Discrete Dynamics in Nature and Society

Discrete Dynamic Gaming Models in Supply Chain Management and Project Management

Guest Editors: Jun Zhuang, Xiaolin Xu, and Gangshu (George) Cai

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Editorial Board

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Editorial

Discrete Dynamic Gaming Models in Supply Chain Management and Project Management

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In the fields of supply chain management and project management, researchers and practitioners have established enormous models to describe, explain, and solve practical problems occurring in the industry. However, the real world is so complex because of the dynamics and uncertainty embedded in a variety of environmental parameters, such as market demand, supply of raw materials, price of raw materials and finished products, the transportation and delivery time, production processing time, and budget. As a result, it is extremely challenging for firms to make their optimal decisions. In addition, the vertical and horizontal competition and coordination among different parties simply make the difficulty grow in an order of magnitude, especially considering the interactive and dynamic social and economic activities. Therefore, applying the discrete dynamic gaming models to industrial practices, while allowing for the randomness of environmental factors and the interactions of different participants, provides a useful and powerful way to resolve many complex practical issues. This special issue is dedicated to such models.

One of the papers studies a three-level supply chain consisting of one manufacturer, one distributor, and one retailer facing a stochastic and sales effort dependent market demand. While the traditional revenue-sharing mechanism cannot coordinate the supply chain perfectly, the authors propose augmented mechanisms based on quantity discount to improve the supply chain performance. Another paper considers the joint production planning of complex supply chains constrained by carbon emission reduction policies. The authors incorporate input-output model to capture the interrelated demand link in an arbitrary pair of two nodes in scenarios with or without carbon emission limits. Two typical carbon emission reduction policies are studied to examine how carbon emission reduction policies affect the profit and carbon emission of a particular supply chain. Another paper explores how the joint carbon emission reduction of the manufacturer and the retailer impacts a dyadic supply chain. A new side-payment contract is designed to coordinate the whole supply chain.

One of the papers investigates a make-to-stock system with controllable demand rate by varying selling price and adjustable service rate by outsourcing production. It shows that the optimal control policy is of threshold type depending on the inventory position. Another paper considers a closed-loop supply chain network equilibrium problem in multiperiod planning horizons with constraints on product lifetime and carbon emission cap. The network consists of four tiers, suppliers, manufacturers, retailers, and consumers. Based on the variational inequality and complementary theory, the optimal behaviors of different tier players are characterized, and the governing closed-loop supply chain network equilibrium model is established. The equilibrium is obtained using a modified project contraction algorithm with fixed steps. One of the papers considers a technical problem on the uncertainty of supply chain network design. As the spatial location areas become extremely large, it is a big challenge to evaluate the covariance matrix determined by the set of location distance even for gridded stationary Gaussian process. To alleviate the numerical challenges, the authors construct a nonparametric estimator, called periodogram of spatial version, to represent the sample property in frequency domain. Under some regularity conditions on the process, the asymptotic unbiasedness property of periodogram as estimator of the spectral density function is investigated and the convergence rate is demonstrated.

Another paper studies the horizontal coalition stability problem of players in a two-stage supply chain consisting of one supplier and multiple dealers. The first stage game is a vertical sequential game with the supplier being the leader and the dealers being the followers. While at the second stage, multiple dealers either play the Nash game separately or form a coalition as a whole. Another paper explores the cost allocation mechanism in public-private partnership projects based on the theory of contracts as reference points. Some managerial insights are provided from the derivation of the optimal investment ratio. One of the papers considers a new scheduling problem where emergency jobs suddenly appear during the processing of a particular job and must be processed immediately upon the completion of the current job. The processing times of all jobs are random and should be completed on a single machine. Two objective functions, the weighted sum of waiting times and the weighted discounted cost of waiting times, respectively, are proposed based on industrial practices (e.g., the surgery problem). Optimal polices are addressed to minimize the corresponding objectives.

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Research Article

Coordinating Three-Level Supply Chain by Revenue-Sharing Contract with Sales Effort Dependent Demand

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Revenue-sharing contract is a kind of mechanism to improve performance or to achieve perfect coordination of supply chain. Considering a three-level supply chain consisting of a manufacturer, a distributor, and a retailer who faces a stochastic and sales effort dependent demand, the paper analyzes the impact of sales effort on supply chain coordination and expounds the reasons why traditional revenue-sharing contract cannot coordinate supply chain in this condition. Given three cases: only the retailer bears the sales effort cost, only the manufacturer bears the sales effort cost, and the retailer bears the sales effort cost with the manufacturer, the paper proposes an improved revenue-sharing contract based on quantity discount policy to coordinate the supply chain. It illustrates that improved revenue sharing contract can coordinate supply chain by implementing it in one transaction or two transactions of three-level supply chain. The model of improved revenue-sharing contract is optimized, respectively, by addition form and multiplication form with sales effort dependent demand. Formulas are given to determine the optimal contract parameters. Finally, numerical experiments are given to test the accuracy of the model of improved revenue-sharing contract.

1. Introduction

Supply chain is made up of several different decision makers who pursue different objectives and may conflict among each other. Then, the contractual supply chain actually faces a "double-marginal utility": each individual of the supply chain makes the decision, respectively, in order to achieve optimum profits. But the decision is not identical with the one in the coordinated supply chain. So we need a kind of coordinative mechanism or contract to make the whole profits of supply chain achieve the optimum and guarantee each company gets more profits than it does in the noncontractual supply chain.

Revenue-sharing contract [1] is a kind of contract to coordinate supply chain. The contract can be described by two parameters (w, φ): the supplier charges the retailers a unit wholesale price w, lower than the unit marginal cost c, in exchange for a percentage $(1 - \varphi)$ of the retailer's revenue. The position $w < c$ guarantees channel coordination, whereas φ determines the distribution of total profits between the supplier and the retailer. In particular, φ is the supply chain profit quota gained by the retailer.

The revenue-sharing contract initially appeared in the industry of video-rent and was later successfully extended to other industries, such as franchise model commonly used in China. From an economic point of view of revenue-sharing contract applications in the video rental industry (1998-1999), Mortimer [2] carried out an empirical study and found that the contract can make different enterprises in the supply chain increase their profits by 3%–6%. Due to the successful applications of revenue-sharing contract in supply chain practice, the study on revenue-sharing contract shows growing trends and the literatures [3–11] are representative. Based on the literature of Cachon and Lariviere [1], Giannoccaro and Pontrandolfo [12] proposed a model based on revenue-sharing contract to coordinate a three-level supply chain. On the basis of Giannoccaro and Pontrandolfo [12], the model was further improved [13–15].

By reviewing the extensive literature available, we can see that most of the literatures do not consider the effort activities of supply chain members that affect market demand. And it is known that there exists a wide gap between market demand and the actual situation. In reality, the effort activity of

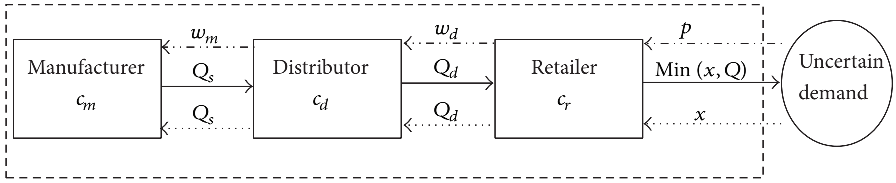

Figure 1: Three-level supply chain.

supply chain members, as important factors affecting market demand [16], is beneficial to the entire supply chain system. It can increase the market demand to some extent [17–19], such as increasing investment in advertising, hiring more sales staff to promote products, and considering customers' personality in product manufacturing. However, it needs cost to do these effort activities. Therefore, if only one supply chain member bears the cost of effort activities, he will choose the effort level which is best beneficial to him, and often this choice can not make the supply chain coordinate [20].

Cachon [21] and Cachon and Lariviere [1] pointed out that the revenue-sharing contract can not coordinate supply chain with retailer's sales effort dependent market demand, the kind of situation many scholars have studied. He et al. [22, 23] proposed a mixed contract based on both revenue-sharing contract and rebate and penalty contract to coordinate supply chain. Hu and Wang [24] proposed an evolution contract based on revenue-sharing contract, and studied this problem in the framework of principal-agent. Qu and Guo [11] proposed an improved revenue-sharing contract considering a hybrid model of supply chain to make the supply chain achieve the coordination of order quantity and effort. J. Chen and J. F. Chen [25] studied the application of revenue-sharing contract in the virtual enterprise.

These contradictions may be resolved when all supply chain members share the effort cost. For example, Wang and Gerchak [26] proposed that a supplier can compensate for the retailer's the effort cost by way of stock subsides, while considering shelf space as an effort variable. Krishnan et al. [27] proposed a contract to coordinate supply chain by sharing effort cost. He [28] and He et al. [22] proposed a flexible quantity contract by sharing effort cost. Xu et al. [20] suggested a return-back contract by sharing effort cost. However, in many cases, due to the fact that the effort of each supply chain member cannot be observed, all kinds of hidden effort moral hazard will make the policy of sharing effort cost fail to coordinate supply chain. Therefore, it is not a desirable way to coordinate supply chain by sharing effort cost among supply chain members.

It should be noticed that the research object of literatures mentioned above is a two-level supply chain. Through literatures searching, we have not found the literatures that take effort into account in three-level supply chain. Therefore, the paper tries to further study in the following aspects. First, the research object is extended to three-level supply chain with sales effort dependent demand, which meets the actual circs better. Second, by analyzing the behavior of the decentralized supply chain under the traditional revenue-sharing contract with sales effort dependent demand, we find that the contract fails to coordinate the supply chain. Third, by supposing only the retailer or the manufacturer bears the effort cost, we propose an improved revenue-sharing contract based on quantity discount policy, which can coordinate the three-level supply chain. Because there are two different transaction phases in the three-level supply chain, two different conditions are discussed: implementing the improved revenue-sharing contract in one transaction phase and in two transaction phases. Further, by supposing the demand and the effort satisfy addition form or multiplication form, we characterize the optimal solution to the three-level supply chain with two decision variables (sales effort and inventory quantity), and numerical examples are given to illustrate the model and the solution process. Finally, by considering the retailer bears the effort cost together with the manufacturer, an improved revenue-sharing contract based on quantity discount policy is also discussed in this paper, which can coordinate the three-level supply.

The rest of this paper is organized as follows. Section 2 introduces model assumptions and notations. Section 3 examines the traditional revenue-sharing contract. In Section 4, we analyze three conditions: only the retailer bears the sales effort, only the manufacturer bears the sales effort, and the retailer bears the sales effort with the manufacturer. Section 5 provides concluding remarks and describes future research.

2. Model Descriptions

The supply chain studied in this paper is made up of one manufacturer (m), one distributor (d), and one retailer (r); see Figure 1. Upstream member provides a single product to downstream member, and the demand is stochastic. Before the sale season, both the distributor and the retailer have only one chance to buy products. All members are risk neutral, and information is symmetric among them.

Suppose p is sales price of unit product; c_i is supply chain member's marginal unit costs ($i = m, d, r$), and $c = c_r + c_d + c_m$; v is salvage value for unsold unit product ($v < c$); Q is order quantity; w_i^j is the wholesale price that upstream member charges downstream member in j condition ($j = c, d; i = m, d$); e is the retailer's effort, and $e \geq 0$; $g(e)$ is the retailer's effort cost when his effort is e, supposing $g(0) = 0$, and $g'(e) > 0$ and $g''(e) > 0$ when $e > 0$; x is the stochastic demand when retailer's effort is e, with probability density function $f(x \mid e)$ and differentiable cumulated distribution function $F(x \mid e)$, and $F(x \mid e)$ is continuously differentiable; because the demand is the increasing function of effort, $\partial F(x \mid e)/\partial e < 0$; Q is the order quantity; when retailer's effort is e, the expectation sale quantity is $S(Q,e) = E\min(Q,x) = Q - \int_0^Q F(x \mid e)dx$, and $\partial S(Q,e)/\partial e > 0$; the expectation unsold quantity is $I(Q,e)$ and $I(Q,e) = Q - S(Q,e)$. In particular, it is reasonable that $w_d^d > w_m^d + c_d, v < c < p, w_m^d + c_r < p$.

3. Analysis of Revenue-Sharing Contract

Supply chain contracts often apply incentive measures to adjust the relationship among the members to coordinate supply chain and to make the entire profit of decentralized supply chain equal to that of centralized supply chain as much as possible. The goal of the analysis is to design the revenue-sharing contract so as to achieve channel coordination (maximum profit for the whole supply chain). Besides, it aims at analyzing whether and how the contract parameter can be modified so as to more evenly share the profit along the chain (win-win condition for the chain partners), guaranteeing channel coordination. So, the maximum profit of centralized supply chain should first be considered as the goal of revenue-sharing contract to coordinate supply chain. The profit function of centralized supply chain can be described as

$$\begin{aligned}\Pi_t(Q,e) &= pS(Q,e) + vI(Q,e) - cQ - g(e)\\ &= (p-v)S(Q,e) - (c-v)Q - g(e).\end{aligned} \tag{1}$$

Supposing e^* is the retailer's optimal effort when given order quantity Q, then e^* should satisfy

$$\frac{\partial \Pi_t(Q,e^*)}{\partial e} = (p-v)\frac{\partial S(Q,e^*)}{\partial e} - g'(e^*) = 0. \tag{2}$$

Supposing Q^* is the retailer's optimal order quantity when given effort e, then Q^* should satisfy $\partial\Pi_t(Q^*,e)/\partial Q = (p-v)(\partial S(Q^*,e)/\partial Q) - (c-v) = 0$; namely, Q^* should satisfy

$$F(Q^* \mid e) = \frac{p-c}{p-v}. \tag{3}$$

Therefore, if revenue-sharing contract can coordinate supply chain, then formulae (2) and (3) are the necessary conditions to achieve supply chain coordination.

Giannoccaro and Pontrandolfo [12] proposed a model of revenue-sharing contract to coordinate three-level supply chain without considering supply chain member's stockout loss and salvage value of unsold unit product. In this paper, we analyze whether the revenue-sharing contract can coordinate three-level supply chain or not with taking salvage value of unsold unit product and the retailer's effort into account. Supposing the retailer would keep a quota ϕ_2 of his revenue, giving the rest $(1-\phi_2)$ to the distributor and this would be balanced by a lower price w_d^c. Similarly, the distributor would keep a quota ϕ_1 of his revenue, giving the rest $(1-\phi_1)$ to the manufacturer and this would be balanced by a lower price w_m^c. Also assume only the retailer bears the effort cost. Then, the expected profit function of the retailer can be described as

$$\begin{aligned}\Pi_r^c(Q,e) &= \phi_2[pS(Q,e) + vI(Q,e)] - c_rQ - w_d^cQ - g(e)\\ &= \phi_2(p-v)S(Q,e) - (c_r + w_d^c - \phi_2 v)Q - g(e).\end{aligned} \tag{4}$$

From (4), we get $\partial\Pi_r^c(Q,e)/\partial e = \phi_2(p-v)(\partial S(Q,e)/\partial e) - g'(e)$. Compared with (2), it can be seen that the retailer's effort e in this condition is smaller than the optimal effort of centralized supply chain, namely, $\partial\Pi_r^c(Q,e)/\partial e < \partial\Pi_t(Q,e)/\partial e$. That is to say, in this condition, the revenue-sharing contract cannot coordinate supply chain. The reasons lie in that the retailer bears all the effort cost of supply chain, but he only get partial profit of the whole supply chain.

Similarly, if only the distributor or the manufacturer bears the sales effort cost, the revenue-sharing contract cannot coordinate the supply chain. Then, we come to a conclusion: the traditional revenue-sharing contract cannot coordinate three-level supply chain with sales effort dependent demand.

In this paper, an improved revenue-sharing contract based on quantity discount policy is proposed to coordinate three-level supply chain when demand depends on effort.

4. The Improved Revenue-Sharing Contract Based on Quantity Discount Policy

From the discussion above, it can be seen that the optimal effort of the supply chain member under revenue-sharing contract is not equal to that of the centralized supply chain. The reasons lie in that the supply chain member only gets partial gains but bears all the effort cost $g(e)$ of the whole supply chain. To solve this problem, we can make his gains be equal to a fixed ratio of the gains of the whole supply chain. Here, we propose an improved revenue-sharing contract based on quantity discount policy to coordinate supply chain. Because there are two different transaction phases: upstream process and downstream process, it needs to consider how to implement this improved contract in three-level supply chain.

4.1. Only Retailer Bearing the Effort Cost

4.1.1. Only Implement between Retailer and Distributor. This problem can be described as follows: we implement the revenue-sharing contract based on quantity policy between the retailer and the distributor and the traditional revenue-sharing contract between the distributor and the manufacturer.

Suppose the distributor provides the retailer the revenue-sharing contract $(\phi_2, w_d^c(Q))$, and

$$w_d^c(Q) = \phi_2(c-v) - (1-\phi_2)\frac{g(e^*)}{Q} - c_r + \phi_2 v. \tag{5}$$

Here, e^* is the optimal effort of the supply chain.

In this condition, the profit function of retailer can be described as

$$\begin{aligned}&\Pi_r^c(Q,e)\\ &= \phi_2[pS(Q,e) + vI(Q,e)] - c_rQ - w_d^c(Q)Q - g(e)\\ &= \phi_2(p-v)S(Q,e) - \phi_2(c-v)Q + (1-\phi_2)g(e^*) - g(e).\end{aligned} \tag{6}$$

According to the paper of Giannoccaro and Pontrandolfo [12], without considering demand dependent effort, the ratio that the retailer's profit shares the profit of the supply chain is ϕ_2. Here, supposing when only the retail bears the effort cost, the ratio that his profit shares the profit of the whole supply chain is also ϕ_2. It is easily known that the effort of the retail would be equal to the optimal effort of the supply chain; namely,

$$\begin{aligned}\Pi_r^c(Q,e^*) &= \phi_2\left[pS(Q,e^*)+vI(Q,e^*)\right]-c_rQ-w_d^c(Q)Q-g(e^*)\\ &= \phi_2(p-v)S(Q,e^*)-\phi_2(c-v)Q-\phi_2 g(e^*)\\ &= \phi_2\Pi_t(Q,e^*).\end{aligned} \tag{7}$$

From (7), it can be seen that the optimal order quantity would be equal to that of the supply chain. Then, the profit function of the distributor can be described as

$$\begin{aligned}\Pi_d^c(Q,e) &= \phi_1\left[(1-\phi_2)(pS(Q,e)+vI(Q,e))+w_d^c(Q)Q\right]\\ &\quad -c_dQ-w_m^cQ\\ &= \phi_1(1-\phi_2)(p-v)S(Q,e)\\ &\quad -\left[c_d+w_m^c-\phi_1(1-\phi_2)v-\phi_1 w_d^c(Q)\right]Q.\end{aligned} \tag{8}$$

Because the distributor is in the middle of the supply chain, to maximize his profit, he will make his optimal order quantity equal to the optimal order quantity of retailer in perfect information symmetry condition; namely,

$$\frac{\partial\Pi_d^c(Q^*,e)}{\partial Q}=\frac{\partial\Pi_r^c(Q^*,e)}{\partial Q}=\frac{\partial\Pi_t(Q^*,e)}{\partial Q}. \tag{9}$$

From (9), it can get

$$w_m^c=(1+\phi_1\phi_2)c-\phi_1c_r-c_d-\left[1-\phi_1(1-\phi_2)\right]v. \tag{10}$$

Take (5) and (10) into the profit function of the supply chain members and we can get

$$\begin{aligned}\Pi_r^c(Q,e) &= \phi_2\Pi_t(Q,e),\\ \Pi_d^c(Q,e) &= \phi_1(1-\phi_2)\Pi_t(Q,e),\\ \Pi_m^c(Q,e) &= (1-\phi_1)(1-\phi_2)\Pi_t(Q,e).\end{aligned} \tag{11}$$

From (11), it can be seen that the profit functions of the supply chain members are all affine functions of the whole supply chain's profit function. So in this condition, the revenue-sharing contract can coordinate the three-level supply chain.

Therefore, if we implement the revenue-sharing contract based on quantity discount policy between the retailer and the distributor and the traditional revenue-sharing contract between the distributor and the manufacturer and the contract parameters which satisfy (5) and (10), the improved revenue-sharing can coordinate the three-level supply chain.

4.1.2. Implement among Supply Chain Members. This problem can be described as follows: we implement both the revenue-sharing contract based on quantity discount policy between the retailer and the distributor, and between the distributor and the manufacturer.

Suppose the distributor provides the retailer the revenue-sharing contract $(\phi_2, w_d^c(Q))$ and the manufacturer provides the distributor the revenue-sharing contract $(\phi_1, w_m^c(Q))$. Here, e^* is the optimal effort of the supply chain, $w_d^c(Q)$ is equal to (5) and

$$w_m^c(Q)=\phi_1(c-v)-\phi_1(1-\phi_2)\frac{g(e^*)}{Q}-\phi_1c_r+\phi_1v-c_d. \tag{12}$$

Here, we can also suppose that, when only the retail bears the effort cost, the ratio that his profit shares the profit of the whole supply chain is also ϕ_2. It is easy to know that the optimal effort and the optimal order quantity of the retailer would be equal to those of the supply chain.

In this condition, the profit function of the distributor can be described as

$$\begin{aligned}\Pi_d^c(Q,e) &= \phi_1\left[(1-\phi_2)(pS(Q,e)+vI(Q,e))+w_d^c(Q)Q\right]\\ &\quad -c_dQ-w_m^c(Q)Q\\ &= \phi_1(1-\phi_2)(p-v)S(Q,e)\\ &\quad -\left[c_d+w_m^c(Q)-\phi_1(1-\phi_2)v-\phi_1w_d^c\right]Q\\ &= \phi_1(1-\phi_2)(p-v)S(Q,e)-\phi_1(1-\phi_2)(c-v)Q.\end{aligned} \tag{13}$$

Similarly, the optimal order quantity of the distributor should satisfy

$$\frac{\partial\Pi_d^c(Q^*,e)}{\partial Q}=\frac{\partial\Pi_r^c(Q^*,e)}{\partial Q}=\frac{\partial\Pi_t(Q^*,e)}{\partial Q}. \tag{14}$$

Take (5) and (12) into the profit function of the supply chain member and we can get

$$\begin{aligned}\Pi_r^c(Q,e) &= \phi_2\Pi_t(Q,e),\\ \Pi_d^c(Q,e) &= \phi_1(1-\phi_2)(\Pi_t(Q,e)+g(e)),\\ \Pi_m^c(Q,e) &= (1-\phi_1)(1-\phi_2)\Pi_t(Q,e)-\phi_1(1-\phi_2)g(e).\end{aligned} \tag{15}$$

From (15), it can be seen that the profit functions of the supply chain members are all affine functions of the whole supply chain's profit function. So in this condition, the revenue-sharing contract can coordinate the three-level supply chain. Comparing (11) with (15), it can be seen that the retailer's profit keeps unchanged, the distributor's profit increases $\phi_1(1-\phi_2)g(e)$ while the manufacturer's profit decreases $\phi_1(1-\phi_2)g(e)$.

Therefore, if we implement the revenue-sharing contract based on quantity discount policy both between the retailer and the distributor and between the distributor and the manufacturer and the contract parameters which satisfy (5) and (12), the improved revenue-sharing contract can coordinate the three-level supply chain.

4.1.3. Model Optimization. From the discussion mentioned above, we can come to a conclusion that the traditional revenue-sharing contract cannot coordinate three-level supply chain with sales effort dependent demand. However, the improved revenue-sharing contract based on quantity discount policy can coordinate three-level supply chain. And, in this condition, the optimal order quantity and optimal effort of retailer are equal to those of the centralized supply chain. Then, it needs to determine the optimal order quantity and the optimal effort to maximize the profit of supply chain, namely, $\max \Pi_t(Q,e)$. The following gives the method to determine the optimal order quantity Q^* and the optimal effort e^*.

Suppose the market demand $X(e,\xi)$ is the function of effort e and random factor ξ, and ξ is independent of e; $f(\xi)$ and $F(\xi)$ are, respectively, probability density function and differentiable cumulated distribution function of ξ. Usually we can use two forms to describe how the effort affects demand: addition form and multiplication form [29]. In this paper, suppose $X(e,\xi) = y(e) + \xi$. Because the influence of effort on marker demand is diminishing marginal utility, we can suppose $y(e)$ is the monotone increasing and concave function of effort e; namely, $y'(e) > 0$, $y''(e) \leq 0$.

When the market demand satisfies $X(e,\xi) = y(e) + \xi$, we can get

$$\begin{aligned} S(Q,e) &= Q - \int_0^Q F(x \mid e)\,dx = Q - \int_{y(e)}^Q F(x - y(e))\,dx \\ &= Q - \int_0^{Q-y(e)} F(t)\,dt. \end{aligned} \tag{16}$$

So the profit function of supply chain can be described as

$$\begin{aligned} \Pi_t(Q,e) &= (p-v)\,S(Q,e) - (c-v)\,Q - g(e) \\ &= (p-c)\,Q - (p-v)\int_0^{Q-y(e)} F(t)\,dt - g(e). \end{aligned} \tag{17}$$

Given effort e, it can be seen that $\Pi_t(Q,e)$ is the concave function of order quantity Q. So the optimal order quantity Q^* should satisfy

$$\frac{\partial \Pi_t(Q^*,e)}{\partial Q} = (p-c) - (p-v)\,F(Q^* - y(e)) = 0. \tag{18}$$

From (18), we can get

$$Q^* = Q^*(e) = y(e) + F^{-1}\left(\frac{p-c}{p-v}\right). \tag{19}$$

Then, the profit function of supply chain can be described as

$$\begin{aligned} &\Pi_t(Q^*,e) \\ &= (p-c)\,Q^* - (p-v)\int_0^{Q^*-y(e)} F(t)\,dt - g(e) \\ &= (p-c)\,y(e) + (p-v)\int_0^{F^{-1}((p-c)/(p-v))} t f(t)\,dt - g(e). \end{aligned} \tag{20}$$

From (20), it can be seen that there is only one variable e in $\Pi_t(Q^*,e)$. Therefore, the next work is to determine the optimal effort e^* to maximize $\Pi_t(Q^*,e)$. From (20), it can get $\partial\Pi_t(Q^*,e)/\partial e = (p-c)y'(e) - g'(e)$, $\partial^2\Pi_t(Q^*,e)/\partial e^2 = (p-c)y''(e) - g''(e)$. Because $g''(e) > 0$ and $y''(e) \leq 0$, $\partial^2\Pi_t(Q^*,e)/\partial e^2 < 0$; namely, $\Pi_t(Q^*,e)$ is the concave function of effort e. That is to say, there is an optimal effort e^* which can make $\Pi_t(Q^*,e)$ achieve the maximum. By $\partial\Pi_t(Q^*,e)/\partial e = 0$, we get

$$(p-c)\,y'(e^*) - g'(e^*) = 0. \tag{21}$$

Then the optimal effort e^* should satisfy (21). Equation (21) is the first-order differential equation of the optimal effort e^*. By solving (21), we can get the expression of e^*.

4.1.4. An Example. Suppose a supply chain is made up of a retailer, a distributor, and a manufacturer, and the supply chain parameters are $c_r = 1$, $c_d = 2$, $c_m = 5$, $v = 3$, $w_d^d = 18$, $w_m^d = 10$, $p = 35$, $\phi_1 = 0.55$, and $\phi_2 = 0.55$.

Here, assume the relationship between demand and effort satisfies addition form, and $g(e) = e^2/2$, $y(e) = e$; ξ satisfies uniform distribution in [50, 100]. It is easy to get $f(\xi) = 1/50$, $F(\xi) = (\xi - 50)/50$, and $F^{-1}(\xi) = 50 + 50\xi$.

(1) The revenue-sharing contract based on quantity discount policy.

From the discussion above, we know that the revenue-sharing contract based on quantity discount policy can coordinate the supply chain whether it is implemented in one or two transactions. According to the formulas mentioned above, it is easy to get the optimal effort, the optimal order quantity, and the profit of the whole supply chain. Here, $e^* = 27$, $Q^* = 119$, and $\Pi_t(Q^*,e^*) = 2272.5$.

(2) The traditional revenue-sharing contract.

Similarly, under the traditional revenue-sharing contract, the optimal effort and order quantity of the retailer and the profit of the whole supply chain are, respectively, $e^r = 14.85$, $Q^r = 107$, and $\Pi_t(Q^r,e^r) = 2202.7$.

(3) The decentralized supply chain (the wholesale price contract).

Similarly, under the traditional revenue-sharing contract, the optimal effort and order quantity of the retailer and the profit of the whole supply chain are, respectively, $e^d = 16$, $Q^d = 91$, and $\Pi_t(Q^d,e^d) = 2137$.

Table 1 shows the parameters of the supply chain in different mode. A means that we implement the revenue-sharing contract based on quantity discount policy only between the retailer and the distributor. B means that we implement the revenue-sharing contract based on quantity discount policy in two transactions of the supply chain.

Table 1 shows that the retailer's effort in the revenue-sharing contract based on quantity discount policy is the highest in the three modes, which can make the supply chain

Table 1: The parameters of supply chain in different modes with only retailer bearing effort cost.

	Revenue-sharing contract based on quantity discount policy A	Revenue-sharing contract based on quantity discount policy B	Traditional revenue-sharing contract	Wholesale price contract
Optimal effort e	27	27	14.85	16
Optimal order quantity Q	119	119	107	91
Profit of the whole supply chain Π_t	2272.5	2272.5	2202.7	2137

maximize its profit; the retailer's effort in traditional revenue-sharing contract is the lowest in the three modes, but his order quantity is still higher than that in wholesale price contract due to the incentive of the revenue-sharing contract. So the profit of the supply chain in the traditional revenue-sharing contract is still higher than that in wholesale price contract. So it can be seen that the traditional revenue-sharing contract is better than the wholesale price contract with sales effort dependent demand.

4.2. Only Manufacturer Bearing the Effort Cost

4.2.1. Only Implement between Manufacturer and Distributor. This problem can be described as follows: we implement the revenue-sharing contract based on quantity policy between the manufacturer and the distributor and the traditional revenue-sharing contract between the distributor and the retailer.

Supposing the manufacturer provides the distributor the revenue-sharing contract $(\phi_1, w_m^c(Q))$, and

$$w_m^c(Q) = -(1-\phi_1)c + [1-(1-\phi_1)(1-\phi_2)]\frac{g(e^*)}{Q} + (1-\phi_1)c_r + c_m. \tag{22}$$

Here, e^* is the optimal effort of the supply chain.

Taking (22) into the manufacturer's profit function, we can get

$$\begin{aligned}\Pi_m^c(Q,e) &= (1-\phi_1)[(1-\phi_2)(pS(Q,e)+vI(Q,e))+w_d^cQ] \\ &\quad + w_m^c(Q)Q - c_mQ - g(e) \\ &= (1-\phi_1)(1-\phi_2)(p-v)S(Q,e) \\ &\quad + [1-(1-\phi_1)(1-\phi_2)]g(e^*) \\ &\quad - (1-\phi_1)[c-c_r-(1-\phi_2)v-w_d^c]Q - g(e).\end{aligned} \tag{23}$$

According to the paper of Giannoccaro and Pontrandolfo [12], without considering demand dependent effort, the ratio that the manufacturer's profit shares the profit of the supply chain is $(1-\phi_1)(1-\phi_2)$. Here, supposing when only the manufacturer bears the effort cost, the ratio that his profit shares the profit of the whole supply chain is also $(1-\phi_1)(1-\phi_2)$. It is easily known that the effort of the manufacturer would be equal to the optimal effort of the supply chain; namely,

$$\begin{aligned}\Pi_m^c(Q,e) &= (1-\phi_1)[(1-\phi_2)(pS(Q,e^*)+vI(Q,e^*))+w_d^cQ] \\ &\quad + w_m^c(Q)Q - c_mQ - g(e^*) \\ &= (1-\phi_1)(1-\phi_2)\Pi_t(Q,e).\end{aligned} \tag{24}$$

From (24), we can get

$$w_d^c = \phi_2 c - c_r. \tag{25}$$

Taking (25) and (22) into the profit function of the supply chain members, we can get

$$\begin{aligned}\Pi_r^c(Q,e) &= \phi_2[\Pi_t(Q,e)+g(e)], \\ \Pi_d^c(Q,e) &= \phi_1(1-\phi_2)\Pi_t(Q,e) - \phi_2 g(e), \\ \Pi_m^c(Q,e) &= (1-\phi_1)(1-\phi_2)\Pi_t.\end{aligned} \tag{26}$$

From (26), it can be seen that the profit functions of the supply chain members are all affine functions of the whole supply chain's profit function. So in this condition, the revenue-sharing contract can coordinate the three-level supply chain.

Therefore, if we implement the revenue-sharing contract based on quantity discount policy between the retailer and the distributor and the traditional revenue-sharing contract between the distributor and the manufacturer and the contract parameters which satisfy (22) and (25), the improved revenue-sharing can coordinate the three-level supply chain.

Compared with the paper of Giannoccaro and Pontrandolfo [12], it can be seen that the retailer's profit increases $\phi_2 g(e^*)$ while that of the distributor decreases $\phi_2 g(e^*)$.

4.2.2. Implement among Supply Chain Members. This problem can be described as follows: we implement both the revenue-sharing contract based on quantity discount policy between the retailer and the distributor and between the distributor and the manufacturer.

Suppose the distributor provides the retailer the revenue-sharing contract $(\phi_2, w_d^c(Q))$ and the manufacturer provides the distributors the revenue-sharing contract $(\phi_1, w_m^c(Q))$. Here, e^* is the optimal effort of the supply chain and

$$w_d^c(Q) = \phi_2 c - (1-\phi_2)\frac{g(e^*)}{Q} - c_r,$$
$$w_m^c(Q) = \phi_1 c + \frac{g(e^*)}{Q} - \phi_1 c_r - c_d. \tag{27}$$

Taking (27) into the profit function of the manufacturer, we can get

$$\begin{aligned}\Pi_m^c&(Q,e)\\ &= (1-\phi_1)\left[(1-\phi_2)(pS(Q,e)+vI(Q,e)) + w_d^c(Q)Q\right]\\ &\quad + w_m^c(Q)Q - c_m Q - g(e)\\ &= (1-\phi_1)(1-\phi_2)(p-v)S(Q,e)\\ &\quad - (1-\phi_1)(1-\phi_2)(c-v)Q\\ &\quad + \left[1-(1-\phi_1)(1-\phi_2)\right]g(e^*) - g(e).\end{aligned} \tag{28}$$

Here, we also suppose that the ratio that his profit shares the profit of the whole supply chain is also $(1-\phi_1)(1-\phi_2)$. It is easy to know that the optimal effort of the manufacturer would be equal to that of the supply chain. In this condition, the profit function of the manufacturer can be described as

$$\Pi_m^c(Q,e) = (1-\phi_1)(1-\phi_2)\Pi_t(Q,e). \tag{29}$$

Taking (27) into the profit function of the retailer, we can get

$$\begin{aligned}\Pi_r^c(Q,e) &= \phi_2\left[pS(Q,e^*) + vI(Q,e^*)\right] - c_r Q - w_d^c(Q)Q\\ &= \phi_2(p-v)S(Q,e^*) - \phi_2(c-v)Q + (1-\phi_2)g(e).\end{aligned} \tag{30}$$

From (30), it is easy to know that the optimal order quantity of the retailer is equal to that of the supply chain. So (30) can be described as

$$\Pi_r^c(Q,e) = \phi_2\Pi_t(Q,e) + g(e). \tag{31}$$

Taking (27) into the profit function of the distributor, it can get

$$\begin{aligned}\Pi_d^c&(Q,e)\\ &= \phi_1\left[(1-\phi_2)(pS(Q,e^*) + vI(Q,e^*)) + w_d^c(Q)Q\right]\\ &\quad - c_d Q - w_m^c(Q)Q - g(e^*)\\ &= \phi_1(1-\phi_2)(p-v)S(Q,e^*) - \phi_1(1-\phi_2)(c-v)Q\\ &\quad - (1+\phi_1(1-\phi_2))g(e).\end{aligned} \tag{32}$$

From (32), it can be seen that the optimal order quantity of the distributor is equal to that of the retailer, so (32) can also be described as

$$\Pi_d^c(Q,e) = \phi_1(1-\phi_2)\Pi_t(Q,e) - g(e). \tag{33}$$

From (29), (31), and (33), it can be seen that the profit functions of the supply chain members are all affine functions of the whole supply chain's profit function. So in this condition, the revenue-sharing contract can coordinate the three-level supply chain. Therefore, if we implement the revenue-sharing contract based on quantity discount policy both between the retailer and the distributor and between the distributor and the manufacturer and the contract parameters which satisfy (27), the improved revenue-sharing can coordinate the three-level supply chain.

4.2.3. Model Optimization. Similarly, suppose the market demand $X(e,\xi)$ is the function of effort e and random factor ξ, and ξ is independent of e; $f(\xi)$ and $F(\xi)$ are, respectively, probability density function and differentiable cumulated distribution function of ξ. Here, suppose $X(e,\xi) = y(e)\cdot\xi$. As the influence of effort on marker demand is diminishing marginal utility, we can suppose $y(e)$ is the monotone increasing and concave function of effort e; namely, $y'(e) > 0$, and $y''(e) \le 0$.

When the market demand satisfies $X(e,\xi) = y(e)\cdot\xi$, we can get

$$\begin{aligned}S(Q,e) &= Q - \int_0^Q F(x \mid e)\,dx = Q - \int_0^Q F\left(\frac{x}{y(e)}\right)dx\\ &= Q - y(e)\int_0^{Q/y(e)} F(t)\,dt.\end{aligned} \tag{34}$$

So the profit function of supply chain can be described as

$$\begin{aligned}\Pi_t(Q,e) &= (p-v)S(Q,e) - (c-v)Q - g(e)\\ &= (p-c)Q - (p-v)y(e)\int_0^{Q/y(e)} F(t)\,dt - g(e).\end{aligned} \tag{35}$$

Given effort e, it can be seen that $\Pi_t(Q,e)$ is the concave function of order quantity Q. So the optimal order quantity Q^* should satisfy

$$\frac{\partial\Pi_t(Q^*,e)}{\partial Q} = (p-c) - (p-v)F\left(\frac{Q^*}{y(e)}\right) = 0. \tag{36}$$

From (36), we can get

$$Q^* = Q^*(e) = y(e)F^{-1}\left(\frac{p-c}{p-v}\right). \tag{37}$$

Then, the profit function of supply chain can be described as

$$\begin{aligned}\Pi_t(Q^*,e) &= (p-c)y(e)F^{-1}\left(\frac{p-c}{p-v}\right)\\ &\quad - (p-v)y(e)\int_0^{F^{-1}((p-c)/(p-v))} F(t)\,dt - g(e)\\ &= (p-v)y(e)\int_0^{F^{-1}((p-c)/(p-v))} tf(t)\,dt - g(e).\end{aligned} \tag{38}$$

Table 2: The parameters of supply chain in different modes with only manufacturer bearing effort cost.

	Revenue-sharing contract based on quantity discount policy A	Revenue-sharing contract based on quantity discount policy B	Traditional revenue-sharing contract	Wholesale price contract
Optimal effort e	19.62	19.62	3.97	3.79
Optimal order quantity Q	1674	1674	339	288
Profit of the whole supply chain Π_t	19246.28	19246.28	7008.33	6544.36

According to the supposition, $\Pi_t(Q^*, e)$ is the concave function of effort e. So the optimal effort e^* of the supply chain should satisfy

$$\frac{\partial \Pi_t(Q^*, e^*)}{\partial e} = (p-v)\, y'(e^*) \int_0^{F^{-1}((p-c)/(p-v))} tf(t)\,dt - g'(e^*) = 0. \tag{39}$$

From (39), it can easy to get that the optimal effort e^* of the supply chain should satisfy

$$(p-v)\, y'(e^*) \int_0^{F^{-1}((p-c)/(p-v))} tf(t)\,dt = g'(e^*). \tag{40}$$

The method to solve the expression of e^* in (40) is the same as the method in (21).

4.2.4. An Example. Suppose a supply chain is made up of a retailer, a distributor, and a manufacturer, and the supply chain parameters are $c_r = 1$, $c_d = 2$, $c_m = 5$, $v = 3$, $w_d^d = 18$, $w_m^d = 10$, $p = 35$, $\phi_1 = \phi_2 = 0.55$, and $a = 100$.

Here, assume the relationship between demand and effort satisfies multiplication form, and $g(e) = ae^2/2$, $y(e) = e$; ξ satisfies uniform distribution in [60, 90]. It is easy to get $f(\xi) = 1/30$, $F(\xi) = (\xi - 60)/30$, and $F^{-1}(\xi) = 60 + 30\xi$.

(1) The revenue-sharing contract based on quantity discount policy.

From the discussion above, we know that the revenue-sharing contract based on quantity discount policy can coordinate the supply chain whether it is implemented in one or two transactions. According to the formulas mentioned above, it is easy to get the optimal effort of the manufacturer, the optimal order quantity of the retailer, and the profit of the whole supply chain. Here, $e^* = 19.62$, $Q^* = 1674$, $\Pi_t(Q^*, e^*) = 19246.28$.

(2) The traditional revenue-sharing contract.

Similarly, under the traditional revenue-sharing contract, the optimal effort of the manufacturer and order quantity of the retailer and the profit of the whole supply chain are, respectively, $e^{mc} = 3.97$, $Q^{rc} = 339$, and $\Pi_t(Q^{rc}, e^{mc}) = 7008.33$.

(3) The decentralized supply chain (the wholesale price contract).

Similarly, under the traditional revenue-sharing contract, the optimal effort of the manufacturer and the order quantity of the retailer and the profit of the whole supply chain are, respectively, $e^{md} = 3.79$, $Q^{rd} = 288$, and $\Pi_t(Q^{rd}, e^{dm}) = 6544.36$.

Table 2 shows the parameters of the supply chain in different modes. A means that we implement the revenue-sharing contract based on quantity discount policy only between the retailer and the distributor. B means that we implement the revenue-sharing contract based on quantity discount policy in two transactions of the supply chain.

Table 2 shows that the manufacturer's effort in the revenue-sharing contract based on quantity discount policy is the highest in the three modes, which can make the supply chain maximize its profit, and the revenue-sharing contract based on quantity discount policy is better than the traditional revenue-sharing contract with effort dependent demand; the manufacturer's effort in traditional revenue-sharing contract is higher than that of the supply chain in the wholesale price contract, and retailer's order quantity is still higher than that in wholesale price contract due to the incentive of the revenue-sharing contract, so the profit of the supply chain in the traditional revenue-sharing contract is still higher than that in wholesale price contract. So it can be seen that the traditional revenue-sharing contract is better than the wholesale price contract with effort dependent demand.

4.3. The Retailer Bearing the Effort Cost with the Manufacturer. Suppose the effort cost that the retailer bears is $\alpha g(e)$ $(0 \le \alpha \le 1)$, then the manufacturer bears $(1-\alpha)g(e)$ effort cost. If $\alpha = 0$, it means that only the manufacturer bears the effort cost; if $\alpha = 1$, it means that only the retailer bears the effort cost.

In this condition, we find that the traditional revenue-sharing contract fails to coordinate the supply chain whether it is implemented in one transaction phase or in two transaction phases of the three-level supply chain. The reason lies in that the distributor does not bear the effort cost, but he can get partial profit of the whole supply chain, which is unfair for the retailer and the manufacturer. So the effort level of the retailer and the manufacturer will be lower than the effort level of the centralized supply chain. Now, the following discusses how to design the improved revenue-sharing implemented in two transaction phases to coordinate the supply chain.

Suppose the distributor provides the retailer the revenue-sharing contract $(\phi_2, w_d^c(Q, e))$ and the manufacturer provides the distributors the revenue-sharing contract $(\phi_1,$

$w_m^c(Q,e)$). Then, the profit functions of the supply chain members can be described as

$$\begin{aligned}\Pi_r^c(Q,e) &= \phi_2(p-v)S(Q,e)\\&\quad-\left[c_r+w_d^c(Q,e)-\phi_2 v\right]Q-\alpha g(e),\\\Pi_d^c(Q,e) &= \phi_1(1-\phi_2)(p-v)S(Q,e)\\&\quad-\left[c_d+w_m^c(Q,e)-\phi_1(1-\phi_2)v\right.\\&\qquad\left.-\phi_1 w_d^c(Q,e)\right]Q,\\\Pi_m^c(Q,e) &= (1-\phi_1)(1-\phi_2)(p-v)S(Q,e)\\&\quad-\left[c_m-w_m^c(Q,e)-(1-\phi_1)(1-\phi_2)v\right.\\&\qquad\left.-(1-\phi_1)w_d^c(Q,e)\right]Q\\&\quad-(1-\alpha)g(e).\end{aligned}\tag{41}$$

If the improved revenue-sharing contract can coordinate the supply chain, the optimal effort level e^* of the retailer and the manufacturer should satisfy (2). Namely, the optimal order (production) quantity Q^* of the retailer and the manufacturer should satisfy (3). Then, we can get

$$\begin{aligned}&\frac{\partial\Pi_r^c(Q,e^*)}{\partial e}=\frac{\partial\Pi_m^c(Q,e^*)}{\partial e}=0,\\&(p-v)\frac{\partial S(Q,e^*)}{\partial e}-g'(e^*)=0,\\&\frac{\partial\Pi_r^c(Q^*,e)}{\partial Q}=\frac{\partial\Pi_m^c(Q^*,e)}{\partial Q}=0,\\&(p-v)\frac{\partial S(Q^*,e)}{\partial Q}-(c-v)=0.\end{aligned}\tag{42}$$

From (42), we can get

$$\begin{aligned}w_d^c(Q,e)&=\phi_2(c-v)-(1-\alpha)\frac{g(e)}{Q}-c_r+\phi_2 v,\\w_m^c(Q,e)&=\phi_1(c-v)+\phi_1(1-\alpha)\frac{g(e)}{Q}-\phi_1 c_r+\phi_1 v-c_d.\end{aligned}\tag{43}$$

Taking (43) into the profit functions of the supply chain members (41), we can get

$$\begin{aligned}\Pi_r^c(Q,e)&=\phi_2\Pi_t(Q,e),\\\Pi_d^c(Q,e)&=\phi_1(1-\phi_2)\Pi_t(Q,e),\\\Pi_m^c(Q,e)&=(1-\phi_1)(1-\phi_2)\Pi_t(Q,e).\end{aligned}\tag{44}$$

We find that (44) equals (11). That is to say, when the retailer bears the effort with the manufacturer, the improved revenue-sharing contract can coordinate the three-level supply chain.

Therefore, if we implement the revenue-sharing contract based on quantity discount policy among the supply chain members and the contract parameters which satisfy (43), the improved revenue-sharing can coordinate the three-level supply chain.

The method to determine the optimal order quantity Q^* and the optimal effort e^* can follow the methods given in Sections 4.1.3 and 4.2.3.

5. Conclusion

The traditional revenue-sharing contract cannot coordinate three-level supply chain with sales effort dependent demand. The paper proposes an improved revenue-sharing contract based on quantity discount policy, which can be implemented in one transaction or two transactions of three-level supply chain, to coordinate the supply chain. The paper also shows that the profit of the supply chain members depends on the value of the contract parameters which is determined by the status of the members in supply chain and their bargaining power. It also gave the method to determine the optimal order quantity and the optimal effort, which can provide the decision makers of supply chain with decision support. But it also should be noticed that this research is carried out in the symmetrical information condition. So the next step is to carry out the research in the asymmetric information condition.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Research Article

Joint Optimal Production Planning for Complex Supply Chains Constrained by Carbon Emission Abatement Policies

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We focus on the joint production planning of complex supply chains facing stochastic demands and being constrained by carbon emission reduction policies. We pick two typical carbon emission reduction policies to research how emission regulation influences the profit and carbon footprint of a typical supply chain. We use the input-output model to capture the interrelated demand link between an arbitrary pair of two nodes in scenarios without or with carbon emission constraints. We design optimization algorithm to obtain joint optimal production quantities combination for maximizing overall profit under regulatory policies, respectively. Furthermore, numerical studies by featuring exponentially distributed demand compare systemwide performances in various scenarios. We build the "carbon emission elasticity of profit (CEEP)" index as a metric to evaluate the impact of regulatory policies on both chainwide emissions and profit. Our results manifest that by facilitating the mandatory emission cap in proper installation within the network one can balance well effective emission reduction and associated acceptable profit loss. The outcome that CEEP index when implementing Carbon emission tax is elastic implies that the scale of profit loss is greater than that of emission reduction, which shows that this policy is less effective than mandatory cap from industry standpoint at least.

1. Introduction

People around the world have gradually realized that carbon dioxide emission has become a global environmental and energy issue as discussed in the World Climate Conference held in 2009 in Copenhagen [1]. To cut carbon emission into certain levels, governments in many countries put forward emission reduction targets confronting pressure both from internationally and domestically environmental concerns [2]. Therefore, governments have designed a series of carbon abatement policies to serve these carbon emission reduction purposes, say mandatory cap, carbon and trade, carbon tax, and carbon offset policies in the report of Congress of the United States [3]. Apparently, a variety of manufacturing industries as one of the main bodies emitting carbon dioxide are definitely subject to those emission reduction regulations.

On the way to confront the carbon emission regulations, firms over a great range of industries have to take a series of actions on reducing their own carbon emission. For firms' response to carbon emission regulations, we can observe from practice or reports that their common actions are often reflected in the replacement of energy-saving equipment, adoption of low-carbon technology for new product development, substitution of fossil fuels and saving in consumption of electricity power, and so forth [4]. For example, manufacturers can renew their old machine with high emission by turning to low-carbon emitted equipments.

Different from those aforementioned means for meeting carbon emission constraints, a few of companies are aware of the possibility in reducing emission by operations adjustment. This is an attractive and plausible solution in the low-carbon economic environment. However, this kind of actions led only by single firm is not easy to achieve the anticipated goals. There are two reasons: a single firm is usually self-interest oriented even when choosing her emission reduction operations strategies, which will be often distorted by the interactions with her supply chain partners; the other is the carbon footprint of final product which is accumulated through the whole supply chain process over installations one by one as well as the pipeline. However, the carbon footprint reflected in the carbon label will affect the market demand due to consumers' low-carbon awareness and in turn influence each firm's profit, eventually.

Although the government has released a couple of carbon emission regulatory policies for changing all carbon emitters' behaviors, especially for firms in industries, we should ask whether those policies can get their initial expected effect in carbon emission control. Upon the intuition, one can infer that some carbon emission regulation may not work well if it is originated from controlling single firm emission behavior but not from supply chain. The interactions among firms in the supply chain usually incur behaviors deviating from the goal of the whole system. In low carbon environment, firms in supply network also have different appeals when confronting various regulatory carbon emission policies. The low-carbon concerns will worsen the conflict of interests among node firms. This is the reason why we focus on supply chain production planning under carbon emission constraints in this study.

In this paper, our goal is to study the impact of various carbon emission regulations on the operations and then the performance of a complex supply chain network that is assembly system like made up of a series of nodes with convergent material flows to the final product. In this system, each node firm's production is constrained by her own emission regulatory policy, say either mandatory cap or carbon tax as we exploit afterwards. In contrast to single firm production planning problem under low-carbon scenario, we turn to focus on a more realistic and typical manufacturing system as we show here, which attracts us to investigate how we manage a set of manufacturing units under low-carbon era compared with traditional setting without low-carbon thinking. This kind of comparison can uncover the impact of carbon emission regulations on the typical supply chain network. Under systemic centralized setting, we formulate the problem as constrained nonlinear programming with n dimensional variables.

On the other hand, we conduct another kind of comparison of performances resulted from two distinctive emission regulatory policies, respectively. We intend to measure and show the difference of two policies' transmission mechanisms through the whole supply network, which is designed to inspire our interest and thinking on the link of supply chain operations and public administrative policies. To achieve this target, we induce some metric index in terms of individual level and system level profits and emissions.

There are several streams of research related to our work. For an extensive survey in general on the operations management in low-carbon environment, see Benjaafar et al. [4] and Xia et al. [1]. Some researchers holding the macroperspective have studied the relationship between carbon emissions and the national economy; see Dhakal [5], Holtz-Eakin and Selden [6], and Zhang and Cheng [7] while many other studies consider the evaluation and influential factors of carbon emissions. Similarly, some researches pay attention to the relationship between macroeconomic factors and carbon emissions. Leng [8] develops a theoretical analysis of environmental economics for analyzing the impact of economic factors on carbon emissions. In this direction, further analysis on how economic factors affect carbon emissions was conducted by Zhu et al. [9] and Sun and Chen [10]. They also propose that the adjustment of the industrial structure, enhancement of energy efficiency, and transformation of the energy structure are among the emphases of energy conservation. Whereas being obviously different from the abovementioned literatures, our study focuses on microfirm and especially the network-level organization operations considering emission constraints.

Another relevant stream in this area focuses on the impact of policy instruments on the carbon emission reduction, and these instruments can be summarized as mandatory emission caps, taxes on emissions, cap-and-trade, and cap-and-offset; see Cropper and Oates [11], Hepburn [12], Webster et al. [13], and Bureau [14]. While these researches study the impact of public policies on the carbon emission reduction, they do not link operations management and public policy making for reducing carbon emission and also do not compare the effectiveness of carbon emission control under different policies.

Another relevant stream to our work are papers that consider the influence of carbon emissions policies on the industrial firms' production and inventory decisions, profits and costs, and so forth. Benjaafar et al. [4] and Benjaafar et al. [15] are among the first who proposed how to achieve the targeted emission reduction effect only from the perspective of manufacturing operations when a single firm was subject to carbon emissions regulations. Pan [16] investigates the carbon emission reduction and production planning of a single company as well as duopoly companies both under the carbon tax and cap-and-trade policies, respectively. But his study does not take the excess inventory generated emissions into account. As a contrast, Kang and Yoon [17] study the case considering the emissions incurred by inventory. For further exploring the relation between profit and emission reductions, Song and Leng [2] exploiting classic single-cycle newsvendor model investigate the optimal quantity ordering under three kinds of emission regulatory policies, that is, mandatory emission cap, emissions tax, and cap-and-trade. Afterwards, Chen et al. [18] research a similar problem, but they adopt the conventional EOQ model by assuming that the market demand rate is constant and shortage is not permitted. Although considering the comparison of performances incurred by different policies, the above researches are limited to a single firm and therefore do not study the impact of carbon emission regulations on supply chains. On the contrary, this is just the gap our work attempts to fill.

