	[0.75, 0.75]	[0.56, 0.67]	[0.67, 0.78]	[0.5, 0.63]	[0.38, 0.5]	[0.5, 0.62]	[0.62, 0.75]
Scheme 8	[0.71, 0.86]	[0.86, 1]	[0.5, 0.62]	[0.62, 0.75]	0.89	[0.78, 0.89]	[0.38, 0.5]	[0.5, 0.63]	[0.75, 0.87]	[0.87, 1]
Scheme 9	[0, 0.14]	[0.14, 0.29]	[0.5, 0.62]	[0.37, 0.5]	[0.56, 0.67]	[0.56, 0.67]	[0.13, 0.25]	0.13	[0.5, 0.62]	[0.37, 0.5]
Scheme 10	[0.29, 0.43]	[0.14, 0.29]	0.87	[0.75, 0.87]	[0.89, 1]	[0.78, 0.89]	[0, 0.13]	0	[0.37, 0.5]	[0.37, 0.5]

Table 10: The combinations of new product scheme 2.

combination	C_1	C_2	C_3	C_4	C_5	evaluation
1	[0.71, 0.86]	[0.5, 0.62]	[0.67, 0.78]	[0.5, 0.63]	[0, 0.12]	[0.47, 0.59]
2	[0.71, 0.86]	[0.5, 0.62]	[0.67, 0.78]	[0.5, 0.63]	0.12	[0.49, 0.59]
3	[0.71, 0.86]	[0.5, 0.62]	[0.67, 0.78]	0.5	[0, 0.12]	[0.44, 0.57]
4	[0.71, 0.86]	[0.5, 0.62]	[0.67, 0.78]	0.5	0.12	[0.46, 0.57]
5	[0.71, 0.86]	[0.5, 0.62]	0.67	[0.5, 0.63]	[0, 0.12]	[0.47, 0.54]
6	[0.71, 0.86]	[0.5, 0.62]	0.67	[0.5, 0.63]	0.12	[0.49, 0.54]
7	[0.71, 0.86]	[0.5, 0.62]	0.67	0.5	[0, 0.12]	[0.44, 0.52]
8	[0.71, 0.86]	[0.5, 0.62]	0.67	0.5	0.12	[0.46, 0.52]
9	[0.71, 0.86]	0.62	[0.67, 0.78]	[0.5, 0.63]	[0, 0.12]	[0.49, 0.59]
10	[0.71, 0.86]	0.62	[0.67, 0.78]	[0.5, 0.63]	0.12	[0.51, 0.59]
11	[0.71, 0.86]	0.62	[0.67, 0.78]	0.5	[0, 0.12]	[0.46, 0.57]
12	[0.71, 0.86]	0.62	[0.67, 0.78]	0.5	0.12	[0.48, 0.57]
13	[0.71, 0.86]	0.62	0.67	[0.5, 0.63]	[0, 0.12]	[0.49, 0.54]
14	[0.71, 0.86]	0.62	0.67	[0.5, 0.63]	0.12	[0.51, 0.54]
15	[0.71, 0.86]	0.62	0.67	0.5	[0, 0.12]	[0.46, 0.52]
16	[0.71, 0.86]	0.62	0.67	0.5	0.12	[0.48, 0.52]
17	[0.43, 0.57]	[0.5, 0.62]	[0.67, 0.78]	[0.5, 0.63]	[0, 0.12]	[0.47, 0.57]
18	[0.43, 0.57]	[0.5, 0.62]	[0.67, 0.78]	[0.5, 0.63]	0.12	[0.49, 0.57]
19	[0.43, 0.57]	[0.5, 0.62]	[0.67, 0.78]	0.5	[0, 0.12]	[0.44, 0.54]
20	[0.43, 0.57]	[0.5, 0.62]	[0.67, 0.78]	0.5	0.12	[0.46, 0.54]
21	[0.43, 0.57]	[0.5, 0.62]	0.67	[0.5, 0.63]	[0, 0.12]	[0.47, 0.52]
22	[0.43, 0.57]	[0.5, 0.62]	0.67	[0.5, 0.63]	0.12	[0.49, 0.52]
23	[0.43, 0.57]	[0.5, 0.62]	0.67	0.5	[0, 0.12]	[0.44, 0.5]
24	[0.43, 0.57]	[0.5, 0.62]	0.67	0.5	0.12	[0.46, 0.5]
25	[0.43, 0.57]	0.62	[0.67, 0.78]	[0.5, 0.63]	[0, 0.12]	[0.49, 0.57]
26	[0.43, 0.57]	0.62	[0.67, 0.78]	[0.5, 0.63]	0.12	[0.51, 0.57]
27	[0.43, 0.57]	0.62	[0.67, 0.78]	0.5	[0, 0.12]	[0.46, 0.54]
28	[0.43, 0.57]	0.62	[0.67, 0.78]	0.5	0.12	[0.48, 0.54]
29	[0.43, 0.57]	0.62	0.67	[0.5, 0.63]	[0, 0.12]	[0.49, 0.52]
30	[0.43, 0.57]	0.62	0.67	[0.5, 0.63]	0.12	[0.51, 0.52]
31	[0.43, 0.57]	0.62	0.67	0.5	[0, 0.12]	[0.46, 0.5]
32	[0.43, 0.57]	0.62	0.67	0.5	0.12	[0.48, 0.5]

Table 11: New products ranking on the basis of Plausibility curves.

Scheme	Q_i^*	Ranking
Scheme 1	0.45	8
Scheme 2	0.5	7
Scheme 3	0.55	5
Scheme 4	0.59	3
Scheme 5	0.59	3
Scheme 6	0.51	6
Scheme 7	0.62	2
Scheme 8	0.71	1
Scheme 9	0.38	10
Scheme 10	0.44	9

product selection and build an index system. The second stage is to determine the indicator weights through AHP and DST. Several experts score the indicators according to scoring criteria. Then the indicator weights given by different experts are calculated through AHP. Then the final weights are calculated through Dempster aggregation rule. In the third stage, all indexes of new schemes are predicted and preprocessed. This problem is forecasted by experts based on the fact that they are familiar with the market, enterprises, and each new scheme at the same time. Even so, experts use interval numbers more confidently to represent the indicators compared to numerical values. In order to facilitate future evaluation, we need to handle the interval values positively and nondifferently. The fourth stage is the evaluation of new schemes based on DST. By analyzing the belief and plausibility function of event $\overline{E} = \{Q_i \geq Q_i^*\}$, we can get the critical value Q_i^* with given confidence level. The critical value is used to express the evaluation of new products and to rank them. In order to prove how the proposed fuzzy hybrid method works, a new products evaluation in Qingdao is taken as an example to make an empirical study, through which the following conclusions can be drawn.

Table 12: Final ranking.

Scheme	confidence level q=0.9		confidence level q=0.8	Ranking
	Q_i^*	$Pl(Q_{ir} \geq Q_i^*)$	Q_i^*	
Scheme 8	0.71	–	–	1
Scheme 7	0.62	–	–	2
Scheme 4	0.59	0	0.61	3
Scheme 5	0.59	0	0.59	4
Scheme 3	0.55	–	–	5
Scheme 6	0.51	–	–	6
Scheme 2	0.5	–	–	7
Scheme 1	0.45	–	–	8
Scheme 10	0.44	–	–	9
Scheme 9	0.38	–	–	10

These indicators were selected by the research team based on years of work experience. The members of the research group are middle-level managers who have more than five years' working experience in the enterprise and have a lot of practical experience of NPD. These attributes are as follows: Technical Difficulty (C1) is the product and methods complexity which highlights the physical possibility of product realization; Product Performance (C2) is the superior quality and new product features, which reflects the product substantial competitive advantage; Market Potential (C3) reflects market demand, growth, and size, which shows an increase in demand for a particular product over time; Project Risk (C4) is about any feature that might potentially interfere with successful completion, including financial risks, managerial risks, envisioning risks, design risks, and execution risks. Project Cost (C5) depicts the consumption of resources. The attributes for NPD which are put forward in this case can be used for a reference to other enterprises.

What is more, the evaluation of new products is a prediction for future problems, which has many risks and uncertainties. Because it is a prediction for future, experts use fuzzy number more confidently compared to using numerical values to assess the possible future performance of every scheme. And a number of experts are invited to predict the future performance of the new schemes, which can improve the prediction accuracy. Therefore, there would be multiple fuzzy numbers for the same index of the same scheme. This adds to the difficulty of the evaluation and ranking of new products. A new product evaluation and ranking method based on DST is proposed in this paper. Dempster-Shafer Theory (DST) is suggested as a proper mathematical framework to deal with the epistemic uncertainty. This method allows experts to use interval numbers to express indicators and obtain all possible combinations as evidence through decomposition and merging approach. Through analyzing the Belief and Plausibility curve of event $\overline{E} = \{Q_{ir} \geq Q_i^*\}$, the critical value Q_i^* is obtained and used as the evaluation results of the scheme and sorted.

Finally, the method based on AHP and DST proposed in this paper has higher reliability of index weights. Weight determination is a key content for MADM problem. Subjective or objective methods were used to determine weight. However, the weight obtained by objective method often does not accord with the actual situation, and the weight obtained by subjective valuation method is often influenced by experts. The method based on DST and AHP is a comprehensive index weighting method which combines subjective and objective methods. If only one expert evaluates the importance of the index, it is difficult to ensure that the index weight is not affected by the expert's personal preferences. Therefore, many experts are often invited to evaluate the indicator weight to improve the accuracy of the evaluation. The method based on AHP and DST proposed in this paper is used to deal with the difference of index weight.

Data Availability

The data used to support the findings of this study are included within the article. No additional data are available.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Research Article

Exploring Multicriteria Elicitation Model Based on Pairwise Comparisons: Building an Interactive Preference Adjustment Algorithm

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Pairwise comparisons have been applied to several real decision making problems. As a result, this method has been recognized as an effective decision making tool by practitioners, experts, and researchers. Although methods based on pairwise comparisons are widespread, decision making problems with many alternatives and criteria may be challenging. This paper presents the results of an experiment used to verify the influence of a high number of preferences comparisons in the inconsistency of the comparisons matrix and identifies the influence of consistencies and inconsistencies in the assessment of the decision-making process. The findings indicate that it is difficult to predict the influence of inconsistencies and that the priority vector may or may not be influenced by low levels of inconsistencies, with a consistency ratio of less than 0.1. Finally, this work presents an interactive preference adjustment algorithm with the aim of reducing the number of pairwise comparisons while capturing effective information from the decision maker to approximate the results of the problem to their preferences. The presented approach ensures the consistency of a comparisons matrix and significantly reduces the time that decision makers need to devote to the pairwise comparisons process. An example application of the interactive preference adjustment algorithm is included.

1. Introduction

Preference judgment is a key issue in multicriteria decision-making (MCDM) methods; MCDM is a generic term given to a collection of systematic approaches and methods developed to support the evaluation of alternatives in a context with many objectives and conflicting criteria [1, 2].

Multicriteria methods can be classified into three groups: first, aggregation methods based on a single criterion of synthesis, whose main representatives are the multiattribute utility theory, the analytic hierarchy process (AHP), and MACBETH. The second group is outranking methods, such as the PROMETHEE and ELECTRE methods. Finally, the third group consists of the interactive methods, such as multiobjective linear programming (PLMO) [3, 4].

Different types of cognitive and behavioral biases play an important role in decision-making (DM). The MCDM aims to help people make strategic decisions according to their preferences and an overarching understanding of the problem [2], and it is subject to various cognitive and procedural deviations [5]. These deviations can occur at all stages of the decision-making process (problem structuring, evaluation criteria and alternatives, and sensitivity analysis), although they can lead to incorrect recommendations.

In additive models, inconsistency can be perceived in two distinct situations of decision maker (DM) judgments: intercriteria and intracriteria evaluations. The intercriteria evaluation involves the elicitation procedure for determining the weights of the criteria, where inconsistencies have been reported such as ratio [6], swing [7], trade-off [2, 8, 9], and fitradeoff [10]. In the intracriterion evaluation, a value function is determined for each criterion. At this stage, some methods consider a simplistic approach by assuming a linear value function, such as in Smarts and Smarter [7], or a

Table 1: Number of pairwise comparisons based on the number of criteria and alternatives.

		Number of Alternatives										Number of Alternatives								
		2	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*		*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Number of Criteria	*2*	2	4	6	8	10	12	14	16	18	*2*	2	6	12	20	30	42	56	72	90
	3	3	6	9	12	15	18	21	24	27	*3*	3	9	18	30	45	63	84	108	135
	4	4	8	12	16	20	24	28	32	36	*4*	4	12	24	40	60	84	112	144	180
	5	5	10	15	20	25	30	35	40	45	*5*	5	15	30	50	75	105	140	180	225
	6	6	12	18	24	30	36	42	48	54	*6*	6	18	36	60	90	126	168	216	270
	7	7	14	21	28	35	42	49	56	63	*7*	7	21	42	70	105	147	196	252	315
	8	8	16	24	32	40	48	56	64	72	*8*	8	24	48	80	120	168	224	288	360
	9	9	18	27	36	45	54	63	72	81	*9*	9	27	54	90	135	189	252	324	405
	10	10	20	30	40	50	60	70	80	90	*10*	10	30	60	100	150	210	280	360	450
		Reciprocal Transitive Matrix										*Traditional Method*								

more sophisticated approach to build a utility function that expresses risk attitudes [2]. Another group of methods builds the value function based on pairwise comparisons (PCs) of several preference statements (questioning), such as the analytic hierarchy process [11–13] and Macbeth [14, 15].

A common cognitive deviation in MCDM methods occurs when there is a PC of a large number of alternatives, which requires a great deal of cognitive effort by the DM [16–20]. In this case, several studies have reported concerns related to the applicability of this type of procedure in situations where the number of criteria and alternatives is quite large [17, 18, 20, 21]. In such situations, the number of PCs made by the DM grows at an alarming rate. The time that analysts spend with DMs is increasingly scarce, and convincing a top executive to spend hours, or even days, making PCs of alternatives and criteria is often not feasible.

This paper focuses on the preference judgment of a DM, based on PCs of a qualitative criterion, to build a value function. For this purpose, an analysis was conducted based on the PC process. The AHP is one of the best known multicriteria decision aid (MCDA) approaches and is thus widely used [1, 22]. AHP is an additive method proposed by Saaty [11–13] and, since its introduction, has attracted increasing attention from researchers [22–26]. The method converts subjective assessments of relative importance to a vector of priorities (value function of one criterion), based on PCs performed within each criterion. The comparative judgments are made using the fundamental scale devised by Saaty [11], and a consistency logic is applied to check the DM's judgments [27].

In this article, we explore the influence of consistency and inconsistency in preference assessments—based on PCs—by performing an experiment with several individuals to assess their preferences based on a given situation. A high level of inconsistency has been verified within the literature (reference). Additionally, we evaluate an alternative procedure to assess the DM's preferences. The preferences of a DM are assessed based on an interactive procedure of asking questions and adjusting preferences, thus reducing the time spent by the DM and assuming an acceptable level of possible inconsistencies.

This paper is organized as follows. The next section presents a review of the literature concerning the causes of inconsistencies in PCs, as well as several solutions. Section 3 describes the experiment conducted to verify the influence of inconsistency. Lastly, a preference adjustment algorithm in PCs is presented.

2. Literature Review

Benitez et al. [28] have found that human beings make more good decisions than bad ones throughout their lifetime and that irrationality is a common cause of bad decisions. They thus proposed a model based on the most likely choice to capture shifts in the decision-making (irrationality) process to reduce biases caused by inconsistent judgments.

A PC uses human abilities, such as knowledge and experience, to compare alternatives and criteria in a pairwise manner and assemble a comparisons matrix [29]. Inconsistency arises when some opinions of a comparisons matrix contradict others. Therefore, it is important to check the consistency of opinions when performing a series of calculations to arrive at the value of the consistency ratio (CR), which indicates the consistency of the comparisons matrix. For the PCs matrix, it is desirable that the CR for any comparisons matrix be less than or equal to 0.10 [25, 30].

The number of n PCs is $n(n-1)/2$; hence grouping and hierarchy structure should be used for larger n [21]. Bozóki et al. [20] performed a controlled experiment with university students (N = 227), which enabled them to obtain 454 PCs matrices. They conducted experiments with matrices of different sizes from different types of multicriteria decision support methods and found that the size of a matrix affects the DM's consistency (measured by inconsistency indices): An increasing matrix size leads to greater inconsistency.

Consider the multicriteria problem, where the DM must make a PC alongside five criteria and ten alternatives. In such a situation, the DM will have to allow time to perform 225 evaluations (Table 1), to compare the alternatives within each criterion. If we insert two more criteria, this number increases to 315 evaluations. Ten criteria with ten alternatives require 495 evaluations. The number of alternatives is certainly the largest source of comparisons, whereas all alternatives are compared considering each criterion. Equation (1) calculates the number of comparisons (CN) in the traditional method [21]:

$$CN = \frac{k}{2}[n(n-1)] \quad (1)$$

where CN is the number of PCs of the traditional method, k is the number of criteria, and n is the number of alternatives.

Alternatively, Equation (2) calculates the number of comparisons in the Reciprocal Transitive Matrix (RTM) Method; this method was initially proposed by Koczkodaj and Szybowski [31] and called Pairwise comparisons simplified.

$$CN^{*} = kn - 1 \quad (2)$$

where CN^{*} is the number of PCs of the RTM Method.

Previous studies have already proposed several solutions to this problem. Weiss and Rao [32], for example, have proposed reducing the required number of questions to each DM through the use of incomplete blocks administered to different responders. Furthermore, Harker [33] developed an incomplete PC technique that aims to reduce this effort by ordering the questions in decreasing informational value and terminating the process when the value of additional questions decreases below a certain level.

A comparisons matrix with a CR equal to zero is representative of a fully consistent DM; this matrix is known as a RTM or a consistent matrix [31, 34]. The process of building a reciprocal transitive comparisons matrix requires the DM to conduct comparisons of only one line of the comparison matrix. The remaining values are determined by obeying the mathematical assumptions of a RTM. The resulting matrix comparison is consistent, based on the comparison made by the DM's criteria or alternatives, with respect to a preselected criterion or alternative.

The process of building a RTM dramatically reduces the number of comparisons performed by the DM, as shown in Table 1. For example, consider a problem with seven criteria and eight alternatives. Using the traditional method, the DM performs 196 comparisons, while using the process of building a RTM reduces the number of comparisons to 49.

The effort of DMs is greatly reduced when building a RTM, which may reflect a more careful evaluation of PCs. On the other hand, an error or very imprecise evaluation of the initial comparison can cause distortion in the decision process [21]. Kwiesielewicz and Uden [35], for example, have found that the relationship between inconsistent and contradictory matrices of data exists as a result of the PC. The consistency check is performed to ensure that judgments are neither random nor illogical. The authors reveal that, even if a matrix successfully passes a consistency test, it can be contradictory. Thus, an algorithm for checking contradictions is proposed.

2.1. Issues with Pairwise Comparisons. Numerous studies have examined problems in the use of the PC [20, 36–38]. Some authors have dedicated their research to these problems. Some problems are related to this work, especially in the use of ratio scales and eigenvalue as a measure of inconsistency.

With regard to the ratio scale problem, Ishizaka et al. [38], Salo and Hämäläinen [39], Donegan et al. [40], and Lootsma [41] proposed new ratio scales to solve problems associated with the use of this type of scale. Goepel [37] and Koczkodaj et al. [41], moreover, conducted research comparing the scales but unanimously determined that the scales have limitations in their maximum value, which restricts the interpretation of the DM. However, using larger scales may increase uncertainties.

In some cases, an unlimited scale is required, especially when comparing measurable entities such as distance and temperature [42].

Another issue is the eigenvalue problem. Some authors agree that while the eigenvalue is used as a good approximation for consistent matrices, there are expressive results regarding the existence of better approximations, such as geometric means [42–44]. Some recent studies have found that the use of geometric means [45] is the only method that satisfies the principles of consistency and is immune to the problem of reversal order [45].

A PC matrix, refereed here as M, presents the relations between n alternatives. M is reciprocal if $m_{ij} = 1/m_{ji}$, for all $i, j = 1, \ldots n$. M is consistent if it satisfies the transitivity property $m_{ij} = m_{ik} x\, m_{kj}$, for all $i, j, k = 1, \ldots n$ [36, 46–48]. Note that while every consistent matrix is reciprocal, the converse is, in general, false [31].

When an n × n matrix M is not consistent, it is necessary to measure the degree of inconsistency. One popular inconsistency index, proposed by Saaty [11], is defined as follows:

$$Ic(M) = \frac{\lambda_{max} - n}{n - 1} \quad (3)$$

λ_{max} is the principal eigenvalue of M.

Let RI denote the average value of the randomly obtained inconsistency indices, which depends not only on n but on the method of generating random numbers, too. The consistency ratio (CR) of M, indicating inconsistency, is defined by [11, 13]

$$CR(M) = \frac{Ic(M)}{RI} \quad (4)$$

If the matrix is consistent, then $\lambda_{max} = n$, so $Ic(M) = 0$ and $CR(M) = 0$.

Saaty [11, 13] introduced eigenvalues to verify the consistency of PC matrices. Furthermore, Saaty considers a matrix to be consistent when $CR(M) \leq 0.1$, meaning that 10% of the deviation of the largest eigenvalue of a given matrix from the corresponding eigenvalue of a randomly generated matrix. Saaty's definition of consistency is good for any array order. The main disadvantage of Saaty's definition of consistency is the rather unfortunate threshold of 10% [47].

Koczkodaj [47] introduced a new way of measuring inconsistencies of a PCs matrix based on the measure of the lowest deviation of an M matrix in relation to a consistent RTM. The interpretation of the measure of consistency becomes easier when we reduce a basic reciprocal matrix to a vector of three coordinates $[a, b, c]$. We know that $b = a * c$

is valid for each RTM; thus, we can always produce three RTM (vectors) by calculating a coordinate of the combination of the two remaining coordinates. These three vectors are $[i, b, c]$, $[a, a * c, c]$ and $[a, b, t]$. The consistency measure (CM) is defined as the distance relative to the nearest RTM, represented by one of these three vectors for a given metric. Considering Euclidean (or Chebysheff) metrics, we have [47]

$$CM = \min\left(\frac{1}{a}\left|a - \frac{b}{c}\right|, \frac{1}{b}\left|b - ac\right|, \frac{1}{c}\left|c - \frac{b}{a}\right|\right) \quad (5)$$

Koczkodaj's consistency index [36, 42, 46, 47, 49–54] not only measures the inconsistency, but it also shows where it is larger, thus guiding the DM to reassess and correct the inconsistency. Additionally, Bozóki and Rapcsák [46] investigated some properties of the PCs matrix inconsistencies of Saaty and Koczkodaj. The results indicate that the determination of inconsistency requires further study.

Considering Saaty's inconsistency index, some questions remain to be answered [46]: What is the relationship between an empirical matrix of human judgments and a randomly generated one? Is an index obtained from several hundred randomly generated matrices the correct reference point to determine the level of inconsistency of the matched comparisons matrix constructed from human decisions for a real decision problem? How can one take matrix size into account more precisely?

Considering Koczkodaj's consistency index, an important issue seems to be the elaboration of the thresholds in higher dimensions or the replacement of the index with a refined rule of classification [46].

2.2. Correcting Inconsistencies. Wadjdi et al. [16] investigated the importance of data collection using forms to ensure the consistency of comparison matrices. The authors have proven that the proposed form can guarantee data consistency in the PCs matrix. The form can also be used in any other techniques, such as Fuzzy-AHP, TOPSIS, and Fuzzy-TOPSIS.

Xu [55] defined the concepts of incomplete reciprocal relation, additive consistent incomplete reciprocal relation, and multiplicative consistent incomplete reciprocal relation and subsequently proposed two goal programming models based on additive consistent incomplete reciprocal relation and multiplicative consistent incomplete reciprocal relation, respectively, to obtain the incomplete reciprocal relation priority vector.

Pankratova and Nedashkovskaya [56] employed a computer simulation to compare these methods of consistency improvement without the participation of a DM. It has been found that, taking an inadmissible inconsistent matrix of comparison with the consistency ratio equal to CR = 0.2 or CR = 0.3, consistency improvement methods can help decrease inconsistency up to admissible level $CR \leq 0.1$ for $n \geq 5$. However, the results reveal that these methods are not always effective. Drawing near to admissible inconsistency does not guarantee closeness to the real priority vector of decision alternatives.

A method for constructing consistent fuzzy preference relations from a set of n - 1 preference data was proposed by [57]. The authors stated that, with this method, it is possible to ensure better consistency of the fuzzy preference relations provided by the DMs. However, this approach differs from our interactive preference adjustment insofar as our analysis seeks to obtain information from the DMs, whereas the previous approach was mathematical.

Voisin et al. [58] have noted that several consistency indices for fuzzy PC matrices have been proposed within the literature. However, some researchers argue that most of these indices are not axiomatically grounded, which may lead to deviations in the results. The authors of the present paper overcome this lack of an axiomatically grounded index by proposing a new index, referred to as the knowledge-based consistency index.

Benítez et al. [59] proposed the use of a technique that provides the closest consistent matrix, given inconsistent matrices, using an orthogonal projection on a linear space. In another paper [60], the same authors proposed a framework that allows for balancing consistency and DMs' preferences, focusing specifically on a process of streamlining the trade-off between reliability and consistency. An algorithm was designed to be easily integrated into a decision support system. This algorithm follows a process of interactive feedback that achieves an acceptable level of consistency relative to the DMs' preferences.

Benítez et al. [61] also proposed a method for achieving consistency after reviewing the judgment of the comparisons matrix decider using optimization and discovering that it approximated the nearest consistent matrix. This method has the advantage of depending solely on n decision variables (the number of elements being compared), being less expensive than other optimization methods, and being easily implemented in almost any computing environment.

Motivated by a situation found in a real application of AHP, Negahban [62] extends previous work on improving the consistency of positive reciprocal comparative matrices by optimizing its transformation into almost consistent matrices. An optimization approach is proposed and integrated into the Monte Carlo AHP framework, allowing it to solve situations where distinct or almost insufficient matrices are generated through the direct sampling of the original paired comparison distributions—a situation that prohibits significant statistical analysis and effective decision-making through the use of the traditional AHP Monte Carlo.

Brunelli et al. [63] found evidence of proportionality between some consistency indices used in the AHP. Having established these equivalences, the authors proposed a redundancy elimination when checking the consistency of the preferences.

Xia and Xu [64] proposed methods to derive interval weight vectors from reciprocal relations to reflect the inconsistency that exists when the DMs' preferences are taken into account for alternatives (or criteria). The authors presented programming models to minimize the inconsistency based on multiplicative and additive consistency.

Benítez et al. [27], again, proposed a formula that provides, in a very simple manner, the consistent matrix closest to a reciprocal (inconsistent) matrix. Additionally, this formula is computationally efficient, since it only uses

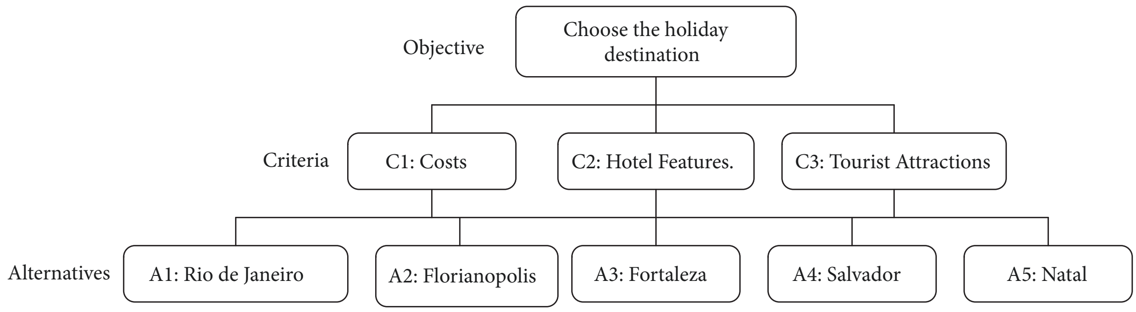


Figure 1: Criteria hierarchy of the holiday destination choice problem.

sums to perform the calculations. A corollary of the main result reveals that the normalized vector of the vector whose components are the geometric means of the rows of a comparisons matrix only produces the priority vector for consistent matrices.

The evaluation of consistency has also been studied using imprecise data. Thus, the theory of fuzzy numbers has been applied to MCDA methods to better interpret the judgment of DMs. The fuzzy AHP has been widely applied, and the evaluation of consistency in such situations has been the object of research. Bulut et al. [65] proposed a fuzzy-AHP generic model (FAHP) with pattern control consistency of the decision matrix for group decisions (GF-AHP). The GF-AHP improved performance using direct numerical inputs without consulting the decider. In practice, some criteria can easily be calculated. In such cases, consulting with the DM becomes redundant.

Ramík and Korviny [66] presented an inconsistency index for matrices of PC with fuzzy entries based on the Chebyshev distance. However, Brunelli [67] showed that the Chebyshev distance may fail to capture inconsistency and, as a result, should be replaced by the most convenient metric. Liu et al. [68] reported that the study of consistency is very important, since it helps to avoid erroneous recommendations. Accordingly, they proposed a definition of reciprocal relationships that privileged triangular fuzzy numbers that can be used to check fuzzy-AHP comparison matrices.

Xu and Wang [69] extended the eigenvector method (EM) to prioritize and define a multiplicative consistency for a list of incomplete fuzzy prioritizations. The authors presented an approach to judge whether this relationship is acceptable or not and subsequently developed a prioritization ratio of consistency to fuzzy incomplete similar to the one proposed by Saaty.

Koczkodaj et al. [49, 51, 54] used incomplete comparison matrices to create a system that monitors the inconsistency of the DM at each step of the PC process, informing the DM if it exceeds a certain threshold of inconsistency. The algorithm is constructed by locating the main reason for the inconsistency. Our research differs from the system proposed, in that it reduces the space in which the DM would be inconsistent by introducing an interactive process where, in the first stage, the DM is asked to complete only a single line of the matrix. Furthermore, with the help of an algorithm, we have captured and corrected deviations from the matrix in relation to the DM's preferences.

A recent study examined the notion of generators of the PCs matrix [31]. The proposed method decreased the number of PCs from $n(n-1)/2$ to $n-1$, which is similar to our approach, except that we use an algorithm that identifies and corrects deviations in the PCs matrix to better capture the DM's preferences.

3. Checking the Influence of Inconsistency

We have performed an experiment of a decision-making situation among students and staffs of a university, using a procedure based on the PCs matrix. The experiment aims to identify the influence of the number of PCs on the comparisons matrix consistency, in accordance with the research conducted by Bozóki et al. [20]. To measure consistency, we use the consistency ratio [11–13].

The experiment consisted of presenting different location options for a summer holiday to DMs. DMs completed a comparisons matrix whose number of alternatives grew as the DM concluded a comparative stage. The alternatives were the cities of Rio de Janeiro, Florianopolis, Salvador, Natal, Fortaleza, Maceió, João Pessoa, Aracaju, Vitória, Recife, Porto Alegre, and Curitiba, as presented in Figure 1.

While the experiment surveyed a sample of 180 people, only 76 answered it completely. Of the 76 who responded in full, only 30 dedicated the answers. The others only completed the questionnaire without any criteria or attention and were very inconsistent, even in the initial stages when the number of alternatives and the cognitive effort were minimal.

The evaluation process began with the comparison of three possible alternatives (cities). Then, 4 alternatives, 5 alternatives, and eventually up to 10 alternatives were evaluated. Individuals reported that, as the number of alternatives increased, their ability to discern the difference between them decreased. Lastly, the surveyed individuals were asked whether they agreed with the ranking obtained; 67% of the individuals reported that they had a different perspective than the presented results.

The results of the experiment indicate that inconsistency in the comparisons matrix increases as the number of alternatives increases, that is, as the comparisons increase (see Figure 2).

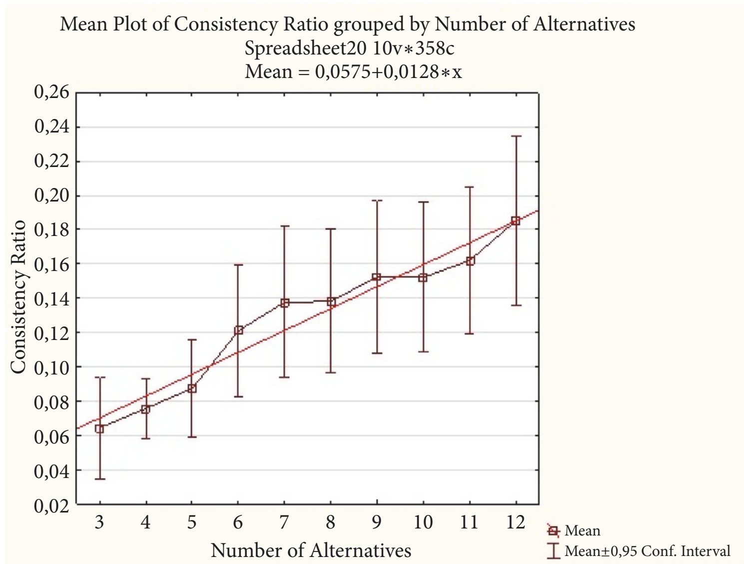


FIGURE 2: Behavior of the consistency ratio in relation to the number of alternatives.

TABLE 2: Results of all evaluations of alternatives.

Cities	CR =0/Rank		CR=0.02/Rank		CR=0.04/Rank		CR=0.06/Rank		CR=0.08/Rank		CR=0.10/Rank	
Rio de Janeiro	0.263	1	0.260	1	0.257	1	0.253	1	0.249	1	0.251	1
Florianópolis	0.052	6	0.051	6	0.051	6	0.050	6	0.051	6	0.057	6
Recife	0.131	3	0.139	3	0.138	3	0.137	3	0.136	3	0.137	3
Salvador	0.037	8	0.037	8	0.036	8	0.036	9	0.038	10	0.046	8
Natal	0.065	5	0.064	5	0.063	5	0.063	5	0.062	5	0.063	5
Fortaleza	0.258	2	0.252	2	0.249	2	0.248	2	0.249	1	0.228	2
Maceió	0.087	4	0.091	4	0.090	4	0.089	4	0.089	4	0.089	4
Joao Pessoa	0.033	9	0.035	9	0.035	9	0.034	10	0.040	8	0.040	9
Aracaju	0.044	7	0.043	7	0.048	7	0.048	7	0.047	7	0.050	7
Vitória	0.029	10	0.028	10	0.032	10	0.039	8	0.038	9	0.038	10

The findings indicate that comparisons of 3, 4, and 5 alternatives do not usually generate problems of consistency when considering CR <0.1, as recommended by Saaty, when using PCs. However, matrices with 6 or more alternatives usually generate CR> 0.1, thus extrapolating the limit.

The results further confirm the research by Bozóki et al. [20]. The cognitive effort to complete a comparisons matrix with more than 5 alternatives is significant, and fifteen successive comparisons are repeated as the problem increases the number of criteria. Thus, we propose a solution based on the reduction in the number of comparisons, verifying and correcting inconsistencies through an interactive algorithm.

The results from the previous experiment were used to explore the influence of inconsistency on the PCs Matrix. We selected 10 alternatives (cities), in which we have observed a high rate of inconsistency in all results. The matrix values were randomly changed so as not to reverse the DM's preferences. We analyzed the inconsistency in each example and then reproduced similar situations of inconsistency for a selected DM.

For example, if the DM compares A_1 and A_2, such as 4, and A_2 and A_3, as 2, to maintain consistency and transitivity, the comparison between A_1 and A_3 will be 6. The inconsistency introduced does not change the order $A_1 > A_2 > A_3$, however, although it would change the comparison between A_1 and A_3 to 5 or 7.

To verify the influence of inconsistencies, we created a consistent matrix formed by the RTM to serve as a reference point for each decision-maker. Therefore, a consistent comparisons matrix was created using selected responses provided by the decision-makers, i.e., the minimal necessary information. By including the additional information from the PC that was already provided by the decision-maker, we were able to generate different levels of inconsistency.

The results of a consistent matrix were compared with five other situations of inconsistency, all within an accepted level. In all situations, the matrices of comparison maintained the first line of evaluation completed by the DMs but, in each case, allowed for a different level of inconsistency (from 0 to 0.1).

The final ranking of the situations was analyzed for a given decision-maker. The results are presented in Table 2.

Table 3: Representation of the comparison indices that will be evaluated.

n odd		n even	
Alternatives	Comparison indices	Alternatives	Comparison indices
$\mathbf{A_2, A_3}$	$I_{2,3}$ or $I_{3,2}$	A_2, A_3	$I_{2,3}$ or $I_{3,2}$
...	...	...	...
$\mathbf{A_{n-3}, A_{n-2}}$	$I_{n-3,n-2}$ or $I_{n-2,n-3}$	A_{n-2}, A_{n-1}	$I_{n-2,n-1}$ or $I_{n-1,n-2}$
$\mathbf{A_{n-1}, A_n}$	$I_{n-1,n}$ or $I_{n,n-1}$	A_{n-1}, A_n	$I_{n-1,n}$ or $I_{n,n-1}$

Legend: A_i: ith alternative; $I_{i,j}$: comparison index of alternative A_i in relation to alternative A_j; n: number of alternatives.

An evaluation of the alternatives was created without inconsistencies and, keeping the first matrix's row fixed, four other evaluations were performed with CR = 0.02; 0.04; 0.06; 0.08; and 0.10 (Table 2).

A straightforward verification of the alternatives position in Table 2 shows that the variation of the CR can influence the final results. Considering that CR is equal to 0, 0.01, and 0.02, there is no change in ranking, although there are slight variations in the priority vector. With CR equal to 0.04, 0.06, 0.08, and 0.10, however, changes in ranking occur. This fact has been observed for all DMs, and it is an indication that inconsistencies directly influence the result; this finding is especially important for additive methods.

A similar result has been observed for all DMs who participated in the present experiment. In all cases, at least rank changed positions. This result leads to the conclusion that we cannot rely only on CR to measure the consistency of the comparisons matrix.

Inconsistency can affect the recommendations made to the DM and can even be detrimental, resulting in a wrong decision. For example, in all ranks, Rio de Janeiro was ranked first; however, when CR = 0.08, Rio de Janeiro was tied with Fortaleza. This tie could generate doubt in the DM and cause him to choose Fortaleza. In this example a wrong decision will most likely not have detrimental consequences, but for strategic decision-making an error could provoke serious consequences.

4. Pairwise Comparisons with Preference Adjustment

All research related to the consistency of a comparisons matrix stems from the impossibility of ensuring that DMs are consistent with their value judgment when making PCs of the alternatives.

In the previous section, we attempted to show how the inconsistency of DMs increases as the number of alternatives increases, and how this inconsistency influences the overall results. Additionally, inconsistency is influenced by the bias in the process of DM's elicitation of preferences.

Thus, we suggest a method that seeks to reduce and correct inconsistencies caused by DMs in the qualitative assessment of PCs.

This method is based on two procedures: First, the DM must access the preferences of a set of alternatives for a given criterion by comparing the set of alternatives to a reference alternative. The compassion process only occurs between the reference alternative, for instance, the best alternative, and the set of available alternatives. Thus, only one line of the comparisons matrix needs to be completed. In a second step, the remaining lines will be filled in with the help of the mathematical assumptions of the RTM. This ensures the consistency of the array and reduces the number of comparisons. The procedure uses the bases of the mathematical presuppositions of the RTM to verify acceptable values of inconsistencies. Later, the interactive algorithm will identify and correct the inconsistencies of the DM.

At this point, we assume that the restructuring process has already been undertaken and that the criteria and alternatives have already been identified. Decision makers subsequently sort alternatives from better to worse.

The proposed procedure reduces the number of preference comparisons demanded by the DM, while increasing the accuracy of their preferences. The DM fills in only the first row of the comparison matrices and a few pairs of alternatives of the other lines. This procedure results in a significant reduction of the number of PCs.

The interactive algorithm selects a few pairs of available alternatives to be evaluated, i.e., additional comparisons between the alternatives, to confirm the preference judgment of the DM with an estimated value in situations without inconsistency.

When DMs complete one row of a comparisons matrix, there are no inconsistencies caused by a lack of reciprocity; only deviations from judgment can occur. Nevertheless, the DM may not have been consistent with his or her own preferences. Thus, a preference adjustment in the comparisons matrix may be performed to avoid possible deviations. One way to accomplish this is proposed below.

4.1. Preference Adjustment Algorithm. The preference adjustment is based on questions posed to the DM. These questions review a comparison between two alternatives. These two alternatives should be chosen according to the number of alternatives.

The number of PCs has been presented previously to ensure that the minimum number of required assessments is performed. Only then can there be a good preference adjustment. Indeed, complete consistency can only be ensured if all comparison indices are evaluated. However, evaluating all comparison indices may be too expensive, thereby diminishing the advantage of reducing the number of PCs.

The number of revisions required depends on the number of alternatives (Table 3).

Table 4: Proposed standard questions to assess consistency.

Indices Range	Question
$1 \leq I_{i,j} \leq 1,5$	Considering the C_k criterion. Do the alternatives Ai and Aj also contribute to the objective?
$1,5 < I_{i,j} \leq 2,5$	There was indecision between the previous question and the next question.
$2,5 < I_{i,j} \leq 3,5$	Considering C_k criterion. Does your judgment slightly favor the alternative A_i on the alternative A_j?
$3,5 < I_{i,j} \leq 4,5$	There was indecision between the previous question and the next question.
$4,5 < I_{i,j} \leq 5,5$	Considering the C_k criterion. Does your judgment strongly favor the alternative A_i over alternative A_j?
$5,5 < I_{i,j} \leq 6,5$	There was indecision between the previous question and the next question.
$6,5 < I_{i,j} \leq 7,5$	Considering the C_k criterion. Does your judgment reveal that the alternative A_i is very strongly favored over alternative A_j, its domination of importance is demonstrated in practice?
$7,5 < I_{i,j} \leq 8,5$	There was indecision between the previous question and the next question.
$8,5 < I_{i,j} \leq 9$	Considering the C_k criterion. Does your judgment show that the A_i alternative overcomes the A_j alternative with the highest degree of certainty??

(i) For a comparisons matrix with an even number of elements n: Do not conduct assessments with comparison indices between the first row and the first column; there will be an odd number of elements to evaluate. It is not possible to form pairs with an odd number of elements without repetition, so n/2 + 1 evaluations would be needed.

(ii) For a comparisons matrix with an odd number of elements n: Again, do not conduct assessments with comparison indices between the first row and the first column; there will be an even number of elements to evaluate. In this case, n/2 evaluations would be needed.

Initially, the preference adjustment may seem costly and the number of evaluations may seem as large, as it is in the traditional PCs Matrix. However, this is not true. The solution to the problem with ten criteria and ten alternatives using RTM would require 145 evaluations (Table 1), 90 PCs, and 55 preference adjustments, whereas in the traditional PCs matrix there would be 450 PCs.

The number of comparison indices is always greater than zero. The comparison indices of the main diagonal may not be evaluated, as each receives a value of 1. The comparison indices of the first row and first column of the comparisons matrix must be completed directly by the DM when he or she makes the PCs, where the first column is a direct result of this comparison.

As the PCs made by the DM are based on an interpretation of Saaty's fundamental scale, the preference adjustment will also rely on this scale. A standard set of questions based on the fundamental scale of Saaty is proposed in Table 4. The questions are presented only for comparison scores higher than zero.

As previously mentioned, to ensure an intuitive judgment by the DM, it is important to rank the alternatives from best to worst based on the criterion in question. Thus, the DM should only be asked to compare ordered alternatives in such a way that the best alternative is always positioned in the first row.

The assessment procedure by the DM is formalized considering the whole procedure of preference adjustment. A standard algorithm that structures all steps of the analysis was created (Figure 3). The algorithm consists of six operations, which are presented below.

(1) *Identify and order the comparison indices to be evaluated:* Let n be the number of alternatives. To identify the comparison indices the rules below should be followed:

The first comparison index to be evaluated is always the one that compares the alternatives A2 and A3; it is $I_{2,3}$ or $I_{3,2}$.

The second comparison index to be evaluated is always the one that compares alternatives A4 and A5.

Repeat this procedure until the nth alternative. If n is positive, the last comparison index necessarily represents the comparison between An-1 and An; also, An-1 does not change, since it has been previously evaluated (Table 3).

(2) *Loop repeat:* After ordering the comparison indices to be evaluated, initiate the evaluation process of ordering until the last index comparison is evaluated, at which point the procedure ends.

(3) *Verification of range:* The beginning of the assessment consists in determining which range from Table 4 will be used to evaluate the comparison index.

(4) *Question:* Ask the DM the questions listed in Table 4.

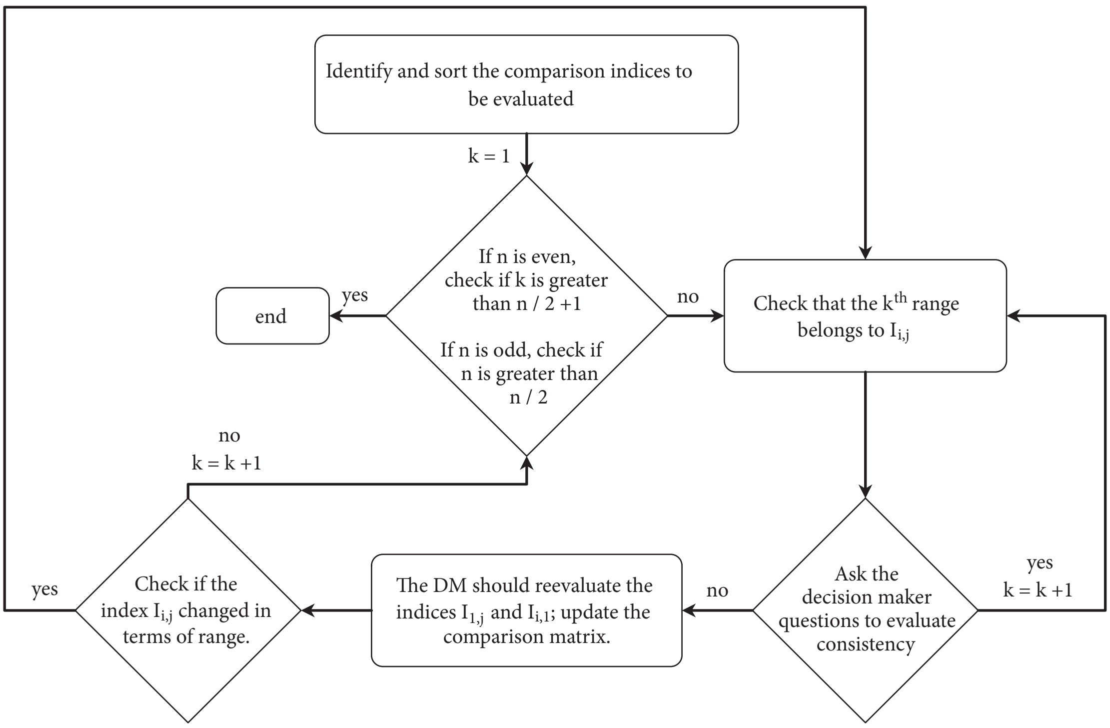
  (i) If the answer is yes, go back to step (2) to start the next evaluation.

  (ii) If the answer is no, proceed to step (5).

(5) *Reevaluation of index of the first line:* The DM should reevaluate the indices $I_{1,j}$ and $I_{i,1}$; update the comparisons matrix accordingly.

(6) *Determine if there was a change in the range:* Check if the index $I_{i,j}$ changed in terms of range, as shown in Table 4.

If there was no change in terms of range, consider that the DM was given the opportunity to reassess the



k: counter.

FIGURE 3: Preference adjustment algorithm.

comparison index of the first line and that, despite his or her negative response in step (4), the range did not change enough to change the landscape of the initial evaluation. Return to step (2) to start the next evaluation.

If there was a change in terms of range, go back to step (3) to conduct a new evaluation of the same index.

An important issue to be considered is the automatic change in another comparison index when the DM revises and changes an index of the first row. Similarly, it is wise for researchers to consider that some indices of comparison, as assessed by the DM, will be subjected to change. When these changes occur, they must be reevaluated. However, the algorithm solves this problem.

If an index in the first row is changed, the other indexes that depend on it are automatically corrected considering the mathematical assumptions of the RTM. Thus, it will correct the deviations in the DM's preferences and simultaneously maintain the consistency of the matrix.

As we did not perform evaluations with A1, start with $I_{2,3}$ or $I_{3,2}$. If the DM changes the value of $I_{1,2}$ and $I_{1,3}$, the underlined comparison indices will change.

Without repeating the lines already evaluated, the DM must evaluate $I_{4,5}$ or $I_{5,4}$, which were not altered by previous reevaluation. If the DM changes the value of $I_{1,4}$ and $I_{1,5}$, the comparison indices underlined in Table 5 will change. It is important to note that none of the possible indices previously evaluated, $I_{2,3}$ or $I_{3,2}$, will have changed.

TABLE 5: Change in the comparisons matrix due to the preference adjustment of $I_{1,4}$ and $I_{1,5}$.

	A1	A2	A3	A4	A5	A6
A1	$I_{1,1}$	$I_{1,2}$	$I_{1,3}$	$\underline{I_{1,4}}$	$\underline{I_{1,5}}$	$I_{1,6}$
A2	$I_{2,1}$	$I_{2,2}$	$\mathbf{I_{2,3}}$	$\underline{I_{2,4}}$	$\underline{I_{2,5}}$	$I_{2,6}$
A3	$I_{3,1}$	$\mathbf{I_{3,2}}$	$I_{3,3}$	$\underline{I_{3,4}}$	$\underline{I_{3,5}}$	$I_{3,6}$
A4	$\underline{I_{4,1}}$	$\underline{I_{4,2}}$	$\underline{I_{4,3}}$	$\underline{I_{4,4}}$	$\underline{I_{4,5}}$	$\underline{I_{4,6}}$
A5	$\underline{I_{5,1}}$	$\underline{I_{5,2}}$	$\underline{I_{5,3}}$	$\underline{I_{5,4}}$	$\underline{I_{5,5}}$	$\underline{I_{5,6}}$
A6	$I_{6,1}$	$I_{6,2}$	$I_{6,3}$	$\underline{I_{6,4}}$	$\underline{I_{6,5}}$	$I_{6,6}$

Finally, no index containing A_6 was evaluated and there is no alternative to pair with A_6. In this case, we must evaluate $I_{5,6}$ or $I_{6,5}$, as shown in Table 3. For this situation (even n), the DM can only change the value of $I_{1,6}$, since $I_{1,5}$ has previously been reevaluated and any change in $I_{1,5}$ would lead to a reassessment of $I_{5,4}$ because its value will be changed. If the DM changes the value of $I_{1,6}$, the comparison indices underlined in Table 6 will change. It is important to note that none of the possible indices previously evaluated ($I_{2,3}$, $I_{3,2}$, $I_{4,5}$, and $I_{5,4}$) will have changed; thus they do not require reassessment.

Table 6: Change in the comparisons matrix due to the preference adjustment of $I_{1,6}$.

	A1	A2	A3	A4	A5	A6
A1	$I_{1,1}$	$I_{1,2}$	$I_{1,3}$	$I_{1,4}$	$I_{1,5}$	$\underline{I_{1,6}}$
A2	$I_{2,1}$	$I_{2,2}$	$\mathbf{I_{2,3}}$	$I_{2,4}$	$I_{2,5}$	$\underline{I_{2,6}}$
A3	$I_{3,1}$	$\mathbf{I_{3,2}}$	$I_{3,3}$	$I_{3,4}$	$I_{3,5}$	$\underline{I_{3,6}}$
A4	$I_{4,1}$	$I_{4,2}$	$I_{4,3}$	$I_{4,4}$	$I_{4,5}$	$\underline{I_{4,6}}$
A5	$I_{5,1}$	$I_{5,2}$	$I_{5,3}$	$I_{5,4}$	$I_{5,5}$	$\underline{I_{5,6}}$
A6	$\underline{I_{6,1}}$	$\underline{I_{6,2}}$	$\underline{I_{6,3}}$	$\underline{I_{6,4}}$	$\underline{I_{6,5}}$	$\underline{I_{6,6}}$

Table 7: Preference evaluation of the holiday destination choice problem for the criterion Tourist Attractions.

	A1	A2	A3	A4	A5
A1	1.00	3.00	3.00	5.00	3.00

5. Application of the Procedure

We have applied the procedure for two situations. First, we used the problems explored in the experiment conducted on item 3 (tourist cities). Then, we tested the procedure with 10 DMs to determine the inconsistency and agreement level with the selected alternative.

5.1. Summer Holiday Case. We applied the proposed procedure to the case of selecting the next city to spend the summer holiday. The alternatives were Rio de Janeiro, Florianopolis, Salvador, and Natal, as presented in Figure 1.

The procedure involved one DM and the support of an analyst to run the interactive algorithm. We will assume three criteria with the following weights: W1 = 0.60, W2 = 0.10, and W3 = 0.30, as previously defined.

Details of the procedure are presented with the Tourist Attractions criterion (Tables 7 and 8). For this situation, alternative A1, Rio de Janeiro, is considered the reference alternative. Continuing, the procedure should be performed with the alternative's comparison matrices considering the three criteria.

Evaluate $I_{2,3} = 1$. Question: Considering the Tourist Attractions criterion, do alternatives A2 and A3 also contribute to the goal? The answer is yes, so the Matrix was not updated.

Evaluate $I_{5,4} = 1,67$. Question: Considering the Tourist Attractions criterion, do alternatives A5 and A4 also contribute to the goal or do your experience and judgment slightly favor alternative A5 over alternative A4? Are you sure about both situations? The DM says that alternatives A5 and A4 also contribute to the goal, $I_{5,4}$ assumes value 1, and we update the comparisons matrix.

$I_{5,4}$ *change in the range;* repeat the question to assess $I_{5,4} = 1$. Ask the DM: Considering the Tourist Attractions criterion, do alternatives A5 and A4 also contribute to the goal or do your experience and judgment slightly favor alternative A2 over alternative A4? The answer is yes, so the Matrix was not updated. At the end of the preference settings, the priority vectors are presented in Table 9.

The results presented in Table 10 indicate that the preference adjustment is an important opportunity for the DM to identify deviations in the comparisons matrix and thus correct them. It was found that the results, with and without adjustment, differ from one another in three ranking positions: Fortaleza moved from third position to fourth, Salvador from fourth to fifth position, and Florianopolis from fifth to third position.

5.2. Additional Experiment. In the experiment, ten DMs of the very inconsistent group were chosen to test the PCs with preference adjustment. The DMs presented CR higher than 0.2, and thus their results were not considered in the experiment.

We begin the experiment reassessing the DM's preferences for the first line of the matrix comparison. Then we apply the preference adjustment algorithm.

At the end of the process we asked two questions: Do you consider the effort and time spent applying the method to be reasonable? Do you consider that the result represents your preferences? For both questions, a five-point scale was used as an option for response: (1) No, not at all; (2) No, not much; (3) More or less; (4) In general, yes; (5) Yes, of course.

For question 1, seven DMs responded, "Yes, for sure," and three responded, "Usually, yes." This indicates that effort declined considerably compared to the traditional method.

For question 2, five DMs responded, "Yes, for sure," three responded, "Usually yes," and two responded, "More or less." This indicates that the results can be considered satisfactory in 80% of the cases.

The application of the proposed approach reveals that the effort required by the DMs to evaluate alternatives and to assess their preferences clearly decreased while attempting to maintain an acceptable level of inconsistency.

6. Conclusion

In this study, we demonstrate that the inconsistency of PC matrices, even within acceptable limits, can influence the results of a decision process. Thus, we proposed the following approach: First, the PC comprises only the first row of the matrix, while the other lines are filled in based on the assumptions of the RTM. Second, we use an algorithm to identify and correct deviations in the preferences of the DMs in the matrices.

Table 8: Additional preference information for the comparisons matrix of the holiday destination choice problem for the criterion Tourist Attractions.

	A1	A2	A3	A4	A5
A1	1.00	3.00	3.00	5.00	3.00
A2	0.33	**1.00**	1.00	1.67	1.00
A3	0.33	1.00	1.00	1.67	1.00
A4	0.20	0.60	0.60	1.00	0.60
A5	0.33	1.00	1.00	**1.67**	1.00

Bold cell, representing the additional value required by DM.

Table 9: Priority vectors of the alternatives for choosing the holiday destination.

	Costs		Hotel Features		Tourist Attractions	
Weight	0.60		0.10		0.30	
	With adjustment	Without adjustment	Without adjustment	With adjustment	Without adjustment	With adjustment
Rio de Janeiro	0.071	0.053	0.506	0.466	0.455	0.484
Florianopolis	0.100	0.158	0.169	0.233	0.152	0.161
Fortaleza	0.166	0.158	0169	0.155	0.152	0.161
Salvador	0.166	0.158	0.101	0.093	0.091	0.097
Natal	0.489	0.474	0.056	0.052	0.152	0.097

Table 10: Comparison of the problem's results of choosing the holiday destination without preference adjustment and with preference adjustment.

Alternatives	Without adjustment		With adjustment	
	Priority Vector	Ranking	Priority Vector	Ranking
Rio de Janeiro	0.243	2°	0.223	2°
Florianopolis	0.124	5°	0.166	3°
Fortaleza	0.162	3°	0.159	4°
Salvador	0.135	4°	0.133	5°
Natal	0.336	1°	0.318	1°

The process of building a RTM, when applied to the PCs matrix, reduces the DM's effort in the PCs. The significant reduction in a DM's effort stems from the building of a RTM, which entails a totally consistent evaluation. Traditionally, PCs allow for certain level of inconsistency: CR less than or equal to 0.10.

The most important issue concerns the measurement of deviations when inconsistencies are allowed. The simulation and the experiments in this work clearly indicate that allowing inconsistencies achieves different results than when using consistent DM evaluations, although this is not always the case. This result does not nullify the value of building the RTM, nor does it discourage the use of a traditional PCs matrix.

Ultimately, the most important aspect is to check the consistency of the evaluation results to confirm that the DM's priorities are reflected in his or her judgment. If the answer is negative, it may be necessary to reassess. In fact, this process is already included in virtually all MCDA methods, yet in the case of the PCs matrix, for which there are many alternatives, this would make decision making even more difficult. High cognitive efforts generate inconsistencies, in addition to requiring a significant amount of time. Thus, it is important to use tools that reduce cognitive effort and ensure satisfactory results.

The preference adjustment algorithm aims to complement the information provided by the DM. The adjustments approximate the results of the problem to the preferences of the DM. The procedures presented in this paper reduce the cognitive effort of the DM, eliminate inconsistencies in the comparison process, and present a recommendation that reflects the preferences of the DM.

More research is needed, however, to verify whether the results that are overly sensitive to inconsistency are linked to the scale used in the assessment, are caused by small differences between alternatives, or are simply the result of errors caused by the DM's high cognitive effort. With regard to the application of the proposed algorithm, an important issue is to examine the use of other scales and the consistency index associated with these scales, such as the scale proposed by Koczkodaj [46].

Data Availability

The data included in the study "Exploring Multicriteria Elicitation Model Based on Pairwise Comparison: Building an Interactive Preference Adjustment Algorithm" are the responsibility of authors Giancarllo Ribeiro Vasconcelos and Caroline Maria de Miranda Mota, who collected the data in an open database. There are no restrictions on access to them.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Research Article

A Novel Comprehensive Model of Suitability Analysis for Matching Area in Underwater Geomagnetic Aided Inertial Navigation

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Geomagnetic aided inertial navigation is a way to use the geophysical field for navigation. It can locate the carrier position by the correlation between geomagnetic data and running track. It is an effective mean to realize autonomous navigation. Matching area suitability is one of the important factors affecting geomagnetic aided inertial navigation. Through the suitability analysis of matching areas, the areas with obvious geomagnetic features and rich information are selected as matching areas, which can effectively improve the real-time and accuracy of geomagnetic aided navigation. However, matching area suitability analysis for geomagnetic aided navigation is a complex process and needs to consider diverse factors, based on which a decision may be made. The area suitability analysis inherently can be considered as a multicriterion decision analysis (MCDA) problem. This paper presented a novel comprehensive model combining principal component analysis (PCA) and analytical hierarchy process (AHP) to evaluate the suitability of the navigation matching area. Firstly, according to the features of the areas, key feature parameters and the corresponding weights are determined by PCA and AHP, respectively. Then comprehensive evaluation values of the navigation matching areas are calculated through the comprehensive model. Finally, experiments were implemented in Bohai Bay; the correlation-matching algorithm is applied to verify the validity of the model in the areas. The experiment results well indicate the consistency between the comprehensive evaluation value and the matching area suitability. It is reasonable to regard the comprehensive evaluation value as a basis for area suitability analysis.

1. Introduction

Geomagnetic aided navigation (GAN) has become a research focus in recent years because of its merits such as all-weather, passive, no-radiation, error bounded, and strong anti-interference ability [1]. The geomagnetic aided inertial navigation system is one of the GAN systems. The principle of geomagnetic aided inertial navigation system is that when a vessel enters the matching areas, a matching algorithm is used to obtain the position information; this information can correct the cumulative error of inertial navigation system (INS) in time to guarantee that the final integrated navigation system output is credible [2–4].

Since each matching area includes different geomagnetic information, the matching effect is different in each area. Selecting the areas that have rich geomagnetic features and good suitability can help improve navigation accuracy and stability, and optimize navigation path planning as well. The area suitability analysis is one of the key technologies for GAN, the objective of which is to identify the areas with the highest potential for meeting the needs of navigation consistently at an acceptable cost [5].

Suitability is an intrinsic property of the geomagnetic matching area. According to the published literature, there are many features describing the suitability of geomagnetic field, such as roughness, gradient deviation, information entropy, standard deviation, and correlation coefficient [6–8]. Many methods for evaluating the suitability of navigation matching areas based on these features have been proposed. Kang took geomagnetic entropy and geomagnetic variance entropy as reference standards of geomagnetic matching area [9]. In view of the problems of one-sided appraisal when using single feature parameter to evaluate geomagnetic map suitability, some multiattribute decision-making methods have been proposed. Zhu proposed a comprehensive evaluation method using entropy technology to enhance the weight, thus overcoming the poor objectivity of traditional fuzzy evaluation method when confirming the weight of index [10]. Li proposed a method for evaluating geomagnetic matching area by synthesizing several geomagnetic features [11]. All the above methods aggregate the feature parameters, but they are limited to the fusion of several parameters with abundant information. However, in practical application, some parameters' information is not rich but can still play a positive role in the evaluation of area suitability. Therefore, it is particularly important to consider the feature parameters extensively. In such a context, the idea of multicriterion decision analysis (MCDA) can be introduced to address this problem. The principle of MCDA is to aggregate decision-making information in a certain way, and then the schemes are sorted and optimized to provide reference for decision-makers (DMS) [12–14].

Among all the feature parameters for the matching area in GAN, some have great influence on the level of suitability. If these parameters can be qualitatively distinguished, that will be beneficial to the weight decision analysis. Principal component analysis is a commonly used multivariate analysis method; its core idea is dimension reduction. This feature of PCA is very meaningful when there are many analysis indicators, especially considering the large amount of calculation. Through principal component analysis, the analysis process and amount of calculations can be greatly reduced. Due to the advantages such as objectivity, simple calculation, and convenient application, PCA has been widely used in mathematical modeling, mathematical analysis, and other disciplines [15–17]. Li [15] introduced PCA into geomagnetic navigation matching area selection, which took each principal component as the object, and the contribution rate was used as the weight to obtain the comprehensive evaluation value.

Analytical hierarchy process (AHP) is a common method in MCDA. In recent years, AHP has been widely used to determine weights [18, 19] and thus can be used to perform weight decision analysis on matching area parameters. The advantage of AHP is that it can provide a quantitative reference for qualitative analysis of human subjective judgments. The disadvantage is that when constructing the judgment matrix, the importance between the parameters depends too much on empirical judgment, not objective enough, so the AHP cannot be directly used to calculate the evaluation value of the original data. It is with this in mind that Wang combined accelerated genetic algorithm and AHP to extract the ranking information of evaluation index sample set and determine the ranking weight of each evaluation index, which could obtain more accurate parameter weights [20].

In addition, there are some other weighting methods, such as Delphi method, entropy method, and coefficient of variation method [21–23]. The Delphi method relies entirely on the judgment of experts, and sometimes it is one-sided and difficult to review whether it is correct or not. The entropy method and coefficient of variation method are objective valuation methods, which can avoid the deviation caused by human factors. However, the entropy method neglects the importance of the index itself, and sometimes the weight of the index will differ greatly from the expected value. The coefficient of variation method does not pay enough attention to the specific significance of the index, and there will be some errors. Although AHP is also a subjective weighting method, its disadvantage is that the construction of evaluation matrix is not objective enough, but when combined with PCA, the PCA-AHP model can provide an objective reference. Therefore, based on the above analysis, the combination of PCA and AHP is more suitable in this case.

This paper combine the principal component analysis and the analytic hierarchy process to establish the PCA-AHP model, and then all parameters in the matching areas are analyzed and fused by the model, which ensures the integrity of the geomagnetic parameters. After the data in the navigation area are processed by PCA, the load coefficients of each principal component can provide objective basis for AHP to construct the judgment matrix, thereby obtaining more accurate weight information.

The structure of this paper is as follows. First, the common geomagnetic feature parameters in the matching areas are described. Then the principle of PCA and AHP is elaborated upon. After that, the PCA-AHP model is proposed. Based on the above, a comprehensive evaluation value of the matching area is calculated. Actual applications verify the proposed method, performances of the proposed method are also discussed, and some conclusions and suggestions are drawn from the experiments and discussion.

2. Geomagnetic Feature Parameters

The geomagnetic field data is stored in the computer in the form of a grid. The features extracted from the geomagnetic field data are called geomagnetic features. Suitability is an intrinsic property of geomagnetic fields. Since geomagnetic fields are expressed in digital geomagnetic maps, this property must be reflected by the geomagnetic grid content and statistical characteristics. Set the size of the geomagnetic map to $M \times N$ (M is the length in the latitudinal direction, N is the length in the longitudinal direction), and $f(i, j)$ is the geomagnetic field value of coordinate (i, j), where (i, j) corresponds to a pair of latitude and longitude coordinates (φ, λ). This paper considers several geomagnetic feature parameters as follows.

2.1. Average Value. Average value $\overline{f}$ represents the average of the magnetic field in the candidate matching areas.

$$\overline{f} = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N} f(i, j). \tag{1}$$

2.2. Standard Deviation. The geomagnetic standard deviation reflects the discretization and the general fluctuation of the geomagnetic field. The larger the standard deviation, the more obvious the fluctuation of the geomagnetic field, which is more favorable for geomagnetic matching. The standard deviation δ is defined as

$$\delta = \sqrt{\frac{1}{MN-1}\sum_{i=1}^{M}\sum_{j=1}^{N}\left(f(i,j)-\overline{f}\right)^{2}}. \quad (2)$$

2.3. Roughness. Geomagnetic roughness reflects the average smooth level and local heave of geomagnetic field in certain area. The bigger the roughness, the richer the geomagnetic information. The roughness r is defined as

$$r = \frac{r_x + r_y}{2}, \quad (3)$$

where

$$r_x = \sqrt{\frac{1}{M(N-1)}\sum_{i=1}^{M}\sum_{j=1}^{N-1}\left[f(i,j)-f(i,j+1)\right]^{2}}, \quad (4)$$

$$r_y = \sqrt{\frac{1}{(M-1)N}\sum_{i=1}^{M-1}\sum_{j=1}^{N}\left[f(i,j)-f(i+1,j)\right]^{2}}. \quad (5)$$

Here r_x denotes the roughness in latitudinal direction and r_y denotes the roughness in longitudinal direction.

2.4. Kurtosis Coefficient. Kurtosis coefficient reflects the data concentration level. The bigger the kurtosis coefficient, the higher the concentration of the data near the average value, and the more difficult for matching; conversely, the more uniform the distribution, the easier for matching. The kurtosis coefficient C_e is defined as

$$C_e = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}\frac{\left(f(i,j)-\overline{f}\right)^{4}}{\delta^{4}} - 3. \quad (6)$$

2.5. Skewness Coefficient. Skewness coefficient reflects the symmetry or skewness of the geomagnetic field, the bigger the value, the higher the asymmetry of the data, and the simpler the matching along the asymmetric direction; the smaller the value, the higher the symmetry of the date, and the easier the mismatching along the symmetric direction. The skewness coefficient C_s is defined as

$$C_s = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}\frac{\left(f(i,j)-\overline{f}\right)^{3}}{\delta^{3}}. \quad (7)$$

2.6. Information Entropy. Information entropy, also known as Shannon entropy, is proposed by Shannon to solve the problem of quantitative measurement of information [8]. Geomagnetic information entropy is used to measure the amount of geomagnetic information. The information entropy H is defined as

$$H = -\sum_{i=1}^{M}\sum_{j=1}^{N} p(i,j)\log_2 p(i,j), \quad (8)$$

where

$$p(i,j) = \frac{f(i,j)}{\sum_{i=1}^{M}\sum_{j=1}^{N} f(i,j)}. \quad (9)$$

3. PCA and AHP Model

3.1. The Principle of Principal Component Analysis. Principal component analysis (PCA) aims to reveal the intrinsic relationship between multifeatures and large sample size by using mathematical dimension reduction ideas. PCA can be used to transform multivariate date into a few uncorrelated synthetic variables and reduce the dimension of observation space, while retaining the most important information. In the progress of multicriterion decision analysis, the complexity of decision-making will increase greatly if the number of variables is large, so it is best to use as few variables as possible. In many cases, there is a certain correlation between the variables. When there is a certain correlation between the two variables, it can be explained that the information reflected by the two variables has a certain overlap. PCA is about excluding redundant variables (closely related variables) and creates as few new variables as possible, so that these new variables are uncorrelated, and these new variables keep as much as possible of the original information.

The detailed steps are as follows:

(1) Establish the evaluation matrix according to the geomagnetic field feature parameters of the matching area (evaluation index). Suppose the number of matching area is m and the evaluation index is n; then the evaluation matrix is

$$\mathrm{X} = \begin{pmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{pmatrix} = \left[x_{ij}\right]_{m\times n} \quad (10)$$
$$\left(i = 1,2,\ldots,m;\ j = 1,2,\ldots,n\right),$$

where x_{ij} represents the evaluation value of the *jth* indicator of the *ith* matching area.

(2) Normalize the evaluation matrix. Different evaluation indexes often have different dimensions and dimension units, which affect the results of data analysis. The purpose of normalization is to eliminate the dimension effect between indexes and to solve the comparability between data indexes. After data normalization, the original data are in the same order of magnitude, which is suitable for comprehensive comparison and evaluation.

$$z_{ij} = \frac{x_{ij} - \overline{x_j}}{\sigma_j}, \quad (11)$$

$$\overline{x_j} = \frac{1}{m}\sum_{i=1}^{m} x_{ij}, \quad (12)$$

$$\sigma_j = \sqrt{\frac{1}{m}\sum_{i=1}^{m}\left(x_{ij} - \overline{x_j}\right)^2}, \quad (13)$$

where $\overline{x_j}$ is the average value of jth index and σ_j is the variance of the jth index. The normalized evaluation matrix is

$$Z = \left[z_{ij}\right]_{m\times n} \quad (i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n). \quad (14)$$

(3) Calculate the correlation coefficient matrix R.

$$R = \left[r_{kj}\right]_{n\times n} \quad (k, j = 1, 2, \ldots, n). \quad (15)$$

(4) Solve the eigenvalues and corresponding eigenvectors of correlation coefficient matrix R. Suppose that $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n \geq 0$ are the eigenvalues of R and $L_{g1}, L_{g2}, \ldots, L_{gi}$ are the corresponding eigenvectors. Thus, the principal component can be obtained, which is represented by F_i.

$$F_i = \sum_{j=1}^{n} L_{gi} z_{ij} \quad (i = 1, 2, \ldots, n). \quad (16)$$

(5) Determine the number of principal components. In order to reduce the workload and the loss of information as much as possible, only the first k principal components are kept. The k value can be determined through the cumulative contribution rate $\alpha(k)$, and the criteria are as follows:

$$\alpha(g) = \frac{\lambda_g}{\sum_{g=1}^{n}\lambda_g}, \quad (17)$$

$$\alpha(k) = \sum_{g=1}^{k}\alpha_g = \sum_{g=1}^{k}\left(\frac{\lambda_g}{\sum_{g=1}^{n}\lambda_g}\right). \quad (18)$$

The greater the contribution rate, the stronger the information about the original variable contained in the principal component. Generally, the k principal components whose cumulative contribution rate is above 80% and whose eigenvalues are greater than 1 are taken as the principal components for the final selection.

(6) After determining the number of principal components, the eigenvectors corresponding to each principal component eigenvalue are calculated according to the eigenvalues of the correlation coefficient matrix, and the influence of each index on the principal component can be known according to the descending order of the eigenvector coefficients. The larger the value, the greater the influence of the indicator on the principal component. If the coefficient is less than 0.1, its influence on the principal component is insignificantly small and can be neglected; the index represented by the coefficient can be eliminated.

Table 1: Matrix judgment scale.

Scale	Meaning
1	Equal importance
3	Weak importance
5	Essential or strong importance
7	Very strong importance
9	Absolute importance
2,4,6,8	The intermediate values of the adjacent judgments mentioned above

If the importance ratio of element i to element j is a_{ij}, then the importance ratio of element j to element i is $a_{ji} = 1/a_{ij}$.

According to the above steps, a complete index system that can comprehensively reflect the information contained in all indexes can be determined and can eliminate the correlation between the indexes and facilitate the operation of AHP.

3.2. The Principle of Analytical Hierarchy Process. The analytic hierarchy process (AHP) was put forward by Saaty in the middle of 1970s [24]. It is a combination of a qualitative and quantitative, systematic and hierarchical analysis method. Because of its practicality and effectiveness in dealing with complex decision-making issues, it soon gained worldwide attention. The principle of AHP is to decompose the problem according to its feature parameters and the ultimate goal to be achieved, and then classify and combine the factors at different levels according to their interrelated effects and membership relations, so as to form a structural model with multilevel analysis. Finally, the problem is attributed to the determination of the relative importance weights of the lowest level relative to the highest level or the arrangement of merits and demerits.

The detailed steps are as follows.

(1) Establish a Hierarchical Structure Model. The general structural model is divided into three layers, the top is the target layer, the bottom is the plan layer, and the middle is the criterion layer.

(2) Construct All the Judgment Matrixes in Each Level. The judgment matrix is constructed by comparing the weights of the criteria with respect to the target by mutual comparison. In the analytic hierarchy process, in order to quantify the importance of each element in the matrix, a matrix judgment scale (1-9 scale) is introduced, as shown in Table 1.

(3) Hierarchical Single Ranking. Hierarchical single ranking refers to the importance ranking of the elements at the current level for a certain element on the previous level. The relative weights of the compared elements for the current level elements are calculated by the judgement matrix.

For the judgement matrix A, calculate the eigenvalues and eigenvectors that satisfy

$$AW = \lambda'_{\max} W, \quad (19)$$

Table 2: Random consistency index.

n	1	2	3	4	5	6	7	8	9	10	11
RI	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49	1.51

where $\lambda'_{\max}$ is the largest eigenvalue of matrix A and W is the normalized eigenvector corresponding to $\lambda'_{\max}$. The component ω_i of W is the weight of the corresponding elements in single order. The detailed steps are as follows:

(a) Normalize each column vector of A.

$$\check{\omega}_{ij} = \frac{a_{ij}}{\sum_{i=1}^{n} a_{ij}}. \tag{20}$$

(b) Sum each row of $\check{\omega}_{ij}$.

$$\check{\omega}_i = \sum_{j=1}^{n} \check{\omega}_{ij} . \tag{21}$$

(c) Normalize $\check{\omega}_i$.

$$\omega_i = \frac{\check{\omega}_i}{\sum_{i=1}^{n} \check{\omega}_i}, \tag{22}$$

$$\omega = (\omega_1, \omega_2, \dots \omega_n)^T . \tag{23}$$

(d) $\lambda'_{\max}$ can be calculated from

$$\lambda'_{\max} = \frac{1}{n}\sum_{i=1}^{n}\frac{(A\omega)_i}{\omega_i}. \tag{24}$$

(4) Consistency index calculation:

Calculating the weight vector from the judgment matrix requires that the judgment matrix has a general consistency, and the consistency index CI needs to be calculated by

$$CI = \frac{\lambda'_{\max} - n}{n-1}. \tag{25}$$

In order to test whether the judgment matrix has satisfactory consistency, it is necessary to find the standard for measuring the consistency index CI of the matrix A, and Saaty introduced the random consistency index RI, as shown in Table 2 [16].

Consistency ratio CR is defined as

$$CR = \frac{CI}{RI}. \tag{26}$$

When $CR < 0.1$, the degree of inconsistency of A is within the allowable range and the eigenvector of A can be used as the weight vector.

(5) Hierarchical total ranking.

Hierarchical total ranking is a process of ranking weights to determine the relative importance of all elements in a layer to the overall objective. Suppose the rank of the M elements $B_1, B_2, \dots, B_m$ in layer B to total objective is $b_1, b_2, \dots, b_m$; the hierarchical single ranking of the N elements in layer C to layer B is $c_{1j}, c_{2j}, \dots, b_{nj}$ $(j = 1, 2, \dots, m)$. Then the hierarchical total ranking of layer C is

$$\begin{aligned} &C_1 : b_1c_{11} + b_2c_{12} + \cdots b_mc_{1m} \\ &C_2 : b_1c_{21} + b_2c_{22} + \cdots b_mc_{2m} \\ &\quad\cdots \\ &C_n : b_1c_{n1} + b_2c_{n2} + \cdots b_mc_{nm}. \end{aligned} \tag{27}$$

That is, the weight of the *ith* element of the C layer to the total objective is

$$\overline{W_i} = \sum_{j=1}^{m} b_j c_{ij}. \tag{28}$$

Suppose that the hierarchical single ranking consistency index of the factors in the C-layer to the B-layer is CI_j, and the random consistency index is RI_j; then the total order consistency ratio is

$$CR' = \frac{a_1CI_1 + a_2CI_2 + \cdots a_mCI_m}{a_1RI_1 + a_2RI_2 + \cdots a_mRI_m}. \tag{29}$$

When $CR' < 0.1$, it means that the hierarchical total ranking passes the consistency check.

3.3. Establishment of PCA-AHP Model. The purpose of establishing the PCA-AHP model is to get a comprehensive evaluation value of navigation area suitability. The flowchart of the model is shown in Figure 1.

There are four main methods to synthesize multi-index: linear synthesis method, geometric synthesis method, mixed synthesis method, and model synthesis method. Simply speaking, they are weighted summation, weighted geometric average, linear weighting, and geometric synthesis [25–28]. In this paper, the comprehensive evaluation values of the matching areas can be determined by

$$T_i = \sum_{j=1}^{n} z_{ij}\overline{W}_i. \tag{30}$$

The larger T_i is, the better the suitability performance of the *ith* candidate matching area is. According to this, each matching area can be sorted.

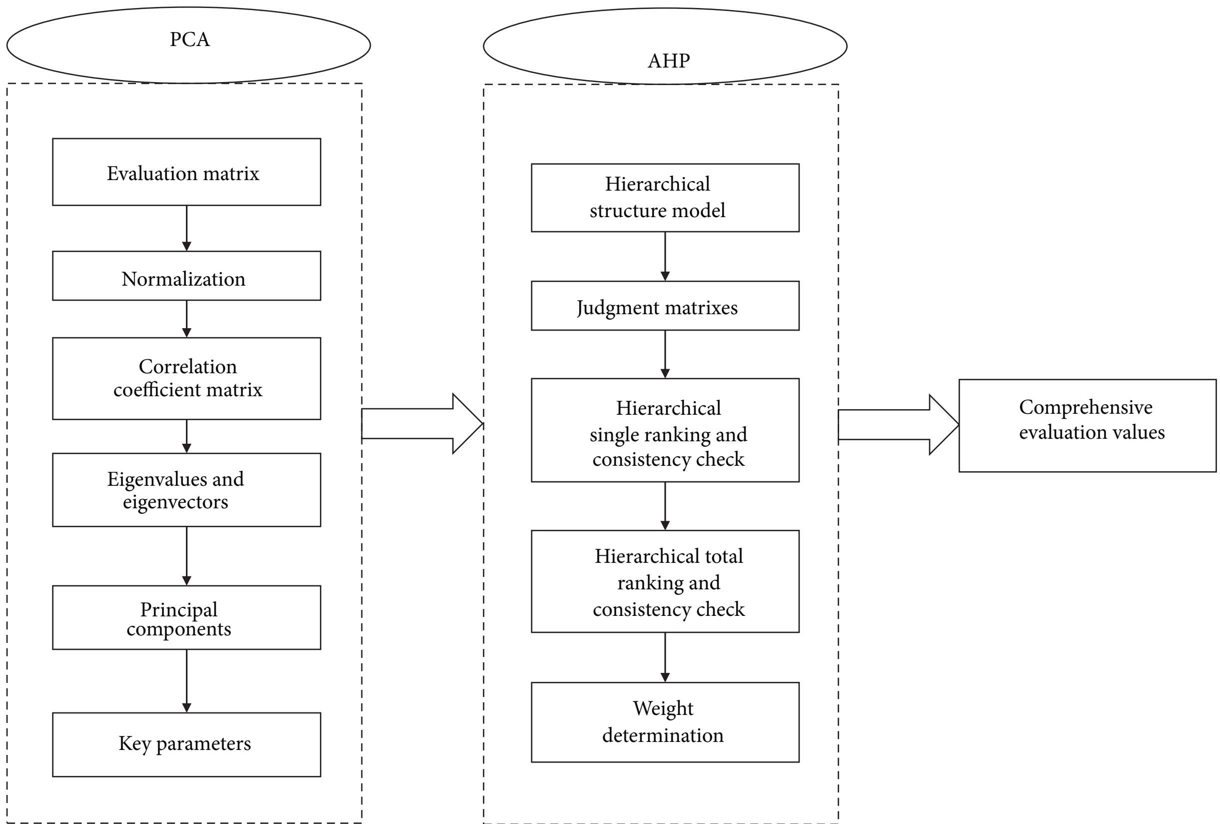

Figure 1: PCA-AHP model.

4. Experiment and Analysis

An experiment was implemented in a water area in Bohai Bay. An underwater reference for simulation is illustrated in Figure 2. The size of the geomagnetic map is about 20km∗10km. Through the geomagnetic background field modeling, the geomagnetic map with ±5nT accuracy is obtained. The PCA and PCA-AHP were used to obtain the comprehensive evaluation values of each matching area, and then the accuracy was verified by experiments.

In order to test the methods mentioned above, seven candidate matching areas of size 15×15 (grid precision: 200m) are selected, which are marked with red squares in Figure 2. The feature parameters of the candidate matching areas are calculated and listed in Table 3.

4.1. Principal Component Analysis. According to the (10)-(15), the normalized evaluation matrix Z and the correlation matrix R can be obtained.

$$Z = \begin{bmatrix} -0.500 & -0.482 & -0.182 & -0.258 & -0.802 & -1.431 & 0.685 \\ -1.162 & -0.278 & -1.067 & -0.945 & -0.802 & 0.065 & -1.376 \\ 0.436 & 1.211 & 0.078 & 0.250 & 1.069 & -0.034 & -0.683 \\ 1.683 & 1.599 & 1.792 & 1.837 & -0.802 & -0.526 & -0.241 \\ 0.706 & -0.461 & 0.779 & 0.638 & -0.802 & -0.599 & -0.623 \\ -0.263 & -0.482 & -0.596 & -0.619 & 1.069 & 1.509 & 1.452 \\ -0.900 & -1.106 & -0.804 & -0.903 & 1.069 & 1.017 & 0.786 \end{bmatrix}. \quad (31)$$

$$R = \begin{bmatrix} 1.000 & 0.763 & 0.969 & 0.975 & -0.227 & -0.332 & -0.163 \\ 0.763 & 1.000 & 0.694 & 0.771 & -0.118 & -0.298 & -0.407 \\ 0.969 & 0.694 & 1.000 & 0.993 & -0.412 & -0.490 & -0.187 \\ 0.975 & 0.771 & 0.993 & 1.000 & -0.397 & -0.484 & -0.249 \\ -0.227 & -0.118 & -0.412 & -0.397 & 1.000 & 0.777 & 0.485 \\ -0.332 & -0.298 & -0.490 & -0.484 & 0.777 & 1.000 & 0.407 \\ -0.163 & -0.407 & -0.187 & -0.249 & 0.485 & 0.407 & 1.000 \end{bmatrix}. \quad (32)$$

Solve the eigenvalues λ_n and eigenvector L_{gi} of the correlation matrix R and calculate the contribution rate $\alpha(g)$ and cumulative contribution rate $\alpha(k)$ by formula (17) and (18), as listed in Table 4.

As can be seen from Table 4, the eigenvalues of the first two principal components are greater than 1 and the cumulative contribution rate is greater than 80%. This means that the first two principal components contain most of the information of the original matrix and can be used to evaluate the suitability of the matching areas.

According to formula (18) and Table 4, it can be found that the absolute value of standard deviation, x-roughness, y-roughness, and roughness has greater load factors in the first principal component. The absolute values of entropy, kurtosis coefficient, and skewness coefficient have greater load factors in the second principal component.

4.2. Index Weights Determination by AHP. According to the principal components and key parameters determined by

Table 3: The parameters in the candidate matching areas.

Area	Standard Deviation /nT	x-Roughness /nT	y-Roughness /nT	Roughness /nT	Entropy /bit	Kurtosis Coefficient	Skewness Coefficient
1	18.513	1.204	2.203	2.511	7.053	-1.454	0.318
2	9.886	1.273	1.233	1.772	7.053	-0.776	-0.298
3	30.721	1.776	2.488	3.057	7.082	-0.821	-0.091
4	46.993	1.907	4.366	4.764	7.053	-1.044	0.041
5	34.242	1.211	3.256	3.474	7.053	-1.077	-0.073
6	21.615	1.204	1.749	2.123	7.082	-0.122	0.547
7	13.303	0.993	1.522	1.817	7.082	-0.345	0.348

Table 4: Eigenvalues, $\alpha(g)$ and $\alpha(k)$.

Eigenvalue λ_n	Eigenvector L_{gi} $(i=1,2,3,4,5,6,7)$ corresponding to λ_g							$\alpha(g)$	$\alpha(k)$
	L_{g1}	L_{g2}	L_{g3}	L_{g4}	L_{g5}	L_{g6}	L_{g7}		
4.192	-0.440	-0.387	-0.461	-0.470	0.274	0.318	0.218	0.599	0.599
1.567	-0.323	-0.235	-0.188	-0.191	-0.587	-0.481	-0.439	0.224	0.823
0.813	0.066	-0.460	0.210	0.101	-0.278	-0.291	0.754	0.116	0.939
0.286	0.194	-0.508	0.217	0.137	-0.351	0.662	-0.279	0.041	0.980
0.133	0.206	-0.560	0.163	0.049	0.602	-0.376	-0.335	0.019	0.999
0.009	-0.785	0.005	0.467	0.377	0.135	0.061	-0.035	0.001	1
1.68E -16	-0.021	-0.114	-0.646	0.754	0.017	-0.011	0.014	0	1

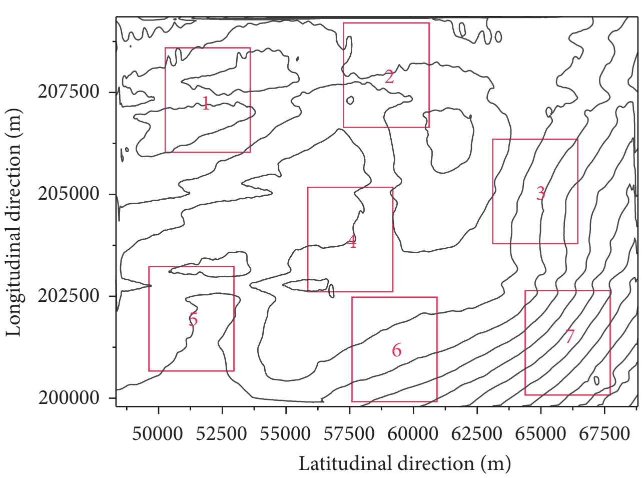

Figure 2: Underwater reference geomagnetic map for simulation.

PCA, a hierarchical structure is established as shown in Table 5.

The judgment matrix A_A is established according to the eigenvalue corresponding to each principal component and rules in Table 1. The larger the eigenvalue, the more important the corresponding principal component. The judgment matrices A_{B1}, A_{B2} are established according to the feature parameters' importance in every principal component and the rules in Table 1. A_A, A_{B1}, A_{B2} are shown in Tables 6, 7, and 8.

From Tables 6, 7, and 8, the comprehensive weight can be obtained, as shown in Table 9.

4.3. Comprehensive Evaluation Value. According to reference [15] and (16)-(18), the comprehensive evaluation values obtained by PCA can be determined by (33); the results are shown in Table 10.

$$T_i' = \sum_{g=1}^{k} \alpha(g) F_i \quad (i = 1, 2, \ldots, m). \tag{33}$$

The comprehensive evaluation value of each matching area determined by PCA-AHP can be obtained by (30); the results are shown in Table 11.

It can be seen from Tables 10 and 11 that the comprehensive evaluation values of the matching regions obtained by PCA and PCA-AHP are quite different. The validity of the two methods will be verified by experiments.

4.4. Consistency Analysis of Comprehensive Evaluation Value and Area Suitability. Generally speaking, the greater the comprehensive evaluation value, the better the area suitability. In order to confirm whether the comprehensive evaluation value can reflect the area suitability, other simulation experiments are carried out in these areas. In the above seven areas, a track with the most stable matching performance under zero noise is simulated. When designing a route, it is necessary to avoid track passing through the highly symmetrical areas. After choosing the track, the navigation

Table 5: Hierarchical structure.

A(Objective level)	B(Criterion level)	C(Index level)
Matching area suitability evaluation	B1 The first principal	C11 standard deviation
		C12 x-roughness
		C13 y-roughness
		C14 roughness
	B2 The second principal	C21 entropy
		C22 kurtosis coefficient
		C23 skewness coefficient

Table 6: Judgment matrix A_A.

A_A	B_1	B_2
B_1	1	5
B_2	1/5	1

$\lambda_{max} = 2$, CI = 0, CR = 0 < 0.1 $\omega = (0.833\ 0.167)^T$

Table 7: Judgment matrix A_{B1}.

A_{B1}	C11	C12	C13	C14
C11	1	3	1	1
C12	1/3	1	1/3	1/3
C13	1	3	1	1
C14	1	3	1	1

$\lambda_{max} = 3.999$, CI $= -2.501 \times 10^{-4}$, CR $= -2.778 \times 10^{-4} < 0.1$ $\omega = (0.3\ 0.1\ 0.3\ 0.3)^T$

Table 8: Judgment matrix A_{B2}.

A_{B2}	C21	C22	C23
C21	1	3	3
C22	1/3	1	1
C23	1/3	1	1

$\lambda_{max} = 2.999$, CI $= -3.334 \times 10^{-4}$, CR $= -5.748 \times 10^{-4} < 0.1$ $\omega = (0.6\ 0.2\ 0.2)^T$

Table 9: Comprehensive weight.

Level	B_1 0.833	B_2 0.167	Comprehensive weight
C11	0.3		0.250
C12	0.1		0.083
C13	0.3		0.250
C14	0.3		0.250
C21		0.6	0.100
C22		0.2	0.033
C23		0.2	0.033

experiment is carried out with the parameters described in Table 12. Random noises with errors of 1 nT, 2 nT, 3 nT, 5 nT, and 8 nT are added to the measured geomagnetic intensity series.

Table 10: Comprehensive evaluation values determined by PCA.

Matching area	Comprehensive evaluation value	Ranking
1	0.3237	4
2	1.0504	2
3	-0.5860	5
4	-2.2633	7
5	-0.6564	6
6	0.8220	3
7	1.3096	1

Table 11: Comprehensive evaluation values determined by PCA-AHP.

Matching area	Comprehensive evaluation value	Ranking
1	-0.3798	5
2	-0.9400	7
3	0.3747	2
4	1.3553	1
5	0.3718	3
6	-0.2048	4
7	-0.5771	6

Table 12: Matching experiment parameters.

Parameter	Value
Initial deviation in latitudinal direction	1500m
Initial deviation in longitudinal direction	1500m
Initial deviation of INS angle	1°
INS angle drift error	0.01°/h

If there is no noise in the measured track, the above areas can achieve good matching accuracy. Therefore, in order to reflect the suitability of each area, geomagnetic noise with errors ranging from 1.0 nT to 8.0 nT is added to the measured data. The matching result is shown in Figure 3.

From Figure 3, it can be clearly seen that the matching suitability of area 4 is the most stable; even if the noise reaches 8 nT, it still maintains a good matching accuracy. Secondly, area 5 keeps a good matching accuracy before the noise reaches 5 nT. Area 3 and area 1 mismatch at 5 nT, but the

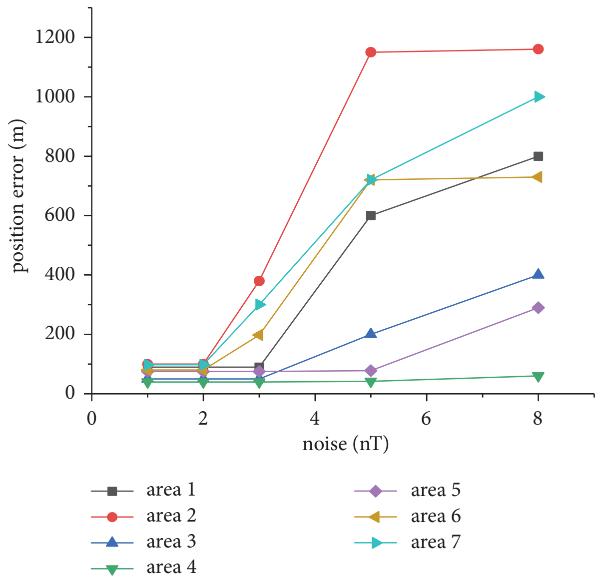

Figure 3: Performance analysis of different matching areas.

matching accuracy of area 5 is slightly higher than that of area 1. When the noise is 3 nT, the accuracies of areas 2, 6, and 7 all begin to deteriorate, but the precision of area 2 decreases rapidly, and the other two areas keep good consistency.

