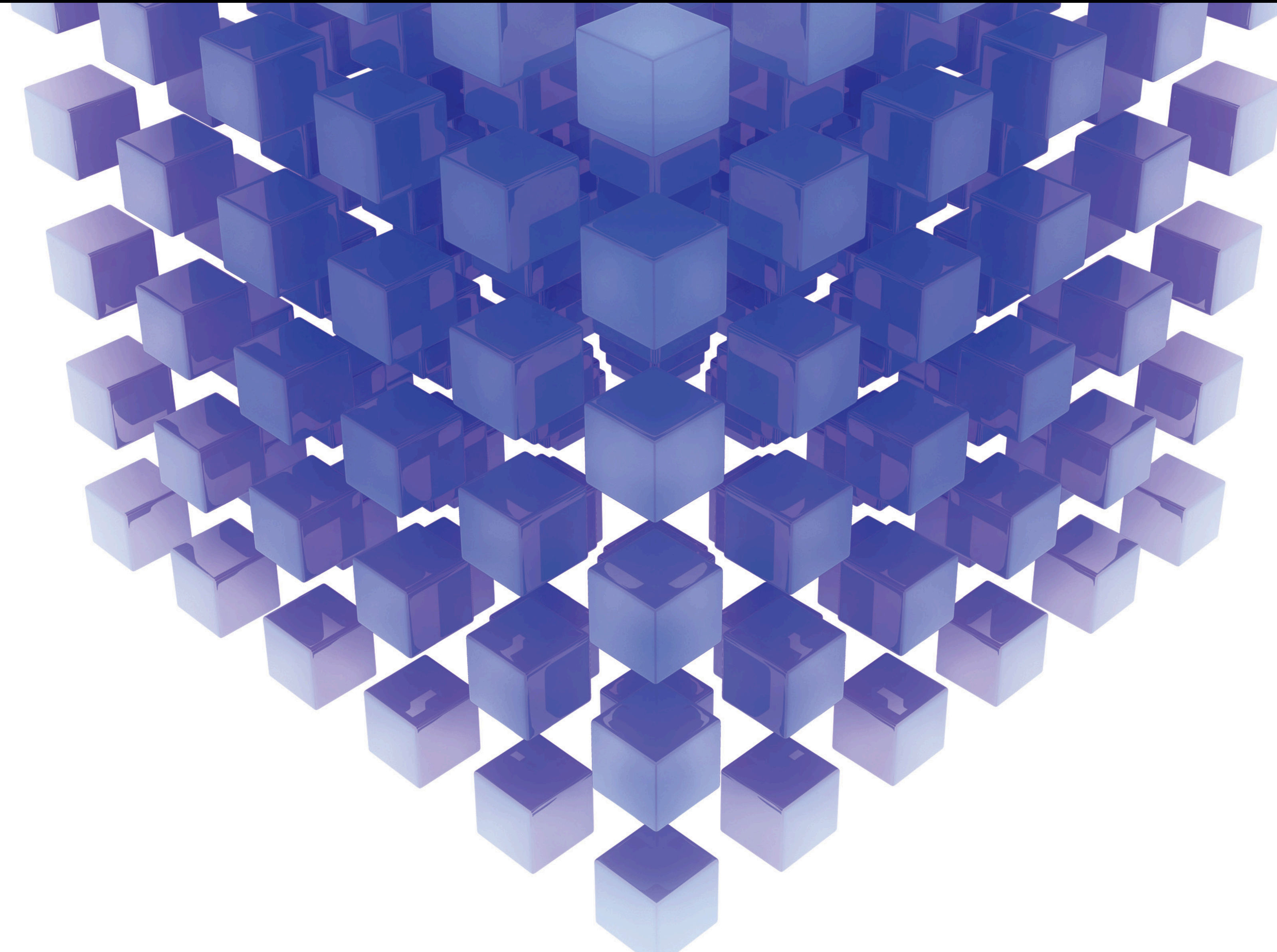
Mathematical Problems in Engineering

Building Mathematical Models for Multicriteria and Multiobjective Applications 2020

Lead Guest Editor: Adiel Teixeira de Almeida
Guest Editors: Love Ekenberg, Juan Carlos Leyva-Lopez, Danielle Costa Morais, and Tomasz Wachowicz



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Editorial

Building Mathematical Models for Multicriteria and Multiobjective Applications 2020

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This is the Fourth Special Issue (SI) dealing with "Building Mathematical Models for Multicriteria and Multiobjective Applications." The first one was published in 2016, and based on its success, another one was published in 2018 and 2019 each and now this one in 2020. The ambition is to henceforth publish an annual Special Issue. This series has been attracting the Multicriteria Decision-Making/Aid (MCDM/A) and multiobjective community, researchers, and practitioners.

The focus of this Special Issue is to demonstrate how MCDM/A and multiobjective methods can be highly useful for decision-makers (DMs) in solving decision problems involving multiple criteria.

This Special Issue offers seven original research papers covering a variety of applications for real-world problems while combining theoretical methodology and mathematical analysis. The authors of these papers are from Brazil, China, and Thailand. Two papers bring new approaches to deal with group decision-making and negotiation. Two papers discuss applications of the FITradeoff method, and three papers are focused on multiobjective applications.

The paper, which deals with a novel group decision-making approach, applies the hesitant fuzzy linguistic term sets (HFLTSs) to elicit the decision-makers' linguistic preferences as they are efficient and flexible in representing uncertainty. The approach considers the advantages of the rough set theory and OWA operators and presents an extended VIKOR method. Another paper brings a novel procedure to pursue aspired procurement negotiation (PN) outcomes using the combined multiple attribute decision-making (MADM) model. This model allows identifying, measuring, and depicting suboptimal situations in the context of an influential network relation map (INRM).

The papers which bring applications of the FITradeoff method are focused on ranking problems. One paper discusses a supplier selection for a wholesaler and retailer company of the construction sector to assist the DM in selecting new suppliers to keep the products and suppliers in line with the company's strategic plans and objectives. The other paper presents the multicriteria decision model that prioritizes sections of Brazilian roads by criticality and the risk of their use for drivers. The goal is to ensure an efficient movement of traffic under stable conditions and minimal traffic congestion, i.e., keep the federal highways safe and prevent accidents.

This Special Issue brings three different approaches concerning multiobjective applications. One paper presents a goal programming approach to nurse scheduling that simultaneously considers workload fairness and individual preferences on working shift and day off assignments. Another paper presents a combination of the Kriging model, optimal Latin hypercube sampling, and particle swarm optimization, an algorithm, EIR2-MOEA, for solving expensive multiobjective optimization problems. It is applied to three sets of standard test functions of varying difficulty and compared with two other competitive infill point criteria. Finally, the last paper presents a design-task-oriented model assignment framework that involves model value selection, multiobjective model establishment, and multiobjective optimization algorithm. It provides a solution for the problem of model assignment in the model repository to the design tasks in Model-Based System Engineering (MBSE).

We hope that the papers presented in this Special Issue will be useful and stimulating for further developments and applications of multicriteria and multiobjective models and that we again have been able to highlight the extensive range of contexts over which these methods can be used.

Conflicts of Interest

The Guest Editors declare that they have no conflicts of interest.

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Research Article

A Novel Procedure to Pursue Aspired Procurement Negotiation Outcomes Using a Combined MADM Model

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In the modern global economy, public and private organizations frequently procure goods and services from external suppliers. As such, negotiations are essential to reach procurement agreements and thus achieve organizational objectives and meet criteria in a timely and economically efficient manner. However, numerous relevant studies have revealed that suboptimal agreements frequently occur in procurement negotiation (PN) settings, which negatively affect the realization of business objectives and criteria. This study proposes the addition of a novel procedure that integrates a combined multiple attribute decision-making (MADM) model into the PN framework to identify, measure, and depict suboptimal situations in the context of an influential network relation map (INRM). This approach enables visualized and systematic information to be continuously provided, thus helping to determine possible improvement initiatives for transitioning suboptimal agreements to aspired levels. A real numerical case study is used to demonstrate the practical application of the proposed procedure. The results reveal that by employing the combined MADM model, the proposed procedure can provide managers with a practical foundation for early identification of the critical factors/dimensions for continuous improvement of a negotiated agreement regardless of how or why a suboptimal agreement initially occurs.

1. Introduction

In the modern cooperative business environment, functions are rarely performed entirely in-house. Public and private organizations must procure goods and services from external suppliers to achieve organizational objectives and criteria in a timely and economically efficient manner [1]. However, managing multiple procurement tasks can be complicated. Procurement typically involves an unclear scope, unforeseen costs, or a long lifespan that requires buyers and sellers to engage in trade-offs of technical, financial, and commercial factors [2]. In such situations, negotiations are commonly used to reach final agreements [3, 4]. Given that procurement is essential to businesses' success, such negotiations play a critical role in corporate management [5–8].

However, although some procurement decision-makers successfully apply such negotiations to clarify trade terms in formulating mutually beneficial agreements [9], numerous studies have indicated that other negotiated agreements are unable to fully support all aspects of procurement in achieving business objectives [10–12]. For example, in most procurement negotiation (PN) settings, negotiators typically hide information to gain advantages [1]. This opacity of information prevents managers from reaching agreements that ensure optimized business objectives [13]. Additionally, most procurement agreements have a defined lifespan. A time constraint is typically imposed on PN completion [1], which can create pressure and reduce a manager's motivation or ability to fully process, evaluate, and determine all possible alternatives in the pursuance of optimization [14, 15]. Instead, managers are forced to finalize negotiations to achieve suboptimal agreements that create variations, including changed orders and claims, during their implementation [4, 5]. These studies have implied that suboptimal agreements occur frequently in the PN setting.

A practical survey by Turner and Keegan [12] indicated that an agreement is merely a tentative settlement, which by its nature is incomplete; further improvements are always required later during the process of implementation. More comprehensively, Ertel [11] analyzed hundreds of complex negotiation projects and concluded that "countless deals that were signed with optimism fall apart during implementation, despite the care and creativity with which their terms were crafted." His study emphasized how decisions made in a negotiated agreement can be affected by future trends, and he also highlighted the need for continuous improvement to facilitate optimal results for the business objectives and criteria. Kujala et al. [3] examined negotiated agreements from the perspectives of sales and implementation and highlighted the importance of improving joint decisions between buyers and sellers over a transaction's lifecycle. Yang et al. [16] argued that negotiated agreements cause the most substantial project delays, which in turn create serious conflicts during and after implementation of such agreements. These studies have suggested that an efficient approach is required to improve the implementation of negotiated agreements.

The relevant literature on how to settle suboptimal negotiated agreement problems can be traced back to 1985, when Raiffa [17] argued that the majority of negotiated agreements have the potential for improvement. His study also proposed a postsettlement settlement concept that encourages negotiators to use the negotiated agreement as a foundation to seek additional value. Based on Raiffa's concepts, Susskind [18] argued that pos-settlement settlement can be applicable under cooperative negotiation, in which the negotiating parties treat each other as partners and share information. William [19], who adopted a macroperspective, suggested that monitoring and evaluation of negotiated agreements should be viewed as an essential part of the negotiation process. Smolinski and Xiong [20] concluded that postsettlement settlement is a critical negotiation competency to manage conflicts in the increasingly complex modern business environment. These studies have introduced several helpful ideas to address the problems that are associated with negotiated agreements. However, based on this study's literature review, these ideas have not been developed into an operational procedure for practical use.

This study proposes a novel procedure that integrates a hybrid multiattribute decision-making (HMADM) model into the PN framework to increase the ability of negotiated agreements to obtain optimal PN outcomes during their implementation. The HMADM model was originally introduced by Tzeng to solve decision problems in interdependent situations [21]. The HMADM model provides theoretical suggestions for how to continuously improve decision implementation toward aspired levels [22–25], and thus, its use is appropriate for this study. The proposed procedure enables the identification, measurement, and depiction of suboptimal agreements in a context of the influence network relation map (INRM). The INRM provides managers with visualized and systematic information to easily analyze index gaps among factors, dimensions, and the overall agreement to pursue the aspired PN outcomes. This study uses a numerical example to demonstrate the functionalities of the proposed procedure. The results of the HMADM model revealed that the proposed procedure can provide managers with a critical foundation for the early identification of the critical factors and dimensions of a negotiated agreement, which are needed to continuously improve outcomes, regardless of how and why a suboptimal agreement initially occurs. The remainder of this study is arranged as follows. Section 2 reviews the literature on the PN process. Section 3 introduces the HMADM model comprising the DEMATEL method, the DEMATEL-based analytic network process, and the modified VIKOR method. Section 4 discusses the proposed procedure. Section 5 demonstrates the operation of the proposed procedure by examining a real-world numerical example and discusses the results. Section 6 provides conclusions and further remarks.

2. Literature Review

In practice, negotiations can be characterized differently depending on specific business situations. PNs can occur between buyers and sellers at numerous points in the procurement process. However, this study emphasizes how to improve negotiated agreements during the implementation of such agreements and when two parties have multiple issues in play. Additionally, this study assumes that the parties involved in PN intend to obtain and implement favorable agreements to produce aspired outcomes and thus fulfil their procurement objectives and criteria.

Typically, PN follows a three-phase framework adopted by procurement professionals worldwide [1] comprising prenegotiation, meeting, and postnegotiation (Figure 1).

Prenegotiation starts with issues that the negotiating parties disagree about, such as time, cost, scope, and quality [26]. In the PN environment, if any of these issues or dimensions change, at least one other issue or dimension is likely to be influenced. For example, if the procurement time is reduced, costs often rise due to the additional resources needed to complete the same scope of procurement in a shorter time. If a budget increase is not possible, both procurement scope and desired quality may be reduced. Negotiators must consider these mutually interdependent situations to prepare a set of alternatives to finalize negotiations in each PN phase [1].

Two types of negotiating strategies are most common: win-lose and win-win. In a win-lose situation, each party seeks a maximized share of a fixed amount of resources. In a win-win negotiation, one party's gain does not necessarily come at the other's expense [13]. In most PN settings, a win-lose strategy typically emerges early in the negotiation process, but communication and information sharing can transform this into a win-win situation [27]. During the meeting phase, negotiators present their initial offers and then decide whether to make concessions based on a package or separately [3]. Determining and offering the right options

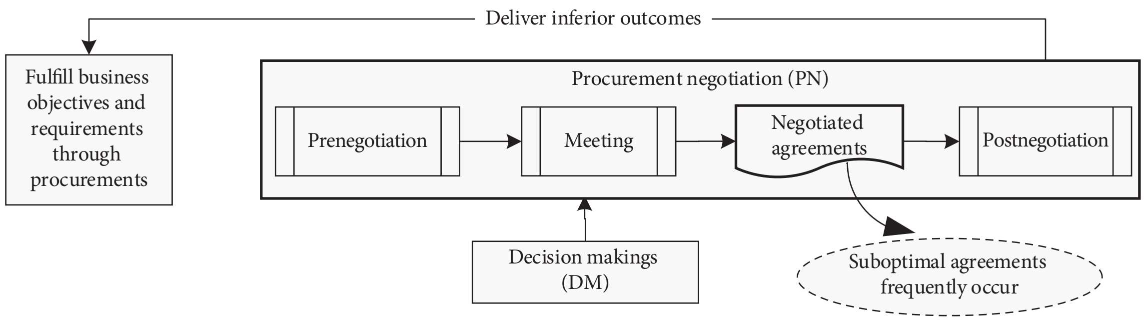

Figure 1: General PN framework.

can provide flexibility and expedite the negotiation process. A range of decision-making methods have therefore been developed to streamline option selection in negotiations [5].

Keeney and Raiffa [28] proposed a utility function to model negotiator preferences. Rubinstein [29] proposed the use of alternating-offer bargaining in his analysis of negotiation game outcomes. Following these studies, many researchers have proposed negotiation decision aids [30]. Teich et al. [31] classified these decision-aid models into four categories: (a) value function-based and concession-based models; (b) value function-based and Pareto-improvement-seeking models; (c) interactive concession-making models; and (d) interactive Pareto-improvement models. Most decision-aid models emphasize the determination of a Pareto-optimal solution through concessions [32] under limited resources (which are presumably subject to constraints). However, limited resources impose difficulties in determining the Pareto frontier to seek optimal solutions, resulting in suboptimal agreements that deliver inferior outcomes in the conventional approach [20]. To fulfil the shortcomings, Tzeng and Huang [33] proposed a hybrid multiple attribute decision-making (HMADM) model to pursue aspired decision outcomes through continuous improvements.

The HMADM model employs a decision-making trial and evaluation laboratory (DEMATEL) technique [34], which allows for interinfluential effects to be identified between the decision factors/dimensions (or objectives/criteria) on an influential network relations map (INRM). Second, this model applies an analytic network process (ANP) [35] to transform the DEMATEL interinfluential values into influential weights (IWs), so as to be able to better prioritize the decision-making criteria, a procedure called a DEMATEL-based ANP (DANP). Third, this model employs an "aspiration level" principle to modify the multicriteria optimization and compromise solution method, named "ViseKriterijumska Optimizacija I Kompromisno Rešenje" in Serbian (called the "modified VIKOR method") [36], to avoid "choosing the best among inferior options/alternatives," [33]. The aspiration level concept was proposed by Simon [37], who argued that actual human decision-making behavior is a sequential process, and a decision-maker is satisfied when a selected alternative aligns with an aspiration level criterion, which is the highly desirable outcome level. Therefore, the modified VIKOR method replaces the traditional max/min approach (choosing relatively good solutions from existing alternatives) with an aspiration level, which enables a shift from ranking and selection when determining the most preferable alternative to attaining performance improvements based on the INRM [36]. By combining these aforementioned concepts and procedures, the HMADM model can provide managers with systematic visual information to allow for the identification of the critical factors needed to enact improvement strategies that would better enable the aspired outcomes to be reached [21].

Currently, behaviors, models, and support systems associated with multiple criteria group decision-making and negotiations remain a prominent research topic for a wide range of applications [38, 39]. For example, Frej et al. [40] proposed a decision support tool based on the FITradeoff method to expertise agreement achievement for multiple criteria group decision-making with situations of partial/incomplete/imprecise information. Their study has also indicated the usefulness of graphical visualization techniques in collective decision-making processes. With these advanced studies, negotiators enable to apply new methods and tools to reduce the level of uncertainties in evaluating, ranking, and selecting alternative offers and counteroffers in obtaining group consensus at different stages within a negotiation process.

In the postnegotiation phase, both parties implement and administer the negotiated agreement in an attempt to achieve the aspired outcomes and to satisfy the determined performance objectives and criteria [11]. However, suboptimal negotiated agreements frequently occur, and the parties involved typically hold meetings to discuss related issues such as performance gaps, possible changes, and improvement requirements [17, 19]. Generally, these meetings can foster closer relationships between the parties and increase mutual trust through information sharing [13]. Additionally, implementing such agreements often takes a long time. In the cloud-computing era, new technologies and management mechanisms emerge on an almost daily basis [41]. Accordingly, in contrast to traditional negotiations that follow a suboptimal agreement, the modern postnegotiation phase provides opportunities to create additional value.

Research has indicated that adding value to a negotiated agreement requires additional effort to evaluate problems and to generate a comprehensive list of potential options from which workable solutions can be selected [13]. Additionally, the negotiating parties must constantly assess the implementation of the agreement and maintain proactive communications to determine possible improvements [16]. As such, the next section proposes a novel procedure to analyze and improve on suboptimal outcomes in the postnegotiation phase within the PN setting.

3. A Novel Procedure to Pursue Aspired PN Outcomes

This section first briefly introduces the essential concepts and computational equations related to the DEMATEL method, DANP, and the modified VIKOR method. Next, it explains the proposed procedure.

3.1. The DEMATEL Method. The DEMATEL method was developed by the Battelle Geneva Institute in 1972 for assessing and solving complex groups of problems. This method uses Boolean operations and Markov processes [34] to measure cause and effect relationships in each dimension or criterion within a system (or subsystem). Quantitative measurements are then mapped onto an INRM-representing problem structure with a visual rout exhibiting the degree and direction in which each dimension or criterion influences the overall system performance. This method has been widely applied to help managers easily obtain valuable information for practical improvements in fields such as security systems and aerospace services [26, 33]. The computational steps for the DEMATEL method are described as follows.

Step 1. Obtain the initial average influence relation matrix **A**. This step uses a team comprising E experts to identify the number of factors or criteria, n, in a system. Each expert measures the degree of influence that factor i has on factor j in achieving system objectives. Typically, the measurement scale ranges from 0 to 4, with 0 representing "absolutely no influence," 1 representing "low influence," 2 representing "medium influence," 3 representing "high influence," and 4 representing "very high influence." Through pairwise comparisons, the results of all expert measurements are denoted as matrix $\mathbf{H}^e = [h_{ij}^e]$, where $e = 1, 2, \ldots, E$, and E is the number of experts. By averaging the matrix $\mathbf{H}^e = [h_{ij}^e]$, the initial average influence relation matrix $\boldsymbol{A}$ can be obtained, as illustrated by

$$\mathbf{A} = \left[a_{ij}\right]_{n\times n}, \tag{1}$$

where $a_{ij} = \sum_{e=1}^{E} (h_{ij}^e/E)$, and h_{ij}^e is the measurement by eth expert in $\mathbf{H}^e$.

Step 2. Obtain the normalized influence relation matrix **D**. By using **A** [ ], the normalized influence relation matrix **D** can be obtained as shown in

$$\mathbf{D} = \frac{\mathbf{A}}{s} = \left[d_{ij}\right]_{n\times n}, \tag{2}$$

where $s = \max\left(\max_{1\le i\le n} \sum_{j=1}^{n} a_{ij}, \max_{1\le i\le n} \sum_{i=1}^{n} a_{ij}\right)$.

Step 3. Obtain the total influence relation matrix **T**. Through the matrix operation of **D**, the total influence relation matrix **T** can be obtained as shown in

$$\mathbf{T} = \mathbf{D}\left(\mathbf{I} - \mathbf{D}\right)^{-1}, \quad \text{when}\, u \longrightarrow \infty, \lim_{u\longrightarrow\infty} \mathbf{D}^u = [0]_{n\times n}, \tag{3}$$

where $\mathbf{I}$ is an identity matrix, $\mathbf{D} = [d_{ij}]_{n\times n}$, $0 \le d_{ij} \le 1$, $0 \le \sum_i d_{ij} \le 1$, $0 \le \sum_j d_{ij} \le 1$ and at least one column or one row of summation but not every column or row equals one; then, $\lim_{u\longrightarrow\infty} \mathbf{D}^u = [0]_{n\times n}$ can be guaranteed.

Step 4. Obtain the INRM. Based on the matrix **T**, the INRM can be constructed using the following substeps (SSs):

SS 4-1. Define each row sum and column sum of matrix $\boldsymbol{T}$ as a respective $n \times 1$ vector, as shown in the following equations:

$$\mathbf{r} = \left[r_i\right]_{n\times 1} = \left[\sum_{j=1}^{n} t_{ij}\right]_{n\times 1}, \tag{4}$$

$$\mathbf{c} = \left[c_j\right]'_{1\times n} = \left[\sum_{t=1}^{n} t_{ij}\right]'_{1\times n} = \left[c_j\right]_{n\times 1}, \tag{5}$$

where r_i indicates the total influence that factor i has on the all other factors and c_j indicates the total influence that the all other factors have on factor j for $i, j = 1, 2, \ldots, n$; the superscript $'$ denotes the transpose.

SS 4-2. Compute $r_i + c_j$ and $r_i - c_j$, $i, j \in \{1, 2, \ldots, n\}$. When $i = j$, $r_i + c_i$ provides an index representing the strength of the total influence that each factor exerts on and receives from the others; that is, $r_i + c_i$ indicates the centrality of the role that factor i plays in the system. In addition, $r_i - c_i$ indicates the net influence that factor i contributes to the system. If $(r_i - c_i)$ is positive, factor i influences the other factors, and if $(r_i - c_i)$ is negative, factor i is influenced by the other factors.

SS 4-3. Plot the dataset $[(r_i + c_i), (r_i - c_i)]$ into the INRM to visualize the structure of the interrelationship among all factors related to the system performance; this plot reveals valuable information for problem solving.

3.2. DANP. DANP can be described as DEMATEL-based ANP. ANP was proposed by Saaty [35] to address interdependence and feedback among the factors, dimensions, or alternatives associated with a decision-making problem. By applying the basic concept of ANP to formulate the influence relation matrix obtained from the DEMATEL method, DANP can derive the influence weights (IWs) among a set of interrelated factors for superior communication of real interdependent problematic situations, improvement alternatives, and decisions [33]. DANP's operational steps are described as follows.

Step 5. Obtain an unweighted supermatrix. This step involves the following three SSs:

SS 5-1. Classify all factors in the total influence relation matrix $\mathbf{T}$ (see Step 3) into the appropriate dimensions (clusters) to form a new matrix, which is referred to as the total influence relation matrix of factors, $\mathbf{T}_C$, as shown in equation (6), where $\sum_{j=1}^{m} m_j = n$, $m < n$, and $\mathbf{T}_c^{ij}$ is an $m_i \times m_j$ matrix.

$$\mathbf{T}_c = \begin{array}{cc} & \begin{array}{ccc} D_1 & D_j & D_m \\ c_{11}\ldots c_{1m_1} & c_{j1}\ldots c_{jm_j} & c_{m1}\ldots c_{mm_m} \end{array} \\ \begin{array}{cc} D_1 & \begin{array}{c} c_{11} \\ c_{12} \\ \vdots \\ c_{1m_1} \end{array} \\ \vdots & \vdots \\ D_i & \begin{array}{c} c_{i1} \\ c_{i2} \\ \vdots \\ c_{1m_i} \end{array} \\ \vdots & \vdots \\ D_m & \begin{array}{c} c_{m1} \\ c_{m2} \\ \vdots \\ c_{mm_m} \end{array} \end{array} & \begin{bmatrix} \mathbf{T}_c^{11} & \cdots & \mathbf{T}_c^{1j} & \cdots & \mathbf{T}_c^{1m} \\ \vdots & & \vdots & & \vdots \\ \mathbf{T}_c^{i1} & \cdots & \mathbf{T}_c^{ij} & \cdots & \mathbf{T}^{im} \\ \vdots & & \vdots & & \vdots \\ \mathbf{T}_c^{m1} & \cdots & \mathbf{T}_c^{mj} & \cdots & \mathbf{T}_c^{mm} \end{bmatrix}_{n\times n | m<n,\ \Sigma_{j-1}^{m} m_j = n} \end{array}. \quad (6)$$

SS 5-2. Normalize each dimension (cluster) of criteria by its total degree of effect to obtain a matrix, which is referred to as the normalized total influence relation matrix of factors, as demonstrated in

$$\mathbf{T}_c^{\alpha} = \begin{array}{cc} & \begin{array}{ccc} D_1 & D_j & D_m \\ c_{11}\ldots c_{1m_1} & c_{j1}\ldots c_{jm_j} & c_{n1}\ldots c_{mm_m} \end{array} \\ \begin{array}{cc} D_1 & \begin{array}{c} c_{11} \\ c_{12} \\ \vdots \\ c_{1m_1} \end{array} \\ \vdots & \vdots \\ D_i & \begin{array}{c} c_{i1} \\ c_{i2} \\ \vdots \\ c_{im_i} \end{array} \\ \vdots & \vdots \\ D_m & \begin{array}{c} c_{m1} \\ c_{m2} \\ \vdots \\ c_{mm_m} \end{array} \end{array} & \begin{bmatrix} \mathbf{T}_c^{\alpha 11} & \cdots & \mathbf{T}_c^{\alpha 1j} & \cdots & \mathbf{T}_c^{\alpha 1m} \\ \vdots & & \vdots & & \vdots \\ \mathbf{T}_c^{\alpha i1} & \cdots & \mathbf{T}_c^{\alpha ij} & \cdots & \mathbf{T}_c^{\alpha im} \\ \vdots & & \vdots & & \vdots \\ \mathbf{T}_c^{\alpha m1} & \cdots & \mathbf{T}_c^{\alpha mj} & \cdots & \mathbf{T}_c^{\alpha mm} \end{bmatrix}_{n\times n | m<n,\ \Sigma_{j-1}^{m} m_j = n} \end{array}, \quad (7)$$

where $\mathbf{T}_C^{\alpha 11}$ is calculated as shown in equation (8). The other elements, $\mathbf{T}_C^{\alpha ij}$, can be obtained using the same method.

$$\mathbf{T}_D^{\alpha} = \begin{bmatrix} \dfrac{t_{c^{11}}^{11}}{d_1^{11}} & \cdots & \dfrac{t_{c^{1j}}^{11}}{d_1^{11}} & \cdots & \dfrac{t_{c^{1m_1}}^{11}}{d_1^{11}} \\ \vdots & & \vdots & & \vdots \\ \dfrac{t_{c^{i1}}^{11}}{d_i^{11}} & \cdots & \dfrac{t_{c^{ij}}^{11}}{d_i^{11}} & \cdots & \dfrac{t_{c^{im_1}}^{11}}{d_i^{11}} \\ \vdots & & \vdots & & \vdots \\ \dfrac{t_{c^{m_1 1}}^{11}}{d_{m_1}^{11}} & \cdots & \dfrac{t_{c^{m_1 j}}^{11}}{d_{m_1}^{11}} & \cdots & \dfrac{t_{c^{m_1 m_1}}^{11}}{d_{m_1}^{11}} \end{bmatrix}_{m\times m} = \begin{bmatrix} t_{c^{11}}^{\alpha 11} & \cdots & t_{c^{1j}}^{\alpha 1j} & \cdots & t_{c^{1m_1}}^{\alpha 11} \\ \vdots & & \vdots & & \vdots \\ t_{c^{i1}}^{\alpha 11} & \cdots & t_{c^{ij}}^{\alpha 11} & \cdots & t_{c^{im_1}}^{\alpha 11} \\ \vdots & & \vdots & & \vdots \\ t_{c^{m_1 1}}^{\alpha 11} & \cdots & t_{c^{m_1 1}}^{\alpha 11} & \cdots & t_{c^{m_1 m_1}}^{\alpha 11} \end{bmatrix}, \quad (8)$$

where $d_{ci}^{11} = \sum_{j=1}^{m1} t_{ij}^{11}$, $i = 1, 2, \ldots, m_1$.

SS 5-3. Transpose $\mathbf{T}_C^{\alpha}$ into an unweighted supermatrix $\mathbf{W}$ according to the dependent relationship in the dimension (cluster), as shown in

$$\mathbf{W} = (\mathbf{T}_c^{\alpha})' = \begin{array}{cc} & \begin{array}{ccc} D_1 & D_j & D_m \\ c_{11}\ldots c_{1m_1} & c_{j1}\ldots c_{im_i} & c_{m1}\ldots c_{mm_m} \end{array} \\ \begin{array}{c} D_1 \\ \vdots \\ D_j \\ \vdots \\ D_m \end{array} \begin{array}{c} c_{11} \\ c_{12} \\ \vdots \\ c_{1m_1} \\ \vdots \\ c_{j1} \\ c_{j2} \\ \vdots \\ c_{jm_j} \\ \vdots \\ c_{m1} \\ c_{m2} \\ \vdots \\ c_{mm_m} \end{array} & \begin{bmatrix} \mathbf{W}^{11} & \cdots & \mathbf{W}^{i1} & \cdots & \mathbf{W}^{m1} \\ \vdots & & \vdots & & \vdots \\ \mathbf{W}^{1j} & \cdots & \mathbf{W}^{ij} & \cdots & \mathbf{W}^{mj} \\ \vdots & & \vdots & & \vdots \\ \mathbf{W}^{1m} & \cdots & \mathbf{W}^{im} & \cdots & \mathbf{W}^{mm} \end{bmatrix}_{n\times n | m<n,\ \sum_{j=1}^{m} m_j = n} \end{array}, \tag{9}$$

where $\mathbf{W}^{11}$ is calculated as indicated in equation (10). The other $\mathbf{W}^{ij}$ can be obtained using the same method. If a blank space or a zero appears in the matrix, the dimension or factor is independent.

$$\mathbf{W}^{11} = (\mathbf{T}_c^{11})' = \begin{array}{cc} & \begin{array}{ccccc} c_{11} & \cdots & c_{i1} & \cdots & c_{m_1 1} \end{array} \\ \begin{array}{c} c_{11} \\ \vdots \\ c_{1j} \\ \vdots \\ c_{1m_1} \end{array} & \begin{bmatrix} t_{c^{11}}^{\alpha 11} & \cdots & t_{c^{i1}}^{\alpha 11} & \cdots & t_{c^{m_1 1}}^{\alpha 11} \\ \vdots & & \vdots & & \vdots \\ t_{c^{1j}}^{\alpha 11} & \cdots & t_{c^{ij}}^{\alpha 11} & \cdots & t_{c^{m_1 j}}^{\alpha 11} \\ \vdots & & \vdots & & \vdots \\ t_{c^{1m_1}}^{\alpha 11} & \cdots & t_{c^{im_1}}^{\alpha 11} & \cdots & t_{c^{m_1 m_1}}^{\alpha 11} \end{bmatrix} \end{array}. \tag{10}$$

Step 6. Obtain the weighted supermatrix. This involves the following three SSs:

SS 6-1. Based on equation (6) in SS 5-1, each dimension in $\mathbf{T}_C$ is grouped as the total influence relation matrix of dimensions $\mathbf{T}_D$ by using

$$\mathbf{T}_D = \begin{bmatrix} t_D^{11} & \cdots & t_D^{1j} & \cdots & t_D^{1m} \\ \vdots & & \vdots & & \vdots \\ t_D^{i1} & \cdots & t_D^{ij} & \cdots & t_D^{im} \\ \vdots & & \vdots & & \vdots \\ t_D^{m1} & \cdots & t_D^{mj} & \cdots & t_D^{mm} \end{bmatrix}_{m\times m} \tag{11}$$

SS 6-2. Normalize each dimension in $\mathbf{T}_D$ with respect to the total degree of the effects and obtain the normalized total influence relation matrix of dimensions $\mathbf{T}_D^{\alpha}$, as shown in

$$\mathbf{T}_D^{\alpha} = \begin{bmatrix} \frac{t_D^{11}}{d_1} & \cdots & \frac{t_D^{1j}}{d_1} & \cdots & \frac{t_D^{1m}}{d_1} \\ \vdots & & \vdots & & \vdots \\ \frac{t_D^{i1}}{d_i} & \cdots & \frac{t_D^{ij}}{d_i} & \cdots & \frac{t_D^{im}}{d_i} \\ \vdots & & \vdots & & \vdots \\ \frac{t_D^{m1}}{d_m} & \cdots & \frac{t_D^{mj}}{d_m} & \cdots & \frac{t_D^{mm}}{d_m} \end{bmatrix}_{m\times m} = \begin{bmatrix} t_D^{\alpha 11} & \cdots & t_D^{\alpha 1j} & \cdots & t_D^{\alpha 1m} \\ \vdots & & \vdots & & \vdots \\ t_D^{\alpha i1} & \cdots & t_D^{\alpha ij} & \cdots & t_D^{\alpha im} \\ \vdots & & \vdots & & \vdots \\ t_D^{\alpha m1} & \cdots & t_D^{\alpha mj} & \cdots & t_D^{\alpha mm} \end{bmatrix}_{m\times m}, \tag{12}$$

where $d_i = \sum_{j=1}^{m} t_D^{ij}$, $i = 1, 2, \ldots, m$.

SS 6-3. Based on equations (9) and (12), the weighted supermatrix can be obtained using equation (13), where $t_{ij}^{\alpha D}$ is a scalar and $\sum_{j=1}^{m} m_j = n$.

$$\mathbf{W}^{\alpha}=\mathbf{T}_{D}^{\alpha}\mathbf{W}=\begin{array}{cc} & \begin{array}{ccccc} D_1 & & D_j & & D_m \\ c_{11}\ldots c_{1m_1} & & c_{i1}\ldots c_{im_i} & & c_{m1}\ldots c_{mm_m} \end{array} \\ \begin{array}{cc} D_1 & \begin{array}{c} c_{11} \\ c_{12} \\ \vdots \\ c_{1m_1} \end{array} \\ \vdots & \vdots \\ D_j & \begin{array}{c} c_{j1} \\ c_{j2} \\ \vdots \\ c_{jm_j} \end{array} \\ \vdots & \vdots \\ D_m & \begin{array}{c} c_{m1} \\ c_{m2} \\ \vdots \\ c_{mm_m} \end{array} \end{array} & \begin{bmatrix} t_D^{\alpha 11}\times\mathbf{W}^{11} & \cdots & t_D^{\alpha i1}\times\mathbf{W}^{i1} & \cdots & t_D^{\alpha m1}\times\mathbf{W}^{m1} \\ \vdots & & \vdots & & \vdots \\ t_D^{\alpha 1j}\times\mathbf{W}^{1j} & \cdots & t_D^{\alpha ij}\times\mathbf{W}^{ij} & \cdots & t_D^{\alpha mj}\times\mathbf{W}^{mj} \\ \vdots & & \vdots & & \vdots \\ t_D^{\alpha 1m}\times\mathbf{W}^{1m} & \cdots & t_D^{\alpha im}\times\mathbf{W}^{im} & \cdots & t_D^{\alpha mm}\times\mathbf{W}^{mm} \end{bmatrix}_{n\times n} \end{array}. \quad (13)$$

Step 7. Limit the weighted supermatrix. This step involves multiplying the weighted supermatrix $\mathbf{W}^{\alpha}=[w_{ij}]_{n\times n}$ by itself, denoted as $(\mathbf{W}^{\alpha})^{z}$, until $([w_{ij}]_{n\times n})^{2}$ converges to some other value that is the same for all instances of j. Using $\lim_{z\longrightarrow\infty}(\mathbf{W}^{\alpha})^{z}$, the IWs of factors can be obtained as follows: $\mathbf{w}=(w_1,\ldots,w_j,\ldots,w_n)$.

3.3. Modified VIKOR. The VIKOR method was proposed by Opricovic [42] to solve problems that involve incommensurable and conflicting factors. Originally, this method focused on analyzing a set of alternatives and selecting a compromise solution closest to the ideal state [36, 43]. The ideal state was defined as a set of maximum or minimum values related to the benefit or cost criteria for all alternatives, if performance values $\{f_{kj}|k=1,2,\ldots,K\}$ exhibited k alternatives in the jth criterion, where higher is better, and then $f_j^{*}=\max\{f_{kj}|k=1,2,\ldots,K\}$ and $f_j^{-}=\min\{f_{kj}|k=1,2,\ldots,K\}$ in the conventional approach (i.e., "max-min" as the benchmark). However, these traditional compromises can lead to situations of "choosing the best among inferior options/alternatives"; hence, Tzeng and Huang [33] proposed the modified VIKOR method to replace the maximum/minimum approach with the "aspired worst" method, in which $f_j^{\text{aspired}}=10$ and $f_j^{\text{worst}}=0$ are set as the aspired level and the worst level, respectively, for criterion j, if the questionnaire scores range from 0 ("complete dissatisfaction/bad") to 10 ("extreme satisfaction/good"). Recently, this method has been used to aid decision-makers in identifying critical gaps in need of further improvement [44]. Details regarding the operational steps of the modified VIKOR method are presented as follows.

Step 8. Obtain an aspired or tolerable level. Assume that a problem with K alternatives denoted as $A_1, A_2, \ldots A_k, \ldots A_K$ is evaluated using n factors. The performance value of alternative A_k with respect to the jth factor is denoted as f_{kj}, and the IW of the jth factor is denoted as w_j obtained using DANP, where $j=1,2,\ldots,n$. Then, the best value (aspired level) f_j^{aspired} and worst value f_j^{worst} are determined for all criteria $j=1,2,\ldots,n$, and the original ratings f_{kj} are transformed into normalized gap-ratings, as shown in

$$r_{kj}=\frac{\left|f_j^{\text{aspired}}-f_{kj}\right|}{\left|f_j^{\text{aspired}}-f_j^{\text{worst}}\right|}. \quad (14)$$

Step 9. Compute the mean of group utility S_k (average value) and maximal regret Q_k (priority improvement). In this step, S_k indicates the synthesized gaps based on the weighted average operations from each factor into specific dimensions and overall. Q_k indicates the maximal gap of k alternatives in each dimension and the overall for the priority improvement. The values of S_k and Q_k are computed as presented in the following equations:

$$\begin{aligned} L_k^{p=1}&=S_k=\sum_{j=1}^{n}w_j r_{kj} \\ &=\sum_{j=1}^{n}w_j\left(\frac{\left|f_j^{\text{aspired}}-f_{kj}\right|}{\left|f_j^{\text{aspired}}-f_j^{\text{worst}}\right|}\right), \quad k=1,2,\ldots K, \end{aligned} \quad (15)$$

where w_j are the influential weights of the criteria from DANP.

$$L_k^{p=\infty}=Q_k=\max_j\left\{r_{kj}|j=1,2,\ldots,n\right\}, \quad k=1,2,\ldots,K. \quad (16)$$

Step 10. Compute the index value. The index values are computed using the relation as shown in

$$R_k=v\frac{\left(S_k-S^{\text{aspired}}\right)}{S^{\text{worst}}-S^{\text{aspired}}}+(1-v)\frac{\left(Q_k-Q^{\text{aspired}}\right)}{Q^{\text{worst}}-Q^{\text{aspired}}}, \quad (17)$$

where $k=1,2,\ldots,K$, and setting "$S^{\text{aspired}}=\min_k S_k$ and $Q^{\text{aspired}}=\min_k Q_k$" and "$S^{\text{worst}}=\max_k S_k$ and $Q^{\text{worst}}=\max_k Q_k$". We can also assume that $S^{\text{aspired}}=0$ and $Q^{\text{aspired}}=0$ (when all criteria are achieved up to the aspiration level, completely and without gaps as the target) and that $S^{\text{worst}}=1$ and $Q^{\text{worst}}=1$ (i.e., all criteria are as in the worst situations); v is presented as the weight of the strategy for paying attention to the average gap S_k (i.e., the maximum group utility in how the gap nears zero), or paying attention $(1-v)$ to punishing gap Q_k which is the maximized gap criterion (i.e., the individual regret/gap) that should be prioritized for improvement. As such, equation (17) can be rewritten as equation (18) to measure the gaps with dual ranking and selecting criteria.

$$R_k = vS_k + (1 - v)Q_k, \quad (18)$$

where $S^{\text{aspired}} = 0$, $S^{\text{worst}} = 1$, $Q^{\text{aspired}} = 0$, and $Q^{\text{worst}} = 1$.

Step 11. Rerank or improve the alternatives for a compromise solution. In this step, the values S_l, Q_l, and R_l for $l = 1, 2, \ldots, K$ are sorted in decreasing order, and a compromise solution is presented based on the alternatives $A^{(1)}, A^{(2)}, \ldots A^{(l)}, \ldots A^{(k)}$ by applying equation (17). Finally, the performance gaps in each factor, each dimension, and the overall system that correspond to the compromise solution are measured using equations (15) and (16).

3.4. Development of the Proposed Procedure. This section introduces how the MADM method is integrated into the PN setting to evaluate the dependency relationships and performance gaps in a negotiated agreement for obtaining the aspired PN outcomes. Figure 2 is a graphical representation of the proposed procedure. As presented in Figure 2, the proposed procedure first organizes a team of experts familiar with the procurement project under negotiation. The experts then identify and determine strategies to improve all factors that may impair the realization of the procurement objectives, dimensions, and criteria during the postnegotiation stage. More specifically, the proposed approach comprises four main stages: (1) identification of factors (criteria) to be improved using the expert team; (2) evaluation of interinfluential effects at all tiers using the DEMATEL method to reveal the degree and direction of each factor's influence upon the aspired objectives or criteria; (3) computation of the IW of each factor or dimension using DANP to determine at which priority level they help reduce the performance gaps; and (4) measurement of the gap indices using the modified VIKOR method. We then integrate the measurements with the weights on the INRM, thus providing managers with systematic tools for determining how to implement continuous improvement. The next section demonstrates the proposed procedure in practice.

4. Application of the Proposed Procedure to a Real Numerical Example: Results and Discussion

This section examines a logistics service project operating in both the public and private sectors to demonstrate the proposed procedure's functionality in a PN setting. To preserve project information confidentiality, all data are transformed into equivalent units.

4.1. Background. The case study organization (hereafter referred to as the buyer) has implemented a logistics transformation policy by outsourcing part of its organic workload to domestic private contractors. This policy requires potential contractors to ensure that support systems are reliable, available, and maintainable by the end users. Due to the limited number of suppliers in the market, such logistics service projects are executed through sole-source procurement by negotiation. One example is a three-year logistics support service for a TA helicopter that the buyer purchases from Company B (hereafter referred to as the seller) in a PN context. During the meeting phase of the negotiation, the seller disagrees with the supportability deployment level of several subsystems requested by the buyer. To meet the end user's deadline, the buyer accepts the seller's offered supportability. However, the compromise-level agreement contains performance gaps that could negatively impact the buyer's operations. Therefore, the buyer asks the seller to implement improvements over the 3-year service period. The seller claims that because this necessitates additional resources, they cannot guarantee improvements to all subsystems. Meanwhile, the seller asks the buyer to submit a proposal for further consideration. The buyer applies this study's proposed procedure to obtain a proposal to satisfactorily settle the problems that result from the suboptimal negotiated agreement.

4.2. Application of the Proposed Procedure. This section describes how the buyer follows the proposed procedure to pursue its aspired PN outcomes.

4.2.1. Identification of Factors to be Improved Using an Expert Team. The buyer forms a team of experts from its departments in charge of logistics (two experts), procurement (one expert), finance (one expert), end users (two experts), and project management (two experts), all of whom have considerable backgrounds in this project. The team members analyze the negotiated agreement and the pertinent information, determining that three main systems (dimensions) of the 12 subsystems (factors or criteria) contain performance gaps that require improvement in the deployment level of supportability (Table 1). Table 1 illustrates the three different levels of supportability under the buyer's consideration on each respective subsystem in terms of its operations. The threshold is the lowest level of supportability (minimized criteria) required by the buyer, the deployment is the highest level of supportability (maximized criteria) that the buyer expects to obtain, and the negotiated level of supportability is what the buyer and the seller agree on. Additionally, the gaps to be improved are the differences between the negotiated level and the deployment level that the buyer requests from the seller during implementation of the agreement.

4.2.2. Evaluation of Interinfluential Effects Using the DEMATEL Method. As illustrated in Table 1, the team members aggregate the analytical results in the 12-by-12 matrix of initial average influence relations **A** (Table 2) by using equation (1). The initial average influence relation matrix is further normalized to obtain matrix **D** (Table 3) by using equation (2). Subsequently, the team members calculate a total influence relation matrix **T** by using equation

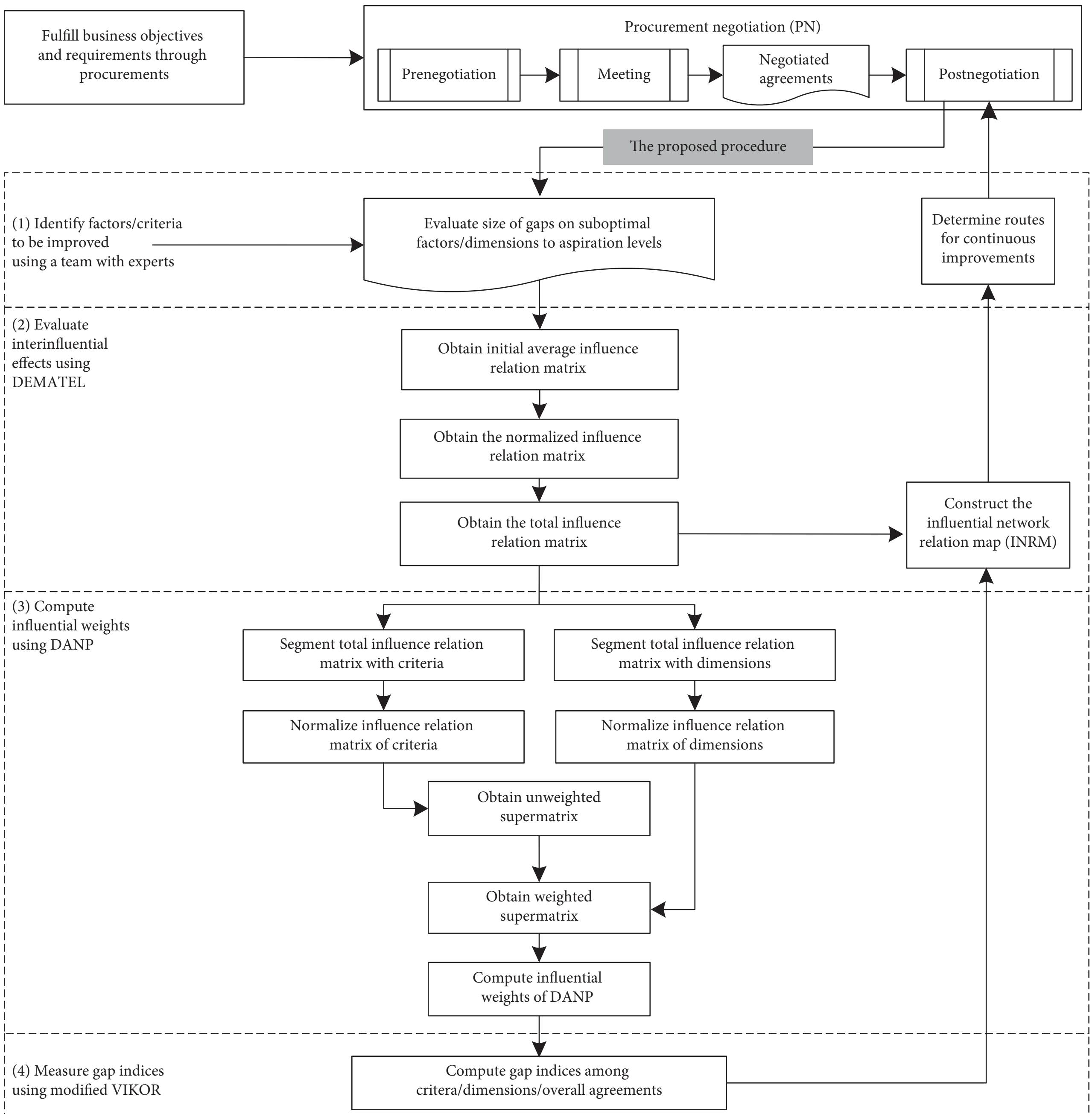

Figure 2: Proposed procedure.

(3). They then use equation (6) to classify the 12 factors and three dimensions to obtain a total influence relation matrix of factors or criteria $\mathbf{T}_C$, after which equation (11) is used to obtain a total influence relation matrix for dimensions $\mathbf{T}_D$. The results are summarized in Table 4. In matrix $\mathbf{T}$, the inconsistency rate of the evaluation results from the experts is only 1.16%, which is much lower than 5%. This result implies that the inclusion of an additional expert would not influence the findings and that the results are significant at a confidence level of 98.84%.

Table 4 illustrates the degree of total influence that a factor exerts on the other factors using equation (4) and the degree of total influence received by a factor from the other factors using equation (5). The results are summarized in Table 5, which also presents role centrality ($r_i + c_i$) and net influence ($r_i - c_i$) for the factors and dimensions. Based on these values, the buyer establishes an INRM illustrating the degrees and directions of interinfluential effects between the 12 factors in the three dimensions associated with the supportability deployment level for the mission operations (Figure 3).

For example, in the airframe dimension (D_1; upper right in Figure 3), the x-coordinate is the role centrality ($r_i + c_i$), and the y-coordinate is the net influence ($r_i - c_i$). First, with reference to Table 5, we determine the values for the airframe (D_1), electrics (D_2), and weapon (D_3) dimensions, which are

Table 1: Expert-identified improvement factors.

Dimensions and factors/criteria		Supportability (%)			
Main systems (dimensions)	Subsystems (factors/criteria)	Threshold	Deployment	Negotiated	Gaps to be improved
Airframe (7)	Landing gear (LG)	76	85	78	7
	Hydraulics system (HS)	78	86	80	6
	Fuel system (FS)	80	88	83	5
	Assistant power units (APU)	78	84	80	4
	Structure system (SS)	78	86	79	7
	Transmission system (TS)	76	84	78	6
	Flight control system (FCS)	75	83	77	6
Electrics (2)	Communication system (CS)	80	84	83	1
	Radio system (RDS)	82	87	85	2
Weapons (3)	Missile system (MS)	79	81	80	1
	Rocket system (RS)	85	88	86	2
	Gun system (GS)	84	89	87	2

Table 2: Initial average influence relation matrix $\mathbf{A} = [a_{ij}]_{n\times n}$ obtained using the DEMATEL method.

A	LD	HS	FS	APU	SS	TS	FCS	CS	RDS	MS	RS	GS
LD	0.0000	1.1250	1.6250	1.6250	3.0000	2.0000	1.1250	0.3750	0.3750	0.3750	0.3750	0.3750
HS	3.6250	0.0000	3.8750	3.7500	2.0000	3.5000	3.5000	1.7500	1.7500	3.1250	3.0000	2.8750
FS	2.6250	3.6250	0.0000	1.3750	1.2500	1.2500	1.3750	1.2500	1.2500	1.2500	1.2500	1.1250
APU	3.5000	3.0000	2.8750	0.0000	2.0000	2.8750	3.1250	1.8750	1.8750	2.8750	2.0000	1.0000
SS	2.3750	1.8750	2.1250	3.0000	0.0000	2.8750	2.2500	2.0000	1.8750	2.8750	2.8750	2.7500
TS	2.0000	2.7500	2.7500	2.8750	2.0000	0.0000	2.8750	1.0000	1.0000	1.8750	1.8750	1.8750
FCS	2.0000	2.1250	2.0000	3.0000	1.0000	2.0000	0.0000	3.0000	3.8750	3.1250	3.0000	2.8750
CS	1.2500	0.3750	1.0000	0.3750	1.8750	1.0000	3.1250	0.0000	4.2500	2.6250	2.6250	3.0000
RDS	1.1250	0.2500	0.8750	0.2500	1.7500	0.8750	3.0000	3.7500	0.0000	2.8750	2.8750	2.8750
MS	0.6250	1.7500	0.8750	1.7500	1.7500	1.0000	1.8750	1.0000	1.0000	0.0000	2.5000	2.5000
RS	0.6250	1.0000	0.8750	0.8750	1.7500	1.0000	1.0000	1.0000	1.0000	2.6250	0.0000	2.8750
GS	0.6250	0.8750	0.8750	0.8750	1.8750	1.0000	1.1250	0.8750	1.0000	2.6250	3.1250	0.0000

Table 3: Normalized influence relation matrix $\mathbf{D} = [d_{ij}]_{n\times n}$ obtained using the DEMATEL method.

D	LD	HS	FS	APU	SS	TS	FCS	CS	RDS	MS	RS	GS
LD	0.0000	0.0344	0.0496	0.0496	0.0916	0.0611	0.0344	0.0115	0.0115	0.0115	0.0115	0.0115
HS	0.1107	0.0000	0.1183	0.1145	0.0611	0.1069	0.1069	0.0534	0.0534	0.0954	0.0916	0.0878
FS	0.0802	0.1107	0.0000	0.0420	0.0382	0.0382	0.0420	0.0382	0.0382	0.0382	0.0382	0.0344
APU	0.1069	0.0916	0.0878	0.0000	0.0611	0.0878	0.0954	0.0573	0.0573	0.0878	0.0611	0.0305
SS	0.0725	0.0573	0.0649	0.0916	0.0000	0.0878	0.0687	0.0611	0.0573	0.0878	0.0878	0.0840
TS	0.0611	0.0840	0.0840	0.0878	0.0611	0.0000	0.0878	0.0305	0.0305	0.0573	0.0573	0.0573
FCS	0.0611	0.0649	0.0611	0.0916	0.0305	0.0611	0.0000	0.0916	0.1183	0.0954	0.0916	0.0878
CS	0.0382	0.0115	0.0305	0.0115	0.0573	0.0305	0.0954	0.0000	0.1298	0.0802	0.0802	0.0916
RDS	0.0344	0.0076	0.0267	0.0076	0.0534	0.0267	0.0916	0.1145	0.0000	0.0878	0.0878	0.0878
MS	0.0191	0.0534	0.0267	0.0534	0.0534	0.0305	0.0573	0.0305	0.0305	0.0000	0.0763	0.0763
RS	0.0191	0.0305	0.0267	0.0267	0.0534	0.0305	0.0305	0.0305	0.0305	0.0802	0.0000	0.0878
GS	0.0191	0.0267	0.0267	0.0267	0.0573	0.0305	0.0344	0.0267	0.0305	0.0802	0.0954	0.0000

(0.8978, 0.1163), (0.8436, 0.0805), and (0.8643, −0.1968), respectively. We then determine their directions based on the degree of total influence of the dimensions according to Table 4, which indicates that the total influence degree of the D_1 on D_2 is 0.1436; conversely, the total influence degree of D_2 on D_1 is 0.1205. The directional arrow is then drawn from D_1 to D_2 because 0.1436 is greater than 0.1205. The influence directions between all dimensions and factors are similarly determined and presented in Figure 3. As Figure 3 illustrates, the interinfluential relationships between the three dimensions are as follows: (1) D_1 influences D_2 and D_3, and (2) D_2 influence D_3. When adopting the same approach, the interinfluential effects visualized on the different tiers of the INRM reveal a structure that allows for analysis of the factors and dimensions that require enhanced scrutiny when determining strategies for improvement.