The most highly related research stream on low-carbon supply chain mainly focus on supply chain design and production system decisions. They argue that structure of the supply chain plays an important role in the chainwide carbon emissions, so it is important to optimize the supply chain design to achieve the emission control goal; see Cachon [19], Ramudhin et al. [20], and Abdallah et al. [21]. Ramudhin et al. [20] established a mixed integer programming model for a green supply chain redesign. On its basis, Ramudhin et al. [22] consider the life cycle assessment in the supply chain design. As a further study developed by Cachon [19], he considers the impact of two aspects supply chain structure and operations on carbon emissions. Another research direction on production decisions of supply chain under the low-carbon policies is primarily derived by Benjaafar et al. [15]. They study how to optimize operations (such as procurement, production, and inventory controlling) in supply chain under emission regulations. Abdallah et al. [21] establish a mix integer programming (MIP) model to minimize carbon emissions. Similar to our study, Zhao and Lv [23] construct a model for some supply chain operations and develop optimal policy to minimize the chainwide overall carbon emissions. The supply chain structures considered in these literatures are relatively simple, and few studies above discuss the relation between system profit and carbon emission of the whole supply chain. Being compared with these literatures, the supply chain structure studied in this paper is more likely to reflect the reality of assembly-system-like supply chain, the characteristics of which lead to more sophisticated operations to handle under low-carbon oriented polies.

In this study, we concentrate on an assembly-system-like complex supply chain, as described in He [24], but with existence of various carbon emission regulations. We take both of production relevant and inventory-incurred carbon emission into account with the supply chain firm suffering random demands from the system outside. Combining the input-output technology, we establish a system production planning model of the supply chain and get the optimal production planning for each firm as well as the whole supply chain. What is more, we demonstrate the impact of the emissions reduction on the production decision-making. And we analyze how to adjust emissions reduction policy for public administration to reach the goal of carbon emissions reduction. Moreover, we introduce the concept of "carbon emissions elasticity of profit" index to evaluate the effect of emissions reduction policy.

The rest of the paper is organized as follows. We begin by addressing the problem this study focuses on and introduce notation and assumptions. In Section 3, we discuss the complex supply chain production planning in the setting without carbon emission regulations. The results here are to be compared with those in other scenarios. In Sections 4 and 5, we investigate the system planning problem considering typical carbon emission constraints of two low-carbon policies, namely, *mandatory carbon emission cap* and *carbon emission tax*, respectively. We present in these two sections how to make the optimal production planning by considering profit optimization and emission requirements simultaneously. In Section 6, we conduct numerical experiments and associated analysis to enrich and examine the solutions in afore sections. Section 7 comes by summarizing the managerial insights and possible extensions for research in the future.

2. Notation, Assumptions, and Problem Formulation

In this paper, we consider supply chains composed of a set of nodes firms, in which between a pair of nodes is the coupled demand according to physical structure of the final product. The integrator of the assembly supply chain makes product

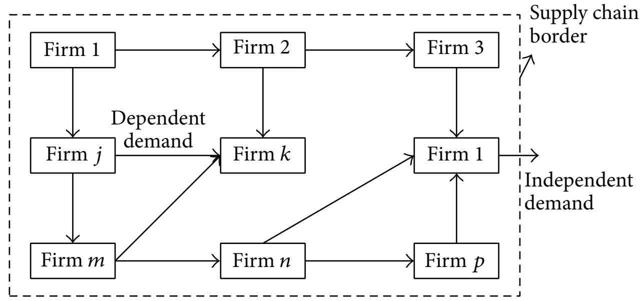

Figure 1: Schematic map of complex supply chain.

with multiprocess and a complex structure. For research simplicity without loss of generality, we assume that the product consists of a set of complementary component parts, each of which is uniquely produced by one node before the assembly. Demand relationship existing between a pair of nodes is determined by corresponding material flows, which are eventually installed by final product configuration. In other words, a node firm is the customer of her preceding node as well as the supplier of her succeeding node meanwhile.

There are two kinds of demands existing for each firm in the supply chain network, which are either from other node firms within or from organizations beyond the boundary of the network. Sorted by the source of the demand, the former is called dependent demand and the latter independent demand; see Jacobs and Chase [25]. There are quantity correlations between these two types of demands according to He et al. [26], which can be modeled by physical input-output analysis including the direct consumption coefficients and the total demand coefficients. The structure of the supply chain and its material flows are shown in Figure 1 as described in literature [27].

Embodied in the supply chain, there are three descriptions featuring the system operations: (1) to fabricate a final product, the production quantity for each company should meet a specific relationship which is determined by the configuration of the components in the final product; (2) each node firm should allocate her own capacity to satisfy relatively deterministic internal dependent demands and stochastic external demand simultaneously; (3) we assume the existence of an integrator as system controller to make a centralized decision by assigning productions distributed among all nodes to maximize total supply chain profit under carbon emission constraints. In this paper, we call above system related demand supply chain (RDSC) that is prevalent in the industry.

The complexity of managing the RDSC is mainly reflected in two aspects: (1) we should consider multinode synchronously to seek a global optimizer; pursuing the maximization of individual own interests, each node firm has his/her own interest appeal; this typical self-interested decentralized decision-making modes usually results in suboptimal performance both for its upstream and downstream companies, deviating from optimal decisions; (2) stochastic demands beyond and within the network boundary are intertwined with each other. The volatility of external random demand is passed through other nodes and then nodes are coupled together across the entire supply chain network, which raises the complexity of the production planning.

Moreover, driven by today's harsh climate change and challenging environmental issues, many countries have enacted a series of laws and regulations to limit carbon emissions. Two representative policies, the mandatory emission cap and carbon tax policies, are selected. Specific content of mandatory emission caps policy is that government requires carbon dioxide emissions of companies cannot exceed a given value within a given period. Carbon tax policy is to levy appropriate taxes according to the amount of carbon dioxide emitted during making business products or services, for the purpose of reducing carbon emissions.

In this paper, the related demand supply chain we researched faces the government's carbon reduction policy mentioned above. In this situation, the RDSC should adjust its production operational decisions. In the condition of mandatory emission caps policy, production of each company cannot be excessive; otherwise it will exceed the government's mandatory carbon emission limits; under carbon tax policy, if the company produces more, it will emit much more carbon emissions, resulting in more carbon taxes, but, on the other hand, company can sell more products and improve its revenue. So it is important to formulate appropriate production volume for the company in the supply chain to balance the relationship between carbon taxes and products revenue, in order to obtain maximum total profit of the supply chain.

We first figure out the optimal production decisions of node firms to maximize the supply chain profit in the condition without considering the carbon emission policy. And we give out analytical solutions of optimal production planning of dependent demand in complex supply chains. Then we consider how to adjust the production strategies of the company to maximize the chainwide profit in the condition of two kinds of carbon emissions policies as we mentioned. What is more, we investigate how to use these policies to achieve good effect of emission reduction while without substantial damages to the profit of the supply chain.

Parameters, Notation, and Assumptions. Parameters used in this paper are shown below.

Parameters

$N = \{1, 2, \ldots, n\}$: collection of all companies (products) in the supply chain

X_i: stochastic market demand of independent demand product; its probability density function is $f_i(x_i^L)$, cumulative distribution function is $F_i(x_i^L)$, and mean value is μ_i

E_i: the actual carbon emissions of each company

C_i: carbon emission quotas of each company

K_i: fixed cost of each product

v_i: variable cost of per unit of product

h_i: inventory cost due to the presence of residual product

s_i: penalty cost due to shortage

e_i^m: carbon dioxide emissions generated by the production of per unit of product

E_i^K: fixed carbon emissions of each production

e_i^h: carbon emissions of per unit remaining inventory

p_i: unit price of each product

ω: carbon tax of per unit of carbon emissions.

Decision Variables

q_i: stock after completion of the production of each company

$Q_M = [\ldots q_i^M \ldots]$: production planning for dependent demand of each firm

$Q_L = [\ldots q_i^L \ldots]$: production planning of independent demand for each firm

$Q_S = [\ldots q_i^S \ldots]$: total production planning for each firm, $q_i^S = q_i^L + q_i^M$.

To make our research clear and simple without loss of generality and essence, we need to give out some assumptions as follows.

Assumption 1. Each node firm only produces one kind of component in supply chain.

Assumption 2. The initial inventory at each node firm is zero.

Assumption 3. Demand relationships between a pair of nodes in the supply chain are determined by the structure of the product and do not change over a single cycle.

Quantitative relationships between the inventory of companies upon completion of each production Q and production planning of independent demand Q_L can be connected by the direct consumption coefficient a_{ij} and matrix $A = \{a_{ij}\}_{n\times n}$. According to the input-output balanced equations, $\sum_{j=1}^{n} a_{ij} \cdot q_j + q_i^L = q_i$, so $Q = A \cdot Q + Q_L$. And we can get the following formula from the relationship between Q and Q_L:

$$Q_L = (I - A) \cdot Q = B \cdot Q, \tag{1}$$

where $B = I - A = \{b_{ij}\}_{n\times n}$.

We can prove that $(I - A)$ is a full-rank matrix and further calculate its inverse matrix. And another form of relationship between Q and Q_L can be rewritten as

$$Q = (I - A)^{-1} Q_L = H \cdot Q_L, \tag{2}$$

where $D = (I - A)^{-1} = \{d_{ij}\}_{n\times n}$, and we call it absolutely necessary coefficient matrix. So it is obvious that the total production planning amount of each company can be derived from the production planning of independent demands, and the relationship between them is expressed as $q_i = \sum_{j=1}^{n} d_{ij} q_j^L$, $i \in N$. Based on this relationship, we take the production planning of independent demands as the decision variable.

3. Production Planning Model of the Supply Chain without Carbon Emission Constraints

In this section, we first consider production planning problem in the same complex supply chain system without emission constraints existing. The results in this setting are to be scaleplate compared with that in other scenarios. There exist two kinds of costs to be considered in this study. One is the inventory holding cost. The company produces more than the market demand and the remaining products needed to be stored. The parameter h_i is the stock cost per unit of product. The other is the penalty cost. When the amount of the product cannot meet the demand of the market, the company should be punished with per unit product penalty cost s_i. We have assumed that there is no initial stock for each node firm. So the ending stock level q_i is equal to the total production planning amount q_i^s which is the sum of production planning of internal dependent demand and external independent demand; that is, $q_i = q_i^s = q_i^L + q_i^M$. And because supply and procurement within the supply chain is determined by the bill of materials (BOM) table, so the production planning of dependent demand of a company is determined by the total production planning amount of its downstream firms. Therefore, we can draw a conclusion that inventory costs and shortage costs are influenced by the relationship between production planning of independent demand and stochastic market demand.

We establish a vector $\Delta C(Q_L) = [\ldots, \Delta c_i, \ldots]$ considering inventory underage and overage simultaneously. We call it difference cost function between the production planning of internal demand and that of external demand, the expression of which is as follows:

$$\Delta c_i = h_i \cdot \left(X_i - q_i^L\right)^- + s_i \cdot \left(X_i - q_i^L\right)^+. \tag{3}$$

The practical significance of the formula above is that the differences' cost function contains the inventory costs and shortage costs. And it can be transformed into piecewise function in the following:

$$\begin{aligned}
\left(X_i - q_i^L\right)^- &= \begin{cases} q_i^L - X_i & \text{where } X_i < q_i^L \\ 0 & \text{where } X_i \geq q_i^L, \end{cases} \\
\left(X_i - q_i^L\right)^+ &= \begin{cases} 0 & \text{where } X_i < q_i^L \\ X_i - q_i^L & \text{where } X_i \geq q_i^L. \end{cases}
\end{aligned} \tag{4}$$

In addition to these two kinds of costs, we consider another two kinds of costs, that is, fixed cost and variable cost. Because we only consider the case of single-cycle, the fixed costs and variable costs per unit of product are constants, and variable costs are positively correlated with the total yields.

In this part, our target is to maximize the profit of the supply chain, and we construct the profit function according to the relationship that profit is equal to the value of revenue minus cost. The profit function is as follows:

$$\begin{aligned}
\pi(Q_L) = \sum_{i=1}^{n} \Bigg\{ & p_i \cdot \min\left(q_i^L, X_i\right) \\
& - \Bigg[h_i \cdot \left(X_i - q_i^L\right)^- + s_i \cdot \left(X_i - q_i^L\right)^+ \\
& + K_i + v_i \cdot \sum_{j=1}^{n} d_{ij} q_j^L \Bigg] \Bigg\}.
\end{aligned} \tag{5}$$

On the right-hand side of the above equation, it is the profit of each firm that located in the curly braces. And the sum of individual company profit constitutes the supply chain profit. The first item in the curly braces is the revenue of the company and the company obtains it by selling the product to the market. We do not consider the revenue from selling product to the other companies inside the supply chain because this part of revenue is purchasing cost for them and this revenue and cost can be offset by the entire supply chain profit. And we can find that sales' volumes of each node firm are the minimum of the production planning of independent demand and the stochastic market demand. If the company produces more than the market demand, then the sales' volumes are determined by the planned production of independent demand. Otherwise the sales' volumes are equal to the market demand. The second part within the curly braces is the cost of the company which contains inventory costs, shortage costs, fixed costs, and variable costs.

If we know the probability density function and the cumulative distribution function of the stochastic market demand for node firm i, we can get the objective function to maximize the profit of the supply chain as follows:

$$\begin{aligned}
\max \Pi = E(\pi) = \sum_{i=1}^{n} \Bigg\{ & \left(p_i + s_i\right) \cdot q_i^L \\
& - \left(p_i + h_i + s_i\right) \int_0^{q_i^L} F\left(X_i\right) dX_i \\
& - K_i - s_i \mu_i - v_i \cdot \sum_{j=1}^{n} d_{ij} q_j^L \Bigg\}
\end{aligned} \tag{6}$$

$$\text{s.t.} \quad q_i^L \geq 0.$$

It is obvious that the objective function is a second-order continuous function. If we want to get the optimal solution to maximize the profit, we should first prove that it is a concave function and then we calculate the first-order derivative of the function for the planned production of independent demand by equating it to zero. By solving the equations system we can obtain the optimal value of production planning. We prove the concavity of the function in the appendix. And the first-order derivative of the function is as follows by letting it be zero:

$$-p_k - s_k + (p_k + h_k + s_k) F_k (q_k^L) + \sum_{i=1}^{n} v_i d_{ik} = 0. \tag{7}$$

From this equation, we can calculate the planning production of independent demand of each company. According to the relationship between planned production of independent demand and the total production of each company, we can calculate out some other kinds of production. And the expression of the planned production of independent demand is as follows:

$$q_k^{L*} = F_k^{-} \left[\frac{p_k + s_k - \sum_{i=1}^{n} v_i d_{ik}}{(p_k + h_k + s_k)} \right]. \tag{8}$$

It is obvious that there is no relation between the optimal planned productions of independent demand of a company with three parameters of other companies in the supply chain: the price, inventory holding cost, and shortage penalty cost per unit product. It is only influenced by these three cost factors of its own, but another finding is that optimal planning production of independent demand is related to the variable cost of per unit product of other companies in the supply chain. To learn more details of the research content as well as proofs shown in Section 3, please refer to literature [27]. And we will do a further study in the section of numerical analysis.

4. Supply Chain Decision under the Mandatory Emission Cap Policy

China's carbon emission occupies nearly 1/4 of the world's emissions, which is the highest in the world. So China has been facing the pressure to cut emissions from the international community in the past. In contrast, China has said it would reduce its "carbon intensity" or the proportion of emissions relative to economic output. Related personnel of NDRC Energy Research Institute said that the NDRC was considering implementing an absolute upper limit of carbon emissions in the next five-year plan and now they were studying what is the appropriate level.

Mandatory emission cap policy is a good method to reduce carbon emissions, but the problem is that companies which are facing pressure from government must have flexibility on production decisions and also have an impact on corporate profits. This section is intended to study the impact of mandatory emission cap policy on the production decisions of supply chain members.

4.1. Joint Optimization Model under Mandatory Emission Cap Policy. In the case of mandatory emission cap policy, the company's total carbon emissions E_i must not exceed government regulations value C_i.

At this time, the expected profit function is the same as the scenarios under no carbon emissions. Under the permit that each company satisfies $E_i \le C_i$, objective function of the supply chain is

$$\begin{aligned} \max_{q_i^L} \prod_{\mathrm{MC}} (Q_L) = \sum_{i=1}^{n} \Bigg\{ & (p_i + s_i) \cdot q_i^L - (p_i + h_i + s_i) \\ & \cdot \int_0^{q_i^L} F_i (x)\, dx - s_i \mu_i - K_i \\ & - v_i \cdot \sum_{j=1}^{n} d_{ij} q_j^L \Bigg\} \end{aligned} \tag{9}$$

$$\text{s.t. } E_i \le C_i$$
$$q_i^L \ge 0$$
$$\forall i \in N,$$

where $E_i = e_i^m \cdot \sum_{j=1}^{n} d_{ij} \cdot q_j^L + E_i^K + e_i^h \cdot \int_0^{q_i^L} F_i(x) dx$.

Company's total carbon emissions are equal to the sum of variable production relevant emissions, fixed production relevant emissions, and excess inventory relevant carbon emissions. Variable production relevant emissions are proportionally increasing in total number of products; that is, $e_i^m \sum_{j=1}^{n} d_{ij} \cdot q_j^L$; fixed production of emissions is E_i^K. The expected value of remaining inventory is $\int_0^{q_i^L} F_i(x)\, dx$, so the expected value of carbon emissions generated by the excess remaining inventory is $e_i^h \cdot \int_0^{q_i^L} F_i(x)\, dx$.

4.2. Interior Point Method to Solve the Problem. For general unconstrained optimization problem, there exists a variety of efficient algorithms currently. People usually transform the constrained problem into an associated unconstrained problem. Specifically, according to the constraint characteristics, they construct some kind of "punishment" function and then add it to the objective function; then the constraint solving problem is transformed into a series of unconstrained problem solving. In this respect, there are many algorithms, such as outside the penalty function method, the penalty function method, and the multiplier method.

As for the planning problem with inequality constraints under mandatory emission cap policy, this paper uses the penalty function method to solve it. In order to make iterative points always possible, the penalty function method builds a high "wall" on the boundary of the feasible region. When the iteration point is near the border, the objective function value suddenly increases, as a punishment to stop the iteration point across the border, and thus the optimal solution is blocked in the feasible region.

Before the use of penalty function method, we construct a penalty function and add it to the objective function. Specific steps are as follows.

Step 1. Make the original problem into a minimization problem:

$$\begin{aligned}\min J_1(Q_L) = \sum_{i=1}^{n}\Bigg\{&-(p_i+s_i)\cdot q_i^L+(p_i+h_i+s_i)\\&\cdot\int_0^{q_i^L}F_i(x)\,dx+s_i\mu_i+K_i\\&+v_i\cdot\sum_{j=1}^{n}d_{ij}q_j^L\Bigg\}\\ \text{s.t. } g_i\left(q_i^L\right) = C_i &- e_i^m\sum_{j=1}^{n}d_{ij}\cdot q_j^L\\&-E_i^K-e_i^h\cdot\int_0^{q_i^L}F_i(x)\,dx\ge 0\\ q_i^L &\ge 0,\\ \forall i &\in N.\end{aligned} \tag{10}$$

Step 2. Structure a new planning with the form $\min_{Q_L\in\mathbb{R}^n}\varphi(Q_L, r^{(k)}) = J_1(Q_L) + r^{(k)}\sum_{i=1}^{n}(1/g_i(Q_L))$, where the penalty factor $r^{(k)} > 0$.

Step 3. Given the initial point ${Q_L}^{(0)} \in \mathbb{R}^n$, the allowable error $\varepsilon > 0$, the penalty factor $\gamma^{(1)} > 0$, and magnificence $b \in (0, 1)$, set $k = 1$.

Step 4. Take ${Q_L}^{(k-1)}$ as the initial point; then solve the following planning problem: $\min_{Q_L\in\mathbb{R}^n}\varphi(Q_L, r^{(k)}) = J_1(Q_L) + r^{(k)}\sum_{i=1}^{n}(1/g_i(Q_L))$; ${Q_L}^{(k)}$ is the minimal point.

Step 5. If $r^{(k)}\sum_{i=1}^{n}(1/g_i(Q_L)) < \varepsilon$, stop calculating; the result is ${Q_L}^{(k)}$.

Step 6. Otherwise $\gamma^{(k+1)} = b\gamma^{(k)}$, set $k = k + 1$ and go back to step four.

5. Supply Chain Decisions under Carbon Tax Policy

Carbon tax originated in the British economist Pigou "Pigou's theory"; the theory is that in order to achieve effective control of pollution and pollutant emissions purposes, the government imposes tax on polluters according to the degree of harm caused. In order to achieve the reduction of carbon dioxide emissions and fossil fuel consumption, the government levies carbon tax on coal, gasoline, natural gas, jet fuel, and other fossil fuel products, according to the proportion of their carbon content. Carbon tax can take many forms and the simplest case is penalty with the linear growth of unit carbon emissions.

In this part, we study the impact of carbon tax policy on supply chain operations. In order to obtain the optimal supply chain profit, it is important to balance the relationship between the revenue and the tax brought by the production of the company. If the companies can sell more products and the revenue may cover the carbon emissions tax, it is clear that companies will choose to produce more. Conversely, companies produce less. We build the profit model of the supply chain under carbon tax policy in this part to study the impact of tax policy on production decisions in the complex supply chain.

Company i needs to pay taxes ω for releasing per unit of carbon emissions. Companies' carbon emissions are $e_i^m\sum_{j=1}^{n}d_{ij}\cdot q_j^L+E_i^K+e_i^h\cdot(X_i-q_i^L)^-$, so companies need to pay taxes $\omega\cdot[e_i^m\sum_{j=1}^{n}d_{ij}\cdot q_j^L+E_i^K+e_i^h\cdot(X_i-q_i^L)^-]$. The carbon tax is a kind of cost in the profit function according to the profit function of no carbon emissions circumstances and the profit function can be expressed as

$$\begin{aligned}\prod_{\text{CT}}(Q_L) = \sum_{i=1}^{n}\Bigg\{&p_i\cdot\min\left(q_i^L, X_i\right)-h_i\cdot\left(X_i-q_i^L\right)^- - s_i\\&\cdot\left(X_i-q_i^L\right)^+-K_i-v_i\cdot\sum_{j=1}^{n}d_{ij}q_j^L-\omega\\&\cdot\Bigg[e_i^m\sum_{j=1}^{n}d_{ij}\cdot q_j^L+E_i^K\\&+e_i^h\cdot\left(X_i-q_i^L\right)^-\Bigg]\Bigg\}.\end{aligned} \tag{11}$$

So the model of maximizing expected profit of entire supply chain can be written as follows:

$$\begin{aligned}\max E\left(\prod_{\text{CT}}\right) = \sum_{i=1}^{n}\Bigg\{&(p_i+s_i)\cdot q_i^L-\left(p_i+h_i+s_i+\omega e_i^h\right)\\&\cdot\int_0^{q_i^L}F_i(x)\,dx-\left(v_i+\omega e_i^m\right)\\&\cdot\sum_{j=1}^{n}d_{ij}q_j^L-\omega E_i^K-s_i\mu_i-K_i\Bigg\}\\ \text{s.t. } q_i^L &\ge 0\\ \forall i &\in N.\end{aligned} \tag{12}$$

First, we assume the profits function $\prod_{\text{CT}}$ is second-order continuous. The presence of optimal solution for the above equation is that the first derivative of $\prod_{\text{CT}}$ for a variety of independent demand q_k^L $(k \in N)$ is zero, and $\prod_{\text{CT}}$ is a concave function.

According to the method of proving $\prod$ to be a concave function, it is easy to prove that the profit function $\prod_{\text{CT}}$ is also concave function.

Then we seek first-order partial derivative of $\prod_{\mathrm{CT}}$ for q_k^L (for all $k \in N$) and set it to zero as follows:

$$-p_k - s_k + \left(p_k + h_k + s_k + \omega e_k^h\right) F_k\left(q_k^L\right) + \sum_{i=1}^{n} d_{ik}\left(v_i + \omega e_i^m\right) = 0. \quad (13)$$

So, we can get the optimal production planning of independent demand:

$$q_k^{L*} = F_k^{-1}\left[\frac{p_k + s_k - \sum_{i=1}^{n} d_{ik}\left(v_i + \omega e_i^m\right)}{\left(p_k + h_k + s_k + \omega e_k^h\right)}\right]. \quad (14)$$

6. Numerical Experiments and Analysis

In this section, we conduct numerical experiments to examine some conclusions or findings in preceding sections. A complex supply chain of one functional product is designed as in Figure 2.

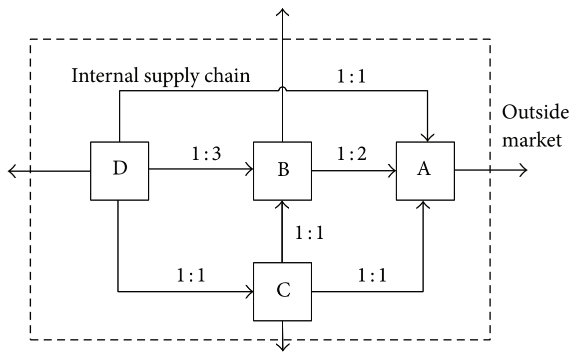

Figure 2: A manufacturing supply chain structure.

Among these products, product A is the final product assembled which only faces random independent demand from external market while the products B, C, and D are all component parts and they are confronted with dependent demand from network nodes and independent demand of external market. According to the demand structure, we can get the direct consumption coefficient matrix A = [0, 0, 0, 0; 2, 0, 0, 0; 1, 1, 0, 0; 1, 3, 1, 0].

Now, independent demand x_i^L is assumed to subject to exponential distribution with parameter λ_i and its mean value μ_i, the probability density function $f_i(x_i^L) = \lambda_i e^{-\lambda_i x_i^L}$, and the cumulative distribution function is $F_i(x_i^L) = 1 - e^{-\lambda_i x_i^L}$.

In seek of further analysis of joint optimization strategy of production for the supply chain, we set up several groups of experiments in this paper through numerical simulation to study the relationship between decision variables and some key parameters under the carbon emissions policy.

6.1. Analysis of Planned Production of Each Company under No Carbon Emissions Constraints. The optimal planned production of independent demand of each company is

$$q_k^{L*} = -\mu_k \ln\left[1 - \frac{p_k + s_k - \sum_{i=1}^{n} v_i d_{ik}}{\left(p_k + h_k + s_k\right)}\right]. \quad (15)$$

As we can see from the formula, the optimal planned production of independent demand of each company only has the correlation with factors of its own including the product market demand, product prices, the inventory cost, penalty cost, and the initial inventory and is not affected by these factors of other companies. According to the absolutely necessary coefficient matrix, we get $d_{ik} = 0$, where $k > i$. It means that when company i is a customer to company k we get $d_{ik} = 0$. At this time, companies' optimal planned production of independent demand will not be influenced by their customers' variable cost per unit of product. And we can get $d_{ik} > 0$, where $k < i$. It means that when company i is a supplier of company k we get $d_{ik} > 0$. Thus, to customers, their optimal planned production of independent demand is influenced by variable cost per unit of product of their suppliers. Besides, planned production of independent demand of customer companies is on the decrease with the increasing of the suppliers' variable cost per unit of product. This phenomenon illustrates the importance of real powerful impact factor—variable cost per unit of product in the supply chain.

(1) This section mainly analyzes optimal planned production of independent demand trends under the given μ, P, H, S, and the changes in production costs, thereby getting analysis of the influence of production cost vector on the system performance. Without loss of generality, we set $\mu = [100, 112, 120, 140]^T$, $P = [10, 6, 4, 1]^T$, $H = [h_1, h_2, h_3, h_4] = [0.6, 0.4, 0.3, 0.05]^T$, and $S = [s_1, s_2, s_3, s_4] = [0.8, 0.4, 0.2, 0.1]^T$. The production planning of independent demand and system profits are shown in Table 1.

From Table 1, it is not difficult to find that when the assembly plant A's production cost increases, its production planning amount related to independent demand decreases and that of its suppliers B, C, and D is unchanged. At this point the system gross profit is decreasing. When the factory B's production costs increases, planned production of independent demand of the factories A and B decreases, while that of C and D remains unchanged, and the total profits of the system decrease. When production cost of A, B and suppliers C increases, the planned production of independent demand of A, B, and C was decreased, but that of raw materials supplier D is unchanged. And we can find that the total profit of the system has also gradually reduced in this case. It is an interesting insight that when the production costs of raw material supplier D increased, the planned production of independent demand of all the companies in the supply chain has reduced, and the total profit system has also gradually reduced. From the above analysis, we can find that the total profit of the system is negatively correlated to the production costs of each company. The planned production of independent demand of downstream company in the supply chain is affected by production costs of its own and its upstream suppliers, and there is a negative correlation between the planned production of independent demand and the production costs. But the planned production of independent demand of the upstream companies in the supply chain is only affected by its own production costs.

TABLE 1: Production planning and system profits sensitivity analysis to V^P.

V^P	Planned production of independent demand	System profit
[0.6, 0.4, 0.4, 0.08]	[104.73, 167.80, 210.30, 305.20]	803.06
[0.8, 0.4, 0.4, 0.08]	[99.85, 167.80, 210.30, 305.20]	782.61
[1.0, 0.4, 0.4, 0.08]	[95.20, 167.80, 210.30, 305.20]	763.11
[1.2, 0.4, 0.4, 0.08]	[90.76, 167.80, 210.30, 305.20]	744.51
[1.4, 0.4, 0.4, 0.08]	[86.50, 167.80, 210.30, 305.20]	726.79
[1.6, 0.4, 0.4, 0.08]	[82.42, 167.80, 210.30, 305.20]	709.90
[1.6, 0.6, 0.4, 0.08]	[74.72, 153.96, 210.30, 305.20]	646.35
[1.6, 0.8, 0.4, 0.08]	[67.58, 141.63, 210.30, 305.20]	588.37
[1.6, 1.0, 0.4, 0.08]	[60.91, 130.54, 210.30, 305.20]	535.49
[1.6, 1.2, 0.4, 0.08]	[54.65, 120.44, 210.30, 305.20]	487.31
[1.6, 1.4, 0.4, 0.08]	[48.77, 111.18, 210.30, 305.20]	443.49
[1.6, 0.4, 0.4, 0.08]	[82.42, 167.80, 210.30, 305.20]	709.90
[1.6, 0.4, 0.6, 0.08]	[71.08, 153.96, 182.91, 305.20]	592.55
[1.6, 0.4, 0.8, 0.08]	[60.91, 141.63, 160.63, 305.20]	489.19
[1.6, 0.4, 1.0, 0.08]	[51.67, 130.54, 141.84, 305.20]	398.06
[1.6, 0.4, 1.2, 0.08]	[43.21, 120.44, 125.60, 305.20]	317.84
[1.6, 0.4, 1.4, 0.08]	[35.42, 111.18, 111.30, 305.20]	247.47
[1.6, 0.4, 0.4, 0.08]	[82.42, 167.80, 210.30, 305.20]	709.90
[1.6, 0.4, 0.4, 0.10]	[78.50, 162.05, 207.27, 285.16]	670.55
[1.6, 0.4, 0.4, 0.12]	[74.72, 156.59, 204.30, 267.64]	632.84
[1.6, 0.4, 0.4, 0.14]	[71.08, 151.38, 201.41, 252.07]	596.70
[1.6, 0.4, 0.4, 0.16]	[67.58, 146.40, 198.59, 238.06]	562.03
[1.6, 0.4, 0.4, 0.18]	[64.19, 141.63, 195.83, 225.32]	528.76
[1.6, 0.4, 0.4, 0.20]	[60.91, 137.06, 193.13, 213.65]	496.82
[1.6, 0.4, 0.4, 0.22]	[57.73, 132.67, 190.50, 202.87]	466.17

(2) This section mainly analyzes rends of planned production of independent demand Q^L in a given μ, V^P, H, and S with the changes of product prices. Thereby we analyze the impact of the product price vector on system performance. Without loss of generality, we order $\mu = [100, 112, 120, 140]^T$, $\nu = [1.6, 0.4, 0.4, 0.08]$, $H = [h_1, h_2, h_3, h_4] = [0.6, 0.4, 0.3, 0.05]^T$, $S = [s_1, s_2, s_3, s_4] = [0.8, 0.4, 0.2, 0.1]^T$, and $K = [30, 21, 16, 10]^T$. Planned production of independent demand and system profits is shown in Table 2.

It can be found in Table 2 that product price changes of each company will only affect the production planning of independent demand but will not affect other companies. With prices gradually decreasing, the production planning for external random demand is to decline. This indicates that the supply chain production operations decision is influenced by product prices vector. Similarly, the total profit of supply chain also goes down with decreasing prices.

TABLE 2: Planned production of independent demand and system profit sensitivity analysis to price P.

Price P	Planned production of independent demand	System profit
[10.0, 6, 4, 1]	[82.42, 167.80, 210.30, 305.20]	709.90
[9.5, 6, 4, 1]	[77.93, 167.80, 210.30, 305.20]	682.33
[9.0, 6, 4, 1]	[73.24, 167.80, 210.30, 305.20]	655.81
[8.5, 6, 4, 1]	[68.31, 167.80, 210.30, 305.20]	630.44
[8.0, 6, 4, 1]	[63.13, 167.80, 210.30, 305.20]	606.36
[7.5, 6, 4, 1]	[57.66, 167.80, 210.30, 305.20]	583.68
[7.0, 6, 4, 1]	[51.88, 167.80, 210.30, 305.20]	562.59
[6.5, 6, 4, 1]	[45.74, 167.80, 210.30, 305.20]	543.28
[10, 6.0, 4, 1]	[82.42, 167.80, 210.30, 305.20]	709.90
[10, 5.5, 4, 1]	[82.42, 159.25, 210.30, 305.20]	666.91
[10, 5.0, 4, 1]	[82.42, 149.98, 210.30, 305.20]	624.98
[10, 4.5, 4, 1]	[82.42, 139.89, 210.30, 305.20]	584.33
[10, 4.0, 4, 1]	[82.42, 128.79, 210.30, 305.20]	545.20
[10, 3.5, 4, 1]	[82.42, 116.47, 210.30, 305.20]	507.93
[10, 3.0, 4, 1]	[82.42, 102.62, 210.30, 305.20]	472.97
[10, 2.5, 4, 1]	[82.42, 86.82, 210.30, 305.20]	440.99
[10, 6, 4.0, 1]	[82.42, 167.80, 210.30, 305.20]	709.90
[10, 6, 3.8, 1]	[82.42, 167.80, 204.85, 305.20]	690.16
[10, 6, 3.6, 1]	[82.42, 167.80, 199.13, 305.20]	670.62
[10, 6, 3.4, 1]	[82.42, 167.80, 193.13, 305.20]	651.30
[10, 6, 3.2, 1]	[82.42, 167.80, 186.82, 305.20]	632.23
[10, 6, 3.0, 1]	[82.42, 167.80, 180.15, 305.20]	613.43
[10, 6, 2.8, 1]	[82.42, 167.80, 173.09, 305.20]	594.93
[10, 6, 2.6, 1]	[82.42, 167.80, 165.58, 305.20]	576.79
[10, 6, 4, 2.0]	[82.42, 167.80, 210.30, 392.80]	838.52
[10, 6, 4, 1.8]	[82.42, 167.80, 210.30, 379.13]	812.29
[10, 6, 4, 1.6]	[82.42, 167.80, 210.30, 363.98]	786.26
[10, 6, 4, 1.4]	[82.42, 167.80, 210.30, 346.99]	760.47
[10, 6, 4, 1.2]	[82.42, 167.80, 210.30, 327.65]	734.99
[10, 6, 4, 1.0]	[82.42, 167.80, 210.30, 305.20]	709.90
[10, 6, 4, 0.8]	[82.42, 167.80, 210.30, 278.45]	685.38
[10, 6, 4, 0.6]	[82.42, 167.80, 210.30, 245.36]	661.68

6.2. The Analysis of Supply Chain Profits and Carbon Emissions under Mandatory Emission Caps Policy. This part is mainly devoted to the changing regularity of the total profit and the total carbon emission of the supply chain with the change of carbon caps C under the given μ, P, H, S, X, and V^P. Now, we give the value of these factors as follows:

$$\begin{gathered} P = [10, 6, 4, 1]^T, \qquad \mu = [100, 112, 120, 140]^T, \\ H = [0.6, 0.4, 0.3, 0.05]^T, \qquad S = [0.8, 0.4, 0.2, 0.1]^T, \\ K = [30, 21, 16, 10]^T, \qquad \nu = [1.6, 0.4, 0.4, 0.08]^T, \\ e^m = [2, 3, 5, 8]^T, \qquad E^K = [20, 14, 12, 10]^T, \\ e^h = [0.5, 0.4, 0.3, 0.1]^T. \end{gathered} \tag{16}$$

We have gotten the optimal planning production of independent demand of each company under the condition of no carbon policy. Because of the existence of carbon discharge during the process of production or stock in the supply chain, we can calculate each company's carbon discharge, the total profit carbon emission of the supply chain system. We call all these values "datum carbon emissions," "datum system profit," and "datum total carbon emissions." This part majorly analyzed how the government should formulate the emission cap of each company when they develop the plans of reduction. Generally when the government develops a specific reduction policy, they will consult annual carbon emission data which we have mentioned above, that is, "baseline carbon emissions." We analyzed a case where the emission cap of each company, based on "baseline carbon emissions," reduces a number of percentage points respectively (one company reduced emission cap while others' is constant). This is equivalent to the government stepped up restrictions on carbon emissions for each company. It will inevitably lead to two consequences. First is that the profit of system will be affected; the second is that the system's total carbon emissions will be changed. And we obtain the change trends of system profit and carbon emission with the emission cap of each company reducing from 1 percent to 80 percent, respectively. The results are shown in Figures 3 and 4.

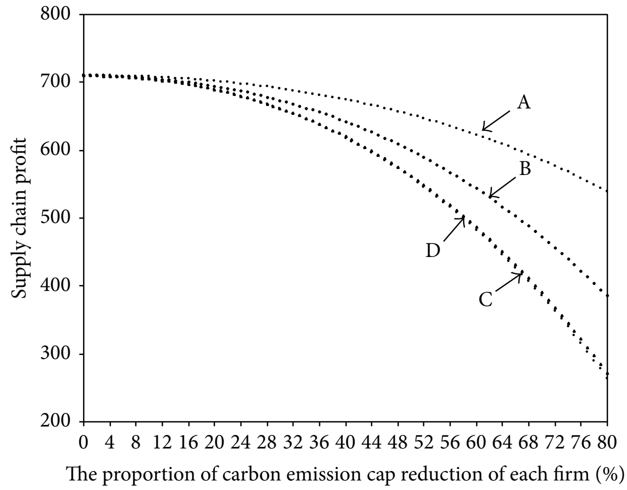

Figure 3: System profit when each company reduces carbon emissions by 1%–80%.

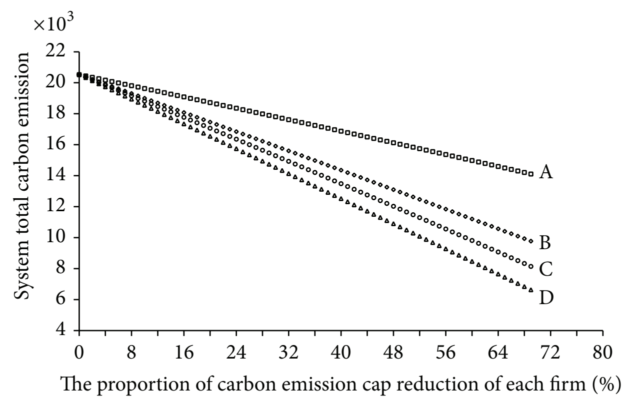

Figure 4: System emissions when each company reduces carbon emissions by 1%–80%.

As can be seen from Figure 3, it is obvious that, for each company, when the government tighten carbon emission cap, the total profit of the supply chain system is reduced. And the reduction volume is different when the government's tightening measures are applied in different company. And it is obvious that when the government strengthens restriction of the carbon emissions of company A, which is the assembly companies of the supply chain, system profit is reduced less than the case that the emission caps of other companies reduce the same percentage. From this perspective, it is better for supply chain when the government tighten carbon emission cap of company A. But from the perspective of the government, it would not be the best decision because government also needs to take the pressure of reduce emissions into account. For further study, we use Figure 4 to explore the changes of the system carbon emissions when the emission cap of each company is reduced by percentage.

We can find a remarkable phenomenon from Figure 4; when the emission cap of company A is reduced by a certain percentage from the "datum carbon emissions" (the emission cap of other companies are unchanged at this time), the supply chain carbon emissions is higher than the case that the emission cap of other company is reduced by the same proportion. From the government perspective, it could not be a perfect consequence. The government hopes that emissions reduction volume reduces more the better. So we can draw a conclusion that when the government can only tighten the emission cap of one company, it will tighten the emission cap of company D by a certain proportion to achieve a higher emission reduction effect, compared with the same reduction percentage of emission caps of other companies. It was totally opposed to the results of Figure 3, in which the supply chain wants the tightening policy to be applied on company A. So the effect of the carbon reduction with tightening emission cap of a company in the supply chain is ambivalent from the perspective of the government and supply chain. We try to introduce a kind of evaluation index to balance the contradiction between the government and the supply chain. We define the indicators for "carbon emission elasticity of profit." Similar to the price elasticity of demand, the value of carbon emission elasticity of profit is the quotient from dividing change rate of system profits relative to the datum supply chain profit by change rate of supply chain carbon emission relative to datum total carbon emissions. If the value is between 0 and 1, it means that the change rate of supply chain profit is less than the change rate of supply chain carbon emission. In this condition, the government will be very satisfied with the high proportion of carbon emissions reduction, and the supply chain is also happy that its profit is not reduced by a high proportion. Therefore, it is a very good result that the value of carbon emission elasticity of profit is between 0 and 1. In addition, the carbon emission elasticity of profit is different when the government tightens the emission cap on different companies. Figure 5 analyzes the change of carbon emission elasticity of profit with the respectively reduction of emission cap of each company by a certain proportion.

As can be seen from Figure 5, carbon emission elasticity of profit is all between 0 and 1 with respective reduction of

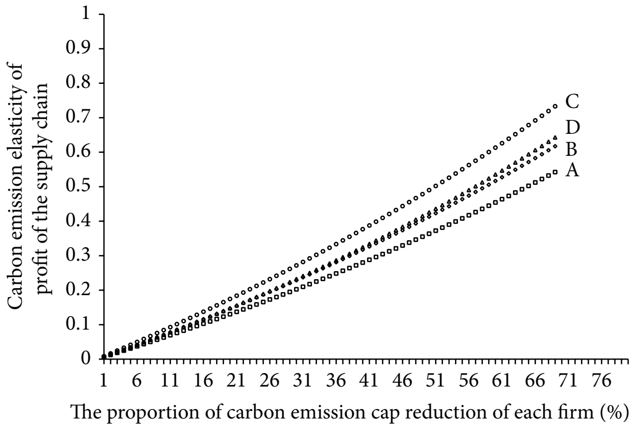

Figure 5: Carbon emission elasticity of profit with the respective reduction of emission cap of each company by a certain proportion.

emission cap of each company by a certain proportion, and the value increases with the increase of the reduction proportion. What is more, we can find that for A company, when its emission cap reduces a certain proportion, the carbon emission elasticity of profit is lower compared with the situation that other companies' emission cap reduce the same percentage. For example, when the reduction proportion of emission cap of company A is 2%, the carbon emission elasticity of profit is 0.012. At this time, the carbon emission elasticity of profit is lowest compared with the situation that other companies' emission cap reduce 2%. In other words, in this situation, if the change rate of carbon emissions was 1%, then the change rate of supply chain profit is only 0.012. The effect is very significant for supply chain and the government. So both sides will be more inclined to reduce a certain proportion of emission cap of company A.

6.3. The Analysis of Supply Chain Profits and Carbon Emissions under Carbon Tax Policy. At this point all companies' optimal planned production of independent demand:

$$q_k^{L*} = -\mu_k \ln\left[1 - \frac{p_k + s_k - \sum_{i=1}^{n} d_{ik}\left(v_i + \omega e_i^m\right)}{\left(p_k + h_k + s_k + \omega e_k^h\right)}\right] - x_k^0. \tag{17}$$

This part mainly analyzes trends of the optimal planned production of independent demand Q, total profit, and actual carbon emissions of supply chain with changes in the carbon tax ω per unit of carbon emissions under the given μ, P, H, S, v, e^m, E^K, and e^h. The values of these factors are as follows:

$$\begin{gathered} \mu = [100, 112, 120, 140]^T, \qquad P = [10, 6, 4, 1]^T, \\ H = [0.6, 0.4, 0.3, 0.05]^T, \qquad S = [0.8, 0.4, 0.2, 0.1]^T, \\ K = [30, 21, 16, 10]^T, \qquad v = [1.6, 0.4, 0.4, 0.08]^T, \\ e^m = [2, 3, 5, 8]^T, \qquad E^K = [20, 14, 12, 10]^T, \\ e^h = [0.5, 0.4, 0.3, 0.1]^T. \end{gathered} \tag{18}$$

Table 3: Sensitivity analysis of production planning of independent demand and the supply chain profit to carbon tax ω of unit carbon emission.

ω	Planned production of independent demand	System profits	System carbon emissions
0	[82.42, 167.80, 210.30, 305.20]	709.90	20526.00
0.008	[80.78, 165.45, 208.69, 298.40]	693.62	20188.94
0.016	[79.17, 163.15, 207.09, 291.91]	677.60	19856.00
0.024	[77.58, 160.89, 205.51, 285.72]	661.84	19530.21
0.032	[76.02, 158.68, 203.95, 279.78]	646.35	19211.21
0.04	[74.48, 156.51, 202.42, 274.09]	631.11	18898.68
0.048	[72.97, 154.38, 200.90, 268.62]	616.11	18592.30
0.056	[71.47, 152.30, 199.41, 263.36]	601.36	18291.81
0.064	[70.00, 150.25, 197.93, 258.29]	586.84	17996.94
0.072	[68.56, 148.24, 196.47, 253.40]	572.56	17707.47
0.08	[67.13, 146.26, 195.02, 248.67]	558.51	17423.16
0.088	[65.72, 144.32, 193.60, 244.10]	544.68	17143.82
0.096	[64.33, 142.41, 192.19, 239.68]	531.08	16869.26
0.104	[62.97, 140.53, 190.80, 235.39]	517.69	16599.29
0.112	[61.62, 138.69, 189.42, 231.22]	504.52	16333.76
0.12	[60.28, 136.87, 188.06, 227.18]	491.55	16072.49
0.128	[58.97, 135.09, 186.72, 223.25]	478.80	15815.36
0.136	[57.67, 133.33, 185.39, 219.44]	466.25	15562.20
0.144	[56.39, 131.60, 184.07, 215.72]	453.90	15312.90
0.152	[55.13, 129.90, 182.77, 212.10]	441.75	15067.33
0.16	[53.88, 128.22, 181.48, 208.57]	429.79	14825.37
0.168	[52.65, 126.56, 180.21, 205.13]	418.03	14586.91
0.176	[51.43, 124.93, 178.95, 201.77]	406.45	14351.85
0.184	[50.23, 123.33, 177.71, 198.49]	395.06	14120.07
0.192	[49.04, 121.75, 176.47, 195.28]	383.86	13891.49
0.2	[47.86, 120.19, 175.25, 192.15]	372.83	13666.01
0.208	[46.70, 118.65, 174.04, 189.09]	361.99	13443.55
0.216	[45.55, 117.13, 172.85, 186.09]	351.32	13224.02
0.224	[44.42, 115.63, 171.66, 183.16]	340.83	13007.34
0.232	[43.30, 114.15, 170.49, 180.28]	330.51	12793.43
0.24	[42.19, 112.70, 169.33, 177.47]	320.36	12582.23
0.248	[41.09, 111.26, 168.18, 174.71]	310.38	12373.66
0.256	[40.01, 109.84, 167.04, 172.00]	300.56	12167.65
0.264	[38.94, 108.43, 165.92, 169.35]	290.91	11964.15
0.272	[37.87, 107.05, 164.80, 166.74]	281.42	11763.08
0.28	[36.82, 105.68, 163.69, 164.19]	272.09	11564.39
0.288	[35.78, 104.33, 162.59, 161.68]	262.92	11368.02
0.296	[34.76, 102.99, 161.51, 159.21]	253.90	11173.91
0.304	[33.74, 101.67, 160.43, 156.79]	245.04	10982.01
0.312	[32.73, 100.37, 159.37, 154.41]	236.33	10792.28
0.32	[31.73, 99.08, 158.31, 152.07]	227.77	10604.66

At this point we can also get the planned production of independent demand and the supply chain profit under different carbon tax. The results are shown in Table 3.

The Effect of Imposing Carbon Tax Per Unit Carbon Emission on Production Planning for Independent Demand. Figure 6 shows that, with the gradual increase of carbon tax of per unit carbon emissions, planned production of independent demand is decreased approximately linearly. And under a given carbon tax, the planned production of independent demand of the company A is the lowest.

The Effect of Imposing Carbon Tax Per Unit Carbon Emissions on Overall Supply Chain Carbon Emission. As can be seen from Figure 7, within a certain range, the system profit and total carbon emission reduce with the increasing of carbon tax. And this phenomenon illustrates that it is indeed an efficient and not complicated mechanism to decline the system's total carbon emissions through increasing the carbon tax. From the government's perspective, the carbon tax policy is a good policy because emissions can be greatly reduced, but from the point of view of supply chain, the carbon tax reduces system profit greatly. Similar to the case of mandatory emission cap policy, this section also build a similar system of carbon emission elasticity of profit, and we compare the result with the case of mandatory emission cap policy. With other parameters remaining unchanged, we take the optimal solution under no carbon emissions limits as the reference point (in this case the carbon tax is 0), and we obtained the profit and carbon emissions of the supply chain in the reference point. Similarly, we obtain the carbon emission elasticity of profit under different carbon tax in Figure 8.

As can be seen from Figure 8, with the gradual increase of carbon tax rate, the carbon emission elasticity of profit increased and then decreased, which are all greater than 1. This shows that the change rate of profit is greater than the change rate of the carbon emissions. It is clear that the supply chain and the government are more difficult to accept the results. Because the profit loss in this situation can not be neglected, and unfortunately the carbon emissions reduction is not comparatively significant. In comparison, a mandatory emissions cap policy is more excellent because regardless of carbon emissions austerity policies imposed on which company, the carbon emission elasticity of profit is less than one which is an optimum result for the government and supply chain. Therefore, for the supply chain we studied, the carbon tax policy is not prior to mandatory emission cap policy to be used in practice.

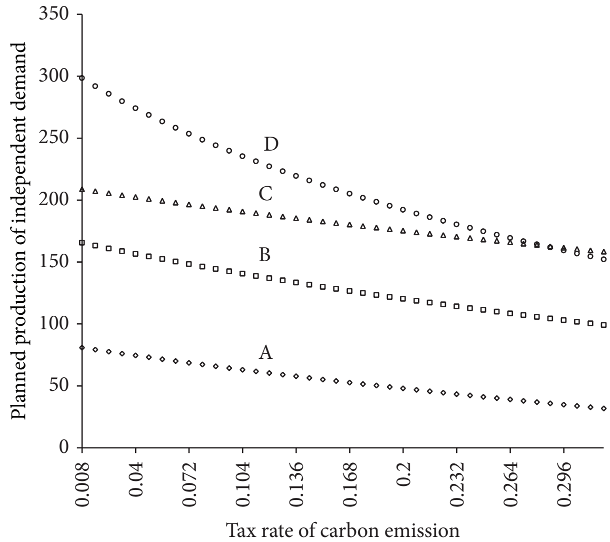

Figure 6: Effect of carbon tax of per unit carbon emissions on planned production of independent demand.

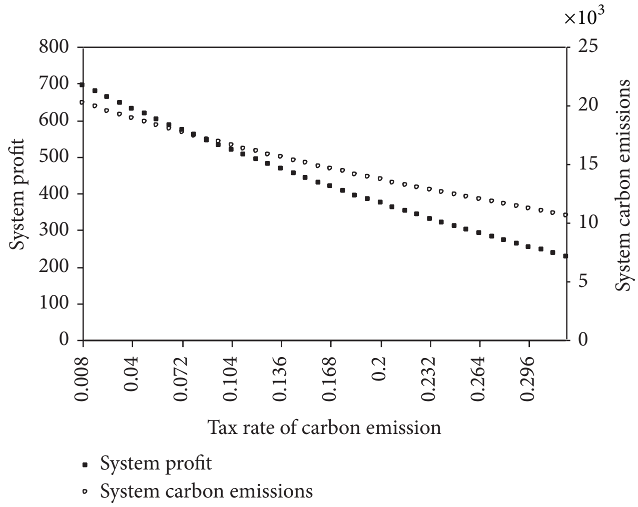

Figure 7: Effect of carbon tax of per unit carbon emissions on supply chain total carbon emission.