By analyzing the variation of matching accuracy with the increase of measured geomagnetic errors, the matching suitability of each area is as follows:

$$4 > 5 > 3 > 6 > 1 > 7 > 2. \quad (34)$$

Comparing (34) with the comprehensive evaluation values in Tables 10 and 11, it can be found that the comprehensive evaluation values obtained by PCA-AHP model are more accurate than the comprehensive evaluation values obtained by PCA and have good consistency with the area matching suitability.

In the PCA-AHP model:

(1) The ranking of comprehensive evaluation value and matching performance for areas 3 and 5 are slightly different. The comprehensive evaluation values of areas 3 and 5 are 0.3747 and 0.3718, and the difference is very small. Because the standard deviation and roughness of area 5 are larger than those of area 3, the matching accuracy of area 5 is slightly better than that of area 3. The reason for the inconsistency is that only seven feature parameters are used to describe candidate matching areas, and the suitability is not evaluated comprehensively. If there were more parameters, the suitability of the matching area can be more comprehensively evaluated, and the probability of inconsistency would be lower. Therefore, the inconsistency in ranking of areas 3 and 5 is acceptable.

(2) The geomagnetic variation standard deviation corresponds best to the geomagnetic matching performance. The standard deviation of area 4 is 46.993 nT, which is much higher than 9.886 nT in area 2. The experimental results also show that the matching stability of area 4 is much higher than that of area 2. The geomagnetic standard deviation of the remaining areas is also in good agreement with the results described in (34). It shows that the standard deviation is extremely related to the area matching suitability.

(3) The geomagnetic roughness is also the largest in area 4, reaching 4.76 nT, which is higher than that in the remaining areas. Area 2 has the smallest roughness of 1.772 nT. The remaining geomagnetic roughness changes are also in good agreement with (34). It shows that the geomagnetic roughness is related to the area matching suitability.

(4) In the experiment water area, the change of geomagnetic entropy is maintained between 7.053 bits and 7.082 bits, which is not obvious and cannot reflect the change of matching suitability very well. Nevertheless, geomagnetic information entropy reflects the richness of geomagnetic features and is an indispensable feature parameter for judging geomagnetic matching suitability; when the regional area difference is large, it is still recommended to be adopted.

(5) The variation of kurtosis coefficient and skewness coefficient is irregular, which cannot directly reflect the change of matching suitability.

In addition, among the seven matching areas, some areas have rich feature information, some feature information is not rich enough, and some areas have similar feature information, with little difference. From the consistency analysis, PCA-AHP can accurately give the evaluation value. Therefore, it is believed that this method can work elsewhere when the conditions are different.

The PCA has a large difference between the comprehensive evaluation values and the test results. There may be two reasons as follows.

(1) The abandoned principal component with small contribution rate may contain important information.

(2) The method of calculating the comprehensive evaluation value or determining the weight is inappropriate.

5. Conclusions

A single geomagnetic feature cannot fully reflect the performance of geomagnetic matching area. To address this problem, a method of multigeomagnetic feature fusion evaluation based on PCA-AHP model is proposed in this paper. The experiment results show that there is an adequate level of consistency between the comprehensive evaluation value and the area matching suitability. It is effective to evaluate the matching suitability of candidate matching areas by using PCA-AHP model. This method can achieve comprehensive evaluation of candidate area suitability by considering multiple geomagnetic features. The conclusions provide a quantitative basis for the analysis of candidate area suitability. In practical application, this method can help select the best matching area from the candidate matching areas, which can be used for the matching area selection of geomagnetic aided navigation.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflict of interest regarding the publication of this paper.

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Research Article

Multicriteria Evaluation of Transport Plan for High-Speed Rail: An Application to Beijing-Xiongan

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Beijing has an enormous transportation challenge: to relieve the extreme congestion that has arisen, largely due to overpopulation. To meet this challenge, the city administration has decided to extend its territory; a new city will be planned and built. This new city, Xiongan, will reduce the burden on the capital. A new high-speed railway (HSR) line is designed to transport millions of people every day within less than an hour. This study applies the potential of Geographical Information Systems (GIS) and multicriteria methods, Analytic Hierarchy Process (AHP) and Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE II), to determine the best alternative of transportation for the new high-speed railway line between Beijing and Xiongan, comparing different ones. The methodology consists of two stages. In the first stage remote sensing datasets such as ASTER DEM and LANDSAT images and GIS software such as ERDAS IMAGINE and ArcGIS have been used to determine settlement distribution, station location, elevation model, slope percentage, vegetation percentage, and route alignment for a new high-speed railway line for better understanding of its spatial distribution pattern over the study area. The second phase of the study focusing on assessing the various alternatives of transportation has been determined, and three approaches to choosing the best alternative have been introduced. In the paper we examine criteria associated with travel and economic criteria: travel time, the number of train stops, public satisfaction with transport, the number of seats per day, connectivity, operating costs, profit, and the payback period. Six alternatives of transportation have been studied. The stops in Guan and stations in the metro's rings have been investigated. In the second stage, the Analytic Hierarchy Process (AHP) and PROMETHEE II methods have been used to select the best alternative. The first approach uses only criteria related to the trip, as the criterion to choose the best alternative is the maximum of the net outranking flows by PROMETHEE II method; the second approach applies two independent criteria: the ratio of normalized operating costs and the normalized net outranking flows, and the ratio of the normalized payback period and the normalized net outranking flows; the third approach includes all defined criteria, and the criterion of choosing the best alternative is the maximum of net outranking flows as calculated by the PROMETHEE II method. The approaches have been analyzed with the purpose of comparing the results. The result indicates that it is expedient to have a station in Guan, which will increase the connection and connectivity among the cities while providing fast mobility options for a large number of inhabitants of Guan city. Furthermore, the result from Remote Sensing and GIS analysis demonstrates that the proposed high-speed railway line will be environmentally sustainable and is economically/socially feasible and that it will certainly attract current and future passengers because of their needs.

1. Introduction

The population of Beijing has increased rapidly, and further migration towards the capital city is increasing at an exponential rate. Due to urbanization, the city is facing challenges such as traffic congestion, rising real estate prices, environmental pollution, overpopulation, and dependency on private vehicles [1]. It feels impelled to address the instability generated by uncontrolled growth and to start planning to deal with the problems caused by the consequent overspill. Many studies characterize the current state of Beijing by words such as "urban disease" and "urban ill"

[2, 3]. Urbanization in China is booming on an unprecedented scale, improving people's lives but also creating massive challenges for the country and people [4]. Development generated by urbanization in China is at its height; most of the new job opportunities occur in new urban areas [5]. The capital city stretches for 100 km wide; it has 18 subway lines and 13 more under construction and has 6 ring roads built around Tiananmen Square in piecemeal fashion following a pattern of concentric circles centered on Tiananmen Square [6]. The populations of Beijing, Tianjin, and Hebei are 22 million, 15.5 million, and 74.3 million, respectively, and their annual growth rates have reached 16.2%, 14.4%, and 11.6%, respectively [7]. Therefore, there is a huge gap in the development of these three areas; Hebei needs to sustain almost four times the population of Tianjin and Beijing.

Beijing decided to extend its municipal administration to Hebei province on the border with Tongzhou to reduce the population burden on the capital [6]. The newly built area "Xiongan" will act as an auxiliary capital, providing population relief to the massive capital city [3]. In the initial phase, the administration decided to plan for 2 to 2.5 million people [8]. For Beijing, it is necessary to solve the problem of choosing a transport connection with the newly formed satellite city of Xiongan. The current project is located in Hebei; it is hoped thereby that it will close the developmental gap by providing job opportunities and a boost to the local economy. Xiongan is planned to be built between the present counties of Xiong, Rongcheng, and Anxin, from which it obtains the name "XiongAn". The newly developed area, also called "Jing-Jin-Ji", will connect Beijing, Tianjin, and Hebei and is designed to increase economic activity and close the gap in development between these three provinces. The key characteristic of the capital city is its strategic position as the center of politics, along with international exchange, culture, and technology and innovation; the noncapital functions will move to the newly built area [3]. The newly selected location is ideal for such a mega plan; as documented by the same white paper, the area is low in population density, has a low level of development and thus ample space for further development, and is highly suited to construction [9].

In the early 20th century, sixteen cities throughout the world surpassed a million in population [10]; after 1950 this rate increased until it reached 411 in 2010 [11]. Migration from rural to urban areas and population growth have increased all over the world. In 2050, 70% of the world population is expected to live in cities, which will cause a threefold increase in urban journeys [12]. Rapid expansion of motorization causes many problems such as congestion, noise, emissions, and safety risk and so carries an economic and social cost [13]. Naturally therefore smart mobility became a topic of huge interest for researchers, who competed in suggesting different measures to achieve smart sustainable cities [10, 14, 15]. Proposed transportation networks need to satisfy a variety of criteria: demand, technical feasibility, detailed quantitative evaluation, clear objectives and constraints, travel time, cost of travel, safety, reliability, accessibility, and the environment [16].

Transportation planning is complex to achieve at the design and planning level. The rapid expansion of cites also tends to cause transportation networks to be spread around large areas, within which the long-distance regional rail network is formed [17]. Travel mode selection is based on high reliability, shorter travel time, higher operational efficiency, and the need for fewer transfers [18]. Cities are expanding rapidly, and noncapital functions tend to move to the suburbs, thereby causing an increase in the numbers of passengers [18]. Many studies have been done for the railway which consider different parameters as per the study outcome, such as [19] shortest travel time and passenger satisfaction [20, 21]. Timetable studies seek to reduce passenger waiting time [22] and passenger transfer [23] and to minimize travel time, energy consumption [24], and precipitation time [25]. Recent studies mainly focus on the level of service level, selecting the shortest route possible, and traffic capacity [26]. Sustainable mobility is a smart tool to reduce congestion, pollutants caused by traffic, and emissions and to improve the quality of the environment [10, 14]. During the Reform and Opening Up, migration increased from rural to urban areas due to many factors, such as higher quality of life and employment opportunities [27], and in search of affordable homes a major percentage of people also moved to nearby areas (Urban Sprawl) [28]. From 1980 till 2010, Chinese migration was only analyzed from rural to urban areas and from west to east [29]. More recently, after the implementation of the high-speed railway (HSR) network, migration is observed from the east to urban areas, now that people work in one city and live in another [30].

The smart sustainable model for transport should reduce the demand for transport and dependency on the car, while designing urban space in such a way that it can be attractive and easily accessible by public transport [31, 32]. Each urban and transport model objective should increase connection and connectivity, urban quality of life, and transit-oriented development (TOD); be multimodal, effective, safe, and reliable; and have less adverse effect on the environment [33]. To change individual or overall transport travel behavior is a long-term complex process [10]. Sustainable urban mobility is likewise a long-term process, the achievement of which can be possible through strategies, initiatives, parameters, goals, and targets; increase the passenger attraction connected with modern mobility; and improve the technological innovation to increase effectiveness of overall system [14, 34]. Transportation bottlenecks are the major impediment to economic growth in Chinese megacities; to vitalize economic options, time-space shrinkages, and sustainable mobility options, HSR is the best solution for the country [35, 36].

HSR is considered a modern 20th century transportation tool [37]; many developed countries in the world have adopted HSR and are still investing in it [38, 39]. Attitudes to HSR in China are quite favorable, and the government is investing huge capital each year to build and extend the length of HSR lines [40]. High-speed railway was launched in early 2008 between Beijing and Tianjin; in 2009 another line came into service between Guangzhou and Wuhan [41]. In 2015, China became a world leader in HSR, with a total of 19,000km [42]. HSR is a hot topic for many researchers in China. Under consideration are issues such as social issues, railway scheduling, operational management [43], efficient stop optimization [23, 43], safety [44, 45], passenger flow, HSR passengers behavior [46], increasing connection and connectivity [16], circuity analysis [47], and capacity optimization [48]. HSR network has quite positive impacts on people's lives, travel behavior, and household mobility; promotes and encourages tourism and travel activities; and strengthens social interactions and employment opportunities [49–53].

China is growing at a rapid pace; many areas have converted to high-rise buildings which have reduced the appeal of living in cities for many average-income people [54]. Previous studies provide evidence that HSR increases the option of living in better and more affordable cities as per individual choice, where people live in one city and work in another. Chinese cities where this has been shown to be the case include Guangzhou-Foshan, Shanghai-Suzhou, Shenzhen-Huizhou, and Beijing-Tianjin [55, 56]. HSR changes the migration patterns in China. For example, between Beijing and Tianjin, people work in Beijing but live in Tianjin because of the affordability of housing in Tianjin as compared to Beijing [54]. HSR has improved Chinese economic development, travel convenience, and opportunities for job creation as observed by local people [57]. It has also secured improvement in regional economies [58] and in local social structure and life [59, 60].

Geographic Information Systems (GIS) and multicriteria analysis (MCA) are methods that can be used to study and analyze transport systems. GIS works as a modeler object, while MCA works as a decision object. By combining them, the optimum result for any project can be achieved [61]. In the present research GIS is used for searching the optimal route, short route analysis, and optimal path analysis, while AHP and PROMETHEE are used to determine the weights of criteria [62]. The combination of GIS and AHP is a strong planning tool; GIS helps to analyze different factors, while AHP is used to assign weight to these factors on the basis of their relative importance, which can be of great help to decision makers and planners [63]. The combination of GIS and AHP is widely used to achieve sustainability in transportation by measuring territorial impact, transport alternatives, accessibility improvements, environmental impacts, and landscape connectivity [64] to focus on alternatives and potential indicators needed for planning the high-speed railway line. Another study was carried out through GIS and AHP for land use allocation, land use planning, and land use proposal deliberation [65]. The combination of PROMETHEE and AHP is used by many studies on transportation to determine mode selection, route selection [66, 67], average waiting time, and operational efficiency [68]. The combination of AHP and PROMETHEE has been used in past studies to determine route selection and mode of transportation and to evaluate different transportation related issues. The criterion for selecting the optimal transport alternative is introduced—the minimal value of the ratio between normalized operating costs and normalized PROMETHHE scores. Combinations of GIS, AHP, and PROMETHEE have been used to determine the mode of transportation, to evaluate transport projects, and to select route [67, 69]. To choose the optimal transport type with regard to safety, access, cost, capacity, speed, and reliability, AHP and PROMETHEE are applied in [67]. In addition [70], transport planning was determined with the help of the multistakeholder, multiobjective, and AHP modeling to calculate minimum costs. The combination of PROMETHEE, AHP, and GIS has been applied for susceptibility landslide mapping [71], improvement of healthcare waste management, centralized location selection for modern waste [72], and the evaluation and selection of ecotourism sites [73]. PROMETHEE and AHP are used to select transportation infrastructure [74] and select the optimal alternative for an intercity train [68]. In [75], AHP and the Multidimensional Cost Model are used to find the optimal road transport path for the purpose of reducing the traffic congestion. In [76] AHP and GIS are applied to determine the accessibility patterns of new housing development, to make cost-benefit analyses with regard to residents' access to facilities.

It may be concluded that the application of the GIS and multicriteria methods such as AHP and PROMETHEE can serve as a basis for creating a model for passenger transportation for the new railway line. The number of stations on the line depends on many factors, such as travel time, number of passengers, frequency of services, and investment. When selecting the best transport alternative, it is necessary to compare different alternatives and choose the best one by given criteria.

It can be summarized that the multicriteria methods are an appropriate tool to make decision and analyze complex problems due to their ability to assess different alternatives on various criteria for possible selection of the suitable alternative. When developing transport technology, it is necessary to take into account both the possibilities of the transport operators and the requirements of the passengers; i.e., it is necessary to assess different criteria. The main methodological steps for the selection of the suitable alternative from set of available alternatives by using multicriteria methods can be summarized as follows: defining the criteria, establishment of alternatives, appropriate data collection, selection of method to solve the problem, choosing the suitable alternatives.

The aim of the present study is to determine the best transportation alternative between compared ones for a new high-speed railway line between Beijing and Xiongan using GIS and multicriteria analysis. The combination of both methods permits assessment of many quantitative and qualitative indicators, at the same time also taking into account the design of the railway line. The paper is organized as follows. In Section 2 we present the research methodology in two stages: the first is the application of ArcGIS and ERDAS IMAGINE to determine the high railway line, and the second is a combination of AHP (shown in Appendix A) and PROMETHEE II (shown in Appendix B) method for choosing the best alternative of transportation. In Section 3, by applying our proposed approach to the Beijing to Xiongan high-speed railway line, the study demonstrates the effectiveness of this methodology by numerical results. Conclusions are given in Section 4. For reference, the appendix provides the basics of the AHP and PROMETHHE II methods.

2. Methodology

The methodology is based on application of GIS and multicriteria methods AHP and PROMETHEE II. The remote sensing of GIS software was used for better understanding and knowledge of its spatial distribution pattern over the study area. The model does not generate input data for the multicriteria model. The AHP method is applied to determine the weights of criteria, and PROMETHEE II technique is used for ranking the alternatives. The methodology includes three approaches to decision making: the first approach is based only on criteria connected to the trip into PROMETHEE II model; the second approach uses complex criteria for ranking; the third approach is based on all defined criteria which are applied to PROMETHEE II model. These approaches serve as a sensitivity analysis of results.

The methodology comprises two stages.

2.1. Stage 1. Stage 1 includes identifying the suitable locations for stations, elevation mode, slope percentage, and the proposal of a new railway line between cities by using ArcGIS and ERDAS IMAGINE. In this analysis, we have utilized high-end satellite datasets such as Landsat-8 and the Advanced Spaceborne Thermal Emission and Reflection Radiometer (ASTER) Digital Elevation Model (DEM). In the first step, they were mosaicked to adequately represent the study area. The ERDAS IMAGINE high-end image processing software [77] and ArcGIS: rigorous GIS analysis software [78] were utilized to create classified output such as land use categories of settlement and vegetation. The ArcGIS software was used to create a systematic grid over the specified buffer area. Various themes such as settlement percentage (%), slope %, elevation, and vegetation % were calculated and produced in the form of various maps to understand the feasibility of new routes, using population data, road network, railway network, and recreational data. Such a GIS investigation systematically reveals that a new proposed line is significantly suitable relative to the environment/cost/socioeconomic aspect of the human settlement of the region. Furthermore, it will adequately help the present and future population and support their needs in a sustainable manner. Additionally, the various themes/layers produced here were not used as an input data for the multicriteria model.

2.2. Stage 2. Stage 2 includes defining alternatives of the organization of trains on the new railway line, defining criteria for selecting the best alternative, ranking the alternatives, and choosing the best one.

Stage 2 includes the following steps.

Step 1. Determining the alternatives: these alternatives for the transport plan of passenger trains are formed according to the number of stops at the stations.

Step 2. Determining the criteria of the alternatives assessment: in the study the following criteria are introduced:

(i) C1: travel time (min) (this is an important factor for attracting passengers).

(ii) C2: number of train stops (this is also a factor relevant to passenger satisfaction).

(iii) C3: transport satisfaction, trains/day (this represents the frequency of transport services).

(iv) C4: number of seats /day (this is important for attracting passengers for trips).

(v) C5: connectivity (this is important for the convenience of traveling and the total journey time from the starting point to the final destination). The values of this criterion are set by 0 or 1. C5=1 if the alternative has connection with metro, and C5=0 otherwise.

(vi) C6: operating costs, millions of US dollars (this factor is important for the carrier).

(vii) C7: profit, millions of US dollars/year (this factor is important for the carrier).

(viii) C8: payback period, years (this specifies the return on the investment).

These criteria are important to assess the transportation and the economic impact on alternatives. The criteria are defined by authors taking into account different conditions of transportation, due to the different numbers of stops in the route, and the need for investment for expansion or new construction of stations.

Within the study three approaches have been applied to determine the best alternative.

(1) First Approach. The optimal alternative is defined according to the impact of criteria connected to the trips (C1-C5). The weight given to each of the criteria is determined by AHP. These weights are fed into PROMETHEE to determine the best alternative by criterion maximum of net outranking flows.

$$\varphi_f^{(1)} \longrightarrow max \tag{1}$$

The research uses PROMETHEE II outranking method which is based on a preference function approach [79]. It is one among the 6 variations of PROMETHEE. PROMETHEE I is used for partial ranking of the alternatives, PROMETHEE II is for complete ranking, PROMETHEE III is applied for ranking based on interval, PROMETHEE IV is for complete or partial ranking of the alternatives with a continuous set of solutions, PROMETHEE V is for problems with segmentation constraints, and PROMETHEE VI provides the decision maker with additional information on his own personal view of his multicriteria problem. PROMETHEE methods have a simple mathematical approach and are user-friendly. The PROMETHEE II method is based on the pairwise comparison of alternatives along each defined criterion. It requires two additional types of information: weights of the criteria and a decision maker's preference functions, which were used for comparing the alternatives. PROMETHEE II is the most frequently applied version because it enables one to find a full-ranking of alternatives.

(2) Second Approach. The best alternative is chosen by taking into account both the impact of the costs presented by criteria C6-C8 and the complex effect of the benefits presented by criteria C1-C5. The criteria C6-C8 are not introduced into the

PROMETHEE model. The impact is examined separately; the optimization is made by using a complex criterion.

In this study we have been using the following complex optimization criteria, which have been applied separately for all the alternatives investigated:

(a) Minimum value of ratio r_{f1} of the normalized operating costs and the normalized scores corresponding to the PROMETHEE II priority [68]:

$$r_{f1} = \frac{c_f}{a_f} \longrightarrow min \quad (2)$$

where c_f are the normalized values of operating costs; a_f is the normalized net outranking flow by PROMETHEE II method for alternative f; $f = 1, \ldots, F$ is the number of alternatives.

The normalized values of operating costs are

$$c_f = \frac{R_f}{\sum_{f=1}^{F} R_f} \quad (3)$$

where R_f is the operating costs for alternative f, USD/year. These costs include the costs for movement of the trains (electricity), for stop-braking and accelerating of the trains, for maintenance and repair of the rolling stock, for salaries for all personnel, for personnel at the stations on the route, and for infrastructure charges.

The normalized values of net outranking flow by PROMETHEE II are determined according to [68]:

$$a_f = \frac{\varphi_f^{(1)} + M}{\sum_{f=1}^{F} \left(\varphi_f^{(1)} + M\right)} \quad (4)$$

where M is a positive integer that should make all net outranking flow $\varphi_f^{(1)}$ positive; $\varphi_f^{(1)} \in [-1; 1]$ is net outranking flow by PROMETHEE II for alternative f.

This criterion takes into account both criteria presented by the impact of the trips and economic criteria.

(b) Minimum value of ratio r_{f2} of the normalized values of the payback period and the normalized scores corresponding to the PROMETHEE II priority.

In the study the building of the new railway line and stations is investigated. Therefore, it is important to include in the optimization criterion the factors that are relevant to the amount of investments, costs, and revenues.

The payback period is one of the simplest investment appraisal techniques. It presents the time at which the initial cash outflow of an investment is expected to be recovered from the cash inflows generated by the investment. The payback period is determined as follows:

$$P_f = \frac{I_f}{W_f - R_f}, \text{ years} \quad (5)$$

where W_f is the revenue of the trips, million USD/year; I_f are the investment costs, million USD.

The normalized values of the payback period are as follows.

$$p_f = \frac{P_f}{\sum_{f=1}^{F} P_f} \quad (6)$$

The criterion of choosing the best alternative is the minimum value of ratio r_{f2} of the normalized values of the payback period, and the normalized scores corresponding to the PROMETHEE II priority are determined as follows:

$$r_{f2} = \frac{p_f}{a_f} \longrightarrow min \quad (7)$$

where p_f are the normalized values of the payback period P_f.

(3) Third Approach. The best alternative is defined according to the impact of all criteria (C1-C8). The weights of the criteria have been determined by AHP. These weights are fed into PROMETHEE II to determine the best alternative by criterion maximum of net outranking flows.

$$\varphi_f^{(2)} \longrightarrow max \quad (8)$$

The value of net outranking flows means PROMETHEE II score.

(4) Choosing the Best Alternative. All three approaches are considered when making the final decision. They are accepted as a tool for sensitivity analysis for suitable alternative.

The alternatives are ranked based on each of the criteria defined by formulas (1), (2), (7), and (8). The ranking of the alternative given by each of the criteria corresponds to the scores under this alternative.

3. Results and Discussion

The numerical experiments of high-speed railway line between Beijing and Xiongan have been conducted to evaluate the proposed methodology.

3.1. First Stage. In the first stage of the study, ArcGIS was used to analyze the two city spatial distribution patterns in detail. In stage 2, the stop for HSR can be located in Beijing and how many stops are really needed was determined. Through ArcGIS, the location for the station was selected on the basis of the availability of land next to the station, the possibility of future extension, the price of land, and subway connections. Furthermore, during the analysis the elevation model, settlement percentage, slope percentage, and vegetation model were determined for the proposed route, extending from Beijing to Guan and then Xiongan. On the basis of an analysis of the average circular distances between each ring road in Beijing, a buffer of 30 km was sketched in order to analyze possible passenger demand and attraction. The stations were considered on the basis of population buffer, such as 30 km in Beijing, but 5 km at Guan and 30 km at Xiongan. The distance between each of the six ring roads in Beijing varies: the average distance between the second and third ring road is 2.4km, between the 3rd and the 4th 2.7km, between the 4th and the 5th 4 km, and between the 5th and the 6th approximately 11 km. The city was designed accordingly, with development being based on ring road allocation. The 6th ring road is considered to be at the border of the city and has relatively less population. The distance

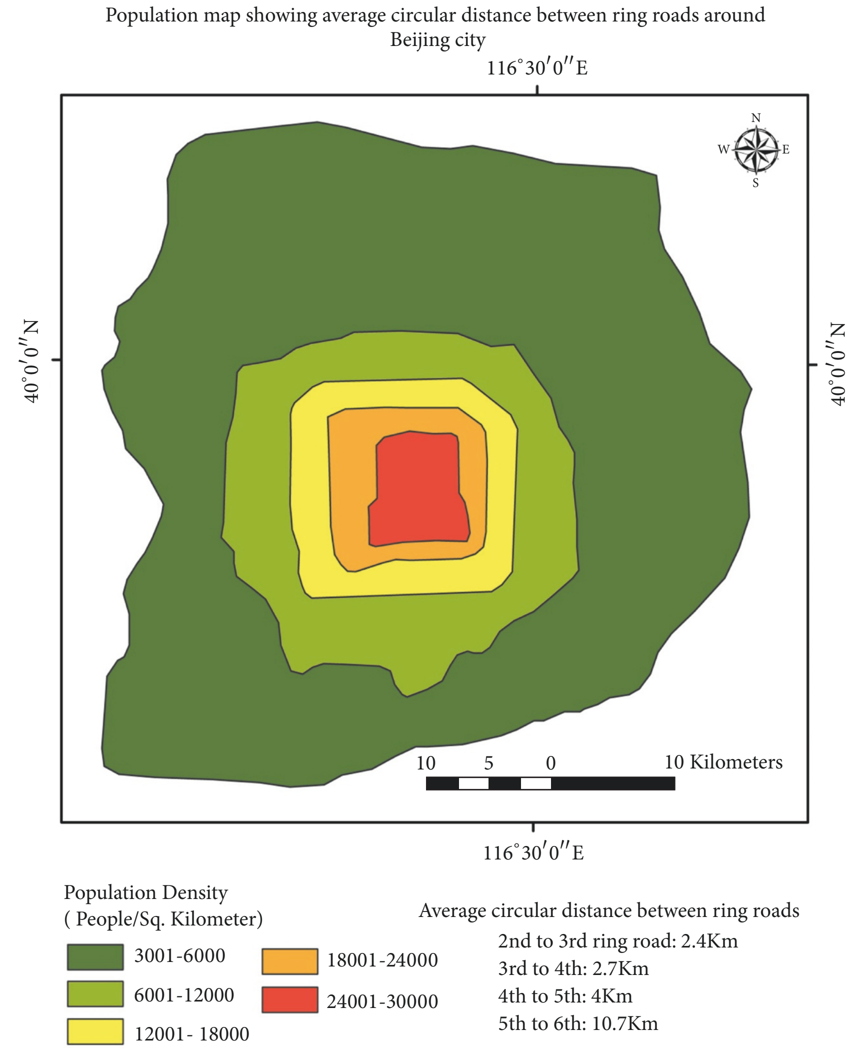

Figure 1: Beijing population index along with circular distance between each ring road.

from the 6th ring road to Xiongan is 80 km, and from the 6th ring road to the center of Beijing 125 km. In the middle there is a tiny town called Guan, which is 30 km from the 6th ring road and 51 km from Xiongan. The purpose of Figure 1 is to clearly indicate the overall idea about distances of each ring road from the center and the population density between each ring road. Figure 2 shows the overall settlement image along with population buffer (passenger attraction area) from Beijing to Guan to Xiongan. The HSR buffer of 5 km is used in the figure for route allocation, while along Beijing and Xiongan 30 km passenger buffer is used.

The total traveling time (as analyzed by Baidu software) from Beijing to Baoding, the area near to that of the study, is around three and a half hours using the local intercity railway line. The current proposed line will follow the last station on the 6th ring road, follow the route which stops and Guan, and then proceed to the destination. The line as currently proposed will deliver passengers there in less than one hour.

The circle is the center of Guan city which shows that the spatial structure of settlement depicted in the form of percent is significantly high near the center. The study from GIS analysis verifies that the population settlement is quite dominated in this area which retains approximately 520,000 inhabitants; there are industries and educational institutions, with huge population density (as shown in Figure 2). The study suggests that if the new line passes through the area near Guan, which will serve a large number of inhabitants, it will increase the passenger attraction between Beijing and Xiongan.

The elevation map is designed around the proposed high-speed railway line, with Xiongan at low and Beijing at high elevation. The variation in elevation between the two is quite gradual and will significantly facilitate the construction of transportation infrastructure which will be a sustainable option for connecting the capital to Xiongan (as shown in Figure 3).

As shown in Figure 4, the settlement is near the proposed buffer line; furthermore the line also takes into account future growth of passenger traffic, specifically in the new stations at Guan and Xiongan. The line will facilitate the day to day activities of huge numbers of people and secure the supply of food to newly built city. The general slope

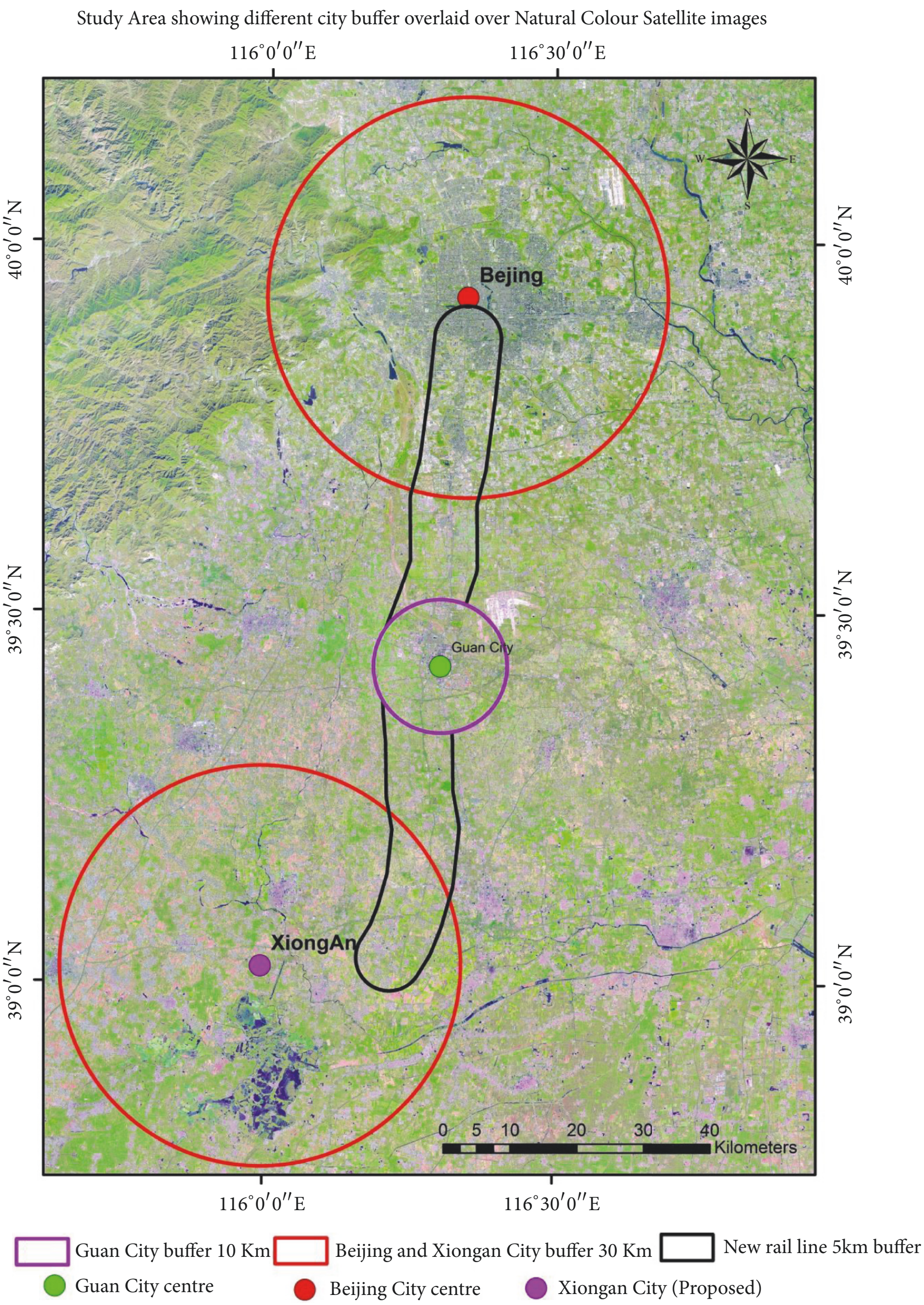

Figure 2: HSR proposed route alignment between Beijing and Xiongan.

percent (Figure 7) trend is moderate to low and therefore economically sustainable (a high slope percent requires more cutting operations) and will support the construction of transportation infrastructure such as rail lines, bridges, and tunnels as shown in Figure 5. The vegetation in study area, as shown in Figure 6, comprises mainly agricultural fields, vineyards, orchards, and pastureland. The spatial distribution pattern of vegetation percent also indicates that the construction of the new rail line will have little adverse effect on natural vegetation. The protection of nature through the protection of biological and landscape diversity is a key issue relative to rendering the project environmentally feasible, as indicated by Figure 5.

3.2. Second Stage. In this paper we examine the intermediate stops in three Beijing stations located in the metro rings and one in Guan. The stops in the metro rings would increase the connectivity and convenience of the trip. The study plans the extension of these stations so as to be able to serve high-speed trains. Guan is situated between Beijing and Xiongan. The

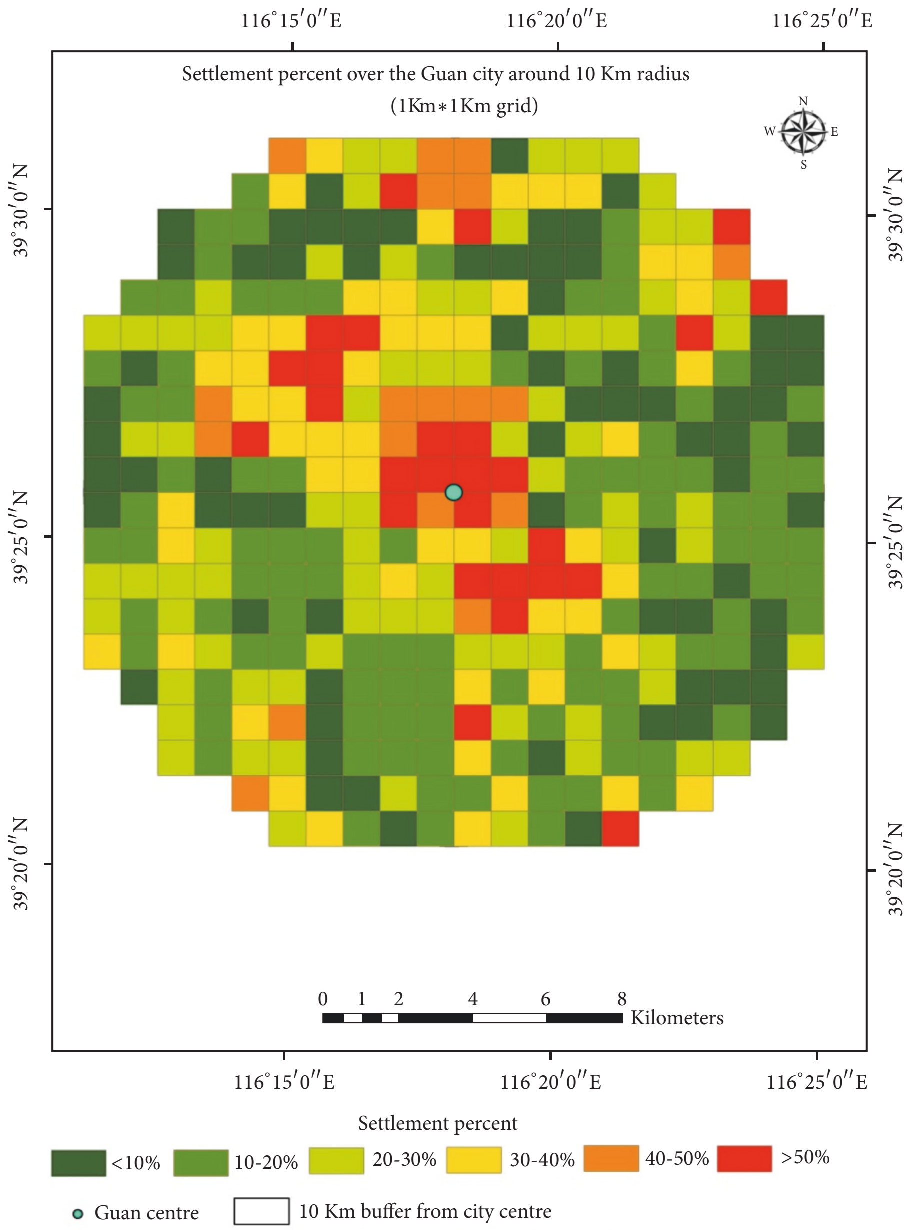

Figure 3: The density of settlement at Guan (1km∗1km grid spacing) from Landsat-8 satellite image at 10 km radius of Guan city.

population of Guan is 516,735 habitants. The construction of the station in Guan would contribute to the expansion of this satellite city while reducing the population of Beijing.

Six different alternatives of transportation have been formed taking into account proposed stations between Beijing and Xiongan. Figure 8 presents these alternatives. They differ in the number of stops and stations for stopping. The alternatives are as follows:

(i) Alternative 1: There are stops in three intermediate stations in Beijing.

(ii) Alternative 2: There are stops in three intermediate stations in Beijing and one in Guan.

(iii) Alternative 3: This alternative provides a direct service with no stops.

(iv) Alternative 4: This service includes one stop in Guan.

(v) Alternative 5: The intermediate stops are at the most distant station in the metro ring (Huangcun railway station) and also in Guan.

(vi) Alternative 6: This service includes one stop in the most distant station in the metro ring.

The location of Guan railway station and also the distances and travel time between railway stations have been determined by using ArcGIS. The track of the railway line is made by using ArcGIS.

Calculations are based on the following conditions:

(i) The high-speed trains are composed of 8 wagons and carry 600 passengers in one direction.

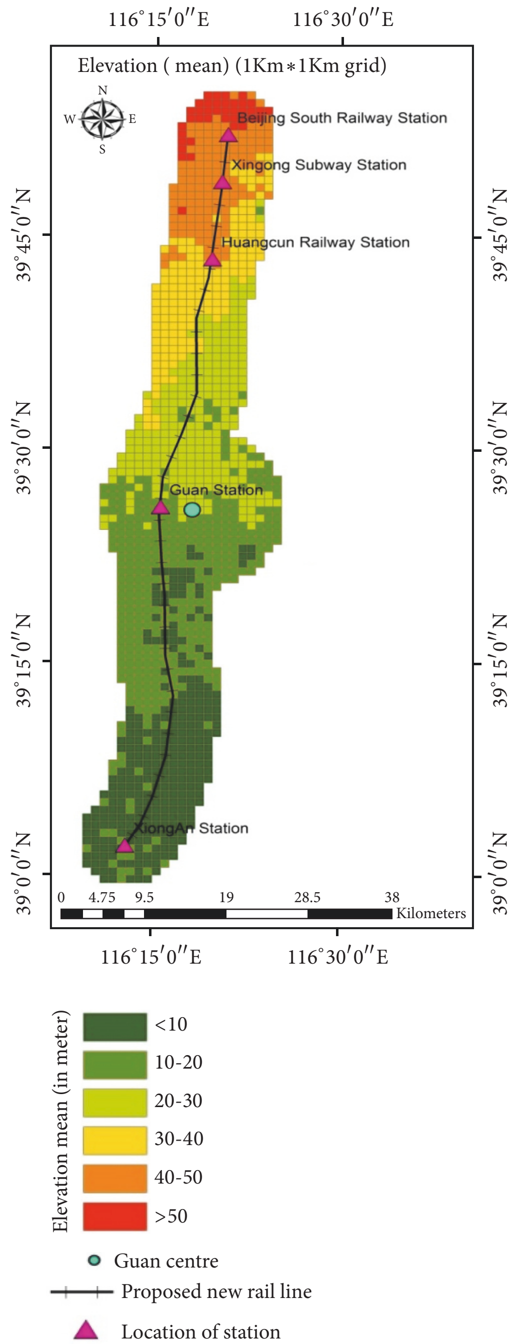

Figure 4: The mean elevation (1km∗1km grid spacing) from ASTER Digital Elevation Model (DEM).

(ii) The investment costs for the new station building are taken to be 1900 million USD. Similar investment was made for stations in new high-speed railway lines elsewhere in China. According to the size of the station, the cost of stations varies, with small stations (3,000 sq. m station building) costing about RMB 40 million (5.7 million USD) to RMB 13 billion (1900 million USD) [80, 81].

(iii) The investment costs for the extension of stations have been accepted by experts. The value is 5% of the value of a new railway station building.

(iv) The costs for electricity to power the trains are 13 USD/km according to Chinese Railways. This data is similar to that for the Beijing to Guangzhou high-speed railway line for Engine Type CRH380A for a train with eight wagons [82]

(v) The costs for braking and accelerating in intermediate stops are determined as 0.2% of costs of movement. This value was accepted according to [83] which documents the research of high-speed train energy consumption. Furthermore, regenerated energy from the trains is assumed to be consumed by other trains

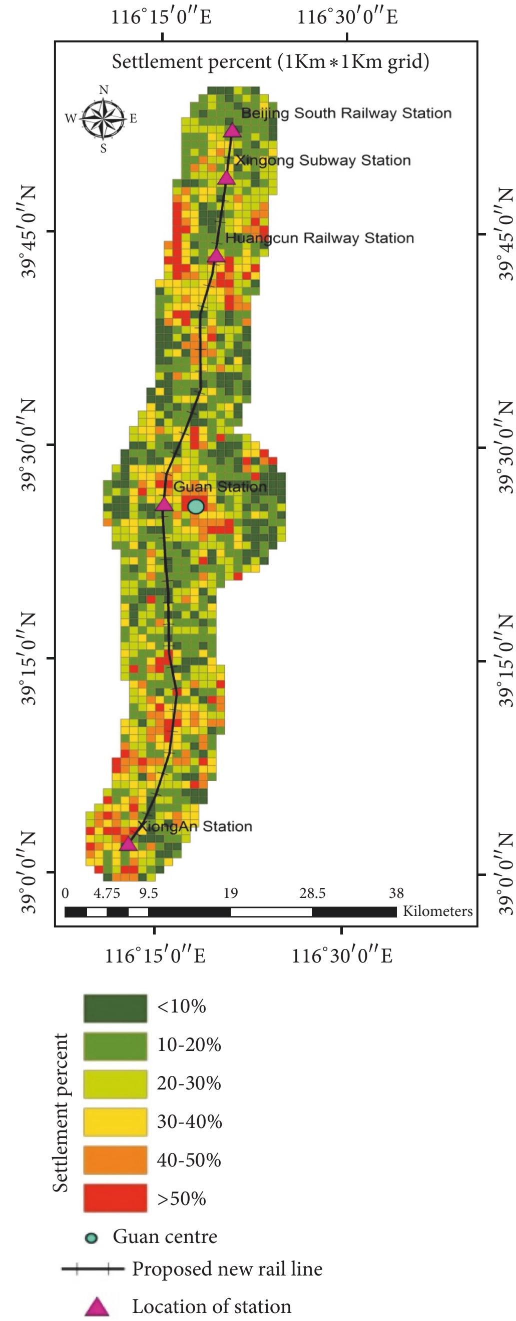

Figure 5: Density of settlement (1km∗1km grid spacing) from Landsat-8 satellite image.

or for auxiliary purposes. For high-speed trains, the reduction in electricity can be used by employing regenerative braking.

(vi) The investment costs for the construction of a new railway high-speed line are the same for all six alternatives and therefore they are not considered.

(vii) In Chinese Railways there are no infrastructure charges. Therefore, operating costs do not include infrastructure charges.

(viii) The data on the salaries of railway personnel both on the trains and at the stations and the data for maintenance and repair of the rolling stock are derived from Chinese Railways.

(ix) The average ticket price for class seats is calculated as an average of 13 USD. This is similar to the Beijing-Tianjin Intercity High-Speed Train which is of similar length and travel time [84].

In the study we have experimented two scenarios according to the capacity utilized on the new high-speed railway line:

(i) Scenario 1: 150/160 pair trains per day.

(ii) Scenario 2: 160/180 pair trains per day.

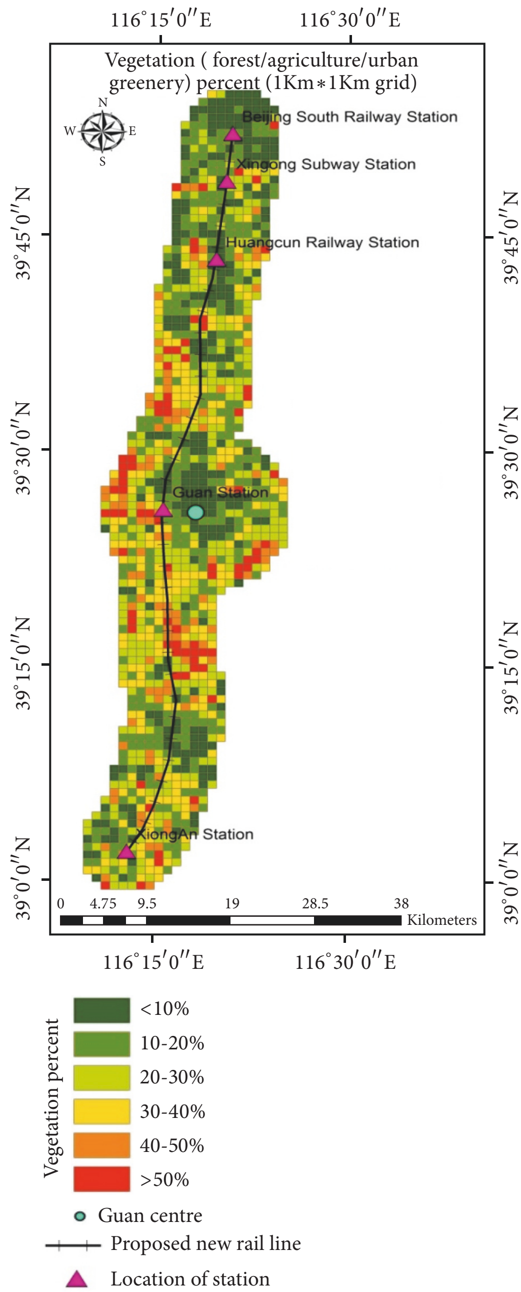

FIGURE 6: The vegetation percent (1km∗1km grid spacing) from Landsat-8 satellite image.

The first value presents the number of pair trains for the alternatives that do not stop in Guan (alternatives 1, 3, and 6); the second value presents the number of pair trains for alternatives including stopping in Guan (alternatives 2, 4, and 5).

Both scenarios do not exceed the maximum capacity of the new high-speed railway line. The scenarios serve as sensitivity analysis of given results.

3.2.1. First Approach. In the first approach, five criteria have been studied. The weights of each of these criteria have been determined using AHP. In this research, a group of experts gave an overall score on the scale of Saaty which is shown in Appendix A (Table 13). The group of experts consists of nine railway specialists from China Railway Corporation and Ministry of Railways with many years of experience. They were asked by the authors to give a general assessment of the pairwise comparison of the criteria using the scale of Saaty. Table 1 presents the pairwise comparison of criteria using Saaty's scale (Appendix A) and received weights. The value of consistency ratio CR=0.09 shows that the assessment of these experts is adequate. The greatest impacts are the criteria travel time (35%), transport satisfaction (30%), and connectivity (20%).

Table 2 shows the data used for the two scenarios. The values of criterion C1 are determined by using ArcGIS. The

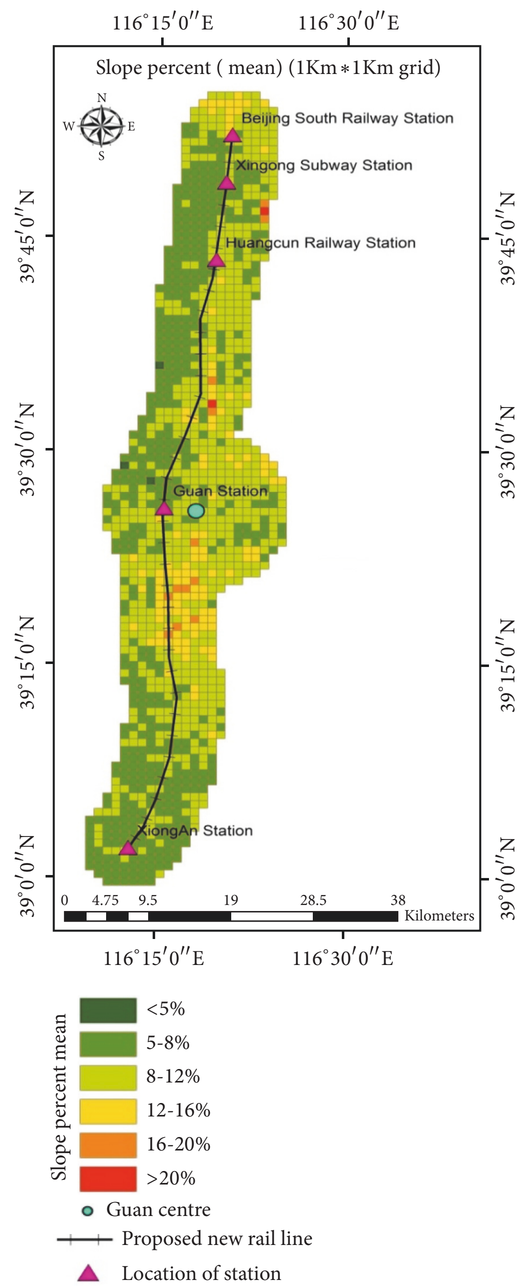

FIGURE 7: The slope percent categories derived (at 1km∗1km grid spacing) from Digital Elevation Model (DEM) image.

TABLE 1: Pairwise comparison of criteria from C1 to C5 and weights.

Criteria	C1	C2	C3	C4	C5	Weights
C1: Travel time, min	1	4	2	5	1	0.35
C2: Number of train stops	1/4	1	1/5	1/2	1/3	0.06
C3: Transport satisfaction, pair trains/day	1/2	5	1	4	2	0.30
C4: Number of seats/day	1/5	2	1/4	1	1/2	0.09
C5: Connectivity	1	3	1/2	2	1	0.20
CR=0.09						

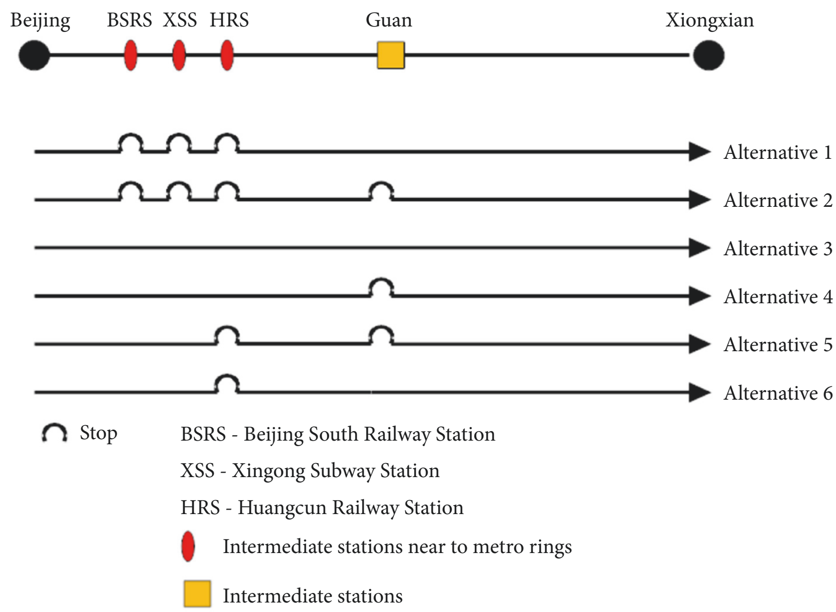

Figure 8: Alternatives between Beijing (start point) and Xiongan (final point).

Table 2: Parameters for the first approach.

Alternative	C1, min	C2	C3, pair trains/day		C4, seats/day		C5
			Scenario 1	Scenario 2	Scenario 1	Scenario 2	
1	36	3	150	160	180000	192000	1
2	38	4	160	180	192000	216000	1
3	30	0	150	160	180000	192000	0
4	32	1	160	180	192000	216000	0
5	34	2	160	180	192000	216000	1
6	32	1	150	160	180000	192000	1
Type of optimization	min	min	max	max	max	max	max
Type of preference function	linear	linear	linear	linear	linear	linear	usual

values of criterion C2 are put according to the number of intermediate stops for each alternative that are presented in Figure 8. These data are the inputs parameters for PROMETHEE II. The criterion of choosing the best alternative is the maximum net outranking flows. PROMETHEE II method uses six types of preference functions. The type of optimization of the criteria and preference function for this study are given it the last rows in Table 2.

Figure 9 presents the ranking of alternatives by first approach using Visual PROMEHEE software. The Visual PROMETHEE uses PROMETHEE II method to find the best solution. Figure 9 contains two parts. The first part presents the PROMETHEE II outranking flows; the second part shows the weight of each criterion. The research has been conducted separately for each scenario. The results are the same. It can be seen that the optimal alternative, in this case, is alternative 5 which involves stopping in Guan.

Table 3 presents the values of PROMETHEE II score (net outranking flows) and the ranking of alternatives. Alternative 5 has the maximal value of net outranking flows and is the best for this approach.

Table 4 shows the sensitivity analysis of the results given by Visual PROMETHEE software. Here we present the limits of changing the weights of the criteria while preserving the best solution. The "weight" column in Table 4 shows the weights obtained from AHP method. The "minimum" and "maximum weight" columns indicate the range of weights, such that the ranking of the alternatives remains unchanged. A larger weight stability interval indicates that it could have large effect on the ranking. It can be seen that the criteria C3 (transport satisfaction, trains/day) and C4 (number of seats/day) have large stability intervals.

3.2.2. Second Approach. The best alternative is chosen by taking into account both the impact of the costs presented by criteria C6-C8 and the complex effect of the benefits presented by criteria C1-C5. The criteria C6-C8 are not introduced into the PROMETHEE II model. The impact of these criteria is examined separately; the choice of the best alternative is made by using a complex criterion. The parameters of both scenarios are given in Table 5. In this approach, all defined criteria, C1-C8, have been taken into

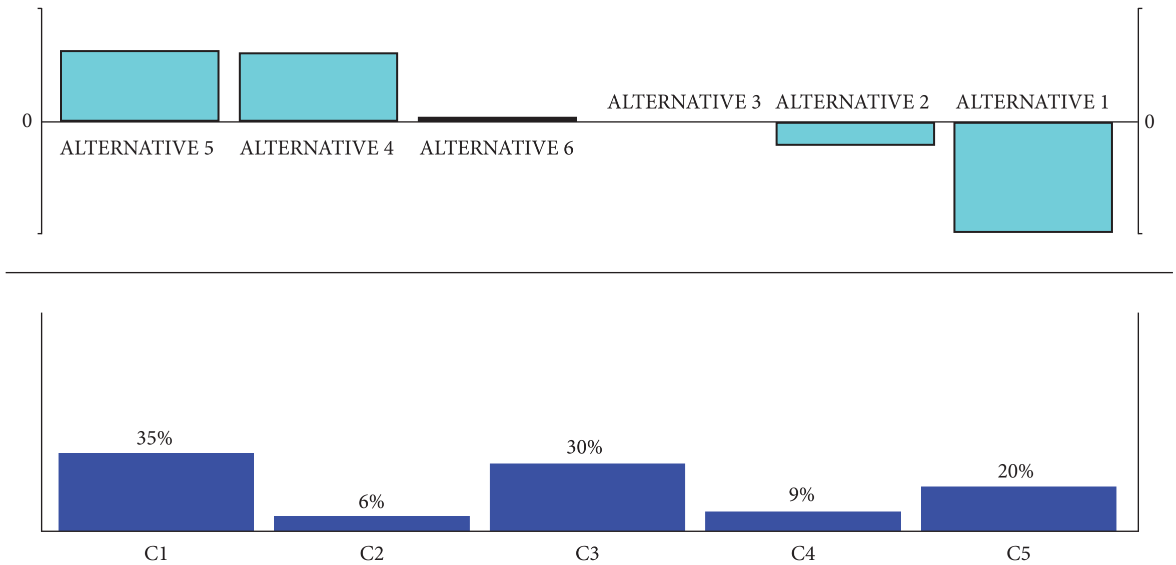

Figure 9: Ranking of alternatives on criteria C1 to C5 (results in Visual PROMETHEE software).

Table 3: Net outranking flows for the first approach.

			Alternatives					
	Criterion	Scenario	1	2	3	4	5	6
First approach	Net outranking flows	1;2	-0.400	-0.084	-0.008	0.238	0.244	0.01
	rank		6	5	4	2	1	3

account. Tables 6 and 7 present the values of criteria for both scenarios. The values of the criteria were determined by formulas (2) and (7). The values of net outranking flows are shown in Tables 6 and 7. It can be seen that in both scenarios the best alternative by criterion r_{f1}—ratio of the normalized operating costs and the normalized net outranking flows—is alternative 5; the values of this criterion for alternatives 5 and 4 are close. Alternative 3 is optional by criterion r_{f2}—ratio of the normalized values of the payback period and the normalized net outranking flows. The last columns of Tables 6 and 7 present the ranking of alternatives for both scenarios.

The real contribution of the second approach is the definition of two new complex criteria for choosing the best alternative. These criteria present the ratio of costs and benefits. The determination of the score of benefits is made by using multicriteria analysis: AHP method to evaluate the criteria and PROMETHEE II method for assessing and ranking the alternatives. In this case is used only criteria C1-C5. The other assessment is made according to the criteria presenting the costs. For this purpose two different criteria have been introduced: normalized operating costs and normalized payback period. The final ranking of alternatives is made by using complex criterion that considered both costs and benefits. Two complex criteria have been defined. The first considers the impact of operating costs, the second takes into account the impact of investment, revenue, and operating cost. These criteria serve the comparison of results.

3.2.3. Third Approach. In this approach, all the criteria have been used.

The type of optimization of the criteria and preference function for this study are given in the last rows in Tables 8 and 9.

Table 10 presents the pairwise comparison of the criteria and received results of weights. The scores are given by the same group of nine experts. The value of consistency ratio CR=0.08 (Appendix A, Table 14) shows that the assessment of the experts is adequate. The greatest impacts are upon the criteria connectivity (19%), travel time (15%), profit (15%), transport satisfaction (16%), and payback period (14%).

The optimization criterion is the maximum of the PROMETHEE II score—maximum net outranking flows. Figures 10 and 11 show the results of optimization using Visual PROMETHEE software for both scenarios. In the first parts of the figures we show the PROMETHEE II net outranking flows; in the second parts we present the weights of criteria used.

In this approach, the best alternative for scenario 1 is alternative 5, for scenario 2, it is alternative 4. It can be seen that for scenario 2 the results for alternative 4 and alternative 5 are close. For both alternatives there is a stop in Guan.

Table 11 shows the sensitivity analysis of the results given by Visual PROMETHEE software. Here we present the limits of changing the weights of the criteria while preserving the optimal solution for both scenarios. It can be seen that the criteria C1 (travel time) and C2 (number of train stops)

Table 4: Limits of changing the weights of the criteria related to the trips while preserving the optimal solution.

Criteria	Weight, %	Minimum weight, %	Maximum weight, %	Range difference, %
C1	35	32.43	35.64	3.21
C2	6	0.60	7.39	6.79
C3	30	13.58	34.17	20.59
C4	9	0.00	14.42	14.42
C5	20	19.60	24.76	5.16

Table 5: Parameters of the alternatives.

	Strategy 1				Strategy 2			
Alternative	Number of pair trains/ day	Million Pass./ year	I_f, million USD	W_f, million USD/ year	Number of pair trains/day	Million Pass./ year	I_f, million USD	W_f, million USD/ year
1	150	65.7	2185	854.1	160	70.08	2185	911.04
2	160	70.08	4085	911.04	180	78.84	4085	1024.92
3	150	65.7	1900	854.1	160	70.08	1900	911.04
4	160	70.08	3800	911.04	180	78.84	3800	1024.92
5	160	70.08	3895	911.04	180	78.84	3895	1024.92
6	150	65.7	1995	854.1	160	70.08	1995	911.04

Table 6: Results for optimization criteria, Scenario 1.

	Scenario 1							
Alternative	p_f	c_f	r_{1f}	r_{2f}	φ_f	a_f	Rank By r_{1f}	Rank by r_{2f}
1	0.128	0.162	1.615	1.277	-0.400	0.100	6	5
2	0.224	0.172	1.130	1.468	-0.084	0.153	5	6
3	0.111	0.161	0.973	0.670	-0.008	0.165	4	1
4	0.208	0.172	0.833	1.007	0.238	0.206	2	3
5	0.213	0.172	0.830	1.028	0.244	0.207	1	4
6	0.116	0.161	0.957	0.691	0.010	0.168	3	2

have large stability intervals for which the ranking remains unchanged.

3.2.4. Choosing the Suitable Alternative. In this research the defined approaches and scenarios serve to make sensitivity analysis in regard to choice of suitable alternative.

Table 12 presents the ranking of the alternatives according to each approach and its scenarios. It contains the rank of each alternative and approach according to the defined criteria of choosing the best alternative. The results of ranking for the first approach are taken from Table 3; the results for the second approach, respectively, from Tables 7 and 8; and the results for the third approach according to Tables 9 and 10. Figure 12 presents a comparison of results.

The results show the following:

(i) Alternative 5 is the best one by the first approach and both scenarios, when applying only criteria connected to transportation C1-C5. The second position is for alternative 4. In this case, the economic criteria are not taken into account, which is the reason for the sustainability of the solution for both strategies.

(ii) The best alternative for the second approach is different for both criteria. The results show that alternative 5 is the best one by using the criterion r_{1f}, the ratio between the normalized operating costs and the normalized net outranking flows corresponding to the PROMETHEE II priority. The best alternative by criterion r_{2f}, the normalized values of the payback period and the normalized net outranking flows corresponding to the PROMETHEE II priority, is alternative 3. The difference in ranking is due to the parameters that are considered in both criteria. The second criterion besides the operating costs considers investment and revenues of transportation. This is the reason for the change in the ranking of alternatives. Alternative 3 has minimum investment costs and payback period for both scenarios.

(iii) The best alternative for the third approach when applying all criteria is alternative 5 for the first scenario; the best alternative for the second scenario is alternative 4. These alternatives differ in the number of stops, but both have a stop at the proposed new station Guan.

Table 7: Results for optimization criteria, Scenario 2.

				Scenario 2				
Alternative	p_f	c_f	r_{1f}	r_{2f}	φ_f	a_f	Rank by r_{1f}	Rank by r_{2f}
1	0.132	0.157	1.570	1.320	-0.400	0.100	6	5
2	0.220	0.177	1.157	1.438	-0.084	0.153	5	6
3	0.115	0.157	0.952	0.697	-0.008	0.165	3	1
4	0.204	0.176	0.854	0.990	0.238	0.206	2	3
5	0.209	0.176	0.850	1.010	0.244	0.207	1	4
6	0.120	0.157	0.935	0.714	0.010	0.168	4	2

Table 8: Parameters for the second approach, Scenario 1.

Criteria	C1	C2	C3	C4	C5	C6	C7	C8
Alternative	min	stop	pair trains/day	seats/day	coef.	million USD/year	million USD/year	year
1	36	3	150	180000	1	413.64	440.46	5
2	38	4	160	192000	1	441.71	469.33	8.7
3	30	0	150	180000	0	412.16	441.94	4.3
4	32	1	160	192000	0	440.15	470.89	8.1
5	34	2	160	192000	1	440.67	470.37	8.3
6	32	1	150	180000	1	412.65	441.45	4.5
Type of optimization	min	min	max	max	max	min	max	min
Preference function	linear	linear	linear	linear	usual	linear	linear	linear

Table 9: Parameters for the second approach, Scenario 2.

Criteria	C1	C2	C3	C4	C5	C6	C7	C8
Alternative	min	stop	pair trains/day	seats/day	coef.	million USD/year	million USD/year	year
1	36	3	160	192000	1	441.19	469.85	4.65
2	38	4	180	216000	1	496.86	528.06	7.74
3	30	0	160	192000	0	439.64	471.4	4.03
4	32	1	180	216000	0	495.16	529.76	7.17
5	34	2	180	216000	1	495.72	529.2	7.36
6	32	1	160	192000	1	440.15	470.89	4.24
Type of optimization	min	min	max	max	max	min	max	min
Preference function	linear	linear	linear	linear	usual	linear	linear	linear

Table 10: Pairwise comparison of criteria and weights.

Criteria	C1	C2	C3	C4	C5	C6	C7	C8	weights
C1: Travel time, min	1	4	2	5	1	1/2	1/2	1/2	0.15
C2: Number of train stops	1/4	1	1/5	1/2	1/5	1/2	1/3	1/3	0.04
C3: Transport satisfaction, pair trains/day	1/2	5	1	3	2	2	1	1	0.16
C4: Number of seats/day	1/5	2	1/3	1	1/5	1/2	1	1/2	0.06
C5: Connectivity	1	5	1/2	5	1	2	1	3	0.19
C6: Operating costs, million USD/year	2	2	1/2	2	1/2	1	1/3	1	0.10
C7: Profit, million USD/year	2	3	1	1	1	3	1	1/2	0.15
C8: Payback Period, years	2	3	1	2	1/3	1	2	1	0.14
CR=0.08									

The investigated approaches for choosing the best alternative between compared ones use different criteria for decision making. They can be used to compare the results given by applying each of them.

It can be seen that alternative 5 is the best for scenario 1 for the three approaches. There is a difference in the results for the second scenario among the different approaches. In this case alternative 3 is the best when using the second approach

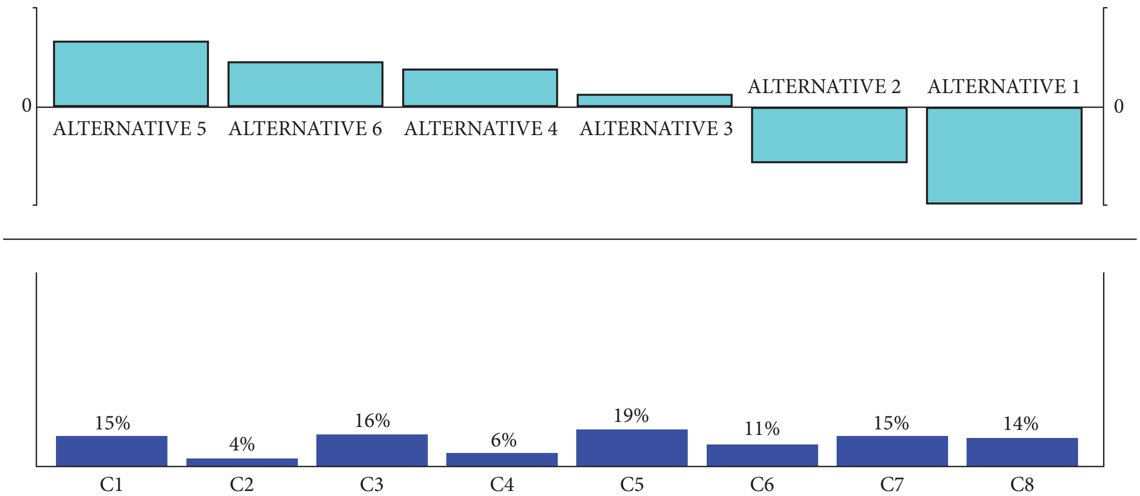

Figure 10: Ranking of alternatives for scenario 1, results in Visual PROMETHEE software.

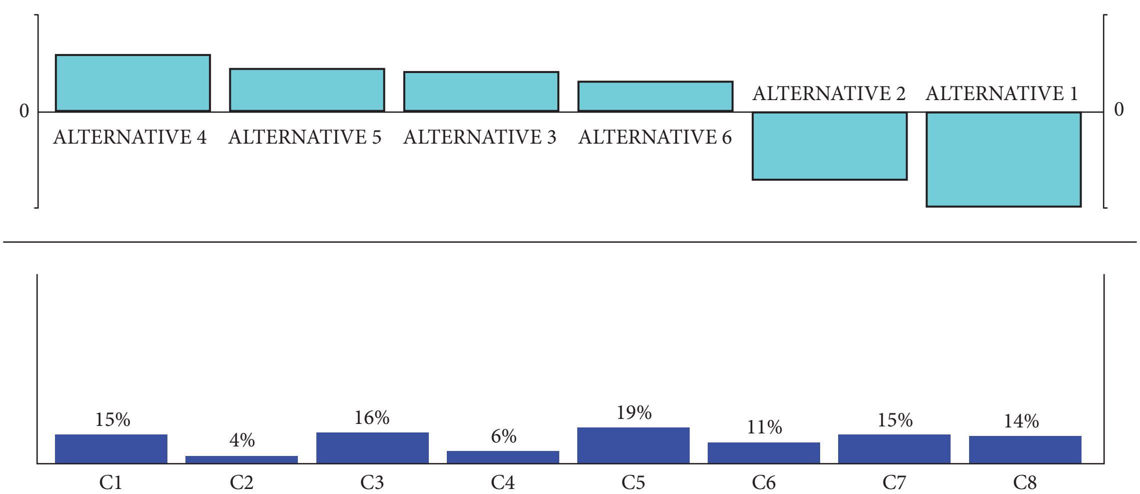

Figure 11: Ranking of alternatives for scenario 2, results in Visual PROMETHEE software.

and criterion r_{2f} to select the best one. This is explained by the little needed investments for this alternative. Alternative 4 is the best one for the third approach.

It can be concluded that with the increment of the number of trains in second scenario, the suitable alternative is changed. The application of the criterion ratio of the normalized values of the payback period and the normalized outranking flows given by PROMETHEE II as the tools of ranking alternatives leads to change in the suitable decision. The change in ranking indicates that additional alternatives could be explored, with combined services in terms of number of stops at intermediate stations; for example, some trains stopped in Guan, and others stopped in both HRS station and Guan; or some trains do not stop anywhere on the route, others stop in Guan, and others stop in both HRS station and Guan.

The first approach considered the effect of only the criteria related to transportation. It could be used for preliminary analyses when there is not information on operating costs and other economic criteria. The second approach takes into account both transportation and economic criteria. It uses two new criteria to assess the alternatives. This approach could be used when the decision maker wishes to make a decision through the cost-benefit ratio. The benefits are determined by transportation criteria by using multicriteria analysis. The costs are determined as operating costs or as a payback period. The third approach takes into account the impact of both transportation criteria and economic criteria in multicriteria model. It could be used to make decision about investigated alternatives.

Finally we can conclude that alternative 5 is defined in most of the investigated cases as the suitable one.

That is why we choose, as the suitable alternative for transportation, the variant where the trains have a stop in metro ring (Huangcun railway station) and also in Guan—alternative 5. This alternative provides an extension of Huangcun railway station and construction of a new railway station, Guan. The extension of Huangcun railway station for high-speed transport will contribute to the convenience of passengers to use the new service. The introduction of a new

Table 11: Limits of changing the weights of all the criteria while preserving the best solution.

Criteria	Weight,%	Scenario 1 Minimum weight, %	Scenario 1 Maximum weight, %	Scenario 1 Range difference, %	Scenario 2 Minimum weight, %	Scenario 2 Maximum weight, %	Scenario 2 Range difference, %
C1	15	10.53	15.56	5.03	9.19	20.06	10.87
C2	4	0.00	5.26	5.26	0.00	12.33	12.33
C3	16	15.44	31.61	16.17	13.25	16.83	3.58
C4	6	5.37	23.47	18.10	2.93	6.93	4.00
C5	19	18.46	20.98	2.52	18.18	26.81	8.63
C6	1	3.78	11.44	7.66	10.33	13.73	3.40
C7	15	14.14	36.69	22.55	12.22	15.63	3.41
C8	14	7.03	14.43	7.40	12.69	16.64	3.95

Table 12: Rank of priorities of the alternatives.

	Criterion	Scenario	Alternatives 1	2	3	4	5	6
First approach	$\varphi 1$	1, 2	6	5	4	2	1	3
Second approach	r_{1f}	1	6	5	4	2	1	3
		2	6	5	3	2	1	4
	r_{2f}	1,2	5	6	1	3	4	2
Third approach	$\varphi 2$	1	6	5	2	3	1	2
		2	6	5	4	1	2	3

station will increase travel and help to expand the satellite cities of Beijing.

In further researches we would expand the scope of the alternatives studied. This study demonstrates the applicability of proposed methodology which uses multicriteria analysis as an appropriate tool for decision making.

4. Conclusion

This research focuses on multicriteria and GIS decision approaches to determine the best alternative. The results from Remote Sensing and GIS analysis show that the proposed high-speed railway line will be economically/socially feasible and that it will satisfy the needs of significant numbers of current and future passengers. The criteria to assess the alternatives of transportation have been determined. The alternatives have been defined according to the stops in the high-speed railway line. A new railway station, Guan, has been proposed. It was found that when taking into account only the criteria related to transportation C1-C5 in the first approach, the greatest impacts are the criteria travel time (35%), transport satisfaction (30%), and connectivity (20%). The greatest impacts are upon the criteria connectivity (19%), travel time (15%), profit (15%), transport satisfaction (16%), and payback period (14%) when taking into account all criteria in the third approach. Two scenarios of passenger transport satisfaction were examined. The methodology proposed in this paper examines three approaches to decision making. The first considers only the effect on the criteria related to trips which were included in the PROMETHEE II model. It was found that alternative 5 is the best one. This alternative provides transportation between Beijing and Xiongan with intermediate stops in Guan and a third ring of the metro to meet the needs of passenger traffic. The effect of both criteria related to the trips and economic criteria have been studied in the second approach. Two criteria for choosing the best alternative have been proposed; one presents the ratio of normalized values of operating costs and normalized values of net outranking flows by PROMETHEE II method; the other presents the ratio of normalized values of payback period and normalized values of net outranking flows by PROMETHEE II method. The results show that alternative 5 is the suitable one by using the criterion of the ratio between the normalized operating costs and the normalized net outranking flows. The suitable alternative by the criterion of the normalized values of the payback period and the normalized net outranking flows is alternative 3. This alternative delivers direct transport without intermediate stops from Beijing to Xiongan. The third approach includes all investigated criteria into the PROMEHEE II model. The results for the two scenarios include stopping in Guan city (alternative 4 for scenario 2 and alternative 5 for scenario 1). The final decision is made by comparing the results of the three approaches. Finally we can conclude that alternative 5 is defined by the different approaches as the suitable one. This alternative presents a stop in metro ring (Huangcun railway station) and also in Guan—alternative 5. This alternative provides an extension of Huangcun railway station and construction of a new railway station, Guan. The result indicates that it is expedient to have a station in Guan, which will increase connection

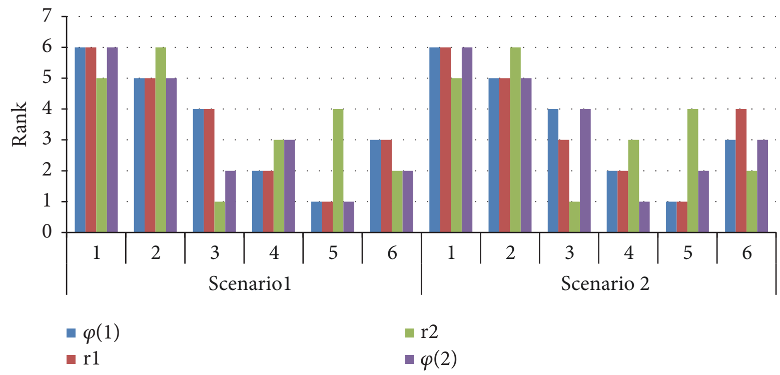

Figure 12: Ranking of the alternatives for scenarios 1 and 2.

Table 13: Saaty's scale for pairwise comparison.

Intensity of importance	Definition
1	Equal importance
3	Moderate importance of one factor over another
5	Strong or essential importance
7	Very strong importance
9	Extreme importance
2,4,6,8	Values for intermediate comparison

and connectivity among cities while providing fast mobility options to the large number of inhabitants of Guan. Our results point to the desirability of a strategy related to high-speed rail transportation between Beijing and Xiongan. This transportation will improve the mobility around and between these cities and the development of the cities themselves. The proposed methodology in this paper can be applied to make research for other conventional and high-speed rail.

Further research should consider more criteria to establish a better and more highly refined model. The investigated problem could be expanded to explore mixed service on the new high-speed railway line.

Appendix

A. AHP Method for Determining the Weights of Criteria

This method uses pairwise comparisons between criteria by Saaty's scale [85] of nine levels as shown in Table 13.

The elements of evaluation matrix A of the pairwise comparison of n criteria consist of (n, n) elements which have the following relationships:

$$a_{ii} = 1; \quad a_{ij} \neq 0; \quad a_{ji} = \frac{1}{a_{ij}} \tag{A.1}$$

where a_{ij} $(i, j = 1, \ldots, n)$ are the elements of the evaluation matrix.

The relative weights are given by the normalized right eigenvector ($W = \{w_1, .., w_n\}^T$) associated with the largest eigenvalue ($\lambda_{\max}$) of the square matrix A.

The largest eigenvalue ($\lambda_{\max}$) is calculated as follows.

$$AW = \lambda_{\max} . W \tag{A.2}$$

$$\lambda_{\max} = \sum_{i=1}^{n} \left[\left(\sum_{j=1}^{n} a_{ij} \right) . W_i \right] \tag{A.3}$$

The adequacy of an expert's assessment is determined by the consistency ratio:

$$CR = \frac{CI}{RI} \leq 0{,}1 \tag{A.4}$$

where CI is the consistency index and RI is a random index. The random matrix is given by Saaty [86]. Its values are shown in Table 14.

The consistency index is as follows.

$$CI = \frac{\lambda_{\max} - n}{n - 1} \tag{A.5}$$

B. PROMETHEE II Method for Ranking Alternatives

This method is based on a comparison of pair-by-pair possible decisions along each criterion. Possible decisions are evaluated according to different criteria, which have to be maximized or minimized. The use of the PROMETHEE II method requires two additional types of information for each criterion i: a weight w_i and a preference function $P_i(a, b)$. Preference function $P_i(a, b)$ depends on a pairwise difference between the evaluations $f_i(a)$ and $f_i(b)$ of alternatives a and b for criterion i. The preference function characterizes the difference for a criterion between the evaluations obtained by two possible decisions into a preference degree ranging from 0 to 1. In order to facilitate the definition of these functions, six basic preference functions have been proposed: usual criterion; quasi criterion; criterion with linear preference; level criterion; criterion with linear preference and

Table 14: Random consistency index (RI).

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48	1.56	1.57	1.59

indifference area; and Gaussian criterion. The explanation and mathematical calculation steps of the PROMETHEE II method are summarized below [79, 87, 88].

Step 1. The preference degree is computed for each pair of possible decisions and for each criterion, the value of e.

Step 2. For each pair of possible decisions, a global preference index $\pi(a,b)$ has to be calculated:

$$\pi(a,b) = \frac{\sum_{i=1}^{n} w_i.P_i(a,b)}{\sum_{i}^{n} w_i} \quad (B.1)$$

where $i = 1, \ldots, n$ is the number of criteria.

Step 3. This step includes ranking of the possible decisions and inclusion of the computing of the outranking flows. For each possible decision the positive outranking flow $\phi^{+}(a)$ and the negative outranking flow $\varphi^{-}(a)$ are computed:

$$\phi^{+}(a) = \frac{\pi(a,b)}{m-1} \quad (B.2)$$

$$\varphi^{-}(a) = \frac{\pi(b,a)}{m-1} \quad (B.3)$$

where j=1,. . ., m is the number of alternatives.

Step 4. This step includes determination of net outranking flows which are used to establish a complete ranking between the possible decisions. The net outranking flow $\varphi(a_j)$ of a_j in the alternatives set m of a possible decision is computed as a difference between $\varphi^{+}(a_j)$ and $\varphi^{-}(a_j)$.

$$\varphi(a_j) = \varphi^{+}(a_j) - \varphi^{-}(a_j) \quad (B.4)$$

For net outranking flow, the following conditions are valid.

$$\varphi(a_j) \in [-1;1] \quad (B.5)$$

$$\sum_{j=1}^{m} \varphi(a_j) = 0 \quad (B.6)$$

The highest value of the net outranking flow shows the best decision.

Data Availability

We have not used any confidential data, and the reference or web-link is given with each parameter or data. Only the assessment survey was done manually, and its hard copy is available.

Conflicts of Interest

The authors declare no conflicts of interest.

Authors' Contributions

The main framework for the research study was developed by Asim Farooq and Svetla Stoilova, while Mowen Xie contributed to the survey and collection of data, and Firoz Ahmad, Asim Farooq, and Svetla Stoilova analyzed the data and modeled the figures. We would like to thank Mark Buck and Edward Williams for the valuable comments and for proofreading the article.

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Research Article

An Outranking Multicriteria Method for Nominal Classification Problems with Minimum Performance Profiles

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In recent years, nominal classification problems have gained importance, especially in the context of strategic management of organizations. In this sense, this paper presents a novel multicriteria nominal classification method, derived from the concepts of PROMETHEE, applied to use in problems characterized by minimum performance profiles (MMP) for the classes. The main advantages of this proposal are criterion and alternative flexibility for classes; robustness, because it uses the concepts of a well-known method (PROMETHEE); and usefulness, because many real situations are characterized by MMP for the classes. Moreover, a real-world example is presented: a retailer's assignment in a bank, showing the applicability of the method. The proposal of a new multicriteria nominal classification method emerges from a need to devise a more flexible and realistic procedure for characterizing classes because the feature of criterion and alternative flexibility for classes has not been addressed in any extant multicriteria nominal classification procedure. The present proposal thereby endeavors to address this deficit in the multicriteria field.

1. Introduction

Multicriteria decision problems represent situations in which the decision maker (DM) confronts at least two alternatives, and the decision aims to achieve multiple objectives that are, most of the time, conflicting [1]. In these problems, the DM may pose the problem by choosing, ranking, classifying, or describing the alternatives. These modes of framing are referred to as problematics [2].

Classification allows a DM to assign alternatives to predefined classes, a process known as supervised assignment, or to nonpredefined classes, which is known as unsupervised classification and typically referred to as clustering. In both cases, according to [3], this problematic has compelling implications in numerous areas related to practical or scientific issues, such as the following fields: inventory classification [4–6]; supplier classification [7]; risk analysis in pipelines [8]; and cooperation classification [9]. For more detailed information on clustering approaches that permit allocation of alternatives into nonpredefined classes, see [10–15].

In the context of a supervised assignment, the predefined classes can either be ordered or not ordered. Sorting applies to cases involving ordered classes, and classification applies to problems involving nonordered classes, also known as nominal classification problems [3]. According to [17], in sorting problems, classes are either represented by the lower and upper bounds of a limiting profile (as in the case of ELECTRE TRI) or by a central profile as in [18].

The method proposed herein enables the allocation of alternatives into predefined classes. The proposal of a new multicriteria nominal classification method based on MPP (minimum performance profiles) emerges from a need to devise a more flexible and realistic procedure for characterizing classes, using concepts already associated with multicriteria methods. It is, however, worth noting that the method is easily adapted to apply to other types of problems with classes characterized by maximum performance profiles, central profiles, or alternatives representing the typical element of a class and pursuant to a proximity index. As such, this proposal's main advantages are as follows: criterion and alternative flexibility for classes; robustness, conceptualized in terms of a well-known method (i.e., PROMETHEE); and usefulness.

The paper is structured as follows. The next section—Theoretical Contributions—highlights the importance of methods devoted to nominal classification problems by outlining several potential applications and presents the gap of the literature which motivated the development of the method proposed herein. The section Materials and Methods comprises two subsections: the first subsection is devoted to describe nominal classification problems and the aim of the proposed method; the second subsection—Proposed Method: Features and Definitions—first presents this proposed method in detail, along with a summary of PROMETHEE concepts, assumptions, and notations, and goes on to describe the conditions and features used in the proposed nominal classification method. The section Application presents an illustrative example and a comparison among several nominal classification methods, and it is followed by a robustness analysis of the proposed method. The following section provides a discussion of the results. Finally, the section Conclusions presents some conclusions and final remarks.

2. Theoretical Contributions

Although [19] assert that, in recent years, nominal classification problems have grown more important, mainly in the context of managing of organizations and institutions, the same authors also acknowledge that this has not yielded a correspondingly vast literature on multiple criteria nominal classification. Indeed, due to its competitiveness, modern society is on a constant quest for patterns or homogeneity aimed at more effective implementation of its policies and strategies.

Five potential applications of multicriteria nominal classification problems are described in [19]. One such application is the problem of identifying or determining the most accurate disease class(es) for a given patient, based on his/her symptoms. Thus, patients assigned to the same class (es) of disease may be subject to identical medical procedure(s). Alternatively, the process of recruiting soldiers could also be handled as a nominal classification problem, as each candidate is assessed according to multiple individual features (i.e., physical fitness, intelligence, motivation, teamwork skills, and mental faculties) and subsequently assigned to one of several special core skill task units, where they will undertake special training courses. Another potential application relies on the fact that alerting people to information about public health events and risks, via social media, should be pursued differently, according to the specific type of user targeted. Whenever possible, users are characterized in terms of various features, such as age, health condition, frequency of travel, and degree of dependence on social networks. Users can then be assigned to one of several social groups, like "younger," "middle-aged," or "elderly." The fourth potential application concerns the problem of assigning responsibilities to risk owners (i.e., a person or entity responsible for managing an assigned risk). This is normally performed in risk management. Finally, the fifth potential application involves the task of determining the type of instrument(s) for issuing environmental policy best suited to manage each environmental issue in a way that achieves desired outcomes strategically, effectively, and efficiently. This is especially important, because policies play a key role in addressing complex environmental and health problems, and consequently, in improving the state of the environment.

In the multicriteria field, many approaches have been proposed to address the sorting problematic. Evidently, the ELECTRE TRI method [20, 21] is "the most popular", according to [16], and "the most used", according to [3], method of ordinal classification and based on limiting or boundary profiles. Adaptations of this method are exemplified by many works, including ELECTRE TRI-C, based on characteristic or central reference profile [22]; ELECTRE TRI-NC, where each class is defined by several central reference actions [23]; and ELECTRE-SORT [24], where classes are defined by central limiting profiles that can also be incomparable. It is possible to cite additional methods, along with the ELECTRE TRI, that deal with ordinal classification: PROMSORT [25]; AHP-Sort [26]; THESEUS [27]; TRICHOM [28]; N-TOMIC [29]; FlowSort [17]; a pairwise comparison-based method [18]; ORCLASS [30]; and a hybrid method based on AHP method, a veto system, and the K-means algorithm [31]. However, there are substantially fewer methods developed to address nominal classification problems than methods, proposed in the last few decades, intended to aid DMs in choosing, ranking, and even sorting problems.

Most current methods designed to handle nominal classifications problems are procedures based on reference actions, also called central profiles. Indeed, [32] argued that such problems usually require determination of whether an alternative a is close or similar to alternative b, or to an alternative representing a typical element of a class—also known as a prototype, and [33] explained preferences for criteria in terms of weights reflecting the importance of the criteria, relative to all classes. As such, the latter mode does not rely on a reference profile, as the weights define the classes. Moreover, [34] treated a problem defining nonordered classes by the least typical representative of each, referred to as the entrance threshold, and [35] defined each class by a given number of features, conditions, or constraints. Problems characterized by MPPs have drawn the attention of multiple researchers. For instance, [36] employed a nominal classification method aimed at enabling a construction company to select managers for different roles (i.e., the classes), according to different competencies and MPPs for classes; [37] applied the NeXClass nominal classification method to the project of assigning military students to one of multiple classes, characterized by MPPs consistent with predefined criteria; and [34, 38, 39] presented a real-world application of a classification method, using MPPs, to a problem in a banking environment.

Indeed, according to a literature review on classification methods, the feature of criterion and alternative flexibility for classes has not been addressed in any extant multicriteria nominal classification procedure, except in the method proposed by [33]. Although the proposal of [33] evaluates the alternatives, according to some criteria, the classification method relies on a binary linear programming approach, akin to a portfolio problem maximizing a valued objective function. The present proposal thereby endeavors to address this deficit in the multicriteria field. The feature of criterion and alternative flexibility for classes will be fully discussed and described in the second subsection of the next section.

3. Materials and Methods

3.1. Nominal Classification Problems. Classification (nominal and ordinal) problems aim to assign alternatives to predefined classes, according to some evaluation criteria, thus, giving the following: a set of n alternatives $A = (a_1, a_2, \ldots, a_i)$, where $i = 1, \ldots, n$; a set of m criteria $G = (g_1, g_2, \ldots, g_j)$, where $j = 1, \ldots, m$; a set of c classes $C = (C_1, C_2, \ldots C_k)$, where $k = 1, \ldots, c$; and c sets of MPPs, for each class k and for all m criteria, $B_k = (b_{k1}, b_{k2}, \ldots, b_{kj})$-; the aim is to assign each alternative in a specific class by evaluating the alternative pursuant to the criteria relevant to the different classes and also according to the MPPs defined for each class. Mathematically, each class C_k is represented by a B_k of MPPs. Further, each class is defined by a unique MPP. These concepts are represented, schematically, in Figure 1.

According to Figure 1, an alternative a_i must meet the minimum performance profile b_{kj} for each criterion g_j, defined for a specific class k, to be able to belong to this class. The problem presented in Figure 1 is a sorting problem, as, when comparing two classes C_k and C_{k-1} for all criteria, the MPP required by C_k is always greater than that required by C_{k-1}.

The main feature distinguishing nominal classification from sorting problems is that, in the first, classes are nonordered regarding the criteria. Figure 2 illustrates this idea for a nominal classification problem characterized by MPPs.

As is observable in Figure 2, the MPP required by some criteria in some classes does not follow an order. To wit, the classes are nonordered. For example, the MPP required for one alternative, to be assigned to class C_k, is greater than the MPP required for the same alternative to be assigned to class C_{k-1} for criterion g_1. However, the MPP requirement in the case of criterion g_2 is greater for the C_{k-1} class than it is for the C_k class.

Regarding the methods applicable to multicriteria nominal classification problems, researchers have proposed some modes of assigning alternatives to classes, including the following: [41] proposed the fuzzy nominal classification method PROAFTN; [33] presented a multicriteria decision method with an additive linear function, based on SMART and with linear constraints; [32] developed a method based on the concepts of concordance and discordance; and [19] proposed a nominal classification method based on the concepts of similarity and dissimilarity. There are certainly more nominal classification proposals, such as those from the following researchers: [42], with TRINONFC; [43], with CLOSORT; [34], with NeXClass; [35], with a method based on selectability/rejectability measures; and [44].

As can be seen, there are numerous potential applications of multicriteria nominal classification problems. This is a clear motivation driving the development of the proposal presented in this paper. The problem stated here consists of assigning an alternative to a specific class, considering a set of alternatives, a set of predefined nonordered classes, and a set of evaluation criteria. Also, for each predefined class, the DM defines a MPP for each evaluation criterion, which represents the minimum requirements for the inclusion of an alternative in this class. In that way, the method proposed here differs from the methodological contributions described previously. Our proposal aims to assign each alternative to the most suitable class, or rather, the alternative that outranks the reference profile with a greater magnitude, thereby ensuring coherent classification is coherent.

The next subsection details the aspects of the proposed method, after presenting the general features of the outranking multicriteria approach—more specifically the PROMETHEE, in which the proposal is based.

3.2. Proposed Method: Features and Definitions. Following an outranking multicriteria approach, where two alternatives a_1 and $a_2 \epsilon A$ are compared, the result must be expressed as a preference. Therefore, a preference function F / F: A × A $\longrightarrow$ (0, 1), representing the intensity of preference of alternative a_1 regarding alternative a_2, must be recognized, such that [2, 45–48].

(i) $F(a_1, a_2) = 0$ means indifference between a_1 and a_2, or no preference of a_1 over a_2;

(ii) $F(a_1, a_2) \sim 0$ means weak preference of a_1 over a_2;

(iii) $F(a_1, a_2) \sim 1$ means strong preference of a_1 over a_2;

(iv) $F(a_1, a_2) = 1$ means strict preference of a_1 over a_2.

