Complexity

Complexity Measures and Models in Supply Chain Networks

Lead Guest Editor: Vladimir Modrak
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Editorial

Complexity Measures and Models in Supply Chain Networks

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Supply chains networks (SCNs) have become in the last two decades increasingly global and are generally considered to be one of leading drivers of business value. SCNs create more and more complex systems that tend to move from tightly coupled to loosely coupled structures. The loosely coupled structures allows for higher flexibility due to low interdependency. Moreover, Brown et al. [1] observed that those companies that swapped their tightly coupled processes for loosely coupled ones achieved performance improvements. On the other hand, loosely coupled SCNs resulted in more complex logistics infrastructures. Also for that reason, complexity is a topical issue in the supply chain literature. While definitions of supply chain complexity may vary due to contextual differences, there is a general consent that supply chain complexity is multifaceted phenomenon that is driven by several sources (see, e.g., [2–6]). Among them, uncertainty, technological intricacy, organizational practice, the number of suppliers, the portfolio of products' structure, and the flow of manufacturing processes can be identified. Naturally, it is difficult to recognize what exactly determines supply chain complexity and which consequences are critical for effective coordination and/or scheduling in the supply chain. The main hurdle in that effort is the lack of the comprehensive principles that govern how supply chains with complex organizational structure and function arise and develop [7]. The positive thing is that there are many partial approaches to the problem, dealing with different perspectives. For example, a promising approach has been used in Portuguese automotive supply chain [8]. In this regard, each novel supply chain complexity measures and models may help in better understanding the yet unknown effects of the possible factors.

This special issue collection on complexity measures and models in supply chain networks encompasses a series of articles, which can be divided into two major categories: (1) optimization models of SCNs as tools for decision-making and management under uncertainty; (2) scheduling problems in supply chain networks. Articles belonging to the first one are introduced in the following four paragraphs.

The reliability-based robust design optimization (RBRDO) model of inventory management system in terms of furniture merchandising company is developed and used in the paper titled "Stochastic Reliability Measurement and Design Optimization of an Inventory Management System." Proposed reliability-based robust design optimization (RBRDO) approach consists of reliability-based design optimization (RBDO) and robust design optimization (RDO). RBRDO considers various uncertainties arising from changes in specifications, transportation delays, raw material availability, manufacturing processes, and operational conditions. The results of the case study showed that RBRDO allows the supply chain company to effectively control reliability of deliveries according to customer requirements.

A model of multichannel household appliance supply chain with price competition and demand uncertainty is presented in the paper titled "Complex Characteristics of Multichannel Household Appliance Supply Chain with the Price Competition." Considering that price competition often leads to the demand and order fluctuation, the authors of this paper focus their research on bullwhip effect (BWE) phenomena in supply chain. For this purpose, a numerical experiment to investigate how the bullwhip effect is affected by the channels' price strategy in different states was invented

and applied. Moreover, the feedback control method was used to control the chaos and the BWE in the supply chain system. Based on the numerical simulation, they found among other findings that a chaotic state of a supply chain system will suffer larger bullwhip effect than a stable system.

The design and operation of complex supply chain processes are considered as NP-hard optimization problems and approached through metaheuristics. In the work titled "Optimization of Consignment-Store-Based Supply Chain with Black Hole Algorithm," an optimization approach of consignment-store-based supply chain with black hole algorithm is applied through a case study in a power plant supply chain company. The optimization of this complex supply chain problem is aimed to minimize the materials handling costs of the whole supply chain. Obtained results in this paper indicate the efficiency of new advanced black hole optimization operators to increase the convergence of the algorithm.

Development of a mathematical model for the evaluation of the distribution of production tasks in several plants to achieve maximum production in the shortest possible time is described in the paper titled "Modelling Decision-Making Processes in the Management Support of the Manufacturing Element in the Logistic Supply Chain." The proposed model focuses on seeking satisfactory solutions by making orders in various locations, which differ in production capacity and manufacturing cost. The model was tested through simulation experiments independently for two manufacturing strategies. Obtained results showed that the model appears to be a valid instrument for optimization of total production cost.

Articles focused on scheduling problems in supply chain networks are briefly characterized in the following three paragraphs.

Consequences of supply chain complexity are quite frequently articulated and complexity sources can be more or less predicted. Nevertheless, those factors cannot be managed without the ability to measure them. Therefore, the main challenge in the complexity metric is to increase their effectiveness by getting a better appreciation of the real problems. In this context, a novel complexity measure of manufacturing systems is proposed in the paper titled "Novel Complexity Indicator of Manufacturing Process Chains and Its Relations to Indirect Complexity Indicators." The principle of the method in this article relies on using the sequences of machine operations for manufacturing of group of product according to the scheduled plan. The authors also analyzed relations between production line balancing rate, number of intercell part flows, intracell part flows, and the complexity measure.

The paper titled "Architecting a System Model for Personalized Healthcare Delivery and Managed Individual Health Outcomes" offers architecture of system model for personalized healthcare delivery and managed individual health outcomes. Its scope is to show an analogy between mass-customized production systems and healthcare delivery systems and to highlight the stochastic evolution of an individual's health state as a key distinguishing feature. Therefore, modelling of healthcare processes requires a systems approach. The research presented provides knowledge-based modelling support for the planning and scheduling of healthcare processes.

In the last but not least research paper titled as "A Multilayer Model Predictive Control Methodology Applied to a Biomass Supply Chain Operational Level," the authors presented their multilayer model predictive control methodology applied to a biomass supply chain operational level. The methodology is composed of two interconnected levels that closely monitor the system state update, in the operational level, and delineate a new routing and scheduling plan in case of an expected deviation from the original one. The authors proved their approach by using an experimental case study. This novel strategy enables the online scheduling of the supply chain transport operation using a predictive approach.

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Research Article

Architecting a System Model for Personalized Healthcare Delivery and Managed Individual Health Outcomes

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In recent years, healthcare needs have shifted from treating acute conditions to meeting an unprecedented chronic disease burden. The healthcare delivery system has structurally evolved to address two primary features of acute care: the relatively short time period, on the order of a patient encounter, and the siloed focus on organs or organ systems, thereby operationally fragmenting and providing care by organ specialty. Much more so than acute conditions, chronic disease involves multiple health factors with complex interactions between them over a prolonged period of time necessitating a healthcare delivery model that is personalized to achieve individual health outcomes. Using the current acute-based healthcare delivery system to address and provide care to patients with chronic disease has led to significant complexity in the healthcare delivery system. This presents a formidable systems' challenge where the state of the healthcare delivery system must be coordinated over many years or decades with the health state of each individual that seeks care for their chronic conditions. This paper architects a system model for personalized healthcare delivery and managed individual health outcomes. To ground the discussion, the work builds upon recent structural analysis of mass-customized production systems as an analogous system and then highlights the stochastic evolution of an individual's health state as a key distinguishing feature.

1. Introduction

1.1. Complexity. The National Academy of Sciences Report on Building a Better Delivery System states that "similar to the supply chains in manufacturing and other industries, the healthcare delivery system is so large and complex that it has become impossible for any individual, or even any single organization, to understand all of the details of its operations [1]." This statement elucidates three key points about the healthcare delivery system. First, that the healthcare delivery system is a *supply chain*. Second, that it is *complex*. Finally, that it *has become* this way, suggesting that it was not this way previously.

A supply chain here refers to a series of care services provided by the healthcare delivery system to the operand, the patient. This healthcare delivery system has evolved to become more complex and as such can be considered a complex adaptive system [2–5]. "A complex adaptive system is a collection of individual agents with freedom to act in ways that are not always totally predictable, and whose actions are interconnected so that one agent's actions changes the context for other agents" [3].

The next section describes how our healthcare system, which developed to treat acute conditions but is now burdened by the treatment of chronic disease, has grown in complexity. This complexity is due to an increase in the number of agents, their roles, and their relative position. These changes have led to the need for greater collaboration and information sharing. Finally, these agents are organized into systems of systems that are continually coevolving.

1.2. Current Healthcare Delivery System. The current healthcare delivery system organically developed to meet "one-off" acute conditions. It evolved to respond to any acute illness or injury that came through the door [6]. The focus of the

Table 1: Healthcare delivery challenges for chronic conditions.

(1) First, by definition, chronic disease is described by a sequence of events that (d)evolve an individual's health state over a duration that is often far longer than any single visit to a healthcare facility.
(2) Second, the sequence of these events do logically depend on each other as described by medical science.
(3) Third, how any individual experiences this chronic condition is often entirely unique given their unique combination of social, behavioral, environmental, and biological risk factors.
(4) Finally, this chronic condition often affects many aspects of an individual's health that are often covered by disparate medical disciplines.

system is on the urgency of *diagnosing* and *curing* the physical anomalies of the individual-patient before they fall into more serious diagnoses [7]. Such acute episodes last on the order of days to weeks, where the individual-patient is considered a passive recipient of treatment [8]. This model of care comes from *the biomedical model*: the dominant allopathic medicine model introduced in the mid-19th century and used until today to diagnose disease [9]. Such acute and urgent care needs enabled the evolution of a centralized infrastructure system.

The model developed during the time when the scientific approach focused on the body as a machine and therefore disease to be the consequence of breakdown in the machine [10]. Therefore, the model is disease-oriented and reductionist, focusing on the identification of physical causes assuming that illness and symptoms arise from an underlying pathophysiology of cellular abnormalities or imbalances in homeostasis [8, 11]. Such a model focus was very useful in addressing the pressing medical problems of the 19th and early 20th centuries, namely, infectious diseases and traumatic illness [12].

In contrast, the current healthcare system is facing an unprecedented chronic disease burden. These conditions, unlike acute conditions, are particularly complex in that they are ongoing and tend to involve multiple factors with multiple interactions between them [13]. Furthermore, they currently represent the leading causes of death and disability in the United States and globally [14, 15]. As of 2012, 50% of all adults had one or more chronic health conditions [16]. In the first time in history, our children's generation is expected to lead shorter life spans than our own [17]. Chronic diseases are also significantly increasing demand for healthcare services and driving up costs. As of 2010, 86% of all healthcare spending was for people with one or more chronic medical conditions [18]. They account for 81% of hospital admissions; 91% of all prescriptions filled; and 76% of all physician visits [19].

Relative to acute conditions, the characteristics of chronic conditions present several new healthcare delivery challenges. Four are identified in Table 1.

To further distinguish between acute and chronic healthcare delivery, this work refers to "individuals" rather than patients. The former addresses people throughout their lives in general whereas the latter addresses their state when they are in a healthcare delivery facility.

Several definitions have been proposed for the term chronic disease [20]. All of which encompass a concept of either (1) unspecified long duration [21, 22] or (2) specified long duration lasting more than 12 months [23–27]. Defining chronic conditions with the key component of *long duration* emphasizes that the changes, in the individual's health state, occur over a period that is much longer than any single visit to a healthcare facility [20].

Goodman et al. demonstrate that most chronic disease definitions include a key component of *need for ongoing medical care* [20, 22–25, 27, 28]. There is, however, an important property to such ongoing medical care, which is typically described by the term *continuity*, emphasizing that the sequence of events logically depend on each other. Haggerty et al. define continuity as *"the degree to which a series of discrete healthcare events is experienced as coherent and connected and consistent with the individual's medical needs and personal context"* [29]. Continuity has been shown to improve chronic disease outcomes [30, 31].

Definitions of chronic diseases are generally very broad and describe key components rather than the specifics of a disease. This is primarily because the experience of a chronic condition may manifest uniquely for each individual based on several factors [32–36]. Our scientific models have been shifting from the classic biomedical model pervasive in acute care to the *biopsychosocial model*, which argues that the causes and consequences of illness exist at multiple levels of organization: biological, psychological, and social [10, 37]. Furthermore, the Institute of Medicine has identified *individual-centered care* as one of the six specific aims of improvement in healthcare and further emphasizes the importance of an individual's unique experience of a chronic condition [38]. This has led to the incorporation of shared decision-making [39] between healthcare providers and the individual. Allowing the individual to become a decision agent in their care significantly increases complexity with the need for cooperation, information sharing, and consensus in order to reach a decision which was classically dictated by the healthcare provider agent alone.

Finally, chronic conditions often affect many aspects of an individual's health that are often covered by disparate medical disciplines. This has led to the need for multiple specialties in treating a person with a chronic condition [40–44]. Requiring a much larger team of healthcare providers including specialists and non-MD clinicians has fundamentally changed the role (i.e., tasks) of the agent (i.e., healthcare provider) and their position in taking care of the patient. Furthermore, several healthcare delivery models (e.g., Collaborative Care Model [45] and Integrated Care Model [46]) have restructured to coordinate and/or co-locate care. Such embedded care models are effective systems within other systems that are likely to co-evolve. Systems embedded within

Table 2: New healthcare delivery system requirements.

(1) Continues to deliver care well after the individual has left the healthcare facility.
(2) Deeply understands the health state of the individual.
(3) Manages individualized health outcomes.
(4) Coordinates numerous practitioners representing many medical specialties.

other systems may appear as healthcare delivery models within the same clinic as described or may be due to the changes from fee-for-service to value-based care [47] which have effectively widened the boundary of a system.

Thus, the characteristics of chronic diseases require much more from the operation of a healthcare delivery system than the way it has operated to address acute conditions within individual visits. Instead, the four characteristics presented in Table 1 present four new requirements on the healthcare delivery system as shown in Table 2.

The current healthcare delivery system, designed from the outset to address acute conditions, is ill-suited to address the four requirements stated above. Furthermore, their fulfillment fundamentally changes, not just the relationships between the individual and the healthcare delivery system, but also the relationships between its many services and resources as well. Addressing and architecting the relationships between services and resources and those between the individual and the healthcare delivery system are critical to managing the complexity arising from these relationships since "the interactions within a complex adaptive system are often more important than the discrete actions of the individual parts" [48]. These relationships suggest the need for architecting a system model for personal healthcare delivery and managed individual health outcomes.

1.3. Mass-Customized Production Systems. While this strategic shift in healthcare delivery systems may appear dramatic, it is not without precedent in other domains. Mass production systems underwent a similar transformation to become mass-customized production systems [49, 50]. In the 1990s, manufacturing became increasingly characterized by a continually evolving and an ever more competitive marketplace. The implementation of lean manufacturing principles had freed excess capacity and thus gave consumers greater influence over the quality, quantity, and variety of products [51, 52]. In order to stay competitive, manufacturing firms had to respond with high variety products achieved through the use of flexible manufacturing systems and reconfigurable manufacturing systems [49, 50]. Reconfigurable manufacturing systems, in particular, required a rearchitecting of production systems in favor of modular machine tools and distributed control systems in the form of multi-agent systems [53–64]. In time, these new architectural developments were situated within quantitative graph theoretic frameworks [65–69] and used to design new mass-customized production systems [70, 71]. This quantitative foundation now lends itself to reapplication for personalized healthcare delivery.

1.4. Paper Contribution. This paper architects a systems model for personal healthcare delivery and managed individual health outcomes. It serves to address the identified need for systems tool in medicine [72, 73]. To support the development, it specifically roots itself in recent work on the architecture of mass-customized production systems and then incorporates features specific to healthcare delivery. This model directly addresses the four requirements derived from the characteristics of chronic diseases. Special attention will be given to the description of an individual's health state and its stochastic evolution in relation to the healthcare delivery system. This is in contrast to many existing works [74, 75], particularly in healthcare discrete-event simulation, where the individual is treated as a stateless passive entity (e.g., a Petri-net token) being pushed or pulled through various healthcare system queues.

The development of an architecture model opens several avenues for future work including cost-benefit analysis, discrete-event simulation, resilience analysis, optimization, and multi-agent systems.

1.5. Paper Outline. The paper first begins with the description of the architecture model (Section 2). Next, An Acute Care Illustrative Example (Section 3) and a Chronic Care Illustrative Example (Section 4) are presented and followed by Discussion (Section 5) and Conclusion (Section 6). The work assumes prerequisite knowledge in model-based systems engineering [76–79], graph theory [80, 81], and discrete-event simulation [82] which is otherwise gained from the cited texts.

2. Development of Architecture Model

The development of the architecture model proceeds in five parts following Figure 1. As found in many systems engineering texts [76, 77], the healthcare delivery system is characterized by its form, function, and concept. Section 2.1 describes the system form as a set of human and technical resources that make up a physical architecture. Section 2.2 describes the system function as a set of system processes that make up a functional architecture. Section 2.3 describes the system concept as an allocated architecture composed of a bipartite graph between the system processes and resources. Section 2.4 then introduces a discrete-event Petri-net model describing the evolution of an individual's health state. Here, the individual represents the primary value-adding operand of the healthcare delivery system. Its introduction addresses the first three requirements identified in the introduction. Finally, Section 2.5 introduces a bipartite graph that links the healthcare system function to the evolution of an individual's health state. To support the discussion, the architecture is presented graphically in SysML as well as quantitatively. The quantitative discussion draws heavily on analogous works on mass-customized production systems [65–71] and may be viewed as an extension of recent work on healthcare human resources management [83].

2.1. Healthcare System Form: Systems Resources and Their Flexible Aggregation. To begin, the healthcare system form

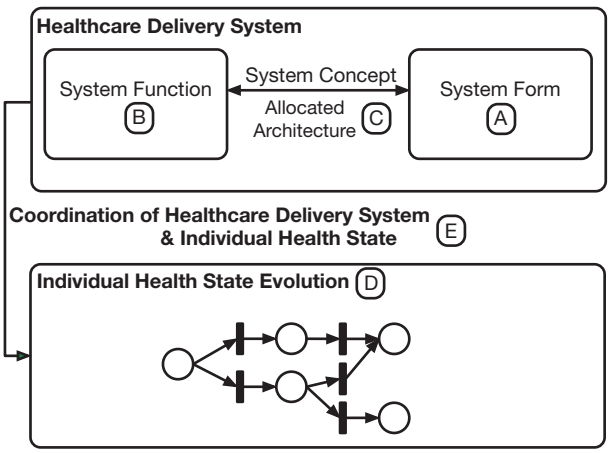

Figure 1: Healthcare System Architecture includes the healthcare delivery system, the individual health state, and their coordination.

[76, 77] (shown as A in Figure 1) is composed of several types of system resources which may be flexibly aggregated. In mass-customized production systems, the system resources were classified as transformation resources (i.e., value-adding machines), storage resources (i.e., independent buffers), and transportation resources (i.e., material handlers) [65–71]. Each of these has their analogous counterparts in the healthcare delivery system. That said, the healthcare delivery system has several essential characteristics which require a finer classification of its system resources. These include the definition of measurement and decision resources as well as the distinction between human and technical resources. The remainder of the discussion on healthcare system form is guided by the SysML block diagram in Figure 2.

Definition 1 (transformation resource). A resource $\mathbb{r}_F \in \mathbb{R}_F$ capable of a transformative effect on its operand (e.g., the health state of an individual). They include *human* transformation resources $r_F \in R_F$ (e.g., surgeon, cardiologist, and psychologist) and *technical* transformation resources $\mathscr{r}_F \in \mathscr{R}_F$ (e.g., operating theatres, chemotherapy infusion room, and delivery room). Transformation resources are the set union of human and technical transformation resources, $\mathbb{R}_F = R_F \cup \mathscr{R}_F$.

Definition 2 (decision resource). A resource $\mathbb{r}_D \in \mathbb{R}_D$ capable of advising the operand, an individual, on how to proceed next with the healthcare delivery system. They include *human* decision resources $r_D \in R_D$ (e.g., oncologist, general practitioner, and surgeon) and *technical* decision resources $\mathscr{r}_D \in \mathscr{R}_D$ (e.g., decision support systems and electronic medical record decision tools). Decision resources are the set union of human and technical decision resources, $\mathbb{R}_D = R_D \cup \mathscr{R}_D$.

Decision resources are analogous to storage resources in previous work on mass-customized production systems [65–71] but are different in two regards. First and fundamentally, in production systems, a shop-floor controller (be it automatic or manual) often dispatches a passive product. Naturally, within the medical community, an individual is viewed as an active stakeholder-participant within the healthcare delivery system rather than a passive entity. In this regard, recent work on "intelligent products" [84–86] in mass-customized production systems is a much more appropriate analogy. Such "intelligent products" are cyber-physical entities that consist of a physical product tied 1-to-1 with an informatic agent that is capable of negotiating and coordinating with the production system. Second, in production systems, intelligent products do not need to be in a specific location to be part of decisions for the next steps of production. In contrast, individuals must often meet healthcare professionals face-to-face in order to determine next steps. Consequently, the analogy to production system storage resources is retained because these decisions must occur at well specified locations in the healthcare delivery system.

Definition 3 (measurement resource). A resource $\mathbb{r}_M \in \mathbb{R}_M$ capable of measuring the operand: here the health state of an individual. They include *human* measurement resources $r_M \in R_M$ (e.g., MRI technician, sonographer, and phlebotomist) and *technical* measurement resources $\mathscr{r}_M \in \mathscr{R}_M$ (e.g., magnetic resonance imaging scanner, ultrasound machine, and Holter monitor). Measurement resources are the set union of human and technical measurement resources, $\mathbb{R}_M = R_M \cup \mathscr{R}_M$.

Measurement resources are analogous to storage resources in previous work on mass-customized production systems [65–71]. Fundamentally speaking, in production systems, a product's state is relatively well-known from the course of its production. Storage resources are required to simply account for a product's location. In contrast, an individual's health state needs to be explicitly ascertained by the healthcare delivery system. The analogy to production system storage resources is retained because naturally this measurement must occur at well specified locations in the healthcare delivery system.

Definition 4 (transportation resource). A resource $\mathbb{r}_N \in \mathbb{R}_N$ capable of transporting its operand: the individual themselves. They include *human* transportation resources $r_N \in R_N$ (e.g., emergency medical technician, clinical care coordinator, and surgical team member) and *technical* transportation resources $\mathscr{r}_N \in \mathscr{R}_N$ (e.g., ambulance, gurney, and wheelchair). Transportation resources are the set union of human and technical transportation resources, $\mathbb{R}_N = R_N \cup \mathscr{R}_N$.

Transportation resources act much like they do in mass-customized production systems. However, healthcare transportation resources are only required when the individual is no longer able to transport themselves unassisted within the healthcare delivery system.

Definition 5 (buffer resource). A resource $r \in \mathbb{R}_B$ where

$$\mathbb{R}_B = \mathbb{R}_F \cup \mathbb{R}_D \cup \mathbb{R}_M. \tag{1}$$

In order to support the discussion of transportation processes, it is useful to introduce the concept of buffer

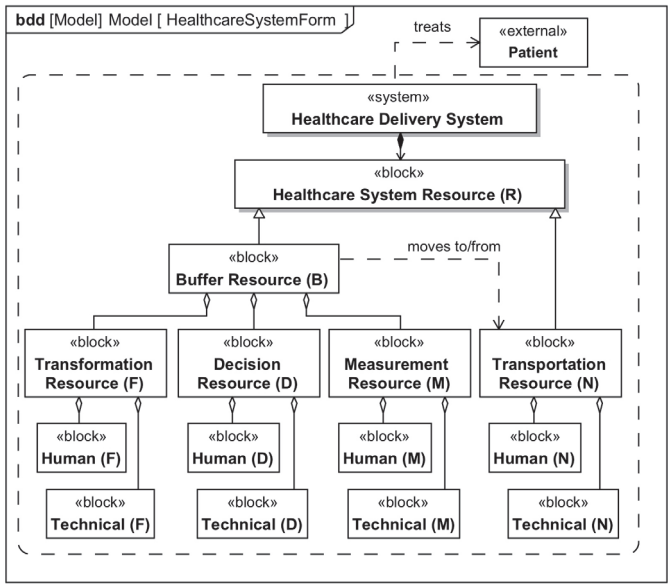

Figure 2: SysML block diagram of healthcare system form.

resources shown in Figure 2 and Definition 5. Collectively, they denote specified locations. In production systems, they were the set union of transformation and storage resources. Here, they are the set union of transformation, measurement, and decisions resources.

Furthermore, it is often useful to view healthcare delivery system resources purely in terms of human and technical classifications.

Definition 6 (human resource). A resource $r \in R$ where

$$R = R_F \cup R_D \cup R_M \cup R_N. \tag{2}$$

Definition 7 (technical resource). A resource $\mathscr{r} \in \mathscr{R}$ where

$$\mathscr{R} = \mathscr{R}_{\mathscr{F}} \cup \mathscr{R}_{\mathscr{D}} \cup \mathscr{R}_{\mathscr{M}} \cup \mathscr{R}_{\mathscr{N}}. \tag{3}$$

The healthcare delivery system resources described thus far allows specific instances to be non-uniquely classified. In the cases where a specific resource is capable of performing several processes, it must be uniquely classified. For example, a surgeon is trained and defined by their transformation ability and not just their decision capability. In order to create a unique classification of these resources, a set of ordered classification rules are implemented.

Definition 8 (rules for classification of healthcare system resources).

Rule 1. If $r \in R$ can *Transform*; then $r \in R_F$. If $\mathscr{r} \in \mathscr{R}$ can *Transform*; then $\mathscr{r} \in \mathscr{R}_{\mathscr{F}}$.

Rule 2. If $r \in R$ can *Decide*; then $r \in R_D$. If $\mathscr{r} \in \mathscr{R}$ can *Decide*; then $\mathscr{r} \in \mathscr{R}_{\mathscr{D}}$.

Rule 3. If $r \in R$ can *Measure*; then $r \in R_M$. If $\mathscr{r} \in \mathscr{R}$ can *Measure*; then $\mathscr{r} \in \mathscr{R}_{\mathscr{M}}$.

Rule 4. Otherwise $r \in R_N$ and $\mathscr{r} \in \mathscr{R}_{\mathscr{N}}$.

These rules effectively sort resources on the basis of their most valuable capabilities. It is assumed that, with respect to value, Transform > Decide > Measure > Transportation. This prioritization is based on healthcare resource hierarchical medical value to the healthcare system. In healthcare delivery systems, the value for these different capabilities has pushed the system to encourage clinicians to "practice at the top of their license."

As many healthcare systems have hundreds or thousands of personnel and equipment, it is useful to form aggregated resources $\overline{\mathbb{R}}$ [65–67, 70, 83].

$$\overline{\mathbb{R}} = A_R \circledast \mathbb{R}, \tag{4}$$

where $\circledast$ is an aggregation operator and A_R is an aggregation matrix [65–67, 70, 83]. These aggregations are flexible and logical in nature and can be changed administratively. For example, an Orthopedic Care Team may be composed of a surgeon, an anesthesiologist, nurses, surgical techs, residents/medical students, and cleaning staff. Naturally, the composition of this aggregation can be changed at a later time. Healthcare resource aggregation is critical for allowing flexibility in the level of abstraction (i.e., individual, teams, departments, clinics, and regions or state) of the system.

In summary, healthcare delivery system resources are the set union of these previously mentioned types of resources.

$$\mathbb{R} = R_F \cup \mathscr{R}_F \cup R_D \cup \mathscr{R}_D \cup R_M \cup \mathscr{R}_M \cup R_N \cup \mathscr{R}_N, \tag{5}$$

$$\mathbb{R} = \mathbb{R}_F \cup \mathbb{R}_D \cup \mathbb{R}_M \cup \mathbb{R}_N, \tag{6}$$

$$\mathbb{R} = R \cup \mathscr{R}. \tag{7}$$

2.2. Healthcare System Function. Healthcare system function [76, 77] (shown as B in Figure 1) is composed of several types of system processes which will ultimately be deployed by the system resources. In mass-customized production systems, the system processes were classified as two types: transformation and transportation [65–71]. Storage processes were considered as transportation processes with nondistinct origin and destination [65–71]. Here, the focus was on *physical* processes that directly interacted with the value-adding operand, the mass-customized product. Analogously, transformation and transportation processes exist similarly in the healthcare delivery system as physical processes on the individual. That said, the healthcare delivery system has several essential characteristics that requires a broader classification. The engineering systems literature often classifies processes into five: transformation, transportation, storage, control, and exchange [87]. Consequently, in healthcare, measurement processes are identified as a type of control process and collaborative decisions are identified as a type of exchange process. It is important to note that these are *cyber-physical* processes in that they require the physical presence of the value-adding operand (i.e., the individual) as well as information flow between the individual and the healthcare delivery system (and its resources). This classification scheme is summarized by the SysML block diagram in Figure 3.

As with mass-customized production systems [65–71], these system processes may be organized to make up a (generic) template model of healthcare delivery system function. These functions are based on a diagnostic model [88] that first examines the patient's complaint (measure), second, attempts to determine its cause (diagnose and decide) and, third, applies a treatment regimen to that cause (treat or transform). Sequentially, these are

(1) *measurement*: understand, quantify or classify individual state,

(2) *decision*: determine what to do for the individual and when,

(3) *transformation*: perform service(s) for the individual,

(4) *transportation*: move the individual between any of these processes.

Figure 4 shows this template service model graphically. Each of these is now described in detail.

Definition 9 (transformation process). A *physical* process $p_F \in P_F$ that transforms the operand: specifically the internal health state of the individual (i.e., treatment of condition, disease, or disorder).

A transformation process typically changes the internal health state of the individual. Such processes include surgical procedures (e.g., amputation, ablation, laparoscopic surgery, and endoscopic surgery) and therapeutic procedures (e.g., pharmacotherapy, chemotherapy, physical therapy, psychotherapy, and laser therapy).

Definition 10 (decision process). A *cyber-physical* process $p_D \in P_D$ occurring between a healthcare system resource and the operand: the individual that generates a decision on how to proceed next with the healthcare delivery system.

Several types of decision processes exist in a healthcare delivery system. Planning is defined as the determination of *which* healthcare system processes need to occur for the individual (e.g., treatment plan and cancer screening plan). Scheduling is defined as *who/what* is going to perform that process and *when* (e.g., individual booking). Furthermore, it is important to distinguish between *intermediate* and *dispatching* decisions where the latter serve to trigger physical activities to the individual and the former do not.

As a physical process, the individual must be physically present at a healthcare system resource (buffer) and in that sense a decision process resembles a storage process. As an informatic (i.e., cyber) process, information is exchanged (in both directions) between the individual and the healthcare system resource to support *collaborative* decision-making [39]. A critical aspect of shared decision-making and information exchange includes the healthcare system resource educating the individual. This enhances the individual's ability to make the most beneficial medical and behavioral decisions. If the individual is incapacitated, then the healthcare system resource makes the decision autonomously.

Definition 11 (measurement process). A *cyber-physical* process $p_M \in P_M$ that converts a physical property of the operand into a cyber, informatic property to ascertain health state of the individual.

Typical healthcare measurement processes acting on individuals include clinical evaluation, diagnostic tests (e.g., blood test, urine test, and stool test) and diagnostic procedures (e.g., medical imaging, endoscopy, and electrocardiography).

As a physical process, the individual must be physically present at a healthcare system resource (buffer) and in that sense a measurement process resembles a storage process. As an informatic (i.e., cyber) process, information is drawn from the individual to ascertain their health state (i.e., diagnose). In mass-customized production systems, the state of each product is relatively well-known from the course of its production. In contrast, an individual's health state evolves stochastically and spontaneously. Understanding an individual's health state is one of the core functions or processes of the healthcare system, which it performs through measurement.

Definition 12 (transportation process). A *physical* process $p_N \in P_N$ that moves individuals between healthcare resources (e.g., bring individual to emergency department and move individual from operating to recovery room).

Although individuals do not typically need to be moved (unless incapacitated), transportation processes are specifically included for the sake of completeness and adherence to the mass-customized production system analogy. This is also performed because it explicitly states the capabilities of the system rather than the utilization of the system by the operand.

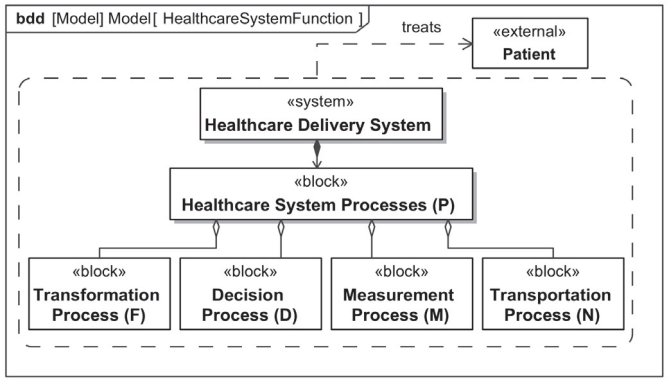

Figure 3: SysML block diagram of healthcare system function.

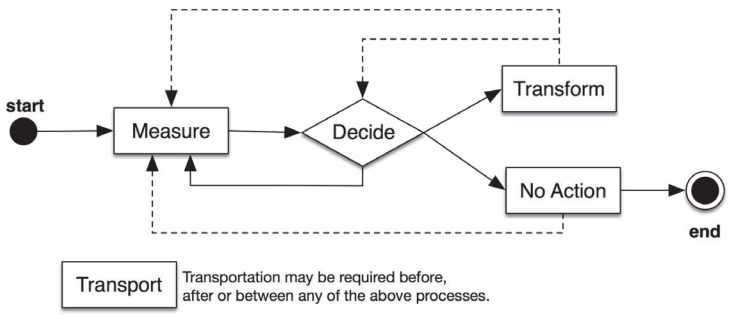

Figure 4: *Healthcare functional model concept of healthcare system processes*. Solid lines represent *typical* process sequences and dotted lines represent *possible* process sequences.

Furthermore, the introduction of the set of buffer resources $\mathbb{R}_B$ implies that there are $\sigma^2(\mathbb{R}_B)$ transportation processes, where the $\sigma()$ notation is introduced to give the size of a set. As a matter of convention, a healthcare process p_{Nu} transports an individual from resource $r_{y_1} \in \mathbb{R}_B$ to resource $r_{y_2} \in \mathbb{R}_B$ such that [65–71]

$$u = \sigma(\mathbb{R}_B)(y_1 - 1) + y_2. \tag{8}$$

Definition 13 (non-transportation process). A combination of non-transportation processes representing transformation, decision, and measurement process, $p_B \in P_B$, is a set union of non-transportation processes.

$$P_B = P_F \cup P_D \cup P_M. \tag{9}$$

As many healthcare systems have hundreds or thousands of processes, it is often useful to form aggregated processes $\overline{P}$.

$$\overline{P} = A_P \circledast P, \tag{10}$$

where $\circledast$ is an aggregation operator and A_P is an aggregation matrix. These aggregations are flexible and logical in nature. Since the healthcare sector has been so heavily influenced by fee-for-service reimbursement strategies, there have been many efforts to codify many of these services or processes to various degrees in various specialties.

In summary, the healthcare system processes are the set union of transformation, decision, measurement, and transportation processes.

$$P = P_F \cup P_D \cup P_M \cup P_N. \tag{11}$$

2.3. Healthcare System Concept (Knowledge Base). Now that healthcare system function and form have been described, the allocation of their constituent processes to their associated resources can be presented. System concept is defined as an allocated architecture composed of a bipartite graph between the system processes and resources (shown as C in Figure 1). This is an integral aspect of many common engineering design methodologies [76, 89–91]. Here, this work builds upon Axiomatic Design Theory, and more specifically for Large Flexible Engineering Systems [68, 92] where this allocation is mathematically formalized in terms of a "design equation" [65–71].

$$P = J_S \odot \mathbb{R}, \tag{12}$$

where J_S is a binary matrix called a "system knowledge base" and $\odot$ is "matrix Boolean multiplication" [65–71].

Definition 14 (system knowledge base [65–71]). A binary matrix J_S of size $\sigma(P) \times \sigma(\mathbb{R})$ whose element $J_S(w, v) \in \{0, 1\}$ is equal to one when event $e_{wv} \in \mathscr{E}_S$ (in the discrete-event systems sense [82]) exists as a system process $p_w \in P$ being executed by a resource $r_v \in \mathbb{R}$.

This system knowledge base definition has been applied to mass-customized production systems [65–71], transportation systems [93–95], water systems [68, 96, 97], and electric power systems [98] and is likely suitable to the healthcare delivery system as another instance of the class of Large Flexible Engineering Systems. It emphasizes the elemental capabilities that *exist* within the system.

It is important to note that the healthcare delivery system knowledge base J_S has a special structure that can be determined from smaller knowledge bases that individually address transformation, decision, measurement, and transportation processes. Using the rules presented in Definition 8, it follows that

$$P_F = J_F \odot \mathbb{R}_F, \tag{13}$$

$$P_D = \left[J_{FD} \;\; J_D\right] \odot \left(\mathbb{R}_F \cup \mathbb{R}_D\right), \tag{14}$$

$$P_M = \left[J_{FM} \;\; J_{DM} \;\; J_M\right] \odot \left(\mathbb{R}_F \cup \mathbb{R}_D \cup \mathbb{R}_M\right), \tag{15}$$

$$P_N = \left[J_{FN} \;\; J_{DN} \;\; J_{MN} \;\; J_N\right] \odot \left(\mathbb{R}_F \cup \mathbb{R}_D \cup \mathbb{R}_M \cup \mathbb{R}_N\right). \tag{16}$$

Consequently,

$$J_S = \begin{bmatrix} J_F & 0 & 0 & 0 \\ J_{FD} & J_D & 0 & 0 \\ J_{FM} & J_{DM} & J_M & 0 \\ J_{FN} & J_{DN} & J_{MN} & J_N \end{bmatrix}. \tag{17}$$

The elemental capabilities that exist within the healthcare delivery system may not always be *available*. In the operational time frame, constraints may apply that effectively eliminate events from the event set. The existence of such constraints is captured within a system events constraints matrix.

Definition 15 (system events constraints matrix [65–71]). A binary matrix K_S of size $\sigma(P) \times \sigma(\mathbb{R})$ whose element $K_S(w, v) \in \{0, 1\}$ is equal to one when a constraint eliminates event e_{wv} from the event set.

Such constraints can be applied on technical resources in the form of breakdowns or maintenance. Similarly, human resources may call in sick or request other types of time off.

The construction of J_S and K_S allows the enumeration of the healthcare system's structural degrees of freedom.

Definition 16 (structural degrees of freedom [65–71]). The set of independent actions $\psi_i \in \mathscr{E}_S$ that completely define the available processes in the system. Their number is given by

$$\mathrm{DOF}_S = \sigma\left(\mathscr{E}_S\right) = \sum_{w}^{\sigma(P)} \sum_{v}^{\sigma(\mathbb{R})} \left[J_S \ominus K_S\right](w, v) \tag{18}$$

$$= \sum_{w}^{\sigma(P)} \sum_{v}^{\sigma(\mathbb{R})} A_S(w, v), \tag{19}$$

where $\ominus$ is Boolean subtraction ($A \ominus B = A \cdot \overline{B}$, where $A \cdot B$ is the Hadamard product or equivalently matrix AND for Booleans. $\overline{B} = \mathrm{NOT}(B)$). These structural degrees of freedom enumerate the capabilities of the healthcare delivery system independent of their sequence. They have been shown to be an essential step in determining the system behavior of several Large Flexible Engineering Systems including mass-customized production systems [65–71], transportation systems [93–95], water systems [68, 96, 97], and electric power systems [98].

From an architectural perspective, the structural degrees of freedom serve to construct a heterofunctional network [68, 92] that describes the structure of the healthcare delivery system. Such a network describes feasible sequences of pairs of structural degrees of freedom called strings. Consider two arbitrary structural degrees of freedom $e_{w_1 v_1}$ and $e_{w_2 v_2}$. Their corresponding string is $z_{\psi_1 \psi_2} = e_{w_1 v_1} e_{w_2 v_2} \in Z$ where $\psi_1 = \sigma(P)(w_1 - 1) + w_2$ and $\psi_2 = \sigma(\mathbb{R})(v_1 - 1) + v_2$ $\forall w_1, w_2 \in \{1, \sigma(P)\}$ and $\forall v_1, v_2 \in \{1, \sigma(\mathbb{R})\}$. The existence of these strings can be captured in a system sequence knowledge base J_ρ.

Definition 17 (system sequence knowledge base [65–71]). A square binary matrix J_ρ is of size $\sigma(P)\sigma(\mathbb{R}) \times \sigma(P)\sigma(\mathbb{R})$ whose element $J_\rho(\psi_1, \psi_2) \in \{0, 1\}$ is equal to one when string $z_{\psi_1 \psi_2}$ exists. It may be calculated directly as

$$J_\rho = \left[J_S \cdot \overline{K}_S\right]^V \left[J_S \cdot \overline{K}_S\right]^{VT}, \tag{20}$$

where $()^V$ is shorthand for vectorization (i.e., vec()).

As before, there may exist sequence-dependent constraints that eliminate some of these two-event strings. These are captured within a system sequence constraints matrix.

Definition 18 (system sequence constraints matrix [65–71]). A square binary constraints matrix K_ρ of size $\sigma(P)\sigma(\mathbb{R}) \times \sigma(P)\sigma(\mathbb{R})$ whose elements $K(\psi_1, \psi_2) \in \{0, 1\}$ are equal to one when string $z_{\psi_1 \psi_2} = e_{w_1 v_1} e_{w_2 v_2} \in Z$ is eliminated.

Unlike K_S where a zero matrix is possible, it has been shown in prior work [65–69] that the system sequence constraints matrix K_ρ has perpetually binding constraints that arise from the functional architecture. In mass-customized production systems, these include, at a minimum, continuity relations that ensure the destination of the first structural degree of freedom is equivalent to the origin of the second [65–69]. Extensive discussions have been provided on the sources of additional sequence-dependent constraints [65–67]. Healthcare delivery systems naturally observe the constraints from continuity relations. They also have many constraints arising from clinical medical practice and administration. Examples of these are discussed in greater detail in Section 5.

Finally, the construction of J_ρ and K_ρ allows the construction of an adjacency matrix A_ρ that describes a heterofunctional network.

$$A_\rho = J_\rho \ominus K_\rho. \tag{21}$$

Definition 19 (system sequence degrees of freedom [65–71]). The set of independent actions $\mathscr{E}_\rho$ that completely defines the available sequence processes in the system. Their number is given by

$$\mathrm{DOF}_\rho = \sigma\left(\mathscr{E}_\rho\right) = \sum_{\psi_1}^{\sigma(P)} \sum_{\psi_2}^{\sigma(\mathbb{R})} \left[J_\rho \ominus K_\rho\right](\psi_1, \psi_2) \tag{22}$$

$$= \sum_{\psi_1}^{\sigma(P)} \sum_{\psi_2}^{\sigma(\mathbb{R})} A_\rho(\psi_1, \psi_2). \tag{23}$$

Here, the nodes represent structural degrees of freedom and the edges represent system sequence degrees of freedom as the feasible sequences between them. The adjacency matrix A_ρ has been shown in prior work to affect the resilience properties of Large Flexible Engineering Systems including mass-customized production systems [65–71], transportation systems [93–95], water systems [68, 96, 97], and electric power systems [98].

In summary, the healthcare system concept is captured in the system knowledge base J_S and the system sequence knowledge base J_ρ to describe the system's capabilities individually and in pairs. This also requires their corresponding constraint matrices K_S and K_ρ. These capability and constraint matrices allow for the construction of a heterofunctional network adjacency matrix A_ρ where the nodes represent the structural degrees of freedom $\mathscr{E}_S$ and the edges represent their feasibility as pairs.