7. Conclusion

In this paper, we focus on how to get the optimal production planning confronting various carbon emission regulations for a complex supply chain comprising nodes firms with coupled internal dependent demand flows and random market demands. We solve the joint production optimization problem of the supply chain under mandatory emission cap and emission tax, respectively, and uncover the impact of these regulatory policies on the profit and total emission of the supply chain. We compare the optimal production quantities, profits, and overall emissions arising, respectively, from three scenarios, that is, no emission policy, mandatory emission cap, and emission tax. One of contributions of this study is that we introduce the "carbon emission elasticity of profit (CEEP)" index to evaluate the influence of carbon emission regulatory policies on profit and total emissions of a supply chain and her node firms. Taking advantage of the CEEP index, we find that under the mandatory emission cap policy if we only decrease mandatory emission cap of the assembly company by a certain proportion, the CEEP index is lowest compared with situations where we instead decrease the cap of other node firms by same scale. Meanwhile, the value of the index is between 0 and 1, namely, nonelastic, which implies we can reach the emission control target at a relatively low price of profit loss. That is obviously acceptable both for the supply chain and public administration. As for the scenario of carbon tax policy, we find that the CEEP is elastic, which in turn indicates it should not be prior to mandatory

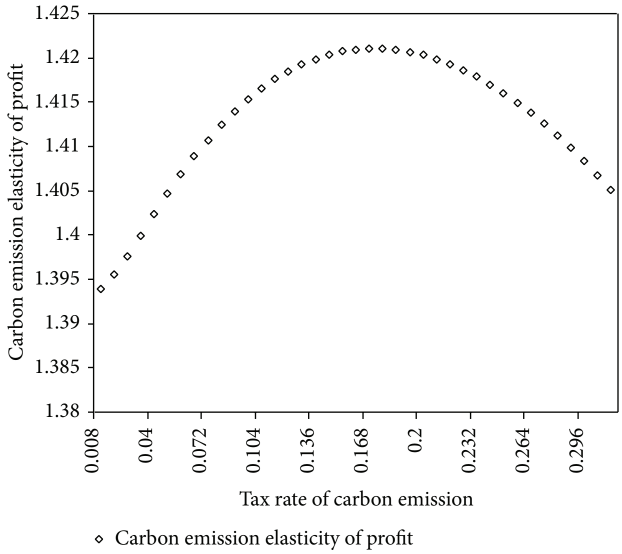

FIGURE 8: Carbon emission elasticity of profit under different carbon tax.

emission cap policy to be adopted and preferred by industry and government.

There may be some extension in the future. For example, one can consider the multiple-horizon production planning problem in the same supply chain structure under low-carbon constraints. One can also add the pricing decision into the consideration.

Appendix

Proof. The objective function of the supply chain

$$\max_{q_i^L \geq 0} \prod = E(\pi) = \sum_{i=1}^{n} \left\{ (p_i + s_i) \cdot q_i^L - (p_i + h_i + s_i) \int_0^{q_i^L} F(X_i)\, dX_i - K_i - s_i \mu_i - v_i \cdot \sum_{j=1}^{n} d_{ij} q_j^L \right\} \quad \text{(A.1)}$$

can be written as

$$\min_{q_i^L \geq 0, i \in N} J(Q_L) = \sum_{i=1}^{n} \left\{ -(p_i + s_i) \cdot q_i^L + (p_i + h_i + s_i) \cdot \int_0^{q_i^L} F_i(x)\, dx + s_i \mu_i + K_i + v_i \cdot \sum_{j=1}^{n} d_{ij} q_j^L \right\}. \quad \text{(A.2)}$$

In order to prove that the target function $\prod$ is the concave function, this needs to prove that function $J(Q_L)$ is convex function.

Construct a function $\phi(x) = \int_0^x F(t)dt$, its second derivative $\phi''(x) = f(x) > 0$; thus $\phi(x)$ is a convex function, to its domain for all $x_1, x_2 > 0$, we have $\phi(\lambda x_1 + \bar{\lambda} x_2) \leq \lambda\phi(x_1) + \bar{\lambda}\phi(x_2)$, so $\int_0^{\lambda x_1 + \bar{\lambda} x_2} F(t)dt \leq \lambda \int_0^{x_1} F(t)dt + \bar{\lambda} \int_0^{x_2} F(t)dt$, where $0 \leq \lambda \leq 1$, $\bar{\lambda} = 1 - \lambda$。So we get $\int_0^{\lambda q_1^i + \bar{\lambda} q_2^i} F_i(x)dx \leq \lambda \int_0^{q_1^i} F_i(x)dx + \bar{\lambda} \int_0^{q_2^i} F_i(x)dx$。

Thus

$$\begin{aligned}
& J\left(\lambda Q_1^L + \bar{\lambda} Q_2^L\right) - \left[\lambda J\left(Q_1^L\right) + \bar{\lambda} J\left(Q_2^L\right)\right] \\
&= \sum_{i=1}^{n} \left\{ -(p_i + s_i) \cdot \left(\lambda q_1^i + \bar{\lambda} q_2^i\right) + (p_i + h_i + s_i) \cdot \int_0^{(\lambda q_1^i + \bar{\lambda} q_2^i)} F_i(x)\, dx + s_i \mu_i + K_i + v_i \cdot \sum_{j=1}^{n} d_{ij} \left(\lambda q_1^j + \bar{\lambda} q_2^j\right) \right\} \\
&\quad - \lambda \sum_{i=1}^{n} \left\{ -(p_i + s_i) \cdot q_1^i + (p_i + h_i + s_i) \cdot \int_0^{q_1^i} F_i(x)\, dx + s_i \mu_i + K_i + v_i \cdot \sum_{j=1}^{n} d_{ij} q_1^j \right\} \\
&\quad - \bar{\lambda} \sum_{i=1}^{n} \left\{ -(p_i + s_i) \cdot q_2^i + (p_i + h_i + s_i) \cdot \int_0^{q_2^i} F_i(x)\, dx + s_i \mu_i + K_i + v_i \cdot \sum_{j=1}^{n} d_{ij} q_2^j \right\} \\
&= \sum_{i=1}^{n} (p_i + h_i + s_i) \left[\int_0^{\lambda q_1^i + \bar{\lambda} q_2^i} F_i(x)\, dx - \lambda \int_0^{q_1^i} F_i(x)\, dx - \bar{\lambda} \int_0^{q_2^i} F_i(x)\, dx \right].
\end{aligned} \quad \text{(A.3)}$$

In this formula, $(p_i + h_i + s_i) > 0$, and we can calculate from the formula and get $\int_0^{\lambda q_1^i + \bar{\lambda} q_2^i} F_i(x)dx - \lambda \int_0^{q_1^i} F_i(x)dx - \bar{\lambda} \int_0^{q_2^i} F_i(x)dx \leq 0$. So $J(\lambda Q_1^L + \bar{\lambda} Q_2^L) - [\lambda J(Q_1^L) + \bar{\lambda} J(Q_2^L)] < 0$; that means $J(Q_L)$ is a convex function. Therefore $\prod$ is a concave function; then there is a global optimum point. □

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Research Article

Game Theoretic Analysis of Carbon Emission Reduction and Sales Promotion in Dyadic Supply Chain in Presence of Consumers' Low-Carbon Awareness

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The paper studies how the combination of the manufacturer's carbon emission reduction and the retailer's emission reduction relevant promotion impacts the performances of a dyadic supply chain in low-carbon environment. We consider three typical scenarios, that is, centralized and decentralized without or with side-payment. We compare measures of supply chain performances, such as profitabilities, emission reduction efficiencies, and effectiveness, in these scenarios. To improve chain-wide performances, a new side-payment contract is designed to coordinate the supply chain and numerical experiments are also conducted. We find the following. (1) In decentralized setting, the retailer will provide emission cutting allowance to the manufacturer only if their unit product profit margin is higher enough than the manufacturer's, and the emission reduction level of per unit product is a monotonically increasing function with respect to the cost pooling proportion provided by the retailer; (2) the new side-payment contract can coordinate the dyadic supply chain successfully due to its integrating sales promotion effort and emission reduction input, which results in system pareto optimality under decentralized individual rationality but achieves a collective rationality effect in the centralized setting; (3) when without external force's regulation, consumers' low-carbon awareness is to enhance consumers' utility and decrease profits of supply chain firms.

1. Introduction

Coming with the emerging low-carbon economy and environmental issues over the world, consumers' awareness of low-carbon products gradually arises. Consequently, the product and services having low-carbon features are anticipated to gain more evaluation value by customers. For example, there are some projects that succeed in influencing people's behaviors by using economic instruments, such as incentives, discounts, education, and information sharing; see Geller et al. [1].

To help consumers identify the carbon footprint of products (including carbon emissions of products from manufacturing with raw materials to disposing the finished products) [2], more and more enterprises begin to use carbon labels to track carbon footprint message of products. The adoption of carbon label increases the transparency of the carbon footprint. Therefore, the carbon emission of the product affects the value and utility perceived by consumers directly and thus affects the market demand for the product. Under the influence of awareness of environment, consumers are even willing to pay a higher price for environmentally friendly products than for common products [3]. Affected by this, many large international companies have begun to emphasis their image of emissions reduction. They publish annual environmental and social responsibility reports regularly (such as Toyota Kirloskar Motor and TOTAL) and set their reduction targets. In short, as carbon emission is becoming one of the significant factors affecting product demand increasingly, carbon cutting has become one of the major issues for most firms.

Meanwhile, in many industries, manufacturers are facing growing challenges from retailers; see Inderst and Wey [4]. In more and more industries, it is the retailer, such as

Wal-Mart, Tesco, and Home Depot, that determines the manufacturing and marketing of products [5]. Retailers in the strong position tend to use their "bargaining power" to take all kinds of unequal trade with suppliers, in order to achieve their aim of delivering the cost and risk, and their actions will cause the whole supply chain to deviate from Pareto optimality [6]. However, enterprises in a supply chain should cooperate while compete. The upstream and the downstream of the supply chain can work together to enhance the product demand, for example, the manufacture focuses on emission reduction while the retailer engages in marketing and promoting related to the products' low-carbon feature. If so, both the manufacturer and the retailer are able to maximize their profits.

Based on the above reasons, we focus on the game between a manufacturer and a retailer in the supply chain considering the impact of carbon cutting and promotion related to the product emission reduction on product demand, under the situation that consumers have the awareness of low carbon. We analyze the decisions of the manufacturer and retailer aiming to maximize their profits. Also, we explore the solution to optimizing the system profit using a new kind of side-payment contract when the manufacturer and the retailer make decisions individually. The new contract can not only increase the profit of the manufacturer and the retailer, but also achieve the rationality and fairness in allocating the incremental profit of the supply chain system.

The studies closely related with our paper focus on supply chain coordination contract designing, corporate decision-making, and supply chain operation under low-carbon environment.

There are many previous studies on supply chain coordination contract design. These contracts include buyback contract, revenue sharing contract, quantity discount contract, contract sales rebates, wholesale price contracts, price discounts contracts, and so forth. To find more analytical and detailed commentary on these contracts as well as a detailed review of these studies, readers can refer to Cachon [7] and Leng and Zhu [8]. In addition, Luo and Zhang [9] study a decentralized supply chain with a single supplier and two retailers improving the effectiveness of the supply chain through the side-payment contract. Tang et al. [10] propose a linear side-payment mechanism, so that retailers offer the best promotional level to achieve supply chain coordination.

From early on, people have begun to discuss the impact of carbon emissions on global warming, but companies pay attention to operation under carbon emissions constraints much later [11]. Under carbon emissions constraints, companies tend to adopt more energy-efficient facilities and equipment or transportation facility to reduce emissions. In fact, the same goal could be gained by adjusting the way of operating, transport, and inventory. Even companies can cut more carbon emission with lower cost by this way [12]. But it did not attract enough attention in the academic and private sectors. The studies of business operations under a low-carbon environmental decision-making which are still tiny at this point include the following aspects.

Firstly, some researchers focus on optimizing the transportation mode of supply chain in order to reduce carbon emissions under low-carbon environment. Transportation is a significant source of carbon footprint in supply chain [13]. Hoen et al. made important research on supply chain transportation model selection. Their research showed that in an established network, adjusting transportation model can reduce carbon emissions greatly [14], but the actual decision depends on the carbon related regulations and other practical issues [15].

Secondly, many researchers focus on supply chain network design in low-carbon environment. Companies are different in pressure, marginal cost, and marginal profit of carbon emission reduction because they are in different regions or different in technological and management ability. Those make it possible for companies to optimize the carbon footprint of product by supply chain network design. The optimization of supply chain network is helpful for cutting emission and saving cost at the same time [16]. Considering carbon trading regulation, Ramudhin et al. [17] and Diabat and Simchi-Levi [18] introduced a mixed integer mathematical model for companies to reveal an optimal strategy to meet their carbon cap. Cachon [19] observed that not only the carbon emissions constrain to retailers but also the cost of consumers should be considered in supply chain network design because the network design would affect not only the cost of retailers but also the cost of consumers.

Thirdly, supply chain coordination considering low-carbon regulations is an important topic. Product carbon footprint is the total emission across its life cycle; it is difficult for enterprises to optimize carbon footprint in supply chain on their own, so cooperation between the upstream and the downstream is essential for carbon emission reduction [20].

In order to minimize the product carbon footprint, it must be decomposed into processes of the supply chain firstly. Caro et al. [21] introduced an effective model to solve this problem.

In the field of supply chain coordination, Du et al. [22] investigated the behavior and decision-making of each member in the emission dependent supply chain with an emission permit supplier and an emission dependent firm considering the "cap-and-trade" regulation. Xia et al. [23] analyzed the joint carbon emission reduction between the manufacturer and the supplier in considering "cap-and-trade" system and introduced transfer payment contract to optimize carbon cutting and the supply chain profit. Yang et al. [24] compared the impact of two carbon regulations, mandatory emission reduction, and carbon tax on carbon emission and the cost of supplier, retailer, and supply chain system by means of system dynamics.

In this study, being different from existing research, we take consumers' low-carbon awareness into consideration and consider emissions reduction and promotion about low-carbon product as the factors affecting the demand function to study promotion decisions of retailers and the carbon emission reduction decisions of manufacturers. Besides, we established a new side-payment contract, with that both sides in the game can achieve the collective optimality when making individual's decisions. The contract design can not only meet demand for increasing revenue but also solve rationality and fairness issues of allocating incremental revenue in the system and then achieve the supply chain coordination.

The rest of the paper is organized as follows. In Section 2, we describe the problem and give out the general model on this problem. In Section 3, centralized decision model is constructed to be benchmarked for afterwards settings. In Sections 4 and 5, we develop the decentralized model via Stackelberg game without or with side-payment. In Section 6, the side-payment contract is used to coordinate the supply chain compared to the centralization scenario. In Section 7, we conduct numerical study to examine models in afore sections. In Section 8, this study comes to the end by summarizing main insights and future extensions.

2. Problem Formulation and General Model

This paper focuses on a dyadic supply chain composed of a manufacturer as follower and a retailer as leader in Stackelberg games. Considering consumer's low-carbon awareness, the manufacturer engaged in carbon emissions reduction (deciding the volume of reduction of per unit product), and the retailer focuses on product promoting related to its low-carbon feature (deciding the promotion level). Also, the retailer decides the distribution of emission reduction cost between manufacturer and retailer (showed as Figure 1).

To simplify the study and without loss of generality, this paper is based on the following assumptions: (1) only one product is involved in the supply chain; (2) stock and inventory backlog is not considered; (3) the retailers and manufacturers earn a fixed profit per unit product (before deducting the investment of promotion and emission reduction); (4) initial emission of per unit of product is 1, so the emission reduction of per unit product is less than 1; the promotion level is also less than 1.

Some parameters involved are given as follows: DS, DC, C, D, and DT denote Stackelberg game model with side-payment, Stackelberg game model without side-payments, centralized decision-making model, the initial dynamic game model, and side-payment self-executing contract model, respectively.

Decision Variables

q: Retailer's promotion level, $0 \leq q < 1$

θ: The proportion of emission reduction cost the retailer takes, $0 \leq \theta < 1$

e: The emission reduction of per unit product, $0 \leq e < 1$.

Parameters

a: Market size without considering carbon emission reduction and promotion, $a > 0$

D: Product demand

b: Constant coefficient, $b > 0$

ρ_m: Manufacturer's profit of per unit product without considering carbon cutting cost, $\rho_m > 0$

ρ_r: Retailer's profit of per unit product without considering promotion cost, $\rho_r > 0$

γ: The coefficient of carbon emission's effect on demand, $\gamma > 0$

δ: The coefficient of promotion's effect on demand, $\delta > 0$

Π_m: The profit of manufacturer

Π_r: The profit of retailer

Π: The profit of the supply chain system.

In the case of the current growing concern about carbon emissions, it has grown up to be an effective mean for enterprises to improve product demand by reducing carbon emissions and promoting products according to its carbon trait.

Since carbon emission of product affects consumer's utility directly, companies can enhance the market demand of product by cutting carbon emission. Laroche et al. [25] figured out that the more consumers cared about environment, the higher price consumers would be willing to pay for green products. Plambeck [26] also indicated that companies could increase product market share and consumer confidence by disclosing the information about carbon emission. Therefore, in a low-carbon environment, in addition to providing quality products and competitive price, reducing carbon emissions and disclosing information related to carbon are also new strategies for companies to increase market demand.

Retailer's sale promotion related to low-carbon trait of product can improve product marketing demands too. Liu et al. [3] pointed out that the main benefit increment of company selling environmentally friendly production resulted from the product demand increment caused by consumer's environmental awareness, and eco-friendly companies should adopt some measures in marketing process to improve consumer's environmental awareness and turn nongreen consumers to green consumers. In practice, large retail stores often adopt advertising and promotion related to the product's low-carbon feature to shape consumer's value and influence the consumption action [27]. Taylor [28] demonstrates that the retailer's promotional effort is one of the crucial factors affecting market demand.

Product demand increases with carbon emission reduction, but the demand increment decreases with emission reduction. It is same as the retailer's promotional effort. Learning from the research of Geylani et al. [5], Yue et al. [29], and Szmerekovsky and Zhang [30], this paper expresses the demand function as $D = a' - b'p - be^{-\gamma}q^{-\delta}$; p represents the sales price. The improvement of consumer's low-carbon awareness is a gradual process since it is affected by consumer cognition. Currently, consumer's low-carbon awareness is not very high. So it is reasonable to assume that the companies' aim of emission reduction and related promotion is not to enhance the sale price and gain higher average profit but to increase market demand. In other words, the sale price and average profit of the low-carbon product are exogenous. Therefore, $p = c_m + c_r + \rho_m + \rho_r$, c_m and c_r denote the cost of per unit product for manufacturer and retailer, respectively.

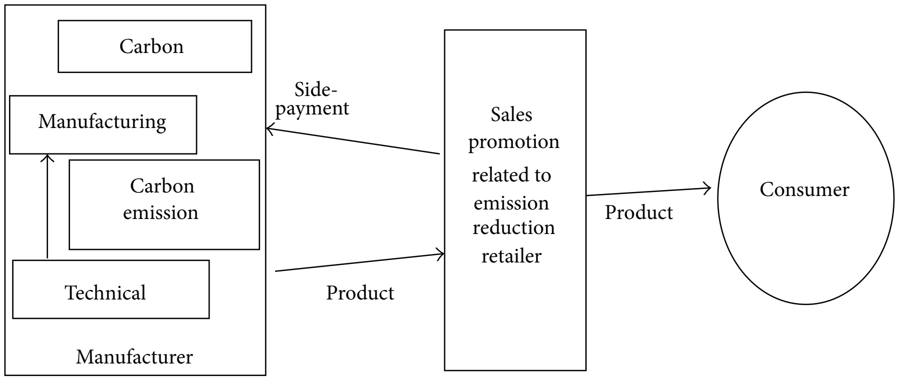

Figure 1: Manufacturer's emission reduction, retailer- promoting cooperation, and competition.

Let $a = a' - b'(c_m + c_r + \rho_m + \rho_r)$; the demand function of product could be expressed as

$$D = a - be^{-\gamma}q^{-\delta}. \quad (1)$$

The costs of carbon emissions reduction and promotion are convex in the emissions reduction of per unit product and promotion level, respectively [31]. So the cost of carbon emissions reduction ($C(e)$) and the cost of promotion ($C(q)$) can be expressed as $(1/2)u_m e^2$ and $(1/2)u_r q^2$, respectively.

3. Centralized Decision-Making Model (C)

The double marginal effect resulted from decentralized decision-making in supply chain usually results in a suboptimal profit of the system. If there is a stimulus conditions or a binding agreement which could make the participators make decision centrally and get more profits, centralized decision-making is possible. Under the situation of centralized decision-making, a system integrator replaces two game participants to make decisions to optimize the total profit of the system. The objective function could be expressed as

$$\max_{0\le e<1,\,0\le q<1} \Pi^C = (\rho_m + \rho_r)\left(a - be^{-\gamma}q^{-\delta}\right) - \frac{1}{2}u_m e^2 - \frac{1}{2}u_r q^2. \quad (2)$$

Since the Hessian matrix is negative definite, the optimal promotion levels and reductions are as follows:

$$q^C = \left[\frac{\delta b(\rho_m + \rho_r)}{u_r}\right]^{(\gamma+2)/(2\delta+2\gamma+4)} \times \left[\frac{\gamma b(\rho_m + \rho_r)}{u_m}\right]^{-\gamma/(2\delta+2\gamma+4)},$$
$$e^C = \left[\frac{\delta b(\rho_m + \rho_r)}{u_r}\right]^{-\delta/(2\delta+2\gamma+4)} \times \left[\frac{\gamma b(\rho_m + \rho_r)}{u_m}\right]^{(\delta+2)/(2\delta+2\gamma+4)}. \quad (3)$$

4. Stackelberg Game without Side-Payments (DC)

In the Stackelberg game between a manufacturer and a leading retailer, the retailer decides the level of sale promotion firstly and the manufacturer determines the emission reduction of per unit product secondly. Objective functions of manufacturer and retailer are shown as follows:

$$\max_{0\le e<1} \Pi_m^{DC} = \rho_m\left(a - be^{-\gamma}q^{-\delta}\right) - \frac{1}{2}u_m e^2, \quad (4)$$

$$\max_{0\le q<1} \Pi_r^{DC} = \rho_r\left(a - be^{-\gamma}q^{-\delta}\right) - \frac{1}{2}u_r q^2. \quad (5)$$

Since $\partial^2\Pi_m^{DC}/\partial e^2 = -\gamma(\gamma+1)\rho_m b e^{-\gamma-2}q^{-\delta} - u_m < 0$, Π_m^{DC} is the concave function of e. Let $\partial\Pi_m^{DC}/\partial e = \gamma\rho_m b e^{-\gamma-1}q^{-\delta} - u_m e = 0$; e could be obtained as follows:

$$e = \left(\frac{\gamma\rho_m b}{u_m}\right)^{1/(\gamma+2)} q^{-\delta/(\gamma+2)}. \quad (6)$$

Substitute (6) into (5) and take the derivative of q; thus we obtain

$$\frac{\partial^2\Pi_r^{DC}}{\partial q^2} = -\frac{2\delta(2\delta+\gamma+2)}{(\gamma+2)^2} b\rho_r \left(\frac{\gamma\rho_m b}{u_m}\right)^{-\gamma/(\gamma+2)} \times q^{(-2\gamma-2\delta-4)/(\gamma+2)} - u_r < 0, \quad (7)$$

so Π_r^{DC} is concave in q.

Let $\partial\Pi_r^{DC}/\partial q = (2\delta/(\gamma+2))b\rho_r(\gamma\rho_m b/u_m)^{-\gamma/(\gamma+2)} q^{(-2\delta-\gamma-2)/(\gamma+2)} - u_r q = 0$; the optimal promotion level is obtained as

$$q^{DC} = \left[\frac{2\delta b\rho_r}{u_r(\gamma+2)}\right]^{(\gamma+2)/(2\delta+2\gamma+4)} \left(\frac{\gamma\rho_m b}{u_m}\right)^{-\gamma/(2\delta+2\gamma+4)}. \quad (8)$$

Substitute q^{DC} into (6); the optimal emission reduction could be expressed as

$$e^{DC} = \left[\frac{2\delta b\rho_r}{u_r(\gamma+2)}\right]^{-\delta/(2\delta+2\gamma+4)} \left(\frac{\gamma\rho_m b}{u_m}\right)^{(\delta+2)/(2\delta+2\gamma+4)}. \quad (9)$$

5. Stackelberg Game with Side-Payments (DS)

Product demand increases with carbon emission reduction. It provides opportunity for both the manufacturer and the retailer to enhance their profit while only the cost of manufacturer increases. In the supply chain dominating by retailer, a side-payment contract requiring the retailer sharing part of manufacturer's cost of emission reduction will encourage the manufacturer to improve its emission reduction. For the retailers, as long as cost increment is lower than profits increment caused by emission reduction, they are willing to share the carbon cutting cost. Assume that the proportion of emission reduction cost taken by the retailer is θ; both players still perform Stackelberg game. The retailer determines the promotion level (q) and the proportion of cost allocation (θ); then manufacturer determines the emission reduction of per unit products (e). The objective functions of both players could be expressed as follow:

$$\max_{0\le e<1} \Pi_m^{DS} = \rho_m \left(a - be^{-\gamma} q^{-\delta}\right) - \frac{1}{2}(1-\theta) u_m e^2, \tag{10}$$

$$\max_{\substack{0\le q<1,\\ 0\le\theta<1}} \Pi_r^{DS} = \rho_r \left(a - be^{-\gamma} q^{-\delta}\right) - \frac{1}{2}\theta u_m e^2 - \frac{1}{2} u_r q^2. \tag{11}$$

With backward induction method, the optimal emission reduction level can be obtained as

$$e = \left(\frac{\gamma b \rho_m}{u_m - \theta u_m}\right)^{1/(\gamma+2)} q^{-\delta/(\gamma+2)}. \tag{12}$$

Obviously, $e = (\gamma b\rho_m/u_m)^{1/(\gamma+2)} q^{-\delta/(\gamma+2)}$ is the particular case of $e = (\gamma b\rho_m/(u_m - \theta u_m))^{1/(\gamma+2)} q^{-\delta/(\gamma+2)}$ when $\theta = 0$. We obtain $\partial e/\partial\theta > 0$, $\partial e/\partial\rho_m > 0$, and $\partial e/\partial q < 0$, $\partial e/\partial u_m < 0$ from (6) and (12). So we draw conclusions as follows.

Conclusion 1. In the Stackelberg game, (1) the emission reduction level (e) decreases with the promotional level (q) whether a side-payment exits or not; (2) with a side-payment, the emission reduction level e increases with the cost sharing proportion (θ).

In (6) and (12), e is not the optimal emission reduction level but the emission reduction reaction function about promotion level (q). It is the decision basis of the manufacturer. From (6) and (12), we know that e increases with θ. It means that it is effective for the retailer to encourage the manufacturer to enhance the emission reduction level by sharing its carbon cutting cost. So it is possible for the two players to cooperate in emission reduction. At the same time, the manufacturer's emission reduction level decreases with the promotion level. It means that "free riding" is possible in cooperation. In other words, the manufacturer might cut down its carbon cutting level while the retailer enhances the promotion level. So the retailer should take measures to prevent or alleviate "free riding." Providing side-payment for emission reduction is one of the effective measures.

Substituting (12) into (11), then the optimal cost-sharing proportion θ, promotion level q, and emission reduction level could be obtained as

$$\theta^{DS} = \begin{cases} \dfrac{2\rho_r - \gamma\rho_m - 2\rho_m}{2\rho_r - \gamma\rho_m}, & \text{if } \rho_r > \dfrac{1}{2}(\gamma+2)\rho_m, \\ 0, & \text{if } \rho_r \le \dfrac{1}{2}(\gamma+2)\rho_m, \end{cases}$$

$$q^{DS} = \left[\frac{\gamma b(2\rho_r - \gamma\rho_m)}{2u_m}\right]^{-\gamma/(2\gamma+2\delta+4)} \times \left[\frac{\delta b(2\rho_r - \gamma\rho_m)}{2u_r}\right]^{(\gamma+2)/(2\gamma+2\delta+4)}, \tag{13}$$

$$e^{DS} = \left[\frac{\gamma b(2\rho_r - \gamma\rho_m)}{2u_m}\right]^{(\delta+2)/(2\gamma+2\delta+4)} \times \left[\frac{\delta b(2\rho_r - \gamma\rho_m)}{2u_r}\right]^{-\delta/(2\gamma+2\delta+4)}.$$

Based on the above analysis, we can get Conclusions 2, 3, and 4 shown as follows.

Conclusion 2. In Stackelberg game, the retailer will share the manufacturer's carbon cutting cost only if $\rho_r > (1/2)(\gamma+2)\rho_m$, and the cost allocation increases with ρ_r but decreases with ρ_m.

Conclusion 2 shows that the retailer will share the emission reduction cost only when its profit of per unit product is higher enough than that of the manufacturer. Since $\partial\theta^{DS}/\partial\rho_r > 0$ and $\partial\theta^{DS}/\partial\rho_m < 0$, the higher ρ_r or lower ρ_m is, the more emission reduction cost the retailer will bear. Cost sharing will encourage the manufacture to enhance carbon cutting level and then improve the product demand. So the retailer will share the manufacturer's cost as long as the cost increment resulted from cost sharing is lower than the profit increment resulted from demand improvement. On the other hand, the higher the ρ_r is the more the retailer benefits from product demand increment and the more he is capable of sharing the manufacturer's cost. So the higher ρ_r is, the higher θ is. On the contrary, the higher ρ_m is, the more profit the manufacturer benefits from product demand improvement, so the lower cost-sharing level the retailer takes.

Since $\partial e^{DS}/\partial\rho_m < 0$ and $\partial e^{DS}/\partial\rho_r > 0$, the optimal emission reduction level e^{DS} decreases with the manufacturer's profit of per unit product ρ_m and increases with the retailer's profit of per unit product ρ_r. In order to balance the players' profit and encourage the manufacturer to improve carbon cutting, the retailer needs to share the manufacturer's emission reduction cost; that is, $\theta^{DS} > 0$.

Actually, the retailer will not share the manufacturer's cost if $\rho_r \le (1/2)(\gamma+2)\rho_m$, and the players will perform the Stackelberg game without side-payments, so the optimal emission level and promotion level will be equal as e^{DC} and q^{DC}, respectively.

Conclusion 3. (1) The optimal promotion level increases with ρ_r and u_m but decreases with ρ_m and u_r whether or not side-payments exist. (2) The optimal emission reduction level increases with u_r but decreases with u_m whether or not a side-payments exist. (3) The optimal emission reduction level increases with ρ_m but decreases with ρ_r without side-payments existing, and it is contrary to the case of side-payments existing.

No matter what kind of game is performed, the emission reduction level reaches the optimal value when the manufacturer's marginal cost of carbon emission is equal to its marginal income. In fact, the manufacturer's profit of per unit of product disregarding emission reduction, ρ_m, is the marginal income. Since ρ_m is fixed, the higher u_m is the lower emission reduction level e is. Similarly, the higher the retailer's promotion cost is, the lower promotion level is, and the more important enhancing emission reduction level is for improving product demand. So the optimal promotion level increases with u_m and decreases with u_r.

The higher retailer's profit of per unit product (ρ_r) is, the more profit retailer gains from market demand improvement, and the greater incentive the retailer has to enhance promotion level. The higher the manufacturer's profit of per unit product (ρ_m) is, the greater the retailer relies on the manufacturer to improve market demand. It is same for the manufacturer's optimal emission reduction level in case of Stackelberg game without side-payment. But the higher ρ_r is, the higher θ^{DS} is. It means that the more carbon cutting cost the retailer will take, the less cost the manufacturer will take by itself. In fact, the manufacturer's actual marginal cost of emission reduction decreases. So the optimal emission reduction level will increase. In contrary, the higher ρ_m is, the lower θ^{DS} is, and the lower optimal emission reduction level is.

Conclusion 4. (1) In Stackelberg game, the optimal level of emission reduction with side-payment is higher than that without side-payment; that is, $e^{DS} > e^{DC}$. (2) In the Stackelberg game, the optimal promotion level with side-payment is higher than that without side-payment if $((2\rho_r - \gamma\rho_m)/2)^{2/(2\gamma+2\delta+4)} > (2\rho_r/(\gamma+2))^{(\gamma+2)/(2\delta+2\gamma+4)}\rho_m{}^{-\gamma/(2\delta+2\gamma+4)}$; that is, $q^{DS} > q^{DC}$; otherwise, $q^{DS} \le q^{DC}$. (3) Cost allocation related to emission reduction between the retailer and the manufacturer results in a Pareto improvement of the supply chain profit; that is, $\Pi^{DS} > \Pi^{DC}$, $\Pi_m^{DS} > \Pi_m^{DC}$, and $\Pi_r^{DS} > \Pi_r^{DC}$.

Conclusion 4 is proved in the appendix.

Conclusion 4 shows that the retailer realizes encouraging the manufacturer to improve emission reduction by sharing its cost, but it is determined by the value of parameters whether the promotion level is improved or not. Since $\Pi^{DS} > \Pi^{DC}$, $\Pi_m^{DS} > \Pi_m^{DC}$, and $\Pi_r^{DS} > \Pi_r^{DC}$, the profits of the supply chain system and those of both players are improved by cost allocation between the players. In other words, the retailer would be stimulated to share the manufacturer's cost as long as $\rho_r > (1/2)(\gamma+2)\rho_m$. So great possibility for cooperation related to carbon cutting between the players exists.

6. Supply Chain Coordination Based on Side-Payment Contract Design

The optimal decisions of the players in case of centralized decision-making and Stackelberg equilibrium with a side-payment or not have been analyzed, respectively. The Pareto equilibrium point of the supply chain is achieved as (q^c, e^c) without considering the cost of resource allocation between the manufacturer and the retailer. The corresponding system profit is the benchmark of the supply chain profit. Obviously, this optimal system profit could not be obtained because of double marginalization between the players in case of decentralized decision-making. But centralized decision-making is possible as long as both players could gain more profit than that in case of noncooperation. Of course, two important conditions, participation constraint and incentive compatibility constraint, are necessary for centralized decision-making. Also, reasonable distribution of the profit increment resulting from centralized decision-making is essential.

In the following section, a new side-payment selfexecuting contract (SSEC) is designed to equalize the game equilibrium between the players in case of decentralized decision-making and that in case of centralized decision-making. Actually, SSEC is firstly proposed by He and Zhao [32], to which one can refer for learning more details in inducing and designing processes of SSEC this theory and mechanism method. Then the Pareto optimality of the system is realized and both the participation constraint and the incentive compatibility constraint are met. Assume the profits of manufacturers and retailers without side-payment self-executing contracts are $\Pi_m^D(q,e)$ and $\Pi_r^D(q,e)$, and those with side-payment self-executing contracts are $\Pi_m^{DT}(q,e)$ and $\Pi_r^{DT}(q,e)$, respectively.

For convenience, a nonlinear function of q and e is assumed as $T(q,e) = xq^2 + ye^2$ and the side-payment function is $\widetilde{T}(q,e) = xq^2 + ye^2 + g$. In which, x and y are given as nonnegative constants, and g is a constant. Then the objective functions of manufacturer and retailer could be defined as follows, respectively (the real shift direction of side-payment is decided by sigh of $\widetilde{T}(q,e)$):

$$\max_{0\le e<1} \Pi_m^{DT}(q,e) = \rho_m\left(a - be^{-\gamma}q^{-\delta}\right) - \frac{1}{2}u_m e^2 + T(q,e) + g,$$

$$\max_{0\le q<1} \Pi_r^{DT}(q,e) = \rho_r\left(a - be^{-\gamma}q^{-\delta}\right) - \frac{1}{2}u_r q^2 - T(q,e) - g. \tag{14}$$

Both $\Pi_m^{DT}(q,e)$ and $\Pi_r^{DT}(q,e)$ could be proved as the concave functions of e and q. A Stackelberg game is played between the retailer and the manufacturer. The retailer as the leader makes decision firstly; then the manufacturer decides the emission reduction level. In this case, the optimal decisions of the players could be obtained as follows:

$$q^{DT} = \left(\frac{\gamma b\rho_m}{u_m - 2y}\right)^{-\gamma/(2\gamma+2\delta+4)} \times \left[\frac{2\delta b\left(\gamma y\rho_m + u_m\rho_r - 2y\rho_r\right)}{(\gamma+2)\left(u_m - 2y\right)\left(u_r - 2x\right)}\right]^{(\gamma+2)/(2\gamma+2\delta+4)},$$

$$e^{DT} = \left(\frac{\gamma b \rho_m}{u_m - 2y}\right)^{(\delta+2)/(2\gamma+2\delta+4)} \times \left[\frac{2\delta b\left(\gamma y \rho_m + u_m \rho_r - 2y\rho_r\right)}{(\gamma+2)\left(u_m - 2y\right)\left(u_r - 2x\right)}\right]^{-\delta/(2\gamma+2\delta+4)}. \quad (15)$$

If the participation constraint and the incentive compatibility constraint are satisfied, the decision-making in Stackelberg game with the new side-payment self-executing contract will be the same as that in centralized decision-making. The participation constraint ensures that both of the players are willing to participate in the cooperation. Incentive compatibility constraint makes sure that the decision-making according to individual rationality equals to the optimal Pareto equilibrium.

Participation Constraint. $\Pi_m^{DT}(q,e) \geq \Pi_m^D(q,e)$; $\Pi_r^{DT}(q,e) \geq \Pi_r^D(q,e)$. Both the manufacturer and the retailer are able to gain more profit in the case of side-payment self-executing contract than that before this contract is adopt.

Incentive Compatibility Constraint. $(q^{DT}, e^{DT}) = (q^C, e^C)$. The game equilibrium in case of side-payment self-executing contracts is the same as that in case of centralized decision-making.

To demonstrate the side-payment function, the value of x, y, and g is required. According to incentive compatibility constraint, and setting $q^{DT} = q^C$, $e^{DT} = e^C$, we can obtain

$$x = \frac{1}{2}u_r - \frac{u_r \rho_r \left(\gamma \rho_m + \rho_m - \rho_r\right)}{(\gamma+2)\left(\rho_m{}^2 - \rho_r{}^2\right)}, \quad y = \frac{u_m \rho_r}{\rho_m + \rho_r}. \quad (16)$$

Thus $\widetilde{T}(q,e) = [(1/2)u_r - u_r\rho_r(\gamma\rho_m + \rho_m - \rho_r)/(\gamma + 2)(\rho_m{}^2 - \rho_r{}^2)]q^2 + (u_m\rho_r/(\rho_m + \rho_r))e^2 + g$. Then, $\widetilde{T}(q,e)$ could be determined as long as the value of g is determined.

Much of the existing literature adopts Nash bargaining theory to distribute the incremental profit of the system between the players in perfect information static game. Nash bargaining theory fits for distributing profit in cooperative game. And a binding agreement is necessary for cooperative game. In fact, since the power structure of the supply chain is unbalanced, the dominant player would not be willing to participate in cooperative game. So the Nash bargaining solution is not always realistic. On the other hand, frequently trades between the upstream and the downstream of the supply chain are similar to the multiperiod bargaining game model. So it is reasonable to adopt Rubinstein bargaining game to simulate the process of profit allocation.

Let θ_r and θ_m denote the discount factors of retailer and manufacturer, respectively. And let δ_r and δ_m denote the share of profit increment resulting from cooperation of the retailer and the manufacturer, respectively ($\delta_r + \delta_m = 1$). Rubinstein [33] has proved that there is a unique subgame refining Nash equilibrium in an indefinite duration alternating offers game which can be expressed as $(\delta_r^*, \delta_m^*) = ((1-\theta_r)/(1-\theta_r\theta_m), (\theta_r - \theta_r\theta_m)/(1-\theta_r\theta_m))$. The discount factor usually can be regarded as the game players' patience degree. The larger the discount factor is, the more patient the game participant is and will have greater influence on the game. And the more patient participant will suffer less lost or opportunity cost resulted from time delay than the less patient participant. [i] In this paper,

$$\begin{aligned} \Delta\Pi_m &= \Pi_m^C\left(q^C, e^C\right) + T\left(q^C, e^C\right) + g - \Pi_m^D\left(q^D, e^D\right) \\ &= \delta_m \Delta\Pi, \\ \Delta\Pi_r &= \Pi_r^C\left(q^C, e^C\right) - T\left(q^C, e^C\right) - g - \Pi_r^D\left(q^D, e^D\right) \\ &= \delta_r \Delta\Pi, \\ \Delta\Pi_C &= \Delta\Pi_m + \Delta\Pi_r. \end{aligned} \quad (17)$$

Based on (17), g could be obtained as follows:

$$\begin{aligned} g = {} & \delta_r\left[\Pi_m^C\left(q^C, e^C\right) + T\left(q^C, e^C\right) - \Pi_m^{DS}\left(q^{DS}, e^{DS}\right)\right] \\ & - \delta_m\left[\Pi_r^C\left(q^C, e^C\right) + T\left(q^C, e^C\right) - \Pi_r^{DS}\left(q^{DS}, e^{DS}\right)\right]. \end{aligned} \quad (18)$$

According to the result of game equilibrium above and (18), the value of g can be determined. Thus, the new side-payment self-executing contract can be identified. In decentralized decision-making situation, the players will make their own optimal decisions based on individual rationality. In order to achieve the centralized decision-making system performance in decentralized decision-making set, anyone of the payers could provide the side-payment to the other player to influence their payoff functions.

7. Numerical Analysis

In the following section, numerical analysis is adopted to explore the impact of γ and δ on the decision-making and profit of the retailer and manufacturer. The related parameters are set as follows.

In order to analyze the impact of γ, let $a = 800$, $b = 40$, $\rho_m = 10$, $\rho_r = 25$, $\delta_r = 0.6$, $\delta_m = 0.4$, $u_m = 10000$, $u_r = 12000$, $\delta = 4$, and $1 \leq \gamma \leq 3$.

In order to analyze the impact of δ, let $a = 800$, $b = 40$, $\rho_m = 10$, $\rho_r = 30$, $\delta_r = 0.6$, $\delta_m = 0.4$, $u_m = 10000$, $u_r = 14000$, $\gamma = 2.2$, and $1 \leq \delta \leq 5$.

The result of numerical analysis is shown from Figure 3 to Figure 11. Shown as from Figure 3 to Figure 11, conclusion 1 to conclusion 4 are proved and some important findings are discovered.

(1) In case of decentralized decision the profits of both players increase because of the new side-payment self-executing contract, and the optimal system profit is equal to that in centralized decision set. It is proved that the requirements of participation constraint, incentive compatibility, and justice of profit allocation are met.

(2) As shown in Figure 2, no matter in decentralized decision or in centralized decision, the optimal emission reduction level of the manufacturer increases with γ. The

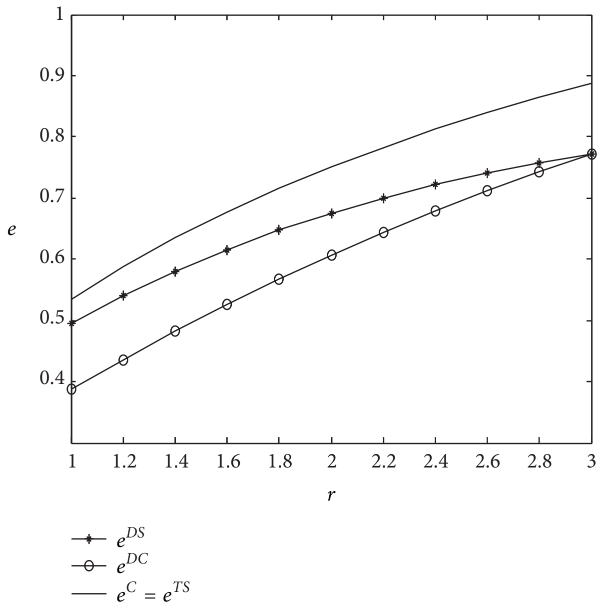

Figure 2: Impact of γ on optimal emission reduction.

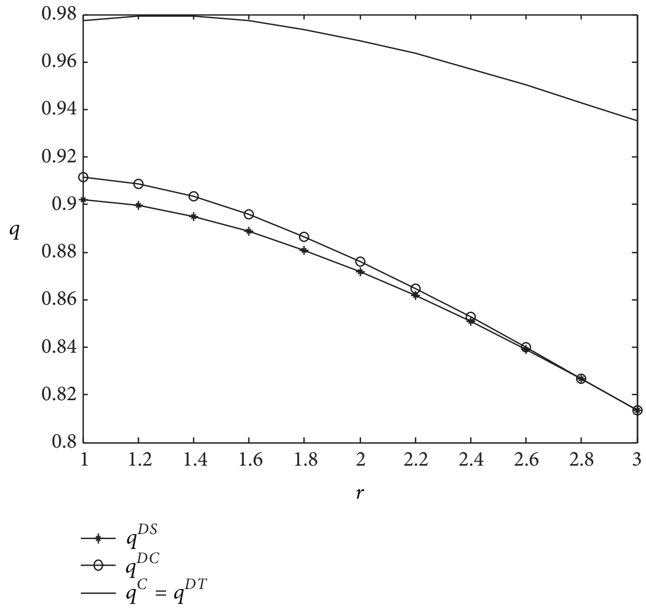

Figure 3: Impact of γ on optimal promotion level.

optimal emission reduction levels in both of the centralized decision and the Stackelberg game with side-payment are higher than those in Stackelberg game without side-payment. From the case without side-payment to which a side-payment contract is executed, the increment of optimal emission reduction level decreases with γ.

The bigger γ is, the greater emission reduction affects product demand and the more effective it is to improve product demand by means of enhancing carbon cutting level. For a rational decision maker, efficiency of profit improving is the only criterion for adopting emission reduction or sale promotion to enhance product demand. So the optimal emission reduction level increases with γ whether in centralized decision or not.

It has been proved that the optimal emission reduction with side-payment is more than that without side-payment in Stackelberg game.

When $\rho_r > (1/2)(\gamma+2)\rho_m$, $\theta^{DS} = (2\rho_r - \gamma\rho_m - 2\rho_m)/(2\rho_r - \gamma\rho_m)$ and $\partial\theta^{DS}/\partial\gamma < 0$. It means that θ^{DS} decreases with γ. In other words, the higher γ is, the lower the proportion of emission reduction cost the retailer will take, and the less effective to encourage the manufacturer to improve emission reduction. Therefore, from the game without a side-payment contract to that with a side-payment contract, the increment of emission reduction decreases with γ until $\rho_r \leq (1/2)(\gamma + 2)\rho_m$ ($\theta^{DS} = 0$).

(3) As shown in Figure 3, the optimal promotion level decreases with γ no matter in case of decentralized decision or in centralized decision.

The bigger γ is, the lower relative importance of promotion is for product demand improvement. So the optimal promotion level decreases with γ.

As shown in Figure 3 and proved previously, the optimal emission reduction level with side-payment is higher than

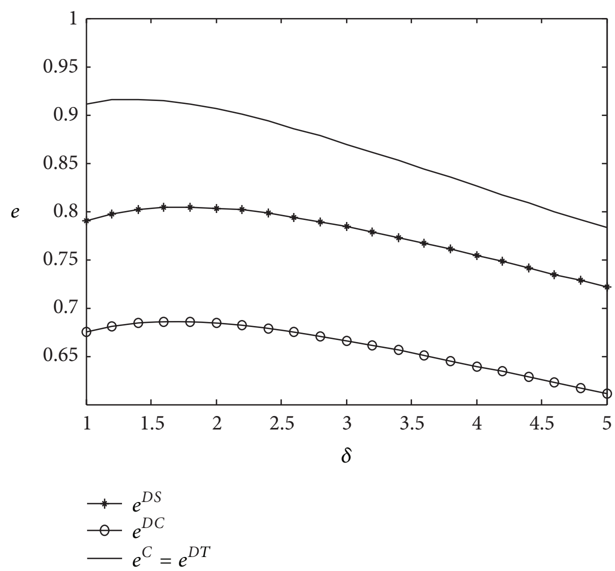

Figure 4: Impact of δ on optimal emission reduction.

that without side-payment if $((2\rho_r - \gamma\rho_m)/2)^{2/(2\gamma+2\delta+4)} \leq (2\rho_r/(\gamma+2))^{(\gamma+2)/(2\delta+2\gamma+4)}\rho_m^{-\gamma/(2\delta+2\gamma+4)}$. Also, an opposite conclusion could be obtained if $((2\rho_r - \gamma\rho_m)/2)^{2/(2\gamma+2\delta+4)} > (2\rho_r/(\gamma+2))^{(\gamma+2)/(2\delta+2\gamma+4)}\rho_m^{-\gamma/(2\delta+2\gamma+4)}$.

At the same time, Figure 3 proves again that the retailer does not share the manufacturer's emission reduction cost if $\rho_r \leq (1/2)(\gamma+2)\rho_m$, and it means that the side-payment is zero.

(4) As shown in Figures 4 and 5, the optimal promotion level of the retailer increases with δ, and the optimal emission reduction level increases first and then decreases with δ

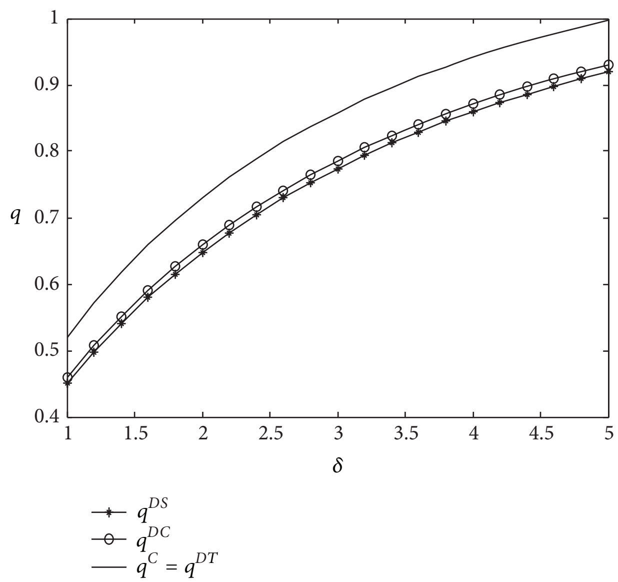

FIGURE 5: Impact of δ on optimal promotion level.

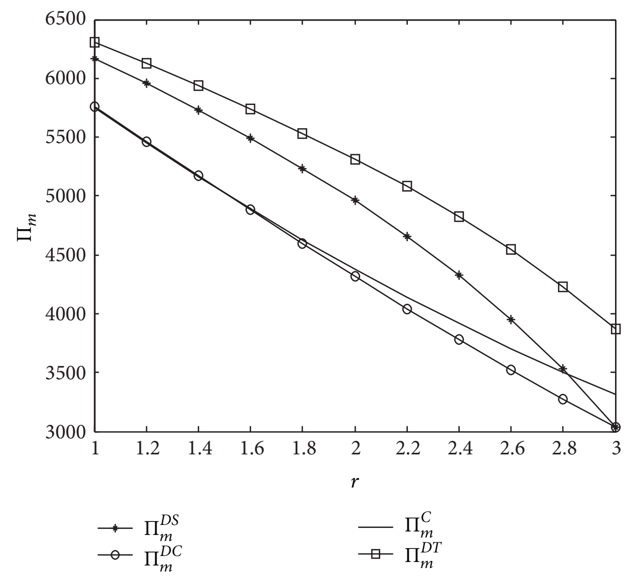

FIGURE 6: Impact of γ on manufacturers profits.

whether in case of decentralized decision or centralized decision. The increment of optimal emission reduction decreases with δ but that of the optimal promotion level increases with δ between different decision-making modes.

Though δ does not affect retailer's payment for the manufacturer's emission reduction cost, it does influence the difference of the optimal emission reduction levels between the game with side-payment and that without side-payment. And the increment of optimal emission reduction decreases with δ but that of the optimal promotion level increases with δ between different decision-making modes. It means that the bigger δ is, the less effective the incentive for encouraging the manufacturer to improve carbon cutting is. So as δ increases, the retailer prefers to invest in promotion but not in sharing the manufacturer's cost. Also the optimal promotion level increases with δ.

(5) As γ increases, the optimal profit of the manufacturer decreases; the optimal profit of the retailer firstly decreases and then increases and the total profit of the supply chain decreases. As δ increases, the optimal profit of the retailer decreases; the optimal profit of the manufacturer decreases firstly and then increases and the total profit of the supply chain decreases.

As γ increases, the optimal emission reduction of the manufacturer increases while its optimal profit decreases. But it does not mean that the manufacturer's profit decreases with the emission reduction level when the exogenous variables, such as γ, are definite. The increasing γ means the improving consumer low-carbon awareness. Companies' carbon cutting stress increases with γ and they have to improve emission reduction to satisfy consumers. But as a result, the marginal cost of emission reduction increases. So the optimal emission reduction level increases with γ, but the optimal profit of the manufacturer decreases with γ. In other words, the manufacturer's optimal emission reduction increases while its optimal profit decreases. It means that the total profit of manufacturer and retailer decreases while the consumer utility increases. That is because the negative externality of carbon emission decreases as emission reduction improves.

As emission reduction increases, the product demand increases and the retailer get more profit without raising promotion level. Since the optimal promotion level decreases with γ and it results in the reduction of product demand, the optimal profit of the retailer decreases first and then increases. In fact, the balance between the increasing emission reduction and the decreasing promotion level is gradual. Similarly, as δ increases, the manufacturer's optimal profit decreases firstly and then increases.

An increasing δ means that promotion's effect on market demand is increasing. In other words, consumer's voice rises in the game between consumer and retailer, so retailer's optimal profit decreases with δ. But when δ is determined, raising promotion level can improve the retailer's profit really.

Then we get to know that as γ and δ increase, consumers' voice rises in the game between them and companies; consumer surplus increases, but the total profit of the supply chain (including manufacturer and retailer) decreases gradually.

(6) As shown from Figure 8 to Figure 11, the optimal profit of the supply chain in centralized decision-making setting is more than that in decentralized decision-making setting. But as shown in Figures 6, 7, 9, and 10, the same conclusion does not always apply to both of the retailer and the manufacture. So, it is not easy for the retailer and the manufacture, when they make a decision according to individual rationality, to achieve the same decision-making effect in centralized decision-making. Then a new contract is necessary for the supply chain to obtain profit when the players make a decision individually as much as that in centralized decision making.

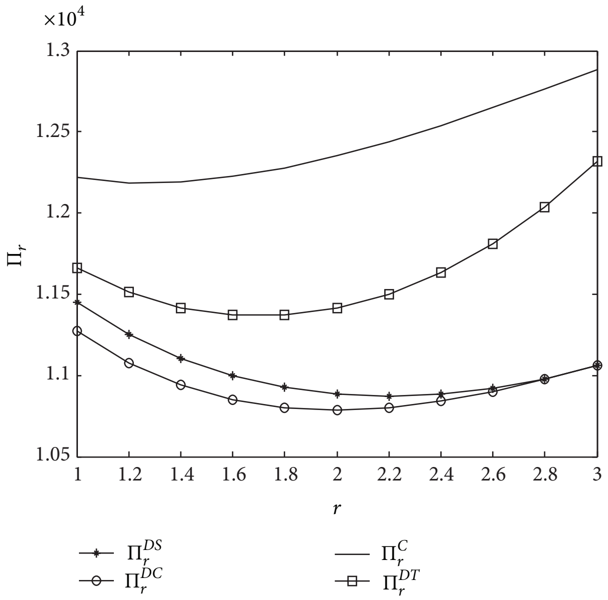

Figure 7: Impact of γ on retailers profits.