It is worth stating that the symbol ~ stands for "close to" in the multicriteria literature [45–48].

Among methods for outranking multicriteria, the PROMETHEE, proposed by [48], is particularly simple and suitable method for achieving accuracy, where multiple evaluation criteria are involved [49]. The PROMETHEE methods use six types of preference functions associated with each criterion, as detailed by [48]. These were based on previous methods, such as ELECTRE III (linear criterion), or on preference modeling structures (usual, U-shape, and level criterion). In most practical applications, the six preference types provide the DM with a sufficient level of flexibility [40]. The six types of criteria and their respective descriptions are provided in Table 1.

As presented in Table 1, most types of preference functions used in PROMETHEE have a double threshold: p and q. Reference [50] has noted the importance of defining the structure of criteria in classification methods, by a double threshold (i.e., preference and indifference thresholds). According to this author, a double-threshold structure prevents improper classification. To wit, the absence of preference and indifference thresholds can lead to improper judgements between strict preference and indifference among alternatives and profiles of classes. In fact, several multicriteria classification methods, such as ELECTRE TRI or NeXClass, rely on the double-threshold structure.

Further, another justification for the double-threshold structure is that it facilitates avoidance of weak outranking

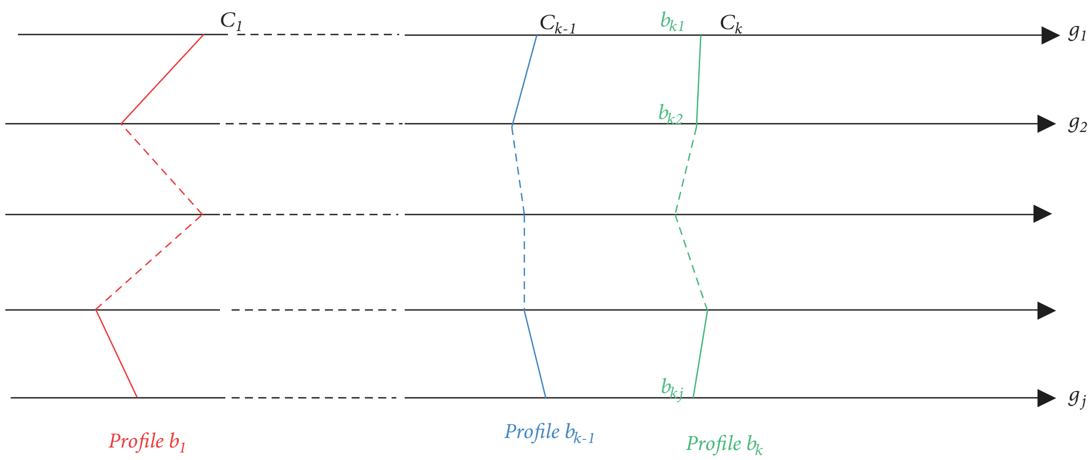

FIGURE 1: Profiles in ordinal classification problem [16].

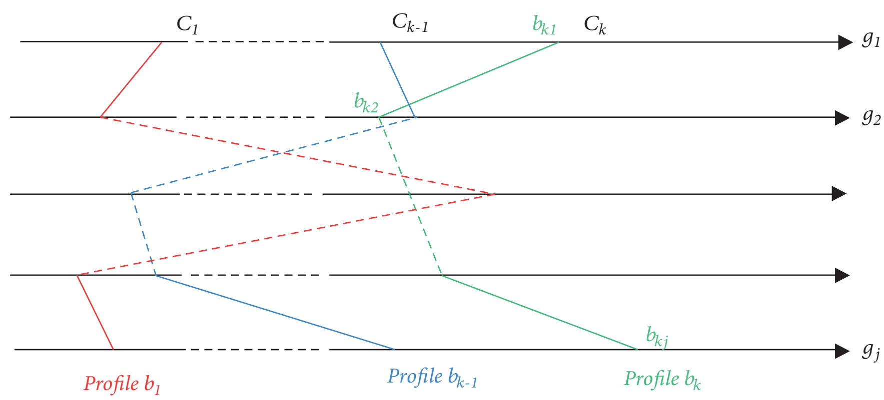

FIGURE 2: Profiles in a nominal classification problem characterized by MPPs.

relations between alternatives and profiles of classes that produce improper assignments to classes. Moreover, given that there must be imprecise and uncertain information about the MPPs, setting indifference and preference thresholds is recommended. Finally, in a case where the DM is absolutely sure about the values for the MPPs, the preference and indifference thresholds can equal zero. Therefore, our approach is flexible in the sense that it can use or not use a double-threshold structure.

For each pair of alternatives a_1 and $a_2 \epsilon A$, one first defines a preference index Π for a_1 regarding a_2 over all the m criteria. Suppose every criterion j ($j = 1, 2, \ldots m$) has been identified as one of the six types considered (Table 1), so the preference functions $F_j(a_1, a_2)$ have been defined for each j. The multicriteria preference index Π for a_1 with regard to a_2 over all the m criteria in the PROMETHEE method is therefore defined as the weighted average of preference functions F_j:

$$\Pi(a_1, a_2) = \frac{\sum_{j=1}^{m} \pi_j F_j(a_1, a_2)}{\sum \pi_j} \quad (1)$$

$F_j(a_1, a_2)$ represents the preference function F of alternative a_1 regarding a_2 over the criterion j.

π_j represents the weight of criterion j.

$\Pi(a_1, a_2)$ represents the intensity of preference of the DM of alternative a_1 over alternative a_2, given all the criteria simultaneously. It is a value between 0 and 1:

(i) $\Pi(a_1, a_2) \sim 0$ denotes a weak preference of a_1 over a_2 for all the criteria,

(ii) $\Pi(a_1, a_2) \sim 1$ denotes a strong preference of a_1 over a_2 for all the criteria.

In classification problems, nominal or ordinal, the outranking relationships are then generated by comparing alternatives to profiles. This comparison, in the approach proposed in this paper, is made through two indices that validate the claim $a_i S B_k$. These indices are defined in the following set of terms.

Definition 1 (intensity of membership). For any alternative a_i from A and any MPP B_k representing class C_k, $\Pi(a_i, B_k)$ represents the intensity of the membership of a_i in B_k; to wit, the amount of evaluation criteria supports this membership.

$$\Pi(a_i, B_k) = \frac{\sum_{j=1}^{m} w_{kj} F_{kj}(a_i, B_k)}{\sum w_{kj}} \quad (2)$$

$F_{kj}(a_i, B_k)$ represents the preference function F of alternative a_i regarding the profile B_k over the criterion j for the class C_k.

Table 1: The six types of preference functions used in PROMETHEE [40].

Criterion	Type	Parameters	Description
Usual	I	None	It is used for qualitative criteria with few evaluation levels (up to 5-point-scale)
Quasi-criterion (U-shape)	II	q parameter (indifference threshold)	It is a special case of level one
Preference threshold (V-shape)	III	p parameter (preference threshold)	It is a special case of the linear criterion when there is no indifference threshold (q)
Pseudo-criterion (Level)	IV	p and q (preference and indifference thresholds)	It is used for qualitative criteria when one needs to differentiate smaller deviations from large ones
Indifference area (Linear)	V	p and q (preference and indifference thresholds)	It is used for quantitative criteria expressed on a continuous scale
Gaussian	VI	Standard deviation	It is more difficult to structure because its threshold value is somewhere between the q indifference threshold and the p preference threshold

Definition 2 (intensity of nonmembership). Having a_i from A and any MPP B_k representing class C_k, $\Pi(B_k, a_i)$ represents the amount of evaluation criteria opposed to the membership of a_i into B_k.

$$\Pi(B_k, a_i) = \frac{\sum_{j=1}^{m} w_{kj} F_{kj}(B_k, a_i)}{\sum w_{kj}} \quad (3)$$

$F_{kj}(B_k, a_i)$ represents the preference function F of the profile B_k regarding an alternative a_i over the criterion j for the class C_k.

The sets of parameters for application of the present nominal classification proposal are as follows:

$W_k = (w_{k1}, w_{k2}, \ldots, w_{kj})$: the set of all criteria weights for class k;

$B_k = (b_{k1}, b_{k2}, \ldots, b_{kj})$: the set of each MPP for each criterion j in class k;

$P_k = (p_{k1}, p_{k2}, \ldots, p_{kj})$: the set of each preference threshold for each criterion j in class k.

$Q_k = (q_{k1}, q_{k2}, \ldots, q_{kj})$: the set of each indifference threshold for each criterion j in class k.

Given that not all criteria are necessarily considered across all classes, and even when they are, they may vary in their preference functions, weights, or thresholds, depending on their relevance to and influence on each class, and the sets F_k, W_k, Q_k, and P_k may differ for each class.

Based on these two indices (intensity of membership and intensity of nonmembership), the assignment of an alternative a_i to a class C_k is determined by the intensity of the assignment $\Pi(a_i, C_k)$ described in the following.

Definition 3 (intensity of the assignment). For any alternative a_i from A and any MPP B_k, representing class C_k, $\Pi(a_i, C_k) = \Pi(a_i, B_k) - \Pi(B_k, a_i)$ represents the intensity of the assignment of a_i to C_k. Thus, in the proposed method, the objective is to $\max \Pi(a_i, C_k)$.

The framework presented in Figure 3 summarizes the proposal through steps divided into three phases (Problem Definition, Evaluation, and Assignment).

The Problem Definition phase comprises the definition as follows:

(i) The set of n alternatives: $A = (a_1, a_2, \ldots, a_i)$, where $i = 1, \ldots, n$, that must be assigned to some nonordered classes;

(ii) The set of c nonordered classes: $C = (C_1, C_2, \ldots, C_k)$, where $k = 1, \ldots, c$;

(iii) The set of m criteria: $G = (g_1, g_2, \ldots, g_j)$, where $j = 1, \ldots, m$, which comprises all the criteria used to define the classes and specifies which alternatives will be evaluated.

For each class k, the following sets are also defined:

(i) MPPs: $B_k = (b_{k1}, b_{k2}, \ldots, b_{kj})$;

(ii) Criteria weights: $W_k = (w_{k1}, w_{k2}, \ldots, w_{kj})$;

(iii) Preference thresholds: $P_k = (p_{k1}, p_{k2}, \ldots, p_{kj})$;

(iv) The indifference thresholds: $Q_k = (q_{k1}, q_{k2}, \ldots, q_{kj})$.

In the Evaluation phase, each single alternative a_i is compared with each MPP B_k, and $\Pi(a_i, B_k)$ is calculated using (2). Further, each MPP B_k is compared with each single alternative a_i, and $\Pi(B_k, a_i)$ is calculated, using (3). Then, $\Pi(a_i, C_k)$ is defined for all alternatives as it bears on all classes.

The Assignment phase is performed via the allocation of each alternative a_i to a specific class C_k as a way of maximizing $\Pi(a_i, C_k)$.

In the proposed method, it is possible to apply different criteria subsets to different classes, given the possibility that some criteria may be applicable to characterizing some classes but unnecessary for other classes. It is worth stating that this is a specific characteristic of nominal classification problems and thus does not apply to sorting problems in which classes are ordered and are characterized by the same criteria. Therefore, a unique set of criteria, including all criteria considered for at least one class, is generated. Thus, the set of criteria weights of a class k, for example, is represented by the set $W_k = (w_{k1}, w_{k2}, \ldots, w_{kj})$. The value of a given criterion weight represents its relevance to each class. So, when a criterion g_2, for example, is neither relevant to nor

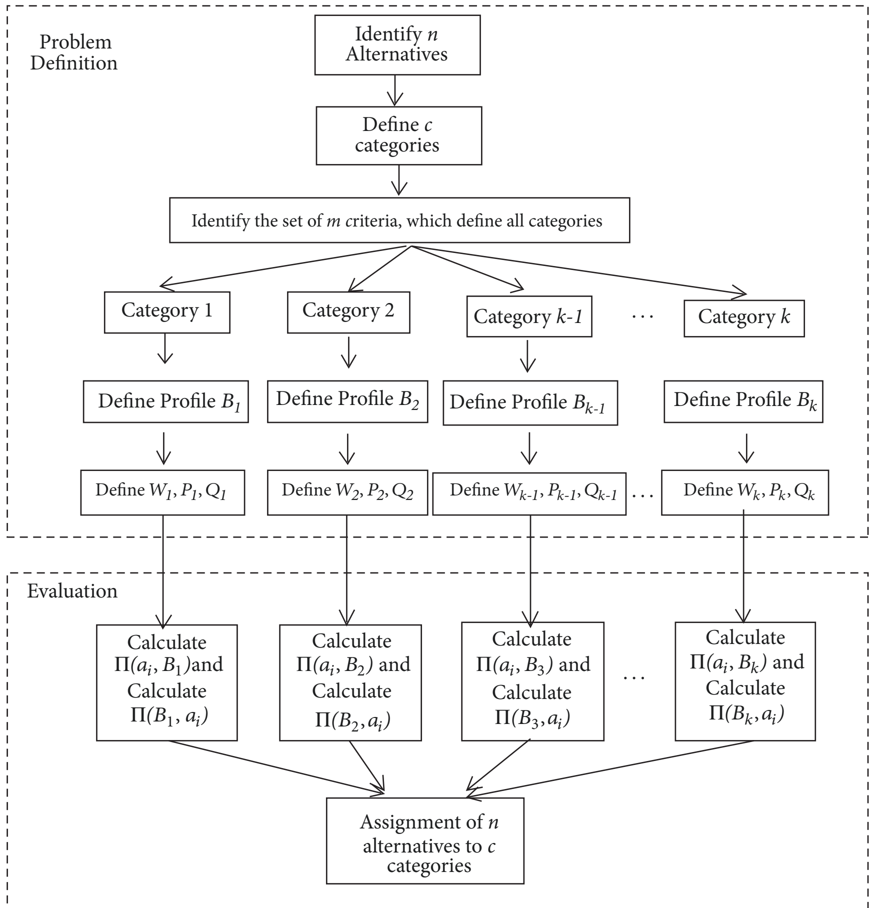

Figure 3: Outranking method for nominal classification.

even considered by a specific class k, w_{k2} assumes a null value. Indeed, other researchers have pointed out the property of criteria flexibility for classes. For instance, [41] claims that criteria weights should be defined in terms of the following two conditions: criterion g_j is not pertinent to the assignment of alternative a_i to class C_k and criterion g_j is the only criterion pertinent to the assignment of alternative a_i to class C_k.

In fact, there are several classification problems where some criteria characterize more than one class and some criteria are specific to one class. In medical diagnosis, for example, patients are assessed on the basis of different symptoms (e.g., fever, pain, headache, and cough) characterizing a very heterogeneous group of diseases (classes). According to the medical evaluation of the patient (alternative), given these various symptoms (criteria), the appropriate treatment is prescribed, to maximize the chances of success [19, 41].

In addition to the criteria flexibility for classes, given that it may not be appropriate to apply the same criteria set to different classes, another important feature in nominal classification problems, exemplified by the proposed method, is the alternative flexibility for classes, which means that some alternatives may be assigned to more than one class, and others may not be assigned to any class. As in the case of medical diagnoses, the patient could have symptoms that characterize different diseases and require different treatments. However, for the disease that represents the worst condition afflicting patient, the correspondent treatment takes priority. In this way, the minimum profile approach is used to identify the class (C_k) to which an alternative a_i gives the maximum contribution (the worse condition in the medical example), using the expression $\max \Pi(a_i, C_k)$, by means of assessing the alternative a_i for the criteria that characterize the class C_k.

Therefore, it is important to present formal and explicit definitions of criteria and alternative flexibility for classes.

Definition 4 (criteria flexibility for classes). For each class C_k, the criterion weight (w_{kj}) can assume the following values:

(a) 0, when the criterion g_j is not pertinent for the assignment of an alternative a_i to class C_k.

(b) $0 < \quad g_j \quad < 1$, when the criterion g_j is not the only pertinent criterion for the assignment of an alternative a_i to class C_k.

(c) 1, when the criterion g_j is the only pertinent criterion for the assignment of an alternative a_i to class C_k.

Definition 5 (alternative flexibility for classes). An alternative $a_i \in A$, will be as follows:

(a) assigned to only one class C_k, if $\Pi(a_i, C_k) \geq 0$ and this is the maximum value when comparing with other $\Pi(a_i, C_l)$, where $l \neq k, l = 1, \ldots, c$.

(b) assigned to $C_k \in S \subset C$, if $\Pi(a_i, C_k) \geq 0$ and it is the maximum value when comparing with other $\Pi(a_i, C_l)$ where $l \neq k$, $l = 1, \ldots, c$ and if $\Pi(a_i, C_t) = \Pi(a_i, C_k)$, $\forall C_t \in S, t = 1, \ldots, s$.

(c) not assigned to any class C_k, if $\Pi(a_i, C_k) < 0\ \forall\ k$, $k=1\ldots c$.

Despite the importance of flexibility, in relation to both criteria and alternatives, there are only a few works described in the literature, such as [33], which approach this flexibility in the context of proposing models for nominal classification. This flexibility is a particularly strong characteristic of the method proposed in the present work.

Other important properties, regarding nominal classification methods, are proposed by Costa et al. (2018) and regard the operations of merging, splitting, adding, and removing.

Definition 6 (merging operation). If two different classes, C_l and C_s, characterized by MPPs B_l and B_s, respectively, are merged to become a new one, C_t, characterized by the MPP B_t, then $B_t = \{B_{t1} = \min\{B_{l1}, B_{s1}\}, B_{t2} = \min\{B_{l2}, B_{s2}\}, \ldots, B_{tj} = \min\{B_{lj}, B_{sj}\}\}$. As a result, all the alternatives previously assigned to classes C_l and C_s will be assigned to this new class C_t.

Definition 7 (splitting operation). If one class C_t, characterized by the MPP B_t, is separated into two different classes, C_l and C_s, characterized by two new MPPs B_l and B_s, respectively, then one of new classes is characterized by the MPP B_{tj}, that is, B_{lj} or $B_{sj} = B_{tj}$, for all criteria j. Consequently, all the alternatives previously assigned class C_t will be assigned to the new classes C_l and C_s.

Definition 8 (adding classes operation). If one class C_t is included in the problem, this operation leads to build a new MPP as well as the set of criteria weights for this class. Such a new class may receive alternatives previously assigned to other classes and alternatives which were previously not assigned to any class.

Definition 9 (removing classes operation). If one class C_t is removed from the problem, alternatives previously assigned to this class may be assigned to one, more than one, or none of the remaining classes.

The next section furnishes a better understanding of the proposed method through the application of the method to a real-world problem.

4. Application

To illustrate the proposal, this paper presents a real-world application that uses real data presented by [34], concerning the problem of assigning retailers to use bank services. The real-world problem involved a Greek bank aiming to reorganize its electronic payment network of retailers equipped with terminals for online payments. To improve service efficiency, the bank wants to assign retailers to four predefined nonordered classes that represent the potential and profitability characteristics, according to specific criteria. The bank uses a two-dimensional evaluation framework, which comprises the retailer's site potential and profitability dimensions to classify retailers.

Following the framework proposed in Figure 2, for this example, in the Problem Definition phase, the following were defined:

(i) 20 alternatives: 20 retailers, $A = (a_1, a_2, \ldots, a_{20})$ and

(ii) 4 nonordered classes, $C = C_1, C_2, \ldots, C_4$, described in Table 2.

The four classification classes (Table 2), defined on the basis of this segmentation, depict the importance of the retailer to the bank.

Further, the classes are also linked to a marketing strategy that the bank will follow as a result of the classification.

(i) 13 criteria, $G = (g_1, g_2, \ldots, g_{13})$, grouped into financial and nonfinancial dimensions, as shown in Table 3;

(ii) 4 profiles, $B_k = (b_{k1}, b_{k2}, \ldots, b_{k13})$; and

(iii) the set of criterion weights $W_k = (w_{k1}, w_{k2}, \ldots, w_{k13})$, which, in this problem, is the same for all classes.

Data regarding the evaluations of alternatives, criterion weights, and MPP of the classes are shown in Table 4.

Figure 4 illustrates the idea of MPPs, considering the minimum performance profiles (b_1 and b_2) required for 2 classes (C_1 and C_2) for 5 of the 13 criteria.

As can be seen in Figure 3, the MPPs for this application do not set boundaries between classes, as expected in nominal classification methods. It worth noting, further, that the profile of class 2 (blue line) is below the profile of class 1 (red line) for criteria g_1, g_2, and g_3, but the profile of class 2 is above the profile of class 1 for criteria g_4 and g_5.

(i) 4 sets of preference thresholds, $p_k = (p_{k1}, p_{k2}, \ldots, p_{k13})$ and

(ii) 4 sets of indifference thresholds, $q_k = (q_{k1}, q_{k2}, \ldots, q_{k13})$.

The data regarding preference and indifference thresholds for each class C_k according to each criterion g_j are shown in Table 5 and in Table 6. The values determined for this problem are exactly the same for all criteria and classes.

The results of the Evaluation phase, where $\Pi\ (a_i, b_k)$, $\Pi\ (b_k, a_i)$, and $\Pi\ (a_i, C_k)$ are calculated, can be seen in Appendix. Finally, the Assignment phase is performed through the allocation of each alternative a_i to a specific class C_k as a way to maximize $\Pi\ (a_i, C_k)$. Table 7 summarizes results of the comparison of this nominal classification proposal with three methods: the NeXClass by [37], the method presented by [35], and the one proposed by [33]—adapted for this example. The methods used in these three papers, as proposed in this paper, aim to help the DM address a nonordinal classification problem. Details about them were presented in the initial sections.

Table 2: Classes for retailer classification [34].

Class specification	C_1	C_2	C_3	C_4
Definition	Retailers with relative low potential and medium to high profitability.	Retailers with relative high potential and medium to high profitability.	Retailers with minimum to high potential and medium to low profitability.	Retailers with medium to low potential and low profitability.
Strategy	Bank will allocate substantial resources to strengthen retailer's potential.	Bank will allocate maximum resources to provide high added value innovative services.	Bank will minimize resource allocation and focus to top retailers of the class.	Bank will screen retailers for potential development, allocating a minimum level of resources.

Table 3: Criteria for the evaluation of retailers [37].

Criterion	Definition	Scale
g_1	Retailer size (average daily sales in 1000 Euros)	1-100
g_2	Intensity of EFT/PoS (percentage of daily sales through EFT/PoS	1-100
g_3	Average value per EFT/PoS transaction (in Euros)	1-100
g_4	Average cost per EFT/PoS terminal (in Euros)	1-100
g_5	EFT/PoS terminal profitability (average monthly revenue per terminal [in Euros]/average monthly cost per terminal [in Euros])	1-100
g_6	Average growth rate (indicator showing monthly increase in transaction ratio)	1-100
g_7	Merchant class (based on bank's merchant type definition, according to merchant activity)	1-100
g_8	Collaboration efficiency (index based on merchants calls to bank support center)	1-100
g_9	Exclusivity (index based on retailer's exclusive collaboration; normally a retailer has installed at the same place EFT/PoS terminals from several competing banks)	1-100
g_{10}	Location (Index based on retailer's distance factors from areas with high traffic)	1-100
g_{11}	Opening hours (index based on retailer's opening hours)	1-100
g_{12}	Training of employees (index expressing employees' expertise on EFT/PoS)	1-100
g_{13}	Alternative channels (index expressing usage degree of bank's alternative payment channels from retailer)	1-100

∗EFT/PoS: Electronic Fund Transfer at Point Sale.
Source: [37].

As can be seen, NeXClass [37] differs in three classifications, [35] in one classification, [33] in one classification, and this proposal in one classification, relative to the current procedure. It is important to note that [33, 35] did not apply thresholds to the problem; however, the structure with a double threshold (preference and indifference thresholds) used in this paper prevents improper classification, as stated before. Although our results are the same as those seen [35] and differ only in one classification from the results of [33], it is extremely important to analyze the results with different data.

5. A Scenario Analysis

Therefore, this paper addresses the robustness of the results obtained by the nominal classification method proposed herein, using this first illustrative example. According to [51], robustness is a key issue in the field of decision-aiding, as well as in operations research. As a result, numerous researchers have recently addressed this issue [51–62] and have proposed the use of performance measures for classification and clustering methods [63, 64]. The term robustness refers to a capacity for withstanding "vague approximations" and/or "zones of ignorance" to maintain certain properties [51].

In general, the values assigned to the parameters in multicriteria methods are not perfectly defined. Indeed, according to [57], a critical challenge faced by analysts utilizing a multicriteria decision aid (MCDA) framework is the elicitation of the criteria weights. In the proposed method, the aim is to provide recommendations concerning the classification of retailers that remain acceptable for a wide range of values of the parameters. Thus, robustness with respect to different scenarios was assessed by changing some preference parameters, such as criteria weights, profiles of classes, and preference and indifference thresholds. As a result, a total of 138 scenarios were tested: the combination of changing the values of 13 criteria weights, four profiles of classes according to each criterion, and preference and indifference thresholds in ±10%, following similar procedures to those presented in [19, 56].

The results of the analysis of 134 scenarios are shown in Table 8. As can be seen, results are unchanged for 19 out of 20 alternatives. The only alternative showing different classifications, according to different values, for the parameters is

TABLE 4: Evaluations of alternatives, criterion weights, and MPP of the classes [37].

	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}	g_{12}	g_{13}
a_1	29	22	28	25	69	25	61	52	25	39	58	61	68
a_2	80	78	88	69	59	30	50	45	48	42	22	15	27
a_3	77	90	88	61	63	28	35	33	51	33	22	28	33
a_4	16	39	26	25	55	25	50	51	43	65	37	38	73
a_5	28	56	51	21	34	8	37	61	30	37	55	66	98
a_6	79	75	80	65	60	25	30	34	22	19	22	18	21
a_7	50	625	54	25	38	21	47	41	40	57	65	65	88
a_8	44	19	31	55	49	29	80	70	73	55	48	29	45
a_9	49	43	28	29	61	22	67	42	25	39	51	62	55
a_{10}	30	25	30	51	55	44	82	84	90	74	32	15	32
a_{11}	30	29	32	87	86	80	77	46	28	49	25	29	33
a_{12}	49	17	54	25	37	21	47	39	42	54	65	55	98
a_{13}	42	14	27	51	43	22	74	67	69	53	40	25	92
a_{14}	25	19	26	90	81	79	70	44	32	45	28	24	30
a_{15}	42	14	27	51	56	46	81	78	82	52	40	25	33
a_{16}	80	77	79	69	65	22	31	37	28	22	19	21	29
a_{17}	21	15	22	86	79	83	68	40	30	41	20	19	25
a_{18}	18	12	25	82	81	79	64	38	29	39	19	15	27
a_{19}	22	18	26	49	51	41	80	80	86	69	24	11	26
a_{20}	41	35	44	29	34	21	47	61	50	57	62	61	98
W_k	10	12	4	13	13	8	10	4	4	8	4	8	2
b_1	75	70	75	60	55	20	25	35	20	15	15	10	20
b_2	15	10	20	75	70	75	60	30	25	35	15	10	20
b_3	15	10	20	45	45	40	75	70	75	60	15	10	20
b_4	55	10	20	15	10	20	35	30	40	70	75	60	55

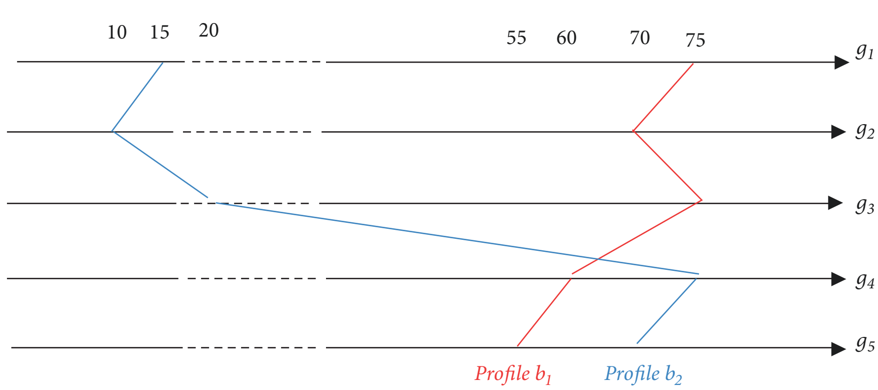

FIGURE 4: Minimum performance profiles (application of Rigopoulos et al., 2010a).

TABLE 5: Preference thresholds for each class, according to each criterion [34].

p_{kj}	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}	g_{12}	g_{13}
C_1	5	5	5	5	5	5	5	5	5	5	5	5	20
C_2	5	5	5	5	5	5	5	5	5	5	5	5	20
C_3	5	5	5	5	5	5	5	5	5	5	5	5	20
C_4	5	5	5	5	5	5	5	5	5	5	5	5	20

Table 6: Indifference thresholds for each class, according to each criterion [34].

q_{kj}	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}	g_{12}	g_{13}
C_1	2	2	2	2	2	2	2	2	2	2	2	2	10
C_2	2	2	2	2	2	2	2	2	2	2	2	2	10
C_3	2	2	2	2	2	2	2	2	2	2	2	2	10
C_4	2	2	2	2	2	2	2	2	2	2	2	2	10

Table 7: Final classification comparison for the five procedures.

	NeXClass [37]	Method presented by [35]	Method presented by [33]	Nominal Classification with MPP – the method proposed	Existing Procedure (Benchmarking)
a_1	C_4	C_4	C_3	C_4	C_3
a_2	C_1	C_1	C_1	C_1	C_1
a_3	C_1	C_1	C_1	C_1	C_1
a_4	C_3	C_4	C_3	C_4	C_4
a_5	C_4	C_4	C_4	C_4	C_4
a_6	C_1	C_1	C_1	C_1	C_1
a_7	C_4	C_4	C_4	C_4	C_4
a_8	C_3	C_3	C_3	C_3	C_3
a_9	C_4	C_4	C_4	C_4	C_4
a_{10}	C_3	C_3	C_3	C_3	C_3
a_{11}	C_2	C_2	C_2	C_2	C_2
a_{12}	C_4	C_4	C_4	C_4	C_4
a_{13}	C_4	C_3	C_3	C_3	C_3
a_{14}	C_2	C_2	C_2	C_2	C_2
a_{15}	C_3	C_3	C_3	C_3	C_3
a_{16}	C_1	C_1	C_1	C_1	C_1
a_{17}	C_2	C_2	C_2	C_2	C_2
a_{18}	C_2	C_2	C_2	C_2	C_2
a_{19}	C_3	C_3	C_3	C_3	C_3
a_{20}	C_4	C_4	C_4	C_4	C_4

a_{13}. Depending on these values, a_{13} can be assigned mainly to classes C_2 (86.23%) and C_4 (86.96%). In addition, in 6.52% of scenarios, a_{13} is assigned to class C_3, which is the correct class, according to the existing procedure based on heuristics. It is worth noting that the method proposed in this paper assigns a_{13} to the same class—C_3 (using the initial values for parameters). Moreover, alternative a_{13} is assigned to more than one class in some scenarios analyzed (alternative flexibility for classes feature). In general, these results show that the proposed method leads to robust classification, according to the changes in the preference parameters.

6. Discussion

The illustrative example presented in the previous section demonstrates the applicability of the proposal for nominal classification problems using MPPs and it can be seen that the results achieved by this proposed method are similar to those determined by other nominal classification methods, including the existing procedure performed by the bank, which can be used as the benchmark. This example had twenty alternatives, to be assigned to four classes—each characterized by one profile—regarding thirteen criteria, accounting for thirteen preference thresholds and thirteen indifference thresholds. It is worth noting that, for this example, the criterion flexibility for classes was not verified, as all criteria influenced the assignment to the four classes. Thus, the profiles of each class were evaluated according to the same thirteen criteria.

From the example, it is possible to illustrate the importance of intensity of the nonmembership Π (b_k, a_i) index for a correct assignment. The alternative a_9, for example, could be assigned to the class C_2 if only the intensity of the membership index Π (a_i, b_k) had been taken into account. However, a_9 was assigned to C_4 after considering the intensity of the nonmembership index Π (b_k, a_i). It demonstrates that, despite the fact that a_9 has good evaluation on some criteria to ensure it belongs to the C_2 class, this alternative did not meet other important (weighted) criteria for C_2, caused, in a balanced way, a_9 to become more pertinent in class C_4; thus, it was allocated to this class. Still, it is important to highlight that the proposed method aims to maximize the

Table 8: Results of the analysis of scenarios.

Alternatives	C_1	C_2	C_3	C_4	The proposed method (initial values)	Existing Procedure (Benchmarking)
a_1				138 (100%)	C_4	C_3
a_2	138 (100%)				C_1	C_1
a_3	138 (100%)				C_1	C_1
a_4				138 (100%)	C_4	C_4
a_5				138 (100%)	C_4	C_4
a_6	138 (100%)				C_1	C_1
a_7				138 (100%)	C_4	C_4
a_8			138 (100%)		C_3	C_3
a_9				138 (100%)	C_4	C_4
a_{10}			138 (100%)		C_3	C_3
a_{11}		138 (100%)			C_2	C_2
a_{12}				138 (100%)	C_4	C_4
a_{13}		119 (86.23%)	9 (6.52%)	120 (86.96%)	C_3	C_3
a_{14}		138 (100%)			C_2	C_2
a_{15}			138 (100%)		C_3	C_3
a_{16}	138 (100%)				C_1	C_1
a_{17}		138 (100%)			C_2	C_2
a_{18}		138 (100%)			C_2	C_2
a_{19}			138 (100%)		C_3	C_3
a_{20}				138 (100%)	C_4	C_4

overall allocation, i.e., to assign the alternative a_i to the class C_k that leads to max $\Pi(a_i, C_k)$.

Another important characteristic of the present proposal concerns alternative flexibility for classes, which refers to an alternative—according to the intensity of assignment parameter $\Pi\ (a_i, C_k)$—that can be in zero, one, or more classes. The first possibility could occur when the $\Pi\ (a_i, C_k)$ for each class is below a DM's given minimum, such that the alternative does not belong to any class of the problem. A real-life example might involve some candidates, under consideration for employment by a company, one or more of whom cannot be assigned to any job vacancy, given the lack of required skills. The second possibility is the most common: an alternative is assignable to one, and only one, class. The last possibility refers to a situation where an alternative could be assigned to more than one class, due to a difference between two or more of the biggest $\Pi\ (a_i, C_k)$ that is too small or possibly even zero. This was the case for some scenarios considered in the robustness analysis, and it would be the case, in the context of the aforementioned real-life example previously presented, where one or some of the candidates have the skills required for more than one job.

To deal with the alternative flexibility, this work proposes assignment thresholds to be discussed and determined by the DM. These thresholds would be in accordance with a minimum-intensity assignment parameter and indifference between more than one of the biggest intensities of assignment parameters.

One can observe that the proposed method requires the definition of several parameters (criteria weights, preference and indifference thresholds, and MPP) which is a common requirement of most multicriteria methods. For this reason, in the last decades, there has been an increase in research dedicated to elicitation of parameters because the elicitation process is one of the most complex and critical tasks facing research and applications within the field of decision analysis [59]. Indeed, this is especially critical because such parameters can change the position of any alternative in a class [9]. Reference [16], for example, proposed a methodology for the ELECTRE TRI that encompasses this problem, by substituting assignment examples by direct elicitation of the parameters of the model. The values of the parameters are inferred via a certain form of regression on assignment examples, which can be extended to apply to our method.

Another important point is that more than one DM may participate in the nominal classification process and consequently a potential conflict can emerge regarding the numerical values of parameters. An interesting discussion regarding group decision process is provided by [60–62]. Finally, it is worth noting that it is possible to incorporate those methodologies related to the elicitation of parameters for group decision in our method.

7. Conclusions

As it can be seen, the type of classification problems which aims to assign alternatives in different classes according to particular characteristics is getting much attention from researchers and practitioners. The method proposed herein has three unique features, namely: flexibility, criterion and

Table 9

	$\Pi(a_i, b_k)$	$\Pi(b_k, a_i)$	$\Pi(a_{ni}, C_k)$
a_1 x b_1	0.8	0.2	0.6
a_1 x b_2	0.7	0	0.7
a_1 x b_3	0.99	0	0.99
a_1 x b_4	0.87	0	0.87
a_2 x b_1	0.8	0.2	0.6
a_2 x b_2	0.6	0.4	0.2
a_2 x b_3	0.31	0.68	-0.37
a_2 x b_4	0.45	0.39	0.06
a_3 x b_1	0.4	0.6	-0.2
a_3 x b_2	0.9	0.1	0.8
a_3 x b_3	0.62	0.37	0.25
a_3 x b_4	0.84	0.16	0.68
a_4 x b_1	0.923	0	0.923
a_4 x b_2	1	0.93	0.07
a_4 x b_3	0.62	0	0.62
a_4 x b_4	0.768	0	0.768
a_5 x b_1	0.8	0.2	0.6
a_5 x b_2	0.6	0.4	0.2
a_5 x b_3	0.68	0.31	0.37
a_5 x b_4	0.68	0.32	0.36
a_6 x b_1	0.4	0.6	-0.2
a_6 x b_2	0.9	0.1	0.8
a_6 x b_3	0.68	0.31	0.37
a_6 x b_4	0.74	0.26	0.48
a_7 x b_1	0.8	0.2	0.6
a_7 x b_2	0.7	0.3	0.4
a_7 x b_3	0.99	0	0.99
a_7 x b_4	0.87	0.13	0.74
a_8x b_1	0.6	0.2	0.4
a_8 x b_2	0.7	0	0.7
a_8 x b_3	0.302	0	0.302
a_8 x b_4	0.87	0	0.87
a_9 x b_1	0.8	0.2	0.6
a_9 x b_2	0.7	0	0.7
a_9 x b_3	0.68	0.31	0.37
a_9 x b_4	0.61	0.26	0.35
a_{10} x b_1	0.2	0.8	-0.6
a_{10} x b_2	0.6	0.292	0.308
a_{10} x b_3	0.68	0.31	0.37
a_{10} x b_4	0.61	0.343	0.267

alternative flexibility for classes; robustness, because it uses concepts of a well-known method (PROMETHEE); and usefulness, many real problems are characterized by MPP for the classes; thus, this novel approach demonstrably addresses this problem.

Moreover, because our method deals with nominal classification problems using the concept of MPP, the alternatives are designed to classes according to the concept of maximizing the overall performance of the assignment taking into account particular characteristics (criteria) of the classes. For instance, suppose that one is analyzing the health condition (class) of a patient (alternative) according to several symptoms (criteria). Using the proposed method, the patient would be assigned to a class in which the treatment would be efficient for all possible diseases.

For future work, given the relative ease of the proposed method and its practical utility, this research may be extended, by using interval operations to deal with imprecise data. Further investigations may account for the study of the proposed assignment thresholds. Yet, some problems may require classifying alternatives by similarity, to allow for comparisons to the profiles of the classes related to a proximity index. An important item to remember is that this proposal is easily modified to address all problems with maximum performance profiles. Finally, another subject for future research is the development of a decision support system (DSS) with the proposed multicriteria method to make it available in a convenient way.

Appendix

Evaluation Phase for the Illustrative Example

See Table 9.

Data Availability

Previously reported real data were used to support this study and are available at [10.3923/jas.2008.443.452]. This prior study is cited at relevant places within the text as reference [34].

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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Research Article

A Novel Multicriteria Decision-Making Method Based on Distance, Similarity, and Correlation: DSC TOPSIS

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Decision-making, briefly defined as choosing the best among the possible alternatives within the possibilities and conditions available, is a far more comprehensive process than instant. While in the decision-making process, there are often a lot of criteria as well as alternatives. In this case, methods referred to as Multicriteria Decision-Making (MCDM) are applied. The main purpose of the methods is to facilitate the decision-maker's job, to guide the decision-maker and help him to make the right decisions if there are too many options. In cases where there are many criteria, effective and useful decisions have been taken for granted at the beginning of the 1960s for the first time and supported by day-to-day work. A variety of methods have been developed for this purpose. The basis of some of these methods is based on distance measures. The most known method in the literature based on the concept of distance is, of course, a method called Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS). In this study, a new MCDM method that uses distance, similarity, and correlation measures has been proposed. This new method is shortly called DSC TOPSIS to include the initials of distance, similarity, and correlation words, respectively, prefix of TOPSIS name. In the method, Euclidean was used as distance measure, cosine was used as similarity measure, and Pearson correlation was used as relation measure. Using the positive ideal and negative-ideal values obtained from these measures, respectively, a common positive ideal value and a common negative-ideal value were obtained. Afterward DSC TOPSIS is discussed in terms of standardization and weighting. The study also proposed three different new ranking indexes from the ranking index used in the traditional TOPSIS method. The proposed method has been tested on the variables showing the development levels of the countries that have a very important place today. The results obtained were compared with the Human Development Index (HDI) value developed by the United Nations.

1. Introduction

Decision-making with the simplest definition is the process of making choices from the available alternatives. Although it was expressed in different forms, basically a decision-making process involves: identification of the objective, selection of the criteria, selection of the alternatives, selection of the weighting methods, determination of aggregation method, and making decisions according to the results.

To be able to adapt to rapidly changing environmental conditions and make effective decisions in parallel with this change can only be possible by using scientific methods that can evaluate a large number of qualitative and quantitative factors in the decision-making process [1]. Many problems encountered in real life fit the definition of multicriteria decision-making. People find their individual preferences while they are present in evaluative judgments in multicriteria decision-making problems. It may not be difficult to decide when there are few criteria or few alternatives. However, as the subject becomes more complex, the information processing capacity of people is restricted, decision-making becomes more difficult, and help may be needed. In such cases, instead of trying to integrate too much knowledge and trying to decide, applying simple rules and procedures and evaluating the problem gradually will make it easier to decide. Such approaches will also facilitate decision-makers to make rational decisions, and the decision will be appropriate within the constraints [1].

Taking more than one criterion, choosing the most appropriate one among the alternatives, or alternating sorting problems is called Multicriteria Decision-Making (MCDM) problems. The MCDM could have been appropriate for the purposes of evaluating the alternatives in a particular order, or alternatively, in order to determine the alternatives [2]. MCDM methods, which are used in a wide range of fields from personal selection problems to economic, industrial, financial, political decision problems, have begun to develop together with the increasingly complex decision-making process from the beginning of 1960's. In particular, MCDM methods are being used to control the decision-making mechanism and to obtain the decision result as quickly and as easily as possible, provided that the target to be achieved is explained by a number of criteria and each of the alternatives has its own advantages. MCDM methods can involve different decision-makers in the decision-making process and allow many factors related to the decision-making problem to be evaluated simultaneously at different levels [1].

TOPSIS, one of the MCDM methods, developed by Hwang and Yoon [3] is a technique to evaluate the performance of alternatives through the similarity with the ideal solution. According to this technique, the best alternative would be one that is closest to the positive-ideal solution and farthest from the negative-ideal solution. The positive-ideal solution is one that maximizes the benefit criteria and minimizes the cost criteria. The negative-ideal solution maximizes the cost criteria and minimizes the benefit criteria. In summary, the positive-ideal solution is composed of all best values attainable of criteria, and the negative-ideal solution consists of all the worst values attainable of criteria [4–6].

The basis of this study is based on the report presented in the statistics conference (istkon 2017) held in Turkey in 2017 [7]. In the report, it was emphasized that TOPSIS method could be formed in different structures by drawing attention to the unit of measurement. In addition, it was noted that ranking indexes may also be different. This study was carried out with the consideration of the issues discussed in the congress and much more.

In this study, a new MCDM method based on the traditional TOPSIS method is proposed. In the proposed method, ideal positive solution approximation or ideal negative solution distance is calculated based on the distance, similarity, and correlation (DSC) measures, unlike the traditional TOPSIS method. For this reason, this new unit of measure proposed to rank alternatives is named as DSC and the new MCDM method developed is called DSC TOPSIS. The main advantage of the proposed unit of measurement is that it does not only rank the alternatives according to the concept of distance but also according to the concepts of similarity and correlation. In other words, the major advantage of the proposed method is that it proposes a stronger unit of measure by considering the three basic concepts that can be used to compare units: distance, similarity, and correlation. Another advantage of the proposed new MCDM method is that it suggests three new different methods that can be used to rank alternatives according to their importance levels. In addition, the impact of the proposed method on the cases where the decision matrix is dealt with by row, column, and double standardization methods is discussed in detail. These methods are applied to the results obtained by the proposed method. Thus, the traditional TOPSIS method and the sorting technique used in this method are compared with the proposed MCDM method and the proposed three new sorting methods. The functioning of the method has been tested to determine the order of development of countries. For this purpose, the indicators of Human Development Index (HDI) calculated by UN Development Programme (UNDP) have been utilized. The results were compared with HDI and traditional TOPSIS values.

There are no studies in the literature that use the concepts of distance, similarity, and correlation together. In this context, this study will be the first. The closest one to the proposed method in this study is Deng's method which was published in the study entitled "A Similarity-Based Approach to Ranking Multicriteria" in 2007. With this study Deng presented a similarity-based approach to ranking multicriteria alternatives for solving discrete multicriteria problems. Then Safari and et al. [5] modified Deng's similarity-based method and they proposed a new MCDM method based on similarity and TOPSIS. And then Safari and Ebrahimi [8] used a similarity-based technique by Deng [4] to rank countries in terms of HDI. They also proposed a solution for resolving a problem which exists in Deng's method. Although the issue discussed in the application part of Safari and Ebrahimi [8] and this study is on HDI data, the ways of handling this data are different. The most important difference between the two studies is the selection of criteria. While Safari and Ebrahimi [8] preferred to use "life expectancy at birth," "mean years of schooling, "expected years of schooling," and "log GNI," which are the indicator of HDI, as criteria, in this study, it was preferred to use "Life," "Education," and "Income" indexes that expressed the dimensions of HDI. At this point, Safari and Ebrahimi's [8] method can be criticized on their preferred criteria. That is, HDI's two indicators of the education dimension are considered as separate criteria in their study; in fact, it gave more weight to the dimension of education than the other dimensions in terms of relative importance. In this context, with this study, a suggestion has been made to this situation mentioned above. Another difference is undoubtedly that the HDI data discussed in both studies are of different years.

In the following sections, the concepts of distance, similarity, and correlation will be mentioned first (Section 2). Then, in Section 3, respectively, the operation of the traditional TOPSIS method will be described; the steps of the MCDM method proposed in the study will be given; and the proposed sorting methods will be explained. In Section 4; the functioning of the proposed MCDM method and ranking techniques will be tested on the variables that indicate the level of development of the countries. For this purpose, brief information about HDI will be given first in this section. In Section 5 evaluation of the results obtained will be made.

2. Distance, Similarity, and Correlation

Since the new unit of measurement proposed in the study, which is developed as an alternative to the unit of measurement used in the traditional TOPSIS method, is based on the concepts of distance, similarity, and correlation, these and related concepts will be briefly explained in the following subsections. Thus, a better understanding of the proposed unit of measurement will be provided.

2.1. Metrics, the Euclidean Distance. Any function d_{ij} is defined as a metric in case it satisfies the following four conditions (metric axioms) for all points [9–11].

(i) If $i = j$ then $d_{ij} = 0$,

(ii) If $i \neq j$, then $d_{ij} > 0$,

(iii) $d_{ij} = d_{ji}$ (*symmetry axiom*),

(iv) $d_{ij} + d_{ik} \geq d_{jk}$ for any triple i, j, k of points (*triangle inequality axiom*),

In mathematics, distance and metric expressions are used in the same sense [12]. The most important and special case of a family of functions, metrics, or distances is known as Euclidian distance. Euclidean distance is defined as the linear distance between two points in the simplest sense.

2.2. Dissimilarity. Any function d_{ij} is defined as "dissimilarity" if the first three of the metric axioms described above are satisfied. Thus, dissimilarity is more general concept. The upper and lower bound of the most dissimilarity functions are 1 and 0, respectively ($0 \leq d_{ij} \leq 1$).

2.3. Similarity. The most common measure used to compare two cases is similarity. The most important reason why similarity is more preferable than distance and dissimilarity is that it is easier for people to find similar aspects when comparing two things. Similar to dissimilarity, the concept of similarity varies between 0 and 1 ($0 \leq s_{ij} \leq 1$). Similarity is the complement of the dissimilarity. This relationship between the two concepts is shown in

$$s_{ij} = 1 - d_{ij} \tag{1}$$

There are a variety of transforms methods for achieving a distance from similarity. Including transformation given in (2), the most preferred ones are $d_{ij} = 1 - s_{ij}$, $d_{ij} = \sqrt{1 - s_{ij}}$, $d_{ij} = (1 - s_{ij})/s_{ij}$, $d_{ij} = \sqrt{2(1 - s_{ij}^2)}$, $d_{ij} = \arccos s_{ij}$, $d_{ij} = -\ln s_{ij}$, etc. [11].

The transformation given in (2) contains special meaning. That is, if many coefficients are converted according to this formula in the range of $[0, 1]$, the structure to be obtained will be a metric or even Euclidean [10].

$$d_{ij} = \sqrt{1 - s_{ij}} \tag{2}$$

2.4. Correlation and Association. The concepts of distance, dissimilarity, and similarity can be interpreted in geometrical terms because they express the relative positions of points in multidimensional space. On the other hand, the association and correlation concepts reveal the relations between the axes of the same space based on the coordinates of the points.

Except for covariance, most of association and correlation coefficients measure the strength of the relationship in the interval of $[-1, 1]$. These coefficients can be easily converted to Euclidean by using (2) [10].

2.5. Some Distances and Similarities. The *power* (p, r) *distance* is a distance on $\mathfrak{R}^n$ defined by

$$\left(\sum_i^m \left| x_i - y_i \right|^p \right)^{1/r} \tag{3}$$

For $p = r \geq 1$, it is the l_p*-metric*, including the *Euclidean, Manhattan* (or magnitude or city block), and *Chebyshev* (or maximum value, dominance) *metrics* for $p = 2$, 1 and ∞, respectively. The case $(p, r) = (2, 1)$ corresponds to the *squared Euclidean distance* [11].

The *covariance similarity* is a similarity on $\mathfrak{R}^n$, defined by

$$\mathrm{cov\_s}_{ij} = \frac{\sum_i^m (x_i - \overline{x})(y_i - \overline{y})}{n} = \frac{\sum_i^m x_i y_i}{n} - \overline{x}.\overline{y} \tag{4}$$

The *correlation similarity,* which is also referred to as Pearson correlation or Pearson product-moment correlation linear coefficient, is a similarity on $\mathfrak{R}^n$, defined by

$$corr\_s_{ij} = \frac{\sum_i^m (x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\left(\sum_i^m (x_i - \overline{x})^2\right)\left(\sum_i^m (y_i - \overline{y})^2\right)}} \tag{5}$$

The *dissimilarities*

$$1 - \left(corr\_s_{ij}\right) \tag{6}$$

$$1 - \left(corr\_s_{ij}\right)^2 \tag{7}$$

are called the *Pearson correlation distance and squared Pearson distance*, respectively.

The *cosine similarity* (or Orchini similarity, angular similarity, normalized dot product) is a similarity on $\mathfrak{R}^n$, defined by

$$\cos\_s_{ij} = \frac{\sum_i^m (x_i - y_i)^2}{\sqrt{\left(\sum_i^m (x_i)^2\right)\left(\sum_i^m (y_i)^2\right)}} = \frac{\langle x, y \rangle}{\|x\|_2 \cdot \|y\|_2} = \cos\phi \tag{8}$$

were ϕ is the angle between vectors x and y.

According to this, the *cosine dissimilarity or cosine distance* are defined by

$$1 - \left(\cos\_s_{ij}\right) \tag{9}$$

2.6. Global Distance. The "global distance" is a measure in which the result of combining the various distance measures is called "local distance" with different methods. The most common of these methods used to combine local distances are "total sum, weighted sum, and weighted average (e.g., geometric mean)" [13].

3. Multicriteria Decision-Making

MCDM problem is a problem in which the decision-maker intends to choose one out of several alternatives on the basis of a set of criteria. MCDM constitutes a set of techniques which can be used for comparing and evaluating the alternatives in terms of a number of qualitative and/or quantitative criteria with different measurement units for the purpose of selecting or ranking [8].

MCDM problems are divided into Multiattribute Decision-Making (MADM) and Multiobjective Decision-Making (MODM) problems. The MADM problems have a predetermined number of alternatives and the aim is to determine the success levels of each of these alternatives. Decisions in the MADM problems are made by comparing the qualities that exist for each alternative. On the other hand, in the MODM problems, the number of alternatives cannot be determined in advance and the aim of the model is to determine the "best" alternative [14]. There are different methods used in the literature for the solution of MCDM problems [15–17] and none of these methods gives a complete advantage over others. The most important advantage of these methods is that they allow us to evaluate quantitative and qualitative criteria together. The most well-known MCDM methods are Weighted Sum Method (WSM), Weighted Product Method (WPM), Analytic Network Process (ANP) method, Analytical Hierarchy Process (AHP) method, ELimination Et Choix Traduisant la REalité (ELECTRE), the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), Vise Kriterijumska Optimizacija I Kompromisno Resenje (VIKOR), Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE), Superiority and Inferiority Ranking (SIR), and so on.

Regardless of the type of decision-making problem, the decision-making process generally consists of the following four basic steps:

(i) determination of criteria and alternatives,

(ii) assignment of numerical measures of relative importance to criteria,

(iii) assigning numerical measures to alternatives according to each criterion,

(iv) numerical values for sorting alternatives.

The MCDM methods have been developed to effectively carry out the fourth stage of this process. There are different methods used in the literature for the solution of MCDM problems. The differences between the methods are due to the approaches that they recommend to make decisions. *In fact, no one has a complete superiority over the other*.

In this study, a new MCDM method based on the traditional TOPSIS method is proposed. In the proposed method, ideal positive solution approximation or ideal negative solution distance is calculated based on the distance, similarity, and correlation measures, unlike the traditional TOPSIS method. For this reason, the method is called DSC TOPSIS. Three different new methods are also proposed in order to rank the alternatives according to their importance levels in DSC TOPSIS or similar MCDM methods. There are several studies in the literature that modify the TOPSIS method. Some of them can be listed as follows. Hepu Deng [4], in his paper named as "*A Similarity-Based Approach to Ranking Multi-Criteria Alternatives*" presented a similarity-based approach to ranking multicriteria alternatives for solving discrete multicriteria problems. Ren and et al. [18] introduced a novel modified synthetic evaluation method based on the concept of original TOPSIS and calculated the distance between the alternatives and "optimized ideal reference point" in the D^+ D^- plane and constructing the P value to evaluate quality of alternative in their study titled "*Comparative Analysis of a Novel M-TOPSIS Method and TOPSIS.*" Cha [19] built the edifice of distance/similarity measures by enumerating and categorizing a large variety of distance/similarity measures for comparing nominal type histograms at the paper titled "*Comprehensive Survey on Distance/Similarity Measures between Probability Density Functions.*" Chakraborty S. and Yeh C.-H. [20], in the study named as "*A Simulation Comparison of Normalization Procedures for TOPSIS*," compare four commonly known normalization procedures in terms of their ranking consistency and weight sensitivity when used with TOPSIS to solve the general MADM problem with various decision settings. Chang et al. [21] adopted the concepts of "Ideal" and "Anti-Ideal" solutions as suggested by Hwang and Yoon [3] and studied the extended TOPSIS method using two different "distance" ideas, namely, "Minkowski's L_λ metric" and "Mahalanobis" distances in their study titled "*Domestic Open-End Equity Mutual Fund Performance Evaluation Using Extended Topsis Method with Different Distance Approaches.*" Hossein and et al. [5] in their paper named as "*A New Technique for Multi Criteria Decision-Making Based on Modified Similarity Method*" modified Deng's similarity-based method. Omosigho and Omorogbe [22] in their study named as "*Supplier Selection Using Different Metric Functions*" examined the deficiencies of using only one metric function in TOPSIS and proposed the use of spherical metric function in addition to the commonly used metric functions. Kuo [23] in his paper named as "*A Modified TOPSIS with a Different Ranking Index*" by proposing w^- and w^+ as the weights of the "*cost*" criterion and the "*benefit*" criterion, respectively, he defined a new *ranking index*.

Since the proposed method is based on the TOPSIS method, the steps of the traditional TOPSIS method will first be explained in the following subsections. And then the details of the proposed MCDM method will be discussed.

3.1. TOPSIS Method

Step 1. Determining the decision matrix:

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \cdots & \cdots & x_{ij} & \cdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{bmatrix} \tag{10}$$

In a decision matrix, lines represent alternatives A_i $(i = 1, 2, \ldots, n)$ and columns refer to criteria C_j $(j = 1, 2, \ldots, m)$.

Step 2. Determining the weighting vector as follows:

$$W = \left(w_1, \cdots, w_j, \cdots, w_m\right) \quad (11)$$

in which the relative importance of criterion C_j with respect to the overall objective of the problem is represented as w_j.

Step 3. Normalizing the decision matrix through Euclidean normalization:

$$x'_{ij} = \frac{x_{ij}}{\left(\sum_{j=1}^{n} x_{ij}^2\right)^{1/2}} \quad (12)$$

As a result, a normalized decision matrix can be determined as

$$X' = \begin{bmatrix} x'_{11} & x'_{12} & \cdots & x'_{1m} \\ x'_{21} & x'_{22} & \cdots & x'_{2m} \\ \cdots & \cdots & x'_{ij} & \cdots \\ x'_{n1} & x'_{n2} & \cdots & x'_{nm} \end{bmatrix} \quad (13)$$

Step 4. Calculating the performance matrix:

The weighted performance matrix which reflects the performance of each alternative with respect to each criterion is determined by multiplying the normalized decision matrix (13) by the weight vector (11).

$$Y = \begin{bmatrix} w_1 x'_{11} & w_2 x'_{12} & \cdots & w_m x'_{1m} \\ w_1 x'_{21} & w_2 x'_{22} & \cdots & w_m x'_{2m} \\ \cdots & \cdots & w_i x'_{ij} & \cdots \\ w_1 x'_{n1} & w_2 x'_{n2} & \cdots & w_m x'_{nm} \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1m} \\ y_{21} & y_{22} & \cdots & y_{2m} \\ \cdots & \cdots & y_{ij} & \cdots \\ y_{n1} & y_{n2} & \cdots & y_{nm} \end{bmatrix} \quad (14)$$

Step 5. Determining the PIS and the NIS:

The positive-ideal solution (PIS) and the negative-ideal solution (NIS) consist of the best or worst criteria values attainable from all the alternatives. Deng [4] enumerated the advantages of using these two concepts as: their simplicity and comprehensibility, their computational efficiency, and their ability to measure the relative performance of the alternatives in a simple mathematical form [8].

For PIS (I^+),

$$I^+ = \left\{\left(\max_i y_{ij} \mid j \in J\right), \left(\min_i y_{ij} \mid j \in J'\right\} \quad (15)$$

$$I^+ = \{I_1^+, I_2^+, \ldots, I_n^+\} \quad (16)$$

And for NIS (I^-),

$$I^- = \left\{\left(\min_i y_{ij} \mid j \in J\right), \left(\max_i y_{ij} \mid j \in J'\right\} \quad (17)$$

$$I^- = \{I_1^-, I_2^-, \ldots, I_n^-\} \quad (18)$$

At both formulas, J shows benefit (maximization) and J' shows loss (minimization) value.

Step 6. Calculating the degree of distance of the alternatives between each alternative and the PIS and the NIS:

The D^+ and the D^- formulas are given in (19) and (20), respectively, by using Euclidean distance.

$$D_i^+ = \sqrt{\sum_{j=1}^{m} \left(y_{ij} - I_j^+\right)^2} \quad (19)$$

$$D_i^- = \sqrt{\sum_{j=1}^{m} \left(y_{ij} - I_j^-\right)^2} \quad (20)$$

Step 7. Calculating the overall performance index for each alternative across all criteria:

$$P_i^1 = \frac{D_i^-}{D_i^- + D_i^+} \quad (21)$$

$0 \leq P_i \leq 1$. The P_i value indicates the absolute closeness of the ideal solution. If $P_i = 1$ then A_i is the PIS; if $P_i = 0$ then A_i is the NIS.

Step 8. Ranking the alternatives in the descending order of the performance index value.

3.2. Proposed Method. The approaches used in the steps of "normalizing the decision matrix," "calculating the distance of positive and negative-ideal solutions," and "calculating the overall performance index for each alternative across all criteria" of the traditional TOPSIS method are open to interpretation and can be examined, improved, or modified. Approaches that can be recommended in these steps are briefly summarized below.

"Normalizing the decision matrix" step (Step 3 at the traditional TOPSIS): The normalization method used to normalize the decision matrix can also be achieved with different normalization formulas. Also, for this step, standardization rather than normalization is one of the methods that can be applied.

"Calculating the distance of positive- and negative-ideal solutions" step (Step 6 at the traditional TOPSIS): alternative to the distance of Euclidean (Minkowski L_2), which is used to calculate the distance of PIS and NIS, is also possible to use many different distance measures such as linear [15], spherical [22], Hamming [22], Chebyshev [21], Dice [24], Jaccard [24], and cosine (Liao and Xu, 2015). The concept of similarity is also considered as an alternative (Zhongliang, 2011 and [4]).

"Calculating the overall performance index for each alternative across all criteria" step (Step 7 at the traditional TOPSIS): different approaches such as the concept of distance can be used instead of the simple ratio recommended in the traditional TOPSIS method since it is considered to be a very simple way by most researchers.

To offer solutions to the weaknesses listed above, in this study instead of

(i) "normalization approach" applied in Step 3 of the traditional TOPSIS method, *column and row-standardization approach*,

(ii) "euclidean distance" used in Step 6, *a new measure based on the concept of "distance, similarity and correlation" (DSC)*,

(iii) "simple ratio" used in Step 7, *new sorting approaches based on the concept of distance* have been proposed.

This method, which could be an alternative to MCDM methods especially TOPSIS, due to the new measure proposed is called DSC TOPSIS. The main objective of

(i) using different standardization methods is to emphasize what can be done for different situations that can be encountered in real life problems;

(ii) developing a new measure for the traditional TOPSIS method in this study is to improve the approach of "evaluating the two alternatives only based on the Euclidean distance to the PIS and NIS values" and make it more valid;

(iii) developing new sorting methods is to criticize the method used in traditional TOPSIS because it is based on simple rate calculation.

At this point, an important issue mentioned earlier must be remembered again. That is, the fact that none of the MCDM methods is superior to the other. Therefore, the new MCDM method proposed in this study will certainly not provide a full advantage over other methods. But a different method will be given to the literature.

Basically, there are three approaches used to compare vectors. These are distance, similarity, and correlation. The concept of distance is the oldest known comparison approach and is the basis of many sorting or clustering algorithms. On the other hand both similarity and correlation are two other important concepts that should be used in vector comparisons. According to Deng [4] mathematically, comparing two alternatives in the form of two vectors is better represented by the magnitude of the alternatives and the degree of conflict between each alternative and the ideal solution, rather than just calculating the relative distance between them [8]. The degree of conflict between each alternative and the ideal solution is calculated by "cosine similarity." On the other hand, it is possible to compare alternatives according to their relationship. In this case, the concept of correlation that evaluates from another point of view and therefore correlation similarity will be introduced. Briefly Euclidean distance, cosine similarity, and correlation similarity arise as the basic metrics that can be used for ranking the alternatives.

In order to express an MCDM problem in "*m*-dimensional real space," alternatives can be represented by A_i vector and PIS and NIS can be represented by I_j^+ and I_j^- vectors, respectively. In this case, the angle between A_i and I_j^+ (I_j^-) in the *m*-dimensional real space, which is shown by θ_i^+ (θ_i^-), is a good measure of conflict between the vectors [8]. These vectors (I_j^+, I_j^-) and the degree of conflict ($\cos\theta_i$) between them are shown in Figure 1 by Deng [4].

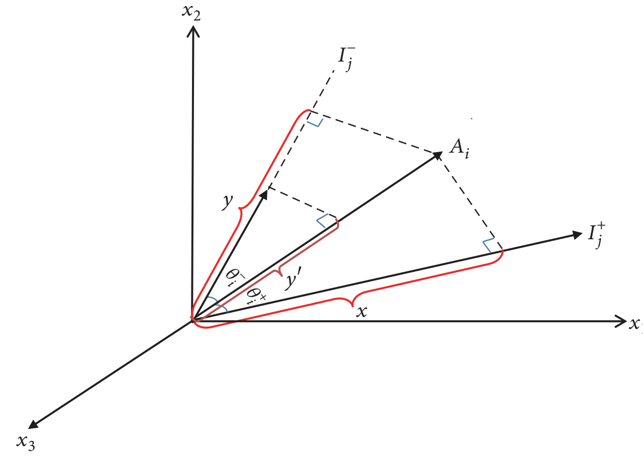

Figure 1: The degree of conflict.

The situation of conflict occurs when $\theta_i \neq 0$, that is, when the gradients of A_i and I_j^+ (I_j^-) are not coincident. Thus the conflict index is equal to "one" as the corresponding gradient vectors lie in the same direction, and the conflict index is "zero" when $\theta_i = \pi/2$ which indicates that their gradient vectors have the perpendicular relationship with each other [4, 8].

In the light of the above-mentioned explanations and causes, the three approaches of "*distance, similarity, and correlation*" that can be used to compare vectors have been exploited in order to combine them in a common pavilion to develop a stronger comparison measure. Thus, the steps detailed below have been carried out.

3.2.1. Proposed Step 3: Standardize the Decision Matrix. Unlike the traditional TOPSIS method, this step has been developed on the basis of standardization. The differences of column and row-standardization were emphasized.

Standardization vs. Normalization. Data preprocessing, which is one of the stages of data analysis, is a very important process. One of the first steps of data preprocessing is the normalization of data. This step is particularly important when working with variables that contain different units and scales.

All parameters should have the same scale for a fair comparison between them when using the Euclidean distance and similar methods. Two methods are usually well known for rescaling data. These are "normalization" and "standardization." *Normalization* is a technique which scales all numeric variables in the range $[0, 1]$. In addition to (12) some other possible formulas of normalization are given below and the other is given in (59):

$$x'_{ij} = \frac{x_{ij}}{median_X} \tag{22}$$

$$x'_{ij} = \frac{x_{ij}}{\max\left(x_{ij}\right)} \tag{23}$$

On the other hand *standardization* is a method which transforms the variables to have zero mean and unit variance, for example, using the equation below:

$$x'_{ij} = \frac{x_{ij} - \mu_X}{\sigma_X} \quad (24)$$

Both of these techniques have their drawbacks. If you have outliers in your data set, normalizing your data will certainly scale the "normal" data to a very small interval. And generally, most of data sets have outliers. Furthermore these techniques can have negative effects. For example, if the data set dealt with outlier values, the normalization of this data will cause the data to be scaled at much smaller intervals. On the other hand using standardization is not bounded data set (unlike normalization). Therefore, in this study standardization method was preferred to use.

Standardization can be performed in three ways: *column-standardization* (variable or criteria), *row-standardization* (observation or alternative), and *double standardization* (both row and column standardization) [25]. In *column-standardization* variables are taken separately and observations for each variable are taken as a common measure, whereas in *row-standardization* the opposite is the case. That is, the observations are handled one by one and the variables are brought to the same measurement for each observation. In addition, it may be the case that both column-standardization and row-standardization are done together. This is referred to as "*double-standardization*."

Selection Appropriate Standardization Method. Different transformations of the data allow the researchers to examine different aspects of the underlying, basic structure. Structure may examine "wholistically" with main effects, interaction, and error present or the main effects can be "removed" to examine interaction (and error) [25].

A row or column effect can be removed with a transformation that sets row or column means (totals) to equal values. Centering or standardization of row or column variables results in a partial removal of the main effects, by setting one set of means to zero. Double centering and double standardization remove both sets of means. Removal of the "magnitude" or "popularity" dimension allows the researcher to examine the data for patterns of "interactive" structure [25].

Centering or standardization within rows removes differences in row means but allows differences in column mean to remain. Thus the "consensus" pattern among rows, characterized by differences in the column means, remains relatively unaffected. Column-standardization also removes only one set of means; column means are set to zero. Centering of data, usually performed on column or row variables, is analogous to analyzing a covariance matrix. Data are sometimes double-centered to remove the magnitude or popularity dimension (Green, 1973).

These are approaches that can be used in sorting and clustering alternatives. According to the structure of the decision problem, it is necessary to determine which standardization method should be applied. That is,

(i) *column standardization:* if observations are important,

(ii) *row-standardization:* if it is important to bring the variables to the same unit,

(iii) *double standardization:* where both observations and variables are important to be independent of the unit preferable.

3.2.2. Proposed Step 6: Calculating the Degree of Distance of the Alternatives between Each Alternative and the PIS and the NIS. In the context described above and in Section 2.5 (using geometric mean approach to combine local distances) the proposed new measure formula is constructed as follows.

$$\text{Proposed\_}d_i = \sqrt[3]{(squared\_d_i) . (corr\_d_i) . (\cos\_d_i)} \quad (25)$$

The three terms in the formula are "*the squared Euclidean distance* ((3), the case $(p, r) = (2, 1)$)," "*the correlation distance* (6)," and "*the cosine distance* (9)," respectively.

As mentioned before, the methods that can be used to determine the differences between vectors are distance, correlation, and similarity. In (25), these concepts were combined with the geometric mean and a stronger measure than the traditional TOPSIS method used to determine the differences between the vectors.

Below column-standardization and row-standardization cases of this proposed measure were discussed, respectively. In the literature, the effects of row-standardization on the TOPSIS method have not been addressed at all. For this reason, the effects of row-standardization on the TOPSIS method in this study were also investigated.

For Column-Standardization. In the case of standardizing the data according to the columns, in other words according to the alternatives, there is no change or reduction in the formulas of the squared Euclidean, correlation, and cosine distances used in the proposed measure. For this reason (25) can be used exactly in the case of column-standardization.

Thus, the degree of distance of the alternatives between each alternative and the PIS and the NIS for column-standardized data are as follows:

$$\text{Proposed\_}D_i^+ = \sqrt[3]{(squared\_d_i^+) . (corr\_d_i^+) . (\cos\_d_i^+)} \quad (26)$$

$$\text{Proposed\_}D_i^- = \sqrt[3]{(squared\_d_i^-) . (corr\_d_i^-) . (\cos\_d_i^-)} \quad (27)$$

For square Euclidean distance, cosine distance, and correlation distance, the D^+ and D^- values and, for correlation similarity and cosine similarity, the S^+ and S^- values are, respectively, as follows.

Square Euclidean (Sq_Euc) Distance for Column-Standardized Data:

$$D_i^+ = \sum_{j=1}^{n} \left(y_{ij} - I_j^+\right)^2 \tag{28}$$

$$D_i^- = \sum_{j=1}^{n} \left(y_{ij} - I_j^-\right)^2 \tag{29}$$

Correlation Similarity (Corr_s) for Column-Standardized Data:

$$S_i^+ = \frac{\sum_j^m \left(y_j - \overline{y}\right)\left(I_j^+ - \overline{I}_j^+\right)}{\sqrt{\left(\sum_j^m \left(y_j - \overline{y}\right)^2\right)\left(\sum_j^m \left(I_j^+ - \overline{I}_j^+\right)^2\right)}} \tag{30}$$

$$S_i^- = \frac{\sum_j^m \left(y_j - \overline{y}\right)\left(I_j^- - \overline{I}_j^-\right)}{\sqrt{\left(\sum_j^m \left(y_j - \overline{y}\right)^2\right)\left(\sum_j^m \left(I_j^- - \overline{I}_j^-\right)^2\right)}} \tag{31}$$

Cosine Similarity (Cos_s) for Column-Standardized Data:

$$S_i^+ = \frac{\sum_j^m \left(y_j - I_j^+\right)^2}{\sqrt{\left(\sum_j^m \left(y_j\right)^2\right)\left(\sum_j^m \left(I_j^+\right)^2\right)}} \tag{32}$$

$$S_i^- = \frac{\sum_j^m \left(y_j - I_j^-\right)^2}{\sqrt{\left(\sum_j^m \left(y_j\right)^2\right)\left(\sum_j^m \left(I_j^-\right)^2\right)}} \tag{33}$$

Correlation Distance (Cor_d) for Column-Standardized Data:

$$\begin{aligned} D_i^+ &= 1 - S_i^+ \\ &= 1 - \frac{\sum_j^m \left(y_j - \overline{y}\right)\left(I_j^+ - \overline{I}_j^+\right)}{\sqrt{\left(\sum_j^m \left(y_j - \overline{y}\right)^2\right)\left(\sum_j^m \left(I_j^+ - \overline{I}_j^+\right)^2\right)}} \end{aligned} \tag{34}$$

$$\begin{aligned} D_i^- &= 1 - S_i^- \\ &= 1 - \frac{\sum_j^m \left(y_j - \overline{y}\right)\left(I_j^- - \overline{I}_j^-\right)}{\sqrt{\left(\sum_j^m \left(y_j - \overline{y}\right)^2\right)\left(\sum_j^m \left(I_j^- - \overline{I}_j^-\right)^2\right)}} \end{aligned} \tag{35}$$

Cosine Distance (Cos_d) for Column-Standardized Data:

$$D_i^+ = 1 - S_i^+ = 1 - \frac{\sum_j^m \left(y_j - I_j^+\right)^2}{\sqrt{\left(\sum_j^m \left(y_j\right)^2\right)\left(\sum_j^m \left(I_j^+\right)^2\right)}} \tag{36}$$

$$D_i^- = 1 - S_i^- = 1 - \frac{\sum_j^m \left(y_j - I_j^-\right)^2}{\sqrt{\left(\sum_j^m \left(y_j\right)^2\right)\left(\sum_j^m \left(I_j^-\right)^2\right)}} \tag{37}$$

For Row-Standardization. If the vectors are standardized according to the rows, some reductions are concerned in the formula of the recommended measure. It is detailed below (in the following, it is expressed as *Case 4*).

Let X and Y be two vectors. When μ_X and μ_Y are the means of X and Y, respectively, and σ_X and σ_Y are the standard deviations of X and Y, the correlation between X and Y is defined as

$$corr\_s_{XY} = \frac{(1/n)\sum_j^n x_j y_j - \mu_X \mu_Y}{\sigma_X \sigma_Y} \tag{38}$$

The numerator of the equation is called the *covariance* of X and Y and is the difference between the mean of the product of X and Y subtracted from the product of the means. If X and Y *are row-standardized*, they will each have a mean of "*0*" and a standard deviation of "*1*," so the (38) reduces to

$$corr\_s_{XY}^* = \frac{1}{n}\sum_j^n x_j y_j \tag{39}$$

While Euclidean distance is the sum of the squared differences, correlation is basically the average product. From the Euclidean distance formula it can be seen that there is further relationship between them.

$$\begin{aligned} Euc\_d_i &= \sqrt{\sum_j^n \left(x_j - y_j\right)^2} \\ &= \sqrt{\sum_j^n x_j^2 + \sum_j^n y_j^2 - 2\sum_j^n x_j y_j} \end{aligned} \tag{40}$$

If X and Y are *standardized*, at (40) the sum of squares will be equal to n ($\sum_j^n x_j^2 = n$ and $\sum_j^n y_j^2 = n$). So (40) reduced to (42).

$$Euc\_d_i^* = \sqrt{n + n - 2\sum_j^n x_j y_j} \tag{41}$$

$$= \sqrt{2\left(n - \sum_j^n x_j y_j\right)} \tag{42}$$

In this case, if the square of the Euclidean distance is taken (the *square distance*) and some adjustments are made, (42) reduced to the formula for the correlation coefficient as follows:

$$d_i^{2*} = 2n\left(1 - corr\_s_i^{\,*}\right) \tag{43}$$

In this way, for *row-standardized* data, the correlation between X and Y can be written in terms of the squared distance between them:

$$corr\_s_i^{\,*} = 1 - \frac{d_i^{2*}}{2n} \tag{44}$$

And the *correlation distance* is as

$$corr\_d_i^{\ *} = 1 - corr\_s_i^{\ *} = \frac{d_i^{2*}}{2n} \quad (45)$$

On the other hand, in the case $\overline{x} = 0$ and $\overline{y} = 0$ the correlation similarity becomes $\langle x, y\rangle / (\|x\|_2 . \|y\|_2)$ which is the formula of *cosine similarity*. So the formula of correlation distance is the formula of *cosine distance* at the same time.

$$\cos\_d_i^{\ *} = corr\_d_i^{\ *} = \frac{d_i^{2*}}{2n} \quad (46)$$

Considering the relationships described above for row-standardized data correlation and cosine similarities are transformed into square distance value. Thus, these three concepts used to measure the differences between the vectors are expressed in the same way. Thus, (25) is converted into the form given in

$$\text{Proposed\_}d_i = \sqrt[3]{d_i^2 . \frac{d_i^2}{2n} . \frac{d_i^2}{2n}} \quad (47)$$

$$= \frac{1}{\sqrt[3]{4n^2}} . d_i^2 \quad (48)$$

So, calculating the degree of distance of the alternatives between each alternative and the PIS and the NIS for row-standardized data are as follows:

$$\text{Proposed\_}D_i^+ = \frac{1}{\sqrt[3]{4n^2}} . d_i^{+2} = \frac{1}{\sqrt[3]{4n^2}} . \sum_{j=1}^{n} \left(y_{ij} - I_j^+\right)^2 \quad (49)$$

$$\text{Proposed\_}D_i^- = \frac{1}{\sqrt[3]{4n^2}} . d_i^{-2} = \frac{1}{\sqrt[3]{4n^2}} . \sum_{j=1}^{n} \left(y_{ij} - I_j^-\right)^2 \quad (50)$$

In row-standardization case the D^+, D^- and S^+, S^- values for square Euclidean distance, correlation similarity, cosine similarity, correlation distance, and cosine distance are as follows. Differently here the reduced states of the correlation and cosine formulas are used (for column-standardization was explained at *Proposed Step 6*, with (30)-(37)). For this reason the correlation S^+ and S^- formulas are equal to the formulas of cosine S^+ and S^-, respectively.

Square Euclidean (Sq_Euc) Distance for Row-Standardized Data. The D^+ and D^- values of the square Euclidean distance for row-standardized data are equal to formulas of (28)-(29), respectively (the D^+ and D^- values of the square Euclidean distance for column-standardized data).

Correlation Similarity (Corr_s) or Cosine Similarity (Cos_s) for Row-Standardized Data:

$$S_i^+ = \frac{1}{n} \sum_{j=1}^{n} y_{ij} I_j^+ \quad (51)$$

$$S_i^- = \frac{1}{n} \sum_{j=1}^{n} y_{ij} I_j^- \quad (52)$$

Correlation Distance (Corr_d) or Cosine Distance (Cos_d) for Row-Standardized Data:

$$D_i^+ = 1 - S_i^+ = 1 - \left(\frac{1}{n} \sum_{j=1}^{n} y_{ij} I_j^+ \right) \quad (53)$$

$$D_i^- = 1 - S_i^- = 1 - \left(\frac{1}{n} \sum_{j=1}^{n} y_{ij} I_j^- \right) \quad (54)$$

Circumstance Critique: although the Euclidean distance formula (28)-(29) is not affected by column or row-standardization, the cosine (34)-(35) and correlation (36)-(37) distance formulas are affected. In order to apply (53)-(54), it is necessary that the performance matrix has a row based average of "0" and a standard deviation of "1." To ensure this condition, see the following.

For the row-standardization case, either the criteria (variables) are not to be weighted, so the weight vector must be treated as a "unit vector," or the performance matrix obtained after weighting the criteria has to be standardized again on a row basis.

For the column-standardization case: whether the criterion weighting is done or not, the performance matrix has to be standardized again on a row basis.

For both cases the PIS and NIS vectors must be standardized on a row basis.

3.2.3. Proposed Step 7: Calculating the Overall Performance Index (Ranking Index P_i) for Each Alternative across All Criteria. In this study three different performance indexes were proposed. The significance of these three new different P_i values were tried to emphasize by taking into account "the P_i value suggested by the traditional TOPSIS method" (21) and "the P_i value proposed by Ren et al. [18]"

$$P_i^2 = \sqrt{\left[D_i^+ - \min\left(D_i^+\right)\right]^2 + \left[D_i^- - \max\left(D_i^-\right)\right]^2} \quad (55)$$

As can be understood from the above formula, Ren and et al. [18] calculated the P_i values using the Euclidean distance. For this reason, this value will be called the *Euclidean P_i* value in the study.

Euclidean P_i value: the D^+, D^- plane is established with D^+ at the x-axis and D^- at the y-axis. The point (D_i^+, D_i^-) represents each alternative $(i = 1, 2 \ldots, \text{n})$ and the $(\min(D_i^+), \max(D_i^-))$ point to be the "optimized ideal reference point" (in Figure 2, point A); then the distance from each alternative to point A is calculated by using (55). The graph of the distance is shown in Figure 2. And finally for ranking the preference order, the value obtained for each alternative is sorted from "*small to big*" because the formulas are based on distance concepts.

Proposed P_i values: The ranking index of the traditional TOPSIS method is obtained as the ratio of the NIS to the sum

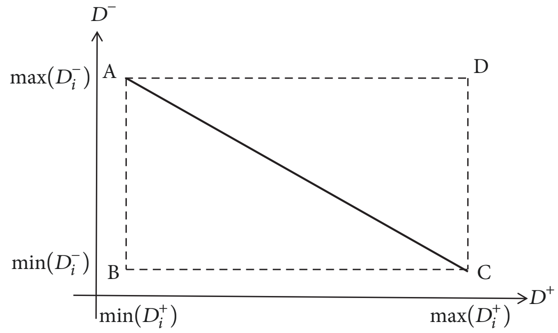

Figure 2: Explanation of the Euclidean distance.

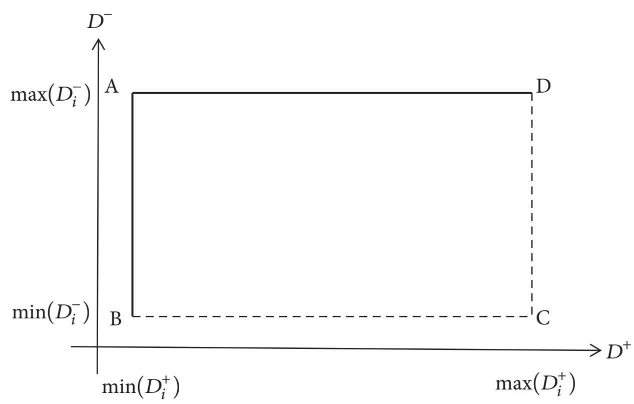

Figure 3: Manhattan distance.

of the NIS and PIS (see (21)). Although it seems reasonable, its validity is questionable and debatable in that it is based on a simple rate calculation. For this reason, researchers have tried various methods to rank alternatives. One of them is the Euclidean P_i value detailed above.

In this study alternative to the Euclidean P_i value (P_i^2) and of course to the traditional P_i value (P_i^1) three different methods have been proposed below. The first two proposed methods include the distances of "*Manhattan*" and "*Chebyshev*" in the L_q distance family, just like the *Euclidean* distance. The latter method is a new global distance measure consisting of the geometric mean of these three distances in the L_q family. The Euclidean, Manhattan, and Chebyshev distances are detailed in Section 2.4. For this reason in this section, it is mentioned that these measures and the new global distance measure are adapted to the (D_i^+, D_i^-) and $[\min(D_i^+), \max(D_i^-)]$ points so that they can sort the alternatives from the best to the less good ones. Just as *Euclidean* P_i (P_i^2) for ranking the preference order, the value obtained for each alternative is sorted from "*small to big.*"

(1) Manhattan P_i value (P_i^3) . : the formula for the proposed index based on the Manhattan distance is given in (56) and the graph of the distance is shown with thick lines in Figure 3.

$$P_i^3 = \left|D_i^+ - \min\left(D_i^+\right)\right| + \left|D_i^- - \max\left(D_i^-\right)\right| \qquad (56)$$

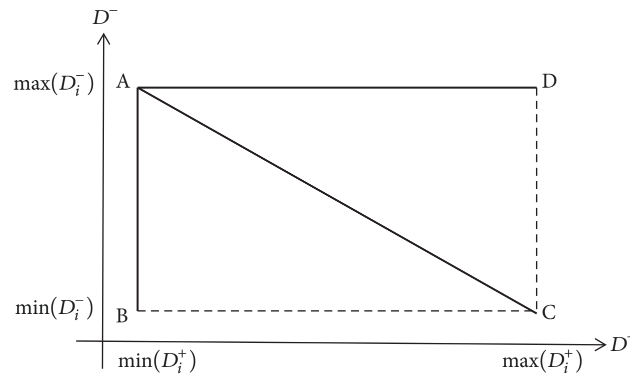

Figure 4: Chebyshev distance.

(2) Chebyshev P_i value (P_i^4) . : the proposed index based on the Chebyshev distance is given in (57). The graph of the distance is shown with thick lines in Figure 4.

$$P_i^4 = \max\left[\left|D_i^+ - \min\left(D_i^+\right)\right|, \left|D_i^- - \max\left(D_i^-\right)\right|\right] \qquad (57)$$

(3) Global Distance P_i value (P_i^5) . : in global distance value Euclidean, Manhattan and Chebyshev are considered as local distance. The global distance measure is based on the geometric average of these three local distance measures (see Section 2.5).

$$P_i^5 = \sqrt[3]{P_i^2 . P_i^3 . P_i^4} \qquad (58)$$

3.3. Formulations of the DSC TOPSIS Method. According to the standardization method used (Structure of the Performance Matrix) and whether the criteria are weighted or not the distance and similarity values mentioned above can be calculated by means of which formulas (Proposed Method Formulas) are shown in Table 1.

Explanations about the cases in Table 1:

(i) Case 1, Case 2, Case 3, and Case 4 are the cases where only one of the column or row-standardization methods are applied, while the remaining last four cases, Case 5, Case 6, Case 7, and Case 8, showed the conditions under which "double standardization" are applied.

(ii) In Case 1, Case 3, Case 5, and Case 7 the "standardized matrix" will be also "performance matrix" because any weighting method is not used.

(iii) In Case 3, Case 4, Case 5, and Case 6 after the row-standardization the "PIS and NIS values" should be also standardized according to "row-standardization." This correction is needed since the PIS and NIS are considered as a single value in the column-standardization and as a vector in row-standardization.

Table 1: Implementation of the proposed method.

Cases	Structure of the Performance Matrix			Proposed Method Formulas					
	Column-Standardized	Row-Standardized	Weighting	Sq_Euc	Corr_s	Cos_s	Cos_d	Cos_d	Proposed D
Case 1	Used	Unused	Unused	Equations (28)-(29)	Equations (30)-(31)	Equations (32)-(33)	Equations (34)-(35)	Equations (36)-(37)	Equations (26)-(27)
Case 2	Used	Unused	Used	Equations (28)-(29)	Equations (30)-(31)	Equations (32)-(33)	Equations (34)-(35)	Equations (36)-(37)	Equations (26)-(27)
Case 3	*Unused*	*Used***	*Unused*	*Equations (28)-(29)*	*Equations (51)-(52)*	*Equations (51)-(52)*	*Equations (53)-(54)*	*Equations (53)-(54)*	*Equations (49)-(50)*
Case 4	*Unused*	*Used***	*Used*	*Equations (28)-(29)*	*Equations (51)-(52)*	*Equations (51)-(52)*	*Equations (53)-(54)*	*Equations (53)-(54)*	*Equations (49)-(50)*
Case 5	*Used (First)*	*Used** (Second)*	*Unused*	*Equations (28)-(29)*	*Equations (51)-(52)*	*Equations (51)-(52)*	*Equations (53)-(54)*	*Equations (53)-(54)*	*Equations (49)-(50)*
Case 6	*Used (First)*	*Used** (Second)*	*Used*	*Equations (28)-(29)*	*Equations (51)-(52)*	*Equations (51)-(52)*	*Equations (53)-(54)*	*Equations (53)-(54)*	*Equations (49)-(50)*
Case 7	Used (Second)	Used* (First)	Unused	Equations (28)-(29)	Equations (30)-(31)	Equations (32)-(33)	Equations (34)-(35)	Equations (36)-(37)	Equations (26)-(27)
Case 8	Used (Second)	Used* (First)	Used	Equations (28)-(29)	Equations (30)-(31)	Equations (32)-(33)	Equations (34)-(35)	Equations (36)-(37)	Equations (26)-(27)

*: Standardize criteria according to row-standardization.

**: First standardize criteria according to row-standardization and then standardize PIS and NIS values according to row-standardization.

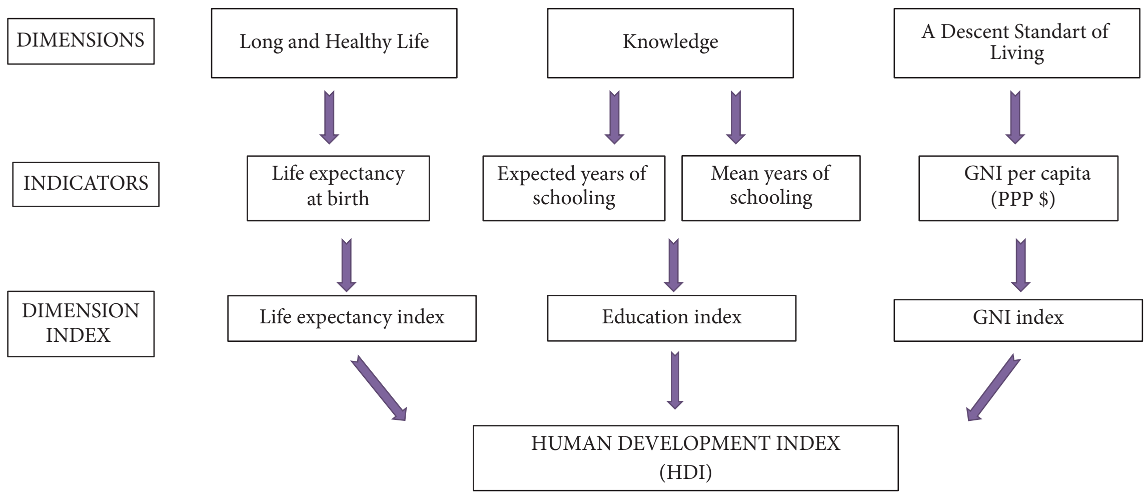

Figure 5: The dimensions and indicators of HDI.

Note. In the application part of the study, Case 5, Case 6, Case 7, and Case 8, where double standardization is applied, will not be included because they are partly addressed in Case 1, Case 2, Case 3, and Case 4, where one-way (row or column) standardization is applied.

The application types that can be used for the cases detailed in Table 1 can be summarized as follows.

Case 1 and Case 2: in these two cases where column-standardization is used, alternatives for each criterion are brought to the same unit. In Case 1, the criteria are handled without weighting and in Case 2 they are weighted. In this study, *equal* weighting method was preferred for comparison. In Case 2 different weighting methods such as Saaty, point, best-worst, entropy, swara, etc. can also be used.

Case 3 and Case 4: for these two cases where row-standardization is used, the criteria for each alternative are brought to the same unit. In Case 3 the criteria are unweighted and in Case 4 they are weighted. For Case 4, it is possible to apply different weighting methods just as in Case 2. At these cases the PIS and NIS values obtained after standardization of the decision matrix are standardized according to row-standardization.

Case 5, Case 6, Case 7, and Case 8: these are the cases where double standardization is used. In Case 5 and Case 6 following the standardization of the decision matrix according to double standardization (first by row, second by column) PIS and NIS values are also standardized according to the rows because the latest row-standardization is applied.

4. Case Study of the Proposed Methods

In this section all of the proposed methods were applied on the HDI data. Therefore, in the following subsections, information about HDI will be given first, and then implementation steps of the proposed method will show on criteria and alternatives that constitute HDI. Finally, the results obtained will be compared with the HDI value.

4.1. Human Development Index. Human development, or the human development approach, is about expanding the richness of human life, rather than simply the richness of the economy in which human beings live [26]. Developed by an economist Mahbub ul Haq, this approach emphasizes that monetary indicators such as Gross National Income (GNI) per capita alone are not enough to measure the level of development of countries. In HDR, the term GNI is defined as "aggregate income of an economy generated by its production and its ownership of factors of production, less the incomes paid for the use of factors of production owned by the rest of the world, converted to international dollars using purchasing power parity (PPP) rates, divided by mid-year population." [27]

It has been pointed out that the opportunities and choices of people have a decisive role in the measurement. These are to live long and healthy, to be educated and to have access to resources for a decent standard of living [28]. The human development approach was transformed into an index with a project supported by UNDP in 1989 and named HDI. In this way, it is aimed to measure the development levels of the world states better. After introducing the HDI, the UNDP published a report in 1990, in which the index was computed for each country as a measure of the nation's human development. Since then UNDP has continued publishing a series of annual Human Development Reports (HDRs) [5].

The first Human Development Report introduced the HDI as a measure of achievement in the basic dimensions of human development across countries. This somewhat crude measure of human development remains a simple unweighted average of a nation's longevity, education, and income and is widely accepted in development discourse. Before 2010 these indicators are used to measure HDI. Over the years, some modifications and refinements have been made to the index. In HDI 20th anniversary edition in 2010, the indicators calculating the index were changed. Figure 5 shows the dimensions and indicators of the HDI index [26].

As can be seen from Figure 1, HDI is a composite index of three dimensions which are a decent standard

Table 2: The first and last five countries with high HDI, I_{Life}, $I_{Education}$, and I_{Income} values.

No	HDI	I_{Life}	$I_{Education}$	I_{Income}
1	Norway	Hong Kong, China (SAR)	Australia	Liechtenstein
2	Switzerland	Japan	Denmark	Singapore
3	Australia	Italy	New Zealand	Qatar
4	Germany	Singapore	Norway	Kuwait
5	Singapore	Switzerland	Germany	Brunei Darussalam
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
184	Eritrea	Guinea-Bissau	Guinea	Madagascar
185	Sierra Leone	Mozambique	Ethiopia	Togo
186	South Sudan	Nigeria	Sudan	Mozambique
187	Mozambique	Angola	Mali	Malawi
188	Guinea	Chad	Djibouti	Guinea

of living, knowledge, and long and healthy life. A decent standard of living dimension which contained GNI per capita, create "GNI Index (Income Index-I_{Income})." Likewise, life expectancy at birth at the long and healthy life generate "Life Expectancy Index (Life Index-I_{Life})." And finally both mean years of schooling and expected years of schooling which is in dimension knowledge create "Education Index ($I_{Education}$)."