2.4. Individual's "Clinical" Health State Evolution. With the architecture of the healthcare delivery system in place, the discussion turns to an individual's health state evolution (shown as D in Figure 1). While it is important to quantify the capabilities of the healthcare delivery system, it is equally critical to introduce the evolution of each individual's health state so as to keep track of individual patient outcomes. Ultimately, this is necessary to meet the requirements presented in Table 2 so as to address healthcare delivery challenges posed by chronic conditions described in Table 1.

It is here that the analogy between a personalized healthcare delivery system and mass-customized production systems firmly takes shape. In mass-customized production systems, each product is assumed to be entirely different from the one before it. For example, Mercedes Benz offered 3.347807348×10^{24} variations on their Mercedes E class model in 2002 [99]. Human individuals are also unique. From the healthcare delivery system's perspective, the International Classification of Diseases (ICD), currently at ICD-10, has 68,000 diagnosis codes [100]. When one considers that 25% of Americans have multiple chronic conditions [26], the number of possible *combinations* is essentially equal to the population. In both cases, there exist a large number of unique operands that utilize different capabilities of their respective systems. Therefore, a systematic approach is required to model each individual.

In terms of modeling each individual, one must distinguish between the bio-physical-chemical *continuous* health state of the individual, often found in systems biology [101, 102] and an individual's clinical health state. The clinical health state is often ascertained by the clinician through differential diagnosis [103, 104]. The process of diagnosis generally includes a form of *discrete* classification such as by type (e.g., Type 1 diabetes versus Type 2 diabetes [105]), stage (e.g., Breast Cancer Stage IA versus Stage IIIC [106]), grade (e.g., Brain Tumor Grade II diffuse astrocytoma versus Grade IV glioblastoma [107]) or class (e.g., Heart Failure Functional Class I versus Heart Failure Functional Class IV [108]). Furthermore, the evolution of that state happens at irregular time intervals and often as a result of specific events be they from the healthcare delivery system (e.g., surgery), the environment (e.g., exposure to allergens), or new behavior (e.g., a new exercise regimen). Therefore, it is more appropriate to use a discrete-event system model to describe the evolution of an individual's *clinical* health state.

To continue the analogy, in mass-customized production systems, the evolution of a product's state from raw good to finished product was described by a deterministic untimed Petri-net called a "Product Net" [67]. Similarly, a "Health Net" is introduced, this time as a fuzzy timed Petri-net, to model an individual's clinical health state.

Definition 20 (Health Net). Given an individual l_i, that is part of a population L, where $L = \{l_1, \ldots, l_{\sigma(L)}\}$, the evolution of their clinical health state can be described as a fuzzy timed Petri-net [109–111]:

$$N_{l_i} = \left\{S_{l_i}, \mathscr{E}_{l_i}, \mathscr{M}_{l_i}, W_{l_i}, D_{l_i}, Q_{l_i}\right\}, \tag{24}$$

where

(i) N_{l_i} is the Health Net;

(ii) S_{l_i} is the set of places describing a set of health states;

(iii) $\mathscr{E}_{l_i}$ is the set of transitions describing health events;

(iv) $\mathscr{M}_{l_i} \subseteq (S_{l_i} \times \mathscr{E}_{l_i}) \cup (\mathscr{E}_{l_i} \times S_{l_i})$ is the set of arcs describing the relations of health states to health events or health events to health states;

(v) W_{l_i} is the set of weights on the arcs describing the health transition probabilities for the arcs;

(vi) D_{l_i} is the set of transition durations;

(vii) Q_{l_i} is the Petri-net marking representing the likely presence of the set of health states as a discrete probabilistic state.

The Petri-net structure leads directly to the definition of its discrete-event dynamics.

Definition 21 (fuzzy timed Petri-net (discrete-event) dynamics [112]). Given a binary input firing vector $U_{l_i}^+[k]$ and a binary output firing vector $U_{l_i}^-[k]$, both of size $\sigma(\mathscr{E}_{l_i}) \times 1$, and the positive and negative components $\mathscr{M}_{l_i}^+$ and $\mathscr{M}_{l_i}^-$ of the Petri-net incidence matrix of size $\sigma(S_{l_i}) \times \sigma(\mathscr{E}_{l_i})$, the evolution of the marking vector Q_{l_i} is given by the state transition function $\Phi(Q_{l_i}[k], U_{l_i}[k])$:

$$Q_{l_i}[k+1] = \Phi\left(Q_{l_i}[k], U_{l_i}^-[k], U_{l_i}^+[k]\right), \tag{25}$$

where $Q_{l_i} = [Q_{S_{l_i}}; Q_{\mathscr{E}_{l_i}}]$ and

$$Q_{S_{l_i}}[k+1] = Q_{S_{l_i}}[k] + \mathscr{M}_{l_i}^{+} U_{l_i}^{+}[k] - \mathscr{M}_{l_i}^{-} U_{l_i}^{-}[k], \quad (26)$$

$$Q_{\mathscr{E}_{l_i}}[k+1] = Q_{\mathscr{E}_{l_i}}[k] - U_{l_i}^{+}[k] + U_{l_i}^{-}[k]. \quad (27)$$

$Q_{S_{l_i}}$ is introduced to probabilistically mark Petri-net places whereas $Q_{\mathscr{E}_{l_i}}$ is introduced to mark the likelihood that a timed transition is currently firing. The transitions are fired based on a scheduled event list that combines the discrete events with a time interval.

Definition 22 (scheduled event list [82]). A tuple $\mathscr{S}_{l_i} = (u_{l_i\psi}[k], t_{k_{l_i}})$ consists of all elements $u_{l_i\psi}[k]$ in firing vectors $U_{l_i}^{-}[k]$ and their associated times $t_{k_{l_i}}$. For every element, $u_{l_{i\psi}^{-}}[k] \in U_{l_i}^{-}[k]$, there exists another element $u_{l_{i\psi}^{+}}[\kappa] \in U_{l_i}^{+}[\kappa]$ which occurs at time $t_{\kappa_{l_i}}$, $d_{\psi_{l_i}}$ time units later. $t_{\kappa_{l_i}} = t_{k_{l_i}} + d_{\psi_{l_i}}$.

The Health Net is a practical representation of an individual's health state evolution from a clinical practitioner's perspective. Health states may include specific health factors (e.g., BMI level and glucose level) or may represent specific outcomes (e.g., pain level and cancer remission). The health events allow for the progression from one health state to the next as has been described in the scientific medical literature. The weights W_{l_i} on the arcs $\mathscr{M}_{l_i}$ are no longer integers but instead probabilities of (1) a health state leading to a health event or (2) a health event leading a health state. The introduction of event timing and fuzzy state evolution are now specifically included to account for the requirements presented in Table 2.

An individual's health events $\mathscr{E}_{l_i}$ may be further classified. $\mathscr{E}_{l_i} = \mathscr{E}_{Fl_i} \cup \mathscr{E}_{\varphi l_i}$. Each health event in $\mathscr{E}_{Fl_i}$ is triggered by the transformation processes P_F in the healthcare delivery system. Each health event in $\mathscr{E}_{\varphi l_i}$ is the result of a stochastic human process P_φ. These stochastic human processes (i.e., the capability of the human body to change health state without a healthcare delivery system trigger) may occur randomly for unknown reasons or it may be mediated by non-healthcare delivery system factors that may be internal or external to the individual, such as injury and social, economic, environmental, or biologic/genetic factors (e.g., car accident, BMI, and gender). Note that in mass-customized productions systems $\mathscr{E}_{\varphi l_i}$ do not exist. Furthermore, while the mass-customized production system describes a production transformation process (e.g., milling and painting) as having a single deterministic outcome, the scientific medical literature describes healthcare transformation processes (e.g., cancer therapy) as having several probabilistic health outcomes (e.g., cancer recurrent or cancer in remission) which would be reflected in the partially marked state Q_{l_i}.

Finally, in mass-customized production systems, the product net had events that occurred instantaneously. In contrast, the Health Net has events with stochastic duration. This is particularly important as an individual's health recovers and degrades at different rates.

With the Health Net model in place, it becomes important to understand how the full evolution of the clinical health states can be partitioned into *episodes*.

Definition 23 (episode). A partition of the Health Net $N_{el_i} = \{S_{el_i}, \mathscr{E}_{el_i}, \mathscr{M}_{el_i}, W_{el_i}, Q_{el_i}\} \subset N_{l_i}$ describing a single noteworthy happening characterized by an underlying condition be it acute or chronic.

The set of episodes are assumed to be collectively exhaustive of the Health Net. $N_{l_i} = \bigcup_j N_{e_j l_i}$. Furthermore, with respect to health events, episodes are mutually exclusive; $\bigcap_j \mathscr{E}_{e_j l_i} = \varnothing$.

The definition of health nets and episodes allows a return to the central premise of the paper summarized in Tables 1 and 2. More specifically, episodes can be classified as either *Acute* or *Chronic*.

Definition 24 (acute condition). Acute condition occurs as an episode (e.g., infection, trauma, and fracture) with a short clinical course that usually responds to treatment where a return to a state of complete-pre-morbid health is the rule [113].

This definition facilitates two assumptions. (1) Acute conditions are mutually exclusive. $\bigcap_j N_{a_j l_i} = \varnothing$, which implies that for n acute conditions

$$\mathscr{M}_{l_i} = \begin{bmatrix} \mathscr{M}_{a_1 l_i} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \mathscr{M}_{a_n l_i} \end{bmatrix}. \quad (28)$$

(2) The duration of an acute episode occurs on the order of duration of a facility visit. This explains why the primary focus of many works on discrete-event simulation in the healthcare delivery system literature [74, 75] is on minimizing transportation and wait times.

Definition 25 (chronic condition). Chronic conditions occur as episodes (e.g., diabetes mellitus, arthritis, cardiovascular disease, and cancer) that have a protracted, usually more than 6 months clinical course (in many cases lifelong), requiring long-term therapy where response is suboptimal and return to a state of complete or pre-morbid normalcy is the exception [113].

Consequently, and unlike acute conditions, chronic conditions are not mutually exclusive. $\bigcap_j N_{c_j l_i} = \varnothing$. Furthermore, the duration of a chronic episode is much longer than duration of a facility visit, and therefore health events may occur both inside and outside the clinic.

In summary, it is important to recognize that the Health Net fulfills three of the requirements in Table 2. It specifically understands the clinical health state of an individual. It also tracks this health state as individuals reach favorable health outcomes. Finally, it recognizes that health events can be part of chronic episodes that are of long duration that can occur well after the individual has left the healthcare facility.

2.5. Linking Healthcare System State with Individual Health State. In order to link the transformation processes of the Healthcare Delivery System to the Individual Clinical Health

State Evolution, a coordination link is necessary (shown as E in Figure 1).

In mass-customized production systems the linking between the production system state and the product state is captured using the product transformational feasibility matrix [65–71], where each transformation process in the production system induces a product event. Analogously, each transformation process in the healthcare delivery system induces its corresponding health event. For each individual, l_i, this feasibility condition can be captured in a binary individual transformational feasibility matrix.

Definition 26 (individual transformation feasibility matrix Λ_{F_i} [65–71]). A binary matrix of size $\sigma(\mathscr{E}_{l_i}) \times \sigma(P_F)$, where $\Lambda_{F_i}(x, j) = 1$ if transformational process p_{F_j} realizes the health event e_{xl_i}.

Since each transformation process realizes exactly one individual health event, the sum of each column of the individual transformational feasibility matrix must equal one. The sum of each row gives the number of times that each transformation process is required by the individual.

Note that in mass-customized production systems, there are typically more unique transformation processes than in all the mass-customized products being produced [114]. In contrast, the healthcare delivery system typically only has transformation processes if they serve to improve individuals' health state. Meanwhile, all the health events in $\mathscr{E}_{\varphi l_i}$ $\forall l_i$ are entirely autonomous of the healthcare delivery system.

2.6. Architecture Model Summary. This section has presented a personalized healthcare delivery system model following the conceptual depiction in Figure 1. The analogy between mass-customized production systems and personalized healthcare delivery is summarized in Table 3 and follows the nomenclature of the conceptual depiction in Figure 1.

3. Acute Care Illustrative Example

Now that the system architecture model has been developed in detail in Section II, it is used to model an acute episode of an ACL injury and repair as an illustrative example. Section 3.1 provides a narrative of an acute episode composed of several health events. Sections 3.2 and 3.3 parse this narrative into quantitative models of the Healthcare Delivery System and the Individual's Health State Evolution, respectively.

3.1. Description of Orthopedic Case. A typical example orthopedic case study of an ACL injury and repair is described below, drawn from a textbook clinical case [115].

Case Study 1. "Adam injured his left knee playing rugby when he fell forwards and sideways while the left foot remained fixed on the ground. He felt immediate pain and was unable to continue with the game. Pain and swelling increased over the next 2 hours. He was seen in an emergency department (ED) and X-rays were negative for fractures. He was prescribed anti-inflammatories, given elbow crutches and advice on ice, rest and elevation. A clinic appointment to see an orthopedic consultant was arranged.

The orthopedic clinician (Ortho) evaluated the individual through a battery of special tests: anterior drawer test and valgus stress instability and active Lachman's test all of which were not conclusive due to pain and swelling. The individual received an urgent MRI scan which showed a rupture of the left ACL and a medial collateral ligament tear. Surgery was performed followed by an ACL post-operative rehabilitation protocol at physical therapy (PT)."

3.2. Modeling the Healthcare Delivery System of the Orthopedic Case. To begin the modeling of the Healthcare Delivery System, system *Form* and *Function* were determined by identifying the resources and processes mentioned in the text of Case Study 1. These were used to construct the system knowledge base J_S as shown in Figure 5.

At this low-level of abstraction, the resources and processes do not reflect the typical practice of clinical operation and are instead aggregated to a higher level using equation (4). The term "orthopedic surgery" now describes an aggregated resource composed of human and technical resources of the orthopedic surgery team, room, and equipment. A similar aggregation is performed on the processes using equation (10) to aggregate the decision and decision support processes. Finally, a careful inspection of J_S in Figure 5 shows that all resources are connected via transportation degrees of freedom. If these transportation capabilities are assumed to be always available, of relatively short duration and of sufficient capacity, then they can be eliminated without loss of generality from the knowledge base so as to focus on the more valuable healthcare delivery capabilities of transformation, decision, and measurement. These steps yield the knowledge base $\bar{J}_{FDM}$ at a higher level of aggregation as shown in Figure 6. For simplicity, the system is assumed to not have any event constraints during this acute episode. $K_S = 0$. The associated number of structural degrees of freedom is calculated from equation (18). $\mathrm{DOF}_S = 14$.

Continuing with the knowledge base $\bar{J}_{FDM}$, the system sequence knowledge base J_ρ is calculated from equation (20). The system sequence constraint matrix K_ρ is typically nonzero because it must reflect continuity relations as constraints [65–69]. However, because the transportation structural degrees of freedom have already been eliminated, such constraints no longer apply. Instead, further constraints may arise from logical sequences in the clinical practice of medicine as described by Figure 4. More specifically, a transformation cannot occur immediately after a measurement; a decision must occur in between. This introduces a total of 375 constraints in K_ρ which eliminates 15 sequence degrees of freedom. The associated heterofunctional network adjacency matrix A_ρ is shown in Figure 7. The associated number of system sequence degrees of freedom is calculated from equation (22). $\mathrm{DOF}_\rho = 181$.

Returning to the narrative of Case Study 1, it can now be rewritten as a string of healthcare delivery system events $\mathscr{E}_S$ as shown in Figure 8. Each event in $\mathscr{E}_S$ has a unique index

Table 3: Summary of the analogy between mass-customized productions systems and personalized healthcare delivery.

System	Mass-customized production system	Personalized healthcare delivery system
(A) System form		
Resources	Buffer (B_S) [Transformation (M) ∪ Independent buffer (B)] ∪ Transportation (H)	Buffer (R_B) [Transformation (R_F) ∪ Decision (R_D) ∪ Measurement (R_M)] ∪ Transportation (R_N)
Resource classification	Fixed	Transform > Decide > Measure > Transportation
(B) System function		
Processes	Transformation (P_μ) ∪ Transportation (P_η)	Transformation (P_F) ∪ Decision (P_D) ∪ Measurement (P_M) ∪ Transportation (P_N)
(C) System concept		
System knowledge base	$J_S = \begin{bmatrix} J_M & \mid & 0 \\ \longleftarrow & J_H & \longrightarrow \end{bmatrix}$	$J_S = \begin{bmatrix} J_F & 0 & 0 & 0 \\ J_{FD} & J_D & 0 & 0 \\ J_{FM} & J_{DM} & J_M & 0 \\ J_{FN} & J_{DN} & J_{MN} & J_N \end{bmatrix}$
System constraint matrix	$K_S = \begin{bmatrix} K_M & \mid & 0 \\ \longleftarrow & K_H & \longrightarrow \end{bmatrix}$	$K_S = \begin{bmatrix} K_F & 0 & 0 & 0 \\ K_{FD} & K_D & 0 & 0 \\ K_{FM} & K_{DM} & K_M & 0 \\ K_{FN} & K_{DN} & K_{MN} & K_N \end{bmatrix}$
Structural degrees of freedom (nodes)	$\mathrm{DOF}_S = \sum_{w}^{\sigma(P)} \sum_{v}^{\sigma(R)} [J_S \ominus K_S](w, v)$	$DOF_S = \sum_{w}^{\sigma(P)} \sum_{v}^{\sigma(\mathbb{R})} [J_S \ominus K_S](w, v)$
System sequence knowledge base	$J_\rho = [J_S \cdot \overline{K}_S]^V [J_S \cdot \overline{K}_S]^{VT}$	$J_\rho = [J_S \cdot \overline{K}_S]^V [J_S \cdot \overline{K}_S]^{VT}$
System sequence constraint matrix	K_ρ	K_ρ
System sequence degrees of freedom (edges)	$\mathrm{DOF}_\rho = \sum_{\psi_1}^{\sigma(P)} \sum_{\psi_2}^{\sigma(R)} [J_\rho \ominus K_\rho](\psi_1, \psi_2)$	$DOF_\rho = \sum_{\psi_1}^{\sigma(P)} \sum_{\psi_2}^{\sigma(\mathbb{R})} [J_\rho \ominus K_\rho](\psi_1, \psi_2)$
(D) Operand Petri-net model	*Product net* (N_{l_i})	*Health net* (N_{l_i})
Places	Product places (S_{l_i})	Health state (S_{l_i})
Transitions	Product event ($\mathscr{E}_{l_i}$)	Health event ($\mathscr{E}_{l_i}$)
Transition duration	Infinitesimal	Fixed duration (D_{l_i})
Arcs	$\mathscr{M}_{l_i}$	$\mathscr{M}_{l_i}$
Arc weight	{0, 1}	Stochastic ("Fuzzy")(W_{l_i})
(E) Coordinating system and operand	*Production*	*Healthcare*
Operand Transformation Feasibility Matrix	Product Transformation Feasibility Matrix (Λ_{μ_i}) of size $\sigma(\mathscr{E}_{l_i}) \times \sigma(P_\mu)$	Individual Transformation Feasibility Matrix (Λ_{F_i}) of size $\sigma(\mathscr{E}_{l_i}) \times \sigma(P_F)$

and its associated combination of process and resource. The transformational events are highlighted in bold.

3.3. Modeling the Individual Health Net Episode of the Orthopedic Case. From the narrative of Case Study 1 and its associated healthcare delivery system events, the Individual Health Net N_l and the Individual Transformation Feasibility Matrix Λ_F can be constructed as shown in Figures 9 and 10, respectively.

The Health Net shows the individual's *health states* at the places (circles) and the individual's *health state transformations* at the transitions (rectangles) which may occur due to the healthcare delivery system events P_F or the stochastic human process P_φ.

The Individual Transformation Feasibility Matrix is constructed by linking the Individual Health Net transitions (i.e., health events) to the corresponding Healthcare Delivery System Transformational Events (i.e., transformation process P_F).

In summary, the Healthcare System Architecture for the acute orthopedic example has been developed and described in terms of the five components in Figure 1. This acute episode case study quantitatively describes the application of this system model for personalized healthcare delivery and managed individual health outcomes.

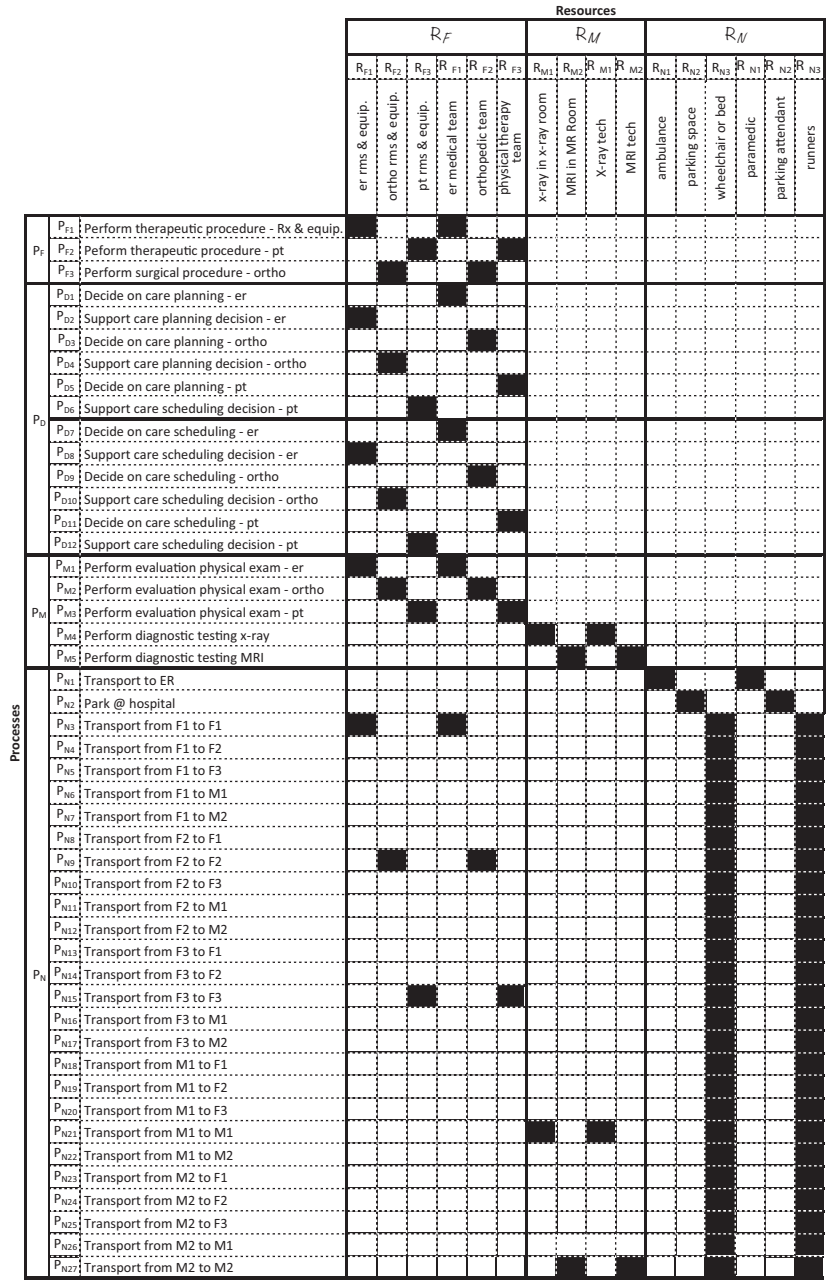

Figure 5: Lowest level healthcare delivery system knowledge base J_S with allocated processes to resources (dark filled) for the acute orthopedic example.

4. Chronic Care Illustrative Example

The system architecture model is used to model a chronic episode of a diabetic case demonstrating the importance of communication between primary and specialized care in the coordinated healthcare of individuals with diabetes. This example specifically illustrates the difference in healthcare transformation processes and individual health state evolution when an episode is misclassified as acute rather than chronic. As before, Sections 4.2 and 4.3 parse this narrative into quantitative models of the Healthcare Delivery System and the Individual's Health State.

4.1. Description of Diabetic Case. An example diabetes case study is described below; it is drawn without modification directly from an example textbook clinical case [116].

Case Study 2. "Juanita is a 66-year old Hispanic individual with a 20-year history of poorly controlled T1DM, chronic

Processes		Resources: R_F			R_M	
		R_{F1} emergency room	R_{F2} orthopedic surgery	R_{F3} physical therapy	R_{M1} X-ray service	R_{M2} MRI service
P_F	P_{F1} Perform therapeutic procedure - Rx & equip.	■				
	P_{F2} Peform therapeutic procedure - pt			■		
	P_{F3} Perform surgical procedure - ortho		■			
P_D	P_{D1} Decide on care planning - er	■				
	P_{D2} Decide on care planning - ortho		■			
	P_{D3} Decide on care planning - pt			■		
	P_{D4} Decide on care scheduling - er	■				
	P_{D5} Decide on care scheduling - ortho		■			
	P_{D6} Decide on care scheduling - pt			■		
P_M	P_{M1} Perform evaluation physical exam - er	■				
	P_{M2} Perform evaluation physical exam - ortho		■			
	P_{M3} Perform evaluation physical exam - pt			■		
	P_{M4} Perform diagnostic testing x-ray				■	
	P_{M5} Perform diagnostic testing MRI					■

Figure 6: Higher level (transformation, decision, and measurement) knowledge base $\bar{J}_{FDM}$ with allocated aggregated processes to aggregated resources (dark filled), for the acute orthopedic example.

w_i v_j	ψ^2 ψ^1	w_1 v_1 1	w_4 v_1 4	w_7 v_1 7	w_{10} v_1 10	w_3 v_2 17	w_5 v_2 19	w_8 v_2 22	w_{11} v_2 25	w_2 v_3 30	w_6 v_3 34	w_9 v_3 37	w_{12} v_3 40	w_{13} v_4 55	w_{14} v_5 70
w_1 v_1	1	■	■	■	■	■	■	■	■	■	■	■	■	■	■
w_4 v_1	4	■	■	■	■	■	■	■	■	■	■	■	■	■	■
w_7 v_1	7	■	■	■	■	■	■	■	■	■	■	■	■	■	■
w_{10} v_1	10	□	■	■	■	□	■	■	■	□	■	■	■	■	■
w_3 v_2	17	■	■	■	■	■	■	■	■	■	■	■	■	■	■
w_5 v_2	19	■	■	■	■	■	■	■	■	■	■	■	■	■	■
w_8 v_2	22	■	■	■	■	■	■	■	■	■	■	■	■	■	■
w_{11} v_2	25	□	■	■	■	□	■	■	■	□	■	■	■	■	■
w_2 v_3	30	■	■	■	■	■	■	■	■	■	■	■	■	■	■
w_6 v_3	34	■	■	■	■	■	■	■	■	■	■	■	■	■	■
w_9 v_3	37	■	■	■	■	■	■	■	■	■	■	■	■	■	■
w_{12} v_3	40	□	■	■	■	□	■	■	■	□	■	■	■	■	■
w_{13} v_4	55	□	■	■	■	□	■	■	■	□	■	■	■	■	■
w_{14} v_5	70	□	■	■	■	□	■	■	■	□	■	■	■	■	■

Figure 7: Adjacency matrix $A_\rho(\phi, \psi)$ composed of sequence events (dark filled), for the acute orthopedic example.

kidney disease, and diabetic amyotrophy. She decided to consult an orthopedic specialist on her own for "terrible leg pains." After a brief workup, which consisted of a magnetic resonance image (MRI) of the knee, a decision was made to perform a total joint replacement on her arthritic right knee. Although the surgeon considered the procedure a "great success," the individual had persistent pain postoperatively, which actually worsened over a 3-month period. The frustrated surgeon could not understand why the individual was complaining of so much pain "when the bone scan and postoperative x-rays showed no evidence of osteomyelitis." The 12 visits of physical therapy also appeared to worsen her discomfort to the point where she became incapacitated by the pain. She was having increasing difficulty with her balance and could not tolerate having any bed sheets come in contact with her feet. Four months after having her knee replacement surgery, she returned to her PCP.

On examination the individual exhibited hyperalgesia, allodynia, and loss of ankle reflexes bilaterally, which was worse on the right (postoperative) extremity. The individual wore a slipper on the right foot to lessen the effects of her painful peripheral diabetic neuropathic pain. The PCP placed the individual on duloxetine, which resulted in a 50% improvement in her overall pain intensity within 3 weeks. Communication between the specialist and the PCP is of utmost importance when managing individuals with diabetes. Had the surgeon discussed this case with the PCP prior to operating he would have realized that a more conservative approach was warranted. Not only was this individual's pain the result of diabetic peripheral neuropathy

Episode	DOF Index	DOF Event	DOF Process	DOF Resource
ACL injury & repair	10	e_{M1F1}	Perform evaluation physical exam - er	by emergency room
	13	e_{M4M1}	Perform diagnostic testing x-ray	by X-ray service
	4	e_{D1F1}	Decide on care planning - er	by emergency room
	7	e_{D4F1}	Decide on care scheduling - er	by emergency room
	1	$\mathbf{e_{F1F1}}$	**Perform therapeutic procedure - Rx & equip.**	**by emergency room**
	7	e_{D4F1}	Decide on care scheduling - er	by emergency room
	11	e_{M2F2}	Perform evaluation physical exam - ortho	by orthopedic surgery
	14	e_{M5M2}	Perform diagnostic testing MRI	by MRI service
	5	e_{D2F2}	Decide on care planning - ortho	by orthopedic surgery
	8	e_{D5F2}	Decide on care scheduling - ortho	by orthopedic surgery
	3	$\mathbf{e_{F3F2}}$	**Perform surgical procedure - ortho**	**by orthopedic surgery**
	8	e_{D5F2}	Decide on care scheduling - ortho	by orthopedic surgery
	12	e_{M3F3}	Perform evaluation physical exam - pt	by physical therapy
	6	e_{D3F3}	Decide on care planning - pt	by physical therapy
	9	e_{D6F3}	Decide on care scheduling - pt	by physical therapy
	2	$\mathbf{e_{F2F3}}$	**Peform therapeutic procedure - pt**	**by physical therapy**

Figure 8: Orthopedic acute episode described in terms of the healthcare delivery system events found in $\bar{J}_{FDM}$. The *healthcare delivery transformational events* are in bold.

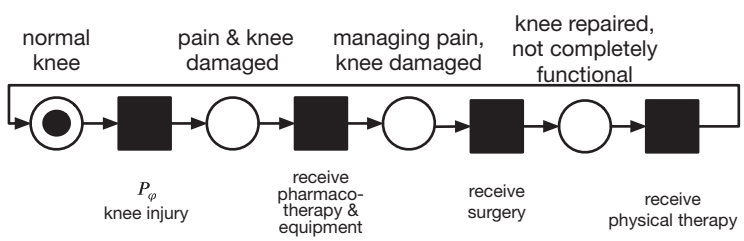

Figure 9: Acute orthopedic Individual Health Net visualizes the health state at the *places* (circles) and the health events, causing the changes in health state, at the *transitions* (rectangles).

rather than "arthritis" but her fasting blood glucose level on the day of surgery was 323 mg per dL. The PCP was unaware that the individual was even hospitalized. Had the specialist been concerned about the individual's preoperative laboratory studies (including her A1C of 12.2%), the surgery would have been canceled until she was medically cleared to undergo the procedure. On the second postoperative day, the individual developed acute renal failure." [116].

This example addresses two possible outcome episodes: (1) *Episode A* describes the events as they happened (i.e., not taking into account the patient's previous history or that they are currently in a chronic episode) and (2) *Episode B* describes the hypothetical scenario if appropriate communication between primary and specialty care had been implemented. Such a scenario would take into account the individual's chronic episode and treat it accordingly.

4.2. Modeling the Healthcare Delivery System of the Diabetes Case. Similar to the first example, the modeling of the Healthcare Delivery System *Form* and *Function* was determined by identifying the resources and processes in the text of Case Study 2. These were used to construct the system knowledge base J_S as shown in Figure 11.

As in the prior case study, the resources and processes at this low-level of abstraction need to be aggregated using equation (4) to a higher level to better reflect the practice of clinical operation. This includes the aggregation of technical and human resources and decision and decision support processes. Transportation capabilities are also assumed to be fully available.

These steps allow the construction of knowledge base $\bar{J}_{FDM}$ at a higher level of aggregation as shown in Figure 12. For simplicity, the system is assumed to not have any event constraints during either episodes. $K_S = 0$. The associated number of structural degrees of freedom is calculated from equation (18). $\mathrm{DOF}_S = 14$.

Continuing with the knowledge base $\bar{J}_{FDM}$, the system sequence knowledge base J_ρ is calculated using equation (20). The assumptions made to the system sequence constraint matrix K_ρ in the first illustrative example are also applied here. Therefore, continuity relations are discounted and logical clinical practice sequence constraints are applied as described by Figure 4. This introduces a total of 648 constraints in K_ρ which eliminates 18 sequence degrees of freedom. The associated heterofunctional network adjacency matrix A_ρ is shown in Figure 13. The associated number of

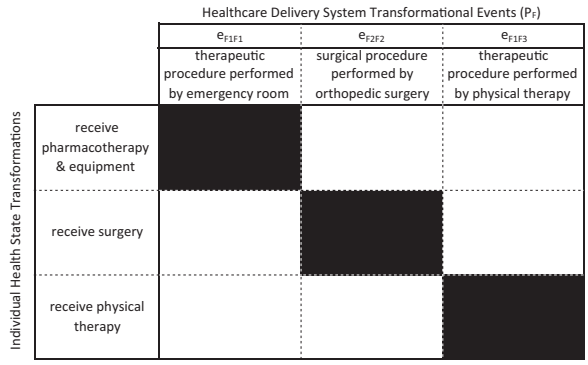

Figure 10: Individual Transformation Feasibility Matrix Λ_F for the acute orthopedic example.

system sequence degrees of freedom is calculated from equation (22). $\mathrm{DOF}_\rho = 178$.

Returning to the narrative of Case Study 2, it can now be rewritten as a string of healthcare delivery system events $\mathscr{E}_S$. Figures 14 and 15 show these strings for Episodes A and B, respectively.

4.3. Modeling the Individual Health Net Episodes of the Diabetic Case. From the diabetes case study narrative and the modeled healthcare delivery system events, the Individual Transformation Feasibility Matrix Λ_F (see Figure 16) and the Individual Health Net showing both Episodes (see Figure 17) can be constructed. The Individual Health Net Episodes show the individual's *health states* at the places (circles) and the individual's *health state transformations* at the transitions (rectangles).

The Individual Transformation Feasibility Matrix is constructed by linking the Individual Health Net transitions (i.e., health events) to the corresponding Healthcare Delivery System Transformational Events (i.e., transformation process P_F).

In summary, the Healthcare System Architecture for the chronic diabetes example has been developed in terms of the five components in Figure 1. This chronic example case study quantitatively shows the importance of communication in the co-management of chronic diseases. In Episode A, a chronic episode was treated as acute, leading to an adverse effect on the individual's health state evolution. In Episode B, the improved health outcome was achieved through coordinated care.

5. Discussion

The acute and chronic care illustrative examples have demonstrated a system model for personalized healthcare delivery and managed individual outcomes. The strengths of the model arise from several network structures that allow for coordinated healthcare while distinguishing between acute and chronic conditions. They are as follows:

(1) The aggregation matrices A_R and A_P.

(2) The system knowledge base J_S.

(3) The system events constraints matrix K_S.

(4) The system sequence constraints matrix K_ρ.

(5) The Health Net N_{l_i}.

Together, these network structures serve to provide appropriate, coordinated, and personalized healthcare to unique individuals. Furthermore, each of these matrices may be viewed as the outcome of a healthcare delivery system design decision. These decisions are now discussed in the context of the five parts of the healthcare system architecture shown in Figure 1.

The aggregation matrices A_R and A_P were introduced so as to view the Healthcare Delivery System *Form and Function* at higher levels of aggregation. The need for physical aggregation reflects how teams of healthcare professionals and groups of technical equipment must often be brought together to form a single operating unit (e.g., surgical team in an operating theatre). Similarly, the need for functional aggregation reflects how many low level system processes are required to perform a single healthcare service (e.g., perform orthopedic surgery). While it is possible to design a healthcare delivery system with constant values of A_R and A_P such rigidity is prohibitively expensive. Healthcare delivery system administrators must often choose new values of A_R so as to form new clinical teams with each shift and assure that the right technical equipment is in the appropriate facilities and rooms. These administrators also choose the values of A_P when they formulate hospital procedures in terms of low-level system processes.

The system knowledge base J_S was introduced so as to view the Healthcare Delivery System *Concept* in terms of the *existence* of capabilities that are the feasible combinations of system processes and resources. Fundamentally, it is a succinct description of what the system can do and how. From a design perspective, the value of J_S is determined by two types of healthcare delivery system administrators: human resource managers and procurement managers. In hiring new personnel, new columns are added to J_S. When these new personnel represent new specializations, new rows are added J_S. Training programs allow each human resource the ability to execute new system processes. Similarly, the procurement of new technical equipment also adds columns to J_S.

The system events constraints matrix K_S was introduced to the Healthcare Delivery System *Concept* to distinguish

Processes		Resources: R_F						R_M						R_N			
		R_{F1} ortho rms & equip.	R_{F2} pt rms & equip.	R_{F3} pcp rms Rx & equip.	R_{F1} orthopedic team	R_{F2} physical therapy team	R_{F3} pcp team	R_{M1} MRI in MR Room	R_{M2} Bone Scintigraphy in BS room	R_{M3} x-ray in x-ray room	R_{M1} MRI tech	R_{M2} BS tech	R_{M3} x-ray tech	R_{N1} parking space	R_{N2} wheelchair or bed	R_{N1} parking attendant	R_{N2} runners
P_F	P_{F1} Perform therapeutic procedure - pt		■			■											
	P_{F2} Perform therapeutic procedure - pcp			■			■										
	P_{F3} Perform surgical procedure - ortho	■			■												
P_D	P_{D1} Decide on care planning - ortho				■												
	P_{D2} Support care planning decision - ortho	■															
	P_{D3} Decide on care planning - pt					■											
	P_{D4} Support care planning decision - pt		■														
	P_{D5} Decide on care planning - pcp						■										
	P_{D6} Support care scheduling decision - pcp			■													
	P_{D7} Decide on care scheduling - ortho				■												
	P_{D8} Support care scheduling decision - ortho	■															
	P_{D9} Decide on care scheduling - pt					■											
	P_{D10} Support care scheduling decision - pt		■														
P_M	P_{M1} Perform evaluation physical exam - ortho	■			■												
	P_{M2} Perform evaluation physical exam - pt		■			■											
	P_{M3} Perform evaluation physical exam - pcp			■			■										
	P_{M4} Perform diagnostic testing MRI							■			■						
	P_{M5} Perform diagnostic testing BS								■			■					
	P_{M6} Perform diagnostic testing x-ray									■			■				
P_N	P_{N1} Park @ hospital													■		■	
	P_{N2} Transport from F1 to F1	■			■										■		■
	P_{N3} Transport from F1 to F2														■		■
	P_{N4} Transport from F1 to F3														■		■
	P_{N5} Transport from F1 to M1														■		■
	P_{N6} Transport from F1 to M2														■		■
	P_{N7} Transport from F1 to M3														■		■
	P_{N8} Transport from F2 to F1														■		■
	P_{N9} Transport from F2 to F2		■			■									■		■
	P_{N10} Transport from F2 to F3														■		■
	P_{N11} Transport from F2 to M1														■		■
	P_{N12} Transport from F2 to M2														■		■
	P_{N13} Transport from F2 to M3														■		■
	P_{N14} Transport from F3 to F1														■		■
	P_{N15} Transport from F3 to F2														■		■
	P_{N16} Transport from F3 to F3			■			■								■		■
	P_{N17} Transport from F3 to M1														■		■
	P_{N18} Transport from F3 to M2														■		■
	P_{N19} Transport from F3 to M3														■		■
	P_{N20} Transport from M1 to F1														■		■
	P_{N21} Transport from M1 to F2														■		■
	P_{N22} Transport from M1 to F3														■		■
	P_{N23} Transport from M1 to M1							■			■				■		■
	P_{N24} Transport from M1 to M2														■		■
	P_{N25} Transport from M1 to M3														■		■
	P_{N26} Transport from M2 to F1														■		■
	P_{N27} Transport from M2 to F2														■		■
	P_{N28} Transport from M2 to F3														■		■
	P_{N29} Transport from M2 to M1														■		■
	P_{N30} Transport from M2 to M2								■			■			■		■
	P_{N31} Transport from M2 to M3														■		■
	P_{N32} Transport from M3 to F1														■		■
	P_{N33} Transport from M3 to F2														■		■
	P_{N34} Transport from M3 to F3														■		■
	P_{N35} Transport from M3 to M1														■		■
	P_{N36} Transport from M3 to M2														■		■
	P_{N37} Transport from M3 to M3									■			■		■		■

Figure 11: Healthcare delivery system knowledge base J_S with allocated processes to resources (dark filled) for the chronic diabetes example.

between the existence and the *availability* of the capabilities. From a design perspective, some availability constraints are planned. These include shift changes for human resources and planned maintenance for technical resources. Other availability constraints may be viewed as unplanned disturbances to the architecture. They include personal and sick leave for human resources and breakdowns for technical resources.