Table 4: Total influence relation matrix of factors $\mathbf{T}$, $\mathbf{T}_C$ by factors, and $\mathbf{T}_D$ by dimensions, obtained using the DEMATEL method.

T (**T** *C*)	LD	HS	FS	APU	SS	TS	FCS	CS	RDS	MS	RS	GS
LD	0.0660	0.0958	0.1112	0.1131	0.1471	0.1208	0.1064	0.0661	0.0693	0.0906	0.0879	0.0833
HS	0.2458	0.1398	0.2490	0.2481	0.2018	0.2361	0.2646	0.1752	0.1841	0.2704	0.2619	0.2485
FS	0.1608	0.1805	0.0842	0.1255	0.1213	0.1196	0.1389	0.1090	0.1139	0.1427	0.1398	0.1311
APU	0.2218	0.2018	0.2021	0.1238	0.1802	0.1998	0.2316	0.1613	0.1690	0.2360	0.2083	0.1743
SS	0.1844	0.1664	0.1755	0.2015	0.1196	0.1943	0.2032	0.1611	0.1651	0.2346	0.2306	0.2193
TS	0.1668	0.1820	0.1845	0.1900	0.1631	0.1044	0.2056	0.1232	0.1299	0.1907	0.1865	0.1786
FCS	0.1743	0.1713	0.1717	0.1991	0.1530	0.1703	0.1447	0.1954	0.2258	0.2480	0.2408	0.2299
CS	0.1189	0.0917	0.1109	0.0974	0.1457	0.1116	0.1950	0.0878	0.2103	0.1984	0.1971	0.2025
RDS	0.1107	0.0844	0.1030	0.0898	0.1379	0.1039	0.1860	0.1856	0.0900	0.1989	0.1978	0.1940
MS	0.0938	0.1214	0.0998	0.1262	0.1267	0.1030	0.1428	0.0976	0.1025	0.1019	0.1705	0.1651
RS	0.0799	0.0892	0.0865	0.0895	0.1158	0.0902	0.1047	0.0862	0.0901	0.1607	0.0853	0.1619
GS	0.0805	0.0867	0.0871	0.0903	0.1199	0.0909	0.1086	0.0836	0.0907	0.1619	0.1737	0.0823

$\mathbf{T}_D$	Airframe (D_1)	Electrics (D_2)	Weapons (D_3)
Airframe (D_1)	0.1686	0.1463	0.1921
Electrics (D_2)	0.1205	0.1434	0.1981
Weapons (D_3)	0.1016	0.0918	0.1404

Note. $IR = (1/n^2)\sum_{i=1}^{n}\sum_{j=1}^{n}(|t_{ij}^{p} - t_{ij}^{p-1}|/t_{ij}^{p}) \times 100 \cong 1.16\% < 5\%$, where t_{ij}^{p} and t_{ij}^{p-1} denote the average influence of the ith criterion on the jth criterion by the experts (samples) for $p = 8$ and $p - 1 = 7$, respectively, and $n = 12$ denotes the number of factors or criteria. Thus, the results are significant at a confidence level of 98.84%, which is greater than the 95% level used to test for significance.

Table 5: Total influence exerted and received for the dimensions and factors obtained using the DEMATEL method.

Dimensions/factors	r_i	c_i	$r_i + c_i$	$r_i - c_i$
Airframe (D_1)			**0.8978**	**0.1163**
LD (C_1)	1.1575	1.7038	2.8613	−0.5463
HS (C_2)	2.7252	1.6111	4.3364	1.1141
FS (C_3)	1.5673	1.6655	3.2328	−0.0981
APU (C_4)	2.3099	1.6943	4.0042	0.6157
SS (C_5)	2.2556	1.7322	3.9878	0.5234
TS (C_6)	2.0055	1.6450	3.6505	0.3606
FCS (C_7)	2.3244	2.0321	4.3565	0.2923
Electrics (D_2)			**0.8436**	**0.0805**
CS (C_8)	1.7674	1.5321	3.2996	0.2353
RDS (C_9)	1.6820	1.6407	3.3227	0.0413
Weapons (D_3)			**0.8643**	**−0.1968**
MS (C_{10})	1.4514	2.2348	3.6862	−0.7834
RS (C_{11})	1.2401	2.1802	3.4203	−0.9400
GS (C_{12})	1.2562	2.0709	3.3271	−0.8148

The values in bold are used to distinguish the total influence between dimensions and factors.

4.2.3. Computation of IWs Using DANP. After the establishment of the INRM, the expert team normalizes the total influence relation matrix of factors and dimensions by using equations (7), (8), (11), and (12), as exhibited in Table 6. Additionally, the expert team transposes the matrices in Table 6 to an unweighted supermatrix by using equations (9) and (10) and then uses equation (13) to obtain a weighted supermatrix, as illustrated in Table 7. Finally, the buyer multiplies the weighted super matrix until it converges into a steady-state condition, with the IWs for the factors and dimensions, as presented in Table 8.

4.2.4. Measurement of Gap Indices Based on the Modified VIKOR Method. During this stage, based on Table 1, the buyer sets the worst-case value f_j^{worst} and best-case value f_j^{aspired} (aspired level) to analyze the three supportability levels, which are denoted by A_1, A_2, and A_3. Subsequently, the buyer uses the IWs from DANP together with the modified VIKOR method to compute the gap indices for the three alternatives and their dimensions and factors by using equations (14)–(18). The computational results, which are summarized in Table 9**,** reveal the extent to which each alternative, dimension, and factor would need to be improved to reach the respective aspiration levels. Based on Table 9, the expert team aggregates the gap indices on the INRM (Figure 3) and arranges meetings including those with external participants to discuss actionable ways to eliminate the gaps. All participants review Tables 1–9 with reference to the INRM to realize the extent of the gaps that require bridging. The meeting results provide the buyer with valuable insights regarding the provision of alternative methods to finalize the improvement solutions with the seller. Table 10 presents an example to illustrate how the different types of information produced by the proposed procedure are used to

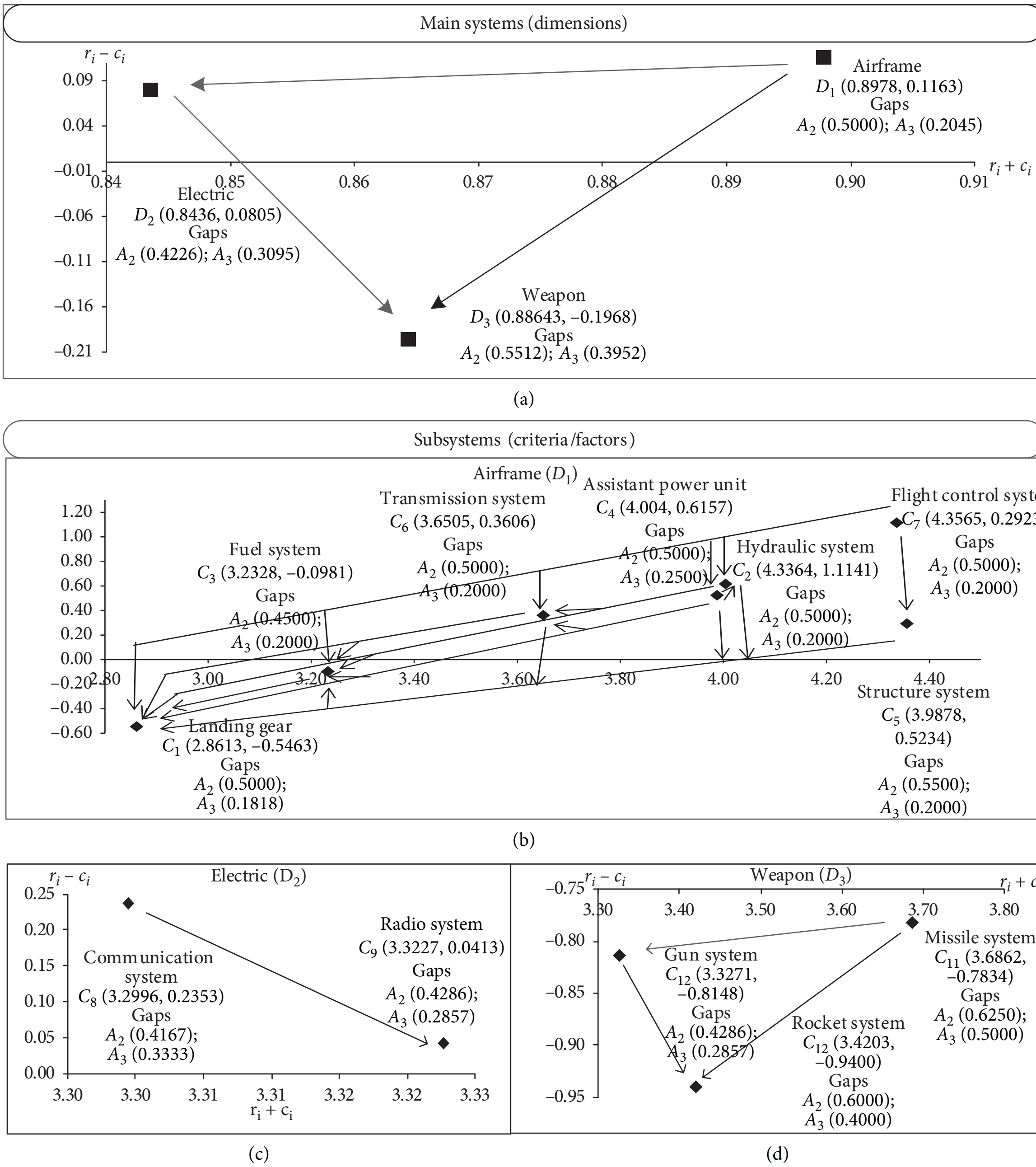

Figure 3: Influential network relation map.

determine the improvement proposals, which also provides the buyer with a basis on which to iterate the proposed procedure in the postnegotiation phase to implement continuous improvements.

5. Discussion and Implications

This study derives several critical results concerning improvements to suboptimal PN agreements. First, according to Table 5, using the DEMATEL method to evaluate the interinfluential effects between the criteria and dimensions associated with the negotiated PN agreement can yield valuable information to assist managers to systematically understand suboptimal situations at different levels. This understanding provides managers with insight into the identification of the critical subsystems and systems that should be prioritized for improvement. For example, in the DEMATEL results illustrated in Table 5, the $(r_i + c_i, r_i - c_i)$ values at the dimension level are (0.8978, 0.1163) for D_1, (0.8436, 0.0805) for D_2, and (0.8643, −0.1968) for D_3. In these values, the $r_i + c_i$ values indicate that the expert team members generally agreed that all three main systems are central to achieving the desired supportability. Additionally, according to the $r_i - c_i$ values, D_1 influences D_2 and D_3, and D_2 influences D_3, implying that improvements to the airframe (D_1) should contribute the greatest improvement to the overall supportability, followed by improvements to the electronics system (D_2) and the weapons system (D_3).

Table 6: Normalized total influence relation matrix of factors $\mathbf{T}_C^{\alpha}$ and dimensions $\mathbf{T}_D^{\alpha}$ derived using DANP.

$\mathbf{T}_C^{\alpha}$	LD	HS	FS	APU	SS	TS	FCS	CS	RDS	MS	RS	GS
LD	0.0869	0.1260	0.1463	0.1487	0.1934	0.1589	0.1399	0.4880	0.5120	0.3460	0.3359	0.3180
HS	0.1550	0.0882	0.1571	0.1565	0.1273	0.1489	0.1669	0.4876	0.5124	0.3463	0.3354	0.3183
FS	0.1728	0.1939	0.0904	0.1348	0.1303	0.1285	0.1492	0.4891	0.5109	0.3449	0.3380	0.3171
APU	0.1630	0.1483	0.1485	0.0910	0.1324	0.1468	0.1701	0.4883	0.5117	0.3815	0.3367	0.2818
SS	0.1481	0.1336	0.1410	0.1618	0.0961	0.1561	0.1632	0.4939	0.5061	0.3428	0.3369	0.3203
TS	0.1394	0.1521	0.1542	0.1588	0.1364	0.0873	0.1719	0.4868	0.5132	0.3431	0.3355	0.3214
FCS	0.1472	0.1446	0.1450	0.1681	0.1292	0.1438	0.1222	0.4639	0.5361	0.3451	0.3350	0.3199
CS	0.1365	0.1053	0.1272	0.1118	0.1672	0.1281	0.2239	0.2945	0.7055	0.3317	0.3296	0.3386
RDS	0.1357	0.1035	0.1262	0.1101	0.1691	0.1274	0.2280	0.6733	0.3267	0.3367	0.3348	0.3285
MS	0.1153	0.1492	0.1227	0.1551	0.1557	0.1266	0.1755	0.4878	0.5122	0.2329	0.3897	0.3774
RS	0.1219	0.1361	0.1319	0.1365	0.1766	0.1375	0.1596	0.4891	0.5109	0.3940	0.2092	0.3969
GS	0.1212	0.1306	0.1312	0.1361	0.1806	0.1369	0.1635	0.4798	0.5202	0.3874	0.4156	0.1970

$\mathbf{T}_D^{\alpha}$	Airframe (D_1)	Electrics (D_2)	Weapons (D_3)
Airframe (D_1)	0.3326	0.2886	0.3788
Electrics (D_2)	0.2608	0.3104	0.4288
Weapons (D_3)	0.3044	0.2750	0.4206

Table 7: Unweighted and weighted supermatrices $\mathbf{W} = (\mathbf{T}_C^{\alpha})'$ and $\mathbf{W}^{\alpha} = \mathbf{T}_D^{\alpha}\mathbf{W}$ derived using DANP.

$\mathbf{W}$	LD	HS	FS	APU	SS	TS	FCS	CS	RDS	MS	RS	GS
LD	0.0279	0.0497	0.0554	0.0523	0.0475	0.0447	0.0472	0.0319	0.0317	0.0335	0.0354	0.0352
HS	0.0404	0.0283	0.0622	0.0476	0.0429	0.0488	0.0464	0.0246	0.0242	0.0433	0.0395	0.0380
FS	0.0469	0.0504	0.0290	0.0476	0.0452	0.0495	0.0465	0.0297	0.0295	0.0356	0.0383	0.0381
APU	0.0477	0.0502	0.0432	0.0292	0.0519	0.0509	0.0539	0.0261	0.0257	0.0451	0.0397	0.0395
SS	0.0620	0.0408	0.0418	0.0425	0.0308	0.0437	0.0414	0.0390	0.0395	0.0452	0.0513	0.0525
TS	0.0510	0.0478	0.0412	0.0471	0.0501	0.0280	0.0461	0.0299	0.0297	0.0368	0.0399	0.0398
FCS	0.0449	0.0535	0.0479	0.0546	0.0524	0.0551	0.0392	0.0523	0.0532	0.0510	0.0464	0.0475
CS	0.1502	0.1501	0.1506	0.1503	0.1520	0.1499	0.1428	0.1006	0.2299	0.1446	0.1450	0.1422
RDS	0.1576	0.1577	0.1573	0.1575	0.1558	0.1580	0.1650	0.2409	0.1115	0.1518	0.1515	0.1542
MS	0.1285	0.1286	0.1281	0.1417	0.1273	0.1274	0.1282	0.1410	0.1431	0.0962	0.1627	0.1600
RS	0.1248	0.1246	0.1255	0.1250	0.1251	0.1246	0.1244	0.1401	0.1423	0.1609	0.0864	0.1717
GS	0.1181	0.1182	0.1178	0.1047	0.1190	0.1194	0.1188	0.1439	0.1396	0.1558	0.1639	0.0814
$\mathbf{W}^{\alpha}$	LD	HS	FS	APU	SS	TS	FCS	CS	RDS	MS	RS	GS
LD	0.0379	0.0375	0.0373	0.0373	0.0375	0.0377	0.0376	0.0364	0.0364	0.0375	0.0373	0.0373
HS	0.0372	0.0374	0.0364	0.0370	0.0371	0.0370	0.0370	0.0360	0.0361	0.0367	0.0370	0.0372
FS	0.0374	0.0373	0.0378	0.0373	0.0375	0.0373	0.0374	0.0365	0.0365	0.0375	0.0372	0.0373
APU	0.0384	0.0383	0.0385	0.0388	0.0382	0.0383	0.0382	0.0376	0.0377	0.0382	0.0386	0.0386
SS	0.0439	0.0445	0.0446	0.0445	0.0446	0.0444	0.0445	0.0446	0.0445	0.0448	0.0444	0.0443
TS	0.0377	0.0378	0.0381	0.0378	0.0377	0.0382	0.0378	0.0371	0.0372	0.0380	0.0378	0.0378
FCS	0.0503	0.0500	0.0501	0.0500	0.0500	0.0500	0.0502	0.0499	0.0498	0.0498	0.0501	0.0500
CS	0.1528	0.1527	0.1527	0.1527	0.1524	0.1527	0.1537	0.1614	0.1447	0.1522	0.1522	0.1527
RDS	0.1612	0.1612	0.1613	0.1613	0.1615	0.1612	0.1602	0.1530	0.1698	0.1609	0.1609	0.1603
MS	0.1370	0.1370	0.1370	0.1359	0.1372	0.1371	0.1371	0.1383	0.1379	0.1403	0.1358	0.1362
RS	0.1353	0.1353	0.1352	0.1351	0.1352	0.1353	0.1353	0.1370	0.1365	0.1343	0.1399	0.1327
GS	0.1311	0.1310	0.1312	0.1323	0.1310	0.1309	0.1309	0.1321	0.1330	0.1299	0.1288	0.1356

Table 8: $\lim_{Z\longrightarrow\infty}(\mathbf{W}^{\alpha})^{Z}$ and IWs obtained using DANP.

$\lim_{Z\longrightarrow\infty}(\mathbf{W}^{\alpha})^{Z}$	LD	HS	FS	APU	SS	TS	FCS	CS	RDS	MS	RS	GS
LD	0.0371	0.0371	0.0371	0.0371	0.0371	0.0371	0.0371	0.0371	0.0371	0.0371	0.0371	0.0371
HS	0.0367	0.0367	0.0367	0.0367	0.0367	0.0367	0.0367	0.0367	0.0367	0.0367	0.0367	0.0367
FS	0.0371	0.0371	0.0371	0.0371	0.0371	0.0371	0.0371	0.0371	0.0371	0.0371	0.0371	0.0371
APU	0.0382	0.0382	0.0382	0.0382	0.0382	0.0382	0.0382	0.0382	0.0382	0.0382	0.0382	0.0382
SS	0.0445	0.0445	0.0445	0.0445	0.0445	0.0445	0.0445	0.0445	0.0445	0.0445	0.0445	0.0445
TS	0.0376	0.0376	0.0376	0.0376	0.0376	0.0376	0.0376	0.0376	0.0376	0.0376	0.0376	0.0376
FCS	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500
CS	0.1526	0.1526	0.1526	0.1526	0.1526	0.1526	0.1526	0.1526	0.1526	0.1526	0.1526	0.1526
RDS	0.1611	0.1611	0.1611	0.1611	0.1611	0.1611	0.1611	0.1611	0.1611	0.1611	0.1611	0.1611

Table 8: Continued.

$\lim_{Z\longrightarrow\infty}(\mathbf{W}^{\alpha})^{Z}$	LD	HS	FS	APU	SS	TS	FCS	CS	RDS	MS	RS	GS
MS	0.1375	0.1375	0.1375	0.1375	0.1375	0.1375	0.1375	0.1375	0.1375	0.1375	0.1375	0.1375
RS	0.1359	0.1359	0.1359	0.1359	0.1359	0.1359	0.1359	0.1359	0.1359	0.1359	0.1359	0.1359
GS	0.1317	0.1317	0.1317	0.1317	0.1317	0.1317	0.1317	0.1317	0.1317	0.1317	0.1317	0.1317
IWs	LD	HS	FS	APU	SS	TS	FCS	CS	RDS	MS	RS	GS
Local	0.0371	0.0367	0.0371	0.0382	0.0445	0.0376	0.0500	0.1526	0.1611	0.1375	0.1359	0.1317
Global				Airframe				Electrics			Weapons	
	LD	HS	FS	APU	SS	TS	FCS	CS	RDS	MS	RS	GS
				0.2812				**0.3137**			**0.4051**	
	0.1319	0.1305	0.1319	0.1358	0.1583	0.1337	0.1778	0.4865	0.5135	0.3394	0.3355	0.3251

The values in bold are used to distinguish the IWs between dimensions and factors.

Table 9: Gap indices for factors, dimensions, and alternatives obtained using the modified VIKOR method.

Dimension/factor	Influential weights according to DANP		Performance measurements with respect to factors of each alternative			f_j^{aspired}	f_j^{worst}	Gap indices on factors/dimensions with respect to each alternative		
	Local	Global	*A*1	*A*2	*A*3			*A*1	*A*2	*A*3
Airframe	**0.2812**							**0.7955**	**0.5000**	**0.2045**
LD	0.1319	0.0371	0.7800	0.8150	0.8500	0.8700	0.7600	0.8182	0.5000	0.1818
HS	0.1305	0.0367	0.8000	0.8300	0.8600	0.8800	0.7800	0.8000	0.5000	0.2000
FS	0.1319	0.0371	0.8300	0.8550	0.8800	0.9000	0.8000	0.7000	0.4500	0.2000
APU	0.1358	0.0382	0.8000	0.8200	0.8400	0.8600	0.7800	0.7500	0.5000	0.2500
SS	0.1582	0.0445	0.7900	0.8250	0.8600	0.8800	0.7800	0.9000	0.5500	0.2000
TS	0.1339	0.0376	0.7800	0.8100	0.8400	0.8600	0.7600	0.8000	0.5000	0.2000
FCS	0.1778	0.0500	0.7700	0.8000	0.8300	0.8500	0.7500	0.8000	0.5000	0.2000
Electrics	**0.3137**							**0.5357**	**0.4226**	**0.3095**
CS	0.4865	0.1526	0.8300	0.8350	0.8400	0.8600	0.8000	0.5000	0.4167	0.3333
RDS	0.5135	0.1611	0.8500	0.8600	0.8700	0.8900	0.8200	0.5714	0.4286	0.2857
Weapons	**0.4051**							**0.7071**	**0.5512**	**0.3952**
MS	0.3394	0.1375	0.8000	0.8050	0.8100	0.8300	0.7900	0.7500	0.6250	0.5000
RS	0.3355	0.1359	0.8600	0.8700	0.8800	0.9000	0.8500	0.8000	0.6000	0.4000
GS	0.3251	0.1317	0.8700	0.8800	0.8900	0.9100	0.8400	0.5714	0.4286	0.2857
Gap indices on alternatives								**0.7900**	**0.5613**	**0.4076**

The values in bold are used to distinguish the influential weights and the gap indices between dimensions and factors.

Table 10: Example of determining an improvement proposal.

Options	Levels	Proposals	Main consideration
1	Dimensions Criteria	Weapons (D_3) ≺ Airframe (D_1) ≺ Electrics (D_2) MS (C_{10}) ≺ RS (C_{11}) ≺ SS (C_5) ≺ HS (C_2) ≺ FCS	Performance gap size
2	Dimensions Criteria	Airframe (D_1) ≺ Weapons (D_3) ≺ Electrics (D_2) SS (C_5) ≺ HS (C_2) ≺ APU (C_4) ≺ MS (C_{10}) ≺ RS (C_{11})	Degree of centrality

Following this approach allows for the level of interinfluence between the factors to be understood. As revealed in Table 5, between the 12 subsystems, the hydraulics system (HS, C_2_4.3364, 1.1141), the assistant power units (APU, C_4_4.0042, 0.6157), and the flight control system (FCS, C_7_4.3565, 0.2923) are the most critical subsystems for improving overall supportability. This reflects the facts that (1) they have higher role centrality than the other factors do, and (2) their net influences are positive.

Because each evaluation dimension and criterion produce essential information for improving negotiated agreements, we strongly suggest that managers use the DEMATEL method to evaluate the interdependencies among the dimensions and criteria of negotiated agreements during their implementation. In addition, as presented in Table 6, the interrelationships generated from the DEMATEL method can also be used as input data for DANP to derive the IWs for each criterion and dimension. Although the assumption of most decision-making methodologies regarding mutual exclusivity and independence on the decision variables may not hold in a PN setting, the proposed procedure employs DANP to enable interdependent negotiation situations to be considered in the decision process and outcomes of the PN setting.

The results of the case study also reveal that combining DANP with the modified VIKOR method can help the buyer gain insights into practical methods to solve suboptimality problems. For example, the proposed procedure measures gap indices in both the main systems (the three dimensions) and the subsystems (12 factors) under the respective alternatives (Table 9). These measurements provide the buyer with additional options when formulating improvement proposals. Taking A_2 as an example (Table 9), the gap indices in the dimension and factor levels indicate that the weapons system (D_3) and its subsystems (i.e., the missile system (MS) and the rocket system (RS)) should be prioritized when determining improvement proposals because of their substantial aspiration level gaps compared with those of the other main systems and subsystems. This finding differs from the implications of the DEMATEL results (Table 5), in which it is assumed that improving the airframe (D_1) should be prioritized. For the buyer, the additional information obtained from the gap indices provides for increased options and flexibility when determining improvement proposals before negotiating with the seller to produce sustainable improvement solutions. For example, in Table 10, option 1 of the improvement proposals is based first on the performance gap size and then on the role centrality. By contrast, option 2 is considered differently. Consideration of the different types of information produced from the proposed procedure can result in various improvement effects and priorities for the main systems and subsystems, which, in turn, provide the buyer with increased flexibility to prepare and pursue the aspired PN outcomes.

The proposed procedure also enables the different tiers of the interinfluential effects and the gap indices to be systemized and visualized in an INRM. The INRM provides the PN participants with a solid basis to manage the dynamics of improvement situations that arise during an agreement's implementation lifecycle. According to Table 9, when the supportability level is increased to A_3, the sequence of the dimension-level gap indices differs from that of A_2, which indicates that the organizations have different perspectives on implementation. Therefore, periodic meetings between the parties are essential. The INRM (Figure 3) is updated by the buyer before each meeting by comparing the undated total influence effects between each dimension and factor. The buyer is then able to inform the seller about the main systems and subsystems that should be prioritized. For example, as exhibited in Figure 3, from the weapons system perspective, the seller should prioritize the MS because maintaining its supportability would positively influence both the gun system (GS) and the RS. By assisting parties to understand their own requirements and by providing alternatives and possible solutions to share with the opposite party, the proposed procedure can ensure practical settlement of negotiated agreements in a PN setting.

6. Conclusions and Remarks

Complicated PNs are usually concluded at a suboptimal level. This study proposes a novel procedure, based on a combined MADM model to help managers create post-negotiation settlements over an agreement's implementation lifecycle.

A numerical example of a logistics service project demonstrates the proposed procedure's merits: (1) interinfluential effects and gap indices are used to support improvement strategies in the pursuit of aspired outcomes regardless of how and why a suboptimal agreement initially occurs; (2) managers obtain detailed and systematic information on a visualized map to identify the core issues causing suboptimality PN problems; and (3) it extends on previous studies of improving negotiated agreements from the conceptual level to a practical operational level. These merits indicate that the proposed procedure can provide a valuable foundation for solving suboptimal agreement problems in the PN setting.

However, this study also has several limitations. First, its conclusions are based on a procurement project between a public organization and private contractor in the case study country. Future research could apply our procedure to other settings, such as those occurring across countries and organizations in different industries, to examine how our procedure functions over a wide range of procurement situations. The resulting comparisons could offer more insights into the applicability of the proposed procedure. Second, the improvement proposals addressed in the example only comprise principle concepts. Future research could be undertaken to identify substantial aspects for improvement. Such work could be formulated as a design problem, and future research could adopt multiple criteria decision-making methods such as the de novo programming for multiple objective decision-making, which might provide valuable results for improving negotiated agreements with changeable objectives and decision spaces. Finally, the proposed procedure assumes that negotiating parties are willing to work cooperatively to resolve suboptimality problems after the negotiated agreement is concluded. In reality, incentives could be applied to the proposed procedure so that it functions more pragmatically to enhance the efficiency and effectiveness of the initial negotiations. Integrating practical incentives and flexible contractual mechanisms to motivate procurement agreement participants to pursue continuous improvement solutions also deserves further study.

Data Availability

The numerical data used to support the findings of this study are included within the article.

Disclosure

The sponsors had no role in the design of the study; in the collection, analysis, or interpretation of the data; in the writing of the manuscript; or in the decision to publish.

Conflicts of Interest

The authors declare no conflicts of interest.

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Research Article

Multicriteria Model Based on FITradeoff Method for Prioritizing Sections of Brazilian Roads by Criticality

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Three of the most important objectives of the Federal Road Police (Patrol) when managing the traffic on federal roads in Brazil are to ensure that there is an efficient movement of traffic under stable conditions and minimal traffic congestion problems that the federal highways are safe and that accidents do not occur. Therefore, multiple matters are relevant for road safety and user security, such as prompt maintenance of these highways, regular monitoring of their state of conservation and their characteristics; controlling the traffic; and preventing and combatting criminal activities on the highways. However, considering the vast network and different conditions of roads, the different types of traffic accidents and their consequences, and different levels of violence on federal roads, it is essential that these roads undergo regular maintenance and regular inspections, are constantly patrolled, and are subject to continuous improvements. Therefore, defining the prioritization and criticality of roads takes on the characteristics of a multicriteria decision, given the multidimensional aspects of the risks inherent in them. Thus, this paper presents a multicriteria decision model for prioritizing road sections, based on their criticality and the risks that users face. The model was applied using the FITradeoff method, due to its flexibility and due to it requiring less cognitive effort from the decision-maker with regard to providing information regarding his/her preferences. A case study was undertaken on a set of the federal roads of the state of Pernambuco (Brazil), covering 22 different sections with different characteristics. As a result, it was possible to rank and identify the most critical sections of a highway. The use of FITradeoff gave support to decision-making on ordering the sections and also let a general analysis of the data be undertaken.

1. Introduction

Worldwide, about 1.35 million people die every year in traffic accidents [1]. Moreover, between 20 and 50 million other people have some degree of physical or psychological sequelae. In addition to the irreparable loss of human lives, the sequelae and trauma of victims, and the immeasurable impact on the families affected and on the communities in which they live, the monetary costs of traffic accidents for society as a whole are high. The social and economic costs amount to 3% of the gross domestic product (GDP) of all countries in the world [2].

In 2016, deaths caused by traffic accidents rose by two positions in the ranking of deaths from all causes due to the increase in road accident fatalities, which, therefore, became the eighth leading cause of death. There were 1.4 million such deaths (2.5% of all deaths), and road accidents were the leading cause of death among young people aged 15 to 29 years. In contrast, when only high-income countries are considered, traffic fatalities are not among the top ten causes of death, thus reinforcing how serious this possible cause of death is for the 15–29-year-old age group, especially in low- and middle-income countries [3].

In Brazil, tens of thousands of people die on roads and tens of millions are injured or disabled every year. In 2015, for instance, 38,651 traffic fatalities were recorded. Brazil was ranked third among the countries with most traffic deaths in absolute terms. When the ratio of the number of deaths to the total number of inhabitants is taken into account, Brazil ranks 14th, with an average of 19.7 deaths per 100,000 inhabitants [1].

Hence, the large number of traffic accidents makes them the leading cause of adverse impacts on humans such as death, injury, and hospitalization, resulting in high economic and social costs. In 2014, approximately 170,000 traffic accidents generated a cost of R$ 12.3 billion for Brazil, of which 64.7% were associated with the victims of such accidents. This included expenditure on health care and the costs arising from loss of production due to injuries or death, and 34.7% were related to vehicles, such as damage to public and private property, loss of cargo, and the cost of removing wrecked and damaged vehicles [4].

Intervening in the road system and making improvements to the roads, in order to reduce the number of deaths and injured victims and thus to reduce the damage and losses from accidents, is a major challenge for transport management and engineering. Therefore, studying the most critical locations is an appropriate way to establish a policy for improving safety levels on roads, thereby prioritizing critical roads [5]. Furthermore, considering that resources are limited, it is not possible to invest in all roads simultaneously, so the most critical roads must be targeted [6].

It is noteworthy that the issue of prioritizing road sections is not restricted only to issues related to traffic accidents, since security issues also affect users of roads. Some of the crimes that can occur within the jurisdiction of federal highways are drug trafficking, arms trafficking, human trafficking, smuggling, environmental crimes, theft of cargo and theft of vehicles, and sexual exploitation of children and adolescents. Therefore, in addition to traffic safety issues on roads, the role of the Federal Road Police (FRP) in Brazil also includes endeavoring to ensure that federal roads are not used to support criminal activities such as the transport of illegal drugs, arms, contraband, and people trafficking nor as to the stage for crimes such as the hijacking of goods and vehicles or hold-ups of long-distance buses and robbing passengers. Similarly, another duty of the FRP is to identify and arrest perpetrators of crime travelling on federal roads in public or private transport including stolen vehicles. In the first half of 2020, about 316 tons of marijuana and about 14 tons of cocaine were seized by the FRP in Brazil, corresponding to about 25% and 15% of all marijuana and cocaine, respectively, seized in Brazil [7]. As to the hijacking of trucks and the theft of cargo, these are crimes that have been shown to be constantly increasing in Brazil, and the FRP has had to devote more and more resources to combat them. In fact, Brazil is one of the countries in which the security of cargo is at the highest risk from criminal activity. The estimated direct cost of the theft of cargo to the Brazilian economy is 442 million USD per year, an amount that has been increasing in the last decade [8].

It is emphasized that some characteristics (condition/state) of the roads can influence issues associated with crime [9]. For example, drivers can avoid driving at certain times on certain roads, for fear of being held up; certain conditions of the road may also facilitate criminal activity, such as having to reduce speed on sections of a road which is in a poor state of repair; road sections that are not regularly policed favor the circulation of narcotics, and raise issues about the control of borders. Finally, when looking to improve the "general conditions" of the road, the objective is to reduce both accidents and criminal actions and thus to improve safety in a more comprehensive way and from a wider perspective than other countries in which the extent of criminal activity on highways is considerably less in volume and value than in Brazil.

One way to reduce the problems related to the costs and severity of the consequences generated by road accidents is by studying the most critical sites to establish a policy of improving the level of safety on the roads, thus guiding what priority to give to the most critical road sections. However, it should be noted that this type of study involving the phenomenon of road safety is complex and should involve detailed aspects of the causality of accidents and injuries, not only data on the number of occurrences. Road crashes are a consequence of multiple factors, which are generally grouped in relation to the infrastructure (road condition), the vehicle, and the driver (human state of health and fitness to drive) [10–15]. Additionally, given that this study addresses issues associated with the scope of the FRP, violence on roads is considered to be another dimension that the FRP must deal with. Such violence can be regarded as a form of criminality, not only when there are acts of aggression and physical assault but also when drivers are under the influence of alcohol and/or drugs or are otherwise incapable of driving safely (e.g., truck drivers driving continuously for longer than the law permits and falling asleep at the wheel, a not uncommon occurrence). Therefore, this involves a multidimensional concept that cannot be captured by a single indicator, which implies that there is a multi-objective feature to the problem [16–18]. It can be seen, from this overview of the duties of the FRP, that traffic accidents and violence on the roads compete for common resources, for example, since the patrol that deals with traffic accident issues also deals with criminality.

Given this structure, a multicriteria approach to deal with this type of problem is frequently taken [6, 19–21]. Within the different approaches used in multicriteria analysis, additive models stand out [22–25]. However, an inherent difficulty when using these methods concerns the elicitation of weights, given the cognitive effort required in this process [19, 20, 24, 26, 27].

Thus, this paper proposes using a multicriteria approach to address the problem [21]. To this end, considering the advantages that will be discussed below, the FITradeoff (flexible and interactive tradeoff) method [24], widely used in the literature [22–25, 28], was used to identify the most critical road sections in the state of Pernambuco and to indicate the amount of future investments needed in order to improve road safety and security.

In view of the difficulty in the elicitation process, a procedure must be chosen, which in addition to mitigating the effort required in the process, also takes proper account of the compensation among the criteria. In this respect, the FITradeoff method [24] stands out because it offers a procedure of flexible elicitation of weights or scale constants based on the traditional axiomatic procedure of tradeoff elicitation, which is one of the most used when considering additive methods. However, unlike the traditional procedure, which takes into account the relationship of indifference, the FITradeoff also admits the relationship of preference which makes the cognitive effort that the decision-maker (DM) needs to make smaller, while, at the same time, the possibility of inconsistencies regarding the DM's judgments is reduced [24].

The article is structured as follows: in the related work section, a literature review on assessing the conditions of roads using a multicriteria approach is presented; in the Materials and Methods section, a framework developed for this application is presented and the FITradeoff method is explained; the subsequent section is devoted to the Case study which was used to analyze the decision problem that prompted this study. The Results and Discussion section presents some insights about the results and some conclusions are drawn in the last section.

2. Related Work

Given the complexity of studies related to the factors that link accidents and road conditions, a multidimensional analysis is often required. Hence, some authors use a multicriteria approach in order to analyze the multiple dimensions of the problem.

The performance indicators of traffic safety are listed in a global index and combined with linguistic descriptors that are based on the knowledge of experts by a fuzzy TOPSIS hierarchical model [16]. The model proved to be useful because it helps the general public to have a good understanding of the results, thereby supporting the desired policy. This was achieved by displaying several road safety performance indicators in a single composite index together with experts' comments.

This is also commented on by Tešić et al. [19], who used data envelopment analysis (DEA) to assess the efficiency of European countries in terms of traffic safety on the basis of the following criteria: laws about driving and alcohol, speed limits, and adequacy of protection systems (e.g., seat belts, airbags, central reservation or protective barriers, condition of vehicles, condition of roads, and provision of trauma management). The study highlighted the importance of selecting indicators that have the greatest influence on the outcome. The result depends on the collection and quality of data, on the method used and on the indicators. In addition, the study pointed out that a greater number of indicators provide a higher quality result so that DMs can precisely define actions and identifying strengths and weaknesses. In contrast, in practice, there is a need to use an index that has a limited number of indicators that provide enough quality to compare countries.

Fancello et al. [29] used a multicriteria ranking method with the objective of comparing different road sections regarding safety conditions. Therefore, an algorithm based on the ELECTRE III method was used to rank road sections in a real study case. Hence, the analysis of intervention priorities was based on the final ranking. In a similar approach, Fancello et al. [20] developed a decision support system (DSS) for road analysis incorporating different indicators. Their main objective was to determine which sections require interventions to improve safety conditions. In multicriteria terms, concordance analysis was used. With the objective of improving both previous methodologies, Fancello et al. [6] combined two multicriteria methods (VIKOR and TOPSIS) for comparison of the concordance analysis. They used eight criteria based on traffic volume and the geometrics of road sections.

Temrungsie et al. [27] used the analytic hierarchy process (AHP) method as a tool for learning and prioritizing the information that can prevent and reduce accidents. The subcriteria in the analysis of this study were framed into 4 main criteria: engineering, economic, socioenvironmental, and safety. To collect the information necessary for the method, a questionnaire was applied to 100 respondents distributed in different expert groups.

However, still using the AHP method, Kanuganti et al. [30] aimed at prioritizing safety requirements of a simple additive weighting category of rural roads in India. Three different methods were analyzed for this purpose: simple additive weighting, AHP, and fuzzy AHP.

In Khorasani et al. study [31], a multicriteria method for assessing the safety performance of 21 European countries was presented. The evaluation of the countries was based on a survey of 11 indicators and the simple additive weighting method was used. To determine the weights, the entropy method applied to the decision matrix was used.

Castro-Nuño and Arévalo-Quijada [32] presented two multidimensional safety indicators that combine a set of criteria related to the economy, demography, and sustainability in urban transport. These indicators were used to determine safety performance in 50 Spanish provinces. Thus, a multicriteria analysis using the PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluation) method was applied along with the entropy method of determining weights.

With the same aim of determining an aggregation index, Rosić et al. [33] used different models with the aim of increasing efficiency in generating a composite safety index in relation to roads. Thus, the authors based the construction of their index on a combined analysis using DEA and TOPSIS to present a model for selecting composite indicators by using the PROMETHEE-RS method. Based on the same idea, Chen et al. [34] presented a methodology for formulating a composite index based on the TOPSIS–RSR entropy methodology.

For Hermans et al. [35], the idea of building a composite road safety indicator was used on the basis of a methodology that applies ordered weighted averaging (OWA) operators. Based on this approach, seven indicators were combined into a single indicator for evaluating different countries. The weights of each indicator were determined by the relative importance of each criterion by means of the AHP method.

In a broader context, Rodrigues et al. [36] presented a road network classification model based on traffic accidents integrated with a geographical information system. In the model, an equation was defined to obtain a road safety index based on the following indicators: severity, damage to property only, and the costs of accidents. In addition to the advantage of classification, the model enabled the analysis of the spatial coverage of accidents with a view to determining the location of regions with the highest accident rates.

In the context of expansion using a DEA model, Shen et al. [37] presented a generalized multiple layer data envelopment analysis (MLDEA) model. The model was applied to a case study that evaluated the performance of safety measures in a set of 19 European countries and used 13 hierarchical performance indicators of safety.

Furthermore, there are several cases of road accidents that have been attributed to human factors due to crime-related activities. A high level of crime may affect road transport in many different ways. For instance, police pursuits of criminals; criminals holding up tourist buses and private cars; intercity or interstate buses and private cars: robbing drivers and passengers. [38]. The study by Barreto et al. [39] reported on criminality with regard to transporting goods in Brazil, where the use of force, violence, and threats to steal goods is most likely to occur on highways or when trucks are parked at key locations on the way to the distribution center.

In this relationship between crime and road safety, Brace et al. [40] suggested that there is a direct correlation between (i) the general negative behavior (e.g., involvement in antisocial behaviors) and risky driving behavior; (ii) criminal behavior and traffic offenses (specifically violence, theft and burglary and recidivist/drink driving, and driving whilst disqualified); (iii) risky traffic behavior contributing to a crash and the criminal record of the perpetrator (particularly for violent crime, vandalism, property crime, and involvement in drug or people trafficking; (iv) involvement in crashes, drink driving, and general criminal record of the perpetrator including theft, car theft, drug and alcohol-related crimes, violence and property damage.

2.1. The Framework Proposed and the Method FITradeoff. The purpose of this section is to address the framework applied to the case study and to discuss the method used. Before presenting these, some issues need to be considered.

First, in a multicriteria decision context, a particularly important issue to deal with is the DM's preferential information. In this paper, the DM was judged to have compensatory rationality and the model presented in the next section incorporates a method suitable for this. As to the DM's rationality, it is important to understand the meaning of the preferential information that the model requires. In connection with this, two terms are commonly used in the literature that refers to intercriteria parameters: weights and scales constant. The former represents the relative importance of criteria and is suitable for non-compensatory decisions. The latter is more suitable for compensatory decisions and represents not only the relative importance but incorporates the scale information required for the DM's tradeoffs over the criteria. Nevertheless, in the context of compensatory decisions, the term weights are commonly used for the sake of linguistic simplicity [21]. So, throughout this paper, when the term 'weights' is used, its meaning is that of scale constants.

The other issue, still in relation to the criteria weights, concerns the choice of the FITradeoff method. As discussed in the literature [26, 41–42], one of the difficulties in using multicriteria methods to support real-life decisions is the process of eliciting weights. Many methods require the DM to provide the values of the weights directly, which often results in imprecise information, which undermines decision support. Although the methods using the trade-off procedure have contributed to minimizing the difficulty of the process of eliciting weights, much information is still required from the DM. Unlike the traditional trade-off procedure that takes into account the indifference relations, the FITradeoff method admits preference relations, which reduces both the cognitive effort that the DM needs to make and the possibility of inconsistencies regarding the DM's judgment [24]. The procedure for eliciting criteria weights in the FITradeoff method is more flexible does not require direct information and requires less information from the DM. In fact, simulations performed by Mendes et al. [43] show that, in 5% of the cases, a unique solution can be found after the criteria weights have been ranked using FITradeoff. Also, in 98% of the cases simulated, the subset of potentially optimal alternatives is reduced after the criteria weights have been ranked.

2.1.1. The Framework. In order to structure the application of FITradeoff in the problem addressed, a framework was developed based on de Almeida et al. [21, 44], based on other real applications of the FITradeoff [25, 45]. The framework is presented in Figure 1.

The first part of this framework consists of structuring the problem, during which DMs and other actors in the process are identified and objectives, criteria, and decision alternatives are defined. In the problem addressed in this article, the DM represents the Federal Road Police (FRP), and the objective is to identify the most critical sections of a high-traffic road, which is supervised by an FRP unit. The criteria were defined based on the objectives and the sections to be evaluated by these criteria were identified from information obtained from the National Department of Transport Infrastructure (NDTI) and also from qualitative evaluations carried out by the DM.

The objective of the second part is to obtain a ranking of the sections in terms of their criticality and for that purpose the FITradeoff method for ranking was used. Although the method is detailed in the next section, it is emphasized here that, it is applied in two stages. In the first stage, based on the DM's preferences, a ranking of the criteria weights is obtained, and in the second stage, considering the DM's preferences with respect to the performance of some alternatives, a ranking of the road sections is obtained.

Then, the method presents the results and the possibility of carrying out some sensitivity analysis. Discussions were held regarding the results, which was of great value since this helped the DM to understand the tool in greater depth. Further details about this are given in the discussion section.

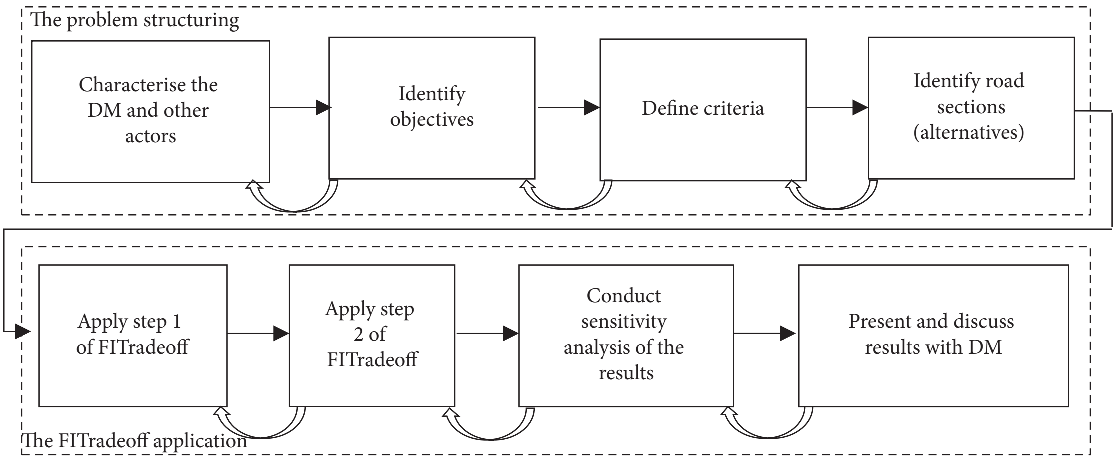

Figure 1: The framework developed for the road section prioritization.

It is important to note that the framework is not a waterfall model. Returning to previous steps is not allowed. In fact, the return to previous steps due to the knowledge, which was built up throughout the process, happened a few times in this application. Also, the framework developed can be replicated in supporting similar problems to the one covered in this paper. Therefore, the criteria and alternatives can be redefined, in addition to which the data used to carry out the assessment of the alternatives can be obtained from other sources.

2.1.2. FITradeoff for Ranking. The decision model that we present in this paper with a view to prioritizing sections of Brazilian roads, according to the criticality related to several aspects associated with the objectives of the institutions responsible for the roads, is based on the method presented by Frej et al. [23].

The decision model calculates the criticality level of each road section ($v(\mathrm{RS}_i)$) using the additive model, described in the following equation:

$$v\left(\mathrm{RS}_i\right) = \sum_{j=1}^{m} w_j v_j\left(\mathrm{RS}_{ij}\right), \quad (1)$$

where RS_i is a road section i, RS_{ij} is the finding about the criticality of a road section i in criterion j, and w_j is the scale constant of criterion j and v_j is the intracriterial function of j, described in the following equation:

$$v_j\left(\mathrm{RS}_{ij}\right) = \frac{\mathrm{RS}_{ij} - \mathrm{RS}_j^-}{\mathrm{RS}_j^+ - \mathrm{RS}_j^-}, \quad (2)$$

where RS_j^+ and RS_j^- are, respectively, the most critical and the least critical findings among all road sections for criterion j.

The FITradeoff method was proposed by de Almeida et al. [24], based on the tradeoff procedure proposed by [46], which has a strong axiomatic structure. The FITradeoff method, however, uses a flexible and interactive procedure that allows the decision model to recommend the best alternative to the DM without needing him/her to provide information at a high cognitive cost. This means that even though the model receives incomplete information, it can provide the DM with a soundly-based recommendation in the context of the objectives set for a choice problematic which does not require all the alternatives to be ranked.

For a ranking problem, however, the FITradeoff adapted by [23] is more appropriate, as it incorporates the concept of pairwise dominance. Thus, it provides information that can be used to obtain a partial or complete ranking, depending on the information provided by the DM in a certain cycle during the implementation of the model, so that he/she can be satisfied with the recommendation provided. In other words, even without a complete ranking, the DM can be satisfied with a partial one that informs him/her of the preferred relationships for the alternatives of his/her greatest interest. The FITradeoff method for the sorting problem can be easily implemented in a decision model using the FITradeoff software which is available at http://fitradeoff.org/.

Initially, the FITradeoff method consists of ranking the criteria according to the value of their scale constants. The criteria ranking step, in the FITradeoff software, can be conducted by two procedures, the holistic evaluation and the pairwise comparison procedure.

In the holistic assessment, the DM is faced with a fictitious consequence, the outcomes of which in each criterion are the least critical (RS_j^-). Then, the DM is asked which criterion, assuming the most critical outcome (RS_j^+), would increase the criticality of the consequence the most. In its first statement, the DM identifies the criterion with the highest scale constant. Then, by asking similar questions, the remaining criteria are ordered so that the order shows which one increases the criticality of the consequence the most after those with a higher impact have been identified, thereby completing the ranking of the criteria.

In the pairwise comparison procedure, two fictitious consequences are presented to the DM so that he/she can identify which is the most critical. Each consequence is represented by the most critical outcome (RS_j^+) in one criterion and the least critical (RS_j^-) in the other criteria, so that, in the two consequences, the criterion with the most critical outcome is different. According to equations (1) and (2), the value of each consequence is the scale constant value of the criterion with the most critical outcome. When the DM chooses the most critical consequence, this means that the value of the scale constant represented by the chosen consequence is greater than the value of the scale constant represented by the other consequence, even though the value of neither is yet known. The rest of the ranking is obtained from the DM's answers to questions about similar pairwise comparisons.

It is important to note that there is an important difference between the two procedures. The holistic evaluation is faster; as the DM only needs to provide $m-1$ statements, while in the ranking procedure by pairwise comparison, this number can be much higher. However, the ranking procedure by holistic evaluation can be cognitively more difficult for the DM, since he/she must consider all the criteria at the same time, especially when he/she defines which is the criterion with the highest scale constant, in the first statement. Thus, the DM can choose the ranking procedure by pairwise comparison if he/she does not feel comfortable about ranking or able to rank the criteria by holistic evaluation.

After ranking the criteria, the model seeks to find a rank among the road sections. The FITradeoff method, Frej et al. [23], solves a linear programming problem for each pair of alternatives (i, k), described by the following equations:

$$\operatorname{Max}(A_i, A_k) = \sum_{j=1}^{m} w_j v_j(A_i) - \sum_{j=1}^{m} w_j v_j(A_k), \quad i \neq k, \tag{3}$$

s.t.

$$w_1 > w_2 > \cdots > w_m, \tag{4}$$

$$\sum_{j=1}^{m} w_j = 1, \tag{5}$$

$$w_j v_j\left(x_j'\right) > w_{j+1}, \quad j = 1 \text{ to } m-1, \tag{6}$$

$$w_j v_j\left(x_j''\right) < w_{j+1}, \quad j = 1 \text{ to } m-1, \tag{7}$$

$$w_j \geq 0, \quad j = 1, 2, \ldots, m. \tag{8}$$

Equation (3) calculates the maximum difference in the value between two alternatives A_i and A_k. In our model, the alternatives A_i and A_k will be road sections. Equation (4) represents the constraints related to the ranking between the scale constants of the criteria. Equation (5) represents the constraint that obliges the scale constants to be normalized. Equations (6) and (7) define the maximum and minimum limits of a scale constant, when compared to a partial value of the scale constant immediately above in the ranking. In our model, initially x_j' and x_j'' assume the values of (RS_j^+) and (RS_j^-), respectively. Equation (8) represents the non-negativity constraint of the scale constants.

According to Frej et al. [23], three situations can occur when comparing two alternatives using the FITradeoff results for ranking. If $\operatorname{Max}(A_i, A_k) < 0$, then A_k dominates A_i. If $\operatorname{Max}(A_i, A_k) < \varepsilon$ and $\operatorname{Max}(A_k, A_i) < \varepsilon$, then A_i and A_k are indifferent, considering the current weight subspace of the problem. If $\operatorname{Max}(A_i, A_k) > 0$ and $\operatorname{Max}(A_k, A_i) > 0$, then A_i and A_k are incomparable considering the current weight subspace of the problem. By using the preference relations obtained from the pairwise comparisons, the model can obtain a partial or complete ranking of alternatives based on the weight subspace obtained with the partial information that the DM has provided.

If the DM is not satisfied with the information obtained with the ranking of the criteria, the FITradeoff method for ranking continues the flexible elicitation procedure. This presents the DM with two consequences in each cycle, similar to the procedure for ranking the criteria by pairwise comparison. However, in this step, in the consequence that represents the criterion with the highest scale constant, an intermediate outcome is presented, between the most and least critical. The DM can choose one of the consequences as being the most critical, or he/she can declare indifference or not answer the question if he/she does not feel able to or prefers not to answer, thus moving on to the next question, if it is still possible to explore the weight subspace.

If the DM chooses one of the consequences or declares indifference, the current subspace of weights is reduced by adjusting the vectors X_j' and X_j'', and solving again the linear programming problems described in equations (3)–(8). These cycles are repeated. The elicitation procedure ends if the model finds a complete order, if the DM is satisfied with the partial result or if it is not possible to obtain more information from the DM.

3. Case Study

The model presented was applied to the problem of prioritizing sections of roads according to the criticality for action planning by the Federal Road Police (FRP) in the state of Pernambuco, Brazil. To do so, several road sections under the jurisdiction of the FRP team were considered, so that by identifying the most critical sections, the planning of actions and investments could be better directed and consequently dealt with in greater detail. As previously noted, it is noteworthy that one of the FRP's functions, besides endeavoring to ensure the safety of road traffic, is to be the police force for the jurisdiction of federal highways, i.e., one duty of the FRP is to prevent and combat crimes and violence that occur on these highways.

For the DM, among the objectives considered in the problem that makes a road section critical are the damage (impacts) to human beings resulting from traffic accidents, issues related to violence/crime, and characteristics/conditions of the road and its traffic. Therefore, several criteria j were chosen, together with the DM, that assess the extent to which these objectives have been achieved: c_1: accident rate (I_n); c_2: index of accidents with fatal victims (I_F); c_3: index of people with serious injuries involved in traffic accidents (I_{SI}); c_4: index of people with minor injuries involved in traffic accidents (I_{MI}); c_5: index of traffic accidents with damage only to property (I_{DP}), c_6: percentage of heavy vehicles in road traffic (such as trucks, buses) $(\%N_{\mathrm{HV}})$; c_7: percentage of motorcycles in road traffic $(\%N_{\mathrm{MC}})$; c_8:

Pavement characteristics/conditions (PAV); c_9: Signaling characteristics/conditions (SIN); c_{10}: Characteristics of the track geometry (GEO); c_{11}: criminality (CRIM). The criterion C_1 is related to the frequency of traffic accidents, c_2 to c_4 are related to objective damage to human beings resulting from traffic accidents. Furthermore, criterion C_5 considers the material losses resulting from a traffic accident. The criteria c_6 to c_{10} are related to objective characteristics/conditions of the road and its traffic, and finally, criterion c_{11} is related to the objective issues related to violence (crime).