In order to calculate the HDI first these three core dimensions are put on a common (0, 1) scale. For this purpose the following equation is used:

$$x'_{ij} = \frac{x_{ij} - \min\left(x_{ij}\right)}{\max\left(x_{ij}\right) - \min\left(x_{ij}\right)} \quad (59)$$

After that, the geometric mean of the dimensions is calculated to produce the HDI (UNDP, 2011). This formula is given in

$$HDI = \sqrt[3]{I_{Life} . I_{Education} . I_{Income}} \quad (60)$$

The world countries are ranked according to the HDI values calculeted from (60).

4.2. Implementation of the Proposed Method. In this section the degree of similarity of the alternatives between each alternative and the PIS and the NIS is calculated with proposed D^{+} and D^{-} value at (26)-(27) for column-standardization and at (49)-(50) for row-standardization, respectively. The overall performance index for each alternative across all criteria is calculated with proposed P_i values at (56)-(57)-(58) used to rank countries in terms of HDI. The HDI value is basically based on the criteria of life expectancy at birth, mean years of schooling, expected years of schooling, and GNI per capita, as detailed above. These criteria were used to calculate the HDI value by being reduced to three dimensions: Income Index, Life Index, and Education Index.

From this point of view, in order to operate the DSC TOPSIS method proposed in this study, the development level of the preferred countries and the problem of deciding to create a new HDI were discussed through these three dimensions. Since 188 countries were considered in the HDR report published in 2016 (prepared with 2015 values), the initial decision matrix created in the implementation part of the study was formed from 188 alternatives. Income Index, Life Index, and Education Index are considered as criteria so that the initial decision matrix was formed from 188 alternatives and 3 criteria. For the tables not to take up too much space, the data and the results of the first five countries in alphabetical order from 188 countries are given. In terms of giving an idea before the steps of the DSC TOPSIS method are implemented, the first and last five countries with the highest and lowest Income Index, Life Index, and Education Index values in addition to HDI value are given in Table 2, respectively.

Table 3: The original data for the first five countries: Decision matrix.

Country	I_{Life}	$I_{Education}$	I_{Income}
Afghanistan	0.626	0.398	0.442
Albania	0.892	0.715	0.699
Algeria	0.847	0.658	0.741
Andorra	0.946	0.718	0.933
Angola	0.503	0.482	0.626
.	.	.	.
.	.	.	.
.	.	.	.
Weights	*0.333*	*0.333*	*0.333*
Mean	0.790	0.639	0.687
St. Dev.	0.127	0.174	0.180

Step 1 (determining the decision matrix). In this matrix 188 countries, which have been investigated in the 2016 HDR, are considered to be alternatives and the three HD dimension formed the criteria. The decision matrix is shown at Table 3 for the first ten countries.

Table 4: Standardized data (or performance matrix of unweighting data) for the first five countries.

Country	Column-Standardization					Row-Standardization				
	I_{Life}	$I_{Education}$	I_{Income}	Mean	St. Dev.	I_{Life}	$I_{Education}$	I_{Income}	Mean	St. Dev.
Afghanistan	-1.289	-1.386	-1.363	*-1.346*	*0.041*	1.391	-0.918	-0.473	*0.000*	*1.000*
Albania	0.801	0.439	0.069	*0.436*	*0.299*	1.410	-0.614	-0.797	*0.000*	*1.000*
Algeria	0.448	0.111	0.303	*0.287*	*0.138*	1.271	-1.172	-0.099	*0.000*	*1.000*
Andorra	1.226	0.456	1.372	*1.018*	*0.402*	0.768	-1.412	0.644	*0.000*	*1.000*
Angola	-2.255	-0.902	-0.338	*-1.165*	*0.805*	-0.535	-0.866	1.401	*0.000*	*1.000*
.	.	.	.	.	.	.	.	.	.	.
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.	.	.	.	.	.	.	.	.	.	.
Weights	*0.333*	*0.333*	*0.333*			*0.333*	*0.333*	*0.333*		
Mean	*0.000*	*0.000*	*0.000*			*1.025*	*-0.780*	*-0.245*		
St. Dev.	*1.000*	*1.000*	*1.000*			*0.595*	*0.604*	*0.750*		

Table 5: Performance matrix for the first five countries.

Country	Column-Standardization					Row-Standardization				
	I_{Life}	$I_{Education}$	I_{Income}	Mean	St. Dev.	I_{Life}	$I_{Education}$	I_{Income}	Mean	St. Dev.
Afghanistan	-0.430	-0.462	-0.454	-0.449	0.014	0.464	-0.306	-0.158	*0.000*	*0.333*
Albania	0.267	0.146	0.023	0.145	0.100	0.470	-0.205	-0.266	*0.000*	*0.333*
Algeria	0.149	0.037	0.101	0.096	0.046	0.424	-0.391	-0.033	*0.000*	*0.333*
Andorra	0.409	0.152	0.457	0.339	0.134	0.256	-0.471	0.215	*0.000*	*0.333*
Angola	-0.752	-0.301	-0.113	-0.388	0.268	-0.178	-0.289	0.467	*0.000*	*0.333*
.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.
Weights	*0.333*	*0.333*	*0.333*			*0.333*	*0.333*	*0.333*		
Mean	*0.000*	*0.000*	*0.000*			*0.342*	*-0.260*	*-0.082*		
St. Dev.	*0.333*	*0.333*	*0.333*			*0.198*	*0.201*	*0.250*		

Step 2 (determining the weighting vector). Since the purpose is to explain the operation of the model rather than the weighting, the criteria are handled without any weighting (*unweighted*) and with "*equally weighted*" in the study. Just as in the traditional TOPSIS method, in the proposed method can also be given different weights for the criteria (will be considered as the future research topic).

"Equal weighting" is a method of giving equal importance to the criteria being considered. Since three criteria are considered in the study, the weight of each criterion is determined as "*1/3 = 0.333*." The weights for the criteria are shown in the bottom row of Table 3.

Step 3 (standardized the decision matrix). Before ranking the countries, to put data on a common scale standardization method are used column-standardization and row-standardization. It was performed with (24). The result is shown in Table 4.

In the column-standardization part of Table 4, while the alternatives (countries) are brought to the same unit for each criterion (Life, Education, and Income), in the row-standardization part, the criteria for each alternative are brought to the same unit.

Step 4 (calculating the performance matrix). The performance matrix is obtained by multiplying the weight vector by the matrix of the standardized values (Table 5).

It should be reminded again at this point that if the criterion weighting is not performed, then the matrix obtained as a result of column or row-standardization (standardized data, Table 4) would also be considered as a "performance matrix."

Step 5 (determining the PIS and the NIS). The PIS and the NIS are attainable from all the alternatives (188 countries) across all three criteria according to (15) and (17), respectively. These values are shown in Table 6.

Step 6 (calculating the degree of distance of the alternatives between each alternative and the PIS and the NIS). After the column and row-standardizations, in this step first the degree of similarities and distances of the alternatives between each alternative and the PIS and the NIS are calculated. And then from these values the degree of *proposed distances* of the alternatives between each alternative and the PIS and the NIS is calculated (Table 7).

Step 7 (calculating the overall performance index). The overall proposed performance index for each alternative

Table 6: PIS (I^+) and NIS (I^-) value.

Weighting Status	PIS and NIS	Column-Standardization					Row-Standardization				
		I_{Life}	$I_{Education}$	I_{Income}	Mean	St. Dev.	I_{Life}	$I_{Education}$	I_{Income}	Mean	St. Dev.
Unweighted	I^+	1.548	1.728	1.746	*1.674*	*0.089*	1.414	1.358	1.401	*1.391*	*0.024*
	I^-	-2.711	-2.491	-2.338	*-2.513*	*0.153*	-1.404	-1.414	-1.414	*-1.411*	*0.005*
Weighted	I^+	0.516	0.576	0.582	*0.558*	*0.030*	0.471	0.453	0.467	*0.464*	*0.008*
	I^-	-0.904	-0.830	-0.779	*-0.838*	*0.051*	-0.468	-0.471	-0.471	*-0.470*	*0.002*

Table 7: PIS (I^+) and NIS (I^-) value for Case 4.

Row-Standardization				
I_{Life}	$I_{Education}$	I_{Income}	Mean	St. Dev.
0.958	-1.380	0.422	*0.000*	*1.000*
1.414	-0.705	-0.710	*0.000*	*1.000*

across all three criteria is calculated based on (21)-(55)-(56)-(57)-(58). The results obtained by applying the DSC TOPSIS method according to the "cases" detailed above and the results of "ranking" performed in Step 8 are given in Tables 8, 9, 10, and 11, respectively. The tables are constructed to contain proposed P_i values (P_i^3-(56), P_i^4-(56), P_i^5-(56)) and their ranking, as well as the values and ranking of P_i^1 (21), P_i^2 (55), HDI, and traditional TOPSIS method (P). Thus, all methods are provided to be implemented together (Tables 8, 9, 10, and 11).

Table 8 shows the results obtained from the application of Case 1. In Case 1, after the column-standardization of the decision matrix was carried out (in other words, after the countries are brought to comparable level for Life, Education, and Income criteria, respectively) the proposed DSC TOPSIS method was applied without any weighting to the criteria.

The application results of Case 2 were given in Table 9. Unlike Case 1 in Case 2, after the column-standardization of the decision matrix was performed, the proposed DSC TOPSIS method was applied with equal weight (different weighting methods can also be used) to the criteria.

Table 10 shows the results obtained from the application of Case 3. Unlike Case 1 in Case 3, after the row-standardization of the decision matrix was made (in other words, after Life, Education, and Income criteria are brought to comparable level for each country) the proposed DSC TOPSIS method was applied without any weight to the criteria.

The application results of Case 4 were given in Table 11. In Case 4, unlike Case 3, after the row-standardization of the decision matrix was performed, the proposed DSC TOPSIS method was applied by giving equal weight to the criteria. As in Case 2, in Case 4, weighting methods other than the equal weighting method can be used.

Step 8 (ranking the alternatives). The alternatives could be ranked in the descending order of the proposed P_i indexes values. In Tables 12 and 13, the first five countries with the highest value and the last five countries with the lowest value are listed according to the ranking results obtained for Case 1, Case 2, Case 3, Case 4, and Case 5, respectively, in alphabetical order. Additionally in Figures 6 and 7, the values of all the indexes computed during the study, mainly HDI, are handled together for all countries. Thus, it is possible to compare development levels of countries according to different indexes. In the graphs, the traditional TOPSIS index is denoted as "***P***" and P_i^1, P_i^2, P_i^3, P_i^4, P_i^5 are denoted as "***P1, P2, P3, P4, P5***," respectively. In accordance with the equal weighting method used in Case 2, although all the P_i values are different for Case 1 and Case 2, the results are given together because the ranks are the same.

If Table 12 is examined, Norway for HDI0, P and P_i^1, France for P_i^2 and P_i^5, Japan for P_i^3, and Korea for P_i^4 have become the most developed countries. Other results in the table can be interpreted similarly.

Figure 6 shows that although there are some differences in the ranking according to development levels of countries, both HDI and other indexes are similar.

According to Table 13 and Figure 7 it is observed that indices give similar results even though there are some differences.

In Figures 6 and 7, the x-axis represents the countries (order numbers 1 to 188 of the 188 world countries sorted in alphabetical order) and the y-axis represents a range of 0 and 1 for each index (for each line in the figures).

4.3. Discussion of Results. Findings obtained from the study: in DSC TOPSIS method, standardization, weighting, and performance indexes were found to be important in order to rank the alternatives. It is possible to interpret the results obtained in the application part of the study as follows.

(i) In order to sort and compare the alternatives, HDI values, the results of the traditional TOPSIS method and the results of the proposed DSC TOPSIS method were given together and a general comparison was made. With the DSC TOPSIS method, a new measure that is more sensitive and stronger than the measure used in the traditional TOPSIS method was proposed. The reason why the proposed unit of measurement is expressed as more sensitive and stronger is the use of similarity and correlation distances in addition to the Euclidean distance used in the traditional TOPSIS method in order to rank the alternatives. In other words, DSC TOPSIS uses three units that can be used to compare alternatives: distance, similarity, and correlation. From the results of the application it was observed that HDI, TOPSIS, and DSC TOPSIS results are not exactly the same and show some differences.

Table 8: The overall performance index P_i values for the first five countries-Case 1.

Country	HDI		**P**		P_i^1		P_i^2		P_i^3		P_i^4		P_i^5	
	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank
Afghanistan	0.479	169	0.281	172	0.061	182	7.035	185	9.938	185	5.197	172	7.136	186
Albania	0.764	75	0.697	70	0.787	78	1.520	32	2.119	40	1.241	27	1.587	32
Algeria	0.745	83	0.667	83	0.793	75	1.761	46	2.468	53	1.402	41	1.827	47
Andorra	0.858	32	0.819	33	0.879	52	1.496	29	1.948	32	1.386	40	1.593	33
Angola	0.533	150	0.338	155	0.166	153	5.396	138	6.304	128	5.303	176	5.650	142

Table 9: The overall performance index P_i value for the first five countries-Case 2.

Country	HDI		**P**		P_i^1		P_i^2		P_i^3		P_i^4		P_i^5	
	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank
Afghanistan	0.479	169	0.281	172	0.061	182	3.382	185	4.778	185	2.499	172	3.431	186
Albania	0.764	75	0.697	70	0.787	78	0.731	32	1.019	40	0.597	27	0.763	32
Algeria	0.745	83	0.667	83	0.793	75	0.847	46	1.186	53	0.674	41	0.878	47
Andorra	0.858	32	0.819	33	0.879	52	0.719	29	0.937	32	0.667	40	0.766	33
Angola	0.533	150	0.338	155	0.166	153	2.594	138	3.031	128	2.549	176	2.716	142

Table 10: The overall performance index P_i value for the first five countries-Case 3.

Country	HDI		**P**		P_i^1		P_i^2		P_i^3		P_i^4		P_i^5	
	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank
Afghanistan	0.479	169	0.006	171	0.079	149	3.609	119	3.954	113	3.590	155	3.714	117
Albania	0.764	75	0.230	114	0.007	170	3.682	169	4.307	142	3.617	169	3.856	152
Algeria	0.745	83	0.086	141	0.597	71	3.438	75	3.561	75	3.436	100	3.478	75
Andorra	0.858	32	0.464	94	0.970	15	2.789	26	2.814	26	2.788	29	2.797	26
Angola	0.533	150	0.875	48	0.706	62	1.527	1	2.158	1	1.113	1	1.542	1

This is the expected result because the measurement units used in each method are different. At this point, which method is preferred depends on the decision-maker's opinion.

(ii) In the DSC TOPSIS method, although the performance indexes for Case 1 and Case 2, which have the objective to bring the alternatives to the same unit for each criterion, have different values, the ranking values are the same. This is expected because the equal weighting method is actually equivalent to the situation where there is no weighting. Of course, if different weights are used for the criteria in Case 2, it is highly probable that both different performance index values and different rankings can be obtained.

(iii) Case 3 and Case 4, which bring the criteria to the same unit for each alternative, refer to situations that are performed without weighting and equal weighting, respectively. The situation described above is also valid here and the results of the indexes are the same.

(iv) Which standardization method is preferred depends on the decision-maker's opinion and the nature of the problem being addressed. The decision problem in this study was the calculation of HDI and the proposed solution method was DSC TOPSIS. When the structure of the decision problem is examined, it can be concluded that it is more reasonable to prefer Case 1 and Case 2 where column-standardization is used. Considering the advantage of the fact that different weighting methods can be tried as well as equal weighting in Case 2, it is obvious that the most reasonable solution would be Case 2.

(v) Which performance index is preferred to rank the alternatives depends on the decision-maker. At this point, it is expected that P^5, which is the average of all performance indexes proposed in the study, is preferred.

5. Conclusions

There are lots of efficient MCDM methods which are suitable for the purpose of ranking alternatives across a set of criteria. None of MCDM methods in the literature gives a complete superiority over the other. Each of them can give different results according to their main purposes. For this reason, the method to be applied should be decided according to the structure of the decision problem and the situation to be investigated. In some cases, the results obtained by trying different MCDM methods are compared and the most reasonable solution is selected.

Table 11: The overall performance index P_i value for the first five countries-Case 4.

Country	*HDI*		***P***		P_i^1		P_i^2		P_i^3		P_i^4		P_i^5	
	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank
Afghanistan	0.479	169	0.006	171	0.441	149	1.203	119	1.318	113	1.197	155	1.238	117
Albania	0.764	75	0.230	114	0.390	170	1.227	169	1.436	142	1.206	169	1.285	152
Algeria	0.745	83	0.086	141	0.511	71	1.146	75	1.187	75	1.145	100	1.159	75
Andorra	0.858	32	0.464	94	0.623	15	0.930	26	0.938	26	0.929	29	0.932	26
Angola	0.533	150	0.875	48	0.622	62	0.509	1	0.719	1	0.371	1	0.514	1

Table 12: Top and bottom five countries with high and low level of development-Case 1 and Case 2.

No	*HDI*	***P***	P_i^1	P_i^2	P_i^3	P_i^4	P_i^5
1	Norway	Norway	Norway	France	Japan	Korea (Republic of)	France
2	Australia	Switzerland	Finland	Korea (Republic of)	France	France	Korea (Republic of)
3	Switzerland	Australia	Netherlands	Japan	Korea (Republic of)	Israel	Japan
4	Germany	Netherlands	Switzerland	Israel	Israel	Japan	Israel
5	Denmark	Germany	United States	Spain	Spain	Spain	Spain
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
184	Burundi	Guinea-Bissau	Central African Republic	Burkina Faso	Burkina Faso	Chad	Burkina Faso
185	Burkina Faso	Mozambique	Guyana	Afghanistan	Afghanistan	Benin	Gambia
186	Chad	Sierra Leone	South Sudan	Gambia	Gambia	Guyana	Afghanistan
187	Niger	Chad	Chad	Niger	Niger	Guinea-Bissau	Niger
188	Central African Republic	Central African Republic	Congo	Guinea	Guinea	Congo	Guinea

Undoubtedly, the oldest and valid method used to rank the alternatives is distance. The concept of distance forms the basis of the TOPSIS method (it was called "traditional TOPSIS" in the study). In TOPSIS (the steps of implementation are given in detail in Section 3.1), the best result and the worst result are determined first. Then, each of the alternatives discussed determines the distance between the best result and the worst result. A general ranking value is calculated by evaluating the best solution distance and the worst solution distance values, and the alternatives are sorted according to the size of these values. It is desirable that the alternatives in the method are as close as possible to the best solution, and the worst solution is away from the other side.

As previously stated, "normalizing the decision matrix (Step 3)," "calculating the distance of positive and negative-ideal solutions (Step 6)," and "calculating the overall performance index for each alternative across all criteria (Step 7)" of the traditional TOPSIS method steps can be examined, improved, or modified. The aim of the study is to draw attention to the above-mentioned weaknesses of the traditional TOPSIS method and to propose new solutions and approaches. For this purpose *"column, row, and double standardization"* were substituted for "normalization approach," *"a new measure based on the concepts of distance, similarity, and correlation (DSC)"* was substituted for "Euclidean distance," and *"new sorting approaches based on the concept of distance (performance index)"* were substituted for "simple ratio." This new MCDM method is called DSC TOPSIS.

The DSC TOPSIS method is particularly remarkable in its *new measure and new performance indexes*. Because in addition to the distance method used in the traditional TOPSIS, this new measure also includes similarity and correlation effects for sorting alternatives. Just as in the distance method, alternatives are compared with positive and negative-ideal solutions in the similarity method. In the correlation method, alternatives are compared according to the concept of relationship. It has been tried to achieve a stronger evaluation criterion by combining these three measurement techniques. In the proposed new performance indexes, distance concepts were used. Thus, more powerful sorting techniques have been tried to be obtained.

Table 13: Top and bottom ten countries with high and low level of development-Case3 and Case 4.

No	*HDI*	***P***	P_i^1	P_i^2	P_i^3	P_i^4	P_i^5
1	Norway	Azerbaijan	Austria	Angola	Angola	Angola	Angola
2	Australia	France	Hong Kong, China (SAR)	United States	United States	United States	United States
3	Switzerland	Lesotho	Azerbaijan	Nigeria	Nigeria	Nigeria	Nigeria
4	Germany	Finland	Libya	Botswana	Botswana	Botswana	Botswana
5	Denmark	Spain	Chad	Equatorial Guinea	Equatorial Guinea	Equatorial Guinea	Equatorial Guinea
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
184	Burundi	Niger	Israel	Malawi	Palau	Argentina	Slovenia
185	Burkina Faso	El Salvador	Argentina	Bolivia (Plurinational State of)	Latvia	Nepal	Armenia
186	Chad	Cabo Verde	Nepal	Palestine, State of	Estonia	Ghana	Liberia
187	Niger	Dominica	Ghana	Denmark	Fiji	Haiti	Burundi
188	Central African Republic	Ethiopia	Haiti	Uganda	Ukraine	Lithuania	Hungary

Figure 6: Comparison of the development levels of countries for all indices, Case 1 and Case 2.

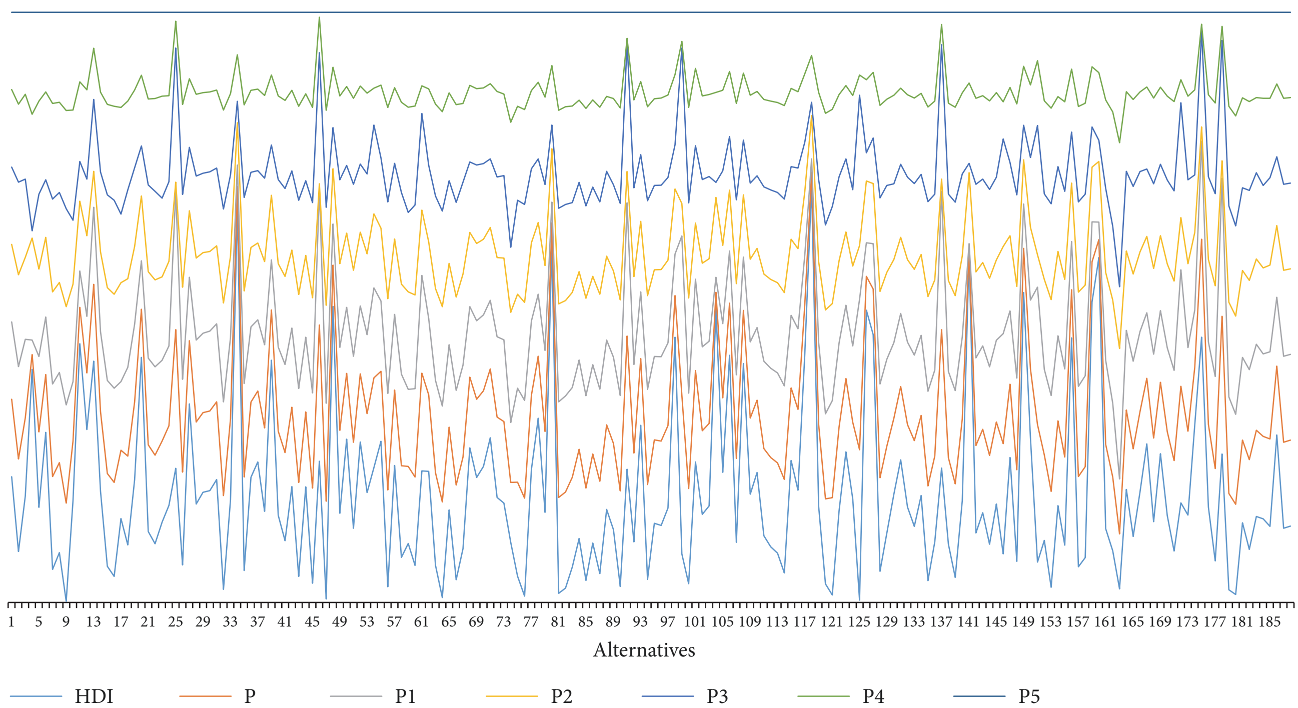

Figure 7: Comparison of the development levels of countries for all indices, Case 3 and Case 4.

The formulation of the DSC TOPSIS method is detailed for the eight different cases defined (Table 1). The first four cases are exemplified on the HDI data in the "*Implementation of the Proposed Method*" subsection (Section 4.2) because they represent the basis of the DSC TOPSIS method (they contain the last four cases partly). HDI is an index developed to rank countries according to their level of development. The index is basically based on Income Index, Life Index, and Education Index variables and is calculated by geometric mean. One of the most important reasons for considering the HDI in the study is that it is not very desirable to calculate the index based on the geometric mean and that there are various criticisms on it. With this paper, the order of development for the countries of the world has been tried to be obtained by a different method. In a more explicit way, a different proposal was made for the calculation method of HDI calculated by UNDP. It was concluded that Case 2 and P^5 performance index could be preferred when using DSC TOPSIS method in the calculation of HDI.

In conclusion, the effect of the proposed DSC TOPSIS method on the following concepts is discussed in all aspects:

(i) different standardization methods,

(ii) different weighting methods,

(iii) different performance indexes.

6. Limitations and Future Research

The proposed DSC TOPSIS method does not have any limitations, and the method can be easily applied to all decision problems where traditional TOPSIS can be applied.

In the study, the standardization of the decision matrix in different ways is also emphasized. According to the preferred standardization type, it is emphasized that the decision matrix will have different meanings and different results will be obtained. The only issue to be considered in the application of the method is to decide the standardization method to be applied. The following topics can be given as examples for future applications.

(i) establishing different versions of the new measurement proposed by the DSC TOPSIS method: e.g., combining the concepts of distance, similarity, and correlation with a different technique instead of a geometric mean or experimenting with different combinations of these concepts together (distance and similarity, distance and correlation, similarity and correlation, etc.),

(ii) suggesting different ranking index (P_i) formulas to sort alternatives,

(iii) experiment of different weighting techniques in DSC TOPSIS method,

(iv) blurring the DSC TOPSIS method: fuzzy DSC TOPSIS.

Data Availability

Data are obtained from "http://hdr.undp.org/en/humandev, United Nations Development Programme Human Development Reports" website.

Conflicts of Interest

The author declares that they have no conflicts of interest.

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Research Article

Evaluation of Enterprise Learning Performance in the Process of Cooperation Innovation Using Heronian Mean Operator

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In order to clarify the achievement and efficiency of enterprise learning in the process of cooperative innovation, a comprehensive criteria framework for the evaluation of enterprise learning performance is constructed taking the learning process and learning results as the construction idea based on organizational learning theory. And this paper proposes a novel dynamic evaluation method considering the interaction between attributes of learning performance. In this study, the criterion framework for the evaluation of enterprise learning performance in the process of cooperative innovation includes learning process performance and learning outcomes performance. The original matrices are given by managers and experts using fuzzy set theory, and a dynamic time sequence weight vector is calculated based on information entropy and time degree. The weight of learning performance attributes under different time series is calculated based on entropy measure method. The interactive information of learning performance attributes is integrated through the weight of learning performance attributes and the three-parameter weighted Heronian mean operator considering the interaction between attributes. And then, the dynamic and comprehensive evaluation result of learning performance in the process of cooperative innovation could be computed by integrating the learning performance information under different time series with time sequence weight vector. Finally, a real case is studied to verify the scientificity and validity of the criteria framework for the evaluation of enterprise learning performance and the method proposed in this study. This study not only helps cooperative enterprises get feedback in time and adjust cooperative relationships and learning styles but also enriches the theory of interorganizational management and provides a theoretical basis for the process of enterprise cooperative innovation.

1. Introduction

With the rapid development of Internet industry and the deep integration of artificial intelligence and Internet of Things, cooperative innovation among enterprises has become an important way for enterprises to gain competitive advantage. Mining and learning new knowledge from cooperative innovators have gradually become a key factor in the long-term development of enterprises [1]. In the process of cooperative innovation, interenterprise learning plays undoubtedly an important way to enhance their competitive advantages [2]. Discernible learning effect has an important impact on adjusting learning content, perfecting learning management mechanism, improving learning management ability, and whether to end current cooperation innovation projects. The evaluation of enterprise learning performance plays a vital tool for enterprises to clarify learning effectiveness and is also an important mechanism to judge whether cooperative goals are achieved and to manage cooperative relationships among enterprises [3]. In the process of cooperative innovation, learning performance evaluation can also help cooperative enterprises get feedback in time and understand the degree of implementation of learning objectives among enterprises [4]. The evaluation results of learning performance determine whether enterprises adjust their interenterprise cooperative relationships and learning styles. The evaluation of enterprise learning performance can effectively realize the intention of cooperative innovation among enterprises and avoid the difficulty of cooperative innovation. With the deep integration of Internet and Internet of Things, resource sharing, complementary advantages, risk sharing, and achievement sharing among enterprises have become a new normal, which further strengthens the cooperative innovation relationship among enterprises [5]. Therefore, in the process of cooperative innovation, enterprise learning performance evaluated scientifically and reasonably has important theoretical value and practical significance.

In the process of cooperative innovation, enterprises can acquire knowledge and skills of other enterprises across organizational boundaries through inter-enterprise learning. Interenterprise learning not only enriches the knowledge source channels and knowledge structure of enterprises but also makes up for the lack of internal learning [6]. With the change of environment and technology, more and more knowledge is acquired through innovative cooperation of different types or fields outside enterprises. Moreover, knowledge could flow across organizational boundaries to innovative members through cooperative innovation activities [1, 2]. Thus, how to manage knowledge among enterprises has become an important connotation of interenterprise learning. Interenterprise learning is a means for enterprises to acquire and use knowledge resources from cooperative innovation partners. The essence of interenterprise learning is the ability to acquire, disseminate, and maintain new knowledge in the cooperative innovation network for improving future performance [7]. Interenterprise learning, as the most valuable learning resource, can enable enterprises to maintain long-term competitive advantage. This kind of learning resource acquires the knowledge and skills needed by enterprise innovation based on learning from other enterprises and generates the new knowledge needed in the process of internalization of external knowledge [4]. Trust is an important behavioral model to establish cooperative relationships among enterprises, so as to realize learning among enterprises [3–8]. In the process of cooperative innovation, the culture of mutual benefit is an important guarantee mechanism for the effective use of interenterprise learning [9]. Interenterprise learning is a bilateral or multilateral learning process, which enables different enterprises to achieve different goals in the process of cooperative innovation [10]. In addition, learning behavior and learning process are the core content of cooperative innovation. However, how to measure the effectiveness of interenterprise learning is the key to long-term cooperative innovation and learning among enterprises. From the function of performance evaluation itself, performance evaluation theory shows that performance evaluation is a management control tool to implement organizational strategy. By comparing the differences between learning outcomes and learning objectives in the process of cooperative innovation based on performance evaluation, enterprises can dynamically track the process of strategy implementation and find problems in management [4, 6–8, 11]. Moreover, learning performance evaluation can provide feedback for the next innovation planning of enterprises by analyzing the deviation of past performance. Discernible learning effect has an important impact on adjusting learning content, perfecting learning management mechanism, improving learning management ability, and whether to end current cooperation innovation projects [3].

In the process of cooperative innovation, the evaluation of enterprise learning performance is a comprehensive evaluation of complex multidimensional factors, which belongs to interorganizational performance. Many scholars have studied the learning mechanism, learning methods, and influencing factors of inter-organizational learning performance [12–20]. Kellogg [13] empirically tested the importance of learning in specific organizational relationships based on high-frequency data from oil and gas drilling. Şengün studied the effect of trust types on knowledge sharing between small and medium-sized manufacturers and retailers in furniture industry cluster, and this research shows that the trust based on competence, reliability, and predictability has no significant correlation with interenterprise learning, while the trust based on goodwill, benevolence, and nonopportunism has a significant positive correlation with interenterprise learning [8]. Subsequently, Şengün and Önder [14] further studied the interactive effect of ability and goodwill trust on interenterprise learning performance and thought that goodwill trust has a positive main effect on interenterprise learning performance, while ability trust has no positive main effect on interenterprise learning performance. Gupta and Polonsky [15] concluded that multinational enterprises could operate and interact more effectively by sharing knowledge reflecting customer structure and suitability with outsourcing enterprises, and learning and knowledge sharing are closely coupled in the product development stage. Saenz et al. [16] thought that interorganizational learning practice is one of the important ways to solve the problems of lack of trust among trading partners and difficulties in strategic collaboration. Gibb et al. [3] investigated the interorganizational learning mechanism in network and believed that interorganizational learning including learning how to compete and how to behave helps to coordinate network learning among horizontal enterprises in order to achieve performance goals. Lis and Sudolska [17] studied the relationship between learning process and learning content among enterprises through internal learning and thought that there was synergy between them. Akbar [10] highlighted that interaction and open communication enable enterprises not only to understand customer needs but also to acquire more knowledge based on twelve interviews with four Finnish SMEs. In terms of theoretical research on interorganizational performance, Ittner and Larcker [18] thought that interorganizational performance evaluation is of great significance and will be the focus of future research. However, the studies on interorganizational cooperation control are gradually increasing, and interorganizational performance evaluation still does not occupy the mainstream. Although some scholars have studied the concept of interorganizational cost and related evaluation methods [19], there are still some deficiencies in the theoretical and practical research of linking interorganizational performance evaluation with interorganizational learning. Then, Zhi and Dai [20] proposed a systematic balanced scorecard framework for strategic alliance performance evaluation and used this framework to study the relationship between interorganizational cooperation and intrafirm performance dimensions. Through qualitative research, Dai and Zhi [4] thought that interorganizational learning in strategic alliances could improve the development ability of enterprises, and the evaluation of learning performance could improve learning behavior. In order to further explore the significance of the theoretical framework of interorganizational learning performance evaluation, Petersen et al. [12] thought that interactive learning and trust evolution have a synergistic relationship, which could improve the efficiency of knowledge transfer in cooperative network.

In terms of evaluation methods, there are few studies on the methods of performance evaluation in the process of cooperative innovation. Kusi-Sarpong et al. [21] used the best-worst multicriteria decision-making model to evaluate sustainable innovation management standards with five Indian manufacturing companies as samples in order to prove the applicability and efficiency of the proposed framework. Song et al. [22] believed that it is very important to apply existing theories and methods to evaluate environmental performance successfully in practice, and environmental performance evaluation provides scientific basis and guidance for formulating environmental protection policies. Arbolino et al. [23] used principal component analysis model to measure ecoindustrial policy and industrial environmental sustainability index by identifying the application strategies of relevant participants through variables. Kazancoglu et al. [24] put forward a performance evaluation model of green supply chain management in cement enterprises and used the technology of fuzzy decision-making test and evaluation laboratory to analyze the causality. Cook et al. [25] extended data envelopment analysis to the performance evaluation of performance incentive plan, and the proposed method was applied to evaluate the performance of decision-making units after the implementation of incentive plan according to achievable goals and representative best practices. Luo et al. [26] used AHP and matter-element analysis to evaluate the performance of Public-Private Partnership. Malings et al. [27] assessed the temporal variability of spatial distribution and air quality in cities using various algorithms for correcting slope measurements, which include linear and quadratic regression, clustering, neural network, Gauss process, and mixed stochastic forest-linear regression model. Santos et al. [28] evaluated the performance of green suppliers using entropy weight method, technique for order preference by similarity to an ideal solution method, and decision-making trial and evaluation laboratory method. Moldavska and Welo [29] proposed a new method to assess the sustainability of manufacturing enterprises and thought that enterprise sustainability assessment in manufacturing industry is a tool framework to guide organizations towards sustainable practices. Lee and Choi [30] decomposed the sequential Malmquist-Luenburger index into two indicators, efficiency change and technology change, and evaluated the environmental performance of Korean manufacturing industry using Malmquist-Luenburger index and generalized directional distance function. In addition, the above traditional performance evaluation methods including AHP [11], BSC [31], and EVA [32] mostly assume that attributes are independent of each other. However, attributes are not always independent, and there are always some interaction relationships, such as complementarity, redundancy, and preference. Even if the C-POWA operator [33], DLWA operator [34], and other related integration operators are used for performance evaluation, the same problem exists. Moreover, most of these methods are based on static information of a single period. Even though these methods involve dynamic related elements, the comprehensiveness of evaluation is still neglected. Although some scholars, such as Su et al. [35], Park et al. [36], and Cao et al. [37], introduced time weight vectors, they established these time sequence weight vectors by direct assignment of subjective randomness, which may lead to unreasonable comprehensive evaluation results.

Although the above studies provide a support for the evaluation of enterprise learning performance in the process of cooperative innovation, there is still a lack of relevant research on the practical application value of enterprise learning performance evaluation. The above studies seldom deal with the content related to the evaluation system of learning performance in the process of enterprise cooperative innovation from a systematic perspective. The above methods of performance evaluation also have great limitations, without considering the interaction between attributes. The evaluation of learning performance in the process of cooperative innovation plays an important role in improving the management mechanism of intraorganizational learning and interorganizational learning [12]. In the process of enterprise cooperative innovation, the evaluation results of learning performance have an important impact on the adjustment of cooperation mode, cooperation content, and cooperation strategy. The application of reasonable method is one of the important guarantees for the accuracy and rationality of the performance evaluation results [22–37]. Therefore, it is necessary to further study the practical application of the evaluation of enterprise learning performance in the process of cooperative innovation by considering the interaction between attributes and determining a scientific and reasonable sequential weight vector.

To address these shortcomings, a comprehensive criteria framework for the evaluation of enterprise learning performance is constructed taking the learning process and learning results as the construction idea based on organizational learning theory. This study further proposes a novel dynamic evaluation method considering the interaction between attributes of learning performance. The practical contribution of this study is that the method and criteria framework proposed in this study not only help to improve the effectiveness and stability of the comprehensive evaluation results of enterprise learning performance in the process of cooperative innovation but also help cooperative enterprises get feedback in time and adjust cooperative relationships and learning styles. In theory, the comprehensive and representative framework proposed in this study not only provides a theoretical basis for the study of factors affecting interorganizational learning and performance evaluation but also enriches the theory of interorganizational management and provides a theoretical basis for the process of enterprise cooperative innovation. In addition, the novel method proposed in this study expands the application scope of fuzzy theory and time series in learning performance evaluation.

The rest of this paper is structured as follows: the criteria framework for the evaluation of enterprise learning performance in the process of cooperative innovation is presented in Section 2. In Section 3, a novel dynamic comprehensive evaluation method considering the interaction between attributes will be proposed to evaluate enterprise learning performance. A case study is furnished in Section 4 to illustrate how the approach could be applied to multicriteria evaluation problems. Conclusions including managerial and practical implications and future research are highlighted in Section 5.

2. Criteria Framework for the Evaluation of Enterprise Learning Performance

A large number of cooperative information data could be produced in the process of cooperative innovation. The selection of cooperative information data has an important impact on the scientificity and reliability for the evaluation results of learning performance. Some studies directly measure interorganizational learning effectiveness criteria, which include knowledge transfer, knowledge accumulation, strategic coherence, relational capital, and learning control [38, 39]. In addition, Lane and Lubatkin thought that the correlation between interorganizational learning performance and capacity accumulation is stronger than that between interorganizational learning performance and business performance, and the capacity accumulation is regarded as the evaluation criterion of interorganizational learning performance [7]. According to the content and evolution characteristics of interorganizational learning, the evolution content of interorganizational learning is the process of clarification, sharing, and internalization of relevant knowledge into another organization based on the perspective of system view.

From different perspectives, scholars put forward many factors that affect the effectiveness of inter-organizational learning [40–44]. Zhu et al. [40] proposed that there are four main factors affecting the effectiveness of interorganizational learning, namely, culture, structure, technology, and absorptive capacity. Among them, absorptive capacity is the basis of technical learning in an organization, and shared knowledge links at any level are conducive to the dissemination of absorptive capacity elements. Zhang et al. [41] thought that the factors affecting the success of alliance learning can be divided into three categories: the availability of alliance knowledge, the effectiveness of knowledge acquisition, and the learning connection based on the availability of alliance knowledge and the effectiveness of knowledge acquisition. Yayavaram et al. [42] thought that the key elements of organizational learning are interaction between partners, high learning goals, trust, and long-term goal orientation. The degrees of trust between organizations, learning intention, partner's knowledge attribute, and organizational learning ability have great influence on knowledge transfer and interorganizational learning [43]. In order to realize the learning opportunities provided by the alliance, collaborators must attach importance to learning and consciously think about how to learn. In the process of learning, the learning ability of alliance enterprises is influenced by organizational knowledge transfer ability, acceptance ability, core competence, and past experience [44]. The transfer and accumulation of knowledge in interorganizational cooperation depend on the factors of learning process. These studies found that learning objectives, tasks, environments, and knowledge sharing about skills and processes are key process factors in interorganizational learning. However, knowledge exists in the specific organizational path of partnership enterprises, which has the characteristics of tacit and embedded [40–42]. Therefore, it is necessary to set reasonable learning objectives and arrange appropriate learning tasks and activities under appropriate circumstances, so as to complete the transfer and accumulation of knowledge. This paper constructs a criteria framework for the evaluation of enterprise learning performance in the process of cooperative innovation. Enterprise learning performance is divided into learning process performance and learning outcomes performance, so as to establish a practical criteria framework for the evaluation of enterprise learning performance, which is shown in Figure 1.

As shown in Figure 1, the criterion framework for the evaluation of enterprise learning performance in the process of cooperative innovation includes learning process performance and learning outcomes performance, in which learning process performance is the learning outcomes of enterprises in cooperation, and learning outcomes performance is the core goal of enterprise core competence development [4, 13–15]. Learning process performance includes learning content, learning management, learning objectives, and learning processes, which affect the choice of learning methods and the formulation of learning mechanism in the process of cooperative innovation [6–8, 14–17]. Learning outcomes performance includes knowledge accumulation, ability accumulation, and performance creation [6–8]. Knowledge accumulation and performance creation mainly represent the outcomes of internal learning, while ability accumulation is the comprehensive outcome of internal and external learning. That is to say, technological progress could be achieved through internal knowledge acquisition and transformation, and internal management process could be optimized, which help to enhance external cooperative management capability. Learning process performance not only directly affects the internal learning outcomes of enterprises but also indirectly affects the external learning outcomes, thus affecting the core objectives of enterprises [10, 15–17, 20]. Learning outcomes performance could provide information feedback based on the rationality of learning content and learning management of enterprises [4, 6–8]. This also reflects the scientificity of the criterion framework established by the integrated system view and the rationality of the evaluation method proposed in this study for the evaluation of enterprise learning performance in the process of cooperative innovation.

The abovementioned criteria for the evaluation of enterprise learning performance in the process of cooperative innovation could be set as qualitative criteria, except for the quantitative criteria of technological progress. For quantitative criteria, there are many ways to calculate them, while quantification of qualitative criteria could generally be divided into two categories. The data acquisition of a class

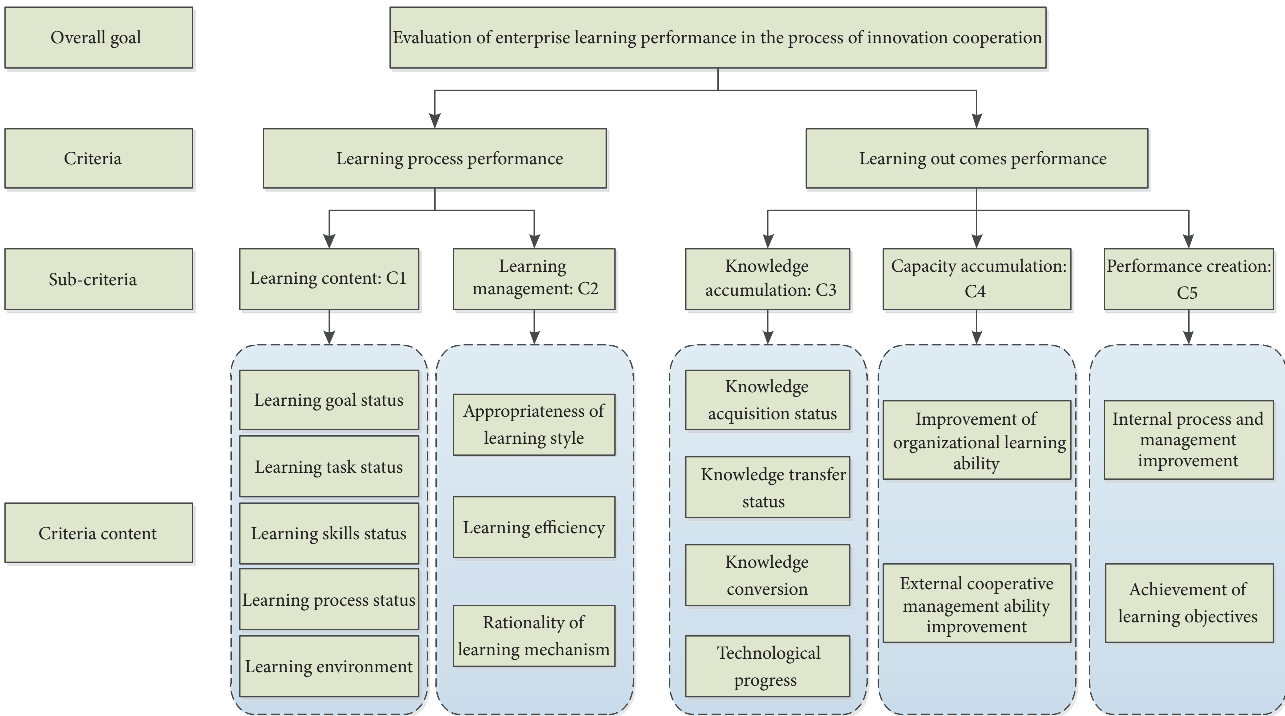

Figure 1: The criteria framework for the evaluation of enterprise learning performance in the process of cooperative innovation.

of criteria could be through questionnaires or the design of its numerical value, and this kind of criterion data could be obtained through the form of questionnaires, such as the appropriateness of learning methods, learning efficiency and so on. The data acquisition of the other criteria is difficult to determine by questionnaire or design. The data of these criteria could be obtained by the form of fuzzy linguistic information or fuzzy numbers of experts and enterprise managers, such as the improvement degree of organizational learning ability and the improvement degree of external cooperative management ability. In addition, with the rapid development of artificial intelligence and Internet, the cooperation among enterprises has been further deepened and developed, and the amount of information generated by cooperation has become more and more huge. Because of the complexity and fuzziness of learning performance evaluation, some evaluation criteria often fail to be given accurate evaluation values. With the introduction of fuzzy set theory, it has certain scientific and practical significance in dealing with fuzziness [45–52]. The method proposed in this study has paid attention to the practical application of the criterion evaluation process, so the above criteria could be quantified using fuzzy set theory.

3. Solution Methodology

3.1. Dimensionless Criteria and Distance Measurement of Fuzzy Numbers. In the process of cooperative innovation, the criteria for the evaluation of enterprise learning performance include qualitative and quantitative factors, and the dimensions of different attributes are different. In order to reduce the difference of evaluation results caused by different measurement units, it is necessary to deal with the criteria of learning performance evaluation in dimensionless way. According to [24, 28, 46–48, 53], different types of criteria have different dimensionless methods which are shown below.

If the evaluation values are clear numbers, many methods could be used to deal with it. For example, the range transformation method could be used to dimensionless quantitative attributes. Let v_{ij} denote the clear number of criterion j of the i system. α_{ij} and β_{ij} are the upper and lower limits of the order parameters of critical points of the learning evaluation system. Let p_{ij} represent the contribution value of variable v_{ij} to learning system, where $p_{ij} \in [0, 1]$. The dimensionless results of the criteria for the evaluation of enterprise learning performance could be expressed as follows:

$$p_{ij} = \begin{cases} \dfrac{\left(v_{ij} - \beta_{ij}\right)}{\left(\alpha_{ij} - \beta_{ij}\right)} & \textit{Benefit criterion} \\[2ex] \dfrac{\left(\alpha_{ij} - v_{ij}\right)}{\left(\alpha_{ij} - \beta_{ij}\right)} & \textit{Cost criterion.} \end{cases} \tag{1}$$

If the evaluation values are interval numbers $[x_{ijh}^L, x_{ijh}^M]$, the dimensionless results $y_{ijh} = [y_{ijh}^L, y_{ijh}^M]$ are assumed. Especially, the original data need to be reciprocated before standardization of cost-based criteria.

For benefit-oriented attributes, there is

$$y_{ijh} = \left[y_{ijh}^L, y_{ijh}^M\right] = \left[\frac{x_{ijh}^L}{\sum_{i=1}^n x_{ijh}^M}, \frac{x_{ijh}^M}{\sum_{i=1}^n x_{ijh}^L}\right]. \tag{2}$$

For cost attributes, there is

$$y_{ijh} = \left[y_{ijh}^{L}, y_{ijh}^{M}\right] = \left[\frac{\left(1/x_{ijh}^{M}\right)}{\sum_{i=1}^{n}\left(1/x_{ijh}^{M}\right)}, \frac{\left(1/x_{ijh}^{L}\right)}{\sum_{i=1}^{n}\left(1/x_{ijh}^{L}\right)}\right]. \quad (3)$$

If the evaluation values are linguistic values, the dimensionless processing of qualitative criteria requires quantitative conversion of qualitative evaluation values [53]. The basis of converting linguistic value into triangular fuzzy number is shown in Table 1, which is applied to the case study in this study. Language variables can be set to multiple levels. The level of variables can be set according to the demand of business managers who carry out learning performance evaluation. Five linguistic variables in Table 1 are only applied to the case study in this study.

If the evaluation values are triangular fuzzy numbers $[x_{ijh}^{L}, x_{ijh}^{M}, x_{ijh}^{N}]$, the dimensionless results $y_{ijh} = [y_{ijh}^{L}, y_{ijh}^{M}, y_{ijh}^{N}]$ are assumed. Especially, the original data need to be reciprocated before standardization of cost-based criteria. For benefit-oriented attributes, there is

$$y_{ijh} = \left[\frac{x_{ijh}^{L}}{\max_i x_{ijh}^{N}}, \frac{x_{ijh}^{M}}{\max_i x_{ijh}^{M}}, \frac{x_{ijh}^{N}}{\max_i x_{ijh}^{L}} \wedge 1\right]. \quad (4)$$

For cost attributes, there is

$$y_{ijh} = \left[\frac{\min_i x_{ijh}^{L}}{x_{ijh}^{N}}, \frac{\min_i x_{ijh}^{M}}{x_{ijh}^{M}}, \frac{\min_i x_{ijh}^{N}}{x_{ijh}^{L}} \wedge 1\right]. \quad (5)$$

The distance and defuzzification formulas of the dimensionless fuzzy numbers, such as two interval numbers $[y_{ijh}^{L}, y_{ijh}^{M}]$ and $[z_{ijh}^{L}, z_{ijh}^{M}]$, are as follows:

$$d\left(y_{ijh}, z_{ijh}\right) = \frac{\sqrt{2}}{2}\sqrt{\left(y_{ijh}^{L} - z_{ijh}^{L}\right)^2 + \left(y_{ijh}^{M} - z_{ijh}^{M}\right)^2}, \quad (6)$$

$$m\left(z_{ijh}\right) = \frac{1}{2}\left(z_{ijh}^{L} + z_{ijh}^{M}\right). \quad (7)$$

For two triangular fuzzy numbers $[y_{ijh}^{L}, y_{ijh}^{M}, y_{ijh}^{N}]$ and $[z_{ijh}^{L}, z_{ijh}^{M}, z_{ijh}^{N}]$, the distance and the formula of defuzzification are as follows:

$$d\left(y_{ijh}, z_{ijh}\right) = \frac{\sqrt{3}}{3}\sqrt{\left(y_{ijh}^{L} - z_{ijh}^{L}\right)^2 + \left(y_{ijh}^{M} - z_{ijh}^{M}\right)^2 + \left(y_{ijh}^{N} - z_{ijh}^{N}\right)^2}, \quad (8)$$

$$m\left(z_{ijh}\right) = \frac{1}{4}\left(z_{ijh}^{L} + 2z_{ijh}^{M} + z_{ijh}^{N}\right). \quad (9)$$

3.2. Three-Parameter Weighted Heronian Mean Operator

Definition 1 (see [49]). Let $p \geq 0$, $q \geq 0$, and p, q not take the value 0 simultaneously. a_i $(i = 1, 2, \ldots, n)$ is a collection of nonnegative numbers. If

$$\mathrm{GHM}^{p,q}\left(a_1, a_2, \ldots, a_n\right) = \frac{1}{p+q}\left(\prod_{i=1, j=1}^{n}\left(pa_i + qa_j\right)^{2/n(n+1)}\right), \quad (10)$$

then GHM is called the geometric Heronian mean operator.

Definition 2 (see [50]). Let $p \geq 0$, $q \geq 0$, and p, q not take the value 0 simultaneously. a_i $(i = 1, 2, \ldots, n)$ is a collection of nonnegative numbers, and $w = (w_1, w_2, \ldots, w_n)^{\mathrm{T}}$ is a weight vector, which satisfies $0 \leq w_j \leq 1$ and $\sum_{j=1}^{n} w_j = 1$. If

$$\mathrm{GWHM}_{w}^{p,q}\left(a_1, a_2, \ldots, a_n\right) = \frac{1}{\sum_{i=1}^{n}\sum_{j=1}^{n} w_i^p w_j^q}\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\left(w_i a_i\right)^p\left(w_j a_j\right)^q\right)^{1/(p+q)}, \quad (11)$$

then GWHM is called the generalized weighted Heronian mean operator.

Definition 3 (see [51]). Let a_i $(i = 1, 2, \ldots, n)$ be a collection of nonnegative numbers, $p \geq 0$, $q \geq 0$, and $r \geq 0$. If

$$\mathrm{GBM}^{p,q,r}\left(a_1, a_2, \ldots, a_n\right) = \left(\frac{1}{n(n-1)(n-2)}\sum_{n} a_i^p a_j^q a_k^r\right)^{1/(p+q+r)}, \quad (12)$$

then GBM is called the generalized Bonferroni mean operator.

Definition 4 (see [51]). Let a_i $(i = 1, 2, \ldots, n)$ be a collection of nonnegative numbers, $p \geq 0$, $q \geq 0$, $r \geq 0$, and $w = (w_1, w_2, \ldots, w_n)^{\mathrm{T}}$ a weight vector, which satisfies $0 \leq w_j \leq 1$, $\sum_{j=1}^{n} w_j = 1$. If

$$\mathrm{GWBM}^{p,q,r}\left(a_1, a_2, \ldots, a_n\right) = \left(\frac{1}{n(n-1)(n-2)}\sum_{i,j,k=1}^{n} w_i w_j w_k a_i^p a_j^q a_k^r\right)^{1/(p+q+r)}, \quad (13)$$

then GWBM is called the generalized weighted Bonferroni mean operator.

Table 1: Fuzzy linguistic variable.

Linguistic variables	Abbreviations	TFNs
Very poor	VP	(0.0,0.0,0.2)
Poor	P	(0.0,0.2,0.4)
Medium	M	(0.2,0.4,0.6)
Medium good	MG	(0.4,0.6,0.8)
Good	G	(0.6,0.8,1.0)
Very good	VG	(0.8,1.0,1.0)

According to the above definition, a novel operator could be obtained; that is, if

$$\mathrm{TPHM}^{p,q,r}\left(a_1, a_2, \ldots, a_n\right) = \left(\frac{1}{\lambda}\sum_{i=1}^{n}\sum_{j=i}^{n}\sum_{k=j}^{n} a_i^p a_j^q a_k^r\right)^{1/(p+q+r)} \quad (14)$$

where $\lambda = \sum_{l=1}^{n}(l(l+1)/2)$, $p \geq 0$, $q \geq 0$, and $r \geq 0$, then TPHM is called the three-parameter Heronian mean operator [50].

Based on the above analysis, let a_i $(i = 1, 2, \ldots, n)$ be a collection of nonnegative numbers, $p \geq 0$, $q \geq 0$, $r \geq 0$, and $w = (w_1, w_2, \ldots, w_n)^{\mathrm{T}}$ a weight vector, which satisfies $0 \leq w_j \leq 1$, $\sum_{j=1}^{n} w_j = 1$. If

$$\mathrm{TPWHM}_w^{p,q,r}(a_1, a_2, \ldots, a_n) = \left(\frac{1}{\lambda}\sum_{i=1}^{n}\sum_{j=i}^{n}\sum_{k=j}^{n}(w_i a_i)^p (w_j a_j)^q (w_k a_k)^r\right)^{1/(p+q+r)}, \quad (15)$$

where $\lambda = \sum_{i=1}^{n}\sum_{j=i}^{n}\sum_{k=j}^{n} w_i^p w_j^q w_k^r$, then TPWHM is called the three-parameter weighted Heronian mean operator [50].

3.3. Time Sequence Weight Vector. Compared with evaluation methods of traditional performance, the influence of time factor should be considered in the process of performance evaluation in this study. In the process of general performance evaluation in reality, decision-makers do not have sufficient information, and the distribution of information is not clear. In order to fully reflect the timeliness of the attribute information for the evaluation of enterprise learning performance in the process of cooperative innovation, a time sequence weight vector is acquired based on the principle of "thick present, thin ancient" [45]. The connotation of this principle is that the closer the attribute information is, the stronger the timeliness is and the closer it is to the real value of the target. The larger the weight coefficient is, the more attention is paid to the new information and the more effective it is. Therefore, in this paper, time weight vectors are obtained by using the method of "thick, present and thin, ancient" based on time degree and information entropy.

Let $I = -\sum_{k=1}^{p} w_k \ln w_k$ be an information entropy, which is used to objectively reflect the amount of information about learning performance evaluation attributes under different time series. The smaller the amount of information, the greater the information entropy [45].

Let $\lambda = \sum_{k=1}^{p}((p-k)/(p-1))\eta_k$ be a time degree, where $\eta = \{\eta_1, \eta_2, \cdots, \eta_p\}$. When $\lambda = 0$, it shows that the evaluators only pay attention to the information of the current moment, which is called the positive time weight vector. When $\lambda = 1$, it shows that the evaluators only pay attention to the oldest information, which is called negative time weight vector. When $\lambda = 0.5$, that is, $V = \{1/p, 1/p, \cdots, 1/p\}$, the evaluators pay equal attention to all time information. Based on the above analysis, the definition of time sequence weight vector is as follows.

Definition 5 (see [45]). Under the condition of given time degree, the time weight η_k $(k = 1, 2, \cdots, p)$ is determined by the criterion of time degree and information entropy maximization. The nonlinear programming model of time sequence weight vector is as follows:

$$\begin{aligned} \max \quad & I = -\sum_{k=1}^{p} w_k \ln w_k \\ \text{s.t.} \quad & \lambda = \sum_{k=1}^{p}\frac{p-k}{p-1}\eta_k, \\ & \sum_{k=1}^{p}\eta_k = 1, \ \eta_k \in [0,1], \ k = 1, 2, \ldots, p. \end{aligned} \quad (16)$$

The time sequence weight vector could be obtained by solving the model based on the relevant software of MATLAB.

In order to fully reflect the advantages of Definition 5, a comparative analysis with Chen et al.'s [52] study is shown as follows.

The distance between two time weight vectors could be expressed as follows:

$$d(\varphi', \varphi'') = \sqrt{\sum_{k=1}^{p}|\varphi_k' - \varphi_k''|^2}. \quad (17)$$

Then the distances between any time weight vector $\varphi = \{\varphi_1, \varphi_2, \cdots, \varphi_p\}$ and positive and negative ideal time weight vectors are $d(\varphi, \varphi^+) = \sqrt{\sum_{k=1}^{p}\varphi_k^2 + (1-\varphi_p)^2}$ and $d(\varphi, \varphi^-) = \sqrt{(1-\varphi_1)^2 + \sum_{k=1}^{p}\varphi_k^2}$. Thus, the approximation degree of the time weight vector to the ideal time weight vector can be expressed as

$$C(\varphi) = \frac{d(\varphi, \varphi^-)}{d(\varphi, \varphi^+) + d(\varphi, \varphi^-)}. \quad (18)$$

According to the idea of "thick past, thin present", the time series weight vector of solving the nonlinear programming model can be obtained by maximizing $C(\varphi)$ as far as possible under the given time scale. The nonlinear programming model can be expressed as

$$\begin{aligned} \max \quad & C(\varphi) = \frac{\sqrt{(1-\varphi_1)^2 + \sum_{k=2}^{p}\varphi_k^2}}{\sqrt{\sum_{k=1}^{p}\varphi_k^2 + (1-\varphi_p)^2} + \sqrt{(1-\varphi_1)^2 + \sum_{k=2}^{p}\varphi_k^2}} \\ \text{s.t.} \quad & \lambda = \sum_{k=1}^{p}\frac{p-k}{p-1}\varphi_k, \quad \sum_{k=1}^{p}\varphi_k = 1, \ \varphi_k \in [0,1]. \end{aligned} \quad (19)$$

From the above analysis, we can see that when $p = 4$, the time weight vector calculated by the formula (19) method in [52] exists a case where the weight of a time point is zero, for example, when $p = 4$ and $\lambda = 0.4$; that is to say, the data comes from four time points and is used to evaluate enterprise learning performance according to four time points. The time weight vector calculated by the formula (16) method used in this study is η_k $(k = 1,2,3,4) = (0.1671, 0.2133, 0.2722, 0.3474)$. However, the time weight vector calculated by the formula (19) method in [52] is φ_k $(k = 1,2,3,4) = (0.2549, 0.2177, 0.0000, 0.5274)$. Obviously, compared with the method in Chen et al.'s study, the method of obtaining time weight vector in this study has distinct advantages. The method in this study can not only reflect objectively the importance of different time points but also avoid the transition of subjective operation evaluation results. In addition, the dynamic comprehensive evaluation results of enterprise learning performance are more objective, authentic, and practical using this method.

3.4. Entropy Measure Method. Entropy weight method is a method to determine weight objectively. According to the basic principle of information theory, information is an ordered measure of the whole system, and entropy is a disordered measure of the whole system [28]. In the process of learning performance evaluation, the smaller the information entropy of evaluation criteria, the larger the amount of information represented by the criteria, the higher the weight. On the contrary, the opposite is true. Some basic steps of entropy measure method used in this paper are given as follows [28].

Step 1. Standardize the original decision matrix according to the formulas (1)-(5).

Step 2. Entropy of each criterion is calculated by

$$e_j = -\frac{1}{\ln m}\sum_{i=1}^{m} p_{ij}\ln p_{ij}, \quad p_{ij} = \frac{r_{ij}}{\sum_{i=1}^{m} r_{ij}}, \ 0\ln 0 = 0. \quad (20)$$

Step 3. The normalized criteria weight is computed by

$$w_j = \frac{1-e_j}{\sum_{j=1}^{n}\left(1-e_j\right)}. \quad (21)$$

3.5. Solution Procedure. In this study, not only a three-parameter weighted Heronian mean operator considering the interaction between attributes is proposed, but also a time sequence weight vector based on time degree and information entropy is extended. Aiming at the problem of dynamic performance evaluation with completely unknown attribute weights and time weights, a dynamic method considering the interaction between attributes for the evaluation of enterprise learning performance in the process of cooperative innovation is proposed in this paper. The specific steps are as follows.

Step 1. In view of the dynamic evaluation for enterprise learning performance, relevant experts on cooperative innovation and interenterprise cooperation are invited and then determine different time series. Each expert evaluates the values of learning performance attributes under different time series. After many rounds of comprehensive feedback and experts' evaluation, the results of experts' evaluation are consistent. The fuzzy evaluation matrix $G(t_k) = (g_{ij}(t_k))_{m\times n}$ of learning performance under different time series is obtained.

Step 2. Determine the entropy weight w_j of the j attribute c_j under the k periods based on entropy weight method of measure information in (20) and (21).

Step 3. Aggregate learning performance attribute information using the formula (15) considering the interaction between attributes, and the comprehensive values a_i^k of learning performance T_i under the k periods are obtained.

Step 4. According to the suggestions of enterprise cooperation managers and experts, a scientific and reasonable time parameter λ is set to solve the nonlinear programming equation (16), and the time weight vector η_k of the k periods is obtained.

Step 5. Aggregate the comprehensive information of learning performance under different periods based on the formula (15); the comprehensive evaluation value Z of learning performance T_i under k periods is obtained.

Step 6. Analyse the comprehensive value a_i^k and Z of enterprise learning performance in the process of cooperative innovation under different periods, so as to find out the learning deficiency in enterprise cooperation, and then enterprise cooperation managers adjust the learning content and learning management.

The solution procedure for the evaluation of enterprise learning performance in the process of cooperative innovation could be shown in Figure 2 including associated techniques. Major steps include (a) structuring a criteria framework to evaluate enterprise learning performance, (b) determining the weight vector w_j of the j attribute c_j, (c) aggregating learning performance attribute information, (d) calculating the time weight vector η_k of the k periods, (e) obtaining the comprehensive evaluation values, (f) analyzing the comprehensive values a_i^k and Z of enterprise learning performance, and (g) adjusting the learning content and learning management.

4. Case Study

4.1. Case Background. ZX company is a medium-sized enterprise in the information and communication technology industry, founded in 2005, located in Beijing, China. ZX company is mainly engaged in research and development (R&D), design, production, and operation of communication equipment and other products. The main products include communication network wiring, information cabinet products, and medical information products. Communication network wiring and information cabinet products include ODN products, optical devices products, wireless

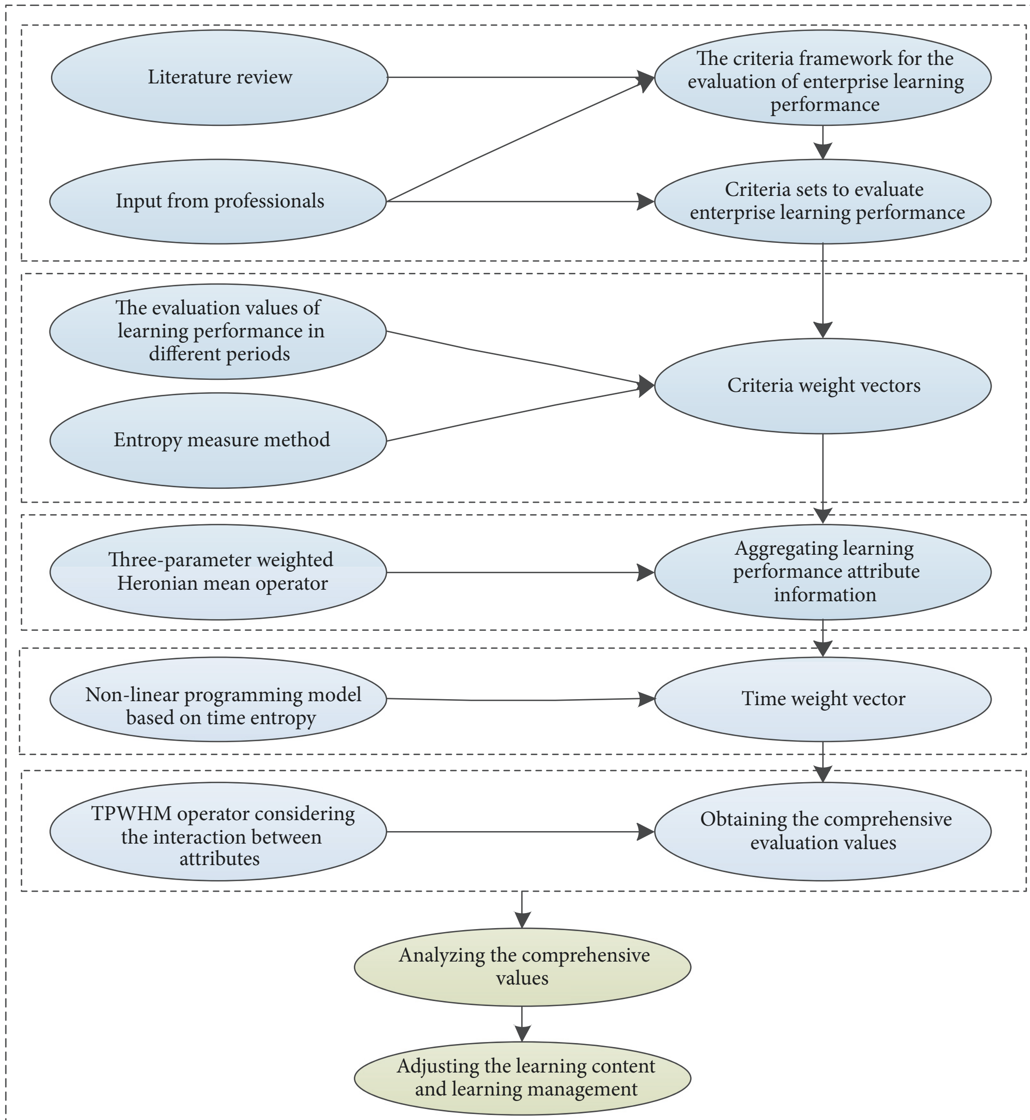

FIGURE 2: The solution procedure for the evaluation of enterprise learning performance in the process of cooperative innovation.

access products, and information cabinet products. There are more than 20 kinds of products, which are widely used in communication network, cloud platform IDC room, railway communication network, and urban rail transit communication network. The main customers include China Telecom, China Mobile, China Unicom, Railway Communications Corporation, and Radio and Television Corporation. In the field of information equipment, the company is committed to providing complete medical information solutions and equipment to hospitals.

ZX company has certain advantages in innovation resources of communication equipment, but its ability of R&D, design, and operation status is general in the field of information and communication. At present, ZX company has been cooperating with HW, which is a large enterprise in the field of information and communication for five years. In the process of cooperative innovation, ZX company's internal processes level, management level, and product R&D, design and operation have improved to a certain extent, but compared with enterprises in the same situation in the industry, there is still a certain gap. At the same time, with the personalized demand of customers, the trend of digital transformation is becoming more and more prominent, and ZX company is facing enormous pressure of customer-oriented communication equipment product innovation. In order to improve the customer service level, technological innovation ability, and market competitiveness of enterprises, ZX company decided to adjust the innovation cooperation with HW enterprise, which involves the adjustment of learning from HW enterprise. For this reason, ZX company needs to carry out a dynamic comprehensive evaluation of learning performance in the process of cooperative innovation with HW enterprise, so as to find out the deficiencies of enterprise learning in the process of cooperative innovation and provide

Table 2: Evaluation matrices of learning performance under different time series.

	C1	C2	C3	C4	C5
T1	G	P	(0.35,0.45)	M	0.35
T2	MG	M	(0.45,0.58)	G	0.45
T3	VP	VG	(0.74,0.85)	G	0.50
T4	M	VG	(0.55,0.65)	VG	0.55
T5	VG	P	(0.60,0.77)	M	0.65

Table 3: Evaluation matrices after standardization and defuzzification.

	C1	C2	C3	C4	C5
T1	0.288	0.078	0.140	0.120	0.538
T2	0.213	0.153	0.180	0.240	0.692
T3	0.018	0.360	0.270	0.240	0.769
T4	0.140	0.360	0.205	0.285	0.846
T5	0.343	0.078	0.235	0.120	1.000

practical guidance for adjusting the learning planning of the next innovation cooperation project.

4.2. Evaluation of Enterprise ZX Learning Performance. Based on the criterion framework for the evaluation of enterprise learning performance in the process of cooperative innovation, the TPWHM operator considering the interaction between attributes, time entropy, and entropy weight method are applied to evaluate the learning performance of ZX company. In order to pre-evaluate the learning performance, five subcriteria in the criteria framework as shown in Figure 1 are applied for fuzzy evaluation, and the practice of learning performance evaluation is carried out as follows.

Step 1. Five subcriteria in the criteria framework are applied by ZX company. Ten evaluation experts in the fields of cooperative innovation and interenterprise cooperation are invited by ZX company. Five different time series are determined by experts according to the cooperation time; that is, each year is regarded as a time series. The attributes of learning performance are evaluated under different time series. Learning performance evaluation criteria are learning content (language variables), learning management (language variables), knowledge accumulation (interval number), ability accumulation (language variables), and performance creation (explicit number). After several rounds of comprehensive feedback and cooperative expert evaluation, the results of expert evaluation tend to be consistent. Five fuzzy evaluation matrices of learning performance attribute information under different time series are obtained as shown in Table 2, and the evaluation matrices after standardization and defuzzification are shown in Table 3.

Step 2. The entropy weight w_j of the j attribute c_j under the k periods is determined based on entropy weight method of measure information in (20) and (21) and is as follows:

$$w_j^{1-5} = (0.1637, 0.1653, 0.1654, 0.1638, 0.3418). \tag{22}$$

Step 3. Learning performance attribute information is aggregated using the formula (15) considering the interaction between attributes, and the comprehensive values a_i^k of learning performance T_i under the k periods are calculated as follows:

$$\begin{aligned} &\mathrm{TPWHM}_w^{1,1,1}(a_1, a_2, \ldots, a_5) \\ &\quad = \left(\frac{1}{\lambda} \sum_{i=1}^{5} \sum_{j=i}^{5} \sum_{k=j}^{5} (w_i a_i)^1 (w_j a_j)^1 (w_k a_k)^1 \right)^{1/3} \end{aligned} \tag{23}$$

$$\text{and } \lambda = \sum_{i=1}^{5} \sum_{j=i}^{5} \sum_{k=j}^{5} w_i^1 w_j^1 w_k^1 = 0.2985.$$

Then

$$\begin{aligned} \mathrm{TPWHM}_w^{1,1,1}(T_1) &= \left(\frac{0.0114}{0.2985} \right)^{1/3} = 0.3364, \\ \mathrm{TPWHM}_w^{1,1,1}(T_2) &= \left(\frac{0.0236}{0.2985} \right)^{1/3} = 0.4294, \\ \mathrm{TPWHM}_w^{1,1,1}(T_3) &= \left(\frac{0.0332}{0.2985} \right)^{1/3} = 0.4809, \\ \mathrm{TPWHM}_w^{1,1,1}(T_4) &= \left(\frac{0.0442}{0.2985} \right)^{1/3} = 0.5289, \\ \text{and } \mathrm{TPWHM}_w^{1,1,1}(T_5) &= \left(\frac{0.0593}{0.2985} \right)^{1/3} = 0.5834. \end{aligned} \tag{24}$$

Step 4. According to the suggestions of enterprise cooperation managers and experts, in order to reflect the importance of recent data, the time degree is set to 0.4 and the nonlinear programming equation (16) is solved. The time weight of the k $(k = 1, 2, \cdots, 5)$ period is obtained as follows:

$$\begin{aligned} &\eta_k (k = 1, 2, \cdots, 5) \\ &\quad = (0.1277, 0.1566, 0.1920, 0.2353, 0.2884). \end{aligned} \tag{25}$$

Step 5. Aggregate the comprehensive information of learning performance under different periods based on the formula (15), and the comprehensive evaluation value Z of learning performance T_i under k periods is computed as follows:

$$Z = \sum_{i=k=1}^{5} \mathrm{TPWHM}_w^{1,1,1}(T_i)\, \eta_k = 0.4952. \tag{26}$$

Step 6. Analyse the comprehensive value a_i^k and Z of enterprise learning performance in the process of cooperative innovation under different periods. From Step 3, learning performance of ZX company shows an upward trend as shown in Figure 3.

From Figure 3, although the learning performance of ZX company has increased from 0.3364 to 0.5834 in the five years

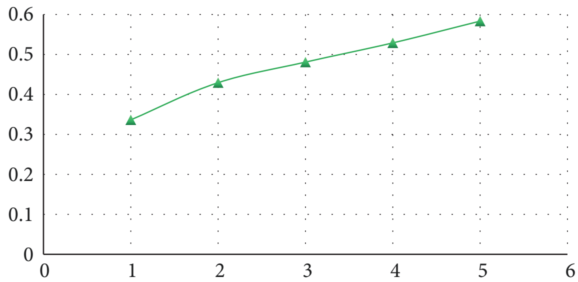

Figure 3: The changing trend of learning performance based on TPWHM operator.

Table 4: The evaluation results of learning performance under different integration operators.

	OWA operator	OWG operator	OWH operator
T1	0.0262	0.0201	0.0164
T2	0.0518	0.0439	0.0396
T3	0.0795	0.0377	0.0138
T4	0.0511	0.0412	0.0350
T5	0.0719	0.0458	0.0332
Z	0.2805	0.1887	0.1379

of innovation cooperation with HW, it is still lower than the passing level. The reason is that ZX company has improved its learning content, knowledge accumulation, ability accumulation, and performance creation by cooperating with HW. The effect of cooperative innovation on the improvement of learning management level is not obvious. The comprehensive evaluation value of learning performance is only 0.4952, which is at a low level. Through interviews with the executive managers of the cooperative innovation projects of ZX company, the actual cooperative learning situation is basically consistent with the results of this evaluation. Therefore, ZX company should adjust learning styles and develop learning mechanism to further improve learning performance.

4.3. Comparative Analysis of Evaluation Results. In order to verify the validity and scientificity of the approach proposed in this study, it is compared with the integration operators which assume that the attributes are independent of each other. OWA operator, OWG operator, and OWH operator are used to evaluate the learning performance of ZX company, respectively. The evaluation results are compared with the result of the TPWHM operator proposed in this study, which considers the interaction between attributes. The evaluation results are shown in Table 4.

From the results of Step 5 and Table 4, it can be seen that TPWHM operator considering the interaction between attributes, OWA operator, OWG operator, and OWH operator are, respectively, used to integrate learning performance attributes information. The results of dynamic comprehensive evaluation of learning performance obtained by these operators are different.

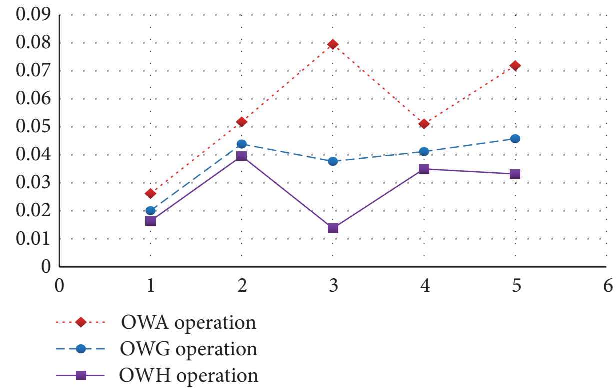

Figure 4: The changing trends of learning performance based on different integration operators.

In the aspect of the changing trends, the changing trends of learning performance evaluation results using TPWHM operator, OWA operator, OWG operator, and OWH operator are shown in Figures 3 and 4. As shown in Figures 3 and 4, it can be seen that the changing trends of learning performance of ZX company evaluated by TPWHM operator, OWA operator, OWG operator, and OWH operator are different. The evaluation results based on TPWHM operator tend to increase gradually, while the results evaluated by the other operators show certain fluctuation, which changes greatly, and the evaluation results of learning performance are unstable. Through interviews with the executive managers of the cooperative projects of ZX company, the actual cooperative learning situation is basically consistent with the results using TPWHM operator. It is more stable and scientific to use TPWHM operator considering the interaction between attributes to evaluate learning performance.

In the aspect of evaluation values, the results evaluated by TPWHM operator considering the interaction between attributes, OWA operator, OWG operator, and OWH operator are, respectively, 0.4592, 0.2805, 0.1887, and 0.1379. The results evaluated by TPWHM operator are better than those based on OWA operator, OWG operator, and OWH operator. The reason is that TPWHM operator considers the interaction between attributes, while OWA operator, OWG operator, and OWH operator assume that attributes are independent when attribute information is integrated. Therefore, the evaluation results in this study are closer to the actual evaluation values and in accord with the actual results of learning performance evaluation.

5. Conclusions

In order to judge whether cooperative goals are achieved and manage cooperative relationships among enterprises, the scientific and reasonable evaluation of learning performance in the process of cooperative innovation has become an important issue. Therefore, from the perspective of system view, this paper constructs a comprehensive and representative criterion framework for the evaluation of enterprise learning performance taking learning process performance and learning outcomes performance as the core ideas in the process of cooperative innovation. In order to consider the interaction between attributes of learning performance and determine a scientific and reasonable time series weight vector, a dynamic comprehensive evaluation approach for evaluating learning performance is proposed to improve the validity and stability of the results of group dynamic comprehensive evaluation.

In this study, the criterion framework for the evaluation of enterprise learning performance in the process of cooperative innovation includes learning process performance and learning outcomes performance. Learning process performance reflects the comprehensive level of learning content and learning management, and learning outcomes performance is the core goal of enterprise core competence development. Learning process performance affects the choice of learning methods and the formulation of learning mechanism in the process of cooperative innovation. Learning outcomes performance is the comprehensive outcome of internal and external learning. That is to say, technological progress could be achieved through internal knowledge acquisition and transformation and internal management process could be optimized, which help to enhance external cooperative management capability. Learning process performance not only directly affects the internal learning outcomes of enterprises but also indirectly affects the external learning outcomes, thus affecting the core objectives of enterprises. Learning outcomes performance could provide information feedback for the rationality of learning content and learning management.

The original matrices are obtained using fuzzy set theory, and time sequence weight vector is calculated based on information entropy and time degree. The weight of learning performance attributes under different time series is calculated based on entropy measure method. The interactive information of learning performance attributes is integrated through the weight of learning performance attributes and the TPWHM operator considering the interaction between attributes. And then, the dynamic and comprehensive evaluation result of learning performance in the process of cooperative innovation could be computed by integrating the learning performance information under different time series with time sequence weight vector. Finally, a real case is studied to verify the scientificity and validity of the criteria framework for enterprise learning performance and the method proposed in this study.

5.1. Management Implications. The criteria framework and dynamic evaluation method considering the interaction between attributes proposed in this study are of both theoretical and practical significance. On the one hand, the criterion framework for the evaluation of enterprise learning performance proposed in this study fully embodies the core objective of learning in the process of cooperative innovation. The criterion framework based on learning process performance and learning outcomes performance has theoretical and practical significance. On the other hand, the dynamic evaluation method of enterprise learning performance fully considers the interaction of complementarity, redundancy, and preference among learning performance attributes under different time series, which accords with the reality where attributes are often interrelated to each other in different degrees. The time sequence weight vector based on information entropy and time degree is more stable, which avoids the unreasonable situation of information integration, and makes the dynamic comprehensive evaluation results more realistic.

The practical contribution of this study is that the method and evaluation criteria framework proposed in this study not only help to improve the effectiveness and stability of the comprehensive evaluation results of enterprise learning performance in the process of cooperative innovation but also help cooperative enterprises get feedback in time and adjust cooperative relationships and learning styles. In theory, this paper proposes a comprehensive and representative criteria framework for the evaluation of enterprise learning performance in the process of cooperative innovation. This framework not only provides a theoretical basis for the study of factors affecting interorganizational learning and performance evaluation but also enriches the theory of interorganizational management and provides a theoretical basis for the process of enterprise cooperative innovation. In addition, the novel method proposed in this study expands the application scope of fuzzy theory and time series in learning performance evaluation.

5.2. Limitations and Future Work. The scope of this study is the evaluation of enterprise learning performance in the process of cooperative innovation. In this study, the criteria framework for enterprise learning performance is constructed taking the learning process and learning outcomes as the core idea, and a novel dynamic evaluation approach considering the interaction between attributes of learning performance is proposed. This paper not only helps cooperative enterprises get feedback in time and adjust cooperative relationships and learning styles but also enriches the theory of interorganizational management and provides a theoretical basis for the process of enterprise cooperative innovation. Although the research goal of this paper was achieved, there are still some limitations which deserve the attention of future research. First of all, the influential criteria should be extended with the change of different types and different forms of cooperative innovation. Secondly, the weight method should be developed based on subjective and objective combination weighting method. In addition, artificial intelligence technology is gradually applied to multicriteria evaluation problems and plays an important role in enlightenment of performance evaluation approach in the future.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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Research Article

Modern Machine Learning Techniques for Univariate Tunnel Settlement Forecasting: A Comparative Study

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Tunnel settlement commonly occurs during the tunnel construction processes in large cities. Existing forecasting methods for tunnel settlements include model-based approaches and artificial intelligence (AI) enhanced approaches. Compared with traditional forecasting methods, artificial neural networks can be easily implemented, with high performance efficiency and forecasting accuracy. In this study, an extended machine learning framework is proposed combining particle swarm optimization (PSO) with support vector regression (SVR), back-propagation neural network (BPNN), and extreme learning machine (ELM) to forecast the surface settlement for tunnel construction in two large cities of China P.R. Based on real-world data verification, the PSO-SVR method shows the highest forecasting accuracy among the three proposed forecasting algorithms.

1. Introduction

Accurate tunnel surface settlement prediction is crucial for construction companies to prevent unexpected disasters, such as tunnel collapse and landslide. For tunnel constructions in large cities, such as metro trains constructions, a precaution alarm of the tunnel settlement helps reduce the risks with affecting nearby people's activities, possible building damage and environment pollution [1]. For rural area tunneling, especially for mountain tunneling, tunnel settlement monitoring prevents landslide that usually can cause construction workers injuries or deaths [2].

Two types of tunnel settlement forecasting methods are available in the literature, namely, model-based methods and artificial intelligence (AI) enhanced methods. Model-based methods build physical or mathematical models based on physics theories and verify the model using physical simulations [3, 4]. However, some physics or mathematical theories are hard to apply directly to real-world situations due to various dependent parameters and, in many cases, serious assumptions have to be made for the physical model can be applied, which may become invalid from time to time.

The fast development of artificial intelligence (AI) provides another option for tunnel settlement prediction. Available AI enhanced methods include pattern sequence forecasting (PSF) [5], support vector regression (SVR) [6], artificial neural networks (ANN) [7], and deep learning neural networks (DLNN) [8]. Between them, the PSF, SVR, and ANN are usually applied to small data analysis, whereas the DLNN methods are more popular for big data analysis and able to provide accurate forecasting results while a long history of the time series data is available.

Two difficulties arise for tunnel settlement forecasting using data-driven methods. First, the time period of tunnel construction is limited, which makes it impossible to collect a long history of data, e.g., up to several years. Moreover, tunnel settlements are usually expected to be fixed within a short period of time, which again makes the collected time series data in short length. Second, the tunnel construction company usually only records relative height of the measuring points, which makes the collected time series data in univariate form. Univariate time series data forecasting is reported to be more difficult than multivariate data forecasting problems [9]. The above two difficulties make the DLNN methods, such as the long short-term memory (LSTM) neural network and its extensions, not suitable for the tunnel settlement forecasting, since for small size data, the DLNN methods usually produce less accurate forecasting results compared with conventional neural networks, such as back-propagation neural network (BPNN), SVR, and extreme learning machine (ELM) [10].

In this study, an extended AI enhanced approach that combines the traditional machine learning techniques with particle swarm optimization (PSO) is proposed. A real-word tunnel surface settlement dataset is employed to verify the performance of the proposing method. In overall, the work that we described in this paper contributes to both the scientific and industrial areas with the following three points:

(1) *Utilizing machine learning techniques for tunnel settlement forecasting*. Tunnel settlement forecasting is a realistic issue in real-world civilization process. However, not many works have been done in this area; especially when the AI enhanced techniques have been rapidly developed, the essentialness of fully utilizing the historical data in tunnel construction process must be emphasized.

(2) *Univariate time series data forecasting with small data size*. The tunnel settlement data, which was employed in this study, was recorded by a metro tunnel construction company located in Shanghai. For each measured tunnel surface point, a time series dataset of size 100 is provided. Moreover, the construction company only records the height of each measured point. However, it is evident that the tunnel settlement is affected by multiple external factors, such as the environmental elements and civilization works. The univariate and small data size properties make the forecasting problem increasingly challenging.

(3) *Extended machine learning approaches are proposed*. The proposed forecasting method modifies the traditional machine learning techniques, such as SVR, BPNN, and ELM, to make them more suitable for tunnel settlement forecasting. A PSO process is added to search for the optimal parameters for various classifiers. In the experiment phase, a comparative analysis is performed to justify the effectiveness of the proposed method.

2. Literature Review

In general, there are two approaches for time series data forecasting, namely, model-based method and data-driven method. Model-based methods utilize mathematical of physical models to perform simulation and usually require multivariate data to be recorded. The extra variables excluding the tunnel surface point heights may include underground water pumping, soil quality measurements, and other assumptions. The forecasting accuracy depends on the validity of the physical assumptions. Shi et al. [11] investigated the soil movement responding to the tunnel excavation in clays through simulations. The soil movements are the main causes of tunnel settlements. Chakeri et al. [12] designed a FLAC3D (Fast Lagrange Analysis of Continua in 3 Dimensions) model to simulate the tunnel excavation process and consequently investigate the ground surface settlement. The proposed FLAC3D is finite-difference approach, based on a number of mathematical assumptions. Strokova [13] surveyed traditional model-based prediction methods for tunnel settlement during construction process. A finite-element based software named “Plaxis” and a mathematical model built based on real-world tunnel settlement data in 2007-2008 at Munich Technical University are utilized for simulation and performance comparison [14]. In summary, the model-based methods provide a white-box modeling for the tunnel settlement problem. The forecasting accuracy of model-based methods is comparable to data-driven approached methods while multiple external variables are available with valid mathematical assumptions.

Data-driven approaches are grey-box or black-box models that involve a complex internal structure, receive a preprocessed version of input dataset, and output integrated forecasting results. Conventional data-driven approaches for time series data forecasting include autoregressive (AR) methods [15], artificial neural networks (ANNs) [16], support vector regression (SVR) [17], deep learning neural networks (DLNNs) [18], and wavelets methods [19]. Ji et al. [20] proposed a least square support vector regression (LSSVR) method for ground surface settlement. Wang et al. [16] reported that by utilizing an adaptive differential evolution (ADE) algorithm to overcome the local extreme issues in optimal weight searching process in BPNN, the traditional BPNN can outperform most existing forecasting methods, such as SVR and AR models. Kuremoto et al. [21] proposed to use a deep belief network with restricted Boltzmann machines to perform time series data forecasting. Wang et al. [22, 23] proposed to use extended echo state network (ESN) to forecast electricity energy consumption in China. Wu and Gao [24] combined AdaBoost algorithm and long short-term memory (LSTM) neural network to forecast financial time series data. Lu et al. [25] introduced another extended LSTM algorithm combining with the differential evolution (DE) method for electricity price forecasting. Yan et al. [26] proposed a multistep forecasting algorithm that integrates convolutional neural network (CNN) with LSTM to forecast single household energy consumption.

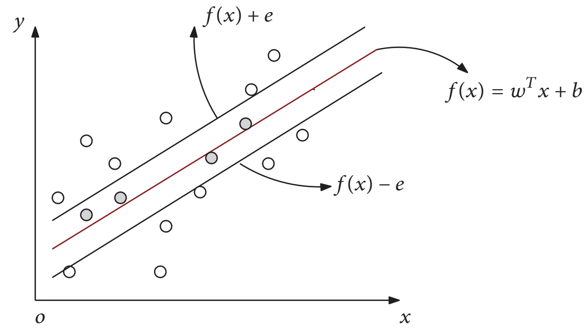

Figure 1: A simple linear regressive plane with insensitive loss variable ε.

3. Proposed Algorithm for Tunnel Settlement Forecasting

3.1. Data Description. Two real-world tunnel settlement datasets were employed for the study of tunnel settlement prediction based on various modern machine learning techniques. Both datasets were collected by a local China tunnel construction company with one of them measuring the tunnel surface settlement of the metro train line 3 construction in Ningbo city, China, and the other one measuring the tunnel surface settlement of a subway construction in Zhuhai city, China. Over 700 ground surface sensors were utilized, measuring the overall settlement on each day during the tunnel construction period. The recording frequency is once per day; and the total number of records for each surface point is around 100, depending on the particular construction progress conditions.

In the experiment phase, in total 10 measured surface points were selected and, for each point, 5/6 of the total recorded length was taken as the training dataset for modern machine learning prediction models, including BPNN, SVR, and ELM. The remaining 1/6 of the total recorded length was used for verification purposes, computing classic error measurement metrics, including root mean square error (RMSE), mean square error (MSE), and mean absolute percentage error (MAPE).

3.2. Back-Propagation Neural Network. BPNN, as one specific form of ANN, represents one of the most classic machine learning techniques, which is continuously employed and improved in various application fields [27–29]. The most critical limitation of BPNN is probably the situation when it is used dealing with big data. For tremendous size data, parallelization of the original BPNN is required [29]. However, when the data size is serious small, the BPNN usually provides high forecasting accuracy with minimal time required compared with other machine learning techniques. Over the past few decades, many extensions of BPNN are proposed. With a preprocessing step, such as the particle swarm optimization (PSO), the extended BPNN becomes more suitable for forecasting and prediction under various working conditions.

3.3. Support Vector Regression. Support vector regression (SVR) is a state-of-the-art and probably the most commonly applied machine learning technique for various purposes in the field of industry engineering, including solar energy generation optimization [30], traffic flow forecasting [31], and molecular dynamics forecasting [32]. Inheriting the core idea from support vector machine (SVM), SVR looks for a hyperplane in high dimension that best represents the data pattern. Figure 1 shows a simple linear support vector regressive plane with insensitive loss variable ε.

LibSVM is an assembled tool-box developed by Chang and Lin, which provides the easy access to use SVR and SVM [25]. For a given set of training data, $Tr = \{(x_i, y_i)\}$, where x_i is the training input and y_i is the objective output value. LibSVM is able to find the objective function $f(x)$ with specified three important parameters: K, C, and γ. K stands for the kernel function that maps the low dimensional input data into high dimensional feature space. C and γ can be optimized by the PSO algorithm.

3.4. Extreme Learning Machine. Extreme learning machine (ELM), proposed by Huang et al. in 2004 and 2006 [33, 34], is reputable by its fast learning speed with low computational resources and simultaneously providing competitive classification results [35–37]. ELM was well known as a single-layer feed-forward neural network (NN) and also has been extended to non-NN forms. Compared to other neural networks in the literature, such as BPNN, multilayer neural networks and SVM, ELM is much faster in terms of training efficiency and provides higher generalized classification accuracy in many proven cases.