The system sequence constraints matrix K_ρ was introduced to the Healthcare Delivery System *Concept* to distinguish between capabilities as individual elements versus as logical pairs. In the healthcare of acute conditions, where

Processes			Resources: R_F			R_M		
			R_{F1} orthopedic surgery	R_{F2} physical therapy	R_{F3} primary care	R_{M1} MRI service	R_{M2} BS service	R_{M3} x-ray service
P_F	P_{F1}	Perform therapeutic procedure - pt		■				
	P_{F2}	Perform therapeutic procedure - pcp			■			
	P_{F3}	Perform surgical procedure - ortho	■					
P_D	P_{D1}	Decide on care planning - ortho	■					
	P_{D2}	Decide on care planning - pt		■				
	P_{D3}	Decide on care planning - pcp			■			
	P_{D4}	Decide on care scheduling - ortho	■					
	P_{D5}	Decide on care scheduling - pt		■				
P_M	P_{M1}	Perform evaluation physical exam - ortho	■					
	P_{M2}	Perform evaluation physical exam - pt		■				
	P_{M3}	Perform evaluation physical exam - pcp			■			
	P_{M4}	Perform diagnostic testing MRI				■		
	P_{M5}	Perform diagnostic testing BS					■	
	P_{M6}	Perform diagnostic testing x-ray						■

Figure 12: Higher level (transformation, decision, and measurement) knowledge base $\overline{J}_{FDM}$ with allocated aggregated processes to aggregated resources (dark filled), for the chronic diabetes example.

		w_3 v_1	w_4 v_1	w_7 v_1	w_9 v_1	w_1 v_2	w_5 v_2	w_8 v_2	w_{10} v_2	w_2 v_3	w_6 v_3	w_{11} v_3	w_{12} v_4	w_{13} v_5	w_{14} v_6
	ψ^2 / ψ^1	3	4	7	9	15	19	22	24	30	34	39	54	69	84
w_3 v_1	3	■	■	■	■	■	■	■	■	■	■	■	■	■	■
w_4 v_1	4	■	■	■	■	■	■	■	■	■	■	■	■	■	■
w_7 v_1	7	■	■	■	■	■	■	■	■	■	■	■	■	■	■
w_9 v_1	9	□	■	■	■	□	■	■	■	□	■	■	■	■	■
w_1 v_2	15	■	■	■	■	■	■	■	■	■	■	■	■	■	■
w_5 v_2	19	■	■	■	■	■	■	■	■	■	■	■	■	■	■
w_8 v_2	22	■	■	■	■	■	■	■	■	■	■	■	■	■	■
w_{10} v_2	24	□	■	■	■	□	■	■	■	□	■	■	■	■	■
w_2 v_3	30	■	■	■	■	■	■	■	■	■	■	■	■	■	■
w_6 v_3	34	■	■	■	■	■	■	■	■	■	■	■	■	■	■
w_{11} v_3	39	□	■	■	■	□	■	■	■	□	■	■	■	■	■
w_{12} v_4	54	□	■	■	■	□	■	■	■	□	■	■	■	■	■
w_{13} v_5	69	□	■	■	■	□	■	■	■	□	■	■	■	■	■
w_{14} v_6	84	□	■	■	■	□	■	■	■	□	■	■	■	■	■

Figure 13: Adjacency matrix $A_\rho(\phi, \psi)$ composed of sequence events (dark filled) for the chronic diabetes example.

the timescale is relatively short, continuity relations dominate K_ρ. Ensuring that patients can move from one "value-adding" healthcare service to another while avoiding lengthy queues is of fundamental importance. Capacity limitations on transportation capabilities are often important. It is for this reason that discrete-event simulation has been featured so prominently in the study of medical emergency healthcare operations [117–120]. In the healthcare of chronic conditions, the timescale is comparatively long. Transportation processes and their associated continuity relations are no longer of prime importance. Instead, sequence-dependent constraints arise from rules of clinical and administrative practice. Clinical practice dictates which types of scans and tests are required for clinical decisions which are required prior to the execution of specific procedures. Such constraints are put in place to assure the quality of medical practice. Further constraints may be placed by healthcare administrators to control costs. These include limitations on the number of scans and clinical consultations.

The Health Net N_{l_i} for a given individual l_i was introduced as a mathematical description of clinical medical science where the individual's health state requires coordination with the healthcare delivery system in order to achieve the desired health outcomes. In the healthcare of acute conditions, where the timescale is relatively short, health events driven by stochastic human processes $\mathscr{E}_{\varphi l_i}$ may not have the chance to occur and so it is reasonable to assume that the health evolution of an individual is purely determined by the health events driven by healthcare transformation processes $\mathscr{E}_{Fl_i}$. In such a way an individual (patient) becomes a passive entity in

Episode	DOF Index	DOF Event	Process	Resource
Uncontrolled Diabetes Progression with Knee Pain	4	e_{M1F1}	Perform evaluation physical exam - ortho	by orthopedic surgery
	12	e_{M4M1}	Perform diagnostic testing MRI	by MRI service
	2	e_{D1F1}	Decide on care planning - ortho	by orthopedic surgery
	3	e_{D4F1}	Decide on care scheduling - ortho	by orthopedic surgery
	1	$\mathbf{e_{F3F1}}$	**Perform surgical procedure - ortho**	**by orthopedic surgery**
	4	e_{M1F1}	Perform evaluation physical exam - ortho	by orthopedic surgery
	13	e_{M5M2}	Perform diagnostic testing - BS	by BS service
	14	e_{M6M3}	Perform diagnostic testing - x-ray	by x-ray service
	2	e_{D1F1}	Decide on care planning - ortho	by orthopedic surgery
	3	e_{D4F1}	Decide on care scheduling - ortho	by orthopedic surgery
	8	e_{M2F2}	Perform evaluation physical exam - pt	by physical therapy
	6	e_{D2F2}	Decide on care planning - pt	by physical therapy
	7	e_{D5F2}	Decide on care scheduling - pt	by physical therapy
	5	$\mathbf{e_{F1F2}}$	**Perform therapeutic procedure - pt**	**by physical therapy**
	11	e_{M3F3}	Perform evaluation physical exam - pcp	by primary care
	10	e_{D3F3}	Decide on care planning - pcp	by primary care
	9	$\mathbf{e_{F2F3}}$	**Perform therapeutic procedure - pcp**	**by primary care**

Figure 14: Diabetes Chronic Episode A described in terms of the healthcare delivery system events found in $\overline{J}_{FDM}$. The *healthcare delivery transformational events* are in bold.

Episode	DOF Index	DOF Event	Process	Resource
Uncontrolled Diabetes Progression with Knee Pain	11	e_{M3F3}	Perform evaluation physical exam - pcp	by primary care
	10	e_{D3F3}	Decide on care planning - pcp	by primary care
	9	$\mathbf{e_{F2F3}}$	**Perform therapeutic procedure - pcp**	**by primary care**

Figure 15: Diabetes Chronic Episode B described in terms of the healthcare delivery system events found in $\overline{J}_{FDM}$. The *healthcare delivery transformational events* are in bold.

the healthcare delivery system. Such is the inherent assumption of many works on discrete-event simulation of medical emergency healthcare operations [117–120]. In contrast, in the healthcare of chronic conditions, the health events driven by stochastic human processes $\mathscr{E}_{\varphi l_i}$ play a prominent role and it becomes important to track the evolution of an individual's health state as had been done in mass-customized production systems [65].

That said, an individual's health state has several features that distinguish it from mass-customized products. First, the state of the mass-customized product is typically completely understood and quantifiable, whereas the true state of the human being's health is typically fuzzy. Consequently, the healthcare delivery must heavily utilize measurement processes to ascertain this state, the fundamental reason for the inclusion of measurement processes in the process classification. Second, individuals (or patients) may be viewed as semiautonomous decision-making rather than passive entities [39]. In that regard, the intelligent (mass-customized) product literature [84–86] may prove a relevant extension of the analogy presented in this paper. Recent work has specifically included a product net at the heart of an intelligent product agent's data structure [70] and so one can expect the Health Net to take a similar role for individuals. As a third distinguishing feature, this model specifically includes decision capabilities because individuals often need to physically meet with clinicians in order for these shared decisions to occur [39].

6. Conclusion

In conclusion, this paper architects a system model for personalized healthcare delivery and managed individual health outcomes. This work is built upon recent structural analysis of mass-customized production systems as an analogous system. It highlights the stochastic evolution of an individual's health state as a key distinguishing feature. In doing so, it systematically addressed the new healthcare delivery system requirements described in Table 2 that were derived from healthcare delivery challenges posed by chronic conditions described in Table 1. The architecture model was then demonstrated for two illustrative examples: one for acute care and another for chronic care. The contrast of the two examples shows inherent complexities of managing personalized healthcare delivery for (potentially multiple) chronic conditions. The development of the architecture model opens

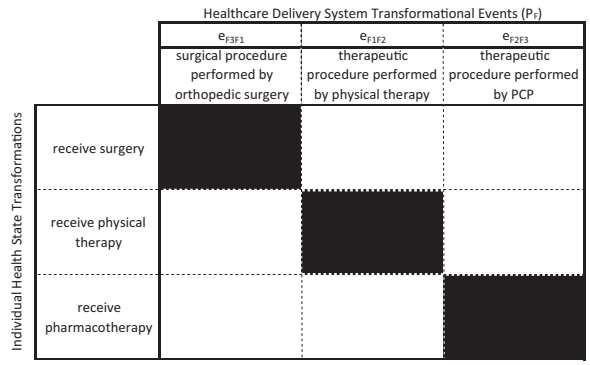

Figure 16: Individual Transformation Feasibility Matrix Λ_F of the chronic diabetes example.

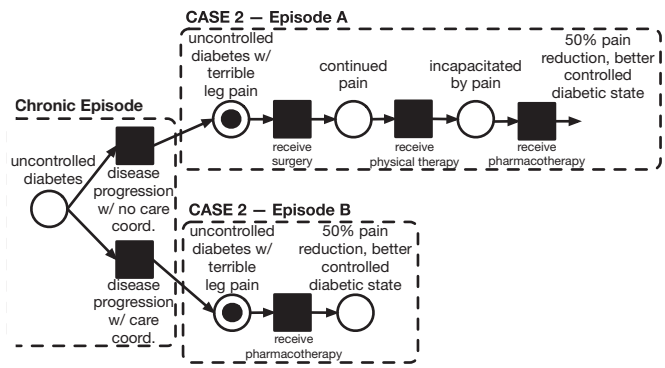

Figure 17: Chronic diabetes Individual Health Net illustrating two possible episodes (A or B) based on level of care coordination when disease progresses for an individual in a chronic episode. Health Net visualizes health states at the *places* (circles) and health state transformations, causing the changes in health state, at the *transitions* (rectangles).

several avenues for future work including discrete-event simulation, resilience analysis, and optimization methods.

The developed model directly addresses the complexity arising from treating chronic diseases and in doing so incorporates the stochastic evolution of an individual's health in relation to the healthcare delivery system. This is in contrast to classic healthcare simulation of an individual as a stateless passive entity. From a healthcare management perspective, such a model architecture captures two key shifts in healthcare: (1) the shift towards allowing human resources to "practice at the top of their license" and (2) taking into consideration patient preferences in the quickly rising literature of shared decision-making.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Research Article

Optimization of Consignment-Store-Based Supply Chain with Black Hole Algorithm

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The globalization of economy and market led to increased networking in the field of manufacturing and services. These manufacturing and service processes including supply chain became more and more complex. The supply chain includes in many cases consignment stores. The design and operation of these complex supply chain processes can be described as NP-hard optimization problems. These problems can be solved using sophisticated models and methods based on metaheuristic algorithms. This research proposes an integrated supply model based on consignment stores. After a careful literature review, this paper introduces a mathematical model to formulate the problem of consignment-store-based supply chain optimization. The integrated model includes facility location and assignment problems to be solved. Next, an enhanced black hole algorithm dealing with multiobjective supply chain model is presented. The sensitivity analysis of the heuristic black hole optimization method is also described to check the efficiency of new operators to increase the convergence of the algorithm. Numerical results with different datasets demonstrate how the proposed model supports the efficiency, flexibility, and reliability of the consignment-store-based supply chain.

1. Introduction

In today's economy, the pressure is on to make the operations of supply chain from purchasing to distribution more and more efficient. The competition is characterized as competition between supply chain networks rather than competition between individual production and service companies. The upstream and downstream linkages of involved organizations like production and service companies, suppliers, 3PL or 4PL providers, wholesalers, and retailers increase the complexity of supply chain networks. This increased complexity led to the implementation of new strategies and tools to make supply chain structure more transparent, while the efficiency and flexibility are increased. One of these tools is the consignment inventory concept. Consignment inventory gives advantages for both the suppliers and the customers since the supplier enjoys the advantages of close connection with customers, decrease of own store capacity, and decreased transportation and packaging costs. Meanwhile, customers using consignment-inventory-based supply can enjoy advantages provided by consignment inventory like low supply risk, decreased supply costs, and transparency of inventories.

The design and operation of consignment inventory or consignment-store-based supply chains include a huge number of problems: facility location, routing, scheduling, budgeting, transportation problem, inventory optimization, assignment, and queuing problems.

The model presented in this work not only combines the facility location of consignment stores and the assignment problems of stores, suppliers, and customers but also takes into account capacity of logistic resources. To the best of our knowledge, the facility location of consignment stores in supply chains and its assignment to customers and suppliers has not been considered in the current literature.

The main contributions of this work include (1) an integrated consignment-store-based supply chain model that combines facility location planning and assignment of involved organizations of the supply chain, (2) a black-hole-optimization-based algorithm, which includes new heuristic operators to increase the convergence, (3) a test of the

Table 1: Selected papers related to topics of the research.

SN	Author(s)	Year	Topics
1	Hallikas and Lintukangas	2016	Risk management in purchasing and supply chain
2	Matopoulos et al.	2016	Modelling in purchasing and supply chain
3	Immonen et al.	2016	Supply chain in B2B services
4	Zhang et al.	2017	In-plant supply chain design, production routing
5	Govindan and Soleimani	2017	Supply chain management in reverse logistics
6	Ma et al.	2016	Integrated supply chain design
7	Diabat and Deskoores	2016	Supply chain optimization with hybrid genetic algorithm
8	Pishvaee and Rabbani	2011	Responsive supply chain network design with heuristics
9	Liu and Chen	2011	Inventory routing in a supply chain with heuristics
10	Chávez et al.	2017	Simulation-based supply chain modelling
11	Dorigatti et al.	2016	Agent-based simulation of collaborative supply chain
12	Ge et al.	2016	Simulation and hybrid optimization of supply chain
13	Ben Othman et al.	2017	Resource scheduling with decision support system
14	Dey et al.	2017	Facility location in supply chain
15	Zahran et al.	2016	Consignment stock modelling with delay-in-payment
16	Hackett	1993	Consignment contracting
17	Bylka	2013	Noncooperative consignment stock strategies
18	Bazan et al.	2014	Consignment stock agreements for two-level supply chain
19	Li et al.	2014	Supply diversification
20	Batarfi et al.	2016	Strategy for dual-channel supply chain
21	Ru and Wang	2010	Consignment contracting
22	Fraser	2016	Schwarzschild radius
23	Piotrowski et al.	2014	Black hole optimization versus other heuristics
24	Dorigo and Gambardella	1997	Ant colony optimization
25	Yang	2014	Nature-inspired optimization algorithms
26	Bhargava et al.	2013	Cuckoo search optimization
27	Lozano et al.	2017	Artificial bee colony algorithm
28	Niknam et al.	2013	Bat-inspired heuristics
29	McKendall Jr. et al.	2006	Simulated annealing heuristics
30	Saha et al.	2014	Gravitation search algorithm
31	Srivastava	2015	Intelligent water drop optimization
32	Bányai et al.	2015	Harmony search optimization
33	Zhang et al.	2008	Random black hole particle swarm optimization
34	Yaghoobi and Mojallali	2016	Black hole algorithm with genetic operators
35	Wang et al.	2016	Black hole base optimization
36	Bouchekara	2013	Black-hole-based optimization technique
37	Wang et al.	2014	Parameter optimization based on black hole algorithm
38	Hatamlou	2013	Data clustering with black hole algorithm
39	Azizipanah-Abarghooee et al.	2014	Power system scheduling with black hole optimization
40	Bouchekara	2014	Power flow optimization with black hole algorithm

modified black hole algorithm with different datasets and test functions based on CEC 2005, and (4) computational results of consignment-store-based supply chain problems with different datasets.

This paper is organized as follows. Section 2 presents a literature review, which systematically summarizes the research background of supply chain, consignment stores, and black hole optimization. Section 3 describes the model framework of the consignment-store-based supply chains. Section 4 presents the black hole optimization and supposes some modification to improve its convergence and enhance its efficiency. Section 5 demonstrates the sensitivity analysis of the algorithm based on CEC 2005 functions. For our study, in Section 6, we focus on the optimization results with numerical analysis. Conclusions and future research directions are discussed in Section 7.

2. Literature Review

Since our study embraces several related research streams, namely, supply chain management, consignment stores, and black hole optimization, we provide a brief review on each stream before to elaborate the model, algorithm, and solution. Table 1 enlists the papers published in these areas related to our research.

Only limited attention has been paid to consignment-store-based supply chain optimization with metaheuristic methods in the literature. There has recently been an

increased interest in performance analysis of supply chains [1–14] and some recent analysis has been targeted specifically towards the integration of consignment stores into supply chain processes [15–21] and optimization with metaheuristics based on swarming algorithms [22–40], especially black hole optimization. Firstly, the relevant terms were defined and the initial searches were conducted. The keywords used in the search were supply chain management, supply, logistics, consignment store, consignment contract, heuristics, metaheuristics, and black hole optimization. The literature sources were found through scientific databases (ScienceDirect, Scopus, and Web of Science) and regular search engines (Google) and included journal articles. Initially, more than 300 articles were identified. This list was narrowed down to 42 titles by selecting journal articles focusing on our research field. It is worth mentioning that the search was conducted in February 2017; therefore new articles may have been published since then.

2.1. Research on Supply Chain Management. Successful production and service processes provide a significant competitive advantage over other participants of the market. Logistics-related operations such as warehousing, transportation, loading unit building, packaging, customer service, and inventory carrying account for up to 95% of the total cost, depending on the corporate sector. For that reason, it is important to take the logistics-related operations and processes under control and turn logistics into the source of competitive edges. Logistics can be divided into four important parts: purchasing, production or service, distribution, and recycling.

Supply chain management influences the efficiency of purchasing, because supplier orientation, supplier dependency, customer orientation, and purchasing strategy have an effect on performance [1]. Supply chain modelling in purchasing is important not just at the functional and operational level but also at the organizational and strategic level [2]. The relationship between purchasing strategies and e-business solutions like business-to-business services has been studied in the literature and provides new knowledge on complex purchasing systems [3]. In-plant supply chain design includes the problems of facility location, production routing, and scheduling [4].

There is a growing interest in the field of closed-loop supply chain design and green supply chain. Reverse logistic systems play an important role in the end-of-life product recycling and influence consumer's return practices for collecting used products, like wastes of electric and electronic equipment [5]. The functional sequences of supply chain can be taken into consideration as an integrated process; the integration of production and distribution planning is a good solution to avoid conflict in sales [6].

The increased complexity of industrial and service processes led to the application of complex solution methods and procedures to optimize the parameters of related supply chains. Heuristic optimization methods are used to solve NP-hard problems of supply chains: integrated supply chain problems can be solved by hybrid algorithms [7], graph-theory-based heuristic supports responsive supply chain design [8], and inventory routing and pricing problems in a supply chain can be solved with Tabu Search [9]. Simulation-based methods support the optimization of deterministic and stochastic models in the field of transportation [10], collaboration analysis for jointly working members [11], and supply chain optimization to find the core parameters of supply chain strategies to ensure cost efficiency [12]. Decision support systems make it possible to find optimal solutions for resource scheduling in supply chain [13] and group decision-making is an effective tool for facility location problems of supply chains [14].

2.2. Research on Consignment Stores. There is a great body of research dealing with consignment policies, consignment stores, and consignment strategies. Within the frame of this chapter, we give a short overview on this reach literature source related to our research. Production companies usually have four types of inventories: raw materials, work-in-process, finished products, and manufacturing supplies. Holding inventories makes it possible to avoid losses of sales, gain quantity discounts, reduce order costs, and achieve efficient production run. Consignment is a special coordination mechanism of inventories, because the owner keeps ownership of his goods and products until they are sold. Consignment stock can improve the supply chain improvement, because vendor uses its buyer's warehouse capacity to store goods. The operation of consignment-store-based supply chain depends on the policy of its operation; therefore, it is important to optimize its operation strategy [15]. The first work investigated the consignment contracting and defined the role of consignment; it can limit the middleman's commitment and increase the profitability of sales [16]. In a later study, the importance of noncooperative stock strategies was underlined and generalized consignment policies were considered to minimize the average total costs by individual decisions [17]. Another topic that has received significant attention in the literature is the analysis of multilevel supply chain with consignment stores. The result of these researches showed the following advantages of using consignment stores in supply chain [18]: improved customer service through decreased reaction time on customer's demands, levelling of customer's demands, decreased inventory on the side of the supplier [19], and cost reduction [20]. The possible control processes of consignment inventory have been studied in the literature; consignment arrangements from the point of view of suppliers and retailers are discussed [21].

2.3. Research on Black Hole Optimization. Black holes were predicted by Einstein's theory of general relativity. If the mass of a dead star's core is more than three times the solar mass, the force of gravity overwhelms all other forces. In a black hole, gravity pulls so much that nothing, not even particles, light, or radiation, can escape from it. The boundary of a black hole is called event horizon, beyond which events cannot be observed and particles cannot move in any direction but only closer to the core of the black hole. This boundary is called Schwarzschild radius, which is given as

$$r_s = \frac{2gM}{c^2}, \tag{1}$$

where g is the gravitational constant, M is the object mass, and c is the speed of light [22].

The black hole optimization (BHO) is based on this phenomenon. Black hole optimization can be described as simplification of the well-known particle swarm optimization using inertia weight. The black hole optimization belongs to heuristics inspired by the laws of nature, like Newton's three laws of motion, his law of gravitation, the ideal gas law, and so on. The laws of nature may be as relevant sources of inspiration for heuristics as living bodies or human-depending phenomenon [23]. Living-bodies-inspired heuristics are, for example, ant colony optimization [24], firefly optimization [25], cuckoo search [26], artificial bee colony optimization [27], bat algorithm [28], or bacterial algorithm. Simulated annealing [29], gravitation search [30], intelligent water drops [31], and black hole algorithms [23] are inspired by physical laws, while harmony search [32] is based on a strongly human-depending attitude.

BHO can be used for hybrid metaheuristics [33]. Genetic operators can increase the convergence and the optimization results of BHO performing a more diverse search in the search space [34] and the convergence of the basic algorithm can be increased by improved measurement of distances in the search space [35]. The technique can be proposed for the optimization of different scientific problems, like pole face optimization of a magnetizer [36], optimization of parameters of least squares support vector machine [37], data clustering [38], scheduling of thermal power systems [39], or power flow optimization [40].

2.4. Analysis of Recent Papers. More than 80% of the articles were published in the last 4 years. This result indicates the scientific potential of this research field including the problems of supply chain, consignment stores, and heuristic optimization. The articles that addressed the optimization of supply chain processes are focusing on conventional manufacturing and service processes and only a few of them aimed to identify the optimization aspects of consignment-store-based supply chain. Therefore, the heuristic optimization of consignment-store-based supply still needs more attention and research, especially in the case of robust, networking cases. It was found that heuristic algorithms are important support tools for design, since a wide range of models determines an NP-hard optimization problem. According to that, the main focus of this research is on the modelling and optimization of consignment-store-based supply chain.

The aim of this paper is to investigate the effect of location of consignment stores on the performance of the whole supply chain. The contribution of this paper to the literature is twofold: description of a consignment-store-based supply chain model including the optimization problem of facility location and assignment and development of a black-hole-based algorithm to solve an integrated optimization problem.

3. Model Framework

The model framework of the consignment-store-based supply chain is a two-level supply chain including suppliers, consignment stores, and customers (Figure 1). The supply chain has m suppliers that produce the needs of p different customers. The supplier and the customer want to set up a consignment store network to support the just-in-time and just-in-sequence supply. Depending on the number and location of consignment stores and consignment agreements, the suppliers are able to ship their products to different consignment stores and the customers are able to buy their needs from different consignment stores. The decision variables of this model are the following: optimal location of the consignment stores, types of consignment agreements between suppliers and customers, and assignment of objects of supply chain and order quantities. These decision variables include an integrated optimization problem: facility location problem and assignment problem.

The decision variables describe the decisions to be made. In this model, the following must be decided: (a) how many products from suppliers through consignment stores to customers should be transported; (b) location of each consignment store. These two decisions represent the above-mentioned assignment and facility location problem. With this in mind, we define q_{ijk} as amount of products transported from ith suppliers through jth consignment store to the kth customer and (x_i^W, y_i^W) as coordinates of the jth consignment store.

The objective function of the problem describes the minimization of the costs of both the suppliers and the customers.

$$\min \quad C = \sum_{i=1}^{m} C_i^S + \sum_{k=1}^{p} C_k^C, \tag{2}$$

where C_i^S represents the costs of suppliers and C_k^C represents the costs of customers.

The first part of the cost function (2) includes the sum of transportation costs among suppliers and consignment stores, the warehousing costs, and the manufacturing costs of all suppliers.

$$C_i^S = \sum_{j=1}^{n} \sum_{k=1}^{p} q_{ijk} \left[c_j^W + c_{ij}^T l_{ij} \left(x_j^W, y_j^W \right) \right] + C_i^M, \tag{3}$$

where c_j^W is the specific warehousing cost in the jth consignment store, c_{ij}^T is the specific transportation cost between the ith supplier and jth consignment store, l_{ij} is the length of the transportation route between the ith supplier and jth consignment store, x_j^W and y_j^W are the coordinates of the jth consignment store, and C_i^M is the manufacturing cost of the ith supplier.

The second part of the cost function (2) includes the transportation costs from consignment stores to customers and the purchasing costs of products.

$$C_k^C = \sum_{i=1}^{m} \sum_{j=1}^{n} q_{ijk} \left(c_i^P + c_{jk}^T l_{jk} \left(x_j^W, y_j^W \right) \right), \tag{4}$$

where c_i^P is the specific purchasing cost from the ith supplier, c_{jk}^T is the specific transportation cost between the jth consignment store and kth customer, and l_{jk} is the length of the

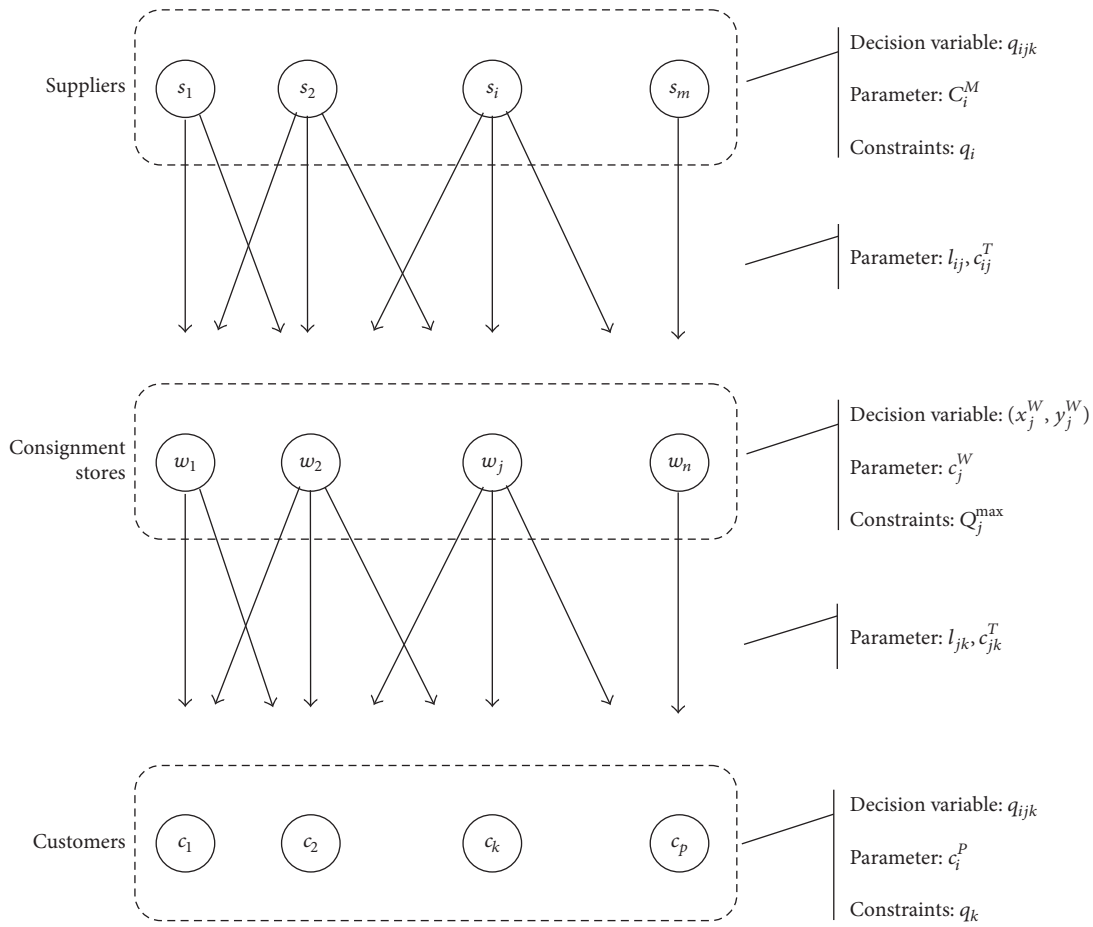

Figure 1: Model framework of consignment-store-based supply chain.

transportation route between the jth consignment store and kth customer.

The values of the products supplied, stored, and bought are limited by the following three constraints.

Constraint 1. Each time interval, no more than the capacity of the consignment store may be transported from suppliers to consignment stores (see the following equation):

$$\sum_{i=1}^{m}\sum_{k=1}^{p} q_{ijk} \le Q_j^{\max} \quad j \in (1, 2, \ldots, n), \tag{5}$$

where $Q_j^{\max}$ is the capacity of the jth consignment store.

Constraint 2. All products manufactured by suppliers should be transported to consignment stores each time interval (see the following equation):

$$\sum_{j=1}^{n}\sum_{k=1}^{p} q_{ijk} = q_i \quad i \in (1, 2, \ldots, m), \tag{6}$$

where q_i is the total amount of product produced by the ith supplier.

Constraint 3. Each time interval, amount of purchased products must reach the total demand of customers (see the following equation):

$$\sum_{i=1}^{m}\sum_{j=1}^{n} q_{ijk} = q_k \quad k \in (1, 2, \ldots, p), \tag{7}$$

where q_k is the total required amount of the kth customer in the planning period.

The decision variables can only assume nonnegative values, so we associate sign restrictions with the above-mentioned decision variables (see the following equation):

$$q_{ijk}, x_j^W, y_j^W \ge 0. \tag{8}$$

4. Black Hole Algorithm

Black holes are places in the outer space where the gravitation force is so high that no particles even light can get out. Black holes are born when stars die. The environment of black holes can be analyzed, but the black holes are invisible. The Schwarzschild radius is the radius of the event horizon. If the distance between a particle (star, proton, electron, photon, etc.) is much higher than the Schwarzschild radius, then the particle can move in any direction. If this distance is larger

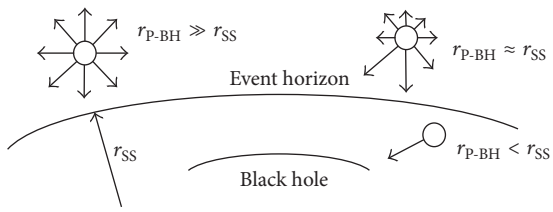

Figure 2: Behavior of particles depending on the distance from black hole.

than the Schwarzschild radius but this difference is not too much, the space-time is deformed, and more particles are moving towards the center of the black hole than in other directions. If a particle reaches the Schwarzschild radius, then it can move only towards the center of the black hole (Figure 2). The black hole optimization is based on this phenomenon of black holes.

The first phase of the black hole optimization is the so called big-bang, when a new generation of stars is generated in the search space. Each star represents one solution of the optimization problem. The coordinates of the ith star in the n-dimensional search space represent the decision variables of the n-dimensional optimization problem.

$$\vec{x}_j^{s_i} = \left(x_1^{S_i}, x_2^{S_i}, \ldots, x_n^{S_i}\right). \tag{9}$$

The second phase of the algorithm is the evaluation of the stars. The stars are evaluated with the value of the objective functions (gravity force represented by the star).

$$f^{S_i} = f^{S_i}\left(x_1^{S_i}, x_2^{S_i}, \ldots, x_n^{S_i}\right). \tag{10}$$

The third phase is to choose one or more black holes. Black holes are the stars with the highest gravity force.

$$f^{\text{BH}} = \max_i \left(f^{S_i}\right). \tag{11}$$

The fourth phase of the algorithm is to move the stars towards the black holes in the search space. There are different operators to calculate the new locations of the stars. The basic operator uses only the gravity force of the black holes and the gravity force of stars is not taken into consideration. There are two main types of operators: the first type calculates the new position of the stars depending on the gravity forces among stars and black holes, and the second type does not take into account the gravity forces (see the following equation):

$$x_j^{S_i}(t + \Delta t) = x_j^{S_i}(t) + \text{Rnd} \cdot \left(x_j^{\text{BH}}(t) - x_j^{S_i}(t)\right). \tag{12}$$

The movement of stars towards the black hole changes the decision variables of the solution represented by the moving star so that the decision variables will move to the decision variables of the best solution represented by the black hole (Figure 3).

Stars reaching the event horizon will be absorbed and a new star is generated in the search space. The radius of the event horizon (the Schwarzschild radius) is calculated as follows:

$$R^{\text{EH}} = \frac{f^{\text{BH}}}{\sum_{i=1}^{n} f^{S_i}}, \tag{13}$$

where R^{EH} is the radius of event horizon, f^{BH} is the gravity force of the black hole, and f^{S_i} is the gravity force of the ith star.

The fifth phase is the evaluation of stars. Stars with the best gravity force will be the new black holes, and the old black holes become stars. This role of this fifth phase is the same as the role of the mutation operator of genetic algorithms: to avoid the local optimum. Termination criteria of the algorithm can be the number of iteration steps, computational time, or the measure of convergence.

If the location of the optimum is inside the event horizon, it is impossible to find it, because all stars inside the event horizon are absorbed. Stephen Hawking published a theoretical argument for the existence of blackbody radiation [41]. Virtual particle-antiparticle pairs, like photons or neutrinos, are being created near the event horizon of the black hole. These particle-antiparticle pairs annihilate each other or one of them falls into the black hole and the other one escapes as Hawking radiation due to quantum effects. The black hole loses a part of its energy and its mass. This Hawking radiation is called black hole evaporation. It is possible to apply this black hole evaporation to search inside the event horizon [42]. This application means that a little change occurs in the location of the black hole, so that a small part of the old event horizon is available for the stars.

$$x_j^{\text{BH}}(t + \Delta t) = x_j^{\text{BH}}(t) + \varepsilon, \quad |\varepsilon| \ll R^{\text{EH}}. \tag{14}$$

Another way to open the event horizon of black holes for the stars to search for the best solution is to decrease the measure of event horizon. The pseudocode shows the developed approach (Pseudocode 1).

The described Pseudocode 1 makes it possible to replicate the implementation. It is also possible to have more than one black hole; in this case, the movement of stars towards the black holes is similar to the gravity search algorithm [43].

5. Sensitivity Analysis

Within the frame of this chapter, the sensitivity analysis of the black hole algorithm is described. The following 10 different benchmark functions were used to evaluate the above-described black hole algorithm:

(i) The n-dimensional nonconvex Shifted sphere function is evaluated on the hypercube $x_i \in [-100, 100]$. It has a global minimum at $f_1(s_1, \ldots, s_i, \ldots, s_d) = b$.

(ii) The n-dimensional shifted Schwefel function is evaluated on the hypercube $x_i \in [-100, 100]$. It has a global minimum at $f_2(s_1, \ldots, s_i, \ldots, s_d) = b$.

(iii) The n-dimensional shifted elliptic function is evaluated on the hypercube $x_i \in [-100, 100]$. It has a global minimum at $f_3(s_1, \ldots, s_i, \ldots, s_d) = b$.

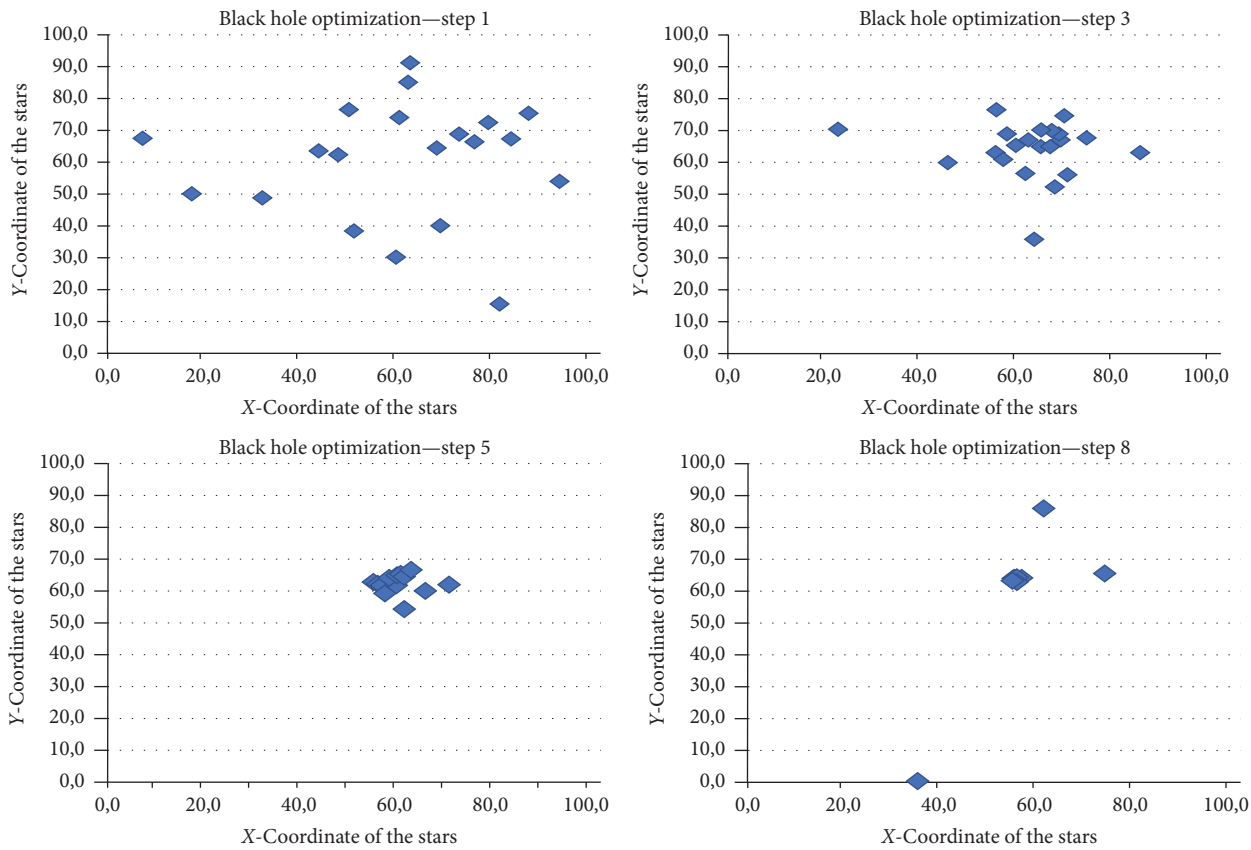

FIGURE 3: *Moving of stars in BHO* (step 1: initialization of stars; step 3: stars are moving in the direction of the black hole; step 5: stars are nearing the event horizon; step 8: some stars cross the Schwarzschild radius, they are absorbed, and new stars are generated in the search space).

```
Input: number of stars, objective function, constraints, sign restrictions, termination criteria
Output: optimal solution
// Initialization
        (1) generate feasible solutions randomly in the n-dimensional search space (9)
// Pre-evaluation
        (2)  for each stars, evaluate the objective function ((2)–(4), (10))
// Loop until the termination criteria satisfy
While (termination criteria satisfy) do
    // Selection of the black hole
        (3)  select the best star that has the best value to become a black hole (11)
    // Hawking radiation
        (4)  change the position of the black hole (14)
    // movement of stars towards the black hole
        (5)  move the stars towards the black holes (12) while constraints ((5)–(8)) are taken into consideration
    // Check the position of stars
        (6)  if star is inside the Schwarzschild radius
                absorb the star and generate a new one in the search space (13)
             end if
    // Evaluation
        (7)  for each stars, evaluate the objective function ((2)–(4), (10))
End of while
```

PSEUDOCODE 1: Pseudocode of BHA.

Table 2: Error values of BHO in the case of 10 benchmark functions after 100 iteration steps.

Evaluation function	Standard BHO	BHO with moving black hole location	BHO with decreasing event horizon	Complex BHO
Shifted sphere function $f_1(x_1,\ldots,x_i,\ldots,x_d)=\sum_{i=1}^{d}(x_i-s_i)^2+b$	1,02$E-7$	1,02$E-7$	9,55$E-8$	4,76$E-8$
Shifted Schwefel function $f_2(x_1,\ldots,x_i,\ldots,x_d)=\sum_{i=1}^{d}\left(\sum_{j=1}^{i}(x_i-s_i)\right)^2+b$	8,82$E-6$	8,34$E-6$	7,21$E-6$	4,02$E-6$
Shifted elliptic function $f_3(x_1,\ldots,x_i,\ldots,x_d)=\sum_{i=1}^{d}\left(10^6\right)^{(i-1)/(d-1)}(x_i-s_i)^2+b$	4,32$E-7$	4,12$E-7$	2,62$E-7$	1,92$E-7$
Styblinski-Tang function $f_4(x_1,\ldots,x_i,\ldots,x_d)=\frac{1}{2}\sum_{i=1}^{d}\left(x_i^4-16x_i^2+5x_i\right)$	2,91$E-6$	1,38$E-6$	6,12$E-7$	4,65$E-7$
Rosenbrock function $f_5(x_1,\ldots,x_i,\ldots,x_d)=\sum_{i=1}^{d-1}\left[100\left(x_{i+1}-x_i^2\right)^2+(x_i-1)^2\right]$	5,50$E-6$	5,44$E-6$	2,92$E-6$	2,02$E-6$
Rastrigin function $f_6(x_1,\ldots,x_i,\ldots,x_d)=10+\sum_{i=1}^{d}\left[x_i^2-10\cos 2\pi x_i\right]$	2,45$E-6$	2,32$E-6$	1,07$E-6$	9,69$E-7$
Ackley function $f_7(x,y)=-20\cdot e^{-0.2\sqrt{0.5(x^2+y^2)}}-e^{(0.5(\cos 2\pi x)+\cos 2\pi y)}+e+20$	1,14$E-7$	1,01$E-7$	9,63$E-8$	7,17$E-8$
Beale function $f_8(x,y)=(1.5-x+xy)^2+(2.25-x+xy^2)^2+(2.625-x+xy^3)^2$	2,29$E-7$	2,26$E-7$	1,91$E-7$	1,21$E-7$
Booth function $f_9(x,y)=(x+2y-7)^2+(2x+y-5)^2$	5,31$E-6$	2,11$E-6$	5,51$E-7$	3,26$E-7$
Goldstein-Price function $f_{10}(x,y)=[1+(x+y+1)^2(19-14x+3x^2-14y+6xy+3y^2)][30+(2x-3y)^2(18-32x+12x^2+48y-36xy+27y^2)]$	3,34$E-8$	8,22$E-9$	6,31$E-9$	3,12$E-9$

(iv) The n-dimensional Styblinski-Tang function is evaluated on the hypercube $x_i \in [-5,5]$. It has a global minimum at $f_4(-2.903534,\ldots,-2.903534) = -39.16599$.

(v) The n-dimensional Rosenbrock function is evaluated on the hypercube $x_i \in [-\infty,\infty]$. It has a global minimum at $f_5(1,\ldots,1,\ldots,1)=0$.

(vi) The n-dimensional Rastrigin function is evaluated on the hypercube $x_i \in [-5.12,5.12]$. It has a global minimum at $f_6(0,\ldots,0,\ldots,0)=0$.

(vii) The 2-dimensional Ackley function is evaluated on the $x_i \in [-5,5]$ square. It has a global minimum at $f_7(0,0)=0$.

(viii) The 2-dimensional Beale function is evaluated on the $x_i \in [-4.5,4.5]$ square. It has a global minimum at $f_8(3,0.5)=0$.

(ix) The 2-dimensional Booth function is evaluated on the $x_i \in [-10,10]$ square. It has a global minimum at $f_9(1,3)=0$.