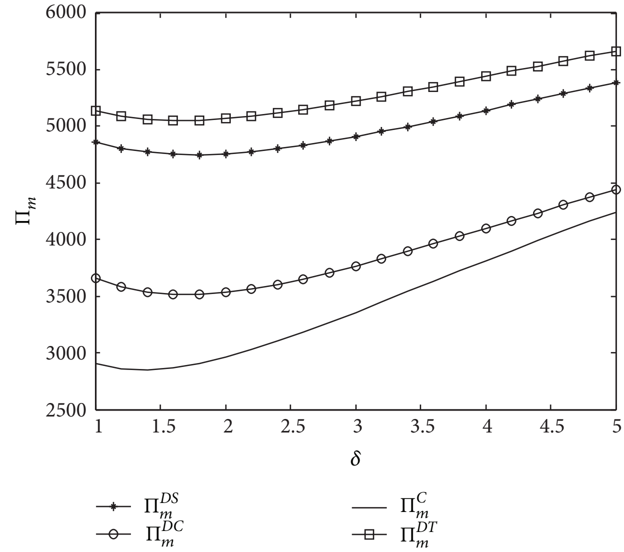

Figure 9: Impact of δ on manufacturers profits.

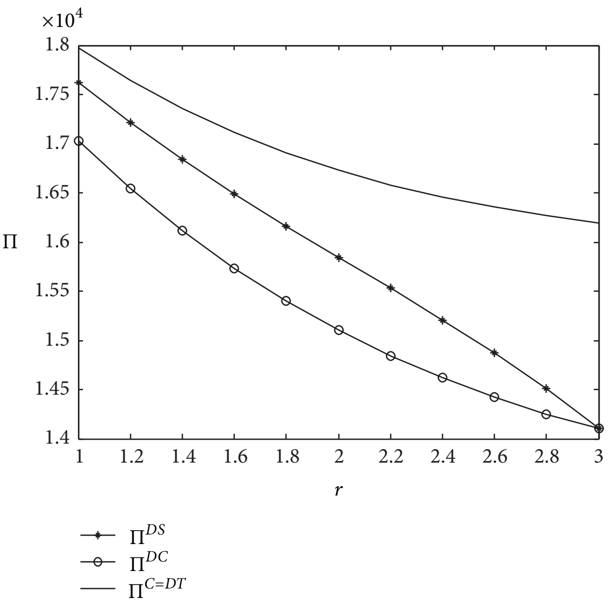

Figure 8: Impact of γ on supply chain profits.

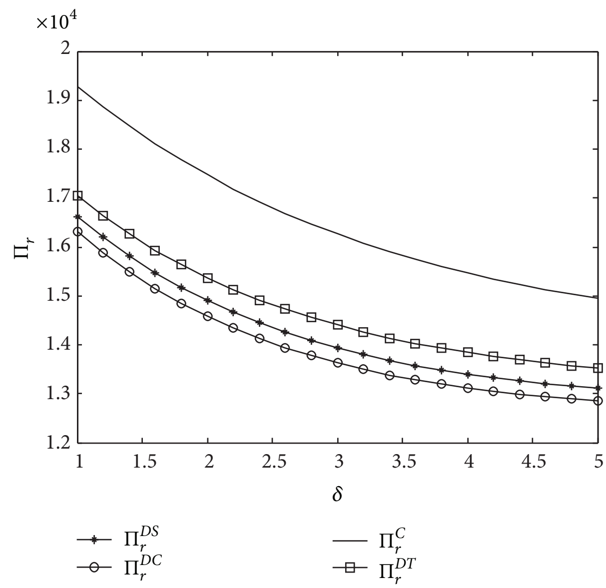

Figure 10: Impact of δ on retailers profits.

8. Conclusion

It is more and more obvious that carbon emission of product affects market demand as consumer's low-carbon consciousness improves. Retailers play the leading roles in many industries and they tend to take advantage of their buyer power to gain more profit. In fact, the retailer and the manufacture can cooperate and compete at the same time as long as cooperation results in profit increase for both of them. One of the approaches to cooperate is that the retailer pays side-payment for the manufacturer's emission reduction. Although it is helpful for improving profits of both of the players, the system profit is not able to achieve Pareto optimality. It is proved that Pareto optimality of the system profit can be gained with the mode of centralized decision, but the allocation of profit must be executed under "fair and reasonable" conditions.

Therefore, this paper analyzes the optimal decisions of the manufacture and the dominant retailer in different game models considering the impact of emission reduction and promotion related to the product's low-carbon feature on product demand. Then, the paper designs a new side-payment self-executing contract to optimize the system profit and resolve the problem of fairness and reasonability in profit allocation. Our study shows that it is effective to encourage the manufacturer to improve emission reduction by means

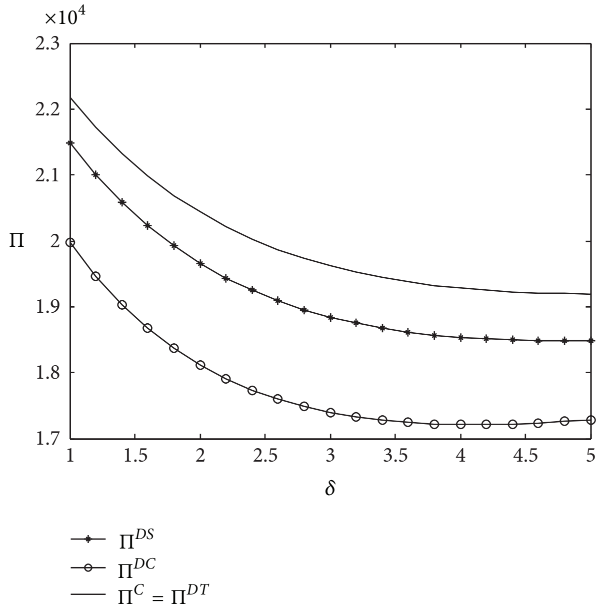

Figure 11: Impact of δ on supply chain profits.

of sharing its cost in carbon cutting. At the same time, the retailer will share the manufacturer's cost only if their marginal profit and consumer's low-carbon consciousness meet certain conditions. Although cost allocation is helpful for improving profits of both of the players, the system profit is not able to achieve Pareto optimality. But the new side-payment self-executing contract is effective in achieving this goal. At the same time, it is an important approach to realize fairness and reasonability in profit allocation. Numerical analysis shows that the optimal profits of companies decrease, the negative external effect of carbon emission decreases, and the consumer utility increases as consumer's low-carbon consciousness increases.

The paper is based on a two-echelon supply chain with deterministic product demand considering only one retailer and one manufacturer. The research on coordination of supply chain network with stochastic product demand considering carbon regulations is the further research direction.

Appendix

(1)

$$\frac{e^{DS}}{e^{DC}} = \left(\left[\frac{\gamma b\left(2\rho_r - \gamma\rho_m\right)}{2u_m}\right]^{(\delta+2)/(2\gamma+2\delta+4)} \times \left[\frac{\delta b\left(2\rho_r - \gamma\rho_m\right)}{2u_r}\right]^{-\delta/(2\gamma+2\delta+4)}\right)$$
$$\times \left(\left[\frac{2\delta b\rho_r}{u_r\left(\gamma+2\right)}\right]^{-\delta/(2\delta+2\gamma+4)} \times \left(\frac{\gamma b\rho_m}{u_m}\right)^{(\delta+2)/(2\delta+2\gamma+4)}\right)^{-1}$$
$$= \left(\frac{2\rho_r - \gamma\rho_m}{2\rho_m}\right)^{(\delta+2)/(2\gamma+2\delta+4)} \times \left(\left[\frac{\left(\gamma+2\right)\left(2\rho_r - \gamma\rho_m\right)}{4\rho_r}\right]^{\delta/(2\gamma+2\delta+4)}\right)^{-1} \tag{A.1}$$

Since $(2\rho_r - \gamma\rho_m)/2\rho_m > 1$, $((2\rho_r - \gamma\rho_m)/2\rho_m)^{(\delta+2)/(2\gamma+2\delta+4)} > ((2\rho_r - \gamma\rho_m)/2\rho_m)^{\delta/(2\gamma+2\delta+4)}$. Consider

$$\frac{e^{DS}}{e^{DC}} = \frac{((2\rho_r - \gamma\rho_m)/2\rho_m)^{(\delta+2)/(2\gamma+2\delta+4)}}{[(\gamma+2)(2\rho_r - \gamma\rho_m)/4\rho_r]^{\delta/(2\gamma+2\delta+4)}}$$
$$\geq \frac{((2\rho_r - \gamma\rho_m)/2\rho_m)^{\delta/(2\gamma+2\delta+4)}}{[(\gamma+2)\left(2\rho_r - \gamma\rho_m\right)/4\rho_r]^{\delta/(2\gamma+2\delta+4)}}$$
$$= \left[\frac{2\rho_r}{\rho_m\left(\gamma+2\right)}\right]^{\delta/(2\gamma+2\delta+4)} \geq 1; \tag{A.2}$$

that is, $e^{DS} > e^{DC}$. Consider

(2)

$$q^{DS} - q^{DC} = \left(\frac{\delta b}{u_r}\right)^{(\gamma+2)/(2\delta+2\gamma+4)} \left(\frac{\gamma b}{u_m}\right)^{-\gamma/(2\delta+2\gamma+4)} \times \left[\left(\frac{2\rho_r - \gamma\rho_m}{2}\right)^{2/(2\gamma+2\delta+4)} - \left(\frac{2\rho_r}{\gamma+2}\right)^{(\gamma+2)/(2\delta+2\gamma+4)} \rho_m{}^{-\gamma/(2\delta+2\gamma+4)}\right]. \tag{A.3}$$

Since

$$\left(\frac{\delta b}{u_r}\right)^{(\gamma+2)/(2\delta+2\gamma+4)} \left(\frac{\gamma b}{u_m}\right)^{-\gamma/(2\delta+2\gamma+4)} > 0, \tag{A.4}$$

$q^{DS} > q^{DC}$ if $((2\rho_r - \gamma\rho_m)/2)^{2/(2\gamma+2\delta+4)} - (2\rho_r/(\gamma+2))^{(\gamma+2)/(2\delta+2\gamma+4)} \rho_m{}^{-\gamma/(2\delta+2\gamma+4)} > 0$; otherwise $q^{DS} \leq q^{DC}$.

(3) In the Stackelberg game with side-payments $(e^{DS}, q^{DS}, \theta^{DS})$ is the unique optimal solution for the retailer. So the retailer's profit in this case is more than the case that the

emission reduction, promotion level, and cost sharing is e^{DC}, q^{DC}, and $\theta = 0$, respectively. Consider

$$\begin{aligned}\max \Pi_r^{DC} &= \rho_r \left[a - b\left(e^{DS}\right)^{-\gamma}\left(q^{DS}\right)^{-\delta} \right] \\ &\quad - \frac{1}{2}\theta^{DS} u_m \left(e^{DS}\right)^2 - \frac{1}{2} u_r \left(q^{DS}\right)^2 \\ &> \rho_r \left[a - b\left(e^{DS'}\right)^{-\gamma}\left(q^{DS'}\right)^{-\delta} \right] - \frac{1}{2} u_r \left(q^{DS'}\right)^2 \\ &= \max \Pi_r^{DC}.\end{aligned} \tag{A.5}$$

For the manufacturer,

$$\begin{aligned}&\max \Pi_m^{DS} - \max \Pi_m^{DC} \\ &\quad = b\rho_m \left[\left(e^{DC}\right)^{-\gamma}\left(q^{DC}\right)^{-\delta} - \left(e^{DS}\right)^{-\gamma}\left(q^{DS}\right)^{-\delta} \right] \\ &\qquad + \frac{1}{2} u_m \left[\left(e^{DC}\right)^2 - \left(1 - \theta^{DS}\right)\left(e^{DS}\right)^2 \right] \\ &\quad = b\rho_m \frac{\gamma + 2}{2} \left(\frac{\delta b}{u_r}\right)^{-2\delta/(2\delta+2\gamma+4)} \left(\frac{\gamma b}{u_m}\right)^{-2\gamma/(2\delta+2\gamma+4)} \\ &\qquad \times \left[\rho_m{}^{-2\gamma/(2\delta+2\gamma+4)} \left(\frac{2\rho_r}{\gamma + 2}\right)^{-2\delta/(2\delta+2\gamma+4)} \right. \\ &\qquad \left. - \left(\frac{2\rho_r - \gamma\rho_m}{2}\right)^{(-2\delta-2\gamma)/(2\gamma+2\delta+4)} \right]\end{aligned} \tag{A.6}$$

since

$$\begin{gathered}\rho_r > \frac{1}{2}(\gamma + 2)\rho_m, \qquad \rho_m < \frac{2\rho_r - \gamma\rho_m}{2}, \\ \frac{2\rho_r}{\gamma + 2} - \frac{2\rho_r - \gamma\rho_m}{2} = \frac{\gamma\left(\gamma\rho_m + 2\rho_m - 2\rho_r\right)}{4 + 2\gamma} < 0,\end{gathered} \tag{A.7}$$

so $\rho_m{}^{-2\gamma/(2\delta+2\gamma+4)}(2\rho_r/(\gamma+2))^{-2\delta/(2\delta+2\gamma+4)} - ((2\rho_r - \gamma\rho_m)/2)^{(-2\delta-2\gamma)/(2\gamma+2\delta+4)} > 0$, $\max \Pi_m^{DS} - \max \Pi_m^{DC} > 0$.

We have $\max \Pi_r^{DS} > \max \Pi_r^{DC}$, $\max \Pi_m^{DS} > \max \Pi_m^{DC}$, and obviously $\max \Pi^{DS} > \max \Pi^{DC}$.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Research Article

Optimal Control of a Make-to-Stock System with Outsourced Production and Price-Sensitive Demand

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We consider a make-to-stock system with controllable demand rate (by varying product selling price) and adjustable service rate (by outsourcing production). With one outsourcing alternative and a choice of either high or low price, the system decides at any point in time whether to produce or even outsource for additional capacity as well as which price to sell the product at. We show in the paper that the optimal control policy is of dynamic threshold type: all decisions are based on the product inventory position which represents the state of the system; there is a state dependent base stock level to decide on production and a higher level on outsourcing; and there is a state dependent threshold which divides the choice of high and low prices.

1. Introduction

A case study of Mattel, the world's largest toy maker, was done by Johnson [1] with a focus on its production capacity management. In particular, Johnson reported that Mattel owned a state-of-the-art die-cast facility in Penang, Malaysia, that was operating at full capacity to produce die-cast toy vehicles. Due to surge of demand for Hot Wheels, a core line of product, Mattel considered several options to expand production capacity, including one through the Vendor Operations Asia Division to outsource production in Asia-Pacific. VOA added flexibility to Mattel's in-house manufacturing capability and was one of the company's most valuable assets. In the meantime Mattel managed demand for the Hot Wheels through a new marketing strategy that changed the assortment mix of cars every two weeks.

In this paper, we consider a single-product make-to-stock system that has the option to increase production capacity by outsourcing to external contract manufacturers. The systems can also manage product demand through adjusting its selling price. For the basic setting of one outsourcing alternative and a choice of either high or low price, the system optimal control problem is to decide at any point in time whether to produce at the in-house facility or to outsource for additional capacity as well as which price to sell the product at. We model the production processes at the in-house and external facilities by exponential times of different means and the demand process by a Poisson process with a price-dependent rate. Thus, mathematically, the problem is optimal control of an $M/M/1$ make-to-stock queue with discretely adjustable production and demand rates.

With the objective to maximize the total discounted profit, we show in the paper that the optimal control policy is of dynamic threshold type: all decisions are based on the product inventory position which represents the state of the system; there is a state dependent base stock level to decide on production and a higher level on outsourcing; and there is a state dependent threshold which divides the choice of high and low prices. Furthermore, we show that, for a given outsourcing production capacity, all three thresholds of the optimal control policy and the associated optimal profit are decreasing in the outsourcing cost. This implies that outsourcing to lower cost facilities will lead to lower inventory holdings and lower selling price but higher profit.

There is a rich literature on the optimal control of $M/M/1$ make-to-stock queues, most of which take demand as exogenous and solve for the optimal control of the production rate. Typically, the optimal policy is of base stock type (produce at the maximum rate when the inventory holding falls below certain level, otherwise halt production), which was first proved by Gavish and Graves [2] and Sobel [3] for the single product and single machine case. Later works of Zheng and Zipkin [4], Wein [5], Veach and Wein [6], and Bertsimas and Paschalidis [7] attempt to extend the base stock policy to the multiple product cases. There are also extensions to the case of single product but with multiple demand classes, like Ha [8–10]. Another direction of extension has been to incorporate more detailed modeling of the production facility. For example, Kapuscinski and Tayur [11] model the production process by a tandem queue, and Feng and Yan [12] and Feng and Xiao [13] deal with unreliable production facilities.

Li [14] and Chen et al. [15, 16] are three works that incorporate controls on both production and demand processes in a make-to-stock queue optimization problem. Li [14] assumes a continuous spectrum of product selling prices and corresponding demand rates and, hence, manages to derive a concave and differentiable profit function in terms of the production rate and the selling price, which yields a qualitative characterization of the optimal policy. Chen et al. [15] allow discrete choices of prices and derive an efficient algorithm to compute the optimal policy as well as its qualitative characterization. Chen et al. [16] consider a make-to-stock manufacturing system with batch production and discrete choices of price and derive the characterization of the optimal control policy. Similar work on a make-to-order queue is done by Ata and Shneorson [17]. Carr and Duenyas [18] study the optimal control of a mixture of make-to-stock and make-to-order queues. Our work adds in another dimension with the outsourcing option to expand production capacity.

The rest of the paper is organized as follows. Section 2 describes precisely the system model and defines an optimization problem that solves for the optimal policy. Section 3 characterizes the optimal threshold policy, proves its global optimality amongst all nonanticipative control policies, and discusses its relationship to the cost of outsourcing. Section 4 briefly discusses the extension to multiple price choices. Section 5 concludes the paper with a summary of the results and possible extensions in the future research.

To streamline presentation of the paper, we state in the main body of the paper all the results without proofs and collect all the proofs in the appendix.

2. Problem Formulation

The make-to-stock system of concern in the paper has an in-house facility with a production rate μ and a unit production cost b. The system can outsource production to an external facility which can produce at a rate a and a per unit cost c. We assume that the existing in-house facility has a lower variable production cost than the external facility, that is, $b < c$, which holds true in the case of Mattel, for example, and is the reason for keeping the in-house facility. We also assume that the production processes at both facilities are random and follow exponential distributions.

The demand process for the product is assumed to be a Poisson process with a price-dependent rate. Specifically, there are two selling prices: high p_1 and low p_2, which correspond to two demand rates: λ_1 and λ_2. We assume that $\lambda_1 < \lambda_2$ and

$$\frac{\lambda_2 p_2 - \lambda_1 p_1}{\lambda_2 - \lambda_1} > b, \tag{1}$$

which indicates that the marginal profit gain from switching price from high to low is greater than the in-house production cost b. Also to ensure system stability, we assume that $\mu + a > \lambda_1$.

When a demand arrives, it is filled from the finished goods inventory if possible; otherwise, it is added to a waiting queue which is served in first-come-first-serve order. The finished goods inventory carries a holding cost of h^+ per unit product per unit time, and the backordering cost for demand unmet at arrival is h^- per waiting demand per unit time. Define the inventory cost function $h(x) = h^+x^+ + h^-x^-$, where $x^+ = \max[x, 0]$ and $x^- = \max[-x, 0]$. We will consider the total discounted profit, assuming a discount factor γ. To ensure that it is more profitable to produce to fill demand than to backlog forever, we assume that $b < c < h^-/\gamma$.

We specify a dynamic control policy for the system by $u = \{\mu(t), a(t), p(t) : t > 0\}$, where $\mu(t) = 0$ or 1 representing in-house production is off or on; similarly, $a(t) = 0$ or 1 representing outsourced production being off or on, and $p(t) = p_1$ or p_2 representing the price charged at time t. A policy u is called *nonanticipatory* if, at all $t > 0$, $\mu(t)$, $a(t)$, and $p(t)$ depend only on information prior to t. Let $\mathscr{U}$ be the collection of all nonanticipatory control policies. Under a given $u \in \mathscr{U}$, denote the total demand sold at price p_i up to time $t > 0$ by $N_i^u(t), i = 1, 2$, the total in-house production by $P_I^u(t)$, and the total outsourced production by $P_O^u(t)$. Then, the product inventory level at time t is given by

$$X^u(t) = x + P_I^U(t) + P_O^u(t) - N_1^u(t) - N_2^u(t), \tag{2}$$

where x is the initial inventory at $t = 0$.

Consequently, the total discounted profit under policy u is

$$V^u(x) = E\left[\int_0^\infty e^{-\gamma t}\left(\sum_{i=1}^{2} p_i dN_i^u(t) - b dP_I^u(t) - c dP_O^u(t) - h(X^u(t))\, dt\right)\right]. \tag{3}$$

A policy $u^* \in \mathscr{U}$ is said to be *optimal* if it solves the following optimization problem:

$$V^{u^*}(x) = \sup_{u\in\mathscr{U}} V^u(x). \tag{4}$$

The optimal solution to this semi-Markov decision problem can be characterized by the following Hamilton-Jacobi-Bellman (HJB) equation (cf. Chapter 7 of [19]):

$$0 = -\gamma V(x) - h(x) + \max_{\alpha=0,\mu} \alpha [V(x+1) - V(x) - b]$$

$$+ \max_{\beta=0,a} \beta \left[V(x+1) - V(x) - c\right] + \max_{j=1,2} \lambda_j \left[V(x-1) - V(x) + p_j\right]. \tag{5}$$

Since $c > b$, we have $V(x+1)-V(x)-c < 0$ when $V(x+1)-V(x)-b < 0$, and thus, $\beta = 0$ when $\alpha = 0$. In essence, we can envisage an effective production process with rates $\mu_0 = 0$, $\mu_1 = \mu$, and $\mu_2 = \mu + a$ corresponding to the unit production costs $b_0 = 0$, $b_1 = b$, and $b_2 = (\mu b + ac)/(\mu + a)$, respectively. As a result, HJB equation (5) can be simplified to

$$\begin{aligned} 0 = & -\gamma V^u(x) - h(x) \\ & + \max_{i=0,1,2} \mu_i \left[V(x+1) - V(x) - b_i\right] \\ & + \max_{j=1,2} \lambda_j \left[V(x-1) - V(x) + p_j\right]. \end{aligned} \tag{6}$$

We are especially interested in a class of control policies which are parameterized by three thresholds: R, D, and S, with $\max\{R, D\} < S$. An (R, D, S) policy decides on production, outsourcing, and pricing in the following manner: (1) when the inventory is above or equal to S, there is no production at the in-house facility and no production outsourcing; (2) when it is below S and above or equal to D, production is on at the in-house facility but there is no outsourcing; (3) when it is below D, production is on at the in-house and outsourced to the external facility; (4) the product sale price is set low at $p(t) = p_2$ when it is above R, and the produce selling price is set low at p_2; otherwise, the price is high at p_1. In Section 3 below, we characterize the best (R, D, S) policy and verify that it satisfies the above HJB equation (6) and, thus, is optimal amongst all policies in $\mathscr{U}$.

3. Optimality of (R, D, S) Policy

The HJB equation (6) can be made more specific when given an (R, D, S) policy. For example, for an (R, D, S) policy with $0 < R < D < S$, it can be simplified to the following equations.

For $x \geq S$,

$$0 = -\gamma V(x) - h^+ x + \lambda_2 \left[V(x-1) - V(x) + p_2\right]; \tag{7}$$

for $D \leq x < S$,

$$\begin{aligned} 0 = & -\gamma V(x) - h^+ x + \lambda_2 \left[V(x-1) - V(x) + p_2\right] \\ & + \mu \left[V(x+1) - V(x) - b\right]; \end{aligned} \tag{8}$$

for $R < x < D$,

$$\begin{aligned} 0 = & -\gamma V(x) - h^+ x + \lambda_2 \left[V(x-1) - V(x) + p_2\right] \\ & + (\mu + a) \left[V(x+1) - V(x) - \frac{\mu b + ac}{\mu + a}\right]; \end{aligned} \tag{9}$$

for $0 \leq x \leq R$,

$$\begin{aligned} 0 = & -\gamma V(x) - h^+ x + \lambda_1 \left[V(x-1) - V(x) + p_1\right] \\ & + (\mu + a) \left[V(x+1) - V(x) - \frac{\mu b + ac}{\mu + a}\right]; \end{aligned} \tag{10}$$

and for $x < 0$,

$$\begin{aligned} 0 = & -\gamma V(x) - h^- x + \lambda_1 \left[V(x-1) - V(x) + p_1\right] \\ & + (\mu + a) \left[V(x+1) - V(x) - \frac{\mu b + ac}{\mu + a}\right]. \end{aligned} \tag{11}$$

Let $V^{(R,D,S)}(x)$ be the profit function of a given (R, D, S) policy. The following lemma characterizes some of its limiting behaviors.

Lemma 1. *The profit function of an (R, D, S) policy has the following limits:*

(1) $\lim_{x\to\pm\infty}[V^{(R,D,S)}(x) - V^{(R,D,S-1)}(x)] = 0$;

(2) $\lim_{x\to\pm\infty}[V^{(R,D,S)}(x) - V^{(R,D-1,S)}(x)] = 0$;

(3) $\lim_{x\to\pm\infty}[V^{(R,D,S)}(x) - V^{(R-1,D,S)}(x)] = 0$;

(4) $\lim_{x\to+\infty}\mathscr{D}V^{(R,D,S)}(x) = -h^+/\gamma$; *and* $\lim_{x\to-\infty}\mathscr{D}V^{(R,D,S)}(x) = h^-/\gamma$.

Our approach is to first find the *best* (R, D, S) policy and then to prove its global optimality.

Definition 2. An (R, D, S) policy is said to be better than an (R', D', S') policy if $V^{(R,D,S)}(x) \geq V^{(R',D',S')}(x)$ for any initial inventory level x and at least for an x, the inequality holds strictly. It is said to be the best (R, D, S) policy if no other (R, D, S) policies are better.

For notational simplicity we also define a first-order difference operator $\mathscr{D}$:

$$\mathscr{D}g(x) = g(x) - g(x-1), \quad \text{for any function } g(x). \tag{12}$$

The following lemma provides means to compare (R, D, S) policies.

Lemma 3. *Comparing with a given (R, D, S) policy, we have the following results:*

(1) *the $(R, D, S+1)$ policy is better if and only if*

$$\mathscr{D}V^{(R,D,S)}(S+1) > b \quad \text{or} \quad \mathscr{D}V^{(R,D,S+1)}(S+1) > b; \tag{13}$$

(2) *the $(R, D+1, S)$ policy is better if and only if*

$$\mathscr{D}V^{(R,D,S)}(D+1) > c \quad \text{or} \quad \mathscr{D}V^{(R,D+1,S)}(D+1) > c; \tag{14}$$

(3) *the $(R+1, D, S)$ policy is better if and only if*

$$\begin{aligned} & \mathscr{D}V^{(R,D,S)}(R+1) > \frac{\lambda_2 p_2 - \lambda_1 p_1}{\lambda_2 - \lambda_1}, \\ \text{or} \quad & \mathscr{D}V^{(R+1,D,S)}(R+1) > \frac{\lambda_2 p_2 - \lambda_1 p_1}{\lambda_2 - \lambda_1}. \end{aligned} \tag{15}$$

The first comparison implies that, under either the (R, D, S) policy or the $(R, D, S+1)$ policy, if the marginal profit at inventory level $x = S$ for producing one more unit of product is higher than the in-house production cost, then the $(R, D, S + 1)$ policy is better than the (R, D, S) policy. Similar implications can be drawn from the other two comparisons.

Lemma 3 leads to the following characterization of the best (R, D, S) policy.

Theorem 4. *If the (R^*, D^*, S^*) policy is the best (R, D, S) policy, then*

(1) *at the best base stock level S^*,*

$$\mathscr{D}V^{(R^*,D^*,S^*)}(S^* + 1) \le b < \mathscr{D}V^{(R^*,D^*,S^*)}(S^*); \tag{16}$$

(2) *at the best outsourcing threshold D^*,*

$$\mathscr{D}V^{(R^*,D^*,S^*)}(D^* + 1) \le c < \mathscr{D}V^{(R^*,D^*,S^*)}(D^*); \tag{17}$$

(3) *at the best price switch threshold R^*,*

$$\mathscr{D}V^{(R^*,D^*,S^*)}(R^* + 1) \le \frac{\lambda_2 p_2 - \lambda_1 p_1}{\lambda_2 - \lambda_1} < \mathscr{D}V^{(R^*,D^*,S^*)}(R^*). \tag{18}$$

Furthermore, we have that $V^{(R^*,D^*,S^*)}(x)$ is concave in x.

Finally, we obtain the main result of the paper based on the characteristics of Theorem 4 as stated above.

Theorem 5. *If the (R^*, D^*, S^*) policy is the best (R, D, S) policy, then its profit function $V^{(R^*,D^*,S^*)}(x)$ satisfies the HJB equation (6). Hence, the (R^*, D^*, S^*) policy is optimal amongst all nonanticipative policies of $\mathscr{U}$.*

The optimal (R^*, D^*, S^*) policy has some important properties which we list below.

Theorem 6. *In the best (R^*, D^*, S^*) policy, $S^* \ge 0$; and $D^* > R^*$ if $(\lambda_2 p_2 - \lambda_1 p_1)/(\lambda_2 - \lambda_1) > c$; otherwise $R^* \ge D^*$.*

Theorem 6 tells us two results as the following. (i) Under the best thresholds (R^*, D^*, S^*) policy, the maximum inventory level S^* is nonnegative; and it is optimal to idle both the in-house and outsourced facilities when the stock level is above the maximum inventory level S^*. (ii) When production is on at both in-house and outsourced facilities and the inventory is increasingly building up, if the marginal profit gain when switching price from high to low is greater than the outsourced production cost c, it is more profitable to first stop outsourced production than to first stop in-house production and then decrease price. The converse is true if the marginal profit gain when switching price from high to low is smaller than the outsourced production cost c.

Theorem 7. *As for fixed sourcing production rate a, suppose that the variable cost c of outsourced production is negotiable. Let $(R^*(c), D^*(c), S^*(c))$ policy be the optimal threshold policy associated with a cost c. Then,*

(1) *the optimal thresholds $R^*(c)$, $S^*(c)$ are piecewise constant, increasing functions of c, but $D^*(c)$ is a piecewise constant, decreasing function of c.*

(2) *as for the optimal profit function, $V^{(R^*(c),D^*(c),S^*(c))}(x; c)$ is decreasing in c.*

Theorem 7 concludes the following results. (i) A lower variable cost from outsourced production will lead to lower product selling price but higher safety stock level and more outsourced production. And (ii) a lower variable outsourced production cost will lead to higher optimal long-term discounted profit.

4. Extension to Multiple Price Choices

In this section, we briefly discuss the extension of the results in the previous section to multiple price choices. Namely, we have now $K \ge 2$ possible prices to choose from for the selling of the product: $p_1 > p_2 > \cdots > p_K > c$, with corresponding K demand arrival rates: $\lambda_1 < \lambda_2 < \cdots < \lambda_K$.

We further assume that the profit rates are also increasingly ordered; that is,

$$\lambda_1 (p_1 - c) < \lambda_2 (p_2 - c) < \cdots < \lambda_K (p_K - c). \tag{19}$$

It can be shown that if the profit rates do not follow this order, a *dominating* subset of the price levels can be chosen to make the other prices unattractive for selection. Readers are referred to Chen, Feng, and Ou [20] for the analysis on dominating prices.

The K price choice model can be optimized by a policy which maximizes the right-hand side of the following Hamilton-Jacobi-Bellman equation:

$$\begin{aligned} 0 = & -h(x) + \max_{\alpha=0,\mu} \alpha [V(x+1) - V(x) - c] \\ & + \max_{\beta=0,a} \mu [V(x+1) - V(x) - c] \\ & + \max_{i=1,\ldots,K} \lambda_i [V(x-1) + p_i - V(x)] - \gamma V(x), \end{aligned} \tag{20}$$

where $V(x)$ is the profit function of the policy.

The natural extension of the (R, D, S) policy as defined at the end of Section 2 is a K-level control policy characterized by $K + 1$ parameters.

Consider $(R_2, \ldots, R_K, D, S)$ with $R_2 < \cdots R_K < S$ and $D < S$. S is the base stock level on and above which there is no production at the in-house facility and also no production outsourcing. And when it is below S and above or equal to D, production is on at the in-house facility but no outsourcing; when it is below D, production is on at the in-house and outsourced to the external facility. The other $K-1$ parameters are price switch thresholds: when the inventory is in the range $(R_i, R_{i+1}]$, $i = 1, \ldots, K$ (assuming $R_1 = -\infty$ and $R_{K+1} = +\infty$), the product is sold at price p_i.

For a given $(R_2, \ldots, R_K, D, S)$ policy, the profit function can be calculated recursively as

$$\begin{aligned} 0 = & -h(x) + \lambda_K [p_K - \mathscr{D}V(x)] - \gamma V(x), \quad x \ge S; \\ 0 = & -h(x) + \mu [\mathscr{D}V(x) - b] \\ & + \lambda_K [p_K - \mathscr{D}V(x)] - \gamma V(x), \quad D \le x < S; \end{aligned}$$

$$0 = -h(x) + \mu[\mathscr{D}V(x) - b] + a[\mathscr{D}V(x) - c] + \lambda_i[p_i - \mathscr{D}V(x)] - \gamma V(x), \quad R_i < x \le R_{i+1}. \tag{21}$$

Qualitatively, the $K+1$-level $(R_2, \ldots, R_K, D, S)$ policies possess the following properties similar to those for the two-price case.

Proposition 8. *Let $V^{(R_2,\ldots,R_K,D,S)}(x)$ be the long run total discounted profit of the K-level $(R_2, \ldots, R_K, D, S)$ policy when the initial inventory is x; then*

(1) *the $(R_2, \ldots, R_K, D, S+1)$ policy is better than the $(R_2, \ldots, R_K, D, S)$ policy if and only if*

$$\mathscr{D}V^{(R_2,\ldots,R_K,D,S)}(S+1) > b \quad \text{or} \quad \mathscr{D}V^{(R_2,\ldots,R_K,D,S+1)}(S+1) > b; \tag{22}$$

(2) *the $(R_2, \ldots, R_K, D+1, S)$ policy is better than the $(R_2, \ldots, R_K, D, S)$ policy if and only if*

$$\mathscr{D}V^{(R_2,\ldots,R_K,D,S)}(D+1) > c \quad \text{or} \quad \mathscr{D}V^{(R_2,\ldots,R_K,D+1,S)}(D+1) > c; \tag{23}$$

(3) *the $(R_2, \ldots, R_K, D, S)$ policy cannot be better than the $(R_2, \ldots, R_K, D, S+1)$ policy if $S < 0$;*

(4) *the $(R_2, \ldots, R_i+1, \ldots, R_K, D, S)$ policy is better than the $(R_2, \ldots, R_i, \ldots, R_K, D, S)$ policy if and only if*

$$\mathscr{D}V^{(R_2,\ldots,R_i,\ldots,R_K,D,S)}(R_i+1) > \frac{\lambda_i p_i - \lambda_{i-1}p_{i-1}}{\lambda_i - \lambda_{i-1}} \tag{24}$$

or

$$\mathscr{D}V^{(R_2,\ldots,R_i+1,\ldots,R_K,D,S)}(R_i+1) > \frac{\lambda_i p_i - \lambda_{i-1}p_{i-1}}{\lambda_i - \lambda_{i-1}}. \tag{25}$$

We also have the following characterization of the best K-level policy.

Theorem 9. *The best K-level $(R_2^*, \ldots, R_K^*, D^*, S^*)$ policy is characterized by the following relations:*

(1) *the profit function $V^{(R_2^*,\ldots,R_K^*,D^*,S^*)}(x)$ is concave in integer value of x;*

(2) *the best base stock level S^* satisfies $S^* \ge 0$ and*

$$\mathscr{D}V^{(R_2^*,\ldots,R_K^*,D^*,S^*)}(S^*+1) \le b < \mathscr{D}V^{(R_2^*,\ldots,R_K^*,D^*,S^*)}(S^*); \tag{26}$$

(3) *at the optimal threshold D^*,*

$$\mathscr{D}V^{(R_2^*,\ldots,R_K^*,D^*,S^*)}(D^*+1) \le c < \mathscr{D}V^{(R_2^*,\ldots,R_K^*,D^*,S^*)}(D^*); \tag{27}$$

(4) *at the best price switch thresholds R_i^*, $i = 2, \ldots, K-1$,*

$$\mathscr{D}V^{(R_2^*,\ldots,R_K^*,D^*,S^*)}(R_i^*+1) \le \frac{\lambda_i p_i - \lambda_{i-1}p_{i-1}}{\lambda_i - \lambda_{i-1}} < \mathscr{D}V^{(R_2^*,\ldots,R_K^*,D^*,S^*)}(R_i^*). \tag{28}$$

The following Theorem 10 characterizes the impact of souring cost c on the optimal discounted profit and its optimal thresholds.

Theorem 10. *As for fixed sourcing production rate a, suppose that the variable cost c of outsourced production is negotiable. Let $(R_2^*(c), \ldots, R_K^*(c), D^*(c), S^*(c))$ policy be the optimal threshold policy associated with a cost c. Then,*

(1) *the optimal thresholds $R_2^*(c), \ldots, R_K^*(c), S^*(c)$ are piecewise constant, increasing functions of c, but $D^*(c)$ is a piecewise constant, decreasing function of c.*

(2) *the optimal profit function $V^{(R_2^*(c),\ldots,R_K^*(c),D^*(c),S^*(c))}(x;c)$ is decreasing in c.*

5. Concluding Remarks

We have analyzed the optimal control of a single-product make-to-stock system that has the option to increase production capacity by outsourcing to external contract manufacturers and the option to vary product selling prices. Idealizing the system by a simple $M/M/1$ make-to-stock queue model with discretely adjustable production and demand rates, we obtain a complete characterization of the optimal threshold control policy and prove beneficial impact of low cost outsourced production to the system. We wish to point out that the results can be easily extended multiple outsourcing alternatives and multiple price choices. It should also be possible to extend to the case of multiple demand classes that compete for the same product. More challenging extensions will be on incorporating fixed cost of outsourcing into the model as well as having multiple product classes in the model.

Appendix

In order to simplify writing, we let

$$\delta_1 = \mu + a + \lambda_1 + \gamma, \qquad \delta_2 = \mu + a + \lambda_2 + \gamma. \tag{A.1}$$

Proof of Lemma 1. First, we attempt to show that $\lim_{x\to+\infty}[V^{(R,D,S)}(x) - V^{(R,D,S)}(x)] = 0$. To this end, we let $\Delta(x) = V^{(R,D,S)}(x) - V^{(R,D,S)}(x)$. By definition, function $\Delta(x)$ can be derived recursively from the recursion of the (R, D, S) policy. In particular, for $x \ge S$,

$$(\lambda_2 + \gamma)\Delta(x) = \lambda_2 \Delta(x-1), \tag{A.2}$$

which leads to

$$\Delta(x) = \frac{\lambda_2+\gamma}{\lambda_2}\Delta(x+1) = \cdots = \left(\frac{\lambda_2+\gamma}{\lambda_2}\right)^n \Delta(x+n),$$
$$\Delta(x) = \left(\frac{\lambda_2}{\lambda_2+\gamma}\right)^{x-S}\Delta(S), \tag{A.3}$$

where $x \geq S$. These equations establish that if $\Delta(S) \geq 0$, then $\Delta(x) \geq 0$ and $\Delta(x) \leq \Delta(x+1)$ for all $x \geq S$, and, further, if $\Delta(S) \leq 0$, then $\Delta(x) \leq 0$ and $\Delta(x) \geq \Delta(x+1)$ for all $x \geq S$. Moreover, a limiting behavior of $\Delta(x)$ is deduced by the fact that

$$\lim_{x\to\infty} \Delta(x) = \lim_{x\to\infty} \left(\frac{\lambda_2}{\lambda_2+\gamma}\right)^{x-S} \Delta(S) = 0. \tag{A.4}$$

That is,

$$\lim_{x\to+\infty} \left[V^{(R,D,S)}(x) - V^{(R,D,S-1)}(x)\right] = 0. \tag{A.5}$$

Next, we are proving a parallel result that

$$\lim_{x\to-\infty} \left[V^{(R,D,S)}(x) - V^{(R,D,S-1)}(x)\right] = 0. \tag{A.6}$$

To this end, we consider $x \leq \min\{R, 0\}$ and define a shift operator T such that $Tf(x) = f(x+1)$ for any function $f(x)$. The inverse of T is expressed formally as $T^{-1}: T^{-1}f(x) = f(x-1)$. Using D we can derive the corresponding characteristic equations for linear recursion (11). Then characteristic equation of recursion (11) is

$$0 = \lambda_1 T^{-1} + (\mu + a) T - \delta_1, \tag{A.7}$$

or

$$0 = \lambda_1 y^{-1} + (\mu + a) y - \delta_1, \tag{A.8}$$

which has two solutions:

$$\begin{aligned} y_1 &= \frac{\delta - \sqrt{\delta_1^2 - 4(\mu+a)\lambda_1}}{2(\mu+a)}, \\ y_2 &= \frac{\delta + \sqrt{\delta_1^2 - 4(\mu+a)\lambda_1}}{2(\mu+a)}. \end{aligned} \tag{A.9}$$

It can be checked that $0 < y_1 < 1 < y_2$.

Then the homogeneous solution to (11) is in the form of

$$\widehat{V}(x) = C_1 y_1^x + C_2 y_2^x, \tag{A.10}$$

where C_1 and C_2 are constants to be determined at $x \to -\infty$. Note that when $x \to -\infty$, y_1^x increases to ∞ exponentially. However, we do not expect $V(x)$ to grow or diminish exponentially when $x \to -\infty$ and, hence, postulate that $C_1 = 0$. To determine a particular solution to (11), we suppose it is $H_1 + H_2 x$ where $H_i, i = 1, 2$, are to be determined from the following equation:

$$\begin{aligned} &-h^- x + (\mu b + ac) - \lambda_1 p_1 \\ &\quad = \left[\lambda_1 T^{-1} + (\mu+a) T - \delta_1\right](H_1 + H_2 x). \end{aligned} \tag{A.11}$$

Comparing the coefficients of constant and linear terms between the two sides yields

$$\begin{aligned} H_2 &= \frac{h^-}{\gamma}, \\ H_1 &= \frac{1}{\gamma^2}\left[\gamma(\lambda_1 p_1 - (\mu b + ac)) + h^-(\mu + a - \lambda_1)\right]. \end{aligned} \tag{A.12}$$

Hence, the general solution to (11) becomes

$$\begin{aligned} V^{(R,S,B)}(x) &= C_2 y_2^x + \frac{h^- x}{\gamma} + \frac{\lambda_1 p_1 - (\mu b + ac)}{\gamma} \\ &\quad + \frac{h^-(\mu + a - \lambda_1)}{\gamma^2}. \end{aligned} \tag{A.13}$$

Denoting by $\widehat{C}_2$ the coefficient of y_2 for $V^{(R,D,S)}(x)$, it is deduced from (A.13) that

$$\Delta(x) = V^{(R,D,S)}(x) - V^{(R,D,S-1)}(x) = \left(C_2 - \widehat{C}_2\right) y_2^x. \tag{A.14}$$

Then we get

$$\lim_{x\to-\infty} \left[V^{(R,D,S)}(x) - V^{(R,D,S-1)}(x)\right] = 0. \tag{A.15}$$

We can prove (2) and (3) by analogy with this logic and therefore omit its proof.

Now we turn to prove $\lim_{x\to+\infty} \mathscr{D}V^{(R,D,S)}(x) = -h^+/\gamma$ and $\lim_{x\to-\infty} \mathscr{D}V^{(R,D,S)}(x) = h^-/\gamma$. Similar to the preceding methodology in the proof of $\lim_{x\to-\infty}[V^{(R,D,S)}(x) - V^{(R,D,S-1)}(x)] = 0$, we can write the general solution of $V^{(R,D,S)}(x)$ for $x > S$ as the following:

$$V^{(R,D,S)}(x) = P\left(\frac{\lambda_2}{\lambda_2+\gamma}\right)^x - \frac{h^+ x}{\gamma} + \frac{\lambda_2 p_2}{\gamma} + \frac{\lambda_2 h^+}{\gamma^2}, \tag{A.16}$$

where P is a constant to be determined at $x \to +\infty$. And for $x < \min\{R, D, 0\}$, we get the profit function $V^{(R,D,S)}(x)$ from (A.13) as follows:

$$\begin{aligned} V^{(R,D,S)}(x) &= C_2 y_2^x + \frac{h^- x}{\gamma} + \frac{\lambda_1 p_1 - (\mu b + ac)}{\gamma} \\ &\quad + \frac{h^-(\mu + a - \lambda_1)}{\gamma^2}. \end{aligned} \tag{A.17}$$

where C_2 is a constant to be determined at $x \to -\infty$ and

$$y_2 = \frac{\delta_1 + \sqrt{\delta_1^2 - 4(\mu+a)(\lambda_1)}}{2(\mu+a)}. \tag{A.18}$$

Combining $\lambda_2/(\lambda_2+\gamma) \in (0,1)$ and $y_2 > 1$, we get

$$\begin{aligned} \lim_{x\to+\infty} \mathscr{D}V^{(R,D,S)}(x) &= -\frac{h^+}{\gamma}, \\ \lim_{x\to-\infty} \mathscr{D}V^{(R,D,S)}(x) &= \frac{h^-}{\gamma}. \end{aligned} \tag{A.19}$$

□

Proof of Lemma 3. For brevity, we provide only the proof that $(R, D, S+1)$ policy is better than (R, D, S) policy if and only if $\mathscr{D}V^{(R,D,S)}(S+1) > b$ or $\mathscr{D}V^{(R,D,S+1)}(S+1) > b$. The proofs of other two essential and sufficient conditions, (2) and (3), are similar, so including them here is not additionally illustrative.

We first prove inequality $\mathscr{D}V^{(R,D,S+1)}(S+1) > b$ implies that $(R, D, S+1)$ policy is better than (R, D, S) policy. For that we focus on the case that $R < D < S$; the other cases can be proved in the same fashion.

Let $\Delta(x) = V^{(R,D,S+1)}(x) - V^{(R,D,S)}(x)$ and make use of (7)–(11) to obtain

$$\begin{aligned} 0 &= -(\lambda_2+\gamma)\Delta(x) + \lambda_2\Delta(x-1), \quad \text{for } x \geq S+1; \\ -\epsilon &= -(\lambda_2+\gamma)\Delta(x) + \lambda_2\Delta(x-1), \quad \text{for } x = S; \\ 0 &= \mu\Delta(x+1) - (\lambda_2+\gamma+\mu)\Delta(x) \\ &\quad + \lambda_2\Delta(x-1), \quad \text{for } D \leq x < S; \\ 0 &= (\mu+a)\Delta(x+1) - \delta_2\Delta(x) \\ &\quad + \lambda_2\Delta(x-1), \quad \text{for } R \leq x < D; \\ 0 &= (\mu+a)\Delta(x+1) - \delta_1\Delta(x) \\ &\quad + \lambda_1\Delta(x-1), \quad \text{for } x \leq R, \end{aligned} \tag{A.20}$$

where $\epsilon = \mu[\mathscr{D}V^{(R,D,S+1)}(S+1) - b]$, $\epsilon > 0$.

We know that $\epsilon > 0$. While in line with Lemma 1, we get that

$$\lim_{x\to-\infty}\left[V^{(R,D,S+1)}(x) - V^{(R,D,S)}(x)\right] = 0. \tag{A.21}$$

Hence, when M is a large enough positive integer and $M > \{S, D, |R|\}$, we can approximately write the following equation for $x = -M$:

$$0 = (\mu+a)\Delta(-M+1) - \delta_1\Delta(-M). \tag{A.22}$$

So, we can verify that the coefficient matrix of the linear equations about variable $\Delta(x)$, $-M \leq x \leq M$ is totally nonpositive. Hence, by solving a finite approximate negative system of linear equations, we can corroborate that $\Delta(x) \geq 0$ for all $-M \leq x \leq M$. Thus we get that $\Delta(x) \geq 0$ for all x when $\to +\infty$. At last, there exists an x such that $\Delta(x) > 0$ because $\epsilon > 0$.

Consequently, the $(R, D, S+1)$ policy is better than (R, D, S) policy. To this end, we reemploy all above equations but change equation with $x = S$ as follows:

$$\begin{aligned} &-\mu\left[DV^{(R,D,S)}(S+1) - b\right] \\ &\quad = \mu\Delta(S+1) - (\mu+\lambda_2+\gamma)\Delta(S) + \lambda_2\Delta(S-1). \end{aligned} \tag{A.23}$$

The rest follows exactly the same as above.

We now prove the reverse. And we suppose $\mathscr{D}V^{(R,D,S)}(S+1) \leq b$, or $\mathscr{D}V^{(R,D,S+1)}(S+1) \leq b$. The above proof steps will lead to $\Delta(x) \leq 0$ (rather than $\Delta(x) > 0$) for any x; that is, $V^{(R,D,S+1)}(x) \leq V^{(R,D,S)}(x)$. □

Proof of Theorem 4. Based on Lemma 1, we find that (1), (2), and (3) are true. So we only need to prove $V^{(R,D,S)}(x)$ is concave. For simplicity, we give the proof for the case of $0 \leq R^* < D^* < S^*$. The other cases can be analogously analyzed. To simplify the notation, we drop the superscript $*$ in the proof. Define $\mathscr{D}^2V(x) = \mathscr{D}V(x) - \mathscr{D}V(x-1)$ and $\mathscr{D}V(x) = V^{(R,D,S)}(x) - V^{(R,D,S)}(x-1)$, then that $V^{(R,D,S)}(x)$ is concave is equivalent to $\mathscr{D}^2V(x) < 0$ for all x.

Based on (7)–(11), we obtain the following system of equations:

$$\begin{aligned} h^+ &= -\lambda_2\mathscr{D}^2V(x) - \gamma\mathscr{D}V(x), \quad \text{for } x \geq S+1 \\ h^+ &= -\lambda_2\mathscr{D}^2V(S) - (\mu+\gamma)\mathscr{D}V(S) + \mu b, \quad \text{for } x = S; \\ h^+ &= \mu\mathscr{D}^2V(x+1) - \lambda_2\mathscr{D}^2V(x) \\ &\quad - \gamma\mathscr{D}V(x), \quad \text{for } D+1 \leq x < S; \\ h^+ &= \mu\mathscr{D}V(D+1) - \delta_2\mathscr{D}V(D) \\ &\quad + \lambda_2\mathscr{D}V(D-1) + ac, \quad \text{for } x = D; \\ h^+ &= (\mu+a)\mathscr{D}^2V(x+1) - \lambda_2\mathscr{D}^2V(x) \\ &\quad - \gamma\mathscr{D}V(x), \quad \text{for } R+1 < x \leq D-1; \\ h^+ &= (\mu+a)\mathscr{D}^2V(R+2) - \lambda_1\mathscr{D}^2V(R+1) \\ &\quad - (\lambda_2-\lambda_1+\gamma)\mathscr{D}V(R+1) + \lambda_2p_2 - \lambda_1p_1; \\ h^+ &= (\mu+a)\mathscr{D}^2V(x+1) - \lambda_1\mathscr{D}^2V(x) \\ &\quad - \gamma\mathscr{D}V(x), \quad \text{for } 1 \leq x \leq R; \\ -h^- &= (\mu+a)\mathscr{D}^2V(x+1) - \lambda_1\mathscr{D}^2V(x) \\ &\quad - \gamma\mathscr{D}V(x), \quad \text{for } x \leq 0. \end{aligned} \tag{A.24}$$

Evaluating the above for $x \geq S+1$, at x and $x-1$ and from their difference, we derive, for $x \geq S+2$,

$$0 = -(\lambda_2+\gamma)\mathscr{D}^2V(x) + \lambda_2\mathscr{D}^2V(x-1). \tag{A.25}$$

Similarly, we derive

$$\begin{aligned} 0 &= -(\lambda_2+\gamma)\mathscr{D}^2V(S+1) + \lambda_2\mathscr{D}^2V(S) \\ &\quad + \mu\mathscr{D}V(S) - \mu b, \quad \text{for } x = S+1; \end{aligned} \tag{A.26}$$

$$\begin{aligned} 0 &= -(\lambda_2+\gamma+\mu)\mathscr{D}^2V(S) + \lambda_2\mathscr{D}^2V(S-1) \\ &\quad - \mu\mathscr{D}V(S) + \mu b, \quad \text{for } x = S; \end{aligned} \tag{A.27}$$

$$\begin{aligned} 0 &= \mu\mathscr{D}^2V(x+1) - (\lambda_2+\gamma+\mu)\mathscr{D}^2V(x) \\ &\quad + \lambda_2\mathscr{D}^2V(x-1), \quad \text{for } D+2 \leq x < S; \end{aligned} \tag{A.28}$$

$$\begin{aligned} \epsilon_3 &= \mu\mathscr{D}^2V(D+2) - \delta_2\mathscr{D}^2V(D+1) \\ &\quad + \lambda_2\mathscr{D}^2V(D), \quad \text{for } x = D+1; \end{aligned} \tag{A.29}$$

$$\begin{aligned} \epsilon_4 &= \mu\mathscr{D}^2V(D+1) - \delta_2\mathscr{D}^2V(D) \\ &\quad + \lambda_2\mathscr{D}^2V(D-1), \quad \text{for } x = D; \end{aligned} \tag{A.30}$$

$$\begin{aligned} 0 &= (\mu+a)\mathscr{D}^2V(x+1) - \delta_2\mathscr{D}^2V(x) \\ &\quad + \lambda_2\mathscr{D}^2V(x-1), \quad \text{for } R+2 < x \leq D-1; \end{aligned} \tag{A.31}$$

$$\epsilon_5 = (\mu + a)\mathscr{D}^2V(R+3) - \delta_2\mathscr{D}^2V(R+2) + \lambda_1\mathscr{D}^2V(R+1); \quad (A.32)$$

$$\epsilon_6 = (\mu + a)\mathscr{D}^2V(R+2) - \delta_2\mathscr{D}^2V(R+1) + \lambda_1\mathscr{D}^2V(R); \quad (A.33)$$

$$0 = (\mu + a)\mathscr{D}^2V(x+1) - \delta_1\mathscr{D}^2V(x) + \lambda_1\mathscr{D}^2V(x-1), \quad \text{for } 2 \le x \le R; \quad (A.34)$$

$$\epsilon_7 = (\mu + a)\mathscr{D}^2V(2) - \delta_1\mathscr{D}^2V(1) + \lambda_1\mathscr{D}^2V(0); \quad (A.35)$$

$$0 = (\mu + a)\mathscr{D}^2V(x+1) - \delta_1\mathscr{D}^2V(x) + \lambda_1\mathscr{D}^2V(x-1), \quad \text{for } x \le 0, \quad (A.36)$$

where $\epsilon_3 = -a[\mathscr{D}V(D+1) - c]$, $\epsilon_4 = a[\mathscr{D}V(D) - c]$, $\epsilon_5 = -(\lambda_2-\lambda_1)\mathscr{D}V(R+1)+(\lambda_2 p_2-\lambda_1 p_1)$, $\epsilon_6 = (\lambda_2-\lambda_1)\mathscr{D}V(R)-(\lambda_2 p_2-\lambda_1 p_1)$, and $\epsilon_7 = h^+ + h^-$.