The traditional ELM algorithm maps the input data samples with the recognized pattern using one single layer of neurons. For any testing sample x, the ELM function mapping can be expressed by

$$f(x) = \sum_{i=1}^{L} w \cdot h(a, b, x), \quad (1)$$

where a, b are tuned parameters and w is the weight vector for hidden neurons, which is fixed during the training phase. The function $f(x)$ represents the recognized pattern of the input data samples. The tuning-free feed-forward training

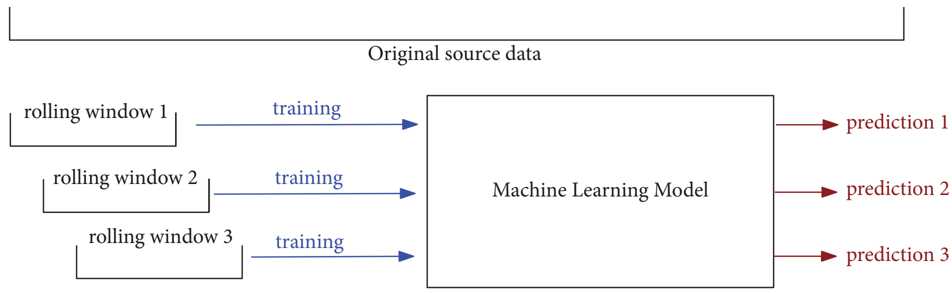

FIGURE 2: Rolling window size determines the length of effective source data samples in the training dataset. For all machine learning techniques, a rolling window size k must be specified for best prediction performance.

strategy of ELM is equivalent to the process of solving a linear equation system that requires very low computational cost.

The basic ELM implementation can be found at http://www.ntu.edu.sg/home/egbhuang/index.html. To achieve the best result using ELM, two important parameters are required to be tuned, which are the number of hidden neurons, and the activation function. The two parameters, again, can be optimized using PSO algorithm.

3.5. Rolling Window Size Selection. Considering the properties of the real-world tunnel settlement data, such as short size, univariate and sparse sampling data points (1 sampling on each day), we select a suitable rolling window size for each machine learning technique in its training process. The univariate training data was reorganized into batches according to the rolling window size and inserted into the machine learning models to predict the next time stamp value (Figure 2). The rolling window size is another important parameter for each machine learning model and basically determines the length of effective source data samples in the training dataset for prediction, since too old data samples usually have less significant influence to the prediction results. According to the data description in Section 3.1, the suitable rolling window size usually lies in the range from 1 to 20.

3.6. Using Particle Swarm Optimization to Find Optimal Parameters for Various Machine Learning Techniques. For all three machine learning techniques that we used in this work, i.e., BPNN, SVR, and ELM, there are important parameters to be tuned, which will seriously impact the final forecasting results [38]. In this study, the PSO is adopted to find the optimal parameters for the three machine learning techniques. The overall algorithms are denoted as PSO-BPNN, PSO-SVR, and PSO-ELM.

Compared to the other optimization search algorithms, such as the genetic algorithm (GA), ant colony algorithm and differential evolution (DE) algorithm, the PSO algorithm is more efficient and able to avoid problems of stagnation behavior and premature convergence [39–41]. Moreover, in the PSO algorithm, the number of parameters is small and the real number coding is adopted. Although the PSO algorithm has shortcomings, such as easy to fall into local extremes, the convergence speed is affected by inertia weight, etc. These shortcomings can be resolved by repeated runs and selecting an appropriate combination of the parameters for the algorithm [42].

Taking PSO-SVR algorithm as an example, the initial parameters of PSO include the number of particles m, inertia weight w, and two learning constants c_1 and c_2. The search of the parameters of PSO depends on the mean absolute percentage error (MAPE) evaluation of SVR results. For a given set of training data $X = \{x_i, t_i\}$ with number of samples n, where x_i stands for the actual data and t_i stands for the forecasting result produced by SVR, the MAPE value is calculated by

$$MAPE = \frac{100}{n} \bullet \sum_{i=1}^{n} \left| \frac{x_i - t_i}{x_i} \right|. \qquad (2)$$

First, we set $m = 200$ and search w in the range $[0.5, 50]$, c_1 and c_2 in the range $[1, 10]$. Based on grid search results with step size 0.1, and with various combinations of parameters for SVR, we select $w = 25$, $c_1 = 1.2$, and $c_2 = 1.6$.

Next, after fixing the parameters of PSO, we look for the optimal parameter combination of SVR using PSO (illustrated in Figure 3). Then the optimal values of C, γ, and k (SVR parameters) are obtained when all particles converge (Figure 4). The detailed steps of the PSO-SVR algorithm are listed in Algorithm 1.

The same process can be applied to search for the optimal parameter combination of BPNN and ELM.

4. Experimental Results

The three machine learning techniques, namely, BPNN, SVR, and ELM, combining with PSO parameter optimization algorithm is applied to a real-world tunnel settlement prediction problem with two datasets collected by a local China tunnel construction company with one of them measuring the tunnel surface settlement of the metro train line 3 construction in Ningbo city, China, and the other one measuring the tunnel surface settlement of a subway construction in Zhuhai city, China. For each tunnel construction project, 5 representative surface points are selected, which are surface point numbers 184, 191, 192, 220, and 230 for the subway construction in

Input: Searching space of vector (C, γ, k), where C ranges from 1 to 10000; γ ranges from -100 to 100; and k is the rolling window size, ranges from 1 to 20.
Output: The optimal values of C, γ, k based on MAPE evaluation of SVR.
Step 1: For each particle p, a location vector l_p and a velocity vector v_p are assigned.
Step 2: For each particle p, the fitness function is evaluated, which is the MAPE value of SVR using this particular particle's location vector.
Step 3: At each iteration, if the fitness function is not satisfied, all particles update their historical optimal location h and global optimal location g according to their current location and velocity.
Step 4: When the maximum iteration is reached, or the MAPE value is less than a pre-defined value, the global optimal location g in the search space is outputted.

Algorithm 1: PSO algorithm looking for the optimal parameter set for SVR.

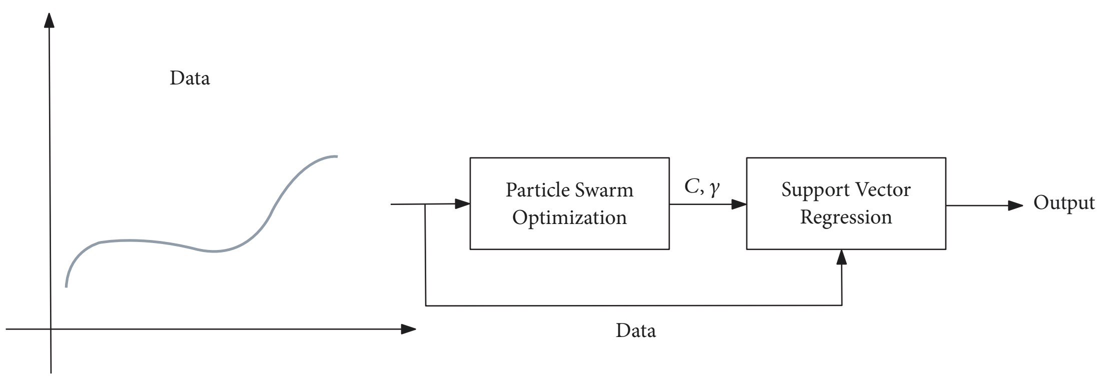

Figure 3: Illustrating the steps of finding optimal parameter combination for SVR.

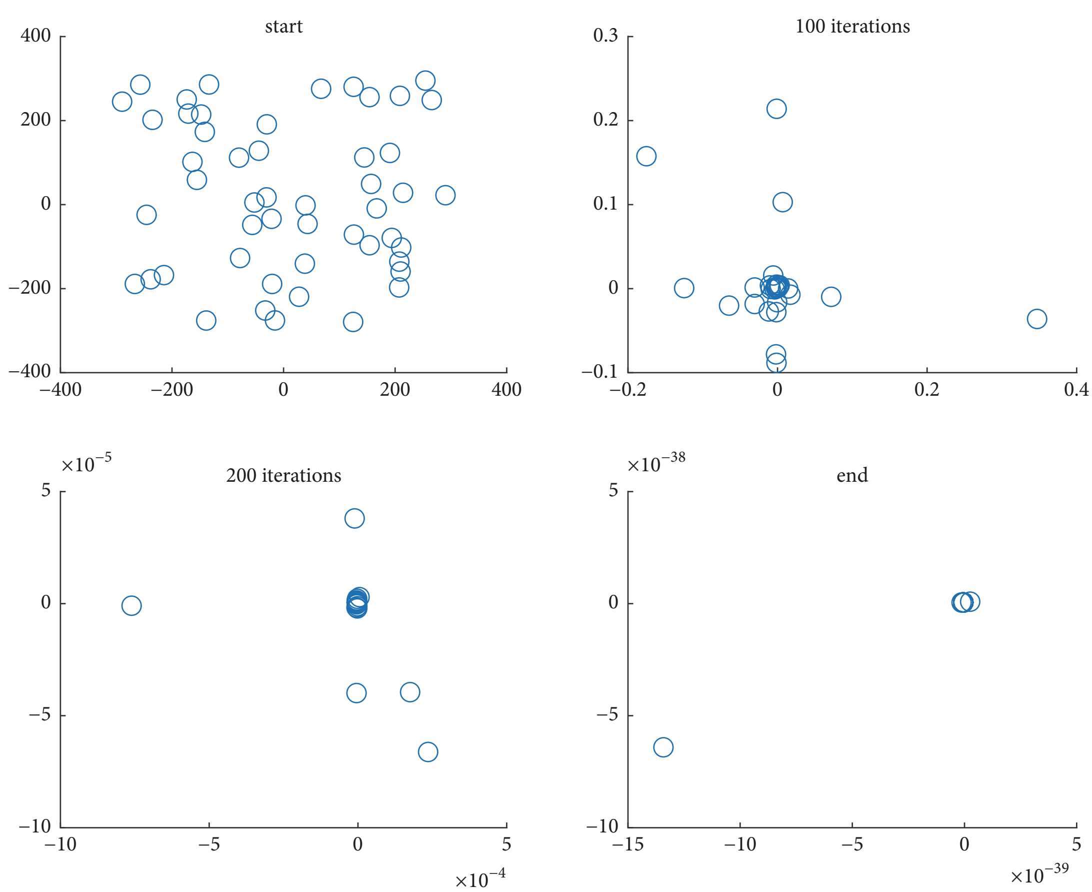

Figure 4: An example of the particles convergence process.

Table 1: Prediction results of all measurement points.

Point number	PSO-SVR			PSO-BPNN			PSO-ELM		
	MSE	RMSE	MAPE %	MSE	RMSE	MAPE %	MSE	RMSE	MAPE %
184	0.000158	0.012562	0.653395	0.072721	0.269669	0.411474	0.246769	0.496759	1.412007
191	0.000381	0.019515	0.120525	0.02051	0.143215	0.611794	0.018798	0.137107	0.642722
192	0.000242	0.015568	1.153007	0.05744	0.239667	11.00573	0.014357	0.119823	3.166807
220	0.002564	0.050634	0.039021	0.398009	0.630879	5.88052	0.39639	0.629595	5.869113
230	0.000128	0.011333	0.141477	0.134172	0.366295	0.757707	0.117423	0.342671	0.969642
554	0.003051	0.055235	1.186884	0.254399	0.504379	1.491434	0.560271	0.748513	2.346384
569	0.001809	0.042533	1.441162	0.378479	0.615206	2.444453	1.046243	1.02286	3.023007
570	0.002652	0.051502	1.116073	0.419282	0.64752	1.932607	0.509027	0.713461	2.194194
571	0.001287	0.035874	0.351681	0.570813	0.755522	0.957628	0.794074	0.891108	1.856434
580	0.003629	0.060241	2.490363	1.560899	1.24936	11.4861	0.509974	0.714125	8.178366

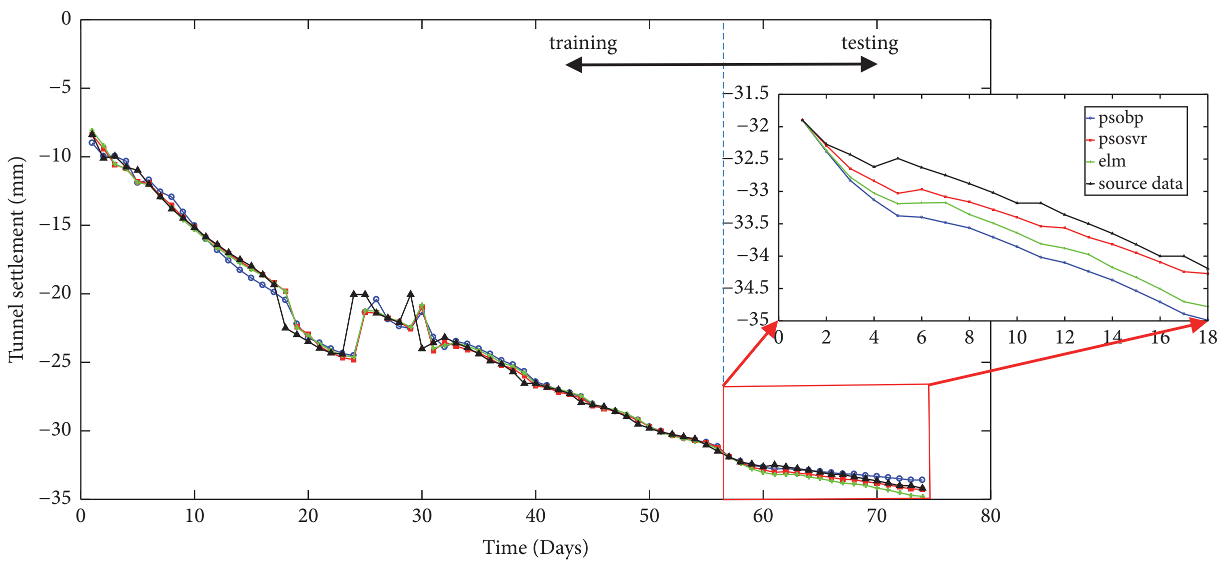

Figure 5: Prediction result of surface point number 184 from Zhuhai subway tunnel construction. The first 5/6 of the source data is used to train the machine learning models. The last 1/6 of the source data is used to compare with the prediction results from various machine learning techniques.

Zhuhai city and surface point numbers 554, 569, 570, 571, and 580 for the metro train line 3 construction in Ningbo city. For each surface point, 5/6 of the total recorded length will be taken as the training dataset and the remaining 1/6 was used for testing purposes, which contains approximately 10 to 20 points.

Figure 5 shows the tunnel settlement prediction for measuring surface point number 184 for the subway construction in Zhuhai city. The actual surface point height decreases most of the time from -7.5mm to -34 mm with some unstable movements because of the underground tunnel construction. In total, there are 75 data points for this particular measurement point. All three machine learning models with parameters optimized by PSO were tested with this measurement point. The first 5/6 of the total dataset is used for training and looking for the best fits of the machine learning models. With each trained model, each rolling window batch will produce predicted value, which is shown in different colors. The results of BPNN are shown in blue color; the results of SVR are shown in pink color; and the results of ELM are shown in green color. For most of the cases, PSO-SVR produces the best RMSE, MSE, and MAPE values according to Table 1, following by BPNN and ELM. Figures 6–9 show the prediction results of surface point numbers 191, 192, 220, and 230, respectively.

Figure 10 shows the tunnel settlement prediction for measuring surface point number 554 for the metro train line 3 construction in Ningbo city. Most of the measuring surface points of this project go up in the first phase of the construction and drop down in the later phase due to the underground human interferes. The surface point movement trend of the first phase is useless for forecasting the testing time period. This is one important reason that we introduce the rolling window in Section 3.5. With proper

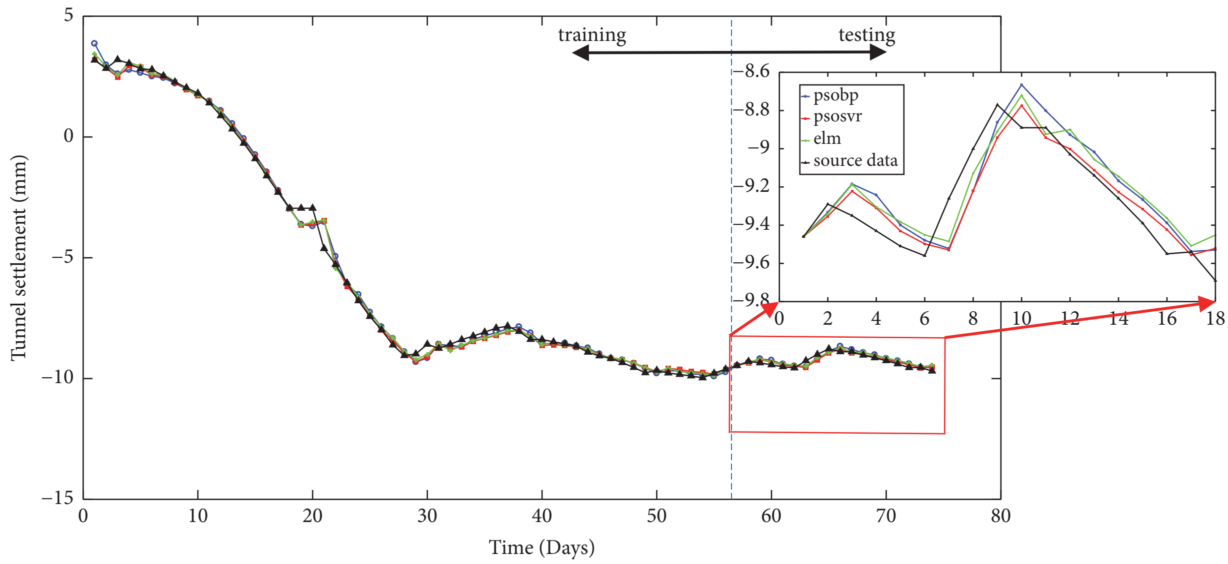

FIGURE 6: Prediction result of surface point number 191 from Zhuhai subway tunnel construction.

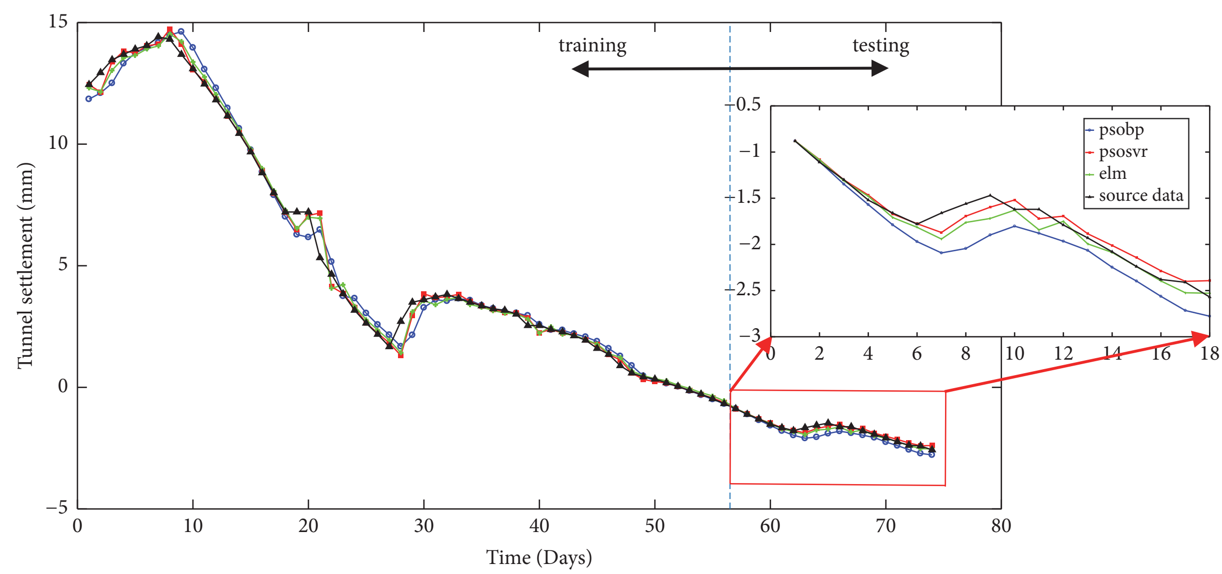

FIGURE 7: Prediction result of surface point number 192 from Zhuhai subway tunnel construction.

rolling window sizes selected, the proposed machine learning framework predicts the tunnel surface point movement based on the most recent movement history and ignores the movement history outside the rolling window. Figures 11–14 show the prediction results of surface point numbers 569, 570, 571, and 580, respectively. Experimental results demonstrate that the proposed approach can well predict tunnel surface point movement with human interferes.

For all measurement points shown above, we list MSE, RMSE, and MAPE results of the three machine learning techniques in Table 1. The experimental results show that the PSO-SVR can most accurately predict the tunnel settlement compared with PSO-BP and PSO-ELM. All RMSE values are less than 0.1 with MAPE values less than 2.5%, which suggests that the proposed PSO-SVR method can be well fitted to real-world tunnel settlement forecasting problems.

5. Conclusion and Limitation

Aiming at preventing serious damage during the tunnel construction process, this study proposes an extended machine learning framework combining different machine learning techniques with PSO to forecast the tunnel surface settlement based on univariate historical data. By evaluating the particular form of the real-world tunnel settlement historical data, three modern machine learning techniques were selected, including BPNN, SVR, and ELM. The PSO algorithm is

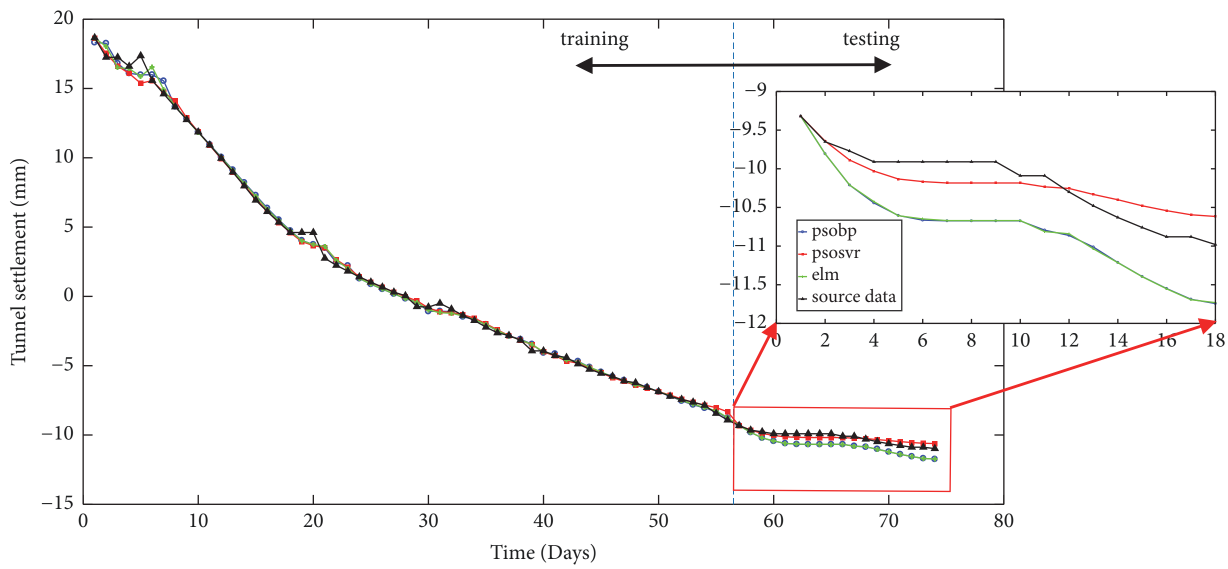

Figure 8: Prediction result of surface point number 220 from Zhuhai subway tunnel construction.

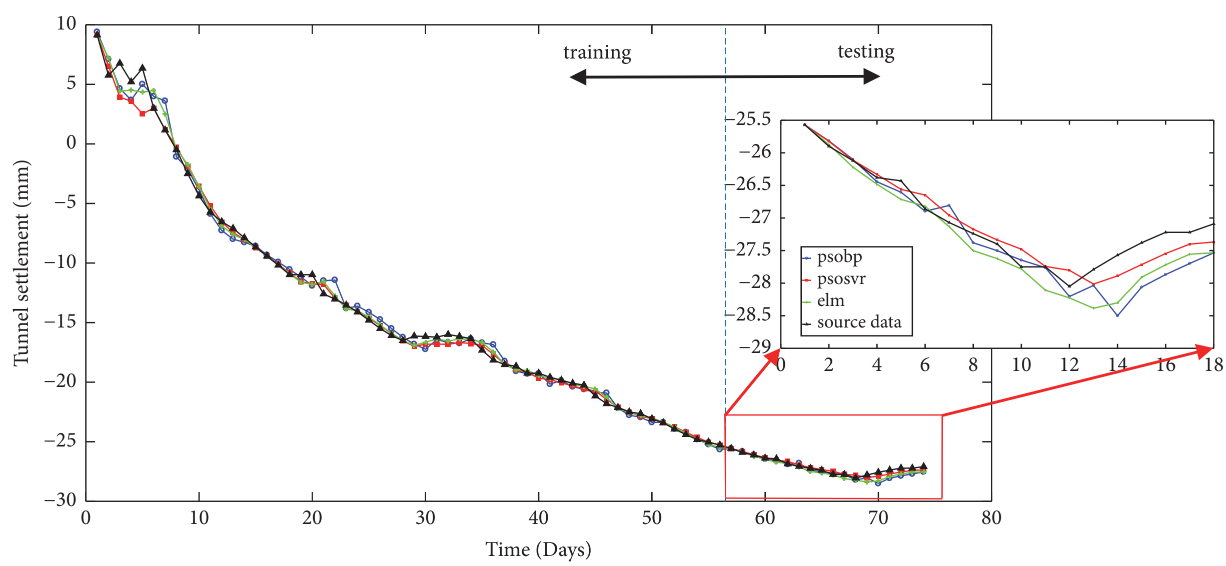

Figure 9: Prediction result of surface point number 230 from Zhuhai subway tunnel construction.

adopted to select the globally optimized parameters for each machine learning technique.

In the experiment phase, two real-world datasets were used for performance comparisons between different machine learning techniques. One dataset records the tunnel surface settlement of a metro train line construction in Ningbo city, China and the other dataset records the tunnel surface settlement of a subway construction in Zhuhai city, China. A comprehensive comparative study is performed, with MSE, RMSE, MAPE values evaluated for each machine learning technique. The overall result suggests that the SVR is most suitable for tunnel settlement forecasting based on the univariate real-world data, followed by BPNN and ELM.

The current work has the following limitations. First, the tunnel settlement data that we used in this study is relatively a small size dataset, which makes the DLNN methods, such as long short-term memory (LSTM) and gated recurrent unit (GRU), not suitable for this study. As a result, instead, three representative nondeep learning techniques, i.e., BPNN, SVR, and ELM, are selected to perform the simulations. More machine learning techniques have to be tested in future study. Second, PSO method is employed to search for the optimal parameter combinations for the

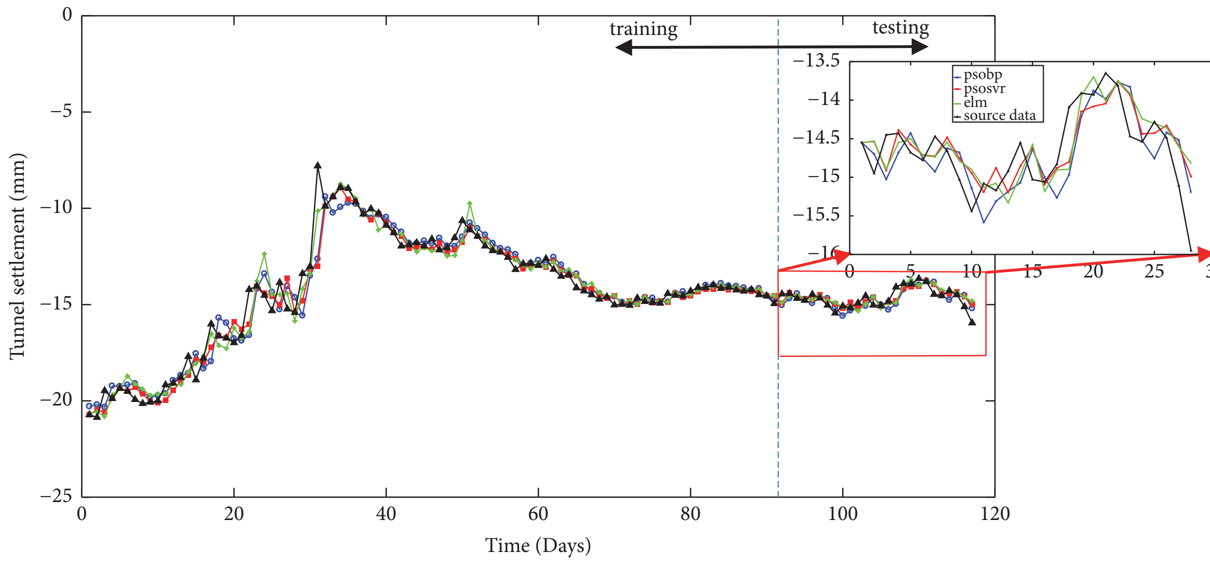

Figure 10: Prediction result of surface point number 554 of the metro train line 3 construction in Ningbo city. The surface point goes up in the first phase of the construction and drops down in the later phase due to the underground human interferes. The irregular movements increase the difficulty for forecasting. However, the proposed method can still well predict the surface point movement due to proper rolling window size settings.

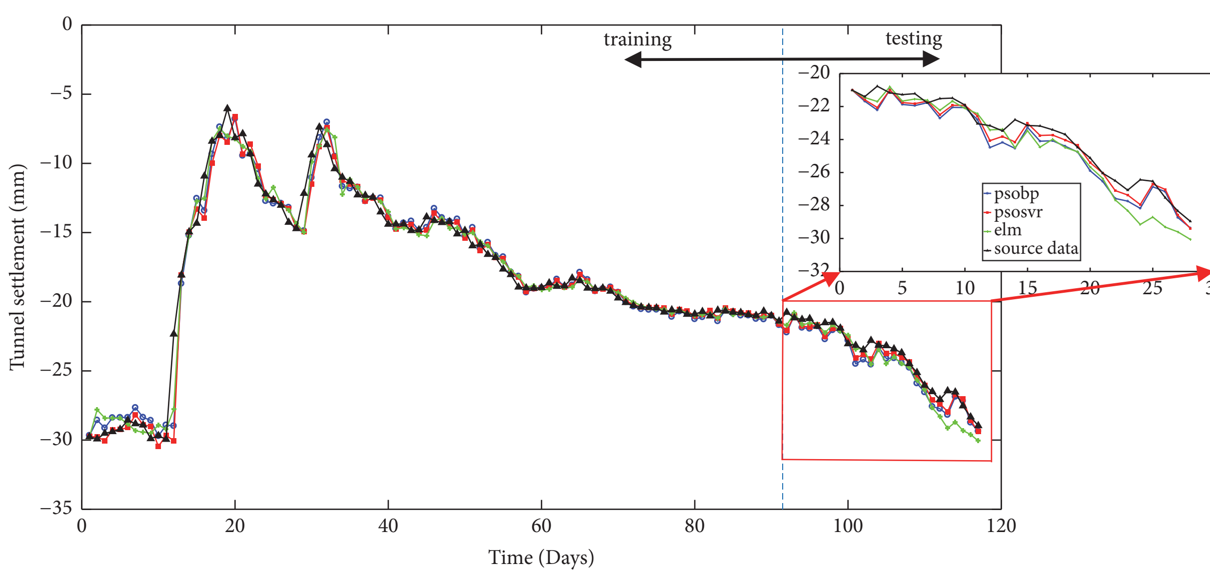

Figure 11: Prediction result of surface point number 569 of the metro train line 3 construction in Ningbo city.

three machine learning methods. More searching algorithms, such as genetic algorithm (GA), ant colony algorithm, and differential evolution (DE) algorithm can be adopted and compared in future study.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding publishing this paper.

Acknowledgments

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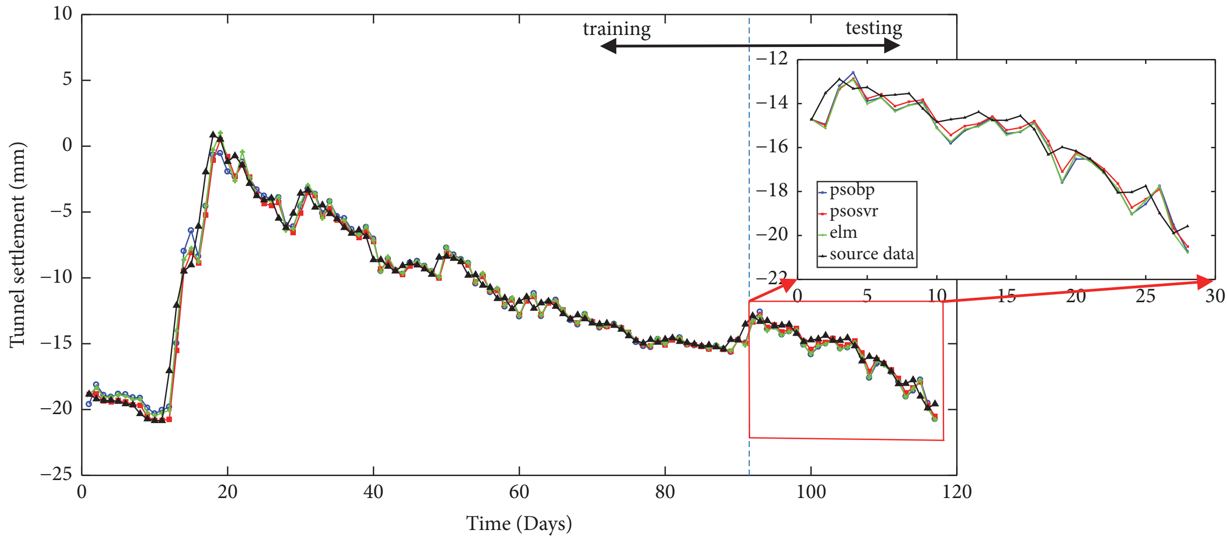

FIGURE 12: Prediction result of surface point number 570 of the metro train line 3 construction in Ningbo city.

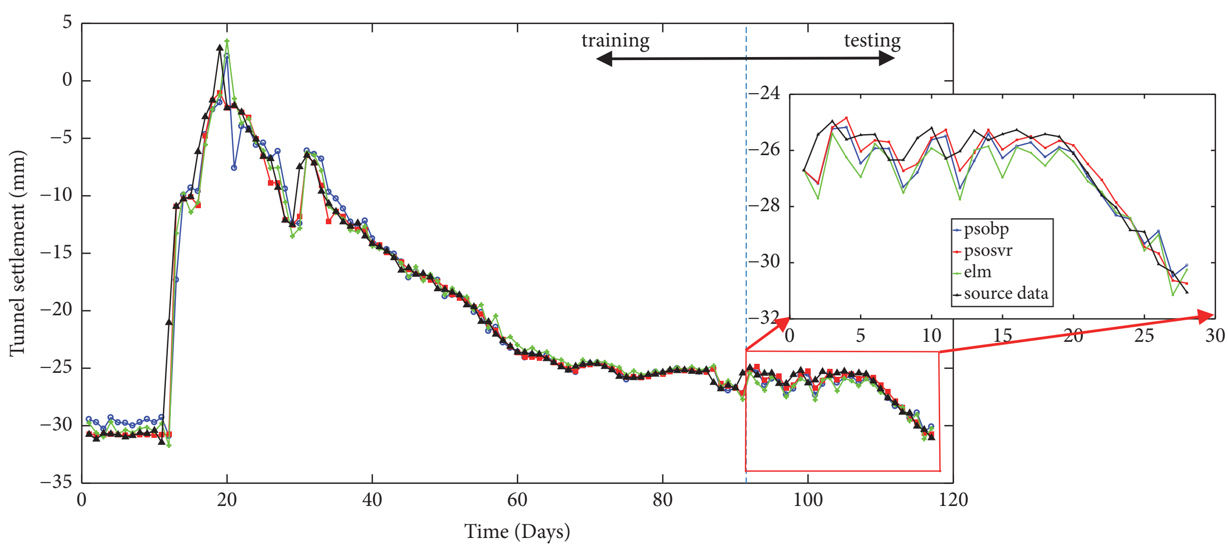

FIGURE 13: Prediction result of surface point number 571 of the metro train line 3 construction in Ningbo city.

and the Foundation of Talent's Start-Up Project in Zhejiang Gongshang University (1120XJ2116016). This work was also partially supported by National Natural Science Foundation of China (Nos. 61850410531, 61602431, and 61374094), Natural Science Foundation of Zhejiang Province (No. LY18F030025), and Shanghai Science and Technology Commission (16DZ1201704).

Supplementary Materials

Two real-world tunnel settlement datasets were employed for the study of tunnel settlement prediction based on various modern machine learning techniques. Both datasets were collected by a local China tunnel construction company with one of them measuring the tunnel surface settlement of the metro train line 3 construction in Ningbo city (section code: NBDT), China, and the other one measuring the tunnel surface settlement of a subway construction in Zhuhai city (section code: ZHSD), China. Over 700 ground surface sensors were utilized, measuring the overall settlement on each day during the tunnel construction period. The recording frequency is once per day and the total number of records for each surface point is around 100, depending on the particular construction progress conditions. Individual attributes descriptions for "TunnelData.cvs": (1) Index. (2) Point ID: each monitored point has an ID. (3) Monitor_date: the date when the data was collected. (4) This_time_change: relative movement (vertical) from the previous record of this point. (5) All_time_change: total movement (vertical) of this point. (6) Project_code: PC0001-PC0008 indicates

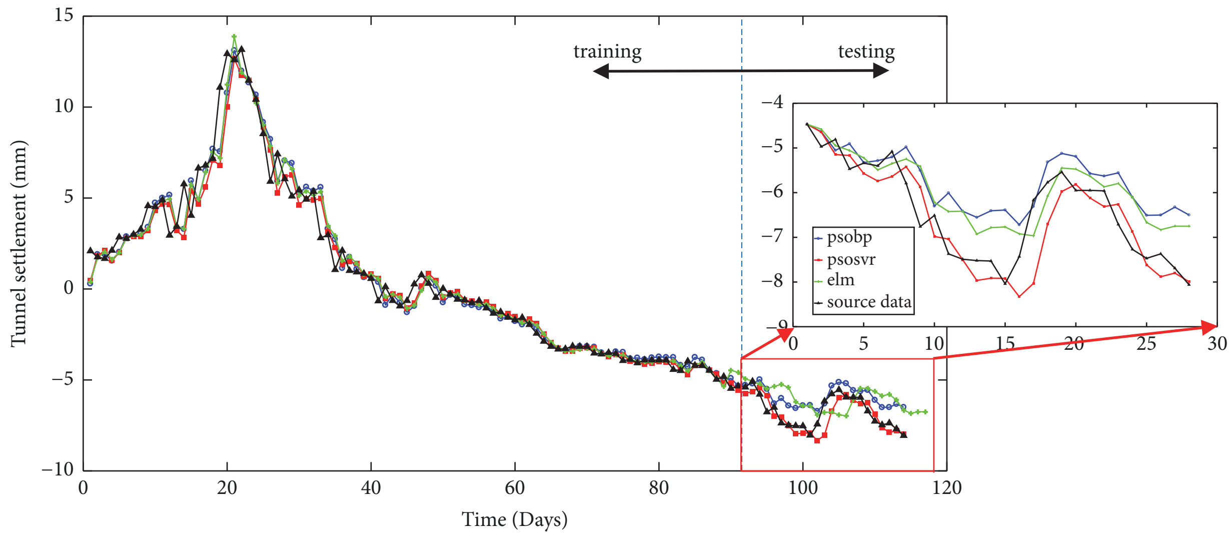

FIGURE 14: Prediction result of surface point number 580 of the metro train line 3 construction in Ningbo city.

8 projects, where in this paper, we only use data with PC0001 (Ningbo) and PC0002 (Zhuhai). (7) Section_code: PC0001 corresponds to NBDT, which stands for the tunnel in Ningbo city, China. PC0002 corresponds to ZHSD, which stands for the tunnel in Zhuhai city, China. (8) Tunnel_code: one city may have multiple tunnels. These are the IDs for tunnels. (9) Point_code: the code for point (another ID with alphabets). (10) Point_type: types of points. (11) Ring_number: the tunnel was constructed by inserting rings. These are the IDs for Rings. (12) Depth: this indicates the depth of the center of ring. (13) Relative_direction: the ring direction. (14) Min_distance_axis: the minimum distance between the actual ring axis and the predefined ring axis. (15) Alarm_rule_id_single: a predefined rule that is used to alarm when the min_distance_axis reaches a threshold. (16) Alarm_rule_id_sum: sum the IDs of all offended rules. (17) Comments. *(Supplementary Materials)*

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Research Article

Evaluating Risks of Mergers & Acquisitions by Grey Relational Analysis Based on Interval-Valued Intuitionistic Fuzzy Information

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Purpose. The purpose of our research is to explore a new grey relational analysis method when information of decision making is interval-valued, intuitionistic, fuzzy, and uncertain in risk analysis of Mergers & Acquisitions. *Design/Methodology/Approach*. We proposed a new method to evaluate risks of Mergers & Acquisitions. The process of our method is to determine the positive and negative ideal solutions of interval-valued intuitional fuzzy uncertain language firstly. Then, calculate grey relational grades of every evaluating value for positive or negative ideal solutions. Third, determine the weights of attributes by a linear programming model if part of attribute information is known. Fourth, calculate grey relational grades of each alternative for the positive or negative ideal solutions. Lastly, calculate relative grey relational grades and sort the alternatives. *Findings*. Our case analysis demonstrated that the new grey relational analysis is an effective tool to evaluate the risks of Mergers & Acquisition when information of decision making is interval-valued, intuitionistic, fuzzy, and uncertain. At the same time, we also bring forward the steps of evaluation. *Originality/Value*. Because risks of Mergers & Acquisitions decide its success or failure to some extent, it is very important to evaluate them by feasible and available method. However, the information of risks is fuzzy and uncertain usually. The new grey relational analysis based on Interval-Valued Intuitionistic Fuzzy Information does not only evaluate risks of Mergers & Acquisitions but also can be widely applied to similar problems of decision making in other fields.

1. Introduction

Mergers & Acquisitions (M&A) are often regarded as part of a corporate searching for value creation and the maximization of shareholder value, including efficient growth, asset redeployment, and market power increase [1]. However, because of information asymmetry and dynamic change of environment, it is difficult to arrive at expectable purpose. Many enterprises did not only realize all kinds of synergetic effects mentioned in enterprises M&A theory, but also even recession occurred. It shows that in the process of enterprise M&A to rapid expansion, many variables objectively exist, which makes the M&A process and act results have considerable uncertainty and risks [2].

At present, risk management has become the core issue of M&A. There are two kinds of studies about risks of M&A. One is about which factors cause risks, another is about how to evaluate them. Many factors influence success of M&A; Xiu and Zhao [3] divided the common risks of M&A into three kinds: risks before M&A, risks during M&A, and risks after M&A [3]. Correct strategy is important and necessary undoubtedly before Mergers & Acquisitions, which can ensure that acquisitions are on the right track. Many activities of M&A failed from wrong strategies, the business which is being purchased does not match its own strategy well [4]. In addition, as time goes on, many conditions in the environment may change, such as law, policy, industry, market, and technology. During M&A, because of moral hazard and adverse selection, if managerial motivations sometimes do not keep up same pace with shareholders, or their private profits have conflictions with shareholders, performance will be under expectation. Zhang (2006) investigated pricing risk of a target company and gave it a brief description. Pricing risk comes from information asymmetry and different valuation techniques. Financial risk also comes out if there exists high financing cost resulting in paying difficulty or tax increases suddenly because an acquirer has insufficient knowledge of tax rules. After M&A, integration becomes a very important issue. Bruce Wasserstein (1998) argued that successful M&A does not only depend on the value created by the acquired company but also depend on integration of post M&A. An acquirer has to face the confliction of technology, organization, management, and culture [5].

Risk identification is the first step of risk evaluation in M&A. Another is to choose an appropriate method of evaluation. There are few papers on how to evaluate risks of M&A. Chen et al. [2] built the coal enterprises M&A risk evaluation system and constructed a risk prediction model based on support vector machine (SVM) [2]. They also employed data envelopment analysis (DEA) model to evaluate M&A risks by collecting 13 enterprises related data during 2004-2008 [6]. Xie and Song [7] used the maximum entropy (ME) method to analyze the risk of Mergers & Acquisitions when only preacquisition information is available [7]. Although few researches focus on evaluating risks of M&A, there are many papers about risk evaluation. Some scholars used grey relational analysis (GRA) to evaluate risk of an affair. Chen et al. [8] established the safety risk evaluation index system for Air Traffic Management (ATM) and used GRA to calculate the correlation coefficients and correlation degree between the safety risk in ATM and the factors [8]. Zheng et al. [9] used GRA method to evaluate the sluice according to the specific conditions of the sick-dangerous sluices and some special managing principle [9]. Akay [10] classified industrial jobs into two categories, low risk and high risk, by using GRA approach, together with the comparisons in terms of classification accuracy between GRA approach and other methods that used the same dataset, including logistic regression, decision tree, neural networks, neural-fuzzy classification, ant colony optimization, memory-based reasoning, and ensemble model. GRA outperforms other alternative methods and yields at least 10% improvement in classification accuracy compared to the best results achieved among the earlier studies [10]. Others combined GRA with some methods. Li and Niu [11] combined the theory of the whole life cycle with AHP and GRA to determine the risk factors and the size of each risk [11]. Su et al. [12] combined the qualitative and quantitative methods to assess risk of projects. They used AHP method of investment appraisal to determine the risk weights and used GRA method to establish a model to assess the correlation of risk [12]. Wang and Shang [13] constructed an applicable evaluation index system of ship financing risks, selected the relevant data of Chinese listed shipping enterprises, used entropy method, factor analysis method, and GRA method to evaluate financing risks of Chinese listed shipping enterprises [13]. Tang et al. [14] introduced the dynamic grey BP neural network model based on GRA to develop a new method for warning of corporate human resources management risk [14]. Qiu et al. [15] used the fuzzy Delphi analytic hierarchy process (FDAHP) and GRA to assess the risk of water inrush [15].

The above literatures show GRA is a very popular and effective method in risk evaluation, which is not only used exclusively but also combined with other methods. Nevertheless, past researches paid more attention to how to evaluate risk based on mass, deterministic data, or information. When the future is not stable and changes dynamically, it is a puzzled problem if we have to depend on experts' knowledge and experience to judge changes of the future without enough information, and they only can give fuzzy and uncertain remarks for risks. In other words, the experts' remarks are fuzzy and uncertain linguistic information for an attribute. In order to solve the problem, we try to convert experts' linguistic remarks into interval-valued intuitionistic fuzzy numbers and then employ GRA to calculate risk order of M&A alternatives. The reason that we choose interval-valued intuitionistic fuzzy method lies in, compared with other methods, that it is a better tool to handle with fuzziness and uncertainty of experts' comments in the process of decision making. The paper contributes to use GRA evaluate risk of M&A under interval-valued intuitionistic fuzzy condition.

The rest of the paper is organized as follows. In the next section, it is preliminary theory about interval-valued intuitional fuzzy uncertain linguistic variable, set, expected and precision function, and distance. In Section 3, we propose a new grey relational analysis based on interval-valued intuitional fuzzy uncertain linguistic information, including how to calculate positive and negative solutions, attribute weight, grey relational grades and relative grey relational grades. In Section 4, it explains how to employ the new grey relational analysis by a case of evaluating the risk of M & A and demonstrates the feasibility and availability of the method. Our conclusions are offered in the final section.

2. Preliminary Theory

Suppose $S = \{s_1, s_2, \ldots, s_n\}$ is a finite, orderly, and discrete set of evaluating risk. For example, a linguistic set with nine risky variables is expressed as $S = \{s_1 =$ extremely high, $s_2 =$ very high, $s_3 =$ high, $s_4 =$ slightly high, $s_5 =$ fair, $s_6 =$ slightly low, $s_7 =$ low, $s_8 =$ very low, and $s_9 =$ extremely low$\}$. In order to reserve all information during the process of decision making, discrete linguistic variable set S can be extended to continuous set $\overline{S} = \{s_\alpha \mid \alpha \in R\}$ [16, 17]. Suppose $s_\alpha, s_\beta \in \overline{S}$, if $\alpha > \beta$, $s_\alpha > s_\beta$; if $\alpha = \beta$, $s_\alpha = s_\beta$. Supposing $\overline{s} = [s_a, s_b]$, $s_a, s_b \in \overline{S}$ which indicate upper limit and lower limit of $\widetilde{s}$, $\widetilde{s}$ is an fuzzy uncertain linguistic variable [16, 18]. In the theory of interval-value intuitional fuzzy uncertain information, the next four definitions are very important.

Definition 1. Suppose $A = \{< [s_{\theta(x)}, s_{\tau(x)}], [\mu_A{}^L(x), \mu_A{}^U(x)], [\nu_A{}^L(x), \nu_A{}^U(x)]\} >| x \in X\}$ is a set of interval-valued fuzzy uncertain linguistic variables. $< [s_{\theta(x)}, s_{\tau(x)}], [\mu_A{}^L(x), \mu_A{}^U(x)], [\nu_A{}^L(x), \nu_A{}^U(x)]\} >$ are interval-valued intuitional fuzzy uncertain linguistic (IVIFUL) variables, where $[s_{\theta(x)}, s_{\tau(x)}]$ are fuzzy uncertain linguistic variables, $[\mu_A{}^L(x), \mu_A{}^U(x)]$ and $[\nu_A{}^L(x), \nu_A{}^U(x)]$ are intervalued membership grade and nonmembership grade respectively that an alternative belongs to the set $[s_{\theta(x)}, s_{\tau(x)}]$ [19].

Definition 2. Supposing that $a = < [s_{\theta(a)}, s_{\tau(a)}], [\mu^L(a), \mu^U(a)], [\nu^L(a), \nu^U(a)] >$ is an interval-valued intuitionistic fuzzy uncertain linguistic variable, expected function $E(a)$ is defined as

$$E(a) = \frac{1}{2} \times \left(\frac{\mu^L(a) + \mu^U(a)}{2} + 1 - \frac{\nu^L(a) + \nu^U(a)}{2} \right) \times s_{(\theta(a)+\tau(a))/2} = s_{(\theta(a)+\tau(a))\times(\mu^L(a)+\mu^U(a)+2-\nu^L(a)-\nu^U(a)))/8} \quad (1)$$

Precision function $P(a)$ is defined as [20]:

$$P(a) = \frac{1}{2} \times \left(\frac{\mu^L(a) + \mu^U(a)}{2} + \frac{\nu^L(a) + \nu^U(a)}{2} \right) \times s_{(\theta(a)+\tau(a))/2} = s_{(\theta(a)+\tau(a))\times(\mu^L(a)+\mu^U(a)+\nu^L(a)+\nu^U(a)))/4} \quad (2)$$

Definition 3. Suppose that $a_1 = < [s_{\theta(a1)}, s_{\tau(a1)}], [\mu^L(a_1), \mu^U(a_1)], [\nu^L(a_1), \nu^U(a_1)] >$ and $a_2 = < [s_{\theta(a_2)}, s_{\tau(a_2)}], [\mu^L(a_2), \mu^U(a_2)], [\nu^L(a_2), \nu^U(a_2)] >$ are two IVIFUL variables, then,

(1) if $E(a_1) > E(a_2)$, then $a_1 \succ a_2$;

(2) if $E(a_1) = E(a_2)$, then,

if $P(a_1) > P(a_2)$, then $a_1 \succ a_2$;

if $P(a_1) = P(a_2)$, then $a_1 \sim a_2$ [21].

Definition 4. Distance between a_1 and a_2 is defined as [22]:

$$d(a_1, a_2) = \frac{1}{6} \times \Big(\frac{(|\theta(a_1) - \theta(a_2)| + |\tau(a_1) - \tau(a_2)|)}{9} + \left|\mu^L(a_1) - \mu^L(a_2)\right| + \left|\mu^U(a_1) - \mu^U(a_2)\right| + \left|\nu^L(a_1) - \nu^L(a_2)\right| + \left|\nu^U(a_1) - \nu^U(a_2)\right| \quad (3)$$

3. Grey Relational Analysis Based on IVIFUL Information

Supposing that there is a multiattribute decision-making problem, $A = \{A_1, A_2, \ldots, A_m\}$ is a set of risky attributes [23], where $w_j \geq 0$, $j = 1, 2, \ldots, n$, $\sum_{j=1}^{n} w_j = 1$. $D = (a_{ij})_{m\times n}$ is a decision-making matrix consisting of evaluating value a_{ij} of the attribute R_j, where, $a_{ij} = < [s_{\theta(a_{ij})}, s_{\tau(a_{ij})}], [\mu^L(a_{ij}), \mu^U(a_{ij})], [\nu^L(a_{ij}), \nu^U(a_{ij})] >$ is an interval-valued intuitionistic fuzzy uncertain linguistic variable [24].

During the process of decision making, if weights of attributes are known, GRA is used to sort alternatives. However, only some of attribute weights are known sometimes. Normally, they belong to the set $PW = \{w_i \geq w_j, i \neq j; w_i - w_j \geq \alpha_i w_i, 1 \geq \alpha_i \geq 0; w_i - w_j \geq w_k - w_l, i \neq j \neq k \neq l; \beta_j \leq w_j \leq \beta_{j+}\varepsilon_j, 0 \leq \beta_j \leq \beta_{j+}\varepsilon_j \leq 1\}$. When faced with specific multiattribute problems, part of attribute-weight information is a subset of PW, denoted by Φ. Because a part of attribute-weight vectors are known, it is necessary to find a way to calculate all attribute-weight vectors. According to GRA, a decision-maker should choose the alternative that has the maximum grey relational grade with positive ideal solution and minimum one with negative ideal solution [25]. A positive and negative ideal solution is denoted by A^+ and A^- respectively [26]. Both of them can be calculated as follows. Firstly, calculate expected function $E(a_{ij})$ and precision function $P(a_{ij})$ of a_{ij} for every alternative by Equation (1) and (2). Then, sort the evaluating value $a_{1j}, a_{2j}, \ldots, a_{mj}$ of the attribute R_j $(j = 1, 2, \ldots, m)$ and choose the maximum value as positive ideal solution a_j^+, the minimum value as negative ideal solution a_j^-. Lastly, we can get positive and negative ideal solution sets A^+ and A^- [27],

$$A^+ = \{a_1^+, a_2^+, \ldots, a_n^+\} = \left\{\max_i a_{i1}, \max_i a_{i2}, \ldots, \max_i a_{in}, \right\} \quad (4)$$

$$A^+ = \{a_1^-, a_2^-, \ldots, a_n^-\} = \left\{\min_i a_{i1}, \min_i a_{i2}, \ldots, \min_i a_{in}, \right\} \quad (5)$$

Grey relational coefficients, between evaluating value a_{ij} of one alternative and a positive ideal solution or negative ideal solution, are calculated as follows [15]:

$$\xi_{ij}^+ = \frac{\min_{1\leq i\leq m}\min_{1\leq j\leq n} d(a_{ij}, a_j^+) + \rho \max_{1\leq i\leq m}\max_{1\leq j\leq n} d(a_{ij}, a_j^+)}{d(a_{ij}, a_j^+) + \rho \max_{1\leq i\leq m}\max_{1\leq j\leq n} d(a_{ij}, a_j^+)} \quad (6)$$

$$\xi_{ij}^- = \frac{\min_{1\leq i\leq m}\min_{1\leq j\leq n} d(a_{ij}, a_j^-) + \rho \max_{1\leq i\leq m}\max_{1\leq j\leq n} d(a_{ij}, a_j^-)}{d(a_{ij}, a_j^-) + \rho \max_{1\leq i\leq m}\max_{1\leq j\leq n} d(a_{ij}, a_j^-)} \quad (7)$$

Because optimal alternative has the maximum grey relational degrade with positive ideal solution and the minimum with negative ideal solution [28], the weights of attributes are calculated by the multiobjective programming model (P-1).

$$\begin{aligned} (P-1)\ \max\quad & \xi^+ = \sum_{j=1}^{n} w_j \xi_{ij}^+, \quad i = 1, 2, \ldots, m \\ \min\quad & \xi^- = \sum_{j=1}^{n} w_j \xi_{ij}^-, \quad i = 1, 2, \ldots, m \\ \text{s.t.}\quad & w \in w_1 + w_2 + \cdots + w_n = 1 \\ & w_j \geq 0 \end{aligned} \quad (8)$$

Because it is same for the importance of every objective, the above model can be translated to single objective programming model (P-2).

$$(P-2)\ \max\quad z=\sum_{i=1}^{m}\sum_{j=1}^{n}w_j\left(\xi_{ij}^{+}-\xi_{ij}^{-}\right)$$
$$\text{s.t.}\quad w\in\Phi \qquad (9)$$
$$w_1+w_2+\cdots+w_n=1$$
$$w_j\geq 0$$

The attribute-weight vector $W=(w_1,w_2,\ldots,w_n)$ can be solved by the above model (P-2).

According to the above analysis, GRA based on IVIFULI is summarized as follows.

Step one: calculate positive and negative solutions A^{+} and A^{-} of interval-valued intuitionistic fuzzy uncertain language by (4) and (5).

Step two: calculate grey relational coefficient between evaluating value of every alternative and positive-negative ideal solutions by (6) and (7).

Step three: if attribute weights are known, go to next step. If only part of attribute weights is known, solve complete attribute-weight vector $W=(w_1,w_2,\ldots,w_n)$ by the model (P-2) according to the known part of attribute information.

Step four: calculate grey relational coefficient of positive or negative ideal solutions for every alternative by following equations.

$$\xi_i^{+}=w_1\xi_{i1}^{+}+w_2\xi_{i2}^{+}+\cdots+w_n\xi_{in}^{+},\quad i=1,2,\ldots,m \qquad (10)$$

$$\xi_i^{-}=w_1\xi_{i1}^{-}+w_2\xi_{i2}^{-}+\cdots+w_n\xi_{in}^{-},\quad i=1,2,\ldots,m \qquad (11)$$

Step five: calculate the relative grey relational grade (RGRG) of positive ideal solution for every alternative [29].

$$\xi_i=\frac{\xi_i^{+}}{\xi_i^{+}+\xi_i^{-}},\quad i=1,2,\ldots,m \qquad (12)$$

Step six: sort all alternatives by RGRGs and choose the best. The bigger the RGRG is, the better one alternative is.

4. Case Analysis

Under the influence of various complex factors, M&A is a kind of risk activity that makes its earning with full of uncertainty. It is available to help decision-makers to understand risk severity by evaluating it objectively. Consequently, risk evaluation is a basis to make a decision of M&A investment, control and transfer risks, and decrease the loss of risks. ZM Company is a state-own corporation group whose developing strategy is to expand share in thermoelectric market. The company is going to acquire a thermopower plant. There are five targets to be chosen, namely, five alternatives. Before purchasing the target, it employed five professional experts to analyze the risks of M&A. The experts put forward that five risks are necessary to be considered, R_1-Political and legal risk; R_2-Market risk; R_3-Information risk; R_4- Financial risk; R_5-Integration risk. On account of the fuzziness and uncertainty in M&A, decision-makers decided to use IVIFUL variable to evaluate the risk of every alternative. The experts gave the evaluating value a_{ij} of the attribute R_j of the alternative A_i according to their experience and knowledge. Decision-making matrix $A=(a_{ij})_{m\times n}$ is shown as in Table 1.

Next, we will sort the alternatives and choose the best by the above-mentioned method. The steps are as follows.

The first step: determine interval-valued intuitional fuzzy positive solution A^{+} and negative solution A^{-}. By (1) and (2), calculate expected function and precision function of the interval-valued intuitional fuzzy linguistic evaluating value. For example, expected functions and precisions of $a_{11},a_{21},\ldots,a_{51}$ are

$$E(a_{11})=s_{4.05},\quad E(a_{21})=s_{3.69},\quad E(a_{31})=s_{4.25},\quad E(a_{41})=s_{5.04},\quad E(a_{51})=s_{5.85}, \qquad (13)$$

and

$$P(a_{11})=s_{4.98},\quad P(a_{21})=s_{4.47},\quad P(a_{31})=s_{7.03},\quad P(a_{41})=s_{7.12},\quad P(a_{51})=s_{8.26}, \qquad (14)$$

Because $E(a_{51})>E(a_{41})>E(a_{31})>E(a_{11})>E(a_{21})$, then $a_{51}>a_{41}>a_{31}>a_{11}>a_{21}$. Therefore, $a_1^{+}=\max_i a_{i1}=a_{51}([s_8,s_9],[0.61,0.74],[0.23,0.36])$, $a_1^{-}=\min_i a_{i1}=a_{21}([s5,s7],[0.43,0.55],[0.15,0.36])$. Similarly, calculate the rest of a_1^{+} $(i=2,3,\ldots,5)$ and a_1^{-} $(i=2,3,\ldots,5)$. Consequently, positive and negative solutions, A^{+} and A, are shown as follows:

$$\begin{aligned}A^{+}=\{&([s_8,s_9],[0.61,0.74],[0.23,0.36])\\&([s_6,s_8],[0.85,0.96],[0.12,0.17])\\&([s_7,s_8],[0.68,0.71],[0.31,0.33])\\&([s_8,s_9],[0.83,0.97],[0.12,0.13])\\&([s_6,s_8],[0.65,0.88],[0.16,0.21])\}\\A^{-}=\{&([s_5,s_7],[0.43,0.55],[0.15,0.36])\\&([s_4,s_6],[0.55,0.60],[0.28,0.41])\\&([s_3,s_4],[0.44,0.55],[0.37,0.50])\\&([s_3,s_4],[0.61,0.74],[0.24,0.33])\\&([s_3,s_4],[0.47,0.66],[0.24,0.33])\}\end{aligned} \qquad (15)$$

Table 1: Decision-making matrix.

	R_1	R_2	R_3	R_4	R_5
A_1	$([s_4, s_6], [0.74, 0.88], [0.14, 0.24])$	$([s_6, s_7], [0.40, 0.57], [0.24, 0.37])$	$([s_6, s_7], [0.66, 0.82], [0.15, 0.27])$	$([s_4, s_6], [0.71, 0.83], [0.16, 0.23])$	$([s_6, s_8], [0.65, 0.88], [0.16, 0.21])$
A_2	$([s_5, s_7], [0.43, 0.55], [0.15, 0.36])$	$([s_7, s_8], [0.53, 0.71], [0.33, 0.35])$	$([s_5, s_6], [0.52, 0.78], [0.15, 0.36])$	$([s_8, s_9], [0.83, 0.97], [0.12, 0.13])$	$([s_4, s_5], [0.61, 0.74], [0.10, 0.38])$
A_3	$([s_6, s_8], [0.56, 0.66], [0.33, 0.46])$	$([s_4 s_6], [0.55, 0.60], [0.28, 0.41])$	$([s_7, s_8], [0.68, 0.71], [0.31, 0.33])$	$([s_3, s_4], [0.61, 0.74], [0.24, 0.33])$	$([s_6, s_7], [0.47, 0.62], [0.35, 0.46])$
A_4	$([s_7, s_8], [0.54, 0.76], [0.23, 0.37])$	$([s_6, s_8], [0.85, 0.96], [0.12, 0.17])$	$([s_3, s_4], [0.44, 0.55], [0.37, 0.50])$	$([s_4, s_5], [0.47, 0.53], [0.36, 0.46])$	$([s_7, s_8], [0.57, 0.64], [0.23, 0.47])$
A_5	$([s_8, s_9], [0.61, 0.74], [0.23, 0.36])$	$([s_5, s_6], [0.63, 0.88], [0.15, 0.27])$	$([s_4, s_5], [0.78, 0.93], [0.11, 0.13])$	$([s_5, s_6], [0.54, 0.67], [0.24, 0.32])$	$([s_3, s_4], [0.47, 0.66], [0.24, 0.33])$

The second step: determine the grey relational coefficients ξ^+ and ξ^- between evaluating value of every alternative and positive ideal solution or negative ideal solution.

$$\xi^+ = \left(\xi_{ij}^+\right)_{5\times5} = \begin{pmatrix} 1.00 & 0.60 & 0.45 & 0.42 & 0.45 \\ 0.47 & 0.47 & 0.67 & 0.49 & 1.00 \\ 0.53 & 0.51 & 0.56 & 1.00 & 0.54 \\ 0.63 & 0.43 & 1.00 & 0.37 & 0.53 \\ 0.78 & 1.00 & 0.42 & 0.33 & 0.59 \end{pmatrix}$$
$$\xi^- = \left(\xi_{ij}^-\right)_{5\times5} = \begin{pmatrix} 0.48 & 0.56 & 0.37 & 0.62 & 1.00 \\ 0.49 & 0.62 & 0.37 & 0.57 & 0.41 \\ 1.00 & 0.54 & 0.46 & 0.33 & 0.60 \\ 0.56 & 1.00 & 0.38 & 1.00 & 0.50 \\ 0.56 & 0.38 & 1.00 & 0.54 & 0.45 \end{pmatrix} \quad (16)$$

The third step: if weight vectors of attributes W are known (if part of weight information is known, go to step six) and $W = (0.05, 0.15, 0.25, 0.25, 0.3)$, calculate the grey relational grade ξ_i^+ or ξ_i^+ of every alternative with positive or negative ideal solution by (8) and (11). The results are shown as follows:

$$\begin{aligned} \xi_i^+ &= 0.49, \\ \xi_i^+ &= 0.68, \\ \xi_i^+ &= 0.65, \\ \xi_i^+ &= 0.60, \\ \xi_i^+ &= 0.56, \\ \xi_i^- &= 0.66, \\ \xi_i^- &= 0.47, \\ \xi_i^- &= 0.51, \\ \xi_i^- &= 0.67, \\ \xi_i^- &= 0.61. \end{aligned} \quad (17)$$

The fourth step: calculate relative grey relational grade of every alternative ξ_i by (12).

$$\begin{aligned} \xi_1 &= 0.43, \\ \xi_2 &= 0.59, \\ \xi_3 &= 0.56, \\ \xi_4 &= 0.47, \\ \xi_5 &= 0.483. \end{aligned} \quad (18)$$

The fifth step: sort the alternatives according to their relative grey relational grades and choose the optimal. Because $R_1 > R_2 > R_4 > R_3 > R_5$, so the optimal one is R_1.

The sixth step: calculate weight vectors W by the model (P-2) if part of weight-vector information of attributes is known. For example, $\Phi = \{0.05 \le w_1 \le 0.25, 0.10 \le w_2 \le 0.45, 0.15 \le w_3 \le 0.35, 0.20 \le w_4 \le 0.40, 0.25 \le w_5 \le 0.65, \}$, calculate the difference between positive solution ξ_{ij}^+ and negative solution ξ_{ij}^- firstly, then according to the model P-2, build a linear programing model as follows:

$$\begin{aligned} (P\text{-}3)\quad \max\quad & z \\ & = 0.3233w_1 - 0.0998w_2 + 0.5258w_3 \\ & \quad - 0.4495w_4 + 0.1519w_5 \\ \text{s.t.}\quad & w_1 + w_2 + w_3 + w_4 + w_5 = 1 \\ & 0.05 \le w_1 \le 0.25 \\ & 0.10 \le w_2 \le 0.45 \\ & 0.15 \le w_3 \le 0.35 \\ & 0.20 \le w_4 \le 0.40 \\ & 0.25 \le w_5 \le 0.65 \end{aligned} \quad (19)$$

By *optimtool* of *Matlab* software, solve the model (P-3) and obtain the weight vectors $W = (0.05, 0.15, 0.15, 0.40, 0.25)$. Then, return to step three and calculate the grey relational grade of every alternative with positive or negative ideal solution,

$$\begin{aligned} \xi_i^+ &= 0.49, \\ \xi_i^+ &= 0.64, \\ \xi_i^+ &= 0.72, \\ \xi_i^+ &= 0.53, \\ \xi_i^+ &= 0.53, \\ \xi_i^- &= 0.66, \\ \xi_i^- &= 0.50, \\ \xi_i^- &= 0.48, \\ \xi_i^- &= 0.76, \\ \xi_i^- &= 0.56. \end{aligned} \tag{20}$$

Next, calculate relative grey relational grade of every alternative by (12),

$$\begin{aligned} \xi_1 &= 0.42, \\ \xi_2 &= 0.56, \\ \xi_3 &= 0.60, \\ \xi_4 &= 0.40, \\ \xi_5 &= 0.49. \end{aligned} \tag{21}$$

Because $\xi_3 = 0.60$ is the biggest, the third alternative R_3 is the best one.

Evidently, when experts describe the information of alternatives with intervalue, intuition, fuzziness and uncertainty, traditional models such as fuzzy comprehensive evaluation or grey rational analysis, can do nothing for evaluating the risk of an alternative. Especially when part of weight information is known, it is a very difficult problem to deduce the weight of every attribute. ZM Company chose the alternative R_3 finally, and at present, R_3, the thermoelectric factory operates very well. Its heating area has reached more than 6.4 million square meters, annual power generation is over 455 million kwh, annual revenue is about 430 million yuan, annual return is 45.15 million yuan, and annual rate of profit is 10.5 percent. The actual situation of R_3 alternative supports IVIFULIV is an effective and feasible appraisal method.

5. Conclusions

When a corporation decides whether it acquires a target company or not, much information may be interval-value, intuitional, fuzzy and uncertain. It is difficult to solve the problem for traditional method of decision making. Consequently, we brought forward a grey relational analysis based on IVIFULIV to sort alternatives. At the same time, we also studied how to calculate the weights based on linear programming model when part of weight information of attributes is known. During the process of analysis, it is important and necessary to determine expected functions, precision functions, positive and negative matrixes, grey relational grades, relative grey relational grades and linear programming model. The case analysis illustrated how to use the method and verified its feasibility and availability. Although the method that we proposed is applied to evaluate risk of M&A, in fact, it is a universal method. In other words, the method adapts to solve multiattribute decision-making problems when the environment is uncertain and complex. Despite many advantages, there are still some limitations about our research. More comparisons should be made with other methods such as fuzzy comprehensive evaluation to demonstrate advantages of IVIFULIV. Moreover, the method is so complex that only professional experts can master and make use of it. Therefore, we will develop a small and visualized platform to employ IVIFULIV for convenient application.

Data Availability

The data used to support the findings of our study are included in Table 1 within the article titled “Evaluating Risks of Mergers & Acquisitions by Grey Relational Analysis Based on Interval-valued Intuitionistic Fuzzy Information”.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Research Article

Interval-Valued Intuitionistic Fuzzy Multiple Attribute Group Decision Making with Uncertain Weights

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The theory of interval-valued intuitionistic fuzzy sets (IVIFSs) has been an impactful and convenient tool in the construction of advanced multiple attribute group decision making (MAGDM) models to counter the uncertainty in the developing complex decision support system. To satisfy much more demands from fuzzy decision making problems, we propose a method to solve the MAGDM problem in which all the information supplied by the decision makers is expressed as interval-valued intuitionistic fuzzy decision matrices where each of the elements is characterized by an interval-valued intuitionistic fuzzy number, and the information about the weights of both decision makers and attributes may be completely unknown or partially known. Firstly, we introduce a consensus-based method to quantify the weights of all decision makers based on all interval-valued intuitionistic fuzzy decision matrices. Secondly, we utilize the interval-valued intuitionistic fuzzy weighted arithmetic (IVIFWA) operator to aggregate all interval-valued intuitionistic fuzzy decision matrices into the collective one. Thirdly, we establish an optimization model to determine the weights of attributes depending on the collective decision matrix and the given attribute weight information. Fourthly, we adopt the weighted correlation coefficient of IVIFSs to rank all the alternatives from the perspective of TOPSIS via the collective decision matrix and the obtained weights of attributes. Finally, some examples are used to illustrate the validity and feasibility of our proposed approach by comparison with some existing models.

1. Introduction

Atanassov [1] introduced intuitionistic fuzzy sets (IFSs) as an extension of conventional fuzzy set proposed by Zadeh in 1965 [2]. Atanassov and Gargov [3] further proposed interval-valued intuitionistic fuzzy sets (IVIFSs) on the basis of IFSs. After their pioneering work, both IFSs and IVIFSs are getting more and more attention and have been hot research issues in a number of fields, such as industrial control [4], pattern classification [5, 6], system modeling [7, 8], and decision making analysis [9–13]. It should be emphasized that multiple attribute decision making (MADM) and multiple attribute group decision making (MAGDM) on IFSs/IVIFSs have been two especially important branches of operations research. A MAGDM problem on IVIFSs can be regarded as a common human activity, which includes a group of experts (decision makers, DMs) to participate in the process of decision making so as to rank all the alternatives on given attributes through a number of decision making matrices provided by all DMs and the weights of both DMs and attributes.

In general, a MAGDM model involves five key parts: (1) quantization of all respective decision making matrices from every DMs, (2) assessing the weights of DMs, (3) aggregating all decision making matrices into a collective one, (4) determining the weights of attributes, and (5) ranking all the alternatives. Up to now, most of relevant studies have put emphasis on (3), (4), and (5). Concerning the topic of (3), a number of aggregation operators on IVIFSs have been successfully proposed in succession from different perspectives [14–19]. For (4), it is desirable first to consider the constraint condition of the given attribute weight information. In general, the given attribute weight information consists of three types, i.e., crisp values, partially known constraint condition, and completely unknown constraint condition. As described in [20, 21], the provided partially known constraint condition may be constructed with the following forms: a weak ranking, a strict ranking, a ranking with multiples, an interval form, a ranking of differences, and an interval-valued intuitionistic fuzzy numbers. Some models, such as multiple-objective programming model [20, 22], fractional programming method [23], nonlinear programming model [24], linear programming model [25], and grey relational analysis [21], have been successfully developed from different perspectives to determine the weight vector of attributes. For more relevant models, please refer to [15, 26, 27]. The primary methods of ranking alternatives include ranking functions [14], TOPSIS-based methods [24], and VIKOR-based methods [28]. On the topic of determining the weights of DMs, Ye [29] presented a method using the ranking functions on IFSs to determine the DMs' weights for MAGDM with completely unknown weight information on DMs. However, this method produces incorrect weight vector of DMs which may lead to unreasonable decision making results. Gupta et al. [30] developed an optimization model to determine DMs' weights where the weight information of DMs is expressed by IVIFNs. Compared with numerous methods on determining attributes weights, the research on assessing the DMs' weights in MAGDM is still in its infancy and remains to be developed.

In view of the above analysis, we shall focus on the issue of MAGDM under interval-valued intuitionistic fuzzy environment where all the information provided by the DMs is characterized by IVIFNs, the information about DMs is completely unknown, and the information about attributes is partially known. The main contributions of this work can be summarized as follows:

(i) A consensus-based method is developed to determine the weights of DMs.

(ii) A multiobjective optimization model is proposed to determine the weights of attributes.

(iii) A TOPSIS-based MAGDM model under interval-valued intuitionistic fuzzy environment is established via the aggregation operator, the weights of DMs, and the weights of attributes.

Overall, in light of the above three aspects, the proposed method delivers a new vision of modeling uncertain group decision making problems from application fields.

The remainder of this paper is organized as follows. In Section 2, we recall some basic concepts and operations. Section 3 proposes a method to solve those MAGDM problems under interval-valued intuitionistic fuzzy environment where all the information provided by the DMs is characterized by IVIFNs, the information about DMs is completely unknown, and the information about attributes is partially known. An example is employed in Section 4 to prove the performance of the proposed method by comparison with some existing algorithms. Section 5 draws a conclusion of this study.

2. Basic Concepts and Operations

2.1. Basic Concepts

Definition 1 (see [1, 3]). Let X be a set and $D[0,1]$ be the set of all closed subintervals of the interval; an IVIFS A on X has the form $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$, where $\mu_A : X \longrightarrow D[0,1], \nu_A : X \longrightarrow D[0,1]$ are two maps satisfying $\sup \mu_A(x) + \sup \nu_A(x) \le 1$ for all $x \in X$. For each IVIFS on X, $\pi_A(x) = [1 - \sup \mu_A(x) - \sup \nu_A(x), 1 - \inf \mu_A(x) - \inf \nu_A(x)]$ is an intuitionistic index of x in A. $\mu_A(x)$, $\nu_A(x)$, and $\pi_A(x)$ denote the membership degree, the nonmembership degree, and the hesitant degree, respectively.

Remark 2. When $\sup \mu_A(x) = \inf \mu_A(x)$ and $\sup \nu_A(x) = \inf \nu_A(x)$, the IVIFS A reduces to an IFS. If $\pi_A(x) = 0$, then an IFS becomes a fuzzy set. In the following part, we utilize (μ, ν, π) (or (μ, ν)) to denote an interval-valued intuitionistic fuzzy number (IVIFN) or an intuitionistic fuzzy number (IFN).

2.2. Operations

Definition 3 (see [31]). Let $X = \{x_1, x_2, \dots, x_n\}$ be the finite universal set and $A, B \in \text{IVIFS}(X)$ be given by $A = \{(x_i, \mu_A(x_i), \nu_A(x_i)) \mid x_i \in X\}$ and $B = \{(x_i, \mu_B(x_i), \nu_B(x_i)) \mid x_i \in X\}$ $(i = 1, 2, \dots, n)$, where IVIFS(X) denotes all the IVIFSs on X. The correlation coefficient between A and B is defined by

$$c(A, B) = \frac{\gamma(A, B)}{(\gamma(A, A), \gamma(B, B))^{1/2}}, \quad (1)$$

where

$$\begin{aligned}\gamma(A, B) = \sum_{i=1}^{n} w_i \big(&\mu_{A_L}(x_i) \cdot \mu_{B_L}(x_i) + \mu_{A_U}(x_i) \\ &\cdot \mu_{B_U}(x_i) + \nu_{A_L}(x_i) \cdot \nu_{B_L}(x_i) + \nu_{A_U}(x_i) \cdot \nu_{B_U}(x_i) \\ &+ \pi_{A_L}(x_i) \cdot \pi_{B_L}(x_i) + \pi_{A_U}(x_i) \cdot \pi_{B_U}(x_i)\big).\end{aligned} \quad (2)$$

The weight vector $w = [w_1, w_2, \dots, w_n]^T$ of x_i $(i = 1, 2, \dots, n)$ satisfies $w_i \ge 0$ $(i = 1, 2, \dots, n)$ and $\sum_{i=1}^{n} w_i = 1$.

Remark 4. Note that the correlation coefficient satisfies the following conditions: (1) $c(A, B) = c(B, A)$; (2) $0 \le c(A, B) \le 1$; and (3) $A = B$ if and only if $c(A, B) = 1$. When A and B reduce to IFSs, the correlation coefficient can be described as

$$c(A, B) = \frac{\gamma(A, B)}{(\gamma(A, A) \cdot \gamma(B, B))^{1/2}}, \quad (3)$$

where $\gamma(A, B) = \sum_{i=1}^{n} w_i[\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i) + \pi_A(x_i)\pi_B(x_i)]$.

Following Definition 3, we introduce a correlation coefficient between two IVIF matrices.

Definition 5. Let $D_1 = [\alpha_{jk}]_{J\times K}$ and $D_2 = [\beta_{jk}]_{J\times K}$ be two interval-valued intuitionistic fuzzy matrices, where the elements of both D_1 and D_2 are expressed by IVIFNs. Then the correlation coefficient between D_1 and D_2 is defined by

$$C(D_1, D_2) = \frac{1}{JK}\sum_{j=1}^{J}\sum_{k=1}^{K} c(\alpha_{jk}, \beta_{jk}), \quad (4)$$

where $c(\alpha_{jk}, \beta_{jk})$ is the correlation coefficient between α_{jk} and β_{jk} (see Definition 3).

Clearly, the above correlation coefficient satisfies the following theorem.

Theorem 6. *For two interval-valued intuitionistic fuzzy matrices $D_1 = [\alpha_{jk}]_{J\times K}$ and $D_2 = [\beta_{jk}]_{J\times K}$, where the elements of both D_1 and D_2 are expressed by IVIFNs, $C(D_1, D_2)$ satisfies the three conditions:*

(i) $C(D_1, D_2) = C(D_2, D_1)$;

(ii) $0 \le C(D_1, D_2) \le 1$;

(iii) $D_1 = D_2$ *if and only if* $C(D_1, D_2) = 1$.

Definition 7 (see [14]). Let α_i $(i = 1, 2, \dots, n)$ be n IVIFNs, where $\alpha_i = ([a_i, b_i], [c_i, d_i])$, $0 \le a_i, b_i, c_i, d_i \le 1$, $b_i + d_i \le 1$, and $1 \le i \le n$. Then the interval-valued intuitionistic fuzzy weighted arithmetic (IVIFWA) operator has the following form:

$$\begin{aligned} \text{IVIFWA}_w(\alpha_1, \alpha_2, \dots, \alpha_n) &= w_1\alpha_1 \oplus w_2\alpha_2 \oplus \cdots w_n\alpha_n \\ &= \left(\left[1 - \prod_{i=1}^{n}(1 - a_i)^{w_i}, 1 - \prod_{i=1}^{n}(1 - b_i)^{w_i}\right],\right. \\ &\left.\left[\prod_{i=1}^{n} c_i^{w_i}, \prod_{i=1}^{n} d_i^{w_i}\right]\right), \end{aligned} \quad (5)$$

where w_i $(i = 1, 2, \dots, n)$ is the weight of α_i satisfying $w_i \ge 0$ and $\sum_{i=1}^{n} w_i = 1$.

2.3. Review of TOPSIS. TOPSIS is a multicriteria decision analysis method, which was firstly introduced by Hwang and Yoon in 1981 [32] with further developments by Yoon in 1987 [33] and Hwang, Lai, and Liu in 1993 [34]. TOPSIS is based on the concept that the chosen alternative should have the shortest geometric distance from the positive ideal solution (PIS) and the longest geometric distance from the negative ideal solution (NIS) [33, 34]. After their pioneering work, TOPSIS has been extensively employed to establish various uncertain decision making models, especially in MADM on IFS/IVIFS.

Concerning TOPSIS within the framework of IVIFS, the maximal IVIFN and the minimum IVIFN are defined by $([1, 1], [0, 0])$ and $([0, 0], [1, 1])$, respectively.

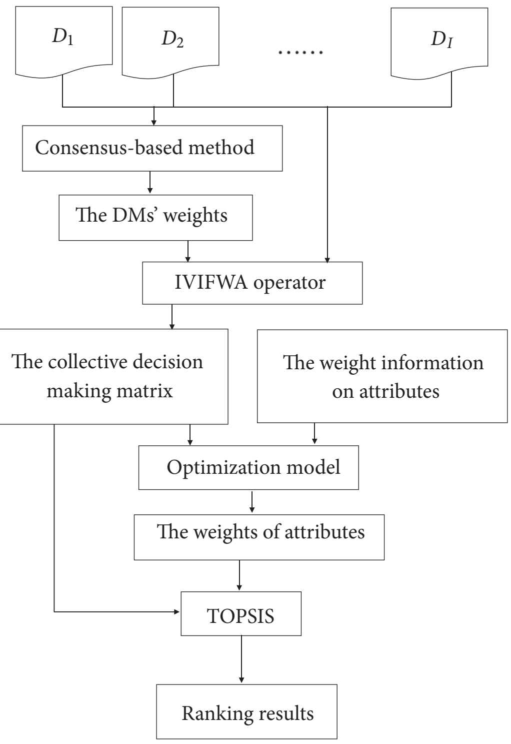

Figure 1: Block diagram of our MAGDM model

3. MAGDM under Interval-Valued Intuitionistic Fuzzy Environment

3.1. Our Proposed MAGDM Model. For a MAGDM problem under interval-valued intuitionistic fuzzy environment, every DM assesses all the alternatives A_j $(j = 1, 2, \dots, J)$ on attributes x_k $(k = 1, 2, \dots, K)$ through a decision making matrix D_i $(i = 1, 2, \dots, I)$ as

$$D_i = \begin{pmatrix} \alpha_{11}^{(i)} & \alpha_{12}^{(i)} & \cdots & \alpha_{1K}^{(i)} \\ \alpha_{21}^{(i)} & \alpha_{22}^{(i)} & \cdots & \alpha_{2K}^{(i)} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{J1}^{(i)} & \alpha_{J2}^{(i)} & \cdots & \alpha_{JK}^{(i)} \end{pmatrix} \quad (6)$$

where $\alpha_{jk}^{(i)} = (\mu_{ij}^{(i)}, \nu_{ij}^{(i)})$ is an IVIFN. Assume that the weight vector of all I DMs is $\omega = [\omega_1, \omega_2, \dots, \omega_I]$ and the weight vector of attributes is $w = [w_1, w_2, \dots, w_K]$, where $\omega_i \ge 0$, $\sum_{i=1}^{I}\omega_i = 1$, $w_k \ge 0$, $\sum_{k=1}^{K} w_k = 1$, $i \in \{1, 2, \dots, I\}$, and $k \in \{1, 2, \dots, K\}$. In this paper, suppose that the information on DMs is completely unknown and the weight information on attributes has the form of linear constraint condition Λ. The block diagram of our MAGDM model is shown as Figure 1. The following part will clearly illustrate this model.

Step 1 (determine the weights of DMs). From the perspective of the majority criterion and consensus [27], those DMs whose decision making matrices have greater consensus with others should be given larger values. Thus, the weights of DMs can be defined as follows:

$$\omega_i = \frac{\theta_i}{\sum_{i=1}^{I}\theta_i}, \quad (7)$$

where θ_i has the form

$$\theta_i = \sum_{i'=1, i'\neq i}^{I} C\left(D_i, D_{i'}\right). \quad (8)$$

Remark 8. By comparison with the majority criterion, all DMs' importance levels have been fully considered and reflected through the obtained weights. This step lays good foundation for making reasonable decision.

Step 2 (calculate the collective decision making matrix D). Depending on all I decision making matrices D_i and their relative weights ω_i, we employ the IVIFWA aggregation operator to get the collective decision making matrix D.

Let

$$D = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1K} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{J1} & \alpha_{J2} & \cdots & \alpha_{JK} \end{pmatrix}. \quad (9)$$

Take α_{jk} $(j = 1, 2, \ldots, J;\ k = 1, 2, \ldots, K)$ as an example. α_{jk} is calculated by $\alpha_{jk} = \text{IVIFWA}_{\omega}(\alpha_{jk}^{(1)}, \alpha_{jk}^{(2)}, \ldots, \alpha_{jk}^{(I)})$.

Step 3 (determine the weight vector of attributes). From the standpoint of TOPSIS and the concept of IVIFS, the positive ideal solution (PIS) can be defined by $A^+ = \{\alpha_1^+, \ldots, \alpha_K^+\}$, where α_k equals $([1,1],[0,0])$ for $k \in \{1, 2, \ldots, K\}$. If we consider certain alternative A_j $(j = 1, 2, \ldots, J)$ with the highest priority, it is easy to establish the following optimization model:

$$\begin{aligned} \max \quad & c\left(\overline{A}_j, A^+\right), \\ \text{s.t.} \quad & w \in \Lambda, \\ & \sum_{k=1}^{K} w_k = 1, \\ & w_k \geq 0, \end{aligned} \quad (10)$$

where $\overline{A}_j$ is the collective value on X for A_j $(j = 1, 2, \ldots, J)$.

Clearly, the bigger the $c(\overline{A}_j, A^+)$, the better the alternative A_j. Solving this model, we can get the optimal solution $w^{(j)} = \left[w_1^{(j)}\ w_2^{(j)}\ \cdots\ w_K^{(j)}\right]^T$. This process is repeated until all the corresponding $w^{(j)}$ is determined. To fully consider all the alternatives as a whole, we define the weight matrix W as follows:

$$W = \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & \cdots & w_1^{(J)} \\ w_2^{(1)} & w_2^{(2)} & \cdots & w_2^{(J)} \\ \vdots & \vdots & \ddots & \vdots \\ w_K^{(1)} & w_K^{(2)} & \cdots & w_K^{(J)} \end{pmatrix}. \quad (11)$$

Moreover, we calculate $\Gamma = (EW)^T(EW)$, where E is defined by

$$E = \begin{pmatrix} c\left(\alpha_{11}, \alpha^+\right) & c\left(\alpha_{12}, \alpha^+\right) & \cdots & c\left(\alpha_{1K}, \alpha^+\right) \\ c\left(\alpha_{21}, \alpha^+\right) & c\left(\alpha_{22}, \alpha^+\right) & \cdots & c\left(\alpha_{2K}, \alpha^+\right) \\ \vdots & \vdots & \ddots & \vdots \\ c\left(\alpha_{J1}, \alpha^+\right) & c\left(\alpha_{J2}, \alpha^+\right) & \cdots & c\left(\alpha_{JK}, \alpha^+\right) \end{pmatrix}, \quad (12)$$

$$\alpha^+ = \left([1,1],[0,0]\right). \quad (13)$$

Let ρ be the normalized eigenvector of Γ. Then w is determined by

$$w = W\rho. \quad (14)$$

Remark 9. As stated above, this optimization model has been established using TOPSIS from the perspective of the criterion of realism decision rule. Thus the attributes' effects have been fully considered and balanced.

Step 4 (calculate $c(\overline{A}_j, A^+)$ $(j = 1, 2, \ldots, J)$). Based on the obtained $\overline{A}_j$, A^+ and the weight vector of attributes w, calculate $c(\overline{A}_j, A^+)$ $(j = 1, 2, \ldots, J)$ via (1).

Step 5 (rank all the alternatives). Depending on the obtained $c(\overline{A}_j, A^+)$ $(j = 1, 2, \ldots, J)$, rank all the values in ascending order, which corresponds to the order of all the alternatives.

3.2. Comparison between Our Proposed Method and Ye's Method. Since the problem of determining the weights of MAGDM under interval-valued intuitionistic fuzzy environment with completely unknown weight information about DMs has been discussed and solved by Ye's method [29], we shall make a comparison between our proposed method and Ye's method.

Here we consider a MAGDM problem which includes three experts, who present their decisions of four alternatives A_j $(j = 1, 2, 3, 4)$ on five attributes x_k $(k = 1, 2, 3, 4, 5)$ through the following three interval-valued intuitionistic fuzzy decision making matrices: D_1, D_2, and D_3. Assume that the weights of three DMs are completely unknown for this decision making problem.

$$D_1 = \begin{pmatrix} ([0,0.2],[0.5,0.5]) & ([0.3,0.3],[0.7,0.7]) & ([0.4,0.6],[0.4,0.4]) & ([0.2,0.2],[0.8,0.8]) & ([0.4,0.4],[0.4,0.6]) \\ ([0.1,0.1],[0.7,0.9]) & ([0.2,0.2],[0.6,0.8]) & ([0,0.2],[0.5,0.7]) & ([0.3,0.3],[0.5,0.7]) & ([0.4,0.4],[0.4,0.6]) \\ ([0,0.2],[0.5,0.7]) & ([0.1,0.1],[0.9,0.9]) & ([0.1,0.3],[0.5,0.7]) & ([0.2,0.2],[0.8,0.8]) & ([0.4,0.4],[0.4,0.6]) \\ ([0.2,0.2],[0.7,0.7]) & ([0,0],[1,1]) & ([0.2,0.4],[0.4,0.6]) & ([0.1,0.3],[0.7,0.7]) & ([0.6,0.6],[0.4,0.4]) \end{pmatrix},$$

$$D_2 = \begin{pmatrix} ([0.5,0.7],[0,0.2]) & ([0.3,0.7],[0,0.2]) & ([0,0.8],[0,0]) & ([0.5,0.7],[0,0.2]) & ([0.6,0.8],[0,0.2]) \\ ([0.6,0.6],[0,0.2]) & ([0.6,0.8],[0.2,0.2]) & ([0.7,0.9],[0.1,0.1]) & ([0.6,0.8],[0,0]) & ([0.9,0.9],[0.1,0.1]) \\ ([0.8,0.8],[0.2,0.2]) & ([0.5,0.7],[0.3,0.3]) & ([0.4,0.8],[0,0.2]) & ([0.3,0.5],[0.3,0.5]) & ([0.2,0.6],[0.1,0.3]) \\ ([0.6,0.8],[0.1,0.1]) & ([0.7,0.9],[0,0]) & ([1,1],[0,0]) & ([0.5,0.7],[0.1,0.1]) & ([0.7,0.9],[0.1,0.1]) \end{pmatrix}, \quad (15)$$

$$D_3 = \begin{pmatrix} ([0.5,0.7],[0.2,0.2]) & ([0.4,0.6],[0.3,0.3]) & ([0.3,0.5],[0.4,0.4]) & ([0.4,0.6],[0.2,0.4]) & ([0.5,0.7],[0.1,0.3]) \\ ([0.5,0.5],[0,0.4]) & ([0.4,0.6],[0.3,0.3]) & ([0,0.6],[0.4,0.4]) & ([0.5,0.5],[0.2,0.4]) & ([0.6,0.6],[0.4,0.4]) \\ ([0.4,0.6],[0.3,0.3]) & ([0.6,0.6],[0,0.4]) & ([0.3,0.5],[0.2,0.4]) & ([0.2,0.4],[0.4,0.6]) & ([0.3,0.5],[0.4,0.4]) \\ ([0.4,0.6],[0.1,0.3]) & ([0.6,0.8],[0.2,0.2]) & ([0,0.6],[0.3,0.3]) & ([0.4,0.6],[0.1,0.3]) & ([0.5,0.7],[0.3,0.3]) \end{pmatrix}.$$

In what follows, we utilize Ye's method [29] and our proposed method to determine the weights of three DMs, respectively.