(x) The 2-dimensional Goldstein-Price function is evaluated on the $x_i \in [-2,2]$ square. It has a global minimum at $f_{10}(0,-1)=3$.

The aim of this evaluation is to analyze the effect of permanently decreased Schwarzschild radius and the changes occurred in the location of black holes.

As Table 2 demonstrates, the permanently decreased Schwarzschild radius and the moving black hole effect decreased the error value after 100 iteration steps. The problem size was fixed as a 10-dimensional problem in the case of f_1 to f_6. The efficiency of the algorithm in the case of different problem sizes (different search space dimensions) is demonstrated in the next chapter. The average error value was reduced by 36% with a deviation of 19% (Table 3).

In order to demonstrate how black hole implementation performs when problem size increases, we tested the algorithm both with test functions and with the consignment-store-based supply chain problem.

As Table 4 shows, the increased size of the problem led to the increase of the required iteration steps to reach the predefined error value that is based on the performance of

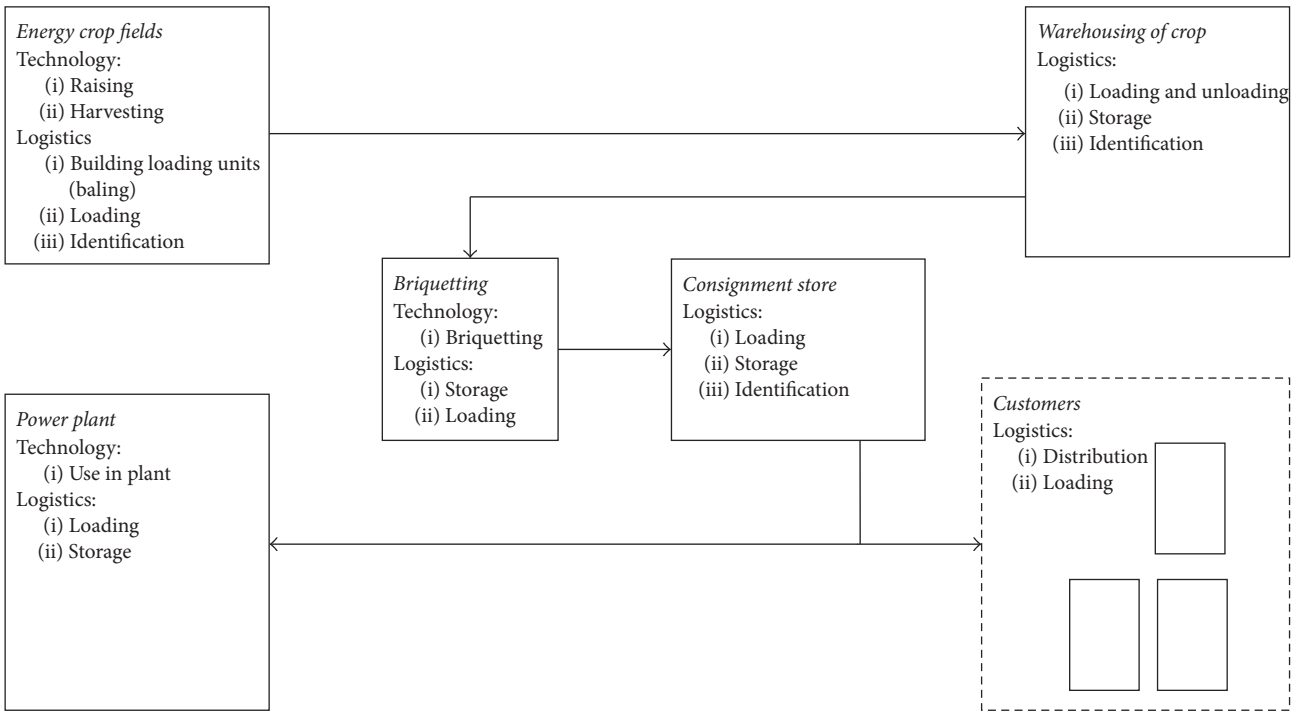

Figure 4: Complex power plant supply chain.

Table 3: Error value decline using moving black holes and permanently decreasing Schwarzschild radius.

Evaluation function	Error value decline
Shifted sphere function	47%
Shifted Schwefel function	46%
Shifted elliptic function	44%
Styblinski-Tang function	16%
Rosenbrock function	37%
Rastrigin function	40%
Ackley function	63%
Beale function	53%
Booth function	6%
Goldstein-Price function	9%

the algorithm in the case of a 10-dimensional search space (Table 2). The highest iteration step was required in the case of the consignment-store-based supply chain problem because of the specific constraints and sign restrictions.

6. Numerical Analysis of Consignment-Store-Based Supply Chain

Within the frame of this chapter, a case study will be analyzed. The aim of this chapter is to analyze the design of an energy crop supply chain of biomass-fired power plants, especially from the point of view of integrated facility location. The model shown in Figure 4 includes the whole energy crop supply chain from harvesting crop in the crop fields through briquetting plants to the distribution for power plants and customers through consignment stores. The following parameters are taken into consideration: the total amount of harvested energy crop, required briquette amount by each power plant and the cluster of customers, transportation distances, specific transportation costs, type of transportation devices (average truck capacity), and location of crop fields, power plants, and customers. The objective function is a cost function based on the transportation processes from energy crop fields to briquetting plants and from briquetting plants to power plants and communal customers. The decision variables are the following: (a) how many energy crops from crop field through consignment stores to customers and power plants should be transported; (b) location of each consignment store.

Figure 5 demonstrates the results of the optimization of the above-mentioned complex supply chain. The datasets represent simple specification of the system so that the results of the optimization can be checked.

In the first case, customers and power plants need the same quantity of briquette; therefore the central position of both consignment stores is correct. The first power plant and the first and second customer's clusters are assigned to the first consignment store, while the second power plant and the third and the fourth customer's clusters are assigned to the second consignment store.

In the second case, the third customer's cluster has a greater demand of briquette. In this case, the second consignment store's location is not centralized in order to minimize the materials handling costs of the whole supply chain.

Table 4: Number of required iteration steps to reach the predefined error value (PEV).

Evaluation function	Problem size $d = 5$	$d = 10$	$d = 25$
Shifted sphere function $f_1(x_1, \ldots, x_i, \ldots, x_d) = \sum_{i=1}^{d}(x_i - s_i)^2 + b$ PEV = 4,76E − 8	94	100	122
Shifted Schwefel function $f_2(x_1, \ldots, x_i, \ldots, x_d) = \sum_{i=1}^{d}(\sum_{j=1}^{i}(x_i - s_i))^2 + b$ PEV = 4,02E − 6	92	100	134
Shifted elliptic function $f_3(x_1, \ldots, x_i, \ldots, x_d) = \sum_{i=1}^{d}(10^6)^{(i-1)/(d-1)}(x_i - s_i)^2 + b$ PEV = 1,92E − 7	81	100	156
Styblinski-Tang function $f_4(x_1, \ldots, x_i, \ldots, x_d) = \frac{1}{2}\sum_{i=1}^{d}(x_i^4 - 16x_i^2 + 5x_i)$ PEV = 4,65E − 7	85	100	162
Rosenbrock function $f_5(x_1, \ldots, x_i, \ldots, x_d) = \sum_{i=1}^{d-1}[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$ PEV = 2,02E − 6	93	100	133
Rastrigin function $f_6(x_1, \ldots, x_i, \ldots, x_d) = 10 + \sum_{i=1}^{d}[x_i^2 - 10\cos 2\pi x_i]$ PEV = 9,69E − 7	84	100	144
Consignment-store-based supply chain problem PEV = 5,63E − 7	89	100	168

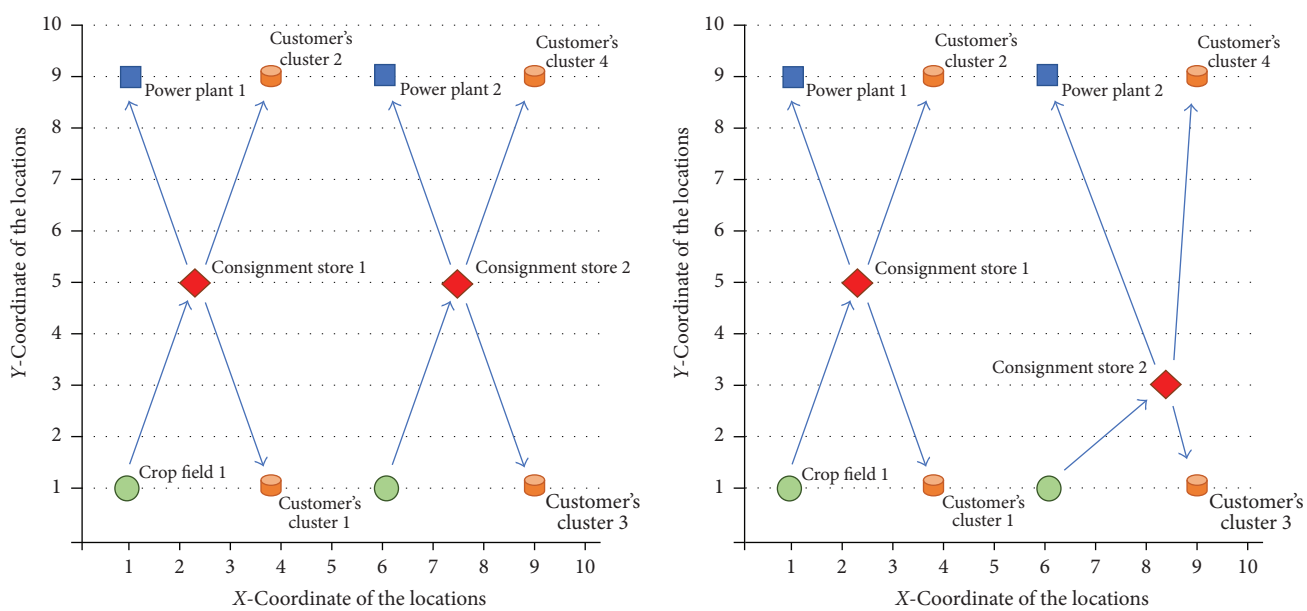

Figure 5: Results of the facility location and assignment problem of energy crop supply chain.

The optimization of this complex supply chain problem can lead to the decrease of different costs, like transportation costs, warehousing costs, and materials handling costs (packaging, loading, unloading, and building of loading units).

7. Conclusions and Further Research Directions

This study developed a methodological approach for design of consignment-store-based supply chains. In this paper, firstly, we reviewed and systematically categorized the recent works presented for consignment-store-based supply chain optimization. Then, motivated by the gaps in the literature, a model for companies performing their purchasing through consignment stores is developed. Two models were proposed: the framework model shows the levels of supply chain, while the second model as a case study focuses on power plant supply. The integrated model included facility location and assignment problems, which were solved with black hole optimization algorithm. The sensitivity analysis showed the efficiency of two advanced BHO operators and a numerical example shows the efficiency of the algorithm.

The scientific contributions of this paper are the following: integrated model for consignment-store-based supply chain, black-hole-optimization-based heuristic algorithm with enhanced convergence through integration of phenomena of real black holes, like dynamic black hole location, and decreased event horizon. The results can be generalized, because the model can be applied for in-plant supply, especially in the case of milk-run-based just-in-sequence supply. The described methods make it possible to support managerial decisions; the operation strategy of the supply chain and the consignment contract can be influenced by the results of the above-described contribution.

However, there are also directions for further research. First, although the transportation routes as distances among the locations are considered in this paper, the capacities of vehicles are not taken into consideration. In further studies, the model can be extended to a more complex model including capacities of vehicles and store capacities of locations. Second, this study only considered the black hole optimization method as possible solution algorithm for the described NP-hard problem. In reality, other heuristic methods can be also suitable for the solution of the problem.

Third, the convergence of the described algorithm can be improved using other operators and the behavior of BHO to other optimization approaches can be tested. However, there is a great body of research dealing with testing of performance of different metaheuristic optimization methods, especially from the point of view of "novel" algorithm, but these tests are sometimes inconsistent. This inconsistency can be caused by the optimization behavior. For example, the comparison of black hole algorithm and particle swarm optimization showed that the performance of BHO is poorer than the performance of PSO [44], while the test in another source showed that the performance of BHO is better than the performance of genetic algorithm or PSO [45]. This should be also considered in the future research.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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Research Article

Stochastic Reliability Measurement and Design Optimization of an Inventory Management System

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Inventory management systems and dynamic reliability measures and controls remain challenging at every stage, especially when time variances and operating conditions are considered. An inventory management system must maintain its adeptness over time while coping with the uncertainty of inventory flow. Unexpected delays during inventory movement can harm the reliability and robustness of the entire system. This paper introduces a method of quantifying the reliability of an inventory management system. Also, a novel, reliability-based robust design optimization model has been developed to optimally allocate and schedule time while considering uncertainty associated with inventory movement. The processes involved include purchasing, shipping, receiving, tracking, warehousing, storage, and turnover. A case study of a furniture company in Saudi Arabia is presented to demonstrate the efficacy of the model.

1. Introduction

The globalization of today's marketplace has brought new opportunities and challenges to industry. This new frontier allows businesses to reach more consumers than ever. However, it also increases the need for reliable, robust global inventory management systems. Technology has become an effective management recordkeeping tool. Many variables associated with managing stock can ultimately determine the success or failure of a business. Thus, an effective inventory management system is essential. Sani [1] revealed that inventory management is used to specify inventory inflows and outflows. It defines the quantity of stock arriving and being registered into the system. It must be able to recognize valuable information. The challenge is to develop a method that aids in evaluating inventory management while coping with unforeseen conditions. Inventory consists of stock, which may include raw materials, work-in-progress, finished, or value-added products or services [1]. Having a good inventory management system aids in maintaining a good record of stock keeping units (SKUs). Hence, maintaining a precise stock level where the system-recorded stock reflects the actual stock is essential to avoid errors [2].

Technology has advanced rapidly, improving several fields of endeavor such as education and business. During the mid-1990s, various companies started to evaluate inventory management system software. This software could document processes and changes as products were transported in and out of any business. In the early 2000s, inventory management solution technology expanded from the small scale to the large. Before this type of industrial innovation, many businesses had to record the products sold and received each day on paper and assemble the resulting documents into a folder for recordkeeping. Orders were placed using handwritten notes, resulting in inefficient business processes.

Inventory management systems are suitable for all types of businesses that deal with tracking and identification of inventory. The main objective of such a system is to determine and maintain the inventory level by preserving records of the products sold and by keeping sales records by date. Companies use inventory management systems to prevent out-of-stock and overstock issues.

An inventory manager might face two major problems: having an overstock of inventory or having a shortage. Excess inventory automatically becomes overstock. This causes problems for businesses and has many negative

consequences. One important effect is an increase in cost. Storage and security fees increase and money is invested in nonessential goods. There is also risk involved in holding excess inventory. There are two costs associated with holding inventory. Carrying (or holding) cost is the cost of storage, insurance, material handling, and so forth. The second type of cost is ordering cost. This is the cost associated with placing orders and receiving goods such as transportation and shipping and receiving and inspecting materials. These two costs represent the core of inventory problems. An order size increase can lead to an increase in the average number of processed goods in inventory and thus increased carrying costs.

On the other hand, stock shortages can also have dire consequences. Business reputations rely on consumer trust. Providing high-quality, affordable goods in a timely and predictable manner is key to success. As a result, a stock shortage can lead to massive losses. These losses can stem from increased production times and purchasing costs, loss of sales, and ultimately the loss of customer goodwill. Inventory decision-making is very risky, as it may impact the supply chain [3]. An inventory management system is necessary to satisfy customer demand at the right time and in a cost-effective manner [4, 5]. Hence, maintaining too little stock may result in production problems while having too much means investing significant money unnecessarily. Inventory is one of the most expensive assets that many companies invest in. It is a critical investment and therefore the heart of any enterprise.

This research introduces a method that can be used to evaluate the reliability of an inventory system with regard to satisfying consumer demand. The resulting measure can be used in organizing and scheduling via reliability-based robust design optimization. The remainder of this section reviews current literature on inventory management system evaluation and design methods.

This paper develops a novel reliability measure and reliability-based robust design optimization (RBRDO) approach. Reliability focuses on the subset stimulation technique to achieve numerical efficiency [6]. Reliability is described as the tendency towards consistency of performance and responsibility or as the fulfillment of a service provider's promises to customers. Reliability deals with the performance and dependability of the service entity in meeting customer requests. A service is considered to be reliable if it repeatedly shows similar results using comparable measures [7]. Moreover, services have to be provided at the time promised in order to maintain credibility [8]. RBRDO provides both cost-effective manufacturing processes and target confidence [9, 10]. Probabilistic constraints are the key constraints in RBRDO. They create several numerical challenges with regard to numerical efficiency, stability, accuracy, and so forth [11–13]. RBRDO is very important for structural optimization because many of its practical applications involve at least two conflicting objectives, typically including low cost and high reliability [14]. The constraints are influenced by both functional and reliability requirements.

Reliability-based robust design optimization (RBRDO) combines of reliability-based design optimization (RBDO) and robust design optimization (RDO). In general, reliability is defined as the ability to start and continue to operate [15]. Robustness was first introduced by Taguchi in 1987 to help find solutions that are less sensitive to unknown variations. Taguchi's definition of a robust design is "a product whose performance is minimally sensitive to factors causing variability." Byrne and Taguchi [16] illustrated the main objective of RDO, which is to find a design with minimum scattering model variance in order to produce results near the mean values of the design parameters. Roos et al. mentioned that RDO can be treated as statistical variability in parameter design [17]. Different methods [18–21] were developed in order to systematically treat uncertainties in engineering analysis and more recently for RBRDO method development. RBRDO can be used a design tool when the function and performance of a product are relatively insensitive to variation [22].

There is limited research available on method development for inventory management system evaluation and design optimization. In [23], Axsäter showed how approximate evaluation can be applied to inventory system policies. Another research [24] investigated hybrid techniques for evaluating life-cycle inventories. In [25], Resurreccion and Santos developed multiobjective prioritization methodologies for inventory system evaluation. The study helped to determine inventory enhancement priorities with user preference and resource availability as new dimensions. Arıkan et al. [26] investigated the interrelation between transportation uncertainties and inventory system performance. In [27] researchers provided a new method which eliminates the unbalanced benefit distributions caused by vendor managed inventory and offers almost equal benefits to the participating firms. A modified particle swarm optimization to solve integrated location and inventory control problems in a two-echelon supply chain network was introduced in [28]. In [29] researchers proposed a long-term extreme price risk measure method for inventory portfolios. Despite numerous inventory management system evaluation studies, further research on inventory systems associated with design optimization and reliability evaluation has rarely been reported. Therefore, this paper presents a model for inventory management system evaluation. A RBRDO approach is developed to help in satisfying reliability requirements in every stage of the system, while coping with uncertainty and minimizing overall handling and management cost. The remainder of this paper is organized as follows: Section 2 introduces methods of evaluating inventory system reliability and presents the methodology developed to design reliable, robust inventory systems. Section 3 evaluates the model described in this paper by applying it to a case study of a furniture company in Jeddah, Saudi Arabia. Conclusions are presented in Section 4.

2. Materials and Methods

This section explains the research design, which includes the evaluation method, optimization model, and data collection procedures. Three key research questions were addressed to achieve the previously mentioned research objectives:

(1) How can all inventory system processes be evaluated using a single, unique measure?

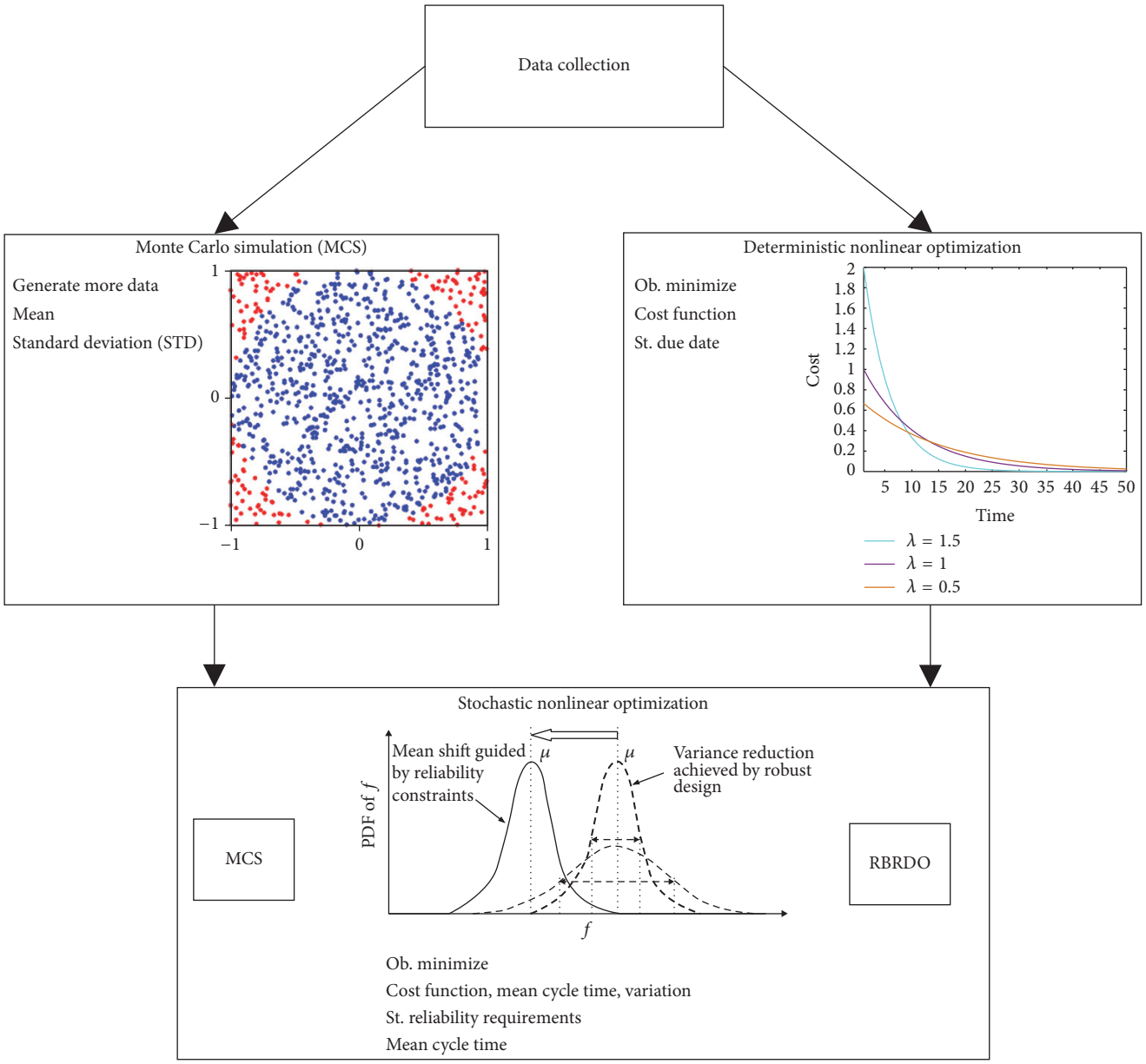

Figure 1: RBRDO procedure applied to an inventory management system.

(2) How to evaluate the reliability of an inventory management system?

(3) How to design a cost-efficient, reliable, robust inventory management system while coping with uncertainty?

Numerous techniques were considered in answering these questions. They include Monte Carlo simulation, linear programming, deterministic nonlinear programing, stochastic nonlinear programing, and RBRDO. Microsoft Excel solver and MATLAB were used for modeling and mathematical simulation. The goal was to investigate the most basic approach that can be used to achieve the objectives of this research. Thus, this investigation started by using basic tools such as linear programing and Excel solver and moved on to advanced technologies such as stochastic nonlinear programing and MATLAB. This helped us to understand a variety of available design and optimization tool capabilities. The methods and tools used to achieve the research objective are shown in Figure 1.

2.1. Inventory Management System Reliability (IMSR) Evaluation Model. Progression and development of an IMSR measure is affected by both internal factors and numerous external factors such as increasing globalization, information availability, global trade, and ecological concerns. The reliability of an inventory system can be defined as its ability to complete all required processes before they are due. Figure 2 shows the IMSR phenomenon.

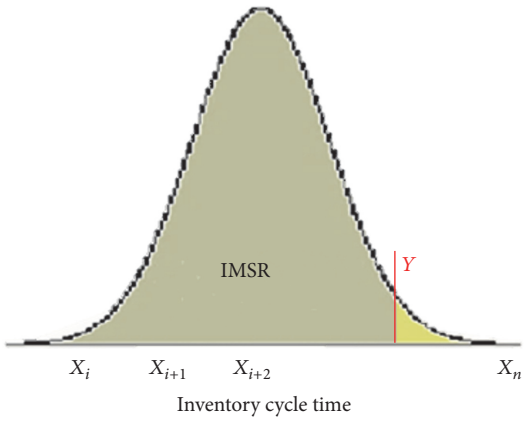

Figure 2: Typical IMSR phenomenon.

The IMSR should be measured and compared using a qualified set of performance measures. In order to control system performance, the process parameters must be kept within a limited range. This makes comparisons of target and actual performances possible. Once comparisons have been made, specific processes can be targeted to improve inventory management performance.

By using an appropriate set of measures, the overall IMSR can be closely monitored for performance. Successive improvements can be applied to each task in the system in order to determine their impacts on the overall IMSR. The IMSRs of suppliers, routes, factories, and the overall inventory management system can be used to achieve customer satisfaction. In general, the IMSR can be defined as the probability of completing all required process at time (X) before the due time (Y) and can be expressed statistically as shown in

$$\text{IMSR} = \Pr\left(X_1 + X_2 + \cdots + X_n \leq Y\right) \tag{1}$$

Mathematically, the IMSRs of suppliers, factory, routes, distribution centers, ports, and the overall inventory management system are represented by the ratio of the total cycle time minus the cumulative delay to the total cycle time. The total cycle time is the cumulative time taken from order placement until receipt and assembly at the customer location. Cumulative delay is the delay due to uncertainty during movement of products through the inventory system. The IMSR can be obtained using

$$\text{IMSR}_{X(i,j)} = \left(Y - \left(f_{X(i,j)} \sum_{j=1}^{J}\sum_{i=1}^{I} \sigma_{X(i,j)} + f_{X(i,j)} \sum_{j=1}^{J}\sum_{i=1}^{I} \tau_{X(i,j)}\right)\right) \times \frac{1}{Y} \tag{2}$$

$$\text{when } Y \leq \left(f_{X(i,j)} \sum_{j=1}^{J}\sum_{i=1}^{I} \sigma_{X(i,j)} + f_{X(i,j)} \sum_{j=1}^{J}\sum_{i=1}^{I} \tau_{X(i,j)}\right),\ Y_{X(i,j)} = 0,$$

where $\text{IMSR}_{X(i,j)}$ represents the reliability performance of node type X (e.g., port, supplier, and route), number i in level j. Y represents the time due and $\sigma_{X(i,j)}$ is the standard deviation (uncertainty) of the distribution function of node type X number i in level j. $\tau_{X(i,j)}$ is the external factor delay affecting the node. When the sum of the delay ($\tau_{X(i,j)}$) and uncertainty ($\sigma_{X(i,j)}$) are larger than the due time (Y), the reliability ($\text{IMSR}_{X(i,j)}$) equals zero.

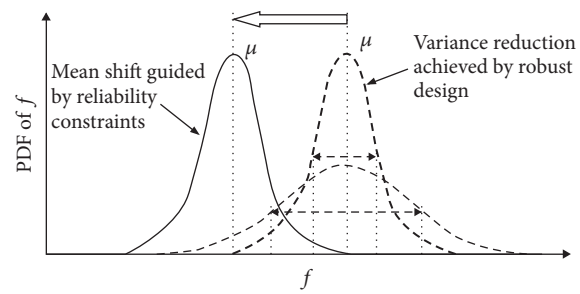

Figure 3: Typical RBRDO phenomenon.

2.2. RBRDO of an Inventory Management System. This subsection presents the RBRDO of an Inventory Management System. In general, this approach characterizes uncertainty variables and failure modes to optimize a design for higher reliability [30]. RBRDO accepts variability and uses the limit state function to separate the stress and strength probability density functions (PDFs) to achieve the desired reliability level. The objective of performing RBRDO on an inventory system is to minimize variation while satisfying inventory system requirements. It also helps to find the best compromise between cost and reliability by taking uncertainties into account. Furthermore, RBRDO is used to shift reliability and reduce variance by designing a multiobjective function that minimizes cost and cost uncertainty factors (variance) with the required reliability target as a constraint, as shown in Figure 3 [31].

The optimal design of an inventory system can be determined using RBRDO, which seeks to find designs that are less sensitive to the uncontrollable variations that are often inherent to the design process. RBRDO outperforms existing deterministic discrete optimization tools when change or uncertainty is involved in optimization problems. Application of RBRDO to the design of an inventory management system has three objectives: minimization of cost, inventory cycle time, and the impact of uncertainty.

RBRDO considers various uncertainties introduced by changes in specifications, transportation delays, raw material availability, manufacturing processes, and operational conditions. Ensuring performance reliability and robustness in terms of time and cost is of vital importance for inventory management systems.

The general formula for the RBRDO of an inventory management system is given by the following:

$$\text{Minimize:}\quad \mathbf{W}_c * f_{X(i,j)} \sum_{j=1}^{J}\sum_{i=1}^{I} C_{X(i,j)} + \mathbf{W}_\mu * f_{X(i,j)} \sum_{j=1}^{J}\sum_{i=1}^{I} \mu_{X(i,j)} + \mathbf{W}_\sigma$$

Table 1: Company design parameters.

Number	Step	Owner	Minimum	Average	Maximum
P_1	Production selection/quote	Sales team	5	7	9
P_2	Order confirmation between customer & company	Sales team	2	4	7
P_3	Placing order for production	Procurement	21	30	45
P_4	Containers ready for shipping	Procurement	7	14	21
P_5	Forwarder collects goods from the port	Procurement	7	10	15
P_6	Goods available in company warehouse	Procurement	1	7	21
P_7	Delivery	Project coordinator	2	4	14
P_8	Installation	Project coordinator	7	9	20
P_9	Inventory + photographs	Project coordinator	2	5	10
	Total		*53*	*90*	*162*

$$
\begin{aligned}
&* \; f_{X(i,j)} \sum_{j=1}^{J} \sum_{i=1}^{I} \sigma_{X(i,j)} \\
\text{subject to:} \quad & \mathrm{IMSR}_{i,j}\left(x_s\right) \geq \mathrm{IMSR}^T, \quad i = 1, 2, \ldots, I \\
& x_s^L \leq x_s \leq x_s^U, \quad j = 1, 2, \ldots, J \\
& x_s \geq 0, \quad s = 1, 2, \ldots, S,
\end{aligned} \tag{3}
$$

where $\mathrm{IMSR}(x_s)$ is the reliability and the reliability target is $\mathrm{IMSR}^T(x_s)$. $C_{X(i,j)}$ is the cost function of node type X, number i, in level j. $\sigma_{X(i,j)}$ represents the standard deviation (delay due to uncertainty) of the inventory cycle time function of node type X, with the indices mentioned earlier. $\mu_{X(i,j)}$ represents the mean of the inventory cycle time function of node type X. $\mathbf{W}$ is the weight attached based on decision maker preference. x_s^L and x_s^U are the lower and upper limits of the design variable, respectively.

2.3. RBRDO of Inventory Management System Summaries. The procedure for RBRDO of an Inventory Management System can be summarized as follows.

Step 1. Collect data.

Step 2. (a) Generate more data points via MCS; (b) generate an initial set of sample points.

Step 3. Perform reliability analysis at the current design $\mathbf{x}$.

Step 4. Execute the developed RBRDO model.

Step 5. (a) Check the convergence criteria. (b) If they converged, the optimum design has been obtained. (c) If they did not converge, repeat Step 2(b) through Step 5.

3. Results and Discussion

3.1. Case Study. The developed method was applied to the inventory management system of a furniture company in Jeddah, Saudi Arabia. This company provides a wide range of furniture from North America, Europe, and the Far East to match client designs within budgets. The inventory management system consists of 9 stages, each aimed at different types of operations. Table 1 shows primary data collected from the company and describes the average, minimum, and maximum number of days that each entity (department) requires for each step in the process.

3.2. Data Collection. This study used primary data collected in coordination with the furniture company to verify and validate the model. Primary data was used via several techniques such as surveys, direct observations, and interviews. This data was collected directly via first-hand experience. Moreover, it included quantitative and qualitative attributes of variables obtained by the sales department.

One of the main obstacles faced during this research was the limited supply of data points. Monte Carlo simulations (MCSs) were used to overcome this problem. The Monte Carlo simulation is a mathematical technique that allows generation of data from limited data resources. The main aim of this technique is to help in decision-making via the range and possible outcomes generated. This method helps to obtain numerical solutions to problems that are too complex to solve analytically. It offers several advantages, including ease of use and the flexibility to use the probabilities generated from the model. In addition, the mathematics required are quite basic. MCSs were used to generate more than 50,000 extra numerical data points, which helped in developing the RBRDO model. The cost function was presented as an exponential distribution. Thus, nonlinear programming was used to solve the model. The exponential distribution $e^{-\lambda}$ is a probability distribution that describes the relationship between cost and time. Figure 4 shows that shipping and processing costs decrease when process durations increase (late due dates).

3.3. Case Study Results. The first step was to model the inventory system using MATLAB R2014a. Then the current performance of the inventory management system was evaluated using (2). The collected data and MCS indicate that the current total cycle time of an order is approximately 107 days, and the IMSR is 88.28%, as shown in Table 2. Thus, if a customer places an order it takes 107 days to complete, and the probability of completing the order on time is 88.28%. Thus,

Table 2: Initial design variables.

P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	Total cycle time (TCT)	Initial reliability
7.1	4.5	33	14	11	11	7.6	13.4	6	107.52	88.2%

Table 3: Optimized points required to reach the delivery target.

P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	Design Max TCT	Design IMSR
5	6.6	21	7.3	7	5.9	8	7	5.8	73.5	95%

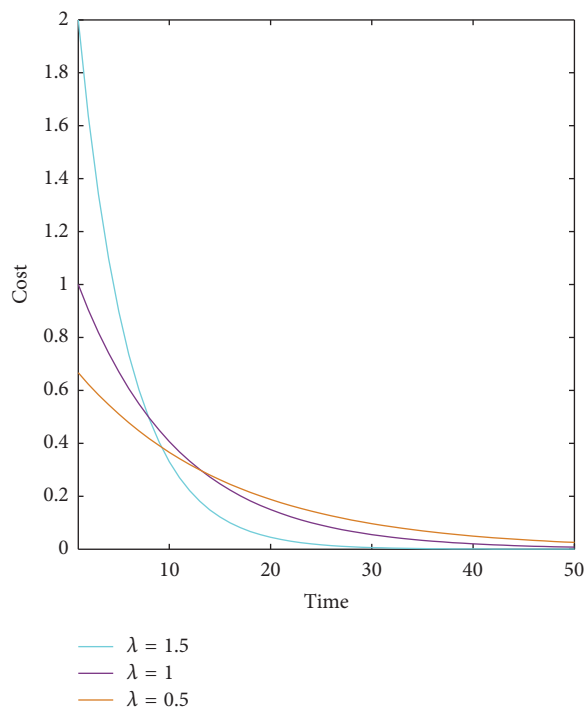

Figure 4: Relationship between cost and due dates.

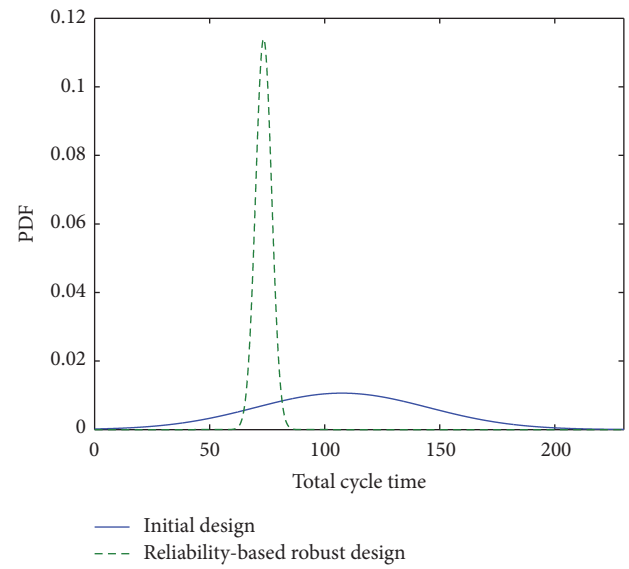

Figure 5: Results from the RBRDO model.

the company should tell their customers that the order will require approximately 107 days. After this initial reliability evaluation, the design optimization model was applied by using (3) to modify the design and find the optimum total cycle time and/or satisfy reliability requirements.

To improve the initial reliability, the company could reduce the cycle time or adjust the schedule or both. For the former, the company could expedite one of the processes by adding more workers to the installation phase (P_8). It could also change delivery methods to reduce P_7. Multiple scenarios have been implemented using MATLAB R2014a and (1)–(3). One such scenario improves the IMSR from 88.2% to 90.16% by adjusting the schedule without changing the actual cycle time. To achieve this, the company should tell the customer their order will require approximately 110 days instead of 107 days. Thus, the model is able to help with project scheduling in order to satisfy reliability requirements.

In another scenario, the company requests a design that can complete all processes in 70 days, with 95% of IMSR. Here, the model helps by assigning durations to each activity in order to satisfy due date and IMSR requirements. RBRDO was applied to this problem, and the results are shown in Table 3.

Table 3 shows the decision variable of each task (time needed to accomplish each task) required to complete the order on time with 95% reliability. For example, delivery (P_7) should not take more than 8 days and installation (P_8) should not take more than 7 days if a 95% IMSR is to be maintained. To determine the best result, model execution (optimization) was repeated several times to produce the optimized points shown in Table 3. The new design satisfies the reliability requirement. Also, the company is 95% sure that the order will be delivered on time. Table 4 shows the cost functions determined using MATLAB.

Table 4 shows the cost of completing the project while satisfying reliability requirements at every iteration during RBRDO. It also shows that the cost of maintaining 95% reliability decreases from $6.092591e^{+06}$ to 173393.6728. Figure 5 compares the PDFs of the initial design and the RBRD. The RBRD has less delay (variation) and a faster cycle time. Also, when comparing initial design and RBRD it is clear that the RBRD has the lowest cycle time while meeting reliability requirements. This optimization model determines the optimal IMSR value required for each stage of the IMS. Thus, we can conclude that developed RBRDO model can minimize costs while satisfying the reliability requirements.

Table 4: Cost functions obtained after each iteration.

Iteration	Cost ($)
0	6.092591e + 06
1	3.407595e + 06
2	2.819751e + 06
3	2.670462e + 06
4	2.603249e + 06
5	2.132391e + 06
6	1.343782e + 06
7	6.404783e + 05
8	5.742168e + 05
9	5.739864e + 05
10	5.405386e + 05
11	5.083312e + 05
12	4.506476e + 05
13	3.197391e + 05
14	2.860919e + 05
15	2.856679e + 05
16	2.850551e + 05
17	2.850489e + 05
18	2.850320e + 05
19	2.160011e + 05
20	2.159386e + 05
21	1.890276e + 05
22	1.806205e + 05
23	1.776129e + 05
24	1.767619e + 05
25	1.763392e + 05
26	1.748977e + 05
27	1.738370e + 05
28	1.734791e + 05
29	1.733999e + 05
30	1.733938e + 05
31	1.733937e + 05
32	1.733937e + 05
33	1.733937e + 05
34	173393.6728

By applying the developed RBRDO model, the required reliability is achieved, all constraints are satisfied, and the impact of uncertainty is minimized as shown in Figure 5.

To investigate how reliability influences the total cost, different scenarios with various reliability rates were simulated. This investigation is summarized in Figures 6 and 7.

Figure 6 shows that the cost increases nonlinearly with the reliability requirements. For example, if the required IMSR is 70% the cost is $27,500, and this cost increases to $100,000 if the reliability requirement increases to 90%. Figure 7 shows that applying the RBRDO achieves the reliability target and reduces variation. Moreover, the RBRDO can shift and adjust the total project cycle time based on the due date and reliability requirements. Figure 7 shows that the variation and cycle time decrease when reliability increases.

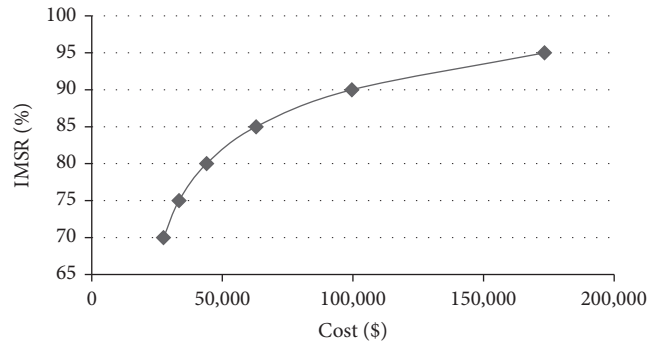

Figure 6: Cost of increasing the IMSR.

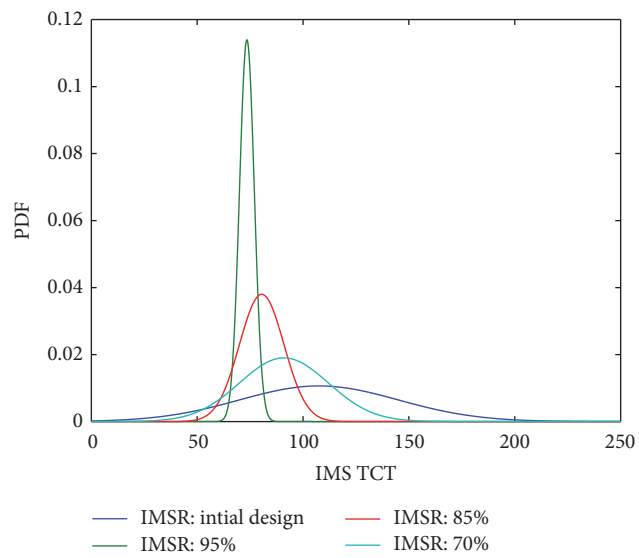

Figure 7: Distributions of the mean TCT at different IMSRs.

For example, a process with 95% reliability has lower cycle time and variation than one with 85% reliability.

Applying the RBRDO allows businesses to control reliability requirements while minimizing variation (impacts of uncertainty), while minimizing total cycle time and cost. This allows identification of activities that should be improved and the level of improvement required to achieve reliability targets. In addition, the proposed model helps to balance cost and reliability requirements, while also minimizing the impacts of product movement uncertainty.

4. Conclusion

IMSs are critical to many businesses. The primary objective of inventory management is to determine and control stock levels to minimize cost and achieve customer satisfaction. To achieve this, multiple design tools were applied to RBRDO development in order to help design a reliable IMS. These techniques include Monte Carlo simulation, deterministic nonlinear programing, and stochastic nonlinear programing. In addition, Microsoft Excel solver and MATLAB were used for modeling and mathematical simulation.