It is important to highlight here the importance of the decision-maker in a multicriteria decision problem since issues that range from defining the decision criteria to eliciting the parameters of the model are relevant to the DM. In other words, the way that criteria are presented here are appropriate for this case study, but that does not mean that they will be relevant in any other problem. Each multicriteria decision problem needs a structuring stage, represented by the first stage of Figure 1. The model presented has the flexibility to adapt to other decision problems, and this includes being used to elicit parameters for which the FITradeoff method is used. It takes into account, partially or totally, the criteria of this case study, as well as other criteria.

The first seven criteria are continuous and quantitative criteria, in order to measure the criticality of the roads. The DM wishes to maximize them because the DM wants to identify which road section has the worst indices (or highest values) that make them critical. The indices described above are calculated from equation (9) to equation (13) associated with road section i.

$$I_n = \frac{10^6 n_i}{365\,(\mathrm{AADT})_i L_i}, \tag{9}$$

$$I_F = \frac{10^6 n_{F_i}}{365\,(\mathrm{AADT})_i L_i}, \tag{10}$$

$$I_{\mathrm{SI}} = \frac{10^6 n_{\mathrm{SI}_i}}{365\,(\mathrm{AADT})_i L_i}, \tag{11}$$

$$I_{\mathrm{MI}} = \frac{10^6 n_{\mathrm{MI}_i}}{365\,(\mathrm{AADT})_i L_i}, \tag{12}$$

$$I_{\mathrm{DP}} = \frac{10^6 n_{\mathrm{MD}_i}}{365\,(\mathrm{AADT})_i L_i}, \tag{13}$$

where AADT_i is the average annual daily traffic; L_i is the length of the road section i in kilometers; n_i is the annual number of accidents in section i; n_{F_i} is the number of traffic accidents with fatality victims; n_{SI_i} is the number of people with serious injuries involved in traffic accidents related to section i; n_{MI_i} is the number of people with minor injuries involved in traffic accidents related to section i; n_{MD_i} is the number of traffic accidents with damage only to property.

Criterion c_6 reflects issues associated with the risk of heavy vehicles being involved in serious accidents and high secondary accidents. As the demand for freight transportation has increased in recent years, traffic agencies have become more interested in monitoring the risk of heavy vehicles being involved in accidents [47].

Criterion c_7 reflects exposure to motorcycle accidents since victims of accidents caused by motorcycles and nonmotorized means of transport (bicycles and carts) have higher accident rates with serious injuries on roads when compared to other types of vehicles [48].

Criterion c_8 (pavement) identifies the characteristics of the road pavement, in which information about the surface condition, speed due to the condition of the pavement, and the presence of critical points are considered and are described on a nominal scale; criterion c_9 (signage) identifies the presence and conditions of horizontal signage (central and lateral lanes), vertical signage (presence of speed signs, indication signs and intersection, and visibility signs and legibility of all signs, which are described on a nominal scale. In criterion c_{10} (road geometry), the conditions of the geometric characteristics of the road are identified, subdivided into the type of road, road profile, presence of additional lane, presence of bridges and viaducts, presence of dangerous curves, and condition the dangerous curve, which are described on a nominal scale. For further details about the qualitative scales of these criteria see CNT [49].

Finally, criterion c_{11} (criminality) considers the number and the frequency and types of road crimes that fall under the jurisdiction of the FRP. Table 1 presents the qualitative scales of criteria from c_8 to c_{11}.

For the case study, 22 road sections are considered. Table 2 shows these road sections, the average annual daily traffic (AADT) of these sections, the length (L) of the road sections in kilometers, and the performance of these sections for the criteria considered.

On being given this information, initially, the DM was informed about the questions that he would be asked and what options of the answers he could provide. He was also advised that he could return to previous phases if he so desired. The FITradeoff method was applied using software downloaded from http://fitradeoff.org/.

The DM was asked to rank the criteria using the holistic evaluation, which is faster. However, he did not feel able to do so due to the high degree of complexity of the problem, the high number of criteria, and his inexperience of working with multicriteria decision methods.

It is important to note that the FITradeoff software is flexible. Therefore the procedure for ranking the criteria can be changed. For this, it was decided to switch to the pairwise comparison procedure for ranking the criteria, with which the DM felt more comfortable, even though he would have to answer more questions to reach the complete ranking of the criteria when compared to the holistic evaluation. Figure 2 shows one of the steps of ranking the criteria by pairwise comparison.

The FITradeoff method asked the DM thirty questions to rank the criteria. However, the DM had no difficulty in providing the answers, so that despite there being a higher number of questions, the procedure was considered quick and clear. The criteria were ranked as follows: CRIM > $I_F > I_{\mathrm{SI}} > I_{\mathrm{MI}} > I_n > I_{\mathrm{DP}} > \mathrm{SIN} > \%N_{\mathrm{MC}} > \mathrm{PAV} > \%N_{\mathrm{HV}} > \mathrm{GEO}$.

Table 1: Qualitative scales of the criteria of pavement, signage, road geometry, and criminality.

Level	Pavement (c_8)	Signage (c_9)	Road geometry (c_{10})	Criminality (c_{11})
1	Very bad conditions	Missing signage in all (or almost all) the section	Very bad geometric characteristics of the road section	Very high level of criminality
2	Bad conditions	Missing signage in parts of the section. Damaged or unclear signage in many parts of the section	Bad geometric characteristics of the road section	High level of criminality
3	Regular conditions	Damaged or unclear signage in many parts of the section	Regular geometric characteristics of the road section	Moderate level of criminality
4	Good conditions	Damaged or unclear signage in some few parts of the section	Good geometric characteristics of the road section	Low level of criminality
5	Very good conditions	Signage in all sections and is very clear	Very good geometric characteristics of the road section	Very low level of criminality

Table 2: Road sections data.

	AADT	L (km)	I_N	I_F	I_{SI}	I_{MI}	I_{DP}	$\%N_{HV}$	$\%N_{MC}$	PAV	SIN	GEO	CRIM
A1	4808	3	3.039	0.000	0.570	2.849	0.570	0.148	0.143	5	3	4	4
A2	5284	3	1.383	0.000	0.346	0.519	0.519	0.128	0.146	5	3	4	4
A3	4808	11	1.295	0.155	0.207	0.829	0.415	0.148	0.143	2	1	4	4
A4	5284	11	0.424	0.047	0.047	0.330	0.094	0.128	0.146	2	1	4	4
A5	4471	9	0.817	0.068	0.272	0.477	0.136	0.178	0.138	4	2	4	4
A6	4693	9	0.843	0.065	0.130	0.714	0.195	0.162	0.140	4	2	4	4
A7	4134	1	0.000	0.000	0.000	0.000	0.000	0.213	0.132	3	4	4	2
A8	4102	1	1.336	0.000	0.000	0.000	1.336	0.206	0.133	3	2	4	2
A9	4134	4	0.166	0.000	0.166	0.166	0.000	0.213	0.070	3	3	3	4
A10	4102	4	0.835	0.000	0.501	0.835	0.000	0.206	0.071	3	3	3	4
A11	4134	7	0.663	0.000	0.000	0.379	0.379	0.213	0.070	3	3	2	3
A12	4102	7	0.668	0.000	0.095	0.859	0.000	0.206	0.071	3	3	2	3
A13	4744	8	0.361	0.000	0.072	0.217	0.144	0.276	0.065	2	3	4	3
A14	4770	8	0.359	0.000	0.000	0.431	0.000	0.281	0.064	2	3	4	3
A15	4723	7	0.580	0.083	0.083	0.663	0.083	0.283	0.064	3	1	2	2
A16	4785	7	0.900	0.164	0.327	0.736	0.164	0.291	0.063	3	1	2	2
A17	2738	53	0.529	0.057	0.227	0.397	0.094	0.354	0.064	4	2	4	2
A18	2743	53	0.603	0.057	0.057	0.452	0.132	0.360	0.063	4	2	4	2
A19	5211	6	0.263	0.000	0.000	0.263	0.088	0.387	0.067	4	2	4	4
A20	5177	6	0.265	0.000	0.265	0.176	0.000	0.391	0.066	4	2	4	4
A21	4147	30	0.198	0.044	0.110	0.110	0.022	0.353	0.071	4	2	3	3
A22	3908	30	0.234	0.023	0.094	0.210	0.023	0.317	0.074	4	2	3	3

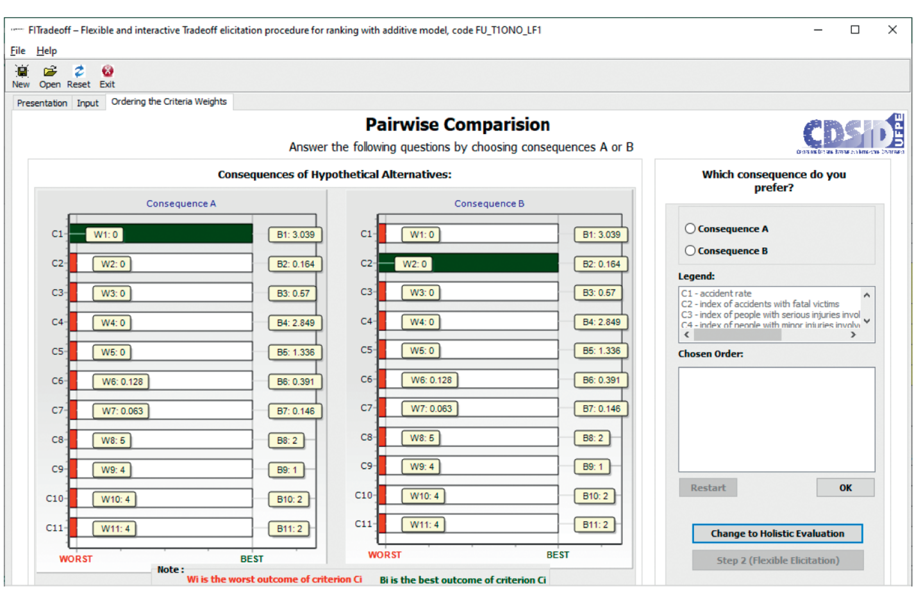

Figure 2: Pairwise comparison procedure for ranking the criteria using FITradeoff software.

After ranking the criteria, the FITradeoff has already provided partial information with only two ranked positions for the alternatives (road sections); it has identified that A9 and A19 were the least critical, occupying the second position, while the other 20 alternatives were in the first position.

The DM considered that this information was insufficient. Thus, the second step of FITradeoff was carried out for which the flexible elicitation procedure was used. In this step, the DM was asked to choose the most critical of two consequences, similar to the first step. In this step, however, the consequence that represented the criterion in a higher position in the criteria ranking presented an intermediate outcome between the least and the most critical. Figure 3 presents FITradeoff's first question for the flexible elicitation procedure, which, by comparing the first ranked criterion with the last, seeks to identify the pattern of the distribution of weights. The subsequent questions compare adjacent criteria in the ranking.

After the DM has provided each answer, the model reduces the weight space according to the information obtained and recalculates the ranking of the road sections according to equations (3)–(8). Table 3 describes the first 13 questions of the flexible elicitation step. The first, second, and third columns show the outcome of the criteria represented by the two consequences presented to DM. The criteria are numbered by their position in the ranking. The fourth column shows the DM's answer, and the last column shows how many levels in the ranking of the road sections have been defined by FITradeoff with the information obtained.

After thirteen questions, the FITradedoff method defined fourteen ranking levels with the information obtained. Although the model did not provide a complete ranking, the DM was satisfied with the result provided after the thirteenth question. Figure 4 shows the last question answered by the DM.

The results identified the ordered ranking of the six most critical road sections, as well as the three least critical. Figure 5 shows the Hasse diagram provided by the FITradeoff software, which presents the preference relations between the alternatives, as well as the ordering levels.

4. Results and Discussion

The results presented by the model identified fourteen ranking levels. The first six positions in the ranking were well defined. The DM found the Hasse diagram quite enlightening. According to Figure 5, the road section 16 (A16) is considered the most critical. It can be seen from Table 2 that this road section has the most critical outcomes for the criteria of criminality and the index of accidents with fatalities. These are the criteria with the highest weights according to the information obtained from the DM.

Despite the incomplete ranking, the DM found the clear definition of the first six levels in the ranking very useful, since these show the roads sections that should be prioritized in future FRP actions.

Road sections 03, 21, and 22 occupy level seven in the ranking. Although there is no clear definition of which road section occupies the seventh level, the diagram shows the preference relations that can be identified between the three alternatives. The DM considered that this was very useful. This was because, despite there being preference relations defined for sections 21 and 22, the position itself had not yet been determined because there was not enough information to define the criticality of these road sections, when compared to road section 03, with an incomparable relation, within the information obtained until the thirteenth question of flexible elicitation. This also occurs at level 11, where it is not possible to establish a preference relationship between road sections 02 and 04.

The greatest lack of information, however, can be identified in position 08, which are occupied by six road sections (01, 05, 11, 12, 13, and 14). However, some relations are defined. Many road sections occupy position 08 due to the difficulty of comparing road section 01 with the other ones which is considered incomparable with all other road sections within this level in the ranking. It can also be seen that with the level of information obtained, it was not possible to establish a preferential relation among road sections 05, 11, and 14.

The DM, however, found the information provided by FITradeoff quite useful, as in a practical decision, the identification of the most critical road sections would be sufficient to prioritize the most important actions and even for lower positions in the ranking, although the information is incomplete, nevertheless, it could guide decisions that are considered less important.

The DM considered it was very useful to have the support of the analyst when using the software which he found easy to understand. He also appreciated the software being flexible. For example, the fact that it is possible to change the procedure for ranking the criteria and to return to previous steps. The use of graphic visualization to support the elicitation process was also considered one of the advantages in the process.

The results obtained from FITradeoff software also provided the possible weight values for each criterion considered in the decision. The limits that define the subspace of weights delimited by the information obtained up to the thirteenth question are shown in Figure 6 and Table 4. These are partial results from the FITradeoff software.

Note that there is a greater concentration of weights in the criteria related to the objectives of combatting crime and the index of accidents with fatal victims. It is important to note that the scale constant does not only consider the relative importance of the decision criterion but also information on the scale of the outcomes of the alternatives in the problem. This means that not only does the relative importance of the criterion affect the value of its scale constant but also the variation in performance between the

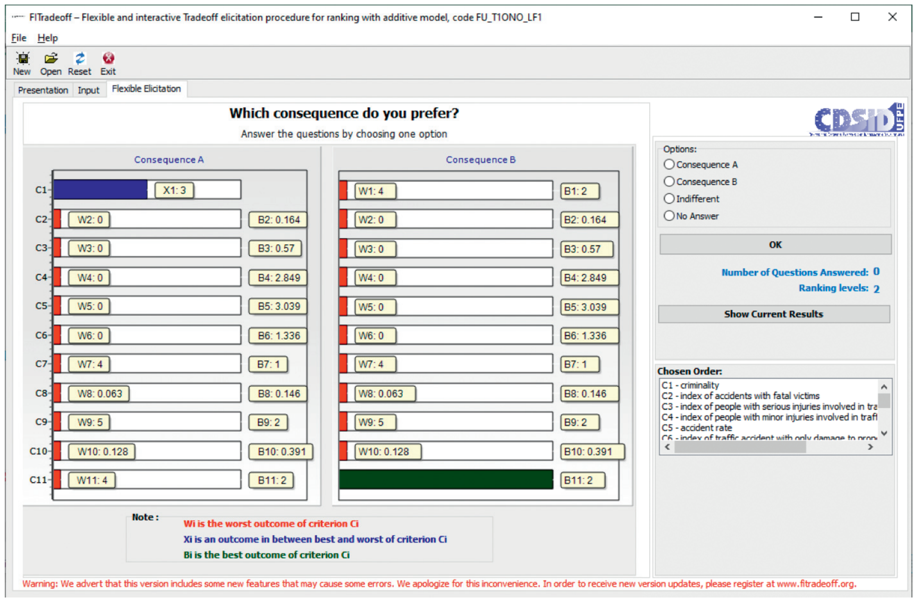

Figure 3: The first question for the flexible elicitation procedure using FITradeoff software.

Table 3: Data on road sections.

Pairwise assessment	Consequence A	Consequence B	DM's answer	Ranking levels
1	3 of C1	2 of C11	A	2
2	3 of C1	0.164 of C2	B	2
3	0.082 of C2	0.57 of C3	Indifferent	4
4	0.0285 of C3	2.849 of C4	A	4
5	1.425 of C4	3.039 of C5	A	7
6	1.52 of C5	1.336 of C6	A	10
7	0.668 of C6	1 of C7	A	10
8	3 of C7	0.146 of C8	B	10
9	0.105 of C8	2 of C9	B	10
10	4 of C9	0.391 of C10	B	10
11	0.26 of C10	2 of C11	B	10
12	2 of C1	0.164 of C2	A	10
13	0.1425 of C3	2.849 of C4	B	14
14	0.712 of C4	3.039 of C5	B	14

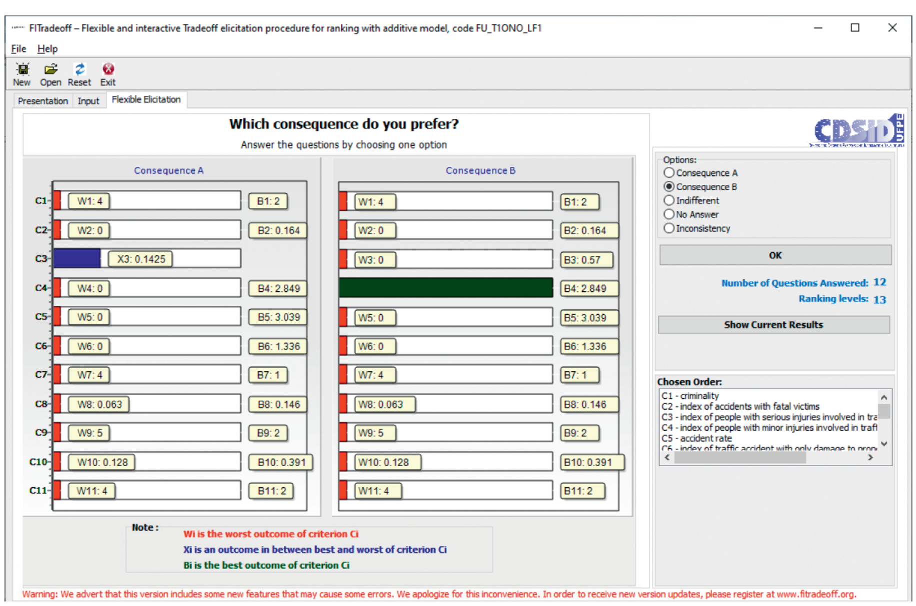

Figure 4: The last question for the flexible elicitation procedure using FITradeoff software.

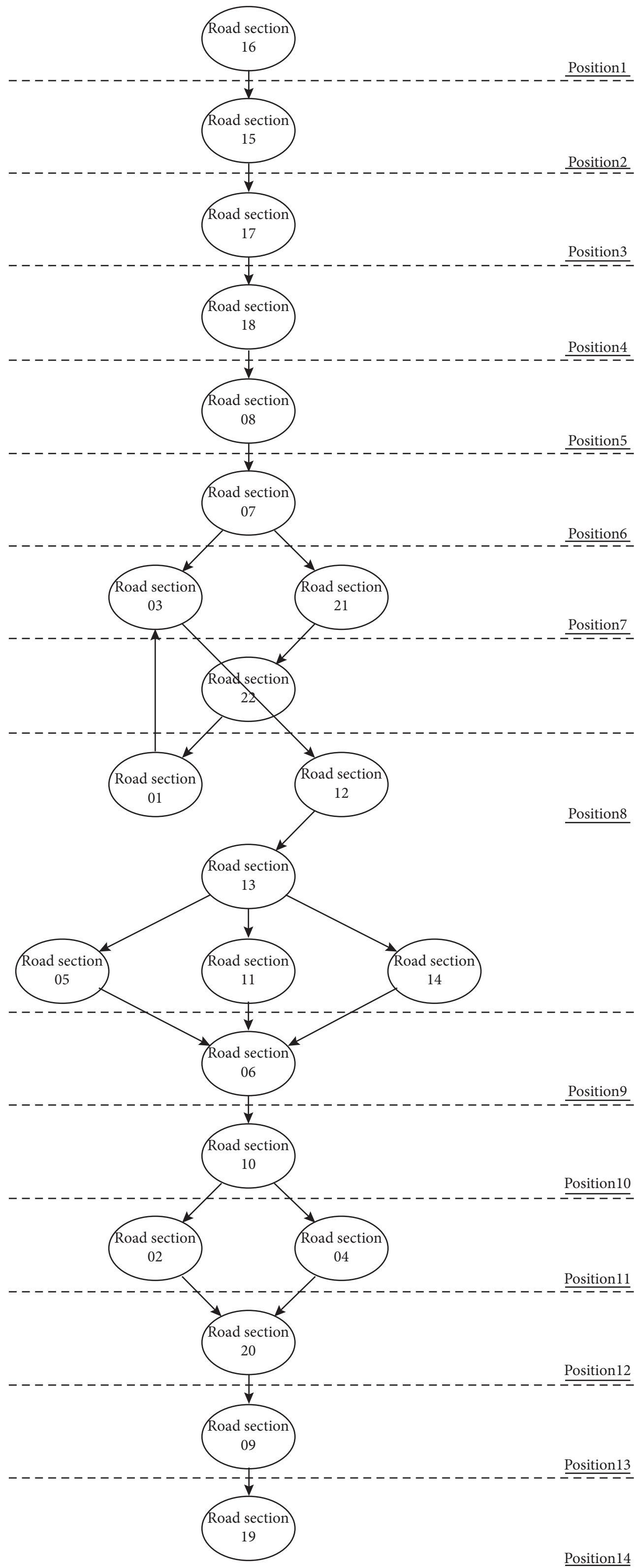

Figure 5: Hasse diagram of the preference relations between the alternatives, as well as the ordering levels provided by the FITradeoff software.

alternative with the lowest and the highest outcome. Scale constants of criteria with a greater range can assume a higher value.

The information obtained on the criminality criterion is related to the extent to which the FRP is commonly called for due to the crimes in each section considered in the problem

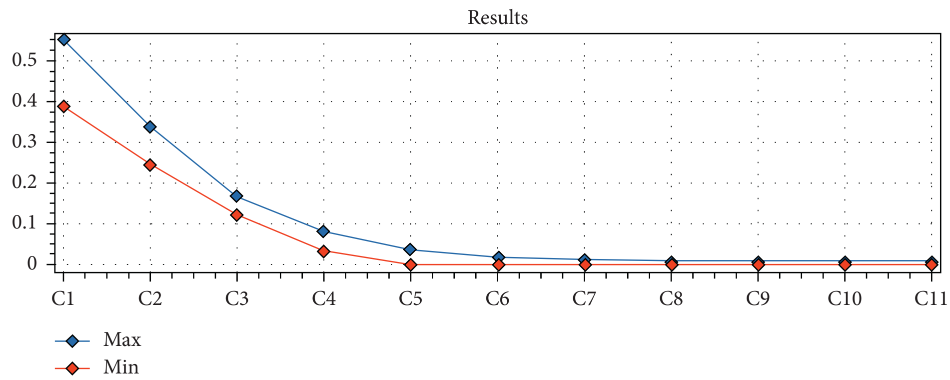

FIGURE 6: The limits that define the subspace of weights delimited by the information obtained up to the thirteenth question.

TABLE 4: The limits that define the subspace of weights delimited by the information obtained up to the thirteenth question.

	C1 (CRIM)	C2 (I_F)	C3 (I_{SI})	C4 (I_{MI})	C5 (I_N)	C6 (I_{DP})	C7 (SIN)	C8 (%N_{MC})	C9 (PAV)	C10 (%N_{HV})	C11 (GEO)
Min k	0.389	0.244	0.122	0.033	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Max k	0.552	0.338	0.169	0.081	0.039	0.019	0.009	0.009	0.009	0.009	0.009

and the extent to which this varies between sections. If all sections have an equally high or equally low level of crime, the variation in crime between sections would not be large and that would make the scale constant have a lower value. The criteria of pavements, signs, geometry, and crime are constructed criteria, which means that it does not make sense to analyze only the difference between classes for each criterion, but what they mean for the DM, which has an impact on the definition of the constant of scale. The meaning of the classes in these criteria for the DM is considered in each step of the FITradeoff method.

Although the criteria related to the characteristics and conditions of the road sections, C7 to C11 in the criteria ranking have very low weights; it can nevertheless be seen that together these criteria can significantly affect the criticality of the decision, considering the information obtained up to the end of the elicitation process. This demonstrates that the objectives represented by these criteria should not be disregarded in the decision.

The possibility of dealing with partial information is another advantage of the model. Despite the problem having eleven criteria, only thirteen questions were necessary before obtaining a satisfactory result for the DM. Compared with models that require complete information about weights and indifference ratios, the DM would have to make at least ten statements of indifference about the adjacent criteria in the ranking. However, it is very difficult for the DM to provide statements of indifference directly.

Considering another scenario that softens the process of obtaining indifference, two approach questions, and a third determination question for each indifference would be asked, totaling $3(n-1)$ statements from DM. In this case, the DM would have to answer thirty questions to define all weights.

When compared to these complete information approaches for obtaining weights, note that, in addition to the model requiring the DM to provide fewer answers, the questions are based mainly on strict preference relations, which are easier to provide, resulting in a faster procedure with fewer inconsistencies.

5. Conclusions

Traffic accidents, in addition to being responsible for one of the biggest causes of death worldwide, represent a considerable economic and social cost for countries, especially middle-income countries. In Brazil, this situation is not different. In 2015, it occupied the fourteenth position in the world ranking that considers the ratio between the number of deaths and the total number of inhabitants.

The risk of accidents can be reduced by first identifying the most critical locations and then establishing a policy to improve the level of safety and security on the roads by prioritizing the most critical road sections. In order to define the most critical locations, it was noticed that several factors influence traffic safety and road security, which causes the problem to be characterized as a multicriteria problem.

After an analysis of the characteristics of the problem, in terms of the decision actors, the data available, and the objectives, the applicability of the FITradeoff method for ranking the most critical road sections was presented as a tool to support the decision.

Hence, a case study was carried out on 22 road sections and 11 evaluation criteria. The variety of criteria considered by the DM well represents the diversity and multiplicity of factors that affect the issue of road safety. The criteria considered included aspects associated with damage (impacts) to humans, issues related to violence, and characteristics/conditions of the road and its traffic.

FITradeoff was applied considering the DM's preferences which were obtained by analysing his answers to the system's questions. The entire process was supported by the software. As presented, after ordering the criteria weights, 2 levels of the ranking were defined and after 13 interactions with the

DM, 14 levels of ranking, and the 6 most critical sections were identified.

The criticality of road sections was represented through a diagram that presents the rank according to the DM objectives. The definition of the six most critical road sections in a precise rank position is provided.

The criticality of the road sections was represented by means of a diagram that presents the ranking according to the DM's objectives. The definition of the six most critical road sections in an accurate ranking position provided the DM with information that allows for proper planning of future actions. Other road sections have an intermediate position in the less defined rank, but still, provide the DM with partial information that can contribute to the planning of actions in sections considered less critical.

Given this result, the DM felt more confident in directing the available resources (such as financial resources, work team, vehicles, radar, and extrasignage) to prevent and mitigate traffic accidents for the prioritized sections. This reflects the confidence that the DM has in the statements he provided since the model requires less information, and such information is obtained by asking the DM questions that he found easier to answer.

Future studies could usefully investigate improving the decision model by examining the DM's evaluation as to the effectiveness and efficiency of using the resources available on the different road sections prioritized. Normally, each preventive/mitigating action directs "energy" towards diminishing or eliminating some dimensions of consequences, e.g., an action of the patrol system regarding the use of police vehicles can reduce the level of violence (criminality). However, this may have little impact on the issues associated with the road pavement, with a view to improving road safety.

Data Availability

All data are available within the paper.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Research Article

A Goal Programming Approach to Nurse Scheduling with Individual Preference Satisfaction

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The use of scheduling optimization tools is essential in creating an efficient nurse shift-rotation schedule. A well-designed nurse scheduling technique can improve nurses' job satisfaction and their intention to stay. This study develops a goal programming approach to nurse scheduling that simultaneously considers workload fairness and individual preferences on working shift and day off assignments. A case study of an operating room at a hospital in Thailand is used to illustrate the model capabilities for solving an actual nurse scheduling problem. The job satisfaction factors defined based on an interview and questionnaire survey are integrated into the model. When compared against the manual scheduling result, the optimal schedules can be implemented to improve the nurse's perception of fairness and preference satisfaction. The analysis of fairness and multiple individual preferences based on a case study investigation is the main contribution of this study.

1. Introduction

The problem of intensified workload and poor working conditions has been identified as one of the major causes of the global nurse shortage predicament [1]. Hospital nurses who do shift work are exposed to understaffing, heavy-workload, and irregular work-scheduling conditions. The overworking and understaffing of nursing staff not only adversely affect the service quality of healthcare operations, but also lead to less patient-nurse interaction [2] and patient safety issues (Liu et al. [3] and Baker et al. [4]). At the same time, the demanding working conditions also impose various negative health consequences on nurses including fatigue, obesity, sleep disorder, and a wide range of chronic diseases [5]. The impact of shift work also includes decreased work-life balance, which can significantly affect nurse's job satisfaction and retention intention [6].

The positive effects of job satisfaction on nurse retention have been addressed in many countries [7–10]. For any profession, the control of job satisfaction in the workplace is an indefinite task due to the variations in job characteristics and individual characteristics. The understanding of the impacts of both types of characteristics is crucial in improving job satisfaction [11]. According to the nurse job satisfaction literature, the work environment is an important job characteristic affecting nurse job satisfaction and retention intention. DeKeyser Ganz and Toren [12] reviews that a well-designed nurse practice environment that promotes nurse engagement in management, staffing adequacy, and positive relations among colleagues is essential for ensuring job satisfaction and preventing the intention to leave. A survey in the underserved areas of Jordan by AbuAlRub et al. [13] suggests that the design of a work environment that promotes job satisfaction and nurse intention to stay should consider the shortcomings of nurses' living conditions. Recent job satisfaction studies tend to include both job and individual characteristics in their investigation. The perceptions of job autonomy and organizational justice have received more research attention than other individual characteristics in recent literature. As shown in Li et al. [14], Giles et al. [15], Mahoney et al. [16], and Koning [17], the perception of job control or autonomy is an important job satisfaction factor that can improve job satisfaction and nurse retention. With autonomy, nurses take part in their shift scheduling process, enabling the direct consideration of their preferences. Organizational justice is another key factor affecting nurse job satisfaction [18]. The recent survey study by Rizany et al. [19] indicates that the implementation of nurse schedules developed by a systematic scheduling method with organizational justice consideration has a positive effect on nurse job satisfaction. The studying results suggest that organizational justice can be achieved by providing an equitable allocation of workload and favorable scheduling outcomes. In a shift-rotation system, many personal preferences can be considered including the preferred shift time, day off, coworkers, etc.

The development of nurse scheduling algorithms has received significant research attention to resolve the job satisfaction and retention issues that are becoming more problematic in different countries. The use of mathematical programming in nurse scheduling tasks helps not only reduce the burdensome and time-consuming manual scheduling efforts, but also allow any change to be made to the initial scheduling easily. Nurse scheduling problems (NSP) have been widely studied in the literature to date. The primary objective of nurse scheduling is to satisfy the hospital's patient demand and staffing policies. For job-satisfaction-enhanced NSP, the fairness and nurse preferences are incorporated into the scheduling models in the form of constraints and objectives. When compared to manual scheduling, which is still widely used by hospitals, the job-satisfaction-enhanced models are capable of producing more fair schedules, as shown by the previous case studies [20, 21]. The manual scheduling task allows schedulers to have the control to assign schedules based on their judgment and may result in an unfair schedule. Manual scheduling is also time-consuming, preventing the rescheduling of nurses shifts should any disruption occurs during the implementation of the proposed schedule. As a result, the original attempt to create a fair and job-satisfaction-enhanced schedule can be compromised. Both single- and multiple-objective techniques have been used in formulating job-satisfaction-enhanced NSP models. Schedulers can maximize or control the level of job satisfaction while being able to pursue other operating objectives. Thus far, there exists a research gap concerning the integration of workforce and individual preferences into nurse scheduling practices. There is also very limited research that combines fairness and preference goals in NSP and use an actual case study to illustrate the practicality of the proposed approach. The details of the literature review are given in the next section. After that, materials and methods are described to gain insights into how the proposed NSP model is developed based on the operating-room-nurse case study. Results and discussion are given to see how the scheduling solutions allocate nurses in response to the simultaneous consideration of multiple scheduling goals. In the end, the conclusion of findings and the main research contributions are provided.

2. Literature Review

The research of NSP has been well-documented in the previous studies for different designing and solving approaches. Especially during the past decade, there has been a growing NSP literature focusing on the job satisfaction of nurses. Many studies in the literature develop nurse scheduling approaches that create positive effects on job satisfaction based on nurse preference. El Adoly et al. [20] proposed a scheduling algorithm that considers shift and day off preferences and uses an actual hospital case study in Egypt for model validation. Thongsanit et al. [22] applied the nurse scheduling technique to balance shift assignments among the nurses. The aim is to improve overall nurse preferences in a hospital in Thailand. Similarly, Cetin and Sarucan [23] aimed for balancing the total work hours among the nurses. They consider factors influencing nurses' preferences such as the desirable shift patterns, weekend day off allocation, and balance between day and night shifts. However, these preferences are based on the group, not individual preferences. Another group of NSP studies makes it possible to specify individual nurse scheduling preferences on shift and day off. Deterministic (Lin et al. [24], Huang et al. [25], and Widyastiti et al. [26]) and stochastic (Bagheri et al. [27]) single-objective linear programming approaches are proposed. The exact or optimization techniques such as Linear programming (LP) or Mixed-integer programming (MIP) have shown to be effective in solving NSP. In more recent NSP studies, the development of fair and preference scheduling using multiobjective optimization techniques such as Goal programming (GP) is observed. The use of GP allows the planners to regard job satisfaction factors as optimization goals. Then, the relative importance of each goal can be assigned to reach the most suitable scheduling solutions. Lim et al. [28] proposed a goal programming-based NSP considering assignment cost, idle time, and nurse preferences simultaneously. In Agyei et al. [29] and Al-Hinai et al. [30] studies, goal programming is used to generate balanced shift assignments for nurses. The undesirable shift patterns or shift precedence is regarded as the main job satisfaction factors. The use of goal programming to reach multi-criteria scheduling decisions is also found in Dumrongsiri and Chongphaisal [31]. The desirable shift work characteristics for an emergency room with nurses' skill heterogeneity in a Thai university hospital are formulated as the GP objectives. Mohammadian et al. [32]'s proposed a GP model to solve an NSP using an emergency room in a hospital in Tehran as a case study. In their case, nurses are entirely homogeneous in skill and experience levels. Their highlight is the formation of seven goals related to the desirable shift work characteristics and nurses' interests. Aside from the skill consideration, these two multi-criteria scheduling approaches aim to maximize job satisfaction based on a different set of hard and soft constraints.

Based on this part of the literature, GP allows decision-makers to find practical solutions based on the desirable scheduling characteristics with target values. The problem-solving is executed by minimizing the unwanted deviation from the target values. At this point, the goals' priority level can be defined based on management interests. Wang et al. [21] developed a weighted goal programming NSP by assigning the overtime restriction as the most important goal. Sundari and Mardiyati [33] formulated a preemptive goal programming NSP, where the goals related to hospital regulations are to be satisfied before nurses' preferences. Another interesting area of NSP literature is the formulating and solving of highly constraint NSP. In this case, previous studies such as Santos et al. [34], Mischek and Musliu [35], and Rahimian et al. [36] use constraint programming (CP) to find feasible solutions rather than optimal solutions based on hard and soft constraints. Hard constraints include hospital regulations and restrictions that need to be satisfied. Soft constraints are those that are formulated based on personal preferences, which can be violated at a penalty.

In the NSP literature, heuristic algorithms are important tools researchers use to solve large-scale NSP. Chiaramonte and Caswell [37] proposed an advanced nurse scheduling algorithm named as competitive nurse rostering and rerostering. The algorithm offers a rerostering ability to generate alternate cost-optimal scheduling solutions that have a minimum negative impact on nurse preferences. An iterated local search is proposed as the solving tool. Zhong et al. [38] develop a two-stage heuristic algorithm to create a nurse schedule with a balanced weekend workload. The algorithm accounts for the individual nurse preferences, patient volume fluctuation, required patient-to-nurse ratio, and other work-rest rules. The solution impact of uncertainty arising from the patient volume fluctuation is shown to be significant for the US hospital case study. The investigation of uncertain impacts is important and has been the subject of an investigation by several other NSP studies. The study of Maass et al. [39] accounts for not only uncertainty arising from patient demand, but also nurse absenteeism. Their model is solved by using the genetic algorithm (GA) to create long-term staffing decisions for different tiers of nursing staff. Leksakul and Phetsawat [40] also use GA to solve NSP that considers the demand for nurse and fair overtime pay for nurses. In Sajadi et al. [41], simulated annealing (SA) and simulation are used in tandem in searching for solutions with less patient wait time. In their NSP, the simulation of emergency room activities in a hospital case study is performed, considering important input data such as patient arrival rate and nurse service time. In the model of Liu et al. [42], the wage of nurses and preference penalty costs are combined into a single cost minimization objective. The computational performances of their proposed heuristic approach are benchmarked against those of several meta-heuristics approaches. Guessoum et al. [43] proposed a two-stage method where the original NSP is reduced in size by a variable fixing heuristic and solvable to optimal or near-optimal by an exact method.

In the current satisfaction-enhanced NSP research pool, the existing models are dedicated to maximizing nurse preferences, desirable scheduling characteristics, and satisfaction. However, there is still a need for more case study research to accommodate the constant change of the NSP context. For nurse scheduling, the exploration of the practicality of the proposed approach based on an actual case study is also important [44]. Regarding job satisfaction factors, there are still limited studies that consider individual preferences together with fairness. This research aims to strengthen the fundamental understanding of the actual nurse scheduling case and to develop a fair and satisfaction-enhanced scheduling tool that can be implemented easily on-site by the existing personnel. Our first step is to conduct field surveys at an operating room department in a private hospital located in Pathum Thani, Thailand. The survey objective is to define the system parameters based on the actual job satisfaction factors, scheduling regulations, and problem environment. The second step is to formulate a mathematical scheduling model, using GP, to achieve all the scheduling goals concerning fairness and job satisfaction. A free add-in optimization software in Microsoft Excel called Opensolver is used, so that it can be implemented by the head nurse without any additional cost or complication.

3. Materials and Methods

3.1. Case Study and Data Collection. This research focuses on the scheduling of operating-room nurses in a private hospital with 200 beds in Pathum Thani, Thailand. A field survey, an interview with the head nurse, and a questionnaire survey with operating room nurses were conducted during December-January 2019/20. The operating room has 17 nurses (1 head nurse and 16 full-time nurses), working in a 3-shift rotation system: morning shift (8:00–16:00), afternoon shift (16:00–24:00), and night shift (0:00–8:00). Currently, the shift schedule is manually generated by the head nurse at the beginning of each month. In the case presented, the scheduling period of 28 days is assumed. The main scheduling task is to assign an adequate number of nurses for each shift across the scheduling period. The scheduling criteria and the relevant hospital's regulations are given in the next section. Aside from these scheduling requirements, based on the survey and interview results, the fairness of workload and job satisfaction of nurses are the areas of interest for management and of this research. The questionnaire survey results indicate that shift and day off preferences are the important parameters contributing to nurses' job satisfaction, mainly because operation room tasks are nonrepetitive and require a variety of skills. However, for the case where nurses are required to perform repetitive tasks over an extended period, job satisfaction factors related to fatigue and job boredom may need to be considered. The job satisfaction effects of job boredom in workforce scheduling research have been investigated in our recent study [45].

To improve job satisfaction, this study develops a nurse scheduling approach that improves nurse's perception of workload fairness and preference fulfillment. A questionnaire is administered to nurses asking them to indicate their preferred shifts and days off across the 28-day scheduling period. Each nurse specifies 8 preferred days off: 4 most preferred days off and 4 second-most preferred days off. A goal programming model for nurse scheduling is formulated to simultaneously address a set of multiple job-satisfaction enhancement objectives: (1) minimizing the unbalanced workload, (2) minimizing the unbalanced preferred shift, and (3) minimizing the unbalanced preferred day off among the nurses. The details of model formulation are as follows.

3.2. Mathematical Model Formulation. The proposed nurse scheduling model is formulated for multishift scheduling over a series of consecutive sequences of days. The heterogeneous characteristics of nurses considered in the model include the experience levels and their preferences on shift and day off. The scheduling model is formulated to account for the shift time and the following case study's conditions and assumptions, which are similar in principle to the previous case study oriented NSP studies (Dumrongsiri and Chongphaisal [31], and Mohammadian et al. [32]).

3.3. Modeling Conditions and Assumptions

(i) The scheduling period is 28 days. In a workday, there are three 8-hour shifts: morning, afternoon, and night shifts.

(ii) The number of nurses in each shift must meet the requirements: 6 for the morning shift, 6 for the afternoon shift, and 2 for the night shift.

(iii) Based on the working experiences, nurses with working experience more and less than 5 years of experience are classified as level-1 and level-2 nurses, respectively.

(iv) In each shift, the number of level-1 (experienced) nurses must be at least half of the total number of nurses.

(v) Nurses must work no less than 22 shifts and no more than 24 shifts in a month.

(vi) Nurses must have at least one day off each week.

(vii) Nurses can work only one shift per day.

(viii) No more than 2 night shifts per week are allowed for each nurse.

(ix) Consecutive night shifts are not allowed.

(x) The head nurse always works on the morning shift from Monday to Saturday and takes a day off on Sunday.

(xi) Indices.

i: number of nurses ($i = 1, \ldots, I$)
j: number of shifts ($j = 1, \ldots, J$; M = morning, A = afternoon, N = night)
k: number of days in the planning horizon ($k = 1, \ldots, K$)

(xii) Input parameters.

R_j: number of required nurses in each shift j.
E_i: level-i nurse (level 1 is an experienced nurse).
$PS_{i,j,k}$: nurses preferences on working shifts and workdays (=1 if nurse i prefers to work in shift j on day t).
$PD_{i,k}$: nurses preferences score on days off (=1 if day k is nurse's i second-most preferred day off, =3 if day k is nurse's i most preferred day off).
WS_{Min}: minimum monthly working shifts allowed.
WS_{Max}: maximum monthly working shifts allowed.
S_{Target}: the target number of working shifts.
PS_{Target}: yhe target number of preferred shift.
PD_{Target}: the target preferred day off scores.

(xiii) Decision variables.

$X_{i,j,k} = 1$ if nurse i is assigned to work in shift j on day k, 0 otherwise.
$Y_{i,k} = 1$ if nurse i is assigned to take a day off on day k, 0 otherwise.
S_i^+, S_i^-: positive and negative deviation of the number of shifts from the target for nurse i.
PS_i^+, PS_i^-: positive and negative deviation of the number of preferred shifts from the target for nurse i.
PD_i^+, PD_i^-: positive and negative deviation of the preferred day off scores from the target for nurse i.

The proposed nurse scheduling model is formulated using the normalized goal programming technique, as shown below. The first goal equation is formulated to balance the number of shifts assigned to nurses. The second and third goal equations aim to consider the individual preferences on shift and day off, respectively, enabling the improvement of autonomy of nurses over their work schedule.

Goal 1. The number of overworked or underworked shifts of each nurse is determined based on the total number of shifts assigned and the target number of working shifts, as shown in equation (1). Both overworking and underworking are regarded as an undesirable scheduling outcome.

$$\sum_{j=1}^{J}\sum_{k=1}^{K} X_{i,j,k} - S_i^+ + S_i^- = S_{\text{Target}}, \quad \forall i. \tag{1}$$

Goal 2. The numbers of preferred and nonpreferred shifts assigned to each nurse are calculated using equation (2). The undesirable scheduling outcome occurs when the number of nonpreferred shifts exceeds the target value.

$$\left(\sum_{j=1}^{J}\sum_{k=1}^{K} PS_{i,j,k} \cdot X_{i,j,k}\right) - PS_i^+ + PS_i^- = PS_{\text{Target}}, \quad \forall i. \tag{2}$$

Goal 3. The number of preferred days off assigned is determined using equation (3). The undesirable scheduling outcome occurs when the score of nonpreferred days off is lower than the target value.

$$\left(\sum_{k=1}^{K} PD_{i,k} \cdot Y_{i,k}\right) - PD_i^+ + PD_i^- = PD_{\text{Target}}, \quad \forall i. \tag{3}$$

The scheduling objective is to simultaneously minimize the undesirable scheduling outcomes related to the three goals. The undesirable outcome of each goal is normalized to its target value. The objective function defined in equation (4) is to minimize the summation of normalized undesirable outcomes as shown below.

$$\begin{aligned}\text{minimize } Z = &\left(\frac{\sum_{i=1}^{I}\left(S_i^+ + S_i^-\right)}{S_{\text{Target}} \cdot I}\right) + \left(\frac{\sum_{i=1}^{I}\left(PS_i^-\right)}{PS_{\text{Target}} \cdot I}\right)\\ &+ \left(\frac{\sum_{i=1}^{I}\left(PD_i^-\right)}{PD_{\text{Target}} \cdot I}\right),\end{aligned} \tag{4}$$

$$\text{subject to} \sum_{i=1}^{I} X_{i,j,k} = R_j, \quad \forall j, k \tag{5}$$

$$WS_{\text{Min}} \le \sum_{j=1}^{J}\sum_{k=1}^{K} X_{i,j,k} \le WS_{\text{Max}}, \quad \forall i \tag{6}$$

$$X_{i,j,k} + X_{i,j+1,k} + X_{i,j+2} \le 1, \quad \forall i, j, k \tag{7}$$

$$\sum_{i=1}^{I} E_i \cdot X_{i,j,k} \ge 0.5 \cdot R_j, \quad \forall j, k, E_i = 1 \tag{8}$$

$$\sum_{j=1}^{J} X_{i,j,k} + Y_{i,k} = 1, \quad \forall i, k \tag{9}$$

$$\sum_{j=N}\sum_{k=1}^{K} X_{i,j,k} \le 2, \quad \forall i \tag{10}$$

$$\sum_{j=N}\sum_{k=k}^{k+1} X_{i,j,k} \le 1, \quad \forall i \tag{11}$$

$$\sum_{i=k}^{k+6} Y_{i,k} \ge 1, \quad \forall i, k \in 1, 8, 15, 21 \tag{12}$$

$$X_{i=\text{Head nurse}, j=M, k} = 1, \quad \forall k \tag{13}$$

$$X_{i=\text{Head nurse}, k=\text{Sunday}} = 1 \tag{14}$$

$$S_i^+, S_i^-, PS_i^+, PS_i^-, PD_i^+, PD_i^- \in Z^+, \quad \forall i. \tag{15}$$

Equation (5) ensures that the number of nurses assigned to each shift meets the requirements. Equation (6) limits the minimum and the maximum number of working shifts per month for nurses. Equation (7) allows nurses to work only one shift per day. Equation (8) ensures an adequate number of experienced nurses in each shift. Equation (9) ensures that there is no shift assignment on a day off. Equation (10) makes certain that the allowable number of night shifts per week is enforced. Equation (11) prohibits the consecutive night shift assignment. Equation (12) ensures that each nurse is entitled to at least one day off per week. Equations (13) and (14) account for the head nurse's fixed schedule. Equation (15) defines deviation variables to be positive integers.

3.4. Case Study Data. The model is validated using the collected case study data. The minimum and the maximum number of shifts per month are 22 and 24, respectively. According to the head nurse, the targets related to the 3 goals are given in Table 1.

The nurses are asked to identify their preferred shifts throughout the planning period of 28 days. The first 14 days of shift preference data are shown in Table 2. For the day off preference, nurse preference data are obtained by using Likert scales rather than binary response scales used by the previous studies [31, 32]. In our questionnaire survey, the nurses are asked to rate how each shift and day off fits their scheduling needs. The ratings provide more scheduling flexibility and a higher chance of maximizing the satisfaction of all nurses. In our case, the most and second-most preferred days off worth 3 and 1 points, respectively. The target preferred day off score (PD_{Target}) of 12 is achieved when the most preferred day off is assigned as the actual day off of every week throughout the 28-day planning period. The first 14 days of the day off preference sheets are shown in Table 3.

4. Results and Discussion

In this part, the problem is divided into three scenarios: normal operation, extended capacity operation, and higher demand for experienced nurses. The number of nurses required for the normal operation over the three shifts is 6, 6, and 2. For the extended capacity operation, the number of nurses required is 9, 6, and 2. It is assumed that the morning-shift capacity is extended to handle high patient demand in the morning. It is worth noting that the scheduling requirements such as the number of shifts allowed and the unallowable shift patterns are formulated as hard constraints in this study. Such model formulation does not provide enough flexibility to conduct an extensive sensitivity analysis. If these hard constraints are reformulated as goal equations and soft constraints, a sensitivity analysis can be performed in a broader sense by varying the design parameters such as the number of nurses available. Here, only the normal and extended capacity scenarios are analyzed.

4.1. Normal Operation Scenario. The goal-programming nurse scheduling model is solved using Opensolver version 2.9.0. The case study problem can be solved to optimality within 5 seconds, using a 2.3 GHz Dual-Core Intel Core i5-8300H operating system. In Table 4, the total numbers of shifts, preferred shifts, and preferred days off assigned to 17 nurses over 28 days are summarized. The actual total number of shifts assigned to nurses based on the schedule that was manually produced in the month before the data collection period is also shown. The details of the optimal

Table 1: The goal targets.

Target number of working shifts (S_{Target})	24 shifts
Target number of preferred shifts (PS_{Target})	20 shifts
Target preferred day off score (PD_{Target})	12 points

Table 2: Nurses' preferred working shifts.

Nurse	EXP	*D1*	*D2*	*D3*	*D4*	*D5*	*D6*	*D7*	*D8*	*D9*	*D10*	*D11*	*D12*	*D13*	*D14*
1	1 (H)	M	M	M	M	M	M	M	M	M	M	M	M	M	M
2	1	M	A	A	A	M	M	M	A	A	A	N	A	A	A
3	1	M	M	M	M	M	N	M	M	M	M	M	M	M	A
4	1	A	A	A	A	A	M	A	A	A	A	M	N	M	M
5	1	M	M	A	A	A	A	N	A	A	A	A	A	M	M
6	1	A	A	A	A	A	N	A	A	A	A	A	M	M	M
7	1	M	M	M	M	M	A	A	A	A	A	A	A	N	A
8	1	A	A	A	A	A	A	M	M	M	M	M	M	M	M
9	1	A	A	M	M	M	M	A	A	A	A	N	A	A	N
10	2	M	M	M	M	M	M	M	M	M	M	M	M	M	M
11	2	A	A	A	A	A	N	A	A	A	N	M	M	M	M
12	2	M	M	A	A	A	M	M	M	M	M	N	A	A	A
13	2	M	M	M	M	M	A	A	A	A	A	N	A	A	A
14	2	A	A	A	A	A	A	A	M	M	M	M	M	M	M
15	2	A	A	A	N	A	M	M	M	M	M	M	A	A	A
16	2	M	M	M	M	M	A	A	A	A	A	A	A	N	A
17	2	N	A	A	A	A	M	M	A	A	A	A	N	A	A

EXP = experience level, H = head nurse, M = morning shift, A = afternoon shift, N = night shift.

Table 3: Nurses' preferred day off.

Nurse	EXP	*D1*	*D2*	*D3*	*D4*	*D5*	*D6*	*D7*	*D8*	*D9*	*D10*	*D11*	*D12*	*D13*	*D14*
1	1 (H)	—	1	—	—	—	—	3	—	1	—	—	—	—	3
2	1	—	—	3	—	—	1	—	—	1	—	—	3	—	—
3	1	1	—	—	—	3	—	—	—	3	—	—	—	1	—
4	1	—	1	—	—	—	—	3	3	—	—	1	—	—	—
5	1	—	—	—	—	—	3	1	—	1	—	—	—	3	—
6	1	—	—	—	3	—	1	—	—	—	1	3	—	—	—
7	1	—	3	—	—	1	—	—	—	1	—	—	—	—	3
8	1	1	—	—	—	—	3	—	1	—	—	—	3	—	—
9	1	—	1	—	—	—	—	3	—	3	—	—	—	—	1
10	2	—	1	—	—	3	—	—	—	—	1	—	3	—	—
11	2	—	—	1	—	—	—	3	—	—	—	1	—	—	3
12	2	3	—	—	—	—	—	1	—	—	3	—	—	1	—
13	2	3	—	—	1	—	—	—	—	3	—	—	1	—	—
14	2	—	3	—	—	—	1	—	1	—	—	—	—	3	—
15	2	1	—	—	—	3	—	—	—	1	—	—	—	—	3
16	2	—	—	1	—	—	3	—	—	—	1	—	3	—	—
17	2	3	—	—	—	—	1	—	3	—	—	—	1	—	—

schedule concerning the deviation from the three goals are also given in the table.

The results mainly suggest that all the scheduling goals can be successfully achieved by using the proposed approach. Concerning the first goal, there are 4 nurses with zero deviation from the workload target of 24 shifts. The deviations from the workload target for all nurses are no more than 2 shifts. The workload among the nurses is reasonably balanced. Regarding the other two goals, the deviations from the target number of preferred shifts and target day off scores are insignificant. However, there is one nurse (nurse 2) who is subject to moderate levels of percent deviation across the three goals, compared to other nurses. This scenario of one or a few people receiving not-too-good scheduling results to maximize the job satisfaction of the entire workforce may occur. This nurse needs to be compensated by a rise in the number of preferred shifts and day off assignments during the next scheduling period. At any rate, for these current optimal solutions, it is reasonable to conclude that all the goals are simultaneously satisfied.

4.2. Extended Capacity Operation Scenario. Based on the results of the extended capacity scenario, the number of nurses required increases from 17 to 20 to cope with the higher patient demand during the morning shift. The results

Table 4: Summary of deviations from the goals of the NSP model for the normal scenario.

Nurse i	Actual total shifts	$G1$: working shifts balancing				$G2$: preferred shift assignments				$G3$: score of preferred day off assignments			
		Total shifts	S_i^-	S Target	% Dev*	Total preferred shifts	PS_i^-	PS Target	% Dev*	Total preferred day off scores	PD_i^-	Pd Target	% Dev*
1	24	23	1	24	0.3	20	0	20	4.3	12	0	12	5.7
2	20	24	0	24	4.1	17	3	20	11.3	10	2	12	11.9
3	20	23	1	24	0.3	18	2	20	6.1	11	1	12	3.1
4	20	23	1	24	0.3	16	4	20	16.6	11	1	12	3.1
5	24	22	2	24	4.6	18	2	20	6.1	11	1	12	3.1
6	23	23	1	24	0.3	20	0	20	4.3	11	1	12	3.1
7	21	23	1	24	0.3	18	2	20	6.1	11	1	12	3.1
8	22	23	1	24	0.3	20	0	20	4.3	11	1	12	3.1
9	24	24	0	24	4.1	20	0	20	4.3	12	0	12	5.7
10	20	24	0	24	4.1	20	0	20	4.3	12	0	12	5.7
11	20	22	2	24	4.6	20	0	20	4.3	12	0	12	5.7
12	24	23	1	24	0.3	19	1	20	0.9	10	2	12	11.9
13	20	23	1	24	0.3	20	0	20	4.3	12	0	12	5.7
14	20	23	1	24	0.3	20	0	20	4.3	12	0	12	5.7
15	24	24	0	24	4.1	20	0	20	4.3	12	0	12	5.7
16	24	22	2	24	4.6	20	0	20	4.3	12	0	12	5.7
17	24	23	1	24	0.3	20	0	20	4.3	11	1	12	3.1
Average					1.9				5.6				5.4

* Dev = percent deviation from the average values.

Table 5: Summary of deviations from the goals of the NSP model for the extend capacity scenario.