Based on the above equations, we know that

$$\mathscr{D}^2V(S) = \frac{h^+ + (\mu+\gamma)\mathscr{D}V(S) - \mu b}{-\lambda_2}, \quad (A.37)$$

and the (R, D, S) policy being the best implies $\mathscr{D}V(S) > b$, therefore we have $\mathscr{D}^2V(S) < 0$.

Equation (A.26) is equivalent to the following equation after taking into account $\mathscr{D}^2V(S)$:

$$(\lambda_2 + \gamma)\mathscr{D}^2V(S+1) = -h^+ - \gamma\mathscr{D}V(S), \quad (A.38)$$

which gives rise to $\mathscr{D}^2V(S+1) < 0$ based on the above result $\mathscr{D}V(S) > b$.

Equation (A.25) shows that when $x \ge S+2$, $\mathscr{D}^2V(x) = (\lambda_2/\lambda_2+\gamma)\mathscr{D}^2V(x-1)$. Thus based on $\mathscr{D}^2V(S+1) < 0$, we have $\mathscr{D}^2V(x) < 0$ for $x \ge S+1$.

By adding the two sides of (A.26) and (A.27), we obtain

$$0 = -(\lambda_2+\gamma)\mathscr{D}^2V(S+1) - (\gamma+\mu)\mathscr{D}^2V(S) + \lambda_2\mathscr{D}^2V(S-1). \quad (A.39)$$

Therefore we have $\mathscr{D}^2V(S-1) < 0$.

To consider $x \le S-2$, we define $\epsilon_8 = -\mu\mathscr{D}^2V(S-1)$, and, for $x = S-2$,

$$\epsilon_8 = -(\lambda_2+\gamma+\mu)\mathscr{D}^2V(S-2) + \lambda_2\mathscr{D}^2V(S-3). \quad (A.40)$$

Of course, $\epsilon_7 > 0$ and $\epsilon_8 > 0$. Besides that, it is easy to prove that $\epsilon_i > 0$ for $i = 3, 4, 5, 6$, and the limit $\lim_{x\to-\infty}\mathscr{D}V^{(R,D,S)}(x) = h^-/\gamma$ implies that $\lim_{x\to-\infty}\mathscr{D}^2V^{(R,D,S)}(x) = 0$. It can be verified that the coefficient matrix of the linear equation about variable $\mathscr{D}^2(R, D, S)(x)$ from $-M$ to $S-2$ is totally nonpositive, where M is a large enough positive integer greater than $\max\{S, |R|\}$. Thus $\mathscr{D}^2V(x) \le 0$ for all $x \le S-2$. It implies that $\mathscr{D}^2V(x) \le 0$ for all x, and thus, $V^{(R,D,S)}(x)$ is concave function of x. □

Proof of Theorem 5. For simplicity, we assume that $R^* < D^* < S^*$, and for the other cases, we can argue them in the same fashion. Based on the concavity of the optimal discounted profit function $\mathscr{D}V^{(R^*,D^*,S^*)}(x)$, we know that $\mathscr{D}V^{(R^*,D^*,S^*)}(x)$ is a decreasing function of x. Hence, for $x \le R^*$,

$$\mathscr{D}V^{(R^*,D^*,S^*)}(x) \ge \mathscr{D}V^{(R^*,D^*,S^*)}(S^*) > \frac{\lambda_2 p_2 - \lambda_1 p_1}{\lambda_2 - \lambda_1}; \quad (A.41)$$

for $x > S^*$,

$$\mathscr{D}V^{(R^*,D^*,d^*)}(x) \le \mathscr{D}V^{(R^*,D^*,S^*)}(S^*+1) \le b; \quad (A.42)$$

for $D^* < x \le S^*$,

$$c \ge \mathscr{D}V^{(R^*,D^*,S^*)}(D^*+1) \ge \mathscr{D}V^{(R^*,D^*,S^*)}(x) \ge \mathscr{D}V^{(R^*,D^*,S^*)}(S^*) \ge b; \quad (A.43)$$

and for $R^* < x \le S^*$,

$$\mathscr{D}V^{(R^*,D^*,S^*)}(R^*+1) \ge \mathscr{D}V^{(R^*,D^*,S^*)}(x) \ge \mathscr{D}V^{(R^*,D^*,S^*)}(S^*), \quad (A.44)$$

which yields

$$\frac{\lambda_2 p_2 - \lambda_1 p_1}{\lambda_2 - \lambda_1} \ge \mathscr{D}V^{(R^*,D^*,S^*)}(x) > b. \quad (A.45)$$

Combining the above five inequalities, we can infer that $V^{(R^*,D^*,S^*)}(x)$ satisfies the Hamilton-Jacobi-Bellman equation (5). It follows, therefore, that the best (R^*, D^*, S^*) policy is globally optimal. □

Proof of Theorem 6. Given an (R, D, S) policy with $S < 0$, we assume that the (R, D, S) policy is optimal. For simplicity, we only consider the case $R < D < S < 0$, and the other case $D < R < S < 0$ can be analogously analyzed. While for (R, D, S) policy with $R < D < S < 0$, we obtain that $\mathscr{D}V_0(x)$, where $\mathscr{D}V_0(x) = \mathscr{D}V^{(R,D,S)}(x)$ for $x \le S$, satisfies the following equations:

$$-h^- = -(\lambda_2+\mu+\gamma)\mathscr{D}V_0(S) + \lambda_2\mathscr{D}V_0(S-1) + \mu b, \quad \text{for } x = S;$$

$$-h^- = \mu\mathscr{D}V_0(x+1) - (\mu+\lambda_2+\gamma)\mathscr{D}V_0(x) + \lambda_2\mathscr{D}V_0(x-1), \quad \text{for } D+1 \le x < S;$$

$$-h^- = \mu\mathscr{D}V_0(D+1) - \delta_2\mathscr{D}V_0(D) + \lambda_2\mathscr{D}V_0(D-1) + ac, \quad \text{for } x = D;$$

$$
\begin{aligned}
-h^- &= (\mu + a)\mathscr{D}V_0(x+1) - \delta_2\mathscr{D}V_0(x) \\
&\quad + \lambda_2\mathscr{D}V_0(x-1), \quad \text{for } R+1 < x < D, \\
-h^- &= (\mu + a)\mathscr{D}V_0(R+2) - \delta_2\mathscr{D}V_0(R+1) \\
&\quad + \lambda_1\mathscr{D}V_0(R) + \lambda_2 p_2 - \lambda_1 p_1, \quad \text{for } x = R+1; \\
-h^- &= (\mu + a)\mathscr{D}V_0(x+1) - \delta_1\mathscr{D}V_0(x) \\
&\quad + \lambda_1\mathscr{D}V_0(x-1), \quad \text{for } x \le R.
\end{aligned} \tag{A.46}
$$

Let $\Delta(x) = \mathscr{D}V_1(x+1) - \mathscr{D}V_0(x)$ where $\mathscr{D}V_1(x) = \mathscr{D}V^{(R+1,D+1,S+1)}(x)$; then we obtain, through a series of subtractions:

$$
\begin{aligned}
0 &= -(\lambda_2 + \mu + \gamma)\Delta(S) + \lambda_2\Delta(S-1), \quad \text{for } x = S; \\
0 &= \mu\Delta(x+1) - (\mu + \lambda_2 + \gamma)\Delta(x) \\
&\quad + \lambda_2\Delta(x-1), \quad \text{for } D+1 \le x < S; \\
0 &= \mu\Delta(D+1) - \delta_2\Delta(D) + \lambda_2\Delta(D-1), \quad \text{for } x = D; \\
0 &= (\mu + a)\Delta(x+1) - \delta_2\Delta(x) \\
&\quad + \lambda_2\Delta(x-1), \quad \text{for } R+1 < x < D; \\
0 &= (\mu + a)\Delta(R+2) - \delta_2\Delta(R+1) \\
&\quad + \lambda_1\Delta(R), \quad \text{for } x = R+1; \\
0 &= (\mu + a)\Delta(x+1) - \delta_1\Delta(x) \\
&\quad + \lambda_1\Delta(x-1), \quad \text{for } x \le R.
\end{aligned} \tag{A.47}
$$

We approximate the above infinite system of linear equations with a finite one involving $-M \le x \le S$ with M being a large enough positive integer greater than $\max\{S, |R|\}$. The last equation in the finite system, corresponding to $x = -M$, is

$$0 = (\mu + a)\Delta(-M+1) - \delta_1\Delta(-M), \tag{A.48}$$

where $\lambda_1\Delta(-M-1)$ is crossed out. As argued before in the proof of Lemma 3, the coefficient matrix of above linear equations about variable $\Delta(x)$, $-M \le x \le S$, is invertible, and, hence, $\Delta(x) = 0$ for $-M \le x \le S$. And then we get $\Delta(x) = 0$ where $M \to +\infty$; that is,

$$\mathscr{D}V^{(R+1,D+1,S+1)}(x+1) = \mathscr{D}V^{(R,D,S)}(x). \tag{A.49}$$

Furthermore, we have the following equation:

$$
\begin{aligned}
&V^{(R+1,D+1,S+1)}(S+1) - V^{(R,D,S)}(S+1) \\
&\quad = -\mathscr{D}V^{(R,D,S)}(S+1) \\
&\qquad + \sum_{y=-M+1}^{S}\Delta(y) + \mathscr{D}V^{(R+1,D+1,S+1)}(-M+1) \\
&\qquad + \left[V^{(R+1,D+1,S+1)}(-M) - V^{(R,D,S)}(-M)\right].
\end{aligned} \tag{A.50}
$$

Lemma 1 implies that

$$\lim_{M\to\infty}\left[V^{(R+1,D+1,S+1)}(-M) - V^{(R,D,S)}(-M)\right] = 0. \tag{A.51}$$

Because of

$$\lim_{M\to\infty}\mathscr{D}V^{(R+1,D+1,S+1)}(-M+1) = \frac{h^-}{\gamma} \tag{A.52}$$

and $\mathscr{D}V^{(R,D,S)}(S+1) \le b$, we get

$$V^{(R+1,D+1,S+1)}(S+1) - V^{(R,D,S)}(S+1) > 0; \tag{A.53}$$

that is, $V^{(R+1,D+1,S+1)}(S+1) > V^{(R,D,S)}(S+1)$. However, this relation invalidates

$$V^{(R+1,D+1,S+1)}(S+1) \le V^{(R,D,S)}(S+1), \tag{A.54}$$

based on the assumption that the (R, D, S) policy is optimal.

If $(\lambda_2 p_2 - \lambda_1 p_1)/(\lambda_2 - \lambda_1) > c$, then we have

$$
\begin{aligned}
\mathscr{D}V^{(R^*,D^*,S^*)}(R^*) &> \frac{\lambda_2 p_2 - \lambda_1 p_1}{\lambda_2 - \lambda_1} \\
&> c \ge \mathscr{D}V^{(R^*,D^*,S^*)}(D^*+1).
\end{aligned} \tag{A.55}
$$

Based on the concavity of the optimal discounted profit function $\mathscr{D}V^{(R^*,D^*,S^*)}(x)$, we get $R^* < D^*$. Similarly, we get $R^* > D^*$ if $(\lambda_2 p_2 - \lambda_1 p_1)/(\lambda_2 - \lambda_1) < c$. □

Proof of Theorem 7. We let

$$V^{(R^*(c),D^*(c),S^*(c))}(x;c) = V^{(R^*(c),D^*(c),S^*(c))}(x;a,c), \tag{A.56}$$

where $(R^*(c), D^*(c), S^*(c))$ means the optimal thresholds policy corresponding to the sourcing production cost c, where sourcing production rate a is fixed.

(1) In the proof, we focus on the case that $0 < R^*(c) < D^*(c) < S^*(c)$; the other cases can be proved in the same fashion. As for a fixed sourcing production rate a, let $\mathscr{D}V(x;c) = \mathscr{D}V^{(R^*(c),D^*(c),S^*(c))}(x;c)$ satisfy the recursions as follows:

$$
\begin{aligned}
h^+ &= -(\lambda_2 + \gamma)\mathscr{D}V(x;c) \\
&\quad + \lambda_2\mathscr{D}V(x-1;c), \quad \text{for } x \ge S^*(c)+1; \\
h^+ &= -(\lambda_2 + \mu + \gamma)\mathscr{D}V(S^*(c);c) \\
&\quad + \lambda_2\mathscr{D}V(S^*(c)-1;c) + \mu b, \quad \text{for } x = S^*(c); \\
h^+ &= \mu\mathscr{D}V(x+1;c) - (\mu + \lambda_2 + \gamma)\mathscr{D}V(x;c) \\
&\quad + \lambda_2\mathscr{D}V(x-1;c), \quad \text{for } D^*(c)+1 \le x < S^*(c); \\
h^+ &= \mu\mathscr{D}V(D^*(c)+1;c) - \delta_2\mathscr{D}V(D^*(c);c) \\
&\quad + \lambda_2\mathscr{D}V(D^*(c)-1;c) + ac, \quad \text{for } x = D^*(c);
\end{aligned}
$$

$$
\begin{aligned}
h^{+} &= (\mu+a)\mathscr{D}V(x+1;c)-\delta_2\mathscr{D}V(x;c)\\
&\quad+\lambda_2\mathscr{D}V(x-1;c), \quad \text{for } R^*(c)+1<x<D^*(c);\\
h^{+} &= (\mu+a)\mathscr{D}V(R^*(c)+2;c)-\delta_2\mathscr{D}V(R^*(c)+1;c)\\
&\quad+\lambda_1\mathscr{D}V(R^*(c);c)+\lambda_2p_2-\lambda_1p_1;\\
h^{+} &= (\mu+a)\mathscr{D}V(x+1;c)-\delta_1\mathscr{D}V(x;c)\\
&\quad+\lambda_1\mathscr{D}V(x-1;c), \quad \text{for } 0<x\le R^*(c);\\
-h^{-} &= (\mu+a)\mathscr{D}V(x+1;c)-\delta_1\mathscr{D}V(x;c)\\
&\quad+\lambda_1\mathscr{D}V(x-1;c), \quad \text{for } x\le 0.
\end{aligned}
\tag{A.57}
$$

And let $\Delta_{1,2}(x;c_1,c_2)=\mathscr{D}V(x;c_2)-\mathscr{D}V(x;c_1)$ and let $\xi=-a(c_2-c_1)$, which can be simplified as $\Delta_1(x)$ without loss of generality. We get $\Delta_1(x)$, satisfying a recursion formulated by subtracting $\mathscr{D}V(x;c_1)$ from $(x;c_2)$ as follows:

$$
\begin{aligned}
0 &= -(\lambda_2+\gamma)\Delta(x)+\lambda_2\Delta(x-1),\\
&\qquad \text{for } x\ge S^*(c_1)+1;\\
0 &= -(\lambda_2+\mu+\gamma)\Delta(S^*(c_1))+\lambda_2\Delta(S^*(c_1)-1);\\
0 &= \mu\Delta(x+1)-(\mu+\lambda_2+\gamma)\Delta(x)+\lambda_2\Delta(x-1),\\
&\qquad \text{for } D^*(c_1)+1\le x<S^*(c_1);\\
\xi &= \mu\Delta(D^*(c_1)+1)-\delta_2\Delta(D^*(c_1))\\
&\quad+\lambda_2\Delta(D^*(c_1)-1); \quad \text{for } x=D^*(c_1);\\
0 &= (\mu+a)\Delta(x+1)-\delta_2\Delta(x)+\lambda_2\Delta(x-1),\\
&\qquad \text{for } R^*(c_1)+1<x<D^*(c_1);\\
0 &= (\mu+a)\Delta(R^*(c_1)+2)-\delta_2\Delta(R^*(c_1)+1)\\
&\quad+\lambda_1\Delta(R^*(c_1)), \quad \text{for } x=R^*(c_1)+1;\\
0 &= (\mu+a)\Delta(x+1)-\delta_1\Delta(x)\\
&\quad+\lambda_1\Delta(x-1), \quad \text{for } 0<x\le R^*(c_1);\\
0 &= (\mu+a)\Delta(x+1)-\delta_1\Delta(x)\\
&\quad+\lambda_1\Delta(x-1), \quad \text{for } x\le 0.
\end{aligned}
\tag{A.58}
$$

In line with Lemma 1, we know that

$$\lim_{x\to-\infty}\Delta(x)=0. \tag{A.59}$$

It can be verified that the coefficient matrix of the above linear equations is totally nonpositive. Hence, by solving a finite approximate negative system of linear equations, we can corroborate that $\Delta(x)\ge 0$; that is,

$$
\begin{aligned}
&\mathscr{D}V^{(R^*(c_1),D^*(c_1),S^*(c_1))}(x;c_2)\\
&\quad\ge \mathscr{D}V^{(R^*(c_1),D^*(c_1),S^*(c_1))}(x;c_1).
\end{aligned}
\tag{A.60}
$$

So we get

$$
\begin{aligned}
&\mathscr{D}V^{(R^*(c_1),D^*(c_1),S^*(c_1))}(S^*(c_1);c_2)\\
&\quad\ge \mathscr{D}V^{(R^*(c_1),D^*(c_1),S^*(c_1))}(S^*(c_1);c_1)>b,
\end{aligned}
\tag{A.61}
$$

which means that the best base stock level $S^*(c_1)+1$ can be better than $S^*(c_1)-1$ and improved when sourcing cost is increased from c_1; that is, $S^*(c_2)\ge S^*(c_1)$. Similarly, we get $R^*(c_2)\ge R^*(c_1)$. Because of thresholds $S^*(c), R^*(c)$ being integers for any sourcing cost c, we know that the optimal thresholds $R^*(c), S^*(c)$ are piecewise constants and increasing functions of c.

On other hand, we will find

$$\mathscr{D}V^{(R^*(c),D^*(c),S^*(c))}(x;c)-c \tag{A.62}$$

is decreasing in c. Let

$$\mathscr{D}V_1(x;c)=\mathscr{D}V^{(R^*(c),D^*(c),S^*(c))}(x;c)-c, \tag{A.63}$$

which satisfies the recursions as follows:

$$
\begin{aligned}
h^{+}+\gamma c &= -(\lambda_2+\gamma)\mathscr{D}V_1(x;c)\\
&\quad+\lambda_2\mathscr{D}V_1(x-1;c), \quad \text{for } x\ge S^*(c)+1;\\
h^{+}+\gamma c &= -(\lambda_2+\mu+\gamma)\mathscr{D}V_1(S^*;c)\\
&\quad+\lambda_2\mathscr{D}V_1(S^*-1;c)+\mu b, \quad \text{for } x=S^*(c);\\
h^{+}+\gamma c &= \mu\mathscr{D}V_1(x+1;c)-(\mu+\lambda_2+\gamma)\mathscr{D}V_1(x;c)\\
&\quad+\lambda_2\mathscr{D}V_1(x-1;c),\\
&\qquad \text{for } D^*(c)+1\le x<S^*(c);\\
h^{+}+\gamma c &= \mu\mathscr{D}V_1(D^*(c)+1;c)-\delta_2\mathscr{D}V_1(D^*(c);c)\\
&\quad+\lambda_2\mathscr{D}V_1(D^*(c)-1;c), \quad \text{for } x=D^*(c);\\
h^{+}+\gamma c &= (\mu+a)\mathscr{D}V_1(x+1;c)-\delta_2\mathscr{D}V_1(x;c)\\
&\quad+\lambda_2\mathscr{D}V_1(x-1;c),\\
&\qquad \text{for } R^*(c)+1<x<D^*(c);\\
h^{+}+\gamma c &= (\mu+a)\mathscr{D}V_1(R^*(c)+2;c)\\
&\quad-\delta_2\mathscr{D}V_1(R^*(c)+1;c)\\
&\quad+\lambda_1\mathscr{D}V_1(R^*(c);c)+\lambda_2p_2-\lambda_1p_1;\\
h^{+}+\gamma c &= (\mu+a)\mathscr{D}V_1(x+1;c)-\delta_1\mathscr{D}V_1(x;a)\\
&\quad+\lambda_1\mathscr{D}V_1(x-1;c), \quad \text{for } 0<x\le R^*(c);\\
-h^{-}+\gamma c &= (\mu+a)\mathscr{D}V_1(x+1;c)-\delta_1\mathscr{D}V_1(x;c)\\
&\quad+\lambda_1\mathscr{D}V_1(x-1;c), \quad \text{for } x\le 0.
\end{aligned}
\tag{A.64}
$$

Let

$$\mathscr{D}V_1(x;c_1) = \mathscr{D}V^{(R^*(c_1),D^*(c_1),S^*(c_1))}(x;c_1) - c_1,$$
$$\mathscr{D}V_1(x;c_2) = \mathscr{D}V^{(R^*(c_1),D^*(c_1),S^*(c_1))}(x;c_2) - c_2. \tag{A.65}$$

And let

$$\Delta_1(x;c_1,c_2) = \mathscr{D}V_1(x;c_2) - \mathscr{D}V_1(x;c_1), \tag{A.66}$$

which can be simplified as $\Delta_1(x)$ without loss of generality. Let $\eta = \gamma(c_2 - c_1)$, then we get that $\Delta_1(x)$ satisfied a recursion formulated by subtracting $\mathscr{D}V_1(x;c_1)$ from $\mathscr{D}V_1(x;c_2)$ as follows:

$$\begin{aligned}
\eta &= -(\lambda_2+\gamma)\Delta_1(x) + \lambda_2\Delta_1(x-1), \\
&\qquad\qquad \text{for } x \geq S^*(c_1)+1; \\
\eta &= -(\lambda_2+\mu+\gamma)\Delta_1(S^*(c_1)) \\
&\quad + \lambda_2\Delta_1(S^*(c_1)-1), \quad \text{for } x = S^*(c_1); \\
\eta &= \mu\Delta_1(x+1) - (\mu+\lambda_2+\gamma)\Delta_1(x) \\
&\quad + \lambda_2\Delta_1(x-1), \quad \text{for } D^*(c_1)+1 \leq x < S^*(c_1); \\
\eta &= \mu\Delta_1(D^*(c_1)+1) - \delta_2\Delta_1(D^*(c_1)) \\
&\quad + \lambda_2\Delta_1(D^*(c_1)-1); \quad \text{for } x = D^*(c_1); \\
\eta &= (\mu+a)\Delta_1(x+1) - \delta_2\Delta_1(x) \\
&\quad + \lambda_2\Delta_1(x-1), \quad \text{for } R^*(c_1)+1 < x < D^*(c_1); \\
\eta &= (\mu+a)\Delta_1(R^*(c_1)+2) - \delta_2\Delta_1(R^*(c_1)+1) \\
&\quad + \lambda_1\Delta_1(R^*(c_1)), \quad \text{for } x = R^*(c_1)+1; \\
\eta &= (\mu+a)\Delta_1(x+1) - \delta_1\Delta_1(x) \\
&\quad + \lambda_1\Delta_1(x-1), \quad \text{for } 0 < x \leq R^*(c_1); \\
\eta &= (\mu+a)\Delta_1(x+1) - \delta_1\Delta_1(x) \\
&\quad + \lambda_1\Delta_1(x-1), \quad \text{for } x \leq 0.
\end{aligned} \tag{A.67}$$

In line with Lemma 1, we know that

$$\lim_{x\to-\infty}\Delta_1(x) = -(c_2-c_1). \tag{A.68}$$

Then the preceding equations form a system of linear equations with unknown variables $\Delta_1(x)$. Let M be a large positive integer that satisfies $M > \max(S^*, |R^*|, |D^*|)$. Then we can get, for $x = -M$,

$$(\gamma+\lambda_1)(c_2-c_1) = (\mu+a)\Delta_1(x+1) - \delta_1\Delta_1(x). \tag{A.69}$$

It can be verified that the coefficient matrix of the above linear equation about the variable $\Delta_1(x)$ with $-M \leq x \leq M$ is totally nonpositive, where M is large enough positive integer. Hence, by solving a finite approximate negative system of linear equations, we get $\Delta_1(x) \leq 0$ for $-M \leq x \leq M$. And then, we have $\Delta_1(x) \leq 0$ for all x when $M \to +\infty$; that is,

$$\begin{aligned}
&\mathscr{D}V^{(R^*(c_1),D^*(c_1),S^*(c_1))}(x;c_2) - c_2 \\
&\quad \leq \mathscr{D}V^{(R^*(c_1),D^*(c_1),S^*(c_1))}(x;c_1) - c_1.
\end{aligned} \tag{A.70}$$

Hence, we get

$$\begin{aligned}
&\mathscr{D}V^{(R^*(c_1),D^*(c_1),S^*(c_1))}(D^*(c_1)+1;c_2) - c_2 \\
&\quad \leq \mathscr{D}V^{(R^*(c_1),D^*(c_1),S^*(c_1))}(D^*(c_1)+1;c_1) - c_1 \leq 0;
\end{aligned} \tag{A.71}$$

that is,

$$\mathscr{D}V^{(R^*(c_1),D^*(c_1),S^*(c_1))}(D^*(c_1)+1;c_2) \leq c_2, \tag{A.72}$$

which implies that $(R^*(c_1), D^*(c_1)-1, S^*(c_1))$ can be better than $(R^*(c_1), D^*(c_1)+1, S^*(c_1))$ and might be improved if the sourcing cost is increased from c_1; that is, $D^*(c_2) \leq D^*(c_1)$. Because of thresholds $D^*(c)$ being integer for any sourcing cost c, we know that the optimal threshold $D^*(c)$ is a piecewise constant, decreasing function of c.

(2) Clearly, for any $u \in \mathscr{U}$, $V^u(x;c_2) < V^u(x;c_1)$. Then,

$$\begin{aligned}
V^{u^*}(x;c_2) &= \sup_{u\in\mathscr{U}} V^u(x;c_2) \\
&\leq \sup_{u\in\mathscr{U}} V^u(x;c_1) = V^{u^*}(x;c_1),
\end{aligned} \tag{A.73}$$

which proves that $V^{u^*}(x;c)$ is strictly decreasing in c. □

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Research Article

The Closed-Loop Supply Chain Network Equilibrium with Products Lifetime and Carbon Emission Constraints in Multiperiod Planning Horizon

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This paper studies a closed-loop supply chain network equilibrium problem in multiperiod planning horizons with consideration of product lifetime and carbon emission constraints. The closed-loop supply chain network consists of suppliers tier, manufacturer tier, retailers tier, and demand markets tier, in which the manufacturers collect used products from the demand markets directly. Product lifetime is introduced to denote the maximum times of manufacturing and remanufacturing, and the relation between adjacent periods is described by inventory transfer. By variational inequalities and complementary theory, the optimal behaviors of all the players are modeled, and, in turn, the governing closed-loop supply chain network equilibrium model is established. The model is solved by modified project contraction algorithm with fixed step. Optimal equilibrium results are computed and analyzed through numerical examples. The impacts of collection rate, remanufacturing conversion rate, product lifetime, and carbon emission cap on equilibrium states are analyzed. Finally, several managerial insights are given to provide decision support for entrepreneurs and government official along with some inspirations for future research.

1. Introduction

Closed-loop supply chain (CLSC) has received increasing attention coming from theorists and entrepreneurs in recent years. From the viewpoint of overall lifecycle, it compromises the forward activities of manufacturing, distribution, and retailing and backward activities of collection and remanufacturing. Remanufacturing, as the significant part of CLSC, is a complicated process in which the core components of used products are disassembled, repaired, and reused [1]. Compared with production from virgin materials, remanufacturing tends to be energy saving, less material consuming and often has a lower impact on the environment. Related studies have shown that the remanufacturing cost of a core component is typically 40–60% less than those of brand-new production by raw materials [2]. Many famous enterprises, such as Xerox, Robert Bosch Tool, Black and Decker, and Hewlett-Packard, have implemented remanufacturing and CLSC strategies successfully [3, 4].

In CLSC operations, the dynamic production, remanufacturing, and pricing strategies in multiperiod planning horizons are of great importance for the firms, which is directly related to their long-term profits. But for the decision-makers, it is much more complicated to make the optimal strategies in several planning horizons than in a single period, because they have to weigh the profit of the current period against those of the subsequent periods. One can easily deduce that if the price is high in the current period, the firm's profit in this period might increase whereas the quantity of the products sold decreases; in turn, in the subsequent periods, the number of remanufacturable products available decreases, therefore reducing the subsequent periods' potential profits. On the other hand, if the price is low in the current period, the profits in this period might decrease, but the manufacturers have more end-of-life products to remanufacture in future periods [5]. Then, in this context, how should the decision-makers make the best choice? Aiming at this issue, the scholars have done a lot of research from the viewpoint of a monopolistic manufacturer, duopoly manufacturers, and single manufacturer-single retailer/collector bilateral monopoly. Their research provides several meaningful management insights for the readers and the practitioners. However, as we know, the actual CLSC is likely to have a wider variety of channel members and complex cooperative and competitive relations. For example, a complete CLSC may have several mutual competitive suppliers, competitive manufacturers, competitive retailers, competitive collectors, and demand markets, in which each kind of players is sorted in the same tier, so the combination of any a player from each tier can form a cooperative chain. From another perspective, some players in the network such as a typical manufacturer or a typical retailer may be included in different CLSC. Therefore, the CLSC usually exists in the form of network structure. According to Hammond and Beullens [6], if and only if all the players in the CLSC network agree to the transaction prices and the transaction amounts of products, the equilibrium state of the CLSC network is achieved. Then, the following questions come up: for such a complex CLSC network, what are the players' optimal decisions in dynamic multiperiod planning horizons? What is the dynamic equilibrium state of the CLSC network?

In addition, the factors of product lifetime are usually neglected in the previous dynamic multiperiod CLSC modeling, and almost all of the literature assumed the used products could be remanufactured only once. But in reality, the core components may undergo multiple cycles of remanufacturing. For example, an end-of-life toner cartridge can be retreaded up to four times before landfill [7]. A single-use camera can be remanufactured approximately 6 times [8]. Many others including the maximum lives of car rites and computer chips are 3 and 4, respectively [9]. Therefore, it is of great importance to take the factors of finite product lifetime into account in dynamic CLSC modeling.

Last but not least, as huge consumptions of fossil energies and accumulative carbon emissions of greenhouse gases have aggravated weather deterioration and environment pollution most recently, more and more attention on how to reduce carbon emissions has been received worldwide. Accordingly, a series of regulation policies are introduced by governments. Kyoto protocol is one of the most famous protocols signed at the third conference of the parties of United Nations Framework Convention on Climate Change held in Kyoto, Japan, in October, 1997. After Kyoto protocol, other policies, for example, EU-ETS, RGGI, are published subsequently [10]. Predictably, in the future the influence on enterprise operations will be more and more coming from carbon emission polices and gradually will be one of the prerequisites in the production/operations management for the firms. Therefore, for the players in CLSC network, how should they arrange their logistic activities when faced with the carbon emission policies?

Motivated by all the above analysis, this paper establishes a multiperiod CLSC network equilibrium model with consideration of product lifetime and carbon emission constraints comprehensively, analyzes the impact of a variety of parameters on CLSC network equilibrium, and obtains several managerial insights. We argue that this study is one of the first studies to explore CLSC network equilibrium in dynamic multiperiod planning horizons, not to mention that the other two important factors are considered simultaneously.

The rest of this paper is organized as follows. In Section 2, we review the literatures related to our research and highlight the contributions of our paper. In Section 3, we give the model assumptions and variable notations. In Section 4, we present our model in which the optimal behavior of various players in the multiperiod CLSC network is described and the governing equilibrium conditions are formulated as a finite-dimension variational inequality problem. In Section 5, we provide a solving algorithm for the model. In addition, by a numerical example, we analyze the equilibrium results and reveal meaningful insights related to product lifetime and carbon emission constraints.

2. Literature Review

Our work is related to three streams of research, and the literature in each research will be reviewed subsequently. We will also point out how our study differs from the existing literature.

The first stream of research related to our work is on the dynamic production, pricing policies, and coordination mechanisms of CLSC in two-period, multiperiod, and infinite planning horizon settings [5, 11–18]. With the consideration of multiple product lifetimes, Geyer et al. [9] investigated the economy of remanufacturing and demonstrated the need to coordinate production cost structure, collection rate, constraints of limited component durability, and finite product life cycles in the process of production operations. Although the above-mentioned literatures lay a solid foundation for future research, they are confined merely to the case of a monopoly manufacturer, duopoly manufacturers, or single manufacturer-single retailer/collector bilateral monopoly. Our study differs from the above-mentioned literature in that we research into dynamic production and pricing decisions in a complex CLSC network.

The second stream of research related to our work is on network equilibrium of CLSC and multiperiod traditional/forward supply chain network equilibrium. Hammond and Beullens [6] construct an oligopolistic CLSC network model including manufacturers and demand markets under WEEE legislation. Yang et al. [19] use variational inequality method to model the CLSC network including suppliers, manufacturers who are involved in the production of a homogeneous product from virgin materials and collected materials, retailers, and recovery centers that collect used products from demand markets and can obtain subsidies from official organizations. Qiang et al. [20] establish a CLSC network model considering the competition, distribution channel investment, and uncertainties. The literature mentioned above deal with static or single-period CLSC network equilibrium problems.

Recently, a few authors explore multiperiod traditional supply chain network equilibrium problems. Cruz and Wakolbinger [21] develop a framework for the analysis of the optimal levels of corporate social responsibility (CSR) activities in a multiperiod supply chain network consisting of manufacturers, retailers, and consumers and describe the problem of carbon emissions. Hamdouch [22] establishes a three-tier equilibrium model with capacity constraints and retailers' purchase strategy from a multiperiod perspective. Cruz and Liu [23] analyze the effects of levels of social relationship on a multiperiod supply chain network with multiple decision-makers associated with different tiers. But to our knowledge, there is no research linking the dynamic multiperiod decisions and CLSC network equilibrium up to now. One intention of our work is to fill the gap.

The third stream of research related to our work is on the production planning with carbon cap schemes and carbon trade schemes. Emission cap scheme, as a traditional regulation, restricts the maximum carbon emission volumes of the firm at a certain time. In contrast, emissions trading schemes provide pollutant emitters with flexibility in how they comply with the regulations. It is not discussed in detail in this paper. We refer readers to the work Gong and Zhou [24] to reach a deeper understanding about production planning with carbon emission schemes. Herein, we will give a brief review on the literature that studies supply chain network equilibrium and CLSC supply chain operations problems with carbon emission constraints. Wu et al. [25] propose an electric power supply chain network equilibrium model with carbon taxes. Based on this model, Nagurney et al. [26] develop a modeling and computational framework that allowed for the determination of optimal carbon taxes applied to electric power plants in electric power supply chain networks. More recently, it has been incorporated into reverse logistics and CLSC modeling. Krikke [27] gives a decision framework for optimizing the combined disposition and location-transport decisions on carbon footprints in CLSC and then applies it to a copier case. Kannan et al. [28] establish a biobjective 0-1 mixed-integer linear programming model for a carbon footprint based reverse logistics network design. The difference between our study and the existing literature lies in that we combine the CLSC network equilibrium with carbon emission constraints.

3. Model Assumptions and Notations

3.1. Model Assumptions. Considering a CLSC network consisting of multiple suppliers, multiple manufacturers, multiple retailers, and multiple demand markets, in which the manufacturers make homogenous products and simultaneously collected used products from demand markets at the end of period t for remanufacturing in the period $t+1$, the criterion of each player in the network is its total profit maximum in multiperiod planning horizontal. The two-period CLSC network comprises two suppliers, two manufacturers, two retailers, and two demand markets which can be described as in Figure 1. We denote a typical raw material supplier by s, a typical manufacturer by m, a typical retailer market by n, and a typical demand market by k. In particular, $m_1(2)$ represents the 1st manufacturer in 2nd period, and we can interpret the other notations in the same way. In the top of Figure 1, the dotted lines between the same manufacturers in different period represent the transfer of inventory between different planning periods. The real lines express the forward transactions between suppliers and manufacturers and manufacturers and retailers, and the thin dotted lines illustrate the reverse transactions between manufacturers and demand markets.

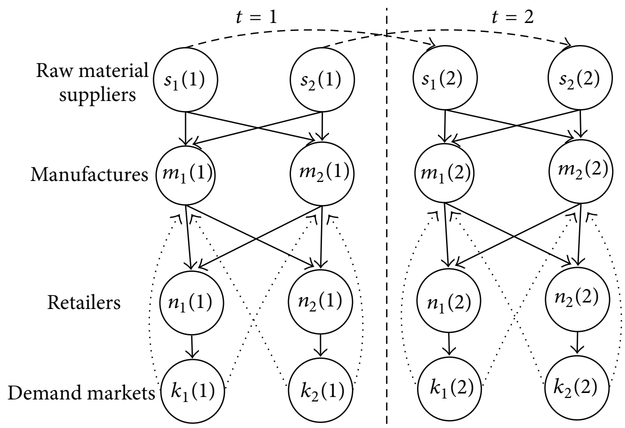


Figure 1: An illustration of a CLSC network with two periods.

Due to the complexity of the problem, we make the following assumptions obeying the economic theories.

Assumption 1. The remanufactured products are the same as new products made by raw materials, and there is no difference between their selling prices in demand markets in each period. This assumption comes from the example of single-use cameras of Eastman Kodak Company. The used cameras are typically upgraded to the quality of new ones, and both products can be perfectly substituted by each other [29].

Assumption 2. Compared with the production by raw materials, the manufacturers firstly choose to remanufacture the collected products because of cost saving. The assumption is commonly used in CLSC management literature to ensure the economy of remanufacturing.

Assumption 3. It is assumed that the manufacturers have carbon emission constraints during manufacturing and remanufacturing in each period. It is different from the case of traditional/forward supply chain, in which the carbon constraints are usually applied for multiperiod of time, for example, one year. However, in CLSC, it will take a relatively long time from product design, production, retailing, collection, and remanufacturing. The average return lags alone are often at least half a year [30].

Assumption 4. The carbon emission volume of remanufacturing a used product is lower than that of manufacturing a new product [27].

Assumption 5. The manufacturers and demand markets are separate in space and the place, and the distance between different tiers is included in the transaction cost functions.

Assumption 6. The number of planning period is larger than product lifetime to illustrate the impact of product lifetime.

Assumption 7. One retailer only copes with one demand market.

Assumption 8. The production functions and transaction cost functions in the model are continuously differentiable and convex. This function property is required to be strictly satisfied in CLSC network equilibrium research [6, 19].

3.2. Variables and Notations. Consider the following:

T: the number of total planning periods;

Z: the product lifetime ($Z \geq 2$), which means that the product can be manufactured once and remanufactured $Z-1$ times;

t: a typical planning period, $t = 1, 2, \ldots, T$;

s: a typical supplier, $s = 1, 2, \ldots, S$;

m: a typical manufacturer, $m = 1, 2, \ldots, M$;

n: a typical retailer, $n = 1, 2, \ldots, N$;

k: a typical demand market, $k = 1, 2, \ldots, K$;

α: the collection rate of used products from demand market k; in this paper, we assume that α is a constant parameter in different periods;

β^r: the average conversion rate of raw materials;

β_i^u: the average remanufacturing conversion rate of used products in period i, $i = 1, \ldots, \min\{t-1, Z-1\}$, which in general satisfies $\beta_i^u \leq \beta_{i-1}^u$, $i = 2, \ldots, \min\{t-1, Z-1\}$. Moreover, in order to simplify the formulation expressions hereinafter, we introduce $\beta_0^u = 1$;

$q_{sm}^r(t)$: the amount of raw materials provided by supplier s to manufacturer m in period t; group all of $q_{sm}^r(t)$ into a SMT-dimension column vector Q^1;

$q_s^r(t)$: the amount of raw materials provided by supplier s to all manufacturers in period t; group all of $q_s^r(t)$ into a ST-dimension column vector q_1^r and $q_s^r(t) = \sum_{m=1}^{M} q_{sm}^r(t)$;

$q_m^r(t)$: the amount of raw materials that manufacturer m uses for production in period t; group all of $q_m^r(t)$ into a MT-dimension column vector q^r;

$q_m^u(t)$: the amount of collected products used in remanufacturing process in period $t\,(t \geq 2)$ and $q_m^u(t) = \sum_{k=1}^{K} q_{km}^u(t-1) = \sum_{n=1}^{N} \sum_{i=1}^{\min\{t-1,Z-1\}} \beta_{i-1}^u \cdots \beta_0^u \alpha^i q_{mn}^r(t-i)$; group all of $q_m^u(t)$ into a $M(T-1)$-dimension column vector q^u;

$q_{mn}(t)$: the transaction amount of new products between manufacturer m and retailer n in period t group all of $q_{mn}(t)$ into a MNT-dimension column vector Q^2;

$q_{mn}^r(t)$: the transaction amount of the new products made from raw materials between manufacturer m and retailer n in period t; group all of $q_{mn}^r(t)$ into a MNT-dimension column vector Q^3;

$q_{km}^u(t)$: the transaction amount of used products between manufacturer m and demand market n in period t; group all of $q_{km}^u(t)$ into a NM-dimension column vector $Q^4(t)$, and group all of $q_{km}^u(t)$ into a NMT-dimension column vector Q^4, and then we have $q_{km}^u(t-1) = \sum_{i=1}^{\min\{t-1,Z-1\}} \beta_{i-1}^u \cdots \beta_0^u \alpha^i q_{mn}^r(t-i)$;

$q_{nk}(t)$: the transaction amount of products between manufacturer m and retailer n in period t; group all of $q_{nk}(t)$ into a NKT-dimension column vector Q^5;

$f_s^r(q_1^r(t))$: production cost of raw material by supplier s in period t;

$f_m^r(t) = f_m^r(\beta_r, q^r(t))$: production cost of product by manufacturer m in period t;

$f_m^u(t) = f_m^u(\beta_i^u, q_{mn}^r(t-i))$: remanufacturing cost of manufacturer m in period t, $i = 1, \ldots, \min\{t-1, Z-1\}$;

$c_{km}^u(t)$: transportation cost of used products undertaken by manufacturer m in period $t\,(t \geq 2)$ from demand market k to manufacturer m;

$c_m(q_m^u(t))$: disposal cost of manufacturer m dealing with the collected products $q_m^u(t)$ in period t $(t \geq 2)$;

$I_m(t)$: the inventory level of manufacturer m in period t; group all of $I_m(t)$ into a MT-dimension column vector I^M;

$H_m(t) = H_m(I_m(t))$: the inventory cost function of manufacturer m in period t;

$\rho_{sm}^r(t)$: the transaction price between supplier s and manufacturer m in period t, an endogenous variable;

$\rho_{mn}(t)$: the transaction price between manufacturer m and retailer n in period t, an endogenous variable;

$\overline{\rho}$: unit disposal cost of used products by manufacturers, a constant;

$c_{md}(t) = c_{md}(\beta_i^u, q_m^r(t))$: transportation cost generated from moving the wastes to landfill by manufacturer m in period t $(t \geq 2)$;

$\rho_{km}^u(t)$: the transaction price of used products between manufacturer m and demand market k in period t $(t \geq 2)$, an endogenous variable;

$c_{sm}^r(t) = c_{sm}^r(q_{sm}^r(t))$: the transaction cost undertaken by the supplier between manufacturer m and supplier s in period t;

$c_{mn}(t) = c_{mn}(q_{mn}(t))$: the transaction cost undertaken by the manufacturer between manufacturer m and retailer n in period t;

$c_n(q_{mn}(t))$: handling cost of retailer n, mainly referring to package and exhibition cost;

$c_{nk}^N(t) = c_{nk}^N(q_{nk}(t))$: the transaction cost between retailer n and demand market k in period t at retailer n;

$c_{nk}^{K}(t) = c_{nk}^{K}(q_{nk}(t))$: the transaction cost between retailer n and demand market k in period t at demand market k;

$\alpha_n(Q^4(t))$: the disutility function of consumer in demand market k when he sells the used products to manufacturers;

$\rho_{nk}(t)$: the price of unit product associated with retailer n and demand market k in period t, and all of $\rho_{nk}(t)$ are the elements of the NT-dimension column vector ρ_3;

$\rho_{4k}(t)$: the purchasing price of unit product associated with demand market k in period t; all of $\rho_{4k}(t)$ in period t are the elements of the K-dimension column vector $\rho_4(t)$, and all of $\rho_{4k}(t)$ belong to KT-dimension column vector ρ_4;

$d_k(\rho_4(t))$: the demand volume of the products with the demand price $\rho_{4k}(t)$ at retailer n in period t, which is a monotonic decreasing function depending on $\rho_{4k}(t)$;

e_x: the carbon emissions when manufacturing and remanufacturing, in which $x = r$ denotes the case of production by unit raw materials and $x = u$ denotes the case of remanufacturing by used products; in general, $e_u < e_r$ is satisfied;

$B_m(t)$: carbon emission cap of manufacturer m in period t; group all of the $B_m(t)$ into a MT-dimension column vector B;

$s_{sm}(t)$: the carbon tax paid by manufacturer m in period t, an endogenous variable; group all of the $s_{sm}(t)$ into a MT-dimension column vector s_s.

4. The Multiperiod CLSC Network Model

4.1. The Optimal Behavior of Suppliers. The supplier s supplies raw material to the manufactures in each period and seeks for the profit maximization in the planning horizons. Based on the notations described above, we can express the criterion for raw material supplier s as follows:

$$\pi_s\left(q_{sm}^{r*}(t), q_s^{r*}(t)\right) = \max\left\{\sum_{t=1}^{T}\sum_{m=1}^{M}\rho_{sm}^{r}(t)\,q_{sm}^{r}(t) - \sum_{t=1}^{T}\sum_{m=1}^{M}c_{sm}^{r}(t) - \sum_{t=1}^{T}f_s^{r}\left(q_1^{r}(t)\right)\right\} \quad (1)$$

$$\text{s.t.} \sum_{m=1}^{M} q_{sm}^{r}(t) \le q_s^{r}(t) \quad (2)$$

$$\left(q_{sm}^{r}(t), q_s^{r}(t)\right) \in R_{+}^{SM+S}, \quad \forall s, m. \quad (3)$$

The constraint (2) expresses that production amount of raw materials by supplier s is higher than that of the transaction between the supplier s and all the manufacturers.

It is assumed that all of the suppliers compete in a noncooperative fashion. Therefore, we can simultaneously express the optimal conditions of the suppliers as the variational inequality: determine $(Q^{1*}, q_1^{r*}, \eta_s) \in R_{+}^{SMT+2ST}$, satisfying

$$\begin{aligned}
&\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{m=1}^{M}\left[\frac{\partial c_{sm}^{r}(t)}{\partial q_{sm}^{r}(t)} - \rho_{sm}^{r}(t) + \eta_s(t)\right] \times \left[q_{sm}^{r}(t) - q_{sm}^{r*}(t)\right] \\
&+ \sum_{t=1}^{T}\sum_{s=1}^{S}\left[\frac{\partial f_s^{r}\left(q_s^{r}(t)\right)}{\partial q_s^{r}(t)} - \eta_s(t)\right] \times \left[q_s^{r}(t) - q_s^{r*}(t)\right] \\
&+ \sum_{t=1}^{T}\sum_{s=1}^{S}\left[q_s^{r}(t) - \sum_{m=1}^{M} q_{sm}^{r}(t)\right] \times \left[\eta_s(t) - \eta_s^{*}(t)\right] \ge 0, \\
&\forall \left(Q^1, q_1^{r}, \eta_s\right) \in R_{+}^{SMT+2ST}.
\end{aligned} \quad (4)$$

In (4), $\eta_s(t)$ is the Lagrange multiplier corresponding to constraint (2) and η_s is the ST-dimension column vector with the elements of $\eta_s(t)$.

4.2. The Optimal Behavior of Manufacturers. The manufacturer m purchases the raw material from suppliers to make products, sells his new products to the retailers in every period, and manages inventory between different periods according to market demand; in the same time, the manufacturer m collects the used products consumed by demand markets at the end of period t, which will be remanufactured in the next period $t+1$.

In the first planning period, the manufacturers just make products with virgin materials; from the second period, the manufacturers can make products with virgin materials and collected products simultaneously. With the consideration of the lifetime Z, the collected products can be remanufactured at most $Z-1$ times. When their remanufactured times arrive $Z-1$, they have no value anymore and must be disposed in an appropriate way.

For convenience, let $\beta_i = \beta_i^{u}\beta_{i-1}^{u}\cdots\beta_1^{u}$, $i = 1, \ldots, \min\{t-1, Z-1\}$. The profit function of manufacturer m can be stated as follows:

$$\begin{aligned}
&\pi_m\left(q_{sm}^{r}(t), q_m^{r}(t), q_m^{u}(t), q_{mn}(t), q_{mn}^{r}(t), q_{nm}^{u}(t), I_m(t)\right) \\
&= \max\left\{\sum_{t=1}^{T}\sum_{n=1}^{N}\rho_{mn}(t)\,q_{mn}(t) - \sum_{t=1}^{T}f_m^{r}(t) - \sum_{t=2}^{T}f_m^{u}(t)\right. \\
&\quad - \sum_{t=2}^{T}c_m\left(q_m^{u}(t)\right) - \sum_{t=1}^{T}\sum_{k=1}^{K}c_{nm}^{u}(t) - \sum_{t=1}^{T}H_m(t)
\end{aligned}$$

$$-\sum_{t=1}^{T}\sum_{m=1}^{M}\rho_{sm}^{r}(t)\,q_{sm}^{r}(t)$$
$$-\sum_{t=1}^{T}\sum_{k=1}^{K}\rho_{nm}^{u}(t)\,q_{nm}^{u}(t)-c_{mn}(t)$$
$$-\overline{\rho}\sum_{t=2}^{T}\sum_{i=1}^{\min\{t-1,Z-1\}}\beta_{i-1}\left(1-\beta_{i}^{u}\right)\alpha^{i}q_{mn}^{r}(t-i)$$
$$-\sum_{t=2}^{T}\sum_{i=1}^{\min\{t-1,Z-1\}}c_{md}(t)$$
$$\left.-\,e_{r}\sum_{t=1}^{T}s_{m}(t)\,q_{m}^{r}(t)-e_{u}\sum_{t=2}^{T}s_{m}(t)\,q_{m}^{u}(t)\right\} \tag{5}$$

$$\text{s.t. } I_{m}(t)+\sum_{n=1}^{N}q_{mn}(t)=\sum_{n=1}^{N}\sum_{i=1}^{\min(t-1,Z-1)}\beta_{i}\alpha^{i}q_{mn}^{r}(t-i)+\beta^{r}q_{m}^{r}+I_{m}(t-1) \tag{6}$$

$$q_{m}^{u}(t)=\sum_{n=1}^{N}q_{nm}^{u}(t-1),\quad \forall t\geq 2 \tag{7}$$

$$\sum_{n=1}^{N}q_{nm}^{u}(t-1)=\sum_{n=1}^{N}\sum_{i=1}^{\min\{t-1,Z-1\}}\beta_{i-1}\alpha^{i}q_{mn}^{r}(t-i)\quad \forall t\geq 2; \tag{8}$$

$$\sum_{n=1}^{N}q_{mn}(t)=\sum_{n=1}^{N}q_{mn}^{r}(t)+\sum_{n=1}^{N}\sum_{i=1}^{\min\{t-1,Z-1\}}\beta_{i}\alpha^{i}q_{mn}^{r}(t-i) \tag{9}$$

$$q_{m}^{r}(t)\leq\sum_{s=1}^{S}q_{sm}^{r}(t) \tag{10}$$

$$q_{sm}^{r}(t),q_{m}^{r}(t),q_{m}^{u}(t),q_{mn}(t),q_{mn}^{r}(t),q_{nm}^{u}(t),I_{m}(t)\geq 0,\quad \forall m,s,n. \tag{11}$$

Constraint (6) describes that the current inventory plus the products amount selling to all retailers is equal to the sum of the products amount made from used products, virgin materials, and the inventory from the prior period. Constraint (7) expresses that the manufacturer m collects the used products in period $t-1$ and remanufactures them in period t. The constraint (8) means that all the remanufacturable used products (in other words, their remanufacturing times are less than $Z-1$) are collected from the consumers by the manufacturer m at the end of period $t-1$. Constraint (9) expresses that, apart from remanufacturing the used products, the manufacturers still need to produce with raw materials in each period.