Case 1 (determine the weights of DMs via Ye's method [29]). Firstly, we get the score matrices S_i $(i = 1,2,3)$ of D_i as follows:

$$S_1 = \begin{pmatrix} -0.4 & -0.4 & 0.1 & -0.6 & -0.1 \\ -0.7 & -0.5 & -0.5 & -0.3 & -0.1 \\ -0.5 & -0.8 & -0.4 & -0.6 & -0.1 \\ -0.5 & -1.0 & -0.2 & -0.5 & 0.2 \end{pmatrix},$$

$$S_2 = \begin{pmatrix} 0.5 & 0.4 & 0.4 & 0.5 & 0.6 \\ 0.5 & 0.5 & 0.7 & 0.7 & 0.8 \\ 0.6 & 0.3 & 0.5 & 0 & 0.2 \\ 0.6 & 0.8 & 1.0 & 0.5 & 0.7 \end{pmatrix}, \quad (16)$$

$$S_3 = \begin{pmatrix} 0.4 & 0.2 & 0 & 0.2 & 0.4 \\ 0.3 & 0.2 & -0.1 & 0.2 & 0.2 \\ 0.2 & 0.4 & 0.1 & -0.2 & 0 \\ 0.3 & 0.5 & 0 & 0.3 & 0.3 \end{pmatrix}.$$

Secondly, we get the average score matrix S^* based on S_1, S_2, and S_3 as below:

$$S^* = \begin{pmatrix} 0.1667 & 0.0667 & 0.1667 & 0.0333 & 0.3000 \\ 0.0333 & 0.0667 & 0.0333 & 0.2000 & 0.3000 \\ 0.1000 & -0.0333 & 0.0667 & -0.2667 & 0.0333 \\ 0.1333 & 0.1000 & 0.2667 & 0.1000 & 0.4000 \end{pmatrix}. \quad (17)$$

Thirdly, we get the weights of three DMs

$$\omega = [-0.1677 \;\; 0.6190 \;\; 0.5487]. \quad (18)$$

Case 2 (determine the weights of DMs via our proposed method). By applying our proposed method to determine the weights of DMs, we get

$$\omega = [0.2908 \;\; 0.3304 \;\; 0.3787]. \quad (19)$$

Remark 10. As indicated in this example, ω_1 equals −0.1677 which completely contradicts the condition of Ye's approach, i.e., $\omega_i \geq 0$ $(i = 1,2,3)$ [29]. This example implies that our method can overcome the deficiencies from [29].

4. Illustrative Example

Here we consider a problem concerning a manufacturing company from [29]. The objective of this problem is to determine the best global supplier for one of its most critical parts used in assembling process. There are four alternatives A_j $(j = 1, 2, 3, 4)$ for choice. Four experts (DMs) provide their own decision making information of four alternatives on five attributes, namely, x_1 (cost), x_2 (quality), x_3 (service), x_4 (supplier's profile), and x_5 (risk factor), through the following four matrices: D_i $(i = 1, 2, 3, 4)$. The constraint conditions of attribute weights can be described by $\Lambda = \{0.1 \le w_1 \le 0.3,\ 0.1 \le w_2 \le 0.3,\ 0.2 \le w_3 \le 0.4,\ 0.2 \le w_4 \le 0.4,\ 0.1 \le w_5 \le 0.3\}$.

$$
\begin{gathered}
D_1 = \begin{pmatrix}
([0.5, 0.6], [0.2, 0.3]) & ([0.3, 0.5], [0.4, 0.5]) & ([0.6, 0.7], [0.2, 0.3]) & ([0.5, 0.7], [0.1, 0.2]) & ([0.1, 0.4], [0.3, 0.5]) \\
([0.3, 0.4], [0.4, 0.6]) & ([0.1, 0.3], [0.2, 0.4]) & ([0.3, 0.4], [0.4, 0.5]) & ([0.2, 0.4], [0.5, 0.6]) & ([0.7, 0.8], [0.1, 0.2]) \\
([0.4, 0.5], [0.3, 0.5]) & ([0.7, 0.8], [0.1, 0.2]) & ([0.5, 0.8], [0.1, 0.2]) & ([0.4, 0.6], [0.2, 0.3]) & ([0.5, 0.6], [0.2, 0.3]) \\
([0.3, 0.5], [0.4, 0.5]) & ([0.1, 0.2], [0.7, 0.8]) & ([0.1, 0.2], [0.5, 0.8]) & ([0.2, 0.3], [0.4, 0.6]) & ([0.2, 0.3], [0.5, 0.6])
\end{pmatrix}, \\
D_2 = \begin{pmatrix}
([0.4, 0.5], [0.2, 0.4]) & ([0.3, 0.4], [0.4, 0.6]) & ([0.6, 0.7], [0.1, 0.2]) & ([0.5, 0.6], [0.1, 0.3]) & ([0.1, 0.3], [0.3, 0.5]) \\
([0.3, 0.5], [0.4, 0.5]) & ([0.1, 0.3], [0.3, 0.7]) & ([0.3, 0.4], [0.4, 0.5]) & ([0.2, 0.3], [0.6, 0.7]) & ([0.6, 0.8], [0.1, 0.2]) \\
([0.4, 0.6], [0.3, 0.4]) & ([0.6, 0.8], [0.1, 0.2]) & ([0.7, 0.8], [0.1, 0.2]) & ([0.4, 0.6], [0.3, 0.4]) & ([0.5, 0.6], [0.2, 0.4]) \\
([0.3, 0.4], [0.4, 0.6]) & ([0.1, 0.2], [0.6, 0.8]) & ([0.1, 0.2], [0.7, 0.8]) & ([0.3, 0.4], [0.4, 0.6]) & ([0.2, 0.4], [0.5, 0.6])
\end{pmatrix}, \\
D_3 = \begin{pmatrix}
([0.4, 0.7], [0.1, 0.2]) & ([0.3, 0.5], [0.3, 0.4]) & ([0.6, 0.7], [0.1, 0.2]) & ([0.5, 0.6], [0.1, 0.3]) & ([0.3, 0.5], [0.4, 0.5]) \\
([0.4, 0.5], [0.2, 0.4]) & ([0.2, 0.4], [0.4, 0.5]) & ([0.4, 0.5], [0.3, 0.4]) & ([0.1, 0.2], [0.7, 0.8]) & ([0.6, 0.7], [0.2, 0.3]) \\
([0.2, 0.4], [0.3, 0.4]) & ([0.6, 0.8], [0.1, 0.2]) & ([0.5, 0.7], [0.1, 0.3]) & ([0.5, 0.7], [0.2, 0.3]) & ([0.6, 0.8], [0.1, 0.2]) \\
([0.3, 0.4], [0.2, 0.4]) & ([0.1, 0.2], [0.6, 0.8]) & ([0.1, 0.3], [0.5, 0.7]) & ([0.2, 0.3], [0.5, 0.7]) & ([0.1, 0.2], [0.6, 0.8])
\end{pmatrix}, \\
D_4 = \begin{pmatrix}
([0.6, 0.7], [0.2, 0.3]) & ([0.3, 0.4], [0.3, 0.4]) & ([0.7, 0.8], [0.1, 0.2]) & ([0.5, 0.6], [0.1, 0.3]) & ([0.1, 0.2], [0.5, 0.7]) \\
([0.4, 0.5], [0.4, 0.5]) & ([0.1, 0.2], [0.2, 0.3]) & ([0.3, 0.4], [0.5, 0.6]) & ([0.2, 0.3], [0.4, 0.6]) & ([0.6, 0.7], [0.1, 0.2]) \\
([0.4, 0.5], [0.3, 0.4]) & ([0.6, 0.7], [0.1, 0.3]) & ([0.5, 0.8], [0.1, 0.2]) & ([0.4, 0.5], [0.2, 0.3]) & ([0.5, 0.6], [0.3, 0.4]) \\
([0.3, 0.4], [0.4, 0.5]) & ([0.1, 0.3], [0.6, 0.7]) & ([0.1, 0.2], [0.5, 0.8]) & ([0.2, 0.3], [0.4, 0.5]) & ([0.3, 0.4], [0.5, 0.6])
\end{pmatrix}.
\end{gathered} \tag{20}
$$

In what follows, we utilize the proposed method to solve this problem.

Step 1. Using (7), we get

$$\omega = \begin{bmatrix} 0.2523 & 0.2503 & 0.2478 & 0.2496 \end{bmatrix}. \tag{21}$$

Step 2. On the basis of the known ω and four decision matrices D_i $(i = 1, 2, 3, 4)$, we get the aggregated decision making matrix D through IVIFWA operator as follows:

$$
D = \begin{pmatrix}
 & x_1 & x_2 & x_3 & x_4 & x_5 \\
A_1 & ([0.4821, 0.6334], [0.1684, 0.2916]) & ([0.3000, 0.4523], [0.3467, 0.4684]) & ([0.6277, 0.7289], [0.1191, 0.2215]) & ([0.5000, 0.6280], [0.1000, 0.2708]) & ([0.1543, 0.3596], [0.3660, 0.5438]) \\
A_2 & ([0.3517, 0.4765], [0.3369, 0.4954]) & ([0.1259, 0.3034], [0.2628, 0.4526]) & ([0.3262, 0.4265], [0.3938, 0.4951]) & ([0.1763, 0.3041], [0.5380, 0.6697]) & ([0.6280, 0.7553], [0.1187, 0.2211]) \\
A_3 & ([0.3557, 0.5053], [0.3000, 0.4232]) & ([0.6280, 0.7787], [0.1000, 0.2213]) & ([0.5600, 0.7789], [0.1000, 0.2211]) & ([0.4265, 0.6062], [0.2214, 0.3224]) & ([0.5269, 0.6631], [0.1864, 0.3133]) \\
A_4 & ([0.3000, 0.4270], [0.3369, 0.4952]) & ([0.1000, 0.2262], [0.6238, 0.7738]) & ([0.1000, 0.2260], [0.5439, 0.7740]) & ([0.2263, 0.3265], [0.4227, 0.5956]) & ([0.2033, 0.3301], [0.5231, 0.6443])
\end{pmatrix}. \tag{22}
$$

Step 3. According to the decision making matrix D, we get

$$E = \begin{pmatrix} 0.8434 & 0.6081 & 0.9339 & 0.8417 & 0.4008 \\ 0.6517 & 0.3362 & 0.6054 & 0.3496 & 0.9358 \\ 0.6917 & 0.9342 & 0.9048 & 0.8008 & 0.8712 \\ 0.5862 & 0.2174 & 0.2216 & 0.4315 & 0.3944 \end{pmatrix},$$
$$W = \begin{pmatrix} 0.2000 & 0.2000 & 0.1000 & 0.3000 \\ 0.1000 & 0.1000 & 0.3000 & 0.1000 \\ 0.4000 & 0.2000 & 0.3000 & 0.2000 \\ 0.2000 & 0.2000 & 0.2000 & 0.3000 \\ 0.1000 & 0.3000 & 0.1000 & 0.1000 \end{pmatrix}. \quad (23)$$

Moreover, we get

$$\Gamma = \begin{pmatrix} 1.8150 & 1.7727 & 1.7459 & 1.7819 \\ 1.7727 & 1.7474 & 1.7042 & 1.7423 \\ 1.7459 & 1.7042 & 1.6854 & 1.7120 \\ 1.7819 & 1.7423 & 1.7120 & 1.7539 \end{pmatrix}. \quad (24)$$

From Γ, we have $\rho = [0.0002 \ \ 0.0009 \ \ 0.0018 \ \ 0.9971]^T$. Finally, we get

$$w = [0.2995 \ \ 0.1004 \ \ 0.2002 \ \ 0.2997 \ \ 0.1002]^T. \quad (25)$$

Step 4. Based on the matrix E and the attribute weight vector w, we get

$$c(\overline{A}_1, A^+) = 0.7931,$$
$$c(\overline{A}_2, A^+) = 0.5487,$$
$$c(\overline{A}_3, A^+) = 0.8094,$$
$$c(\overline{A}_4, A^+) = 0.4106. \quad (26)$$

Since $c(\overline{A}_3, A^+) > c(\overline{A}_1, A^+) > c(\overline{A}_2, A^+) > c(\overline{A}_4, A^+)$, the ranking order of four alternatives is $A_3 \succ A_1 \succ A_2 \succ A_4$ and the most desirable one is A_3.

By applying the methods from [18, 20, 29, 30, 35–37] to solve the above MAGDM problem, the decision results are shown as Table 1. (Note that determining the weights of both DMs and attributes has been partially or not been considered in [18, 30, 36, 37]; we employ the weights derived from our proposed method to these models for the above decision problem.)

Remark 11. As shown in Table 1, seven methods get the same decision results except for [18]. The reason is that this method utilizes a different aggregation operator which plays an important role in the process of decision making. What is more, the validity and feasibility of our proposed method have been verified by comparison with seven existing models.

Table 1: Decision results with different models.

Models	Decision results
[18]	$A_1 \succ A_3 \succ A_2 \succ A_4$
[20]	$A_3 \succ A_1 \succ A_2 \succ A_4$
[29]	$A_3 \succ A_1 \succ A_2 \succ A_4$
[30]	$A_3 \succ A_1 \succ A_2 \succ A_4$
[35]	$A_3 \succ A_1 \succ A_2 \succ A_4$
[36]	$A_3 \succ A_1 \succ A_2 \succ A_4$
[37]	$A_3 \succ A_1 \succ A_2 \succ A_4$
Our proposed method	$A_3 \succ A_1 \succ A_2 \succ A_4$

5. Conclusions and Discussions

In this paper, we have proposed a method to solve the MAGDM problem in which all the information supplied by the decision makers is expressed as interval-valued intuitionistic fuzzy decision matrices where each of the elements is characterized by an interval-valued intuitionistic fuzzy number, and the information about the weights of both decision makers and attributes may be completely unknown or partially known. The main merits of this method cover three aspects. Firstly, the problem of determining the weights of DMs and attributes has been solved by the proposed consensus-based method and the proposed multiobjective model, respectively. Secondly, a complete mathematical formulation of MAGDM has been established, and its advantages have been proved by two examples. In addition, we have defined the correlation coefficient between two interval-valued intuitionistic fuzzy matrices which develops basic theories on IVIFSs.

It should be noted that we just consider the situation where the information about DMs is completely unknown. In the future, we will consider the situations where the weights information about both DMs and attributes is expressed with various constraint conditions. Meanwhile, we will employ the proposed method to model some uncertain decision making problems from some concrete applied fields, such as medical decision making, social economic, and financial assessment.

Data Availability

All data generated or analyzed during this study are included in this published article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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Research Article

Research on Multiobjective Topology Optimization of Diesel Engine Cylinder Block Based on Analytic Hierarchy Process

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There are alternating impact loads for the diesel engine cylinder block. The topology optimization of the extreme single-working condition cannot guarantee its overall mechanical performance, and the traditional multiworking condition optimization has the problem that the weight coefficients are difficult to determine. Thus, a multiobjective topology optimization method based on analytic hierarchy process is proposed. Firstly, the static, dynamic characteristics and structure efficiency are calculated by the finite element analysis which indicates the direction of topology optimization for the cylinder block. The hierarchical structure model of topology optimization, including 12 weighting coefficients, is constructed considering static multiworking condition stiffness and dynamic multiorder natural frequency. The comprehensive evaluation function for the cylinder block is established by the compromise programming method and the weight coefficients are determined based on analytic hierarchy process. The optimization mathematical model is established and the multiobjective topology optimization of the cylinder block is carried out. The optimization results show that the proposed method can take into account structural multiworking condition performance, which has obvious advantages over the single objective topology optimization. The simulation results show that the static and dynamic characteristics are improved to some extent and the overall mechanical performance of the new model is more uniform with a 5.22% reduction in weight. It shows that the topology structure of the cylinder block is more reasonable.

1. Introduction

With the rapid and sustained development of automobile manufacturing industry all over the world, automobile ownership has increased greatly and the energy and environment issues are becoming more and more prominent. The energy conservation and emission reduction have become an inevitable trend in the development of automobile industry. The diesel engine, as one of the core components in engineering vehicles, is developing towards high-power-density, high-speed, and lightweight [1]. The cylinder block is the main structure and the heaviest part of the diesel engine; it must have sufficient stiffness and strength to support a variety of loads. At present, the design and optimization for the cylinder block mainly adopt traditional method combining finite element analysis (FEA) with engineering experience to check its strength and stiffness [2, 3]. The method is heavy and cumbersome, and it is difficult to effectively play structural bearing capacity [4].

The topology optimization method can provide lightweight and efficient structure form in the conceptual design stage, which has been widely concerned [5–7]. The single objective topology optimization for a V-type twelve-cylinder diesel cylinder block is carried out in [8] and the structural performance is improved. Jia et al. [9] get the optimal topology structure of a single cylinder block in the extreme working condition by using the topology and shape optimization. To achieve a low vibration design for a four-cylinder block, Du et al. [10] obtained the layout of the inner ribs by the topology optimization. Thus, the application of topology optimization for the cylinder block has made some progress and the research mainly focuses on the extreme working condition [11, 12]. However, there are alternating impact loads in the working process

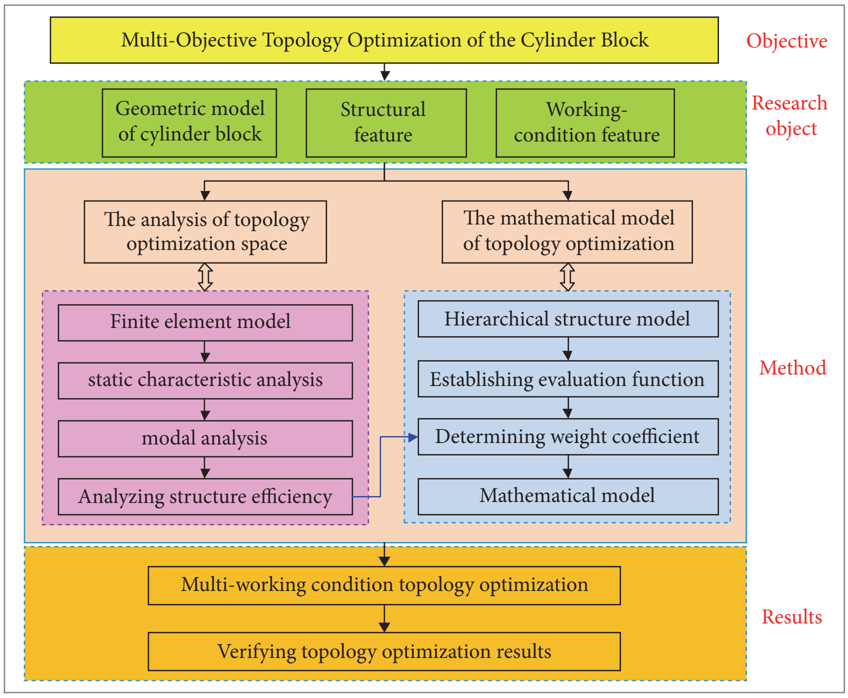


FIGURE 1: The block diagram of multiobjective topology optimization.

of diesel engine. If the explosion of each cylinder for a multicylinder block is regarded as an extreme condition, the topology optimization of the cylinder block belongs to the typical multiworking condition problem. The traditional single objective optimization usually only ensures that the mechanical properties are optimal in a certain working condition while the overall mechanical property may be reduced to a lower value in other working conditions; that is to say, the topology optimization result for the cylinder block will oscillate between different working conditions and the overall mechanical property cannot be guaranteed. In addition, the dynamic characteristics of the cylinder block also need to be considered in the process of optimization.

The multiobjective topology optimization can consider simultaneously several objective functions in the design process [13–15] and the optimal solution can be obtained for each objective function. The intelligent algorithms are used to solve directly to avoid decision of multiobjective weight coefficients [16–18]. However, the calculation for complex structures will cost a lot of time and high economic costs because of numerical instability during the process of topology optimization [19, 20]. Therefore, it is necessary to establish a comprehensive evaluation function to consider several objectives as a whole. But if the weight coefficient of each working condition is decided by the engineering experience, the function will not reflect the overall structural performance in optimization. So the method of determining weight coefficients is the key of the multiobjective topology optimization and whose essence is the multicriterion decision-making problem.

The analytic hierarchy process (AHP) proposed by Saaty [21, 22] is a systematic analysis method for determining qualitatively and quantitatively the relative importance of a set of activities in a multicriteria decision-making problem. The method can effectively analyze the nonsequential relationship between multiobjective criterion systems by combining mathematical processing with subjective judgment, which has been widely used in the field of resource system analysis, economic management, education management, social science, and so on [23, 24]. The AHP is applied to determinate the weight coefficients of the external economic evaluation model to ensure that the wind power engineering project is constructed and developed in a scientific manner [25]. A multiobjective evolutionary structure optimization method is proposed by combining the AHP and evolutionary structural optimization, which improves the optimization effect [26]. Therefore, it has obvious advantages to bring AHP into the decision of weight coefficients for the multiobjective topology optimization.

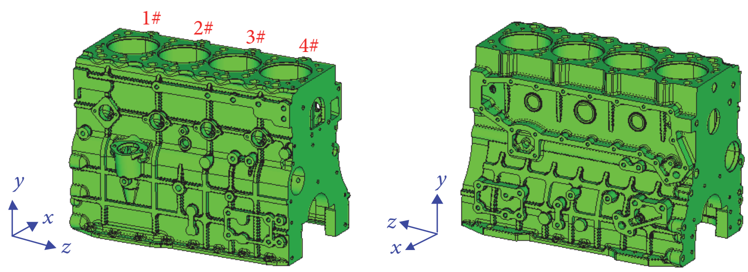
Under the above background, this paper presents a multiobjective topology optimization method based on AHP which is applied to a certain four-cylinder diesel engine cylinder block.

2. Multiobjective Topology Optimization Method

The multiobjective topology optimization method of diesel engine cylinder block based on AHP in this paper is mainly divided into four steps, as shown in Figure 1. The first is to introduce the structural geometry characteristic and working condition of the cylinder block in Section 3. Secondly, the topology optimization space is determined on

Table 1: The mechanical property of HT300.

Material	Elastic modulus/ GPa	Poisson's ratio	Density/ kg•m^{-3}	Tensile strength/ MPa
HT300	143	0.27	7300	300



(a) Front (b) Back

Figure 2: The cylinder block structure.

the basis of analyzing the static characteristics, vibration mode, and structure efficiency of each working condition in Section 4. Then in Section 5, the hierarchical structure model of topology optimization is constructed considering the static multiworking condition stiffness and dynamic multiorder natural frequency. The comprehensive evaluation function is established by the compromise programming method which can more accurately evaluate the structural overall performance. The weight coefficients are determined by AHP and the mathematical model is established. Finally, multiobjective topology optimization of cylinder block is carried out and the optimization effect is verified in Section 6.

3. Structure Analysis

3.1. Structure Feature. The four-cylinder diesel engine cylinder block, as shown in Figure 2, is a box-type structure obtained by casting and machining and widely used in the heavy engineering vehicle. In order to achieve the lightweight, the topology structure of the cylinder block has been modified many times through finite element analysis and manual experience, but the structure is still too cumbersome and unsatisfactory. Its dimensions are 526.7mm long, 326.1mm wide, and 387.8mm high with a weight of 88.97 kg. The material is gray cast iron HT300 and the mechanical property is shown in Table 1.

As the main structure of the diesel engine, it is covered with various stiffening ribs, convex plates, bearing holes, oil channel holes, water-cooled jacket, and so on. So its mechanical property is directly related to the working efficiency of the diesel engine and it has to possess sufficient strength and stiffness to support a variety of loads.

3.2. Working Condition. For the diesel engine cylinder block, its working condition is a cyclic process including four processes of intake, compression, power, and exhaust. The firing order of cylinder block is 1-3-4-2 and the rotation speed of crank is 3000rpm. Therefore, there are alternating and high-speed impact loads for the cylinder block, and the loads are very complicated, including the explosion pressure, the wall pressure from crank-link mechanism, the bolt pretightening force between cylinder block and cylinder head, the reaction force of bearing block and thermal load, etc. It is considered that the heat generated at the moment of gas explosion is first transmitted to the cylinder liner, and then to the cylinder wall, the cylinder liner and the water-cooled jacket bear a large amount of heat during the heat transfer process. In order to simplify calculation in this paper, the thermal load on the cylinder wall is ignored. So, the main loads considered are shown in Figure 3.

And the freedom constraints are applied to the six contact faces (*a-f*) at the bottom of the cylinder block as shown in Table 2, where T_x, T_y, and T_z mean that the displacements of x, y, and z direction are limited, R_x, R_y, and R_z mean that the rotation angles of x, y, and z direction are limited. According to the basic parameters of the cylinder block, the corresponding extreme load values at the moment of each cylinder explosion are calculated, as shown in Table 3. The bolt pretightening force is different in different position of the bolt hole and the number of bolt holes is a great many; only the maximum bolt pretightening force is listed.

4. Optimization Space Analysis

4.1. Static Characteristics Analysis. In order to obtain the topology optimization space, the static and dynamic characteristics of the cylinder block during the working process are obtained by the FEA. The first is to carry out the static characteristics analysis at the moment of each cylinder explosion.

The geometry model is imported into the finite element software, and the bolt hole, chamfer, and oil pipeline are simplified. According to the working condition of the cylinder block in Section 3.2, the finite element model, consisting of the tetrahedral and hexahedral mixing elements, is established as shown in Figure 4. The displacement and stress distribution are calculated and the results are shown in Table 4. It can be seen that the first working condition is

TABLE 2: The constraints of the cylinder block.

Constraint face	*a*	*b*	*c*	*d*	*e*	*f*
Displacement freedoms	T_y	T_z	T_y	T_x / T_y	T_z	T_x / T_y
Rotation freedoms				R_x / R_y / R_z		

TABLE 3: The extreme load value of each cylinder at the time of explosion.

The extreme Loads	No. of explosion			
	1	2	3	4
Bolt pre-tightening force /N	69007	67586	69474	75693
Reaction force of bearing block/N	63750	63336	47300	63752
The wall pressure /N	17695	17695	17695	17695
The explosion pressure /MPa	17	17	17	17

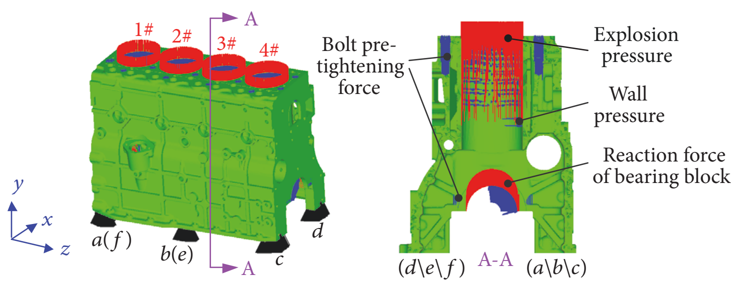


FIGURE 3: The loads and boundary conditions of cylinder block.

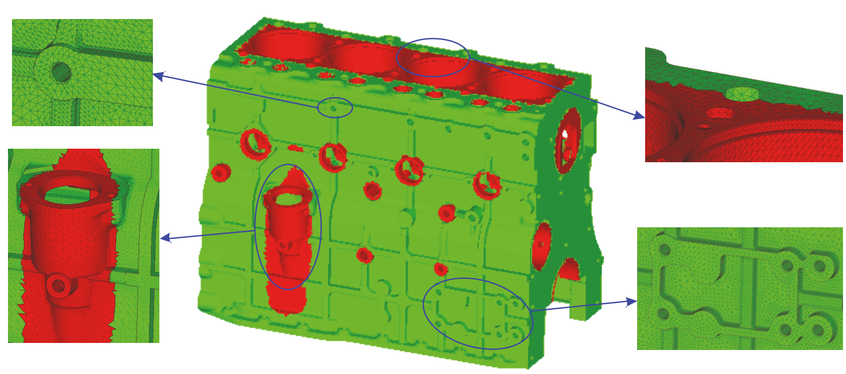


FIGURE 4: The finite element model.

the worst and corresponding displacement and stress distribution cloud charts are shown in Figure 5. The maximum stress is 217.9MPa located at the bolt hole while most of the rest region is about 80 MPa, which is much smaller than the material ultimate strength (300 MPa). It indicates that the cylinder block has optimization space in the worst condition.

4.2. Modal Analysis. The static analysis can only reflect structural stiffness and strength and cannot reflect its vibration performance. Modal analysis is the basis for the dynamic design, analysis, and optimization in modern mechanical products. The structural natural frequencies and vibration modes can be obtained by the modal analysis to evaluate its vibration characteristics.

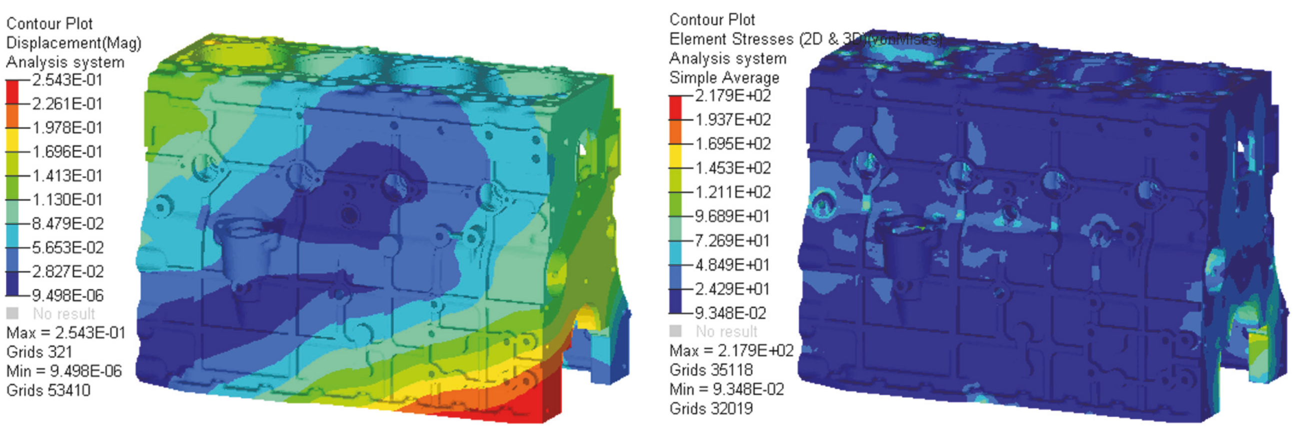
TABLE 4: Results of FEA for the cylinder block.

No. of explosions	Max stress(MPa)	Max displacement (mm)
1	217.9	0.254
2	168.8	0.223
3	168.4	0.214
4	199.2	0.246

The constrained modal of the cylinder block is analyzed by the FEA and the top 6-order natural frequencies and corresponding vibration modes are shown in Figure 6 and

Table 5: The top 6-order natural frequencies and vibration modes.

Orders	Frequency	Vibration mode
1	264Hz	First-order torsional vibration around the X axis
2	493 Hz	First-order bending vibration around the Z axis
3	531 Hz	Second-order torsional vibration around the X axis
4	562 Hz	The skirt vibrates with torsion along the X direction
5	778 Hz	Whole bending torsional vibration
6	1038 Hz	Whole torsional vibrating around the X axis



(a) Displacement distribution (b) Stress distribution

Figure 5: Results of FEA under extreme working condition.

Table 5. It can be seen from Figure 6 that the cylinder block firstly appears whole torsional vibration while the whole bending vibration appears in the higher frequency range, which show that the torsional stiffness is less than the bending stiffness for the cylinder block. In addition, the relative displacement near the four corners is large and it is necessary to improve the freedom constraints to lower the extent of the vibration.

To further evaluate its dynamic performance, the working frequency is calculated by (1). The cylinder block studied in this paper is a four-stroke reciprocating piston engine, the crankshaft turns twice, and the cylinder body completes a working cycle, including four times vibration of intake, compression, power, and exhaust. So the corresponding working frequency f is 100Hz calculated, which is much smaller than the first-order natural frequency for the cylinder block. It indicates that the resonance will not occur in working.

$$f = \frac{2 \cdot n}{60} \tag{1}$$

where n is the rotation speed of crank, n=3000rpm.

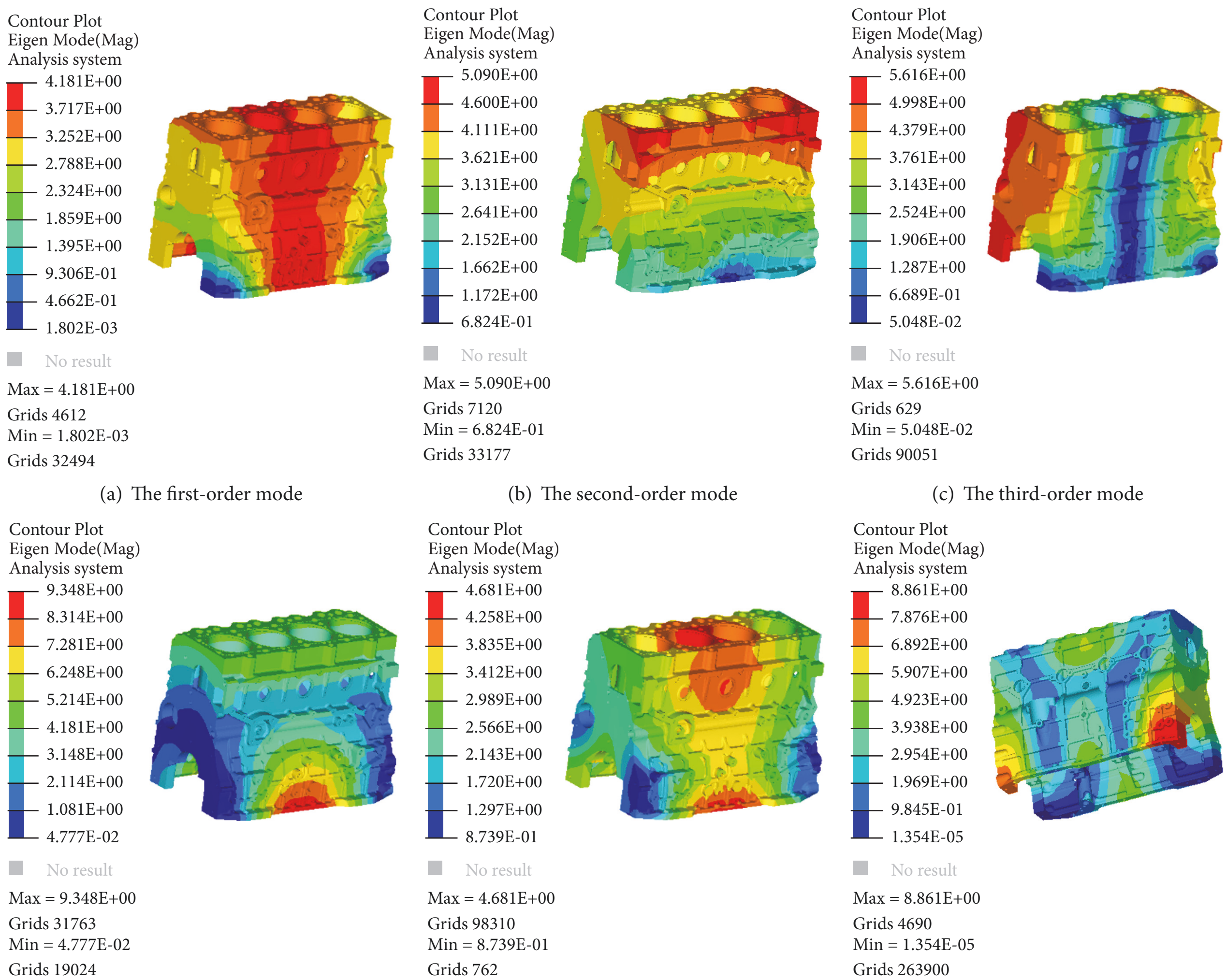
4.3. Structure Efficiency Analysis. Structure efficiency [27] refers to the structural comprehensive characterization of the strength and stiffness per unit weight in the case of meeting the load-bearing property. It is commonly used to evaluate the structural overall performance. The greater structural efficiency, the higher the material utilization, while the smaller the structural efficiency, the larger the optimization space.

In this paper, the structure efficiency of the cylinder block is calculated under four extreme working conditions. The calculation formula of the structure efficiency index η_i is shown in (2). In terms of the multiworking condition topology optimization, its physical meaning is as follows: the value is greater, indicating that the material utilization is higher and the working condition is worse. On the contrary, it shows that the working condition is safer and the optimization space is larger.

$$\eta_i = \frac{\sigma_{i\max} \cdot d_{i\max}}{m} \times 100\% \tag{2}$$

where η_i is the structure efficiency index under the ith working condition, $\sigma_{i\max}$ and $d_{i\max}$ are the maximum stress and maximum displacement under the ith working condition, and m is the structural weight.

Substituting the analysis results of Table 4 into (2), the structure efficiency of the cylinder block is calculated as shown in Figure 7. It can be seen that the cylinder block has the highest structure efficiency at the moment of the first cylinder explosion, and followed by the fourth cylinder, the second cylinder and the third cylinder. It shows that the first cylinder explosion is the worst working condition, and the third cylinder explosion is the safest condition.



(a) The first-order mode (b) The second-order mode (c) The third-order mode

(d) The fourth-order mode (e) The fifth-order mode (f) The sixth-order mode

Figure 6: Results of modal analysis.

Figure 7: The structure efficiency of each cylinder.

Based on the analysis mentioned in Figures 5–7, the cylinder block studied in this paper can meet the stiffness and strength requirements under the worst working condition. The overall stress value (80 MPa) is much lower than material ultimate strength (300 MPa), which indicates that the cylinder block has surplus material and topology optimization space. Its working frequency (100 Hz) is much lower than the first-order natural frequency (264 Hz), and the resonance does not occur. In addition, the importance for four working conditions is sorted: the first cylinder, the fourth cylinder, the second cylinder, and the third cylinder. Therefore, the first cylinder and the fourth cylinder should be focused when determining the weighting coefficients in multiobjective topology optimization. And the material near the second cylinder and the third cylinder should be considered when improving the topology structure.

5. Topology Optimization Mathematical Model Based on AHP

5.1. The Hierarchical Structure Model. The topology optimization for the diesel engine cylinder block belongs to the typical multiworking condition problem. It is necessary to

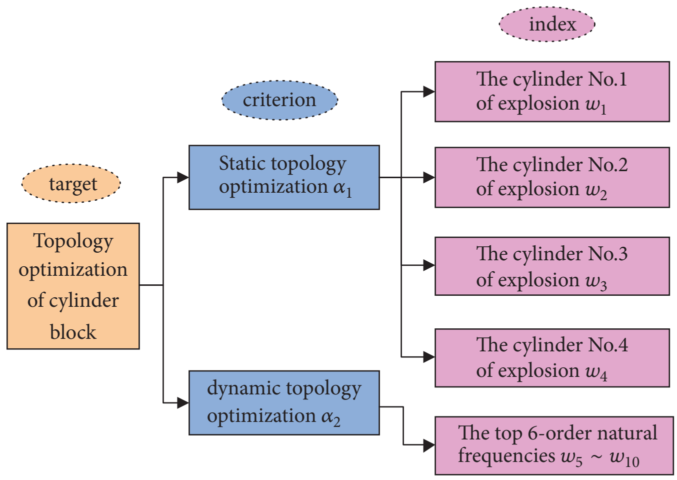


Figure 8: The hierarchical structure model of topology optimization.

take into account the structural performance requirements, including static and dynamic characteristics. For the static characteristics, structural stiffness has to be considered at the moment of each cylinder explosion. And the top 6-order natural frequencies need to be concerned for dynamic characteristics. Therefore, the hierarchical structure model of topology optimization for the cylinder block is established based on static multiworking condition stiffness and dynamic multiorder natural frequency as shown in Figure 8. It can be seen from the figure that the multiobjective topology optimization of the cylinder block includes 12 weighting coefficients that are static and dynamic topology optimization α_1, α_2 in the criterion layer, static multiworking condition stiffness $w_1 \sim w_4$, and dynamic multiorder natural frequency $w_5 \sim w_{10}$.

5.2. Comprehensive Evaluation of the Cylinder Block. The linear weighting method is usually used to transform the multiobjective problem into a single-objective problem for the traditional multiobjective topology optimization. However, the linear weighting method is to calculate weight average value for all functions and it cannot reflect the prominent influence from some certain functions, which does not guarantee that all functions obtain the relative optimal solution. The compromise programming method [28] can get a group of better relative optimal solutions by calculating the sensitivity of all functions to design variables and adjusting each objective to balance each other. From the hierarchical structure model shown in Figure 8, the topology optimization for the cylinder block includes ten optimization objectives, and the static and dynamic multiobjective optimization problem is converted into the single-objective optimization problem by the compromise programming method.

5.2.1. Static Multiworking Condition Stiffness. The topology optimization oriented by stiffness maximization is to research material distribution form in the design domain to maximize the structural stiffness. In this paper, the static stiffness of the cylinder block under four extreme conditions is studied, which belongs to the multiworking condition stiffness problem. In this paper, the objective function of static multiworking condition stiffness is obtained by the compromise programming method as shown in (3). $C(\rho)$ is the comprehensive evaluation value of the static stiffness, and the smaller the value, the larger the structural overall stiffness.

$$\min_{\rho} C(\rho) = \left\{ \sum_{i=1}^{m} w_i^q \left[\frac{C_i(\rho) - C_i^{\min}}{C_i^{\max} - C_i^{\min}} \right]^q \right\}^{1/q} \quad (3)$$

where ρ is the relative density in the variable density topology optimization and m is the total number of working conditions, m=4. w_i is the weight coefficient of the ith working condition while q is the penalty coefficient (q≥2). $C_i(\rho)$ is the structural compliance of the ith working condition. $C_i^{\max}$ and $C_i^{\min}$ are the maximum and minimum compliance of the ith working condition, respectively.

5.2.2. Dynamic Multiorder Natural Frequency. The topology optimization of dynamic multiorder natural frequency is usually targeted at maximizing the low-order natural frequency, and the material remove ratio is taken as boundary. However, if only one low-order natural frequency is used as the optimization objective, the eigenvalues of other adjacent higher order natural frequency may be reduced because of the gradual material remove in the structure. It will result in the interchange of the low-order natural frequencies and the convergence of topology optimization will be influenced. The average frequency method [29] can consider simultaneously the multiorder natural frequency by defining a smooth objective function and improve the convergence, which is widely used in dynamic topology optimization. In this paper, the objective function of dynamic multiorder natural frequency is defined by the average frequency method as shown in (4). $\Lambda(\rho)$ is the comprehensive evaluation value of the top few order natural frequency and the larger the value, the larger the top few order natural frequency.

$$\max \Lambda(\rho) = \lambda_0 + s\left(\sum_{j=1}^{n} \frac{w_j}{\lambda_j - \lambda_0}\right)^{-1} \quad (4)$$

where ρ is the relative density in the variable density topology optimization. λ_j is the jth order natural frequency. λ_0 and s as given parameters are used to adjust the function value, usually λ_0=0, s=1. w_j is the weight coefficient of the jth order natural frequency while n is the order of low-order natural frequency that need to be optimized, n=6.

In addition, the low-order natural frequency is usually paid to attention during the optimization process and the lower the order, the higher the degree of attention. According to this principle, aiming at reducing the complexity of the weighting coefficients determined by the analytic hierarchy process, the weight coefficients $w_5 \sim w_{10}$ of the top 6 natural frequencies are taken as 0.3, 0.2, 0.2, 0.1, 0.1, and 0.1, respectively. So, the 12 unknown weighting coefficients in the hierarchical structure model are reduced to six.

5.2.3. Comprehensive Evaluation Function. The comprehensive evaluation function of multiobjective topology optimization, considering both the static multiworking condition stiffness and the dynamic multiorder natural frequency, is established by the compromise programming method as shown in (5). By adjusting the position of $C_i(\rho)$ and Λ_ρ in the function, the comprehensive evaluation function can uniformly guide the convergence direction of the optimization. And the smaller the value, the better the overall performance of the cylinder block.

$$\min F(\rho) = \left\{\alpha_1^2\left[\sum_{i=1}^{m} w_i \frac{C_i(\rho) - C_i^{\min}}{C_i^{\max} - C_i^{\min}}\right]^2 + \alpha_2^2\left[\frac{\Lambda^{\max} - \Lambda(\rho)}{\Lambda^{\max} - \Lambda^{\min}}\right]^2\right\}^{1/2} \quad (5)$$

where $F(\rho)$ is the objective function value and $\Lambda^{\min}$ and $\Lambda^{\max}$ represent minimum and maximum natural frequencies, respectively. Other variables have the same meaning as (3) and (4).

5.3. The Weighting Coefficients. The comprehensive evaluation function of multiobjective topology optimization, shown in (5), has six unknown weighting coefficients including α_1, α_2, $w_1 \sim w_4$. These unknown weighting coefficients are calculated based on the analytic hierarchy process in this paper. The concrete calculating flow chart is shown in Figure 9. The subjective judgment is scaled based on the measure theory and the judgment matrix is established.

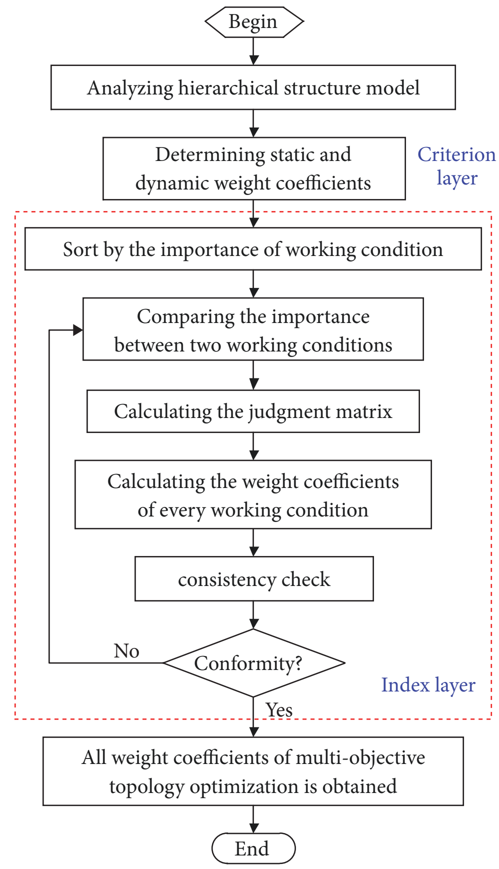


FIGURE 9: The calculating flow chart of the weight coefficients.

Then the all weighting coefficients are calculated through the consistency check.

5.3.1. Criteria Layer Decision. There are static stiffness topology optimization and dynamic natural frequency topology optimization in the criterion layer, and the corresponding weighting factors are, respectively, α_1, α_2. The cylinder block suffers from the alternating impact loads when different cylinder explodes and its stiffness performance directly affects the working reliability. But for the vibration characteristics, it can be seen from Section 4.2 that the maximum working frequency is 100 Hz, which is much smaller than the first-order natural frequency of 264 Hz. Therefore, the static multiworking condition stiffness is more important in the topology optimization for the cylinder block. So the weight coefficients α_1, α_2 are defined as 0.6 and 0.4, respectively.

5.3.2. Index Layer Decision. Firstly, it is necessary to determine the importance of four working conditions. According to the structure efficiency shown in Figure 7, the importance is sorted: the first cylinder, the fourth cylinder, the second cylinder, and the third cylinder. So the weight coefficients are ranked as shown in

$$w_1 > w_4 > w_2 > w_3 \quad (6)$$

Table 6: Meanings of relative scale.

Relative scale	Meanings
1	Two elements have equal importance
3	The former is slightly important than the latter between two elements
5	The former is obviously important than the latter between two elements
7	The former is strongly important than the latter between two elements
9	The former is extremely important than the latter between two elements
2, 4, 6, 8	Indicating the intermediate value above judgment
Reciprocal	If the important ratio between the elements i and j is x, the important ratio between the elements j and i is $1/x$.

Then, according to the standard meaning table of relative scale in the AHP shown in Table 6, the relative importance ratio of four working conditions is determined and the judgment matrix W is constructed as shown in

$$W = \begin{bmatrix} \frac{w_1}{w_1} & \cdots & \frac{w_1}{w_i} & \cdots & \frac{w_1}{w_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{w_j}{w_1} & \cdots & \frac{w_j}{w_i} & \cdots & \frac{w_j}{w_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{w_n}{w_1} & \cdots & \frac{w_n}{w_i} & \cdots & \frac{w_n}{w_n} \end{bmatrix} = \begin{bmatrix} w_{11} & \cdots & w_{1i} & \cdots & w_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ w_{j1} & \cdots & w_{ji} & \cdots & w_{jn} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ w_{n1} & \cdots & w_{ni} & \cdots & w_{nn} \end{bmatrix} \quad (7)$$

where n is the number of the weight coefficients, w_i and w_j $(i, j = 1, 2, \ldots, 6)$ represent the weight coefficients and $w_{ji} = w_j/w_i$ denotes the relative importance of w_j to w_i.

According to the results in Section 4, the first cylinder explosion is the worst condition and it is obviously more important than the third working condition and slightly more important than the fourth working condition, so the weight coefficients w_{13}, w_{14} are determined as 5 and 2, respectively. The importance of the second working condition is between the third working condition and the fourth working condition, so the weight coefficient w_{12} is defined as 4. In the same way, the relative importance ratio of the four working conditions is obtained and the judgment matrix is constructed:

$$W = \begin{bmatrix} 1 & 4 & 5 & 2 \\ \frac{1}{4} & 1 & 2 & \frac{1}{3} \\ \frac{1}{5} & \frac{1}{2} & 1 & \frac{1}{4} \\ \frac{1}{2} & 3 & 4 & 1 \end{bmatrix}. \quad (8)$$

The judgment matrix W is right multiplied by a vector $\boldsymbol{\omega} = (w_1, w_2, w_3, w_4)^{\mathrm{T}}$ consisting of all the weight coefficients, as shown in

$$\begin{gathered} W\boldsymbol{\omega} = \lambda\boldsymbol{\omega} \Longrightarrow \\ (W - \lambda I)\,\boldsymbol{\omega} = 0 \end{gathered} \quad (9)$$

Substituting the judgment matrix W into (9), the maximum eigenvalue $\lambda_{\max} = 4.0484$ is calculated and the corresponding eigenvector normalized is $\boldsymbol{\omega} = (0.49, 0.12, 0.08, 0.31)^{\mathrm{T}}$. So, all weight coefficients are obtained for multiobjective topology optimization of the cylinder block.

In order to ensure the accuracy and reliability of the judgment matrix and avoid influence of individual subjective factor, the consistency test of the judgment matrix is carried out in terms of (10). The consistency ratio $C.R.$ of the judgment matrix W, calculated by (10), is 0.0179, which is less than 0.1. Therefore, it is considered that the judgment matrix has a satisfactory consistency and the four weight coefficients can well reflect the importance of each working condition.

$$C.R. = \frac{C.I.}{R.I.} \quad (10)$$

where $C.I.$ is the consistency index, $C.I. = (\lambda_{\max} - n)/(n-1)$. $R.I.$ is the mean random consistency index, whose value can be obtained directly by referring to the standard random consistency index $R.I.$-n table in the analytic hierarchy process, as shown in Table 7. $C.R.$ is the random consistency ratio and the inconsistency is acceptable when $C.R. < 0.1$.

In addition, the computing platform of weight coefficient for multiworking condition topology optimization (TOWC) is built in *Matlab* to improve the computational efficiency of the method as shown in Figure 10. According to the number of working conditions and the importance of each working

Table 7: The standard random consistency index *R.I.*-*n*.

n	1	2	3	4	5	6	7	8	9	10
R.I.	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

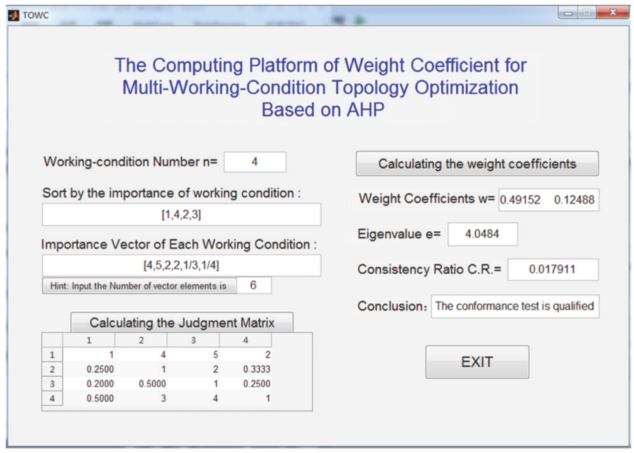


Figure 10: The computing platform of weight coefficient.

condition, the platform can automatically construct the judgment matrix, output the weight coefficients, and verify its consistency. Taking the cylinder block as an example, the operation steps are as follows.

Step 1. Enter the number of working conditions n=4.

Step 2. Rank the importance of each working condition [1, 4, 2, 3].

Step 3. Refer to Table 6, and enter the relative importance between two working conditions expressed in vector form. Before entering the vector, you can click the prompt button to get the number of elements you need to input. The elements in the vector are expressed in sequence as the importance of the first working condition to other working conditions, and the importance of the second working condition to other working conditions and so on. In this paper, six elements need to be input for four working conditions of cylinder block. Based on above analysis, the corresponding vector is [4, 5, 2, 2, 1/3, 1/4].

Step 4. Click the control button "*calculating the Judgment Matrix*" and the button "*calculating the Weight Coefficients*" in turn, the judgment matrix and weight coefficients are calculated, and the consistency is checked. If it is satisfied, the weight coefficients are output, or else, the relative importance between two working conditions needs to be modified in Step 3. Finally, for multiobjective topology optimization of the cylinder block, the weight coefficients (0.49, 0.12, 0.08, 0.31) are output.

5.4. Mathematical Model

(1) Objective. The main objective for the cylinder block is to improve the static and dynamic characteristics in the actual working process. The comprehensive evaluation function can consider both the static multiworking condition stiffness and the dynamic multiorder natural frequency. Therefore, the comprehensive evaluation function shown in (5) is taken as the optimization objective in this paper.

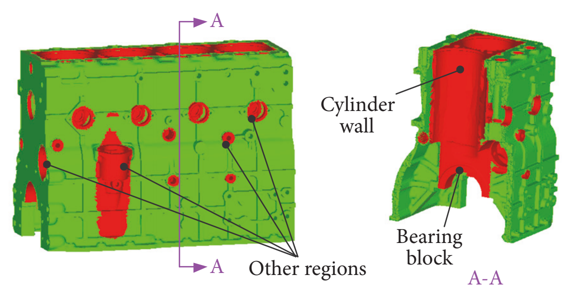
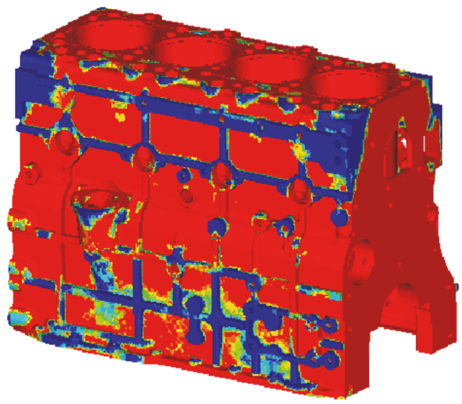


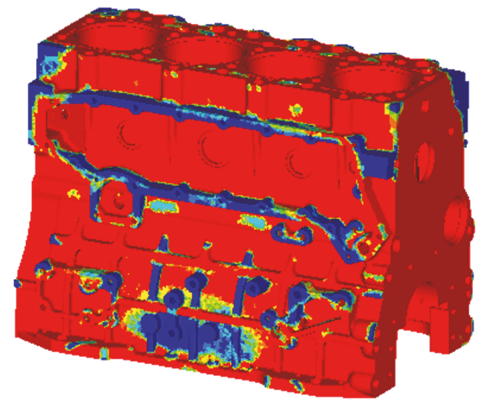
Figure 11: Optimized region and non-optimized region.

(2) Design Variable. The classical variable density topology optimization is applied for the cylinder block, and the design variable is set to the relative density of each element in the optimized area. Since the cylinder wall is to cooperate with the cylinder liner and the cylinder head, it is regarded as a nonoptimized area. In addition, the other area connected with the fuel injection pump, supercharger, radiator, bearing block, etc. is also set as nonoptimized area. In Figure 11, the red region represents the nonoptimized region while the green region represents the optimized region.

(3) Constraint Condition. In the process of topology optimization, it is necessary to ensure that the structure satisfies the equilibrium equation with the continuous material removal in local area. And the relative density of each element is controlled between 0 and 1. In addition, the maximum material remove rate of the cylinder block is set at 10%.



(a) Front



(b) Back

FIGURE 12: Result of the multiobjective topology optimization.

Thus, the mathematical model of multiobjective topology optimization is established as shown in

$$\begin{aligned} \text{Find} \quad & \rho = (\rho_1, \cdots, \rho_n) \\ \min \quad & F(\rho) = \left\{ 0.6^2 \cdot \left[\sum_{i=1}^{4} w_i \frac{C_i(\rho) - C_i^{\min}}{C_i^{\max} - C_i^{\min}} \right]^2 + 0.4^2 \left(\frac{\Lambda_{\max} - \Lambda(\rho)}{\Lambda_{\max} - \Lambda_{\min}} \right)^2 \right\}^{1/2} \\ \text{Subject to} \quad & \mathbf{K}(\rho)\,\mathbf{u} = \mathbf{P} \\ & V(\rho) \le 0.9 \cdot V_0 \\ & 0 < \rho_{\min} \le \rho_i \le 1 \end{aligned} \tag{11}$$

where $F(\rho)$ is the comprehensive evaluation function value. $\mathbf{K}(\rho)$ is the stiffness matrix of finite element model and it is the function of relative density ρ. $\mathbf{u}$ is the displacement vector and $\mathbf{P}$ is the force vector. $V(\rho)$ is the objective volume value and V_0 is the initial volume value. $\rho_{\min}$ represents the minimum relative density in all elements and ρ_i is the relative density of ith element. Other variables have the same meaning as (3)~(5).

6. Results and Discussion

6.1. Topology Optimization Result. The finite element model of the cylinder block is imported into the topology optimization software, and the load and boundary condition are the same as those in Section 3.2. The multiobjective topology optimization mathematical model established by (11) is used for the cylinder block and the result is shown in Figure 12, where the areas from blue to red mean that materials become more and more important. According to the result, the areas where materials can be removed are mainly concentrated on stiffening ribs, convex plates, the side edges, and the inner support plates of cylinder block. Refer to the result of stress analysis and modal analysis in Section 4, the new model is obtained as shown in Figure 13, where the partial area is removed, the thickness and height of the ribs are changed in some areas, and the lightening holes are added in the inner support plate. Its weight has been reduced from 88.97 kg to 84.33 kg, accounting for about 5.22%.

6.2. Comparing with Single Objective Topology Optimization. In order to verify the effectiveness of the multiobjective topology optimization proposed in this paper, the single objective topology optimization of four extreme working conditions for the cylinder block is studied, respectively. For the mathematical model, only the objective is replaced with the minimum structural compliance and other variables remain unchanged as shown in

$$\min c(\rho) = \mathbf{u}^T \mathbf{K} \mathbf{u} = \sum_{e=1}^{n} (\rho_e)^p \, \mathbf{u}_e^T \mathbf{K}_e \mathbf{u}_e \tag{12}$$

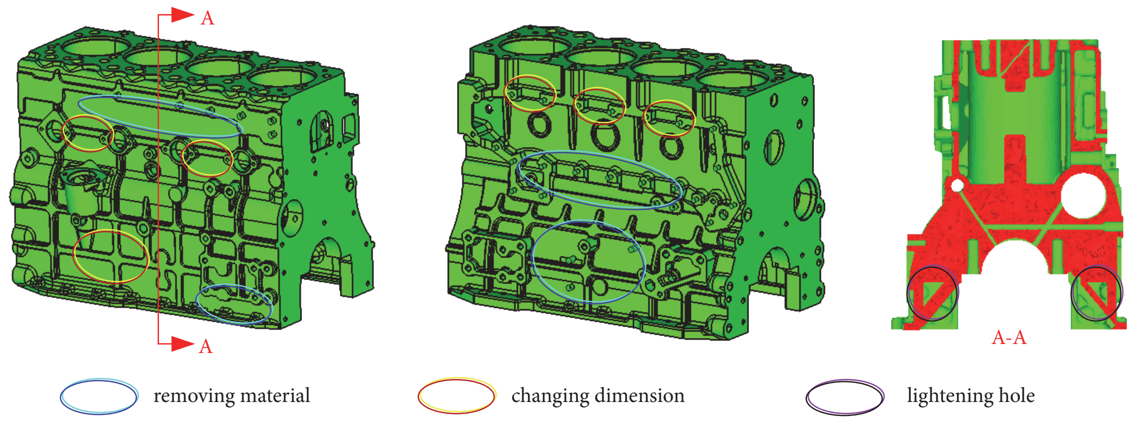


Figure 13: New model of the cylinder block.

where $c(\rho)$ is structural compliance, p is penalty factor, $p > 1$. $\mathbf{u}_e$, $\mathbf{K}_e$ are the displacement vector and the element stiffness matrix corresponding to the eth element. Other variables have the same meaning as (11).

The results are shown in Figure 14. It shows that the material is removed in the vicinity of the fourth cylinder when the first cylinder explodes as shown in Figure 14(a). Similarly, the material is removed in the vicinity of the first cylinder when the fourth cylinder explodes as shown in Figure 14(b). So, it is very clear that the optimization result is different when different working condition is selected. That is to say, the topology optimization of single working condition usually only ensures that structural mechanical property reaches to optimal in the selected working condition, while the mechanical property of other working conditions may be reduced to a lower level.

By comparing with the results, it is necessary to comprehensively consider all working conditions in topology optimization for the cylinder block. If the optimization result of single working condition in a certain working condition is accepted, the structural overall mechanical property may decrease sharply. Therefore, it shows that the method proposed in this paper has obvious advantages comparing with the single objective topology optimization.

6.3. Mechanical Properties Analysis of New Model. The finite element analysis for the new model is used to obtain its static and dynamic characteristics to verify the optimization effect. The calculation process is the same as Section 4 and the displacement and stress distribution are shown in Table 8, and the top 6-order nature frequencies are shown in Table 9.

According to Tables 4 and 8, the comparison of mechanical performance including displacement and stress is, respectively, shown in Figures 15 and 16. From the comparison, the overall stress and displacement of four working conditions keep the same level, and the maximum displacement and maximum stress are slightly reduced in the first and the fourth working condition and others are slightly raised. The first working condition is still the worst and its displacement and stress distribution are shown in Figure 17. The distribution trend of displacement and stress is the same as the original model and the stress is about 80 MPa in most of region, which is much smaller than material ultimate strength (300 MPa). It indicates that the stiffness and strength of the cylinder block can meet working requirements. Comparing Table 5 with Table 9, the 1st natural frequency of new cylinder model is increased by 4 Hz and other order natural frequencies remain basically unchanged, which indicates that the vibration characteristics of the new model meet working requirements.

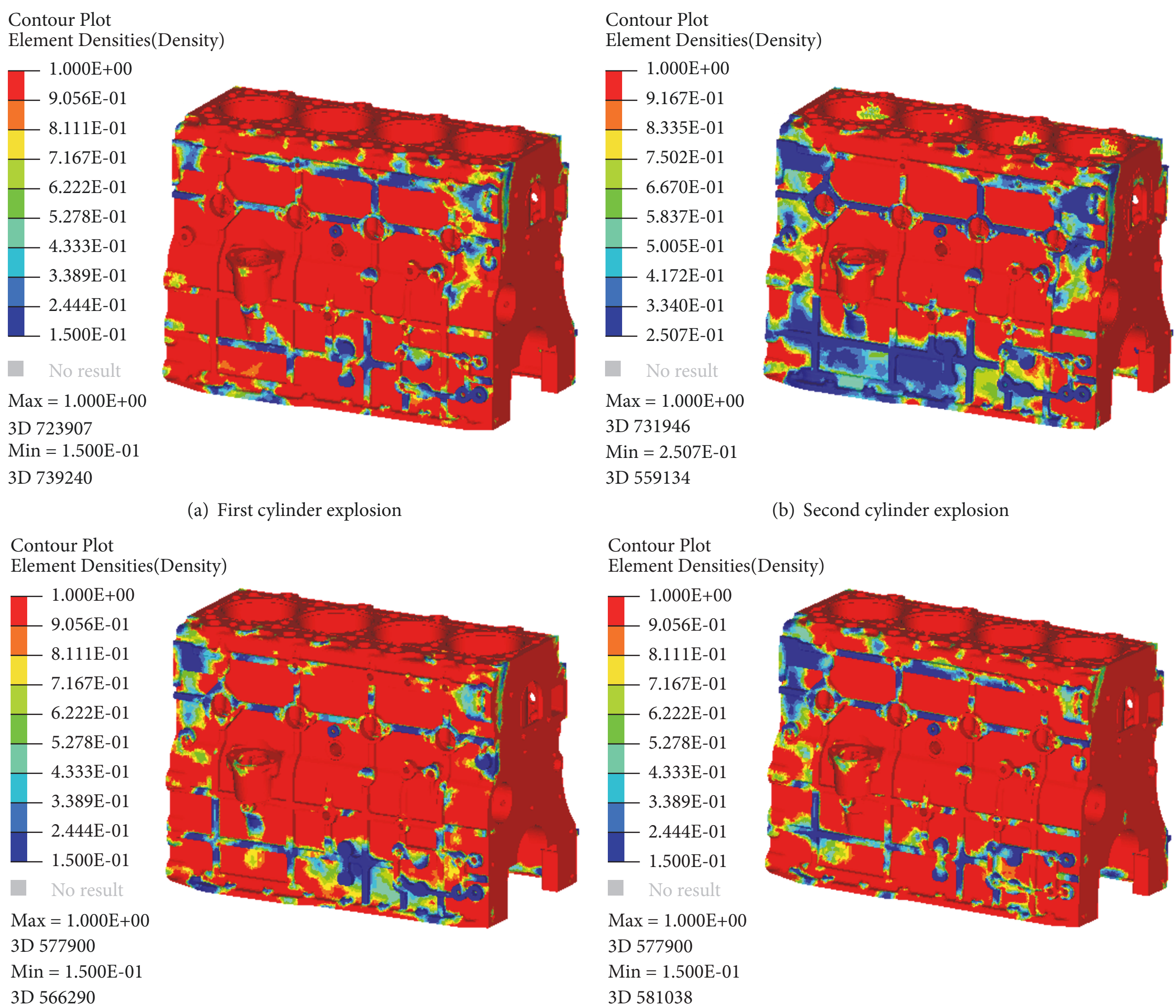
Table 8: Results of FEA for the new model.

No. of explosions	Max stress (MPa)	Max displacement (mm)
1	210.4	0.251
2	177.2	0.202
3	178.3	0.219
4	185.5	0.245

To reflect intuitively the comprehensive performance of the new model in the explosion of each cylinder, structure efficiency is calculated and the results are shown in Table 10. It can be seen from the table that the structure efficiency is increased in the first, second, and third working condition indicating that the material utilization rate becomes higher. The structure efficiency of the fourth working condition is decreased which indicates that the safety becomes higher. In addition, the variance of the original model and the new model are calculated, which are 0.0108 and 0.0079, respectively. It denotes that the mechanical performance of the new model is more uniform. In general, the topology structure of the cylinder block becomes more reasonable by the multiobjective topology optimization.

Table 9: The top 6-order modal analysis results for the new model.

Orders	1	2	3	4	5	6
Frequency/Hz	268	495	534	575	785	1018



(a) First cylinder explosion

(b) Second cylinder explosion

(c) Third cylinder explosion

(d) Fourth cylinder explosion

Figure 14: Results of single working topology optimization.

Figure 15: Comparison of the displacement.

Table 10: Comparison of structure efficiency (MPa•mm•kg^{-1}).

Structure efficiency	No. of explosions			
	1	2	3	4
Original model	62.21%	42.31%	40.51%	55.08%
New model	62.62%	42.45%	46.30%	53.89%
Variation	+0.41%	+0.14%	+5.79%	-1.19%

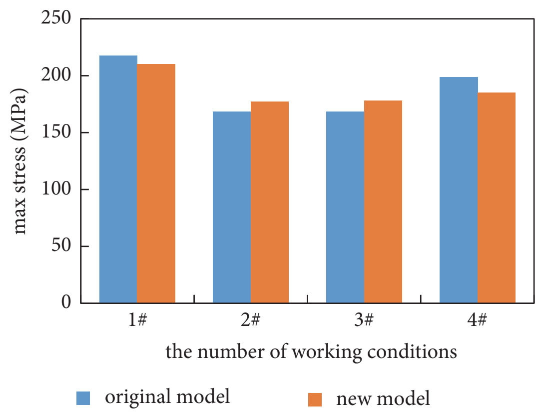
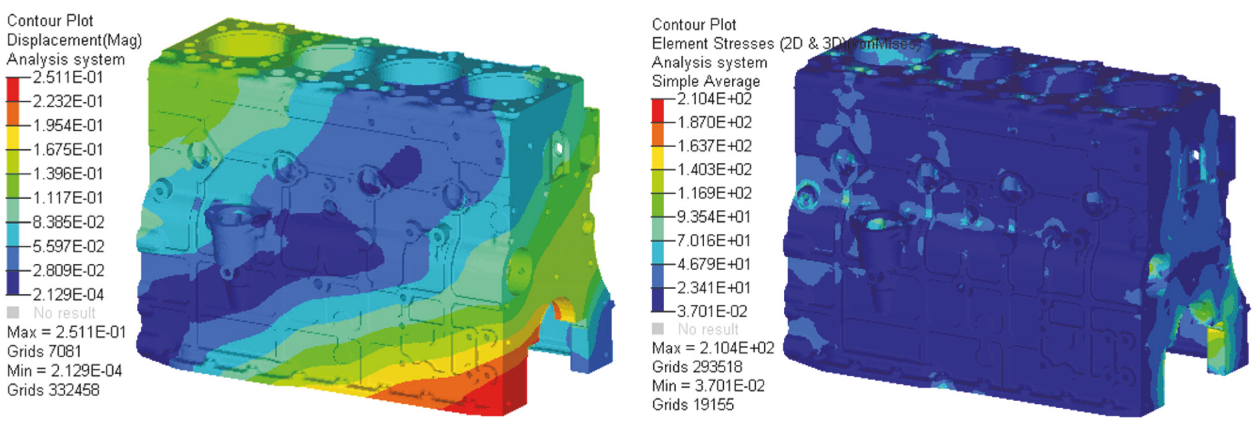


Figure 16: Comparison of the stress.



(a) Displacement distribution (b) Stress distribution

Figure 17: Results of FEA for the new model under extreme working condition.

7. Conclusion

This paper proposes a multiobjective topology optimization method based on AHP. The comprehensive evaluation function for the cylinder block is established by the compromise programming method and the weight coefficients are determined based on AHP. The method is applied to the diesel engine cylinder block and several important conclusions are as follows:

(1) There are alternating impact loads for the diesel engine cylinder block. The traditional single-working condition topology optimization cannot guarantee its overall mechanical performance. The comprehensive evaluation function for the cylinder block is established by the compromise programming method, which can more accurately evaluate the structural performance.

(2) By constructing the hierarchical structure model of topology optimization including 12 weighting coefficients, the establishing process of the comprehensive evaluation function for cylinder block becomes more hierarchical and the determination of the weight coefficients has a theoretical guidance. The method is equally suitable for other multiobjective optimization.

(3) According to the simulation results, the overall structural performance of the cylinder block is improved with a 5.22% reduction in weight. Comparing the structure efficiency variances of the original model and the new model, it can be seen that the mechanical performance becomes more

uniform under different conditions, which shows that the topology structure of the cylinder block is more reasonable.

Data Availability

The [DATA TYPE] data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no financial and personal relationships with other people or organizations that can inappropriately influence their work, and there is no professional or other personal interest of any nature or kind in any product, service, and/or company that could be construed as influencing the position presented in this manuscript.

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Research Article

A Novel TOPSIS Method Based on Improved Grey Relational Analysis for Multiattribute Decision-Making Problem

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Multiattribute decision-making (MADM) problem is difficult to assess because of the large number of attribute indices and the diversity of data distribution. Based on the understanding of data dispersion degree, a new grey TOPSIS method for MADM is studied. The main idea of this paper is to redefine the grey relational analysis through the dispersion of data distribution and redesign the TOPSIS by using the improved grey relational analysis. As a classical multiattribute decision analysis method, traditional TOPSIS does not consider the data distribution of the degree of dispersion and aggregation when it is compared with the optimal and worst alternative solutions. In view of the limitations of traditional TOPSIS, this paper has made two major improvements to TOPSIS. Firstly, the new grey relational analysis is applied to evaluate the grey positive relational degree between each alternative and the optimal solution and compute the grey negative relational degree between each alternative and the worst solution. Secondly, the weights of every attribute index about the optimal and worst solutions are put forward based upon the distance standard deviation and the average distance. Finally, the comprehensive grey TOPSIS is utilized to analyze the ranking of weapon selection problem. The numerical results verify the feasibility of the improved grey relational analysis and also highlight the practicability of the grey comprehensive TOPSIS.

1. Introduction

In recent years, with the development of society, the research on multiattribute decision-making (MADM) is becoming more and more concerned about its simplicity, validity, and accuracy [1–9]. Generally speaking, multiattribute decision-making problem often involves complicated external environment and many different attributes. The TOPSIS (techniques for order preference by similarity to ideal solution) is a useful and powerful method for dealing with multiattribute decision-making (MADM) problems which is presented by Hwang and Yoon [10]. The classical TOPSIS approach is based on the suggestion of the chosen alternative which requires consideration of the distance between the positive ideal solution and the negative ideal solution. The most suitable alternative should be closest to the positive ideal solution and the farthest from the negative ideal solution. At present, with the deepening realization about TOPSIS, some extended TOPSIS techniques have been used widely for MADM problems to overcome the vagueness in the judgments made by the assessors [11–21]. In order to overcome the problem of index weight calculation, AHP is the main auxiliary method of TOPSIS, and fuzzy sets are introduced together to facilitate the quantification of the two pairs of indicators [11–13]. Some researchers have improved TOPSIS by using the intuitionistic fuzzy number, triangular fuzzy number, vague set, intuitionistic fuzzy entropy, and other uncertainty tools, which can solve MADM problems with intuitionistic fuzzy environment or linguistic fuzziness [7, 14–18].

In fact, in the process of multiattribute decision-making with TOPSIS, there are two main problems to solve. On the one hand, the weights of different indexes need to be determined when the alternatives are compared with the optimal or worst schemes. As a relatively simple method, analytic hierarchy process (AHP) has been successfully applied to the weight calculation of TOPSIS [11–13, 19], but subjective interference also exists due to expert scoring and pairwise comparison. The technique of analytic network process (ANP) is used to obtain the relative weights to reduce the number of pairwise comparison on the basis of AHP [20]. So the ANP is the development and supplement of AHP. In addition, the method of comprehensive evaluation, principal component analysis, and other methods are all important methods to calculate weights, which are applied in TOPSIS [21–23]. On the other hand, the optimal or worst value for every attribute should be selected, and a quantitative measure method should be constructed to compare with each other. Fuzzy mathematics provides a quantitative calculation method for describing uncertain things [11–15, 18, 19], but there is a lot of uncertainty and subjectivity in the designing of fuzzy sets. Professor Deng proposed the grey system theory which is one of powerful methods for data analysis with partly known and partly unknown information in uncertain environment [24, 25]. Distance standard deviation is an important numerical characteristic which reflects the distribution of data. Very recently, it is worth noting that distance standard deviation is introduced into the grey confidence interval estimation for small samples [26]. This study shows that distance standard deviation contributes to describe the degree of dispersion between different samples for grey system. However, in the grey relational analysis, the value of distance standard deviation has not been explored. In view of this, this paper will introduce the distance standard deviation factor in the grey relational analysis, so as to take full account of the numerical characteristics of different series of data.

Based on the deepening realization about the generality of multiattribute decision-making problems and the core of TOPSIS method, part of the work has been completed in the following. In Section 2, the sample distance standard deviation is defined between different system behavior sequences, and then the improved grey relational analysis based on sample distance standard deviation is constructed, which paved the way to further discussion. In Section 3, the innovative new TOPSIS method is proposed. The degree of aggregation between different data is served as an important decision factor for TOPSIS method. The grey relational analysis is improved on the basis of the distance standard deviation and applied to quantify the correlation between different sequence data. The distance standard deviation is also used to calculate the weights between the sequence data and the optimal scheme or the worst case. Section 4 gives three applications of the developed approach, and the numerical steps and results are clearly shown. The last section summarizes the whole paper.

2. The Improved Grey Relational Analysis Based on Dispersion Degree

At present, the grey relational analysis has been applied more commonly to many scientific decision-making problems, such as computer science, engineering and other related fields. It provides a tool to evaluate grey relational degree based on distances between reference sequence and comparative sequence. In fact, only considering the maximum and minimum distance between data is not enough. Generally speaking, the sample aggregation characteristics reflect the inherent character of data, which should be considered in the calculation of grey relational degree. The numerical characteristics of the attribute data distribution can reflect the degree of dispersion of the data distribution. If the degree of dispersion of the data is small, the distribution is relatively concentrated, and this attribute is relatively important. And the standard deviation of sample distance of data coincides with this characteristic. Therefore, the sample distance standard deviation as an important concept that embodies the degree of data distribution density will be brought into the calculation of grey relational degree.

Definition 1. For the given system behavior sequence is as follows:

$$\begin{aligned} X_0 &= \left(x_0(1), x_0(2), \dots, x_0(n)\right), \\ X_1 &= \left(x_1(1), x_1(2), \dots, x_1(n)\right), \\ &\dots, \\ X_i &= \left(x_i(1), x_i(2), \dots, x_i(n)\right), \\ &\dots, \\ X_m &= \left(x_m(1), x_m(2), \dots, x_m(n)\right). \end{aligned} \tag{1}$$

The distance between $x_0(k)$ and $x_i(k)$ is defined as follows:

$$d\left(x_0(k), x_i(k)\right) = \left|x_0(k) - x_i(k)\right|. \tag{2}$$

For $\xi \in (0,1)$, and for every i, j, let

$$\gamma\left(x_0(k), x_i(k)\right) = \begin{cases} \dfrac{\xi \max_i s\left(X_0, X_i\right)}{\left|x_0(k) - x_i(k)\right| + \xi \max_i s\left(X_0, X_i\right)}, & \text{if } \min_i s\left(X_0, X_i\right) > 0 \text{ and } s\left(X_0, X_i\right) \neq s\left(X_0, X_j\right) \\ \eta^{\alpha[\max_k d(x_0(k)-x_i(k)) - \min_k d(x_0(k)-x_i(k))]} e^{-\beta d(x_0(k), x_i(k))}, & \text{if } \min_i s\left(X_0, X_i\right) = 0 \text{ or } s\left(X_0, X_i\right) = s\left(X_0, X_j\right), \end{cases} \tag{3}$$

$$\gamma\left(X_0, X_i\right) = \frac{1}{n} \sum_{k=1}^{n} \gamma\left(x_0(k), x_i(k)\right), \tag{4}$$

where $s(X_0, X_i) = \sqrt{(1/(n-1))\sum_{k=1}^{n}(d(x_0(k), x_i(k)) - m(X_0, X_i))^2}$ denotes the sample distance standard deviation between X_0 and X_i, and $m(X_0, X_i) = (1/n)\sum_{k=1}^{n} d(x_0(k), x_i(k))$ is defined as the average distance between X_0 and X_i.

Then $\gamma(X_0, X_i)$ is called grey relational degree with sample distance standard deviation, in which ξ and η are called distinguishing coefficient, $\eta \in (0,1), \alpha, \beta \in (0,1]$, and ξ's value satisfies the following requirements:

$$\xi = \begin{cases} \dfrac{\max_i s(X_0, X_i)}{\min_i s(X_0, X_i) + \max_i s(X_0, X_i)}, & \text{if } \min\limits_i s(X_0, X_i) > 0 \\ \text{constant, and constant} \in (0,1), & \text{if } \min\limits_i s(X_0, X_i) = 0. \end{cases} \quad (5)$$

Theorem 2. *The improved grey relational degree $\gamma(X_0, X_i)$ with sample distance standard deviation $s(X_0, X_i)$ satisfies the following three axioms of grey correlation:*

(1) *The property of normality: $0 < \gamma(X_0, X_i) \le 1$.*

(2) *The property of closeness: the greater the $\gamma(X_0, X_i)$, the closer of X_0 to X_i.*

(3) *The property of pair symmetry: $\gamma(X_i, X_j) = \gamma(X_j, X_i)$.*

Proof. It is obvious that $s(X_0, X_i) \ge 0$ for system behavior sequence $X_0, X_1, \ldots, X_i, \ldots, X_m$. So the theorem can be proved in two cases.

(1) When $\min_i s(X_0, X_i) = 0$ or $s(X_0, X_i) = s(X_0, X_j)$, from (3), we can receive the result that $\gamma(x_0(k), x_i(k)) = \eta^{\alpha[\max_k d(x_0(k)-x_i(k)) - \min_k d(x_0(k)-x_i(k))]} e^{-\beta d(x_0(k), x_i(k))}$, $0 < \gamma(x_0(k), x_i(k)) \le 1$ for every k, then in these circumstances, we can obtain the result that $0 < \gamma(X_0, X_i) \le 1$ by (4). $\gamma(X_0, X_i) = 1$, if and only if $X_0 = X_i$.

For system behavior sequences X_0 and X_i, $\eta^{\alpha[\max_k d(x_0(k)-x_i(k)) - \min_k d(x_0(k)-x_i(k))]}$ is a constant. Because $e^{-\beta d(x_0(k), x_i(k))}$ is monotonically decreasing and $\gamma(x_0(k), x_i(k))$ is monotonically decreasing, the property of closeness is received.

At last, because of $|x_i(k) - x_j(k)| = |x_j(k) - x_i(k)|$, the property of pair symmetry is obvious.

(2) If $\min_i s(X_0, X_i) > 0$ and $s(X_0, X_i) \ne s(X_0, X_j)$, then $\max_i s(X_0, X_i) \ge \min_i s(X_0, X_i) > 0$. So, $0 < \gamma(X_0, X_i) < 1$ by (3). Because $\min_i s(X_0, X_i) > 0$, we can receive the result that $X_0 \ne X_i$ for every i, then $\gamma(X_0, X_i) \ne 1$ in this case.

Because $\max_i s(X_0, X_i)$ and $\min_i s(X_0, X_i)$ are constant which determined by system behavior sequence $X_0, X_1, \ldots, X_i, \ldots, X_m$. It is obvious that the closeness is established.

Because $|x_i(k) - x_j(k)| = |x_j(k) - x_i(k)|$, the property of pair symmetry can be received. □

Remark 3. The new grey relational analysis takes full advantage of the inherent regularity of data distribution and introduces the dispersion of data distribution into them. The distance standard deviation is more likely to reflect the overall dispersion of the data distribution than the distance.

Remark 4. In the case of $\min_i s(X_0, X_i) = 0$ or $s(X_0, X_i) = s(X_0, X_j)$, that is, there are two system behavior sequence X_0 and X_i that are parallel or overlapped, the distance between X_0 and X_i is fully considered by (3). Expressly if two system behavior sequence data are parallel, then the smaller the distance is, the greater the similarity is, and the greater the grey relational degree is.

3. The Novel TOPSIS with Improved Grey Relational Analysis to Solve Multiattribute Decision-Making Problem

This section tries to improve traditional TOPSIS with new grey relational analysis mentioned above. The degree of dispersion and aggregation between different data will serve as an important decision factor for TOPSIS. The above-mentioned grey relational analysis will be used to calculate the grey relational degree between the alternative scheme and the optimal or worst case scheme. At the same time, the standard deviation of distance will be taken as the decisive factor of weight calculation in order to determine the weight of each index for MADM problems.

Step 1. Standardization of the value of original decision matrix $X = (x_{ij})_{n\times m}$: this will help eliminating the influence of dimension and magnitude in data processing. The original decision matrix $X = (x_{ij})_{n\times m}$ can be expressed as follows:

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{bmatrix}_{n\times m}, \quad (6)$$

where x_{ij} is the jth attribute value of the ith evaluation object. For multiattribute decision-making problem, the normalized decision matrix is $Y = (y_{ij})_{n\times m}$ after standardization of $X = (x_{ij})_{n\times m}$.

For the benefit attribute

$$y_{ij} = \frac{x_{ij} - \min_i x_{ij}}{\max_i x_{ij} - \min_i x_{ij}}. \quad (7)$$

For the cost attribute

$$y_{ij} = \frac{\max_i x_{ij} - x_{ij}}{\max_i x_{ij} - \min_i x_{ij}}. \quad (8)$$

Step 2. To pick out the optimal and worst alternatives, the optimal alternative is denoted as $\overline{X} = (\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_m) \in R^m$, where

$$\overline{x}_j = \begin{cases} \max\limits_i x_{ij}, & \text{if } x_{ij} \text{ is benefit attribute} \\ \min\limits_i x_{ij}, & \text{if } x_{ij} \text{ is cost attribute,} \end{cases} \quad (9)$$
$$j = 1, 2, \ldots, m.$$

The worst alternative denoted as $\underline{X} = (\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_m) \in R^m$, where

$$\underline{x}_j = \begin{cases} \max_i x_{ij}, & \text{if } x_{ij} \text{ is cost attribute} \\ \min_i x_{ij}, & \text{if } x_{ij} \text{ is benefit attribute,} \end{cases} \quad j = 1, 2, \ldots, m. \tag{10}$$

For the normalized decision matrix $Y = (y_{ij})_{n\times m}$, the normalized optimal alternative can be denoted as $\overline{Y} = (\overline{y}_1, \overline{y}_2, \ldots, \overline{y}_m) \in R^m$, and the normalized worst alternative can be denoted as $\underline{Y} = (\underline{y}_1, \underline{y}_2, \ldots, \underline{y}_m) \in R^m$. It is not difficult to calculate that $\overline{Y} = (1, 1, \ldots, 1) \in R^m$, and $\underline{Y} = (0, 0, \ldots, 0) \in R^m$ by (7) and (8), respectively.

Step 3. To calculate the sample distance standard deviation for different attribute, the symbol s_{1j} used to denote the sample distance standard deviation between the jth attribute vector and the jth attribute optimal value, where

$$s_{1j} = \frac{1}{n-1}\sqrt{\sum_{i=1}^{n}\left(d\left(y_{ij}, \overline{y}_j\right) - m_{1j}\right)^2}, \quad j = 1, 2, \ldots, m, \tag{11}$$

$$m_{1j} = \frac{1}{n}\sum_{i=1}^{n} d\left(y_{ij}, \overline{y}_j\right), \quad j = 1, 2, \ldots, m. \tag{12}$$

The symbol s_{2j} is used to denote the sample distance standard deviation between the jth attribute vector and the jth attribute worst value, where

$$s_{2j} = \frac{1}{n-1}\sqrt{\sum_{i=1}^{n}\left(d\left(y_{ij}, \underline{y}_j\right) - m_{2j}\right)^2}, \quad j = 1, 2, \ldots, m, \tag{13}$$

$$m_{2j} = \frac{1}{n}\sum_{i=1}^{n} d\left(y_{ij}, \underline{y}_j\right), \quad j = 1, 2, \ldots, m. \tag{14}$$

Step 4. To compute the grey relational degree r_{ij}^{+} and r_{ij}^{-}. There are some different cases based on (3).

(1) When $\min_j s_{1j} > 0$ and $s_{1j} \neq s_{1k}$ for every k, $k = 1, 2, \ldots, m$, the symbol r_{ij}^{+} is the grey positive relational degree between y_{ij} and $\overline{y}_j$, where

$$r_{ij}^{+} = \frac{\xi \max_j s_{1j}}{\left|y_{ij} - \overline{y}_j\right| + \xi \max_j s_{1j}}. \tag{15}$$

At the same case, the symbol r_{ij}^{-} is the grey negative relational degree between y_{ij} and $\underline{y}_j$, where

$$r_{ij}^{-} = \frac{\xi \max_j s_{2j}}{\left|y_{ij} - \underline{y}_j\right| + \xi \max_j s_{2j}}. \tag{16}$$

(2) When $\min_j s_{1j} = 0$ or $s_{1j} = s_{1k}$, for every k, $k = 1, 2, \ldots, m$, the grey positive relational degree r_{ij}^{+} can be computed as follows:

$$r_{ij}^{+} = \eta^{\alpha[\max_j d(y_{ij}, \overline{y}_j) - \min_j d(y_{ij}, \overline{y}_j)]} e^{-\beta d(y_{ij}, \overline{y}_j)}. \tag{17}$$

And the grey negative relational degree r_{ij}^{-} between y_{ij} and $\underline{y}_j$ can be received as follows:

$$r_{ij}^{-} = \eta^{\alpha[\max_j d(y_{ij}, \underline{y}_j) - \min_j d(y_{ij}, \underline{y}_j)]} e^{-\beta d(y_{ij}, \underline{y}_j)}. \tag{18}$$

Step 5. To determine weights $\overline{w}_j$ and $\underline{w}_j$ for the grey positive relational degree and grey negative relational degree, similar to the above, there are two cases below.

(1) When $\min_j s_{1j} > 0$, $s_{1j} \neq s_{1k}$, $\min_j s_{2j} > 0$, and $s_{2j} \neq s_{2k}$ for every k, $k = 1, 2, \ldots, m$, $\overline{w}_j$ and $\underline{w}_j$ can be obtained as follows:

$$\overline{w}_j = \frac{1/s_{1j}}{\sum_{j=1}^{m}\left(1/s_{1j}\right)}, \quad j = 1, 2, \ldots, m, \tag{19}$$

$$\underline{w}_j = \frac{1/s_{2j}}{\sum_{j=1}^{m}\left(1/s_{2j}\right)}, \quad j = 1, 2, \ldots, m, \tag{20}$$

where the sample distance standard deviation s_{1j} and s_{2j} can be determined by (11) and (13).

(2) Otherwise, the weights $\overline{w}_j$ and $\underline{w}_j$ can be calculated as follows:

$$\overline{w}_j = \frac{1/m_{1j}}{\sum_{j=1}^{m}\left(1/m_{1j}\right)}, \quad j = 1, 2, \ldots, m, \tag{21}$$

$$\underline{w}_j = \frac{1/m_{2j}}{\sum_{j=1}^{m}\left(1/m_{2j}\right)}, \quad j = 1, 2, \ldots, m, \tag{22}$$

where the average distance m_{1j} and m_{2j} can be determined by (12) and (14).

The smaller the distance standard deviation of attribute data or the average distance is, the stronger the regularity of attribute data is, and the bigger its weight is. On the contrary, the larger the distance standard deviation of attribute data or the average distance is, the larger the distribution range of attribute data is, the weaker the regularity is, and the smaller the weight is. It is obvious to find that the greater the $\overline{w}_j$ is, the closer alternative attribute value and the optimal alternative attribute value are. Similarly, the weight $\underline{w}_j$ has the same nature. The greater the $\underline{w}_j$ is, the closer alternative attribute value and the worst value are.

Step 6. To calculate the grey TOPSIS grade, the grey positive relational grade R_i^{+} between the ith evaluation alternative and the optimal solution can be received as follows:

$$R_i^{+} = \sum_{j=1}^{m} \overline{w}_j r_{ij}^{+}, \quad i = 1, 2, \ldots, n. \tag{23}$$

The grey negative relational grade R_i^- between the ith evaluation alternative and the worst solution can be received as follows:

$$R_i^- = \sum_{j=1}^{m} \underline{w}_j r_{ij}^-, \quad i = 1, 2, \ldots, n. \tag{24}$$

And the comprehensive grey TOPSIS evaluation result R_i is defined as follows:

$$R_i = \frac{R_i^+}{R_i^+ + R_i^-}, \quad i = 1, 2, \ldots, n. \tag{25}$$

It is obviously that the comprehensive grey TOPSIS evaluation result R_i reflects the closeness between alternative and positive ideal solution and negative ideal solution. The larger the R_i^+ is, the closer R_i is to positive ideal solution, and the smaller the R_i^- is, the farther R_i is to negative ideal solution. The reason behind this is that after the first dimensionless standardized processing of the original data, the improved TOPSIS method is used for evaluation, and the obtained grey relational degree and weights have the function of adaptive adjustment with the change of data. On the one hand, the grey positive relational degree can reflect the closeness between the data and the same attribute data of the positive ideal solution, while the grey negative relational degree can reflect the closeness between the data and the same attribute data of the negative ideal solution. On the other hand, the weights of different attributes can feedback the importance of the attributes based on the numerical characteristics of the attribute data.

4. Numerical Results

In this section, three examples will be used to demonstrate the effectiveness of the methods proposed above. The first example shows that the improved grey relational analysis is more comprehensive than the traditional grey relational analysis, which can avoid some unreasonable results. The second one is used to illustrate the rationality of the improved grey relational analysis. The novel TOPSIS with improved grey relational analysis will be applied for assessment of weapon system in the second example. In addition, Example 3 will illustrate that the method of this paper can avoid the occurrence of the rank reversal phenomenon, which is superior to the traditional TOPSIS method.

Example 1. The following example will try to solve the grey relational degree with improved method in this paper. Take four system behavior sequences for example, $X_1 = \{0.1, 0.9, 0.1, 0.9, 0.1\}$, $X_2 = \{1, 0, 1, 0, 1\}$, $X_3 = \{0.2, 1, 0.2, 1, 0.2\}$, and $X_4 = \{0.9, 0, 0.9, 0, 0.9\}$. Figure 1 illustrates the changing trend curves for different system behavior sequences.

Solution

Step 1. Because the data of four system behavior sequences are in a relatively small range $[0, 1]$, the data preprocessing can be ignored.