This research contributed primarily to IMS evaluation and design optimization. Equations (1) and (2) can be used to evaluate the reliability of each task in an inventory system, as well as overall IMS reliability. Equation (3) helps to design a cost-efficient, reliable, and robust system while coping with uncertainty. In other words, the RBRDO model can help with task scheduling to satisfy reliability requirements, as well as with determining time limits for each activity to satisfy due date and reliability requirements. The furniture company case study verified and validated the models. The case study showed that RBRDO can be used to design a reliable and robust IMS, while minimizing the impact of uncertainty. This positively impacts customer satisfaction. The model helps to determine the optimal time for each task, reducing delays and increasing the level of client trust and satisfaction.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this article.

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Research Article

A Multilayer Model Predictive Control Methodology Applied to a Biomass Supply Chain Operational Level

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Forest biomass has gained increasing interest in the recent years as a renewable source of energy in the context of climate changes and continuous rising of fossil fuels prices. However, due to its characteristics such as seasonality, low density, and high cost, the biomass supply chain needs further optimization to become more competitive in the current energetic market. In this sense and taking into consideration the fact that the transportation is the process that accounts for the higher parcel in the biomass supply chain costs, this work proposes a multilayer model predictive control based strategy to improve the performance of this process at the operational level. The proposed strategy aims to improve the overall supply chain performance by forecasting the system evolution using behavioural dynamic models. In this way, it is possible to react beforehand and avoid expensive impacts in the tasks execution. The methodology is composed of two interconnected levels that closely monitor the system state update, in the operational level, and delineate a new routing and scheduling plan in case of an expected deviation from the original one. By applying this approach to an experimental case study, the concept of the proposed methodology was proven. This novel strategy enables the online scheduling of the supply chain transport operation using a predictive approach.

1. Introduction

The use of renewable energy sources has been promoted as a way to avoid the increase of carbon dioxide concentration in the atmosphere. Legislative guidelines such as the Kyoto Protocol were created with this purpose [1]. Also, the continuous rising of fossil fuels prices has triggered the interest in these sources of energy [2, 3]. Among the available options, forest biomass has gained interest in the last years [1, 4–6].

The biomass supply chain involves several stakeholders, like raw material suppliers, transportation companies, and production facilities, among others, which work together in order to bring the materials from their source to the consumers [7]. Regarding the processes, this supply chain encompasses the harvesting and collection, chipping, transportation, storage, and final conversion. Biomass can be used to provide heat, electricity, and biofuels [8–11]. Also, compared to other renewable sources of energy, biomass has the possibility of storage and, consequently, producing energy on demand [12]. In the renewable energy context, the biomass is considered as a "carbon neutral" source having a neutral balance in the carbon cycle [13]. In this sense and when compared to fossil fuels, forest biomass can contribute to decreasing carbon emissions [12] and the dependence on imported energy [2].

The transportation is the operation that requires the highest parcel of costs, being responsible for 25–40% of the total value [14]. Besides the economic aspect, this drive intensive characteristic emphasizes the energetic and environmental impact of transportation [15, 16]. As such, this work will focus on the transportation process.

Forest biomass has also some drawbacks: it is a seasonal energy source, has low energy density, is usually spread over large areas, and is composed of a set of interconnected stakeholders, making the decision process more difficult [2, 7, 12, 17, 18]. Associated with this, forest biomass supply chains present a variability and uncertainty that turns the planning of their operations more complex. The management procedure in a supply chain is usually differentiated into three levels, namely, strategic, tactical, and operational [19]. In this work, only the operational level is focused on. It must be noted that the processes dynamics are stochastic in nature. During operation, several adverse situations may occur such as natural phenomena, equipment breakdown, and low quality of material, which disturb the system performance and, consequently, invalidate the original delineated plan [12, 15, 20].

As such, the sustainable and robust management of these supply chains at a cost-effective manner is essential to the expansion and growth of these systems [21]. Thus, it is important to efficiently use the resources at the minimum cost possible [22] trying also to reduce the impacts and a continuous feedstock supply [7]. This is also the main challenge of supply chains [9, 15], where complex decision-making processes are needed to achieve short-, medium-, and long-term goals [23].

There are several planning tools available in the literature to optimize the operational performance of biomass supply chains. In [24], a flow-shop approach to handle operations scheduling problems is proposed. The strategy was tested and demonstrated through a small real example and verified improvements compared to the traditional decision-making process. Also regarding schedule optimization, [25] proposed an approach for planning of sequential tasks scheduling in the harvesting and handling operations in geographically dispersed fields. In [26], the authors applied the classical vehicle routing problem to the biomass collection problem in order to determine the routes with minimal costs to the vehicle fleets involved in the biomass supply chain. In [27], three mixed integer programming formulations were presented to solve the truck scheduling problem. For more detailed information, [28] presented a survey on the models developed for biomass supply chains from an operations research perspective, and [18] presented the modelling approaches to optimize economic, social, and environmental criteria in these supply chains. Furthermore, other optimization procedures have been described in the literature for not only the operational but also tactical and strategic levels [29–31].

Despite the available planning tools, the management of these supply chains continues to be executed based on the empirical knowledge and experience of the decision-maker [24]. In case of deviation from the plan and due to these supply chains interlaced nature, such as synchronized schedules and time windows constraints, it becomes very complex to assess the repercussions of the deviation and to replan based on the current conditions.

In this work, a methodology that allows following the biomass supply chain's operational level during a working day, regarding the transportation process and continuously predicting if the initially proposed schedule will be attained, is proposed. In case of deviation from that goal, the proposed framework makes use of the model predictive control (MPC) approach to automatically update the plan to a viable solution that aims to satisfy the demand within the proposed time frame at the lowest cost possible. With this proposal, it is intended to make the decision-making problem automatic and reactive to possible disturbances that might occur in the system with repercussions in the different levels. This contrasts with the static approach commonly found in the literature.

The paper is organized as follows: Section 2 presents the biomass supply chain, describing its processes and the decisions taken in the different management levels; Section 3 details the operational model based on the developed multi-layer model predictive control strategy; Section 4 illustrates the proposed methodology through a simple example; and, finally, Section 5 presents the main conclusions and insights into future work.

2. Problem Statement

Forest-based supply chains are complex systems with several processes and stakeholders involved. Although some processes are common to all these chains, the biomass supply chain has specific requirements and phases not present in the remaining ones. The biomass supply chain starts in the forest area, within the forest stands, where the trees and branches are harvested and forwarded into piles disposed at the roadside. These wood sources are then transformed into small wood chips by the chipping process. Those wood chips are loaded into a truck which will transport them to a mill. In the case of a biomass for bioenergy supply chain, the wood chips arriving to the power plant are converted into energy to the energy market. It should be noted that the chipping process occurs simultaneously to the truck loading process as the chipper machine is itself assembled to the truck. Also, intermediate storage stages can be considered.

Periodically, the power plant demands a certain amount of energy to be delivered at the end of the week. Based on that, the management team has to define the amount of wood that needs to flow inside the supply chain to comply with the demand. This management is divided into several levels, mostly differentiated by the time scale. The tactical level, usually concerning decisions with times from weeks to months, addresses issues of resources dimensioning, that is, the number of resources to be used and the amount of material that flows inside the system. On the other hand, the operational level, as the name indicates, addresses decisions related to routing and scheduling of operations at smaller time scales. At a higher time scale, from months to years, there is also the strategic management level. However, this level is more related to forest management issues. In the present work, only decisions regarding the operational level during a working day will be considered. In this sense, decisions related to material flow and pile-client association are already provided as a result of the tactical level optimization. The operational level, here considered, will deal with the trucks' fleet routing and scheduling questions.

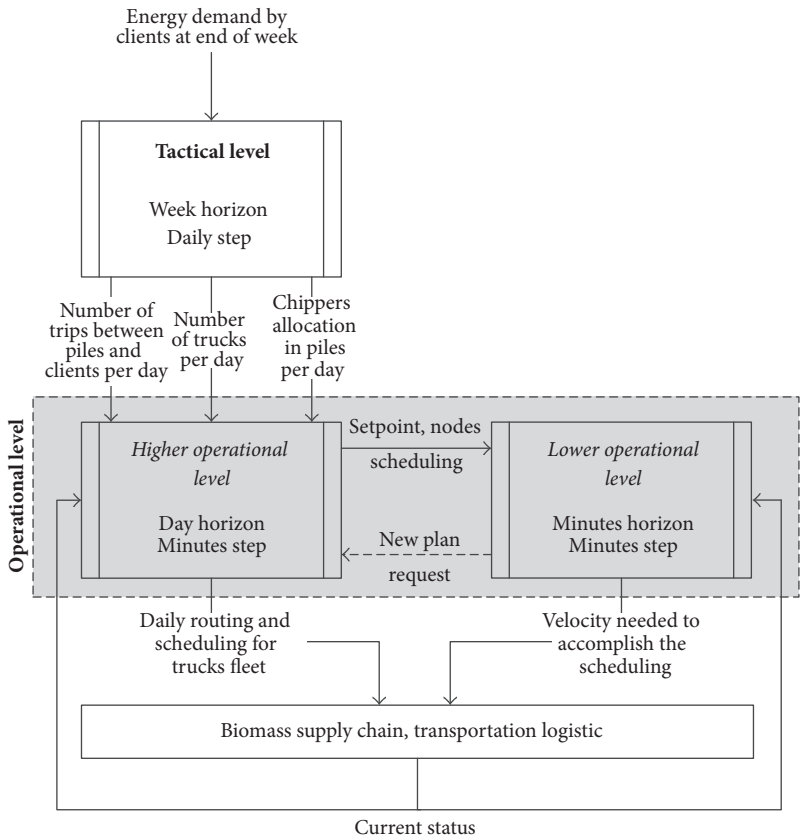

Figure 1: Multilayer operational level proposed methodology.

The management of the operational level is usually performed in a static way, using planning tools to generate an initial plan at the beginning of the day. However, no monitoring of the processes' evolution is verified. Consequently, if there are deviations from the plan, no corrective actions are applied during that day and will accumulate for the following day, possibly leading to an unfeasible solution to the weekly goal.

In this work, a tool that considers the online replanning of the biomass supply chain transportation process is proposed, inspired by the model predictive control technique. The objective is to create a framework that deals with the system evolution during the day, with the possibility of taking the advantages of the MPC technique to forecast possible deviations from the plan and in an automatic and efficient way reacting beforehand and replanning all the involved decisions.

3. Proposed Methodology

Due to its complexity, the overall architecture of significant size supply chains should be described by means of multilayer hierarchical connection between the most fundamental elements. This is true in all its different dimensions: corporative, economical, and logistic. Regarding the latter, strategic, tactical, and operational levels are stacked one after the other, where the operational level is the ground level. Those three levels are highly intertwined and information flows among them. The tactical level is based on the long-term business vision provided by the strategic component and the operational level behaviour defined in a shorter time scale as a function of the tactical level decisions.

The current work addresses uniquely the biomass supply chain operational level, particularly the transportation logistic problem. The overall supply chain regulation paradigm here proposed, with the information flow between the tactical and operational levels, is revealed at Figure 1.

In the current supply chain control methodology, it is assumed that the tactical level provides the information that will steer the decision-making process that will take place in the operational level. This information regards the number of trips between piles and clients and the number of trucks and chippers available.

The tactical level operates at a daily time frame, where the control actions are planned according to the current system states. The planning process is many times carried out aiming to minimize the costs regarding the raw material transportation and both the chipping machines and trucks usage [32–34]. All these variables are defined, per day, along a time horizon of one week. At the beginning of each day, new tactical actions are planned according to the current system states. This information is then conveyed to the operational level whose main objective is to establish the set of working schedules and deliver them to the field working teams.

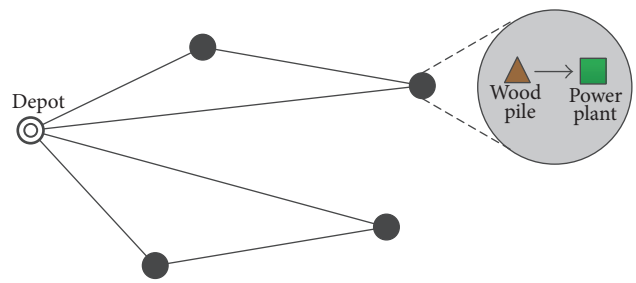

Figure 2: Example of geographic dispersion of nodes.

In order to define both the routing and scheduling of the trucks fleet, the operational level must have a deep insight about the supply chain structure and its timing requirements. Conceptually, the actual supply chain problem is composed of nodes scattered along a given geographical area. All the trucks depart, at the beginning of the day, from the same node, designated by depot. Moreover, they should also arrive to this same node when their working schedule ends. Besides the depot, the supply chain network is composed of a set of nodes, denoted by working nodes, which should be visited during the daily schedule. Each working node in the networks can be further decomposed into a lower level structure constituted by one wood pile, one client, and a job. This concept is illustrated in Figure 2, where two distinct routes are considered. Both depart and arrive to the depot, while traversing two working nodes.

Also in Figure 2, one of the nodes is expanded in order to show its internal activity. This activity assumes the following steps: the truck moves to the wood pile, where it is loaded with wood chips. After the loading process, it carries on toward the client for the unloading process. After this task has been accomplished, the truck leaves the working node toward the next one. Notice that the pairing between wood piles and clients has already been performed, in an optimal context, by the tactical level [34].

Delivering a proper working plan to the field operatives is the responsibility of the operational level. As can be seen from Figure 1, the operational level is divided into two distinct layers: a higher layer and a lower layer. The higher layer operational level component is committed to making schedules and routes taking into consideration some important operational constraints. First, and for each working node, the loading and unloading time windows must be known. That is, each network working node has an admissible time interval, where it is expected to be visited by a truck. For example, the chipping machines are scheduled to be in a particular wood pile during some time window and the clients are only able to process unloading tasks in an alternative time span. For this reason, a truck must arrive at a node at the time instant that permits concluding its intrinsic job. It should be noted that the expected loading, driving, and unloading times for a given working node are known. Moreover, during the present work, it is considered that, after entering the node, the situation is deterministic. That is, the exit time is equal to the arrival time plus the time expended during the loading, driving, and unloading tasks.

The working plan is delivered, at the beginning of the day, to each field operative element. Each schedule sheet provides information to the truck driver regarding the nodes to be visited, their order of appearance, and an estimation of the arrival time. The routing problem faced by the higher layer operational level boils down to a classical travelling salesman problem (TSP) with variable number of salespersons. In this case, each truck can be viewed as a different salesman and the set of cities to be visited are the network nodes. It is well known that those classes of problems are NP-hard and the complexity in finding a solution increases quickly with the number of nodes in the network. In fact, if there are N nodes in the network, then the number of possible paths is equal to the factorial of $(N-1)$. For this reason, the performance of any search algorithm will be severely degraded by the addition of new working nodes in the network.

The lower layer operational level controller will be responsible for closely monitoring the process evolution by means of measuring the relevant state variables such as the position and state of each vehicle and the nodes visited. The trucks' geographical positions are assumed to be provided by a global positioning system (GPS). It is also assumed that the nodes' geographical locations are equal to the one of the wood piles. Hence, the positions of all the nodes in the network are known.

From the available field data, the lower layer operational controller will be able to forecast possible unaccomplished jobs due to truck delays. It is then possible to define a feedback control strategy aiming to maintain the initial target set of jobs even in the presence of disturbances. Those disturbances can span from simple delays in the delivery to a particular machine breakdown or adverse weather conditions, among others. This type of anticipative reaction from a closed loop system is highly desired, since it prevents large deviations in the system state variables. In control theory, this type of paradigm is known by model predictive control (MPC) and, as will be seen in Section 3.2, it is embedded in the lower layer of the operational level controller. If the lower layer predicts that the job is impossible to be accomplished, a new routing and scheduling plan is requested to the higher layer of operational level. Section 3.1 will be devoted to describing and analysing this higher layer by presenting its mathematical formulation.

Before ending, it is worth noticing that if no solution can be achieved by the operational level, then, at the end of the working day, a report is provided to the tactical level with information regarding the jobs that were unaccomplished. The tactical level will use this information to correct the setpoints delivered to the next days, hence avoiding cumulative deviations at the end of the week. However, this tactical replanning is out of the scope of the current work.

3.1. Operational Level: Higher Layer Control. Daily, the tactical level delivers both the wood pile/client pairs and the available resources to the operational level. The former will become the nodes in the operational network and the latter regards the maximum number of available machines that can be used during the tasks execution. The operational level is decomposed into two hierarchical sublevels: a higher layer,

which is responsible for defining the trucks fleet routing and scheduling, and a lower level, which keeps track of the generated schedule and predicts the impact of truck delays in the supply chain. This section is devoted to describing the higher layer of the operational level control.

The routing and scheduling result from an optimization process that takes place at this level, particularly by solving a minimization mixed integer programming problem that describes the behaviour of the supply chain at this level. For this reason, the higher layer operational level model will be presented as an optimization problem.

Let $q \in \mathbb{N}^*$ be the number of working nodes and let $q+1$ be the total number of network nodes including the depot. As previously referred, the depot is a very special node, since all the vehicles are supposed to depart from it and are expected to return to it after the last task takes place. Let this node take the index 0. The set $\mathcal{Q} = \{0, \ldots, q\}$ contains the reference to each network node beginning with the depot. Also, let m be the number of trucks available in a given working day and let $\mathcal{K} = \{1, \ldots, m\}$ be the set of those machines.

Each working node can be in one of three possible states: unvisited, partially visited, or fully visited. The first state is achieved when no vehicle has arrived to it or departed from it, and the second state is achieved when a truck has entered the node but has not left it yet. Finally, the node is said to be fully visited when a previously arrived truck leaves toward a new node. In other words, it is considered that a node is fully visited when a particular truck enters it and departs from it. It is worth noticing that each node is supposed to be visited only once and that the depot is the only node that remains partially visited from the beginning of the truck tours up to the end. Moreover, it will be assumed that the depot can only change its state to "fully visited" when the number of trucks arriving at it will be equal to the number of vehicles that have departed from it. That is, the net flow of trucks in this node must be equal to zero.

When a given node $i \in \mathcal{Q} \setminus \{0\}$ is traversed by one of the m trucks, it will become a fully visited point and will be added to the set $\mathcal{P}$. According to what has been previously stated, the last node to be included in $\mathcal{P}$ will always be the depot. Moreover, let $\mathcal{J}$ be the set of the "partially visited nodes" and let $\mathcal{L}$ be the set of "unvisited nodes." In this context, $\mathcal{Q} = \mathcal{P} \cup \mathcal{J} \cup \mathcal{L}$. Moreover, define the set $\mathcal{R} = \mathcal{L} \cup \mathcal{J}$ as the set of nodes that remain to be "fully visited," that is, the nodes that are in the state of being "partially visited" or "unvisited." The higher layer operational supervisor will be filled with the actual network status, at an asynchronous rate T, triggered by demand of the lower layer operational supervisor whenever collapse in one of the jobs is expected. If this is the case, the higher operational supervisor will compute and submit an alternative working plan considering the new system state. For this reason, the content of the above defined sets can be changed. If the system state sampling occurs at nT, with $n \in \mathbb{N}$, then the sets $\mathcal{P}(n)$, $\mathcal{J}(n)$, and $\mathcal{L}(n)$ contain the "fully visited," "partially visited," and "unvisited" notes at that time instant.

Whenever a particular vehicle arrives at a node, this point will become a partially visited node and will be the new starting point for this truck. Let $s_k \in \mathcal{R}(n)$ be this new starting node for truck $k \in \mathcal{K}$. Then, $\mathcal{S}(n) = \{s_1, \ldots, s_m\}$ represents the set of the starting nodes associated with each truck. Due to the node's number of visits constraint, $s_i = s_j$ if and only if $i = j$.

For the current problem formulation, it is supposed that, at a given time instant, one is able to locate all the resources in the network. That is, the nodes position, their relative distances, and the trucks' positions are known. Moreover, the absolute time is known with a resolution of one minute. Let $t \in [0, 1439]$ be this time where 0 regards 0 h 00 m and 1493 regards 23 h 59 m. Also, let us assume that the distance d_{ij} between two distinct nodes in the graph, $i, j \in \mathcal{Q}$, is known and that the average time, required to perform the trip between those two points, is constant and equal to γ_{ij}. In order to perform the required task in node i, the truck must spend some time on it. Let the required working time at node $i \in \mathcal{Q} \setminus \{0\}$ be constant and equal to α_i.

In this frame of reference, the current decision variables are $x_{ij}^k(n) \in \{0, 1\}$, which take the value of 1 if, from the present time instant n up to the end of the working day, it is expected from vehicle k to visit node $j \in \mathcal{L}(n) \cup \{0\}$ after finishing the task at node $i \in \mathcal{P}(n)$. The value of zero will be used to denote otherwise. Moreover, the expected time at which the activity performed by truck k starts at node $i \in \mathcal{L} \cup \{0\}$, from the present time instant n upwards, is denoted by $t_i^k(n)$ and is also a decision variable. For the depot node, this starting activity concerns the truck's departure time. However, for the remaining nodes, this variable indicates the truck's arrival time. Due to imposed time window constraints at the working nodes, this $t_i^k(n)$ must be within the time interval that spans from $\hat{\eta}_i \in [0, 1439]$ up to $\check{\eta}_i \in [0, 1439]$ with $\hat{\eta}_i < \check{\eta}_i$. These lower and upper time intervals depend on the chipping machine schedule for that node and on the last available unloading time slot at the client.

An important aspect of the schedule plan is to derive the time at which each of the vehicles is expected to arrive at the depot. Thus, an additional decision variable $\tau^k(n)$ is associated with each truck and denotes the time at which it arrives at the depot. This variable has to be less or equal to the maximum working hours per day of truck k. Let w^k be that value.

In order to find the best working plan, at a given time instant nT, the system model is formulated as a linear programming problem, where the cost function is described in (1). This cost function has two terms: the leftmost represents the total distances travelled by the set of all vehicles and the rightmost is the total time required to perform the full set of operations.

$$\min \left(\sum_{i \in \mathcal{R}(n)} \sum_{j \in \mathcal{R}(n)} \sum_{k \in \mathcal{K}} d_{ij} \cdot x_{ij}^k(n) + \sum_{i \in \mathcal{R}(n)} \sum_{k \in \mathcal{K}} t_i^k(n) \right). \quad (1)$$

Aiming to model the supply chain operational behaviour, a set of constraints are added to the above linear programming problem. For example, the following equation addresses

the case where it is required that all trucks, belonging to $\mathscr{K}$, must always leave the actual entering working node.

$$\sum_{i\in\mathscr{L}(n)\cup\{s_k\}} x_{ij}^k(n) = \sum_{i\in\mathscr{L}(n)\cup\{0\}} x_{ji}^k(n) \quad \forall j\in\mathscr{L}(n),\ \forall k\in\mathscr{K}. \tag{2}$$

That is, for each node j, in the set of nodes that remain to be fully visited, if a truck is entering this node from any node not fully visited, excluding the depot, then it must leave to any of the remaining nodes.

Moreover, each node can only be visited one time during the working day. This condition is imposed by the following expression:

$$\sum_{j\in\mathscr{L}(n)\cup\{0\}} \sum_{k\in\mathscr{K}} x_{ij}^k(n) = 1 \quad \forall i\in\mathscr{L}(n). \tag{3}$$

The next pair of constraints implies that the already visited nodes cannot be visited again; that is, none of the trucks can leave from those nodes to another and cannot go from another to those nodes.

$$\begin{aligned} &\sum_{j\in\mathscr{Q}} \sum_{k\in\mathscr{K}} x_{ij}^k(n) = 0 \quad \forall i\in\mathscr{P}(n), \\ &\sum_{i\in\mathscr{Q}} \sum_{k\in\mathscr{K}} x_{ij}^k(n) = 0 \quad \forall j\in\mathscr{P}(n). \end{aligned} \tag{4}$$

The following inequality refers to the start condition of the trucks. This forces each truck to leave from its starting point.

$$\sum_{j\in\mathscr{L}(n)\cup\{0\}} x_{s_k j}^k(n) \le 1 \quad \forall k\in\mathscr{K}. \tag{5}$$

Furthermore, since all the trucks must end at the depot,

$$\sum_{i\in\mathscr{L}(n)\cup\{s_k\}} x_{i0}^k(n) \le 1 \quad \forall k\in\mathscr{K}. \tag{6}$$

An additional decision variable associated with node $i\in\mathscr{Q}$ and denoted as $u_i(n)$ is also added in order to prevent trajectory loops. Those cycles are avoided by means of the following constraint:

$$u_i(n) - u_j(n) + l\cdot\sum_{k\in\mathscr{K}} x_{ij}^k(n) \le l-1 \quad \forall i,j\in\mathscr{L}(n), \tag{7}$$

where l represents the number of nodes present in set $\mathscr{L}(n)$ at sampling time n.

In order to define the time at which each task takes place $t_i^k(n)$, it is necessary to attend the travel time between nodes i and j, γ_{ij}, the time spent in node i, α_i, and the upper time window associated with that node, $\check{\eta}_i$. That is,

$$\begin{aligned} &t_i^k(n) + \left(\gamma_{ij}+\alpha_i\right) - t_j^k(n) \\ &\quad \le \left(\check{\eta}_i+\gamma_{ij}+\alpha_i\right)\cdot\left(1-x_{ij}^k(n)\right) \\ &\qquad \forall i\in\mathscr{L}(n)\cup\mathscr{S}(n),\ j\in\mathscr{L}\setminus\{0\},\ k\in\mathscr{K}. \end{aligned} \tag{8}$$

The following constraints indicate the nature of decision variables:

$$\begin{aligned} &\hat{\eta}_i \le t_i^k(n) \le \check{\eta}_i \quad \forall i\in\mathscr{Q}, \\ &u_i(n) \ge 0 \quad \forall i\in\mathscr{Q}, \\ &x_{ij}^k(n)\in\{0,1\} \quad \forall i,j\in\mathscr{Q},\ \forall k\in\mathscr{K}. \end{aligned} \tag{9}$$

Finally, the time at which the truck returns to the depot must be constrained to be within the interval:

$$t_i^k(n) + \left(\gamma_{i0}+\alpha_i\right)\cdot x_{i0}^k(n) \le \tau^k(n) \le w^k \quad \forall i\in\mathscr{L}(n),\ k\in\mathscr{K}. \tag{10}$$

The above-defined problem will be solved once at the beginning of the working day and whenever there is a demand for a new plan from the lower layer operational controller. The following section will describe, in further detail, the operation of this lower level operational component.

3.2. Operational Level: Lower Layer Control. During the working day, the position of each truck is checked by means of a GPS tracking system. Thus, the truck's relative position regarding the nodes is known. It will be assumed that this information is regularly sampled with a period T_s equal to 10 minutes. This time period was chosen based on the dynamic system characteristics. At each sampling time, nT_s for $n\in\mathbb{N}$, it is considered to have full information concerning the current system state.

The purpose of the lower layer operational control is to keep track of the vehicles and to provide information to the driver regarding the expected average speed he is expected to follow. By forecasting each truck position in the network, this layer is able to provide speed estimates in order to accomplish the working plan. Those predictions are generated according to a model of the truck's position.

This philosophy is in line with the concept of model predictive control (MPC). In particular, the aim of this work is to provide an alternative formulation of the MPC paradigm in the context of supply chains regulation. The MPC unveils its full power if a sufficient accurate system process model is available. This model is used to perform system predictions, leading to an anticipative controller reaction if relevant system state deviations are likely to occur. Classically, the MPC is formulated as a quadratic optimization problem subject to actuators constraints, where the decision variables are the sequence of possible actuation which minimizes the prediction error. The objective function usually encompasses a term that penalizes any setpoint deviations and a second term that leads to control effort minimization. According to the MPC strategy, only the first computed control action is placed into the actuators. When the system status is updated, a new set of control actions is calculated based on this new information.

The MPC model can be translated into the current supply chain control problem. First, according to the current position and time of a particular truck and by resorting to the truck's position model, it will be possible to predict if it will

Table 1: Geographical distance (km) between the considered nodes.

	Vila Real	Mirandela	Bragança	Chaves	Braga
Vila Real	0	59.4	118	66.6	105
Mirandela	59.4	0	61.3	51.5	159
Bragança	118	61.3	0	110	217
Chaves	66.6	51.5	110	0	128
Braga	105	159	217	128	0

arrive to the next node at the time defined by the higher layer operational plan. The desired truck speed, which will be sent to the truck driver, will be computed during this step, taking into consideration the route speeds limits. If the lower layer controller predicts that the time provided by the working plan will not be attained, an alert will follow to the upper level to generate a new scheduling plan. Information regarding the predicted delay in the task accomplishment will be provided.

Following the MPC concept, the objective function to minimize in the lower level takes two terms into consideration: a first term that aims to minimize the remaining distance to the destination node (the quadratic formulation is used to avoid the overpassage of the destination node) and a second term that penalizes the control effort, that is, penalizes abrupt changes in the velocity. The objective function considered in this level is presented in the following equation:

$$\begin{aligned} \min \quad & \left[d_{jp}^{k}(n+h \mid n)\right]^{2} \\ & + \sum_{i=1}^{h}\left[v^{k}(n+i)-v^{k}(n+i-1)\right]^{2}, \end{aligned} \tag{11}$$

where v^k refers to the velocity of truck k, h refers to the control horizon, and n is the current time sample. Note that the number of available trucks is not a decision variable of this level, being provided by the tactical level as depicted in Figure 1. In this formulation, the control horizon is considered as equal to the prediction horizon. This allows better distributing the control actions along the horizon and avoiding future deep changes. In this work, h is assumed to be variable regarding each truck and task and is computed by

$$h = \left\lceil \frac{t_i^k(n) - nT_s}{T_s} \right\rceil. \tag{12}$$

The predictive component is incorporated into the objective function by means of the remaining distance to travel regarding the next node j based on the current position p of a truck k, $d_{jp}^k(n)$. This distance is computed according to the remaining distance to node j, at the previous time instant, plus the distance travelled in that time interval. Mathematically, this can be formulated as

$$d_{jp}^{k}(n+h \mid n) = d_{jp}^{k}(n) - T_s \cdot \sum_{i=1}^{h} v^{k}(n+i). \tag{13}$$

Note that v^k has to be a positive value; that is,

$$v^k \geq 0 \quad \forall k \in \mathscr{K}. \tag{14}$$

In order to provide a proof of concept of the proposed control methodology, a case study will be considered in the next section.

4. Illustrative Example

In this section, a biomass supply chain example is considered to demonstrate the applicability and advantages of the proposed methodology. It should be noted that the dimension of the problem can be easily scalable, and a small scale example is here presented for simplicity of analysis. A total of 5 nodes including the depot are considered, representing a set of cities in the North of Portugal: Vila Real (depot), Mirandela, Bragança, Chaves, and Braga. The distances between each pair of network nodes are presented in Table 1.

Two trucks will be considered during a working day, and each truck spent approximately 45 minutes within the node to complete the defined job (loading the wood chips, drive toward the power plant, and unload). The initial travel time is considered as equal to the time needed to travel the presented distance assuming a constant mean speed of 60 km/h.

The schedule definition process has two stages: a setup stage, where the initial plan is established, and a tracking stage, where the above plan is followed in terms of completion. The former is handled by the higher layer, described in the previous section, and the latter by the lower layer of the operational level control. The setup stage can occur in any time period before the instant at which the lower layer takes charge. For example, considering the above problem definition and assuming that the lower layer should start at 8:00, the initial plan produced by the higher layer optimization problem must be generated at any time instant prior to 8:00. For example, let us assume that, at 7:00, the higher layer was executed and was able to generate the routing and scheduling plan for the two trucks as depicted in Figure 3. Each coloured triangle represents a particular truck route. The time tags added to each node indicate the expected truck's arrival time to the respective node. This is true for all nodes with the exception of the depot, where the time schedule, printed in white background, is the departure time and the remaining coloured tags represent the depot arrival time for the respective truck.

From Figure 3, it is possible to conclude that the higher layer initial schedule assumes that one of the trucks performs the route that visits the nodes 1-4-5-1 and the other executes the sequence 1-2-3-1. Both trucks depart from the depot at the same time, in this case 8:00, but will arrive at the depot at different time instants. The first will be ending its work schedule at 14:29 and the second around one hour earlier.

Table 2: Control of the average speed (km/h) and the expected delay (min) according to systems states update.

Sample n	Computed speed (km/h)							Delay (min)
	$n+1 \mid n$	$n+2 \mid n$	$n+3 \mid n$	$n+4 \mid n$	$n+5 \mid n$	$n+6 \mid n$	$n+7 \mid n$	
0	58	58	57	57	57	57	57	0
1	57	57	57	57	56	56		0
2	62	62	62	62	62			0
3	76	76	76	76				0
4	80	80	80					8
5	80	80						18

Figure 3: Routing and scheduling solution provided at the beginning of the day. Each coloured triangle represents a truck's route.

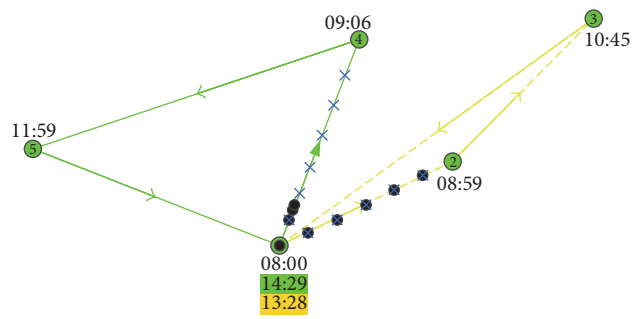

Figure 4: Trucks' position evolution during the working schedule (the blue crosses indicate the trucks' expected position and the black dots the trucks' actual position).

Exactly at 8:00, the lower level operational layer controller will be activated. This layer will sample the trucks' positions, with a sampling period of 10 minutes. Based on this data and resorting to the trucks movement dynamic model, a set of desired truck velocities are computed in order to attain the target nodes at the time instants closest possible to the ones delivered by the initial schedule. The predicted delay at a particular node will be the difference between the expected time of arrival to that node and the time initially scheduled. If this delay is above a predefined threshold, then the lower layer is resumed and the higher layer is asked to provide an alternative schedule plan in order to mitigate further impacts of this delay in the supply chain tasks.

In order to illustrate the behaviour of the lower layer and its connection to the higher layer, consider the situation where both trucks depart from the depot at 8:00. The lower layer will be tracking the position of the trucks at regular time intervals and will be providing suggestions to each truck driver regarding the actual expected speed for the rig. Figure 4 depicts the evolution of both trucks' positions during the first six samples. As can be observed, the truck whose route is represented with the dashed line has a position progress that consistently provides confidence on the accomplishment of its task. However, regarding the other truck, something went wrong after 20 minutes of its departure, since the estimated arrival time at node 4 is constantly postponed. This increasing delay can be due to several situations such as a truck's mechanical malfunction, route problems, and traffic jams. In this case study, let us consider that a malfunction has occurred.

Figure 5: Routing and scheduling solution provided by the higher operational layer due to a predicted delay greater than the threshold.

Table 2 presents the predicted speeds computed by the lower layer operational controller for the truck that presents problems, namely, the truck represented by the solid line. The one-step-ahead speed prediction will be the one delivered to the truck driver. Moreover, the expected time delay, considering the system states update at each sampling time, is also presented.

As can be observed from the table, at sample time $n = 5$, the accumulation of expected delays is near 20 minutes. Assume, for the sake of simplicity, that 10 minutes is the threshold level limit from which a new schedule should be generated. In this context and assuming that the faulty truck cannot carry on due to some malfunction, the higher operational layer control is triggered and the schedule plan presented in Figure 5 is delineated. It is worth noticing that

this new plan is generated assuming that the operational trucks will finish their current tasks.

Moreover, the truck's driver is only informed about the next job at the end of the previous job. This allows changing the plan internally without the knowledge of the driver. This will avoid constant changes in the overall schedule.

With the presented approach, the operations occurring during the day are closely monitored and corrective actions are automatically applied beforehand to avoid deviations in the tasks accomplishment and, consequently, their propagation. Thus, the proposed methodology has been proven to be advantageous when compared to the traditional used strategies.

5. Conclusions

Biomass supply chains are complex systems, where their interconnected character hinders the disturbance propagation analysis along the processes. This task is even more complicated when no decision support tool is available. In this work, a methodology based on model predictive control was proposed to control the operational level of a biomass supply chain regarding the transportation operation. The proposed approach allows following the system state during the working day. At each time step, it predicts if the planned schedules and, consequently, the final goal of client satisfaction will be achieved. At each iteration and considering the current status of the system, if necessary, an alternative control sequence is generated to overcome possible deviations from the previous plan. With this approach, the system is automatically controlled reacting to disturbances in the plan accomplishment, avoiding errors accumulation. In order to test the feasibility of the proposed method, an application example was presented. The simulation results show that the system is able to react to delays, replanning the operations in order to comply with the daily goal. Due to the nature of these systems and their discrete event behaviour, in future work, it is intended to propose a hybrid event driven MPC approach.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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Research Article

Modelling Decision-Making Processes in the Management Support of the Manufacturing Element in the Logistic Supply Chain

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This paper highlights the problems of mathematical modelling for a specific element of the logistic supply chain, that is, the manufacturing system. The complex manufacturing system consisting of a determined number of parallel subsystems is modelled. The fact that the same manufacturing procedure can be carried out in various locations is emphasised. Control algorithms as well as manufacturing strategies are explained. The equations of state are introduced. The two-stage criterion lets us use the result data generated by the simulator of the production system as the initial data for further processing; however, the main goal remains to minimise the time of the course of production. The precisely elaborated case study implements initial data obtained by preceding simulation procedures carried out in manufacturing systems consisting of three, four, and five subsystems.

1. Introduction

Nowadays, design and optimisation of logistic and production systems is a demanding and important area. Businesses are pushed to improve their performance, decrease production prices, and increase variability of their products or shorten delivery times. If a company fails to adapt to these requirements, it cannot normally survive in the contemporary market environment. To survive in the highly competitive global economy, manufacturing systems must be able to adapt to new circumstances [1]. Production systems may vary in terms of their complexity. Moreover, many current production companies have geographically separate production sites and factories are often located in different countries in which there are different labour costs which results in a different price per unit time of production. Moving production from one plant to another may be one way to reduce costs, that is, to increase profits. Manufacturing costs seem to be the most important factor but there are also others which managers must take into account. One of them is stability of the region where the manufacturing plant is located. The next one worth considering is reliability of the supply chain. It is also necessary to consider transport availability, distance from customers, and so on. As it is clear from the above and other contexts, a number of various indicators must be taken into account for complexity assessment of selected general process structures while designing the structure or optimising manufacturing tasks [2]. The structure of production processes also depends on the production needs of specific products that may show high variability. Organising production processes is closely related to the process maps and procedures responsible for manufacturing products from individual components [3]. A key prerequisite for the effectiveness of the above and other production systems is the precise definition of the interaction between the machining process and the machine tools [4]. Also, requirements grow for monitoring possible disruptions of production systems [5]. Additionally, parts and quick recovery of the production process in case of faults and other risks linked to the general input-output model which is a production system are to be considered [6]. Manufacturing of the product generally occurs as a series of individual actions that are performed

manually, mechanically, or a combination of these. Optimisation requires continuously processed orders on individual projects. It can also be a serial production of one product in which you can easily define a set of key performance indicators and manage and automate production [7, 8]. Today, the standard starting point for the design and optimisation of manufacturing systems is simulation. Computer simulations allow you to test different variants of production quickly. It is possible to test many consequences of the changes in the production process and choose the most effective way of making orders in a short time [9]. Simulations can be used both before the draft design of the production system and for the purpose of optimising the production system and reengineering of production processes, respectively. In both cases it is necessary to treat simulation results as baseline data and information for designing or redesigning the considered system. In addition to defining the structure of production systems (in production and logistics systems especially of discreet nature, see, e.g., [10–12]) simulations are useful for planning production and its sustainability and continuity [13, 14]. Specifically, the simulation can help coordinate the needs of various departments and discovery and management bottlenecks and improve resource allocation, distribution of production between production lines or plants, testing strategies, performance measurement, and so on [15, 16]. Everything mentioned may result in lower production costs and increase efficiency ultimately. The basis for the simulation is mathematical models. Creating simulation mathematical models allows us to use a variety of methods and approaches. This could include a multiagents approach [17, 18], Petri nets [19], object-oriented approach [20], or many others. For the purposes of logistics and production systems, which are the main focus of this article, heuristic approaches can be effectively used. Additionally, it is possible to define a mathematical simulation model in the selected set of cases. Such a model allows us to obtain information for particular specific optimisation solutions (see, e.g., [21–23] or [24]). If there is a set of production lines, factories, and so on, it is also possible to use some of the theoretical background in the field of modular systems. The goal of every company is to make a profit and search for new markets and, at the same time, look for ways of minimising manufacturing costs. One of them is locating production in certain destinations where labour costs are considerably lower. Nevertheless, manufacturing capacities of companies can be limited to make products in time which results in placing production load in a more competitive location. Customers usually do not need all ready orders at once. Products they require are to be delivered to their logistic centers subsequently, for example, for further processing.

Production systems have various structures; however, they are defined by certain common features. One of them is the tool replacement problem. Tools are replaced either individually or within replacement of the so-called tool stand. Usually, there is a tendency to replace only fully worn out tools. A problem which occurs in the discussed system is the need to replace the stand with tools even if the tools in it are not fully worn out. So far manufacturing problems of this type have been solved on condition that either fully worn out tools or the unnecessary ones are subject to replacement [25, 26].

The main goal of this article is to define and simulate the mathematical model for the evaluation of the distribution of production in several plants within one complex company to achieve maximum production in the shortest possible time and then minimise costs of manufacturing. Individual companies may be geographically located in different countries with different labour costs and a production time unit. Individual companies are considered as autonomous systems with the same production possibilities. The model takes into account three variants consisting of 3, 4, or 5 companies which can make orders according to vacancies. To compare all possible variants, the mathematical methods of permutations without repetition are used. The basic criteria for the comparison of efficiencies of the production system as a whole are the time and cost of manufacturing time unit.