Similar to [6, 20], all the manufactures compete in a noncooperative fashion, and each manufacturer seeks to maximize his profit based on the other manufacturers' optimal decisions, that is, deciding the amount of raw materials purchased from the suppliers, the used products collected from the consumers, the products sold to retailers, and the inventory transferring to next period in every period. Therefore, the optimal conditions of all the manufacturers can be described by the following variational inequality: determine $(q^{r*},q^{u*},Q^{1*},Q^{2*},Q^{3*},Q^{4*},I^{M*},\gamma_1^*,\gamma_2^*,\gamma_3^*,\gamma_4^*,\gamma_5^*)\in\Omega^{M}$, satisfying

$$\sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{s=1}^{S}\left[\rho_{sm}^{r*}(t)-\gamma_{5m}^{*}(t)\right]\times\left[q_{sm}^{r}(t)-q_{sm}^{r*}(t)\right]$$
$$+\sum_{t=1}^{T}\sum_{m=1}^{M}\left[\frac{\partial f_{m}^{r}(t)}{\partial q_{m}^{r}(t)}-\beta^{r}\gamma_{1m}^{*}(t)+\gamma_{5m}^{*}(t)+e_{r}s_{m}^{*}(t)\right]\times\left[q_{m}^{r}(t)-q_{m}^{r*}(t)\right]$$
$$+\sum_{t=2}^{T}\sum_{m=1}^{M}\left[\frac{\partial c_{m}\left(q_{m}^{u*}(t)\right)}{\partial q_{m}^{u}(t)}+\gamma_{2m}^{*}(t-1)+e_{u}s_{m}^{*}(t)\right]\times\left[q_{m}^{u}(t)-q_{m}^{u*}(t)\right]$$
$$+\sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{n=1}^{N}\left[\frac{\partial c_{mn}(t)}{\partial q_{mn}(t)}+\gamma_{1m}^{*}(t)+\gamma_{4m}^{*}(t)-\rho_{mn}^{*}(t)\right]\times\left[q_{mn}(t)-q_{mn}^{*}(t)\right]$$
$$+\sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{m=1}^{M}\left[\frac{\partial c_{nm}^{u*}(t)}{\partial q_{nm}^{u}(t)}+\rho_{nm}^{u*}(t)-\gamma_{2m}^{*}(t)-\gamma_{3m}^{*}(t)\right]\times\left[q_{nm}^{u}(t)-q_{nm}^{u*}(t)\right]$$
$$+\sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{m=1}^{M}\left[\sum_{i=1}^{\min\{t,N-1\}}\left[\frac{\partial f_{m}^{u*}(t+i-1)}{\partial q_{mn}^{r}(t)}+\frac{c_{md}(t-i)}{\partial q_{mn}^{r}(t)}\right]-\sum_{i=0}^{\min\{t+1,Z-1\}}\beta_{i}\alpha^{i}\gamma_{4m}^{*}(t+i)-\sum_{i=1}^{\min\{t+1,Z-1\}}\beta_{i}\alpha^{i}\gamma_{1m}^{*}(t+i)+\overline{\rho}\times\sum_{i=1}^{\min\{t+1,Z-1\}}\beta_{i-1}\left(1-\beta_{i}^{u}\right)\alpha^{i}+\sum_{i=1}^{\min\{t+1,Z-1\}}\beta_{i-1}\alpha^{i}\gamma_{3m}^{*}(t+i-1)\right]\times\left[q_{mn}^{r}(t)-q_{mn}^{r*}(t)\right]$$
$$+\sum_{t=1}^{T}\sum_{m=1}^{M}\left[\frac{\partial H_{m}^{*}(t)}{\partial I_{m}(t)}+\gamma_{1m}^{*}(t)-\gamma_{1m}^{*}(t+1)\right]\times\left[I_{m}(t)-I_{m}^{*}(t)\right]$$

$$+\sum_{t=1}^{T}\sum_{m=1}^{M}\left[\sum_{n=1}^{N}\sum_{i=1}^{\min\{t-1,Z-1\}}\beta_i\alpha^i q_{mn}^{r*}(t-i)+\beta^r q_m^{r*}(t)+I_m^*(t-1)-\sum_{n=1}^{N}q_{mn}^*(t)-I_m^*(t)\right]\times\left[\gamma_{1m}(t)-\gamma_{1m}^*(t)\right]$$
$$+\sum_{t=1}^{T}\sum_{m=1}^{M}\left[\sum_{n=1}^{N}q_{nm}^{u*}(t)-q_m^{u*}(t+1)\right]\times\left[\gamma_{2m}(t)-\gamma_{2m}^*(t)\right]$$
$$+\sum_{t=2}^{T}\sum_{m=1}^{M}\left[\sum_{n=1}^{N}q_{nm}^{u*}(t-1)-\sum_{n=1}^{N}\sum_{i=1}^{\min\{t-1,Z-1\}}\beta_{i-1}\alpha^i q_{mn}^{r*}(t-i)\right]\times\left[\gamma_{3m}(t)-\gamma_{3m}^*(t)\right]$$
$$+\sum_{t=1}^{T}\sum_{m=1}^{M}\left[\sum_{n=1}^{N}q_{mn}^{r*}(t)+\sum_{n=1}^{N}\sum_{i=1}^{\min\{t-1,Z-1\}}\beta_i\alpha^i q_{mn}^{r*}(t-i)-\sum_{n=1}^{N}q_{mn}^*(t)\right]\times\left[\gamma_{4m}(t)-\gamma_{4m}^*(t)\right]$$
$$+\sum_{t=1}^{T}\sum_{m=1}^{M}\left[\sum_{s=1}^{S}q_{sm}^{r*}(t)-q_m^{r*}(t)\right]\times\left[\gamma_{5m}(t)-\gamma_{5m}^*(t)\right]$$
$$\geq 0$$
$$\forall\left(q^r,q^u,Q^1,Q^2,Q^3,Q^4,I^M,\gamma_1,\gamma_2,\gamma_3,\gamma_4,\gamma_5\right)\in\Omega^M, \tag{12}$$

where $\Omega^M = \{(q^r, q^u, Q^1, Q^2, Q^3, Q^4, I^M, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5) \in R_+^{MT+M(T-1)+MST+2MNT+KMT+MT} \times R^{4MT} \times R_+^{MT}\}$.

In (12), $\gamma_{1m}(t)$, $\gamma_{2m}(t)$, $\gamma_{3m}(t)$, $\gamma_{4m}(t)$, and $\gamma_{5m}(t)$ are the Lagrange multipliers corresponding to constraints (6), (7), (8), (9), and (10), and γ_1, γ_2, γ_3, γ_4, and γ_5 are the MT-dimension column vector with the elements of $\gamma_{1m}(t)$, $\gamma_{2m}(t)$, $\gamma_{3m}(t)$, $\gamma_{4m}(t)$, and $\gamma_{5m}(t)$, respectively.

From the forth term of (12), we can find that when the transaction amount $q_{mn}^*(t) > 0$, $\rho_{mn}^*(t) = (\partial c_{mn}^*(t)/\partial q_{mn}(t)) + \gamma_{1m}^*(t) + \gamma_{4m}^*(t)$.

In the model, we consider the manufacturers' carbon emission constraints from the government in each period, and then the manufacturers' behaviors can be characterized by the following equation:

$$B_m(t)-\left(e_r q_m^r(t)+e_u q_m^u(t)\right)\begin{cases}\geq 0, & s_{sm}(t)=0,\\ =0, & s_{sm}(t)>0.\end{cases} \tag{13}$$

Equation (13) describes that if the manufactures' carbon emission amount is lower than $B_m(t)$ in period t, the carbon tax is zero; on the other hand, if the manufactures' carbon emission amount is equal to $B_m(t)$ in period t, the manufacturers must pay taxes.

The above equilibrium condition corresponds to the following variational inequality: determine $s_s^* \in R_+^{MT}$, satisfying

$$\sum_{t=1}^{T}\sum_{m=1}^{M}\left[B_m^*(t)-\left(e_r q_m^{r*}(t)+e_u q_m^{u*}(t)\right)\right]\times\left[s_{sm}(t)-s_{sm}^*(t)\right]\geq 0,\quad \forall s_s\in R_+^{MT}. \tag{14}$$

In a word, the manufacturers' optimal behaviors must meet (10) and (14) simultaneously.

4.3. *The Optimal Behavior of Retailers.*

The retailers need to decide how many products to purchase from every manufacturer and to sell to the consumers in demand markets to meet the random demand in markets.

For retailer n, the profits can be defined as follows:

$$\pi_n\left(Q^2,Q^5\right)=\max\left(\sum_{t=1}^{T}\rho_{nk}(t)\,q_{nk}(t)-\sum_{t=1}^{T}c_n(t)-\sum_{t=1}^{T}\sum_{m=1}^{M}\rho_{mn}(t)\,q_{mn}(t)-\sum_{t=1}^{T}c_{nk}^{N}(t)\right) \tag{15}$$

$$\text{s.t. } q_{nk}(t)\leq\sum_{m=1}^{M}q_{mn}(t) \tag{16}$$

$$q_{mn}(t),q_{nk}(t)\geq 0,\quad \forall m,n,k. \tag{17}$$

We also assume that the retailers compete in a noncooperative manner. The optimal behavior of each retailer is to maximize his profit when the other retailers' behaviors are given. So, the optimal conditions of all retailers can be described by variational inequality: determine $(Q^{2*}, Q^{5*}, \mu^*) \in \Omega^N$, satisfying

$$\sum_{t=1}^{T}\sum_{n=1}^{N}\left[\frac{c_{nk}^{N*}(t)}{\partial q_{nk}(t)}-\rho_{nk}^*(t)+\mu_n^*(t)\right]\times\left[q_{nk}(t)-q_{nk}^*(t)\right]$$
$$+\sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{m=1}^{M}\left[\rho_{mn}^*(t)+\frac{\partial c_n^*(t)}{\partial q_{mn}}-\mu_n^*(t)\right]$$

$$\times \left[q_{mn}(t) - q_{mn}^{*}(t)\right] + \sum_{t=1}^{T}\sum_{n=1}^{N}\left[\sum_{m=1}^{M} q_{mn}(t) - q_{nk}(t)\right] \times \left[\mu_n(t) - \mu_n^{*}(t)\right] \geq 0$$

$$\forall \left(Q^2, Q^5, \mu\right) \in \Omega^N, \tag{18}$$

where $\Omega^N = \{(Q^2, Q^5, \mu) \in R_{+}^{MNT+NKT+NT}\}$.

In (18), $\mu_n(t)$ is the Lagrange multiplier corresponding to constraints (16) and μ is the NT-dimension column vector with the elements of $\mu_n(t)$.

From the first term of (18), we find that when the transaction volume $q_{nk}(t) > 0$, $\rho_{nk}^{*}(t) = (c_{nk}^{N*}(t)/\partial q_{nk}(t)) + \mu_n^{*}(t)$.

4.4. The Optimal Behavior of Demand Markets. In the forward supply chain, the consumers of each demand market decide to purchase the products from the retailer or not based on the appreciation level of products and also decide the purchasing price and amount.

According to literature [19], the transaction between the demand market and the retailer should satisfy a complement condition; that is, for some demand market k,

$$d_k\left(\rho_4(t)\right) \begin{cases} = \sum_{n=1}^{N} q_{nk}(t), & \rho_{4k}(t) > 0, \\ \leq \sum_{n=1}^{N} q_{nk}(t), & \rho_{4k}(t) = 0. \end{cases} \tag{19}$$

Equation (19) illustrates that if the product amount supplied by each retailer is equal to that of demand in markets, then the transaction can proceed and the transaction price is positive. The demand amount depends on the transaction price paid by consumers in demand market k.

And also similar to [19], in the reverse supply chain, the demand markets' behavior should be characterized by the following equation:

$$\alpha_k\left(Q^{4*}(t)\right) \begin{cases} = \rho_{km}^{u*}(t), & q_{km}^{u}(t) > 0, \\ \geq \rho_{km}^{u*}(t), & q_{km}^{u}(t) = 0, \end{cases} \tag{20}$$

where $\alpha_n(Q^4(t))$ is the disutility of consumers in the demand market n at period t. Equation (20) expresses that the higher collection price paid by manufacturers, the more used products the manufacturers collect. Moreover, the collected amount is lower than or equal to that of the products sold to the demand market, so the following constraint should be satisfied:

$$\sum_{m=1}^{M} q_{km}^{u}(t) \leq \sum_{n=1}^{N} q_{nk}(t). \tag{21}$$

Let $\lambda_k(t)$ denote the Lagrange multiplier corresponding to (21), group all of $\lambda_k(t)$ into a KT-dimension column vector λ.

Besides the purchasing price paying to the retailers, the consumers must pay the transaction cost to obtain the products; hence, we have

$$\rho_{nk}^{*}(t) + c_{nk}^{K*}(t) \begin{cases} = \rho_{4k}(t), & q_{nk}^{*}(t) > 0, \\ \geq \rho_{4k}(t), & q_{nk}^{*}(t) = 0. \end{cases} \tag{22}$$

Equations (19), (20), (21), and (22) can be described by the following variational inequality: determine $(Q^{4*}, Q^{5*}, \rho_4^{*}, \lambda^{*}) \in \Omega^K$, satisfying

$$\begin{aligned} &\sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{m=1}^{M}\left[\alpha_n\left(Q^{4*}(t)\right) - \rho_{km}^{u*}(t) + \lambda_k^{*}(t)\right] \times \left[q_{km}^{u}(t) - q_{km}^{u*}(t)\right] \\ &+ \sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{k=1}^{K}\left[\rho_{nk}^{*}(t) + c_{nk}^{K*}(t) - \lambda_k^{*}(t)\right] \times \left[q_{nk}(t) - q_{nk}^{*}(t)\right] \\ &+ \sum_{t=1}^{T}\sum_{k=1}^{K}\left[\sum_{n=1}^{N} q_{nk}^{*}(t) - d_k\left(\rho_4^{*}(t)\right)\right] \times \left[\rho_{4k}(t) - \rho_{4k}^{*}(t)\right] \\ &+ \sum_{t=1}^{T}\sum_{k=1}^{K}\left[\sum_{n=1}^{N} q_{nk}^{*}(t) - \sum_{m=1}^{M} q_{km}^{u*}(t)\right] \times \left[\lambda_k(t) - \lambda_k^{*}(t)\right] \geq 0, \\ &\forall \left(Q^4, Q^5, \rho, \lambda\right) \in \Omega^K, \end{aligned} \tag{23}$$

where $\Omega^K = \{(Q^4, Q^5, \rho_4, \lambda) \in R_{+}^{NMT+NKT+2NT}\}$.

From the first term of inequality (23), we can conclude that when the transaction amount of collected products $q_{nm}^{u}(t) \geq 0$, the transaction price of collected products between manufacturers and demand markets $\rho_{nm}^{u*}(t) = \alpha_n(Q^{4*}(t)) + \lambda_n^{*}(t)$. In fact, since $\sum_{m=1}^{M} q_{mn}(t) > \sum_{m=1}^{M} q_{nm}^{u}(t)$ and $\lambda_n^{*}(t) = 0$, we can get $\rho_{nm}^{u*}(t) = \alpha_n(Q^{4*}(t))$.

4.5. The Equilibrium Conditions of Multiperiod CLSC Network

Theorem 1. *In equilibrium of the multiperiod CLSC network, the suppliers' optimal behaviors, as expressed by variational inequality* (4)*, the manufacturers' optimal behaviors, as expressed by variational inequality* (12) *and* (14)*, the retailers' optimal behaviors, as expressed by variational inequality* (18)*, and the consumers' optimal behaviors, as expressed by variational inequality* (23)*, must be satisfied simultaneously. One can state it in the following variational inequality: determine*

$(q_1^{r*}, q^{r*}, q^{u*}, Q^{1*}, Q^{2*}, Q^{3*}, Q^{4*}, I^{M*}, s_s^*, \rho^*, \eta_s^*, \gamma_1^*, \gamma_2^*, \gamma_3^*, \gamma_4^*, \gamma_5^*, \mu^*, \lambda^*) \in \Omega$, *satisfying*

$$\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{m=1}^{M}\left[\frac{\partial c_{sm}^{r*}(t)}{\partial q_{sm}^{r}(t)} + \eta_s^*(t) - \gamma_{5m}^*(t)\right] \times \left[q_{sm}^{r}(t) - q_{sm}^{r*}(t)\right]$$
$$+ \sum_{t=1}^{T}\sum_{s=1}^{S}\left[\frac{\partial f_s^{r*}(q_s^r(t))}{\partial q_s^r(t)} - \eta_s^*(t)\right] \times \left[q_s^r(t) - q_s^{r*}(t)\right]$$
$$+ \sum_{t=1}^{T}\sum_{m=1}^{M}\left[\frac{\partial f_m^{r*}(t)}{\partial q_m^r(t)} - \beta^r\gamma_{1m}^*(t) + e_r s_m^*(t) + \gamma_{5m}^*(t)\right] \times \left[q_m^r(t) - q_m^{r*}(t)\right]$$
$$+ \sum_{t=2}^{T}\sum_{m=1}^{M}\left[\frac{\partial c_m(q_m^{u*}(t))}{\partial q_m^u(t)} + \gamma_{2m}^*(t-1) + e_u s_m^*(t)\right] \times \left[q_m^u(t) - q_m^{u*}(t)\right]$$
$$+ \sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{n=1}^{N}\left[\frac{\partial c_{mn}^*(t)}{q_{mn}(t)} + \frac{\partial c_n^*(t)}{\partial q_{mn}} + \gamma_{1m}^*(t) + \gamma_{4m}^*(t) - \lambda_k^*(t) - \mu_n^*(t)\right] \times \left[q_{mn}(t) - q_{mn}^*(t)\right]$$
$$+ \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{m=1}^{M}\left[\frac{\partial c_{km}^{u*}(t)}{\partial q_{km}^u(t)} - \gamma_{2m}^*(t) - \gamma_{3m}^*(t) + \alpha_n\left(Q^{4*}(t)\right) + \lambda_k^*(t)\right] \times \left[q_{km}^u(t) - q_{km}^{u*}(t)\right]$$
$$+ \sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{m=1}^{M}\left[\sum_{i=1}^{\min\{t,N-1\}}\left[\frac{\partial f_m^{u*}(t+i-1)}{\partial q_{mn}^r(t)} + \frac{c_{md}(t-i)}{\partial q_{mn}^r(t)}\right] - \sum_{i=0}^{\min\{t+1,Z-1\}}\beta_i\alpha^i\gamma_{4m}^*(t+i) - \sum_{i=1}^{\min\{t+1,Z-1\}}\beta_i\alpha^i\gamma_{1m}^*(t+i) + \overline{\rho}\sum_{i=1}^{\min\{t+1,Z-1\}}\beta_{i-1}\left(1-\beta_i^u\right)\alpha^i + \sum_{i=1}^{\min\{t+1,Z-1\}}\beta_{i-1}\alpha^i\gamma_{3m}^*(t+i-1)\right] \times \left[q_{mn}^r(t) - q_{mn}^{r*}(t)\right]$$
$$+ \sum_{t=1}^{T}\sum_{m=1}^{M}\left[\frac{\partial H_m^*(t)}{\partial I_m(t)} + \gamma_{1m}^*(t) - \gamma_{1m}^*(t+1)\right] \times \left[I_m(t) - I_m^*(t)\right]$$
$$+ \sum_{t=1}^{T}\sum_{m=1}^{M}\left[B_m(t) - \left(e_r q_m^{r*}(t) + e_u q_m^{u*}(t)\right)\right] \times \left[s_{sm}(t) - s_{sm}^*(t)\right]$$
$$+ \sum_{t=1}^{T}\sum_{n=1}^{N}\left[\frac{c_{nk}^{N*}(t)}{\partial q_{nk}(t)} + c_{nk}^{K*}(t) - \lambda_k^*(t) + \mu_n^*(t)\right] \times \left[q_{nk}(t) - q_{nk}^*(t)\right]$$
$$+ \sum_{t=1}^{T}\sum_{k=1}^{K}\left[\sum_{n=1}^{N} q_{nk}^*(t) - d_k\left(\rho_4^*(t)\right)\right] \times \left[\rho_{4k}(t) - \rho_{4k}^*(t)\right]$$
$$+ \sum_{t=1}^{T}\sum_{s=1}^{S}\left[q_s^{r*}(t) - \sum_{m=1}^{M} q_{sm}^{r*}(t)\right] \times \left[\eta_s(t) - \eta_s^*(t)\right]$$
$$+ \sum_{t=1}^{T}\sum_{m=1}^{M}\left[\sum_{n=1}^{N}\sum_{i=1}^{\min\{t-1,Z-1\}}\beta_i\alpha^i q_{mn}^{r*}(t-i) + \beta^r q_m^{r*}(t) + I_m^*(t-1) - \sum_{n=1}^{N} q_{mn}^*(t) - I_m^*(t)\right] \times \left[\gamma_{1m}(t) - \gamma_{1m}^*(t)\right]$$
$$+ \sum_{t=1}^{T}\sum_{m=1}^{M}\left[\sum_{n=1}^{N} q_{nm}^{u*}(t) - q_m^{u*}(t+1)\right] \times \left[\gamma_{2m}(t) - \gamma_{2m}^*(t)\right]$$
$$+ \sum_{t=2}^{T}\sum_{m=1}^{M}\left[\sum_{n=1}^{N} q_{nm}^{u*}(t-1) - \sum_{n=1}^{N}\sum_{i=1}^{\min\{t-1,Z-1\}}\beta_{i-1}\alpha^i q_{mn}^{r*}(t-i)\right] \times \left[\gamma_{3m}(t) - \gamma_{3m}^*(t)\right]$$
$$+ \sum_{t=1}^{T}\sum_{m=1}^{M}\left[\sum_{n=1}^{N} q_{mn}^{r*}(t) + \sum_{n=1}^{N}\sum_{i=1}^{\min\{t-1,Z-1\}}\beta_i\alpha^i q_{mn}^{r*}(t-i) - \sum_{n=1}^{N} q_{mn}^*(t)\right] \times \left[\gamma_{4m}(t) - \gamma_{4m}^*(t)\right]$$
$$+ \sum_{t=1}^{T}\sum_{m=1}^{M}\left[\sum_{s=1}^{S} q_{sm}^{r*}(t) - q_m^{r*}(t)\right]$$

$$\times \left[\gamma_{5m}(t) - \gamma_{5m}^{*}(t)\right] + \sum_{t=1}^{T}\sum_{n=1}^{N}\left[\sum_{m=1}^{M} q_{mn}(t) - q_{nk}(t)\right] \times \left[\mu_n(t) - \mu_n^{*}(t)\right] + \sum_{t=1}^{T}\sum_{k=1}^{K}\left[q_{nk}^{*}(t) - \sum_{m=1}^{M} q_{km}^{u*}(t)\right] \times \left[\lambda_k(t) - \lambda_k^{*}(t)\right] \geq 0,$$
$$\forall \left(q_1^r, q^r, q^u, Q^1, Q^2, Q^3, Q^4, I^M, s_s, v, \rho, \eta_s, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \mu, \lambda\right) \in \Omega, \tag{24}$$

where $\Omega = \Omega^S \times \Omega^M \times \Omega^N \times \Omega^K$.

4.6. Solving Algorithm and Parameter Setting. For easy formulation hereinafter, we group the terms of the multiplication signs in inequality (24) into a column vector $F(y) = \{F_{smt}^1, F_{st}^2, F_{mt}^3, F_{m(t-1)}^4, F_{mnt}^5, F_{mnt}^6, F_{kmt}^7, F_{mt}^8, F_{mt}^9, F_{nt}^{10}, F_{nt}^{11}, F_{st}^{12}, F_{mt}^{13}, F_{mt}^{14}, F_{mt}^{15}, F_{mt}^{16}, F_{mt}^{17}, F_{nt}^{18}, F_{nt}^{19}\}_{\forall s,m,n,k,t}$ and introduce $y = (q_1^r, q^r, q^u, Q^1, Q^2, Q^3, Q^4, I^M, s_s, v, \rho, \eta_s, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \mu, \lambda) \in \Omega$; thus, we can rewrite the variational inequality problem (24) in the standard form: determine $y^* = (q_1^{r*}, q^{r*}, q^{u*}, Q^{1*}, Q^{2*}, Q^{3*}, Q^{4*}, I^{M*}, s_s^*, v^*, \rho^*, \eta_s^*, \gamma_1^*, \gamma_2^*, \gamma_3^*, \gamma_4^*, \gamma_5^*, \mu^*, \lambda^*)$, satisfying

$$F(y)^T \cdot (y - y^*) \geq 0, \quad \forall y \in \Omega. \tag{25}$$

In order to solve the variational inequality (25), we have several algorithms to select, for example, the modified project contraction algorithm [31], logarithmic-quadratic proximal prediction-correction method [32]. In this paper, we employ the modified project contraction algorithm to solve the problem for two reasons. Firstly, the modified project contraction algorithm provides an iterative solution framework that can deal with variational inequalities defined on polyhedral sets and get the optimal solutions and Lagrangian multipliers simultaneously. So, it satisfies all the requirements of a solution method for our model. Secondly, the design of its iterative steps is much simpler than that of logarithmic-quadratic proximal prediction-correction method. Set the related parameters of the modified project contraction algorithm as follows: the original value of decision variables and Lagrange multipliers is set to 1; the convergence criterion, for example, the absolute value of difference of decision variables and Lagrange multipliers between two steps, is lower than or equal to 10^{-8}.

5. Numerical Examples

Now, we consider a CLSC network with two suppliers, two manufacturers, two retailers, and two demand markets, the lifetime of products is two or three, and the planning horizon is five. In this construction, we give some numerical examples to illustrate the efficiency of our model and analyze the influence of some parameters to obtain managerial insights. The related cost functions are given as follows.

Suppliers' raw material producing cost functions:

$$f_s^r\left(q_1^r(t)\right) = (1+4t)\, q_s^r(t)^2 + q_s^r(t) + 1, \quad s = 1,2; \quad t = 1,2,3,4,5; \tag{26}$$

manufacturers' production cost functions using raw material:

$$f_m^r(t) = (0.5+1.5t)\left(\beta^r q_m^r(t)\right)^2 + \beta^r q_m^r(t)\, \beta^r q_{3-m}^r(t) + 2\beta^r q_m^r(t), \quad m = 1,2; \quad t = 1,2,3,4,5; \tag{27}$$

manufacturers' remanufacturing cost functions using collected products:

$$f_m^u(2) = 4.5\left(\beta_1^u \alpha \sum_{n=1}^{2} q_{mn}^r(1)\right)^2 + 1.5\left(\beta_1^u \alpha \sum_{n=1}^{2} q_{mn}^r(1)\right), \quad m = 1,2;$$
$$f_m^u(t) = (4.5-0.5t) \times \left(\beta_1^u \alpha \sum_{n=1}^{2} q_{mn}^r(t-1) + 2\delta \beta_2^u \beta_1^u \alpha^2 \sum_{n=1}^{2} q_{mn}^r(t-2)\right)^2 + 1.5\left(\beta_1^u \alpha \sum_{n=1}^{2} q_{mn}^r(t-1) + 2\delta \beta_2^u \beta_1^u \alpha^2 \sum_{n=1}^{2} q_{mn}^r(t-2)\right), \quad m = 1,2; \quad t = 3,4,5; \tag{28}$$

the manufacturers' inventory cost functions:

$$H_m(t) = \left(2 + (-1)^t 0.5\right) I_m(t), \quad m = 1,2; \quad t = 1,2,3,4,5; \tag{29}$$

the manufacturers' disposal cost functions dealing with the collected products:

$$c_m\left(q_m^u(t)\right) = 2.5 q_m^u(t)^2 + q_m^u(t) + 2, \quad m = 1,2; \quad t = 2,3,4,5; \tag{30}$$

the transportation cost functions from demand markets to manufacturers:

$$c_{nm}^u(t) = 0.5 q_{nm}^u(t)^2 + 3 q_{nm}^u(t) + 1, \quad m = 1,2; \quad n = 1,2; \quad t = 2,3,4,5; \tag{31}$$

the manufacturers' transportation cost functions to move the waste of disposal products to landfill:

$$\begin{aligned} c_{md}(2) = 0.5\left[(1-\beta_1^u)\,\alpha\sum_{n=1}^{2}q_{mn}^r(1)\right]^2 \\ + 3.5(1-\beta_1^u)\,\alpha\sum_{n=1}^{2}q_{mn}^r(1), \\ c_{md}(t) = 0.5\Bigg[\delta(1-\beta_2^u)\,\beta_1^u\alpha^2\sum_{n=1}^{2}q_{mn}^r(t-2) \\ + (1-\beta_1^u)\,\alpha\sum_{n=1}^{2}q_{mn}^r(t-1)\Bigg]^2 \\ + 3.5\Bigg[\delta(1-\beta_2^u)\,\beta_1^u\alpha^2\sum_{n=1}^{2}q_{mn}^r(t-2) + (1-\beta_1^u)\,\alpha \\ \times\sum_{n=1}^{2}q_{mn}^r(t-1)\Bigg] \\ \text{for } t = 3,4,5;\quad m = 1,2; \end{aligned} \tag{32}$$

the consumers' disutility functions in demand markets:

$$\begin{aligned} \alpha_n\left(Q^4(t)\right) = 0.5\sum_{m=1}^{2}\sum_{n=1}^{2}q_{nm}^u(t), \\ \text{for } n = 1,2;\quad t = 1,2,3,4,5; \end{aligned} \tag{33}$$

the transaction cost functions undertaken by the supplier between suppliers and manufacturers:

$$\begin{aligned} c_{sm}^r(t) = 0.5\left(q_{sm}^r(t)\right)^2 + q_{sm}^r(t), \\ \text{for } s = 1,2;\quad m = 1,2;\quad t = 1,2,3,4,5; \end{aligned} \tag{34}$$

the transaction cost functions undertaken by the manufacturer between manufacturers and retailers:

$$\begin{aligned} c_{mn}(t) = \left(q_{mn}(t)\right)^2 + 3q_{mn}(t) + 0.5, \\ \text{for } m = 1,2;\quad n = 1,2;\quad t = 1,2,3,4,5; \end{aligned} \tag{35}$$

the disposal functions of retailers:

$$\begin{aligned} c_n\left(Q^2\right) = 0.25\left(\sum_{m=1}^{M}q_{mn}(t)\right)^2, \\ \text{for } n = 1,2;\quad t = 1,2,3,4,5; \end{aligned} \tag{36}$$

the transaction cost functions undertaken by retailer n to demand market k:

$$\begin{aligned} c_{nk}^N(t) = 0.5\left(q_{nk}(t)\right)^2 + q_{nk}(t) + 1, \\ n = 1,2;\quad k = 1,2;\quad t = 1,2,3,4,5; \end{aligned} \tag{37}$$

the transaction cost functions undertaken by demand markets purchasing products from retailers:

$$\begin{aligned} c_{nk}^K(t) = q_{nk}(t) + 0.5, \\ n = 1,2;\quad t = 1,2,3,4,5; \end{aligned} \tag{38}$$

the demand functions of the consumer markets:

$$\begin{aligned} d_k\left(\rho_t^D\right) = -(2.2-0.4t)\,\rho_{3-k}^D(t) - 2\rho_k^D(t) + 120, \\ k = 1,2,\quad t = 1,2,3,4,5. \end{aligned} \tag{39}$$

In order to illustrate the impact of lifetime on equilibrium results, we introduce δ, a Boolean parameter, to represent two cases of lifetime values: $\delta = 0$ denotes that the product lifetime is two, and $\delta = 1$ denotes that the product lifetime is three. The total planning horizon is five.

5.1. Numerical Example 1. When $\delta = 0$, for example, the product lifetime is 2, $\overline{\rho} = 1$, $\beta^r = 1$, $\beta_1^u = 0.9$, and $\beta_2^u = 0.6$, we analyze the effect of the collection rate α on the equilibrium results given in Table 1, in which the data in each column are the equilibrium solutions corresponding to a value of α and the data in each row are the optimal values of main variables.

From Table 1, we can find that when α increases, the volume of raw material q_s^r (equal to q_m^r) supplied to manufacturers, the used products q_m^u collected by manufacturers, and the transaction q_{mn} between manufacturers and retailers increase too, so the increase of α stimulates the transaction of the network. Moreover, in order to obtain and remanufacture more used products, the manufacturers and retailers should sell more products and hence collect more used products at the end of each period, so that the transaction price ρ_n between retailers and demand markets is decreasing all the time except the last period. This point characterizes the dynamic network fully in which the players are foresighted rather than shortsighted.

From Table 1, we also find that $I_m(t)$ decreases when α increases. It is because of the transaction volume increasing, and the manufacturers have no motivation to transfer more inventories to next period, and $I_m(5) = 0$ means all the surplus products are sold out in the last period.

In the following, we analyze the impact of α increasing on the profits of players in the CLSC network. The suppliers' and retailers' profits increase because of their transaction volume with manufacturers increasing. The profits of manufacturers increase at the beginning but then decrease, and the maximum values appear at about the point 0.15 of α, so we must explain this interesting phenomena. When α increases, the amount of collected products also increases; therefore, the collection price increases; that is, the manufacturers must pay more to demand markets for obtaining extra used products. When the collection price reaches a threshold, the cost saving via remanufacturing cannot make up the increased collection cost, so the manufacturers' profits reach the maximum and then decrease when the price continues to increase.

5.2. Numerical Example 2. When $\delta = 1$, for example, the product lifetime is 3, $\overline{\rho} = 1$, $\beta^r = 1$, $\beta_1^u = 0.9$, and $\beta_2^u = 0.6$, we

Table 1: $\delta = 0$, the CLSC network equilibrium results when α increases.

α	0.05	0.1	0.15	0.2	0.25	0.3
q_s^r, q_m^r $s = 1, 2;$ $m = 1, 2$	3.0927	3.1929	3.2862	3.3723	3.4510	3.5219
	1.9756	2.0357	2.0917	2.1434	2.1906	2.2332
	1.5540	1.5970	1.6369	1.6738	1.7076	1.7380
	1.2753	1.3087	1.3398	1.3685	1.3948	1.4184
	1.1344	1.1617	1.1871	1.2106	1.2321	1.2514
q_m^u $m = 1, 2$	0.0162	0.0463	0.0901	0.1469	0.2161	0.2965
	0.0460	0.1060	0.1781	0.2612	0.3552	0.4614
	0.0737	0.1595	0.2537	0.3517	0.4480	0.5358
	0.1339	0.2871	0.4638	0.6695	0.9106	1.1957
q_{mn}^r $m = 1, 2;$ $n = 1, 2$	0.1620	0.2317	0.3002	0.3672	0.4322	0.4941
	0.4599	0.5299	0.5937	0.6530	0.7104	0.7690
	0.7374	0.7977	0.8455	0.8792	0.8961	0.8929
	1.3393	1.4353	1.5460	1.6737	1.8212	1.9928
	1.8174	1.6534	1.4854	1.3112	1.1281	0.9325
q_{mn} $m = 1, 2;$ $n = 1, 2$	0.1620	0.2317	0.3002	0.3672	0.4322	0.4941
	0.4672	0.5507	0.6342	0.7191	0.8076	0.9024
	0.7581	0.8454	0.9257	0.9967	1.0559	1.1006
	1.3725	1.5071	1.6602	1.8319	2.0229	2.2339
	1.8776	1.7826	1.6942	1.6125	1.5379	1.4706
I_m $m = 1, 2$	2.7687	2.7295	2.6858	2.6378	2.5866	2.5337
	3.8245	3.7055	3.5901	3.4751	3.3564	3.2288
	3.9037	3.7070	3.5359	3.3906	3.2718	3.1809
	2.5003	2.1451	1.7837	1.4118	1.0241	0.6136
	0	0	0	0	0	0
ρ_n $n = 1, 2$	30.6697	30.4936	30.3205	30.1512	29.9870	29.8307
	33.5850	33.3687	33.1526	32.9328	32.7037	32.4584
	37.4783	37.2456	37.0315	36.8421	36.6842	36.5651
	41.8530	41.4803	41.0565	40.5809	40.0521	39.4675
	48.5833	48.8597	49.1170	49.3546	49.5717	49.7674
ρ_{km}^u $k = 1, 2$	0.0162	0.0463	0.0901	0.1469	0.2161	0.2965
	0.0460	0.1060	0.1781	0.2612	0.3552	0.4614
	0.0737	0.1595	0.2537	0.3517	0.4480	0.5358
	0.1339	0.2871	0.4638	0.6695	0.9106	1.1957
	0	0	0	0	0	0
$\pi_s, s = 1, 2$	32.2546	34.7208	37.0795	39.3119	41.3978	43.3149
$\pi_m, m = 1, 2$	75.9138	77.0719	77.1677	75.8899	72.8294	67.4213
$\pi_n, n = 1, 2$	18.6856	19.5621	20.9267	22.8044	25.2330	28.2688
Total profit	126.8549	131.3548	135.1739	138.0062	139.4602	139.0050

analyze the impact of the collection rate α on the equilibrium results. The equilibrium results are shown in Table 2.

In Table 2, the changing trends of some variables such as q_s^r, q_m^r, q_m^u, q_{mn}, I_m, and profits of suppliers, manufacturers, and retailers are similar to those in Table 1 depending on α, and, in turn, we will pay more attention to the differences between the two tables.

In respect to the profits of suppliers and retailers, there are changes but not large in the course of α increasing. But the profits of manufacturers change dramatically. The maximum value of manufacturers' profit is 77.1677 in Table 1, which is at about the point 0.1 of α and larger than the corresponding value 76.3586 in Table 2. This phenomenon shows that the smaller the product lifetime is, the more profits the manufacturers can earn, which means that the environmental and resource targets are different from those of the enterprise to seek maximized profits.

For CLSC, the maximum total profits are 139.4602 and 136.4576 at the points 0.25 and 0.2 of α when $\delta = 0$ and $\delta = 1$, respectively. The maximum profits of the whole CLSC and the manufacturers appear in different points of α, so the objects of the manufactures and the whole CLSC are also different. If the manufacturers can give up a part of interests, then the performance of CLSC will be better.

Moreover, we compare the CLSC profits in two cases of $\delta = 0$ and $\delta = 1$ when α is set to be a same value. It can be seen that when $\alpha \leq 0.1$, the CLSC profit with $Z = 3$ is larger than that with $Z = 2$. However, when $\alpha \geq 0.15$, the quantitative relationship between the two is opposite. The phenomenon shows clearly that more product lifetime is not always a benefit to the CLSC. Too high collection rate will result in the fact that the collection burden of the manufacturers aggravates and the operational efficiency of the CLSC reduces, although it contributes to the cost saving by remanufacturing. Therefore, the government should set the collection rate in a suitable region based on the manufacturers' collection cost structure and benefits from remanufacturing.

Table 2: $\delta = 1$, the CLSC network equilibrium states when α increases.

α	0.05	0.1	0.15	0.2	0.25	0.3
q_s^r, q_m^r $s = 1, 2$; $m = 1, 2$	3.1073 1.9844 1.5603 1.2802 1.1384	3.2281 2.0568 1.6120 1.3205 1.1713	3.3470 2.1282 1.6630 1.3601 1.2037	3.4627 2.1976 1.7126 1.3987 1.2353	3.5734 2.2641 1.7600 1.4356 1.2655	3.6772 2.3263 1.8045 1.4702 1.2938
q_m^u $m = 1, 2$	0.0177 0.0483 0.0778 0.1358	0.0530 0.1175 0.1802 0.2946	0.1070 0.2087 0.3105 0.4802	0.1805 0.3222 0.4721 0.6971	0.2738 0.4572 0.6677 0.9499	0.3870 0.6116 0.8996 1.2440
q_{mn}^r $m = 1, 2$; $n = 1, 2$	0.1766 0.4748 0.7564 1.3243 1.8031	0.2652 0.5637 0.8500 1.3965 1.6190	0.3568 0.6475 0.9476 1.4729 1.4261	0.4512 0.7242 1.0498 1.5537 1.2244	0.5476 0.7912 1.1573 1.6393 1.0139	0.6450 0.8452 1.2711 1.7302 0.7944
q_{mn} $m = 1, 2$; $n = 1, 2$	0.1766 0.4828 0.7780 1.3590 1.8637	0.2652 0.5876 0.9022 1.4760 1.7492	0.3568 0.6957 1.0394 1.6087 1.6365	0.4512 0.8054 1.1899 1.7583 1.5268	0.5476 0.9144 1.3538 1.9264 1.4218	0.6450 1.0194 1.5306 2.1145 1.3234
I_m $m = 1, 2$	2.7541 3.7888 3.8363 2.4679 0	2.6977 3.6272 3.5392 2.0666 0	2.6333 3.4665 3.2343 1.6486 0	2.5603 3.3094 2.9224 1.2136 0	2.4783 3.1598 2.6053 0.7622 0	2.3871 3.0230 2.2853 0.2951 0
$\pi_s, s = 1, 2$	32.6103	35.6032	38.6502	41.7106	44.7290	47.6346
$\pi_m, m = 1, 2$	75.7360	76.3586	75.2496	71.6741	64.7021	53.2055
$\pi_n, n = 1, 2$	18.5700	19.4041	20.8727	23.0729	26.1039	30.0609
Total profit	126.9163	131.3659	134.7725	136.4576	135.5350	130.9010

5.3. Numerical Example 3. When $B_m(1) = 3$, $\delta = 0$, $\overline{\rho} = 1$, $\beta^r = 1$, $\beta_1^u = 0.9$, and $\beta_2^u = 0.6$, we analyze the impact of the collection rate α on the equilibrium results. The profits of various players in CLSC are shown in Figure 2.

From Figure 2, we can make a conclusion that as α increases, the profits of suppliers and retailers increase while the profit of manufacturers increases at the beginning and then decreases. This is the same as the above analysis in Table 1. The maximum value of the profit of manufacturer differs at another point of α; that is, the maximum value point in Table 1 is at $\alpha = 0.15$ but at $\alpha = 0.1$ in Figure 2. This is because of the existence of carbon emission cap in the 1st period; the profits of all players are lower than that without carbon emission cap by comparing Figure 2 with Table 1.

5.4. Numerical Example 4. When $\alpha = 0.25$ and $\delta = 0$, $\overline{\rho} = 1$, $\beta^r = 1$, $\beta_1^u = 0.9$, and $\beta_2^u = 0.6$, we analyze the effect of different carbon emission caps of the first 3 periods on the equilibrium results shown in Table 3.

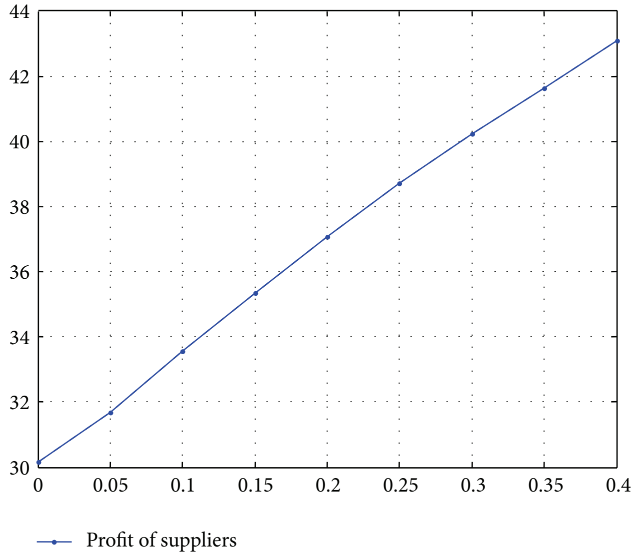
Table 3 presents main variables associated with different carbon caps. As mentioned above, s_m are the Lagrange multiplier associated with the carbon cap, and $s_m > 0$ means that the carbon emission reaches the cap. We compare the first 2 columns in Table 3 and note that when the carbon emission cap of the 1st period becomes smaller, Lagrange multiplier associated with the 1st period is larger, while the amounts of raw material supplied to the manufacturers are larger except in the 1st period due to the carbon emission constraints. The amount of collected products of the manufacturers is smaller in all planning periods.

The profits of all players in the network decrease when the cap decreases. So, the carbon emission constraint is harmful to the CLSC in the economic aspect without considering the environmental target.

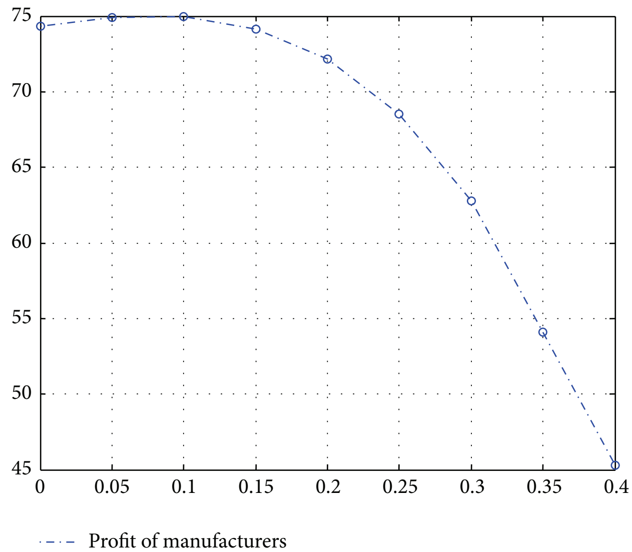
Now, we compare the 1st column with the 3rd and 4th columns, in which the carbon emission cap in the 1st period is fixed, and in the 2nd period it appears and then becomes smaller. The amount of raw materials supplied to the manufacturers is larger than the 3rd period because there are no constraints since this period. The amount of collected products by the manufacturers is larger in the 1st period and smaller in the other periods. The profits of all players in the network decrease when the cap of the 2nd period decreases.

Finally, we compare the 3rd column with the 5th and 6th columns. The amount of raw materials supplied to the manufacturers is larger than the 4th period with no constraints since this period. The amount of collected products by the manufacturers is larger in the 2nd period and smaller in other periods. The profits of all players in the network also decrease.

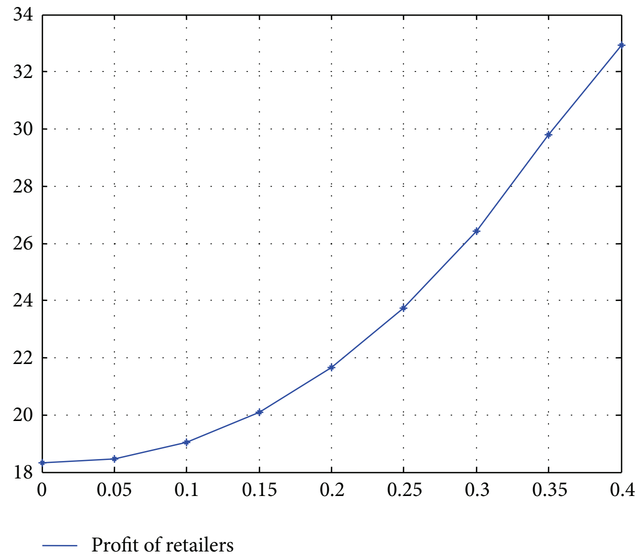
From the analysis above, we can make the following conclusion: when setting carbon emission caps in some periods, the supply amounts of raw materials in these periods



(a) Profit of suppliers



(b) Profit of manufacturers



(c) Profit of retailers

Figure 2: Profits of various players in CLSC.

become lower, while the amounts of collected products in the previous period are larger with the caps; in other period, the collected used products of the manufacturers are smaller. The profits of all players in the CLSC network decrease as the caps become smaller. With the carbon emission caps appearing in Table 3, the decision-makers consider the whole planning horizon comprehensively to obtain the optimal strategy, but not merely a single period.

6. Conclusion

In this paper, we studied the multiperiod CLSC network including manufacturers, retailers, and demand markets taking the product lifetime into account. By using the variational inequalities and complementary theory, we established the CLSC network equilibrium model and solved the model by LQP P-C algorithm. In the end, we provided four numerical examples to illustrate the validity of the modeling framework with or without carbon emission cap, and then we explored the managerial insights in practice. Through the analysis in the numerical examples, we make conclusions as follows.

(1) The increase of product lifetime will enhance the burden of manufacturers and then decrease their profits; the other players in the CLSC are almost not affected; the objects of the manufactures and the whole CLSC are different in general.

(2) The increase of α stimulates the transaction of the network, but it is not always a benefit to the CLSC.

Table 3: $\delta = 0$ and $\alpha = 0.25$, the CLSC network equilibrium results of various emission caps.

$B_m(1), m = 1, 2$	3.2	3	3.2	3.2	3.2	3.2
$B_m(2), m = 1, 2$	∞	∞	2.3	2	2.3	2.3
$B_m(3), m = 1, 2$	∞	∞	∞	∞	1.8	1.6
q_s^r, q_m^r $s = 1, 2;$ $m = 1, 2$	3.2 2.2079 1.7200 1.4044 1.2400	3 2.2218 1.7298 1.4121 1.2463	3.2 2.1975 1.7207 1.4050 1.2405	3.2 1.8906 1.7430 1.4223 1.2546	3.2 2.2025 1.6221 1.4108 1.2452	3.2 2.2131 1.4108 1.4234 1.2555
q_m^u $m = 1, 2$	0.2045 0.3462 0.4383 0.9018	0.1952 0.3390 0.4306 0.8947	0.2050 0.3454 0.4378 0.9012	0.2187 0.3214 0.4221 0.8849	0.1951 0.3559 0.4282 0.8967	0.1739 0.3784 0.4078 0.8870
s_m $m = 1, 2$	2.0993 0 0	3.7721 0 0	2.1126 0.1436 0	2.5025 4.3694 0	2.2441 0.2132 1.8580	2.5256 0.3622 5.8363
$\pi_s, s = 1, 2$	39.8379	38.7104	39.7563	37.5784	38.8658	37.1742
$\pi_n, n = 1, 2$	70.3260	68.5201	70.1648	65.7157	68.0905	63.8557
$\pi_m, m = 1, 2$	24.3986	23.7460	24.3539	23.0658	23.9874	23.2293
Total profit	134.5625	130.9765	134.2750	126.3599	130.9437	124.2592

The government should set the collection rate in a suitable region based on the manufacturers' collection cost structure and benefits from remanufacturing.

(3) The existence of carbon emission cap will lead to the fact that the profits of all players are lower than those with no carbon emission cap.

(4) When setting carbon emission caps in some periods, the supply amounts of raw materials become lower in these periods, while the amount of collected products is larger in the previous period with the caps; the endogenous carbon tax is larger when the cap of carbon emission is lower.

We hope that the insights and corresponding results obtained in this paper may be useful and helpful in theory and practice and enhance the operating quality of enterprises in CLSC.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Research Article

Spectral Methods in Spatial Statistics

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When the spatial location area increases becoming extremely large, it is very difficult, if not possible, to evaluate the covariance matrix determined by the set of location distance even for gridded stationary Gaussian process. To alleviate the numerical challenges, we construct a nonparametric estimator called periodogram of spatial version to represent the sample property in frequency domain, because periodogram requires less computational operation by fast Fourier transform algorithm. Under some regularity conditions on the process, we investigate the asymptotic unbiasedness property of periodogram as estimator of the spectral density function and achieve the convergence rate.

1. Introduction

Recently, spatial statistics have attracted a number of researches from statisticians, geostatisticians, engineers, and econometricians [1–4]. The covariance clustering phenomena of neighborhood observations have always been the fundamental feature of spatial statistics, which means that closer observations have larger correlation and the correlation decays to zero for far separated ones. Therefore, the location information of spatial sited data plays a key role in constructing the probabilistic or statistical covariance function [5]. Using maximum likelihood estimation for the regularly spaced Gaussian data, the calculation of the covariance matrix inverse or determinant needs $O(n^3)$ operations with the sample size n. As n becomes large, it is difficult to implement statistical inference due to computational limitations. Fortunately, spectral method is a powerful tool to analyze this complex spatial structure using fewer calculations with $O(n \log n)$ operations [6, 7].

This spectral method is widely applied in time series analysis [8], but little research has been done in spatial analysis. The spectral analysis, also known as harmonic analysis, is mainly based on the study of the spectral density function, which represents the underlying process property analog to covariance function. By spectral representation theorem, a positive definite covariance function in spatial domain is the Fourier transform of spectral density function in frequency domain. Operations performed on one domain have the corresponding operations on the other domain. In spatial domain, we usually use moment estimator for statistical inference. Under some regular assumptions, like ergodic or stationary, the nonparametric moment method provides asymptotically unbiased estimator. Because of the relationship between spatial and frequency domain, we can use the Fourier transform of moment estimator to make inference. The most important estimator in this sense is periodogram, which is asymptotically unbiased estimator of the spectral density function in time series analysis. Because the periodogram is the square of discrete Fourier transform (DFT), which decomposes the original sequence into components on different frequencies, in this case, the fast Fourier transform (FFT) provides a method to compute the DFT taking $O(n \log n)$ operations.

However, it is unknown whether the periodogram estimators have asymptotic unbiased property for spatial process. In this paper, based on the spatial version periodogram, we prove that the result still holds in spatial analysis and obtain that the convergence rate is a function of the sampling domain.

The periodogram method to study properties of regular stationary time series can be traced to Whittle [9], Ronsenblatt [10], and Priestly [11]. In some cases, the autocorrelation of time series decays very slowly, such that a pair of observations, when even separated far away, are still highly correlated. To overcome this difficulty, Heyde and Gay [12] introduced a smoothed version periodogram to estimate time series with long-range dependence. Geweke and Porter-Hudak [13] employed the log-periodogram as the response variable to construct a regression model for long memory time series.

The paper is organized as follows. Section 2 presents the basic definitions of periodogram and spectral density of spatial version. In Section 3, we give the main theorems on the asymptotic properties of periodogram. Concluding remarks and further research will be given in Section 4. Throughout the paper, the following notation will be adopted: $O(1)(O_p(1))$ denotes a term (a random variable) that is bounded (in probability); $o(1)(o_p(1))$ denotes a term (a random variable) that converges to zero (in probability); and $\mathbb{R}^d$ denotes the Euclidean space with dimension d.