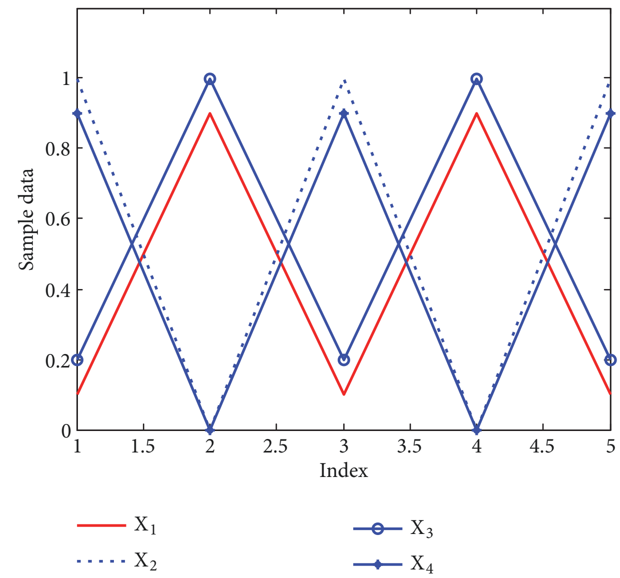

FIGURE 1: Four system behavior sequences.

According to (2), compute the distance $d(x_1(k), x_i(k)) = |x_1(k) - x_i(k)|$, respectively, $(k = 1, 2, \ldots, 5)$. The symbol $d(X_1, X_i) = (d(x_1(1), x_i(1)), d(x_1(2), x_i(2)), \ldots, d(x_1(5), x_i(5)))$ is used to represent the distance vector between X_1 and X_i, $i = 2, 3, 4$.

$$\begin{aligned} d(X_1, X_2) &= (0.9, 0.9, 0.9, 0.9, 0.9), \\ d(X_1, X_3) &= (0.1, 0.1, 0.1, 0.1, 0.1), \\ d(X_1, X_4) &= (0.8, 0.9, 0.8, 0.9, 0.8). \end{aligned} \tag{26}$$

Step 2. Compute the average distance $m(X_1, X_i)$ and the sample distance standard deviation $s(X_1, X_i)$.

$$\begin{aligned} m(X_1, X_2) &= 0.9, \\ m(X_1, X_3) &= 0.1, \\ m(X_1, X_4) &= 0.84, \\ s(X_1, X_2) &= 0, \\ s(X_1, X_3) &= 0, \\ s(X_1, X_4) &= 0.0548. \end{aligned} \tag{27}$$

Step 3. According to (3) and (4), the improved grey relational degree $\gamma(X_1, X_i)$ based on discrete degree of data in statistics can be solved as follows:

$$\begin{aligned} \gamma(X_1, X_2) &= 0.6376, \\ \gamma(X_1, X_3) &= 0.9512, \\ \gamma(X_1, X_4) &= 0.6500. \end{aligned} \tag{28}$$

The results show that $\gamma(X_1, X_3) > \gamma(X_1, X_4) > \gamma(X_1, X_2)$, which indicates X_1 is more relevant to X_3 compared with

Table 1: The comparison between the improved and classical grey relational analysis.

Grey relational degree	$\gamma(X_1, X_2)$	$\gamma(X_1, X_3)$	$\gamma(X_1, X_4)$
The classical grey relational analysis	0.4074	1	0.4270
The improved grey relational analysis	0.6376	0.9512	0.6500

Table 2: The original multiattribute decision data [27].

	M_{max}	M'_{max}	O_{max}	H_{max}	R_{max}	N	P	T	H_{min}	G_0	R_{min}
X_1	6	750	6	24	100	8	0.75	20	0.3	1000	3
X_2	5.5	2300	5	27	90	8	0.8	15	0.025	1000	5
X_3	4.4	1200	5	27	75	6	0.76	20	0.025	1664	5
X_4	3	420	1	24.5	32	1	0.75	40	1	2375	8
X_5	2	400	2	5	12	3	0.75	10	0.5	220	1
X_6	2.2	400	2	3	8	3	0.8	10.4	0.05	85	1
X_7	2.2	410	2	3	8	3	0.7	10	0.05	85	0.5

others. The two sets of data X_1 and X_3 are parallel and no-overlap, so they can be considered different, and the grey relational degree cannot be valued at one. Obviously, the valuing and sorting of the improved grey relational analysis results are superior to the classical method shown in Table 1. In this example, some parameters are taken as follows: $\xi = 0.5$, $\eta = 0.8$, $\alpha = 0.5$, and $\beta = 0.5$.

Example 2. In this example, seven surface-to-air missile weapon system alternatives $X_1, X_2, \ldots, X_7$ will be evaluated by grey TOPSIS method in the following. The multiattribute weapon system is evaluated from the combination of traditional grey relational analysis and TOPSIS in reference paper [27], but no convincing calculation index weight is given. In order to obtain the best selection of weapon systems, [28] developed fuzzy analytic hierarchy process by entropy weight to evaluate the weight. Metin, Serkan, and Nevzat [29] utilized AHP and TOPSIS to assess weapon systems. AHP is more difficult because of the comparison between the two factors, then this paper from the distribution of different indicators to consider the dispersion degree and facilitate the given weight. A good surface-to-air missile weapon system depends on a lot of attributes, such as the maximum speed of missile (M_{max}), the maximum speed of target (M'_{max}), the maximum overload of target (O_{max}), the highest boundary of killing range (H_{max}), the farthest boundary of killing range (R_{max}), the number of targets that can simultaneity be shot by one weapon system (N), the single shot kill probability of missiles (P), the reaction time of missile weapon system (T), the lowest boundary of killing range (H_{min}), the launching weight of missiles (G_0), and the nearest boundary of killing range (R_{min}) as listed in Table 2 [27]. Based on the above facts, the optimal selection of multiple weapons is very important and complicated for evaluation and assessment of weapon system.

Solution

Step 1. The normalized decision matrix $Y = (y_{ij})_{n\times m}$ can be calculated by (7) and (8) as follows:

$$Y = \begin{bmatrix} 1.0000 & 0.1842 & 1.0000 & 0.8750 & 1.0000 & 1.0000 & 0.5000 & 0.6667 & 0.7179 & 0.6004 & 0.6667 \\ 0.8750 & 1.0000 & 0.8000 & 1.0000 & 0.8913 & 1.0000 & 1.0000 & 0.8333 & 1.0000 & 0.6004 & 0.4000 \\ 0.6000 & 0.4211 & 0.8000 & 1.0000 & 0.7283 & 0.7143 & 0.6000 & 0.6667 & 1.0000 & 0.3105 & 0.4000 \\ 0.2500 & 0.0105 & 0 & 0.8958 & 0.2609 & 0 & 0.5000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.2000 & 0.0833 & 0.0435 & 0.2857 & 0.5000 & 1.0000 & 0.5128 & 0.9410 & 0.9333 \\ 0.0500 & 0 & 0.2000 & 0 & 0 & 0.2857 & 1.0000 & 0.9867 & 0.9744 & 1.0000 & 0.9333 \\ 0.0500 & 0.0053 & 0.2000 & 0 & 0 & 0.2857 & 0 & 1.0000 & 0.9744 & 1.0000 & 1.0000 \end{bmatrix} \quad (29)$$

Step 2. Use (9) and (10) to determine the optimal alternative $\overline{Y}$ and worst alternative $\underline{Y}$.

$$\overline{Y} = (1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1), \\ \underline{Y} = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0). \quad (30)$$

Step 3. The average distance and distance standard deviation are calculated, respectively, by (11)-(14).

$$\begin{aligned}M_1 &= (0.5964, 0.7684, 0.5429, 0.4494, 0.5823, 0.4898, 0.4143, 0.2638, 0.2601, 0.3639, 0.3810),\\ M_2 &= (0.4036, 0.2316, 0.4571, 0.5506, 0.4177, 0.5102, 0.5857, 0.7362, 0.7399, 0.6361, 0.6190),\\ S_1 &= (0.4189, 0.3733, 0.3952, 0.4921, 0.4422, 0.3943, 0.3436, 0.3566, 0.3752, 0.3810, 0.3706),\\ S_2 &= (0.4679, 0.6896, 0.4059, 0.5041, 0.4766, 0.3949, 0.3904, 0.6225, 0.6399, 0.4812, 0.4511),\end{aligned} \tag{31}$$

where $M_1 = (m_{11}, \ldots, m_{1,11})$, $M_2 = (m_{21}, \ldots, m_{2,11})$, $S_1 = (s_{11}, \ldots, s_{1,11})$, $S_2 = (s_{21}, \ldots, s_{2,11})$.

Step 4. To determine the indicator weights,

$$\begin{aligned}\overline{w} &= (0.0849, 0.0952, 0.0900, 0.0722, 0.0804, 0.0902, 0.1035, 0.0997, 0.0947, 0.0933, 0.0959),\\ \underline{w} &= (0.0942, 0.0639, 0.1086, 0.0874, 0.0925, 0.1116, 0.1129, 0.0708, 0.0689, 0.0916, 0.0977),\end{aligned} \tag{32}$$

where $\overline{w} = (\overline{w}_1, \ldots, \overline{w}_{11})$, $\underline{w} = (\underline{w}_1, \ldots, \underline{w}_{11})$.

Step 5. The grey positive relational degree r_{ij}^+ and grey negative relational degree r_{ij}^- can be obtained by (15) and (16), $i = 1, 2, \ldots, 7$; $j = 1, 2, \ldots, 11$. Then, we can get the grey positive relational grade R_i^+ and grey negative relational grade R_i^-.

$$\begin{aligned}R_1^+ &= 0.6370,\\ R_2^+ &= 0.7606,\\ R_3^+ &= 0.5232,\\ R_4^+ &= 0.2856,\\ R_5^+ &= 0.4550,\\ R_6^+ &= 0.5826,\\ R_7^+ &= 0.5248,\\ R_1^- &= 0.3835,\\ R_2^- &= 0.3510,\\ R_3^- &= 0.4171,\\ R_4^+ &= 0.8114,\\ R_5^- &= 0.6250,\\ R_6^- &= 0.6078,\\ R_7^- &= 0.6838.\end{aligned} \tag{33}$$

Step 6. At last, the comprehensive grey TOPSIS evaluation results can be derived as follows:

$$\begin{aligned}R_1 &= 0.6242,\\ R_2 &= 0.6842,\\ R_3 &= 0.5564,\\ R_4 &= 0.2604,\\ R_5 &= 0.4213,\\ R_6 &= 0.4894,\\ R_7 &= 0.4342.\end{aligned} \tag{34}$$

For the sake of convenience, the data in Table 3 retains three significant digits. Obviously, the presented method in this paper is consistent with the sorting results of the reference method [27] and TOPSIS. According to the comprehensive grey TOPSIS evaluation results, the order is $R_2, R_1, R_3, R_6, R_7, R_5, R_4$. In general, the ranking results of the methods are consistent, but locally, the change of evaluation values is different by Figure 2. The attribute value of the single shot kill probability of missiles (P) of X_5, X_6, and X_7 is relatively close in Table 2. And the weights $\overline{w}_7 = 0.1035$ and $\underline{w}_7 = 0.1129$ are the biggest in the same kind of attribute index weights. And the weights of attribute indexes with large difference are relatively small. These lead to a close ranking of X_5, X_6, and X_7.

Example 3. For the multiattribute missile selection problem, this example will delete or add alternative to evaluate whether there is a rank reversal phenomenon based on the original data of Example 2. The traditional TOPSIS method usually uses Euclidean distance to measure, and the weights of different attributes are fixed, which easily causes the phenomenon of rank reversal [30]. In the decision-making process of the proposed method, when the alternatives are compared with

Table 3: The comparison between the presented method and reference method.

quantization results	R_1	R_2	R_3	R_4	R_5	R_6	R_7
The presented method	0.624	0.684	0.556	0.260	0.421	0.489	0.434
The reference method [27]	0.638	0.716	0.586	0.281	0.425	0.474	0.431
TOPSIS	0.689	0.787	0.632	0.274	0.442	0.497	0.451

Table 4: The ranking results of the comprehensive grey TOPSIS evaluation in this paper.

	Normalized decision matrix											evaluation results	Rank
	$M_{\max}$	$M'_{\max}$	$O_{\max}$	$H_{\max}$	$R_{\max}$	N	P	T	$H_{\min}$	G_0	$R_{\min}$		
Drop X_7 out of alternative set													
X_1	1	0.1842	1	0.875	1.0000	1	0	0.6667	0.7179	0.6004	0.7143	0.5557	2
X_2	0.875	1	0.8	1	0.8913	1	1	0.8333	1	0.6004	0.4286	0.6387	1
X_3	0.6	0.4211	0.8	1	0.7283	0.7143	0.2	0.667	1	0.3105	0.4286	0.4982	3
X_4	0.25	0.0105	0	0.8958	0.2609	0	0	0	0	0	0	0.2339	6
X_5	0	0	0.2	0.0833	0.0435	0.2857	0	1	0.5128	0.941	1	0.3908	5
X_6	0.05	0	0.2	0	0	0.2857	1	0.9867	0.9744	1	1	0.4698	4
Add a new alternative X_8 to the original alternative set													
X_1	1	0.1842	1	0.875	1	1	0.5	0.6667	0.7179	0.6004	0.6667	0.6145	2
X_2	0.875	1	0.8	1	0.8913	1	1	0.8333	1	0.6004	0.4	0.6733	1
X_3	0.6	0.4211	0.8	1	0.7283	0.7143	0.6	0.6667	1	0.3105	0.4	0.5467	3
X_4	0.25	0.0105	0	0.8958	0.2609	0	0.5	0	0	0	0	0.2577	8
X_5	0	0	0.2	0.0833	0.0435	0.2857	0.5	1	0.5128	0.941	0.9333	0.4134	7
X_6	0.05	0	0.2	0	0	0.2857	1	0.9867	0.9744	1	0.9333	0.4794	5
X_7	0.05	0.0053	0.2	0	0	0.2857	0	1	0.9744	1	1	0.4301	6
X_8	0.05	0	0.22	0	0	0.3571	1	1	0.9744	1	1	0.4940	4

the positive ideal solution and the negative ideal solution, the attributes have two sets of weights, and the weights can be dynamically adjusted with the change of the density of data distribution. Now we drop X_7 out of alternative set based on the second example. Then we add a new alternative $X_8 = \{2.2, 400, 2.1, 3, 8, 3.5, 0.8, 10, 0.05, 85, 0.5\}$ to the original set of Example 2. The main results are shown in Table 4.

In the case of dropping X_7 out of alternative set, the indicator weights are updated to the following results:

$$\overline{w} = (0.0884, 0.0955, 0.0908, 0.0803, 0.0855, 0.0900, 0.0758, 0.1019, 0.0953, 0.0994, 0.0971),$$
$$\underline{w} = (0.0907, 0.0916, 0.0910, 0.0895, 0.0903, 0.0909, 0.0886, 0.0922, 0.0915, 0.0920, 0.0917). \quad (35)$$

In the second case, the weights $\overline{w}_j$ and $\underline{w}_j$ for the grey positive relational degree and grey negative relational degree can be determined as follows:

$$\overline{w} = (0.0849, 0.0974, 0.0922, 0.0698, 0.0795, 0.0937, 0.0988, 0.1008, 0.0969, 0.0921, 0.0939),$$
$$\underline{w} = (0.0923, 0.0642, 0.1150, 0.0940, 0.0896, 0.1264, 0.1022, 0.0697, 0.0690, 0.0865, 0.0911). \quad (36)$$

The above weights are two sets of weights given after considering both positive and negative ideal solutions, which can be adjusted automatically according to the characteristics of data distribution.

Based on Example 2, the ranking results of the original alternative set are

$$R_2 \succ R_1 \succ R_3 \succ R_6 \succ R_7 \succ R_5 \succ R_4. \quad (37)$$

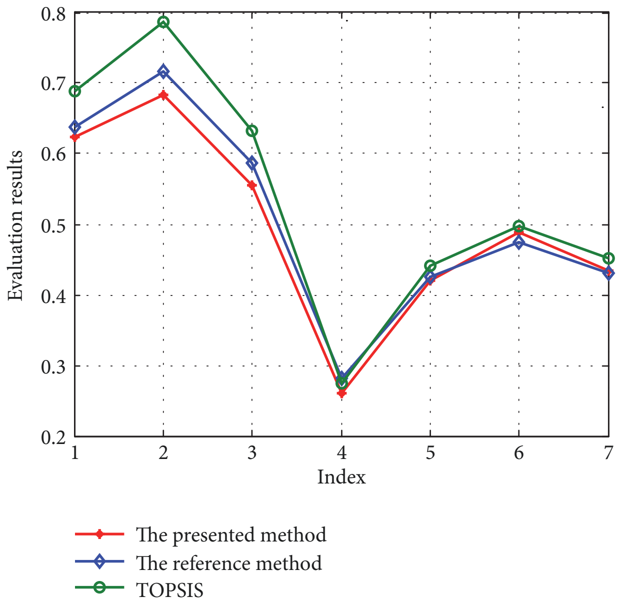

FIGURE 2: Evaluation results and trend.

Similarly, we can obtain the following results with dropping X_7 out of alternative set:

$$R_2 \succ R_1 \succ R_3 \succ R_6 \succ R_5 \succ R_4. \quad (38)$$

Then for the last case, we can receive the following ranking:

$$R_2 \succ R_1 \succ R_3 \succ R_8 \succ R_6 \succ R_7 \succ R_5 \succ R_4. \quad (39)$$

It is not difficult to find that the ranking of the original alternatives is unchanged under the two different circumstances, so there is no rank reversal. We believe that no rank reversal is a normal phenomenon, so the novel TOPSIS method in the paper is more in line with people's thinking logic.

Since the improved TOPSIS method in this paper has two sets of attribute weights during the evaluation process, the weights can be dynamically adjusted with the change of data distribution density no matter in reducing the evaluation objects or increasing the evaluation objects. The final result of the evaluation presented to us is that the original data will not have the phenomenon of rank reversal. Correspondingly, the traditional TOPSIS method is prone to rank reversal because the attribute weights cannot be adjusted dynamically. The specific information under the two different circumstances is shown in Table 4.

5. Conclusions

The traditional TOPSIS method is limited because it only takes the distance between the data into account, without introducing the degree of data dispersion. In addition, the classical grey relational analysis method does not study the aggregation and dispersion degree of different series of data. Based on this, a novel grey TOPSIS method developed from an improved grey relational analysis and a new indicator weight determination method, which well evaluated different objects. The improved grey relational analysis method makes full use of the discrete degree of the different sequence data and considers the overall distribution of the data, so as to avoid the shortage of the maximum and minimum distance of data. At the same time, the concept of sample distance standard deviation is proposed to solve the problem of different indicator weights determination. And there examples have successfully tested the effectiveness of the proposed method. Especially, the ranking of multiple weapons with MADM typical features verifies the feasibility for practical MADM problem. In the process of solving MADM, the present method in this paper overcomes the rank reversal problem of classical TOPSIS. It is worth noting that the proposed TOPSIS method depends on quantitative test data. As a future work, qualitative data and semantic fuzziness will also be introduced into the improved TOPSIS method to solve more complex multiattribute decision problems. In addition, we will reveal the necessity of introducing grey relational analysis into MADM problems further.

Data Availability

Data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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Research Article

A New Model for Deriving the Priority Weights from Hesitant Triangular Fuzzy Preference Relations

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Fuzzy preference relation is a common tool to express the uncertain preference information of decision maker in the process of decision making. However, the traditional fuzzy preference relation will fail under hesitant fuzzy environment as the membership has a single value. In addition, it is very difficult to obtain the precise membership values. Therefore, a new model of fuzzy preference relation is proposed in this paper. Firstly, the concept of hesitant triangular fuzzy preference relation is defined and its properties are investigated based on the concepts of hesitant fuzzy set, hesitant triangular fuzzy set, fuzzy preference relation, and hesitant fuzzy preference relation. Then, the steps of applying this novel model are offered for the case of determining the weights of failure modes. Finally, an example is used to illustrate the proposed model.

1. Introduction

Having received extensive attention in the last few decades, preference relation is a widely used and effective tool to express the preference of decision makers over the alternatives in decision making [1, 2]. There are two kinds of preference relations, i.e., fuzzy preference relation [2] and multiplicative preference relation [3]. Saaty [4] originally proposed the multiplicative preference relation and used the 1/9-9 scale to measure the intensity of the pairwise comparison between two different alternatives. The fuzzy preference relation was proposed by Orlovsky [2], and the 0-1 scale was employed to describe the assessment information of decision maker by comparing the alternatives. Since the emergence of preference relation, extensive research has been conducted and some meaningful conclusions have been drawn, including the consensus models and methodologies [5, 6], the consistency methods [7, 8], the priority methodologies [9–11], and the incomplete values-determined methods [12, 13]. Previous researches have shown that fuzzy preference relation and multiplicative preference relation have been extended to more general forms of fuzzy set, such as interval-valued environment [11, 14] and linguistic environment [15]. To depict the intensities of both preferences and non-preferences, Xu [16] introduced the intuitionistic fuzzy values based on intuitionistic fuzzy set (IFS) [17], where intuitionistic fuzzy values are composed of a membership degree, a nonmembership degree, and a hesitancy degree [18]. The exact values of the three membership degrees in IFS are however difficult to be determined in some practical applications. Atanassov and Gargov [19] introduced the interval-valued intuitionistic fuzzy sets (IVIFSs) which permit decision makers to use interval values rather than point values to express their information. Despite the superiority of IVIFSs in describing fuzziness and uncertainty compared with other preference relations, IVIFSs are not applicable to the situation of several possible preference values commonly seen in many practical problems. For this limitation, Torra and Narukawa [20, 21] introduced the hesitant fuzzy sets (HFSs), a novel and recent extension of fuzzy sets. Because the membership function of HFSs involves a set of possible values, HFSs are considered as a more powerful tool to manage the imprecise and vague information of decision maker in the process of decision making.

Since the proposing of HFS, it has attracted much attention, and a lot of relevant studies have been conducted in the last few years. Torra [21] discussed the relationship between HFS and other kinds of fuzzy sets and showed that the envelope of HFS was actually an IFS. Xia and Xu [22] gave the mathematical representation of HFS and defined some useful operations and aggregation operators for hesitant fuzzy information. Xu and Xia [23] proposed a variety of measures for hesitant fuzzy sets, e.g., distance, similarity, and correlation. Xu and Xia [24] also introduced the concepts of entropy and cross-entropy for hesitant fuzzy information and discussed the relationships among them. In order to cluster the massive evaluation information provided by different experts, Chen and Xu [25] proposed a series of correlation coefficient formulae for HFSs and applied them to calculating the degrees of correlation among HFSs. Rodríguez, R.M., et al. [26] made an overview on hesitant fuzzy sets including concept, extension, aggregation operators, measures, and applications like decision making, evaluation, and clustering. To deal with the case of quantitative settings, Zhao and Lin [27] introduced the concept of hesitant fuzzy linguistic term set (HFLTS). The theory of HFLTSs is very useful in objectively dealing with situations in which experts are hesitant in providing linguistic assessments. Recently, HFLTS has garnered considerable attention from researchers. Wei et al. [28] proposed a hesitant fuzzy LWA operator, a hesitant fuzzy LOWA operator, and comparison methods for the HFLTS. Liu and Rodríguez [29] presented a new representation of the HFLTS by means of a fuzzy envelope to carry out the computing with words processes. Liao et al. [28] investigated the correlation measures and correlation coefficients of HFLTSs and proposed several different types of correlation coefficients for HFLTSs such as intuitionistic fuzzy sets and hesitant fuzzy sets. To incorporate distribution information of hesitant fuzzy sets, Wu and Xu [30] defined the concept of possibility distribution for an HFLTS and proposed the corresponding operators and consensus measure. For further applications of HFLTSs to decision making, Zhu and Xu [31] developed a concept of hesitant fuzzy linguistic preference relations (HFLPRs) as a tool to collect and present the decision makers' preferences.

Inspired by the superiority of FHS in expressing human's hesitancy and based on the concept of fuzzy preference relation, fuzzy preference relations [32] are the most common tools to express decision makers' preferences over alternatives in decision making. A lot of studies have been done about them, such as the consistency methods [8], the group consensus methods [33], the priority methods [34, 35], and the incomplete values-determined methods [12]. Although different kinds of preference relations have already been successfully applied for decision making under hesitant fuzzy information, there have been few papers which have considered hesitation in a linguistic environment. Zhu and Xu [31] defined the concept of hesitant fuzzy linguistic preference relation (HFLPR). Zhang and Wu [36] defined the multiplicative consistency of an HFLPR. Wang and Xu [37] presented some consistency measures for an extended HFLPR (EHFLPR). In group decision making (GDM), preference relations are popular and powerful techniques for decision maker preference modeling [38]. The use of hesitant information in pairwise comparisons enriches the flexibility of qualitative decision making, and hesitant fuzzy linguistic preference relations (HFLPR) are one of the most commonly used. Aiming to deal with the linguistic preferences given by the decision makers, various linguistic models have been presented, such as the 2-tuple linguistic model, the symbolic model, the fuzzy number based model, the type-2 fuzzy sets based model, and the granular method [39–43]. Taking into account that the supplied preferences should satisfy some transitive properties and a consensus reaching process, Wu and Xu [44] developed separate consistency and consensus processes to deal with HFLPR individual rationality and group rationality. Furthermore, in the context of probability hesitant fuzzy preference relations (PHFPR), Wu and Xu [45] proposed an optimization based consistency improvement process to deal with the inconsistencies in a given PHFPR. In the proposed approaches, consensus measures based on the distances between the individuals were computed on three levels: an alternative pair level, an alternatives level, and a preference relations level. Hesitant fuzzy linguistic term sets (HFLTSs) can represent a much broader range of linguistic data. Wu and Xu [46] developed compromise solutions for multiple-attribute group decision making (MAGDM) using HFLTS and proposed two models to derive a compromise solution for the MAGDM problems. One is based on the VIKOR method and the other is based on the TOPSIS method.

It is obviously that hesitant fuzzy preference relation can aggregate more useful information to describe the uncertain and valueless situations in a more deep and objective way than fuzzy preference relation. But the set of the possible values of hesitant fuzzy elements are exact and crisp. However, under many conditions, in the process of trying to determine the relative importance via pairwise comparison, it is inadequate or insufficient to use the membership with a set of possible exact and crisp values due to the complexities and uncertainties of real problems. In addition, the estimation is inaccurate due to the inherent vague thought of human. Besides, the hesitant fuzzy elements of the hesitant fuzzy set are still crisp numbers, which are not easy to obtain because of human's hesitancy in actual life. Fortunately, triangular fuzzy is a method of transforming vague and uncertain linguistic variables into definite values. Furthermore triangular fuzzy number can solve the contradiction that the performance of the evaluated object cannot be measured accurately but can only be evaluated by natural language. So, the membership of hesitant fuzzy preference relation with a set of possible triangular fuzzy values is more objective than that with a set of exact and crisp values. In this paper, hesitant triangular fuzzy preference relation is proposed to overcome the limitation of HFPR. The rest of this paper is organized as follows. In Section 2, an important algorithm of fuzzy AHP is introduced, along with some basic knowledge on hesitant fuzzy set, hesitant triangular fuzzy set, fuzzy preference relation, and hesitant fuzzy preference relation. In Section 3, the model of hesitant triangular preference relation is proposed and the steps for applying the model are offered. In Section 4, an example is used to illustrate the proposed model. And finally, conclusions are given.

2. Preliminaries

In this section, an important algorithm of fuzzy AHP is introduced, along with some basic knowledge on hesitant fuzzy set (HFS), hesitant triangular fuzzy set (HTFS), fuzzy preference relation (FPR), and hesitant fuzzy preference relation (HFPR).

2.1. Fuzzy AHP. In the following, Chang's extent analysis method [47] is explained. Let $X = (x_1, x_2, \cdots, x_n)$ be an object set and $U = (g_1, g_2, \cdots, g_m)$ be a goal set. According to Chang's method, the object is considered one by one, and extent analysis is carried out for each goal. Therefore, there are m extent analysis values for each object, shown as below:

$$\widetilde{M}_{xi}^1, \widetilde{M}_{xi}^2, \cdots, \widetilde{M}_{xi}^m, \quad i = 1, 2, \cdots, n, \tag{1}$$

where $\widetilde{M}_{xi}^j$ $(i = 1, 2, \cdots, n, \; j = 1, 2, \cdots, m)$ are triangular fuzzy numbers.

Next, the steps of Chang's extent analysis are demonstrated.

Step 1. The value of fuzzy synthetic extent with respect to the i^{th} object is defined as

$$S_i \approx \sum_{j=1}^{m} \widetilde{M}_{xi}^j \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} \widetilde{M}_{xi}^j \right]^{-1}. \tag{2}$$

For obtaining $\sum_{j=1}^{m} \widetilde{M}_{xi}^j$, the fuzzy addition operation of m extent analysis values is performed on a particular matrix, i.e.,

$$\sum_{j=1}^{m} \widetilde{M}_{xi}^j = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j \right) \tag{3}$$

and to obtain $\sum_{i=1}^{n} \sum_{j=1}^{m} \widetilde{M}_{xi}^j$, the fuzzy addition operation of $\widetilde{M}_{xi}^j$ $(j = 1, 2, \cdots, m)$ values is performed, i.e.,

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \widetilde{M}_{xi}^j = \left(\sum_{i=1}^{n} l_i, \sum_{i=1}^{n} m_i, \sum_{i=1}^{n} u_i \right) \tag{4}$$

then, the inverse of above vector is calculated, i.e.,

$$\left[\sum_{i=1}^{n} \sum_{j=1}^{m} \widetilde{M}_{xi}^j \right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i} \right). \tag{5}$$

Step 2. Assuming that $\widetilde{M}_1$ and $\widetilde{M}_2$ are two triangular fuzzy numbers, the degree of possibility of $\widetilde{M}_2 = (l_2, m_2, u_2) \geq \widetilde{M}_1 = (l_1, m_1, u_1)$ is defined as

$$V\left(\widetilde{M}_2 \geq \widetilde{M}_1\right) = \sup \left[\min \left(\widetilde{M}_1(x), \widetilde{M}_2(x) \right) \right]. \tag{6}$$

This expression can be equivalently expressed as follows:

$$V\left(\widetilde{M}_2 \geq \widetilde{M}_1\right) = hgt\left(\widetilde{M}_1 \cap \widetilde{M}_2\right) = \widetilde{M}_2(d)$$
$$= \begin{cases} 1, & \text{if } m_2 \geq m_1, \\ 0, & \text{if } l_1 \geq u_2, \\ \dfrac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)}, & \text{otherwise}. \end{cases} \tag{7}$$

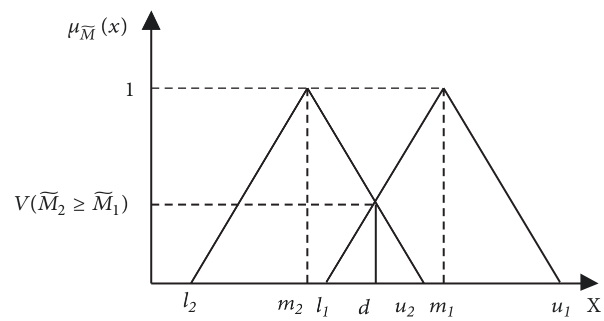

Figure 1: The intersection between $\widetilde{M}_1$ and $\widetilde{M}_2$.

Here d is the abscissa value of the highest intersection point D between $\widetilde{M}_1$ and $\widetilde{M}_2$, as shown in Figure 1. In order to compare $\widetilde{M}_1$ and $\widetilde{M}_2$, both values of $V(\widetilde{M}_1 \geq \widetilde{M}_2)$ and $V(\widetilde{M}_2 \geq \widetilde{M}_1)$ are required.

Step 3. The degree of possibility for a convex fuzzy number to be greater than k convex fuzzy numbers $\widetilde{M}_i$ $(i = 1, 2, \cdots, k)$ can be defined by

$$V\left(\widetilde{M} \geq \widetilde{M}_1, \widetilde{M}_2, \cdots, \widetilde{M}_k\right) = \min V\left(\widetilde{M} \geq \widetilde{M}_i\right), \quad i = 1, 2, \cdots, k. \tag{8}$$

Step 4. $W = (\min V(S_1 \geq S_k), \min V(S_2 \geq S_k), \cdots, \min V(S_n \geq S_k))^T$ is the weight vector for $k = 1, 2, \cdots, n$.

2.2. Concepts of HFS, HTFS, and FPR

Definition 1 (see [20, 21]). Let X be a fixed set; a hesitant fuzzy set on X is in terms of a function h that when applied to X returns a subset of $[0, 1]$, which can be represented as the following mathematical symbol by Xia and Xu [22]:

$$E = \left(\langle x, h_E(x) \rangle \mid x \in X \right), \tag{9}$$

where $h_E(x)$ is a set of values in $[0, 1]$, denoting the possible membership degree of the element $x \in X$ to the set E. For convenience, Xia and Xu [22] took $h = h_E(x)$ as a hesitant fuzzy element (HFE) and H as the set of all HFE.

Definition 2 (see [22]). For a HFE h, $s(h) = (1/\#h) \sum_{\gamma \in h} \gamma$ is called the score function of h, where $\#h$ represents the number of elements in h. For two HFEs h_1 and h_2, if $s(h_1) > s(h_2)$, then $h_1 > h_2$; if $s(h_1) = s(h_2)$, then $h_1 = h_2$.

Definition 3 (see [27]). Let X be a fixed set; a hesitant triangular fuzzy set (HTFS) on X is in terms of a function that when applied to each x in X returns a subset of values in $[0, 1]$, which can be expressed by

$$E = \left(\left\langle x, \widetilde{h}_{E(x)} \right\rangle \mid x \in X \right), \tag{10}$$

where $\widetilde{h}_{E(x)}$ is a set of possible triangular fuzzy values in $[0, 1]$, denoting the possible membership degrees of the element $x \in X$ to the set E. For convenience, Zhao and Lin [27] called

$\tilde{h}_{E(x)} = \tilde{h} = (\gamma^L, \gamma^M, \gamma^R)$ as a hesitant triangular fuzzy element (HTFE).

Based on Definition 3 and the operational principle of HFS, Zhao and Lin [27] defined some new operations on HTFE $\tilde{h} = (\gamma^L, \gamma^M, \gamma^R)$, $\tilde{h}_1 = (\gamma_1{}^L, \gamma_1{}^M, \gamma_1{}^R)$, and $\tilde{h} = (\gamma_2{}^L, \gamma_2{}^M, \gamma_2{}^R)$:

(1) $\tilde{h}^\lambda = \bigcup_{\gamma\in\tilde{h}}\{((\gamma^L)^\lambda, (\gamma^M)^\lambda, (\gamma^R)^\lambda)\}$;

(2) $\lambda\tilde{h} = \bigcup_{\gamma\in\tilde{h}}\{(1-(1-\gamma^L)^\lambda, 1-(1-\gamma^M)^\lambda, 1-(1-\gamma^R)^\lambda)\}$;

(3) $\tilde{h}_1 \oplus \tilde{h}_2 = \bigcup_{\gamma_1\in\tilde{h}_1,\gamma_2\in\tilde{h}_2}\{(\gamma_1{}^L + \gamma_2{}^L - \gamma_1{}^L\gamma_2{}^L, \gamma_1{}^M + \gamma_2{}^M - \gamma_1{}^M\gamma_2{}^M, \gamma_1{}^R + \gamma_2{}^R - \gamma_1{}^R\gamma_2{}^R)\}$;

(4) $\tilde{h}_1 \otimes \tilde{h}_2 = \bigcup_{\gamma_1\in\tilde{h}_1,\gamma_2\in\tilde{h}_2}\{(\gamma_1{}^L\gamma_2{}^L, \gamma_1{}^M\gamma_2{}^M, \gamma_1{}^R\gamma_2{}^R)\}$.

Based on the above definition, Zhao and Lin [27] also defined the score function of $\tilde{h}$:

$$s(\tilde{h}) = \frac{1}{\#\tilde{h}}\sum_{\gamma\in\tilde{h}}\gamma = \left(\frac{1}{\#\tilde{h}}\sum_{\gamma\in\tilde{h}}\gamma^L, \frac{1}{\#\tilde{h}}\sum_{\gamma\in\tilde{h}}\gamma^M, \frac{1}{\#\tilde{h}}\sum_{\gamma\in\tilde{h}}\gamma^R\right), \quad (11)$$

where $\#\tilde{h}$ is the number of triangular fuzzy values in $\tilde{h}$, and $s(\tilde{h})$ is a triangular fuzzy value in $[0,1]$. For two HTFEs $\tilde{h}_1$ and $\tilde{h}_2$, if $s(\tilde{h}_1) \geq s(\tilde{h}_2)$, then $\tilde{h}_1 \geq \tilde{h}_2$.

Definition 4 (see [27]). Let $\tilde{h}_j$ $(j = 1,2,\cdots,n)$ be a collection of HTFEs. The hesitant triangular fuzzy weighted averaging (HTFWA) operator is a mapping $\tilde{H}^n \longrightarrow \tilde{H}$.

$$HTFWA(\tilde{h}_1,\tilde{h}_2,\cdots,\tilde{h}_n) = \bigoplus_{j=1}^{n}(\omega_j\tilde{h}_j) = \bigcup_{\tilde{\gamma}_1\in\tilde{h}_1,\tilde{\gamma}_2\in\tilde{h}_2,\cdots,\tilde{\gamma}_n\in\tilde{h}_n}\left\{\left(1-\prod_{j=1}^{n}(1-\gamma_j{}^L)^{\omega_j}, 1-\prod_{j=1}^{n}(1-\gamma_j{}^M)^{\omega_j}, 1-\prod_{j=1}^{n}(1-\gamma_j{}^R)^{\omega_j}\right)\right\} \quad (12)$$

where $\omega = (\omega_1,\omega_2,\cdots,\omega_n)^T$ represents the weight vector of $\tilde{h}_j$ $(j = 1,2,\cdots,n)$, and $\omega_j > 0$, $\sum_{j=1}^n \omega_j = 1$. In particular, if $\omega = (1/n, 1/n, \cdots, 1/n)^T$, then the HTFWA operator degrades to the hesitant triangular fuzzy averaging (HTFA) operator.

$$HTFA(\tilde{h}_1,\tilde{h}_2,\cdots,\tilde{h}_n) = \bigoplus_{j=1}^{n}\left(\frac{1}{n}\tilde{h}_j\right) = \bigcup_{\tilde{\gamma}_1\in\tilde{h}_1,\tilde{\gamma}_2\in\tilde{h}_2,\cdots,\tilde{\gamma}_n\in\tilde{h}_n}\left\{\left(1-\prod_{j=1}^{n}(1-\gamma_j{}^L)^{1/n}, 1-\prod_{j=1}^{n}(1-\gamma_j{}^M)^{1/n}, 1-\prod_{j=1}^{n}(1-\gamma_j{}^R)^{1/n}\right)\right\} \quad (13)$$

Definition 5 (see [27]). Let $\tilde{h}_j$ $(j = 1,2,\cdots,n)$ be a collection of HTFEs. The hesitant triangular fuzzy weighted geometric (HTFWG) operator is a mapping $\tilde{H}^n \longrightarrow \tilde{H}$.

$$HTFWG(\tilde{h}_1,\tilde{h}_2,\cdots,\tilde{h}_n) = \bigoplus_{j=1}^{n}(\tilde{h}_j)^{w_j} = \bigcup_{\tilde{\gamma}_1\in\tilde{h}_1,\tilde{\gamma}_2\in\tilde{h}_2,\cdots,\tilde{\gamma}_n\in\tilde{h}_n}\left\{\left(\prod_{j=1}^{n}(\gamma_j{}^L)^{w_j}, \prod_{j=1}^{n}(\gamma_j{}^M)^{w_j}, \prod_{j=1}^{n}(\gamma_j{}^R)^{w_j}\right)\right\} \quad (14)$$

where $\omega = (\omega_1,\omega_2,\cdots,\omega_n)^T$ represents the weight vector of $\tilde{h}_j$ $(j = 1,2,\cdots,n)$, and $\omega_j > 0$, $\sum_{j=1}^n \omega_j = 1$. In particular, if $\omega = (1/n, 1/n, \cdots, 1/n)^T$, then the HTFWG operator degrades to the hesitant triangular fuzzy geometric (HTFG) operator.

$$HTFG(\tilde{h}_1,\tilde{h}_2,\cdots,\tilde{h}_n) = \bigoplus_{j=1}^{n}(\tilde{h}_j)^{w_j} = \bigcup_{\tilde{\gamma}_1\in\tilde{h}_1,\tilde{\gamma}_2\in\tilde{h}_2,\cdots,\tilde{\gamma}_n\in\tilde{h}_n}\left\{\left(\prod_{j=1}^{n}(\gamma_j{}^L)^{1/n}, \prod_{j=1}^{n}(\gamma_j{}^M)^{1/n}, \prod_{j=1}^{n}(\gamma_j{}^R)^{1/n}\right)\right\}. \quad (15)$$

Definition 6 (see [48]). Let $X = \{x_1, x_2, \cdots, x_n\}$ be a finite set of alternatives; then $R = (r_{ij})_{n\times n}$ is called an additive FPRR on the product set $X \times X$ with the membership function u_R: $X \times X \longrightarrow [0,1]$, $u_R(x_i, x_j) = r_{ij}$, satisfying $r_{ij} + r_{ji} = 1$, $i, j = 1, 2, \cdots, n$.

Usually, a matrix $R = (r_{ij})_{n\times n}$ denotes the preference relation between two alternatives, and r_{ij} denotes the preference degree of x_i over x_j. In particular, $r_{ij} = 0$ implies that x_j is totally preferred to x_i, and $r_{ij} = 0.5$ indicates that there is no difference between x_i and x_j.

2.3. Concepts of Hesitant Fuzzy Preference Relations. Let $X = \{x_1, x_2, \cdots, x_n\}$ be a fixed set of alternatives; then $A = (a_{ij})_{n\times n}$ is called the fuzzy preference relation matrix [2] on $X \times X$ with $a_{ij} \geq 0$, $a_{ij} + a_{ji} = 1$, $i, j = 1, 2, \cdots, n$, where a_{ij} denotes the degree that the alternative x_i is prior to x_j. In particular, $a_{ij} = 0.5$ means the same importance of x_i and x_j, which can be expressed by $x_i = x_j$; $0 \leq a_{ij} < 0.5$ means that x_j is more important than x_i, expressed by $x_i \prec x_j$, and the smaller the value of a_{ij}, the more important x_j to x_i; on the contrary, $0.5 < a_{ij} \leq 1$ means that x_i is more important than x_j, denoted by $x_i \succ x_j$, and the larger the value of a_{ij}, the more important x_i to x_j. Obviously, the value of a_{ij} indicates that fuzzy preference relation is a certain value between 0 and 1. If a decision group containing several

Table 1: 0.1-0.9 scale values and the means.

Fuzzy number	Meaning
0.5	Equally preferred
0.6	Moderately preferred
0.7	Strongly preferred
0.8	Very strongly preferred
0.9	Extremely preferred
Other values between 0 and 1	Intermediate values used to present compromise

experts is authorized to estimate the degrees of x_i preferred to x_j, there are several possible values, because of the fuzziness and hesitancy of experts and the unpredictable information. For instance, some experts provide a_{ij}^1, some provide a_{ij}^2, and the others provide a_{ij}^3. Therefore, it is clear that $a_{ij}^1, a_{ij}^2, a_{ij}^3 \in [0,1]$, and a_{ij} is called HFE $a_{ij} = \{a_{ij}^1, a_{ij}^2, a_{ij}^3\}$, which implies the preference relation of x_i over x_j. From the above analysis, it can be concluded that HFPR is a novel generalization of fuzzy preference relation, whose fundamental units can be evaluated using the 0.1-0.9 scale described in Table 1.

Definition 7 (see [49, 50]). Let $X = \{x_1, x_2, \cdots, x_n\}$ be a fixed set; then a hesitant fuzzy preference relation A on X is expressed by a matrix $A = (a_{ij})_{n\times n} \subset X \times X$, where $a_{ij} = \{a_{ij}^s, s = 1, 2, \cdots, l_{a_{ij}}\}$ is a HFE denoting all possible degrees to which x_i is preferred to x_j. Additionally, a_{ij} should satisfy the following requirements.

$$a_{ij}^{\sigma(s)} + a_{ji}^{\sigma(l_{a_{ij}}-s+1)} = 1, \quad i, j = 1, 2, \cdots, n, \tag{16}$$

$$a_{ii} = \{0.5\}, \quad i = 1, 2, \cdots, n, \tag{17}$$

$$l_{a_{ij}} = l_{a_{ji}}, \quad i, j = 1, 2, \cdots, n \tag{18}$$

where the values in a_{ij} are assumed to be arranged in an increasing order, $a_{ij}^{\sigma(s)}$ denotes the s^{th} largest value in a_{ij}, and $l_{a_{ij}}$ denotes the number of values in a_{ij}. Obviously, (16) is the reciprocal condition, which means that if $a_{ij} = \{a_{ij}^s, s = 1, 2, \cdots, l_{a_{ij}}\}$ is known, then a_{ji} can be easily obtained by $a_{ji} = \{1 - a_{ij}^s, s = 1, 2, \cdots, l_{a_{ij}}\}$; (17) implies that there is no difference between x_i and x_j. If there is a single value in a_{ij}, $(i, j = 1, 2, \cdots, n)$, then the hesitant fuzzy preference relation degrades to the usual fuzzy ones.

3. Proposed Methodology

3.1. Hesitant Triangular Fuzzy Preference Relations (HTFPR) Model. In many practical situations, due to the incompleteness and uncertainties of information as well as the hesitancy of humans, it is not easy for decision makers to express their preferences between two alternatives with exact and crisp values in traditional HFS. Hence, triangular fuzzy set can model real-life decision problems in a more suitable and sufficient way than real numbers.

Definition 8. Let $X = \{x_1, x_2, \cdots, x_n\}$ be a fixed set; then an additive hesitant triangular fuzzy preference relation $\widetilde{H}$ on X is expressed by a matrix $\widetilde{H} = (\widetilde{h}_{ij})_{n\times n} \subset X \times X$, where $\widetilde{h}_{ij} = \{(\gamma_{ij}^L, \gamma_{ij}^M, \gamma_{ij}^R)^s, s = 1, 2, \cdots, l_{\widetilde{h}_{ij}}\}$ is a hesitant triangular fuzzy element (HTFE) denoting all possible degrees to which x_i is preferred to x_j. Moreover, $\widetilde{h}_{ij}$ satisfies the following requirements.

(1)

$$\begin{gathered}\left(\gamma^L\right)_{ij}^{\sigma(s)} + \left(\gamma^R\right)_{ji}^{\sigma(l_{h_{ij}}-s+1)} = 1, \\ \left(\gamma^M\right)_{ij}^{\sigma(s)} + \left(\gamma^M\right)_{ji}^{\sigma(l_{h_{ij}}-s+1)} = 1, \\ \left(\gamma^R\right)_{ij}^{\sigma(s)} + \left(\gamma^L\right)_{ji}^{\sigma(l_{h_{ij}}-s+1)} = 1, \\ i, j = 1, 2, \cdots, n;\end{gathered} \tag{19}$$

(2)

$$\widetilde{h}_{ii} = \left\{\left(\gamma_{ii}^L, \gamma_{ii}^M, \gamma_{ii}^R\right)\right\} = \{(0.5, 0.5, 0.5)\} = \{0.5\}, \quad i = 1, 2, \cdots, n; \tag{20}$$

(3)

$$l_{\widetilde{h}_{ij}} = l_{\widetilde{h}_{ji}}, \quad i, j = 1, 2, \cdots, n. \tag{21}$$

Here the values in $\widetilde{h}_{ij}$ are assumed to be arranged in an increasing order, $\widetilde{h}_{ij}^{\sigma(s)}$ denotes the s^{th} largest value in $\widetilde{h}_{ij}$, and $l_{\widetilde{h}_{ij}}$ denotes the number of values in $\widetilde{h}_{ij}$. Obviously, (19) is the reciprocal condition, which means that if $\widetilde{h}_{ij} = \{(\gamma_{ij}^L, \gamma_{ij}^M, \gamma_{ij}^R)^{\sigma(s)}, s = 1, 2, \cdots, l_{\widetilde{h}_{ij}}\}$ is known, then $\widetilde{h}_{ji}$ can be obtained easily by $\widetilde{h}_{ji} = \{(1-\gamma_{ij}^R, 1-\gamma_{ij}^M, 1-\gamma_{ij}^L)^{\sigma(l_{\widetilde{h}_{ij}}-s+1)}, s = 1, 2, \cdots, l_{\widetilde{h}_{ij}}\}$. Equation (20) implies that there is no difference between x_i and x_j. If there is a single value in $\widetilde{h}_{ij}$, $(i, j = 1, 2, \cdots, n)$, then hesitant triangular fuzzy preference relation degrades to the usual triangular fuzzy ones.

Example 9. Let $X = \{x_1, x_2, x_3\}$; a decision group containing several experts is authorized to provide the preference values of x_1 over x_2. There are several possible results among the experts. Some experts provide (0.2, 0.3, 0.4), while others provide (0.3, 0.4, 0.5). Then according to Definition 6, the degree of x_2 is preferred to x_1, which should be (1-0.5 = 0.5, 1-0.4 = 0.6, 1-0.3 = 0.7) and (1-0.4 = 0.6, 1-0.3 = 0.7, 1-0.2 = 0.8), respectively. In this case, $\widetilde{h}_{12} = \{(0.2, 0.3, 0.4), (0.3, 0.4, 05)\}$ is called a HTFE, denoting the preference information about x_1 preference to x_2. The preference information about x_2 over x_1 is denoted by a HTFE $\widetilde{h}_{21} = \{(0.5, 0.6, 0.7), (0.6, 0.7, 0.8)\}$. Similarly, let $\widetilde{h}_{13} = \{(0.5, 0.6, 0.7)\}$ and $\widetilde{h}_{23} = \{(0.2, 0.4, 0.5), (0.5, 0.6, 0.7), (0.5, 0.7, 0.8)\}$; then, we can correspondingly obtain $\widetilde{h}_{31} = \{(0.3, 0.4, 0.5)\}$ and $\widetilde{h}_{32} = \{(0.2, 0.3, 0.5), (0.3, 0.4, 0.5)(0.5, 0.6, 0.8)\}$.

Based on above analysis, the hesitant triangular fuzzy preference relation $\widetilde{H} = (\widetilde{h}_{ij})_{n\times n} \subset X \times X$ is presented in Table 2.

3.2. The Steps of HTRPR. The proposed hesitant triangular fuzzy preference relation model can be used to determine

Table 2: The hesitant triangular fuzzy preference relation $\widetilde{H}$.

	x_1	x_2	x_3
x_1	{0.5}	{(0.2, 0.3, 0.4) , (0.3, 0.4, 0.5)}	{(0.5, 0.6, 0.7)}
x_2	{(0.5, 0.6, 0.7) , (0.6, 0.7, 0.8)}	{0.5}	{(0.2, 0.4, 0.5) , (0.5, 0.6, 0.7) , (0.5, 0.7, 0.8)}
x_3	{(0.3, 0.4, 0.5)}	{(0.2, 0.3, 0.5) , (0.3, 0.4, 0.5) , (0.5, 0.6, 0.8)}	{0.5}

the weights of failure modes of a work system. The steps are shown below.

Step 1. A committee comprised of several experts is set up to identify the potential failure modes of a work system and to provide their preference information via pairwise comparison. The assessment preference values are denoted by hesitant triangular fuzzy elements h_{ij} $(i, j = 1, 2, \cdots, n)$, and the HTFPR matrix $\widetilde{H} = (\widetilde{h}_{ij})_{n\times n}$ is constructed.

Step 2. HTFWA (or HTFWG) operator is used to aggregate all h_{ij} $(j = 1, 2, \cdots, n)$ which correspond to the alternative x_i.

Step 3. The score values $s(\widetilde{h}_i)$ of x_i $(i = 1, 2, \cdots, n)$ are calculated using (11).

Step 4. Chang's fuzzy AHP method is used to obtain the weights of the failure modes. Based on the fuzzy synthetic extent proposed by Chang [31], $\sum_{j=1}^{m} \widetilde{M}_{xi}^{j}$ can be treated as $s(\widetilde{h}_i)$, and $\sum_{i=1}^{n}\sum_{j=1}^{m} \widetilde{M}_{xi}^{j}$ can be treated as $\sum_{i=1}^{n} s(\widetilde{h}_i)$. In this case, the value of HTF synthetic extent with respect to the i^{th} alternative can be expressed by

$$S_i' \approx s\left(\widetilde{h}_i\right) \otimes \left[\sum_{i=1}^{n} s\left(\widetilde{h}_i\right)\right]^{-1}, \quad (22)$$

where ⊗ denotes the extended multiplication of two hesitant triangular fuzzy numbers, i.e.,

$$\sum_{i=1}^{n} s\left(\widetilde{h}_i\right) = \left(\sum_{i=1}^{n}\frac{1}{\#\widetilde{h}}\sum_{\gamma\in\widetilde{h}}{\gamma_i}^{L}, \sum_{i=1}^{n}\frac{1}{\#\widetilde{h}}\sum_{\gamma\in\widetilde{h}}{\gamma_i}^{M}, \sum_{i=1}^{n}\frac{1}{\#\widetilde{h}}\sum_{\gamma\in\widetilde{h}}{\gamma_i}^{R}\right), \quad (23)$$

and

$$\left[\sum_{i=1}^{n} s\left(\widetilde{h}_i\right)\right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n}\left(1/\#\widetilde{h}\right)\sum_{\gamma\in\widetilde{h}}\gamma^{R}}, \frac{1}{\sum_{i=1}^{n}\left(1/\#\widetilde{h}\right)\sum_{\gamma\in\widetilde{h}}\gamma^{M}}, \frac{1}{\sum_{i=1}^{n}\left(1/\#\widetilde{h}\right)\sum_{\gamma\in\widetilde{h}}\gamma^{L}}\right). \quad (24)$$

Similarly, (6), (7), and (8) can be equivalently expressed as

$$V\left(s\left(\widetilde{h}_2\right) \geq s\left(\widetilde{h}_1\right)\right) = \sup\left[\min\left(s\left(\widetilde{h}_1(x)\right), s\left(\widetilde{h}_2(x)\right)\right)\right], \quad (25)$$

$$V\left(s\left(\widetilde{h}_2\right) \geq s\left(\widetilde{h}_1\right)\right) = hgt\left(s\left(\widetilde{h}_1\right) \bigcap s\left(\widetilde{h}_2\right)\right) = s\left(\widetilde{h}_2(d)\right) = \begin{cases} 1, & \text{if } \frac{1}{\#\widetilde{h}}\sum_{\gamma\in\widetilde{h}}{\gamma_2}^{M} \geq \frac{1}{\#\widetilde{h}}\sum_{\gamma\in\widetilde{h}}{\gamma_1}^{M}, \\ 0, & \text{if } \frac{1}{\#\widetilde{h}}\sum_{\gamma\in\widetilde{h}}{\gamma_1}^{L} \geq \frac{1}{\#\widetilde{h}}\sum_{\gamma\in\widetilde{h}}{\gamma_2}^{R}, \\ \dfrac{\left(1/\#\widetilde{h}\right)\sum_{\gamma\in\widetilde{h}}{\gamma_1}^{L} - \left(1/\#\widetilde{h}\right)\sum_{\gamma\in\widetilde{h}}{\gamma_2}^{R}}{\left(\left(1/\#\widetilde{h}\right)\sum_{\gamma\in\widetilde{h}}{\gamma_2}^{M} - \left(1/\#\widetilde{h}\right)\sum_{\gamma\in\widetilde{h}}{\gamma_2}^{R}\right) - \left(\left(1/\#\widetilde{h}\right)\sum_{\gamma\in\widetilde{h}}{\gamma_1}^{M} - \left(1/\#\widetilde{h}\right)\sum_{\gamma\in\widetilde{h}}{\gamma_1}^{L}\right)}, & \text{otherwise.} \end{cases} \quad (26)$$

and

$$V\left(s\left(\widetilde{h}\right) \geq s\left(\widetilde{h}_1\right), s\left(\widetilde{h}_2\right), \cdots, s\left(\widetilde{h}_k\right)\right) = \min V\left(s\left(\widetilde{h}\right) \geq s\left(\widetilde{h}_i\right)\right), \quad i = 1, 2, \cdots, k \quad (27)$$

thus, $W = (\min V(S_1' \geq S_k'), \min V(S_2' \geq S_k'), \cdots, \min V(S_n' \geq S_k'))^T$ is the weight vector for $k = 1, 2, \cdots, n$ of failure modes.

4. Case Illustration

This section provides an example of a practical case involving the assessment of the potential failure modes of a kind of typical amusement rides-roller coaster, to illustrate the proposed model. Then a comparative analysis is conducted between the proposed model and the hesitant fuzzy preference relation [49] to show its superiority.

Amusement rides are a very popular recreational activity and take many forms in structure and movement. For example, a device or vehicle may be driven or guided by a power equipment or operator on a track or slide, and sometimes it walks through or bounces on an air bounce, funhouse, or maze [51]. Usually, amusement rides are classified into two categories, i.e., "fixed site ride" and "mobile ride" [51, 52] which are operated in fixed site and mobile forms, respectively [53]. Generally, the injuries and failure of amusement ride are considered to be infrequent by public, but are noteworthy when they occur. Carrying out quantitative and qualitative safety assessment of amusement ride draws much attention of the public and is important for continuous improvement [54–56]. As a kind of typical amusement ride, roller coaster is very popular among young people and is located at outdoor midways generally, as well as big amusement parks or theme parks. It is classified as 'fixed site ride', and its safe and reliable operation is also worthy of researching. To find out the major potential failure modes and derive the priority of them, it is important to conduct the safety management of roller coasters.

4.1. The Proposed Method. In the following, the proposed method is used to obtain the metal-frame major potential failure modes and its priority of roller coaster.

Step 1. An expert team consisting of three cross-functional members was organized to identify the failure modes of a roller coaster and to prioritize them via pairwise comparison. According to the expert team, the major potential failure modes were identified as weld metal cracking, corrosion, joint angle loosening, deformation, and wear denoted by x_i $(i = 1, 2, 3, 4, 5)$. Because of the team members' limited expertise in the problem domain, lack of knowledge or data, and other reasons, it is difficult for experts to reach a consensus to evaluate these five potential failure modes and their relative importance weights precisely. So, the values of preference relations taking the form of HTFE by experts are more reasonable for real practice. Then, the expert team compare these five potential failure modes and provide their preference values denoted by $\widetilde{H} = (\widetilde{h}_{ij})_{5\times 5}$, as expressed in Table 3, where $\widetilde{h}_{ij}$ $(i, j = 1, 2, 3, 4, 5)$ are in the form of HTFPRs.

Step 2. The preference relation information given in matrix $\widetilde{H}$ and the HTFWA (or HTFWG) operator are used to aggregate all h_{ij} $(j = 1, 2, \cdots, n)$ which correspond to the failure modes x_i. Taking the failure modes weld metal cracking x_1 as an example, since $\omega = (1/n, 1/n, \cdots, 1/n)^T$, we have

$$\begin{aligned}
\widetilde{h}_1 &= HTFWA = HTFA\left(\widetilde{h}_{11}, \widetilde{h}_{12}, \widetilde{h}_{13}, \widetilde{h}_{14}, \widetilde{h}_{15}\right) \\
&= \bigoplus_{j=1}^{5}\left(\frac{1}{5}\widetilde{h}_{1j}\right) = \bigcup_{\widetilde{\gamma}_1\in\widetilde{h}_1,\widetilde{\gamma}_2\in\widetilde{h}_2,\widetilde{\gamma}_3\in\widetilde{h}_3,\widetilde{\gamma}_4\in\widetilde{h}_4,\widetilde{\gamma}_5\in\widetilde{h}_5} \left\{\left(1 - \prod_{j=1}^{5}\left(1-\gamma_{1j}{}^{L}\right)^{1/5}, 1 - \prod_{j=1}^{n}\left(1-\gamma_{1j}{}^{M}\right)^{1/5}, 1 - \prod_{j=1}^{n}\left(1-\gamma_{1j}{}^{R}\right)^{1/5}\right)\right\} \\
&= \{\{0.5\}, \{(0.5, 0.7, 0.8), (0.6, 0.8, 0.9)\}, \{(0.7, 0.8, 0.9), (0.6, 0.9, 1.0)\}, \{(0.6, 0.7, 0.8)\}, \{(0.6, 0.8, 0.9)\}\} \\
&= \{(0.6272, 0.7175, 0.8179), (0.6562, 0.7540, 1), (0.6481, 0.7395, 0.8415), (0.6755, 0.7732, 1)\}.
\end{aligned} \tag{28}$$

Step 3. The score values $s(\widetilde{h}_i)$ $(i = 1, 2, 3, 4, 5)$ of the overall hesitant triangular fuzzy preference relation values $\widetilde{h}_i$ $(i = 1, 2, 3, 4, 5)$ are calculated:

$$\begin{aligned}
s\left(\widetilde{h}_1\right) &= \frac{1}{\#\widetilde{h}_1}\sum_{\gamma\in\widetilde{h}_1}\gamma_1 \\
&= \left(\frac{1}{\#\widetilde{h}_1}\sum_{\gamma\in\widetilde{h}_1}\gamma_1{}^{L}, \frac{1}{\#\widetilde{h}_1}\sum_{\gamma\in\widetilde{h}_1}\gamma_1{}^{M}, \frac{1}{\#\widetilde{h}_1}\sum_{\gamma\in\widetilde{h}_1}\gamma_1{}^{R}\right) \\
&= (0.6518, 0.7461, 0.9149),
\end{aligned} \tag{29}$$

$$s\left(\widetilde{h}_2\right) = (0.3471, 0.4344, 0.5174),$$

$$s\left(\widetilde{h}_3\right) = (0.4902, 0.5589, 0.6591),$$

$$s\left(\widetilde{h}_4\right) = (0.3480, 0.4262, 0.5176),$$

$$s\left(\widetilde{h}_5\right) = (0.3708, 0.4654, 0.5695).$$

Step 4. The weights of the failure modes are obtained.

Firstly, by applying (22), we have

$$\begin{aligned}
S_1' &= s\left(\widetilde{h}_1\right) \otimes \left[\sum_{i=1}^{5} s\left(\widetilde{h}_i\right)\right]^{-1} \\
&= (0.6518, 0.7461, 0.9149) \otimes \left(\frac{1}{3.2612}, \frac{1}{2.707}, \frac{1}{2.2963}\right) \\
&\approx (0.1999, 0.2756, 0.3984), \\
S_2' &= (0.3471, 0.4344, 0.5174) \otimes \left(\frac{1}{3.2612}, \frac{1}{2.707}, \frac{1}{2.2963}\right) \\
&\approx (0.1064, 0.1605, 0.2253), \\
S_3' &= (0.4902, 0.5589, 0.6591) \otimes \left(\frac{1}{3.2612}, \frac{1}{2.707}, \frac{1}{2.2963}\right) \\
&\approx (0.1503, 0.2065, 0.287),
\end{aligned}$$

Table 3: The hesitant triangular fuzzy preference relation matrix $\widetilde{H}$ of failure modes.

	weld metal cracking	corrosion	joint angle loosening	deformation	wear
weld metal cracking	{0.5}	{(0.5, 0.7, 0.8), (0.6, 0.8, 0.9)}	{(0.7, 0.8, 0.9), (0.6, 0.9, 1.0)}	{(0.6, 0.7, 0.8), (0.7, 0.8, 0.9)}	{(0.6, 0.8, 0.9)}
corrosion	{(0.1, 0.2, 0.4), (0.2, 0.3, 0.5)}	{0.5}	{(0.3, 0.4, 0.5), (0.6, 0.7, 0.8)}	{(0.3, 0.4, 0.5)}	{(0.2, 0.4, 0.5), (0.3, 0.4, 0.5)}
fatigue	{(0, 0.1, 0.4), (0.1, 0.2, 0.3)}	{(0.2, 0.3, 0.4), (0.5, 0.6, 0.7)}	{0.5}	{(0.6, 0.7, 0.8), (0.7, 0.8, 0.9)}	{(0.6, 0.7, 0.8)}
deformation	{(0.1, 0.2, 0.3), (0.2, 0.3, 0.4)}	{(0.5, 0.6, 0.7)}	{(0.1, 0.2, 0.3), (0.2, 0.3, 0.4)}	{0.5}	{(0.2, 0.3, 0.5), (0.4, 0.5, 0.6)}
wear	{(0.1, 0.2, 0.4)}	{(0.5, 0.6, 0.7), (0.5, 0.6, 0.8)}	{(0.2, 0.3, 0.4)}	{(0.4, 0.5, 0.6), (0.5, 0.7, 0.8)}	{0.5}

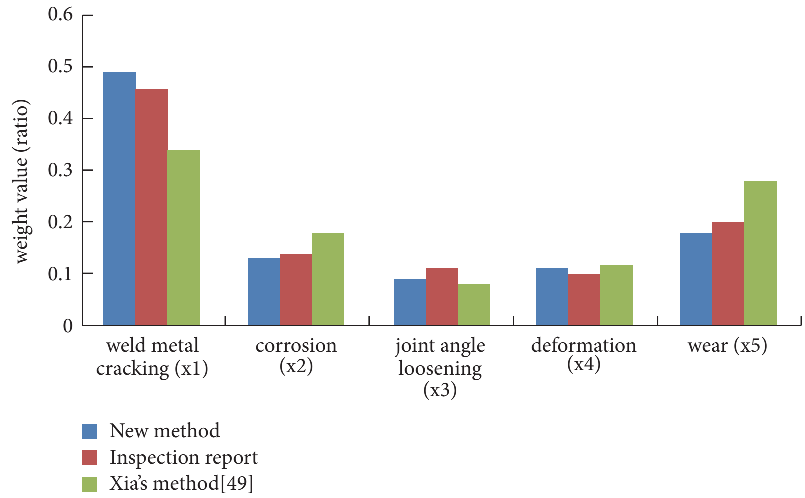

FIGURE 2: Comparison of the new method, Xia's method, and the inspection reports.

$$\begin{aligned} S_4' &= (0.348, 0.4262, 0.5176) \\ &\quad \otimes \left(\frac{1}{3.2612}, \frac{1}{2.707}, \frac{1}{2.2963}\right) \\ &\approx (0.1067, 0.1575, 0.2251), \\ S_5' &= (0.4593, 0.5414, 0.6522) \\ &\quad \otimes \left(\frac{1}{3.2612}, \frac{1}{2.707}, \frac{1}{2.2963}\right) \\ &\approx (0.1408, 0.2, 0.284). \end{aligned} \tag{30}$$

Then, using (25) and (26), we have

$$\begin{aligned} &V\left(S_1' \geq S_2'\right) = 1, \\ &V\left(S_1' \geq S_3'\right) = 1, \\ &V\left(S_1' \geq S_4'\right) = 1, \\ &V\left(S_1' \geq S_2'\right) = 1, \\ &V\left(S_2' \geq S_1'\right) = 0.26, \\ &V\left(S_2' \geq S_3'\right) = 0.62, \\ &V\left(S_2' \geq S_4'\right) = 1, \\ &V\left(S_2' \geq S_5'\right) = 0.68, \\ &V\left(S_3' \geq S_1'\right) = 0.19, \\ &V\left(S_3' \geq S_2'\right) = 1, \\ &V\left(S_3' \geq S_4'\right) = 1, \\ &V\left(S_3' \geq S_5'\right) = 1, \\ &V\left(S_4' \geq S_1'\right) = 0.21, \\ &V\left(S_4' \geq S_2'\right) = 0.98, \\ &V\left(S_4' \geq S_3'\right) = 0.61, \\ &V\left(S_4' \geq S_5'\right) = 0.67, \\ &V\left(S_5' \geq S_1'\right) = 0.37, \\ &V\left(S_5' \geq S_2'\right) = 1, \\ &V\left(S_5' \geq S_3'\right) = 0.95, \\ &V\left(S_5' \geq S_3'\right) = 1. \end{aligned} \tag{31}$$

Finally, using (27), the weight vector of these five potential failure modes can be obtained, i.e., $W_{FM} = (0.49, 0.13, 0.09, 0.11, 0.18)^T$. Thus, the priority ranking of the five potential failure modes follows the order of $FM_{x_1} \succ FM_{x_5} \succ FM_{x_2} \succ FM_{x_4} \succ FM_{x_3}$, as shown in Figure 2. The result indicates that the most important failure mode is the *weld metal cracking*, followed by *wear, corrosion, deformation,* and *joint angle loosening,* and the *weld metal cracking* should be given the top priority for correction.

4.2. The Hesitant Fuzzy Preference Relation Method. In the following, the hesitant fuzzy preference relation method [49] is used to obtain the metal-frame major potential failure modes and its priority for the same case as above. Based on Table 3, the same expert team gave the hesitant fuzzy preference relation matrix H of failure modes as shown in Table 4. And using the hesitant fuzzy preference relation method, we can obtain the weight vector of these five potential failure modes as $W_{FM} = (0.42, 0.36, 0.16, 0.21, 0.38)^T$ (the detailed solution process can be referred to in [49]). Thus, the priority ranking of the five potential failure modes follows the order of $FM_{x_1} \succ FM_{x_5} \succ FM_{x_2} \succ FM_{x_4} \succ FM_{x_3}$, as shown in Figure 2.

Table 4: The hesitant fuzzy preference relation matrix H of failure modes.

	weld metal cracking	corrosion	joint angle loosening	deformation	wear
weld metal cracking	{0.5}	{0.5, 0.6, 0.7, 0.8, 0.9}	{0.6, 0.7, 0.8, 0.9}	{0.6, 0.7, 0.8, 0.9}	{0.6, 0.8, 0.9}
corrosion	{0.1, 0.2, 0.3, 0.4, 0.5}	{0.5}	{0.4, 0.5, 0.6, 0.7, 0.8}	{0.3, 0.4, 0.5}	{0.2, 0.3, 0.4, 0.5}
joint angle loosening	{0.1, 0.2, 0.3, 0.4, 0.5}	{0.2, 0.3, 0.4, 0.5, 0.6}	{0.5}	{0.6, 0.7, 0.8, 0.9}	{0.6, 0.7, 0.8}
deformation	{0.1, 0.2, 0.3, 0.4}	{0.5, 0.6, 0.7}	{0.1, 0.2, 0.3, 0.4)}	{0.5}	{0.2, 0.3, 0.4, 0.5, 0.6}
wear	{0.1, 0.2, 0.4}	{0.5, 0.6, 0.7, 0.8}	{0.2, 0.3, 0.4}	{0.4, 0.5, 0.6, 0.7, 0.8}	{0.5}

4.3. Comparative Analysis of the Two Methods. According to the inspection reports of failure modes of roller coasters coverage for the latest five-year period by the China Special Equipment Inspection Institute, the normalized proportions of these five failure modes are 0.46, 0.14, 0.11, 0.10, and 0.20, which means the priority ranking of them follows the order of $FM_{x_1} \succ FM_{x_5} \succ FM_{x_2} \succ FM_{x_3} \succ FM_{x_4}$, as shown in Figure 2. And Figure 2 shows that the results of the new method and Xia's method are pretty similar to those of the inspection reports, except for the priority ranking of FM_{x_3} and FM_{x_4}. The difference of weight values (ratio) of FM_{x_3} and FM_{x_4} is very slight, which has no practical significance and can be neglected from the point of view of engineering. Furthermore, it is obvious from Figure 2 that the gaps between the new method and the inspection reports are smaller than that between the Xia's method and the inspection reports. Through the above comparative analysis, we can see that the new method is even more reasonable for reality.

5. Conclusion

In this paper, hesitant triangular fuzzy preference relation is proposed and is then applied to determine the priority of potential failure models of a work system. The known hesitant triangular fuzzy weighted averaging aggregation operators are adopted to aggregate the individual preferences and the score function is applied to calculate the score values of each alternative, based on which, the weights of each alternative are obtained using Chang's fuzzy AHP method. Therefore, the priority of potential failure models in both qualitative and quantitative settings is obtained. Finally, an illustrative example is provided to demonstrate the proposed method. In the future, we shall focus on the application of the proposed model in decision making to other areas.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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Research Article

Quantitative Risk Evaluation Model of the Multilevel Complex Structure Hierarchical System in the Petrochemical Industry

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In the petrochemical production system, the high-risk items malfunction may lead to major accidents so that the risk level of the items has become the highest focus of attention for the enterprises in petrochemical industry. Based on structural composition and risk relationship, a risk evaluation framework of the petrochemical production system can generally be divided into subsystems (SS), components and parts (CP), failure modes (FM), risk types, and risk factors. So it is a characteristic of multilevel, complex structure, and lack of evaluation criteria that the evaluated object has in the process of risk evaluation. However, there are few targeted modeling and calculation methods to carry out quantitative risk evaluation in the face of the evaluated object. In order to achieve risk quantitative evaluation of the complex structure hierarchical system, a multilevel Borda model (MLBM) is presented innovatively by us based on the traditional Borda method in this study. Moreover, the MLBM are applied to realize quantitative risk evaluation of the main structure system of truss type crane on the offshore platform. In this case study, the equivalent risk value (ERV) and risk priority number (RPN) of the evaluated object with multilevel, complex structure, and inadequate evaluation criteria are calculated and the risk ties in the RPN are effectively reduced. Then, the quantitative risk results can clarify the risk level and distribution of the high-risk items throughout the production system and provide data support for the development of risk control measures to better protect the production safety. Hence, the feasibility and practicability of the method are verified with the case study. The MLBM can be used to solve other comprehensive evaluation problems with a complex hierarchical structure as well.

1. Introduction

In the petrochemical industry, there are many large-scale integrated and tandem production systems which have high risk, high cost, high technology, and other characteristics. Therefore, a safe and reliable operation has always been the focus of attention. The petrochemical production process has the characteristics of complex conditions such as flammable, explosive, high temperature and pressure, and strict process. The petrochemical production system is composed of hundreds of types of equipment and large-scale integrated systems, but also has a certain series of characteristics of the process of the production system. Statistical analysis on petrochemical industry in the explosion, fire, and other major causes in the world for nearly 30 years includes the following: equipment failure accounted for 41%, operating errors accounted for 20%, unknown reasons accounted for 18%, natural disasters accounted for 6%, design errors accounted 4%, and so on [1]. If the risk level and distribution of the equipment in the petrochemical production system are known, the high-risk items can be identified. This leads to develop risk control measures which can prevent nearly half of the major accidents. Especially in recent years, the global security and environmental issues are of great importance to the petrochemical industry, a major accident may lead to a business bankruptcy. Therefore, in order to achieve the purpose of reducing the probability of occurrence of major production accidents and controlling the operating costs of enterprises, petrochemical enterprises have an urgent need for effective quantitative assessment techniques for failure mode risk (FMR) [2].

In many industrial areas, quantitative risk evaluation methods have been developed to achieve quantitative management of various risks and more effective risk control. Li [3] proposed a quantitative risk evaluation method for long-distance pipeline based on fuzzy fault tree analysis. Xu [4] has proposed a petrochemical plant for the overall qualitative analysis and local quantitative analysis of the assessment method. Zhang [5] had carried out quantitative risk analysis (QRA) and its application in submarine pipeline integrity management. Zhao et al. [6] have demonstrated a quantitative risk analysis method for storage tanks based on the domino effect. C. R. Pitcher et al. [7] have illustrated a simple quantitative risk evaluation method for the limited capacity of the sustainability of the detection device on the impact of the habitat of the sea. Dan and Guix [8] have displayed a Monte Carlo simulation method based on the evaluation of human factors in quantitative risk evaluation. Collins and Davey et al. [9] have presented a new quantitative risk evaluation method to analyze the effects of microbial corrosion (MIC) carbon steel pipelines. However, the current quantitative risk evaluation methods are mostly pipelines, tanks, and other types of structural static equipment. At present, there have been still very a few relevant research oriented systems or equipment with multilevel and complex structure.

But a variety of comprehensive evaluation methods had been studied, including Analytic Hierarchy Process (AHP), Analytic Network Process (ANP), Borda, Delphi, and Rank Sum Ration (RSR) [10, 11]. Among them, the Borda method is a kind of classic postgroup evaluation method, proposed by C. de Borda in 1784, who is the first to solve the voting problem that has been widely used. It is now in the group decision-making, program demonstration, man-made economic evaluation, quality assessment, and many other areas [12]. The basic idea is that the Borda values of the n evaluated objects are determined by comparing the priority number of the n evaluated objects given by the m evaluators, that is, the Borda value of the group being evaluated and then sorted out according to the size of the group, Borda value which is the order of the object to be evaluated. Although the traditional Borda method can effectively solve the risk evaluation of multiple objects in the same one level, it cannot be directly used in the risk evaluation of the above-mentioned multilevel and complex structure problem [13]. Therefore, by analyzing the hierarchical structure and the interobject correlation in the risk evaluation process with the ideas, advantages of the traditional Borda method, an innovative approach should be presented to achieve a quantitative evaluation of multilevel complex structure hierarchical system in this study.

2. Risk Hierarchy Analysis on the Complex Structural System

In the petrochemical industry, failure mode, effects, and criticality analysis (FMECA) is a common method for impact analysis and hazard analysis of system or equipment failure modes. According to the FMECA report (FR) of the petrochemical production system, the system or equipment contains a number of important functional items, including subsystems, equipment, components, and parts, and each of which has one or more failure modes. At the same time, in the risk evaluation, a failure mode will correspond to a variety of risk type, including safety risk (SR), environmental risk (ER), economic loss risk (ELR), and maintenance cost risk (MCR). Each risk type corresponds to multiple risk factors (RF), including Consequence Severity (CS) S_s, Occurrence Frequency (OF) S_p, and Detection Difficulty (DD) S_d. As we know, different people or groups can produce different FMECA analysis reports. Finally, examples table for FMECA report data of the risk factors is shown in Table 1.

The corresponding qualitative description of the S_s, S_p, and S_d is determined with some quantitative scale in Tables 2, 3, and 4.

In this study, the fuzzy linguistic of risk factors is eliminated based on the Tables 2, 3, and 4. Thus a quantization value of the risk factor is obtained with the calculation method as follows [14]:

$$K(x) = \frac{\sum_{i=0}^{n}(b_i - c)}{\sum_{i=0}^{n}(b_i - c) - \sum_{i=0}^{n}(a_i - d)} \quad (1)$$

The membership function of each comment is showed in Figure 1. In the process of defuzzification, the values of c and d are always constant, namely, $c = 0$, $d = 10$. The values of a_1 and b_1 are the extreme points of the two ends of fuzzy linguistic description.

For example, the "Moderate" in the fuzzy linguistic in Table 3 is treated by defuzzification. The relevant parameter values are determined according to the membership function diagram, and then by substituting (1), the definite number $K(x)$ of "Moderate" is calculated.

$$\begin{aligned} K(x) &= \frac{[b_0 - c] + [b_1 - c]}{\{[b_0 - c] + [b_1 - c]\} - \{[a_0 - d] + [a_1 - d]\}} \\ &= \frac{[8 - 0] + [6 - 0]}{\{[8 - 0] + [6 - 0]\} - \{[4 - 10] + [6 - 10]\}} \\ &= 0.583 \end{aligned} \quad (2)$$

And then, the quantitative transformation results of the fuzzy linguistics by defuzzification for S_s, S_p, and S_d, and other fuzzy linguistic descriptions were obtained with this same way above, as shown in Table 5.

Based on the above anlysis and description with the system, subsystem, parts, failure modes risk types and risk factors, the risk evaluation problem of petrochemical production system or equipment is a comprehensive evaluation object with multilevel, complex structure and multicriteria evaluation, as shown in Figure 2.

As shown in the Figure 2, this is a huge problem of system evaluation of diversification, multilevel and complex structure with the relationship among failure mode, risk category, and risk factors and the structural composition of equipment, subsystem, and component. By taking full

Table 1: Examples table for FMECA report data of the risk factors.

		Risk types											
		SR			ER			ELR			MCR		
FM	RN Nnumber (RN) RF	S_s	S_p	S_d	S_s	S_p	S_d	S_s	S_p	S_d	S_s	S_p	S_d
Failure mode 1	FR1												
	FR2												
	. . .												
	FRn												
Failure mode 2	FR1												
	FR2												
	. . .												
	FRn												
	FR1												
. . .	. . .												
Failure mode n													

Table 2: A scale reference table of the S_s.

Severity description of the CS	**Qualitative interpretation of the CS**	**Quantitative scale score**
Very high	A heavy casualty is caused	10
-	One person is killed and many others are injured	9
High	Many people are seriously injured	8
-	One person is seriously injured	7
Moderate	Many people are injured	6
-	One person is injured	5
-	One person is slightly injured.	4
Low	A number of people are frightened	3
-	One person is frightened	2
Very low	Not causing any injuries	1

Table 3: A scale reference table of the S_p.

Frequency description of the OF	**Quantitative interpretation of the OF**	**Quantitative scale score**
Very high	≥1/2	10
-	1/3	9
High	1/8	8
-	1/20	7
Moderate	1/80	6
-	1/400	5
-	1/2000	4
Low	1/15,000	3
-	1/150,000	2
Very low	1/1,500,000	1

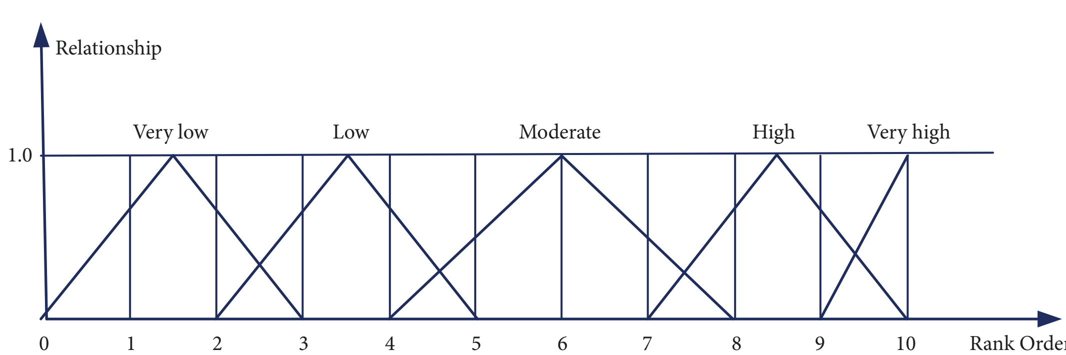


Figure 1: Relation graph of membership function.

Table 4: A scale reference table of the S_d.

Difficulty description of the DD	Quantitative interpretation of the DD	Quantitative scale score
Very high	Absolutely not detected	10
-	Impossible to be tested	9
High	Difficultly detect	8
-	The rate of detection is very low	7
Moderate	The rate of detection is low	6
-	Moderate detection rate	5
-	The detection rate is higher	4
Low	The rate of detection is high	3
-	The rate of detection is very high	2
Very low	Very easy to be detected	1

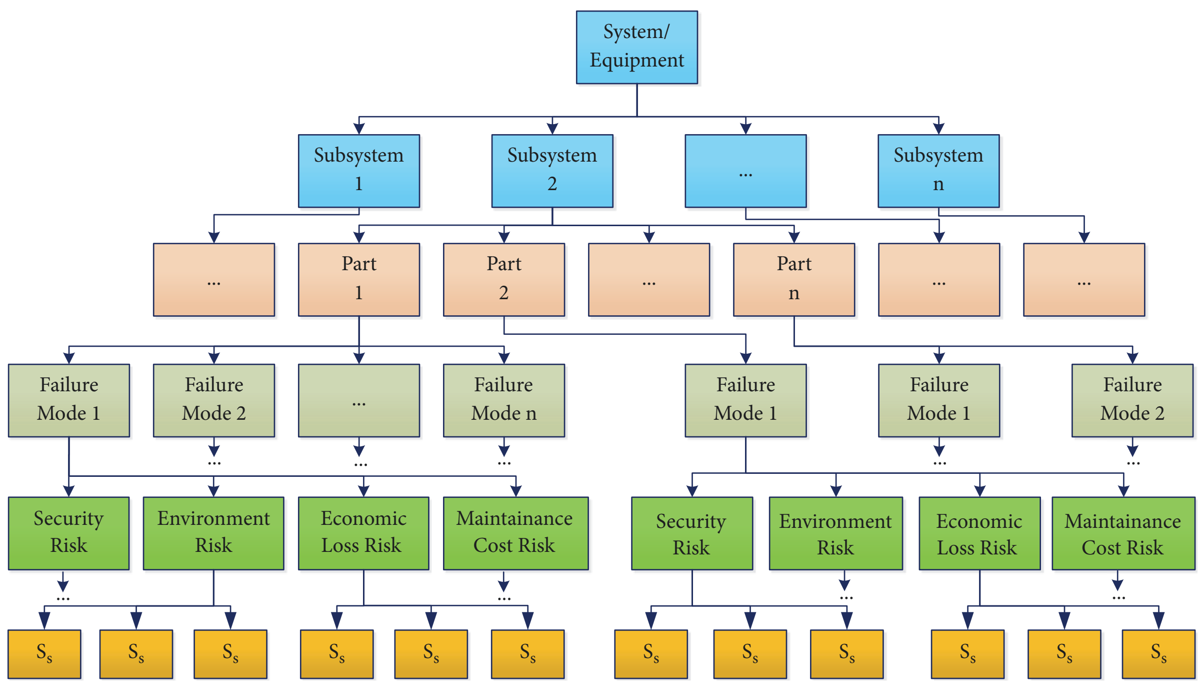


Figure 2: A relationship tree of the risk evaluation problem with petrochemical production system or equipment.

account of the actual situation and characteristics of the evaluated objects in the analogous structural system, it is summarized as a universal evaluation problem, a multilevel complex structure problem of evaluation object in this study. The undertaken problem can be described as follows. Under the known conditions of the basic evaluation criteria, there is a diversified, multilevel, and complex structure relationship between the target object and the basic evaluation criterion. The possibility of exploration is to establish a new mathematical model by using the basic evaluation criterion, hierarchical structure, and complex relationship to carry out quantitative representation of the evaluated objects and determine their priority number.

In Figure 2, the risk evaluation for system or equipment in the petrochemical industry is a multilevel and complex structure evaluation problem. In order to solve the problem of multilevel evaluation effectively, a novelty method is proposed to achieve the quantitative risk evaluation of petrochemical production system.

Table 5: The quantitative transformation results of the fuzzy linguistics.

Fuzzy linguistic description	Crisp number by defuzzification
Very low	0.196
low	0.370
moderate	0.583
high	0.804
Very high	0.952

3. Establishment of Mathematical Model of MLBM

The detailed algorithm of the MLBM is established as follows.

3.1. Modeling Parameter Setting. There are some modeling parameters to be defined to describe calculation process and content of the MLBM [15].

L: the total number of objects to be evaluated, $L \geq 2$.

N^l: the total number of objects to be evaluated in the Level l.

M^l: the total number of evaluation criteria in Level l.

R_i^l: the i-th object in Level l to be evaluated.

R_{ij}^l: the j-th subobject to be evaluated in the Level l-1 is associated with the i-th object in Level l.

Q_i^l: the total number of objects to be evaluated in the lower level of the i-th object in Level l.

k^l: the evaluation criteria of Level l, k^l= 1,2,...,M^l.

$C^k(R_i^l)$: the scoring of R_i^l in Level l under the evaluation criteria k^l.

$C^k(R_{ij}^l)$: the scoring of R_{ij}^l in Level l-1 of the R_i^l in Level l.

$N^k(R_i^l)$: the total number of the evaluated objects whose score is higher than the R_i^l in Level l under the evaluation criteria of k^l.

$B(R_i^l)$: the Borda value of the object to be evaluated in Level l.

$B(R_{ij}^l)$: the Borda value of the object to be evaluated R_{ij}^l in Level l-1.

$O(R_i^l)$: the ranking value of the object to be evaluated in Level l.

l=1,2,. . . L; i=1,2,. . .,N^l; j=1,2,. . ., Q_i; k^l=1,2,. . ., M^l.

3.2. Algorithm and Process. The MLBM is a bottom-up analysis and evaluation process. Therefore, in the whole evaluation process, it needs to be evaluated from the bottom to the up, and then the evaluation value of the target layer will be obtained. The steps and contents of the MLBM are listed as follows.

(Calculating the Borda value and order of the evaluated objects in the first level)

Step 1. Establish the hierarchical structure tree of the object to be evaluated. This step is based on the subordinate relationship, hierarchical relationship, and relevance of the object being evaluated.

Step 2. Determine the target number of Level L; determine the total number of the first level of evaluated objects N^1 and the total number of evaluation criterion M^1.

Step 3. Determine the score $C^k(R_i^1)$ of the first level evaluated object R_i^1 in the evaluation criterion k^1, where i= 1, 2,. . .,N^1;k^1 = 1, 2,. . .,M^1.

Step 4. Calculate the total number of $N^k(R_i^1)$ in all the evaluated objects R^1 in the first level, which is higher than the evaluated R_i^1.

$$N^k\left(R_i^l\right) = \left|\left\{j : C^k\left(R_i^l\right) < C^k\left(R_j^l\right),\ j = 1, 2, \ldots, N^1,\ j \neq i\right\}\right| \quad (3)$$

Step 5. Calculate the Borda value $B(R_i^1)$ of evaluated R_i^1 in the first level.

$$B\left(R_i^1\right) = \sum_{k=1}^{M^1}\left(N - N^k\left(R_i^1\right)\right) \quad (4)$$

(Calculating the Borda value above the second level and sort)

Step 6. Determine the objects to be evaluated and the total number N^l in Level l and Level l-1, N^{l-1}(l≥2):

(1)The total number of objects to be evaluated in Level l is N^l.

(2)The total number of subobjects Q_i is associated with the i-th evaluated object R_i^l in the Level l; then the total number of subobjects in the Level l-1 is

$$N^{l-1} = \sum_{i=1}^{N^l} Q_i \quad (5)$$

Step 7. Determining the evaluation criteria for Level l and its total number M^l.

The method of determining the criteria for each level and its total number is as follows:

(1) If the level specifies the evaluation criteria, the prescribed evaluation criteria are adopted, and the total number of evaluation criteria for the level is the actual number of the specified rules.

(2) If the level does not specify the evaluation criteria, the evaluation criteria are constructed according to the hierarchical structure and the relevance of the object to be evaluated.

The specific method of constructing the evaluation criteria is as follows.

There are no evaluation criteria in Level l; then compare all the evaluated R_i^l in Level l (measured or calculated Borda values), each object of comparison is one of the subobjects R_i^{l-1} associated with the object R_i^l in the Level l-1. The subobjects R_i^{l-1} of all the objects R_i^l in the Level l are sequentially compared with each other. That is, the object R_i^l in the Level l-1 of the object R_i^l to be evaluated in Level l is sorted out and combined in the above-described manner. One combination is an evaluation criterion of the Level l; the total number of evaluation criteria for Level l is the value of the combination of subobjects at Level *l-1*.

Assume the total number of objects to be evaluated in Level l is N^l; the number of subobjects corresponding to the i-th evaluated subobject R_i^l is Q_i. There are a total of $\prod_{i=1}^{P} Q_i$ kinds of mixed combinations in the Level l; namely, the total number of evaluation criteria in the first level is

$$M^l = \prod_{i=1}^{P} Q_i \tag{6}$$

Note: in the hierarchy structure tree, the bottom of the evaluation object must have established evaluation criteria.

Step 8. Determine the score of R_i^l to be evaluated, $C^k(R_i^l)$

For the evaluation criteria of the regulation and construction, in the evaluation process, the evaluation method of the evaluated object is different; the specific difference is as follows:

(1) If the Level l specifies the evaluation criteria, $C^k(R_i^l)$ is the score of the evaluated object R_i^l in Level l under the k^l.

(2) If the level does not specify the evaluation criteria, the number k combination method in the evaluated objects that compared in Level l is C^k, which is

$$C^k = \left[C^k\left(R_{1j}^l\right), C^k\left(R_{2j}^l\right), \ldots, C^k\left(R_{Pj}^l\right)\right] \tag{7}$$

where $C^k(R_{ij}^l) \in \{B(R_{ij}^l), j = 1, 2, \ldots, Q_i\}$. Therefore, $C^k(R_i^l)$ is the score or Borda value of the j-th subobject in the Level *l-1* of the i-th object of the k combination in Level l, which is $C^k(R_i^l) = C^k(R_{ij}^l) = B(R_{ij}^l)$.

Step 9. Calculate the total number $N^k(R_i^l)$ of all objects R^l in the Level l, which is higher than the evaluated R_i^l.

According to different criteria, the calculation method of $N^k(R_i^l)$ is shown as follows:

(1)If the level has the established criteria,

$$N^k\left(R_i^l\right) = \left|\left\{j : C^k\left(R_i^l\right) < C^k\left(R_j^l\right),\ j = 1, 2, \ldots, Q_i,\ j \neq i\right\}\right| \tag{8}$$

(2)If the level does not have the criteria,

$$N^k\left(R_i^l\right) = N^k\left(R_{ij}^l\right) = \left|\left\{j : C^k\left(R_{ij}^l\right) < C^k\left(R_{uv}^l\right), u = 1, 2, \ldots, P; v = 1, 2, \ldots, Q_u\right\}\right| \tag{9}$$

where $|\bullet|$ represents the number of collection elements.

Step 10. Calculate the Borda value, $B(R_i^l)$.

The Borda value of the evaluated object R_i^l in Level l can be calculated, which is calculation formula as follows:

$$B\left(R_i^l\right) = \sum_{k=1}^{M}\left[N - N^k\left(R_i^l\right)\right] \tag{10}$$

Step 11. Determine whether it reaches the target Level L.

If the target Level L is reached, the calculation will be stopped. If the target Level L is not reached, it will be cycled from Step 6 to Step 10 until the target Level L is executed.

Step 12. Output the evaluated object $O(R_i^L)$.

Determine the respective order $O(R_i^L)$ of the object R_i^L in the target Level L, $i = 1, 2, \ldots, N^L$, which is the order (Risk ranking or RPN) of the subject in the level.

Through the summary of the above calculation steps and contents, the computational flow chart of the MLBM is developed, as shown in Figure 3.

4. Case Study

Taking the main structure system of the truss type crane (no. LIUHUA10-1PGC) as an example on the oil and gas production platform, the MLBM is used to evaluate the risk value and the priority relation of its failure modes, main parts, and subsystems. Firstly, the failure modes, risk types, and risk factors for the main structure system are analyzed by using FMECA [16], because different experts have certain differences in personnel composition, expertise, and on-site experience, which will lead to differences in the analysis reports of FMECA. In order to improve the objectivity of the analysis results, we organized 3 expert groups to perform FMECA separately and independently so that 3 FMECA reports were obtained. Secondly, the risk hierarchy structure tree (RHT) of the main structure system was established based on its structural diagrams of mechanical system, as shown in Figure 4.