Complexity has always been a part of each manufacturing environment. Therefore, there has always been a need to classify them as complex systems where numerous phenomena take place [27, 28]. The complexity of the selected system is described by its behaviour. All components interact in multiple ways and follow the rules which result from the specification and model of the system. The system is characterised by its interdependencies which are created step by step in accordance with a software engineering approach. Programming complexity is a measure of the interactions of the various elements of the software created for meeting the article objectives. The mutual interdependencies are regulated by the emerging need to make the order matrix elements at the given stages. The state of the discussed system emerges from a collection of preceding interactions of work stands. The system receives orders at random so it is never known in advance what kind of order customers require. Once the order is set, the whole course of calculations depending on each other should be initiated to deliver the solution to meet the stated criteria. These correlated relationships let us create a differentiated structure that can, as a system, interact with its subsystems which are placed in various locations. The organised aspect of this form of complexity emerges from the need to meet accepted criteria without any operator's guidance at the decision-making level. This duty is taken over by heuristic algorithms. The number of machines does not have to be very large for a particular subsystem to have emergent properties. The system implements specified rules which can be invoked to explain the origin of complexity. Tools in stands which are subject to replacement are an important factor of complexity. The system is highly sensitive to initial conditions. The size of the input influences the level of complexity; that is, the state of the system is a function of the number of order matrix elements. In our case, complexity can be treated as a measure of the total number of correlations in each subsystem whose properties are understood as a state of the system. A complex system is defined by its different attributes and presents problems in both mathematical modelling and subsequent simulation testing. The study of the complex system investigates how relationships between tools in work stands and buffer stores, understood

as interconnected components, give rise to the collective behaviours of the subsystems in the frame of the whole manufacturing system. The equations from which the model of the complex system is developed derive from information theory and represent organised but unpredictable behaviours of the manufacturing system that can be considered fundamentally complex. Simulation of company processes requires a thorough analytical approach. The method presented in this article is proposed for an automotive industry supplier. The output of the analysed global company is represented by various parts used during a car assembly process. The multinational company has its headquarters in the heart of Europe and, from there, customers' orders are directed to dedicated plants for manufacturing. It is possible to adjust manufacturing procedures so that they can be the same in all plants. For comparison reasons it is assumed that all analysed plants have the same arrangements of machines and tools when required. This enables making any order matrix element in any available plant. It is assumed that quality issues are met in every plant of the system. The problem exists because the price of work differs as each macrologistic area is characterised by its autonomous labour costs. However, it is impossible to locate all production output in the plant which would make orders as cheap as possible as there are time constraints. Knowing the initial state of orders as well as time bounds for ordered products the search for the cheapest manufacturing arrangement is initiated. The approach emphasised in the article is an autonomous one as there are no general methods for optimising manufacturing tasks. The model of the system is significant as it eliminates excessive costs, allowing the operator of the system to distribute the orders to the plants whose configuration increases the profits of the global company by minimising the total manufacturing costs. The article focuses on the information approach to the simulation process of the logistic manufacturing system. This kind of approach results from software engineering which has led to creating the dedicated software to carry out required calculations leading to reaching the satisfactory solution.

The proposed model represents one of the possible solutions for optimising production in case of dislocated companies which are at disposal. It is significant as it is possible to use it for a particular realistic case described by the defined structure and, of course, it can be treated as one of the scientific bases for solving similar examples (e.g., adaptation of the model for other structures, implementation of other decision-making algorithms, and expansion of other evaluation parameters, for example, from the set of key performance indicators (KPI) of the company).

2. Mathematical Model

Symbols used for logistic system modelling presented in this section are explained in Symbols shown at the end of the paper. They remain in accordance with the standards of mathematical models used for the description and simulation of logistic, production, trade, and other systems.

It is assumed that manufacturing processes take place in the complex system which consists of A subsystems. Each αth subsystem, $\alpha = 1, \ldots, A$, carries out exactly the same production operations; however, their initial state may differ. Orders can be made in any αth manufacturing plant.

The separate manufacturing plant has a serial form and consists of work stands assembled in sequence. The stands are equipped with tools which are subject to wear throughout the manufacturing process. When a tool in a stand is completely worn out, the manufacturing process is continued with the use of another tool in the same stand as long as either its life or the manufacturing route allows it. It is assumed that tools cannot be replaced separately. When all tools in the given stand are worn out or the manufacturing process cannot be continued through this stand, the discussed stand is subject to replacement. There are buffer stores between the work stands.

The manufacturing plant described above can be multiplied which results in creating exactly the same manufacturing system in various locations. It is assumed that M customer can order N types of products.

Let us assume orders to be made are shown in the matrix of orders:

$$Z^k = \left[z^k_{m,n}\right], \quad k = 1, \ldots, K,\ m = 1, \ldots, M,\ n = 1, \ldots, N, \tag{1}$$

where $z^k_{m,n}$ is the nth order of the mth customer at the kth stage.

If the nth product cannot be made for the mth customer then $z^k_{m,n} = -1$; otherwise $z^k_{m,n} \geq 0$.

Products are made from various types of charges which leads to introducing the adjustment matrix of charges to products:

$$\Omega = \left[\omega_{l/(m,n)}\right], \quad l = 1, \ldots, L,\ m = 1, \ldots, M,\ n = 1, \ldots, N, \tag{2}$$

where $\omega_{l/(m,n)}$ is the adjustment of the nth order of the mth customer to the lth charge (the number of the charge element).

The discussed manufacturing system takes the form shown in the matrix of structure:

$$E = \left[e^{\alpha}_{i,j}\right], \quad \alpha = 1, \ldots, A,\ i = 1, \ldots, I,\ j = 1, \ldots, J, \tag{3}$$

where $e^{\alpha}_{i,j}$ is the ith tool in the jth work stand in the αth manufacturing plant.

The route matrix is introduced:

$$D = \left[d_{j,n}\right], \quad j = 1, \ldots, J,\ n = 1, \ldots, N, \tag{4}$$

where $d_{j,n}$ is the number of the ith tool in the jth work stand used for making the nth type product.

The base life matrix is introduced:

$$G = \left[g_{i,j}\right], \quad i = 1, \ldots, I, \; j = 1, \ldots, J, \tag{5}$$

where $g_{i,j}$ is the number of base elements of a product which can be made by the ith tool in the jth work stand before it becomes worn out.

Let us associate the matrix of coefficients with the previous matrix of life:

$$\begin{gathered} \Psi = \left[\psi_{(m,n)/(i,j)}\right], \\ m = 1, \ldots, M, \; n = 1, \ldots, N, \; i = 1, \ldots, I, \; j = 1, \ldots, J, \end{gathered} \tag{6}$$

where $\psi_{(m,n)/(i,j)}$ is the coefficient determining how many units of the nth type order for the mth customer can be made by the ith tool in the jth tool stand.

At the same time $\psi_{(m,n)/(i,j)} \geq 1$ if the nth type product is made by the ith tool in the jth work stand; otherwise $\psi_{(m,n)/(i,j)} = 0$.

The base coefficient for the calculation purpose $\psi_0 = 1$.

The life matrix for the nth product of the mth customer takes the following form:

$$\begin{gathered} G = \left[g_{(m,n)/(i,j)}\right], \\ m = 1, \ldots, M, \; n = 1, \ldots, N, \; i = 1, \ldots, I, \; j = 1, \ldots, J, \end{gathered} \tag{7}$$

where $g_{(m,n)/(i,j)}$ is the number of nth products for the mth customer elements which can be made by the ith tool in the jth work stand.

At the same time $g_{(m,n)/(i,j)} = \psi_{(m,n)/(i,j)} \cdot g_{i,j}$, $m = 1, \ldots, M$, $n = 1, \ldots, N$, $i = 1, \ldots, I$, $j = 1, \ldots, J$.

The structure vector of base capacity of buffer stores is introduced:

$$B = \left[b_j^{\alpha}\right], \quad \alpha = 1, \ldots, A, \; j = 1, \ldots, J-1, \tag{8}$$

where b_j^{α} is the base capacity of the buffer store placed behind the jth tool stand in the αth manufacturing system. At the same time $b_j^{\alpha} > 0$ if the buffer is active; otherwise $b_j^{\alpha} = -1$.

It is assumed that each αth manufacturing plant has the same active structure of the buffer store system; however, their capacity may differ.

Let us introduce the matrix of coefficients of the buffer stores:

$$\begin{gathered} \Psi_b = \left[\psi_{j,n}^{\alpha}\right], \\ \alpha = 1, \ldots, A, \; j = 1, \ldots, J-1, \; n = 1, \ldots, N, \end{gathered} \tag{9}$$

where $\psi_{j,n}^{\alpha}$ is the coefficient of the jth buffer store in case of storing the nth type product in the αth manufacturing system.

The state of the buffer store is expressed as follows:

$$\begin{gathered} B^k = \left[b_{\alpha/j}^k\right], \\ \alpha = 1, \ldots, A, \; j = 1, \ldots, J-1, \; k = 1, \ldots, K, \end{gathered} \tag{10}$$

where $b_{\alpha/j}^k$ is the state of the jth buffer store in the αth manufacturing plant at the kth stage.

The manufacturing system is always defined in its current state shown in the base matrix of state:

$$\begin{gathered} S^k = \left[s(\alpha)_{i,j}^k\right], \\ \alpha = 1, \ldots, A, \; k = 1, \ldots, K, \; i = 1, \ldots, I, \; j = 1, \ldots, J, \end{gathered} \tag{11}$$

where $s(\alpha)_{i,j}^k$ is the base state of the ith tool in the jth stand in the αth manufacturing system at the kth stage.

The state of the system is recalculated and shown in the matrix of state:

$$S_{\alpha}^k = \left[s(\alpha)_{(m,n)/(i,j)}^k\right], \quad \alpha = 1, \ldots, A, \; k = 1, \ldots, K, \; i = 1, \ldots, I, \; j = 1, \ldots, J, \; m = 1, \ldots, M, \; n = 1, \ldots, N, \tag{12}$$

where $s(\alpha)_{(m,n)/(i,j)}^k$ is the state of the ith tool in the jth stand in the αth manufacturing system in case of making the nth product for the mth customer at the kth stage. At the same time $s(\alpha)_{(m,n)/(i,j)}^k = \psi_{n/(i,j)} \cdot s(\alpha)_{i,j}^k$.

The base flow capacity of the manufacturing system is defined in the base matrix of flow capacity:

$$\begin{gathered} P^k = \left[p(\alpha)_{i,j}^k\right], \\ \alpha = 1, \ldots, A, \; k = 1, \ldots, K, \; i = 1, \ldots, I, \; j = 1, \ldots, J, \end{gathered} \tag{13}$$

where $p(\alpha)_{i,j}^k$ is the base capacity of the ith tool in the jth stand in the αth manufacturing system at the kth stage.

The flow capacity of the system is recalculated and shown in the matrix of flow capacity:

$$P_{\alpha}^k = \left[p(\alpha)_{(m,n)/(i,j)}^k\right], \quad \alpha = 1, \ldots, A, \; k = 1, \ldots, K, \; i = 1, \ldots, I, \; j = 1, \ldots, J, \; m = 1, \ldots, M, \; n = 1, \ldots, N, \tag{14}$$

where $p(\alpha)^k_{(m,n)/(i,j)}$ is the flow capacity of the ith tool in the jth stand in the αth manufacturing system in case of making the nth product for the mth customer at the kth stage. At the same time $p(\alpha)^k_{(m,n)/(i,j)} = \psi_{n/(i,j)} \cdot p(\alpha)^k_{i,j}$.

The flow capacity of the ith tool in the jth stand in the αth manufacturing system in case of making the nth product for the mth customer at the kth stage is expressed as follows:

$$p(\alpha)^k_{(m,n)/(i,j)} = \psi_{(m,n)/(i,j)} \cdot g_{i,j} - \psi_{(m,n)/(i,j)} \cdot s(\alpha)^k_{(m,n)/(i,j)}. \tag{15}$$

The matrix of manufacturing times takes the following form:

$$T^{\mathrm{pr}} = \left[\tau^{\mathrm{pr}}_{(m,n)/(i,j)}\right], \quad m = 1,\ldots,M,\ n = 1,\ldots,N,\ i = 1,\ldots,I,\ j = 1,\ldots,J, \tag{16}$$

where $\tau^{\mathrm{pr}}_{(m,n)/(i,j)}$ is the manufacturing time of one unit of the nth product for the mth customer by the ith tool of the jth work stand.

The vector of replacement times takes the following form:

$$T^{\mathrm{repl}} = \left[\tau^{\mathrm{repl}}_j\right], \quad j = 1,\ldots,J, \tag{17}$$

where τ^{repl}_j is the replacement time of tools in the jth work stand.

Let us introduce the vector of unit time costs:

$$C_{\mathrm{UTC}} = [c_\alpha], \quad \alpha = 1,\ldots,A, \tag{18}$$

where c_α is the cost of one time unit of operating the αth plant.

2.1. Equations of State. The state of the complex manufacturing system changes after either any decision concerning producing any nth product for the mth customer in any αth manufacturing plant or replacing any jth stand in the αth manufacturing plant:

$$S^0_\alpha \longrightarrow \cdots \longrightarrow S^k_\alpha \longrightarrow \cdots \longrightarrow S^K_\alpha. \tag{19}$$

The state of the ith tool in the jth stand in case of making the nth product for the mth customer changes consequently throughout the manufacturing course:

$$s(\alpha)^0_{(m,n)/(i,j)} \longrightarrow \cdots \longrightarrow s(\alpha)^k_{(m,n)/(i,j)} \longrightarrow \cdots \longrightarrow s(\alpha)^K_{(m,n)/(i,j)}, \tag{20}$$

which can be written in the following form:

$$s(\alpha)^k_{(m,n)/(i,j)} = \begin{cases} s(\alpha)^{k-1}_{(m,n)/(i,j)} & \text{in case of not making the } n\text{-th product for the } m\text{-th customer by the } i\text{-th tool in the } j\text{-th stand in the } \alpha\text{-th manufacturing system at the } k\text{-th stage,} \\ s(\alpha)^{k-1}_{(m,n)/(i,j)} + x(\alpha)^k_{(m,n)/(i,j)} & \text{otherwise,} \end{cases} \tag{21}$$

where $x(\alpha)^k_{(m,n)/(i,j)}$ is the amount of the nth product for the mth customer made by the ith tool in the jth stand in the αth manufacturing system at the kth stage.

The state of the base ith tool in case of replacement the jth stand changes as follows:

$$s(\alpha)^k_{i,j} = \begin{cases} s(\alpha)^{k-1}_{i,j} & \text{if the } j\text{-th stand is replaced,} \\ 0 & \text{otherwise.} \end{cases} \tag{22}$$

The order matrix Z^k changes after every decision concerning production activity:

$$Z^0 \longrightarrow Z^1 \longrightarrow \cdots \longrightarrow Z^k \longrightarrow \cdots \longrightarrow Z^K. \tag{23}$$

The order matrix is modified after every decision about production:

$$z^k_{m,n} = \begin{cases} z^{k-1}_{m,n} - x^k_{m,n} & \text{in case of making the } n\text{-th product for the } m\text{-th customer at the } k\text{-th stage,} \\ z^{k-1}_{m,n} & \text{otherwise,} \end{cases} \tag{24}$$

where $x^k_{m,n}$ is the number of units of the nth product for the mth customer made at the kth stage.

2.2. Control Algorithms. There is a need to seek satisfactory solutions by making orders in various locations as they generate various manufacturing costs. Comparing costs allows us to determine where the nth order should be manufactured to minimise production costs. In order to control the course of manufacturing there is also a need to introduce sample heuristic algorithms. First of all, it is necessary to determine the αth manufacturing plant to place an order in. Secondly, the nth product for the mth customer to be made is to be chosen. The flow capacity means how many elements of the order matrix can be passed through the manufacturing plant before production capabilities of the subsystem are used up. First of all, the operator of the system is obliged to choose

the αth plant, $\alpha = 1, \ldots, A$. For example, if the algorithm of the minimal flow capacity of the production plant is set it means that a plant which can make the fewest number of products is required; then the search for the αth plant meeting this condition is initiated. Having detected such a plant, the algorithm of the maximal order is implemented which means that the biggest order to be made through the available route in the αth plant is required. Such an approach is explained in detail in [21, 29].

2.2.1. The Algorithm of the Maximal Flow Capacity of the Production Plant. This algorithm chooses the αth manufacturing plant on condition that it is characterised by the maximal coefficient of flow capacity $\xi_\alpha^k = \sum_{i=1}^{I} \sum_{j=1}^{J} p(\alpha)_{(m,n)/(i,j)}^k$. To determine the λth manufacturing plant where $1 \le \lambda \le A$ condition (25) must be met:

$$\left(q_{\alpha\_\max}^k = \xi_\lambda^k\right) \Longleftrightarrow \left[\xi_\lambda^k = \max_{1\le\alpha\le A} \xi_\alpha^k\right]. \tag{25}$$

2.2.2. The Algorithm of the Minimal Flow Capacity of the Production Plant. This algorithm chooses the αth manufacturing plant on condition that it is characterised by the minimal coefficient of flow capacity $\xi_\alpha^k = \sum_{i=1}^{I} \sum_{j=1}^{J} p(\alpha)_{(m,n)/(i,j)}^k$. To determine the λth plant where $1 \le \lambda \le A$ condition (26) must be met:

$$\left(q_{\alpha\_\min}^k = \xi_\lambda^k\right) \Longleftrightarrow \left[\xi_\lambda^k = \min_{1\le\alpha\le A} \xi_\alpha^k\right]. \tag{26}$$

2.2.3. The Algorithm of the Maximal Order. This algorithm chooses the order matrix element characterised by the maximal value of $\gamma_{m,n}^k$. To produce the order $z_{\mu,\eta}^k$, $1 \le \mu \le M$, $1 \le \eta \le N$, the following condition must be met where $\gamma_{m,n}^k = z_{m,n}^k$:

$$\left(q_{z\_\max}^k = z_{\mu,\eta}^k\right) \Longleftrightarrow \left[\gamma_{\mu,\eta}^k = \max_{\substack{1\le m\le M\\ 1\le n\le N}} \gamma_{m,n}^k\right]. \tag{27}$$

2.2.4. The Algorithm of the Minimal Order. This algorithm chooses the order matrix element characterised by the minimal value $\gamma_{m,n}^k$. To produce the order $z_{\mu,\eta}^k$, $1 \le \mu \le M$, $1 \le \eta \le N$, the following condition must be met where $\gamma_{m,n}^k = z_{m,n}^k$:

$$\left(q_{z\_\min}^k = z_{\mu,\eta}^k\right) \Longleftrightarrow \left[\gamma_{\mu,\eta}^k = \min_{\substack{1\le m\le M\\ 1\le n\le N}} \gamma_{m,n}^k\right]. \tag{28}$$

2.3. Manufacturing Strategies. In order to make orders in the αth subsystem, the following strategies are proposed:

(i) The immediate strategy: a new order is introduced into the manufacturing system as soon as such a possibility emerges.

(ii) The delayed strategy: a new order is introduced into the manufacturing system only then when the preceding one leaves the system.

2.4. Criteria. There are a few criteria to be taken into account; however, the most decisive one in terms of manufacturing under time pressure seems to be the minimal manufacturing time criterion so it is chosen for further consideration. As orders are made in various plants there are corresponding aspects of this problem to be analysed. One of them is a different location of each plant which is connected with different manufacturing costs. It is assumed that each manufacturing process is carried out in accordance with the two-stage criteria specified below. First of all, the criterion of minimising the manufacturing time is introduced:

$$Q_T = \sum_{\alpha=1}^{A} \sum_{k=1}^{K} \sum_{i=1}^{I} \sum_{j=1}^{J} y'(\alpha)_{i,j}^k \cdot \tau_{(m,n)/(i,j)}^{\mathrm{pr}} \longrightarrow \min, \tag{29}$$

where

$$y'(\alpha)_{i,j}^k = \begin{cases} 1 & \text{if the } i\text{-th tool in the } j\text{-th stand in the } \alpha\text{-th manufacturing plant} \\ & \text{is implemented in the manufacturing process at the } k\text{-th stage,} \\ 0 & \text{otherwise.} \end{cases} \tag{30}$$

Having generated the best solution from the point of view of minimising time, there is a need to minimise manufacturing costs by means of the criterion of minimising manufacturing costs:

$$Q_C = \sum_{\alpha=1}^{A} c_\alpha \longrightarrow \min. \tag{31}$$

3. Case Study

In order to verify correctness of the assumptions presented in the paper hereby it is necessary to carry out the simulation process with the use of sample data. Apart from obtaining results concerning the minimal manufacturing time it is possible to present other associating results for each simulation process, that is, total replacement time of tools in work stands, lost flow capacity of tools due to premature replacement, and remaining capacity of tools in case of completing making the whole order.

The following general assumptions for the case study are taken into account:

(i) All available manufacturing plants are at the same initial state as follows:

$$\underset{1\le\alpha\le A}{\exists} S_\alpha^0 = S^0. \tag{32}$$

(ii) There is enough charge material for making the whole order matrix elements.

The simulator used for simulating making order receives data drawn at random from a specified range:

$$Z^0 = \begin{bmatrix} 24 & 32 & 50 & 51 & 29 \\ 39 & 24 & 33 & 28 & 33 \\ 46 & 47 & 48 & 33 & 39 \\ 46 & 43 & 51 & 30 & 26 \\ 37 & 43 & 48 & 44 & 49 \end{bmatrix};$$

$$G = \begin{bmatrix} 5 & 9 & 9 & 6 & 6 \\ 10 & 9 & 7 & 9 & 10 \\ 8 & 6 & 6 & 9 & 6 \end{bmatrix};$$

$$S^0 = \begin{bmatrix} 0 & 4 & 6 & 1 & 0 \\ 7 & 6 & 3 & 7 & 6 \\ 1 & 5 & 2 & 6 & 0 \end{bmatrix};$$

$$\Psi = \begin{bmatrix} 1 & 3 & 2 & 1 & 3 \\ 1 & 3 & 1 & 2 & 1 \\ 3 & 3 & 3 & 3 & 3 \\ 3 & 2 & 2 & 3 & 3 \\ 2 & 1 & 2 & 1 & 2 \end{bmatrix};$$

$$\Psi_b = \begin{bmatrix} 4 & 5 & 2 & 5 & 4 \\ 2 & 2 & 2 & 5 & 3 \\ 5 & 4 & 5 & 2 & 3 \\ 3 & 2 & 4 & 4 & 2 \\ 2 & 1 & 2 & 5 & 2 \end{bmatrix};$$

$$\Omega = \begin{bmatrix} 2 & 2 & 2 & 1 & 1 \\ 2 & 2 & 2 & 2 & 1 \\ 2 & 2 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 2 & 2 \end{bmatrix};$$

$$D = \left[\begin{array}{ccccc:ccccc:ccccc:ccccc:ccccc} 1 & 1 & 2 & 3 & 1 & 1 & 3 & 3 & 1 & 1 & 3 & 1 & 1 & 1 & 3 & 3 & 2 & 1 & 1 & 3 & 2 & 2 & 1 & 2 & 3 \\ 3 & 3 & 3 & 2 & 1 & 2 & 2 & 2 & 2 & 3 & 3 & 3 & 3 & 3 & 2 & 2 & 1 & 3 & 2 & 2 & 2 & 2 & 1 & 2 & 3 \\ 1 & 1 & 3 & 3 & 1 & 1 & 1 & 3 & 3 & 3 & 1 & 3 & 1 & 3 & 1 & 2 & 1 & 2 & 1 & 2 & 2 & 1 & 3 & 2 & 3 \\ 1 & 1 & 1 & 2 & 1 & 3 & 3 & 3 & 3 & 3 & 3 & 2 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 2 & 2 & 1 & 3 & 2 \\ 1 & 2 & 3 & 1 & 2 & 2 & 2 & 1 & 3 & 3 & 3 & 2 & 2 & 2 & 2 & 2 & 3 & 1 & 3 & 1 & 3 & 2 & 1 & 2 & 1 \end{array}\right];$$

$$T_{\mathrm{pr}} = \begin{bmatrix} 5 & 3 & 5 & 4 & 5 & \vdots & 4 & 5 & 4 & 3 & 4 & \vdots & 3 & 4 & 5 & 3 & 4 & \vdots & 4 & 3 & 4 & 4 & 4 & \vdots & 4 & 5 & 5 & 3 & 5 \\ 4 & 3 & 5 & 4 & 4 & \vdots & 5 & 5 & 3 & 3 & 3 & \vdots & 4 & 5 & 3 & 3 & 3 & \vdots & 5 & 3 & 4 & 5 & 4 & \vdots & 5 & 3 & 5 & 4 & 5 \\ 3 & 5 & 4 & 5 & 4 & \vdots & 4 & 3 & 4 & 5 & 4 & \vdots & 4 & 3 & 3 & 3 & 5 & \vdots & 5 & 5 & 4 & 3 & 3 & \vdots & 5 & 5 & 3 & 4 & 5 \\ 3 & 5 & 3 & 5 & 5 & \vdots & 4 & 3 & 4 & 4 & 3 & \vdots & 3 & 4 & 4 & 4 & 3 & \vdots & 3 & 4 & 4 & 4 & 3 & \vdots & 3 & 5 & 3 & 3 & 5 \\ 4 & 5 & 3 & 4 & 5 & \vdots & 4 & 4 & 3 & 3 & 5 & \vdots & 5 & 3 & 3 & 4 & 4 & \vdots & 5 & 5 & 4 & 3 & 3 & \vdots & 5 & 3 & 4 & 5 & 5 \end{bmatrix};$$

$$T_{\mathrm{repl}} = \begin{bmatrix} 9 & 8 & 8 & 8 & 8 \\ 7 & 9 & 7 & 9 & 9 \\ 7 & 9 & 9 & 9 & 7 \end{bmatrix};$$

$$B = \begin{bmatrix} -1 & -1 & -1 & 0 \end{bmatrix};$$

$$C_{\mathrm{UTC}} = \begin{bmatrix} 5 & 6 & 4 & 7 & 8 \end{bmatrix}. \tag{33}$$

The simulator implemented for processing the data either generates them at random or accepts them from the file or the operator of the system introduces them manually. It is possible to alternate data if necessary. In our case the data are obtained from the ready file which was subject to thorough verification [26]. Figure 1 presents the sample way of introducing initial data for the subsequent simulation process.

The data in Figure 1 are subject to the subsequent simulation procedure which generates results shown in Table 1. The main criterion remains the minimal manufacturing time; however, results of the total replacement time, the lost flow capacity, and the remaining capacity are given for comparison reasons. First of all, the strategies of immediate manufacturing and delayed manufacturing are employed; then the algorithm of the maximal flow capacity of the αth production plant $q^k_{\alpha\_\max}$ (max_flow) and the algorithm of the minimal flow capacity of the αth production plant $q^k_{\alpha\_\min}$ (min_flow) are introduced. Finally, the algorithm of the maximal order $q^k_{z\_\max}$ (max_order) and the algorithm of the minimal order $q^k_{z\_\min}$ (min_flow) are introduced. Additionally, results of the 1000 simulations for both manufacturing strategies are presented. The input conditions are the same for simulation experiments in case of the system consisting of either 3 or 4 or 5 manufacturing plants which are subject to the subsequent thorough analysis.

In the case of the three-plant system the total minimal manufacturing time (1997 time units) is obtained by means of carrying out 1000 simulations with the use of the so-called immediate strategy; however, it does not guarantee achieving the lowest value of the total replacement time as well as the lowest value of the lost flow capacity and the highest value of the remaining capacity.

In the case of the four-plant system the total minimal manufacturing time (1509 time units) is also obtained by means of carrying out 1000 simulations with the use of the so-called immediate strategy. This time, it achieves the lowest value of the total replacement time as well as the lowest value of the lost flow capacity. Unfortunately, it delivers the worst value of the remaining capacity.

In the case of the five-plant system the total minimal manufacturing time (1242 time units) is again obtained by means of carrying out 1000 simulations with the use of the immediate strategy. It also reaches the lowest value of the total replacement time as well as the lowest value of the lost flow capacity. Again, it delivers the worst value of the remaining capacity. In conclusion, all best results are obtained by means of the immediate manufacturing strategy.

Let us assume that the results shown in Table 2 enable us to take the total minimal manufacturing times for granted in each manufacturing system, that is, consisting of 3, 4, and 5 available plants on the condition that they are to be used simultaneously. The total minimal manufacturing time in each system consisting of A subsystems is the longest time in the αth subsystem belonging to this system according to the table of results.

The times shown in Table 2 represent sequences of making orders in each αth plant. The sequences were obtained by means of simulation processes carried out for the data of this case study. Each result of the simulation process generates the best sequence of orders made in the separate plants of each system. For illustration reasons Table 3 shows time scheduling in case of implementing the time minimisation criterion for the 5-plant system. It forms the basis for further analysis.

The sequences for minimising the total manufacturing time are given in Table 4 and, at the same time, they are abbreviated to the adequate shortcuts where ρ^{β}_{A} is the βth sequence in the system consisting of A identical plants (it must be emphasised that for the needs of the case study the initial state is the same for each αth subsystem).

The basis for improving the sample time scheduling approach is the minimal total manufacturing time obtained

Table 1: Results of simulation experiments.

Number of plants	Criterion	Immediate manufacturing					Delayed manufacturing				
		max_order max_flow	max_order min_flow	min_order max_flow	min_order min_flow	1000 sim	max_order max_flow	max_order min_flow	min_order max_flow	min_order min_flow	1000 sim
3	Total manufacturing time	2045	2045	2140	2140	**1997**	2196	2196	2223	2223	2124
	Total replacement time	1487	1487	1576	1576	1395	1392	1392	1503	1503	**1309**
	Lost flow capacity	7,00	7,00	4,50	4,50	9,83	7,17	7,17	4,50	4,50	**1,50**
	Remaining capacity	185,33	185,33	**207,83**	**207,83**	133,83	189,17	189,17	**207,83**	**207,83**	130,50
4	Total manufacturing time	1553	1553	1613	1613	**1509**	1648	1648	1704	1704	1609
	Total replacement time	1286	1286	1337	1337	**1163**	1236	1236	1338	1338	1119
	Lost flow capacity	6,17	6,17	5,67	5,67	**0,83**	8,17	8,17	5,67	5,67	1,17
	Remaining capacity	258,17	258,17	251,67	251,67	192,00	**282,17**	**282,17**	251,67	251,67	193,83
5	Total manufacturing time	1320	1317	1326	1358	**1242**	1381	1373	1390	1416	1308
	Total replacement time	1129	1151	1137	1159	**1005**	1163	1159	1158	1172	1003
	Lost flow capacity	7,50	8,00	5,83	7,00	**0,33**	7,17	6,33	6,83	6,83	1,50
	Remaining capacity	303,83	**354,33**	278,50	293,33	239,83	318,17	332	296,5	308,50	240,00

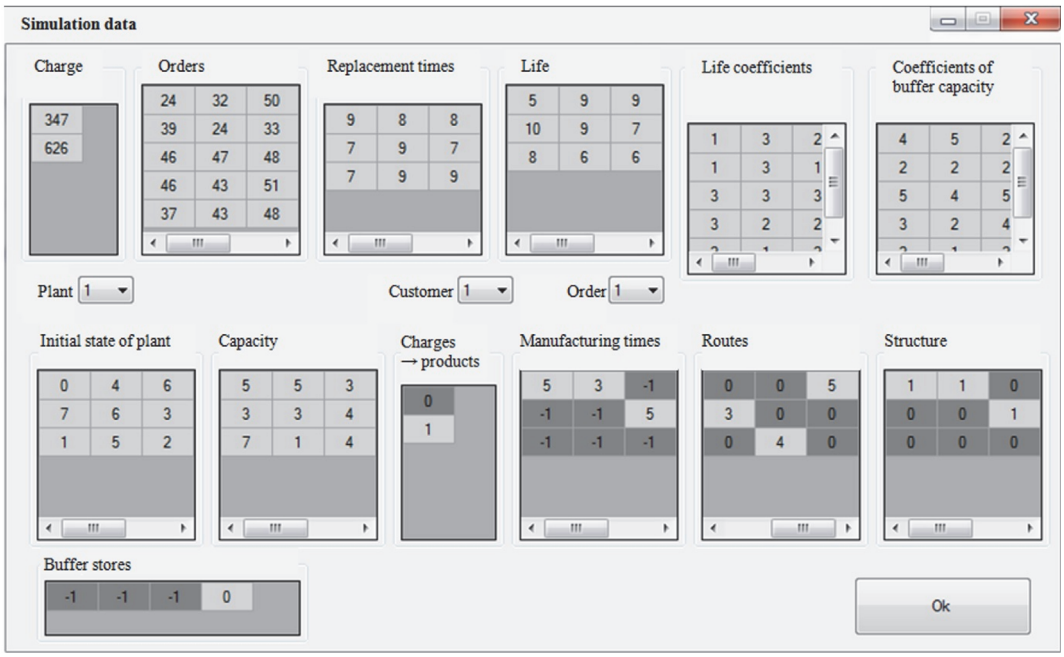

Figure 1: Sample initial data introduction for $\alpha = 1$, $m = 1$, and $n = 1$.

Table 2: Manufacturing times in autonomous systems.

α	System		
	3 plants	4 plants	5 plants
1	1968	1466	**1242**
2	**1997**	1489	**1242**
3	1957	1498	1196
4	×	**1509**	1235
5	×	×	1168
Total manufacturing time in the system	**1997**	**1509**	**1242**

Table 3: Time scheduling in case of implementing the time minimisation criterion.

Order number	$z^0_{1,1}$	$z^0_{1,2}$	$z^0_{1,3}$	$z^0_{1,4}$	$z^0_{1,5}$
Plant number	1	5	3	4	4
Start	119	442	0	813	0
End	353	640	363	1235	161
Order number	$z^0_{2,1}$	$z^0_{2,2}$	$z^0_{2,3}$	$z^0_{2,4}$	$z^0_{2,5}$
Plant number	2	5	2	1	4
Start	900	0	624	540	462
End	1242	134	917	722	698
Order number	$z^0_{3,1}$	$z^0_{3,2}$	$z^0_{3,3}$	$z^0_{3,4}$	$z^0_{3,5}$
Plant number	3	5	1	3	1
Start	733	901	708	344	336
End	996	1168	970	503	554
Order number	$z^0_{4,1}$	$z^0_{4,2}$	$z^0_{4,3}$	$z^0_{4,4}$	$z^0_{4,5}$
Plant number	3	5	2	1	4
Start	979	622	0	0	675
End	1196	922	334	134	828
Order number	$z^0_{5,1}$	$z^0_{5,2}$	$z^0_{5,3}$	$z^0_{5,4}$	$z^0_{5,5}$
Plant number	3	2	4	1	5
Start	488	319	144	909	120
End	751	650	479	1242	469

by means of implementing the immediate manufacturing strategy: 1997 for the 3-plant system, 1509 for the 4-plant system, and 1242 for the 5-plant system (see Table 1). The results for the 5-plant system are shown graphically as a result of the simulation process (Figure 2).

The simulation results are associated with system plants; however, it is possible to move the manufacturing process to any other plant in the system as they perform the same production operations. Following this kind of reasoning it was assumed that it is possible to transfer the sequence of orders adjusted to a certain plant on the basis of the simulation results to any other plant in the discussed system. To carry out this process it is necessary to implement permutation without repetitions in order to find the solution which can minimise manufacturing costs to meet the criterion of minimising costs. It became possible only by comparing the results obtained by the method using the permutations without repetitions. In case of 3 plants there are 3! = 6 possible solutions (see Table 5), 4 plants generate 4! = 24 possible solutions (see Table 6), and, finally, 5 plants deliver 5! = 120 possible solutions (see Table 7).

In the 3-plant system total manufacturing costs are minimised in the case of the combination of production

Table 4: Sequences leading to minimisation the total manufacturing time.

Number of plant	Sequence in the 3-plant system	Production time
1	$\rho_3^1 = z_{5,1}^k z_{3,3}^k z_{1,3}^k z_{5,4}^k z_{4,4}^k z_{2,4}^k z_{3,5}^k z_{1,5}^k z_{2,2}^k$	1968
2	$\rho_3^2 = z_{4,5}^k z_{1,4}^k z_{1,2}^k z_{5,3}^k z_{4,1}^k z_{3,2}^k z_{4,2}^k z_{2,1}^k$	1997
3	$\rho_3^3 = z_{5,2}^k z_{3,1}^k z_{2,5}^k z_{4,3}^k z_{5,5}^k z_{1,1}^k z_{2,3}^k z_{3,4}^k$	1957
Number of plant	Sequence in the 4-plant system	Production time
1	$\rho_4^1 = z_{2,2}^k z_{4,1}^k z_{1,2}^k z_{3,4}^k z_{1,1}^k z_{1,5}^k z_{4,4}^k z_{2,3}^k$	1466
2	$\rho_4^2 = z_{3,2}^k z_{2,4}^k z_{4,2}^k z_{4,3}^k z_{5,4}^k z_{4,5}^k$	1489
3	$\rho_4^3 = z_{3,3}^k z_{1,4}^k z_{3,1}^k z_{5,2}^k z_{5,1}^k z_{3,5}^k$	1498
4	$\rho_4^4 = z_{5,5}^k z_{5,3}^k z_{2,5}^k z_{2,1}^k z_{1,3}^k$	1509
Number of plant	Sequence in the 5-plant system	Production time
1	$\rho_5^1 = z_{4,4}^k z_{1,1}^k z_{3,5}^k z_{2,4}^k z_{3,3}^k z_{5,4}^k$	1242
2	$\rho_5^2 = z_{4,3}^k z_{5,2}^k z_{2,3}^k z_{2,1}^k$	1242
3	$\rho_5^3 = z_{1,3}^k z_{3,4}^k z_{5,1}^k z_{3,1}^k z_{4,1}^k$	1196
4	$\rho_5^4 = z_{1,5}^k z_{5,3}^k z_{2,5}^k z_{4,5}^k z_{1,4}^k$	1235
5	$\rho_5^5 = z_{2,2}^k z_{5,5}^k z_{1,2}^k z_{4,2}^k z_{3,2}^k$	1168

Table 5: Manufacturing costs in the three-plant system (all data).

Plant	Unit cost	Combination number of production sequences											
		1		2		3		4		5		6	
1	5	ρ_3^1	9840	ρ_3^1	9840	ρ_3^2	9985	ρ_3^2	9985	ρ_3^3	9785	ρ_3^3	9785
2	6	ρ_3^2	11982	ρ_3^3	11742	ρ_3^1	11808	ρ_3^3	11742	ρ_3^1	11808	ρ_3^2	11982
3	4	ρ_3^3	7828	ρ_3^2	7988	ρ_3^3	7828	ρ_3^1	7872	ρ_3^2	7988	ρ_3^1	7872
Total costs		29650		**29570**		29621		29599		29581		29639	

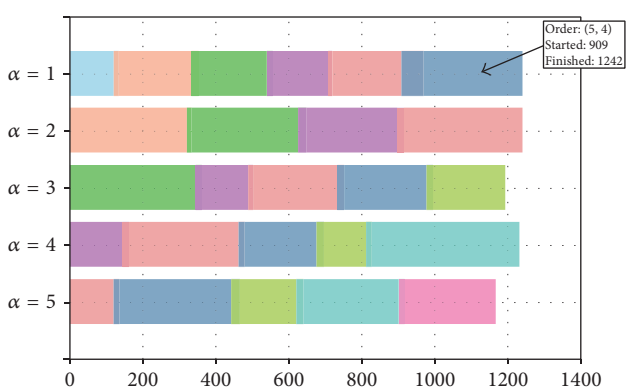

Figure 2: Results of the simulation process for $A = 5$.

sequence number 2 by 80 monetary units comparing them with the best result for 1000 simulations (see Table 1).

In the 4-plant system total manufacturing costs are minimised in the case of the combination of production sequence number 17 by 97 monetary units comparing them with the best result for 1000 simulations (see Table 1).

On the basis of comparison results shown in Table 7, the course of the manufacturing process in the five-plant system was illustrated. Total manufacturing costs are minimised in the case of the combinations of production sequence numbers 15 and 33 which are the best in terms of seeking the minimal costs of making orders. As seen in Table 7, implementing these two combinations resulting from searching for the solution to minimise the total manufacturing costs lowers them by 131 monetary units comparing them with the best result for 1000 simulations (see Table 1). Finding the time minimising combinations numbers 15 and 33 enables us to adjust the best sequence of orders to the plants which minimise the summarised manufacturing time shown in Table 8.

The results obtained on the basis of the data implemented for the purpose of the case study let us come to a conclusion that under the very theoretical conditions specified in the article it can be expected that the greater number of manufacturing plants in the system leads to bigger financial savings.

4. Conclusions

It can be expected that carrying out more simulation experiments for a certain number of manufacturing plants in the system may deliver even a shorter total manufacturing time which is the time in the system characterised by the longest manufacturing time in its subsystem. This result forms the basis for improving the total manufacturing costs

Table 6: Manufacturing costs in the four-plant system (all data).

Plant	Unit cost	Combination number of production sequences											
		1		2		3		4		5		6	
1	5	ρ_4^1	7330	ρ_4^1	7330	ρ_4^1	7330	ρ_4^1	7330	ρ_4^1	7330	ρ_4^1	7330
2	6	ρ_4^2	8934	ρ_4^2	8934	ρ_4^3	8988	ρ_4^3	8988	ρ_4^4	9054	ρ_4^4	9054
3	4	ρ_4^3	5992	ρ_4^4	6036	ρ_4^4	6036	ρ_4^2	5956	ρ_4^2	5956	ρ_4^3	5992
4	7	ρ_4^4	10563	ρ_4^3	10486	ρ_4^2	10423	ρ_4^4	10563	ρ_4^3	10486	ρ_4^2	10423
Total costs		32819		32786		32777		32837		32826		32799	
Plant	Unit cost	Combination number of production sequences											
		7		8		9		10		11		12	
1	5	ρ_4^2	7445	ρ_4^2	7445	ρ_4^2	7445	ρ_4^2	7445	ρ_4^2	7445	ρ_4^2	7445
2	6	ρ_4^3	8988	ρ_4^3	8988	ρ_4^4	9054	ρ_4^4	9054	ρ_4^1	8796	ρ_4^1	8796
3	4	ρ_4^4	6036	ρ_4^1	5864	ρ_4^1	5864	ρ_4^3	5992	ρ_4^3	5992	ρ_4^4	6036
4	7	ρ_4^1	10262	ρ_4^4	10563	ρ_4^3	10486	ρ_4^1	10262	ρ_4^4	10563	ρ_4^3	10486
Total costs		32731		32860		32849		32753		32796		32763	
Plant	Unit cost	Combination number of production sequences											
		13		14		15		16		17		18	
1	5	ρ_4^3	7490	ρ_4^3	7490	ρ_4^3	7490	ρ_4^3	7490	ρ_4^3	7490	ρ_4^3	7490
2	6	ρ_4^4	9054	ρ_4^4	9054	ρ_4^1	8796	ρ_4^1	8796	ρ_4^2	8934	ρ_4^2	8934
3	4	ρ_4^1	5864	ρ_4^2	5956	ρ_4^2	5956	ρ_4^4	6036	ρ_4^4	6036	ρ_4^1	5864
4	7	ρ_4^2	10423	ρ_4^1	10262	ρ_4^4	10563	ρ_4^2	10423	ρ_4^1	10262	ρ_4^4	10563
Total costs		32831		32762		32805		32745		**32722**		32851	
Plant	Unit cost	Combination number of production sequences											
		19		20		21		22		23		24	
1	5	ρ_4^4	7545	ρ_4^4	7545	ρ_4^4	7545	ρ_4^4	7545	ρ_4^4	7545	ρ_4^4	7545
2	6	ρ_4^1	8796	ρ_4^1	8796	ρ_4^2	8934	ρ_4^2	8934	ρ_4^3	8988	ρ_4^3	8988
3	4	ρ_4^2	5956	ρ_4^3	5992	ρ_4^3	5992	ρ_4^1	5864	ρ_4^1	5864	ρ_4^2	5956
4	7	ρ_4^3	10486	ρ_4^2	10423	ρ_4^1	10262	ρ_4^3	10486	ρ_4^2	10423	ρ_4^1	10262
Total costs		32783		32756		32733		32829		32820		32751	

Table 7: Manufacturing costs in the five-plant system (chosen data).