Nurse i	Actual total shifts	$G1$: working shifts balancing				$G2$: preferred shift assignments				$G3$: score of preferred day off assignments			
		Total shifts	S_i^-	S Target	% Dev*	Total preferred shifts	PS_i^-	PS Target	% Dev*	Total preferred day off scores	PD_i^-	PD Target	% Dev*
1	24	24	0	24	0.8	20	0	20	1.3	12	0	12	7.6
2	20	24	0	24	0.8	16	4	20	19.0	10	2	12	10.3
3	20	24	0	24	0.8	20	0	20	1.3	12	0	12	7.6
4	20	23	1	24	3.4	20	0	20	1.3	10	2	12	10.3
5	24	24	0	24	0.8	20	0	20	1.3	10	2	12	10.3
6	23	24	0	24	0.8	20	0	20	1.3	12	0	12	7.6
7	21	24	0	24	0.8	20	0	20	1.3	9	3	12	19.3
8	22	24	0	24	0.8	20	0	20	1.3	12	0	12	7.6
9	24	24	0	24	0.8	19	1	20	3.8	9	3	12	19.3
10	20	24	0	24	0.8	20	0	20	1.3	10	2	12	10.3
11	20	23	1	24	3.4	20	0	20	1.3	11	1	12	1.3
12	24	24	0	24	0.8	20	0	20	1.3	12	0	12	7.6
13	20	24	0	24	0.8	20	0	20	1.3	12	0	12	7.6
14	20	23	1	24	3.4	20	0	20	1.3	11	1	12	1.3
15	24	24	0	24	0.8	20	0	20	1.3	12	0	12	7.6
16	24	23	1	24	3.4	20	0	20	1.3	11	1	12	1.3
17	24	24	0	24	0.8	20	0	20	1.3	12	0	12	7.6
18	—	24	0	24	0.8	20	0	20	1.3	12	0	12	7.6
19	—	24	0	24	0.8	20	0	20	1.3	12	0	12	7.6
20	—	24	0	24	0.8	20	0	20	1.3	12	0	12	7.6
Average					1.3				2.3				8.4

* Dev = percent deviation from the average value.

are summarized in Table 5. The working shift balance objective is well achieved. The shift and day off assignments are also quite consistent with the nurse preferences. In the optimal solution, there is only one nurse whose preferred-shift deviation is up to 4. Other nurses' shift preferences are well satisfied. With the increased number of nurses, it may be easier to proportionately distribute the workload. Solving this problem scenario consumes only 5 seconds. The computational performance of the proposed scheduling approach is also tested using larger problem sizes. It is found that the optimal solution with 50 nurses can be achieved within 20 seconds.

4.3. Higher Demand for Experienced Nurses Scenario. This scenario is created under the assumption that more experienced nurses are required on Monday and Tuesday mornings due to the need to cope with a large number of

Table 6: Summary of deviations from the goals of the NSP model for the scenario with busy days.

Nurse i	Actual total shifts	$G1$: working shifts balancing				$G2$: preferred shift assignments				$G3$: score of preferred day off assignments			
		Total shifts	S_i^-	S_{Target}	% Dev*	Total preferred shifts	PS_i^-	PS_{Target}	% Dev*	Total preferred day off scores	PD_i^-	PD_{Target}	% Dev*
1	24	22	2	24	4.6	20	0	20	6.3	12	0	12	23.6
2	20	24	0	24	4.1	19	1	20	0.9	9	3	12	7.3
3	20	24	0	24	4.1	19	1	20	0.9	4	5	12	58.8
4	20	24	0	24	4.1	17	3	20	9.7	4	5	12	58.8
5	24	24	0	24	4.1	19	1	20	0.9	12	0	12	23.6
6	23	24	0	24	4.1	17	3	20	9.7	10	4	12	3.0
7	21	24	0	24	4.1	20	0	20	6.3	12	0	12	23.6
8	22	24	0	24	4.1	19	1	20	0.9	10	2	12	3.0
9	24	24	0	24	4.1	20	0	20	6.3	6	9	12	38.2
10	20	23	1	24	0.3	20	0	20	6.3	7	2	12	27.9
11	20	22	2	24	4.6	20	0	20	6.3	11	1	12	13.3
12	24	22	2	24	4.6	15	5	20	20.3	12	0	12	23.6
13	20	22	2	24	4.6	16	4	20	15.0	12	0	12	23.6
14	20	23	1	24	0.3	19	1	20	0.9	11	1	12	13.3
15	24	22	2	24	4.6	20	0	20	6.3	12	1	12	23.6
16	24	22	2	24	4.6	20	0	20	6.3	10	1	12	3.0
17	24	22	2	24	4.6	20	0	20	6.3	11	0	12	13.3
Average					3.8				6.4				22.5

*% Dev = percent deviation from the average value.

Table 7: Comparison among the manual and optimal nurse schedules.

	$G1$: no. of working shifts balancing		$G2$: no. of preferred shift assignments		$G3$: score of preferred day off assignments	
	Average	STD	Average	STD	Average	STD
Manual	22	1.84	—	—	—	—
Normal capacity	23.06	0.64	19.17	1.24	11.32	0.68
Extended capacity	23.8	0.4	19.75	0.89	11.15	1.06
Higher demand for experienced nurses	23.04	0.94	18.82	1.60	9.70	2.78

STD = standard deviation.

patients. The number of experienced nurses required during these peak-patient-demand shifts increases from 3 to 5. The model is solved again, and the results are summarized in Table 6. Under this scenario, experienced nurses are subject to more workloads. For most of them, their number of working shifts reaches the maximum allowable level. The ability to maximize nurse preference is now restricted by the requirement for more experienced nurses during peak time. When compared to all the previous scenarios, the average values of percent deviation for all the goals are higher. This is because experienced nurses can no longer take leaves on Monday and Tuesday, and their day off preference cannot be completely satisfied. However, under this scenario, the proposed model still offers a better working-shift balance, compared to the manual scheduling results.

The comparison of performance measures among the manual schedule and optimal schedules is made. The average and standard deviation associated with each goal's performance measure are shown in Table 7. The optimal schedules can be benchmarked against the manual schedule on the aspect of working shift balance only. Before this study, the scheduling of working shifts did not incorporate the preferred shifts and day off information.

The proposed model outperforms the actual schedule in terms of solution quality and execution time. For manual scheduling, it usually required about 1 week to prepare a 1-month schedule. Regarding the working shifts balancing objective, even for the higher-demand scenario, the optimal schedule yields a significantly lower standard deviation suggesting that the model provides a more balanced workload assignment than the manual scheduling. This optimal schedule is also achieved in light of the preferred shift and day off consideration.

5. Conclusions

The nurse scheduling tool is essential in creating an efficient nurse shift-rotation schedule. Manual scheduling is time-consuming and prone to overlooking some of the desirable scheduling characteristics. This study develops a nurse scheduling tool that can proportionally assign shifts to nurses while maximizing their individual preferences on shift and day off. The scheduling outcomes are expected to improve the nurse's perception of fairness and job satisfaction, which positively influences nurse retention. This study contributes to the practicality aspect of the existing NSP literature by illustrating how to formulate and solve a job-satisfaction enhanced model based on an actual case study. The collection of operational and preference data is obtained via field and questionnaire surveys with the nurses at the operating room department in a private hospital in Pathum Thani, Thailand. The proposed model employs the goal programming technique, enabling the consideration of multiple goals: (1) minimizing the unbalanced workload, (2) minimizing the unbalanced preferred shift, and (3) minimizing the unbalanced preferred day off among the nurses. The three problem scenarios with slightly different problem sizes are solved to optimality to show its performance based on the solution quality and computational time. The optimal solutions obtained from the proposed approach show a significant improvement compared to the manually made schedule on the aspect of workload balance. The use of a scheduling optimization approach also allows any change in the operational and preference parameters to be made and solving the new optimal schedules easily. On the job satisfaction factor aspect, this study strengthens the existing NSP literature by considering both workforce- and individual-level preferences. The workload fairness is regarded as the workforce-level desirable characteristics. The case study research that simultaneously considers both preference levels is limited.

The proposed goal programming nurse scheduling model should be able to assist the head nurse in assigning a more balanced workload while maximizing nurses' shift and day off preferences. The proposed scheduling approach can be modified in many ways when applying to other NSP. The shift balancing objective can be modified to consider other scheduling characteristics such as the heterogeneity of tasks, nurse income, and the interrelation between nurses. The hard constraints in the proposed model, such as the number of nurses in each shift and the minimum and the maximum numbers of working shifts per month, can be formulated as soft constraints or goal equations to improve the scheduling flexibility and the ability to fulfill management policy. The proposed model is based on Microsoft Excel and can be modified and implemented without additional software cost and programming difficulty.

Future studies should engage more in the practical application of the NSP approach. More NSP studies that investigate and incorporate the job satisfaction factors of nurses and uncertainties based on the actual case study are needed. According to the existing NSP literature, the differences in job satisfaction factors from case to case can be observed. The operational uncertainties such as patient demand and nurse availability experienced in each case should be addressed with the proper model formulation and solving approaches. The lack of uncertainty consideration is one of the limitations of the current study. The fairness in workload and preferred shift and day off assignments in this study is short-term, over just 28 days. In our future study, the scheduling approach should be able to account for the historical scheduling outcomes to generate the solution with long-term fairness.

Data Availability

Data are available upon request to the corresponding author.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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Research Article

A Supplier Selection Model for a Wholesaler and Retailer Company Based on FITradeoff Multicriteria Method

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The problem approached is regarding supplier selection in a wholesaler and retailer of the construction sector. The current approach used in the company analyzed for the supplier selection problem is unable to evaluate the many issues that the company considers must be addressed. Thus, the Flexible and Interactive Tradeoff (FITradeoff) ranking method was chosen to deal with this problem. It seeks to elicit criteria weights in a decision problem, using partial information about the decision maker's (DM's) preferences in order to choose the most attractive alternative, based on the ranking of the suppliers provided. Based on the Hasse diagram developed when applying the method, the DM makes some observations regarding the suppliers' positions in the ranking. Bar graphics and radar graphics were also used and these enable the DM to evaluate each supplier better. It was perceived that the method is applicable and well accepted by the DM. The graphic part helps the DM in understanding the analysis of the problem, offering no barriers to proceed with the tradeoff. Thus, for this situation of selecting a supplier in a wholesaler and retailer of the construction sector, the DM felt comfortable with the proposed approach using the FITradeoff multicriteria ranking method and with the results presented, and the proposed approach is shown to be adequate.

1. Introduction

The process of selecting partner companies in the supply chain is an important strategic factor in achieving maximum results when using intercompany relationship management and collaboration, but, prior to forming alliances with suppliers, it is necessary to identify which of them are potentially beneficial. Given a scenario where long-term relationships with partners are established to maximize company results, it is observed that establishing the criteria and the way in which partners (suppliers) are chosen are very important issues when constructing supply chains and distribution channels, and, consequently, for generating the advantages that can arise from closer relationships between buyers and suppliers.

The supplier selection process is very important for medium and small organizations since they have limited resources and operational capacity. Therefore, by ensuring that they select suitable suppliers, they can obtain more significant gains for the business, thus increasing the performance and profitability. It is usually the case that small- and medium-sized companies only use a single criterion, often the "price/cost," to select and classify the type of relationship they wish to have with suppliers, even though this is only one of the components of the decision-making problem. In contemporary supply chain management (SCM), multiple criteria are used to evaluate potential suppliers against a single cost factor [1]. Another important issue for these companies is the weighting of selection criteria, where the DM tends to assign the same degree of importance to all the criteria or has difficulties understanding what factors are strategically important for the business.

It is known that supplier selection is a multicriteria decision-making/aiding (MCDM/A) problem that depends on a wide range of factors that involve both quantitative and qualitative factors, such as quality, cost, capacity, delivery, and technical potential [2–4]. MCDM/A problems consist of decisions in which there are at least two alternative actions to evaluate with regard to two or more attributes [5–7], which is the situation in most supplier selection processes. These decisions are made to meet multiple goals that often conflict with each other. In order to support the decision-making process in a multicriteria problem, multicriteria methods that lead to solving the problem must be used. Many decisions in business organizations are based on MCDM/A approaches, which have been applied in many situations and different contexts, such as in newsvendor problem [8], maintenance strategy [9], portfolio selection of projects [10], and risk analysis of hydrogen pipelines [11].

The supplier selection process usually involves evaluating suppliers in relation to a set of criteria previously defined by the company, in which the supplier or suppliers that best meet the criteria will be selected. In addition, there must be a rigid process for evaluating the selected suppliers in order to support the management of relationships in the long term and to ensure productivity, profitability, and success in achieving the objectives expected as a result of the relationships between a company and its suppliers [12]. According to Araújo et al. [13], the quality of the suppliers selected depends on the quality of the selection process. Thus, the types of criteria, the information level that is required and available, the type of information (complete or partial), the problematic approach (ranking, selection of one supplier, or selection of a subgroup of suppliers), and other factors must be considered when selecting the methodology used for the supplier selection problem, since each criterion is adequate for a set of specific conditions (Araújo et al. [3]). Palha et al. [14] state that criteria used in the selection of suppliers vary in relation to the priority given by the clients as a result of a tradeoff process; also some of these criteria could be subjective and probabilistic.

Many supplier selection multicriteria models have been proposed in the management literature for dealing with supplier selection problem [1, 15]. Nevertheless, when using additive methods, establishing statements of indifference between consequences, caused by the application of traditional methods, is a difficult task for some DMs, and they may not be able to provide the necessary information [7]. In addition, the process used to obtain the criteria weights by traditional methods require a lot of time from the DM [15]. Considering this context, approaches that use partial information are required [16–18].

Thus, given the supplier selection problem in a deterministic context, this study aims to propose a multicriteria model to support the supplier selection process in a company that is a retailer and wholesaler of waterproofing products. The model is based on the Flexible and Interactive Tradeoff (FITradeoff) method for the ranking order problem [17, 18]. It is worth mentioning that a multicriteria model for a supplier selection in a food company using the FITradeoff method for the choice problematic is already presented in the literature [18]. The main characteristic of this methodology in relation to the traditional Tradeoff method [5] is that FITradeoff works with partial information about the DM's preferences [16]. Besides, our study presents graphical visualization analysis, which was drawn from a recent study [19–21] that uses the neuroscience approach, to investigate how DMs understand MCDM/A problems represented by graphics. This feature contributes mainly to how graphics can be used to support the DM to tackle an MCDM/A problem. It is a flexible tool that can improve a Decision Support System (DSS).

The paper is organized as follows: Section 2 contains a brief literature review about supplier selection processes. Section 3 describes how the FITradeoff method is used in multicriteria decision problems. Section 4 presents the proposed supplier selection model and its application in the study company, and the last section contains the conclusions of this study and makes suggestions for future lines of research.

2. Supplier Selection Process

Supplier selection is one of the most critical activities of the procurement process in a SCM because a wrong choice can lead to the supply chain as a whole suffering loss, thus directly affecting the performance of the organizations involved [13, 18]. In this context, suppliers should be selected by using formalized methods that ensure that the selection process is reviewed and evaluated and therefore the company's decision-making process is more readily able to choose the suppliers that best align with the company's objectives and supply chain [22].

According to Chen and Chao [12], the supplier selection process is an important issue in SCM and there are two important aspects to the supplier selection problem: (1) setting the criteria for evaluating suppliers and (2) designing the process or method of selecting suppliers. According to Wibowo and Deng [23], supplier selection processes are complex and challenging owing to DMs' having diverse opinions, to uncertainty and imprecision being present, and to the cognitively demanding nature of the decision-making process.

This process has become increasingly complex due to factors such as globalization and keeping pace with exploiting how the Internet can be used, which has broadened the range of potential alternatives to choose from. However, this process needs to be agile to meet the needs of companies and their clients, and there are some laws or standards that require both transparency when activities are being selected and compliance with new procedures. This leads to companies involving more agents (analysts, specialists, and DMs) in the selection process, who may have divergent objectives [24].

Sarvestani et al. [25] emphasize that the problem of supplier selection has attracted much attention in recent years and is a class of problems that have been widely studied by many researchers. Since there are usually several factors that must be taken into account while selecting suppliers, various multiple-criteria decision-making techniques have been used to evaluate and rank suppliers to ease the selection process [25].

Ho et al. [1], De Boer et al. [26], and Chen [27] have conducted extensive reviews of the literature on multicriteria decision-making methods used to support supplier selection. They highlighted the following approaches:

(i) Methods of mathematical programming: an objective mathematical function is developed for the selection problem, where the solution is given by maximization or minimization [27–30].

(ii) Multicriteria model: the DM evaluates a set of alternatives in relation to several decision criteria using a systematic method. Weighting methods are commonly used. In this case, the DM assigns subjective weights to each criterion and the overall assessment is based on the sum of the performances in the criteria of each supplier multiplied by the weight of the criteria [31, 32]. The AHP (analytic hierarchy process) [33–35] and the ANP (analytic network process) [36, 37] are other highlighted methods.

(iii) Data envelopment analysis (DEA): the DM evaluates the alternatives to the benefit criteria (outputs) and the cost criteria (inputs). The alternative is chosen using the ratio of the weighted sum of its outputs to the sum of its inputs [38–40].

(iv) Total cost of ownership (TCO): all measurable costs incurred during the life cycle of the purchased item are incorporated into how a supplier is chosen [41–43].

(v) Fuzzy theory: linguistic values are expressed in fuzzy numbers, which are used to evaluate and assign weights to the criteria [44, 45].

(vi) Artificial intelligence: the decision problem is modeled and solved by means of a computational system [46–48].

The models based on these methods aim to encompass the maximum number of possible criteria and reduce the subjectivity of the decision. De Boer et al. [26] argue that factors such as the number of suppliers available, the importance of the purchase and/or the relationship with the supplier, and the existing amount of uncertainty determine the most appropriate method to be used, depending on the purchase characteristics (new purchase, modified rebuy, straight rebuy, or strategic rebuy). It is observed that most of the models combine more than one technique for structuring and solving supplier selection problems [1, 27].

Another fundamental aspect is to establish the most important criteria in a decision problem. Araújo et al. [13] identified 21 different criteria applied to the supplier selection problem in the recent literature. Ho et al. [1] also identified from their review of the literature what the most popular criteria were which DMs considered in order to evaluate and select the most appropriate supplier. Hundreds of criteria were proposed, among which the most commonly chosen criterion is quality, followed by delivery, price/cost, manufacturing capability, service, management, technology, research and development, finance, flexibility, reputation, relationship, risk, safety, and environmental impact. As can be seen, price/cost is not the most widely adopted criterion. Ho et al. [1] further state that the traditional single-criterion approach based on lowest cost bidding is no longer sufficiently supportive of or robust in contemporary SCM.

Araújo et al. [3] carried out an extensive literature review on 119 papers, as to supplier selection models, which were published between 1973 and 2015. They identified three key points: (1) Many factors are usually considered when selecting suppliers, as it is important for the process of building decision-making models to consider several criteria that aid finding solutions for supplier selection problems. (2) Choosing adequate criteria for supplier selection depends on the needs and priorities of the purchasing company; thus, each model should be built according to that company's needs. (3) Criteria that evaluate the degree of a supplier's engagement are not often found in the literature, although satisfying such criteria is essential to establishing good partnerships.

Because each organization has its own strategic interests, the decision problem needs to be structured within its specific context. Thus, by considering the DM's rationality, the ranking problem, and the need for a more flexible method for setting the parameters, the supplier selection problem in this study is built based on a multicriteria additive aggregation approach, considering the FITradeoff method, as presented in the following section.

3. FITradeoff Method

Flexible and Interactive Tradeoff (FITradeoff) [16] is a method developed to elicit a criteria-scaling constant in the context of Multiattribute Value Theory (MAVT) [5, 7]. In the MAVT context, given a set of alternatives, the first alternative in the ranking is the one that presents the highest global value, as illustrated in equation (1), where k_i is the scaling constant for criterion i and $v_i(x_i)$ is the marginal value function in criterion i.

$$V(A) = \sum_{i=1}^{n} k_i v_i(x_i). \tag{1}$$

In the FITradeoff method, the first step is the intracriteria evaluation. After that, the ranking of scaling constants is conducted. In this step, the decision-maker (DM) has to order the scaling constants based on the range of consequences presented in each criterion. Thus, the first inequality is generated, as illustrated in equation (2), where n is the number of criteria. It is worth mentioning that the intracriteria evaluation and the ranking of scaling constants are common steps in most methods in the MAVT context.

$$k_1 > k_2 > \cdots > k_n, \tag{2}$$

and, after that, the elicitation process is conducted in the FITradeoff method in order to reduce the scaling constant space (or weight space). In the FITradeoff method, the exact value of the criteria weights is not obtained. Instead of that, a weight space is obtained from the preferences expressed by DM during the decision process.

In this context, in the elicitation process, the DMs have to express their strict preferences for some comparisons of consequences. Thus, an intermediate value of consequence, in the criterion which presents the highest value of scaling constant (k_1), is compared to the highest value of consequence $v_2\,(x_2^{\text{upper}})$ in the criterion associated with the adjacent scaling constant (k_2). In the FITradeoff method, the interval scale is applied; thus the best consequence in a criterion presents value function equal to one, and the worse consequence presented value functions equal to 0. Thus, given the situation in which the DM prefers to receive the best consequence in Criterion 2 rather than an intermediate consequence in Criterion 1, the inequality illustrated in equation (3) is obtained. Therefore, after each comparison, an inequality is also generated in order to represent the preference expressed by the DM. It is worth mentioning that other consequences, that is, in other criteria, are not compared since the comparison is in pairs, following the scaling constant order.

$$k_1 v_1\left(x_1^{\text{low}}\right) \leq k_2. \tag{3}$$

An important characteristic of the elicitation conducted in the FITradeoff method is that the DM does not have to identify the exact point of indifference between the consequences, as required in the Tradeoff method [5]. In other words, the FITradeoff elicitation process requires only strict preferences, since this method uses concepts of partial information. Therefore, it is considered an advantage of the FITradeoff method, since identifying the exact point of indifference can lead to 67% of inconsistencies in the results [7].

After each interaction with the DM, the inequality obtained is included in a linear programming problem (LPP). Thus, the LPP model is processed and the relation between alternatives is updated. In other words, from this LPP model, the scaling constant space is reduced, and some alternatives become dominated from others, as described in [17]. The LPP model is illustrated by the system of equations (4)–(8), where A_1 represents alternative 1 in the set of alternatives.

$$\text{Max Dominance}\,(A_1, A_2) = \sum_{i=1}^{n} k_i v_i\,(A_1) - \sum_{i=1}^{n} k_i v_i\,(A_2),$$

s.t.,

$$\tag{4}$$

$$k_1 > k_2 > \ldots > k_n, \tag{5}$$

$$k_i v_i\left(x_i^{\text{low}}\right) \leq k_{i+1}, \tag{6}$$

$$k_i v_i\,(x_i^{\text{upper}}) \geq k_{i+1}, \tag{7}$$

$$\Sigma_{i=1}^{n} k_i = 1. \tag{8}$$

In this context, the DM participates in the whole decision process expressing his/her preferences and evaluating the partial results. Therefore, the FITradeoff method is considered as an interactive method.

The FITradeoff method is implemented by a Decision Support System (DSS), both for the choice problematic [16] and for the ranking problematic [17]. Thus, in the DSS, the holistic evaluation can be conducted to assist the DM during the decision process. Several types of visualization are presented in the DSS, such as bar graphic, bubble graphic, and radar graphic. In the FITradeoff DSS for ranking problematic, the Hasse diagram is also presented to illustrate partial rankings during the decision process.

Therefore, based on graphical visualizations, the DM can define dominance relations between the alternatives in order to reduce the decision process. For the choice problematic, the DM can evaluate the Potentially Optimal Alternatives (POAs), being the set of alternatives which remains in the decision process. Thus, if the DM wishes, he/she can select the final alternative in the group and finalize the FITradeoff process. On the other hand, for the ranking problematic, for alternatives that are incomparable in the partial ranking and can be evaluated in order to define the complete ranking, if the DM wishes, he/she can define a dominance relation between them, obtaining the complete ranking.

In this context, the FITradeoff presents another advantage compared to the traditional tradeoff method, which is the flexibility provided by the holistic evaluation process. Thus, from the holistic evaluation, the decision process can be concluded before the final alternative or the complete ranking is obtained by the LPP model in the elicitation process. The FITradeoff DSS is available by request at http://www.fitradeoff.org. Figure 1 illustrates the FITradeoff interaction process.

4. Case Study

The case study was carried out in a company that operates in the retail and wholesale trading of waterproofing products and services and that has been involved in the civil construction market since 1989. The company is located in the northeast of Brazil, where it has wholesale units and retail stores in two major cities in that region and distributes products to four states. The company stands out in its market of selling waterproofing solutions, for which it aims to offer quality solutions and to be unique in the market. In addition, as one of its management pillars, the company guarantees to search for and supply quality and innovative solutions in waterproofing products in order to satisfy the specific needs of its clients. Therefore, the company needs to have suppliers that are aligned with its strategic objectives. In addition, the supplier selection process should consider the main aspects of service and commercialization that the company expects from its suppliers so that it is better able to lead its market because the excellence of its waterproofing solutions differentiates it from its competitors.

The process of selecting suppliers in the company is carried out based on only two criteria: the cost of products and commercial conditions. This may involve the risk that partnerships may no longer be realized or that partnerships may be entered into based solely on the financial aspects. This also jeopardizes the duration of relationships in the supply chain of the company, since the market

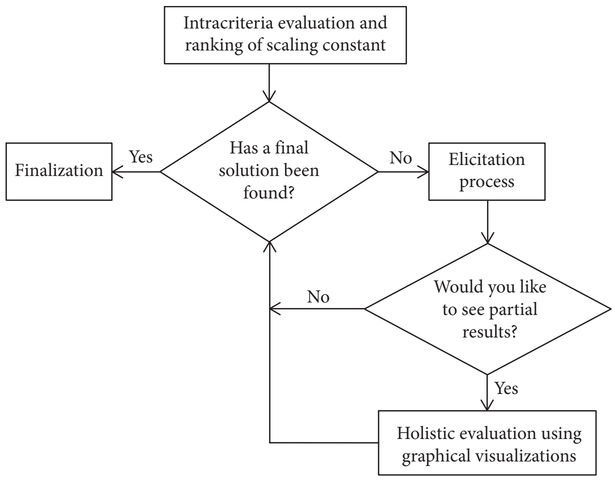


Figure 1: FITradeoff process.

requires not only financially competitive products but also other business features that clients value such as availability, reliable delivery time, product quality, productivity, minimizing losses, warranty, after-sales service, and packaging.

Using this process, the company perceives that the selected suppliers are not aligned with all the aspects that the company aims to have with its business partners, thus making it important for it to develop a "most appropriate supplier" selection model.

4.1. Proposed Model for Supplier Selection. The proposed model, based on FITradeoff method, provides assistance to the DM in relation to the process of selecting new suppliers, with the objective of keeping the suppliers in line with the company's plans and objectives. This will contribute to the growth and development of business and its market share of waterproofing products. The DM is the director of the company, a civil engineer, who leads the procurement, commercial, and technical activities of the organization. The proposed model for supplier selection is presented in Figure 2.

The steps of the model shown in Figure 2 are described in detail in the following points.

4.1.1. Identify New Supplier(s). It refers to identifying new suppliers or products to find new requirements or improve the company's current supply performance. Suppliers are classified into homogeneous groups that supply similar materials (stickers, additions, healing agents and release agents, asphalt emulsions, grouts and special mortars, asphalt blankets, polymeric and elastomeric membranes, auxiliary products, etc.) to facilitate the analysis of the participation of each such group of products in the company's results.

4.1.2. Classify the Supplier's Product Mix. The classification will be based on the participation of each group of products in the analysis of the contribution margin of the ABC curve (a method for classifying materials, according to their cost, based on the frequency of distribution and the Pareto method) for the company's revenue. The supplier selection process will be conducted according to the class of products mix for a given group of materials: mix of class A products (corresponds to 20% of the company's products, accounting for 80% of the company's revenue), mix of class B products (corresponds to 30% of the company's products, accounting for 15% of the company's revenue), and mix of class C products (corresponding to 50% of the company's products, accounting for 5% of the company's revenue). Suppliers will be classified in the product mix classes as follows:

(i) Class A suppliers have at least one item classified in product group A

(ii) Class B suppliers have at least one item classified in product group B and no items in class A

(iii) Class C suppliers do not have products classified in classes A and B

The proposed model was applied to the suppliers classified with a product mix in class A (class A suppliers) for a specific product group. The company's strategic suppliers are included in this class, for which the company has an interest in establishing long-term purchase agreements. In this first stage, 20 suppliers were classified and denominated as S1 to S20.

4.1.3. Prequalify Suppliers. A supplier prequalification process was carried out to identify the suppliers that meet the minimum requirements for submission to the multicriteria selection process. This procedure is necessary to reduce the risk of selecting an inadequate supplier. Criteria used in the prequalification and selection of suppliers are based on studies in [49–51]. The criteria used in the prequalification phase were chosen because they are aligned with the company's business, as shown in Table 1.

The supplier must meet the minimum requirements for each of these qualifying criteria according to its classification (based on the product mix to be supplied) in order to qualify for the supplier selection stage.

The 20 selected suppliers were analyzed against the prequalifying criteria, shown in Table 1, by means of analyzing the documents containing the information required for each criterion evaluated. All 20 suppliers met the minimum requirements and were thus accepted for the supplier selection stage.

4.1.4. Select the Evaluation Criteria. Five criteria were selected by the DM as the most important ones to be used for evaluating the previously selected suppliers, aligned with the expected results of the company. Table 2 presents these criteria.

The evaluation matrix of the suppliers on these criteria is presented in Table 3.

4.1.5. Apply the Multicriteria Method (FITradeoff Method) and the Rank the Suppliers.

In order to solve the supplier selection problem, the FITradeoff method for ranking problems was applied [17].

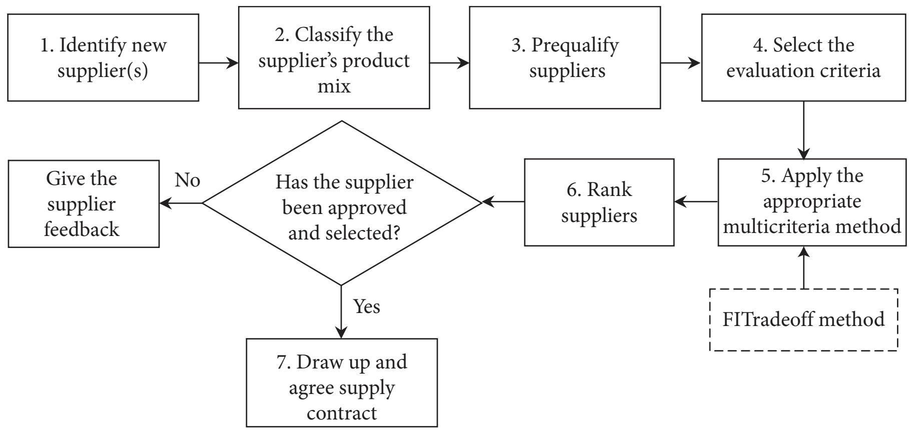


Figure 2: Proposed model for supplier selection.

Table 1: Prequalifying criteria.

Criterion	Description
Technical capacity	Measured by validating that the technical documents in support of the content of the contract show that the supplier is able to provide the materials and services proposed in the supply contract
Financial status	Measured by examining the company's latest accounting returns and relating these to the supplier's financial position in order to be able to assess the potential of the supplier to maintain a long-term partnership under the trading conditions to be negotiated
Infrastructure	Measured by examining the documentary evidence of the industrial structure and service that the supplier will offer
Customer service	Measured by a company document that details the communication channels and telephone and electronic support, which the supplier will offer to the company's customers as the means by which their questions about use, problems, warranty, reverse logistics, and other issues can be made and answered

As discussed in Section 3, the first step is the intracriteria evaluation, which is performed in the previous steps and the outcome is Table 3. After that, it is conducted to ranking of scaling constants. Thus, according to the DM's preferences, the scaling constants concerning criteria C1, C4, and C5 hold the same position in the ranking, followed by the scaling constant of Criterion C2, as well as concluding with the scaling constant of the criterion C3, as illustrated in equation (9).

$$k_{C1} = k_{C4} = k_{C5} > k_{C2} > k_{C3}. \quad (9)$$

After this step, a partial ranking was obtained, as illustrated in Figure 3: the Hasse diagram. Based on this diagram, it was possible to note that supplier S13 was already indicated as the best solution, as it was still placed first in the ranking. In the second and fourth positions, many alternatives were presented. Despite using the Hasse diagram, many relations could be identified; for example, S6 and S20 had equal performances in position 2; S6 dominates S3, and S3 dominates S4. In this context, in order to further complete the ranking, the elicitation process should be conducted.

Hence, the elicitation step was carried out in order to generate more inequalities and specify more relations between the alternatives. In this context, after the second elicitation question in the DSS FITradeoff was answered, the ranking was updated, and this led to the positions being increased from four to seven. Moreover, after the fourth elicitation question, the ranking presented nine positions. The second elicitation question corresponded to comparison of a supplier which had 50% of performance in the criterion Product Quality (named consequence A) *versus* consequence B, a supplier that had 100% of performance in the criterion Flexibility (named consequence B). Given this pairwise comparison, the DM preferred consequence B. Moreover, the fourth question compared a supplier which had 100% of performance in the criterion Product Quality *versus* a supplier that had 100% of performance in the criterion Flexibility. For this other pairwise comparison, the DM preferred consequence A ("100% of performance in the criterion Product Quality").

Figure 4 presents the ranking found after the second and fourth questions. The major differences occurred in the middle of the ranking, specifically in position 3, where S4 dropped down to position 4, and in position 6, which was split in two. Moreover, it was possible to observe that the first and last alternatives had been defined; namely, S13 remained in first position, S6 and S20 were tied in second position, and S11 remained in last position.

Regarding the main proposition of this study, that is, to generate the ranking of suppliers, further investigations in positions three, seven, and eight might be conducted in order to explore the dominance relations between these alternatives and to generate a more detailed ranking. Thus, based on the Hasse diagram obtained after the fourth question, as illustrated in Figure 5, some comments were

Table 2: Supplier selection criteria.

Criterion	Description	Scale	Objective
Product quality (C1)	Presentation of quality tests on the supplied product, carried out internally and externally, and on the suitability of the results for the specified quality	(1) Does not present internal or external quality tests (2) It features internal quality testing that does not meet specifications (3) Features in-house quality testing that meets specifications (4) Features external quality tests that meet specifications (5) Features internal and external quality tests that meet specifications	Maximize
Flexibility (C2)	This concerns commercial flexibility. Degree of adaptation of the commercial conditions of the products and services to the company's needs	(1) Supplier offering a single commercial condition (2) Supplier offering more than one fixed commercial condition (3) Supplier that negotiates with the client up to 50% of the commercial conditions (4) Supplier that negotiates with the client from 50% to 90% of the commercial conditions (5) Supplier that negotiates with the client 90% to 100% of the commercial conditions	Maximize
Relationship (C3)	Degree of supplier relationship with company strategy	(1) Supplier that only carries out commercially related actions (2) Supplier that only aligns prices to its customer's strategy (3) Supplier that aligns prices and deadlines to its customer's strategy (4) Supplier that aligns prices, terms, and operations to its customer's strategy (5) Supplier that aligns prices, terms, operations, information, and development of new products to its customer's strategy	Maximize
Time (C4)	Time (t) between the sending of the order and the invoicing and shipping of the purchase orders	(1) Supplier that has the delivery time until 5 days ($t \leq 5$) (2) Supplier that has the delivery time between 5 and 10 days ($5 < t \leq 10$) (3) Supplier that has the delivery time between 10 and 15 days ($10 < t \leq 15$) (4) Supplier that has the delivery time between 15 and 20 days ($15 < t \leq 20$) (5) Supplier that has the delivery time more than 20 days ($t > 20$)	Minimize
Contribution margin (C5)	% contribution of products to the company's results	(1) Supplier that has the % contribution of products (c) to the company's results less than 20% (c ≤ 20% (2) Supplier that has the % contribution of products (c) to the company's results between 20% and 40% ($20\% < c \leq 40\%$) (3) Supplier that has the % contribution of products (c) to the company's results between 40% and 60% ($40\% < c \leq 60\%$) (4) Supplier that has the % contribution of products (c) to the company's results between 60% and 80% ($60\% < c \leq 80\%$) (5) Supplier that has the % contribution of products (c) to the company's results more than 80%.	Maximize

made: in the third position, the alternatives S14 and S17 dominated S19, which do not have a relation with S3 and S15; besides, S3 and S15 were tied. Moreover, from evaluating this diagram, it was observed that, in position seven, S2 dominates S16, and S12 dominates S16 and S7; and in position eight S9 dominates S8 and S18 dominates S1. Therefore, based on these observations, it could be concluded that alternatives S3, S15, S14, and S17 present the best performance in position three, S2 and S12 in position seven, and S9 and S18 in position eight.

Table 3: Evaluation of the suppliers.

Suppliers	C1	C2	C3	C4	C5
S1	3	5	3	4	1
S2	5	1	3	5	3
S3	4	4	5	2	4
S4	4	4	4	2	3
S5	3	4	4	2	3
S6	5	5	5	3	5
S7	3	5	5	4	2
S8	3	3	4	4	2
S9	3	4	4	5	3
S10	4	4	5	3	3
S11	3	2	3	5	3
S12	3	4	5	3	2
S13	5	5	5	2	4
S14	5	3	3	2	4
S15	4	4	5	2	4
S16	4	2	3	4	2
S17	5	4	4	3	4
S18	4	3	3	5	2
S19	5	3	3	3	4
S20	5	5	5	3	5

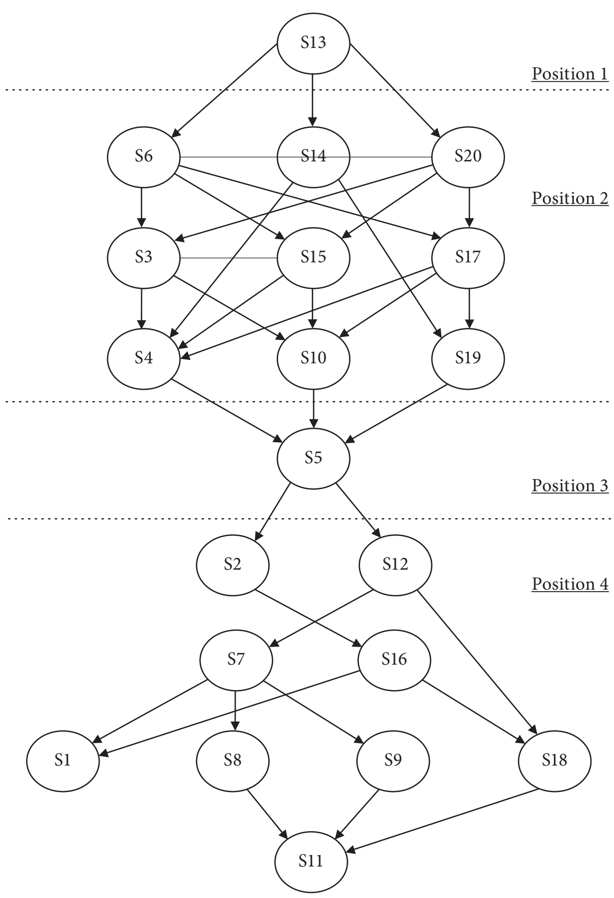
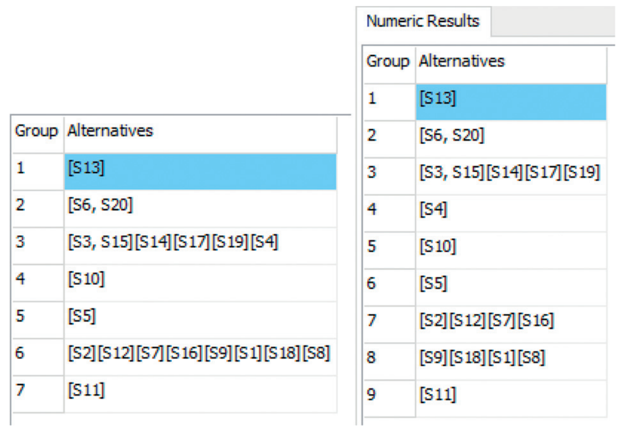


Figure 3: Hasse diagram after the step of rank ordering the criteria.



Group	Alternatives
1	[S13]
2	[S6, S20]
3	[S3, S15][S14][S17][S19][S4]
4	[S10]
5	[S5]
6	[S2][S12][S7][S16][S9][S1][S18][S8]
7	[S11]

Numeric Results

Group	Alternatives
1	[S13]
2	[S6, S20]
3	[S3, S15][S14][S17][S19]
4	[S4]
5	[S10]
6	[S5]
7	[S2][S12][S7][S16]
8	[S9][S18][S1][S8]
9	[S11]

Figure 4: Rank order after second and fourth elicitation questions.

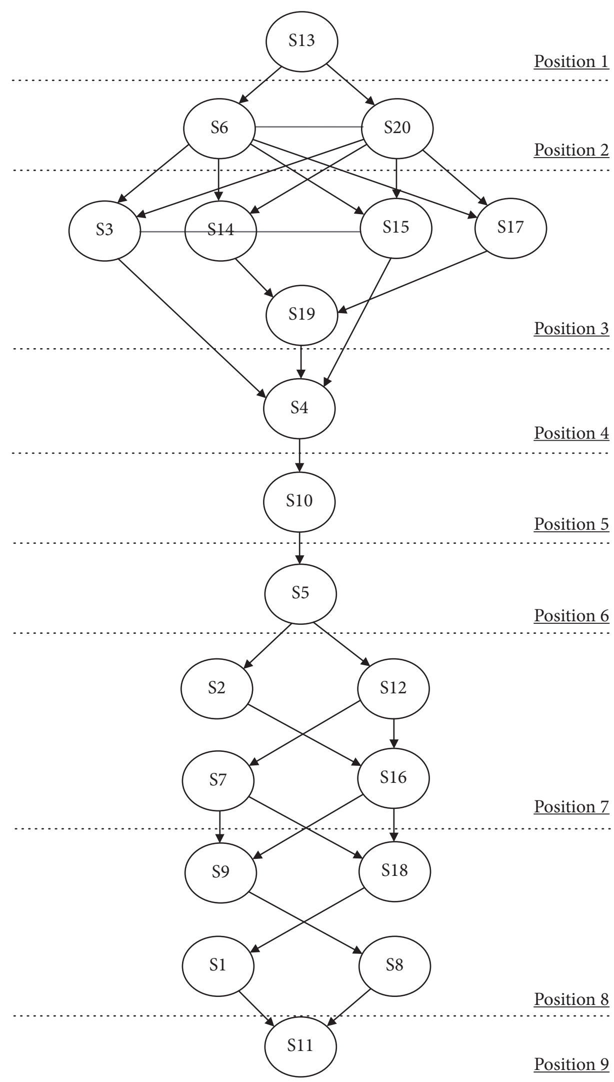


Figure 5: Hasse diagram after the fourth elicitation question.

In order to investigate these alternatives in more depth, bar graphics and radar graphics were used. Bubble graphics were not used since they did not perform well in the evaluation of MCDM/A problems in a previous study [20, 21]. In bar graphics, the height of the columns represents the performance of alternatives in each criterion and in radar graphics the criteria are presented at the vertices and the best performance is reached at these extremities.

Thus, the first pair of graphics (see Figure 6) was constructed to evaluate the alternatives in the third position, S3, S14, S15, and S17. As alternatives S3 and S15 presented the same performance, S15 was excluded to simplify the representation. It is worth mentioning that these graphics represent the evaluation of the suppliers (in Table 3), after applying the interval scale.

Based on Figure 6, it could be observed that all the alternatives presented the same performance in the last criterion (contribution margin–C5), which was therefore excluded in order to simplify the analysis. Based on the minimum level of confidence needed to select the best alternative in a bar graphic developed by [5], a bar graph with three alternatives and five criteria presented 22% of probability of success [19, 20]. However, a bar graph with three alternatives and four criteria presented 57% of probability of success [19, 20]. Thus, based on these results, another bar graphic was constructed, as illustrated in Figure 7.

In criteria that had highest scaling constants, Product Quality (C1) and time (C4), alternative S14 presented the highest performance, followed by alternatives S17 and S3, respectively. In criterion flexibility (C2), alternatives S3 and S17 were tied, and, for C3 (relationship), alternative S3 presented highest performance. Given the MAVT context, in which trade-offs between the performance of the alternatives and scaling constants should be evaluated, the DM can observe that S14 presented a better performance in the criteria with highest scaling constants; however, it presented the lowest performance in criteria C2 and C3, raising doubts over its superiority in the supplier selection problem.

On the other hand, comparing S3 with S17, they were equivalent in C1 (Product Quality) and C4 (time). Thus, considering only C2 (flexibility) and C3 (relationship), it was possible to observe that S3 presented the highest performance, supposing to be better than S17 in the graphic. Now, from comparing S3 and S14, C4 can be excluded, and S14 wins in C1 but presents high disadvantage in C2 and C3, raising doubts in the supplier selection problem.

In order to reinforce the discussion, a radar graphic was constructed to represent individually the performance of each alternative in the problem, as illustrated in Figure 8. Based on the radar graphic, the highest area covered is S3 followed by S17 and S14. Thus, the DM can assume based on the graphical evaluation that alternative S3 dominates S17, and alternative S17 dominates S14.

A second pair of graphics (see Figure 9) was developed to evaluate the alternatives in position seven, S2 and S12. Based on these graphics, it was possible to identify S12 as the best alternative. In Criterion C1, S2 presented the highest performance, but S12 presented high performances in the other criteria, excluding C5 in which the performances of those were low. Given a situation in which the performances of alternative S12 in the criteria C4 and C5 can be aggregated, they become approximately equivalent to the performance of S2 in Criterion C1. Thus, it can be suggested that S12 is the best alternative following the MAVT. Moreover, based on the radar graphic, the area covered by S12 is higher than that covered by S2.

The last evaluation was developed for alternatives S9 and S18 presented in position eight. Based on Figure 10 and using compensatory rationality, alternative S18 presented better performance than S9 only in criterion C1, and it is equivalent to the performance of S9 in criterion C4. Therefore, it was possible to suggest that alternative S9 is better than S18 in the problem.

Therefore, based on the graphical visualization (see Figures 6–10), the ranking can be updated by the holistic evaluation process. Also, as highlighted in the FITradeoff section, if the DM desires, he/she can stop the process at any point and obtain the final partial or complete ranking.

For this study, we identify that the order of the alternatives placed in the first 10 positions of the ranking is considered sufficient for the DM. Therefore, at this point of the decision process, the DM decided to stop the process and considered the partial ranking obtained using the elicitation process and the holistic evaluation, namely, S13, S6 = S20; S3 = S15, S17, S14, S19, S4, S10, S5, and S12. Also, the scaling constant space obtained from the LPP model (in the system of equations (4)–(8)), following the preferences expressed by the DM during the FITradeoff process, presented the highest value of scaling constant for criteria C1, C4, and C5, with a value between 0.2 and 0.24; Criterion C2 presented scaling constant value between 0.13 and 0.2; and the criterion C3 presented scaling constant value between 0 and 0.09, as shown in Figure 11. Unlike the Tradeoff method, the FITradeoff method generates a scaling constant space instead of the exact value of the scaling constants. Also, if the DM has provided different preferences for the pairwise comparisons, the scaling constant space would be different, since the LPP model would present different inequalities to accommodate these preferences expressed.

Unlike the Tradeoff method, the FITradeoff method generates a scaling constant space instead of the exact value of the scaling constants. Thus, for the problem under study, based on the preferences expressed by the DM, the first three criteria present the highest values of scaling constant, varying between approximately 0.24 and 0.29. If the DM expresses different preferences in the process, the space would be different to accommodate the preferences expressed.

4.1.6. Establish Supply Contract. After applying the selection model and obtaining a ranking of the suppliers, the DM should analyze whether or not to approve the supplier. Thus, if the supplier is approved, it is recommended that a supply contract be drawn up.

4.2. Discussion. On applying the proposed supplier selection model, the importance of the supplier selection tool became evident. The FITradeoff method assists the DM in solving the problem based on a structured Linear Programming Program (LPP) model instead of following the DM's empirical

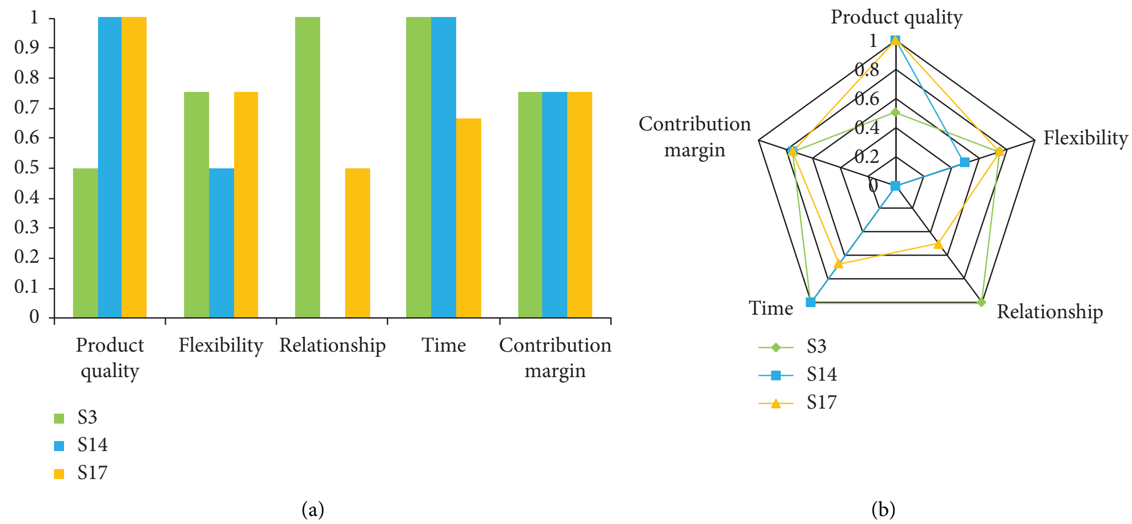


FIGURE 6: Graphics to evaluate alternatives in position three.

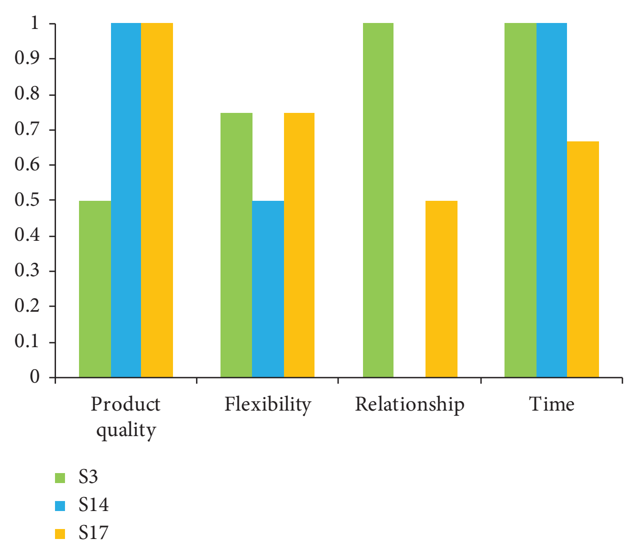


FIGURE 7: Bar graphic without criterion C5.

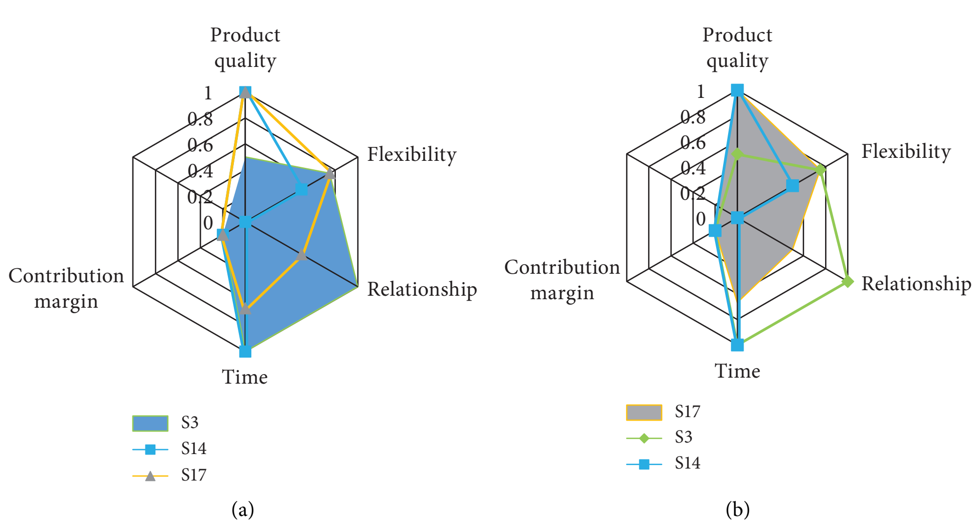
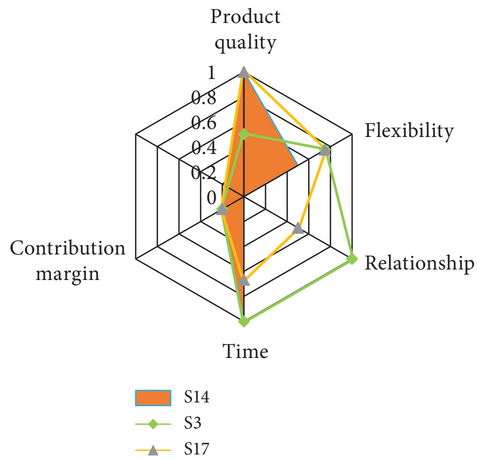
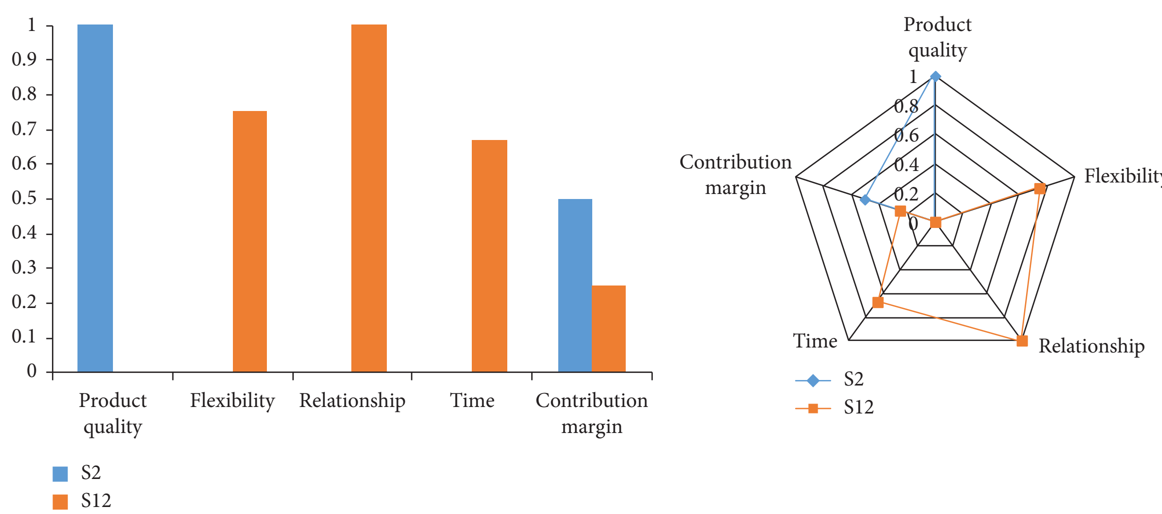


FIGURE 8: Continued.



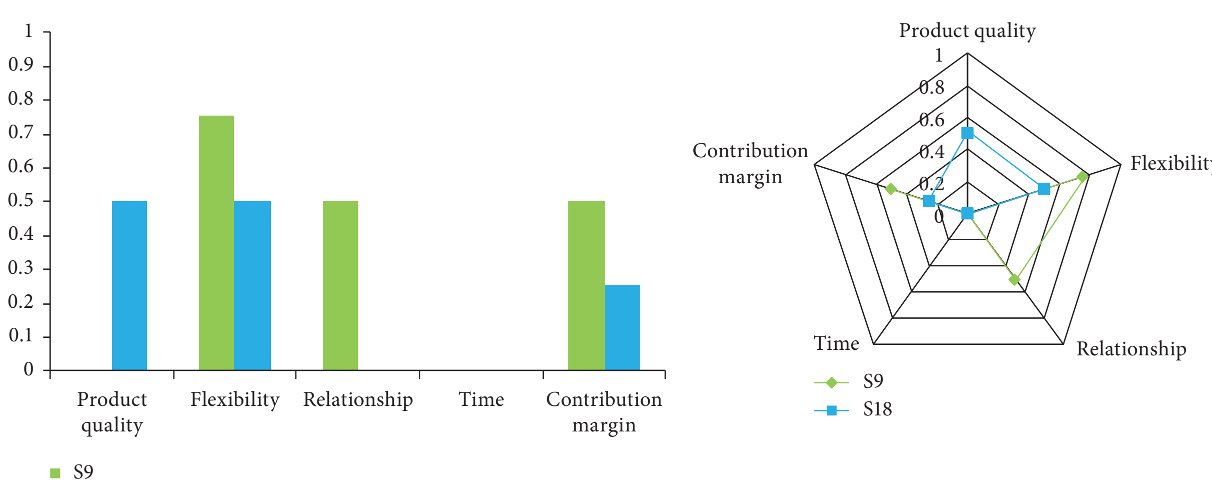
(c)

Figure 8: Radar graphic to compare alternatives S3, S14, and S17.