In the *Level 1* of the RHT of the main structure system in Figure 4, there are three risk factors: Consequence Severity S_s, Occurrence Frequency S_p, and Detection Difficulty S_d. In the *Level 2* of the RHT, there are four risk types relating to risk factors, including safety risk (SR), environmental risk (ER), economic loss risk (ELR), and maintenance cost risk (MCR). Based on semantic vagueness results of the risk factors in 3 FMECA reports, the quantization value of the risk factors corresponding to each failure mode is obtained by the Fuzzy Set Theory, as shown in Table 6 [17].

In Figure 4, the *Level 2* of the RHT of the main structural system is the four risk types (SR, ER, ELR, and MCR) of each failure mode. Their risk evaluation criteria have been set in the *Level 2* based on 3 risk factors of S_s, S_p, and S_d. And the quantitative value of the risk factors corresponding to each failure mode in the Table 6 is regarded as the risk score under the evaluation criterion above. In the MLBM, equivalent risk value (ERV) is Borda value and risk ranking that is obtain by comparing the Borda value is the RPN. And then, the ERV and RPN of the SR, ELR, and MCR for 13 failure modes are obtained by the MLBM in Figure 5, because the failure modes of the main structure system generally do not cause environmental consequences. The ERV and RPN of the ER of the failure modes were not considered in this case study.

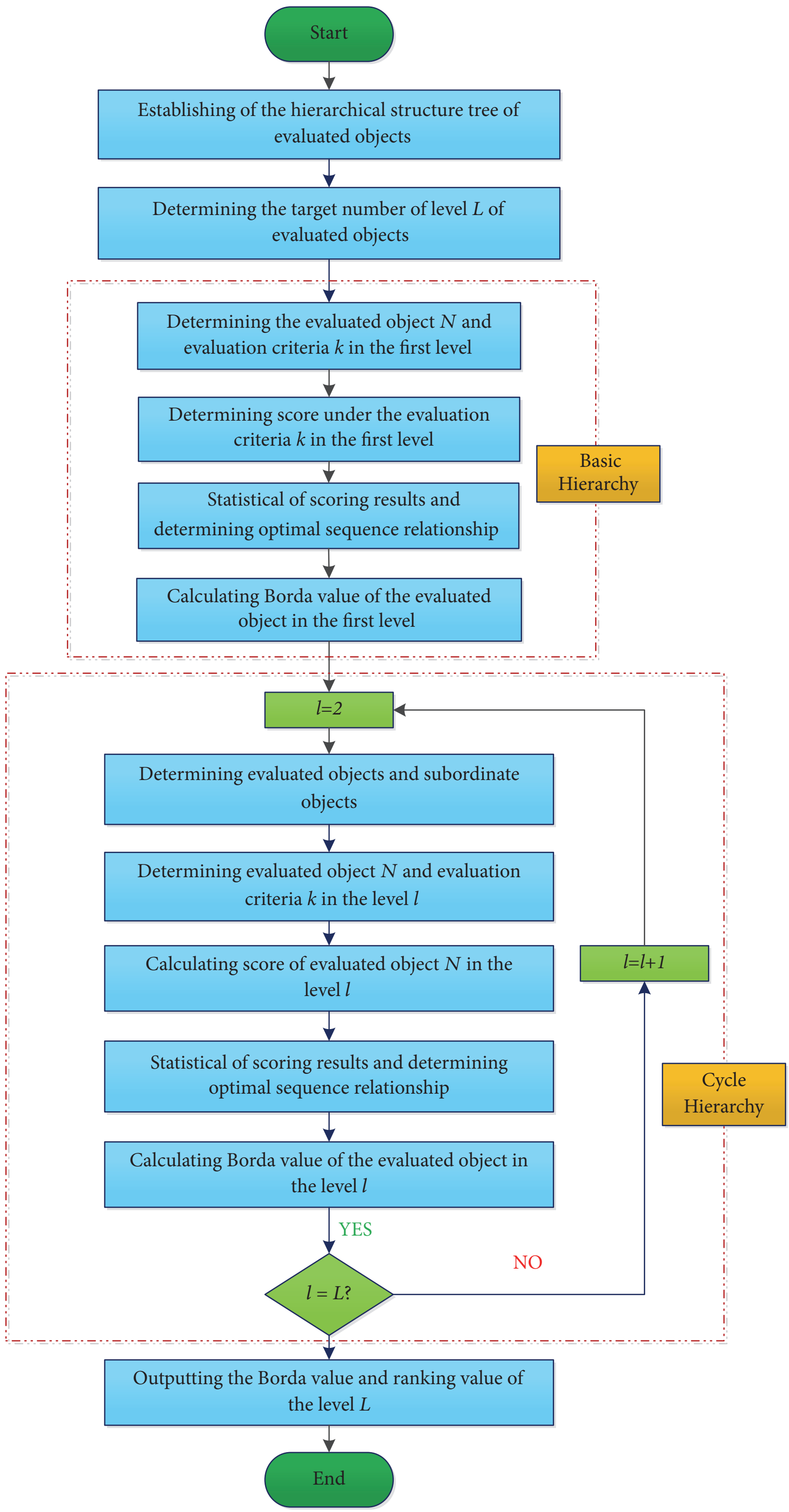


FIGURE 3: Computational flowchart of the MLBM.

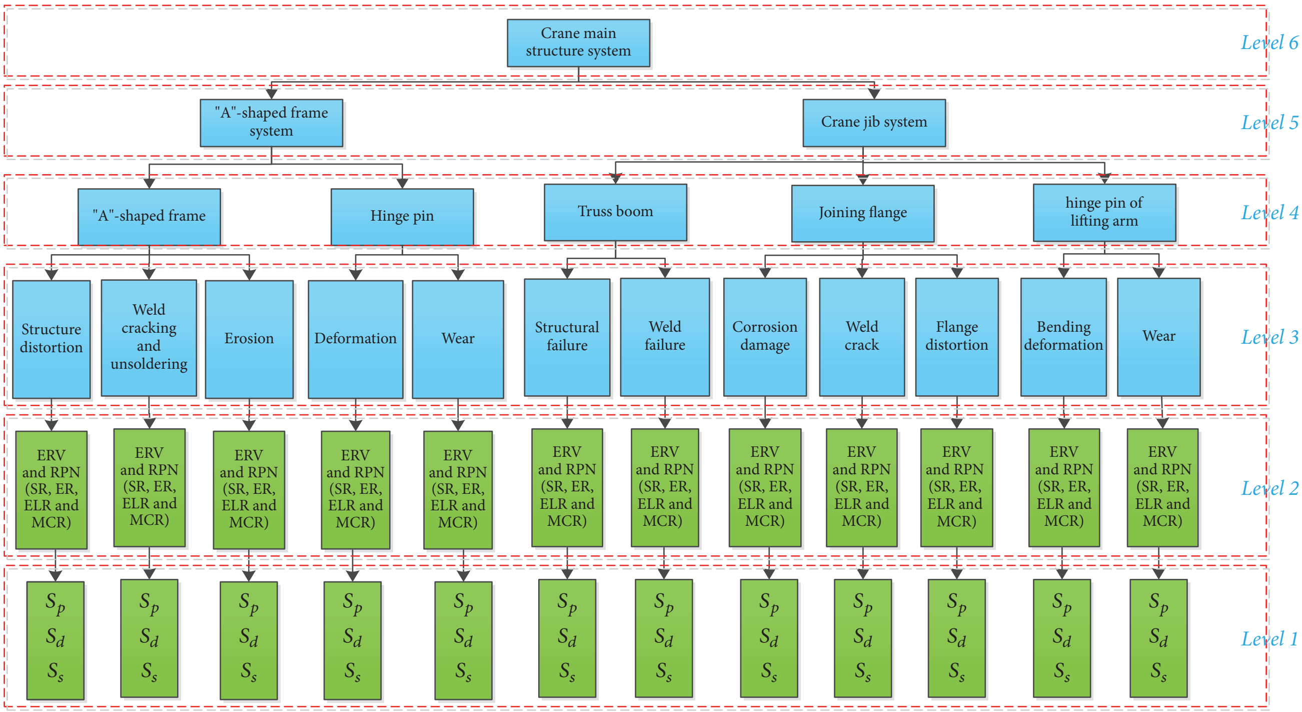


Figure 4: The RHT of the main structure system of the truss type crane.

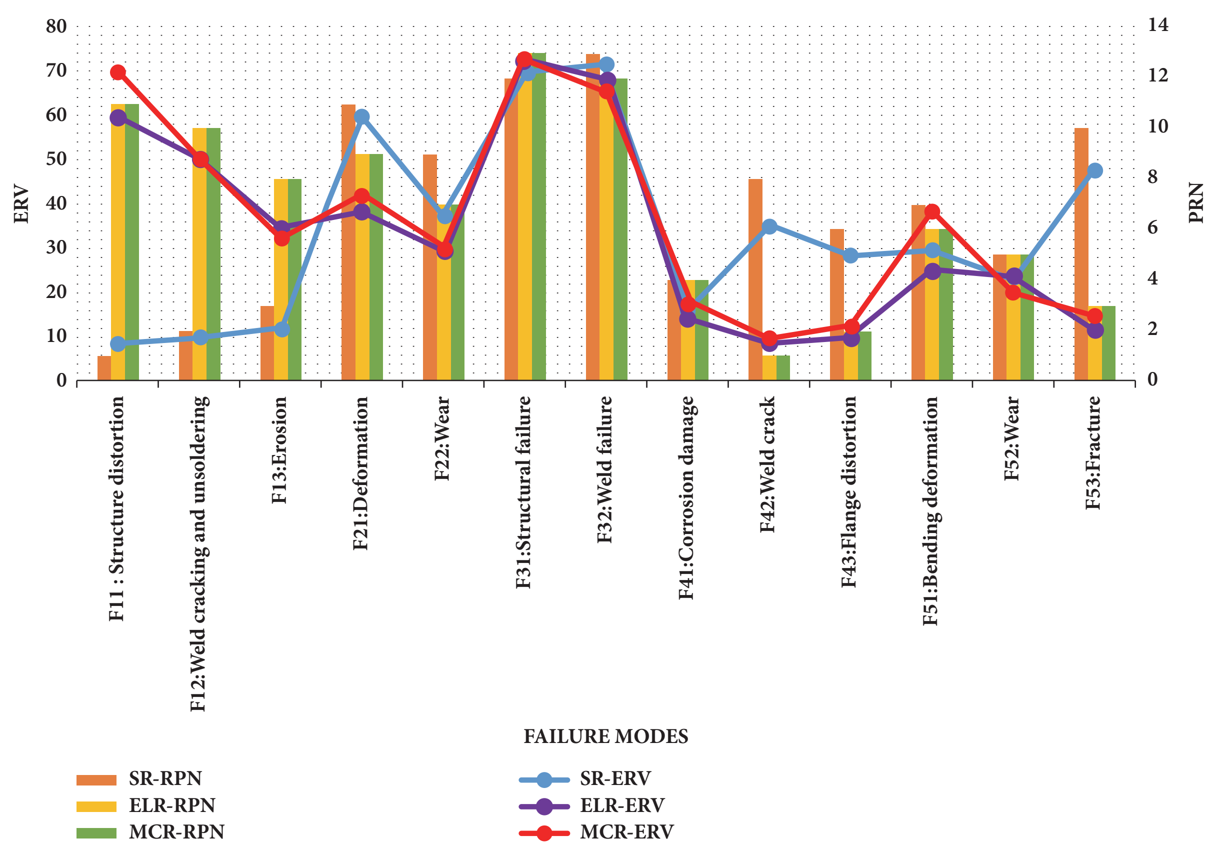
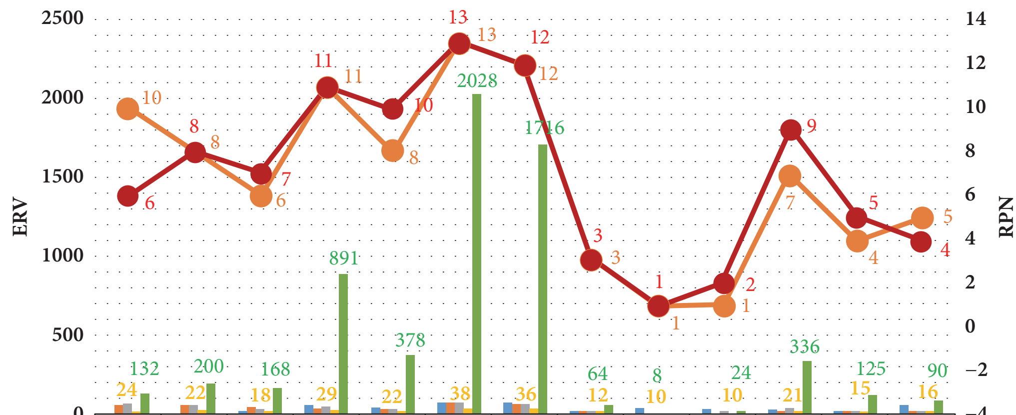


Figure 5: ERV and RPN of three risk types of each failure mode.

Table 6: A summary of the quantitative values of the risk factor.

SS	Parts	FM	RN	S_p	S_d	SR/S_s	ER/S_s	ELR/S_s	MCR/S_s
"A"-shaped frame system	"A"-shaped frame	F11: Structure distortion	FR1	0.583	0.583	0.196	0.196	0.804	0.952
			FR2	0.583	0.583	0.196	0.196	0.804	0.952
			FR3	0.583	0.370	0.196	0.196	0.804	0.952
		F12:Weld cracking and unsoldering	FR1	0.583	0.583	0.196	0.196	0.804	0.952
			FR2	0.804	0.583	0.196	0.196	0.583	0.804
			FR3	0.583	0.370	0.196	0.196	0.804	0.952
		F13:Erosion	FR1	0.583	0.370	0.196	0.196	0.583	0.583
			FR2	0.804	0.583	0.196	0.196	0.583	0.583
			FR3	0.804	0.583	0.196	0.196	0.583	0.583
	Hinge pin	F21:Deformation	FR1	0.583	0.370	0.583	0.196	0.804	0.804
			FR2	0.804	0.370	0.583	0.196	0.804	0.804
			FR3	0.583	0.370	0.583	0.196	0.804	0.370
		F22:Wear	FR1	0.583	0.370	0.196	0.196	0.583	0.583
			FR2	0.583	0.583	0.583	0.196	0.583	0.583
			FR3	0.804	0.583	0.196	0.196	0.583	0.370
Crane jib system	Truss boom	F31:Structural failure	FR1	0.804	0.583	0.583	0.196	0.583	0.952
			FR2	0.804	0.370	0.583	0.196	0.804	0.952
			FR3	0.804	0.370	0.804	0.196	0.583	0.952
		F32:Weld failure	FR1	0.804	0.583	0.370	0.196	0.583	0.583
			FR2	0.804	0.583	0.583	0.196	0.804	0.583
			FR3	0.804	0.583	0.804	0.196	0.583	0.583
	Joining flange	F41:Corrosion damage	FR1	0.196	0.370	0.583	0.196	0.804	0.804
			FR2	0.370	0.370	0.583	0.196	0.583	0.804
			FR3	0.196	0.196	0.583	0.196	0.804	0.804
		F42:Weld crack	FR1	0.370	0.196	0.583	0.196	0.370	0.370
			FR2	0.583	0.370	0.583	0.196	0.370	0.370
			FR3	0.370	0.196	0.583	0.196	0.370	0.370
		F43:Flange distortion	FR1	0.370	0.370	0.583	0.196	0.370	0.370
			FR2	0.370	0.370	0.583	0.196	0.370	0.370
			FR3	0.370	0.196	0.583	0.196	0.370	0.370
	Hinge pin of lifting arm	F51:Bending deformation	FR1	0.370	0.196	0.370	0.196	0.804	0.952
			FR2	0.583	0.370	0.583	0.196	0.804	0.804
			FR3	0.370	0.370	0.583	0.196	0.804	0.370
		F52:Wear	FR1	0.804	0.370	0.196	0.196	0.370	0.370
			FR2	0.804	0.583	0.370	0.196	0.583	0.370
			FR3	0.804	0.583	0.196	0.196	0.370	0.370
		F53:Fracture	FR1	0.196	0.196	0.952	0.196	0.804	0.952
			FR2	0.196	0.370	0.952	0.196	0.804	0.952
			FR3	0.196	0.370	0.952	0.196	0.804	0.952



	F11	F12	F13	F21	F22	F31	F32	F41	F42	F43	F51	F52	F53
SR-ERV	8.52	10	11.85	60.01	37.42	70.18	71.62	15.75	35.11	28.36	29.51	22.65	48.07
ELR-ERV	59.71	50.23	34.2	38.46	29.86	72.41	68.12	14.28	8.92	10.26	25.13	23.6	11.72
MCR-ERV	70.09	50.32	32.03	42.37	30.25	73.22	65.42	17.83	9.61	12.38	38.51	20.13	14.77
FMR-ERV (Borda)	24	22	18	29	22	38	36	12	10	10	21	15	16
FMR-ERV (MBSVM)	132	200	168	891	378	2028	1716	64	8	24	336	125	90
FMR-RPN (Borda)	10	8	6	11	8	13	12	3	1	1	7	4	5
FMR-RPN (MBSVM)	6	8	7	11	10	13	12	3	1	2	9	5	4

FAILURE MODES

Figure 6: ERV and RPN of integrated risk of each failure mode with traditional Borda and the MLBM.

According to the quantitative risk results about SR, ER, and MCR in the Figure 5, we can see that different failure modes might cause different types of risk consequent. We all know that the companies of different backgrounds and industries will focus on different types of risks. From the ERV and RPN of SR, ER, and MCR, the companies can focus directly on the type of risk they care about. F32: Weld failure has the highest SR, and the highest ELR and MCR are F31: structural failure.

In the Figure 4, there are 13 failure modes in the *Level 3* of the RHT of the main structural system. Because the ERV among the failure modes can be directly compared and sorted out when they are under same type of risk, the evaluation criteria in the *Level 3* can be identified as SR, ER, and MCR. So the total of the evaluation criteria is 3 in *Level 3*. Moreover, their ERV calculated in the last step can be taken as the risk scores under evaluation criterion of SR, ER, and MCR. Thus, the ERV and RPN of integrated risk of ach failure mode have been calculated by using traditional Borda method and the MLBM, respectively, in Figure 6 [18].

In the Figure 6, we can see that there are some risk ties of the failure modes with traditional Borda method, but few risks ties with the MLBM. From the Figure 6, F31: structural failure and F32: Weld failure have higher risk level than others according to the ERV and PRN of 13 failure modes. Therefore, in the system inspection process, these two failure modes should be excluded, which will be easier to avoid major failures and accidents in petrochemical production.

The *Level 4* of the RHT of the main structural system includes the "A"-shaped frame, Hinge pin, Truss boom, Joining flange, and Hinge pin. For the components or parts, the lower level displays a variety of different failure modes; it can not specify its evaluation criteria. According to the failure modes corresponding to the components or parts, the evaluation criteria are set in the *Level 4*. By arranging all the failure modes, the total number of evaluation criteria is 108 for calculating the *Level 4*. At the same time, the Borda value of each failure mode is calculated in the previous step taken as its risk score under each evaluation criterion in the current level. The ERV and RPN of the "A"-shaped frame, Hinge pin, Truss boom, Joining flange, and Hinge pin of the lifting arm are calculated as shown in Figure 7.

From Figure 7, we can know that there are not any risk ties for all of parts. The ERV and RPN about the Truss boom and Hinge pin are higher than the other parts. During their routine maintenance and inspection, the Truss boom and Hinge pin should be paid more attention to improve their reliability and security.

In Figure 4, the *Level 4* in the RHT of the main structural system is the "A"-shaped frame system and the Crane jib system. For the subsystems, their lower level objects are components or parts so that their respective ERV can be compared with each other. Therefore, the evaluation criteria of the *Level 4* are the risk value of the components or parts. Through various permutations and combinations, there are six evaluation criteria in the *Level 4*. The Borda value of each component or part calculated in the previous step are taken as the risk score under six evaluation criteria. Quantitative risk evaluation results of the "A"-shaped frame system and the Crane jib system are calculated, as shown in Figure 8.

In Figure 8, the ERV of the "A"-shaped frame system is higher than the Crane jib system. During overhauling for the crane, the "A"-shaped frame system will be a key target, and the other systems are secondary. Therefore, quantitative risk

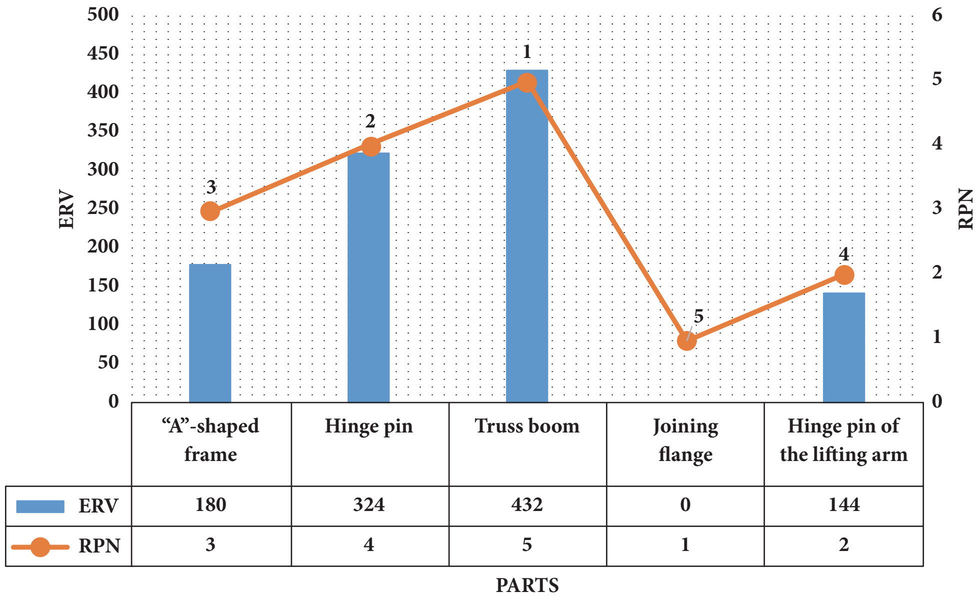


Figure 7: Quantitative risk evaluation results of the parts.

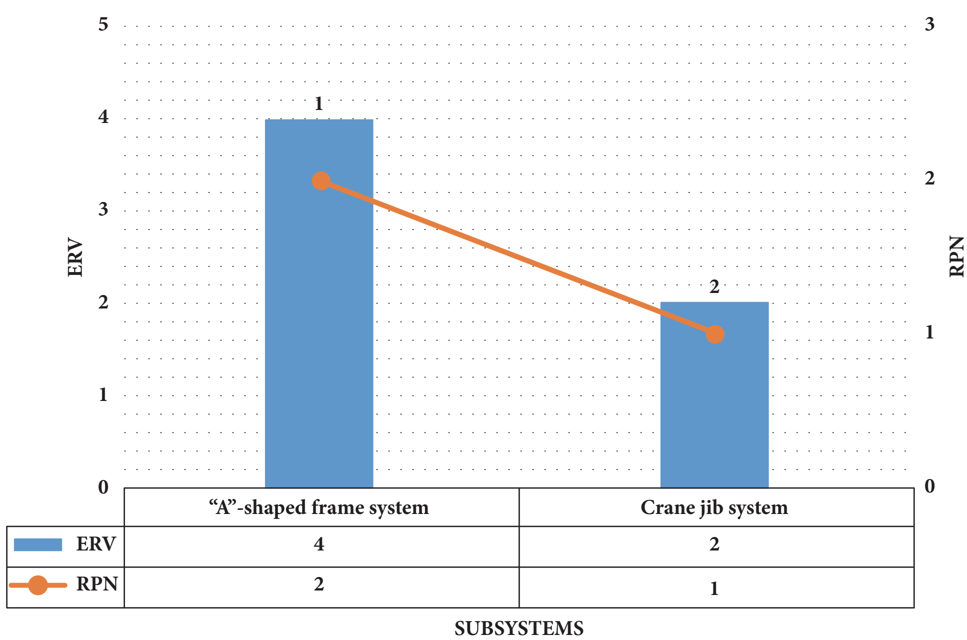


Figure 8: Quantitative risk evaluation results of the subsystem.

evaluation results can better guarantee the safety of the whole system.

5. Conclusion

The MLBM is a modified Borda method based on scoring and ranking among objects to be evaluated. This method is proposed to aim at solving the comprehensive evaluation problem which is multilevel, complex structure and lack of evaluation criteria of the middle level and achieving quantitative calculation results of the objects. In this study, the MLBM is applied to the quantitative risk evaluation of failure of the petrochemical production system. Not only are the risk value and risk ranking of the items, including subsystems, equipment, components, parts, and failure modes, in the petrochemical production system carried out, but also the risk tie of their RPN is reduced effectively to get a more accurate risk ranking. Based on the MLBM, the ERV and RPN of the failure modes are calculated in the overall system. And then, the high-risk failure modes can be selected according to their ERV and RPN. Thus, it can help the optimization of preventive maintenance program, evaluation of running status, and fault diagnosis of devices and equipment in the petrochemical production system. Moreover, the quantitative risk evaluation method based on the MLBM can be applied to analyze the ERV and RPN of each parts, components, and subsystem. And the quantitative risk evaluation results of these items can help to optimize their importance ranking and classification, in order for scientifically screening out important functional items to help the implementation of Reliability Centered Maintenance and asset integrity management (AIM) technology. Further, it is possible to clarify the risk level and distribution of the high-risk items throughout the production system and provide data support for the development of risk control measures to better protect the safety of production according to the quantification ERV and RPN of failure modes, components, parts, subsystems, and systems.

The MLBM can be used not only to solve the multilevel complex structure problem in the quantitative risk evaluation of petrochemical industry but also to provide a reference for other multilevel complex structure comprehensive evaluation problems. The MLBM is a method of fixed analysis and calculation process. In next study, it will be considered that the development of MLBM is carried out as computer program module in order to improve its computational efficiency and better promotion, and the application of the method is outreached and improved according to different objects to be evaluated in other fields.

Data Availability

The data of used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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Research Article

Multi-Target Strike Path Planning Based on Improved Decomposition Evolutionary Algorithm

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This study proposes a path-finding model for multi-target strike planning. The model evaluates three elements, i.e., the target value, the aircraft's threat tolerance, and the battlefield threat, and optimizes the striking path by constraining the balance between mission execution and the combat survival. In order to improve the speed of the Multi-Objective Evolutionary Algorithm Based on Decomposition (MOEA/D), we use the conjugate gradient method for optimization. A Gaussian perturbation is added to the search points to make their distribution closer to the population distribution. The simulation shows that the proposed method effectively chooses its target according to the target value and the aircraft's acceptable threat value, completes the strike on high value targets, evades threats, and verifies the feasibility and effectiveness of the multi-objective optimization model.

1. Introduction

The air defense system has been significantly improved in modern warfare [1]. Air strikes from low altitude play an important role in modern warfare. The use of ground clutter cover and the existence of radar low altitude blind spots can greatly reduce the enemy's air defense capability and improve the penetration probability; however, the high risk and high operational difficulty of low altitude penetration limit the usability of low altitude penetration [2]. Path planning becomes one of the key technologies for precision strike.

Traditional models for path planning mainly evaluate the threat level due to terrain and air defense radar. These models minimize the threat through multi-objective algorithm [3], but seldom consider the situation of multi-target striking. In an actual combat, simply flying from a point A to a point B is not enough. Aircrafts often need to pass through multiple targets or areas extremely important for mission completion, such as the mission assembly area, the refueling area, and the targets to destroy. The ratio between the target value and its associated air defense capability varies for different targets. Therefore, it is necessary to plan the flying path that maximizes the probability of the target elimination as well as the aircraft's chance of survival.

In recent years, [4–6] proposed several modeling approaches that can plan flying paths connecting multiple target regions. The basic theory is to model the path planning problem as a traveling salesman problem (TSP) after the targets are chosen. Although these approaches can make the flying track pass through multiple target areas, there are two problems. Firstly, they do not rank the target according to its value. Secondly, path evaluation and aircraft safety are not considered. The path planned by these models will inevitably pass through the chosen target areas, even if the threat cost to these areas has exceeded the upper limit of the aircraft or a target is simply inaccessible. To solve the aforementioned problems, this paper proposes a path planning modeling approach for multi-target strike missions. It evaluates the balance among the aircraft might, the target value, and the threat level. It links the targets selection and the path evaluation. The evaluation is used to complete path planning under more complex mission conditions.

In our approach, the evolutionary algorithm is used to solve path planning [7, 8], and the decomposition multi-objective evolutionary algorithm (MOEA/D) [9] is used to solve the model. The simulation results show that the model is capable of completing the path planning effectively after weighing the target value and the threat.

2. Methodology

Given known conditions of the battlefield terrain, the air defense threat and the target location, an aircraft faces two kinds of threats: the terrain obstacle and the struck by the air defense system en route to target. Most of the targets in a battlefield are under the protection of an air defense system. The bombing aircraft is inevitably subject to a high level of air defense threats. The aircraft's risk tolerance is higher for higher value targets, and vice versa.

2.1. Function of Path. Within the mission domain, an aircraft flies from point A to point B before leaving the battlefield. Connecting point A and point B, the path of aircraft can be represented by Fourier series with AB as the axis,

$$y_i = \sum_{i=1}^{n} k_i \sin(\omega_i x_i), \tag{1}$$

where w_i, $(n = 1, 2, 3, \ldots, n)$ are the angular frequencies and k_i, $(n = 1, 2, 3, \ldots, n)$ are the corresponding amplitudes. The low frequency part of sine function is taken for the maneuverability. The minimum turning radius of an aircraft is $r_{\min}$.

$$r_{\min} = \frac{v^2}{g} \cdot \tan\theta_{\max}, \tag{2}$$

where v is the speed of the aircraft, g is the acceleration of gravity, and $\theta_{\max}$ is the maximum roll angle of the aircraft. Setting $v = 720 km/h$ and $\theta_{\max} = 75^{\circ}$ gives a minimum turning radius $r_{min} = 1.1$km. The curvature radius of the path should be larger than the minimum turning radius r_{min}:

$$r_{\min} \leq \left| \frac{\left(1 + k_{\max}^2 \times \cos^2(w)^{3/2}\right)}{k_{\max} \times \sin(w)} \right|. \tag{3}$$

The maximum amplitude is $k_{\max}$. Set $k_{\max} = 6$; the range of path angular frequency can be obtained according to Figure 1.

According to Figure 1, the range of available w is about

$$(0.04 + k\pi, 1.30 + k\pi) \cup (1.85 + k\pi, 3.10 + k\pi). \tag{4}$$

The amplitude range is 0<k<6. Within these frequency and amplitude ranges, we use the method in [1] to randomly generate a matrix of n×m. If m is the population size and n is the length of individual chromosome, k_i in each row of the amplitude matrix constitutes a chromosome. The gene location of chromosomes corresponds to the angular frequency, and the gene value is the corresponding amplitude. We next bring the angular frequency w and amplitude k into the path function and compute a fight path.

2.2. Constraint Analysis for Path Planning. The constraints on the path planning consist of four factors formulated as follows.

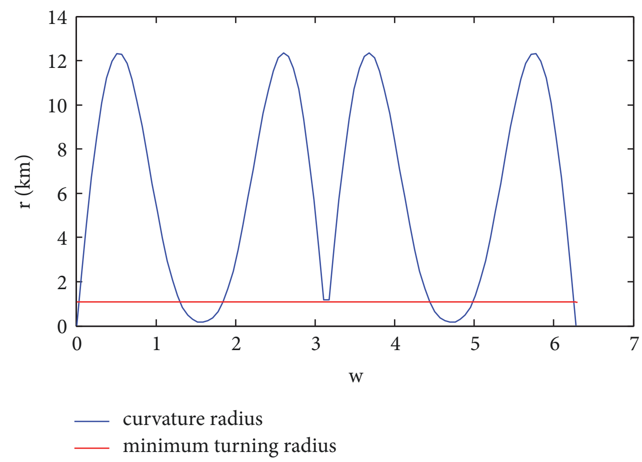

Figure 1: Curvature radius and minimum turning radius function.

(1) The path's length and offset: the path offset p is the distance between the path point (x_i, y_i) and the straight line from A to B.

$$p = \sum_{i=1}^{n} \frac{|x_i - y_i|}{\sqrt{2}}. \tag{5}$$

The path length S is

$$S = \int_{x_1}^{x_i} \sqrt{1 + y^2}\, dx. \tag{6}$$

(2) Terrain obstacles: if a single terrain obstacle can be transformed into a set of circles with different radii, the distance between the path point (x_i, y_i) and each circle $o_i(x_{oi}, y_{oi})$ should be greater than the radius r_{oi}.

$$\sqrt{(x_i - x_{oi})^2 + (y_i - y_{oi})^2} \geq r_i \tag{7}$$

(3) The threats from the air defense system: the air defense threats can be divided into two levels: the general threat area where the aircraft can be detected and shot down, and the serious threat area where the risk of being shot down dramatically increases. We set the threat values for each zone based on the zone's level of threat.

(4) The constraint on the targets: we set the coordinates of the targets to (x_{hi}, y_{hi}). In the case of dropping aerial bombs, an aircraft needs to face the target to ensure a successful bombing. The P is the horizontal distance between the aircraft location and the target point.

$$p = \sqrt{\frac{2h}{g}} \cdot v \tag{8}$$

where h is the altitude and v is the aircraft velocity. Since the aircraft is not guaranteed to face the target after entering the bombing zone, it needs to make some maneuvers to adjust its heading. Considering the various headings when entering the bombing zone, the aircraft should adjust its heading at the latest $R_{\min}$ from the target point,

$$R_{\min} = \sqrt{r_{\min}^2 + p^2}. \tag{9}$$

Therefore, in the initial stage of bombing, the aircraft should be in a ring with an outer diameter of R_{max} and an inner diameter of R_{min}; $R_{max} = R_{min} + 1$. Let (x_h, y_h) be the target point; the path point (x_i, y_i) should satisfy

$$R_{min} \le \sqrt{(x_i - x_h)^2 + (y_i - y_h)^2} \le R_{max} \tag{10}$$

2.3. Fitness Function Design. In the construction of fitness function, the constraints in path planning should be divided into "elastic constraints" and "hard constraints." Elastic constraints include the air defense threat and flight offset. Hard constraints refer to rigid constraints such as terrain constraints and target locations. For elastic constraints, the fitness of a single point has little effect on the full path. The evaluation of the path quality mainly depends on the superposition of each point's elastic fitness value, so the fitness value of a single point is small. For hard constraints, the single-point fitness value has a greater influence on the path evaluation. If a certain point does not satisfy one of the hard constraints, the whole path will be abolished.

2.3.1. Fitness Design of a Flight Path. The fitness of a path is formulated as follows:

$$W_q = p + s + z \tag{11}$$

where P is the path offset, s is the distance of the path, and z is the penalty value of a terrain barrier.

If $\sqrt{(x_i - x_{oi})^2 + (y_i - y_{oi})^2} \ge r_i$ then z=∞; the path is to be abolished if it passes through the terrain obstacle area.

2.3.2. Threat Fitness Design. The path threat fitness is

$$W_t = W_f - J, \tag{12}$$

where W_f is the penalty value of an air defense threat and J is the reward value from bombing. Positive W_t means that threat is higher than reward, the risk of bombing the target is too high, and the path should be abolished, and vice versa.

(1) Design of penalty value for air defense threats: we set the serious threat area of air defense r and the general threat area R. The range of threat W_t is P~50 when the aircraft is in any of the serious threat areas, Q~P when it is in the general threat area, and 0~Q when it is outside the radar effective detection range.

When $L < r$,

$$W_{ij} = 50 - (50 - P) \times \frac{L}{r}. \tag{13}$$

When $r < L < R$,

$$W_{ij} = P - (P - Q) \times \left(\frac{L - r}{R - r}\right). \tag{14}$$

When $R < L$,

$$W_{ij} = Q \times \left(1 - \left(\frac{L - R}{L}\right)\right). \tag{15}$$

The threat value is the summation of each path point threat

$$W_f = \sum_{\substack{i=1 \\ j=1}}^{n} W_{ij}. \tag{16}$$

(2) Target reward value design: in order to highlight the progressive relationship between the target points, a design method of target reward value combined with the analytic hierarchy process (AHP) is proposed.

When the path point (x_i, y_i) is located at the bombardment location satisfying $R_{max} \ge \sqrt{(x_i - x_h)^2 + (y_i - y_h)^2} \ge R_{min}$, the reward value of this point is

$$J_i = C_i \cdot W_{ft}, \tag{17}$$

where W_{ft} is the acceptable threat value of the mission, which is determined by the aircraft's survival and penetration ability, and $C = \{C_1, C_2, C_3 ... C_4\}$, $C_1 + C_2 ... + C_N = 1$ are the target values.

There are many ways to determine the target value. According to different sources of original data, they can be divided into two categories: subjective weighting method and objective weighting method. The difference between the two methods is whether the subjective factors of decision maker are considered. In military field, the evaluation of targets involves many factors, such as tactical intention, commander consideration, and mission impact. These make it difficult to quantify the targets mathematically. Therefore, it is more appropriate to use the subjective weighting method. The subjective weighting method includes Delphi Method, AHP, Binomial Coefficient, and Decision Alternative Ratio Evaluation System (DARE). AHP [10] is a multi-objective decision analysis method combining qualitative and quantitative analysis. In particular, it quantifies the decision-makers' judgment based on experience when the target (factor) structure is complex and lacks necessary data. Compared with other methods, the AHP has the advantages of conciseness, practicability, and high efficiency. At the same time, it also has some limitations, such as excessive influence of subjective factors, low precision, and the inability to generate new solutions. But in the problem of determining the target value, there are the following characteristics. (1) The subjective factors of the planners are very important. (2) The error of the target accuracy has little effect on the fitness value. (3) There is a high requirement for the speed. (4) The problem only needs to rank the original targets and does not need new solutions. In summary, the AHP method is used to determine the target value.

First, the planner compares the targets in pairs. The comparison results are then used to evaluate the relative importance between the pair targets through a score table. The importance is ranked from 1 to 7, where 1 means less important and 7 means extremely important. For N=4, the target comparison table is shown in Table 1.

We can see that Table 1 is an m=4 order square matrix. We calculate the weight according to an analytic hierarchy process.

Table 1: Target comparison table.

Bomb target	Goal 1	Goal 2	Goal 3	Goal 4
Goal 1	C_{11}	C_{12}	C_{13}	C_{14}
Goal 2	C_{21}	C_{22}	C_{23}	C_{24}
Goal 3	C_{31}	C_{32}	C_{33}	C_{34}
Goal 4	C_{41}	C_{42}	C_{43}	C_{44}

First we find the 1/m power $\overline{C_i}$ of the product of every row element in matrix,

$$\overline{C_i} = \prod_{j=1}^{m} \left(C_{ij}\right)^{1/m}. \tag{18}$$

The target weight $\overline{C_i}$ is

$$C_i = \frac{\overline{C_i}}{\sum_{i=1}^{m} \overline{C_i}}. \tag{19}$$

A single target reward value can be obtained from (17), (18), and (19),

$$J_i = \frac{\prod_{j=1}^{m} \left(C_{ij}\right)^{1/m}}{\sum_{i=1}^{m} \prod_{j=1}^{m} \left(C_{ij}\right)^{1/m}}. \tag{20}$$

The target reward value of the entire path is

$$J = \sum_{i=1}^{N} J_i. \tag{21}$$

2.4. Model Optimization. A multi-target strike path optimization model is established based on the above analysis:

$$\begin{gathered} \min\left(W_q\right); \\ \min\left(W_t\right); \\ \sqrt{\left(x_i - x_{oi}\right)^2 + \left(y_i - y_{oi}\right)^2} \geq r_{\mathrm{oi}} + 1; \\ R_{\max} \leq \sqrt{\left(x_i - x_h\right)^2 + \left(y_i - y_h\right)^2} \leq R_{\min}; \\ W_f \leq W_{ft}; \end{gathered} \tag{22}$$

3. The Hybrid Conjugate Gradient Method for MOEA/D Algorithm

In the actual multi-target path planning, the target position and parameters, battlefield environment, etc. often change rapidly. A hybrid conjugate gradient method (MCGM) MOEA/D algorithm is proposed to solve the problem of reprogramming in finite time when the task parameters change.

3.1. MOEA/D Algorithm. MOEA/D algorithm simplifies the multi-objective problem into several single-objective problems by using a decomposition strategy [11]. Compared with the traditional evolutionary algorithm, the subproblems do not need to be optimized repeatedly, and the adjacent subproblems can be optimized with each other to improve the efficiency of the algorithm [12, 13]. However, because the algorithm needs a large population size, the convergence speed is low when the subproblem becomes complex.

3.2. The Conjugate Gradient Method. The conjugate gradient method [14] is a deterministic algorithm based on the steepest descent method [15] and the Newton method [16]. Compared with Newton's method, it does not need to compute Hesse matrix and has good quadratic termination, but it is easy to fall into a local minimum when solving optimization problems.

The algorithm flow is as follows.

Step 1. Set the algorithm precision ε, $k = 1$, and select the initial search point $x^{(1)}$.

Step 2. The algorithm termination condition is $\|\nabla f(x^{(k)})\| \leq \varepsilon$. If it is established, the algorithm will terminate; otherwise

$$d^{(k)} = -\nabla f\left(x^{(k)}\right) + \beta_{k-1} d^{(k-1)} \tag{23}$$

β_{k-1} :

$$\beta_{k-1} = \begin{cases} 0, & k = 1 \\ \dfrac{\left\|\nabla f\left(x^{(k)}\right)\right\|^2}{\left\|\nabla f\left(x^{(k-1)}\right)\right\|^2}, & k > 1 \end{cases} \tag{24}$$

Step 3. One-dimensional search is carried out to solve a_k by minimizing:

$$\min \varphi(a) = f\left(x^{(k)} + a d^{(k)}\right) \tag{25}$$

Set $x^{(k+1)} = x^{(k)} + a_k d^{(k)}$;

Step 4. Set $k = k + 1$; turn to step 2.

3.3. The Improved MOEA/D Algorithm

3.3.1. Analysis of Improved MOEA/D Algorithm. According to the characteristics of the two algorithms, an improved MOEA/D algorithm is proposed by combining the two algorithms.

At present, some studies have incorporated the conjugate gradient method into the evolutionary algorithm. For example, [17, 18] proposed the conjugate gradient method as a search operator to improve the search speed of the algorithm. In [17], the conjugate gradient method is used to search all individuals in the population, which results in excessive computation and a loss of speed advantage. In [18], in order to reduce the computational complexity, the population central individual (population mean individual) is used as the initial search point. Although this method improves the efficiency of the algorithm, in the case of large population, a single central individual cannot represent the entire population, and the solution is easy to fall into local optimal solutions.

To overcome the shortcomings of the above methods, this paper proposes an initial search point selection method with Gauss perturbations. The algorithm improves the speed while retaining the excellent convergence.

3.3.2. Selection of Initial Search Points for Adding Gauss Perturbation. In statistics, the larger the size of the data is, the closer the data distribution is to normal distribution. Under the condition of a large population, the population distribution can be regarded as normal distribution. Gauss perturbation refers to the stochastic perturbations that satisfy normal distribution. In this paper, a Gaussian perturbation is added to the selection of initial search points, and the number of search points is properly increased, so that the set of search points has the probability distribution characteristics of the population.

Given the population size m, the individual $a_i \in R, (i = 1, 2, ...m)$, the central individual a_0 is the average of individual fitness in the population,

$$a_0 = \frac{1}{m}\sum_{i=1}^{m} a_i. \quad (26)$$

Assuming that the population satisfies the normal distribution of (a_0, σ^2) and that a_0 is the average of individual population, $\mu = a_0$, σ^2 can be obtained from the variance formula

$$\sigma^2 = \frac{1}{m}\sum_{i=1}^{m} (a_i - a_0)^2. \quad (27)$$

The initial search point a_0^{i} is generated after adding Gaussian perturbations to the center population

$$a_0^{\mathrm{i}} = a_0 + gauss\left(a_0, \sigma^2\right) \quad (28)$$

Repeat 10 times to generate the initial search point set $\{a_0^{\mathrm{i}}\}(i = 1, 2...10)$. The set $\{a_0^{\mathrm{i}}\}$ thus has the probability distribution characteristics of the population, which not only ensures the diversity of sampling, but also avoids searching for all individuals, so that the algorithm will easily fall into local optimum while increasing the speed.

3.3.3. The Improved MOEA/D Algorithm Flow. The improved MOEA/D algorithm flow is as follows.

Step 1. Algorithm initialization.

Step 2. Neighbor selection among individuals.

Step 3. Mutual optimization among neighbors.

Step 4. Calculate the fitness of individuals and calculate the population center a_0.

Step 5. The initial search point set a_0^i is generated, and the conjugate gradient method is used to search. After satisfying the termination condition, the search result set a_n^i is obtained.

Table 2: Target value comparison.

bomb target	target 1	target 2	target 3	target 4
target 1	1	1	3	1/3
target 2	1	1	3	1/3
target 3	1/3	1/3	1	1/5
target 4	3	3	5	1

Table 3: Simulation parameter table.

Task parameters
Target coordinates: (35,20) (45,60) (55,70) (80,90)
Target value: C_1=0.20 C_2=0.20 C_3=0.07 C_4=0.520
Acceptable threat value: 60
Radar coordinates: (20,30) (56,80)
Radar threat radius: R=15 R=20
Radar severe threat radius: r=7 r=6
Terrain threat coordinates:(60,20) (55,40) (70,40)
Terrain threat radius: R=15 R=15 R=20
Flight speed: 720km/h
Minimum turning radius of aircraft: 1.1km
Operational radius: 500km
Algorithm parameter setting
Population size: 150
Neighbor size: 30
Maximum number of iterations: 500
Number of decision variables: 34

Step 6. a_n^i is used to generate new individuals and calculate fitness values, and the individuals whose fitness values are lower than the new individuals will be replaced.

Step 7. Determine whether the loop ends or not by using the termination condition; if not, turn to step 2 to repeat the cycle until the end of the loop.

4. Simulation Results and Analysis

In order to verify the planning ability of the model under different situations, four simulation experiments are designed to four situations.

4.1. Setting of Simulation Parameters. The battlefield scope is set at 100 km × 100 km, and the target comparison evaluation table is shown in Table 2.

The target weight is $C_1 = 0.20$, $C_2 = 0.20$, $C_2 = 0.20$, $C_3 = 0.07$, $C_4 = 0.520$.

The Simulation parameters are shown in Table 3 .

4.2. Algorithm Performance Analysis. The improved MOEA/D algorithm and the traditional MOEA/D algorithm are used to solve the model and make the first path planning in MATLAB 2014. If the algorithm Pareto curve is smooth and no longer changing, the algorithm has converged. In order

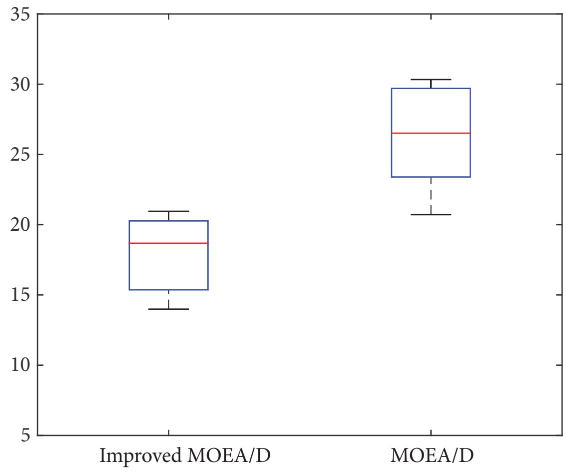

FIGURE 2: Convergence time box diagram.

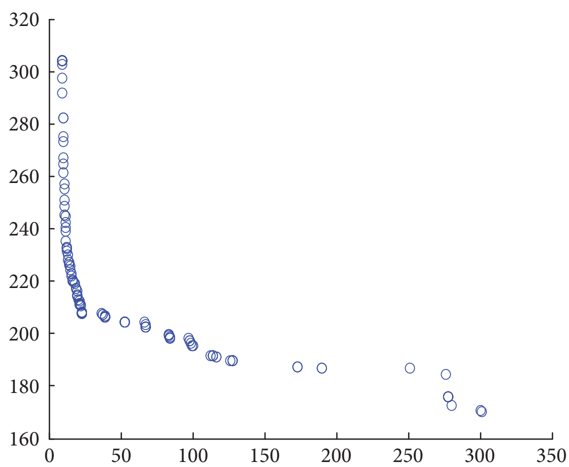

FIGURE 3: The x-axis represents the fitness of path and uses the "Wp" in the formula (11) to indicate. The y-axis represents the threat fitness and uses the "Wt" in the formula (12) to indicate.

to compare the convergence speeds of the two algorithms, 20 simulation experiments were carried out to record the convergence time of the two algorithms, and a box diagram of the convergence time was shown in Figure 2.

Figure 2 shows that the median convergence time of the improved MOEA/D algorithm is 18s, while that of the traditional MOEA/D algorithm is 26s. On the stability of the receiving speed, the upper and lower bounds of the improved MOEA/D algorithm are 14s-21s and the difference is 7s, while the upper and lower bounds of the MOEA/D algorithm are 21s-31s and the difference is 10s. In summary, the improved MOEA/D algorithm has better convergence speed and stability.

Taking the median convergence rate of the algorithm as the data, the Pareto front-end diagrams of the improved MOEA/D algorithm and the traditional MOEA/D algorithm are shown in Figures 3 and 4.

By analyzing Figures 3 and 4, we can see that the shape of Pareto curves of the two algorithms is basically similar, and

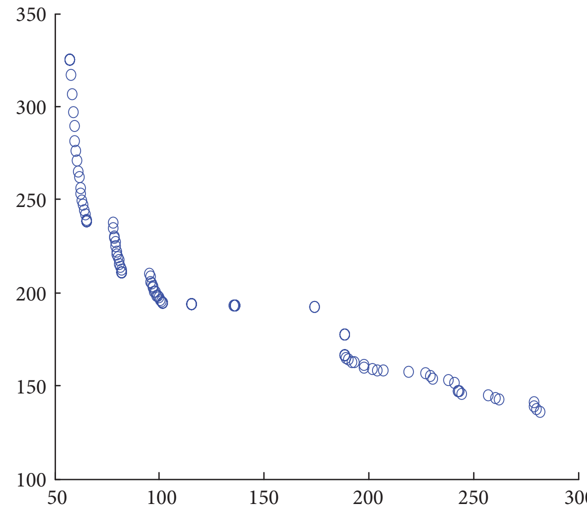

FIGURE 4: MOEA/D algorithm Pareto front-end diagram.

the improved MOEA/D algorithm has better continuity and smoothness.

4.3. Path Analysis. The four situations correspond to (1) striking high-threat and low-value targets, (2) target points in an insurmountable terrain barrier, (3) striking remote target points, and (4) striking high-threat high-value targets. The path planning modeling approach proposed in [6] is used as the first and second situation simulation experiment control group. It verifies the importance of the aircraft performance and the path threat to selected targets. The third and fourth experiments verify the path planning ability under different conditions of the model.

4.3.1. The First Situation Path Analysis. We use the Pareto front-end diagram to analyze the first path planning solved by the improved MOEA/D algorithm. If the range problem is a given priority, the optimal flight path distance of the aircraft is 144 km and the threat value is 148. If the priority is given to the aircraft safety, the flight path distance of the aircraft is 295 km and the threat value is 18. In order to balance the two factors, the median individuals of the population are selected to plot the path. The path planning map is shown in Figure 5.

The circle in the figure represents the radar threat. The outer circle is the general threat area. The inner circle is the serious threat area. If an aircraft enters the serious threat area, it will have a greater probability of being shot down. The interior part filled with oblique lines is the overlooking section of terrain obstacles at flight altitude. The fly path must not pass through this area. The points (C_1, C_2, C_3, C_4) in the figure indicate the target that the mission needs to bomb, and the path should reach the area where it can be bombed. Analysis of Figure 5 shows that the path effectively evades the threat, shortens the range as much as possible, and passes through the targets C_1, C_2, and C_4. Because of the low value of the target and the serious threat, the bombing cost of the aircraft is too high for C_3, so the model abandons the target

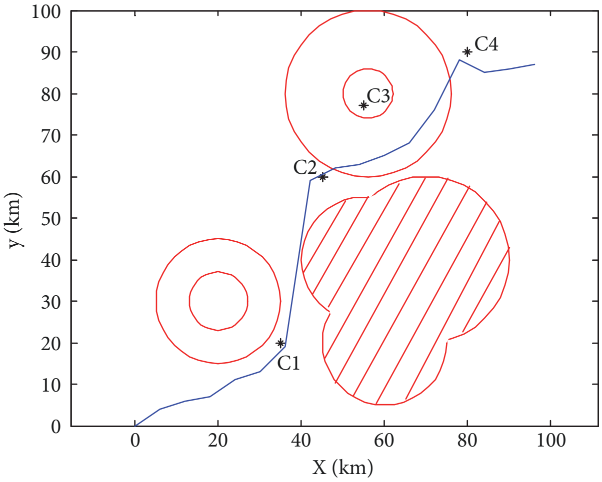

FIGURE 5: The path planning result in the first situation.

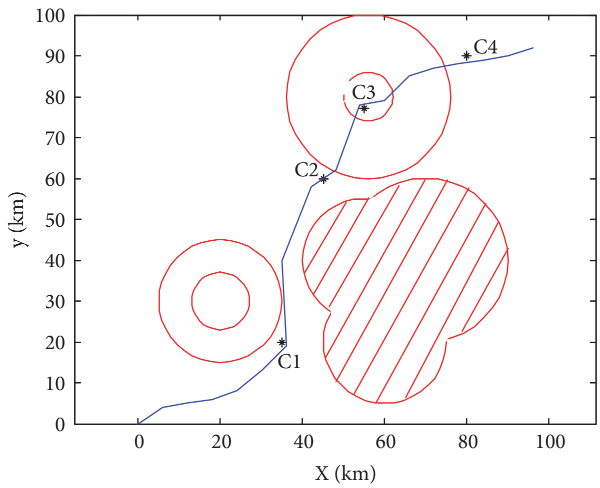

FIGURE 6: The path planning result of the control group in the first situation.

C_3. The calculated flight path threat value is 36, which is less than 60 that the aircraft can tolerate.

Figure 6 shows the path planned by the model of the control group. The planned path also evades the threats and passes through the targets C_1, C_2, C_3, and C_4. The calculated flight path threat value is 104, which is far more than the aircraft's tolerance, 60. If the flight path is executed, the aircraft will be shot down with a high probability.

4.3.2. The Second Situation Path Analysis. Remove the target C_3 from the serious threat area and place C_3 in terrain obstacle which the aircraft is difficult to overcome. The coordinates are (62, 57) and the paths planned by the two approaches are shown in Figures 7 and 8, respectively.

In Figure 7, although striking C_3 does require an aircraft to go through the serious threat area, the aircraft cannot exceed the terrain obstacles, so striking C_3 is given up.

As shown in Figure 8, the path entered the terrain obstacle directly. This is inadvisable.

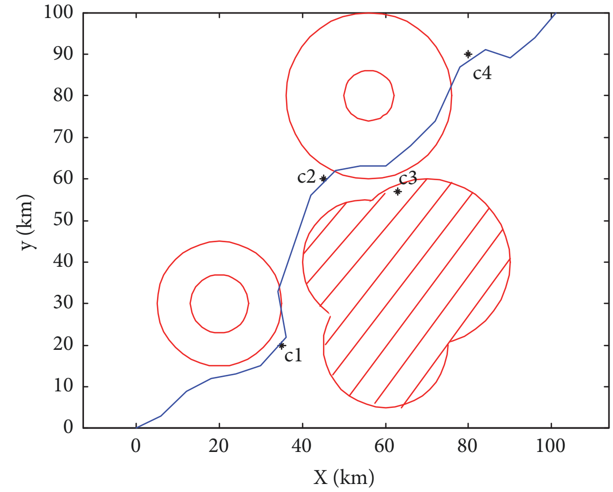

FIGURE 7: The path planning result for the second situation.

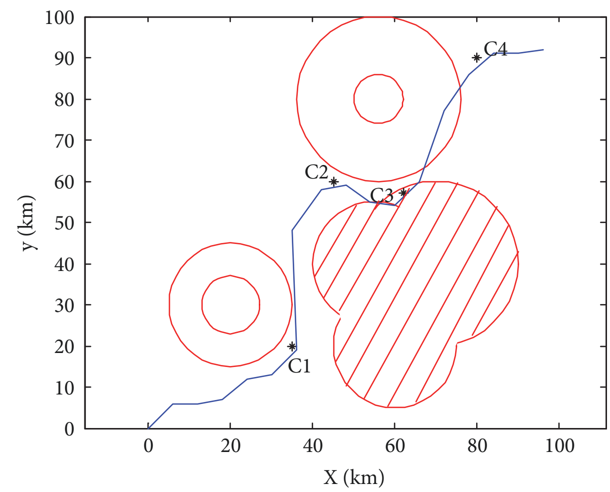

FIGURE 8: The path planning result of control group in the second situation.

4.3.3. The Third Situation Path Analysis. Move the target point C_3 in the serious threat area to a coordinate point (100, 70) that is not threatened but is relatively far from the central axis of the path. The path has been replanned, shown in Figure 9.

Figure 9 shows that although the target C_3 is relatively remote, the path can still pass through the area that leads to the elimination of C_3.

4.3.4. The Fourth Situation Path Analysis. If C_3 is a high value target, we can adjust the acceptable threat value and the value of target point C_3. Here we set the targets C_3=0.55 C_1=0.15 C_2=0.15 C_4 =0.15, and the affordable threat value W_{ft} = 150. The simulated flight path planning is shown in Figure 10.

In Figure 10, comparing the results of the first and the second path planning, the model plans a different path under the condition that the basic battlefield environment remains unchanged, but the target value and the aircraft's tolerance of

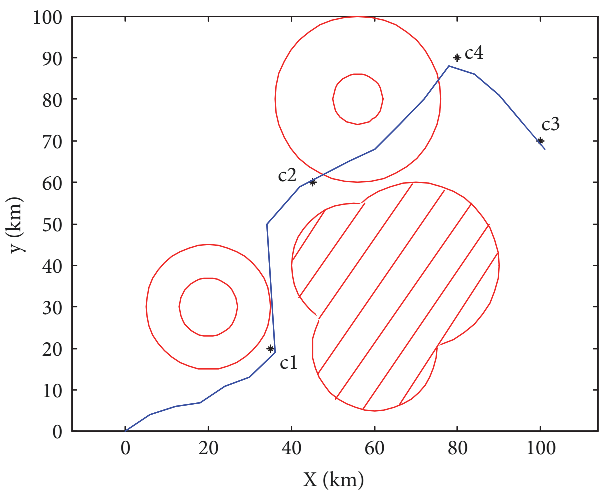

Figure 9: The path planning result in the third situation.

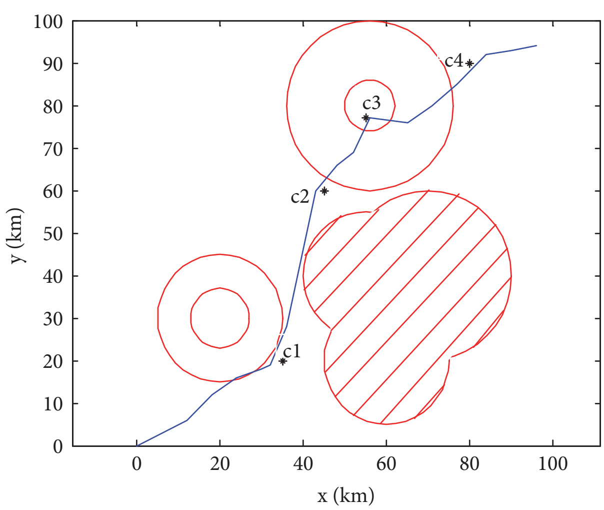

Figure 10: The path planning result in the fourth situation.

threat have changed. When the aircraft is faced with a high-threat and low-value target C_3, it abandons the target. When the aircraft has an improved ability to withstand threats and C_3 becomes a high-value target, the model plans a path which includes C_3.

By analyzing the first and second simulation experiments, the importance of selecting targets based on the value of target, aircraft performance, and threats was proved by the comparison with the control group. Compared with the path of the control group that strikes all targets, the modeling approach proposed in this paper has more practicability. The third and fourth experiments show that the path planned by the model in this paper can strike any target on the premise of ensuring the safety of the aircraft.

5. Conclusion

This paper discusses and solves the flying path planning problem for multi-target missions. The proposed method evaluates the weight of each target using the analytic hierarchy process (AHP) and determines the value for each target after considering the aircraft's acceptable threat tolerance. The improved MOEA/D algorithm is used to solve the problem of target selection and path planning. The simulation results show that the improved algorithm can converge quickly and better. By adjusting the weight of the target value and the acceptable threat tolerance, we can obtain a path that satisfies the mission requirements. We use a simple multi-target bombing mission as an example, but the method can be easily extended to other situations with different constraints on the target region and fly parameters.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Authors' Contributions

Ming Zhong planned the work, completed the simulation experiment, and drafted the main part of the paper. RenNong Yang contributed to error analysis. Huan Zhang and Jun Wu contributed to setup type.

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Research Article

A Hybrid Multiple Criteria Decision Making Model for Selecting the Location of Women's Fitness Centers

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Location selection is a critical problem for businesses that can determine the success of an organization. Selecting the optimal location from a pool of alternatives belongs to a multiple criteria decision making (MCDM) problem. This study employed a hybrid MCDM technique to select locations for women's fitness centers in Taiwan. In the beginning, the fuzzy Delphi method was utilized to obtain selection criteria from interviewed senior executives. In the second stage, the decision making trial and evaluation laboratory (DEMATEL) was employed to extract interdependencies between the selection criteria within each perspective. On the basis of interdependencies between the selection criteria, the analytic network process (ANP) was used to get respective weights of each criterion. Finally, the technique for order preference by similarity to ideal solution (TOPSIS) was ranking the alternatives. To demonstrate application of the proposed model and illustrate a location selection problem, a case was conducted. The capabilities and effectiveness of the proposed model are revealed.

1. Introduction

Location selection is a vital issue in service industries. The quality of location decisions could be improved by a comprehensive location selection model, thereby attracting consumers and positively influencing market share and profitability [1]. The sports industry has been growing in Taiwan, and many organizations now provide sports facilities. In 2015, the government of Taiwan proposed a series of policies to promote female sports participation [2]. Female sports awareness and demand have risen, and female consumers have become a notable potential market. Female consumers are a particularly vital target for sports and fitness centers [3].

Past studies related to location selection problem by using MCDM approaches suffer from several limitations, including (1) not clearly describing how to screen the criteria, (2) not considering the interdependencies among perspectives or criteria in the hierarchy, (3) not applying any quantitative methods to specifically identify the interdependent relations between perspectives or criteria, and (4) not enhancing an optimal solution in a shorter time. To overcome the drawbacks of past studies, this research developed a new hybrid MCDM model including the fuzzy Delphi method, DEMATEL, ANP, and TOPSIS in order to support location selection decisions for women's fitness centers in Taiwan.

A suitable process can help decision makers reduce poor decisions. The fuzzy Delphi method can generate favorable selection criteria [4, 5] and more objective selection criteria can be screened through the fuzzy Delphi method. In this study, we first used the fuzzy Delphi method, which gathers expert opinions, to investigate constructive means of modifying selection criteria. When making decisions, it is important to properly address whether the criteria are dependent on or independent of each other. Many researchers have applied DEMATEL to solve complicated system problems. DEMATEL can make better decisions because it can confirm interdependencies between criteria and help develop a map reflecting their relative relationships [6]. ANP, an extension of the analytic hierarchy process (AHP), is a comprehensive decision making approach that is appropriate to use for both

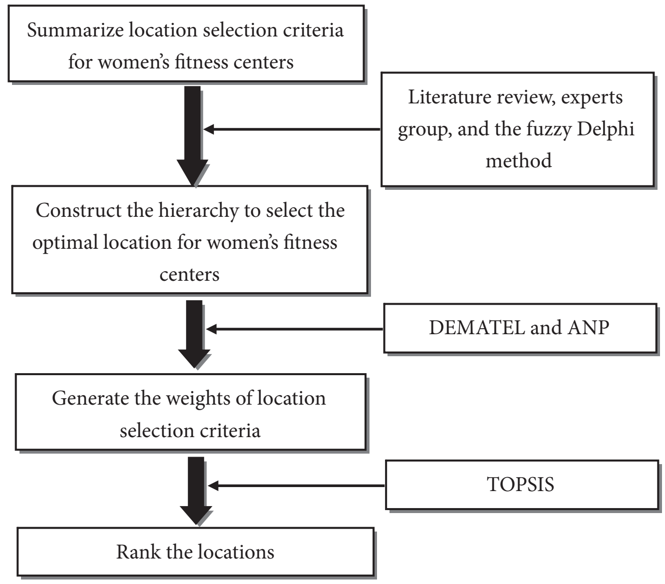

Figure 1: Steps of the proposed selection model.

quantitative and qualitative data and can also overcome the interdependent problems [7]. Several conventional MCDM methods are based on the independence assumption, but ANP accounts for the dependence assumption between criteria, and it is more adapted to applications [8].

Horng et al. [9] concluded that combining DEMATEL and ANP creates a helpful tool. Pai [10] identified the advantages of DEMATEL and ANP. DEMATEL is a useful approach for analyzing cause-effect relationships between criteria. However, DEMATEL cannot generate the weights of criteria. By contrast, ANP can provide criteria priorities. Büyüközkan and Güleryüz [11] described how DEMATEL can be utilized to determine the dependencies of criteria. Integrating ANP and DEMATEL yields successful results in strategic decision making. However, ANP takes more time. This is because interdependent relationships between criteria may increase the number of pairwise comparisons [11]. On the basis of relevant research, we utilized DEMATEL in this study to identify the interdependencies between the criteria within each perspective. According to the interdependencies between the selection criteria, ANP was then applied to get the weights of each. Bongo and Ocampo [12] pointed out that TOPSIS, which ranks alternatives based on both positive and negative ideal solutions, was an appropriate ranking method. Nie et al. [13] also concluded that TOPSIS give a reasonable alternative ranking. Abdel-Basset et al. [14] pointed that ANP needs many pairwise comparison matrices on the basis of the interdependence of the selection criteria. To overcome this drawback, TOPSIS was for ranking alternatives. The selection procedure is shown in Figure 1.

Combining the fuzzy Delphi method, DEMATEL, ANP, and TOPSIS, this study sought to achieve better location selection decisions in a shorter time; this distinguishes this study from others in the literature. The rest of this study is organized as follows. The literature is briefly reviewed. Next, the use of the fuzzy Delphi method, DEMATEL, ANP, and TOPSIS is proposed. Section 7 presents the integrated method for selecting the optimal location. Section 8 concludes the paper.

2. MCDM for Location Selection

Several literature review studies have been presented to examine the location selection problem. Chang et al. [15] applied the fuzzy Delphi method, ANP, and TOPSIS to help managers to select the locations for Taiwanese service apartments. Chen and Tsai [1] proposed a data mining framework using the rough set theory (RST) to support location selection decisions. Aksoy and Yetkin Ozbuk [16] employed the preference selection index (PSI) to rank hotels by location. Chen et al. [17] used the preference ranking organization method for enrichment evaluations (PROMETHEE) based on the cloud model and AHP to select location for a large charging power station. Komchornrit [18] used the confirmatory factor analysis (CFA), the measuring attractiveness by a categorical based evaluation technique (MACBETH), and PROMETHEE to select a dry port location. Lee et al. [19] integrated the interpretive structural modeling (ISM), fuzzy ANP, and the vlsekriterijumska optimizacija i kompromisno resenje (VIKOR) to select the photovoltaic solar plant locations. Nie et al. [20] applied an extended weighted aggregated sum product assessment (WASPAS) to select solar-wind power station locations. Pamučar et al. [21] applied the geographical information systems (GIS), the best worst method (BWM), and the multiattributive ideal-real comparative analysis (MAIRCA) to select the locations for wind farms. Cheng [22] offered a new autocratic multiattribute group decision making approach to select locations for hotels. Deveci et al. [23] proposed a WASPAS based on TOPSIS with an interval type-2 fuzzy MCDM model for selecting a car sharing station location. Sennaroglu and Varlik Celebi [24] employed AHP, PROMETHEE, and VIKOR to select location for a military airport. Stević et al. [25] used rough BWM and rough WASPAS to select potential locations for roundabout construction.

Although main studies have examined finding ideal locations, the literature does not have useful guidance for creating a decision making model for selecting locations for women's fitness centers. There is a lack of published paper in this field. Moreover, past studies related to location selection problem suffer from many limitations. Firstly, some of them did not clearly describe how to screen the criteria. Some authors treat perspectives or criteria as independent but they are often dependent on each other in real world. Although some authors consider the perspectives or criteria as interdependent, they did not utilize quantitative methods to clearly identifying the cause-effect relationships. Lastly, some studies did not enhance an optimal solution in a shorter time.

To address this, we used the fuzzy Delphi method to modify the selection criteria. Then, DEMATEL was applied to identify the interdependencies among the criteria within each perspective. ANP, which captures the interdependencies, was applied to obtain the weights of the selection criteria. TOPSIS was for ranking the alternatives. Based on the fuzzy Delphi method, DEMATEL, ANP, and TOPSIS, this study achieved more favorable location selection decisions. No similar research has combined these methodologies. Therefore, this study introduces an integrated model to fill the gap in this area.

3. Fuzzy Delphi Method

The Delphi method, a traditional forecasting approach, does not require large samples and is employed to generate a professional consensus [26]. Consensus is obtained by conducting up to four consultation rounds and deriving information such as medians, averages, and deviations from the previous rounds [27]. However, the Delphi method was criticized for low convergence of expert opinions and high execution costs [28]. Hsu et al. [29] described how the Delphi method is an expert opinion survey approach that also exposes weaknesses. In some situations, expert judgments cannot be properly reflected in quantitative terms. Some ambiguity results because of differences in meanings or interpretations of expert opinions. Dong et al. [30] also identified the shortcomings of the traditional Delphi method, namely long feedback time, high cost, and low convergence rate, as well as the possibility of distorting expert opinions.

Recognizing the drawbacks of the traditional Delphi method, several researchers [4, 31, 32] have improved it in a fuzzy environment. Kuo and Chen [28] concluded that the fuzzy Delphi method reduced the cost and time of the investigation. Chao and Kao [33] claimed that the main advantage of the fuzzy Delphi method was saving time by reducing the number of surveys while including the opinions of experts. This present study used the fuzzy Delphi method proposed by Klir and Yuan [34] to screen the selection criteria. We first collected expert opinions from questionnaires and established the triangular fuzzy number T_i. Y_{ij} was the evaluation value of expert j for criterion i. On the basis of the work of Hsu et al. [29], we used the simple center of gravity method to calculate the defuzzified value of each criterion S_i. Last, selection criteria were screened out by setting the threshold. In this study, the threshold value was set by experts through discussion.

$$T_i = (L_i, M_i, U_i) \tag{1}$$

$$L_i = \min_j \{Y_{ij}\} \tag{2}$$

$$U_i = \max_j \{Y_{ij}\} \tag{3}$$

$$M_i = \left(\prod_{j=1}^{n} Y_{ij}\right)^{1/n} \tag{4}$$

$$S_i = \frac{L_i + M_i + U_i}{3} \tag{5}$$

4. DEMATEL

DEMATEL is applied to identify the interdependent relationships between various criteria. It is executed as detailed herein [35–38].

Step 1 (gather expert opinions). Experts are designated the task of completing pairwise comparisons to determine the relative influence of pairs of criteria on a scale of 0 (no influence) to 4 (very high influence). Let x_{ij} denote the extent to which expert evaluation criterion i influences criterion j. When i=j, all elements existing on the diagonal become 0. For each expert, we establish an $n{\times}n$ positive matrix in the form $X^k = [X_{ij}^k]$ for $1 \le k \le H$; here, k and n are the number of experts and criteria, respectively, yielding $X^1, X^2, \ldots, X^H$ as the H experts' individual matrixes. To integrate these distinct matrixes, we construct the average matrix $A = [a_{ij}]$:

$$a_{ij} = \frac{1}{H}\sum_{k=1}^{H} x_{ij}^k \tag{6}$$

Step 2 (normalize the generated initial direct relation matrix). We normalize the generated initial direct relation matrix D as $A \times S$, and $S = 1/\max_{1\le i\le n} \sum_{j=1}^{n} a_{ij}$. All elements in D take a value in the 0-1 range.

Step 3 (obtain the total relation matrix). We can determine the total relation matrix T to be given by $T = D(I - D)^{-1}$, with I being the identity matrix. Let c and r, respectively, be the $1 \times n$ and $n \times 1$ vectors denoting the sums of columns and rows of T. If r_i is the sum of row i in T, r_i captures the indirect effects and direct effects exerted by criterion i on other criteria. Similarly, if c_j is the sum of column j in T, c_j represents the indirect effects and direct effects exerted by criterion j from the other criteria. For j=i, (r_i+c_j) expresses the net effect—that is, both exerted and received—by criterion i. This means that (r_i+c_j) is a measure of the influence of criterion i on the system. Conversely, $(r_i - c_j)$ expresses the total effect criterion i adds to the system. Thus, when (r_i-c_j) is larger (smaller) than 0, criterion i is a net cause (net receiver) [39, 40].

Step 4 (establish the threshold). T clarifies the influence of various criteria on each other. Decision makers must therefore establish a threshold to exclude negligible influence relationships. In this study, this threshold was established by experts through discussion [41].

5. ANP

ANP comprises the following four important steps [7].

Step 1 (construct the hierarchy as well as problem structure). On the basis of the goals, perspectives, criteria, and alternatives, the process can create the problem structure within a hierarchy containing multiple levels of elements and connections. This step can be realized on the basis of appropriate methods, for example, brainstorming by decision makers and a literature survey.

Step 2 (establish the perspectives and criteria weights). Through the application of pairwise comparisons, the decision makers determine the interdependency of perspectives and criteria by using Saaty 1-9 as the relative scale. To establish a pairwise comparison, Saaty [7] recommended that all decision makers' preferences be integrated as the geometric mean and proposed consistency ratio (CR) as a measure of the pairwise comparison matrix's consistency [42]:

$$\mathrm{CR} = \frac{\mathrm{CI}}{\mathrm{RI}} \quad \text{with } \mathrm{CI} = \frac{\lambda_{\max} - n}{n-1} \tag{7}$$

Here, CI and RI are the consistency index and random index, respectively, and n and $\lambda_{\max}$ are the number of criteria in the matrix and the maximum (or principal) eigenvalue of the matrix, respectively. The matrix consistency is deemed acceptable if CR < 0.1.

Step 3 (build the supermatrix and solve it). ANP utilizes a supermatrix to account for the outer and inner dependencies among criteria and perspectives. The weights of the criteria and perspectives derived in Step 2 are applied to form the supermatrix column. Then, the supermatrix is multiplied by itself until all the row elements converge to the same value in each column of the matrix, thus ensuring matrix stability. The resulting matrix is termed the limiting matrix.

$$W_{limit} = \lim_{x \longrightarrow \infty} \left(W_{weighted}\right)^{x} \cong \left(W_{weighted}\right)^{2k+1} \tag{8}$$

Here, k is an arbitrarily large number.

Step 4 (choose the optimal alternative). On the basis of the built limiting matrix and the weights of the alternatives (vs. the criteria), the net weight of each alternative can be summed. Then, the identified alternatives are ranked per their weights.

6. TOPSIS

In TOPSIS, which was developed by Hwang and Yoon in 1981 [43], the optimal alternative is the one that is the closest to and farthest from the positive ideal solution (A^*) and negative ideal solution (A^-), respectively. TOPSIS is executed as detailed herein.

Step 1 (build the standardized appraisal matrix).

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \tag{9}$$

Here, i, x_{ij}, and j represent the alternatives, ith alternative under the jth criterion to be assessed, and selection criteria, respectively.

Step 2 (build the weighted standardized appraisal matrix). The product of the weights of the selection criterion $w = (w_1, w_2, \ldots, w_n)$ and the standardized appraisal matrix is expressed as follows:

$$\begin{aligned} v &= \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1n} \\ v_{21} & v_{22} & \cdots & v_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ v_{m1} & v_{m2} & \cdots & v_{nm} \end{bmatrix} \\ &= \begin{bmatrix} w_1 r_{11} & w_2 r_{12} & \cdots & w_n r_{1n} \\ w_1 r_{21} & w_2 r_{22} & \cdots & w_n r_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ w_1 r_{m1} & w_2 r_{m2} & \cdots & w_n r_{mn} \end{bmatrix} \end{aligned} \tag{10}$$

Step 3 (determine both the positive and negative ideal solutions).

$$\begin{aligned} A^* &= \left\{v_1^*, v_2^*, \ldots, v_j^*, \ldots, v_n^*\right\} \\ &= \left\{\left(\max_i v_{ij} \mid j \in J\right) \mid i = 1, \ldots, m\right\}, \\ A^- &= \left\{v_1^-, v_2^-, \ldots, v_j^-, \ldots, v_n^-\right\} \\ &= \left\{\left(\min_i v_{ij} \mid j \in J\right) \mid i = 1, \ldots, m\right\}. \end{aligned} \tag{11}$$

Step 4 (for all alternatives, estimate the Euclidean distance separating the positive (S_i^*) and negative (S_i^-) ideal solutions).

$$\begin{aligned} S_i^* &= \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_i^*\right)^2}, \quad i = 1, \ldots, m, \\ S_i^- &= \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_i^-\right)^2}, \quad i = 1, \ldots, m. \end{aligned} \tag{12}$$

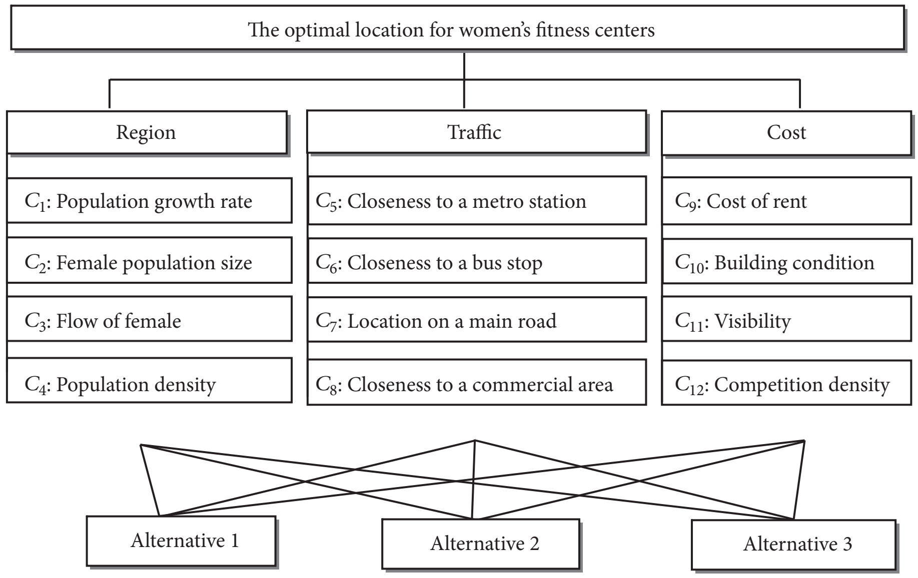

Figure 2: Hierarchy to select optimal location for women's fitness centers.

Step 5 (for all alternatives, calculate the relative distance to the positive ideal solution).

$$C_i^* = \frac{S_i^-}{S_i^* + S_i^-} \tag{13}$$

For C_i^* approaching 1, alternative A_i becomes closer to A^* and farther from A^-.

Step 6 (rank the alternatives on the basis of C_i^*). Larger index values (C_i^*) signify higher performance.

7. Empirical Application

We employed the fuzzy Delphi method, DEMATEL, ANP, and TOPSIS to select the optimal locations for women's fitness centers in Taiwan. The steps of the selection process were as follows.

Step 1 (construct the hierarchy as well as problem structure). The selection criteria for the location were collected from other relevant studies and discussions with executives of women's fitness centers. Using a purposive sampling method, the executives were contacted and asked to recommend other professionals with sufficient experience in selecting locations. Using the fuzzy Delphi method, questionnaires were distributed regarding selection criteria based on a 9-point Likert scale (1=most unimportant and 9=most important). A semistructured questionnaire was used to screen criteria for location selection and assess their importance. The questionnaire also provided space for the executives to suggest additional criteria. Questionnaires were received from 48 senior executives. The data were converted into triangular fuzzy numbers to calculate defuzzified values and screen the criteria for the purpose of assuring content validity.

Based on discussion with senior executives, top 12 criteria were obtained: population growth rate [1], female population size (executive proposed), flow of female (executive proposed), population density [1, 19], closeness to a metro station [1, 15, 16, 22, 23], closeness to a bus stop [1, 15, 16, 22, 23], location on a main road [1], closeness to a commercial area [15], cost of rent [23], building condition (executive proposed), visibility [1], and competition density [1, 15]. Based on past research [1, 15] and discussions with senior executives, the hierarchy was constructed as shown in Figure 2.

Step 2 (identify the relationships within each perspective through DEMATEL). DEMATEL was used to identify the relationships between the selection criteria within each perspective and their influence on each other. The members of the decision making committee, including two managers of a women's fitness center, were interviewed to determine the influential relationships to use for ranking each criterion. The initial direct relation matrix A was obtained from equation (6) as depicted in Tables 1–3. The total relation matrix T is illustrated in Tables 4–6.

To obtain an appropriate relationship, threshold values were chosen after discussion with the same two senior executives. Thus, the interdependencies between the selection criteria within each perspective were developed as shown in Figures 3–5.

Step 3 (establish the perspectives and criteria weights). Each perspective's priority weight is depicted in Table 7. The CR of each pairwise comparison was < 0.1; the consistency is acceptable.

Pairwise comparisons are applied based on the interdependencies of the selection criteria. A pairwise comparison within the Region perspective with respect to Population growth rate is presented in Table 8. Following this method,

Table 1: The initial direct relation matrix within Region perspective.

Criteria	Population growth rate	Female population size	Flow of female	Population density
Population growth rate	0.0000	4.0000	3.0000	3.5000
Female population size	3.5000	0.0000	4.0000	4.0000
Flow of female	3.5000	4.0000	0.0000	4.0000
Population density	3.0000	4.0000	4.0000	0.0000

Table 2: The initial direct relation matrix within Traffic perspective.

Criteria	Closeness to a metro station	Closeness to a bus stop	Location on a main road	Closeness to a commercial area
Closeness to a metro station	0.0000	3.5000	3.0000	4.0000
Closeness to a bus stop	4.0000	0.0000	3.5000	4.0000
Location on a main road	3.0000	3.0000	0.0000	4.0000
Closeness to a commercial area	4.0000	4.0000	3.5000	0.0000

Table 3: The initial direct relation matrix within Cost perspective.

Criteria	Cost of rent	Building condition	Visibility	Competition density
Cost of rent	0.0000	4.0000	3.5000	3.0000
Building condition	4.0000	0.0000	3.0000	3.5000
Visibility	3.5000	1.5000	0.0000	2.5000
Competition density	2.5000	2.0000	4.0000	0.0000

Table 4: The total relation matrix within Region perspective.

Criteria	Population growth rate	Female population size	Flow of female	Population density
Population growth rate	6.6667	**7.9140**	**7.4194**	**7.6667**
Female population size	**7.3840**	**8.2121**	**7.9938**	**8.2320**
Flow of female	**7.3840**	**8.4701**	**7.7357**	**8.2320**
Population density	7.1367	**8.2149**	**7.7544**	**7.7266**
The threshold value is 7.3000				

Table 5: The total relation matrix within Traffic perspective.

Criteria	Closeness to a metro station	Closeness to a bus stop	Location on a main road	Closeness to a commercial area
Closeness to a metro station	**4.2651**	**4.3533**	**4.1566**	**4.7913**
Closeness to a bus stop	**4.8174**	**4.4040**	**4.4551**	**5.1048**
Location on a main road	**4.3058**	**4.1669**	3.7961	**4.6153**
Closeness to a commercial area	**4.8174**	**4.6620**	**4.4551**	**4.8468**
The threshold value is 4.0000				

Table 6: The total relation matrix within Cost perspective.

Criteria	Cost of rent	Building condition	Visibility	Competition density
Cost of rent	**1.8757**	**1.7803**	**2.2064**	**1.9404**
Building condition	**2.1534**	**1.5081**	**2.1855**	**1.9717**
Visibility	**1.6792**	1.2828	**1.4985**	**1.5022**
Competition density	**1.7346**	1.3903	**1.8934**	**1.4098**
The threshold value is 1.4000				

Table 7: The pairwise comparisons of perspectives.

	Region λ_{max}=3.0099	Traffic C.R.=0.0075	Cost	Priority weights
Region	1.0000	2.6458	7.3485	0.6491
Traffic	0.3780	1.0000	3.7417	0.2709
Cost	0.1361	0.2673	1.0000	0.0800

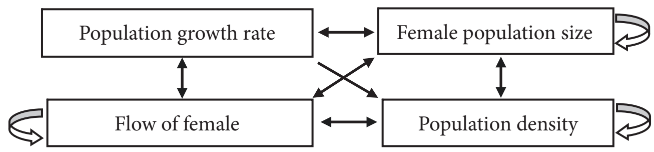

Figure 3: The interdependencies between the selection criteria within the Region perspective.

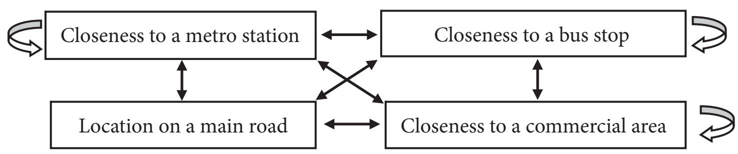

Figure 4: The interdependencies between the selection criteria within the Traffic perspective.

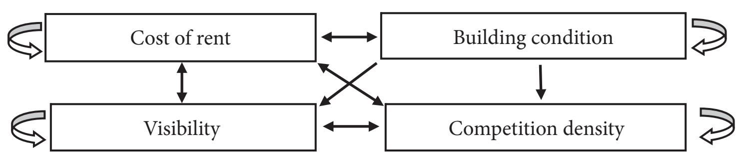

Figure 5: The interdependencies between the selection criteria within the Cost perspective.

we derived every criterion weight and obtained the supermatrix.

Step 4 (build the supermatrix and solve it). The criteria weights derived in Step 3 were used to obtain the column of the supermatrix as depicted in Table 9. The limiting matrix is indicated in Table 10. On the basis of the data presented in Tables 7 and 10, we aggregated the total weight of each criterion as presented in Table 11.

Step 5 (build the standardized and weighted standardized appraisal matrix). We used equation (9) to obtain the standardized appraisal matrix, as displayed in Table 12. The weighted standardized appraisal matrix is depicted in Table 13.

Step 6 (determine both the positive and negative ideal solutions). The positive ideal solution and negative ideal solution were defined according to equation (11) as

$$A^{*} = (0.0625, 0.1787, 0.0784, 0.0921, 0.0400, 0.0686, 0.0239, 0.0396, 0.0179, 0.0072, 0.0090, 0.0060),$$
$$A^{-} = (0.0625, 0.1246, 0.0784, 0.0647, 0.0400, 0.0467, 0.0207, 0.0323, 0.0303, 0.0056, 0.0070, 0.0060). \quad (14)$$

Step 7 (for all alternatives, estimate the Euclidean distance separating the positive and negative ideal solutions). By equation (12), the Euclidean distance for each alternative can be measured.

Step 8 (for all alternatives, calculate the relative distance to the positive ideal solution). The C_i^* value of each alternative was calculated by equation (13).

Step 9 (choose the optimal alternative). According to the results presented in Table 14, the preferred location was selected. Alternative 2 was clearly the optimal location, followed by Alternatives 1 and 3. We provided the results to the case company, and they selected the location as per our suggestion.

8. Conclusion

Location selection is a MCDM problem. In this study, a novel hybrid MCDM model effectively selected the optimal location for women's fitness centers in Taiwan through the fuzzy Delphi method, DEMATEL, ANP, and TOPSIS with

Table 8: The pairwise comparisons within Region perspective with respect to Population growth rate.

	Female population size λ_{max}=3.0260	Flow of female C.R.=0.0197	Population density	Priority weights
Female population size	1.0000	2.6458	2.8284	0.5731
Flow of female	0.3780	1.0000	1.7321	0.2544
Population density	0.3536	0.5774	1.0000	0.1725

Table 9: The supermatrix before convergence.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}
C_1	0.0000	0.2750	0.2532	0.0000								
C_2	0.5731	0.3795	0.3977	0.3725								
C_3	0.2544	0.1684	0.1677	0.2956								
C_4	0.1725	0.1771	0.1814	0.3319								
C_5					0.2617	0.2617	0.2318	0.2550				
C_6					0.3966	0.3966	0.1840	0.4044				
C_7					0.1672	0.1672	0.0000	0.1667				
C_8					0.1745	0.1745	0.5842	0.1739				
C_9									0.4758	0.6586	0.6436	0.6386
C_{10}									0.2332	0.0893	0.0000	0.0000
C_{11}									0.1355	0.1246	0.2740	0.2806
C_{12}									0.1555	0.1275	0.0825	0.0807

Table 10: The supermatrix after convergence (Limiting matrix).

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}
C_1	0.1669	0.1669	0.1669	0.1669								
C_2	0.4141	0.4141	0.4141	0.4141								
C_3	0.2093	0.2093	0.2093	0.2093								
C_4	0.2097	0.2097	0.2097	0.2097								
C_5					0.2559	0.2559	0.2559	0.2559				
C_6					0.3679	0.3679	0.3679	0.3679				
C_7					0.1432	0.1432	0.1432	0.1432				
C_8					0.2330	0.2330	0.2330	0.2330				
C_9									0.5524	0.5524	0.5524	0.5524
C_{10}									0.1414	0.1414	0.1414	0.1414
C_{11}									0.1772	0.1772	0.1772	0.1772
C_{12}									0.1290	0.1290	0.1290	0.1290

Table 11: The total weight of each selection criterion.

	Weights from perspectives	Weights from supermatrix after convergence	Total weights
C_1	0.6491	0.1669	0.1083
C_2	0.6491	0.4141	0.2688
C_3	0.6491	0.2093	0.1358
C_4	0.6491	0.2097	0.1361
C_5	0.2709	0.2559	0.0693
C_6	0.2709	0.3679	0.0997
C_7	0.2709	0.1432	0.0388
C_8	0.2709	0.2330	0.0631
C_9	0.0800	0.5524	0.0442
C_{10}	0.0800	0.1414	0.0113
C_{11}	0.0800	0.1772	0.0142
C_{12}	0.0800	0.1290	0.0103

Table 12: The standardized appraisal matrix.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}
A_1	0.5774	0.5861	0.5774	0.5625	0.5774	0.5541	0.5345	0.5867	0.6860	0.6332	0.5970	0.5774
A_2	0.5774	0.6646	0.5774	0.6765	0.5774	0.4683	0.5774	0.6273	0.6050	0.5970	0.6332	0.5774
A_3	0.5774	0.4634	0.5774	0.4754	0.5774	0.6882	0.6172	0.5121	0.4042	0.4925	0.4925	0.5774

Table 13: The weighted standardized appraisal matrix.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}
A_1	0.0625	0.1576	0.0784	0.0766	0.0400	0.0552	0.0207	0.0370	0.0303	0.0072	0.0085	0.0060
A_2	0.0625	0.1787	0.0784	0.0921	0.0400	0.0467	0.0224	0.0396	0.0267	0.0068	0.0090	0.0060
A_4	0.0625	0.1246	0.0784	0.0647	0.0400	0.0686	0.0239	0.0323	0.0179	0.0056	0.0070	0.0060

Table 14: The results of TOPSIS.

	S_i^*	S_i^-	C_i^*	Rank
A_1	0.0322	0.0365	0.5310	2
A_2	0.0237	0.0612	0.7209	1
A_3	0.0611	0.0254	0.2937	3

group decision making. The fuzzy Delphi method was used to gather information, which effectively addressed the vagueness and imprecision of the expert judgments to identify the location selection criteria. DEMATEL was then employed to identify the interrelationships between criteria within each perspective. On the basis of DEMATEL results, ANP was used to build the criteria weights. To avoid the excessive calculation and additional pairwise comparisons characteristic of ANP, TOPSIS was used to rank the alternatives. TOPSIS eliminated procedures that are typically performed in ANP, enabling the system to achieve a conclusion in a shorter time.

No known study has previously combined the fuzzy Delphi method, DEMATEL, ANP, and TOPSIS. This study therefore contributes to the literature by combining these four approaches for the first time to develop a novel selection model to assist with the location selection process. To validate the effectiveness of the proposed model, two managers of the case company were interviewed. They selected the location per our suggestion and the performance of the new branch is better than expected. In practice, it takes about two and half years to recoup the costs of starting up a branch. In this case company, managers recouped the amount their invested in two years. The results confirmed that the proposed model can help managers improve their decision making processes. This study provided reliable and validated selection criteria on which women's fitness center managers can make optimal location selections. The criteria identified in this study should be considered as comprising guidelines on which managers can make optimal location decisions.

This study had several limitations. First, respondents were senior executives of women's fitness centers in Taiwan. Further studies should be conducted in different countries to account for cultural variation. However, some criteria have a qualitative structure that could not be measured precisely. The fuzzy extensions of DEMATEL or ANP can be considered in the proposed model for dealing with vagueness or imprecision. Lastly, comparative analyses with different MCDM approaches on the same example are not conducted in this paper.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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