Plant	Unit cost	Combination number of production sequences											
		11		12		13		14		15		16	
1	5	ρ_5^1	6210	ρ_5^1	6210	ρ_5^1	6210	ρ_5^1	6210	ρ_5^1	6210	ρ_5^1	6210
2	6	ρ_5^3	7176	ρ_5^3	7176	ρ_5^4	7410	ρ_5^4	7410	ρ_5^4	7410	ρ_5^4	7410
3	4	ρ_5^2	4968	ρ_5^2	4968	ρ_5^5	4672	ρ_5^5	4672	ρ_5^2	4968	ρ_5^2	4968
4	7	ρ_5^4	8645	ρ_5^5	8176	ρ_5^2	8694	ρ_5^3	8372	ρ_5^3	8372	ρ_5^5	8176
5	8	ρ_5^5	9344	ρ_5^4	9880	ρ_5^3	9568	ρ_5^2	9936	ρ_5^5	9344	ρ_5^3	9568
Total costs		36343		36410		36554		36600		**36304**		36332	
Plant	Unit cost	Combination number of production sequences											
		17		18		19		20		21		33	
1	5	ρ_5^1	6210	ρ_5^1	6210	ρ_5^1	6210	ρ_5^1	6210	ρ_5^1	6210	ρ_5^2	6210
2	6	ρ_5^4	7410	ρ_5^4	7410	ρ_5^5	7008	ρ_5^5	7008	ρ_5^5	7008	ρ_5^4	7410
3	4	ρ_5^3	4784	ρ_5^3	4784	ρ_5^2	4968	ρ_5^2	4968	ρ_5^3	4784	ρ_5^1	4968
4	7	ρ_5^5	8176	ρ_5^2	8694	ρ_5^3	8372	ρ_5^4	8645	ρ_5^4	8645	ρ_5^3	8372
5	8	ρ_5^2	9936	ρ_5^5	9344	ρ_5^4	9880	ρ_5^3	9568	ρ_5^2	9936	ρ_5^5	9344
Total costs		36516		36442		36438		36399		36583		**36304**	

Table 8: Manufacturing costs in case of the immediate strategy in the 5-plant system.

Plant number	Production sequence	End time	Unit cost	Total costs
Best results for 1000 simulations				
1	$z^0_{4,4}z^0_{1,1}z^0_{3,5}z^0_{2,4}z^0_{3,3}z^0_{5,4}$	1242	5	6210
2	$z^0_{4,3}z^0_{5,2}z^0_{2,3}z^0_{2,1}$	1242	6	7452
3	$z^0_{1,3}z^0_{3,4}z^0_{5,1}z^0_{3,1}z^0_{4,1}$	1196	4	4784
4	$z^0_{1,5}z^0_{5,3}z^0_{2,5}z^0_{4,5}z^0_{1,4}$	1235	7	8645
5	$z^0_{2,2}z^0_{5,5}z^0_{1,2}z^0_{4,2}z^0_{3,2}$	1168	8	9344
Total manufacturing costs				**36435**
Permutations without repetitions, combination 15				
1	$z^0_{4,4}z^0_{1,1}z^0_{3,5}z^0_{2,4}z^0_{3,3}z^0_{5,4}$	1242	5	6210
2	$z^0_{1,5}z^0_{5,3}z^0_{2,5}z^0_{4,5}z^0_{1,4}$	1235	6	7410
3	$z^0_{4,3}z^0_{5,2}z^0_{2,3}z^0_{2,1}$	1242	4	4968
4	$z^0_{1,3}z^0_{3,4}z^0_{5,1}z^0_{3,1}z^0_{4,1}$	1196	7	8372
5	$z^0_{2,2}z^0_{5,5}z^0_{1,2}z^0_{4,2}z^0_{3,2}$	1168	8	9344
Total manufacturing costs				**36304**
Permutations without repetitions, combination 33				
1	$z^0_{4,3}z^0_{5,2}z^0_{2,3}z^0_{2,1}$	1242	5	6210
2	$z^0_{1,5}z^0_{5,3}z^0_{2,5}z^0_{4,5}z^0_{1,4}$	1235	6	7410
3	$z^0_{4,4}z^0_{1,1}z^0_{3,5}z^0_{2,4}z^0_{3,3}z^0_{5,4}$	1242	4	4968
4	$z^0_{1,3}z^0_{3,4}z^0_{5,1}z^0_{3,1}z^0_{4,1}$	1196	7	8372
5	$z^0_{2,2}z^0_{5,5}z^0_{1,2}z^0_{4,2}z^0_{3,2}$	1168	8	9344
Total manufacturing costs				**36304**

by means of seeking for such a combination of adjusting the sequence of production to plants which minimise the total manufacturing costs. It also seems reasonable to implement other criteria, that is, either the criterion of minimising lost capacity or the criterion of maximising the remaining capacity or the criterion of minimising the replacement time. Nevertheless, the assumptions for the mentioned criteria differ from the ones presented in the case study, emphasising both the criterion of minimising the manufacturing time and the criterion of minimising manufacturing costs. Moreover, it is necessary to analyse the behaviour of the system in case of various initial states obtained at random from data characterised by the same range for drawing.

Symbols

α: The number of the subsystem, $\alpha = 1, \ldots, A$

$b^k_{\alpha/j}$: The state of the jth buffer store in the αth manufacturing plant at the kth stage

b^α_j: The base capacity of the buffer store placed behind the jth tool stand in the αth manufacturing system

c_α: The cost of one unit time of operating the αth plant

C_{UTC}: The vector of unit time costs

B: The structure vector of base capacity of buffer stores

B^k: The state of the buffer store

$d_{j,n}$: The number of the ith tool in the jth work stand used for making the nth type product

D: The route matrix

$e^\alpha_{i,j}$: The ith tool in the jth work stand in the αth manufacturing plant

E: The matrix of structure

$g_{i,j}$: The number of base elements of a product which can be made by the ith tool in the jth work stand before it becomes worn out

G: The base life matrix

k: The stage number resulting from the preceding decision in the system, $k = 1, \ldots, K$

m: The number of the customer, $m = 1, \ldots, M$

n: The type of order, $n = 1, \ldots, N$

$p(\alpha)^k_{i,j}$: The base capacity of the ith tool in the jth stand in the αth manufacturing system at the kth stage

P^k: The base matrix of flow capacity

P^k_α: Matrix of flow capacity

Q_C: The criterion of minimising manufacturing costs

Q_T: The criterion of minimising the manufacturing time

$q^k_{\alpha\_\max}$: The algorithm of the maximal flow capacity of the αth production plant

$q^k_{\alpha\_\min}$: The algorithm of the minimal flow capacity of the αth production plant
$q^k_{z\_\max}$: The algorithm of the maximal order
$q^k_{z\_\min}$: The algorithm of the minimal order
$s(\alpha)^k_{i,j}$: The base state of the ith tool in the jth stand in the αth manufacturing system at the kth stage
$s(\alpha)^k_{(m,n)/(i,j)}$: The state of the ith tool in the jth stand in case of making the nth product for the mth customer
S^k: The kth state of the system
S^k_α: The kth state of the αth system manufacturing subsystem
$x^k_{m,n}$: The number of units of the nth product for the mth customer made at the kth stage
$x(\alpha)^k_{(m,n)/(i,j)}$: The amount of the nth product for the mth customer made by the ith tool in the jth stand in the αth manufacturing system at the kth stage
$z^k_{m,n}$: The nth order of the mth customer at the kth stage
Z^k: The matrix of orders in the kth stage
$\tau^{\mathrm{pr}}_{(m,n)/(i,j)}$: The manufacturing time of one unit of the nth product for the mth customer by the ith tool of the jth work stand
τ^{repl}_j: The replacement time of the jth work stand
T^{pr}: The matrix of manufacturing times
T^{repl}: The vector of replacement times
ξ^k_α: The coefficient of flow capacity of the production plant
Ψ_b: The matrix of coefficients of the buffer stores
$\psi^\alpha_{j,n}$: The coefficient of the jth buffer store in case of storing the nth type product in the αth manufacturing system
$\psi_{(m,n)/(i,j)}$: The coefficient determining how many units of the nth type order for the mth customer can be made by the ith tool in the jth tool stand
Ψ: The matrix of coefficients
$\omega_{l/(m,n)}$: The adjustment of the nth order of the mth customer to the lth charge
Ω: The adjustment matrix of charges to products.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Research Article

Novel Complexity Indicator of Manufacturing Process Chains and Its Relations to Indirect Complexity Indicators

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Manufacturing systems can be considered as a network of machines/workstations, where parts are produced in flow shop or job shop environment, respectively. Such network of machines/workstations can be depicted as a graph, with machines as nodes and material flow between the nodes as links. The aim of this paper is to use sequences of operations and machine network to measure static complexity of manufacturing processes. In this order existing approaches to measure the static complexity of manufacturing systems are analyzed and subsequently compared. For this purpose, analyzed competitive complexity indicators were tested on two different manufacturing layout examples. A subsequent analysis showed relevant potential of the proposed method.

1. Introduction

There is no doubt that manufacturing systems are one of the most complex processes. Moreover, the complexity has a tendency to increase due to dynamic changes in global market environment. For example, growing demand for customized products motivates companies to implement a transition toward a mass-customized manufacturing. According to Blecker et al. [1] mass customization induces a high complexity level in a context of various customer requirements and a steadily changing environment. There can be a positive side resulting from high complexity, but usually it brings negative long-term consequences to the business survival. Therefore, an investigation of complexity manufacturing process chains helps to understand and control the nonlinear behaviour of manufacturing systems [2]. Fredendall and Gabriel [3] in this context point out that by "measuring the system's complexity, the managers can identify problems in the system that are hindering the production flow." Isik [4] specifies other negative consequences of complexity related to logistics activities as high operational costs, customer dissatisfaction, time delay in delivery, excess inventory, or inventory shortage. In general, complexity is not easy to measure, since it is difficult to define precisely. Obviously, there are many useful complexity definitions related to manufacturing systems (see, e.g., [5–9]). In addition, several approaches were proposed during the past decades to analyze the manufacturing complexity. Those approaches differ especially in terms of types of complexity. There are two basic types of complexity in relation to the domain of the application: physical and functional [10]. The complexity viewed in terms of functional domain is defined as a measure of uncertainty in achieving the functional requirements. In the physical domain, manufacturing complexity is frequently classified into two types, static and dynamic. Dynamic complexity can be, in simplified manner, defined as uncertainty of the manufacturing system's behaviour in a certain time period [11]. Static complexity is characterized as a function of the structure of the system [12]. Due to the fact that description of dynamic behaviour of manufacturing systems would require establishing complicated analytical equations, more extensive effort was made to develop static complexity metrics. Nevertheless, the debate on the effectiveness of the static complexity measures is still alive. The reason lies in the fact that static models of manufacturing process chains are generated for several purposes, for which different optimality criteria are specified. Moreover, some indicators of the static complexity can be seen as more or less alternative measures and other ones allow reflecting

only specific properties of manufacturing process models. The main challenge in the complexity metric is to increase their effectiveness by getting a better appreciation of the real problems that manufacturers often encounter. Achieving this objective was the main motivation of the presented research to develop the novel method to measure the static complexity by using Shannon information theory.

2. Related Work

Firstly, it is useful to provide a working definition of complex system to specify a context in which this article focuses on the issues. We understand complex system as "a system with numerous components and interconnections, interactions or interdependencies that are difficult to describe, understand, predict, manage, design, and/or change" [13]. This definition inherently assumes that complexity of such systems arises not only from the size of the system but also from the interrelationships of the system components and the unpredictable behaviour of its drivers.

Basically, there are two approaches to constructing complexity measures of manufacturing systems: algorithmic and probabilistic. Algorithmic complexity is object of study in algorithmic information theory and is based on the idea that simple tasks can be done by short algorithm while complex tasks require long computer programs [14]. The probabilistic approach to complexity is analyzing the system's regularities as a basis for determining complexity [15]. The probabilistic approach can be divided into entropy-based complexity measures and axiomatic design-based complexity measures. In axiomatic design theory, the design process is described in terms of the mapping between four domains: the customer domain, the functional domain, the physical domain, and the process domain. Accordingly, complexity is defined as a measure of uncertainty in achieving the specified functional requirements derived from customer needs [16].

The most accepted methods for complexity measures are related to information theory of entropy. The entropy is commonly associated with the amount of order, disorder, or chaos in a thermodynamic system and was first introduced by Clausius [17]. Later it was studied from statistical aspect mainly by Shannon [18]. The entropy-based complexity measure for manufacturing process chains was first adapted and introduced by Frizelle and Woodcock [19]. Complexity in another view is characterized as the number of system elements and relations among them (see, e.g., [20, 21]). Suh [22] confirmed that the manufacturing design complexity may also be seen in terms of variability, disorder, uncertainty, or entropy and proposed early complexity measurement for product design stages. Frizelle and Richards [23] proposed so-called *dynamic entropy model* divided into structural complexity and operational complexity. Fujimoto et al. [24] proposed an information entropy-based measure of complexity for assembly planning. More recently, Hu et al. [25] applied entropy function to quantify the complexity of manufacturing processes and their configurations with examples in machining processes. Elmaraghy et al. [26] developed a set of complexity indices to compare layout alternatives at early design stages. Zhang [27] focuses on modelling static entropy-based complexity in manufacturing systems. His approach provides insight into the inherent complexity of system components and structure. Fisher et al. [28] pointed out that when looking downstream, uncertainties in demand variability create problems in planning, scheduling, and control. Entropy-based approaches to manufacturing complexity were presented in papers by Sivadasan et al. [29] and Deshmukh Abhijit [30]. Based on this theoretical background, complexity is considered as a random variable with different states and corresponding probabilities for each state. Isik [31] presented quite similar approach to operational complexity in manufacturing considering actual and scheduled demand using a deviation of outcomes from the expected outcome value for a definition of state's intervals.

From drivers' point of view, internal and external complexity are mainly considered [32–34]. According to Serdarasan [35] complexity drivers are more or less manageable. Anderson [36] noted that organizational complexity has been traditionally viewed as a structural variation rate. The concept of complexity has also been treated in manufacturing research by analyzing operations processes (see, e.g., [6, 37, 38]). Several research works were conducted on a relation between complexity and manufacturing strategy [39–41].

Inspired from the mentioned literature, our approach presented in this paper is complementary to the approaches mentioned above.

3. Description of Existing Metrics to Measure Static Complexity

In order to identify differences between the proposed method and existing metrics, three similar metrics will be further described and mutually compared including the proposed one. The first of these methods is developed by Deshmukh [12], who comprehensively defined the term static complexity of manufacturing systems and determined avoidable properties of static complexity metrics.

3.1. Metric by Deshmukh. He developed for this purpose three static complexity measures that differ in numbers of input variables. The first of them includes only number of machines; the second one incorporates number of operations and number of parts. The last of them is dedicated for flexible manufacturing systems assuming that the flexible manufacturing systems have the maximum entropy caused by multiple types of parts n, operations m, and machines r. Then, the maximum static complexity can be expressed as follows [12]:

$$H_s = \log m^2 nr. \tag{1}$$

As an example, if we consider manufacturing system consisting of 5 machines, 10 operations, and 20 parts, then static complexity of manufacturing system using (1) equals 4 bits.

Moreover, he defined significant properties of static complexity measures. According to him, any static complexity measure should be able to satisfy the following conditions.

Rule #1. Static complexity should increase with the number of parts and number of machines and operations required to process the part mix.

Rule #2. Static complexity should increase with increase in sequence flexibility for the parts in the production batch.

Rule #3. Static complexity should increase as sharing of resources by parts increases.

Rule #4. If the original part mix is split into two or more groups, then the complexity of processing should remain constant.

These rules will be further used to validate presented metrics whether they follow the conditions or not.

3.2. Metric by Frizelle. The metric proposed by him adopts the concept of Shannon's information entropy. Its mathematical expression can be formulated in simple way as [18]

$$H_s = -\sum_{i=1}^{S} p_i \cdot \log_2 p_i, \tag{2}$$

where S represents the number of possible states the system can be in and p_i is probability of system being in state i.

The principle of his method is based on relation between products and machines according to the scheduled plan. So, the previous equation (2) was modified as follows [38]:

$$H_S = -\sum_{j=1}^{M} \sum_{i=1}^{S} p_{ij} \log_2 p_{ij}, \tag{3}$$

where M represents the number of machines, S is the number of possible planned states the machine j can be in, p_{ij} is probability that the machine j is in state i.

As an example, let us have the manufacturing system produce only one part with one machine. Then, probability of machine being in working state is calculated as ratio between working time of given machine and total manufacturing lead time.

3.3. Metric by Zhang. Zhang [42] proposed measuring static complexity of manufacturing systems using Shannon's information entropy [22]. In his complexity model it is assumed that probabilities of any machine j being in any state i are those that reflect number of operations on available machines. Accordingly, he modified (2) as follows:

$$H_s = -\sum_{j=1}^{M} \sum_{i=1}^{S_j} p_{ij} \log_2 p_{ij}, \tag{4}$$

where M is the number of machines, S_j represents the number of possible planned states the machine j can be in (increased by one scheduled idle state), and p_{ij} is probability of any machine j being in any state i.

3.4. The Proposed Metric. The proposed method to quantify static complexity measures of manufacturing process is equally based on Shannon's information theory. We adopt (2) by changing a meaning of probabilities of machine states in the following way:

$$H_s = -\sum_{j=1}^{M} \sum_{k=1}^{P} p_{jk} \cdot \log_2 p_{jk}, \tag{5}$$

where p_{jk} is probability that part k is being processed on an individual machine j according to scheduling order, P represents the number of parts produced in manufacturing process chain (MPC), and M is the number of all machines of all types.

Moreover, the following is assumed:

(1) Machines in a given manufacturing process chains are organized in serial and/or parallel manner. Then, probability that part k is being processed on an individual machine j is calculated in the following way. When a part is processed on machines in serial manner, then p_{jk} equals $1/M_s$, where M_s is number of machines organized in serial manner. If a part is processed on machines in parallel manner, then $p_{jk} = 1/M_p$, where M_p is number of machines organized in parallel. In case we have serial/parallel arrangement of machines and a part is processed on one of the parallel machines, then p_{jk} equals $1/M_s \cdot M_p$.

(2) If there are identical MPCs (the same type and number of machines producing the same type and amount of parts), then static complexity of manufacturing system is calculated only for one MPC by the proposed method.

To show applicability of the indicator, the following example can be used (see Figure 1). We have serial/parallel arrangement of machines processing one part P.

Then, probabilities p_{jk} that the part is being processed by machines M_j can be calculated as shown in Figure 2.

When the methods described above are mutually compared from the viewpoint of mechanism design, then some significant differences are identified (see in Table 1).

As it can be seen from Table 1, the proposed method seems to be the most comprehensive instrument to measure static complexity of manufacturing systems. In spite of the previous methods, this method includes parts scheduling. It takes into account the probability of parts being processed on individual machine according to scheduling order.

4. Testing of Described Metrics

As metric by Deshmukh is already verified, the three other metrics will be tested by the above described rules using the theoretical examples shown in Figures 3–8. Prior to testing, we will assume also operation time (set in 10 minutes), since metric by Frizelle needs to know this item.

4.1. Testing Complexity Indicators by Rule #1. First, let us denote value of static complexity for MPC with j machines and k parts as $H_{j,k}$.

Table 1: Mutual comparison of static complexity methods.

Static complexity metrics	Number of machines	Number of parts	Number of operations	Flexible routings	Workplace organization	Part scheduling
Metric by Deshmukh	√	√	√	x	x	x
Metric by Frizelle	√	√	√	√	x	x
Metric by Zhang	√	√	√	√	√	x
Proposed metric	√	√	√	√	√	√

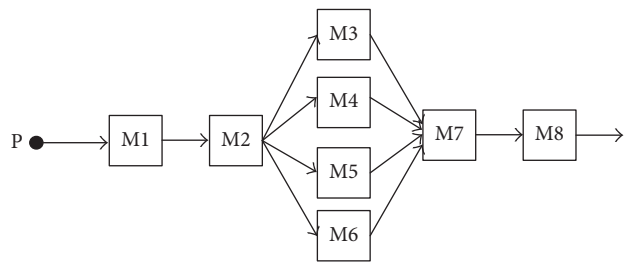

Figure 1: Serial/parallel arrangement of machines processing one part.

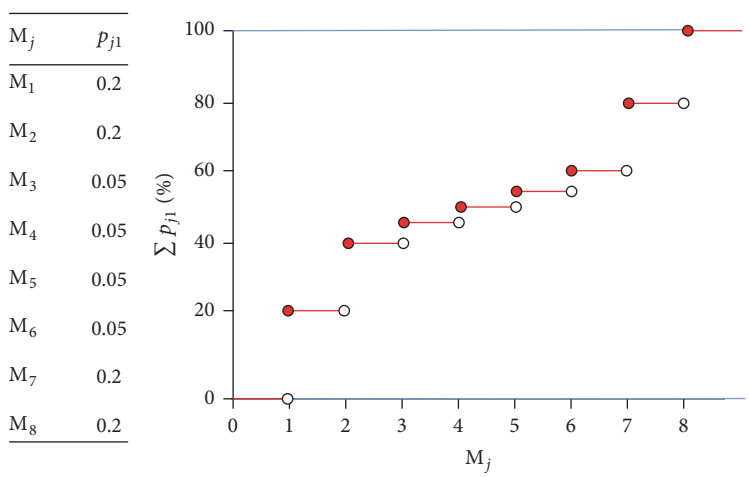

Figure 2: Values of probabilities p_{jk} and their distribution function.

The rule is proposing (in case of the three metrics) two conditional statements:

(I) If k is constant, j is increasing and machines are arranged only in serial manner and then $H_{j-1,k} < H_{j,k}$.

(II) If j is constant, k is increasing and machines are arranged only in serial manner and then $H_{j,k-1} < H_{j,k}$.

4.1.1. Testing of Statement I for the 3 Complexity Metrics. Let us test two MPCs shown in Figure 3 with static complexities $H_{1;1}$ and $H_{2;1}$.

Applying (3), (4), and (5), according to metrics by Frizelle and Zhang and the proposed approach, we obtain the results shown in Table 2.

Table 2: Static complexity values and their proof.

Complexity	Metric by Frizelle	Metric by Zhang	Proposed metric
$H_{1;1}$	0 bits	1 bit	0 bits
$H_{2;1}$	1 bit	2 bits	1 bit
Proof	$H_{2,1} > H_{1,1}$	$H_{2,1} > H_{1,1}$	$H_{2,1} > H_{1,1}$

For example, the complexities according to the proposed metric are calculated using (5) as follows:

$$H_{1,1} = -p_{1,1}\log_2 p_{1,1} = -1\log_2 1 = 0 \text{ bits}$$
$$H_{2,1} = -p_{1,1}\log_2 p_{1,1} + \left(-p_{2,1}\log_2 p_{2,1}\right) = 0{,}5 + 0{,}5 = 1 \text{ bit}. \quad (6)$$

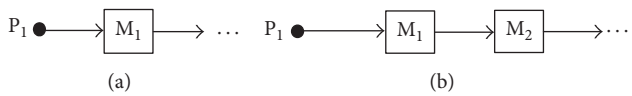

Figure 3: MPC consisting of (a) one machine and one part and (b) two machines and one part.

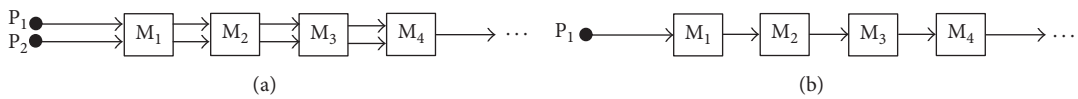

Figure 4: MPC consisting of (a) four machines and two parts and (b) four machines and one part.

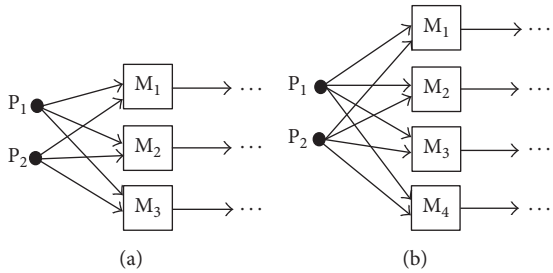

Figure 5: MPC consisting of (a) three machines and two parts and (b) four machines and two parts.

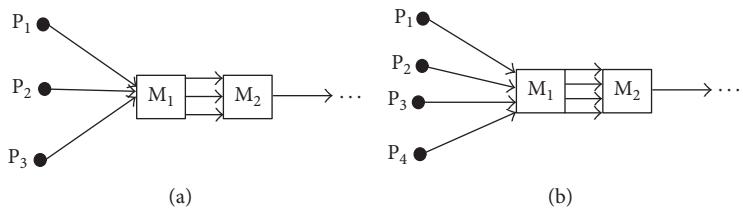

Figure 6: MPC consisting of (a) two machines and three parts and (b) two machines and four parts.

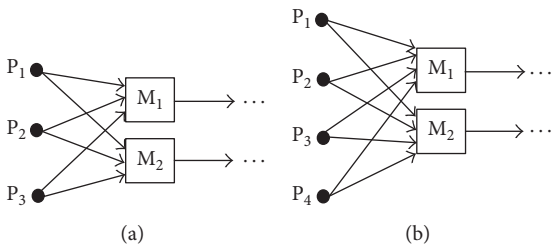

Figure 7: MPC consisting of (a) two machines and three parts and (b) two machines and four parts.

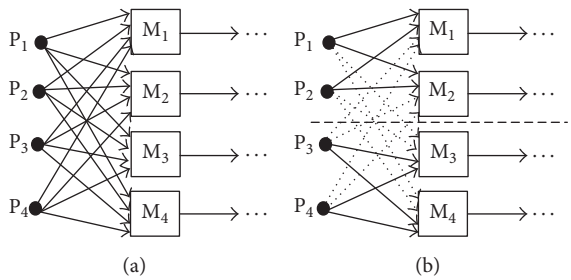

Figure 8: MPC consisting of (a) two machines and four parts and (b) two machines and four parts divided into two groups with one machine and two parts for one group.

Table 3: Static complexity values and their proof.

Complexity	Metric by Frizelle	Metric by Zhang	Proposed metric
$H_{4;1}$	2 bits	4 bits	2 bits
$H_{4;2}$	1,89 bits	6,34 bits	4 bits
Proof	$H_{4,2} < H_{4,1}$	$H_{4,2} > H_{4,1}$	$H_{4,2} > H_{4,1}$

Table 4: Proofs of Rule #1.

Complexity indicator	Proofs for Rule #1 (a)	Proofs for Rule #1 (b)
Metric by Frizelle	True	False
Metric by Zhang	True	True
Proposed metric	True	True

Table 5: Static complexity values and their proof.

Complexity	Metric by Frizelle	Metric by Zhang	Proposed metric
$H_{3;2}$	1,5 bits	4,75 bits	2,86 bits
$H_{4;2}$	1,89 bits	6,34 bits	4 bits
Proof	$H_{4,2} > H_{3,2}$	$H_{4,2} > H_{3,2}$	$H_{4,2} > H_{3,2}$

Table 6: Proofs of Rule #2.

Complexity indicator	Proofs for Rule #2
Metric by Frizelle	True
Metric by Zhang	True
Proposed metric	True

4.1.2. Testing of Statement II for the 3 Complexity Metrics. Let us test two MPCs shown in Figure 4 with static complexities $H_{4;2}$ and $H_{4;1}$.

Applying (3), (4), and (5), according to metrics by Frizelle and Zhang and the proposed approach, we obtain the results shown in Table 3.

Summarily, the results of the proofs are depicted in Table 4.

4.2. Testing Complexity Indicators by Rule #2. The rule proposes the following conditional statement:

> If k is constant, j is increasing and machines are arranged only in parallel manner and then $H_{j-1,k} < H_{j,k}$.

Let us test two MPCs shown in Figure 5 with static complexities $H_{3;2}$ and $H_{4;2}$.

Applying (3), (4), and (5), according to metrics by Frizelle and Zhang and the proposed approach, we obtain the results shown in Table 5.

The results of the proofs are summarized in Table 6.

4.3. Testing Complexity Indicators by Rule #3. The rule is proposing two conditional statements:

Table 7: Static complexity values and their proof.

Complexity	Metric by Frizelle	Metric by Zhang	Proposed metric
$H_{2;3}$	1 bit	4 bits	3 bits
$H_{2;4}$	0,93 bits	4,64 bits	4 bits
Proof	$H_{2,4} < H_{2,3}$	$H_{2,4} > H_{2,3}$	$H_{2,4} > H_{2,3}$

Table 8: Static complexity values and their proof.

Complexity	Metric by Frizelle	Metric by Zhang	Proposed metric
$H_{2;3}$	1 bit	4 bits	3 bits
$H_{2;4}$	0,93 bits	4,64 bits	4 bits
Proof	$H_{2,4} < H_{2,3}$	$H_{2,4} > H_{2,3}$	$H_{2,4} > H_{2,3}$

Table 9: Proofs of Rule #3.

Complexity indicator	Proofs for Rule #3 (a)	Proofs for Rule #3 (b)
Metric by Frizelle	False	False
Metric by Zhang	True	True
Proposed metric	True	True

(I) If j is constant, k is increasing and machines are arranged only in serial manner and then $H_{j,k-1} < H_{j,k}$.

(II) If j is constant, k is increasing and machines are arranged only in parallel and then $H_{j,k-1} < H_{j,k}$.

4.3.1. Testing of Statement I. Let us test two MPCs shown in Figure 6 with static complexities $H_{2;3}$ and $H_{2;4}$.

Applying (3), (4), and (5), according to metrics by Frizelle and Zhang and the proposed approach, we obtain the results shown in Table 7.

4.3.2. Testing of Statement II. Let us test two MPCs shown in Figure 7 with static complexities $H_{2;3}$ and $H_{2;4}$.

Applying (3), (4), and (5), according to metrics by Frizelle and Zhang and the proposed approach, we obtain the results shown in Table 8.

Summarization of the proof results is shown in Table 9.

4.4. Testing Complexity Indicators by Rule #4. The proposition of this rule is as follows:

> If k is constant, j is constant and MPC is split into two groups, and then $H_{j,k} = H_{j/2,k} + H_{j/2,k}$.

Let us test three MPCs shown in Figure 6 with static complexities $H_{4;4}$, $H_{2;4}$, and $H_{2;4}$.

Applying (3), (4), and (5), according to metrics by Frizelle and Zhang and the proposed approach, we obtain the results shown in Table 10.

Summarily, the results of the proofs are depicted in Table 11.

The comparison showed that metrics by Deshmukh and Zhang and the proposed indicator satisfies the rules.

Table 10: Static complexity values and their proof.

Complexity	Metric by Frizelle	Metric by Zhang	Proposed Metric
$H_{2;4}$	0,93 bits	4,64 bits	4 bits
$H_{4;4}$	1,85 bits	9,28 bits	8 bits
Proof	$H_{4,4} = H_{2,4} + H_{2,4}$	$H_{4,4} = H_{2,4} + H_{2,4}$	$H_{4,4} = H_{2,4} + H_{2,4}$

Table 11: Proofs of Rule #4.

Complexity indicator	Proofs for Rule #4
Metric by Frizelle	True
Metric by Zhang	True
Proposed metric	True

However, all the metrics will be further applied on MPC layouts previously used in case studies by Zhang [42] and Yan and Irani [43]. Prior to the application, these two independent layouts and their alternatives in the subsequent section are described.

5. Description of Compared Layouts

Here two layout types are described in this section. The first type (Layout #1) consists of two groups. The first group represents 3 alternatives of 2-cell design layouts and the second group is representing 2 alternatives of 3-cell design layouts. The second type (Layout #2) includes two alternatives of MPC, one arranged as job shop and the other as flow shop.

5.1. Layout #1. MPC1 in Figure 9 is divided into two cells with 23 machines, where first production cell consists of 11 machines and second cell is created by 12 machines. Parts P1–P4 and P7–P11 are processed in first cell; other parts marked as P12–18 are processed only in the second cell, while parts P5 and P6 are partially processed in the second cell and finalized in the first cell.

Manufacturing process chain marked as MPC2 (see in Figure 10) is organized into two cells with 25 machines, while 16 machines are located in cell #1 and 9 machines in cell #2. Parts with numbers from 1 to 11 and 18 are processed in the first cell, while the remaining parts are processed in the second cell.

MPC3 in Figure 11 is similar to MPC2 with machine organization in cells, but it differs that in the second cell machine M1 is redundant and therefore removed. Parts P15 and P16 start to be produced in the first cell and they continue to the second cell for finalization.

MPC4 shown in Figure 12 contains 3 cells. Here there are 26 machines, while 8 machines are located in the first cell, 10 machines are located in the second cell, and 8 machines are located in the third cell. The first cell produces parts with numbers 1, 3, 7, 8, 9, and 11, while part P3 is finalized in the second cell. Parts P2, P4–6, P10, P15, P16, and P18 are produced in the second cell, but parts P15 and P16 are finalized in the third cell along with parts P12–14, P17, and P19.

MPC5 in Figure 13 is divided into 3 cells containing together 24 machines, where the first cell includes 5 machines, the second cell includes 15 machines, and the third cell is comprised of four machines. Parts P1, P3, P7, P8, P9, and P11 are produced in the first cell, while parts P1, P3, P7, P8, and P9 are finalized in the second cell. Parts P2, P4–6, P10, P14, P15, P16, and P18 are machined in the second cell, but parts P15 and P16 are finally produced by machines in the third cell. Remaining parts P12-13, P17, and P19 are machined only in the third cell.

Table 12: Sequence of operations for 19 parts.

Part number	Sequence of operations and their times in minutes
P1	M1 (96), M4 (36), M8 (36), M9 (72)
P2	M1 (36), M4 (120), M7 (20), M4 (120), M8 (24), M7 (20)
P3	M1 (96), M2 (48), M4 (36), M7 (120), M8 (36), M9 (72)
P4	M1 (96), M4 (36), M7 (120), M9 (72)
P5	M1 (96), M6 (72), M10 (200), M7 (120), M9 (72)
P6	M6 (36), M10 (120), M7 (60), M8 (36), M9 (24)
P7	M6 (72), M4 (36), M8 (48), M9 (48)
P8	M3 (144), M5 (120), M2 (48), M6 (72), M4 (36), M8 (48), M9 (48)
P9	M3 (144), M5 (120), M2 (48), M6 (72), M4 (36), M8 (48), M9 (48)
P10	M4 (120), M7 (20), M4 (120), M8 (24)
P11	M6 (72)
P12	M11 (192), M7 (150), M12 (80)
P13	M11 (192), M12 (60)
P14	M11 (288), M7 (180), M10 (360)
P15	M1 (15), M7 (70), M11 (54), M10 (45), M11 (54), M12 (30)
P16	M1 (15), M7 (70), M11 (54), M10 (45), M11 (54), M12 (30)
P17	M11 (192), M7 (150), M12 (80)
P18	M6 (108), M7 (180), M10 (360)
P19	M12 (60)

Layout #1 and its input data were taken from the chapter written by Yan and Irani [43]. These authors studied the impact of 2-cell and 3-cell design layout on MPC performance by comparing their process structure properties. This MPC produces 19 parts (P) by 12 machine types (M). Machine sequence and their operational times for all parts are in Table 12.

5.2. Layout #2. Layout #2 is taken from Zhang's study case [42]. Both, MPC6 for job shop and MPC7 for flow shop, consist of 20 machines of 4 types, while machining time per

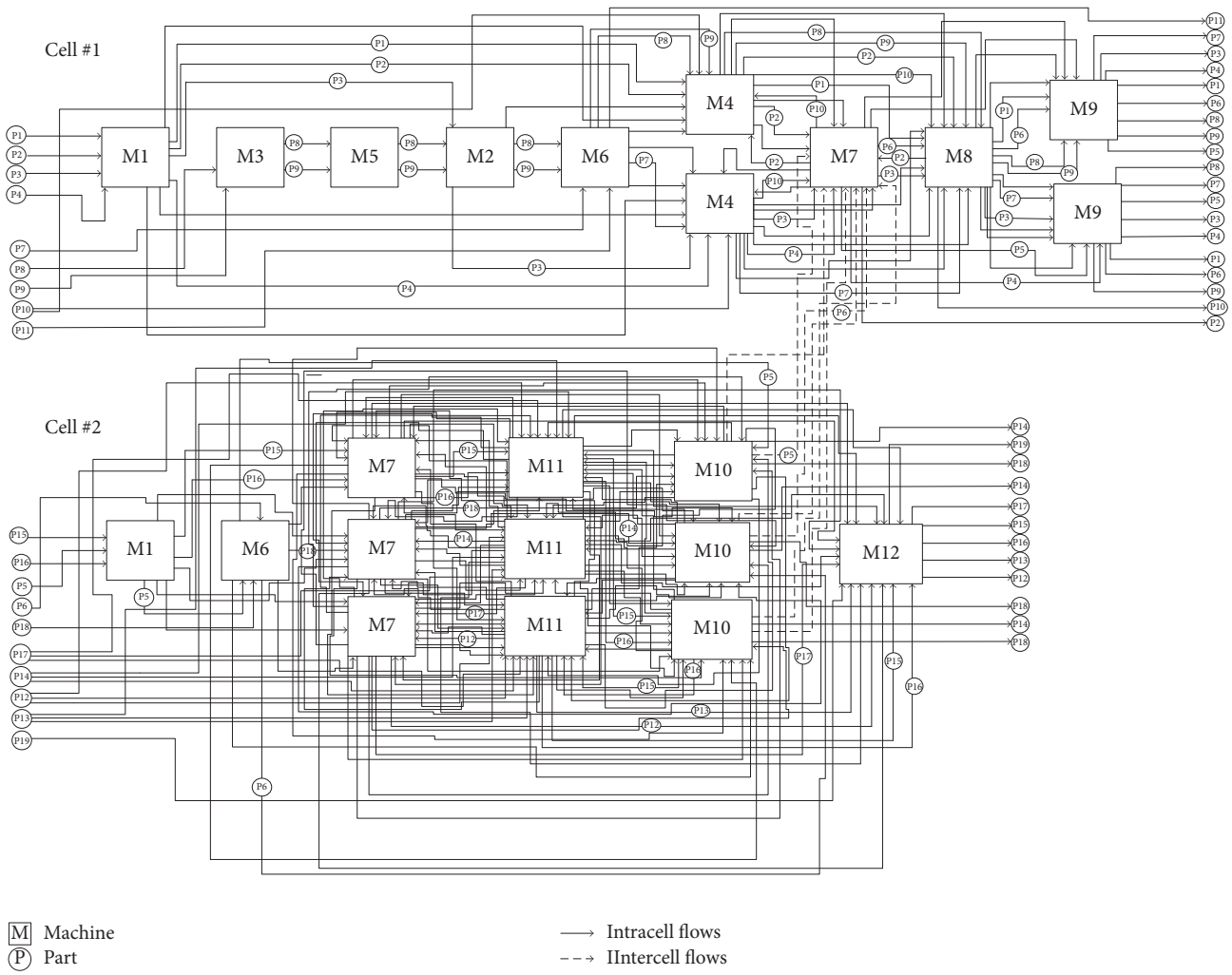

Figure 9: MPC1 and its part flows divided into 2 cells.

Table 13: Comparison of static complexity values for Layout #1.

Layout #1	Metric by Deshmukh	Metric by Frizelle	Metric by Zhang	Proposed metric
MPC1	6,45 bits	18,6 bits	62 bits	47,8 bits
MPC2	6,48 bits	17,7 bits	65,8 bits	50,7 bits
MPC3	6,48 bits	17,4 bits	60,9 bits	53,9 bits
MPC4	6,46 bits	21,7 bits	61,4 bits	44,8 bits
MPC5	6,5 bits	19,1 bits	57,2 bits	47,1 bits

part is 10 minutes. These MPCs produce 100 products and each product passes through one of each machine type.

The first alternative of job shop production in Figure 14 is characterized by arrangement of machines of the same type by free mode.

Transformed layout, in Figure 15, into flow shop production consists of 5 lines. Every line contains four machines of each type. So, the 100 parts are regrouped into five lines, each producing 20 parts.

6. Comparison of Performed Complexity Measures

Calculated values of static complexity are summarized in Tables 13 and 14 for both Layouts #1 and #2 according to the four metrics.

Prior to analyses of results, one must clearly understand the desired aim of the MPC models. Therefore, the following assumptions are formulated.

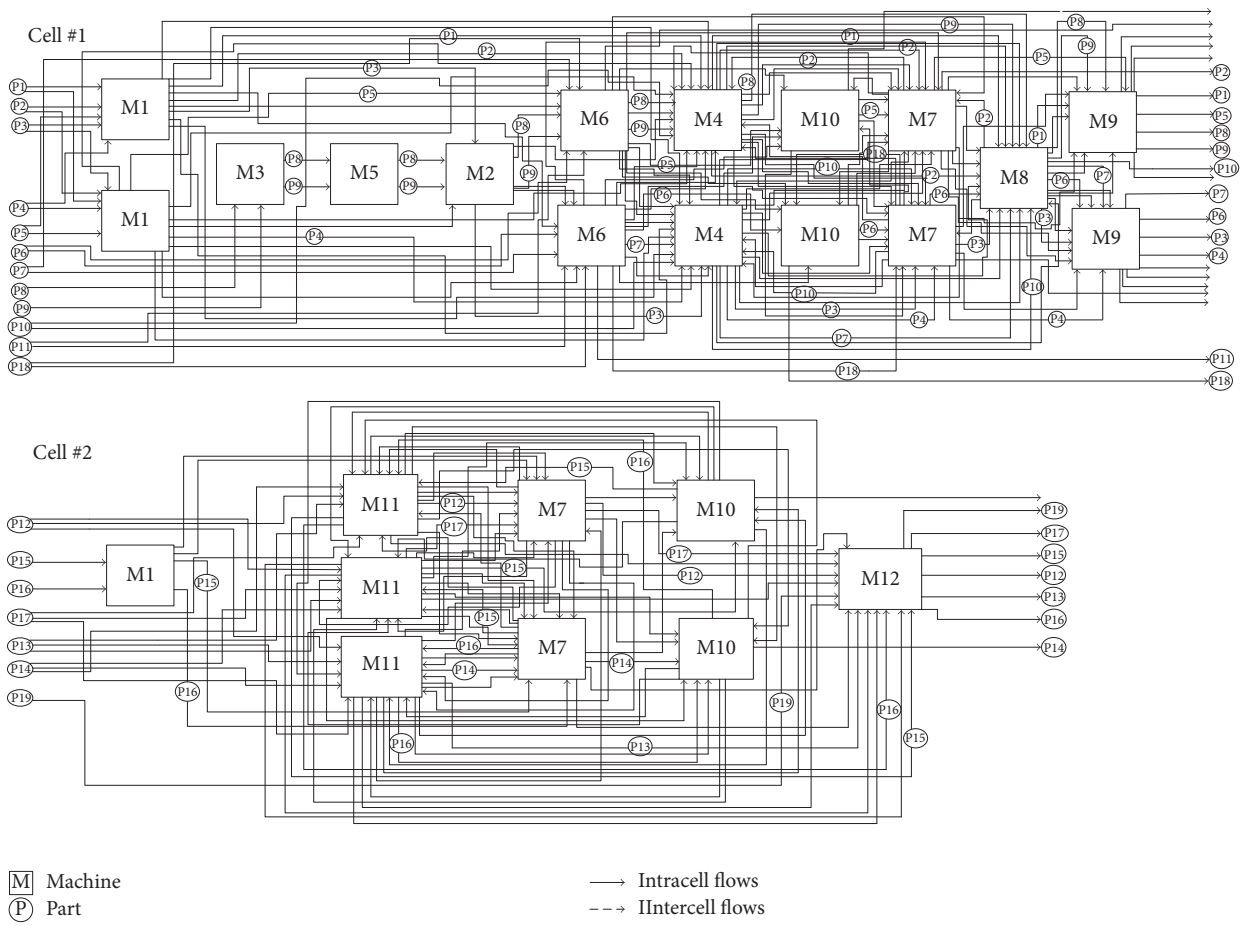

Figure 10: MPC2 and its part flows divided into 2 cells.