(a) (b)

Figure 9: Graphics to evaluate alternatives in position seven.



(a) (b)

Figure 10: Graphics to evaluate alternatives in position eight.

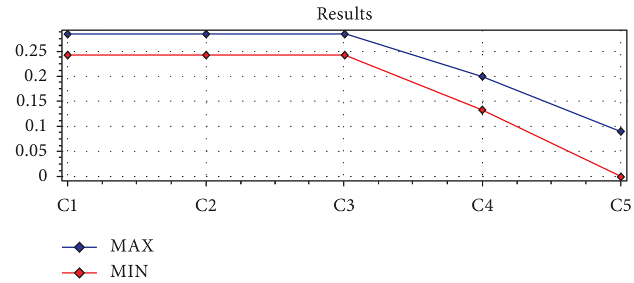


Figure 11: Scaling constant space.

knowledge about the situation faced. Therefore, the selection and approval of suppliers will be more judicious, which will generate greater confidence in the selection.

Some studies were developed regarding the theme supplier selection, as presented before. The study of Palha et al. [14] also presented a holistic model; nevertheless, the focus of their study was not a selection model but a categorization of services to be supplied in the civil construction. Araújo et al. [13] proposed a MCDM/A model for ranking suppliers in the food industry. Nevertheless, in their problem they had a group of decision-makers involved in the selection process, being necessary to use a multicriteria method for group decision. A study using the FITradeoff method for a choice problem for supplier selection in a food manufacturer has already been developed [18]. Nevertheless, it had only five alternatives, presenting a typical case of a choice problematic.

In our study, although we use the same method, it is applied to a ranking problem which is a more robust problematic since this leads the DM to have more information than the choice problematic provides. The actual problem involved 20 alternatives, which shows that this is a complex MCDM/A problem with a large number of consequences for the DM evaluate. The problem is in the context of a waterproofing company, and the ranking was obtained enabling the DM to assess the alternatives placed in the first position of the ranking in order to assist the decision about which one was to be selected. Our study presented graphical visualization analysis, which contributes to supporting the DM during an MCDM/A problem. Also, these visualizations are a flexible tool which can improve the FITradeoff DSS.

Thus, the proposed model is an important managerial tool because it reveals what suppliers are the most suitable due to the fact that they meet the company's selection criteria. Therefore, FITradeoff proved to be a flexible and efficient method for the supplier selection problem, in which companies must define the direction and frequency of this process, depending on the type of alliance to be formed with their suppliers.

5. Conclusions

This study sought to propose a supplier selection model for a wholesaler and retailer of the construction sector, in order to assist the DM in the process of selecting new suppliers, with the objective of keeping the products and suppliers in line with the company's strategic plans and objectives. This process is crucial to the competitiveness and sustainability of the company's business and its supply chain as a whole.

This paper has demonstrated that, in a supplier selection process, several important criteria that are aligned with the strategic interests of the company must be included in the problem of deciding on the best supplier(s). Thus, the supplier selection process is a multicriteria decision problem. Several multicriteria decision models have been proposed in the literature to elicit DMs' preferences in supplier selection problems. However, traditional compensatory methods have become tiring, tedious, and difficult for DMs and have led to many inconsistencies when evaluating alternatives as they consistently fail to meet DMs' preferences, leading to DMs abandoning the methodologies or to the results having no credibility [16–18].

In order to overcome these problems of inconsistencies in the evaluation of alternatives, interactive decision models can lead to less inconsistency in the results by working with partial information on DMs' preferences. Providing partial information requires less effort from the DM during the elicitation process. The FITradeoff method used in this study is for ranking problems and uses tradeoff judgments provided by DM for an easier decision-making process. Elicitation is conducted interactively with the DM using a DSS, which provides graphical visualization to assist the DM in the analysis of preliminary results. Some DMs would like to use the graphical visualization and others do not. However, in a general way, the graphics are presented in the DSS to assist the DMs.

This study contributes to the empirical research on the applicability of the FITradeoff method to solve a supplier selection problem in the retail and wholesale sector, considering the ranking problematic. Thus, this study can demonstrate that the FITradeoff method can be applied to aid the DM in solving a supplier ranking problem in a more flexible way. Using a questionnaire applied through FITradeoff DSS, fewer and easier questions could be answered by the DM, providing graphs that facilitated the process of analyzing the ranking position of suppliers. Data visualization also supported the DM to understand and make his decision process faster and more reliable, considering the aspects that the company judged important to be approached.

One limitation of this study may be the analyst's lack of skill and knowledge in the interpretation of the data to guide the DM, in which neuroscience studies can be used to support the advising process performed by the analyst [19–21].

As a suggestion for future studies, it could be to identify standard criteria for selecting suppliers in companies in the same segment as the company in this study and to analyze how the proposed model would behave for companies in the same segment where the decision is made by a group of DMs.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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Research Article

A Design-Task-Oriented Model Assignment Method in Model-Based System Engineering

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In model-based system engineering (MBSE), reuse of existing models in the development of a new system can be advantageous. Automatic assignment of existing models to each design task within a design task set has been proven to be feasible. However, while several studies have discussed the significance of models in MBSE and methodologies for models reuse, solving the model reusability problem through a model assignment method has not been discussed. Additionally, a significant challenge in model assignment is to address the conflict between the maximization of the model value summations, which are yielded by assigning the models to a design task set, and the minimization of the execution cycle of the task set. This study (a) proposes a design-task-oriented model assignment method that establishes a multiobjective model, based on a model assignment integration framework, and (b) designs a differential-evolution-combined adaptive nondominated sorting genetic algorithm-II to provide an optimal tradeoff between maximizing the total model values and minimizing the execution cycle of the task set. By comparing the performance of the algorithm in resolving the assignment of models to a design task set with those of two conventional algorithms in a phased-array radar development project, the algorithm's performance and promotion of system development are verified to be superior. The new method can be applied for developing model scheduling software for MBSE-compliant product development projects to improve using effects of the models and development cycle.

1. Introduction

Considering the increasingly sophisticated customer demands and the growing requirements for increased product development capabilities—given that products are more integrated and intelligent in various industries, including aviation and space—the traditional product development mode is no longer satisfactory [1]. Model-based system engineering (MBSE) accomplishes the development of complex products with a new mode and is capable of forecasting product behaviors, thereby improving the productivity of the product development process. Arguably, the developments of aviation and space systems, which are regarded as the most complex cyber-physical systems (CPSs) in the industry [2], have adopted the MBSE approach to facilitate the implementation of all the phases of the product lifecycle [3–8]. The widespread use of MBSE has tended to shift emphasis from data management to model management throughout the entire product's lifecycle [9].

MBSE is the formalized application of modeling used to support system requirements, design, analysis, and verification and validation activities, beginning with the conceptual design phase and continuing throughout the development and later lifecycle phases [10]. The output of the MBSE activities is a coherent model of the system (i.e., system model), whereby the emphasis is placed on the evolution and the refinement of the model using model-based methods and tools [11]. Therefore, the model plays the most important role in each stage of the product's lifecycle.

In the concept and design phases, a shared system model is needed to support the exchange of information across various aspects. Accordingly, this system model serves as the core model of the system to provide information and maintain consistency with domain-specific models. For example, the Space Systems Working Group of International Council on Systems Engineering developed the CubeSat reference model for mission-specific CubeSat teams [6]. The German Aerospace Center presented the conceptual data model as an abstraction of domain-specific models [5]. In the development of mechatronic systems, Barbieri et al. used the conceptual model as the information source for domain-specific models of every functional module [12]. Researchers from nine leading Chinese academic and industrial institutions have gathered to discuss the definition and application of MBSE-compliant product meta-models [9].

MBSE has also found application in the early tender phases of complex CPS development, wherein the complex CPS customer can use the MBSE approach to generate a model-based request for tenders and pass it on to the supplier who can use the model to perform system development. Australia's Defense Science and Technology Organization pioneered the adoption of a whole-of-system analytical framework for information transfer across the contractual interface [8]. The integrated design methodology proposed in [12] also demonstrated that the MBSE approach can be used to fulfill stakeholder requirements by adopting them in the system requirements. In addition to the above stages, MBSE can also be used as an effective method for product manufacturing system planning in the manufacturing phase [13, 14].

At a higher level, a model-based system analysis framework is needed to provide the capability to access, integrate, and transform disparate data into actionable information for the design and analysis of complex systems. The United States (U.S.) Air Force (AF) established the digital thread initiative [3, 15] that generates an engineering analytical framework based on an authoritative digital surrogate representation throughout the entire product's lifecycle.

The most commonly used modeling language in MBSE is the system modeling language (SysML), a general-purpose graphical modeling language that supports analysis, specification, design, verification, and validation of complex systems [11, 16], and has been adopted by many MBSE projects [1, 9, 12, 13, 17]. Despite the fact that it is accepted by the Object Management Organization as a standard modeling language, SysML is not easy to adapt for system engineers who have not been exposed to object-oriented concepts because, like the unified modeling language (UML), it emphasizes familiarity with these concepts. For ease of use, organizations have developed some modeling languages, including the modeling and analysis of real-time embedded (MARTE) systems [18], architecture analysis and design language (AADL) [2], domain-specific modeling language (DSML) [19], and others. Based on these languages, some powerful MBSE platforms and tools have been developed. For example, Thales' ARCADIA™ and Capella™ workbench [20, 21] support requirement analyses and system design in the areas of transportation, aviation, space, and radars, while Tucson Embedded Systems' AWESUM™ tool suite supports the U.S. Army's joint common architecture project [2]. In addition, the U.S. Department of Defense's high-performance computing modernization program has developed a computational research engineering acquisition tools environment for air vehicles [3] and realized the digital thread of the U.S. AF.

With the increasing application of the MBSE approach in the industry, numerous models built using various modeling languages have been stored by various organizations adopting MBSE. However, the models are often not reused effectively. When faced with a new development project, the development teams often create new models, rather than reusing the existing models available within each discipline. This repetition of work amounts to an unnecessary expenditure of cost and time for the project. To reuse existing models, appropriate models should be identified and assigned to each of the design tasks in the development project. While literature on model reuse has described approaches for applying development environments [22], ontologies [23], and model repositories [24, 25], no previous studies have discussed model assignments. Therefore, this study focuses on the establishment of a design-task-oriented model assignment method to support model reuse in MBSE. The study includes the following main components.

(i) An integration framework capable of assigning the models in the repository to the design task set is established. In existing MBSE approaches, simply integrating the tools into the product lifecycle development environment [2, 3] does not enable model reuse because it does not consider how the stored models are assigned to the design tasks of the project. The framework proposes an optimization scheme for matching the models and the design tasks to support the needs of the MBSE platform and tools for models reuse.

(ii) The value of each model for a design task is quantified. This captures the suitability of the model for the task. The literature regarding model management in MBSE and software engineering has focused on model management platform [26], model repository building methods [27], and so on, but has not provided a method for the evaluation of model value for the task. This study applies an advantage-number-based analytical technique to evaluate the models to be assigned from the perspective of value and, accordingly, preferentially filters the models to establish a design-task-oriented preferred model set.

(iii) A mathematics model for design-task-oriented model assignment and an optimization algorithm are proposed. This study suggests a multiobjective model of design-task-oriented model assignments to minimize the task set execution cycle and maximize the actual value summation of the models. Additionally, to solve the proposed multiobjective model, the study has designed a differential-evolution-combined adaptive nondominated sorting genetic algorithm-II (DA-NSGA-II).

Finally, based on a case study, the new algorithm is proven to have better performance and promotion of system development than the traditional nondominated sorting genetic algorithm-II (NSGA-II) and particle swarm optimization (PSO).

The rest of this paper is organized as follows. The second section describes a design-task-oriented model assignment integration framework. In the third section, a multiobjective model of model assignment is established according to the quantification of model value. The fourth section proposes an improved NSGA-II to solve the multiobjective model. A case study based on a phased-array radar development project is discussed in the fifth section. Finally, the sixth section focuses on the study's conclusion and the potential avenues for future work.

2. Design-Task-Oriented Model Assignment Integration Framework

In the field of MBSE, model reuse is able to improve the efficiency and reduce the cost of system development. It is a feasible method to assign the existing models to the tasks within a design task set under the condition of discipline matching between the models and the tasks. Using the method, a suitable model is selected for each task according to the model value for a design task and the execution cycle of the task set following the assignment of the models to the task.

Owing to the different properties of the model for different tasks, such as integrality and reliability, the same model would yield different values when applied to various tasks. A model is considered to yield a high value if it can improve the execution effect of a task that it is assigned to. By contrast, the same model is considered to yield low value if it would worsen the execution effect of another task that it is assigned to. In general, model assignment attempts to achieve the highest possible summation of the models' actual values once the models are assigned to the tasks.

In addition, the execution cycle of a single task is different when different models are applied. For a given task set restricted by the temporal relation of the tasks, the applied model assignment strategy determines the execution cycle of the task set. Thus, to complete the task set as soon as possible, another objective of the model assignment is the minimization of the task set execution cycle.

For a simple system, the existing models can be manually assigned to each task in the design task set with ease. However, when the developed system is relatively complex, the model values for a task cannot be directly measured due to the complex model properties and wide task ranges. Thus, it is difficult to directly compare the values of a set of models for a particular task. Moreover, in the development of a complex system, a single model assignment scheme cannot simultaneously satisfy the requirements of maximizing the total model value and minimizing the task set execution cycle. The model assignment solutions must be optimized to determine the optimum scheme. However, the optimization cannot be performed by a human due to the large number of models and tasks in a complex system.

Consequently, a framework for model assignment integration must be established in the field of MBSE in order to quantitatively evaluate the model values and perform an optimal tradeoff between the maximization of the total model values and the minimization of the execution cycle. By doing so, an optimum solution is obtained allowing the assignment of models to the design task set. The current study establishes a design-task-oriented model assignment integration framework, as presented in Figure 1. The key features of the framework are as follows.

(i) A model repository needs to be defined, and the existing models of previous projects should be saved in the repository.

(ii) The design task set is derived from task planning.

(iii) The values of the models from the model repository, with respect to the design tasks, are evaluated quantitatively. Next, a preferred model set is generated after an optimal selection of models based on the quantitative values of the models.

(iv) According to the preferred model set value and cycle matrices concerning the design task set, a multiobjective model that relates the maximization of the models' actual value summation and the minimization of the task set execution cycle is established.

(v) The model assignment scheme is obtained by a multiobjective optimization algorithm.

3. Modeling of Design-Task-Oriented Multiobjective Model Assignment

3.1. Calculation of Model's Actual Value for Task. Based on the model assignment integration framework, models are selected from the model repository. This process involves the calculation of the actual values of different models for each of the specified tasks.

The given development project applies the mode of MBSE assuming that the task set $T = \{T_1, T_2, \cdots, T_n\}$ has n models. Additionally, the model repository related to the task set has l models, with $M = \{M_1, M_2, \cdots, M_l\}$ as the model set and $S = \{s_1, s_2, \cdots, s_k\}$ as set of the models' attributes.

Definition 1. The value of the model Mi for task Tj is v_{ij}.

$$v_{ij} = \sum_{g=1}^{k} w_g c_j^i (s_g), \tag{1}$$

where $g = 1, 2, \cdots, k$, $i = 1, 2, \cdots, l$, $j = 1, 2, \cdots, n$, w_g is the weight of the model's attribute, and $c_j^i(s_g)$, which is expressed by the five-demarcation method as $\{1, 3, 5, 7, 9\}$, is the value of the attribute s_g of model Mi after it is assigned to task Tj.

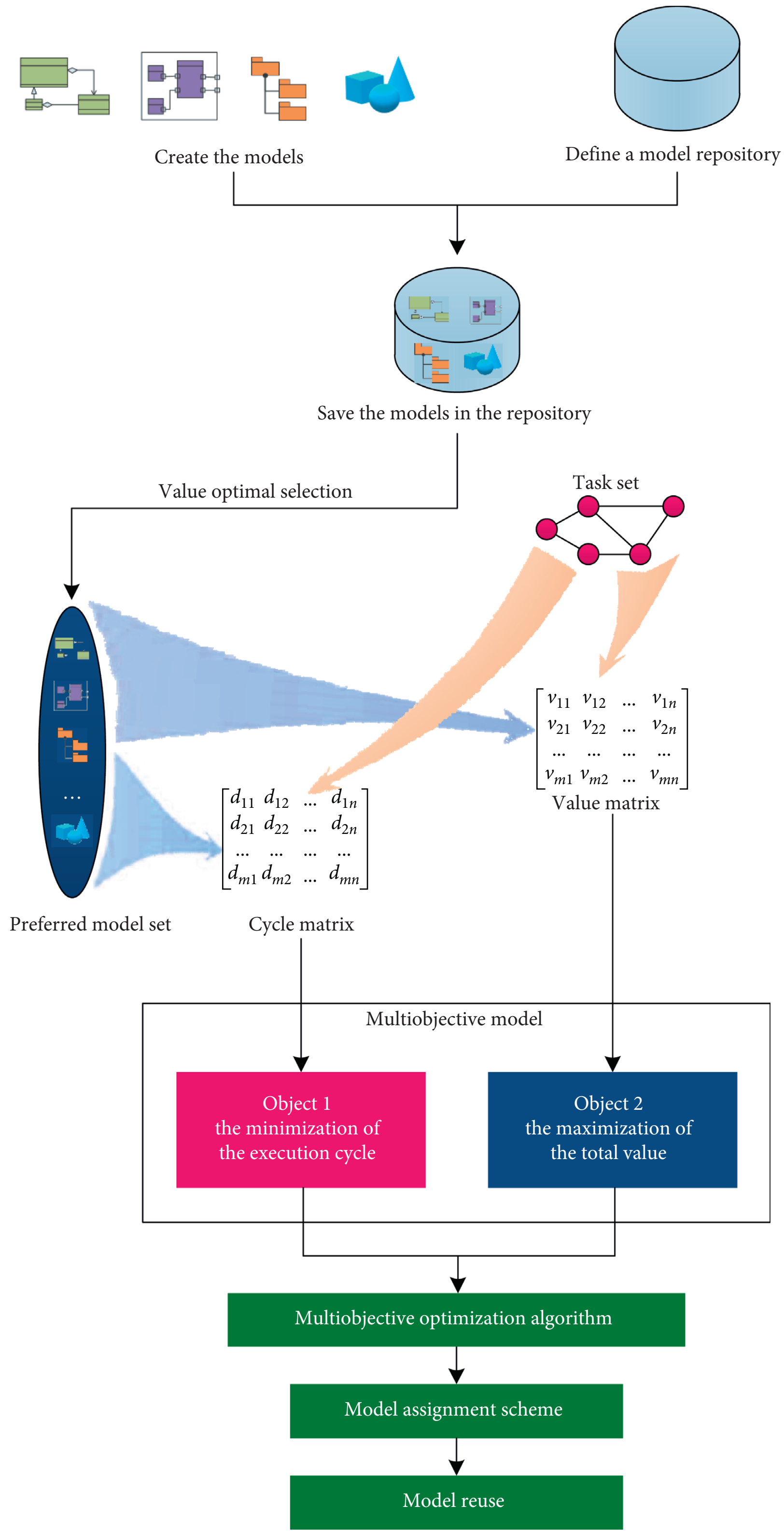

Figure 1: Design-task-oriented model assignment integration framework.

Here, the model's attribute weight w_g is determined using a method based on the significance of the Pawlak attribute in the decision table according to rough set theory [28]. To list the model values of the decision table, select the original assessment data of l' models from the model repository as the universe of discourse U, condition attribute set S as the set of model attributes $\{s_1, s_2, \cdots, s_k\}$, formulate the decision attribute set D as the set of model values $\{v\}$, and assign the field value according to {high, middle, low}. The decision table is thus obtained.

Table 1: Decision table of model values.

Universe of discourse U	Model attributes				Value
	s_1	s_2	$\cdots$	s_k	v
1	Middle	Low	$\cdots$	High	Low
2	Middle	High	$\cdots$	Middle	Middle
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
l'	Low	High	$\cdots$	High	High

The significance of model attribute s_g in Table 1 varies with decision attribute v. In order to determine the significance, we investigate how the decision table classification varies with the removal of the model attribute from the decision table. Generally, if model attribute s_g is deleted from condition attribute set S, then the impact of deleting s_g on the classification ability of S relative to decision attribute v increases with the value of $\gamma_{\mathrm{IND}(S)}(v) - \gamma_{\mathrm{IND}(S-s_g)}(v)$, namely, s_g becomes more significant for S relative to decision attribute v.

Definition 2. The significance of the model attribute s_g for the condition attribute set S relative to the decision attribute v is formulated as

$$\mathrm{sig}\left(s_g, S; v\right) = \gamma_{\mathrm{IND}(S)}(v) - \gamma_{\mathrm{IND}\left(S-s_g\right)}(v) = \frac{\left|\mathrm{pos}_{(S)}(v)\right| - \left|\mathrm{pos}_{\left(S-s_g\right)}(v)\right|}{|U|}, \quad g = 1, 2, \cdots, k, \tag{2}$$

where $\gamma_{\mathrm{IND}(S)}(v)$ is the approximation quality of v by S, $\gamma_{\mathrm{IND}(S-s_g)}(v)$ is the approximation quality of v by $S - s_g$, $\mathrm{pos}_{(S)}(v)$ is the S- positive region of v, and $\mathrm{pos}_{(S-s_g)}(v)$ is the $(S - s_g)$- positive region of v.

Therefore, the weight of the model attribute s_g is obtained as follows:

$$w_g = \frac{\mathrm{sig}\left(s_g, S; v\right)}{\sum_{g=1}^{k} \mathrm{sig}\left(s_g, S; v\right)}. \tag{3}$$

3.2. Model Selection Based on Advantage Number Analysis. To improve the development efficiency, an optimal selection must be conducted before the model assignment to filter out parts of the preferred models for specific tasks, given that not all the models would be applied in all the tasks. In this case, the model selection uses an approach based on advantage number analysis.

According to Definition 1, the values of the l models $M_1, M_2, \cdots, M_l$, for task Tj provide the value vector $\mathbf{V_j} = (v_{1j}, v_{2j}, \cdots, v_{lj})^T$. Set the highest value component of $\mathbf{V_j}$ as the ordinal number one, the second-highest value component as the ordinal number two, and so on. Further, $\mathbf{V_j}$ can be converted in the ordinal number vector $\mathbf{R_j} = (r_{1j}, r_{2j}, \cdots, r_{lj})^T$ of the values of l models for task Tj, whereby r_{ij} $(i = 1, 2, \cdots, l)$ denotes the order of model Mi in the model value list for task Tj.

Correspondingly, in the task set, n tasks produce n ordinal number vectors, and all the ordinal number vectors would constitute a model value ordinal number matrix with $l \times n$ dimensions.

$$\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ r_{l1} & r_{l2} & \cdots & r_{ln} \end{bmatrix}. \tag{4}$$

Based on the advantage number analysis method, the respective advantage number of each element in matrix $\mathbf{R}$ would be

$$a_{ij} = l + 1 - r_{ij}. \tag{5}$$

The ordinal number r_{ij} can be converted to an advantage number a_{ij}. Apparently, a lower ordinal number generates a higher model value on the specific task and a bigger advantage number. Therefore, the model value ordinal number matrix can be converted to a model value advantage number matrix.

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{l1} & a_{l2} & \cdots & a_{ln} \end{bmatrix}. \tag{6}$$

By summing up all the advantage numbers on each line in $\boldsymbol{A}$, the advantage number summation of the specific model corresponding to the line is obtained.

$$A_i = \sum_{j=1}^{n} a_{ij}, \quad i = 1, 2, \ldots, l. \tag{7}$$

Upon selection of a proper threshold λ, all the $A_i \geq \lambda$ models are the assignable models selected from the model repository. Assuming that the number of the selected models is m, the value matrix of m selected models for n tasks of the task set can form an $m \times n$ dimensional matrix.

$$\mathbf{V} = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1n} \\ v_{21} & v_{22} & \cdots & v_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ v_{m1} & v_{m2} & \cdots & v_{mn} \end{bmatrix}. \tag{8}$$

3.3. Establishment of a Multiobjective Model of Model Assignment. Without considering possible constraints on resources, the problem associated with the design-task-oriented model assignment in MBSE involves n tasks within a development project that applies an MBSE mode. Precedence relations exist between some of the tasks that prohibit the onset of the task Tj ($j = 2, 3, \cdots, n$) before all of its precedence tasks T_h ($h \in P_j$) are completed. Secondly, the model repository involves m selected models for all the specified tasks. Task Tj ($j = 1, 2, \cdots, n$) has to choose one of the selected models to be performed, and the model cannot be ceased or changed to another model form during the task once the model is assigned selectively. The actual value of model Mi is different after it has been assigned to different tasks. Meanwhile, for task Tj, the cycle varies depending on the application of different models. Thus, the cycle matrix of m selected models for n tasks in the task set can be expressed as

$$\mathbf{D} = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & d_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ d_{m1} & d_{m2} & \cdots & d_{mn} \end{bmatrix}. \tag{9}$$

The model assignment must meet the two objectives, namely, the minimization of the task set execution cycle and the maximization of the models' actual value summation.

A decision variable can be introduced as

$$x_{ijt} = \begin{cases} 1, & \text{model } M_i \text{ is assigned to task } T_j \text{ and task } T_j \text{ is accomplished at phase } t, \\ 0, & \text{others.} \end{cases} \tag{10}$$

Thus, the design-task-oriented model assignment in MBSE is presented as a multiobjective model according to

$$\min F_n = \sum_{i=1}^{m} \sum_{t=EF_n}^{LF_n} t \cdot x_{ijt}, \tag{11}$$

$$\max V = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijt} v_{ij}, \tag{12}$$

such that

$$\sum_{i=1}^{m} \sum_{t=EF_j}^{LF_j} x_{ijt} = 1, \quad j = 1, 2, \cdots, n, \tag{13}$$

$$\sum_{i=1}^{m} \sum_{t=EF_h}^{LF_h} t \cdot x_{iht} \le \sum_{i=1}^{m} \sum_{t=EF_j}^{LF_j} \left(t - d_{ij}\right) x_{ijt}, \quad j = 2, 3, \cdots, n; h \in \mathscr{P}_j, \tag{14}$$

$$x_{ijt} \in \{0, 1\}, \quad i = 1, 2, \cdots, m; j = 1, 2, \cdots, n; \quad t = EF_j, \cdots, LF_j. \tag{15}$$

Equations (11) and (12) are objective functions that, respectively, denote the minimization of the task set execution cycle and the maximization of the models' actual value summation (after the models are assigned to the tasks). Equation (13) indicates that each task requires only a single model and can be executed only once. Equation (14) indicates that task Tj can only be started if all the precedence tasks T_h are accomplished. Equation (15) defines the range of values of the variable. Correspondingly, a solution of the multiobjective model is achieved after the determination of each model's assignment manner and each task's completion time Fj.

4. Algorithm for the Solution of the Multiobjective Model Assignment

The design-task-oriented model assignment problem is a multiobjective optimization problem. Many optimization algorithms have been developed to solve the optimization problem [29, 30], such as NSGA-II presented by Deb et al. [31, 32] and PSO developed by Kennedy and Eberhart [33]. The algorithm was proved to be superior to other evolutionary algorithms regarding the overall fitness [34]. In MBSE projects, the algorithm was applied to aviation [15], space [7, 35], software [36], and manufacturing [37]. In these applications, the researchers of NSGA-II achieved improvements.

This study introduces the DA-NSGA-II algorithm to solve the proposed design-task-oriented model assignment. The algorithm framework is shown in Figure 2 and the operating procedures are described below:

Procedure 1: establishment of the initial population. The codes of the chromosomes are generated based on the different task assignment manners of each selected model. The code length equals the task quantity n, and the code bits denote the corresponding tasks. The value range of each code bit ranges from 1 to m, while the actual value is decided by the model code that is being assigned to the specific task represented by code bits. According to the coding rule, the initial population is obtained to utilize the individuals that are randomly generated according to the population size to meet the constraints.

Procedure 2: the first fast nondominated sorting. Upon the calculation of each individual's execution cycle of the task set and the actual value summation of the models, the fitness values of the individuals are obtained. Each individual's ordinal number and crowding distance are then obtained by fast sorting the individuals in a nondomination manner based on the fitness value.

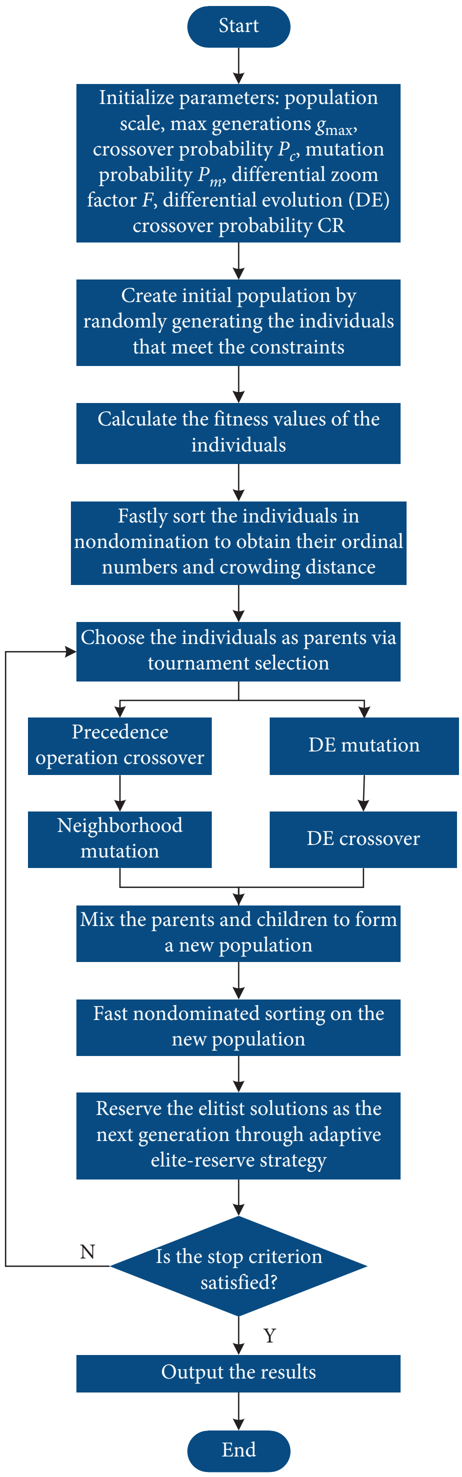

Figure 2: DA-NSGA-II framework.

Procedure 3: crossover and mutation. Consider some of the individuals selected via tournament selection as parents. First, the children with an equal number of the parents are achieved with precedence operation crossover (POX) and neighborhood mutation. To increase the diversity of the children, a differential evolution (DE) algorithm is used on the parents to generate the second batch of children with an equal number of the parents.

Procedure 4: the second fast nondominated sorting. The two batches of children obtained based on crossover, mutation, and DE are introduced in the population, and each individual's ordinal number and crowding distance can then be obtained by executing the second fast nondominated sorting on the new population.

Procedure 5: we introduce an adaptive algorithm in the elite-reserve solution and reserve the elitist individuals from the lowest ordinal number to the highest one as the next generation until the number of individuals reaches the defined population size. In the cases of individuals with the same ordinal numbers, those with larger crowding distances are preferred to be reserved. The reservation rate is

$$\rho = \frac{1}{1+e^{-t}}, \tag{16}$$

where t indicates the number of iterations.

Procedure 6: we output the results once the stop criterion is satisfied, which, in our case, is the maximum number of generations. Alternatively, the execution jumps to Procedure 3.

5. Case Study

5.1. Assignment Solution of Models to the Design Task Set in a Phased-Array Radar Development Project. The MBSE method was applied in a phased-array radar development project. The project's task set involved 14 tasks: T_1 for logical architecture decomposition, T_2 for the simulations of the main lobe and the side-lobe characteristics of the radar antenna, T_3 for interference suppression simulations, T_4 for clutter suppression simulations, T_5 for the assignment of component functions, T_6 for component interface definitions, T_7 for amplitude–phase consistency design, T_8 for radio frequency (RF) modeling, T_9 for RF simulations, T_{10} for scan matching simulations, T_{11} for small-scale modeling, T_{12} for small-scale simulations, T_{13} for radar cross section simulations, and T_{14} for virtual system integration analysis. The temporal relations among tasks are precedence restrictions (i.e., start–end relations) (see Figure 3 for the temporal relations).

There are 15 models whose disciplines match the task set. Each model involves five attributes, that is, model integrity (s_1), simulation operating efficiency (s_2), simulation confidence (s_3), model compatibility (s_4), and model interoperability (s_5). See Table 2 for the execution cycles of each task when separate models are applied.

The objective of the case is to find the optimal solution that assigns the models to the design tasks in the task set based on the proposed method. We first perform an optimal selection of the models. To calculate the model values, the weights of the model attributes have to be determined as a prerequisite. Selecting 12

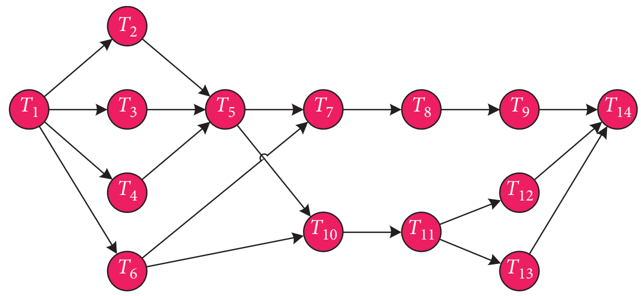

Figure 3: Temporal relations between model-based design tasks of a phased-array radar.

Table 2: Execution cycles of each task when separate models are applied (unit: day).

Model	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}
M_1	3	5	18	9	4	6	8	3	2	4	8	10	11	7
M_2	3	4	11	11	5	9	8	4	4	4	9	9	9	8
M_3	2	6	14	10	3	7	7	3	3	7	6	11	10	6
M_4	3	5	16	13	4	8	10	5	5	5	8	8	12	8
M_5	5	7	12	10	6	5	9	6	2	5	11	12	11	7
M_6	4	9	9	8	4	7	8	2	2	3	7	14	9	12
M_7	3	5	11	12	4	6	8	4	3	4	10	9	13	10
M_8	6	8	17	15	6	4	11	3	1	6	8	10	10	5
M_9	5	6	13	9	5	10	9	6	3	4	12	7	15	11
M_{10}	4	8	12	7	7	9	6	3	4	5	7	15	17	9
M_{11}	5	7	15	14	4	6	10	2	1	3	11	9	14	7
M_{12}	6	4	13	11	5	7	9	4	5	6	10	13	10	9
M_{13}	2	3	14	12	6	8	11	5	2	7	8	8	8	6
M_{14}	4	6	10	8	5	5	7	3	4	5	9	11	12	7
M_{15}	3	5	12	9	3	6	12	5	3	4	9	9	16	10

original data of model value assessment as the universe of discourse U, the conditional attribute set indicates the model attributes $\{s_1, s_2, s_3, s_4, s_5\}$, and the decision attribute set indicates the model value $\{v\}$, thus establishing the decision table with the model values, as shown in Table 3.

According to (2) and (3), the weights of the model attributes s_1, s_2, s_3, s_4, and s_5 would be

$$\begin{aligned}
w_1 &= \frac{\mathrm{sig}(s_1, S; v)}{\sum_{g=1}^{5}\mathrm{sig}(s_g, S; v)} = \frac{0.5}{1.584} = 0.316,\\
w_2 &= \frac{\mathrm{sig}(s_2, S; v)}{\sum_{g=1}^{5}\mathrm{sig}(s_g, S; v)} = \frac{0.167}{1.584} = 0.105,\\
w_3 &= \frac{\mathrm{sig}(s_3, S; v)}{\sum_{g=1}^{5}\mathrm{sig}(s_g, S; v)} = \frac{0.167}{1.584} = 0.105,\\
w_4 &= \frac{\mathrm{sig}(s_4, S; v)}{\sum_{g=1}^{5}\mathrm{sig}(s_g, S; v)} = \frac{0.417}{1.584} = 0.263,\\
w_5 &= \frac{\mathrm{sig}(s_5, S; v)}{\sum_{g=1}^{5}\mathrm{sig}(s_g, S; v)} = \frac{0.333}{1.584} = 0.210.
\end{aligned} \tag{17}$$

According to Definition 1, model values can be obtained for various tasks. For instance, the 15 model values for task T_1 are listed in Table 4.

Therefore, the value vector of the 15 models for task T_1 is $\mathbf{V}_1 = (5.099, 5.205, 4.889, 5.311, 3.835, 4.787, 5.731, 3.731, 4.679, 4.997, 3.945, 4.571, 5.627, 3.521, 4.995)^T$, and it would be transformed to the ordinal number vector $\mathbf{R}_1 = (5, 4, 8, 3, 13, 9, 1, 14, 10, 6, 12, 11, 2, 15, 7)^T$. According to (4), the ordinal number matrix $\boldsymbol{R}$ composed of the ordinal number vectors of all the tasks from the task set can be formulated. According to (5), the advantage number matrix $\boldsymbol{A}$ can be realized based on the conversion from the ordinal number matrix. Finally, based on (7), the advantage number summations of each model can be achieved, as listed in Table 5.

According to the experience, choosing the threshold $\lambda = 106$, the assignable selected model set in the model repository shall be $\{M_1, M_2, M_4, M_5, M_6, M_8, M_9, M_{11}, M_{12}, M_{13}\}$. The value matrix of the 10 models in the selected model set for the 14 tasks in the task set can be expressed as Table 6.

We attempted to solve the assignment of each selected model to the task set by utilizing DA-NSGA-II according to the objective functions of (11) and (12), based on the assumption of the following parameters: 200 for the initial

Table 3: Decision table with a phased-array radar's model values.

Universe of discourse U	Conditional attributes S					Decision attribute D
	s_1	s_2	s_3	s_4	s_5	v
1	Middle	Low	Middle	Middle	Middle	Low
2	Middle	High	Middle	Middle	Middle	Middle
3	Middle	High	Middle	Middle	High	High
4	Middle	Low	Low	Middle	Low	Low
5	Middle	Low	Middle	Middle	Low	Middle
6	Middle	High	Middle	High	Middle	High
7	Low	High	High	Middle	Low	Middle
8	Low	Low	Middle	Middle	Low	Low
9	High	High	Middle	Middle	Middle	High
10	Middle	Low	Middle	High	Low	Middle
11	Middle	Low	Middle	Low	Low	Low
12	Middle	High	High	Middle	Low	High

Table 4: Model values for task T_1.

Model	s_1 0.316	s_2 0.105	s_3 0.105	s_4 0.263	s_5 0.210	v_{i1}
M_1	3	7	5	7	5	5.099
M_2	7	5	7	1	7	5.205
M_3	3	3	7	7	5	4.889
M_4	5	5	3	7	5	5.311
M_5	1	9	7	3	5	3.835
M_6	7	1	5	5	3	4.787
M_7	5	3	1	7	9	5.731
M_8	3	1	9	1	7	3.731
M_9	5	7	9	3	3	4.679
M_{10}	9	5	3	1	5	4.997
M_{11}	7	3	1	1	5	3.945
M_{12}	1	3	7	5	9	4.571
M_{13}	5	1	1	9	7	5.627
M_{14}	3	3	9	1	5	3.521
M_{15}	5	9	1	5	5	4.995

Table 5: Advantage number summations of each studied model.

M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}	M_{11}	M_{12}	M_{13}	M_{14}	M_{15}
108	140	99	129	110	131	98	109	119	101	107	132	118	85	105

Table 6: Value matrix of the selected models for all tasks.

Model	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}
M_1	5.099	3.309	5.209	3.523	6.155	4.573	4.047	4.575	5.523	2.155	5.099	6.153	1.839	6.469
M_2	5.205	5.311	5.417	5.835	4.785	6.151	4.467	4.891	4.681	4.787	4.781	5.835	7.203	6.151
M_4	5.311	5.941	8.151	4.993	3.733	3.103	5.839	8.151	6.361	6.995	3.311	4.047	3.521	3.101
M_5	3.835	4.891	3.627	7.205	6.575	4.259	2.575	3.733	5.627	5.413	4.259	3.627	4.679	5.519
M_6	4.787	4.997	4.891	3.521	4.785	7.729	1.841	6.783	4.997	2.259	7.835	6.993	5.523	6.571
M_8	3.731	2.889	5.627	3.733	4.467	5.419	3.311	1.209	5.519	7.731	6.151	6.677	5.627	2.679
M_9	4.679	5.519	4.783	4.465	6.363	4.783	5.731	4.891	5.207	5.417	4.471	6.257	1.419	4.261
M_{11}	3.945	5.205	6.365	6.255	3.735	6.361	3.101	6.889	4.785	1.735	2.577	2.995	5.625	4.365
M_{12}	4.571	3.521	6.153	5.419	5.101	4.045	6.571	3.209	4.049	4.573	7.309	7.099	5.099	7.415
M_{13}	5.627	5.523	3.101	4.573	5.521	6.889	3.313	4.155	4.993	4.787	4.365	6.677	4.363	2.261

Table 7: Objective values corresponding to the Pareto optimal solution set with DA-NSGA-II.

Object	Solution 1	Solution 2	Solution 3	Solution 4	Solution 5	Solution 6	Solution 7	Solution 8	Solution 9	Solution 10	Solution 11	Solution 12	Solution 13	Solution 14	Solution 15	Solution 16	Solution 17	Solution 18
Execution cycle	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55
Value summation	77.6	81.7	85.4	88.6	90.1	92.3	93.9	94.7	95.2	95.6	96.1	96.3	97.2	97.9	98.1	98.9	99.1	99.3

Table 8: Models for various tasks demonstrated by solution 10 in Table 7.

T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}
M_{13}	M_{13}	M_6	M_5	M_1	M_6	M_{12}	M_4	M_4	M_8	M_6	M_2	M_2	M_{12}

population size, 200 for maximum generations, probability of crossover $\mathrm{Pc} = 0.9$, probability of mutation $\mathrm{Pm} = 0.1$, differential zoom factor $F = 0.5$, and probability of DE crossover $\mathrm{CR} = 0.7$. A set of Pareto optimal solutions were obtained, and the objective values are presented in Table 7.

In the phased-array radar development project, we can select one of the solutions from the Pareto optimal solution set by considering the practical constraint of the execution cycle and the expectation of the of the model value summation to determine the eventual model assignment scheme. For example, the assignment scheme demonstrated by solution 10 is listed in Table 8.

5.2. Algorithms Comparison

5.2.1. Performances Comparison. The traditional NSGA-II and PSO have also been used to solve the model assignment model to verify the performance of the proposed algorithm. The same parameters are preset in both NSGA-II and DA-NSGA-II. The initial population size and maximum generations of PSO are equal to those of DA-NSGA-II, with the remaining PSO parameters set as follows: acceleration constants $c_1 = 0.8$ and $c_2 = 0.8$; inertia weight $\omega_{\max} = 1.2$ and $\omega_{\min} = 0.1$. The comparison diagram of the Pareto fronts generated by the three algorithms is shown in Figure 4.

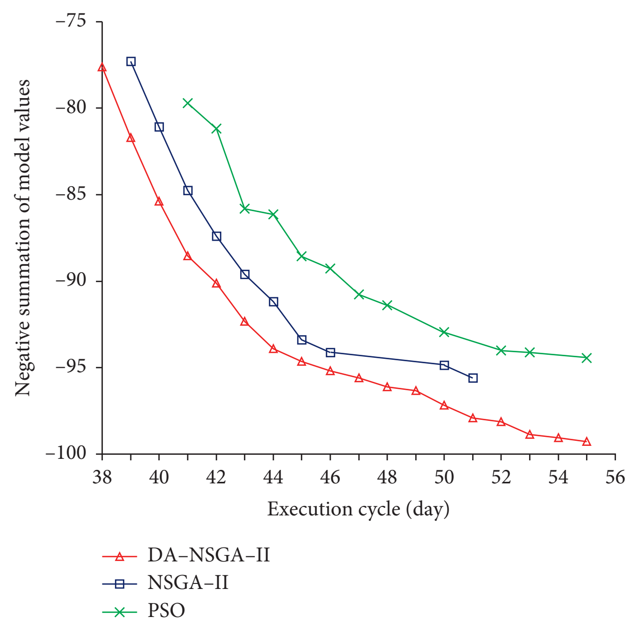

Figure 4: Comparison of the Pareto fronts generated by DA-NSGA-II, NSGA-II, and PSO.

From Figure 4, we can always find superior solutions in the Pareto front of DA-NSGA-II by comparing to the Pareto front of the traditional NSGA-II and PSO. This proves that the convergence of DA-NSGA-II is superior to that of NSGA-II and PSO.

To conduct a quantitative evaluation on the performance of DA-NSGA-II, the study applied the S- and M-measures as the evaluating indicators. Based on the solutions of the model assignment with DA-NSGA-II, NSGA-II, and PSO, Table 9 presents a comparison of S- and M-measures of the three Pareto optimal solutions achieved by executing separately the three algorithms 10 different times.

Table 9 demonstrates that the mean value of the S-measure in the DA-NSGA-II case is lower than those associated with NSGA-II and PSO, and the mean value of the M-measure in the DA-NSGA-II case is higher than those obtained from the execution of NSGA-II and PSO. It also shows that the standard deviations of the S- and M-measures in the DA-NSGA-II case are lower than those in the NSGA-II and PSO cases. These results indicate that the distribution uniformity and the range of the solution outcomes generated by the DA-NSGA-II algorithm are superior to those of the NSGA-II and PSO algorithms-based solutions, and the outputs of DA-NSGA-II are more stable than those of NSGA-II and PSO.

Figure 5 compares the execution cycles and model value summations outputted by DA-NSGA-II, NSGA-II, and PSO from generations 1 to 200. The value of each execution cycle is the minimum execution cycle in the Pareto solution set outputted by the algorithms at each generation (Figure 5(a)). Each model value summation is the maximum summation of the model values in the Pareto solution set outputted by the algorithms at each generation (Figure 5(b)). The execution cycle and model value summations achieved by DA-NSGA-II begin to converge before generation 20, while those of DA-NSGA-II begin to converge at generations 21 and 37, respectively. PSO is not able to attain the optimal values until generations 101 and 135, respectively. These results indicate the faster convergence rate of DA-NSGA-II compared to those of NSGA-II and PSO.

5.2.2. Optimizing Effects Comparison. In this section, we compare the optimizing effects via DA-NSGA-II, NSGA-II, and PSO. First, the execution cycles of the task set corresponding to the model assignment schemes optimized by the three algorithms are compared. The comparison is performed between solutions with equal model value summations. However, from DA-NSGA-II Pareto optimal solution set, we cannot determine a solution with model value summation exactly equal to that of the Pareto optimal solution from NSGA-II or PSO. Therefore, solution sets A and B in the Pareto front generated by DA-NSGA-II are calculated via interpolation. The model value summations corresponding to the solutions in solution set A (B) are equal

Table 9: Comparison of the performances of the three algorithms.

Evaluation index	DA-NSGA-II		NSGA-II		PSO	
	Mean value	Standard deviation	Mean value	Standard deviation	Mean value	Standard deviation
S-measure	0.8201	0.1602	0.8784	0.4468	1.2693	0.8716
M-measure	23.7390	2.5971	21.0179	3.6404	23.2693	3.3199

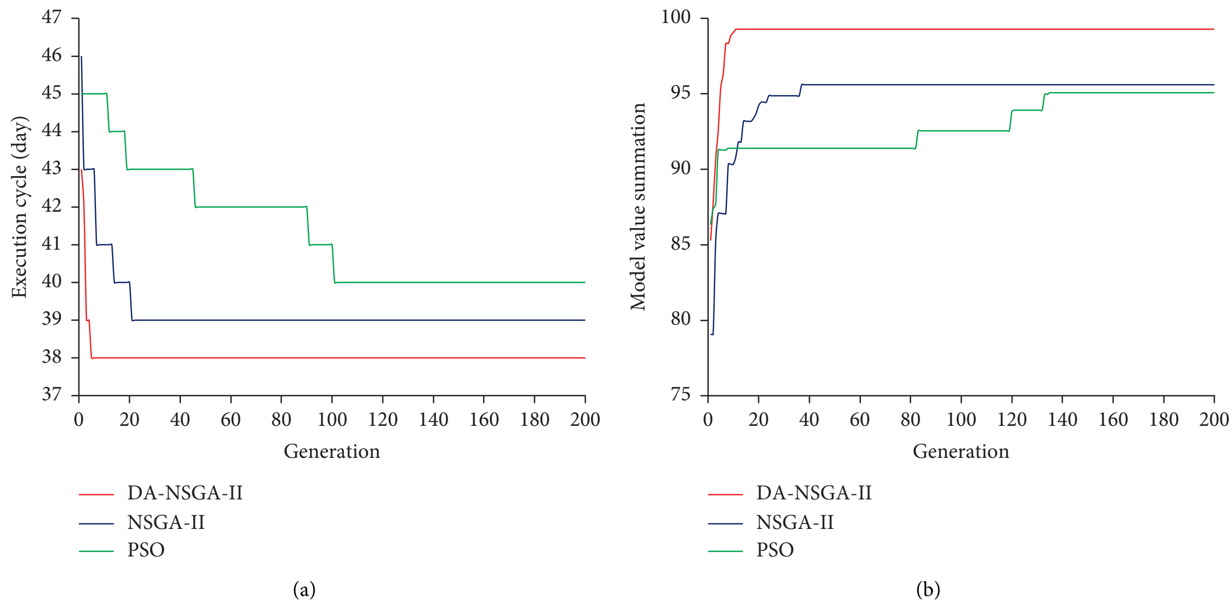

Figure 5: Comparison of the objective values outputted by DA-NSGA-II, NSGA-II, and PSO across multiple generations. (a) Execution cycles of the three algorithms from generations 1 to 200. (b) Model value summations of the three algorithms from generations 1 to 200.

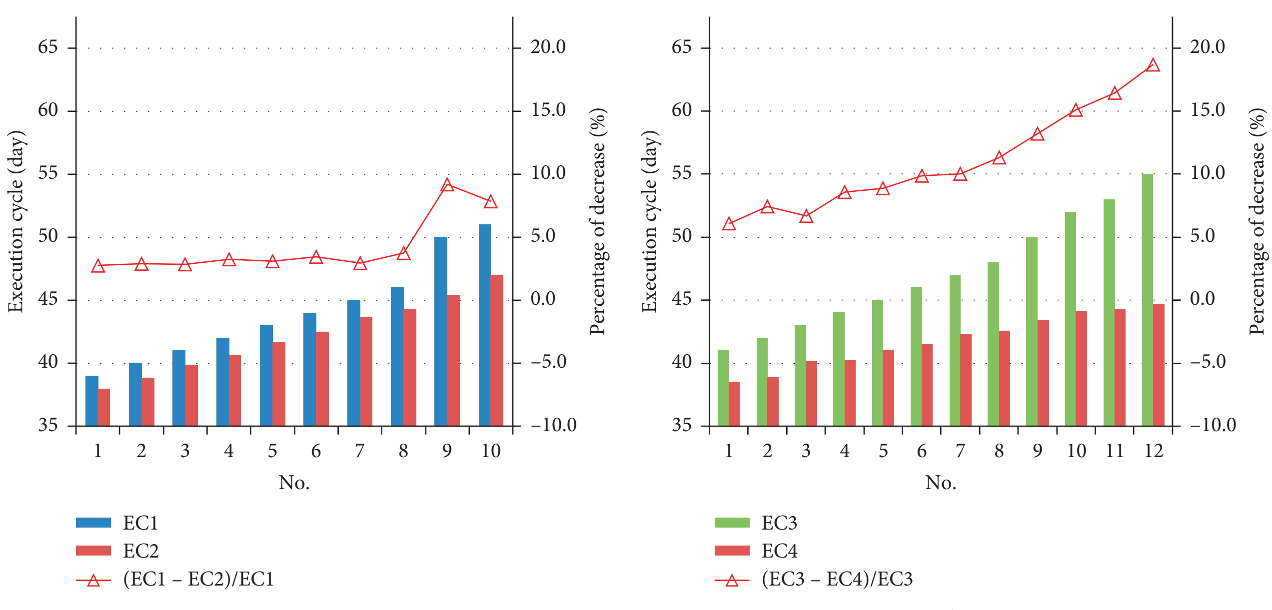

Figure 6: Comparison of the task set execution cycles optimized by the three algorithms, assuming equal model value summations. (a) Comparison of EC2 obtained by DA-NSGA-II and EC1 obtained by NSGA-II. Numbers 1–10 indicate the model value summations in the Pareto optimal solution set from NSGA-II of 77.3, 81.1, 84.8, 87.4, 89.6, 91.2, 93.4, 94.1, 94.9, and 95.6, respectively. (b) Comparison of EC4 determined by DA-NSGA-II and EC3 determined by PSO. Numbers 1–12 indicate the model value summations in the Pareto optimal solution set from PSO of 79.7, 81.2, 85.8, 86.1, 88.6, 89.3, 90.8, 91.4, 93.0, 94.0, 94.1, and 94.4, respectively.

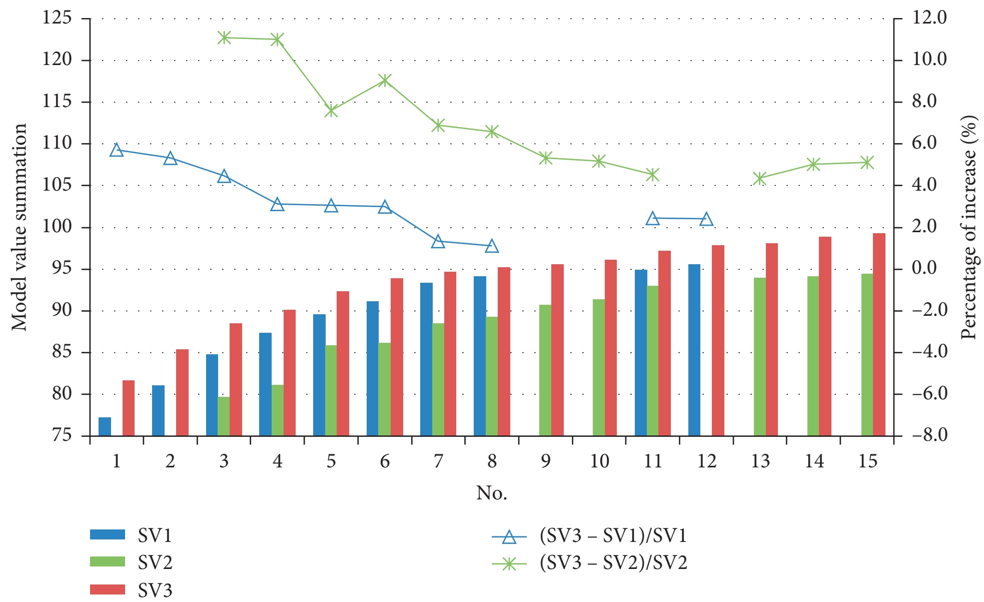

Figure 7: Comparison of the model value summations optimized by the three algorithms under equal task set execution cycles. Numbers 1–15 indicate the task set execution cycles of 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 51, 52, 53, and 55 (unit: day), respectively.

to those of the Pareto optimal solutions based on NSGA-II (PSO). We denote the execution cycle of the task set related to the Pareto optimal solution from NSGA-II, set A, PSO, and set B as EC_1, EC_2, EC_3, and EC_4, respectively. Figure 6 presents the comparison between EC_1 and EC_2 and between EC_3 and EC_4, under the condition that the corresponding summations of the model values are equal.