2. Spectral Domain

We consider a weakly stationary Gaussian process with mean zero in which data are drawn from the regularly spaced grid on d-dimensional Euclidean space, $Z = \{Z(\mathbf{s}); \mathbf{s} \in \mathbb{R}^d\} \in \mathbb{R}$ (see Figure 1). Here, we also call Z a random field on $\mathbb{R}^d$. A random field Z is weakly stationary if its mean is constant, and it has finite covariance function C, such that $C(\mathbf{h}) = \mathrm{Cov}(Z(\mathbf{s}+\mathbf{h}), Z(\mathbf{s}))$, for any single location $\mathbf{h}$ belonging to domain $D \subset \mathbb{R}^d$. Let $X(\boldsymbol{\omega})$ be an orthogonal process on a regular gridded lattice, which is mean zero and has independent increments

$$E\left[\left(X\left(\boldsymbol{\omega}_4\right)-X\left(\boldsymbol{\omega}_3\right)\right)\left(X\left(\boldsymbol{\omega}_2\right)-X\left(\boldsymbol{\omega}_1\right)\right)\right]=0, \quad (1)$$

for $\boldsymbol{\omega}_4 > \boldsymbol{\omega}_3 > \boldsymbol{\omega}_2 > \boldsymbol{\omega}_1$, with variance $E(|dX(\boldsymbol{\omega})|^2) = f(\boldsymbol{\omega})\, d\boldsymbol{\omega}$, where $f(\boldsymbol{\omega})$ is the spectral density function. By spectral representation theorem [14], the real-valued weakly stationary random field $Z(\mathbf{s})$ with mean 0 has representation

$$Z(\mathbf{s}) = \int_{\mathbb{R}^d} \exp\left\{i\boldsymbol{\omega}'\mathbf{s}\right\} dX(\boldsymbol{\omega}). \quad (2)$$

Then the covariance function can be represented as

$$\begin{aligned}
&\mathrm{Cov}\left(Z(\mathbf{s}), Z(\mathbf{s}+\mathbf{h})\right)\\
&\quad= \mathrm{Cov}\left(Z(\mathbf{0}), Z(\mathbf{h})\right)\\
&\quad= E\left[\int_{\mathbb{R}^2} \exp\left\{i\boldsymbol{\omega}'\mathbf{0}\right\} dX(\boldsymbol{\omega}) \int_{\mathbb{R}^2} \exp\left\{i\boldsymbol{\omega}'\mathbf{h}\right\} dX(\boldsymbol{\omega})\right]\\
&\quad= \int_{\mathbb{R}^d} \exp\left\{i\boldsymbol{\omega}'\mathbf{h}\right\} E\left(|dX(\boldsymbol{\omega})|^2\right)\\
&\quad= \int_{\mathbb{R}^d} \exp\left\{i\boldsymbol{\omega}'\mathbf{h}\right\} f(\boldsymbol{\omega})\, d\boldsymbol{\omega}.
\end{aligned} \quad (3)$$

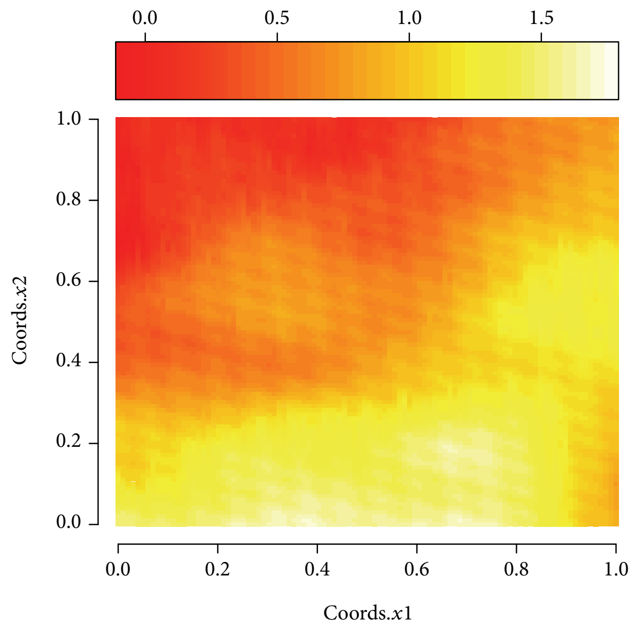

FIGURE 1: Simulation of regular spaced lattice data.

If the covariance function satisfies $\int_{\mathbb{R}^2} |C(\mathbf{x})| d\mathbf{x} < \infty$, the spectral density f is the Fourier transform of the covariance function

$$f(\boldsymbol{\omega}) = \frac{1}{(2\pi)^d} \int_{\mathbb{R}^d} \exp\left\{-i\boldsymbol{\omega}'\mathbf{h}\right\} C(\mathbf{h})\, d\mathbf{h}. \quad (4)$$

If we further assume an additional assumption as in Fuentes [7],

(a) $\sum_{\mathbf{s}\in D} [1+\parallel \mathbf{s} \parallel] |C(\mathbf{s})| < \infty$,

where $\|\cdot\|$ is the Euclidean norm given by $\|\mathbf{s}\| = \sqrt{s_1^2 + \cdots + s_d^2}$, then the spectral density of Z has uniformly bounded first-order derivative. To simplify our discussion, we illustrate the idea by random fields on a two-dimensional grid; that is, $Z = \{Z(\mathbf{s}); \mathbf{s} = (s_1, s_2), s_1 = 1, \ldots, n_1; s_2 = 1, \ldots, n_2\}$, in what follows.

We estimate the spectral density function on the lattice by periodogram of spatial version

$$I_N(\boldsymbol{\omega}) = \frac{1}{(2\pi)^2} \frac{1}{n_1 n_2} \left| \sum_{s_1=1}^{n_1} \sum_{s_2=1}^{n_2} Z(\mathbf{s}) \exp\left\{-i\mathbf{s}'\boldsymbol{\omega}\right\} \right|^2, \quad (5)$$

where $N = n_1 n_2$ is the total sample size; $i = \sqrt{-1}$; and $\boldsymbol{\omega} = (\omega_1, \omega_2)$ is the spatial frequencies, computed at $\omega_1 = 2\pi l_1/n_1$ and $\omega_2 = 2\pi l_2/n_2$ for

$$\begin{aligned}
(l_1, l_2) = &\left\{\left\lfloor -\frac{n_1-1}{2} \right\rfloor, \ldots, n_1 - \left\lfloor \frac{n_1}{2} \right\rfloor\right\}\\
&\times \left\{\left\lfloor -\frac{n_2-1}{2} \right\rfloor, \ldots, n_2 - \left\lfloor \frac{n_2}{2} \right\rfloor\right\}.
\end{aligned} \quad (6)$$

In time series study, under some regular conditions on the stationary process, the periodogram is asymptotically unbiased estimator of spectral density; that is, $I_n(\omega) \rightarrow f(\omega)$ as sample size $n \rightarrow \infty$, where the bias is of order $O(n^{-1})$. However, the periodogram is not a consistent estimator of the spectral density [8].

3. Main Results

Analogous to one-dimensional equally spaced time series observations, define the DFT on two-dimensional grid as

$$\begin{aligned} &J(\omega_1, \omega_2) \\ &= \frac{1}{2\pi\sqrt{n_1 n_2}} \sum_{s_1=1}^{n_1} \sum_{s_2=1}^{n_2} Z(s_1, s_2) \exp\{-i(\omega_1 s_1 + \omega_2 s_2)\} \\ &= J_{\mathscr{R}}(\omega_1, \omega_2) + i J_{\phi}(\omega_1, \omega_2), \end{aligned} \tag{7}$$

where $J_{\mathscr{R}}(\omega_1, \omega_2)$ and $J_{\phi}(\omega_1, \omega_2)$ denote the real and imaginary parts, respectively, and

$$\begin{aligned} J_{\mathscr{R}}(\omega_1, \omega_2) &= \frac{1}{2\pi\sqrt{n_1 n_2}} \sum_{s_1=1}^{n_1} \sum_{s_2=1}^{n_2} Z(s_1, s_2) \cos(\omega_1 s_1 + \omega_2 s_2), \\ J_{\phi}(\omega_1, \omega_2) &= \frac{1}{2\pi\sqrt{n_1 n_2}} \sum_{s_1=1}^{n_1} \sum_{s_2=1}^{n_2} Z(s_1, s_2) \sin(\omega_1 s_1 + \omega_2 s_2). \end{aligned} \tag{8}$$

Since it is known that

$$E(I_N(\boldsymbol{\omega})) = E(J(\boldsymbol{\omega}))^2 = E(J_{\mathscr{R}}(\boldsymbol{\omega}))^2 + E(J_{\phi}(\boldsymbol{\omega}))^2, \tag{9}$$

the expectation of periodogram is equal to the variance of real and imaginary parts of DFT. To achieve this aim, we first show some asymptotic properties of Dirichlet kernel $D_N(\omega) = \sin(N\omega/2)/\sin(\omega/2)$ in time series. By simple calculation, we have

$$\begin{aligned} &\frac{1}{2\pi N} \int_{-\pi}^{\pi} D_N(u-\omega) D_N(v+\omega)\, d\omega = 0, \\ &\qquad \text{for } 0 < |u-v| < 2\pi, \\ &\frac{1}{2\pi N} \int_{-\pi}^{\pi} D_N(u-\omega) D_N(u+\omega)\, d\omega = O\left(\frac{1}{N}\right), \\ &\qquad \text{for } 0 < |u| < \pi. \end{aligned} \tag{10}$$

Also, Anderson [15] shows that

$$\frac{1}{2\pi N} \int_{-\pi}^{\pi} D_N(u-\omega) D_N(u+\omega) f(\omega)\, d\omega = o(1) \qquad \text{for } 0 < |u| < \pi. \tag{11}$$

Also, the Fejer kernel defined as $K_N(\omega) = D_N^2(\omega)$ holds an equation

$$\frac{1}{2\pi N} \int_{-\pi}^{\pi} K_N(\omega)\, d\omega = 1. \tag{12}$$

Using the above formula (10), we obtain the next theorem on asymptotic properties of real and imaginary parts of DFT.

Theorem 1. *Under Assumption (a), the variance value of real and imaginary parts of DFT is given by*

$$\begin{aligned} E\left(J_{\mathscr{R}}^2(\omega_1, \omega_2)\right) &= A + \cos(\omega_1(n_1+1) + \omega_2(n_2+1)) B, \\ E\left(J_{\phi}^2(\omega_1, \omega_2)\right) &= A - \cos(\omega_1(n_1+1) + \omega_2(n_2+1)) B, \end{aligned} \tag{13}$$

where

$$\begin{aligned} A &= \frac{1}{2(2\pi)^2 N} \\ &\quad \times \iint_{\Pi^2} K_{n_1}(\omega_1 - \lambda_1) K_{n_2}(\omega_2 - \lambda_2) f(\lambda_1, \lambda_2)\, d\lambda_1 d\lambda_2, \\ B &= \frac{1}{2(2\pi)^2 N} \\ &\quad \times \iint_{\Pi^2} \prod_{j=1}^{2} D_{n_j}(\omega_j + \lambda_j) D_{n_j}(\omega_j - \lambda_j) \\ &\qquad \times f(\lambda_1, \lambda_2)\, d\lambda_1 d\lambda_2, \end{aligned} \tag{14}$$

where $\Pi^2 = (-\pi, \pi) \times (-\pi, \pi)$.

Proof. Consider

$$\begin{aligned} &E\left(J_{\mathscr{R}}^2(\omega_1, \omega_2)\right) + iEJ_{\mathscr{R}}(\omega_1, \omega_2) J_{\phi}(\omega_1, \omega_2) \\ &= EJ_{\mathscr{R}}^2(\omega_1, \omega_2)\left(J_{\mathscr{R}}(\omega_1, \omega_2) + iJ_{\phi}(\omega_1, \omega_2)\right) \\ &= \frac{1}{(2\pi)^2 N} E\left\{\sum_{s_1=1}^{n_1} \sum_{s_2=1}^{n_2} \sum_{t_1=1}^{n_1} \sum_{t_2=1}^{n_2} Z(s_1, s_2) Z(t_1, t_2) \times \cos(\omega_1 s_1 + \omega_2 s_2) \times \cos(\omega_1 t_1 + \omega_2 t_2)\right\} \\ &\quad + i \frac{1}{(2\pi)^2 N} E\left\{\sum_{s_1=1}^{n_1} \sum_{s_2=1}^{n_2} \sum_{t_1=1}^{n_1} \sum_{t_2=1}^{n_2} Z(s_1, s_2) Z(t_1, t_2) \times \sin(\omega_1 s_1 + \omega_2 s_2) \times \sin(\omega_1 t_1 + \omega_2 t_2)\right\} \\ &= \frac{1}{(2\pi)^2 N} \sum_{s_1, t_1} \sum_{s_2, t_2} C(s_1 - t_1, s_2 - t_2) e^{i(\omega_1 t_1 + \omega_2 t_2)} \\ &\qquad \times \cos(\omega_1 s_1 + \omega_2 s_2) \end{aligned}$$

$$= \frac{1}{(2\pi)^2 N} \times \iint_{\Pi^2} \sum_{s_1,t_1}\sum_{s_2,t_2} f(\lambda_1,\lambda_2) e^{i[(s_1-t_1)\lambda_1+(s_2-t_2)\lambda_2]+i(\omega_1 t_1+\omega_2 t_2)} \times \cos(\omega_1 s_1+\omega_2 s_2)\, d\lambda_1 d\lambda_2$$

$$= \frac{1}{(2\pi)^2 N} \times \iint_{\Pi^2} \sum_{s_1,s_2} \cos(\omega_1 s_1+\omega_2 s_2) e^{i(s_1\lambda_1+s_2\lambda_2)} \times \sum_{t_1,t_2} e^{i(\omega_1-\lambda_1)t_1+i(\omega_2-\lambda_2)t_2} f(\lambda_1,\lambda_2)\, d\lambda_1 d\lambda_2$$

$$= \frac{1}{2(2\pi)^2 N} \iint_{\Pi^2} \sum_{s_1,s_2} \left(e^{i(\omega_1 s_1+\omega_2 s_2)} + e^{-i(\omega_1 s_1+\omega_2 s_2)}\right) \times e^{i(s_1\lambda_1+s_2\lambda_2)} \times e^{i(n_1+1)(\omega_1-\lambda_1)/2+i(n_2+1)(\omega_2-\lambda_2)/2} \times \frac{\sin n_1(\omega_1-\lambda_1)/2}{\sin(\omega_1-\lambda_1/2)} \frac{\sin n_2(\omega_2-\lambda_2)/2}{\sin(\omega_2-\lambda_2)/2} \times f(\lambda_1,\lambda_2)\, d\lambda_1 d\lambda_2$$

$$= \frac{1}{2(2\pi)^2 N} \times \iint_{\Pi^2} \Bigg\{ e^{i(n_1+1)\omega_1+i(n_2+1)\omega_2} \times \prod_{j=1}^{2} \frac{\sin n_j(\omega_j+\lambda_j)/2}{\sin(\omega_j+\lambda_j)/2} \frac{\sin n_j(\omega_j-\lambda_j)/2}{\sin(\omega_j-\lambda_j)/2} + \left(\frac{\sin n_1(\omega_1-\lambda_1)/2}{\sin(\omega_1-\lambda_1)/2}\right)^2 \times \left(\frac{\sin n_2(\omega_2-\lambda_2)/2}{\sin(\omega_2-\lambda_2)/2}\right)^2 \Bigg\} \times f(\lambda_1,\lambda_2)\, d\lambda_1 d\lambda_2$$

$$= \frac{1}{2(2\pi)^2 N} \times \iint_{\Pi^2} K_{n_1}(\omega_1-\lambda_1) K_{n_2}(\omega_2-\lambda_2) \times f(\lambda_1,\lambda_2)\, d\lambda_1 d\lambda_2 + \frac{1}{2(2\pi)^2 N} \cos[(n_1+1)\omega_1+(n_2+1)\omega_2] \times \iint_{\Pi^2} \prod_{j=1}^{2} D_{n_j}(\omega_j+\lambda_j) D_{n_1}(\omega_j-\lambda_j) f(\lambda_1,\lambda_2)\, d\lambda_1 d\lambda_2$$
$$+ i\frac{1}{2(2\pi)^2 N} \sin[(n_1+1)\omega_1+(n_2+1)\omega_2] \times \iint_{\Pi^2} \prod_{j=1}^{2} D_{n_j}(\omega_j+\lambda_j) D_{n_1}(\omega_j-\lambda_j) f(\lambda_1,\lambda_2)\, d\lambda_1 d\lambda_2. \tag{15}$$

Applying the similar calculation procedure to $E(J_\phi^2(\omega_1,\omega_2)) + iEJ_{\mathscr{R}}(\omega_1,\omega_2)J_\phi(\omega_1,\omega_2)$, we can get the result that $E(J_\phi^2(\omega_1,\omega_2)) = A - \cos(\omega_1(n_1+1)+\omega_2(n_2+1))B$. □

By the conclusion of Theorem 1 and formula (9), it follows that

$$E(I_N(\boldsymbol{\omega})) = E\left(J_{\mathscr{R}}^2(\boldsymbol{\omega})\right) + E\left(J_\phi^2(\boldsymbol{\omega})\right) = \frac{1}{(2\pi)^2 N} \iint_{\Pi^2} K_{n_1}(\omega_1-\lambda_1) K_{n_2}(\omega_2-\lambda_2) \times f(\lambda_1,\lambda_2)\, d\lambda_1 d\lambda_2. \tag{16}$$

Therefore, the bias of periodogram $I_N(\boldsymbol{\omega})$ to spectral density is

$$b(\boldsymbol{\omega}) = E[I_N(\boldsymbol{\omega}) - f(\boldsymbol{\omega})] = \frac{1}{(2\pi)^2 N} \iint_{\Pi^2} K_{n_1}(\omega_1-\lambda_1) K_{n_2}(\omega_2-\lambda_2) \times f(\lambda_1,\lambda_2)\, d\lambda_1 d\lambda_2 - f(\boldsymbol{\omega}). \tag{17}$$

The next theorem evaluates the bias in (17), such that we can prove the asymptotic unbiasedness property of periodogram for regular lattice random fields.

Theorem 2. *Under Assumption (a), for $\boldsymbol{\omega} \in \Pi^2 = (-\pi,\pi) \times (-\pi,\pi)$, we have*

$$b(\omega_1,\omega_2) = \min\left\{O\left(n_1^{-1}\right), O\left(n_2^{-1}\right)\right\}. \tag{18}$$

Proof. By formula (17), it follows that

$$b(\omega_1,\omega_2) = \frac{1}{(2\pi)^2 n_1 n_2} \iint_{-\pi}^{\pi} K_{n_1}(\omega_1-\lambda_1) K_{n_2}(\omega_2-\lambda_2) \times (f(\lambda_1,\lambda_2) - f(\omega_1,\omega_2))\, d\lambda_1 d\lambda_2$$
$$= \frac{1}{(2\pi)^2 n_1 n_2} \iint_{-\pi}^{\pi} K_{n_1}(\omega_1-\lambda_1) K_{n_2}(\omega_2-\lambda_2) \times \left\{(\boldsymbol{\lambda}-\boldsymbol{\omega})^T \nabla f(\boldsymbol{\omega}) + \frac{1}{2}(\boldsymbol{\lambda}-\boldsymbol{\omega})^T \nabla^2 f(\boldsymbol{\omega}^*)(\boldsymbol{\lambda}-\boldsymbol{\omega})\right\} d\lambda_1 d\lambda_2, \tag{19}$$

where $\nabla f(\boldsymbol{\omega}) = (f_{\omega_1}, f_{\omega_2})^T$ and $\nabla^2 f(\boldsymbol{\omega}^*)$ denote the first- and second-order derivatives of $f(\boldsymbol{\omega})$ at $\boldsymbol{\omega}$ and $\boldsymbol{\omega}^*$, respectively, where $\|\boldsymbol{\omega}^* - \boldsymbol{\omega}\| < \|\boldsymbol{\lambda} - \boldsymbol{\omega}\|$. By the fact that $\|\nabla^2 f(\boldsymbol{\omega}^*)\| < K$ and by integration range $\|\boldsymbol{\lambda} - \boldsymbol{\omega}\|^2 = (\lambda_1 - \omega_1)^2 + (\lambda_2 - \omega_2)^2 < K$, we have

$$\begin{aligned}
&\|b(\boldsymbol{\omega})\| \\
&\leq \frac{K}{(2\pi)^2 n_1 n_2} \int_{-\pi-\omega_1}^{\pi-\omega_1} \int_{-\pi-\omega_2}^{\pi-\omega_2} K_{n_1}(\lambda_1) K_{n_2}(\lambda_2) \\
&\quad \times \left(f_{\omega_1}\lambda_1 + f_{\omega_2}\lambda_2 \right) d\lambda_1 d\lambda_2 \\
&\quad + \frac{K}{(2\pi)^2 n_1 n_2} \int_{-\pi-\omega_1}^{\pi-\omega_1} \int_{-\pi-\omega_2}^{\pi-\omega_2} K_{n_1}(\lambda_1) K_{n_2} \\
&\qquad \times (\lambda_2) \|\boldsymbol{\lambda}\|^2 d\lambda_1 d\lambda_2 \\
&\leq \frac{K}{(2\pi)^2 n_1 n_2} \\
&\quad \times \left\{ \left| f_{\omega_1} \right| \int_{-\pi-\omega_1}^{\pi-\omega_1} K_{n_1}(\lambda_1) \lambda_1 d\lambda_1 \int_{-\pi-\omega_2}^{\pi-\omega_2} K_{n_2}(\lambda_2) d\lambda_2 \right. \\
&\qquad \left. + \left| f_{\omega_2} \right| \int_{-\pi-\omega_2}^{\pi-\omega_2} K_{n_2}(\lambda_2) \lambda_2 d\lambda_2 \int_{-\pi-\omega_1}^{\pi-\omega_1} K_{n_1}(\lambda_1) d\lambda_1 \right\} \\
&\quad + O\left(\frac{1}{n_1 n_2} \right) \\
&\leq \frac{K}{(2\pi) n_1} \left\{ \int_{-\pi-\omega_1}^{-\eta} + \int_{-\eta}^{\eta} + \int_{\eta}^{\pi-\omega_1} K_{n_1}(\lambda_1) \lambda_1 \, d\lambda_1 \right\} \\
&\quad + \frac{K}{(2\pi) n_2} \left\{ \int_{-\pi-\omega_2}^{-\eta} + \int_{-\eta}^{\eta} + \int_{\eta}^{\pi-\omega_2} K_{n_2}(\lambda_2) \lambda_2 \, d\lambda_2 \right\} \\
&\quad + O\left(\frac{1}{n_1 n_2} \right).
\end{aligned} \tag{20}$$

Also,

$$\begin{aligned}
&\frac{K}{(2\pi) n_1} \int_{-\eta}^{\eta} K_{n_1}(\lambda_1) \lambda_1 d\lambda_1 \\
&\qquad + \frac{K}{(2\pi) n_2} \int_{-\eta}^{\eta} K_{n_2}(\lambda_2) \lambda_2 d\lambda_2 = 0,
\end{aligned} \tag{21}$$

because the integrand is odd, and the remaining terms hold as

$$\begin{aligned}
&\frac{K}{(2\pi) n_1} \left\{ \int_{-\pi-\omega_1}^{-\eta} + \int_{\eta}^{\pi-\omega_1} \right\} K_{n_1}(\lambda_1) \lambda_1 d\lambda_1 = O\left(n_1^{-1} \right), \\
&\frac{K}{(2\pi) n_2} \left\{ \int_{-\pi-\omega_2}^{-\eta} + \int_{\eta}^{\pi-\omega_2} \right\} K_{n_2}(\lambda_2) \lambda_2 d\lambda_2 = O\left(n_2^{-1} \right),
\end{aligned} \tag{22}$$

because $K_N(u) < 1/\sin^2(u/2)$, for $0 < u < \pi$. □

The idea of Theorem 2 can be directly extended to random fields on d-dimensional regular gridded lattice. In conclusion, the periodogram of spatial version is asymptotically unbiased estimator of the spectral density function, in which the convergence rate is the minimum of sampling dimension, that is, $\min\{n_1^{-1}, \ldots, n_d^{-1}\}$.

4. Conclusions and Future Research

In this paper, we consider the nonparametric method to estimate the spectral density function. The spectral density function represents the population property and describes the behavior of underlying process. By constructing a moment estimator periodogram in frequency domain, which is a Fourier transform of covariance function in space domain, the spectral density can be estimated to be asymptotically unbiased. The spectral method outperforms traditional spatial method, because it provides significant computational advantages. In addition, we obtain the convergence rate as the minimum of spatial sampling dimension in the lattice. However, the consistency and asymptotic normality has not been established in this paper, which is important in statistical inference, hypothesis testing, and interval forecasts. On the other hand, by using the Whittle likelihood [9], we can approximate the maximum likelihood method [16] in spatial domain by Whittle estimator in frequency domain and thus estimate the specific parameter in the spectral density.

However, our asymptotic results are based on increasing domain asymptotics, allowing the fact that, as the sample size increases, the distance increases becoming sufficiently large, and thus the observations are nearly independent. However, if one is concerned to predict the value of a point in a bounded area, it is necessary to consider fixed domain asymptotics rather than increasing domain asymptotics. Specifically, even for regularly sited data, results from maximum likelihood estimate only exist in very limited cases. The difficulty under the fixed domain asymptotics is that there is at least one function of parameters that cannot be consistently estimated even for regularly gridded Gaussian process. This is indeed the case for Gaussian random fields with isotropic exponential covariance function $C(\mathbf{h}) = \sigma^2 \exp(-|\mathbf{h}|/\alpha)$, where σ^2 is the sill parameter and α is the range parameter. Ying [17] proved that when $d = 1$, the maximum likelihood estimator of σ^2/α is strongly consistent, but not possible for either σ^2 or α. Therefore, it needs more researches on the fixed domain asymptotics in spatial statistics.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Research Article

Horizontal Coalition Stability Analysis of Supply Chain Entities Based on Sequential Game

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Aiming to find the effect of the same status entities' horizontal coordination on supply chain, this paper studied the coalition stability of dealers in a two-stage supply chain with one supplier and multiple dealers. First, a vertical sequential game model is built, where the supplier is leader and the multiple dealers are followers. In the second stage of the game, multiple dealers face two selections: playing Nash game with each other or developing a coalition. Then, according to the results acquired by comparing the dealers' profits which depend on their coalition situations, the criterion of coalition stability is developed. Finally, numerical simulation is used to verify the validity of the model, and some insights are obtained. For example, if the sensitivity coefficient T of the market price is fixed, dealers' coalition tends to be stable with the increasing of the substitution rate k in a reasonable range; the supplier's optimal wholesale price is constant with and without dealer's coalition, but dealers' coalition causes demand to decrease, which leads to the decrease of the supplier's profit too. The result of this paper provides an important reference for the formation of dealers' coalition in IT or automobile supply chain.

1. Introduction

With the development of market economy, many new-style supply chains come into being and become popular, such as automobile supply chain, IT supply chain, and franchise supply chain consisting of one supplier and multiple dealers. Like other supply chains, the entities in these supply chains compete with each other for maximizing their own profits. Taking an automobile supply chain consisting of one carmaker and several 4*S* dealers as an example, the competition among supply chain entities exists not only in different statuses of the supply chain, such as the competition between carmaker and 4*S* dealers, but also in the same status of the supply chain, such as the competition among multiple 4*S* dealers. Reasonable competition can improve the work efficiency of the supply chain, while irrational competition results in waste of resources, cost increase, price war, and so forth. Meanwhile, there is cooperation among entities of the supply chain, which also can be divided into two situations as the competition. Although full cooperation among entities can give the whole supply chain maximum return [1], all entities participating in the full cooperation are often hard to be stable because the benefit and foresight of each entity are different. Balancing the competition and cooperation among the entities holds great significances: (1) coordinating the relationship among the entities can help the entities to acquire the optimal profits and guarantee the stability of supply chain; (2) coordinating the relationship among the entities could promote the core competitiveness of supply chain. Therefore, the coordination for the relationship among the supply chain entities is a meaningful issue.

The coordination of supply chain is always a focus in the study of supply chain and is involved in many fields of supply chain, such as lead time variation control [2], supply chain coordination strategies about errors [3], and assessment of contracts' coordinating power [4]. The coordination of entities' relationship in a supply chain mainly includes vertical coordination of entities in different statuses and horizontal coordination of entities in the same status. At present, numerous researches focus on vertical coordination. For example, Weng introduced a quantity discount as an incentive in the basis of the traditional newsboy model, in order to coordinate the relationship between manufacturers and consumers and guide consumers to buy the appropriate number of products [5]. Chaharsooghi and Heydari proposed a coordination model to obtain the joint determination of order quantity and reorder point based on credit option in a two-stage supply chain [6]. Arkan and Hejazi designed a coordination mechanism based on a credit period to coordinate orders in a two-echelon supply chain and to achieve channel coordination and a win-win outcome [7]. Wu considered two channel policies for both competing supply chains, vertical integration (VI) and manufacturer's Stackelberg (MS), to achieve the supply chain coordination [8].

Another important coordination method among entities is horizontal coordination, namely, the coordination among the entities in the same status of a supply chain. Through the establishment of horizontal coalition and reduction of the internal competition in a supply chain, entities in the horizontal coalition can obtain more profits, and the horizontal coalition will have a greater say throughout the supply chain. And stability is the key of the horizontal coordination among entities in the supply chain and the increase of the supply chain profit. Therefore, it is necessary to analyze the stability of the horizontal coalition in a supply chain. Now, the research on the problem of horizontal coalition stability of supply chain has drawn more attention. Mahesh studied the formation of agents' dynamic coalition and the condition of the coalition stability in a competitive market [9]. Then Mahesh studied coalition stability in a supply chain consisting of multiple vendors and an assembler [10]. Krajewska et al. studied horizontal cooperation among freight carriers, analyzed the profit margins resulting from horizontal cooperation among freight carriers, and discussed the possibilities of sharing these profit margins fairly among the partners [11]. Karray investigated the effects of horizontal joint promotions (HJP), initiated by competing retailers on the supply chain's strategies and profits, and studied market conditions conducive for profitable joint retail promotions under different channel structures [12]. Seok and Nof discussed collaborative capacity sharing (CCS) among manufacturers at the same horizontal layer in supply networks which can be used to minimize manufacturers' lost sales as well as maximize their production capacity utilization in a long-term period against lumpy demand [13].

As for coordination mechanism among entities, some researches adopted supply chain contract [14]. Cao et al. studied the coordination mechanism of supply chain which has one manufacturer and multiple retailers in the case of the presence of disturbance of the cost and demand of product and obtained the optimal contract to distribute the profits [15]. Pezeshki et al. established a dyadic supply chain model, examined the model under both full information and partial information updating situations, and proposed a coordinating contract for each case [16]. Sajadieh et al. developed a joint optimal policy to acquire more benefits for the cases with less unpredictable lead times, lower shortage prices, and no transportation cost [17]. Jiang et al. discussed three contract mechanisms, wholesale price (WP), pairwise revenue sharing (PRS), and spanning revenue sharing (SRS), to coordinate a three-stage supply chain with competing manufacturers [18].

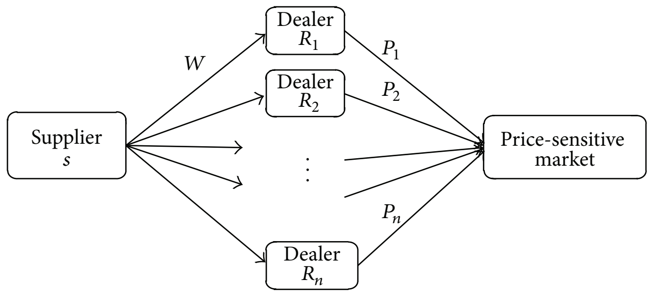

Figure 1: Supply chain structure.

Researches mentioned above mainly focus on theories or methods of supply chain vertical coordination, while the research on the horizontal coordination, especially the horizontal coalition of entities in an actual supply chain, is not enough. Furthermore, there are few studies about the distinguishing characteristics of stable supply chain coalition. Thus, the research on supply chain horizontal coalition is necessary. Based on these observations, this paper studies the horizontal coalition stability of a two-stage supply chain consisting of a supplier and multiple dealers by using the game theory and acquires supply chain coalition stability criterion by comparing the profits of dealers with and without coalition. First, the vertical sequential game model which consists of one supplier as a leader and multiple dealers as the followers is described. Second, each dealer can operate alone and compete with others or join the coalition which has a unified price, so the dealers will play Nash game or form a coalition with each other. Then, the criterion of coalition stability is acquired by comparing each dealer's benefit with coalition and without coalition. Only if all dealers' profits with coalition are more than those without coalition, the dealers' coalition will be stable. Finally, the main conclusions of this paper are verified by numerical simulation.

2. Model Description

The two-stage supply chain consisting of one supplier S and n dealers $(R_1 \cdots R_n)$ is shown in Figure 1. The supplier sells products to downstream dealers facing a price-sensitive market. The supplier manufactures the products at a marginal cost C and sells them to the dealers at a wholesale price W. Each dealer i runs at a marginal cost c_i and a retail price P_i $(i = 1, \dots, n)$.

According to [19], the market demands of two differentiated products are $f_A = a - kp'_A + bp'_B$ and $f_B = a - kp'_B + bp'_A$. Based on these two functions, the demand function of dealer i can be acquired:

$$D_i = \begin{cases} a - TP_1 + k\left(P_2 + \cdots + P_n\right) & i = 1 \\ a - TP_i + k \sum\limits_{j=1, j\neq i}^{n} P_j & 1 < i < n \\ a - TP_n + k \sum\limits_{j=1}^{n-1} P_j & i = n. \end{cases} \quad (1)$$

In formula (1), parameter a denotes the saturation value of market demand of one dealer, so the total saturation value of market demand is na. Parameter T denotes market price sensitivity, and the price sensitivity becomes greater and greater with the increase of T. There is a positive correlation between T and the ratio of the saturation value a and dealer i's retail price P_i. The increase of retail price leads to the decrease of market demand, so $T > 0$. Parameter k denotes the substitution rate between one dealer's product and other dealers' products. The larger the k is, the higher the substitution rate will be, and $0 < k < 1$.

From formula (1), the coefficient matrix of the demand function can be obtained:

$$M = \begin{bmatrix} -T & k & \cdots & k \\ k & -T & \ddots & \vdots \\ \vdots & \ddots & \ddots & k \\ k & \cdots & k & -T \end{bmatrix}. \quad (2)$$

If $b = [P_1, P_2, \cdots P_n]^T$, (1) can be simplified as

$$D = [D_1, D_2, \ldots, D_n]^T = a + Mb. \quad (3)$$

Lemma 1. *If $P_i / \sum_{j=1}^{n} P_j > k/(T+k)$, market demand of dealer i will be less than the saturation value of market demand of one dealer; in other words, $D_i < a$.*

Proof. Equation (1) can be written as

$$D_i = a - TP_i + k\left(\sum_{j=1}^{n} P_j - P_i\right). \quad (4)$$

Making the left side of formula (4) be less than a, the following formula can be deduced:

$$k\sum_{j=1}^{n} P_j < (T+k) P_i. \quad (5)$$

Because $T+k > 0$ and $\sum_{j=1}^{n} P_j > 0$, the above formula can be transformed into

$$\frac{P_i}{\sum_{j=1}^{n} P_j} > \frac{k}{T+k}. \quad (6)$$

From the derivation process above, it is concluded that if $P_i / \sum_{j=1}^{n} P_j > k/(T+k)$, $D_i < a$.

The proof is completed. □

Lemma 2. *If $T/k > n-1$, the total market demand of all dealers' products D_A is less than the total saturation value of market demand na.*

Proof. The total market demand of all dealers' products is

$$D_A = D_1 + D_2 + \cdots + D_n = na - [T - (n-1)k] \sum_{i=1}^{n} P_i. \quad (7)$$

Only when $[T-(n-1)k] \sum_{i=1}^{n} P_i > 0$, can inequation $D_A < na$ be obtained. With $\sum_{i=1}^{n} P_i > 0$, if $T - (n-1)k > 0$, which is equal to $T/k > n-1$, the following conclusions can be obtained: $D_A < na$.

The proof is completed. □

The supplier's profit function can be expressed as

$$\pi_S = D_A (W - C) = \left(na - [T - (n-1)k] \sum_{i=1}^{n} P_i\right)(W - C). \quad (8)$$

Each dealer i's profit function is

$$\begin{aligned} \pi_{R_i} &= D_i (P_i - W - c_i) \\ &= \left[a - TP_i + k\left(\sum_{j=1}^{n} P_j - P_i\right)\right](P_i - W - c_i). \end{aligned} \quad (9)$$

3. Vertical Sequential Game between the Supplier and Dealers

In the above supply chain model, the supplier is monopolistic because just one supplier sells products to multiple dealers. There is a vertical sequential game between supplier (the leader) and dealers (the followers) in the supply chain. The game is divided into two steps and backward induction is usually used to solve the sequential game. First, the supplier sets the wholesale price W to achieve its maximal profit based on the dealers' optimal reaction function of wholesale price $P_i(W)$. Then, each dealer i sets the retail price based on the wholesale price W and the optimal reaction function $P_i(W)$.

It is important to notice that each dealer considers not only the supplier's wholesale price W but also the impact of other dealers' retail prices when pricing its product. All dealers have the same status in the supply chain, and they can set their retail prices simultaneously. Now, the wholesale price W can be regarded as a market environment parameter. The pricing process of each dealer is affected by other dealers' pricing processes, and all dealers' pricing processes are a typical Nash game. Thus, a horizontal Nash game happens in the dealers' pricing process of the vertical sequential game.

Theorem 3. *The profit function of each dealer i is a concave function of the price P_i.*

Proof. From formula (9), the first-order partial derivative and second-order partial derivative of π_{R_i} can be obtained:

$$\begin{aligned} \frac{\partial \pi_{R_i}}{\partial P_i} &= a - 2TP_i + k\left(\sum_{j=1}^{n} P_j - P_i\right) + T(W + c_i), \\ \frac{\partial^2 \pi_{R_i}}{\partial P_i^2} &= -2T. \end{aligned} \quad (10)$$

According to the definition in formula (1), $T > 0$, so $\partial^2 \pi_{R_1}/\partial P_1^2 = \partial^2 \pi_{R_2}/\partial P_2^2 = \cdots = \partial^2 \pi_{R_n}/\partial P_n^2 = -2T < 0$.

In other words, the second-order derivative of π_{R_i} is less than 0. Therefore, the profit function of each dealer i π_{R_i} is a concave function of P_i. □

Theorem 4. *The necessary condition of the conclusion that the profit function of dealer i π_{R_i} can reach its maximum value at the stationary point is $a - k\sum_{j=1,j\neq i}^{n} P_j + T(W + c_i) > 0$; the necessary condition of the conclusion that the profit function of dealer $i\pi_{R_i}$ can reach its maximum value at the boundary of the profit curve is $a - k\sum_{j=1,j\neq i}^{n} P_j + T(W + c_i) < 0$; the necessary condition of the conclusion that the profit function of dealer i π_{R_i} can reach its maximum value at the overlapping point of the stationary point and the lower boundary is $a - k\sum_{j=1,j\neq i}^{n} P_j + T(W + c_i) = 0$.*

Proof. According to Lemma 1, an inequality constraint exists in the following optimization problem of each dealer i: $P_i/\sum_{j=1}^{n} P_j > k/T + k$. So, this profit optimization problem can be written as

$$\begin{gathered}\max_{P_i} \pi_{R_i}(P_i)\\ \text{s.t.} \frac{P_i}{\sum_{j=1,j\neq i}^{n} P_j} - \frac{k}{T} > 0\\ P_i > 0.\end{gathered} \tag{11}$$

Using KKT conditions [20], the Lagrange function of the above optimization problem can be written as

$$\begin{aligned}L(P_i,\lambda) = {} & \left[a - TP_i + k\left(\sum_{j=1}^{n} P_j - P_i\right)\right](P_i - W - c_i)\\ & + \lambda\left(\frac{P_i}{\sum_{j=1,j\neq i}^{n} P_j} - \frac{k}{T}\right) \quad (\lambda \geq 0).\end{aligned} \tag{12}$$

To acquire the optimal value, the following conditions must be satisfied:

$$\begin{gathered}P_i\frac{\partial L(P_i,\lambda)}{\partial P_i} = 0,\\ \lambda\frac{\partial L(P_i,\lambda)}{\partial \lambda} = 0.\end{gathered} \tag{13}$$

Because P_i is retail price and is always greater than 0, the equations above can be simplified as

$$\begin{gathered}\frac{\partial L(P_i,\lambda)}{\partial P_i} = 0,\\ \lambda\frac{\partial L(P_i,\lambda)}{\partial \lambda} = 0.\end{gathered} \tag{14}$$

Formula (14) can be divided into three situations to be discussed, respectively, according to the different values of λ and $\partial L(P_i,\lambda)/\partial\lambda$: (1) $\lambda = 0$, $\partial L(P_i,\lambda)/\partial\lambda > 0$; (2) $\lambda > 0$, $\partial L(P_i,\lambda)/\partial\lambda = 0$; (3) $\lambda = 0$, $\partial L(P_i,\lambda)/\partial\lambda = 0$. □

Situation 1 ($\lambda = 0$, $\partial L(P_i,\lambda)/\partial\lambda > 0$). Formula (14) can be transmitted into

$$\begin{gathered}\frac{\partial L(P_i,\lambda)}{\partial P_i} = 0,\\ \frac{\partial L(P_i,\lambda)}{\partial \lambda} > 0,\\ \lambda = 0.\end{gathered} \tag{15}$$

Putting formula (12) into formula (15), then

$$\begin{aligned}\frac{\partial L(P_i,\lambda)}{\partial P_i} &= a - 2TP_i + k\sum_{j=1,j\neq i}^{n} P_j + T(W + c_i)\\ &= \frac{\partial \pi_{R_i}}{\partial P_i} = 0\\ &\frac{P_i}{\sum_{j=1,j\neq i}^{n} P_j} - \frac{k}{T} > 0.\end{aligned} \tag{16}$$

Now, the first-order derivative of dealer i's profit function π_{R_i} is equal to 0. The max/min value of π_{R_i} can be obtained at stationary point. According to Theorem 3, the function curve of π_{R_i} is concave, so the maximum value of π_{R_i} can be obtained.

Put the equation of (16) into its inequality, and

$$a - k\sum_{j=1,j\neq i}^{n} P_j + T(W + c_i) > 0. \tag{17}$$

Formula (17) is the necessary condition based on which the maximum value of π_{R_i} can be obtained at stationary point.

Situation 2 ($\lambda > 0$, $\partial L(P_i,\lambda)/\partial\lambda = 0$). Formula (14) can be transmitted into

$$\begin{gathered}\frac{\partial L(P_i,\lambda)}{\partial P_i} = 0,\\ \frac{\partial L(P_i,\lambda)}{\partial \lambda} = 0,\\ \lambda > 0.\end{gathered} \tag{18}$$

Put formula (12) into formula (18), and

$$\begin{gathered}\frac{\partial L(P_i,\lambda)}{\partial P_i} = \frac{\partial \pi_{R_i}}{\partial P_i} + \frac{\lambda}{\sum_{j=1,j\neq i}^{n} P_j} = 0,\\ \frac{P_i}{\sum_{j=1,j\neq i}^{n} P_j} - \frac{k}{T} = 0.\end{gathered} \tag{19}$$

According to $\lambda > 0$ and the first equation of formula (19), $\partial\pi_{R_i}/\partial P_i < 0$. Therefore, the profit curve of dealer i is a monotone decreasing curve and the maximum value of π_{R_i} can be obtained on the lower boundary of P_i.

Put the second equation of (19) into the first, so

$$\frac{\lambda}{\sum_{j=1,j\neq 1}^{n} P_j} = -a + k \sum_{j=1,j\neq i}^{n} P_j - T\left(W + c_i\right). \quad (20)$$

Because of $\lambda / \sum_{j=1,j\neq 1}^{n} P_j > 0$, formula (20) can be rewritten as

$$a - k \sum_{j=1,j\neq i}^{n} P_j + T\left(W + c_i\right) < 0. \quad (21)$$

Formula (21) is the necessary condition based on which the maximum value of π_{R_i} can be obtained on the curve boundary.

Situation 3 ($\lambda = 0$, $\partial L(P_i, \lambda)/\partial\lambda = 0$). Formula (14) can be transmitted into

$$\begin{gathered} \frac{\partial L\left(P_i, \lambda\right)}{\partial P_i} = 0, \\ \frac{\partial L\left(P_i, \lambda\right)}{\partial \lambda} = 0, \\ \lambda = 0. \end{gathered} \quad (22)$$

Putting formula (12) into formula (22), then

$$\begin{gathered} \frac{\partial L\left(P_i, \lambda\right)}{\partial P_i} = a - 2TP_i + k \sum_{j=1,j\neq i}^{n} P_j + T\left(W + c_i\right) = \frac{\partial \pi_{R_i}}{\partial P_i} = 0, \\ \frac{P_i}{\sum_{j=1,j\neq i}^{n} P_j} - \frac{k}{T} = 0. \end{gathered} \quad (23)$$

The first-order derivative of dealer i's profit function π_{R_i} is equal to 0 according to the first equation of formula (23), and the lower boundary of the profit curve is just the stationary point of it according to the second equation of (23).

Solving (23), the following formula can be obtained:

$$a - k \sum_{j=1,j\neq i}^{n} P_j + T\left(W + c_i\right) = 0. \quad (24)$$

Formula (24) is the necessary condition based on which the maximum value of π_{R_i} can be obtained at the overlapping point of the stationary point and the lower boundary. It is important to notice that the curve boundary mentioned above is the point which is unlimitedly close to the lower boundary of P_i because of the inequality condition of (9): $P_i / \sum_{j=1,j\neq i}^{n} P_j - k/T > 0$. Based on the above analysis, it can be seen that there are three kinds of optimal value: the stationary point, the lower boundary, the overlapping point of the stationary point, and the lower boundary. The last two cases are something impractical because the market demand of dealer iD_i will be equal to the saturation value of market demand a in these two cases. In fact, there is a great difference between D_i and a. Consequently, there will be a further study on the first case in this paper.

Developing the equation of (16), $\partial\pi_{R_i}/\partial P_i = 0$, linear equations with n variables can be obtained as

$$\begin{gathered} -2TP_1^* + k\left(P_2^* + \cdots + P_n^*\right) = -a - T\left(W + c_1\right) \\ kP_1^* - 2TP_2^* + k\left(P_3^* + \cdots + P_n^*\right) = -a - T\left(W + c_2\right) \\ \vdots \\ k\left(P_1^* + \cdots + P_{n-1}^*\right) - 2TP_n^* = -a - T\left(W + c_n\right). \end{gathered} \quad (25)$$

Nash equilibrium existing in the pricing strategies of the dealers will be calculated by solving formula (25). Cramer's rule can be used to solve (25). The determinant of the equations' coefficients is

$$Q = \begin{vmatrix} -2T & k & \cdots & k \\ k & -2T & \ddots & \vdots \\ \vdots & \ddots & \ddots & k \\ k & \cdots & k & -2T \end{vmatrix}. \quad (26)$$

Making $b_1 = [-a - T(W + c_1), -a - T(W + c_2), \ldots, -a - T(W + c_n)]^T$ and replacing the column i of Q with b_1, Q_i can be obtained:

$$Q_i = \begin{vmatrix} \cdots & \underset{\text{column } i}{b_1} & \cdots \end{vmatrix}. \quad (27)$$

According to Cramer's rule, the optimal retail price of dealer i is

$$P_i^* = \frac{Q_i}{Q} = f_i(W). \quad (28)$$

W is regarded as a market parameter in the process of solving Nash equilibrium, and the expression of each P_i^* contains W. In the vertical sequential game, P_i^* is a function of W. Based on the expression (28) of all retail prices, the supplier's optimal wholesale price W^* can be found.

Putting (25) and (28) into formula (8), then

$$\pi_S = \left[na - T\sum_{i=1}^{n} f_i(W) + (n-1)k\sum_{i=1}^{n} f_i(W) \right](W - C). \quad (29)$$

Making $d\pi_S/dW = 0$, the supplier's optimal wholesale price W^* can be found by solving it. Then putting W^* back into (28), each dealer's optimal retail price P_i^* can be obtained.

4. Coalition Stability Concept

In Bertrand model, perfect market competition results in that the retail price of one product is equal to its marginal cost, so all manufacturers have no profit. The equilibrium solution of Bertrand model is impractical because no one will work for free. However, if the condition "perfect market competition" is replaced with "cooperation among manufacturers," everything will be different. On this occasion, all manufacturers can develop a joint pricing as long as they have an agreement on benefit segmentation. Thus, the cutthroat price war in traditional Bertrand model can be avoided, and all manufacturers' profits are greater than zero.

Similar to Bertrand model, the supply chain studied in this paper also faces a price-sensitive market, but the dealers are not involved in perfect competition, and market sharing does not depend exactly on price. These dealers may develop a coalition for their common interests. Assuming that all dealers in the coalition have a unified retail price, an equilibrium point should be found as the unified price to balance the price and demand. Here, the unified pricing does not mean that the dealers can raise the retail price without restraint because the increase of the price will result in the decrease of market demand.

The establishment of a stable coalition needs the following condition: all dealers' profits with coalition are more than those without coalition. If any dealer suffers loss, the coalition will be considered as an unstable coalition.

Definition 5. Suppose that there are n dealers $R_1, R_2, \ldots, R_n$ who sell homogeneous products bought from a supplier. These dealers sell their products at prices $P_1, P_2, \ldots, P_n$ and get profits $\pi_{R_1}, \pi_{R_2}, \ldots, \pi_{R_n}$, respectively. If they develop a coalition and make P as their unified price, they will get $\pi'_{R_1}, \pi'_{R_2}, \ldots, \pi'_{R_n}$. Then, if $\pi_{R_1} < \pi'_{R_1}, \pi_{R_2} < \pi'_{R_2}, \ldots, \pi_{R_n} < \pi'_{R_n}$ or, in other words, all dealers' profits with coalition are more than those without coalition, the coalition will be regarded as a stable coalition.

Inference 1. If dealers' coalition is stable, the profit allocation among all entities in the entire supply chain achieves Pareto optimality.

Proof. This inference can be proved in two cases. (1) If one or more dealers want more profits when dealers' coalition is stable, they can use two methods: unilateral price increase or unified price increase. However, these two approaches are not infeasible. A unilateral price increase leads to the collapse of the coalition, so each dealer's profit will return to its profit without coalition. According to Definition 5, each dealer's profit without coalition is less than that with coalition, so a unilateral price increase cannot benefit the dealers. Raising unified price is also not a good idea because the unified price calculated by using backward induction is the optimal solution of the sequential game and a positive growth of it will reduce all dealers' profits. In summary, the dealers have no way to increase their profits any more when their coalition is stable. (2) If the suppliers want more profits when dealers' coalition is stable, they can also use two methods: raising wholesale price or destroying dealers' coalition. The former is not available because the wholesale price is also calculated by using backward induction and changing it will lead to a loss of the supplier. The only way in which the suppliers can improve their profit is breaking up dealers' coalition. According to Definition 5, if dealers' coalition is broken, all their profits are less than the profits with coalition. The increase of the suppliers' profit results in the loss of the dealers, which is in accordance with Pareto optimality. In a word, the profit allocation among all entities in the entire supply chain achieves Pareto optimality when dealers' coalition is stable. □

5. Coalition Stability Analyses

According to Definition 5, judging the coalition stability needs a comparison of every dealer's profit with and without coalition. The latter has been studied above, so we need to analyze the former.

All the dealers' profits with coalition can also be analyzed in the framework of the sequential game mentioned above, but Nash game will disappear when all dealers' prices become unified. Because all the dealers enjoy the same retail price P now, according to (8), the profit function of the supplier can be written as

$$\pi_{SC} = n\left[a - TP + (n-1)kP\right]\left(W_C - C\right). \tag{30}$$

If the supplier's wholesale price W is fixed, single dealer's profit function in (9) cannot be used to solve the optimal unified price P^* because every dealer will have their own optimal retail price in response to W and the unified price cannot be solved in that case. All dealers should be regarded as a whole in the process of coalition optimization and its profit function is

$$\pi_{RC} = \left[a - TP + (n-1)kP\right]\left(nP - nW_C - \sum_{i=1}^{n} c_i\right). \tag{31}$$

Theorem 6. *If $T > (n-1)k$, the total profit function of all dealers π_{RC} is a concave function of P.*

Proof. The first-order and second-order derivatives of formula (31) are

$$\begin{aligned}\frac{d\pi_{RC}}{dP} &= -\left[T - (n-1)k\right]\left(nP - nW_C - \sum_{i=1}^{n} c_i\right) \\ &\quad + n\left[a - TP + (n-1)P\right] \\ &= -2n\left[T - (n-1)k\right]P + n\left[T - (n-1)k\right]W_C \\ &\quad + \left[T - (n-1)k\right]\sum_{i=1}^{n} c_i + na, \\ \frac{d^2\pi_{RC}}{dP^2} &= -2n\left[T - (n-1)k\right].\end{aligned} \tag{32}$$

If $T > (n-1)k$, $d^2\pi_{RC}/dP^2 < 0$ can be deduced. The second-order derivative of π_{RC} is less than zero, so π_{RC} is a concave function of P.

Making $d\pi_{RC}/dP = 0$, then

$$\begin{aligned}&-2n\left[T - (n-1)k\right]P + n\left[T - (n-1)k\right]W_C \\ &\quad + \left[T - (n-1)k\right]\sum_{i=1}^{n} c_i + na = 0.\end{aligned} \tag{33}$$

With further derivation, the optimal price P^* can be obtained as

$$P^* = \frac{n\left[T-(n-1)k\right]W_C + na + \left[T-(n-1)k\right]\sum_{i=1}^{n} c_i}{2n\left[T-(n-1)k\right]}. \quad (34)$$

Putting formula (34) into (30), the supplier's profit can be obtained:

$$\pi_{SC} = \frac{1}{2}\left\{na - n\left[T-(n-1)k\right]W_C - \left[T-(n-1)k\right]\sum_{i=1}^{n} c_i\right\}\left(W_C - C\right). \quad (35)$$

□

Theorem 7. *If the condition $T > (n-1)k$ in Theorem 6 is satisfied, the supplier's profit function π_{SC} is a concave function of W.*

Proof. The first-order and second-order derivatives of W in formula (35) are

$$\frac{d\pi_{SC}}{dW_C} = \frac{na}{2} + \frac{n\left[T-(n-1)k\right]C}{2} - n\left[T-(n-1)k\right]W_C - \frac{\left[T-(n-1)k\right]}{2}\sum_{i=1}^{n} c_i,$$
$$\frac{d^2\pi_{SC}}{dW_C^2} = -n\left[T-(n-1)k\right]. \quad (36)$$

According to the condition $T > (n-1)k$ in Theorem 6, $d^2\pi_{RC}/dP^2 < 0$ can be deduced. The second-order derivative of π_{SC} is less than zero, so π_{SC} is a concave function of W.