The bar chart in Figure 6(a) compares EC_1 and EC_2 with equal model value summations from the Pareto optimal solution sets generated by DA-NSGA-II and NSGA-II, respectively, while the line graph presents the percentage decrease of EC_2 compared to EC_1. EC_2 is 2.8–9.2% lower than EC_1. Similarly, the bar chart in Figure 6(b) compares EC_3 and EC_4 with equal model value summations from the Pareto optimal solution sets generated by DA-NSGA-II and PSO, respectively, while the line graph presents the percentage decrease of EC_4 compared to EC_3. EC_4 is 6.1–18.7% lower than EC_3. Thus, the design efficiency of the model assignment solution optimized by DA-NSGA-II is higher than those optimized by NSGA-II and by PSO. The efficiency improved on average by 4.2% and 11.0%, respectively.

Second, the model value summations corresponding to the model assignment solutions optimized by the three algorithms are compared. We define the model value summation of the Pareto optimal solution generated by NSGA-II, PSO, and DA-NSGA-II as SV_1, SV_2 and SV_3, respectively. Figure 7 compares SV_1, SV_2, and SV_3 under the condition of equal task set execution cycles.

The bar chart in Figure 7 compares SV_1, SV_2, and SV_3 for equal task set execution cycles from the Pareto optimal solution sets obtained by the three algorithms, while the line graph presents the percentage increase of SV_3 compared to SV_1 and that of SV_3 compared to SV_2. SV_3 is 1.1–5.7% greater than SV_1, with an average increase of 3.2%, while SV_3 is 4.4–11.1% greater than SV_2, with an average increase of 6.8%. Therefore, the model assignment solution optimized by DA-NSGA-II promotes the design task set to a greater extent compared with those optimized by NSGA-II and PSO.

6. Conclusions

The proposed design-task-oriented model assignment framework that involves model value selection, multiobjective model establishment, and the multiobjective optimization algorithm provides a solution for the problem of model assignment in the model repository to the design tasks in MBSE.

In MBSE, design-task-oriented model assignment based on model value quantification involves the use of multiobjective optimization. This study applies an advantage-number-based analytical technique to quantify the models' values and consequently obtains the value matrix of the models. The goals of the multiobjective model of model assignment, which is established based on the models' cycle and value matrices on the specified tasks, are the minimization of the task set execution cycle and the maximization of the models' actual value summation.

The proposed DA-NSGA-II algorithm increases the diversity of the population and reserves more elite solutions based on the introduction of the DE algorithm and the adaptive elite-reserve solution. The convergence, distribution uniformity, and ranges of the solution outcomes of the algorithm are superior to those of the traditional NSGA-II and PSO. Moreover, the algorithm is more conducive to improving the design efficiency and effect than NSGA-II and PSO. The algorithm is therefore favorable for the solution of problems that involve design-task-oriented model assignments.

Further research should focus on the following two additional perspectives. (1) In the current study, the establishment of a multiobjective model assumed a precondition of unlimited resources. However, resources that include human resources, cash, cost, and other factors are constrained in a realistic product development project. Future research should execute the model establishment over resource-constrained conditions to further enhance and broaden the applicability of the algorithm. (2) For an MBSE-compliant product development project, model scheduling software is capable of improving the usage effects of the models and the development cycles. According to the findings of this study, one can develop a model scheduling software based on the proposed multiobjective model and DA-NSGA-II algorithm.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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Research Article

A Novel Group Decision-Making Approach for Hesitant Fuzzy Linguistic Term Sets and Its Application to VIKOR

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This paper develops a novel group decision-making (GDM) approach for solving multiple-criteria group decision-making (MCGDM) problems with uncertainty. The hesitant fuzzy linguistic term sets (HFLTSs) are applied to elicit the decision makers' linguistic preferences due to their distinguished efficiency and flexibility in representing uncertainty. However, the existing context-free grammar for linguistic description cannot allow generating the linguistic expressions completely free to limit the richness of HFLTSs, and the related methods for dealing with HFLTSs also have limitations in aggregating HFLTSs with different lengths and types. Therefore, this paper proposes extended context-free grammar and a novel GDM approach for HFLTSs, considering the advantages of the rough set theory and OWA operators. The rough set theory can manage the uncertainty existing in the fuzzy representation and deal with HFLTSs represented by the 2-tuple fuzzy linguistic model to get rough number sets. The OWA operator can aggregate these sets with different numbers of elements into an interval simply and objectively. Then, an extended VIKOR method based on the proposed GDM approach for HFLTSs is presented to solve the MCGDM problems. Finally, two examples are given to illustrate the applicability and validity of the developed GDM approach and the hesitant VIKOR method through sensitivity and comparison analysis with other existing approaches.

1. Introduction

Decision-making is a common activity for human beings to select the desirable alternatives in many different fields such as evaluation [1], selection [2], and improvement [3]. Such problems are always presented as multicriteria decision-making (MCDM) problems. The complexity and importance of the real-world decision problems make the inclusion of multiple points of view necessary in order to achieve a solution from the knowledge provided by a group of experts [4]. Therefore, group decision-making (GDM) is a usual technique in MCDM practice. These problems having complex processes where several criteria must be satisfied to find the desirable alternative by multiple experts or decision makers (DMs) are called multiple criteria group decision-making (MCGDM) problems. How to solve MCGDM problems under fuzzy environment has been a challenging and attention-attracting topic in recent decades.

The judgments of experts are often vague and uncertain and cannot be expressed with exact numerical information. Since the introduction of the fuzzy set by Zadeh [5], the fuzzy set and its extensions have been widely used to express and model the fuzzy and vague information in the decision-making process. Fuzzy sets require a positive membership for each element and support the favoring evidence only. Intuitionistic fuzzy sets (IFSs) introduced by Atanassov [6] support both favoring and opposing evidences by means of the membership function and non-membership function and have the advantage that permits the experts having a margin of error in establishing the membership for each element. Type-2 fuzzy sets [7] and fuzzy multisets [8] are other extensions of fuzzy sets. Type-2 fuzzy sets have the advantage that permits the membership of an element having some possible distributions on possible values, where the membership of each element is defined as a fuzzy set. When defining the membership of a given element, fuzzy multisets deal with uncertainty by allowing several values. However, the biggest difficulty of establishing the membership degree in the GDM process is that experts may have a set of possible values. Aiming at such a situation, Torra [9] introduced hesitant fuzzy sets (HFSs) in terms of a function that returns a set of membership values for each element in the domain. Recently, HFSs have been widely used in solving MCDM problems due to their distinguished efficiency and flexibility in modeling uncertainty and vagueness in the decision- making process. Zhang [10] presented hesitant fuzzy power aggregation operators for multiple attribute group decision-making. Xia and Xu [11] presented some aggregation operators of hesitant fuzzy information for GDM. Zhang and Wei [12] extended the VIKOR method based on HFSs for the decision-making problem. Some measures for HFSs have been presented for decision-making [13–15].

Based on the HFSs and fuzzy linguistic approach, Rodríguez et al. [16] introduced the concept of hesitant fuzzy linguistic term sets (HFLTSs) for richer expressions in MCDM. HFLTS complies with the situation that experts prefer adopting imprecise linguistic terms to express their judgments. It avoids the restriction for preference flexibility caused by using a single term or interval linguistic terms. The aim of this paper is to use and improve the operating method of HFLTSs to solve MCGDM problems under the linguistic environment.

The commonly used linguistic description approaches for HFLTSs are the ordered structure approach and the context-free grammar approach. Rodríguez et al. [16] presented how to generate comparative linguistic expressions by using context-free grammar. Context-free grammar can generate different linguistic expressions depending on the specific problem. Based on traditional context-free grammar, we consider similar but extended context-free grammar to support the completely free expression. For the linguistic GDM, the process of computing with words (CWW) is indispensable. In the HFLTS environment, free expression brings convenience for describing experts' preferences. However, the CWW processes for HFLTSs become more complex due to each element being an arbitrary linguistic term subset. Some distance and similarity measures for HFLTSs were put forward and applied to solve MCDM problems [17–19], but these approaches assume that the HFLTSs have the same length. To aggregate or compute HFLTSs with different lengths in solving decision-making problems, many methods were put forward as shown in Table 1.

From the above reviews, we can conclude that the existing operating methods of HFLTSs with different lengths can be classified into three main categories. (1) The first category is the envelope-based method, and the envelope of the HFLTS is a linguistic interval. The HFLTSs can be aggregated or compared as intervals. Rodríguez et al. [16] first proposed the envelope concept to compare HFLTSs based on their envelopes, which are numerical intervals. The introduction of the concept of *envelope* can simplify the comparison operation and other operations. However, it is unreasonable to judge one HFLTS is absolutely superior to another if they have common elements. Although several extended research studies have been conducted, there still exist limitations of the envelope-based method. The one disadvantage of it is that the linguistic interval finally obtains crisp values, losing the initial fuzzy representation. The other disadvantage is that it seems unreasonable to support selecting the preferences from the predefined term sets completely free. If we give hesitant linguistic expressions out of context-free grammar, the envelope-based method may fail to work efficiently. For example, $\{s_1, s_2, s_3\}$ and $\{s_1, s_3\}$ have the same envelope $[s_1, s_3]$. (2) The second category includes the fuzzy envelope-based method and α-cut method. Liu and Rodriguez [23] proposed a fuzzy envelope for the HFLTS. The fuzzy envelope can retain the vagueness of comparative linguistic expressions to a certain extent, but determining the parameters of the fuzzy membership function is fairly complicated, and considerable requisite calculations are required for an MCDM problem in the context of HFLTSs [32]. (3) The third category is the term-adding method, which is to extend the short HFLTSs by adding some linguistic terms until they have the same length as others. The disadvantage of the term-adding method is that it would change the information of the original hesitant fuzzy elements by filling some artificial values.

To support extended context-free grammar and operate HFLTSs with different lengths, we propose a novel GDM approach in the HFLTS environment. The contribution of this approach is transforming the HFLTSs into rough intervals by taking advantage of the 2-tuple linguistic representation model and the rough set theory. In the aggregation phase, the obtained rough numbers are grouped into an interval using the ordered weighted averaging (OWA) operator. In the exploitation phase, an extended VIKOR method based on the proposed GDM approach for HFLTSs is presented.

The 2-tuple linguistic representation model proposed by Herrera and Martínez [33] is based on the concept of symbolic translation, which is composed of a linguistic term and a real number. The main advantage of this representation is to be continuous in its domain. Therefore, it can express any counting of information in the universe of the discourse. Herrera and Martínez [33] pointed out that the computational technique based on the symbolic translation can deal with the 2-tuples without loss of information, such as comparison and aggregation. In recent studies, the 2-tuple linguistic representation model has been used in MCGDM problems successfully, e.g., material selection [34], product management [35], and computer network security system evaluation [36]. Rough set theory proposed by Pawlak [37] is a mathematical approach to manage uncertain data or problems of the information systems. Its main advantage is that it requires no external

Table 1: Review of the operating methods of HFLTSs with different lengths.

Authors	Methods used for operating HFLTSs	Contributions to the field of the HFLTS
Rodríguez et al. [16]	Introducing the concept of *envelope* for an HFLTS to compare two HFLTSs	First introducing the concept of *envelope*
Farhadinia [13]	Using the envelope for each HFLTS to aggregate the preference of experts based on the 2-tuple linguistic representation model	Considering extended context-free grammar close to human beings' cognitive models
Montes et al. [20]	Carrying out the computing with words processes using the envelope of an HFLTS and the 2-tuple linguistic representation model	Presenting a practical application in decision-making of a 2-tuple linguistic fuzzy model with hesitant information
Wu et al. [21]	Proposing a maximum support degree model based on the envelope of the HFLTS	Proposing a new linguistic group decision model by combining the linguistic distributions and HFLTSs
Boyacı [22]	Obtaining the envelope for each HFLTS and proposing the pessimistic and optimistic collective preference relations for comparison	Using the HFLTS-based additive ratio assessment (ARAS) method
Liu and Rodriguez [23]	Introducing a fuzzy envelope for the HFLTS whose representation is a fuzzy membership function	Presenting a new representation of the hesitant fuzzy linguistic term sets using a fuzzy envelope
Chen and Hong [24]	Aggregating the fuzzy sets in each HFLTS into a fuzzy set and performing α-cut to these aggregated fuzzy sets to get intervals	Considering the pessimistic attitude and the optimistic attitude of the decision maker
Lee and Chen [25]	Adopting 1-cut of hesitant fuzzy linguistic term sets for dealing with fuzzy decision-making	Proposing a new fuzzy group decision-making method based on the proposed likelihood-based comparison relations of HFLTSs
Dong et al. [26]	Proposing a new fuzzy envelope of the HFLTS by using a Bonferroni mean operator	Proposing a new fuzzy envelope of the HFLTS and a new cosine similarity measure for HFLTSs
Zhu and Xu [27]	Introducing a method to add linguistic terms to HFLTSs to make sure that two HFLTSs have the same number of linguistic terms	Developing a concept of hesitant fuzzy linguistic preference relations (HFLPRs) as a tool to collect and present the DMs' preferences
Liao et al. [17]; Liao and Xu [28]; Liao et al. [29]	Extending the short hesitant fuzzy linguistic element by adding the linguistic term which is the average of the maximal term and the minimal term	Proposing several different types of correlation coefficients for HFLTSs and cosine distance and similarity measures for HFLTSs
Liao et al. [30]	Enlarging the shorter HFLTS by adding a linguistic term which is between the maximum and the minimum linguistic term	Proposing two distinct methods to compare the HFLTSs and investigating the ELECTRE II method in the HFLTS environment
Lei et al. [31]	Extending the shorter HFLE by adding the maximum value	Proposing the behavioral multigranulation decision-theoretic rough set over two universes with HFL information

parameters and uses the information presented in the given data only. The OWA operator proposed by Yager [38] provides an aggregation result lying between the max and min operators and has received increasing attention. The weight associated with each data depends on the position it takes in the descending arrangement of the data rather than the particular data. Due to the advantage that the OWA operator can provide a wide family of aggregation functions and aggregate a set of values regardless of their numbers, we apply the OWA operator to aggregate the elements in an HFLTS and multiple HFLTSs in the GDM process.

HFLTS has been combined with many MCDM methods, such as ELECTRE [39], extended ELECTRE [40], TOPSIS [41, 42], and VIKOR [28, 43]. The VIKOR method for compromise ranking determines a compromise solution by providing a maximum "group utility" for the "majority" and a minimum of an "individual regret" for the "opponent," which is an effective tool for MCDM, particularly in a situation where the decision maker is not able or does not know how to express his/her preference at the beginning of system design. We pay attention to apply the proposed group decision-making approach to solve MCGDM problems using the VIKOR method.

This paper focuses on dealing with MCGDM problems in the context of linguistic evaluation using HFLTSs and the VIKOR method. The rest of the paper is organized as follows. In Section 2, we briefly review the concepts of HFLTSs and 2-tuple linguistic representation models and introduce how to apply the 2-tuple linguistic representation model to compute with the hesitant fuzzy linguistic information. In Section 3, a novel group decision-making approach for HFLTSs is presented based on the rough set theory and the OWA operator. In Section 4, we give out an extended VIKOR method based on the proposed GDM approach for HFLTSs. In Section 5, two application examples are provided to illustrate the efficiency of the proposed GDM approach and the extended VIKOR, respectively, and the results are compared with other existing methods. Finally, conclusions are drawn in Section 6.

2. Preliminaries

In this section, some concepts and operations of HFLTSs and the 2-tuple linguistic representation model are briefly reviewed, and then, how to apply the 2-tuple linguistic representation model for computing with the hesitant fuzzy linguistic information is introduced.

2.1. Concept and Basic Operations of HFLTSs

Definition 1 (see [16]) Let S be a linguistic term set, $S = \{s_0, \ldots, s_g\}$; an HFLTS, H_s, is an ordered finite subset of the consecutive linguistic terms of S.

The empty HFLTS and the full HFLTS for a linguistic variable ϑ are defined as follows:

(1) The empty HFLTS: $H_S(\vartheta) = \{\}$,

(2) The full HFLTS: $H_S(\vartheta) = S$

Definition 2 (see [16]). Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set; H_S^0, H_S^1, and H_S^2 are three arbitrary HFLTSs on S. H_S^C is the complement set of H_S^0. Three operations are defined as follows:

(1) $H_S^C = S - H_S^0 = \{s_i \mid s_i \in S \text{ and } s_i \notin H_S^0\}$

(2) $H_S^1 \cap H_S^2 = \{s_i \mid s_i \in H_S^1 \text{ and } s_i \in H_S^2\}$

(3) $H_S^1 \cup H_S^2 = \{s_i \mid s_i \in H_S^1 \text{ or } s_i \in H_S^2\}$

Due to the present decision-making problems having higher uncertainty, experts in the decision-making group might hesitate among different linguistic terms to express their preferences. Context-free grammar is close to human beings' cognitive model and can generate comparative linguistic expressions. Rodríguez et al. [16] pointed out how to generate comparative linguistic expressions by using context-free grammar. Rodríguez et al. [4] considered similar but extended context-free grammar to that defined in Rodríguez et al. [16] which might generate comparative linguistic expressions similar to the expressions used by experts in GDM problems. Extended context-free grammar refers to a set containing a single term or several adjacent linguistic terms and cannot support the arbitrarily linguistic term mix. One special case is omitted, that is, experts may be hesitant to choose a better evaluation or a worse evaluation. Therefore, this paper improves extended context-free grammar to introduce the binary relation "or."

Definition 3. Let G_H be improved context-free grammar and $S = \{s_0, \ldots, s_g\}$ be a linguistic term set. The elements of $G_H = (V_N, V_T, I, P)$ are defined as follows:

V_N = {<*primary term*>, <*composite term*>, <*unary relation*>, <*binary relation*>, <*conjunction*>}

V_T = {*lower than, greater than, at least, at most, between, or, and,* $s_0, s_1, \ldots, s_g$}

$I \in V_N$

For context-free grammar, G_H, the production rules are as follows:

P = {I ::= <*primary term*>|<*composite term*>

<*composite term*> ::== <*unary relation*><*primary term*>|<*binary relation*>

<primary *term*><*conjunction*><*primary term*>

<primary *term*> ::== $s_0|s_1|s_g$

<unary *relation*> ::== *lower than*|*greater than*|*at least*|*at most*

<binary *relation*> ::== *between*|*or*

<conjunction> :== *and*}

Definition 4. Let E_{GH} be a function that transforms linguistic expressions $ll \in S_{ll}$ obtained by context-free grammar G_H into a HFLTS H_S, where S is the linguistic term set used by G_H and S_{ll}:

$E_{GH}: S_{ll} \longrightarrow H_S$

The comparative linguistic expressions generated by G_H can be converted into HFLTSs by means of the following:

(1) $E_{GH}(s_i) = \{s_i \mid s_i \in S\}$

(2) E_{GH} (at most s_i) $= \{s_j \mid s_j \in S \text{ and } s_j \le s_i\}$

(3) E_{GH} (lower than s_i) $= \{s_j \mid s_j \in S \text{ and } s_j < s_i\}$

(4) E_{GH} (at least s_i) $= \{s_j \mid s_j \in S \text{ and } s_j \ge s_i\}$

(5) E_{GH} (greater than s_i) $= \{s_j \mid s_j \in S \text{ and } s_j > s_i\}$

(6) E_{GH} (between s_i and s_j) $= \{s_k \mid s_k \in S \text{ and } s_i \le s_k \le s_j\}$

(7) E_{GH} (s_i or $s_j, \ldots, s_k$) $= \{s_i, s_j, \ldots, s_k\}$

Example 1. Let S be a linguistic term set.

$S = \{s_0$: nothing, s_1: very low, s_2: low, s_3: medium, s_4: high, s_5: very high, s_6: perfect$\}$

Three experts give their opinions aiming at the same evaluation object based on improved context-free grammar: ll^1: between low and high; ll^2: low or high; and ll^3: between medium and very high. According to the function E_{GH}, three different HFLTSs are obtained:

$H_S^1 = \{s_2, s_3, s_4\}$, $H_S^2 = \{s_2, s_4\}$, and $H_S^3 = \{s_3, s_4, s_5\}$

2.2. Computing with HFLTSs Using the 2-Tuple Fuzzy Linguistic Representation Model.

Let $S = \{s_i: i = 0, 1, 2, \ldots, g\}$ be a finite and ordered discrete linguistic term set, where s_i represents a possible value for a linguistic variable. The 2-tuple fuzzy linguistic representation model deals with linguistic information by introducing a new parameter called symbolic translation. The concept of symbolic translation is described in Definition 5. It is used to make the information representation continuous in its domain, and it is the foundation of the computation techniques of the 2 tuples. The concept and basic operations of the 2-tuple fuzzy linguistic representation model are as follows.

Definition 5 (see [33]) Let β be a value representing the result of an aggregation of the indices of a set of labels assessed in the linguistic term set S, i.e., the result of a symbolic aggregation operation $\beta \in [0, g]$, being $g+1$, the cardinality of S. Let $i = round\,(\beta)$ and $\alpha = \beta - i$ be two values such that $i \in [0, g]$ and $\alpha \in [-0.5, 0.5)$; then, α is called a *symbolic translation*.

The linguistic representation model 2-tuple (s_i, α_i), $s_i \in S$ and $\alpha_i \in [-0.5, 0.5)$, is developed from the above concept:

(1) s_i represents the linguistic label center of the information

(2) α_i is a numerical value expressing the value of the translation from the original result β to the closest index label, i, in the linguistic term set S, i.e., the symbolic translation

Definition 6 (see [33]) Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set and $\beta \in [0, g]$ be a value representing the result of a symbolic aggregation; then, the 2-tuple that expresses the equivalent information to β is obtained with the function Δ:

$$\begin{aligned} &\Delta\colon\ [0, g] \longrightarrow S \times [0.5, -0.5), \\ &\Delta(\beta) = (s_i, \alpha_i), \\ &\begin{cases} s_i, & i = round\,(\beta), \\ \alpha_i = \beta - i, & \alpha_i \in [-0.5, 0.5), \end{cases} \end{aligned} \tag{1}$$

where $round\,(\cdot)$ is the usual round operation, s_i has the closest index label to β, and α_i is the value of the symbolic translation.

Contrarily, let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set and (s_i, α_i) be a 2-tuple. There is always a Δ^{-1} function:

$$\begin{aligned} &\Delta^{-1}\colon\ S \times [0.5, -0.5) \longrightarrow [0, g], \\ &\Delta^{-1}(s_i, \alpha_i) = i + \alpha_i = \beta. \end{aligned} \tag{2}$$

The original linguistic evaluation variable can be converted into a linguistic 2-tuple by adding value zero as symbolic translation: $s_i \in S \Longrightarrow (s_i, 0)$.

Example 2. The decision information in Example 1 can be transformed into the following 2-tuple information:

$H_S^1 = \{(s_2,\ 0), (s_3,\ 0), (s_4,\ 0)\}$, $H_S^2 = \{(s_2,\ 0), (s_4,\ 0)\}$, $H_S^3 = \{(s_3, 0), (s_4, 0), (s_5, 0)\}$

$\Delta^{-1}(H_S^1) = \{2, 3, 4\}$, $\Delta^{-1}(H_S^2) = \{2, 4\}$, $\Delta^{-1}(H_S^3) = \{3, 4, 5\}$

3. A Novel Group Decision-Making Approach for Hesitant Fuzzy Linguistic Term Sets

This section describes a novel GDM approach based on HFLTSs. Aggregation operators are the most widely used tool for combining individual preference information into overall preference information in the GDM process. The traditional operators are arithmetic average operators and geometric average operators. These operators consider the DMs' preferences, and the weights are always determined subjectively. The OWA operator is a parameterized way of aggregating from "and" to "or." The associated weights can be determined objectively. The classic method for determining the weights is quantifier-guided aggregation. Three fuzzy linguistic preferences, for the most (fuzzy majority), at least half, and as much as possible, are considered in this paper.

After obtaining the 2-tuple sets in Section 2.2, the rough set theory is introduced to transform these sets into rough numbers sets, and the obtained rough numbers sets can be aggregated into an interval using an OWA operator. Then, the GDM problem in the context of HFLTSs degenerates into an information aggregation problem for interval numbers. The framework of the proposed group decision-making approach for hesitant fuzzy linguistic term sets is shown in Figure 1.

3.1. Elicitation of Linguistic Expressions in Decision-Making. Let X be a set of evaluation objects, $X = \{x_i \mid 1 \le i \le n\}$, let C be a set of evaluation criteria, $C = \{c_j \mid 1 \le j \le m\}$, and let E be a set of experts, $E = \{e_k \mid 1 \le k \le l\}$. According to the given linguistic term set, expert e_k uses proposed context-free grammar to give out the linguistic expression $ll_j^k(x_i)$ concerning the criterion c_j for evaluating x_i. The linguistic expression $ll_j^k(x_i)$ can be transformed into an HFLTS $H_S^{jk}(x_i)$ using the transformation function E_{GH}. The hesitant GDM information is presented as shown in Table 2.

3.2. Rough Number Enabled HFLTS Information Processing. Experts in the decision-making group have diversified opinions on the evaluated objects. Moreover, hesitant linguistic information given by all experts may have different lengths. Therefore, translating all the HFLTSs into the information with the same length is a critical procedure for information aggregation. Computing the average value of all the elements in an HFLTS is unreasonable obviously, ignoring the uncertainty of each element. Each linguistic term in the predefined linguistic term set can be deemed as a class. The rough numbers can give the lower and upper approximations of the target class to describe the uncertainty of the class appearing in a group decision-making problem.

Example 3. Let HFLTSs in Example 1 be the information given by three experts with respect to $H_S^{jk}(x_i)$. $H_S^{j1}(x_i) = \{s_2, s_3, s_4\}$, $H_S^{j2}(x_i) = \{s_2, s_4\}$, and $H_S^{j3}(x_i) = \{s_3, s_4, s_5\}$.

Taking s_2 for an example, s_2 in this group decision-making information has fuzziness and uncertainty for the inconsistent judgments of all experts. The boundary region of s_2, i.e., the difference between the lower and upper approximations of s_2, can imply that the knowledge about this term is better. Rough number is a concept proposed by Zhai et al. [44] for managing the imprecise design information, which is derived from the basic notions of the rough set. The basic notions of rough sets are as follows.

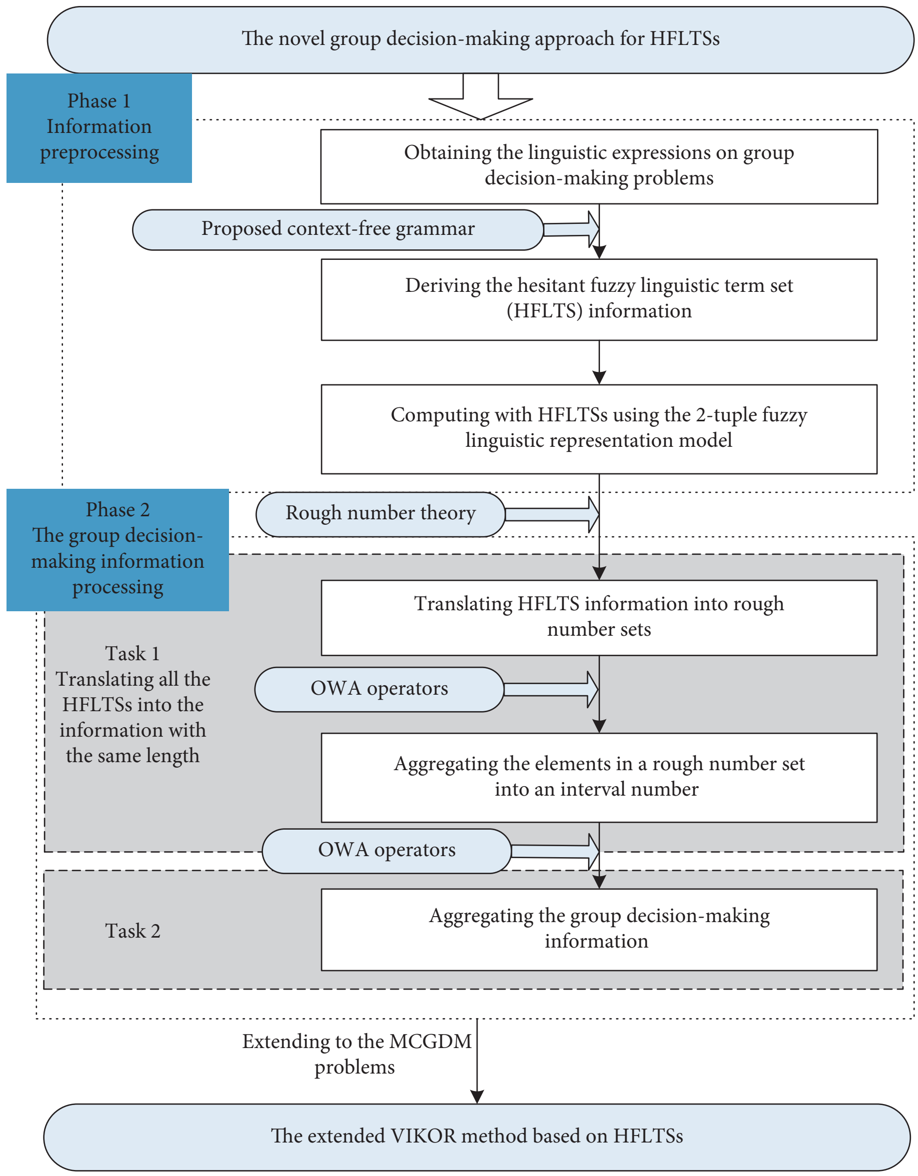

Figure 1: The framework of the proposed group decision-making approach for HFLTSs.

Table 2: Hesitant fuzzy linguistic term sets given by the experts.

Object x_i	Criteria					
	c_1	c_2	...	c_j	....	c_m
e_1	$H_S^{11}(x_i)$	$H_S^{21}(x_i)$	...	$H_S^{j1}(x_i)$	...	$H_S^{m1}(x_i)$
e_2	$H_S^{12}(x_i)$	$H_S^{22}(x_i)$	...	$H_S^{j2}(x_i)$	...	$H_S^{m2}(x_i)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
e_k	$H_S^{1k}(x_i)$	$H_S^{2k}(x_i)$	...	$H_S^{jk}(x_i)$	...	$H_S^{mk}(x_i)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
e_l	$H_S^{1l}(x_i)$	$H_S^{2l}(x_i)$	...	$H_S^{jl}(x_i)$	....	$H_S^{ml}(x_i)$

Rough set theory (RST) is an effective mathematical tool to deal with subjective and vague information using only the given information, which does not require any external information or additional subjective adjustment for data analysis. Furthermore, RST excels in handling imprecise information especially when the data set is small in size and other tools like statistics are not suitable [45]. RST uses the lower and upper approximations to form the approximation of a target set and expresses vagueness using the boundary region of a set. This is indeed the unique advantage of the rough set theory in dealing with vagueness and uncertainty.

Let U be a universe containing all the objects, and all the objects can be categorized into n classes. Assume that set R is the collection of these classes, $R = \{C_1, C_2, \ldots, C_n\}$. Let Y be an arbitrary object of U. If these classes are ordered in the manner of $C_1 < C_2 < \cdots < C_n$, then for any class $C_i \in R$, $1 \le i \le n$, the lower approximation of C_i can be defined as

$$\underline{Apr}(C_i) = \cup\{Y \in U \mid R(Y) \le C_i\}. \tag{3}$$

The upper approximation of C_i can be defined as

$$\overline{Apr}\,(C_i) = \cup\left\{Y \in U \,\middle|\, R(Y) \geq C_i\right\}. \tag{4}$$

The boundary region of C_i can be expressed as

$$\begin{aligned} Bn\,d\,(C_i) &= \cup\{Y \in U \mid R(Y) \neq C_i\} \\ &= \{Y \in U \mid R(Y) > C_i\} \cup \{Y \in U \mid R(Y) < C_i\}. \end{aligned} \tag{5}$$

$\underline{Lim}\,(C_i)$ and $\overline{Lim}\,(C_i)$ represents the lower limit and upper limit for C_i, respectively, which are defined as follows:

$$\underline{Lim}\,(C_i) = \frac{1}{M_L} \sum R(Y) | Y \in \underline{Apr}\,(C_i), \tag{6}$$

where M_L is the number of objects contained in the lower approximation of C_i.

$$\overline{Lim}\,(C_i) = \frac{1}{M_U} \sum R(Y) | Y \in \overline{Apr}\,(C_i), \tag{7}$$

where M_U is the number of objects contained in the upper approximation of C_i.

The rough boundary interval of C_i is the interval between the lower limit $\underline{Lim}\,(C_i)$ and the upper limit $\overline{Lim}\,(C_i)$, which is denoted as $RBn\,d\,(C_i)$:

$$RBn\,d\,(C_i) = \overline{Lim}\,(C_i) - \underline{Lim}\,(C_i). \tag{8}$$

Accordingly, the vague class C_i can be expressed by its lower limit and upper limit as follows:

$$RN\,(C_i) = [\underline{Lim}\,(C_i), \overline{Lim}\,(C_i)]. \tag{9}$$

The above definitions of the rough boundary interval and rough number can be used to deal with the imprecise evaluation information in group decision-making problems.

Example 4. $H_S^{j1}(x_i) = \{s_2, s_3, s_4\}$, $H_S^{j2}(x_i) = \{s_2, s_4\}$, and $H_S^{j3}(x_i) = \{s_3, s_4, s_5\}$ in Example 3 can be represented by the 2-tuple linguistic representation model first, and then, each element can be defined by its rough number to quantify and analyze the subjective evaluations:

$H_S^{j1}(x_i) = \{(s_2, 0), (s_3, 0), (s_4, 0)\}$, $H_S^{j2}(x_i) = \{(s_2, 0), (s_4, 0)\}$, and $H_S^{j3}(x_i) = \{(s_3, 0), (s_4, 0), (s_5, 0)\}$

$\Delta^{-1}(H_S^{j1}(x_i)) = \{2, 3, 4\}$, $\Delta^{-1}(H_S^{j2}(x_i)) = \{2, 4\}$, and $\Delta^{-1}(H_S^{j3}(x_i)) = \{3, 4, 5\}$

$\Delta^{-1}(H_S^{j1}(x_i)) = \{[\underline{Lim}\,(2), \overline{Lim}\,(2)], \; [\underline{Lim}\,(3), \overline{Lim}\,(3)], [\underline{Lim}\,(4), \overline{Lim}\,(4)]\}$

where $\underline{Lim}\,(2) = (2+2)/2 = 2$;
$\overline{Lim}\,(2) = (2+3+4+2+4+3+4+5)/8 = 3.375$

$\underline{Lim}\,(3) = (2+3+2+3)/4 = 2.5$;
$\overline{Lim}\,(3) = (3+4+4+4+5)/5 = 4$

$\underline{Lim}\,(4) = (2+3+4+2+4+3+4)/7 = 3.143$;
$\overline{Lim}\,(4) = (4+4+4+5)/4 = 4.25$

$\underline{Lim}\,(5) = (2+3+4+2+4+3+4+5)/8 = 3.375$;
$\overline{Lim}\,(5) = 5/1 = 5$

$\Delta^{-1}(H_S^{j1}(x_i)) = \{[2, 3.375], [2.5, 4], [3.143, 4.25]\}$

$\Delta^{-1}(H_S^{j2}(x_i)) = \{[2, 3.375], [3.143, 4.25]\}$

$\Delta^{-1}(H_S^{j3}(x_i)) = \{[2.5, 4], [3.143, 4.25], [3.375, 5]\}$

3.3. Rough Information Aggregation Based on the OWA Operator. The following job is to aggregate the elements in a rough number set into an interval number. As the elements in the rough number sets for all $\Delta^{-1}(H_S^{jk}(x_i))$, $1 \leq k \leq l$, are different, the traditional averaging operators with given weights are not flexible and reasonable. The OWA operator provides a parameterized family of aggregation operators that includes the maximum (or), the minimum (and), and the average, as special cases. The basic notions of OWA operators are as follows.

Definition 7 (see [38]). A mapping F from $I^n \longrightarrow I$ is called an OWA operator of dimension n if associated with F is a weighting vector W:

$$W = [w_1, w_2, \ldots, w_n]^T \tag{10}$$

such that (1) $w_i \in [0, 1]$, $1 \leq i \leq n$, and (2) $\sum_{i=1}^{n} w_i = 1$.

And

$$F(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j = w_1 b_1 + w_2 b_2 + \cdots + w_n b_n, \tag{11}$$

where b_j is the jth largest element in the collection $a_1, a_2, \ldots, a_n$.

The most important issue of applying OWA operators is to determine the associated weights. Yager [38] presented a formula to calculate the weighting function for the OWA aggregation operator by using the linguistic quantifier proposed by Zadeh [46]. Yager [47] distinguished three categories of relative quantifiers: regular increasing Monotone (RIM) quantifier, regular decreasing monotone (RDM) quantifier, and regular unimodal (RUM) quantifier. The procedure used for generating the weights from the quantifier depends upon the type of the quantifier provided. In the case of the RIM quantifier, the weights for OWA operators are generated as

$$w_j = Q\left(\frac{j}{n}\right) - Q\left(\frac{j-1}{n}\right) \quad \text{for } j = 1, 2, \ldots, n, \tag{12}$$

where w_j is associated with b_j, which is the jth largest element in the collection $a_1, a_2, \ldots, a_n$.

We consider three fuzzy linguistic preferences: for the most (fuzzy majority), at least half, and as much as possible. These preferences indicate the degree to which the decision maker is satisfied with the number of criteria solved. The linguistic quantifier $Q(x)$ is shown as Figure 2.

For the "for the most," the linguistic quantifier $Q(x)$ is defined as [48]

$$Q(x) = \begin{cases} 0, & \text{for } x \leq 0.3, \\ 2x - 0.6, & \text{for } 0.3 < x < 0.8, \\ 1, & \text{for } x \geq 0.8. \end{cases} \tag{13}$$

For the "at least half," the linguistic quantifier $Q(x)$ is defined as

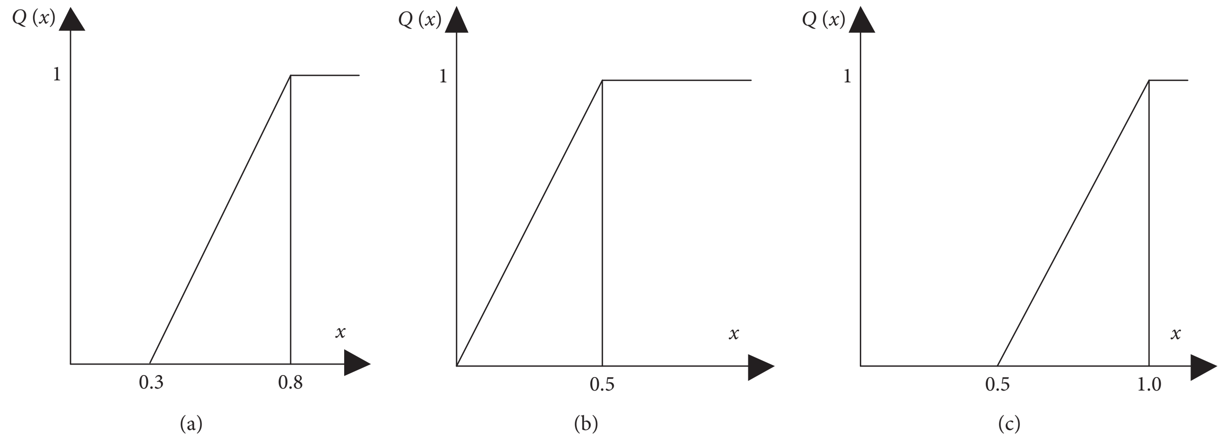

Figure 2: The linguistic quantifier $Q(x)$. (a) "Most." (b) "At least half." (c) "As much as possible."

$$Q(x)=\begin{cases}0, & \text{for } x\le 0,\\ 2x, & \text{for } 0<x<0.5,\\ 1, & \text{for } x\ge 0.5.\end{cases} \tag{14}$$

For the "as much as possible," the linguistic quantifier $Q(x)$ is defined as

$$Q(x)=\begin{cases}0, & \text{for } x\le 0.5,\\ 2x-1, & \text{for } 0.5<x<1,\\ 1, & \text{for } x\ge 1.\end{cases} \tag{15}$$

Assume that the transformed information $\Delta^{-1}(H_S^{jk}(x_i))$, which indicates information expert e_k given concerning the criterion c_j for evaluating the object x_i, is in the form of $\{[\underline{a}_{ij}^{k1},\overline{a}_{ij}^{k1}],[\underline{a}_{ij}^{k2},\overline{a}_{ij}^{k2}],\ldots,[\underline{a}_{ij}^{kr},\overline{a}_{ij}^{kr}]\}$. The elements in the rough number set can be aggregated into $[\underline{a}_{ij}^{k},\overline{a}_{ij}^{k}]$ based on the OWA operator. Since the elements in the rough number set are arranged in the ascending order, $\underline{a}_{ij}^{k}$ and $\overline{a}_{ij}^{k}$ can be determined by the following equations:

$$\underline{a}_{ij}^{k}=\sum_{h=1}^{r} w_{r-h}\cdot\underline{a}_{ij}^{kh},\quad 1\le h\le r, \tag{16}$$

$$\overline{a}_{ij}^{k}=\sum_{h=1}^{r} w_{r-h}\cdot\overline{a}_{ij}^{kh},\quad 1\le h\le r, \tag{17}$$

where w_{r-h} can be obtained according to equation (12).

Example 5. $\Delta^{-1}(H_S^{j1}(x_i))=\{[2,\ 3.375],\ [2.5,\ 4],\ [3.143,\ 4.25]\}$,

$\Delta^{-1}(H_S^{j2}(x_i))=\{[2,\ 3.375],\ [3.143,\ 4.25]\}$,

$\Delta^{-1}(H_S^{j3}(x_i))=\{[2.5,\ 4],\ [3.143,\ 4.25],\ [3.375,\ 5]\}$ in Example 4.

Considering the fuzzy linguistic preference "for the most," the above information can be transformed into the following:

If the rough number set has three elements, $w_1=1-Q(2/3)=0.27$, $w_2=Q(2/3)-Q(2/3)=0.67$, and $w_3=Q(1/3)=0.06$.

If the rough number set has two elements, $w_1=1-Q(1/2)=0.6$ and $w_2=Q(1/2)=0.4$.

$\Delta^{-1}(H_S^{j1}(x_i))=[2.20358,\ 3.59625]$,

$\Delta^{-1}(H_S^{j2}(x_i))=[2.4572,\ 3.725]$,

$\Delta^{-1}(H_S^{j3}(x_i))=[2.72611,\ 4.1275]$.

Concerning criterion c_j for evaluating object x_i, the group decision-making information can be obtained by considering the experts' weights and weights associated with OWA operators.

For $\Delta^{-1}(H_S^{jk}(x_i))=[\underline{a}_{ij}^{k},\overline{a}_{ij}^{k}]$, let we_k be the weight of expert e_k and wo_k^{ij} be the weight associated with the OWA operator according to the order of information e_k given. w_k^{ij} is the normalized weight for the sum of we_k and wo_k^{ij}:

$$w_k^{ij}=\frac{we_k+wo_k^{ij}}{\sum_{k=1}^{l}\left(we_k+wo_k^{ij}\right)},\quad 1\le i\le n,\ 1\le j\le m. \tag{18}$$

The aggregated evaluation information is defined as $\Delta^{-1}(H_S^{j}(x_i))$, $\Delta^{-1}(H_S^{j}(x_i))=[\underline{a}_{ij},\overline{a}_{ij}]$.

$$\underline{a}_{ij}=\sum_{k=1}^{l} w_k^{ij}\cdot\underline{a}_{ij}^{k}, \tag{19}$$

$$\overline{a}_{ij}=\sum_{k=1}^{l} w_k^{ij}\cdot\overline{a}_{ij}^{k}. \tag{20}$$

The final decision matrix A for the MCGDM problem is as follows:

$$A_{n\times m} = \begin{bmatrix} [\underline{a}_{11}, \overline{a}_{11}] & [\underline{a}_{12}, \overline{a}_{12}] & \cdots & [\underline{a}_{1m}, \overline{a}_{1m}] \\ [\underline{a}_{21}, \overline{a}_{21}] & [\underline{a}_{22}, \overline{a}_{22}] & \cdots & [\underline{a}_{2m}, \overline{a}_{2m}] \\ \vdots & \vdots & \vdots & \vdots \\ [\underline{a}_{n1}, \overline{a}_{n1}] & [\underline{a}_{n2}, \overline{a}_{n2}] & \cdots & [\underline{a}_{nm}, \overline{a}_{nm}] \end{bmatrix}. \tag{21}$$

4. Extended VIKOR Method for MCGDM Based on Hesitant Fuzzy Linguistic Term Sets

The VIKOR method introduces the multicriteria ranking index based on the particular measure of closeness to the ideal solution [49]. According to Opricovic and Tzeng [50], the multicriteria measure for compromise ranking is developed from the Lp-metric utilized as an aggregating function in a compromise programming method. For an alternative $x_i\,(1 \le i \le n)$, the evaluating value of the jth criterion $(1 \le j \le m)$ is denoted as f_{ij}. The Lp-metric has the following form [51]:

$$L_{p,i} = \left\{ \sum_{j=1}^{n} \left[\frac{w_j\left(f_j^* - f_{ij}\right)}{f_j^* - f_j^-} \right]^p \right\}^{1/p}, \quad 1 \le p \le \infty,\ i = 1, 2, \ldots, n, \tag{22}$$

where $f_j^* = \max_i f_{ij}$ and $f_j^- = \min_i f_{ij}$.

In the VIKOR method, $L_{1,i}$ (or S_i) and $L_{\infty,i}$ (or R_i) are used to formulate ranking measurements. The solution gained by min S_i is with a maximum group utility, and the solution gained by min R_i is with a minimum individual regret of the opponent [50]. The compromise solution is a feasible solution that is the closest to the ideal, and compromise means an agreement established by mutual concessions. v is introduced as the compromise parameter between the group utility and the individual regret. $Q_i = vf(S_i) + (1 - v)f(R_i)$. $v > 0.5$ represents concerning the group utility (or the majority). $v < 0.5$ represents concerning the individual regret.

The VIKOR method is used to treat the decision matrix to calculate S, R, and Q and then obtain the candidate ranking order. According to the summarized steps of the VIKOR method [50], the extended VIKOR approach proposed in this paper has the following five steps.

Step 1: determine the positive ideal solution A^* and the negative ideal solution A^- of the final decision matrix $A = ([\underline{a}_{ij}, \overline{a}_{ij}])_{n\times m}$.

Criteria set B represents the "larger-the-better" category, and criteria set C represents the "small-the-better" category.

$$\begin{aligned} A^* &= \left\{a_j^* \mid 1 \le j \le m\right\}, \quad \text{where } a_j^* = \begin{cases} \max_{i=1}^n \overline{a}_{ij} \,\middle|\, c_j \in B, \\ \min_{i=1}^n \underline{a}_{ij} \,\middle|\, c_j \in C, \end{cases} \\ A^- &= \left\{a_j^- \mid 1 \le j \le m\right\}, \quad \text{where } a_j^- = \begin{cases} \min_{i=1}^n \underline{a}_{ij} \,\middle|\, c_j \in B, \\ \max_{i=1}^n \overline{a}_{ij} \,\middle|\, c_j \in C. \end{cases} \end{aligned} \tag{23}$$

Step 2: compute the values of $[\underline{S}_i, \overline{S}_i]$ and $[\underline{R}_i, \overline{R}_i]$ by the following formulas:

$$\underline{S}_i = \sum_{c_j \in B} \frac{wc_j\left(a_j^* - \overline{a}_{ij}\right)}{\left(a_j^* - a_j^-\right)} + \sum_{c_j \in C} \frac{wc_j\left(\underline{a}_{ij} - a_j^*\right)}{\left(a_j^- - a_j^*\right)}, \tag{24}$$

$$\overline{S}_i = \sum_{c_j \in B} \frac{wc_j\left(a_j^* - \underline{a}_{ij}\right)}{\left(a_j^* - a_j^-\right)} + \sum_{c_j \in C} \frac{wc_j\left(\overline{a}_{ij} - a_j^*\right)}{\left(a_j^- - a_j^*\right)}, \tag{25}$$

$$\underline{R}_i = \max_j \begin{cases} \dfrac{wc_j\left(a_j^* - \overline{a}_{ij}\right)}{\left(a_j^* - a_j^-\right)}, & \text{for } c_j \in B, \\[2ex] \dfrac{wc_j\left(\underline{a}_{ij} - a_j^*\right)}{\left(a_j^- - a_j^*\right)}, & \text{for } c_j \in C. \end{cases} \tag{26}$$

$$\overline{R}_i = \max_j \begin{cases} \dfrac{wc_j\left(a_j^* - \underline{a}_{ij}\right)}{\left(a_j^* - a_j^-\right)}, & \text{for } c_j \in B, \\[2ex] \dfrac{wc_j\left(\overline{a}_{ij} - a_j^*\right)}{\left(a_j^- - a_j^*\right)}, & \text{for } c_j \in C, \end{cases} \tag{27}$$

where wc_j is the weight of criteria c_j.

Step 3: compute the values of $[\underline{Q}_i, \overline{Q}_i]$:

$$\underline{Q}_i = v\frac{\underline{S}_i - S^-}{S^* - S^-} + (1 - v)\frac{\underline{R}_i - R^-}{R^* - R^-}, \tag{28}$$

$$\overline{Q}_i = v\frac{\overline{S}_i - S^-}{S^* - S^-} + (1 - v)\frac{\overline{R}_i - R^-}{R^* - R^-}, \tag{29}$$

where $S^- = \min_i \underline{S}_i$, $S^* = \max_i \overline{S}_i$ $R^- = \min_i \underline{R}_i$, $R^* = \max_i \overline{R}_i$, and v is the weight of the decision-making strategy of the maximum group utility. $v > 0.5$ represents "voting by majority rule," $v \approx 0.5$ represents "by consensus," and $v < 0.5$ represents "with veto." Selection of v depends on the decision strategy of experts, and it may influence the compromise solution.

Step 4: rank the alternatives, sorting the values of $[\underline{Q}_i, \overline{Q}_i]$, $[\underline{S}_i, \overline{S}_i]$, and $[\underline{R}_i, \overline{R}_i]$ in the ascending order, and then obtain three ranking lists.

For any two rough numbers, $RN_1 = [L_1, U_1]$ and $RN_2 = [L_2, U_2]$, where L_1 and L_2 represent their lower limits and U_1 andU_2 represent their upper limits, the ranking rules of two rough numbers are given as follows [52]:

(1)

(a) If $U_1 > U_2$ and $L_1 \geq L_2$ or$U_1 \geq U_2$ and $L_1 > L_2$, then $RN_1 > RN_2$

(b) If $U_1 = U_2$ and $L_1 = L_2$, then $RN_1 = RN_2$

(2) Let $M_1 = (L_1 + U_1)/2$ and $M_2 = (L_2 + U_2)/2$.

(a) If $L_2 > L_1$ and $U_1 > U_2$: if $M_1 \leq M_2$, then $RN_1 < RN_2$; if $M_1 > M_2$, then $RN_1 > RN_2$

(b) If $L_1 > L_2$ and $U_2 > U_1$: if $M_1 \leq M_2$, then $RN_1 < RN_2$; if $M_1 > M_2$, then $RN_1 > RN_2$

Step 5: propose a compromise solution:

Definition 8. For any two interval numbers $A = [\underline{A}, \overline{A}]$ and $B = [\underline{B}, \overline{B}]$, the distance between A and B, $D(A, B)$, is defined as

$$D(A, B) = \sqrt{\frac{1}{2}\left[(\overline{B} - \overline{A})^2 + (\underline{B} - \underline{A})^2\right]}. \tag{30}$$

(1) If the following two conditions are satisfied, $x^{(1)}$ is the best compromise solution. $x^{(1)}$ is the object ranked first in the $[\underline{Q}_i, \overline{Q}_i]$ list.

Condition 1: acceptable advantage: $D([\underline{Q}_i^{(1)}, \overline{Q}_i^{(1)}], [\underline{Q}_i^{(2)}, \overline{Q}_i^{(2)}]) \geq DQ$, $DQ = 1/(n-1)$
Condition 2: acceptable stability in decision-making: $x^{(1)}$ must also be the best object ranked according to $[\underline{S}_i, \overline{S}_i]$ or/and $[\underline{R}_i, \overline{R}_i]$

(2) If one of the conditions is not satisfied, then a set of compromise solutions is obtained:

① If only condition 2 is not satisfied, $x^{(1)}$ and $x^{(2)}$ are both compromise solutions

② If condition 1 is not satisfied, maximized X can be obtained according to $D([\underline{Q}_i^{(1)}, \overline{Q}_i^{(1)}], [\underline{Q}_i^{(X)}, \overline{Q}_i^{(X)}]) < 1/(n-1)$, and $x^{(1)}, x^{(2)}, \ldots, x^{(X)}$ are all near to the best compromise solution

5. Illustrative Examples

5.1. Example 1. In this section, a numerical example adopted from Rodríguez et al. [4] is provided to validate the effectiveness of the proposed GDM approach based on HFLTSs. A conference committee, composed of 3 researchers $E = \{e_1, e_2, e_3\}$, wants to grant the best paper award in an international conference. There are four selected papers, $X = \{$John's paper, Mike's paper, David's paper, Frank's paper$\}$. The linguistic term set suitable to express such assessments shown in Rodríguez et al. [4] can be given as follows:

$S = \{$neither (s_0); very low (s_1); low (s_2); medium (s_3); high (s_4); very high (s_5); absolute (s_6)$\}$

Step 1: transform the preferences provided by experts in Rodríguez et al. [4] into HFLTSs:

$$P^1 = \begin{pmatrix} - & \{s_0, s_1\} & \{s_5\} & \{s_0, s_1\} \\ \{s_5, s_6\} & - & \{s_4, s_5\} & \{s_0, s_1, s_2, s_3\} \\ \{s_2\} & \{s_0, s_1, s_2\} & - & \{s_5, s_6\} \\ \{s_4, s_5, s_6\} & \{s_4, s_5, s_6\} & \{s_0, s_1, s_2, s_3\} & - \end{pmatrix},$$
$$P^2 = \begin{pmatrix} - & \{s_0, s_1, s_2\} & \{s_4, s_5, s_6\} & \{s_0, s_1, s_2\} \\ \{s_4, s_5, s_6\} & - & \{s_4\} & \{s_1\} \\ \{s_0, s_1\} & \{s_2\} & - & \{s_5, s_6\} \\ \{s_4, s_5\} & \{s_5\} & \{s_0, s_1, s_2\} & - \end{pmatrix},$$
$$P^3 = \begin{pmatrix} - & \{s_4, s_5, s_6\} & \{s_4, s_5\} & \{s_2\} \\ \{s_0, s_1, s_2\} & - & \{s_4, s_5, s_6\} & \{s_4, s_5, s_6\} \\ \{s_0, s_1, s_2\} & \{s_0, s_1, s_2\} & - & \{s_4\} \\ \{s_4\} & \{s_0, s_1, s_2\} & \{s_1\} & - \end{pmatrix}. \tag{31}$$

Step 2: deal with the assessment information based on the HFLTS using the 2-tuple linguistic representation model:

$$\Delta^{-1}(p^1) = \begin{pmatrix} - & \{0,1\} & \{5\} & \{0,1\} \\ \{5,6\} & - & \{4,5\} & \{0,1,2,3\} \\ \{2\} & \{0,1,2\} & - & \{5,6\} \\ \{4,5,6\} & \{4,5,6\} & \{0,1,2,3\} & - \end{pmatrix},$$
$$\Delta^{-1}(p^2) = \begin{pmatrix} & \{0,1,2\} & \{4,5,6\} & \{0,1,2\} \\ \{4,5,6\} & & \{4\} & \{1\} \\ \{0,1\} & \{2\} & & \{5,6\} \\ \{4,5\} & \{5\} & \{0,1,2\} & \end{pmatrix},$$
$$\Delta^{-1}(p^3) = \begin{pmatrix} - & \{4,5,6\} & \{4,5\} & \{2\} \\ \{0,1,2\} & - & \{4,5,6\} & \{4,5,6\} \\ \{0,1,2\} & \{0,1,2\} & - & \{4\} \\ \{4\} & \{0,1,2\} & \{1\} & - \end{pmatrix}. \tag{32}$$

Step 3: quantify the uncertainty in GDM information based on the rough set theory:

$$\Delta^{-1}(p^1) = \begin{pmatrix} - & \{[0,2.38],[0.5,3.17]\} & \{[4,4.83],[4.6,5.25],[4.83,6]\} & \{[0,1],[0.5,1.5]\} \\ \{[2.83,5.5],[3.63,6]\} & - & \{[4,4.67],[4.4,5.33]\} & \{[0,2.75],[0.67,3.14],[1,4],[1.4,4.5]\} \\ \{[1,2]\} & \{[0,1.14],[0.5,1.6],[1.14,2]\} & - & \{[4.75,5.5],[5.29,6]\} \\ \{[4,4.67],[4.4,5.33],[4.67,6]\} & \{[1.75,5],[2.83,5.33],[3.29,6]\} & \{[0,1.25],[0.6,1.67],[1,2.33],[1.25,3]\} & - \end{pmatrix},$$
$$\Delta^{-1}(p^2) = \begin{pmatrix} - & \{[0,2.38],[0.5,3.17],[0.8,4.25]\} & \{[4.6,5.25]\} & \{[0,1],[0.5,1.5],[1,2]\} \\ \{[1.75,5.2],[2.83,5.5],[3.63,6]\} & - & \{[4,4.67]\} & \{[0.67,3.14]\} \\ \{[0,1],[0.5,1.5]\} & \{[1.14,2]\} & - & \{[4.75,5.5],[5.29,6]\} \\ \{[4,4.67],[4.4,5.33]\} & \{[2.83,5.33]\} & \{[0,1.25],[0.6,1.67],[1,2.33]\} & - \end{pmatrix},$$
$$\Delta^{-1}(p^3) = \begin{pmatrix} - & \{[1.33,5],[1.86,5.5],[2.38,6]\} & \{[4,4.83],[4.6,5.25]\} & \{[1,2]\} \\ \{[0,3.63],[0.5,4.14],[1,4.67]\} & - & \{[4,4.67],[4.4,5.33],[4.67,6]\} & \{[1.83,5],[2.29,5.5],[2.75,6]\} \\ \{[0,1],[0.5,1.5],[1,2]\} & \{[0,1.14],[0.5,1.6],[1.14,2]\} & - & \{[4,5.29],[4.75,5.5],[5.29,6]\} \\ \{[4,4.67]\} & \{[0,3.29],[0.5,3.83],[1,4.4]\} & \{[0.6,1.67]\} & - \end{pmatrix}. \tag{33}$$

Step 4: obtain the assessment information with the same length using the OWA operator.

Considering the fuzzy linguistic preference "for the most," the final assessment information obtained corresponding to the three researchers is as follows:

$$\Delta^{-1}(p^1) = \begin{pmatrix} - & [0.2,2.7] & [4.6,5.25] & [0.2,1.2] \\ [3.15,5.7] & - & [4.16,4.93] & [0.74,3.45] \\ [1,2] & [0.4,1.5] & - & [5,5.7] \\ [4.31,5.19] & [2.57,5.28] & [0.7,1.89] & - \end{pmatrix},$$
$$\Delta^{-1}(p^2) = \begin{pmatrix} - & [0.38,3.02] & [4.45,5.18] & [0.4,1.4] \\ [2.59,5.45] & - & [4,4.67] & [0.67,3.14] \\ [0.2,1.2] & [1.14,2] & - & [5,5.7] \\ [4.16,4.93] & [2.83,5.33] & [0.46,1.6] & - \end{pmatrix},$$
$$\Delta^{-1}(p^3) = \begin{pmatrix} - & [1.75,5.4] & [4.24,5] & [1,2] \\ [0.4,4.03] & - & [4.31,5.11] & [2.19,5.4] \\ [0.4,1.4] & [0.4,1.5] & - & [4.58,5.47] \\ [4,4.67] & [0.42,3.69] & [0.6,1.67] & - \end{pmatrix}. \tag{34}$$

Step 5: obtain the preference relation based on the OWA operator according to the approach proposed in Rodríguez et al. [4]:

$$P_C = \begin{pmatrix} - & [0.41,3.08] & [4.4,5.14] & [0.38,1.38] \\ [2.03,5.08] & - & [4.13,4.87] & [0.81,3.48] \\ [0.38,1.38] & [0.44,1.53] & - & [4.89,5.64] \\ [4.13,4.88] & [2,4.85] & [0.57,1.66] & - \end{pmatrix}. \tag{35}$$

Step 6: compute the pessimistic and optimistic collective preference for each alternative. The linguistic interval for each alternative is shown in Table 3.

Step 7: order the set of alternatives and select the best one as the solution to the GDM problem:

$x_2 > x_4 > x_1 > x_3$.

The best solution is similar with the work of Rodríguez et al. [4]. This example can illustrate the effectiveness of the proposed GDM approach for HFLTSs. However, the envelope-based method for computing with HFLTSs in Rodríguez et al. [4] cannot support experts to select nonadjacent linguistic terms. Next, we changed the preference provided by the three experts and compared the results obtained by the commonly used envelope-based method and the proposed GDM approach. We changed the preference referring to more than two linguistic terms to contain only two linguistic terms using the binary relation "or." The revised preference provided by the first expert is as follows:

$$p^{1\prime} = \begin{pmatrix} - & \text{at most } vl & vh & \text{at most } vl \\ \text{at least } vh & - & \text{between } h \text{ and } vh & n \text{ or } m \\ l & n \text{ or } l & - & \text{greater than } h \\ h \text{ or } a & h \text{ or } a & n \text{ or } m & - \end{pmatrix}. \tag{36}$$

The corresponding HFLTSs of the three experts are as follows:

TABLE 3: Linguistic intervals for all alternatives in Example 1.

John	Mike	David	Frank
p_1^R	p_2^R	p_3^R	p_4^R
$[(s_2, -0.27), (s_3, 0.2)]$	$[(s_2, 0.32), (s_4, -0.52)]$	$[(s_2, -0.1), (s_3, -0.15)]$	$[(s_1, 0.23), (s_4, -0.2)]$

$$p^{1\prime} = \begin{pmatrix} - & \{s_0, s_1\} & \{s_5\} & \{s_0, s_1\} \\ \{s_5, s_6\} & - & \{s_4, s_5\} & \{s_0, s_3\} \\ \{s_2\} & \{s_0, s_2\} & - & \{s_5, s_6\} \\ \{s_4, s_6\} & \{s_4, s_6\} & \{s_0, s_3\} & - \end{pmatrix},$$
$$p^{2\prime} = \begin{pmatrix} - & \{s_0, s_2\} & \{s_4, s_6\} & \{s_0, s_2\} \\ \{s_4, s_6\} & - & \{s_4\} & \{s_1\} \\ \{s_0, s_1\} & \{s_2\} & - & \{s_5, s_6\} \\ \{s_4, s_5\} & \{s_5\} & \{s_0, s_2\} & - \end{pmatrix}, \quad (37)$$
$$p^{3\prime} = \begin{pmatrix} - & \{s_4, s_6\} & \{s_4, s_5\} & \{s_2\} \\ \{s_0, s_2\} & - & \{s_4, s_6\} & \{s_4, s_6\} \\ \{s_0, s_2\} & \{s_0, s_2\} & - & \{s_4\} \\ \{s_4\} & \{s_0, s_2\} & \{s_1\} & - \end{pmatrix}.$$

The envelope of $p^{1\prime}$, $p^{2'}$, and $p^{3'}$ does not change, and also, the ranking result of the envelope-based method does not change. However, the ranking result of the proposed GDM approach for HFLTSs changes as the change of preference given by the three experts. The linguistic interval for each alternative after preference change is shown in Table 4. The revised alternative order is $x_2 > x_4 > x_3 > x_1$. By contrast, we can see that the ranking position of x_3 and the position of x_1 have changed. The comparison illustrates that the proposed GDM approach for HFLTSs has higher sensitivity to the change of preference information and has much wider applicability under different linguistic decision-making environments.

5.2. Example 2. A real example of selecting logistics service suppliers is adopted in this section. Company W is a small and medium-sized electric product manufacturer. Its main products are refrigerators, freezers, and air conditioners. To focus on the core competition ability, reduce cost, and improve customer service, the company decides to adopt a logistics outsourcing strategy. After preliminary screening, five candidates (i.e., alternatives), x_1, x_2, x_3, x_4, and x_5, remain for further evaluation. Seven evaluation criteria are considered: $C = \{c_1 =$ quality assurance, $c_2 =$ operation efficiency, $c_3 =$ logistics technology level, $c_4 =$ logistics facility level, $c_5 =$ price, $c_6 =$ management ability, $c_7 =$ development potential level$\}$. c_5 is the "small-the-better" criterion, and the other criteria are in the "larger-the-better" category. The weight vector of the criteria set is $W = (0.21, 0.19, 0.12, 0.14, 0.17, 0.09, 0.08)$. A committee composed of 3 experts $E = \{e_1, e_2, e_3\}$ evaluated the alternative service suppliers. The weights of the 3 experts are $\{we_1 = 0.4, we_2 = 0.3, we_3 = 0.3\}$.

Step 1: obtain the preferences provided by experts based on proposed context-free grammar, and transform the linguistic expressions into HFLTSs according to Definition 4. The predefined linguistic terms set is S. The hesitant evaluation information given by three experts is shown in Tables 5–7. $S = \{$neither (s_0); very low (s_1); low (s_2); medium (s_3); high (s_4); very high (s_5); absolute $(s_6)\}$.

Step 2: deal with the evaluation information based on the HFLTS using the 2-tuple linguistic representation model. Taking Table 4 as an example, the information of $\Delta^{-1}(H_S^{j1}(x_i))$ corresponding to this table is shown in Table 8.

Step 3: transform the information of $\Delta^{-1}(H_S^{jk}(x_i))$ into unified interval numbers for simplifying the group information aggregation.

The information of $\Delta^{-1}(H_S^{jk}(x_i))$ is treated by using the rough set theory. The experts' evaluation information represented as rough numbers is shown in Tables 9–11. The set of rough numbers can be integrated into a rough number based on the OWA operator. The possible numbers of elements in the rough number set in Tables 9–11 are 2, 3, and 4. Since the elements in the rough number set are arranged in the ascending order, the corresponding weights associated with elements for different fuzzy linguistic preferences are shown in Table 12.

Considering the fuzzy linguistic preference "for the most," the evaluation information in the form of interval numbers is shown in Tables 13–15.

Step 4: aggregate the information in Tables 13–15 into the final decision matrix considering both the expert's weights and the OWA weights associated with each $[\underline{a}_{ij}^k, \overline{a}_{ij}^k]$ for $1 \le i \le n$ and $1 \le j \le m$. The aggregated decision information for different fuzzy linguistic preferences is shown in Tables 16–18, respectively.

5.2.1. Sensitivity Analysis. Parameter v is the weight of the strategy of the "majority of attributes," and it plays an important role in determining the set of compromise solutions. As seen in Table 19, $[\underline{Q}_i, \overline{Q}_i]$ is determined by the value of v. When v is changed, $D([\underline{Q}_i^{(1)}, \overline{Q}_i^{(1)}], [\underline{Q}_i^{(2)}, \overline{Q}_i^{(2)}])$ is also changed. Aiming at the aggregated decision information obtained by the fuzzy linguistic preference "for the most," when $0 \le v \le 0.8789$, $D([\underline{Q}_i^{(1)}, \overline{Q}_i^{(1)}], [\underline{Q}_i^{(2)}, \overline{Q}_i^{(2)}]) \ge 0.25$, and only x_3 is the compromise solution; when $0.8789 < v \le 1$, $D([\underline{Q}_i^{(1)}, \overline{Q}_i^{(1)}], [\underline{Q}_i^{(2)}, \overline{Q}_i^{(2)}]) < 0.25$ and $D([\underline{Q}_i^{(1)}, \overline{Q}_i^{(1)}], [\underline{Q}_i^{(3)}, \overline{Q}_i^{(3)}]) \ge 0.25$, and both x_3 and x_4 are the compromise solutions. From the results in Table 19, we can see that the threshold values of v are different for different fuzzy

Table 4: Linguistic intervals for all alternatives after preference change in Example 1.

John	Mike	David	Frank
p_1^R	p_2^R	p_3^R	p_4^R
$[(s_2, -0.32), (s_3, 0.21)]$	$[(s_2, 0.45), (s_4, 0.62)]$	$[(s_2, 0.06), (s_3, 0)]$	$[(s_1, 0.34), (s_4, 0.05)]$

Table 5: The HFLTS information given by expert e_1 in Example 2.

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
x_1	$\{s_2\}$	$\{s_0, s_1, s_2\}$	$\{s_3, s_4\}$	$\{s_4, s_5\}$	$\{s_3, s_4\}$	$\{s_4\}$	$\{s_2, s_3, s_4\}$
x_2	$\{s_3, s_4\}$	$\{s_2\}$	$\{s_2, s_3, s_4\}$	$\{s_0, s_1, s_2\}$	$\{s_4, s_5\}$	$\{s_2, s_3, s_4\}$	$\{s_2, s_3\}$
x_3	$\{s_4, s_5\}$	$\{s_4, s_5, s_6\}$	$\{s_0, s_1, s_2, s_3\}$	$\{s_5, s_6\}$	$\{s_3\}$	$\{s_2, s_3, s_4\}$	$\{s_3, s_4\}$
x_4	$\{s_2, s_3, s_4\}$	$\{s_5, s_6\}$	$\{s_3, s_4\}$	$\{s_0, s_1, s_2\}$	$\{s_3\}$	$\{s_4, s_5\}$	$\{s_5\}$
x_5	$\{s_2, s_3\}$	$\{s_0, s_1, s_2\}$	$\{s_2, s_3, s_4, s_5\}$	$\{s_4, s_5\}$	$\{s_2, s_3\}$	$\{s_0, s_1, s_2\}$	$\{s_2, s_3\}$

Table 6: The HFLTS information given by expert e_2 in Example 2.

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
x_1	$\{s_3, s_4\}$	$\{s_2, s_3, s_4\}$	$\{s_2\}$	$\{s_0, s_1, s_2\}$	$\{s_2, s_3\}$	$\{s_0, s_1, s_2\}$	$\{s_3, s_4\}$
x_2	$\{s_5, s_6\}$	$\{s_0, s_1, s_2\}$	$\{s_2, s_3\}$	$\{s_2, s_3\}$	$\{s_0, s_1\}$	$\{s_3, s_4\}$	$\{s_0, s_1, s_2\}$
x_3	$\{s_6\}$	$\{s_5, s_6\}$	$\{s_4, s_5, s_6\}$	$\{s_2, s_3\}$	$\{s_2\}$	$\{s_5, s_6\}$	$\{s_2, s_3, s_4\}$
x_4	$\{s_0, s_1, s_2\}$	$\{s_2, s_3\}$	$\{s_2, s_3, s_4, s_5\}$	$\{s_5, s_6\}$	$\{s_3, s_4\}$	$\{s_2\}$	$\{s_2, s_3, s_4\}$
x_5	$\{s_2, s_3, s_4\}$	$\{s_3\}$	$\{s_2, s_3\}$	$\{s_0, s_1, s_2\}$	$\{s_3, s_4\}$	$\{s_0, s_1, s_2\}$	$\{s_5, s_6\}$

Table 7: The HFLTS information given by expert e_3 in Example 2.

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
x_1	$\{s_0, s_1, s_2, s_3\}$	$\{s_2, s_3\}$	$\{s_2, s_3, s_4\}$	$\{s_3, s_4\}$	$\{s_2, s_3, s_4\}$	$\{s_5, s_6\}$	$\{s_3\}$
x_2	$\{s_3, s_4\}$	$\{s_3\}$	$\{s_4, s_5, s_6\}$	$\{s_4, s_5\}$	$\{s_0, s_1, s_2\}$	$\{s_3, s_4\}$	$\{s_4, s_5\}$
x_3	$\{s_4, s_5, s_6\}$	$\{s_3, s_4\}$	$\{s_2, s_3\}$	$\{s_2, s_3, s_4\}$	$\{s_1, s_2\}$	$\{s_2, s_3\}$	$\{s_0, s_1, s_2\}$
x_4	$\{s_2, s_3, s_4, s_5\}$	$\{s_2, s_3\}$	$\{s_2, s_3, s_4\}$	$\{s_3, s_4\}$	$\{s_0, s_1, s_2\}$	$\{s_2, s_3, s_4\}$	$\{s_5\}$
x_5	$\{s_2, s_3, s_4\}$	$\{s_4\}$	$\{s_4, s_5\}$	$\{s_2, s_3, s_4\}$	$\{s_3, s_4\}$	$\{s_5, s_6\}$	$\{s_0, s_1, s_2\}$

Table 8: The information of $\Delta^{-1}(H_S^{j1}(x_i))$ corresponding to Table 4.

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
x_1	{2}	{0, 1, 2}	{3, 4}	{4, 5}	{3, 4}	{4}	{2, 3, 4}
x_2	{3, 4}	{2}	{2, 3, 4}	{0, 1, 2}	{4, 5}	{2, 3, 4}	{2, 3}
x_3	{4, 5}	{4, 5, 6}	{0, 1, 2, 3}	{5, 6}	{3}	{2, 3, 4}	{3, 4}
x_4	{2, 3, 4}	{5, 6}	{3, 4}	{0, 1, 2}	{3}	{4, 5}	{5}
x_5	{2, 3}	{0, 1, 2}	{2, 3, 4, 5}	{4, 5}	{2, 3}	{0, 1, 2}	{2, 3}

linguistic preferences. The value of v can differentiate the possible solution sets $\{x_3\}$ and $\{x_3, x_4\}$.

Step 5: select the appropriate supplier using the proposed VIKOR method.

Aiming at the aggregated decision information according to the "for the most," the positive ideal solution A^* and the negative ideal solution A^- are determined first, and then $[\underline{S}_i, \overline{S}_i]$, $[\underline{R}_i, \overline{R}_i]$, and $[\underline{Q}_i, \overline{Q}_i]$ are computed according to equations (25)–(30):

$A^* = \{5.44, 5.11, 4.23, 4.44, 0.60, 4.91, 4.76\}$

$A^- = \{1.23, 1.07, 1.41, 1.20, 3.51, 0.65, 1.25\}$

According to the verification rules of VIKOR, if the acceptable advantage and acceptable ability are satisfied, the best rank can be assigned as a compromise solution. Let $v = 0.5$, which represents selecting the appropriate object in compromise. First, the two conditions of the first case are considered.

Condition 1: $D([\underline{Q}_i^{(1)}, \overline{Q}_i^{(1)}], [\underline{Q}_i^{(2)}, \overline{Q}_i^{(2)}]) = \sqrt{0.5 \times [(0.208 - 0.000)^2 + (0.862 - 0.519)^2]} = 0.323$. $DQ = 1/(5-1) = 0.25$.

Condition 1 is satisfied.

Condition 2: $x^{(1)}$ is the best object ranked according to $[\underline{S}_i, \overline{S}_i]$ and $[\underline{R}_i, \overline{R}_i]$, and condition 2 is satisfied.

Therefore, $x^{(1)}$, i.e., x_3, is the compromise solution.

Considering all the possible values of v, the final results are shown in Table 19.

Table 9: Evaluation information of expert e_1 represented as rough numbers.

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
x_1	{[1.25, 2.8]}	{[0, 2.13], [0.5, 2.43], [1.4, 2.67]}	{[2.5, 3.5], [3, 4]}	{[2.33, 4.33], [2.71, 5]}	{[2.6, 3.4], [3, 4]}	{[4, 5]}	{[2, 3.17], [2.75, 3.4], [3.17, 4]}
x_2	{[3, 4.17], [3.5, 4.75]}	{[1.25, 2.33]}	{[2, 3.63], [2.5, 4.17], [3, 4.75]}	{[0, 2.43], [0.5, 2.83], [1.25, 3.2]}	{[1.33, 4.5], [1.86, 5]}	{[2, 3.29], [2.75, 3.5], [3.29, 4]}	{[1.25, 3.2], [1.6, 4]}
x_3	{[4, 5], [4.5, 5.5]}	{[3.67, 5], [4.2, 5.5], [4.71, 6]}	{[0, 2.89], [0.5, 3.25], [1.25, 3.57], [1.83, 4.2]}	{[3.17, 5.5], [3.57, 6]}	{[2, 3]}	{[2, 3.57], [2.5, 4.2], [2.8, 5]}	{[1.83, 3.5], [2.38, 4]}
x_4	{[1.4, 3.13], [1.86, 3.8], [2.33, 4.33]}	{[3, 5.5], [3.5, 6]}	{[2.6, 3.71], [3.13, 4.25]}	{[0, 3], [0.5, 3.5], [1, 4]}	{[1.8, 3.33]}	{[3, 4.33], [3.33, 5]}	{[3.8, 5]}
x_5	{[2, 2.88], [2.5, 3.4]}	{[0, 2], [0.5, 2.5], [1, 3]}	{[2, 3.5], [2.5, 4], [3, 4.5], [3.5, 5]}	{[2.29, 4.33], [2.63, 5]}	{[2, 3.17], [2.75, 3.4]}	{[0, 2.13], [0.5, 2.83], [1, 3.75]}	{[1.25, 3.6], [1.6, 4.67]}

Table 10: Evaluation information of expert e_2 represented as rough numbers.

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
x_1	{[1.83, 3.33], [2.14, 4]}	{[1.4, 2.67], [1.86, 3.33], [2.125, 4]}	{[2, 3]}	{[0, 2.71], [0.5, 3.17], [1, 3.6]}	{[2, 3], [2.6, 3.4}	{[0, 3], [0.5, 3.6], [1, 4.25]}	{[2.75, 3.4], [3.17, 4]}
x_2	{[3.8, 5.5], [4.17, 6]}	{[0, 1.6], [0.5, 2], [1.25, 2.33]}	{[2, 3.63], [2.5, 4.17]}	{[1.25, 3.2], [1.6, 4]}	{[0, 1.86], [0.5, 2.6]}	{[2.75, 3.5], [3.29, 4]}	{[0, 2.43], [0.5, 2.83], [1.25, 3.2]}
x_3	{[5, 6]}	{[4.2, 5.5], [4.71, 6]}	{[2.14, 5], [2.5, 5.5], [2.89, 6]}	{[2, 3.57], [2.5, 4.2]}	{[1.67, 2.33]}	{[3.17, 5.5], [3.57, 6]}	{[1.25, 3], [1.83, 3.5], [2.38, 4]}
x_4	{[0, 2.6], [0.5, 2.89], [1.4, 3.13]}	{[2, 3.5], [2.5, 4.25]}	{[2, 3.33], [2.6, 3.71], [3.13, 4.25], [3.33, 5]}	{[2.5, 5.5], [3, 6]}	{[1.8, 3.33], [2.17, 4]}	{[2, 3.33]}	{[2, 3.8], [2.5, 4.25], [3, 4.67]}
x_5	{[2, 2.88], [2.5, 3.4], [2.88, 4]}	{[1.5, 3.5]}	{[2, 3.5], [2.5, 4]}	{[0, 2.63], [0.5, 3], [1.25, 3.33]}	{[2.75, 3.4], [3.17, 4]}	{[0, 2.13], [0.5, 2.83], [1, 3.75]}	{[2.17, 5.5], [2.71, 6]}

Table 11: Evaluation information of expert e_3 represented as rough numbers.

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
x_1	{[0, 2.14], [0.5, 2.5], [1.25, 2.8], [1.83, 3.33]}	{[1.4, 2.67], [1.86, 3.33]}	{[2, 3], [2.5, 3.5], [3, 4]}	{[1.5, 4], [2.33, 4.33]}	{[2, 3], [2.6, 3.4], [3, 4]}	{[2.4, 5.5], [3, 6]}	{[2.75, 3.4]}
x_2	{[3, 4.17], [3.5, 4.75]}	{[1.6, 3]}	{[3, 4.75], [3.29, 5.5], [3.63, 6]}	{[2, 4.5], [2.43, 5]}	{[0, 1.86], [0.5, 2.6], [0.8, 3.67]}	{[2.75, 3.5], [3.29, 4]}	{[2, 4.5], [2.43, 5]}
x_3	{[4, 5], [4.5, 5.5], [5, 6]}	{[3, 4.71], [3.67, 5]}	{[1.25, 3.57], [1.83, 4.2]}	{[2, 3.57], [2.5, 4.2], [2.8, 5]}	{[1, 2], [1.67, 2.33]}	{[2, 3.57], [2.5, 4.2]}	{[0, 2.38], [0.5, 2.71], [1.25, 3]}
x_4	{[1.4, 3.13], [1.86, 3.8], [2.33, 4.33], [2.6, 5]}	{[2, 3.5], [2.5, 4.25]}	{[2, 3.33, [2.6, 3.71], [3.13, 4.25]}	{[1.5, 4.5], [2, 5]}	{[0, 2.17], [0.5, 2.6], [1, 3]}	{[2, 3.33], [2.33, 4], [3, 4.33]}	{[3.8, 5]}
x_5	{[2, 2.88], [2.5, 3.4], [2.88, 4]}	{[2, 4]}	{[3, 4.5], [3.5, 5]}	{[1.25, 3.33], [1.6, 4], [2.29, 4.33]}	{[2.75, 3.4], [3.17, 4]}	{[1.57, 5.5], [2.13, 6]}	{[0, 2.71], [0.5, 3.17], [1.25, 3.6]}

Table 12: The successive element's weight in one set for different fuzzy linguistic preferences.

	Set with 2 elements	Set with 3 elements	Set with 4 elements
For the most	0.6, 0.4	0.27, 0.67, 0.06	0.1, 0.5, 0.4, 0
At least half	0, 1	0, 0.33, 0.67	0, 0, 0.5, 0.5
As much as possible	1, 0	0.67, 0.33, 0	0.5, 0.5, 0, 0

Table 13: The final evaluation information obtained from e_1 for "for the most."

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
x_1	[1.25, 2.8]	[0.42, 2.36]	[2.70, 3.70]	[2.48, 4.60]	[2.76, 3.64]	[4.00, 5.00]	[2.57, 3.37]
x_2	[3.20, 4.40]	[1.25, 2.33]	[2.40, 4.06]	[0.41, 2.74]	[1.54, 4.70]	[2.58, 3.47]	[1.39, 3.52]
x_3	[4.20, 5.20]	[4.09, 5.40]	[0.75, 3.34]	[3.33, 5.70]	[2.00, 3.00]	[2.38, 4.08]	[2.05, 3.70]
x_4	[1.76, 3.65]	[3.20, 5.70]	[2.81, 3.93]	[0.40, 3.40]	[1.80, 3.33]	[3.13, 4.60]	[3.80, 5.00]
x_5	[2.20, 3.09]	[0.40, 2.40]	[2.65, 4.15]	[2.43, 4.60]	[2.30, 3.26]	[0.40, 2.70]	[1.39, 4.03]

Table 14: The final evaluation information obtained from e_2 for "for the most."

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
x_1	[1.95, 3.60]	[1.75, 3.19]	[2, 3]	[0.40, 3.07]	[2.24, 3.16]	[0.40, 3.48]	[2.92, 3.64]
x_2	[3.95, 5.70]	[0.41, 1.91]	[2.20, 3.85]	[1.39, 3.52]	[0.20, 2.16]	[2.97, 3.70]	[0.41, 2.74]
x_3	[5.00, 6.00]	[4.40, 5.70]	[2.43, 5.40]	[2.20, 3.82]	[1.67, 2.33]	[3.33, 5.70]	[1.71, 3.40]
x_4	[0.42, 2.83]	[2.20, 3.80]	[2.75, 3.89]	[2.70, 5.70]	[1.95, 3.60]	[2.00, 3.33]	[2.40, 4.15]
x_5	[2.39, 3.30]	[1.50, 3.50]	[2.20, 3.70]	[0.41, 2.92]	[2.92, 3.64]	[0.40, 2.70]	[2.39, 5.70]

Table 15: The final evaluation information obtained from e_3 for "for the most."

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
x_1	[0.75, 2.58]	[1.58, 2.93]	[2.40, 3.40]	[1.83, 4.13]	[2.46, 3.33]	[2.64, 5.70]	[2.75, 3.40]
x_2	[3.47, 4.00]	[1.60, 3.00]	[3.23, 5.33]	[2.17, 4.70]	[0.38, 2.46]	[2.97, 3.70]	[2.17, 4.70]
x_3	[4.40, 5.40]	[3.68, 4.20]	[1.48, 3.82]	[2.38, 4.08]	[1.27, 2.13]	[2.20, 3.82]	[0.41, 2.64]
x_4	[2.00, 3.95]	[2.20, 3.80]	[2.47, 3.64]	[1.70, 4.70]	[0.40, 2.51]	[2.28, 3.84]	[3.80, 5.00]
x_5	[2.39, 3.30]	[2.00, 4.00]	[3.20, 4.70]	[1.55, 3.84]	[2.92, 3.64]	[1.79, 5.70]	[0.41, 3.07]

Table 16: The aggregated decision information for "for the most."

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
x_1	[1.23, 2.88]	[1.22, 2.79]	[2.35, 3.35]	[1.57, 3.94]	[2.47, 3.35]	[2.31, 4.91]	[2.72, 3.43]
x_2	[3.41, 4.52]	[1.07, 2.33]	[2.49, 4.23]	[1.20, 3.47]	[0.60, 2.89]	[2.84, 3.62]	[1.25, 3.51]
x_3	[4.44, 5.44]	[4.03, 5.11]	[1.41, 3.94]	[2.55, 4.38]	[1.63, 2.43]	[2.50, 4.30]	[1.42, 3.25]
x_4	[1.42, 3.47]	[2.43, 4.24]	[2.69, 3.83]	[1.44, 4.44]	[1.43, 3.14]	[2.40, 3.87]	[3.40, 4.76]
x_5	[2.32, 3.23]	[1.22, 3.22]	[2.62, 4.12]	[1.43, 3.75]	[2.71, 3.51]	[0.65, 3.24]	[1.29, 4.06]

Table 17: The aggregated decision information for "at least half."

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
x_1	[1.77, 3.47]	[1.80, 3.40]	[2.80, 3.80]	[2.31, 4.56]	[2.90, 3.85]	[3.04, 5.34]	[3.06, 3.84]
x_2	[4.04, 5.36]	[1.38, 2.64]	[3.12, 5.12]	[1.88, 4.30]	[1.29, 4.11]	[3.25, 3.97]	[1.91, 4.35]
x_3	[4.85, 5.85]	[4.65, 5.79]	[2.22, 4.93]	[3.14, 5.33]	[1.85, 2.69]	[3.09, 5.27]	[2.12, 3.78]
x_4	[2.15, 4.24]	[3.04, 5.19]	[3.15, 4.41]	[2.25, 5.25]	[1.83, 3.59]	[2.96, 4.50]	[3.66, 4.93]
x_5	[2.70, 3.72]	[1.61, 3.61]	[3.26, 4.76]	[2.21, 4.49]	[3.09, 3.88]	[1.46, 4.68]	[2.05, 5.13]

Table 18: The aggregated decision information for "as much as possible."

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
x_1	[0.85, 2.65]	[0.76, 2.47]	[2.15, 3.15]	[1.02, 3.51]	[2.18, 3.12]	[1.64, 4.28]	[2.48, 3.32]
x_2	[3.12, 4.04]	[0.78, 2.14]	[2.22, 3.90]	[0.78, 3.05]	[0.32, 2.46]	[2.48, 3.42]	[0.84, 3.09]
x_3	[4.20, 5.20]	[3.49, 4.49]	[0.87, 3.54]	[2.29, 4.02]	[1.41, 2.30]	[2.24, 3.94]	[0.90, 2.90]
x_4	[0.89, 3.05]	[2.20, 3.90]	[2.31, 3.53]	[0.94, 3.94]	[1.01, 2.84]	[2.23, 3.60]	[3.01, 4.49]
x_5	[2.08, 2.96]	[0.86, 2.86]	[2.24, 3.74]	[0.97, 3.32]	[2.35, 3.28]	[0.38, 2.83]	[0.86, 3.53]

Table 19: The results of the VIKOR method based on the proposed GDM approach.

(a) The values S, R, and Q and the preference ranking order in the case of "for the most"

	$[\underline{S}_i, \overline{S}_i]$	$[\underline{R}_i, \overline{R}_i]$	$[\underline{Q}_i, \overline{Q}_i]$
x_1	[0.435, 0.859]	[0.128, 0.210]	$[0.451-0.036v,\ 1-0.023v]$
x_2	[0.274, 0.763]	[0.131, 0.190]	$[0.471-0.270v,\ 0.867-0.017v]$
x_3	[0.122, 0.536]	[0.060, 0.120]	$[0,\ 0.399+0.150v]$
x_4	[0.227, 0.754]	[0.098, 0.201]	$[0.254-0.116v,\ 0.937-0.099v]$
x_5	[0.408, 0.876]	[0.123, 0.183]	$[0.421-0.042v,\ 0.819+0.181v]$
Ranking order	$x_3 \succ x_4 \succ x_2 \succ x_5 \succ x_1$	$x_3 \succ x_4 \succ x_5 \succ x_2 \succ x_1$	$x_3 \succ x_4 \succ x_2 \succ x_5 \succ x_1$ Compromise solution set $\{x_3\}$, $0 \le v \le 0.8789$ $\{x_3, x_4\}$, $0.8789 < v \le 1$

(b) The values S, R, and Q and the preference ranking order in the case of "at least half"

	$[\underline{S}_i, \overline{S}_i]$	$[\underline{R}_i, \overline{R}_i]$	$[\underline{Q}_i, \overline{Q}_i]$
x_1	[0.440, 0.829]	[0.123, 0.210]	$[0.504-0.045v,\ 1-0.049v]$
x_2	[0.254, 0.804]	[0.136, 0.190]	$[0.578-0.354v,\ 0.887+0.032v]$
x_3	[0.077, 0.521]	[0.034, 0.120]	$[0,\ 0.489+0.072v]$
x_4	[0.198, 0.746]	[0.083, 0.190]	$[0.279-0.125v,\ 0.889-0.044v]$
x_5	[0.376, 0.868]	[0.110, 0.180]	$[0.431-0.053v,\ 0.830+0.170v]$
Ranking order	$x_3 \succ x_4 \succ x_2 \succ x_5 \succ x_1$	$x_3 \succ x_4 \succ x_5 \succ x_2 \succ x_1$	$x_3 \succ x_4 \succ x_2 \succ x_5 \succ x_1$ Compromise solution set $\{x_3\}$, $0 \le v \le 0.8113$ $\{x_3, x_4\}$, $0.8113 < v \le 1$

(c) The values S, R, and Q and the preference ranking order in the case of "as much as possible"

	$[\underline{S}_i, \overline{S}_i]$	$[\underline{R}_i, \overline{R}_i]$	$[\underline{Q}_i, \overline{Q}_i]$
x_1	[0.410, 0.865]	[0.123, 0.210]	$[0.410-0.024v,\ 1-0.010v]$
x_2	[0.268, 0.740]	[0.120, 0.189]	$[0.387-0.190v,\ 0.857-0.033v]$
x_3	[0.120, 0.533]	[0.063, 0.120]	$[0,\ 0.389+0.161v]$
x_4	[0.207, 0.745]	[0.104, 0.208]	$[0.279-0.163v,\ 0.987-0.156v]$
x_5	[0.399, 0.873]	[0.117, 0.185]	$[0.366-0.005v,\ 0.830+0.170v]$
Ranking order	$x_3 \succ x_4 \succ x_2 \succ x_5 \succ x_1$	$x_3 \succ x_4 \succ x_5 \succ x_2 \succ x_1$	$x_3 \succ x_4 \succ x_2 \succ x_5 \succ x_1$ Compromise solution set $\{x_3\}$, $0 \le v \le 0.8614$ $\{x_3, x_4\}$, $0.8614 < v \le 1$

Table 20: The original and changed information on evaluation for x_4 by e_1 and e_2.

		c_1	c_2	c_3	c_4	c_5	c_6	c_7
The original information	$x_4(e_1)$	$\{s_2, s_3, s_4\}$	$\{s_5, s_6\}$	$\{s_3, s_4\}$	$\{s_0, s_1, s_2\}$	$\{s_3\}$	$\{s_4, s_5\}$	$\{s_5\}$
	$x_4(e_2)$	$\{s_0, s_1, s_2\}$	$\{s_2, s_3\}$	$\{s_2, s_3, s_4, s_5\}$	$\{s_5, s_6\}$	$\{s_3, s_4\}$	$\{s_2\}$	$\{s_2, s_3, s_4\}$
The changed information	$x_4(e_1)$	$\{s_2, s_4\}$	$\{s_5, s_6\}$	$\{s_3, s_4\}$	$\{s_0, s_2\}$	$\{s_3\}$	$\{s_4, s_5\}$	$\{s_5\}$
	$x_4(e_2)$	$\{s_0, s_2\}$	$\{s_2, s_3\}$	$\{s_2, s_5\}$	$\{s_5, s_6\}$	$\{s_3, s_4\}$	$\{s_2\}$	$\{s_2, s_4\}$

An obvious advantage of the GDM approach for HFLTSs proposed in this paper is that it can support experts to give arbitrarily term mix to increase the richness of linguistic expressions. In this section, we change some HFLTSs on the evaluation of x_4 given by experts e_1 and e_2. Suppose the two experts tend to use the binary relation "or" and give non-adjacent linguistic terms due to the uncertainty of the market. The original and changed information on the evaluation of x_4 by e_1 and e_2 is shown in Table 20. Aiming at the changed evaluation information, the aggregation result obtained by the envelope-based approach would not change for that the envelopes of the changed HFLTSs are the same as those of the original HFLTSs. However, the aggregation result obtained by the novel group decision-making approach proposed in this paper is different from Tables 13–15 under conditions of three fuzzy linguistic preferences. Furthermore, the VIKOR results of the proposed method based on the changed information will also be different from Table 19, and the related result is shown in Table 21. We can see that the compromise solutions remain unchanged, but the threshold values of v vary for different fuzzy linguistic preferences.

5.2.2. Comparisons and Discussion. To illustrate the effectiveness of the proposed group decision-making approach for HFLTSs and the related VIKOR, we use the above case study to analyze the envelope-based approach for HFLTSs. We take the envelope-based approach proposed by Rodríguez et al. [4] as a comparable approach. The HFLTS information

Table 21: The results of the proposed VIKOR method based on the changed information.

(a) The values S, R, and Q and the preference ranking order in the case of "for the most"

	$[\underline{S}_i, \overline{S}_i]$	$[\underline{R}_i, \overline{R}_i]$	$[\underline{Q}_i, \overline{Q}_i]$
Ranking order	$x_3 \succ x_4 \succ x_2 \succ x_5 \succ x_1$	$x_3 \succ x_4 \succ x_5 \succ x_2 \succ x_1$	$x_3 \succ x_4 \succ x_2 \succ x_5 \succ x_1$ Compromise solution set $\{x_3\}$, $0 \le v \le 0.7905$ $\{x_3, x_4\}$, $0.7505 < v \le 1$

(b) The values S, R, and Q and the preference ranking order in the case of "at least half"

	$[\underline{S}_i, \overline{S}_i]$	$[\underline{R}_i, \overline{R}_i]$	$[\underline{Q}_i, \overline{Q}_i]$
Ranking order	$x_3 \succ x_4 \succ x_2 \succ x_5 \succ x_1$	$x_3 \succ x_4 \succ x_5 \succ x_2 \succ x_1$	$x_3 \succ x_4 \succ x_2 \succ x_5 \succ x_1$ Compromise solution set $\{x_3\}$, $0 \le v \le 0.4807$ $\{x_3, x_4\}$, $0.4807 < v \le 1$

(c) The values S, R, and Q and the preference ranking order in the case of "as much as possible"

	$[\underline{S}_i, \overline{S}_i]$	$[\underline{R}_i, \overline{R}_i]$	$[\underline{Q}_i, \overline{Q}_i]$
Ranking order	$x_3 \succ x_4 \succ x_2 \succ x_5 \succ x_1$	$x_3 \succ x_4 \succ x_5 \succ x_2 \succ x_1$	$x_3 \succ x_4 \succ x_2 \succ x_5 \succ x_1$ Compromise solution set $\{x_3\}$, $0 \le v \le 0.8926$ $\{x_3, x_4\}$, $0.8926 < v \le 1$

Table 22: The aggregated decision information of the comparative method.

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
x_1	[1.7, 2.9]	[1.2, 2.9]	[2.4, 3.4]	[2.5, 3.8]	[2.4, 3.7]	[3.1, 4.0]	[2.6, 3.7]
x_2	[3.6, 4.6]	[1.7, 2.3]	[2.6, 4.3]	[1.8, 3.2]	[1.6, 2.9]	[2.6, 4.0]	[2.0, 3.3]
x_3	[4.6, 5.6]	[4.0, 5.4]	[1.8, 3.9]	[3.2, 4.5]	[2.1, 2.4]	[2.9, 4.3]	[1.8, 3.4]
x_4	[1.4, 3.7]	[3.2, 4.2]	[2.4, 4.3]	[2.4, 3.8]	[2.1, 3.0]	[2.8, 3.8]	[4.1, 4.7]
x_5	[2.0, 2.6]	[2.1, 2.9]	[2.6, 4.4]	[2.2, 3.8]	[2.6, 3.6]	[1.5, 3.2]	[2.3, 3.6]

Table 23: The results of the VIKOR method based on the comparative GDM approach.

	$[\underline{S}_i, \overline{S}_i]$	$[\underline{R}_i, \overline{R}_i]$	$[\underline{Q}_i, \overline{Q}_i]$
x_1	[0.433, 0.848]	[0.135,0.195]	[0.558–0.114v, 0.912–0.085v]
x_2	[0.311, 0.725]	[0.140, 0.167]	[0.588–0.307v, 0.749–0.085v]
x_3	[0.099, 0.491]	[0.040, 0.120]	[0, 0.469 + 0.052v]
x_4	[0.247, 0.689]	[0.095, 0.210]	[0.322–0.126v, 1–0.214v]
x_5	[0.446, 0.850]	[0.150, 0.180]	[0.646–0.184v, 0.823 + 0.177v]
Ranking order	$x_3 \succ x_4 \succ x_2 \succ x_5 \succ x_1$	$x_3 \succ x_4 \succ x_5 \succ x_2 \succ x_1$	$x_3 \succ x_4 \succ x_2 \succ x_5 \succ x_1$ Compromise solution set $\{x_3\}$, $0 \le v \le 1$

given by three experts as shown in Tables 5–7 can be translated into interval information according to the envelope-based approach. The interval information is modeled by the 2-tuple linguistic representation model and then aggregated by the weighted averaging operator. The final decision information of $\Delta^{-1}(H_S^{j1}(x_i))$ is shown in Table 22.

The decision result of Table 20 using the proposed VIKOR approach is shown in Table 23. The ranking order of the alternatives is $x_3 \succ x_4 \succ x_2 \succ x_5 \succ x_1$, which is the same as the ranking order in Table 20. However, the compromise solution set remains unchanged with the change of v from 0 to 1 in Table 23, which has a lower sensitivity.

To reveal the features of the proposed VIKOR method, the rough TOPSIS method [53] is also applied in the case study. The rough TOPSIS method is used to deal with the aggregated decision information for different fuzzy linguistic preferences (Tables 16–18). Table 24 shows the final ranking result of the rough TOPSIS.

In the case of "for the most," if the weight $0 \le v \le 0.8789$, the compromise solution obtained by the rough TOPSIS is the same with the compromise solution of the extended VIKOR. If the weight $0.8789 < v \le 1$, the compromise solution obtained by the rough TOPSIS is different from the compromise solution of the extended VIKOR. In the case of "at least half," if the weight $0 \le v \le 0.8113$, the compromise solution obtained by the rough TOPSIS is the same with the compromise solution of the extended VIKOR. In the case of "as much as possible," if the weight $0 \le v \le 0.8614$, the compromise solution obtained by the rough TOPSIS is the same with the compromise solution of the extended VIKOR. The rough TOPSIS can only obtain the distinct solution, and it cannot obtain the compromise solutions. The VIKOR

Table 24: The evaluation result of the rough TOPSIS approach.

	Closeness coefficient in the rough TOPSIS		
	In the case of "for the most"	In the case of "at least half"	In the case of "as much as possible"
x_1	0.4054	0.4040	0.4073
x_2	0.4804	0.4713	0.4928
x_3	0.6290	0.6463	0.6242
x_4	0.4949	0.5147	0.4977
x_5	0.4043	0.4186	0.4114
Ranking order	$x_3 \succ x_4 \succ x_2 \succ x_1 \succ x_5$	$x_3 \succ x_4 \succ x_2 \succ x_5 \succ x_1$	$x_3 \succ x_4 \succ x_2 \succ x_5 \succ x_1$

method focuses on ranking and selecting from a set of alternatives in the presence of conflicting criteria, and it determines a compromise solution that could be accepted by the decision makers. Thus, the obtained compromise solution is more robust and could be accepted by the decision makers.

6. Conclusions

Many practical issues in various fields can be formulated into MCGDM problems, which refer to the rank given alternatives by a group of decision makers. The challenge of solving these problems is intensified when considering the uncertain information environment. Decision makers may hesitate among several linguistic terms when expressing their preferences. HFLTS is an efficient fuzzy model to express hesitant information, but the flexibility brings obstacle of aggregating HFLTSs with different lengths. Therefore, reliable GDM methods in the hesitant environment have to be studied and extended to solve the MCGDM problems.

In the context of HFLTSs, this paper increases the richness of linguistic expressions by giving out improved context-free grammar and proposes a novel GDM approach based on the 2-tuple fuzzy linguistic representation model, the rough set theory, and the OWA operator. Finally, the GDM approach is extended, and a hesitant VIKOR approach is presented. The case study shows that (1) the novel GDM approach is much more flexible due to the fact that it allows decision makers to select nonadjacent linguistic terms and retains the vagueness in the decision-making information. (2) The novel GDM approach is sensitive to the variation of the information input. (3) Different fuzzy linguistic preferences for the OWA operator affect the associated weights and then contribute to different aggregation results. (4) The weight of strategy v in the hesitant VIKOR has an effect on the compromise solution, and its selection may influence the optimal solution or the number of compromise solutions. Further work may continue to apply the group decision-making approach for HFLTSs to other decision-making methods or solve specific problems.

Data Availability

The data in the case study are subjective and simulated, which are given by the invited experts. The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Research Article

A Kriging Model-Based Expensive Multiobjective Optimization Algorithm Using R2 Indicator of Expectation Improvement

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Most of the multiobjective optimization problems in engineering involve the evaluation of expensive objectives and constraint functions, for which an approximate model-based multiobjective optimization algorithm is usually employed, but requires a large amount of function evaluation. Aiming at effectively reducing the computation cost, a novel infilling point criterion EIR2 is proposed, whose basic idea is mapping a point in objective space into a set in expectation improvement space and utilizing the R2 indicator of the set to quantify the fitness of the point being selected as an infilling point. This criterion has an analytic form regardless of the number of objectives and demands lower calculation resources. Combining the Kriging model, optimal Latin hypercube sampling, and particle swarm optimization, an algorithm, EIR2-MOEA, is developed for solving expensive multiobjective optimization problems and applied to three sets of standard test functions of varying difficulty and comparing with two other competitive infill point criteria. Results show that EIR2 has higher resource utilization efficiency, and the resulting nondominated solution set possesses good convergence and diversity. By coupling with the average probability of feasibility, the EIR2 criterion is capable of dealing with expensive constrained multiobjective optimization problems and its efficiency is successfully validated in the optimal design of energy storage flywheel.

1. Introduction

Science and engineering practice possess a large number of multiobjective optimization problems (MOP) [1] whose objectives often conflict with each other and need to be optimized simultaneously, where the Pareto set is desired. Evolutionary Algorithm (EA) [2] has proven to be very suitable for MOP, some typical algorithms include NSGA-II [3], SPEA2 [4], MOEA/D [5], and MOPSO [6]. However, the success of these algorithms depends heavily on a large number of function evaluations and can only be applied in cases that function evaluation is cheap computationally. To handle some expensive MOPs, it is a desire to develop some efficient algorithms.

To reduce the number of expensive evaluation required, approximate/surrogate model [7], which mimic the input-output relationship of expensive function but is cheap-to-evaluate, is often employed. Polynomial response surfaces [8], support vector machines [9], neural networks [10], and Kriging [11] are commonly used approximate models.

Currently, there are two ways to integrate the approximate model with EA. The first one is to first construct a static global approximate model based on all sample points already evaluated, and the EA algorithm then searches for the optimal solution. However, with limited computational resources, it is hard to guarantee the approximate model's accuracy over the entire design space. The second one utilizes an approximate model in a dynamic manner by first constructing a rough approximate model with a fraction of sample points generated by some Design of Experiments (DoE) [12] techniques, and then maximizing some infilling point criterion (IPC) [13], a new point is located by EA and is evaluated by expensive function. This process is repeated until resources are exhausted. The second manner has the advantage of high efficiency in using computation resources and is used in this paper.

The infilling point criterion [14] plays a critical role in the approximate model-based optimization method. Expectation Improvement (EI), proposed in Efficient Global Optimization (EGO) [15] by Jone, may be one of the most researched method in the literature. It uses the Kriging model as an approximate model and considers not only the predicted value but also modeling uncertainty, achieving a good balance between exploration and exploitation in the optimization process. Other IPCs include statistical lower bound [16] and probability of improvement [17].

However, those criterions are proposed originally only for single optimization and have to be modified for MOPs. Knowles first introduced EI into the field of multiobjective optimization and presented ParEGO [18], where the Kriging model is used and multiple objectives are converted into a single objective through a parameterized scalarizing weight vector, and then followed by the direct application of EGO, the weight vector changed randomly as iteration proceeded. Joan pointed out ParEGO is apt to favor model accuracy rather than finding nondominated solutions and demands more iterations before convergence and hence proposed Pareto Expected Improvement index(PEI) [19], which considers both EI and the probability of being Pareto optimal solution. Multi-EGO [20] builds the Kriging model and calculates separately EI for each objective and calls MOGA [21] as a solver to output a set of candidate points, from which a point is selected as an infilling point. MOEA/D-EGO [22] borrowed the ideas from ParEGO, and it generated not a single but a set of weight vectors that distributed uniformly in the objective space to form single objective optimization subproblems involving EI, optimizing jointly those subproblems produces multiple infill points and in this way parallelly infilling is achieved. Recently, some IPCs compatible with multiobjective optimization were proposed, such as MaxMin improvement [23] and expectation hypervolume improvement [24]. Svenson defined MaxMin improvement in the analytic form when the number of the objective function is two and suggested using the Monte Carlo method (MCM) [25] to calculate MaxMin value when the number of the objective is more than two.

In this paper, a novel IPC is proposed for expensive multiobjective optimization. The outstanding feature of the IPC is to map one point in the objective space to a set in the expectation improvement space, where the fitness of the point is defined as a Quality indicator (QI) [26] value of the set. In this way, QI plays a role in guiding optimization towards an infilling point, rather than evaluating the nondominated solution set after optimization. There are a large number of candidate QIs in the literature, such as IGD [27, 28], ε-indicator [29], SDE indicator [30], and Hypervolume (HV) [31]. However, these QIs do not meet the requirements of easy-using and low-computation cost to some extent. For example, both IGD and ϵ-indicator demand Pareto front (PF) as the reference set, which is impossible in practical problems; HV does not require PF but a reference point, for which, if set unreasonably, will mislead the optimization search direction [32]. Besides, HV possesses high computational complexity, which increases exponentially with the number of objectives. Some QIs have a specific bias, for example, ε-indicator is more convergence-oriented, and as a result, optimization process may suffer from premature convergence, while SDE prefers diversity [33] which may make the optimization period too long [34]. R2 indicator [35, 36], having some desire properties in common with HV but is cheaper to evaluate, attracted widespread interest in recent years. In addition, it is weakly monotonic [37] and does not require PF to be provided; hence, it is adopted in this article.

The rest of paper is organized as follows. In Section 2, the mathematical background of Kriging, EI, and R2 indicator is reviewed. Then, the proposed EIR2 indicator and framework of the whole optimization algorithm are presented in Section 3. To evaluate the performance of the proposed IPC, a series of experiments over three benchmark test function suites against two competitors IPCs is conducted in Section 4, their results are measured by three performance metrics and then analyzed in Section 5, and followed by Section 6, where the proposed EIR2 is modified and applied to a real-world constrained expensive MOP, an optimization design of an energy storage flywheel. Lastly, Section 7 gives the conclusion and future research direction.

2. Background

2.1. Multiobjective Optimization Problem. Without loss of generality, a multiobjective optimization problem is formulated as follows:

$$\begin{aligned} \min \quad & \mathbf{F}(\mathbf{x}) = \left[f_1(\mathbf{x}), \cdots, f_m(\mathbf{x})\right]^T \\ \text{s.t.} \quad & g_i(\mathbf{x}) \le 0, \quad i = 1, \cdots, p \\ & h_j(\mathbf{x}) = 0, \quad j = 1, \cdots, q \\ & \mathbf{x}^{\min} \le \mathbf{x} \le \mathbf{x}^{\max}, \end{aligned} \tag{1}$$

where $\mathbf{F}(\mathbf{x})$ represents the objective function vector which contains m objective functions, g_i and h_j stand for inequality and equality constraint function with the total number p and q, respectively, and $\mathbf{x} = [x_1, \cdots, x_d]^T$ stands for design vector with lower bound $\mathbf{x}^{\min}$ and upper bound $\mathbf{x}^{\max}$. A solution $\mathbf{u}$ is said to dominate another one $\mathbf{v}$ if and only if $f_i(\mathbf{u}) \le f_i(\mathbf{v}) \forall i = 1, \cdots, m$ and $f_i(\mathbf{u}) < f_i(\mathbf{v}) \exists i = 1, \cdots, m$, written as $\mathbf{u} \prec \mathbf{v}$. A solution $\mathbf{x}^*$ is called Pareto optimal if there is no $\mathbf{x}$ satisfying $\mathbf{x} \prec \mathbf{x}^*$(such $\mathbf{x}^*$ is also referred to as nondominated by $\mathbf{x}$). The set of all the Pareto optimal solutions is called the Pareto Set (PS), whose image in the objective space is called Pareto Front (PF).

In practice, objective functions and constraint functions may come from finite element simulation; hence, they have no explicit analytic expressions and are usually computationally expensive to evaluate. Under this condition, we are interested in identifying a set of nondominated solutions, called approximated Pareto Set/Front, to represent the true Preto Set/Front, meanwhile consuming as less computing resources as possible.

2.2. Kriging Model and Expectation Improvement. Kriging model is an unbiased estimation model with smallest estimated variance, excellent high-dimensional, and nonlinear fitting capability. It assumes the relationship between input $\mathbf{x}$ and predicted output $\hat{y}$ as follows:

$$\hat{y}(\mathbf{x}) = \mu + z(\mathbf{x}), \tag{2}$$

where μ is a constant and $z(\mathbf{x}) \sim N(0, \sigma^2)$ is the Gaussian random variable, which represent global trend and local deviation of the output separately.

Having a set of known observations $S = \{(\mathbf{x}^i, y^i), i = 1, \cdots, N\}$, our job is to search for the best values of μ and σ when predicting at a new point $\mathbf{x}$. In the design space, the relationship between two different points $\mathbf{x}^i$ and $\mathbf{x}^j$ is characterized by correlation functions, such as the Gaussian function:

$$R\left(z\left(\mathbf{x}^i\right), z\left(\mathbf{x}^j\right); \boldsymbol{\theta}\right) = \prod_{k=1}^{d} \exp\left(-\theta_k \left|x_k^i - x_k^j\right|\right), \tag{3}$$

where hyperparameter $\boldsymbol{\theta} = [\theta_1, \cdots, \theta_d]^T$ controls the smoothness of prediction function and is usually determined through maximizing a likelihood function of R by some intelligent optimization algorithms such as particle swarm optimization. According to optimization theory, analytical form of μ and σ are obtained:

$$\begin{aligned} \hat{\mu} &= \frac{\mathbf{1}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{y}}{\mathbf{1}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{1}}, \\ \hat{\sigma}^2 &= \frac{(\mathbf{y} - \mathbf{1}\hat{\mu})^{\mathrm{T}}\mathbf{R}^{-1}(\mathbf{y} - \mathbf{1}\hat{\mu})}{N}, \end{aligned} \tag{4}$$

where $\mathbf{y} = [y_1, \cdots, y_d]^T$ is known as the output vector and $\mathbf{R}$ is a matrix of covariance between any two points in S, that is, $\mathbf{R}_{i,j} = R(z(\mathbf{x}^i), z(\mathbf{x}^j))$. Finally, at the new point $\mathbf{x}$, we get the predicted value and variance:

$$\begin{aligned} \hat{y}(\mathbf{x}) &= \hat{\mu} + \mathbf{r}^{\mathrm{T}}\mathbf{R}^{-1}(\mathbf{y} - \mathbf{1}\hat{\mu}), \\ \hat{s}^2(\mathbf{x}) &= \hat{\sigma}^2\left[1 - \mathbf{r}^{\mathrm{T}}\mathbf{R}\mathbf{r} + \frac{(1 - \mathbf{1}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{r})^2}{\mathbf{1}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{1}}\right], \end{aligned} \tag{5}$$

where $\mathbf{r}$ is a correlation vector between point $\mathbf{x}$ and all points in S. Unlike other approximate models such as response surface [38] and neural network [39], which can only give out the predicted value, Kriging is capable of outputting predicted variance as a byproduct, which can be regarded as a predicting uncertainty at $\mathbf{x}$.