Making $d\pi_S/dW_C = 0$, then

$$W_C^* = \frac{a}{2\left[T-(n-1)k\right]} + \frac{C}{2} - \frac{1}{2n}\sum_{i=1}^{n} c_i. \quad (37)$$

Putting formula (37) into (34), then

$$P^* = \frac{3a}{4\left[T-(n-1)k\right]} + \frac{C}{4} + \frac{1}{4n}\sum_{i=1}^{n} c_i. \quad (38)$$

Formulas (37) and (38) are the equilibrium solutions of the supplier and dealers in the vertical game with coalition. Putting these two formulas into (35), the supplier's profit can be obtained:

$$\pi_{SC} = \left\{\frac{na}{4} - \frac{n\left[T-(n-1)k\right]C}{4} - \frac{T-(n-1)k}{4}\sum_{i=1}^{n} c_i\right\} \times \left(\frac{a}{2\left[T-(n-1)k\right]} - \frac{C}{2} - \frac{1}{2n}\sum_{i=1}^{n} c_i\right). \quad (39)$$

Making all the retail prices of (9) be equal to unified price P, the profit function of each dealer j can be written as

$$\pi_{R_j} = \left\{a - \left[T-(n-1)k\right]P\right\}\left(P - W - c_j\right). \quad (40)$$

Putting formulas (37) and (38) into (40), the profit of each dealer j is

$$\pi_{R_j} = \left\{\frac{a}{4} - \frac{\left[T-(n-1)k\right]C}{4} - \frac{\left[T-(n-1)k\right]}{4n}\sum_{i=1}^{n} c_i\right\} \times \left(\frac{a}{4\left[T-(n-1)k\right]} - \frac{C}{4} + \frac{3}{4n}\sum_{i=1}^{n} c_i - c_j\right). \quad (41)$$

According to Definition 5, coalition stability can be judged by the comparison between all dealers' profits with coalition and without coalition. □

6. Numerical Simulations

In the numerical simulations, a two-stage supply chain consists of one supplier and 10 dealers; namely, $n = 10$ is employed. The supplier's marginal cost C is set to 1000, and ten dealers' marginal sale costs are $c_1 = 110$, $c_2 = 120$, $c_3 = 130$, $c_4 = 140$, $c_5 = 150$, $c_6 = 160$, $c_7 = 170$, $c_8 = 180$, $c_9 = 190$, and $c_{10} = 200$. The saturated market demand of each dealer is 2000, in other words, $a = 2000$. With T, k changing, the dealers' retail prices and profits, the supplier's wholesale price and profit, and market demand with and without coalition can be calculated. Partial data is showed in Table 1. Subscript "N" means "without coalition," and subscript "C" means "with coalition." D_C denotes the total market demand of 10 dealers, and each dealer's market demand is 1/10 of D_C because they set the same retail price.

Analyzing the data shown in Table 1, the following conclusions can be summarized.

(1) If $(T, k) = (1, 0.03)$, $(1, 0.05)$, $(1, 0.1)$, and $(2, 0.12)$, all dealers' profits with coalition are greater than those without coalition, which accords with Definition 5, so the coalition is stable in these cases. However, if $(T, k) = (1, 0.1)$, the supplier's cost $C = 1000$ and its wholesale price $W = 10423$. The wholesale price of the supplier is about ten times as much as its cost, and the ratio of wholesale to cost will become greater with the increase of T. This case accords with a luxury sale situation, and the price-sensitive model in this paper is unsuitable for it because the consumers of luxury may not be price-sensitive.

(2) Simulation results show that the supplier's optimal wholesale price is constant with and without coalition. It is clear that the profit of the supplier depends on two factors: wholesale price W and total market demand D_A. If the supplier increases the wholesale price W, it will lead to the increase of the unified price P, which makes the total market demand D_A decrease and the supplier's profit reduce. There is no motivation for the supplier to change its wholesale price, and the wholesale price is kept constant with and without coalition.

(3) Dealers' coalition leads to a loss of the supplier. If dealers' coalition is stable, the unified price P will be higher than all dealers' retail prices they set

TABLE 1: The variety of decision variables and profits influenced by T, k.

	(T, k)					
	(1, 0.01)	(1, 0.03)	(1, 0.05)	(1, 0.1)	(2, 0.1)	(2, 0.12)
P_1	1902.4	2259.5	2814	11413	1582.3	1803.1
P_2	1907.4	2264.5	2818.9	11417	1587.2	1808
P_3	1912.4	2269.4	2823.7	11422	1592.1	1812.8
P_4	1917.4	2274.4	2828.6	11427	1596.9	1817.7
P_5	1922.3	2279.2	2833.5	11432	1601.8	1822.5
P_6	1927.3	2284.2	2838.4	11436	1606.7	1827.4
P_7	1932.3	2289.1	2843.2	11441	1611.6	1832.3
P_8	1937.3	2294	2848.1	11446	1616.4	1837.1
P_9	1942.2	2299	2853	11451	1621.3	1842
P_{10}	1947.2	2303.9	2857.9	11456	1626.2	1846.8
P	1937.1	2343.5	3016	15289	1652.4	1919.2
W_N	1521.4	1792.4	2040.7	10423	1331.6	1509.5
W_C	1521.4	1792.4	2040.7	10423	1331.6	1509.5
π_{SN}	1295200	2649300	5462004	80712280	780310	163550
π_{SC}	1237000	2291600	4233100	44392000	604740	119390
π_{RN}						
π_{R1}	73456	127580	214640	774690	39599	67469
π_{R2}	70758	123980	209920	765490	36768	63741
π_{R3}	68110	120430	205250	756350	34043	60120
π_{R4}	65512	116940	200640	747270	31422	56604
π_{R5}	62965	113490	196070	738240	28907	53194
π_{R6}	60469	110100	191560	729270	26496	49891
π_{R7}	58023	106760	187110	720350	24190	46693
π_{R8}	55627	103470	182700	711480	21990	43601
π_{R9}	53282	100230	178350	702680	19894	40615
π_{R10}	50988	97040	174050	693920	17903	37735
π_{RC}						
π_{R1}	72524	127600	227010	2240800	38444	70241
π_{R2}	70151	124700	223590	2236100	36620	67898
π_{R3}	67779	121810	220180	2221400	34796	65554
π_{R4}	65407	118920	216770	2226700	32973	63211
π_{R5}	63034	116030	213360	2221900	31149	60867
π_{R6}	60662	113130	209950	2217200	29325	58524
π_{R7}	58289	110240	206530	2212500	27501	56180
π_{R8}	955917	107350	203120	2207800	25678	53837
π_{R9}	53545	104460	199710	2203100	23854	51493
π_{R10}	51172	101570	196300	2198400	22030	49150
D_N						
D_1	271	357	463	880	281	367
D_2	266	352	458	875	271	357
D_3	261	347	453	870	261	347
D_4	256	342	448	864	251	336
D_5	251	337	443	859	240	326
D_6	246	332	438	854	230	316
D_7	241	327	433	849	220	306
D_8	236	322	427	843	210	295
D_9	231	317	422	838	199	285
D_{10}	226	312	417	833	189	275
D_A	2484	3343	4402	8566	2353	3210
D_C	2370	2890	3410	4710	1820	2340

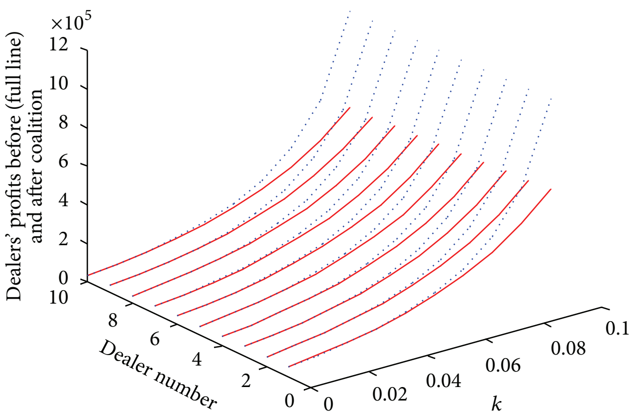

Figure 2: Dealers' profits with and without coalition ($T = 1$).

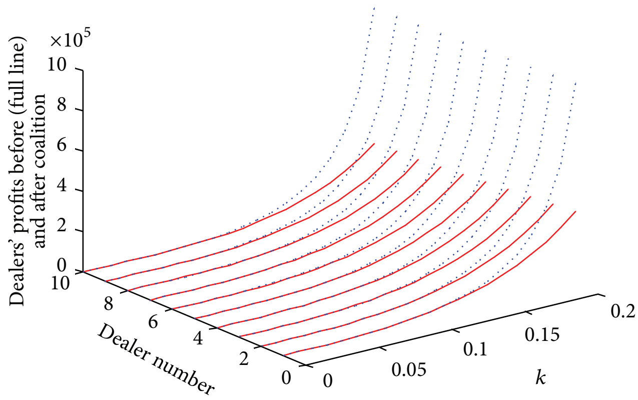

Figure 3: Dealers' profits with and without coalition ($T = 2$).

without coalition. The increase of retail price results in the decrease of demand, and the wholesale price is constant with and without coalition, so the supplier's profit with coalition is less than without coalition. In other words, dealers' coalition has a negative impact on the supplier.

Making $T = 1$, $0 < k < 0.09$ and $T = 2$, $0 < k < 0.2$, the ratio of wholesale price to cost is less than ten in these two value ranges. Moreover, market situations can be described by these two value ranges adequately and completely, so the profit curves of the dealers with and without coalition are drawn as shown in Figures 2 and 3.

In Figures 2 and 3, solid lines and dotted lines are intertwined at first. The dealers' profits with coalition (dotted lines) are gradually higher than those without coalition (solid lines), and the coalition becomes stable with the increase of k. If market price sensitivity T is fixed, the bigger the substitution rate k is (in the above ranges), the more stable the dealers' coalition is. Simulation results show that point $k = 0.03$ is a breakthrough point from stable coalition to unstable one if $T = 1$. In other words, all dealers' profits with coalition are more than those without coalition when $k \geq 0.03$. Similarly, $k = 0.11$ if $T = 2$, $k = 0.209$ if $T = 3$, $k = 0.314$ if $T = 4$, and $k = 0.423$ if $T = 5$ are also the breakthrough points. Obviously, the critical point k increases with the increase of T.

7. Conclusions

Horizontal coalition stability of a two-stage supply chain, which consists of one supplier and multiple dealers, is studied in this paper. Vertical sequential game and horizontal Nash game are used to analyze the profits of the supplier and dealers with and without coalition. If the profits of the supplier and dealers in different coalition situations (different values of T and k) are known, the coalition stability can be judged by the criterion that the profits of all dealers with coalition should be more than those without coalition. Unified pricing is a coordination mechanism for dealers, and the following conclusions can be obtained from numerical simulations: (1) the profits of the supplier and dealers cannot increase at the same time. If the coalition is stable, the increase of dealers' profits results in the loss of the supplier; (2) through the analysis on vertical sequential game, the supplier's optimal wholesale price is constant with and without coalition. If the dealers' coalition is stable, the unified price of dealers with coalition will be higher than all dealers' retail prices without coalition; (3) if market price sensitivity T is fixed, the increase of the substitution rate k in a reasonable range will lead to the increase of the difference between each dealer's profit without coalition and with coalition, and the dealers will be more likely to ally with each other.

The conclusions mentioned above provide an important reference to horizontal coalition decision making of entities in the same status in supply chain. For example, these conclusions can help 4S dealers to decide when and how to form a coalition based on different market situations to acquire the optimal profits.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Research Article

Cost Allocation in PPP Projects: An Analysis Based on the Theory of "Contracts as Reference Points"

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In recent years, the demand for infrastructure has been largely driven by the economic development of many countries. PPP has proved to be an efficient way to draw private capital into public utility construction, where ownership allocation becomes one of the most important clauses to both the government and the private investor. In this paper, we establish mathematical models to analyze the equity allocation problem of PPP projects through a comparison of the models with and without the effects of the theory of "contracts as reference points." We then derive some important conclusions from the optimal solution of the investment ratio.

1. Introduction

In recent years, the demand for infrastructure has been largely driven by the economic development of many countries. Hence governments, especially those of developing countries, face great pressure to find new ways to meet the financial needs of an increasing number of infrastructure projects. PPP (public-private-partnership) is a collaboration through which the public and private sectors both bring their complementary skills into an infrastructure project and receive different levels of benefit based on their proportion of investment and responsibility [1]. PPP has proved to be an efficient way to draw private capital into public utility construction as it not only solves financial problems by utilizing private capital to oversee the designing, financing, building, and operation of the project, but also enhances the level and the efficiency of management [2].

Under PPP mode, how to allocate the asset ownership becomes one of the most important clauses to both the government and the private investor. Usually, the allocation of profit depends on two factors. First, like other cooperation partnerships, the government and the private sector gain earnings in proportion to their investment scales. In addition, the level of risks that each sector undertakes also determines the profit that each can obtain. There are many papers which discuss how to allocate different risk factors between the two parts, many of which suggest that the risks each sector is responsible for should be written in the contract. Once a risk occurs, the government and the private sector can investigate and affix the responsibility for it according to the contract. In the early stages of research, scholars did not think that the issue of risk allocation should follow a common rule. Furthermore, some scholars insisted that one of the key aspects of PPP is that the private sector should assist the government in accomplishing the infrastructure program [3]. However, in the practical application of PPP mode, many cases show that risk allocation should follow some principles. In summary, the cost, the risk, and the profit are three main provisions that should be made in the contract. In fact, risk allocation has become one of the most important issues in current research and many scholars have put forward different views on it. Rutgers and Haley believe that the risks should be assigned to the sector that can control it well [4]. On the other hand, Crampes and Estache analyzed the influence of risk preference to risk allocation. When both the government and the private enterprise were risk neutral, there was no necessity to discuss the issue. However, if the private sector was risk averse, and the government was risk neutral, most of the risks should be allocated to the public sector [5]. Bing et al. explored preferences in risk allocation.

Their paper shows that some macro- and microlevel risks should be retained within the public sector or shared with the private sector. The majority of risks in PPP/PFI projects, especially those in the mesolevel risk group, should be allocated to the private sector [6]. Xu et al. developed a fuzzy synthetic evaluation model for determining equitable risk allocation between the government and the private sector, which assists PPP project practitioners in transforming the risk allocation principles from linguistic terms to a more usable and systematic quantitative-based analysis through the fuzzy set [7]. Medda believes that the process of risk allocation between the two sectors is a bargaining process, and the process could be modeled with a final offer arbitration game. Her paper shows that when guarantees have a higher value than financial loss, we are confronted with strategic behavior and potential moral hazard problems [8]. Ke et al. conducted a two-round Delphi survey to identify the preference of risk allocation in China's PPP projects. They concluded that the government should take the majority of responsibility for thirteen of the risks, whereas the private sector should take responsibility for ten of them, with the rest shared between the two parts [9]. To sum up, there are two principles that are generally accepted: the risk should be assigned to the sector that can control it efficiently, and authority, responsibility, and profit must match.

Equity allocation is an issue that western scholars have paid little attention to and which only a handful of Chinese researchers have conducted studies on. Besley and Ghatak analyzed how ownership matters in public good provision and showed that the party that can better value the benefits generated by the project should own it [10]. Chinese scholars Zhang et al. believed that PPP is a cooperation based on contracts, so control rights become the key influence factor in determining the efficiency of PPP projects. On the basis of this idea, a new analysis frame was proposed [11]. In a subsequent paper, Zhang et al. again pointed out that the key factor of PPP cooperation efficiency is the allocation of control rights from the perspective of incomplete contracts and established mathematical model to analyze the relationship between the allocation of control rights and the cooperation efficiency in the context of PPP projects. This paper showed that the key to enhance PPP efficiency is allocating different degrees of control rights to the public sector or the private one based on different parameters [12]. Ye et al. discussed the essence of control rights in PPP projects and insisted that control rights constitute not only a form of choice for the cooperation between public and private, but also a condition of obligation for both parties [13]. Further, they presented an interest distribution model based on risk adjustment. The model takes both the investment scale and risk allocation into account to allocate the profits fairly and proportionately. However, in this paper the authors only proposed the expressions of the profit of the two parties. The optimal allocation proportion was not calculated [14]. Hu et al. also established a benefit allocation model using the value method of SHAPELY to maximize the participants' profit and showed through numerical analysis how optimality can be obtained under this model [15]. In summary, by reviewing previous literatures, we can find that many studies have discussed the risk and cost allocation of PPP project. Some of them realized that both of the two factors played an important role in equity allocation, but none of them deduced the optimal cost allocation proportion.

One of the key defining features of PPP is the contractual cooperation of the government and the private enterprise to maximize the benefits they expect to get, while sharing both complementary advantages and risk. From this perspective, we can consider the PPP participants as constituting a form of strategy alliance, which is a topic that most papers analyze through game theory. Initially, scholars only took two players into account, before expanding it to include three or more players [16, 17]. However, all of these papers established the game model based on the hypothesis of personal rationality. In recent years, many psychology experiments have shown that people would be influenced by their mental perception, including the need for fairness and their dependence on reference points [18, 19]. In 2006, Hart and Moore proposed a new method of analyzing the processes of decision-making in trading by using contracts as reference points, which takes into consideration the parties' feelings of entitlement. A participant's ex post performance depends on whether he earns what he is entitled to as permitted by the contract. If he is shortchanged, he will shade on performance [20]. Gao and Xu applied this theory to analyze the utility of the alliance's members under four different situations [21]. Similarly, it is necessary to consider the feelings of entitlement of PPP participants.

To enhance the current models, based on the mathematic descriptions of risk and cost in previous literatures, we analyze the equity allocation problem of PPP projects by comparing the results with and without the effect of the theory of "contracts as reference point." We assume that the risks are assigned to the sector that can control it efficiently through the negotiation, and to maximize the total surplus, we derive the optimal cost allocation proportion. The finding shows that the satisfaction of the equity allocation will influence the utility of the participants of PPP projects. The less the satisfaction, the less the cost the participants will pay. In the following parts of the paper, we first set up the basic model and give some assumptions. Then we calculate the optimal investment scale when the theory of "contracts as reference points" is not considered. Subsequently, we take the theory into consideration to solve the optimal proportion. Finally, we derive some important conclusions based on the two results.

2. Basic Model and Some Assumptions

We assume that the government plans to build an infrastructure under PPP mode. The project is jointly funded by the government and the private sector. Thus the proportion of the profit distribution is supposed to be decided by the investment scale and risk taking scale. Here we assume that the investment ratio of the government is k $(0 < k < 1)$ and the risk ration that the government takes is ω_g, while the relevant ratios for the private sector are $1 - k$ and ω_s. λ $(0 < \lambda < 1)$ reflects the importance of investment in profit allocation, and $1-\lambda$ is the importance of risk taking. The profit of the project is R. The private sector has an opportunity utility, which refers to the utility that the private enterprise could get when he does not join the program. Thus, if the government wants to draw the private sector to invest in the project, the public sector should assure the private sector of getting the opportunity profit from the project. Here we assume that the opportunity profit is $\overline{R}$. So the basic object programming model is

$$V = \left[\text{Max } f_1(k), \text{Max } f_2(k)\right], \tag{1}$$

where $f_1(k)$ refers to the profit function of government,

$$f_1(k) = R\left[\lambda k + (1-\lambda)\omega_g\right], \tag{2}$$

and $f_2(k)$ refers to the profit function of the private sector

$$f_2(k) = R\left[\lambda(1-k) + (1-\lambda)\omega_s\right]. \tag{3}$$

In the following parts, we will compare the models with and without the effect of the theory of "contracts as reference points." First, we analyze the simple model without the contract reference point dependence.

3. The Equity Allocation Model without "Contracts as Reference Point" Theory

In this situation, $f_2(k) \geq \overline{R}$, that is, $f_2(k) = R[\lambda(1-k) + (1-\lambda)\omega_s] \geq \overline{R}$. We assume that the public sector here is the agent of the social commonage. So the government should not pursue the economic profit. The profit that the government want to earn is $f_1(k) \geq 0$, that is, $f_1(k) = R[\lambda k + (1-\lambda)\omega_g] \geq 0$.

Here we take the total surplus as the objective function, and the utility function is additive, that is, $F(k) = \sum_{j=1}^{2} f_j(k)$. So the objective function is

$$F(k) = \text{Max} \sum_{j=1}^{2} f_j(k). \tag{4}$$

And the constraint conditions are

$$\begin{gathered} R\left[\lambda(1-k) + (1-\lambda)\omega_s\right] \geq \overline{R}, \\ R\left[\lambda k + (1-\lambda)\omega_g\right] \geq 0. \end{gathered} \tag{5}$$

By applying the Lagrange function, we can solve the single-objective programming problem. Its Lagrange function is

$$L(k,\lambda_1,\lambda_2) = f_1(k) + f_2(k) + \lambda_1 f_1(k) + \lambda_2\left[f_2(k) - \overline{R}\right]. \tag{6}$$

So it can be expressed as

$$\begin{aligned} L(k,\lambda_1,\lambda_2) = {} & \left\{R\left[\lambda k + (1-\lambda)\omega_g\right]\right\} \\ & + \left\{R\left[\lambda(1-k) + (1-\lambda)\omega_s\right]\right\} \\ & + \lambda_1\left\{R\left[\lambda k + (1-\lambda)\omega_g\right]\right\} \\ & + \lambda_2\left\{R\left[\lambda(1-k) + (1-\lambda)\omega_s\right] - \overline{R}\right\}. \end{aligned} \tag{7}$$

The first order conditions are

$$\begin{aligned} \frac{\partial L}{\partial k} &= \lambda_1 - \lambda_2 = 0, \\ \frac{\partial L}{\partial \lambda_1} &= R\left[\lambda k + (1-\lambda)\omega_g\right] = 0, \\ \frac{\partial 0}{\partial \lambda_2} &= R\left[\lambda(1-k) + (1-\lambda)\omega_s\right] - \overline{R} = 0. \end{aligned} \tag{8}$$

Thus, the optimal solution of k is

$$k = 1 - \frac{\overline{R} - R(1-\lambda)\omega_s}{R\lambda}. \tag{9}$$

From this solution, we can draw some important conclusions.

(1) $(\partial k/\partial R) = (\overline{R}/R^2\lambda) > 0$, which means that the government will invest more with the growth of the expected profit of the PPP project. Although we assume that the government does not pursue economic benefits, if the profit is more than expected, the government would get more equity to get more control rights and guarantee a proper social benefit of the infrastructure program to avoid the private sector from earning too much profit.

(2) $(\partial k/\partial \omega_s) = ((1-\lambda)/\lambda) > 0$. From this characteristic, we can see that the higher the risk the private company takes, the more the cost which the government pays. Generally speaking, the purpose of the government in launching the program is to bring social benefit to the public. So the government may prefer to take less risk in the program to guarantee social profit. If the private sector is willing to take more risk, the government should be ready to cut down its equity to make up for the private sector's venture cost.

(3) $(\partial k/\partial \lambda) = ((\overline{R} - \omega_s R)/R\lambda^2)$, when $\omega_s < (\overline{R}/R)$, $(\partial k/\partial \lambda) > 0$; when $\omega_s > (\overline{R}/R), (\partial k/\partial \lambda) < 0$. This means that when the risk ratio that the private enterprise takes is in a certain range, with the growth of the importance of investment, the government is willing to pay more cost. On the contrary, when the risk ratio that the private enterprise takes exceeds a certain value, with the growth of the importance of investment, the government will not be willing to pay much in the program. The reason for this is the government has to invest more to get the profit control right if the private company is not willing to take much risk, so that the government could balance its profit and loss.

In conclusion, if the risk sharing ratio is constant, with the growth of the expected profit of the PPP project, the government will invest more in it. When the risk of the program is low, the government will be willing to pay more cost. When the program faces high risk, the government is inclined to transfer the risk to the private company who can control it. Thus a risk-averse government can choose the investment ratio dynamically to defend the high risk.

4. The Equity Allocation Model with "Contracts as Reference Point" Theory

According to Hart's theory, a trade is partially contractible. Correspondingly, the joint utility is divided into two parts. The first part is called perfunctory performance, which is within the letter of the contract and can be observed. So this part of the utility is judicially enforced because of the contract. The second part is consummate performance. This kind of utility cannot be observed because this performance is within the spirit of the contract, so it cannot be enforced by the contract ex ante. Hart insists that one party of the trade will be willing to provide all the consummate performance if he feels that he is getting what he is entitled to. The entitlement is measured relative to the contract. But when he feels shortchanged, he will withhold some part of the consummate performance, that is, the degree of satisfaction of each party decided by the contract, which is the reference point that is considered in this paper. Gao and Xu applied Hart's theory in analyzing the utility function of strategy alliance member. In their paper, when the members in a strategy alliance distribute the profit, their incomes include perfunctory performance and consummate performance. They gave a simple example in their work. Here are two members in the alliance. u_i $(i = 1, 2)$ represents the perfunctory performance (common benefit), which are written in the contract. α_i $(i = 1, 2)$ represents the difference between reference point and common benefits, whereas σ_i $(i = 1, 2)$ refers to the opportunist benefit. The opportunist benefit only occurs when a party's speculation behavior encroaches on the other's benefit. θ denotes that one dollar of α brings θ dollars loss in spirit, that is, for each dollar that a party feels shortchanged by, he shades his performance so that the other party's payoff falls by θ dollars, where $0 \leqslant \theta \leqslant 1$. Here we assume that $\theta_1 = \theta_2 = \theta$. The utility functions of alliance members are

$$\begin{aligned} U_1 &= u_1 - \sigma_2 - \max\{\theta\alpha_1 - \sigma_1, 0\}, \\ U_2 &= u_2 - \sigma_1 - \max\{\theta\alpha_2 - \sigma_2, 0\}. \end{aligned} \tag{10}$$

Based on this model, the utility of a member is equal to the common benefit defined by the contract, minus the probable opportunist profit encroached on by the other member, and minus the difference between the opportunist profit encroached on from the other member and the benefit he thinks he should get in addition to the common profit. Here we want to emphasize that when both of the two members' α is equal to zero or not, the total utility of this alliance is equal to the total profit minus the sum of α_i $(i = 1, 2)$. To simplify the analysis, we ignore the opportunist benefit, that is,

$$\begin{aligned} U_1 &= u_1 - \theta\alpha_1, \\ U_2 &= u_2 - \theta\alpha_2. \end{aligned} \tag{11}$$

In Hart's paper, he took θ to be exogenous and constant. However, we think that θ should be endogenous and variable. To set up our model, we give some assumptions.

Assumption 1. Both of the government and the private sector's opportunist benefits are equal to zero.

Assumption 2. θ is relevant to the investment amount. So it is different between the government and the private sector.

Assumption 3. For the government, $\theta_g = \beta k^2$. Here $\partial\theta_g/\partial k > 0, \partial^2\theta_g/\partial k^2 > 0$ means that the more the government pays, the more the loss is, when it thinks the profit allocation is unfair, and the faster the loss increases. Similarly, for the private sector, $\theta_s = \beta(1-k)^2$.

On basis of above assumptions, the objective function is

$$\begin{aligned} \max &\left\{R\left[\lambda k + (1-\lambda)\,\omega_g\right] - \theta_g\alpha_1\right\} \\ &+ \left\{R\left[\lambda(1-k) + (1-\lambda)\,\omega_s\right] - \theta_s\alpha_2\right\}. \end{aligned} \tag{12}$$

To maintain the alliance, the constraint conditions are

$$\begin{aligned} R\left[\lambda k + (1-\lambda)\,\omega_g\right] - \theta_g\alpha_1 &\geq 0, \\ R\left[\lambda(1-k) + (1-\lambda)\,\omega_s\right] - \theta_s\alpha_2 &\geq \overline{R}. \end{aligned} \tag{13}$$

To solve the problem, we calculate its $K-T$ conditions by bringing in the Lagrange multipliers μ_1 and μ_2

$$\begin{aligned} &2\beta\alpha_1 k - 2\beta\alpha_2(1-k) + \mu_1\left(2\beta k\alpha_1 - R\lambda\right) \\ &\quad + \mu_2\left[-2\beta\alpha_2(1-k) + R\lambda\right] = 0, \\ &\mu_1\left\{R\left[\lambda k + (1-\lambda)\,\omega_g\right] - \theta_g\alpha_1\right\} = 0, \\ &\mu_2\left\{\overline{R} - R\left[\lambda(1-k) + (1-\lambda)\,\omega_s\right] - \theta_s\alpha_1\right\} = 0, \\ &\mu_1, \mu_2 \geq 0. \end{aligned} \tag{14}$$

From the above equation set, we can get the optimal investment scale.

Proposition. Under the assumption of "contracts as reference point" theory, the optimal investment scale of the government in the PPP project is

$$k = \frac{\alpha_2}{\alpha_1 + \alpha_2}, \quad (\alpha_1, \alpha_2 > 0). \tag{15}$$

Accordingly, the optimal scale for the private sector is

$$1 - k = \frac{\alpha_1}{\alpha_1 + \alpha_2}, \tag{16}$$

where α_i $(i = 1, 2)$ represents the difference between the reference point and common benefits. And the risk that the government takes satisfies

$$\omega_g \in [0, h]. \tag{17}$$

Here

$$h = \frac{\overline{R}(\alpha_1 + \alpha_2)^2 + \beta\alpha_2\alpha_1^2 - R\lambda\alpha_1^2 - R\lambda\alpha_1\alpha_2}{R(1-\lambda)(\alpha_1 + \alpha_2)^2}. \tag{18}$$

From the proposition, we draw some conclusions.

(1) When taking Hart's theory into consideration, the optimal investment ratio is only relevant to the parameters α_i $(i = 1, 2)$, that is, the satisfaction degree to the result of ownership allocation. In this paper, we take the total utility as the objective function. This result illustrates that, in cases where self-interest is guaranteed, the satisfaction degree of the participants in the PPP project decides whether the total utility of the system can reach the maximum value. One's optimal investment proportion is the ratio of the other's difference between the reference point and common benefits in the sum of the participants' difference between the reference point and common benefits. $(\partial k/\partial \alpha_1) < 0$ means that if the government is discontent with the allocation result, it will invest less cost in the project. The same explanation is also suitable for the private sector.

(2) From expressions (17) and (18), we can see that when the risk that the government takes is below a certain value, the alliance can be maintained and the negotiation between the two parties could be successful. Comparing with the result of part 3, the parties' utilities of the project are decided not only by the economic benefits, but also by the satisfaction degree of the ownership allocation result. Thus, to avoid discontentment, the parties will adjust not only the cost they pay but also the risk they take, to make up for the psychological loss they may suffer. Since the government cares for the social benefit more, the risk that the public sector is willing to take has a limit. If the government takes too much risk, the social benefit of the project will suffer a loss. However, the economic benefit for the private sector should be guaranteed, so the risk that the private sector takes should be beyond a certain ratio.

(3) $(\partial h/\partial R) < 0$ means that the higher the profit that the project earns, the less the risk that the government is happy to take. This is because when the profit is beyond expectations, the discontent degree may increase. So the motivation of investing declines. $(\partial h/\partial \overline{R}) > 0$ illustrates that the higher the opportunity benefit of the private sector is, the higher the risk the government may take. This is because the willingness of investment will increase if the private sector has more desire to obtain ownership of the project. So the range of risk that the government takes will be wider.

5. Conclusion

In this paper, we establish mathematic models to analyze the equity allocation problem of PPP projects, by comparing the models with and without the effect of the theory of "contracts as reference point." Subsequently, we derive some important conclusions from the optimal solution of the investment ratio.

Without the assumption of reference point dependence, the optimal allocation of the participants' investment proportions is decided by the risk they take, the profit of the project, the expectation profit, and the importance of investment. If the government is risk averse, the optimal contract should be flexible, so that the government can choose the investment ratio dynamically to defend the high risk.

On the contrary, if we take the reference point into consideration, the satisfaction degree of the participants in the PPP project decides whether the total utility of the system can reach the maximum value. The less the satisfaction is, the less the cost the participants will pay. Besides, the risk that the parties are willing to share is affected by many factors. The parties' utilities of the project are decided not only by the economic benefits, but also by the satisfaction degree of the ownership allocation result. Thus, to avoid discontentment, the parties will adjust not only the cost they pay but also the risk they take, to make up for the psychological loss they may suffer.

In summary, the satisfaction of the equity allocation will influence the utility of the participants of PPP projects. Thus, it is necessary to take the mental factor into consideration in decision-making processes. In this paper, we take the total surplus as the objective function. However, the decision process may be more complicated. Usually, both of the public and the private sectors are inclined to maximize their own utility when they make decision. Thus, the game model may be more appropriate to describe their decision process. In our future work, we will pay more attention to the game problem between the participants to make our study even more practical.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Research Article

Stochastic Machine Scheduling to Minimize Waiting Time Related Objectives with Emergency Jobs

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We consider a new scheduling model where emergency jobs appear during the processing of current jobs and must be processed immediately after the present job is completed. All jobs have random processing times and should be completed on a single machine. The most common case of the model is the surgery scheduling problem, where some elective surgeries are to be arranged in an operation room when emergency cases are coming during the operating procedure of the elective surgeries. Two objective functions are proposed to display this practice in machine scheduling problem. One is the weighted sum of the waiting times and the other is the weighted discounted cost function of the waiting times. We address some optimal policies to minimize these objectives.

1. Introduction

Machine scheduling problems have attracted researchers for decades since they play an important role in various applications from the areas of operations research, management science, and computer science. There are lots of works of the literature published on these problems. However, most of them study the deterministic case where all the information about jobs and machines is completed without any uncertainty; for example, job processing times are assumed to be exactly known in advance. But it is really hard to know the exact values of these parameters in practical situations and thus such an assumption is hardly justifiable. Instead, one sometimes can only roughly estimate the values of these parameters or their probability distributions. As Albert Einstein said, "As far as the laws of mathematics refer to reality, they are not certain, as far as they are certain, they do not refer to reality."

In addition, the majority of machine scheduling problems studied in the literature assume that all jobs are prepared for processing before scheduling. In reality, however, it is a common phenomenon that new jobs may come randomly from time to time. The necessity to account for random coming of jobs is a key drive of developing stochastic approaches to scheduling problems.

In this paper, we study the single machine stochastic scheduling problem with emergency jobs. In our study, we focus on optimal policies to schedule current jobs with some objectives on a single machine when emergency jobs appear during the processing of current jobs and must be processed immediately after the present job is completed. All processing times of jobs are not known. The most common case of the model is the surgery scheduling problem, where some elective surgeries are to be arranged in an operation room when emergency cases are coming during the operating procedure of the elective surgeries. Those emergency cases must be operated as soon as possible. However, since the present surgery occupies the operation room, they should wait to operate until the current surgery is completed.

As indicated by Rothkopf and Smith [1], there are two basic classes of delay costs considered in scheduling problems. The first class includes linear costs, which consider no discount of the value of money over time, whereas the other class involves exponential functions to represent the discounts as a function of time. In our research, two objective functions are expected to minimize. One is the weighted sum of jobs' waiting time and the other is the weighted discounted function of jobs' waiting times. Our first objective belongs to the first class, while our second objective function belongs to the second class. We will give optimal policies to solve those problems.

The remainder of this paper is organized as follows. In Section 2, the related works of the literature will be reviewed. Notations and some definitions of the model are provided in Section 3. And specially, Section 3.1 investigates the optimal policy to schedule jobs with the objective to minimize the total weighted waiting time of jobs. The optimal policy for scheduling jobs to minimize the discount cost of the waiting time of jobs is included in Section 3.2. Concluding remarks and further research are presented in Section 4.

2. Literature Review

There has been an enormous amount of work on single machine scheduling. We do not intend to do a complete review of results in the area and restrict our attention to works of the literature directly related to the matter of this paper.

The first is scheduling jobs on machines minimizing the total weighted completion time of jobs. It has attracted a great deal of attention, partly because of its importance as a fundamental problem in scheduling and also because of new applications, for instance, in manufacturing, compiler optimization, and parallel computing. Researchers have studied it extensively in different environments, such as allowing different release times of jobs, multimachine scheduling, and online scheduling. We here just review the scheduling problem on a single machine and all jobs have the same release times.

Perhaps the most famous scheduling policy with this objective in the deterministic case is Smith's rule, also known as the WSPT (weighted shortest processing time) rule. It schedules jobs in the order of nonincreasing ratio $\{w_j/p_j\}$, where p_j denotes the deterministic processing time of job j and w_j is the weight assigned to job j, and it is shown to be optimal for scheduling jobs on a single machine by Smith [2]. Extending this rule to stochastic environment, firstly Rothkopf [4, 5] develops the WSEPT (weighted shortest expected processing time) rule, which schedules jobs in the order of nonincreasing ratio $\{w_j/\mathbb{E}[P_j]\}$. Rothkopf [4, 5] proves that the WSEPT rule is optimal for single machine scheduling with identical release dates. The technique adopted is to reduce the problem to the deterministic case by taking the expectations of the processing times. This policy, however, fails in the multimachine case with general processing times, even if the weights are identical; that is, $w_i \equiv w$. For more details about this research, we refer readers to see [3].

Another stream related to our research is to minimize total weight discounted cost function, such as Rothkopf [4, 5]. Recently, Cai et al. [6] consider the problem of scheduling a set of jobs on a single machine subject to random breakdowns. They study the preemptive-repeat model, which addresses the situation where, if a machine breaks down during the processing of a job, the work done on the job prior to the breakdown is lost and the job will have to be started from the beginning again when the machine resumes its work. They obtain the optimal policy for a class of problems to minimize the expected discounted cost from completing all jobs. For more results in the area, see the survey [3].

What differentiates our paper from the above works of literature is that we consider a new scheduling problem where the emergency jobs are constantly coming and should be placed as soon as possible. To the best of our knowledge, there is no previous research studying this problem. Therefore, we open up a new research direction for the scheduling problem. In addition, different from previous works of the literature considering the completion time of jobs, we focus on the waiting time of jobs. It is more reasonable in our scheduling environment.

3. Models and Optimal Policies

We here consider the stochastic scheduling problem with emergency jobs. Specially, what we want to study is how to schedule current jobs on a single machine when emergency jobs appear during the processing of jobs and must be processed immediately after the present job is completed. Without loss of generality, in the following we assume that the emergent stream of jobs has a Poisson process with rate λ. The processing time Z_j for the emergency job j is arbitrarily distributed with the mean μ.

Let $J = \{1, 2, \ldots, n\}$ be a set of jobs to be processed nonpreemptively on a single machine. In other words, if a job starts to process, other jobs must wait to be processed until the present job is completed. Assume that the processing time P_j of current job j is a random variable. We denote W_j as the waiting time of job j and let γ_j be the unit waiting cost of job j. In the following, we will propose two objectives and study the optimal polices to minimize those functions.

3.1. Objective: $\mathbb{E}[\sum \gamma_i W_i]$. In this subsection, we will first focus on the objective function to minimize total weighted waiting cost, written as $\mathbb{E}[\sum \gamma_i W_i]$ for short. We here will address that the well-known policy in stochastic single machine scheduling, WSEPT, is also an optimal policy for our model.

Theorem 1. *WSEPT, that is, sequencing jobs in nonincreasing order of $\gamma_j/\mathbb{E}[P_j]$, is an optimal nonpreemptive policy to minimize total weighted waiting cost.*

Proof. Consider the schedule $1, 2, \ldots, n$, denoted as Π. For the first job 1, its waiting time is 0. Based on the fact that new jobs' coming follows the Poisson process, it is easy to know that the expected number of emergency jobs, before job 2 starts to process, is $(\lambda/(1-\lambda))\mathbb{E}[P_1]$. Therefore, the expected waiting time of job 2 is $(1 + \lambda\mu/(1-\lambda))\mathbb{E}[P_1]$, since each new job has mean processing time μ. Let C_j be the completion time of job j. Similarly, we can obtain that

$$\begin{gathered} \mathbb{E}[W_i] = \mathbb{E}[C_{i-1}] + \frac{\lambda\mu\mathbb{E}[P_{i-1}]}{1-\lambda}, \\ \mathbb{E}[C_{i-1}] = \mathbb{E}[W_{i-1}] + \mathbb{E}[P_{i-1}], \\ \mathbb{E}[P_0] = 0. \end{gathered} \tag{1}$$

Thus, we have

$$\begin{aligned}\mathbb{E}\left[\sum_{i=1}^{n}\gamma_i W_i\right] &= \sum_{i=1}^{n}\gamma_i\mathbb{E}\left[W_i\right] \\ &= \sum_{i=1}^{n}\gamma_i\left(\mathbb{E}\left[C_{i-1}\right]+\frac{\lambda\mu\mathbb{E}\left[P_{i-1}\right]}{1-\lambda}\right) \\ &= \sum_{i=1}^{n}\gamma_i\mathbb{E}\left[W_{i-1}\right]+\sum_{i=1}^{n}\gamma_i\left(\frac{\lambda\mu-\lambda+1}{1-\lambda}\right)\mathbb{E}\left[P_{i-1}\right] \\ &= \sum_{i=1}^{n}\gamma_i\mathbb{E}\left[W_{i-2}\right]+\sum_{i=1}^{n}\gamma_i\left(\frac{\lambda\mu-\lambda+1}{1-\lambda}\right)\mathbb{E}\left[P_{i-2}\right] \\ &\quad +\sum_{i=1}^{n}\gamma_i\left(\frac{\lambda\mu-\lambda+1}{1-\lambda}\right)\mathbb{E}\left[P_{i-1}\right] \\ &= \left(\frac{\lambda\mu-\lambda+1}{1-\lambda}\right)\sum_{i=1}^{n}\gamma_i\sum_{k=1}^{i-1}\mathbb{E}\left[P_k\right].\end{aligned} \tag{2}$$

Without loss of generality, we assume that jobs are ordered in nonincreasing order of $\gamma_j/\mathbb{E}[P_j]$; that is, $\gamma_1/\mathbb{E}[P_1] \geq \gamma_2/\mathbb{E}[P_2] \geq \cdots \geq \gamma_n/\mathbb{E}[P_n]$. If this sequence, denoted by Π for brevity, is not an optimal policy, there must exist job j in some optimal policy such that job $j+1$ is processed before job j. Without loss of generality, let $1,2,\ldots,j-1,j+1,j,j+2,\ldots,n$, denoted by Π', be such optimal policy and we hence have $\Phi(\Pi) > \Phi(\Pi')$. Comparing these two policies and observing that the waiting cost of jobs $1,2,\ldots,j-1,\ j+2,\ldots,n$ is not changed, we obtain

$$\begin{aligned}&\Phi\left(\Pi\right)-\Phi\left(\Pi'\right) \\ &\quad= \left(\frac{\lambda\mu-\lambda+1}{1-\lambda}\right) \\ &\qquad\times\left(\sum_{i=j}^{j+1}\gamma_i\sum_{k=1}^{i-1}\mathbb{E}\left[P_k\right]-\gamma_{j+1}\sum_{k=1}^{j-1}\mathbb{E}\left[P_k\right]\right. \\ &\qquad\left.-\gamma_j\sum_{k=1}^{j-1}\mathbb{E}\left[P_k\right]-\gamma_j\mathbb{E}\left[P_{j+1}\right]\right) \\ &\quad= \left(\frac{\lambda\mu-\lambda+1}{1-\lambda}\right)\left(\gamma_{j+1}\mathbb{E}\left[P_j\right]-\gamma_j E\left[\mathbb{E}_{j+1}\right]\right)<0.\end{aligned} \tag{3}$$

Therefore, $\Phi(\Pi) < \Phi(\Pi')$, which leads to a contradiction. With the adjacent pairwise interchange, we can know that WSEPT is an optimal policy. This completes the proof. □

Remarks. (1) In fact, the optimal policy for the objective with minimizing total expected waiting cost is equal to that with minimizing total expected completion time. This can be shown according to the following equation:

$$\begin{aligned}\mathbb{E}\left[\sum\gamma_i C_i\right] &= \mathbb{E}\left[\sum\gamma_i\left(W_i+P_i\right)\right] \\ &= \mathbb{E}\left[\sum\gamma_i\left(W_i\right)\right]+\sum\gamma_i\mathbb{E}\left[P_i\right].\end{aligned} \tag{4}$$

It is well known that WSEPT optimally minimizes the total expected completion time for single machine problem, which coincides with the result we obtained. But here we consider a new scheduling model. Hence, this theorem extends the well-known optimal policy to a new scheduling environment.

(2) The theorem shows that it is simple to construct the policy to minimize the total weighted waiting time of jobs. The only information for the optimal policy is the weight and the expected processing time of jobs, while the exact distribution of processing times of jobs is needed so as to obtain optimal policy with simple structure in most works of the literature in stochastic scheduling.

3.2. Objective: $\sum\gamma_i\mathbb{E}[1-\exp(-\alpha W_i)]$. In this section, we study the objective of weighted discounted cost of the waiting times; that is, $\sum\gamma_i\mathbb{E}[1-\exp(-\alpha W_i)]$. Here α is the discount factor. Before giving the optimal policy for our model, we will release two lemmas.

Lemma 2. *Consider*

$$\sum_{x=0}^{\infty}\frac{\lambda^x}{x!}=e^{\lambda}. \tag{5}$$

Proof. Let x be a random variable and be followed with Poisson distribution. We then have

$$\sum_{x=0}^{\infty}e^{-\lambda}\cdot\frac{\lambda^x}{x!}=1. \tag{6}$$

By multiplying e^{λ} to both sides of the equality, we have $\sum_{x=0}^{\infty}(\lambda^x/x!)=e^{\lambda}$. □

The other lemma which will be used is stated as follows.

Lemma 3. *For any a, if x is followed with Poisson distribution, then* $\mathbb{E}[e^{ax}]=e^{\lambda(e^a-1)}$.

Proof. Since x is followed with Poisson distribution, we obtain

$$\begin{aligned}\mathbb{E}\left[e^{ax}\right] &= \sum_{x=0}^{\infty}e^{ax}\cdot\frac{\lambda^x e^{-\lambda}}{x!}=e^{-\lambda}\sum_{x=0}^{\infty}\frac{\lambda^x e^{ax}}{x!} \\ &= e^{-\lambda}\sum_{x=0}^{\infty}\frac{\left(\lambda e^a\right)^x}{x!}=e^{-\lambda}e^{e^a\lambda}=e^{\lambda(e^a-1)}.\end{aligned} \tag{7}$$

□

We now can prove our second main theorem as follows.

Theorem 4. *Sequencing jobs with the nonincreasing order of*

$$\frac{\gamma_j}{1-f_j} \tag{8}$$

are an optimal policy to minimize the objective $\sum \gamma_i \mathbb{E}[1-\exp(-\alpha W_i)]$, *where*

$$f_j = \mathbb{E}\left[e^{(-\alpha-\lambda+\lambda E[e^{-\alpha Z_1}])P_j}\right]. \tag{9}$$

Proof. Assume that $1, 2, \ldots, n$ is a feasible schedule. Let Y_j be the random variable that denotes the total time of processing job j and processing emergence jobs arriving during the processing of job j. And let N_j denote the random variable as the number of emergence jobs during the processing of job j. Observing that the emergence jobs arrive according to the Poisson distribution with rate λ, we know that random variable $(N_j \mid P_j)$ has a Poisson distribution with mean $(\lambda/(1-\lambda))P_j$. Thus,

$$\begin{aligned}
\mathbb{E}\left[e^{-\alpha Y_j}\right] &= \mathbb{E}\left[e^{-\alpha(P_j+N_jZ_j)}\right] \\
&= \mathbb{E}\left[\mathbb{E}\left[e^{-\alpha(P_j+N_jZ_j)} \mid P_j\right]\right] \\
&= \mathbb{E}\left[e^{-\alpha P_j}\mathbb{E}\left[e^{(-\alpha Z_j)\cdot N_j} \mid P_j\right]\right] \\
&= \mathbb{E}\left[e^{-\alpha P_j}e^{(\lambda/(1-\lambda))P_j(\mathbb{E}[e^{-\alpha Z_j}]-1)}\right] \\
&= \mathbb{E}\left[e^{-\alpha P_j-(\lambda/(1-\lambda))P_j+\mathbb{E}[e^{-\alpha Z_j}]P_j}\right] = f_j.
\end{aligned} \tag{10}$$

Based on the fact that all $\{Y_k\}_{k=1}^{k=n}$ are independent random variables, we thus can know that

$$\mathbb{E}\left[e^{-\alpha W_j}\right] = \mathbb{E}\left[e^{-\sum_{k=1}^{j-1}\alpha Y_k}\right] = \prod_{k=1}^{j-1} f_k. \tag{11}$$

Therefore, the objective function under sequence $j_1, j_2, \ldots, j_n$ results is

$$\sum_{i=1}^{n} r_{j_i}\left(1-\prod_{k=1}^{i-1} f_k\right). \tag{12}$$

Without loss of generality, we assume that jobs are ordered in nonincreasing order of $\gamma_j/(1-f_j)$; that is, $\gamma_1/(1-f_1) \geq \gamma_2/(1-f_2) \geq \cdots \geq \gamma_n/(1-f_n)$. If this sequence, denoted by Π, is not an optimal policy, there must exist j in some optimal policy such that job $j+1$ is processed before job j. Without loss of generality, let $1, 2, \ldots, j-1, j+1, j, j+2, \ldots, n$, denoted by Π', be such optimal policy and we hence have $\Phi(\Pi) > \Phi(\Pi')$. Comparing these two policies and observing that the waiting cost of jobs $1, 2, \ldots, j-1, j+2, \ldots, n$ is not changed, we obtain

$$\begin{aligned}
&\Phi(\Pi) - \Phi(\Pi') \\
&\quad = \sum_{i=j}^{j+1} r_i\left(1-\prod_{k=1}^{i-1} f_k\right) - r_{j+1}\left(1-\prod_{k=1}^{j-1} f_k\right) \\
&\qquad - r_j\left(1-f_{j+1}\prod_{k=1}^{j-1} f_k\right) \\
&\quad = -r_j\left(1-f_{j+1}\right)\prod_{k=1}^{j-1} f_k + r_{j+1}\left(1-f_j\right)\prod_{k=1}^{j-1} f_k \\
&\quad = \left(r_{j+1}\left(1-f_j\right) - r_j\left(1-f_{j+1}\right)\right)\prod_{k=1}^{j-1} f_k < 0.
\end{aligned} \tag{13}$$

Therefore, we have $\Phi(\Pi) < \Phi(\Pi')$, which leads to a contradiction. With the adjacent pairwise interchange, we can know that nonincreasing order of $\gamma_j/(1-f_j)$ is an optimal policy. This completes the proof. □

Remark. The structure of the optimal policy is a little complex comparing to the optimal policy in the last section. In addition to weight and processing time of jobs, this optimal policy is also related to the parameters λ, Z_1, α. It is reasonable since the objective function is really complex. It is, however, also easy to calculate or estimate those values with the help of computer. Besides, if the processing time of current and emergent jobs is followed with some special distribution, the optimal policy will have very simple structure. One example is stated as follows.

Example. Assume that $P_i \sim \exp(\nu_j)$ and $Z_i \sim \exp(\mu)$. We can easily calculate that

$$\begin{aligned}
f_j &= \mathbb{E}\left[e^{(-\alpha-\lambda+\lambda E[e^{-\alpha Z_1}])P_j}\right] = \mathbb{E}\left[e^{(-\alpha-\lambda+\lambda(\mu/(\mu+\alpha)))P_j}\right] \\
&= \mathbb{E}\left[e^{(-\alpha-(\lambda\alpha/(\mu+\alpha)))P_j}\right] = \frac{\nu_j}{\nu_j+\alpha+(\lambda\alpha/(\mu+\alpha))}.
\end{aligned} \tag{14}$$

It is easy to verify that the optimal policy sequences jobs in nonincreasing order of $\gamma_j(\nu_j+\alpha+(\lambda\alpha/(\mu+\alpha)))$.

4. Conclusions and Future Research

In this paper, we consider a new stochastic scheduling model based on the emergency jobs. We give optimal policies for two classes of objectives: one is the total waiting time of jobs and the other is the weighted sum of an exponential function of the waiting times.

One can consider other objectives in scheduling area, such as the weighted number of late jobs. Specially, we can study the environment that each job j has a due date D_j, which follows the exponential distribution. We believe that it is reasonable to study this case. Taking the surgery scheduling as an example again, we can consider the due date for each elective surgery. One reason is that each elective surgery should have a due date to wait for operating. Otherwise, the patient will die or will leave to search for another hospital for help. The other reason is that in some sense this date can also be as the commitment to the patients. And sometimes this commitment is consistent with patients' condition. Therefore, it can be interpreted as a random variable. In addition, Denton et al. [7] consider the overtime to the model and study how case sequencing affects patient waiting time, operating room idling time (surgeon waiting time), and operation room overtime. Therefore, maybe we can study the objective which includes both the weighted number of late jobs and overtime:

$$\mathbb{E}\left[\sum_{k=1}^{K} o_k \max\left\{0, \sum_{i\in S_k} P_i - \Delta_k\right\} + \sum_j \gamma_j \left(W_j - D_j\right)^+\right], \tag{15}$$

where o_k is the unit overtime cost, Δ_k is the width of the kth regular time, and S_k is the subset of cases scheduled in the kth day.

Another direction we can study is to take the waiting cost of emergency jobs into consideration. The reason is that emergency surgery which cannot be treated in time will result in substantial losses. It reflects that the unit waiting time cost of emergency jobs is very large in the model. It, thus, is necessary to operate these cases as soon as possible. Clearly, sometimes it is nonsense if the new coming jobs follow the Poisson process, since its influence on the time line is the same. Thus, in this case, maybe we can take another distribution for the emergency jobs' coming. Sometimes it is reasonable since in practice emergency jobs' appearance is related to the timing. For example, Summala and Mikkola [8] show that the fatal accidents appear more frequently during 3am–5am and 2pm-3pm, which is very close to emergency surgeries.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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