With Kriging, the value at a new point $\mathbf{x}$ is treated as a Gaussian random variable, that is, $y(\mathbf{x}) \sim N(\hat{\mu}, \hat{s}^2)$. An improvement quantity is designed as $I(y(\mathbf{x}), y_{\min}) = \max(y(\mathbf{x}) - y_{\min}, 0)$, where $y_{\min}$ is the smallest y value in S. It is noted that only those $y(\mathbf{x})$ that are less than $y_{\min}$ produce improvement. The expectation of improvement (EI) [40] has a closed form, that is,

$$\mathrm{EI}(y(\mathbf{x}), y_{\min}) = \begin{cases} (y_{\min} - \hat{y}(\mathbf{x}))\Phi\left(\dfrac{y_{\min} - \hat{y}(\mathbf{x})}{\hat{s}(\mathbf{x})}\right) + \hat{s}\phi\left(\dfrac{y_{\min} - \hat{y}(\mathbf{x})}{\hat{s}(\mathbf{x})}\right), & \text{if } \hat{s} > 0, \\ 0, & \text{if } \hat{s} = 0, \end{cases} \tag{6}$$

where Φ and ϕ are the probability distribution and probability density functions, respectively.

2.3. R2 Indicator. The R2 indicator [41] is a unitary indicator, which was proposed to evaluate the performance of the nondominated solution set Q, given as

$$R2(Q, U) = -\frac{1}{|U|}\sum_{u \in U} \max_{q \in Q}\{u(q)\}, \tag{7}$$

where U is a discrete and finite set of utility functions u, for which there are many options to choose, such as weight sums and penalty-based boundary intersection [42].

3. Proposed Method

3.1. Expectation Improvement R2 Indicator. The main difficulty in extending EI infilling criterion to MOPs lies in the fact that we have to specify $y_{\min}$ before calculating the EI value, which is impractical as no such solution exists as being optimal with respect to every objective, but a set of nondominated solution found during previous optimization process. It is now highly desired to find an infilling point having large EI values with respect to all points in the current nondominated set. This motivates us to treat each nondominated solution as current optima and then calculate the EI value in each objective, resulting in a vector function:

$$H(\mathbf{x}, \mathbf{p}) = [\mathrm{EI}_1(\mathbf{x}, \mathbf{p}), \cdots, \mathrm{EI}_m(\mathbf{x}, \mathbf{p})]^T, \tag{8}$$

where $\mathbf{p} = [p_1, \cdots, p_m]^T$ is a point in current nondominated set P and $\mathrm{EI}_j(\mathbf{x}, \mathbf{p}) = \mathrm{EI}(y_j(\mathbf{x}), p_j)$. By this function, we obtain a vector consisting of all EI values for which p_j is deemed as $y_{\min}$ in objective j. Obviously, we need to build m Kriging models, each corresponding to one objective function. After continually applying function H to all nondominated solutions, we get a set composed of $|P|$ points, defined as

$$Q(\mathbf{x}) = \{\mathbf{q} \mid \mathbf{q} = H(\mathbf{x}, \mathbf{p}), \mathbf{p} \in P\}, \tag{9}$$

where $\mathbf{q} = [q_1, \cdots, q_m]^T$. In essence, we mapped point $\mathbf{x}$ in design space to a set $Q(\mathbf{x})$ in another space, called expectation improvement space, where the R2 indicator of $Q(\mathbf{x})$ is defined as fitness of $\mathbf{x}$ being selected as the infilling point. The mapping from objective space to the EI space is illustrated in Figure 1.

Combining EI and R2 indicator, a novel indicator for MOP, Expectation Improvement R2 Indicator (EIR2), is defined as follows:

$$\mathrm{EIR2}(\mathbf{x}, P, U) = -\frac{1}{|U|}\sum_{u\in U}\max_{\mathbf{q}\in Q(\mathbf{x})}\{u(\mathbf{q})\}. \tag{10}$$

As EIR2 is a scalar indicator, some single optimization algorithm can be employed to determine the optimal solution as the infilling point. Besides, EIR2 actually defines an indicator template in which other utility functions can be specified. In this paper, Tchebyche function is employed as utility function because it can tackle MOPs with convex or concave Pareto front, defined as

$$u_{\boldsymbol{\lambda}}(\mathbf{q}) = -\max_{j\in\{1,\cdots,m\}}\lambda_j\left|z_j^* - q_j\right|, \tag{11}$$

where $\lambda = [\lambda_1, \cdots, \lambda_m]^T$ is weight vector satisfying $\lambda_j \geq 0$ and $\sum_{j=1}^{m}\lambda_j = 1$. These weight vectors are uniformly distributed in the objective space and are collected in the weight set Λ. $\mathbf{z}^* = [z_1^*, \cdots, z_m^*]^T$ is the ideal point, that is, $z_j^* = \min(q_j), \forall \mathbf{q}\in Q$. As all EI values are nonnegative, a natural selection is $z^* = \mathbf{0}$. To summarize, the EIR2 indicator has the form

$$\mathrm{EIR2}(\mathbf{x}, P, \Lambda) = \frac{1}{|\Lambda|}\sum_{\boldsymbol{\lambda}\in\Lambda}\min_{\mathbf{p}\in P}\left\{\max_{j=1,\cdots,m}\left\{\lambda_j \times \mathrm{EI}_j(\mathbf{x}, \mathbf{p})\right\}\right\}. \tag{12}$$

The point that maximizes the EIR2 indicator is recognized as an infilling point for expensive evaluation.

3.2. Weight Vectors Generating. The utility function is actually an optimization subproblem parameterized by the weight vector, by which the optimization process can be controlled. If the geometry characteristics of PF are known, such as being convex/concave, a set of weight vectors can be distributed consistent with the PF in the objective space, by which a more accurate approximate PF with uniform distribution is more likely to be obtained. However, there is often no information about the PF in priori, which is common for practical MOP; hence, a better choice in this situation is to treat all objectives equally important and generate a set of weight vectors evenly distributed in the objective space.

To help generate weight vector λ, an auxiliary set is introduced $L = \{0/H, 1/H, \cdots, H/H\}$, where H is an user-specified positive integer. We can chose randomly $m-1$ element from L as the first $m-1$ elements of λ and assign the value of λ_m to ensure $\sum_{i=1}^{m}\lambda_i = 1$. Exhausting every possible combination, there are totally C_{H+m-1}^{m-1} weight vectors generated. Figure 2 illustrates all 21 weight vectors generated using parameter $m = 3$ and $H = 5$.

3.3. Comparison with Other IPCs. The novelty of EIR2 is that fitness assessment of point is carried out not in the objective

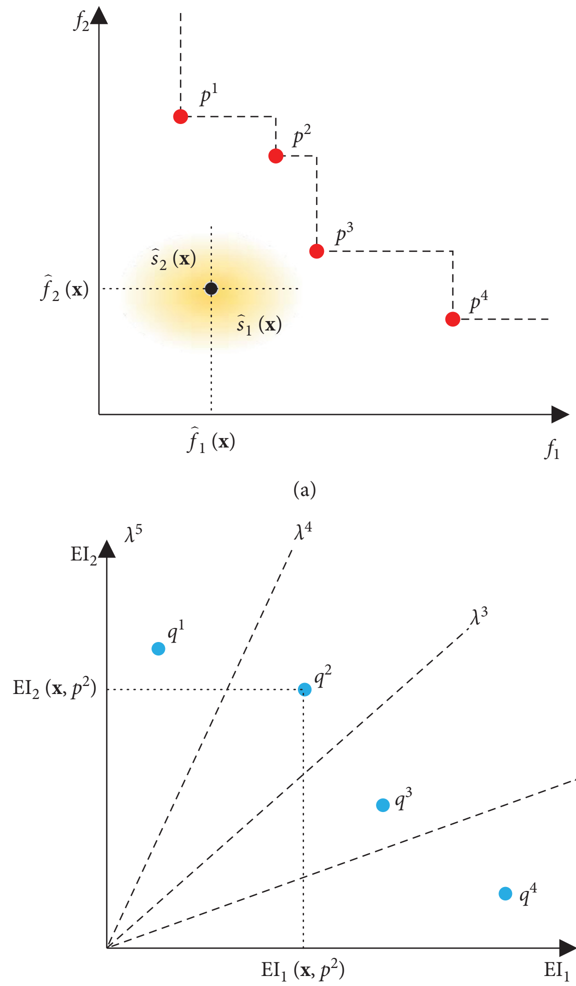

FIGURE 1: (a) A point with its predicted value and standard deviation in 2D objective space and (b) its corresponding mapping set in 2D EI space.

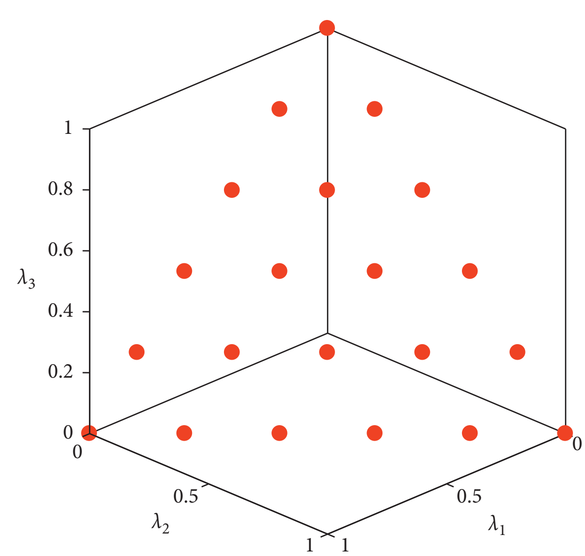

FIGURE 2: Evenly distributed weight vectors in objective space with $m = 3$ and $H = 5$.

space but in the EI space, where the R2 indicator is calculated for the mapping set of the point. Since the EI indicator has a closed function form, so does EIR2 regardless of the number of objective functions. In addition, the computational complexity of EIR2 is $O(m * |P| * |\Lambda|)$, which is considerably less than that of hypervolume [26].

ParEGO also uses the Chebyshev function; however, its purpose is to integrate multiple objectives into one single objective before applying the EGO algorithm. Therefore, in essence, the infilling point criterion used in ParEGO is EI.

With regard to the MaxMin indicator, despite analytic form being existing, its calculation has to resort to dividing the objective space into many blocks on which tedious multivariable integration is carried out. Even worse, the number of blocks increases exponentially with the growth of the number of objective. So, in practice, we only apply this analytic form when $m = 2$ while using the Monte Carlo method when $m > 2$.

Figure 3 shows contour plots of three IPCs, namely, ParEGO, MaxMin, and EIR2 applied in test function TEST2 defined in Table 1, where lighter color denotes higher value. In these plots, 20 known sample points are represented by red points, and blue points represent the true Pareto set, which are composed of three disconnected point subsets. It is easy to find that ParEGO can identify only one Pareto subset out of three, while MaxMin and EIR2 successfully locate the three Pareto subsets. In the region, where no Pareto solution exists, the function value of EIR2 is almost constant, while the function landscape of MaxMin shows a ladder-like trend.

Deutz [43] recently proposed an IPC, also called EIR2, which is however very different from ours, proposed in this article. The main difference is that the R2 indicator is calculated in different spaces. Deutz's R2 indicator is defined in objective space and the expectation of R2 indicators is carried out through multivariate integration. Therefore, the analytic form can only be obtained when $m = 2$ but does not exist when $m \geq 3$, where the Monte Carlo method has to be adopted. In contrast, EIR2 in this paper calculates the R2 indicator in the EI space and hence having the analytic form for any m. Most importantly, our EIR2 involves no multivariable integration and hence is cheap to evaluate.

3.4. Overview of the EIR2-EMOA. Combining the devised EIR2 infilling point criterion with the Kriging model, optimal Latin hypercube sampling method [44], and particle swarm optimization [45], an optimization algorithm for multiobjective problems with expensive functions is proposed, referred to as EIR2-MOEA. The detailed procedure is shown in Algorithm 1.

In the initial stage of EIR2-MOEA, a set of sample points of small size need to be selected for establishing the initial Kriging model. As a design of experiment method, the optimal Latin Hypercube Sampling (LHS) method is utilized as it has flexibility in setting the number of sampling points and can ensure these points filling in the entire design space.

When maximizing the EIR2 value and looking for optimal hyperparameters of the Kriging model, an optimization method is required. However, the landscapes of these two functions are nonlinear, for which traditional gradient-based optimization algorithms are prone to falling into local minima if the initial point is not selected wisely and has to be rerun multiple times. Particle swarm optimization algorithm, as an intelligent optimization algorithm, does not need information about gradient and has fewer parameter settings, high efficiency of search, and lower complexity, hence is employed to find the optimal parameters.

4. Experiment Setup

In order to assess the performance of the proposed algorithm, we applied it to three sets of standard test function suites and compared it with another two algorithms, ParEGO and MaxMin.

4.1. Test Problems. Three benchmark function suites, namely, simple set, ZDT set, and DTLZ set, are selected to verify the performance of the IPCs in different aspects, whose definitions are shown in Table 1. The simple set includes three biobjective optimization problems TEST1, TEST2, and FON with the number of variables less than three, whose corresponding PFs are convex, concave, and discontinuous, respectively. The ZDT set contains three biobjective test functions, ZDT1, ZDT2, and ZDT3, with convex, concave, and discontinuous PF, respectively. We set the number of variables to 5 to test IPCs' performance on problems having more decision variables. DTLZ2, DTLZ5, and DTLZ7 form the DTLZ test set, which all have three objectives and five decision variables, and they are selected to assess the performance of the IPCs when the number of objectives is large.

4.2. Performance Metric. We hope that the approximated Pareto front found close enough to the true Pareto front and is evenly distributed in the objective space, that is, to meet the requirement on convergence and diversity. For this purpose, we choose three performance metrics to quantify the performance of the approximated Pareto front.

(1) Inverted Generational Distance (IGD) [28]:

$$\mathrm{IGD}(T, P) = \frac{1}{|T|} \sum_{\mathbf{x} \in T} \min_{\mathbf{p} \in P} \{d(\mathbf{x}, \mathbf{p})\}, \tag{13}$$

where T and P are true and approximated Pareto set, respectively, $|T|$ is the number of solutions in T, and $d(\mathbf{x}, \mathbf{p})$ represents the Euclidean distance between $\mathbf{x}$ and $\mathbf{p}$ in objective space. IGD measures the average distance between the true and approximated Pareto set. Therefore, the smaller IGD value is preferred.

(2) HyperVolume (HV) [31]:

$$\mathrm{HV}(P) = \mathrm{Vol}\Big(\bigcup_{\mathbf{x} \in P} [f_1(\mathbf{x}), r_1] \times \dots [f_m(\mathbf{x}), r_m]\Big), \tag{14}$$

where $\mathbf{r} = [r_1, \cdots, r_m]^T$ represents reference point which is dominated by all solutions in P and Vol

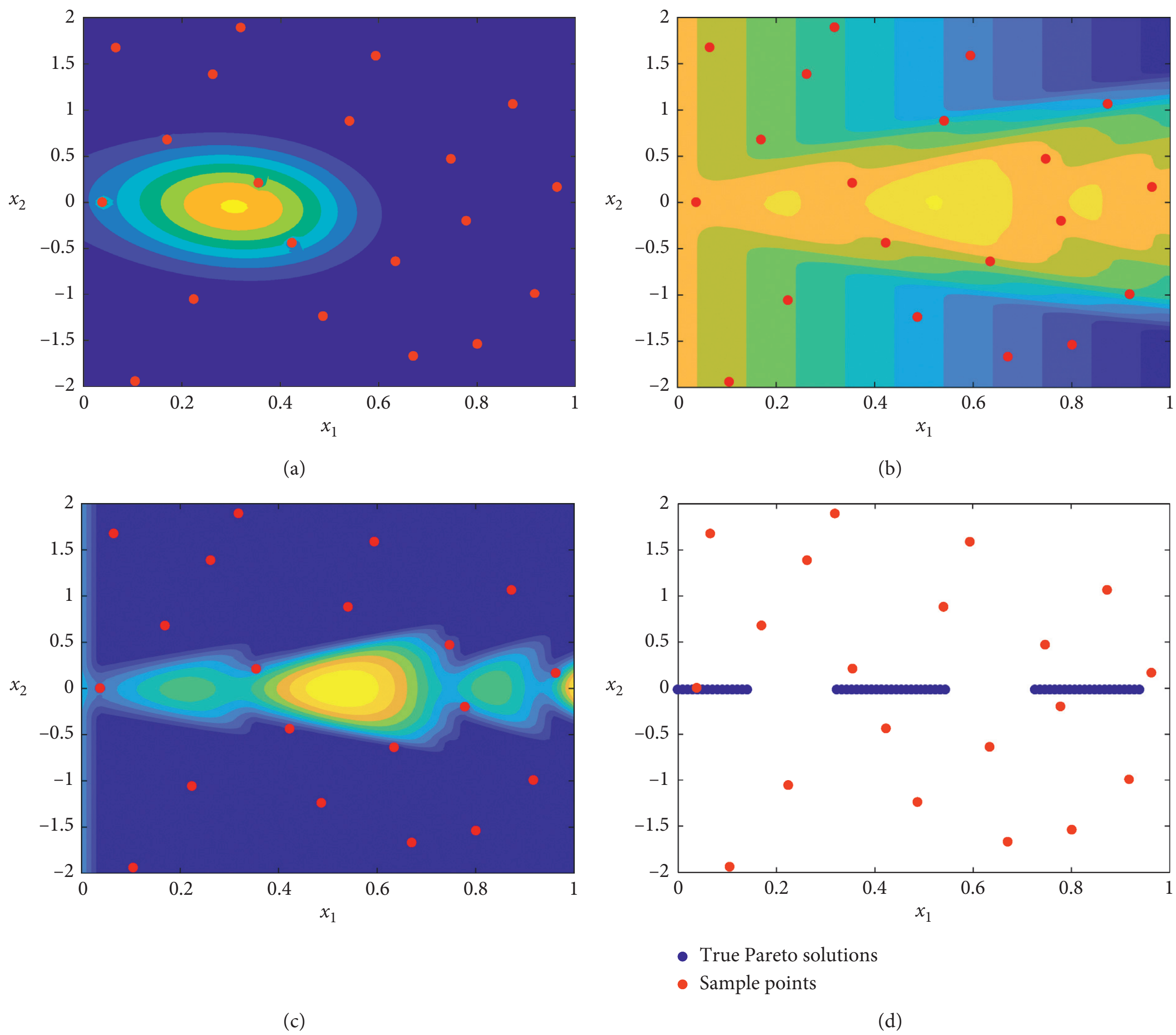

Figure 3: The landscape of infilling point criteria in 2D decision space for TEST2 by (a) ParEGO with weight vector [0.8, 0.2], (b) MaxMin, and (c) EIR2 with 20 uniformly distributed weight vectors. (d) Both the true Pareto set and 20 sample points generated by the optimal Latin hypercube sampling method.

stands for Lebesgue measure. Geometrically, HV is the volume enclosed by the approximated Pareto set P and the reference point $\mathbf{r}$. HV can reflect both convergence and uniformity. If the approximate Pareto set performances well with respect to both aspects, then the HV value will be high.

(3) Nondominated solution ratio (NR):

$$\mathrm{NR}(P) = \frac{|P|}{N_{\max}}, \tag{15}$$

where $|P|$ is the number of nondominated solutions and $N_{\max}$ the maximum number of evaluations by expensive function. NR is used to measure the proportion of the nondominated solutions out of the total sample points evaluated by expensive functions. In other words, NR represents the efficiency of IPC in using computing resources. The larger the NR value, the more nondominated solutions found by the IPC and the higher the utilization efficiency of computing resources.

4.3. Parameter Setting. We hope to evaluate the performance of each IPC with limited computing resources, that is, for each test function we set the maximum number of evaluations involving expensive functions $N_{\max}$, shown in Table 2. The size of the initial sample set is specified to be a fraction of $N_{\max}$, namely, $\alpha N_{\max}$, where α is a proportional coefficient which is 1/3 in this paper. To ensure fairness, all three IPCs start with the same initial sample point set generated by optimal LHS in each run, and ten runs are executed for each test function.

Reference points are required for calculating the HV value. All biobjective optimization problems share the same reference point [1.1, 1.1] except TEST1 for which [20.5, 20.5] is used; all three-objective optimization problems are assigned the same reference point [1.1, 1.1, 1.1] except DTLZ7, where [0.95, 0.95, 6.6] is utilized.

Table 1: The definition of test functions.

Name	Dimension	Boundary	Objective function	Comments on PF
TEST1	2	$x_1 \in [0.4, 1.6]$ $x_2 \in [2, 5]$	$f_1 = (x_1 - 2)^2 + (x_2 - 1)^2$ $f_2 = x_1^2 + (x_2 - 6)^2$	Convex, simple
TEST2	2	$x_1 \in [0, 1]$ $x_2 \in [-2, 2]$	$f_1 = x_1$ $f_2 = 1 + x_2^2 - x_1 - 0.1\sin(5\pi x_1)$	Convex, disconnected
FON	3	$x_i \in [-4, 4]$	$f_1 = 1 - \exp(-\sum_{i=1}^{n}(x_i - (1/\sqrt{3}))^2)$ $f_2 = 1 - \exp(-\sum_{i=1}^{n}(x_i + (1/\sqrt{3}))^2)$	Nonconvex
ZDT1	5	$x_i \in [0, 1]$	$f_1 = x_1$ $f_2 = z(1 - \sqrt{x_1/z})$ $z = 1 + (9/(n-1))\sum_{i=2}^{n} x_i$	Convex
ZDT2	5	$x_i \in [0, 1]$	$f_1 = x_1$ $f_2 = z(1 - (x_1/z)^2)$ $z = 1 + (9/(n-1))\sum_{i=2}^{n} x_i$	Nonconvex
ZDT3	5	$x_i \in [0, 1]$	$f_1 = x_1$ $f_2 = z(1 - \sqrt{x_1/z} - (x_1/z)\sin(10\pi x_1))$ $z = 1 + (9/(n-1))\sum_{i=2}^{n} x_i$	Convex, disconnected
DTLZ2	5	$x_i \in [0, 1]$	$f_1 = (1+g)\cos(\pi x_1/2)\cos(\pi x_2/2)$ $f_2 = (1+g)\cos(\pi x_1/2)\sin(\pi x_2/2)$ $f_3 = (1+g)\sin(\pi x_1/2)$ $g = \sum_{i=\{3,4,5\}}(x_i - 0.5)^2$	Nonconvex
DTLZ5	5	$x_i \in [0, 1]$	$f_1 = (1+g)\cos(\pi\theta_1/2)\cos(\pi\theta_2/2)$ $f_2 = (1+g)\cos(\pi\theta_1/2)\sin(\pi\theta_2/2)$ $f_3 = (1+g)\sin(\pi\theta_1/2)$ $\theta_i = (\pi/(4(1+g)))(1 + 2gx_i), i = 1, 2$ $g = \sum_{i=\{3,4,5\}}(x_i - 0.5)^2$	Convex
DTLZ7	5	$x_i \in [0, 1]$	$f_1 = x_1$ $f_2 = x_2$ $f_3 = (1+g)h$ $g = 1 + 3\sum_{i=\{3,4,5\}} x_i$ $h = 3 - \sum_{i=1}^{2}[(f_i/(1+g))(1 + \sin(3\pi f_i))]$	Disconnected

As for particle swarm optimization, the population size is set to 100, the evolution generation is 100, and the scaling factor is 1.2. Other parameters of MaxMin and ParEGO are taken as the recommended values in original papers.

5. Result and Discussion

For each test function, ten independent runs are performed, and the HV, IGD, and NR indicators of the resulting approximated Pareto set are calculated. Finally, their mean and standard deviation of the ten runs are calculated and listed, respectively, in Tables 3–5. For each test function, among the three IPCs, the winning one is shown in boldface with a gray background.

5.1. Simple Tests. TEST1, TEST2, and FON are all biobjective test functions but have, respectively, convex, piecewise continuous, and concave Pareto front. The comparison of mean and std value for HV, IGD, and NR metric can be found at the top part of Tables 3–5, respectively. It is shown that ParEGO achieved the worst performance on all three performance metrics, while EIR2 and MinMax achieve similar results but both significantly better than ParEGO. Specifically, for TEST1 and TEST2, EIR2 is better than MaxMin in terms of HV and IGD but is slightly worse concerning NR. For FON, EIR2 ranked first on all three performance metrics.

Figure 4 plots the approximated Pareto front obtained by the three IPCs on TEST1, TEST2, and FON test function, respectively, with its HV value being median in ten runs. It is evident that the number of nondominated solutions obtained by ParEGO is small and they are not evenly distributed, which is more obvious on TEST2 and FON. This phenomenon can be foreseen because ParEGO lacks a mechanism to guarantee the uniform distribution of the solutions, that is, its weight vector is randomly generated at each iteration. In contrast, the approximated Pareto fronts obtained by EIR2 and MaxMin seem

Input:
$N_{\max}$: Maximum number of evaluations by expensive function
α: Ratio of the number of initial samples to $T_{\max}$
H: Integer used to generate weight vector
Output:
P: Nondominated solution set
(1) Initialization: Obtain design variable limits $\mathbf{x}^{\max}$ and $\mathbf{x}^{\min}$, number of objective m, etc. according to MOP to be solved. Set $S = \varnothing$ and $P = \varnothing$.
(2) Generate $\alpha N_{\max}$ sample points in design space $[\mathbf{x}^{\min}, \mathbf{x}^{\max}]$ using optimal Latin hypercube sampling.
(3) **for** $k = 1$ to $\alpha N_{\max}$ **do**
(4) Calculate the expensive objective function values $\mathbf{y}^k = \mathbf{F}(\mathbf{x}^k)$ for sample point $\mathbf{x}^k$
(5) $S = S \cup \{(\mathbf{x}^k, \mathbf{y}^k)\}$, i.e., add sample point to S
(6) **end for**
(7) Generate $c = C_{H+m-1}^{m-1}$ weight vectors in objective space and save them in set $\Lambda = \left\{\lambda^1, \cdots, \lambda^c\right\}$.
(8) Find non-dominated solutions in S and put them into P
(9) **for** $k = 1$ to $(1-\alpha)N_{\max}$ **do**
(10) **for** $k = 1$ to m **do**
(11) Using PSO to find the best hyperparameter θ
(12) Build Kriging model of the m-th objective function based on S
(13) **end for**
(14) According to EIR2 indicator, apply PSO to find the best infilling point $\mathbf{x}^{\text{inf}}$
(15) Calculate expensive objective function $\mathbf{y}^{\text{inf}} = \mathbf{F}(\mathbf{x}^{\text{inf}})$
(16) Update $S = S \cup \{(\mathbf{x}^{\text{inf}}, \mathbf{y}^{\text{inf}})\}$
(17) Find the non-dominated solutions in S and put them into P
(18) **end for return** Output P as approximated Pareto set

Algorithm 1: EIR2-MOEA.

Table 2: Maximum number of evaluations involving expensive functions.

	TEST1	TEST2	FON	ZDT1	ZDT2	ZDT3	DTLZ2	DTLZ5	DTLZ7
$N_{\max}$	45	60	90	90	90	120	210	210	210

Table 3: The mean and standard deviation of the HV value of approximated Pareto set found by ParEGO, MaxMin, and EIR2-EMOA on all test functions.

	ParEGO		MaxMin		EIR2	
	Mean	Std	Mean	Std	Mean	Std
TEST1	272.2024	$1.71E+00$	278.8626	$1.03E-01$	**279.0228**	$8.64E-02$
TEST2	0.6900	$8.02E-03$	0.7227	$1.49E-03$	**0.7233**	$7.44E-04$
FON	0.4583	$3.10E-02$	0.5324	$2.21E-03$	**0.5364**	$3.08E-03$
ZDT1	0.8468	$5.53E-03$	0.8463	$5.01E-03$	**0.8643**	$1.08E-03$
ZDT2	0.5057	$2.14E-02$	0.5121	$4.63E-03$	**0.5312**	$1.04E-03$
ZDT3	1.2925	$8.52E-03$	1.2946	$4.72E-02$	**1.3230**	$1.14E-03$
DTLZ2	0.6499	$9.69E-03$	0.7078	$5.60E-03$	**0.7378**	$2.85E-03$
DTLZ5	0.4098	$5.02E-03$	0.4151	$3.90E-03$	**0.4358**	$3.45E-04$
DTLZ7	1.5410	$2.31E-02$	1.6333	$2.72E-02$	**1.6858**	$3.39E-03$

to have completely covered the true Pareto fronts and have good distribution performance, regardless of the nature of the true Pareto front of the test function.

Overall, EIR2 and MaxMin have comparable performance and are much better than ParEGO on simple test functions with a low number of variables.

5.2. ZDT Tests. The middle part of Tables 3–5 presents the IGD, HV, and NR indicator comparison on ZDT test suites. Figure 5 illustrates the approximated Pareto fronts obtained by each IPC on ZDT1~ZDT3 whose HV value being median in ten runs. It is clear that EIR2 is the best in terms of HV, IGD, and NR indicators for all ZDT test functions. Specifically, the HV value and IGD value of EIR2 are much better than that of ParEGO and MaxMin. It is worth noting that the advantages of EIR2 are quite obvious for IGD and NR indicators. For example, for ZDT3 EIR2 gets an NR value of 0.5438, while the other two algorithms only have 0.1924 and 0.3686; meanwhile, it gets an IGD value of 0.0143 while the other two 0.0544 and 0.0591.

According to Figure 5, it is seen that both ParEGO and MaxMin successfully converged to the true Pareto front of

Table 4: The mean and standard deviation of the IGD value of approximated Pareto set found by ParEGO, MaxMin, and EIR2-EMOA on all test functions.

	ParEGO		MaxMin		EIR2	
	Mean	Std	Mean	Std	Mean	Std
TEST1	0.4347	$4.88E-02$	0.1939	$5.19E-03$	**0.1908**	$4.49E-03$
TEST2	0.0415	$1.02E-02$	**0.0097**	$5.06E-04$	0.0097	$5.49E-04$
FON	0.0640	$3.47E-02$	0.0121	$1.05E-03$	**0.0109**	$1.98E-03$
ZDT1	0.0211	$2.42E-03$	0.0232	$4.17E-03$	**0.0096**	$8.15E-04$
ZDT2	0.0285	$1.51E-02$	0.0249	$3.26E-03$	**0.0126**	$1.16E-03$
ZDT3	0.0591	$9.69E-03$	0.0544	$1.21E-02$	**0.0143**	$1.79E-03$
DTLZ2	0.0999	$5.40E-03$	0.0992	$6.36E-03$	**0.0700**	$4.21E-03$
DTLZ5	0.0289	$3.01E-03$	0.0334	$4.89E-03$	**0.0084**	$4.72E-04$
DTLZ7	0.1393	$1.70E-02$	0.0851	$1.25E-02$	**0.0576**	$4.23E-03$

Table 5: The mean and standard deviation of the NR value of approximated Pareto set found by ParEGO, MaxMin, and EIR2-EMOA on all test functions.

	ParEGO		MaxMin		EIR2	
	Mean	Std	Mean	Std	Mean	Std
TEST1	0.6689	$5.78E-02$	**0.8711**	$2.52E-02$	0.8644	$2.86E-02$
TEST2	0.2883	$2.95E-02$	**0.6183**	$2.66E-02$	0.6017	$2.54E-02$
FON	0.1867	$4.50E-02$	0.4200	$3.22E-02$	**0.4367**	$7.33E-02$
ZDT1	0.2611	$1.76E-02$	0.2556	$3.19E-02$	**0.4522**	$3.31E-02$
ZDT2	0.2478	$6.87E-02$	0.2033	$2.10E-02$	**0.4311**	$3.77E-02$
ZDT3	0.1242	$2.17E-02$	0.1283	$1.77E-02$	**0.2958**	$2.95E-02$
DTLZ2	0.2752	$1.81E-02$	0.2733	$2.49E-02$	**0.4381**	$2.00E-02$
DTLZ5	0.1652	$1.27E-02$	0.1262	$1.64E-02$	**0.3529**	$1.09E-02$
DTLZ7	0.1924	$2.43E-02$	0.3686	$6.87E-02$	**0.5438**	$2.53E-02$

ZDT1 and ZDT2; however, they failed in making them evenly distributed. This phenomenon is even worse on ZDT3 whose true Pareto front consists of five segmented curves. The majority of solutions found by ParEGO and MaxMin are clustered in one segment, while only a few solutions located in other segments. In contrast, the approximated Pareto set obtained by EIR2 are uniformly and equally distributed among all five segments.

In summary, EIR2 is capable of finding more non-dominated solutions than ParEGO and MaxMin using the same amount of computing resources and keeping them evenly distributed along the approximated Pareto front.

5.3. DTLZ Tests. The DTLZ test function suites are used aiming at testing an algorithm's ability to three-objective optimization problems. The metric comparisons are listed at the bottom part of Tables 3–5, respectively. Figure 6 shows each IPC's results on DTLZ2, DTLZ5, and DTLZ7 with their HV value being median in ten runs. Same as the ZDT case, on the three test functions, the EIR2 is still significantly ahead of its two competitors in terms of all three performance metrics. To be more specific, take DTLZ2 as an instance, the NR value of EIR2 is 0.3529, which is nearly three times 0.1262 of MaxMin and more than twice 0.1652 of ParEGO, which shows again EIR2 is more efficient in using computation resources compared to other IPCs. The larger HV value and smaller IGD value obtained by EIR2 also indicate its capacity in achieving strong convergence ability.

The same conclusion is also reached by inspecting Figure 6. Specifically, ParEGO has difficulty in converging to the true Pareto front, while MaxMin, despite converged to PF, suffered from being not able to spread its approximated Pareto front uniformly distributed along the true Pareto front which is a 1/8 sphere of radius 1, and there are only a few solutions in the middle part and most of the solutions are found in three edges. In contrast, the approximated PF obtained by EIR2 is adequate to converge to and cover entirely and evenly the 1/8 spheres surface.

6. Engineering Application

In addition to boundary constraints, most MOPs also contain equality or inequality constraints. We assume that constraint functions are expensive; hence, the Kriging model need to be established for each constraint. On the purpose of measuring the degree to which the solution satisfies the constraints, probability of feasibility (PoF) [46, 47] is introduced as follows:

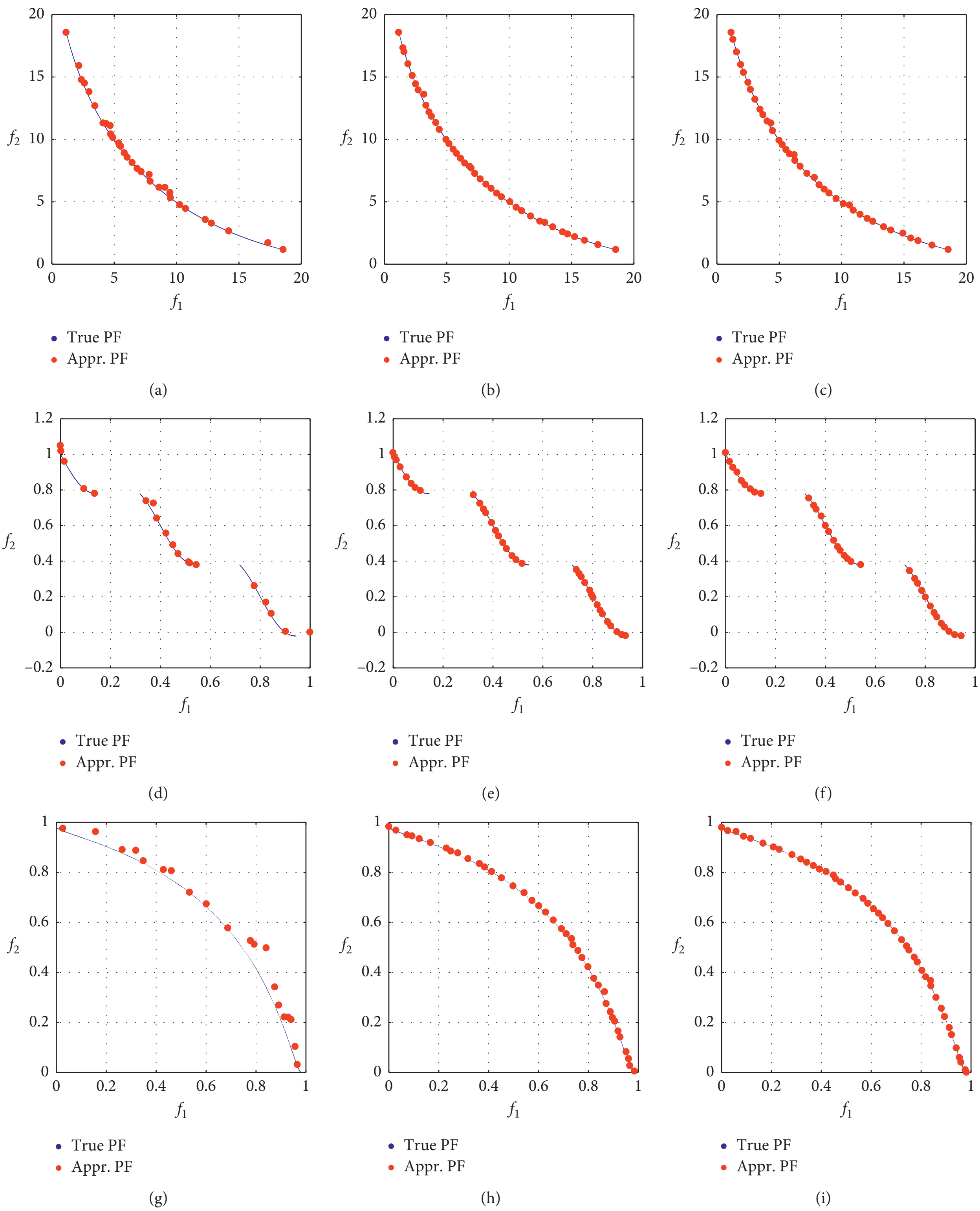

FIGURE 4: Approximated Pareto front found by three infilling criterions ParEGO(*left*), MaxMin(*middle*), and EIR2(*right*) on test function TEST1 (a∼c), TEST2 (d∼f), and FON (g∼i).

$$\mathrm{PoF}_i(\mathbf{x}) = P\left(g_i(\mathbf{x}) \le 0\right) = \Phi\left(-\frac{\widehat{g}_i(\mathbf{x})}{\widehat{s}_i(\mathbf{x})}\right), \qquad (16)$$

where $\widehat{g}_i(\mathbf{x})$ and $\widehat{s}_i(\mathbf{x})$ is the predicted value and standard deviation of constraint function $g_i(\mathbf{x})$. Based on PoF, the average probability of feasibility (APoF) is defined as follows:

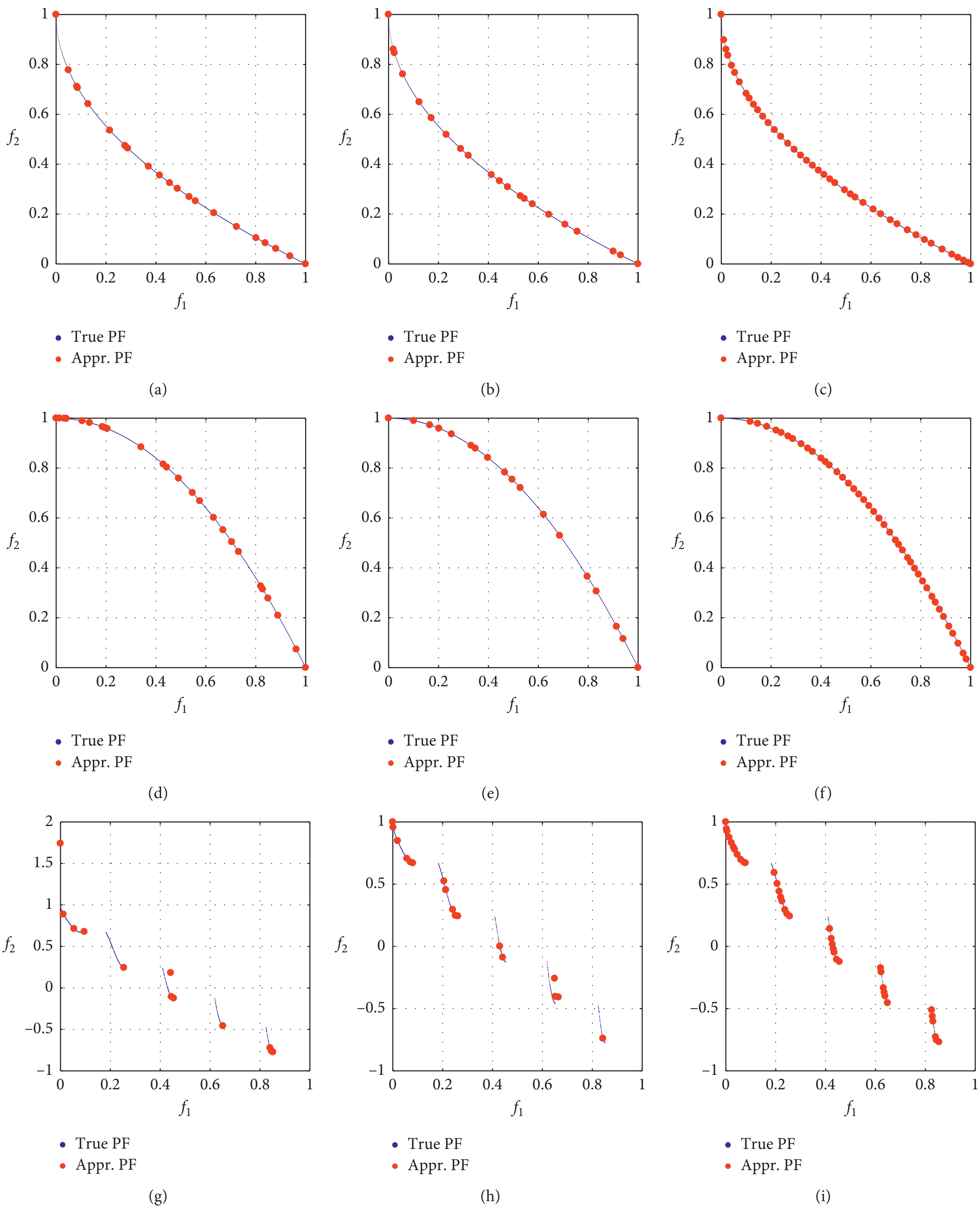

Figure 5: Approximated Pareto front found by three infilling criterions ParEGO(*left*), MaxMin(*middle*), and EIR2(*right*) on test function ZDT1 (a~c), ZDT2 (d~f), and ZDT3 (g~i).

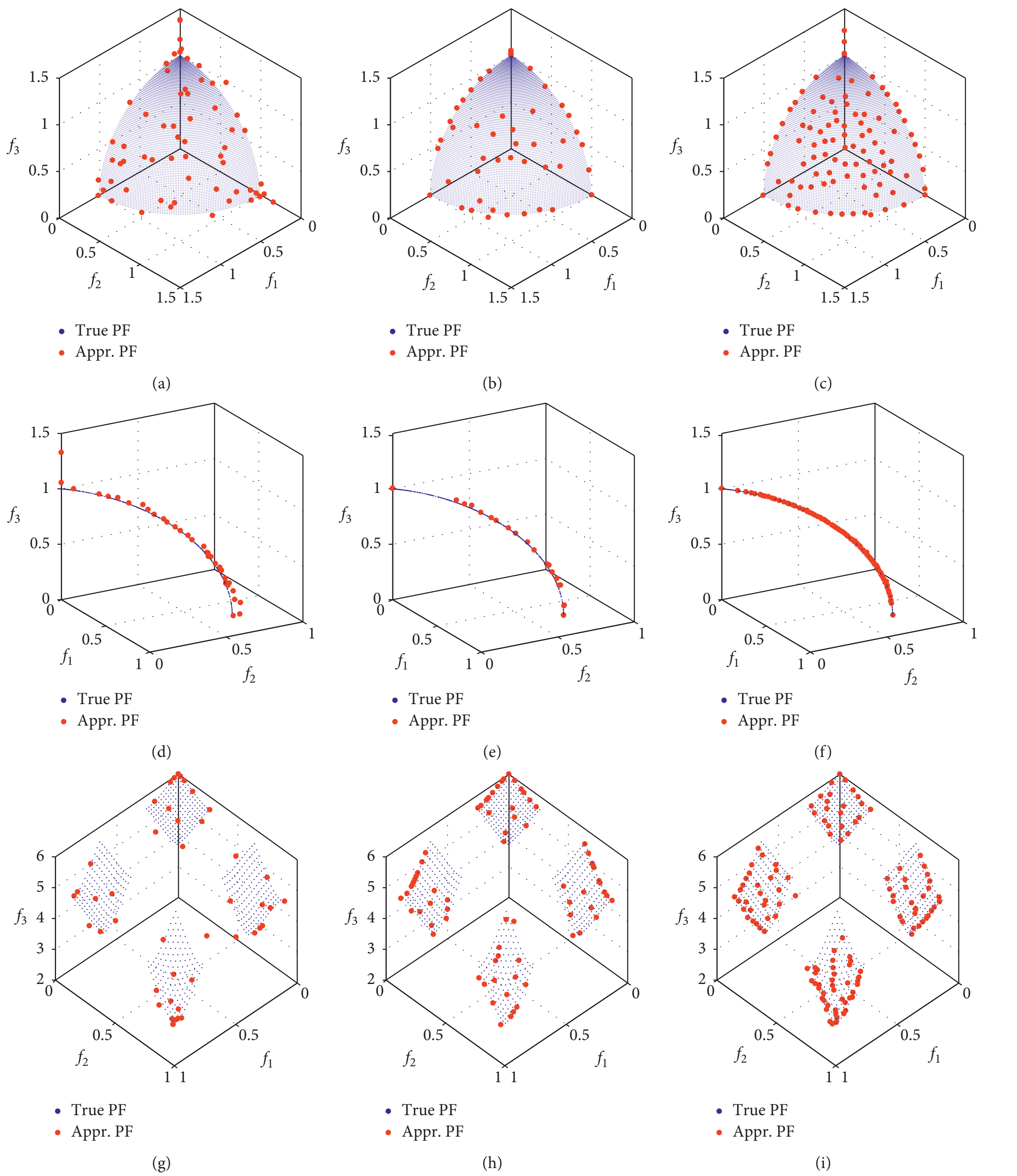

FIGURE 6: Approximated Pareto front found by three infilling criterions ParEGO(*left*), MaxMin(*middle*) and EIR2(*right*) on test function DTLZ1 (a ~ c), DTLZ3 (d ~ f), and DTLZ5 (g ~ i).

$$\text{APoF} = \frac{1}{v}\sum_{i=1}^{v}\text{PoF}_i(\mathbf{x}), \tag{17}$$

where v is the number of all inequality constraints. The APoF value of an infeasible solution is low, but it will increase as more constraints get satisfied. Clearly, the value of APoF varies in interval $[0, 1]$. The closer the value of APoF is to 1, the higher the probability that the solution is feasible, and hence more preferred.

In combination with APoF, EIR2 is extended to constrained MOPs

$$\begin{aligned}\text{CEIR2}(\mathbf{x}) &= \text{EIR2}(\mathbf{x}, P, \Lambda) \times \text{APoF}(\mathbf{x}) \\ &= \frac{1}{|\Lambda|}\sum_{\lambda\in\Lambda}\min_{\mathbf{p}\in P}\left\{\max_{j=1,\cdots m}\left\{\lambda_j \times \text{EI}_j(\mathbf{x},\mathbf{p})\right\}\right\} \\ &\quad\times \frac{1}{v}\sum_{i=1}^{v}\Phi\left(-\frac{\widehat{g}_i(\mathbf{x})}{\widehat{s}_i(\mathbf{x})}\right).\end{aligned} \tag{18}$$

In this section, the proposed CEIR2 indicator is applied to the optimization design of energy storage flywheel [48] to test its effectiveness.

The energy storage flywheel [49] uses a flywheel in high-speed rotating to store kinetic energy and convert it into electrical energy when needed. Given a constant speed, the maximum energy storage is proportional to the moment of inertia of flywheel. Larger moment of inertia can be obtained by adjusting the geometric parameters of the flywheel, but at the same time on the risk of increasing the total mass. In this paper, our job is to maximize the flywheel rotational inertia as well as minimize total mass.

Figure 7 is a cross-sectional profile of an energy storage flywheel. There are six decision variables: $r_1 \sim r_4$ and t_1 and t_2, representing the radial sizes and thickness of the hub and spoke of the flywheel, respectively. Three inequality constraints involve restrictions on radial stress, radial deformation, and hoop stress. The overall optimization problem is expressed as follows:

$$\begin{aligned}\min \quad & \mathbf{F}(\mathbf{x}) = [-I(\mathbf{x}), M(\mathbf{x})] \\ \text{s.t.} \quad & g_1(x) = \sigma_r^{\max}(\mathbf{x}) - [\sigma_r] \le 0 \\ & g_2(x) = \sigma_\theta^{\max}(\mathbf{x}) - [\sigma_\theta] \le 0 \\ & g_3(x) = \Delta_r^{\max}(\mathbf{x}) - [\Delta_r] \le 0,\end{aligned} \tag{19}$$

where $\mathbf{x} = [r_1, r_2, r_3, r_4, t_1, t_2]$, whose upper and lower boundary are listed in Table 6. $I(\mathbf{x})$ and $M(\mathbf{x})$ represent the rotational inertia and total mass of the flywheel, respectively. $\sigma_r^{\max}$, $\sigma_\theta^{\max}$, and $\Delta_r^{\max}$, respectively, represent the maximum radial stress, the maximum hoop stress, and the maximum radial deformation of the outer ring when the flywheel rotates, accompanied by their corresponding allowable values, $[\sigma_r]$, $[\sigma_\theta]$, and $[\Delta_r]$.

Although formulas of the objective functions $I(\mathbf{x})$ and $M(\mathbf{x})$ can be obtained directly from the geometric parameters, the calculation of the constraint functions has to resort to finite element analysis, considering the complexity of flywheel structure and external force conditions; hence, this problem is a constrained expensive multiobjective optimization problem.

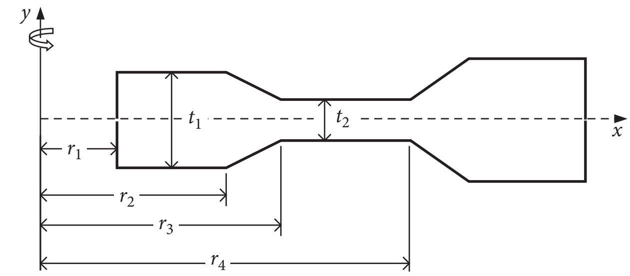

Figure 7: Diagram of energy storage flywheel.

Table 6: Range of decision variables.

Range (mm)	x_1	x_2	x_3	x_4	x_5	x_6
$\mathbf{x}^{\max}$	114.22	128.19	159.94	203.12	36.75	8.35
$\mathbf{x}^{\min}$	104.22	118.19	149.94	193.12	26.75	4.35

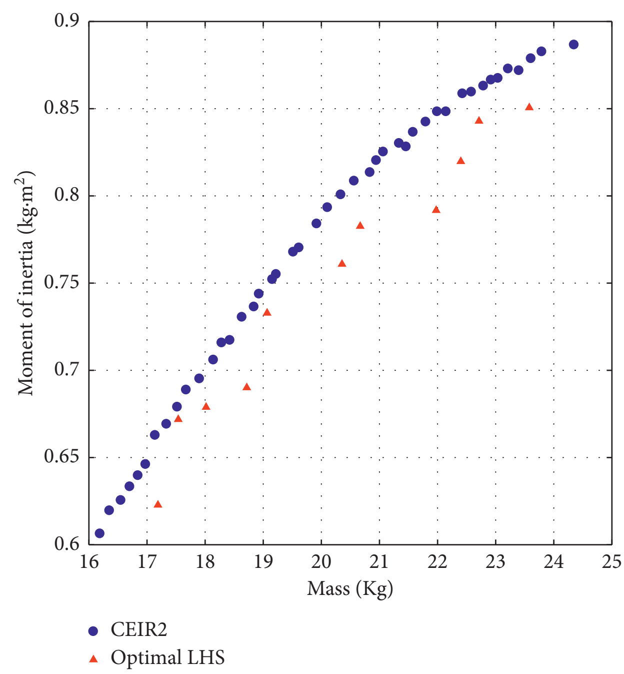

Figure 8: Nondominated solutions obtained by CEIR2 and optimal LHS method for the design of the energy storage flywheel.

The material used for the flywheel is Ti-6Al-4V titanium alloy, with corresponding allowable stress $[\sigma_r] = [\sigma_\theta] =$ 800 MPa. We set $[\Delta_r] = 10$ mm at the working rotation speed $\omega = 20,000$ rpm. ANSYS is employed for calculating values of objective and constraint functions.

As for parameter setting, the maximum number of function evaluations is 360, α is set to 1/4, and other parameters remain unchanged as for the test function case.

The resulting feasible nondominated solutions are obtained by applying CEIR2 to this engineering problem, as shown in Figure 8. For comparison, we use the optimal LHS method to get 900 sample points at once and then evaluate values of the expensive objective and constraint functions, out of which the feasible nondominated solutions are also shown in Figure 8.

CEIR2 found a total of 44 feasible nondominated solutions, which have a large distribution range in the objective space, namely, $I(\mathbf{x}) \sim [16.1, 24.7]$ kg and $M(\mathbf{x}) \sim [0.68, 0.89]$ $\mathrm{kg} \cdot \mathrm{m}^2$, and uniformly distributed along the approximated PF. In comparison, the optimal LHS spend 900 expensive function evaluations but found only 11 feasible nondominated solutions, which are inferior to that of CEIR2 both in terms of convergence and diversity, indicating CEIR2's advantage in using computing resources.

7. Conclusion and Future Work

In this paper, a novel infilling point criterion EIR2 is proposed, which is combined with the Kriging approximate model, optimal Latin hypercube sampling, and PSO algorithm for dealing with expensive multiobjective optimization problems. The basic idea of EIR2 is to map a point in the objective space to a set in the EI space, and the R2 indicator of the set is defined as the fitness of this point. Due to its analytic form, the computation required for this criterion is very low. The proposed algorithm is applied to three suites of standard test functions. The results show that the algorithm can get more nondominated solutions with higher convergence and dispersion under the equal computing resources. Combined with the average probability of feasibility indicator, EIR2 can also be applied to constrained expensive MOPs. In the engineering optimization of energy storage flywheel, the nondominated solutions having a wider range and diversity are obtained exhausting relatively low computation cost.

Future work will have to extend EIR2 to a parallel version that is capable of finding multiple infilling points [50–54] in one iteration with the help of parallel computers to further accelerate the speed of the optimization process. Besides, regarding the parameterization of the utility function, we use a fixed set of uniformly distributed weight vectors. This strategy runs smoothly if the geometry of PF is relatively regular, such as a simplex-like shape, but may struggle [55] if the true PF turns out to be irregular, such as having a sharp peak of a low tail or discontinuous, because multiple weight vectors cloud corresponds to one same solution. As a result, the diversity of the nondominated solution set cannot be guaranteed. A potential promising strategy [56–58] is to dynamically adjust the weight vectors during the optimization process so that the solutions are directed towards each part of the PF. Future work will be incorporating this strategy to our IPC to further improve its ability to maintain diversity.

Abbreviations

MOPs: Multiobjective Optimization Problems
MOEA: Multiobjective Optimization Evolutionary Algorithm
EI: Expectation Improvement
EGO: Efficient Global Optimization
IPC: Infilling Point Criterion
DoE: Design of Experiments
MCM: Monte Carlo Method
EIR2: Expectation Improvement R2 Indicator
IGD: Inverted Generational Distance
LHS: Latin hypercube sampling
HV: Hypervolume
NR: Nondominated solution ratio
PS: Pareto set
PF: Pareto front
PSO: Particle swarm optimization
PoF: Probability of Feasibility
APoF: Average Probability of Feasibility
QI: Quality indicators.

Data Availability

The source code of the EIR2-MOEA algorithm proposed in this article can be obtained by contacting the corresponding author.

Conflicts of Interest

The authors declare no conflicts of interest.

Authors' Contributions

Both authors have contributed equally to this paper and have read and approved the final manuscript.

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