Table 14: Comparison of static complexity values for Layout #2.

Layout #2	Metric by Deshmukh	Metric by Frizelle	Metric by Zhang	Proposed metric
MPC6	7,3 bits	10 bits	133 bits	4,32 bits
MPC7	7,3 bits	10 bits	22 bits	2,00 bits

For Layout #1, consider the following.

Assumption Number 1. It is expected that the static complexity of alternatives with three cells is lower than the static complexity of alternatives with two cells. This expectation results from the finding that "production scheduling for two cell solutions is harder than for three cell solutions" [43]. Because it is known that (see, e.g., Rintanen [44]) "harder scheduling problems typically involve uncertainty," it can be stated that harder scheduling brings more complexity to the manufacturing system than easier scheduling.

For Layout #2, consider the following.

Assumption Number 2. The lower complexity of flow shop production is expected, compared to the job shop. This assumption is more or less generally known. For example, Morton and Pentico [45] argue that flow of parts through the shop is usually less complex than that for closed or open job shops.

Table 15: Comparison of complexity indicators according to the specified assumptions.

	Assumption Number 1	Assumption Number 2
Metric by Deshmukh	√	N/A
Metric by Frizelle	x	N/A
Metric by Zhang	x	√
Novel metric	√	√

The aim of testing was to verify whether these metrics follow these tendencies. In this order, Table 15 offers the overview of how the indicators reflect the assumptions.

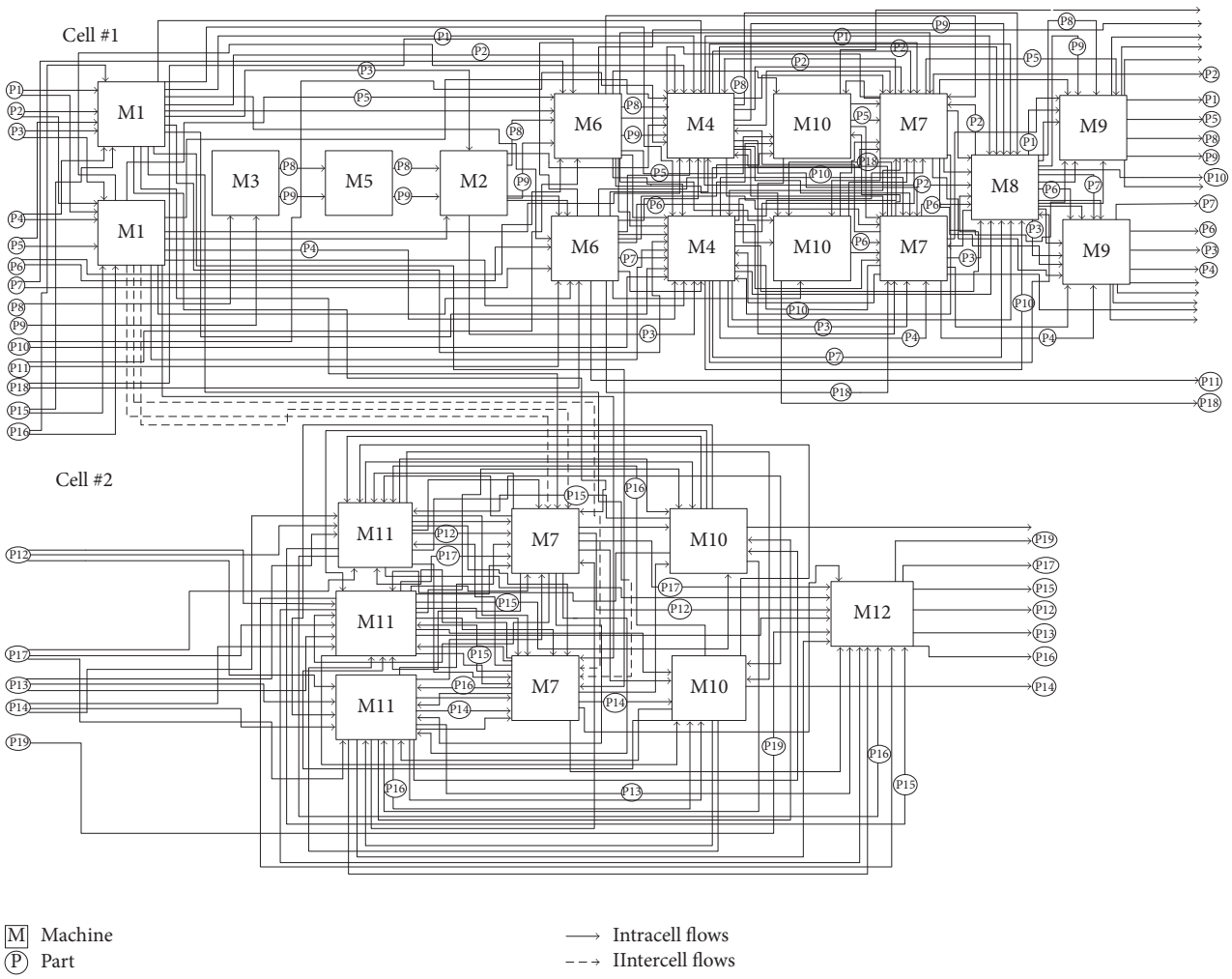

Figure 11: MPC3 and its part flows divided into 2 cells.

Based on the results, the tendencies by Assumption Number 1 were confirmed only by Deshmukh's metric and proposed method. The tendency by Assumption Number 2 was confirmed only by Zhang's metric and novel method, while the remaining metrics are not applicable (N/A).

7. Comparison of the Proposed Method with Indirect Indicators

The comparison shown above yielded positive findings about the proposed method. However, each additional verification of this metric may contribute to its objectivity or versatility. For this reason, the next section is focused on the evaluation of mutual relations between the proposed method and other indirect complexity indicators, such as the production line balancing rate and the number of intercell flows and intracell flows. For this purpose only Layout #1 will be used.

7.1. Description of Indirect Indicators

7.1.1. Production Line Balancing Rate. Production line balancing rate is the quota that measures the average situation of every cycle time in working procedure on processing line. Production line balancing rate is calculated as follows [46]:

$$P = \frac{\sum_{j=1}^{n} t_j}{m * \max\left(T_i\right)}, \tag{7}$$

where t_j is the expression of standard work time of the j job elements, n represents the number of the work elements, m is the number of total lines (cells) in MPCs, T_i represents the work time in the line, and $\max(T_i)$ is the biggest line operating time.

7.1.2. Number of Intercell and Intracell Flows. Part flows of manufacturing system can be classified into intercell and intracell types. It is also known as intercell and intracell layout problems. Intercell flows are expressed as movements between the cells. Intracell flows present the connection between machines at workstations [47]. It is quite frequently stated that intercell flows impact on manufacturing system negatively [48–55]. It is due to the fact that intercell trips are difficult for scheduling and controlling.

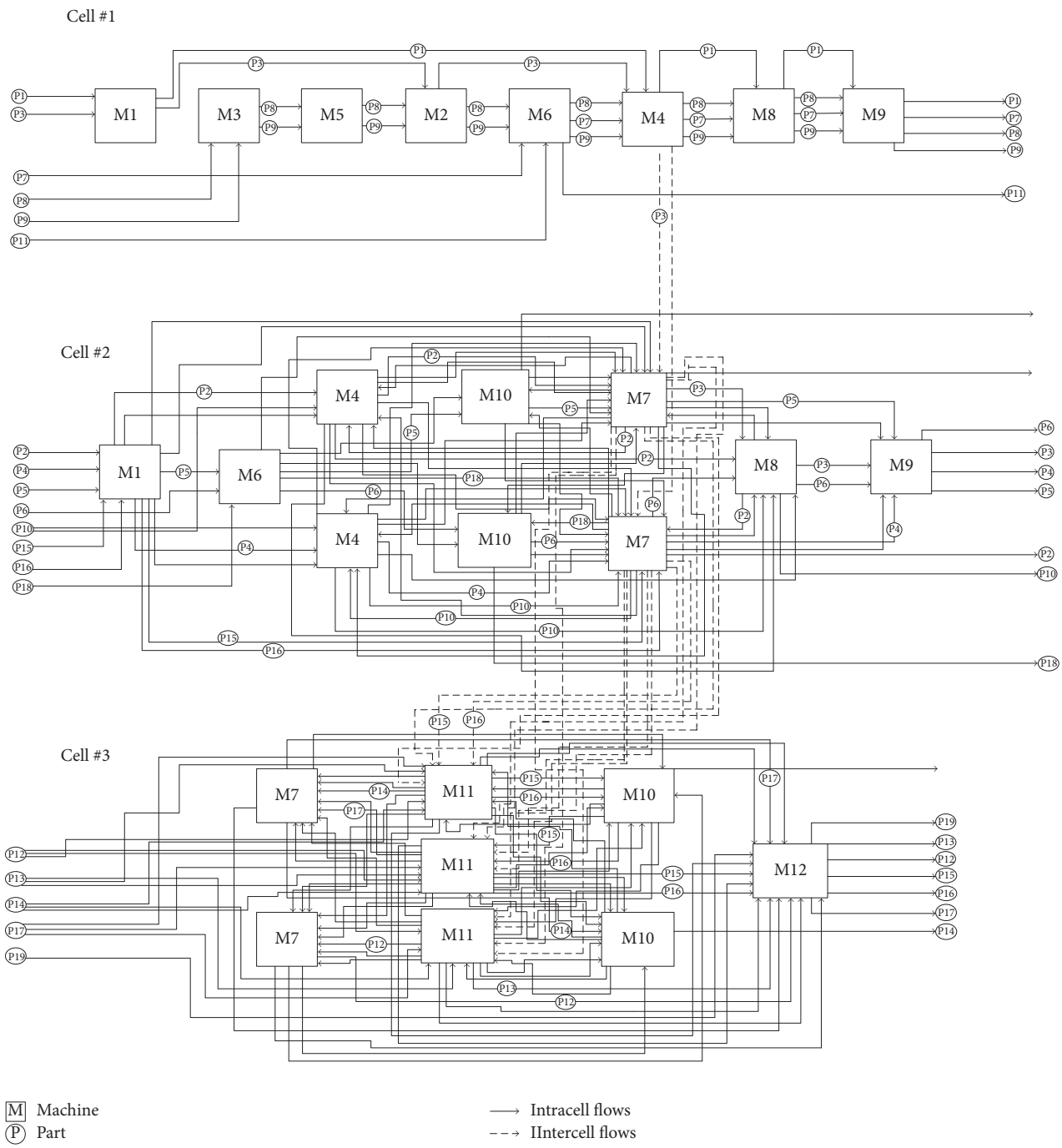

Figure 12: MPC4 and its part flows divided into 3 cells.

7.2. Comparison of the Proposed Method with Indirect Complexity Indicators. The results of indirect indicators calculation are shown in Table 16.

Based on the results in Table 16 one can state that there are significant mutual relations between these indicators and static complexity. Specifically, the following prepropositions can be formulated.

Preproposition Number 1. In the case of well-balancing lines, the static complexity is lower and vice versa.

Preproposition Number 2. The greater number of intercell flows has greater impact on the static complexity than the smaller one.

Preproposition Number 3. The greater number of intracell flows has greater impact on the static complexity than the smaller one.

When verifying the prepropositions by individual MPCs we can conclude that the previous prepropositions were confirmed for all of these three indicators and for all MPCs

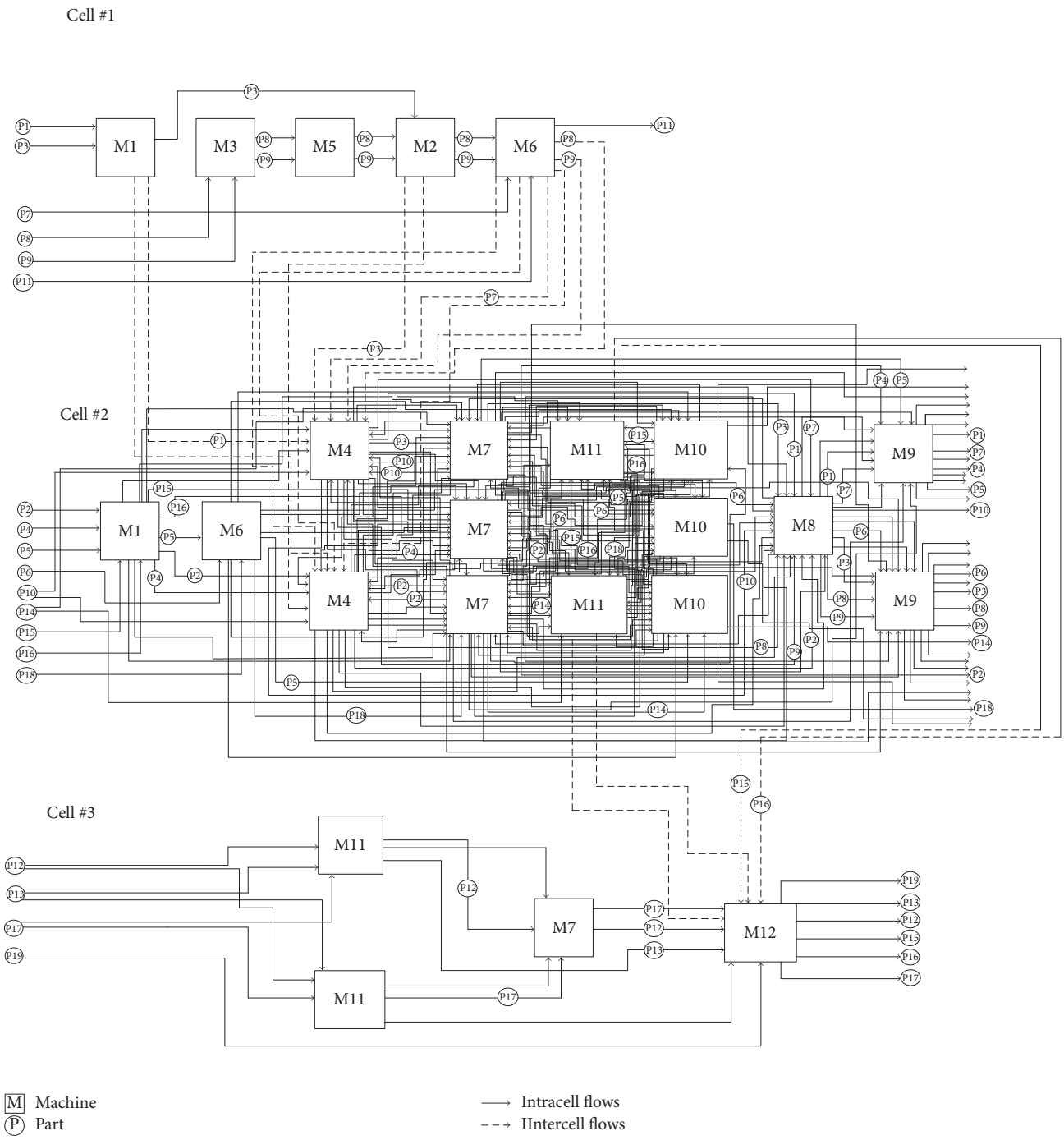

Figure 13: MPC5 and its part flows divided into 3 cells.

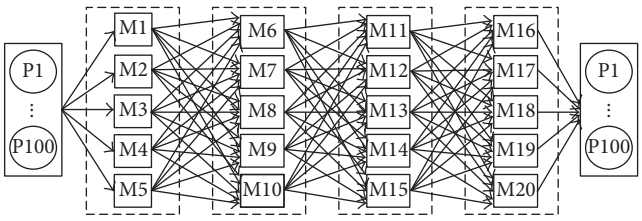

Figure 14: Job shop production with alternative part flows.

Table 16: Comparison of indirect indicators with static complexity measure.

Layout #1	Static complexity	Production line balancing rate	Number of intercell flows	Number of intracell flows
MPC1	47,8 bits	92,4%	15	244
MPC2	50,7 bits	78,7%	0	262
MPC3	53,9 bits	78,2%	8	326
MPC4	44,8 bits	85,6%	6	141
MPC5	47,1 bits	50,7%	14	217

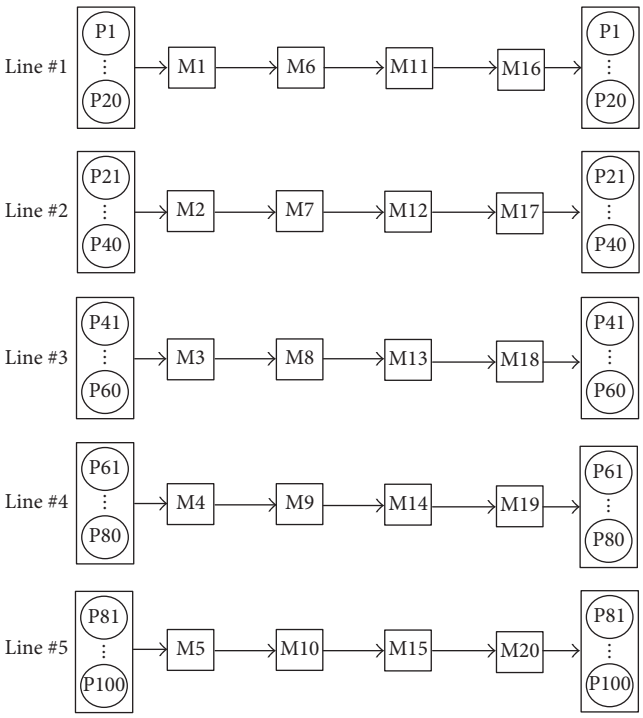

Figure 15: Flow shop production divided into 5 lines.

excluding one situation. It is specifically for number of intercell flows in MPC1, where the number of intercell flows should cause the highest complexity, but in this case the value of static complexity is the lowest.

It can be explained by the fact that it is the specific situation. The second line (cell) contains three machines of M10 type, while in the first line there is no machine of M10 type. But, this layout is the most preferable from the perspective of production line balancing rate.

8. Conclusion

The presented paper offers promising findings for improving the static complexity measurement and assessment of manufacturing systems. It was also proved that the four criteria defined for validation of static complexity have to be respected and we underline their importance for a future research focused on static complexity issues in manufacturing environment.

At the same time, it can be stated that indirect complexity indicators seem to be helpful tools to assess the property of manufacturing systems. It can be also anticipated that production line balancing rate more greatly affects the complexity mitigation, compared to number of intercell part flows and intracell part flows. Nonetheless, it will be necessary to go through a number of simulation experiments with different manufacturing systems to verify these prepropositions.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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Research Article

Complex Characteristics of Multichannel Household Appliance Supply Chain with the Price Competition

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This paper studies the complex characteristics caused by the price competition in multichannel household appliance supply chains. We consider a two-level household appliance supply chain system consisting of a manufacturer with an Internet channel and a retailer with a traditional channel and an Internet channel. Each channel's price-setting follows the bounded rational decision process in order to obtain the optimal profit or more market share. Considering that the price competition often leads to the demand and order fluctuation, we also investigate the bullwhip effect of the multichannel supply chains on the basis of the order-up-to-inventory policy. From the numerical simulation, we find a system in a chaotic state will suffer larger bullwhip effect than a stable system, and the manufacturer's Internet channel is helpful to mitigate the bullwhip effect. Our results provide some useful managerial inspirations for the household manufacturer and retailers. Firstly, each channel should make their retail price with a suitable price adjustment speed in the stable region, and each time pricing cannot exceed the domain of attraction. Secondly, the manufacturer can adopt a more radical pricing strategy in their Internet channel to mitigate the bullwhip effect. Thirdly, the price adjustment should be reviewed and be appropriately reduced if the price adjustment is too large.

1. Introduction

With the widespread e-commerce and the increasingly fierce competition all over the world, more and more manufacturers and traditional retail firms open up the Internet supply channel as an important means to expand the market. In China, as of December 2015, the size of netizens reached 688 million, including 413 million online shopping users whose ratio had got up to 60%. In the first half of 2016, the scale of Chinese online retail sales accounted for 14.8% of social total retail sales. For Chinese household appliances market, the manufacturer firstly formed the direct retail channel on the Internet via opening up an independent network platform or setting up flagship stores on the popular network shopping platform, such as Tmall and JD. Compared with the traditional retail channel, the manufacturer direct retail channel can cut cost and provide a lower price to the customers. In order to dominate the market with a big share, many traditional retailers such as GOME and Suning also construct the network channel, cooperating with their traditional retail channels [1].

In this paper, we focus on a household appliance supply chain, which is composed of a manufacturer with an Internet channel and a retailer with a traditional channel and an Internet channel. The coordination of the manufacturer and retailer in a dual-channel supply chain has been analyzed for many times. [2–4]. The price competition between the manufacturer's Internet channel and the retailer's multichannels has become a common and significant issue in supply chain management. In order to gain much more profit and marker share, retailer and other supply chain firms which distribute the substitute products often lower their price as much as possible to compete with the opponent channels. Unfortunately, the price variability is the main cause of bullwhip effect (BWE) and leads to the demand variability which causes bullwhip effect [5].

In the recent two years, the channel selection and strategy in multichannel supply chain have become a research hotspot. Melis et al. [6] investigated the impact of the multichannel retail mix on online store choice from the perspective of the online experience. Their results show that multichannel shoppers, at the start of online grocery shopping, tended to select

the online store belonging to the same chain as their preferred offline store. When the online grocery shopping experience increased, the multichannel shoppers' focus shifted from a comparison within a chain across channels to a comparison across chains within the online channel. Liu et al. [7] studied channel choice decisions in a multichannel supply chain under a strategy where there was an ex ante commitment made on the retail price markup. They assumed the market demand was uncertain and dependent on the price and sales efforts. They found that the manufacturer would increase inventory quantity for direct sales when the retail price rose, and the increase in demand fluctuation only affected the degree of channel preference but did not change the manufacturer's channel choice. They thought adding a direct channel was a marketing strategy, rather than a competitor of the retail channel, and could help the supply chain expand the market demand. Wang et al. [1] investigated the channel selection in a supply chain with a multichannel retailer, from the role of channel operating costs. They established a linear demand model to explore the channel selection and pricing strategy in a multichannel supply chain and found that the gap between the online and offline channels' operating costs was critical to the retailer's choice of its channel selection strategy. They thought that small product differentiation was more favorable to the manufacturer in a retailer-lead supply chain and the manufacturer could benefit from a rise in the wholesale price and increasing demand in the retail channel.

Most of the research on the coordination of multichannel supply chain is based on the condition of complete information or the complete rationality. However, the retail channels cannot get all the information about their consumers. They can make price decision based on the limited information and experience and have to make dynamic game with the opponents in the market. In recent years, several scholars have studied dual-channel supply chain based on the application of bounded rationality and complex dynamics. Li and Ma [8] analyzed the complexity of the dual-channel supply chain system with delay decision. Ma and Xie [9] gave a comparison and complex analysis on dual-channel supply chain under different channel power structures and uncertain demand.

Most of the existing literatures on multichannel supply chain or supply chain management focus on the impact of the price game on the profit. In fact, the order and inventory of the supply chain are also affected by the price competition. Because the price variability is one of the major causes of the bullwhip effect, the price-sensitive demand streams have also been a common model in the research on bullwhip effect [7, 10–12]. Ma et al. [13] proposed an analytical framework that incorporates two parallel supply chains interacting price-sensitive demands and explored their interactions to determine the bullwhip effect.

However, the existing research work does not demonstrate the complex characteristics of multichannel household appliance supply chain when there are price competitions among the channels in the dynamics system. Instead, we investigate the price making process of the traditional and Internet channels and the BWE of the whole supply chain. In this paper, we consider a two-level household appliance supply chain system consisting of a manufacturer with an Internet channel and a retailer with a traditional channel and an Internet channel. In practice, many household appliance supply chain systems meet the premise of our model; for instance, in Chinese market, Haier, Midea, and Gree, are famous household appliance firms. They all market their products via the large retail chain enterprises, such as GOME, Suning, and their own Internet direct sales channel. The retailers make pricing decisions affected not only by the wholesale price but also by the price of the manufacturer's direct channel.

The objective of our research is to investigate the complex dynamics characteristics of the multichannel price game system and the bullwhip effect in the household appliance supply chain management. On the basis of the assumption that both retailers employ the order-up-to-inventory policy and the bounded rational price forecasting method, we investigate how the bullwhip effect is affected by the pricing strategy in multichannel supply chains with price-sensitive demands.

This study contributes to the existing retailing and multichannel literature in several ways. To the best of our knowledge, we are the first to systematically investigate the complex characteristics of a multichannel household appliance supply chain and the bullwhip effect of the whole supply chain affected by the interaction coming from the channel price adjustment. From a managerial point of view, our results provide useful insights for the household manufacturer who operates an Internet retail channel and retailers who have adopted a multichannel strategy. More specifically, our research provides the following: (i) the household appliance manufacturer and its retailer should manage their retail channel with a suitable price adjustment speed in the stable region, and each of their pricing decisions cannot exceed the domain of attraction. (ii) A greater speed adjustment of the manufacturer's Internet channel is helpful to mitigate the bullwhip effect. (iii) The feedback control on the retailer's Internet channel is an effective method to control the chaos of price game system and mitigate the bullwhip effect of the supply chain.

This paper is organized as follows. In Section 2 we describe the problem and construct the model of the multichannel supply chains. Section 3 analyzes the complexity of the supply chain system; then, in Section 4, we investigate the bullwhip effect and the impact of the price adjustment speeds through the numerical simulation. The feedback control method is used in the system to control the chaos and the BWE, in Section 5. Finally, Section 6 represents the conclusions and insights of this study.

2. The Model

2.1. Assumptions. The supply chain model is based on the following assumptions:

(a) The system constructs multichannel supply chains including a household appliance manufacturer with an Internet channel and a retailer with a traditional channel and its Internet channel.

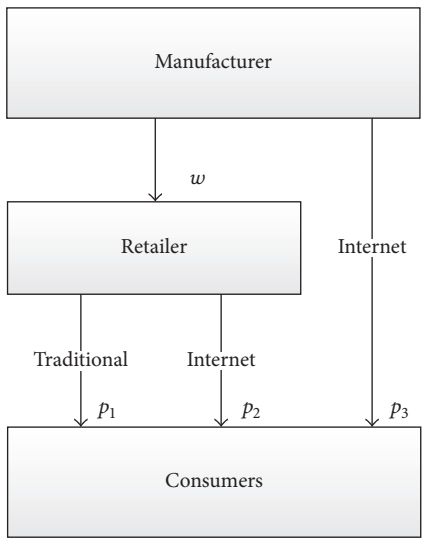

Figure 1: The supply chain model with Internet channel.

(b) The bounded rational retailer and the manufacturer sell homogeneous product to the same market on the basis of price competition.

(c) Every channel of the supply chains could only get partial market information about the market and its rivals and makes price decision with the bounded rationality expectation.

(d) All channels employ the order-up-to-inventory policy with different lead time. The retailer's two channels have longer lead time than that of the manufacturer.

(e) The market's demand for the product is equal to every channel's sales volume.

(f) The manufacturer (M) determines the wholesale price w to the retailer (R) while the retailer's two channels set their prices according the wholesale price, respectively.

2.2. Note Expression. In practice, firms of supply chain, no matter the manufacturer or the retailer, can give an estimate about every parameter appeared in the following model. The unit cost of the product is the basic manufacturing cost elements of firms. The wholesale price w is usually determined by the contract signed by both the manufacturer and retailer. Under normal circumstances, firms of household appliance supply chain can obtain information about the parameters from their financial statements. For ease of reference, the notations of the key parameters as well as their meaning are listed in Key Notations.

2.3. Model Construction. We consider a two two-echelon supply chains system consisting of a manufacturer and a retailer with dual-channel. As shown by Figure 1, channel 1 is the retailer's traditional retail channel; channel 2 is the Internet channel operated by retailer and channel 3 is the Internet channel of the manufacturer.

In the physical channel, the manufacturer distributes the products to the retailer with the unit wholesale price w and retails similar products through the network direct sales channel with the unit price of p_3. According to the wholesale price and the price of the manufacturer's direct channel, the retailer sells the products in its physical channel and network platform, with the unit price of p_1 and p_2, respectively. Generally, the customers are price-sensitive and consider the comparison of every channel's price and nonprice factors to make purchase decisions. The demand functions of the traditional and Internet channel can be written as

$$\begin{aligned} d_1 &= \delta_1 a + \varepsilon_1 - b_1 p_1 + \beta(p_2 - p_1) + \gamma(p_3 - p_1), \\ d_2 &= \delta_2 a + \varepsilon_2 + \beta(p_1 - p_2) - b_2 p_2 + \eta(p_3 - p_2), \\ d_3 &= (1 - \delta_1 - \delta_2) a + \varepsilon_3 + \gamma(p_1 - p_3) + \eta(p_2 - p_3) \\ &\quad - b_3 p_3. \end{aligned} \tag{1}$$

The parameter a ($a > 0$) represents the possible largest demand; δ_1 is the potential market share of traditional channel; δ_2 is the potential market share of the retailer's Internet channel. The customer's possible largest market demand of every channel is uncertain and affected by factor ε_i, $i = 1, 2, 3$. ε_i follows normal distribution; its mean is zero; the variance is σ_i, $i = 1, 2$. $1 - \delta_1 - \delta_2$ is the market share of the manufacturer's Internet channel ($0 < \delta_i < 1$, $i = 1, 2, 3$). b_i denotes the price-sensitivity coefficient of the product; γ, β, η are the substitutability coefficients of the channel i and p_i is the sales price of channel i. This means that each channel has their loyal customers. Here, we denote by d_i the market demand of the first retailer while we will denote by p_i the prices at time t of channel i. We assume that retailer wholesales products from the manufacturer in accordance with the wholesale price w and sells products to customers through the traditional retail channel and its Internet channel, respectively.

The expected profit function of manufacturers and retailers in dual-channel supply chain can be expressed as follows:

$$\begin{aligned} \pi_1 &= d_1(p_1 - w), \\ \pi_2 &= d_2(p_2 - w), \\ \pi_3 &= d_3(p_3 - c) + (d_1 + d_2)(w - c). \end{aligned} \tag{2}$$

In Chinese household appliance market, brick-and-mortar store is the traditional channel of the manufacturer. With the development of the Internet economics, many household appliance manufacturers, for instance, Haier and Midea, open up their online channels to sell products directly. In recent years, some household appliance chain stores, for instance, Suning and GOME, set up online sales channel; they not only chase profit, but also compete for market share with the manufacturers on the online channel. The retailer's international channel seeks the profit, and, meanwhile, it takes the market share as an important aim of their channel.

Their utility functions based on the profit and the market share are as follows:

$$\begin{aligned} u_1 &= \pi_1, \\ u_2 &= v\pi_2 + (1-v)\, e_2, \\ u_3 &= \pi_3. \end{aligned} \tag{3}$$

Here, v is the parameter of the preference for the profit. e_2 is the market share of the retailer's Internet channel (channel 2).

All channel's marginal utilities can be written as follows (see the appendix):

$$\begin{aligned} \frac{\partial u_1}{\partial p_1} &= a\delta_1 + p_2\beta + p_3\gamma - 2p_1(\beta+\gamma+\theta_1) \\ &\quad + w(\beta+\gamma+\theta_1), \\ \frac{\partial u_2}{\partial p_2} &= a\delta_2 + p_1\beta + p_3\eta - 2p_2(\beta+\eta+\theta_2) \\ &\quad + vw(\beta+\eta+\theta_2), \\ \frac{\partial u_3}{\partial p_3} &= a(1-\delta_1-\delta_2) + p_1\gamma + p_2\eta - 2p_3(\gamma+\eta+\theta_3) \\ &\quad + w(\gamma+\eta) + c\theta_3. \end{aligned} \tag{4}$$

2.4. Decision-Making Mechanism. From a long-term perspective, the price game among the multiple channels is a dynamical process. Each channel will adjust their retail price timely on the basis of their marginal utilities and current prices. However, in the actual market environment, each household appliance retail channel can only get partial information about their competitors. Therefore, we assume the retailer and the manufacturer are both bounded rational and they set prices with bounded rational expectations. That is to say, the next period retail price of channel i is determined by the current price and marginal utility [14, 15]. From the view of channel 1, when the marginal utility is positive ($\partial u_1/\partial p_1 > 0$), the increase of the traditional retail price will benefit the retailer. So the next period price of the retailer's traditional channel $p_{1,t}$ should increase $\alpha_i p_{i,t}(\partial u_{i,t}/\partial p_i)$ on the basis of $p_1(t)$.

Their dynamic pricing strategies are as follows:

$$p_{i,t+1} = p_{i,t} + \alpha_i p_{i,t} \frac{\partial u_{i,t}}{\partial p_i}, \quad i = 1,2,3. \tag{5}$$

α_i (≥ 0) is the price adjustment parameter of channel i ($i = 1,2,3$). Their value depends on the power of the retail channel chasing greater utility. A large adjustment parameter means the retailer wants to get the maximum utility as soon as possible. Meanwhile, this channel should have stronger ability to regulate and control the prices.

According to (4) and (5), we can get a multichannel competitive discrete dynamic system:

$$\begin{aligned} p_{1,t+1} &= p_{1,t} + \alpha_1 p_{1,t}(a\delta_1 + p_{2,t}\beta + p_{3,t}\gamma \\ &\quad - 2p_{1,t}(\beta+\gamma+\theta_1) + w(\beta+\gamma+\theta_1)), \\ p_{2,t+1} &= p_{2,t} + \alpha_2 p_{2,t}(a\delta_2 + p_{1,t}\beta + p_{3,t}\eta \\ &\quad - 2p_{2,t}(\beta+\eta+\theta_2) + vw(\beta+\eta+\theta_2)), \\ p_{3,t+1} &= p_{3,t} + \alpha_3 p_{3,t}(a(1-\delta_1-\delta_2) + p_{1,t}\gamma + p_{2,t}\eta \\ &\quad - 2p_{3,t}(\gamma+\eta+\theta_3) + w(\gamma+\eta) + c\theta_3). \end{aligned} \tag{6}$$

3. System Analysis of the Multichannel Supply Chains

3.1. Equilibrium Solutions. In system (6), we let $p_{i,t+1} = p_{i,t}$ ($i = 1,2,3$);

$$\begin{aligned} &\alpha_1 p_{1,t}(a\delta_1 + p_{2,t}\beta + p_{3,t}\gamma - 2p_{1,t}(\beta+\gamma+\theta_1) \\ &\quad + w(\beta+\gamma+\theta_1)) = 0, \\ &\alpha_2 p_{2,t}(a\delta_2 + p_{1,t}\beta + p_{3,t}\eta - 2p_{2,t}(\beta+\eta+\theta_2) \\ &\quad + vw(\beta+\eta+\theta_2)) = 0, \\ &\alpha_3 p_{3,t}(a(1-\delta_1-\delta_2) + p_{1,t}\gamma + p_{2,t}\eta \\ &\quad - 2p_{3,t}(\gamma+\eta+\theta_3) + w(\gamma+\eta) + c\theta_3) = 0. \end{aligned} \tag{7}$$

We can get at most eight fixed points. According to the actual household appliance supply chains in China, we set values to some parameters of the system before we solve (7). Let $a = 4$, $c = 1$, $w = 1.1$, $v = 0.5$, $\theta_1 = 0.6$, $\theta_2 = 0.6$, $\theta_3 = 0.6$, $\delta_1 = 0.4$, $\delta_2 = 0.35$, $\eta = 0.2$, $\gamma = 0.1$, and $\beta = 0.1$ in the following sections, unless otherwise specified. Then the eight equilibrium solutions of system can be expressed as follows: $E_1 = (0,0,0)$, $E_2 = (0,0,1.1375)$, $E_3 = (0,1.05278,0)$, $E_4 = (1.43889,0,0)$, $E_5 = (0,1.11986,1.20749)$, $E_6 = (1.5073,0,1.2317)$, $E_7 = (1.5753,1.2278,0)$, and $E_8 = (1.658,1.3105,1.32303)$.

The fixed points E_k ($k = 1,2,3,4,5,6,7$) are all boundary equilibrium, and E_8 is the Nash equilibrium.

3.2. Stability Analysis. In order to analyze the stability of the equilibrium point, we firstly calculate the Jacobian matrix system (6):

$$J(E) = \begin{pmatrix} j_{11} & j_{12} & j_{13} \\ j_{21} & j_{22} & j_{23} \\ j_{31} & j_{32} & j_{33} \end{pmatrix}, \tag{8}$$

$$\begin{aligned} j_{11} &= 1 + a\alpha_1\delta_1 + p_2\alpha_1\beta + p_3\alpha_1\gamma \\ &\quad - 4p_1\alpha_1(\beta+\gamma+\theta_1) + w\alpha_1(\beta+\gamma+\theta_1), \\ j_{22} &= 1 + a\alpha_2\delta_2 + p_1\alpha_2\beta + p_3\alpha_2\eta \\ &\quad - 4p_2\alpha_2(\beta+\eta+\theta_2) + vw\alpha_2(\beta+\eta+\theta_2), \end{aligned}$$

$$
\begin{aligned}
j_{33} &= 1 + a\alpha_3\left(1-\delta_1-\delta_2\right) + p_1\alpha_3\gamma + p_2\alpha_3\eta \\
&\quad - 4p_3\alpha_3\left(\gamma+\eta+\theta_3\right) + w\alpha_3\left(\gamma+\eta\right) \\
&\quad + c\alpha_3\theta_3. \\
j_{12} &= p_1\alpha_1\beta, \\
j_{13} &= p_1\alpha_1\gamma, \\
j_{21} &= p_2\alpha_2\beta, \\
j_{23} &= p_2\alpha_2\eta, \\
j_{31} &= p_3\alpha_3\gamma, \\
j_{32} &= p_3\alpha_3\eta,
\end{aligned} \tag{9}
$$

The corresponding characteristic equation of (8) can be simplified as the following forum, according to $|J - \lambda I| = 0$:

$$f(\lambda) = \lambda^3 + A\lambda^2 + B\lambda + C = 0. \tag{10}$$

Jury stability criterion is the necessary and sufficient condition of asymptotic stabilization at the equilibrium point. So all parameters of system (6) should meet the following conditions in accordance with the Jury stability criterion:

$$
\begin{aligned}
C^2 &< 1, \\
1 + A + B + C &> 0, \\
-1 + A - B + C &< 0, \\
(AC - B)^2 - \left(C^2 - 1\right)^2 &< 0.
\end{aligned} \tag{11}
$$

The region of stability in the 3D space of the channels' adjustment speeds is plotted in Figure 2, which represents that fixed point of system (6) is asymptotically stable with the values of $(\alpha_1, \alpha_2, \alpha_3)$ in this region. If the values of $(\alpha_1, \alpha_2, \alpha_3)$ are out of the stable region, even if only one parameter is not in the stable region, the system will lose its stability.

As shown in Figure 2, the stability region of system is determined by all three channels' adjustment speed parameters, but it is obvious that range of α_1 is less than that of α_2 and α_3. That is to say, the adjustment speed range of the retailer's traditional channel is less than that of retailer's Internet channel and the manufacturer's Internet channel, while the two Internet channels are similar to each other from the view of the price adjustment speed.

Figure 3 shows three 2D parameter basin diagrams with respect to the two of all three parameters, when the rest one is fixed. Regions in the parameter plane converging to stable cycles of a particular period are plotted in every graph, and each color corresponds to a given period. The green is for the stable state; the orange is for two periodic cycles; the red is for four periodic cycles; and the yellow is for eight periodic cycles. Mostly, as the parameter α_i grows up from the green area passing through orange, red, and yellow areas, the system turns into chaos through flip bifurcation.

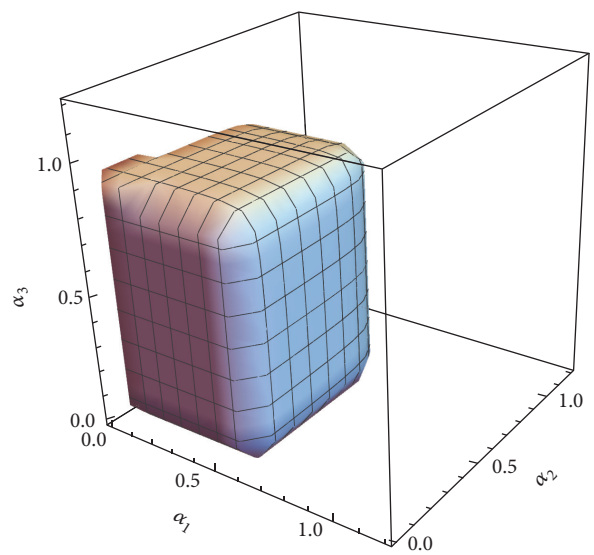

Figure 2: The stability region of system (6).

When (α_1, α_2), (α_2, α_3), or (α_1, α_3) from the orange go to the gray area directly, system (6) enters into chaos through Neimark-Sacker bifurcation. In particular, for (α_1, α_2) and (α_1, α_3), there is a strange road, from which system enters into chaos through Neimark-Sacker bifurcation and enters into period doubling state again and into chaos through flip bifurcation ultimately. Figure 3(b) can be regarded as the magnified version of the special bifurcation area.

3.3. The Bifurcation and the Largest Lyapunov Exponent. Figure 4 shows the bifurcation diagram with respect to the parameter α_1 (the price adjustment speed of the traditional channel), while the other adjustment parameters are fixed ($\alpha_2 = 0.5$, and $\alpha_3 = 0.5$).

In Figure 4, the bifurcation scenario occurred; if α_1 is small then there exists a stable equilibrium point (Nash). As we can see from the bifurcation diagram, the Nash equilibrium of price is locally stable for small values of α_1. As α_1 increases, the Nash equilibrium of price becomes unstable, and the price process enters into chaos via many period doubling bifurcations. It means that if the bounded rational retailer takes a large value of price adjustment speed in the traditional channel, the system would lead to complex dynamics.

From Figure 4, we can get some stability features of the system via the LLE corresponding to the bifurcation diagram. LLE is the largest Lyapunov exponents, which can express the features of the dynamic system. It is the first time that LLE is equal to 0, when $\alpha_1 = 0.73$, and at this point the bifurcation takes place. Once LLE > 0, the system falls into the chaotic state.

Figure 5 shows the basin of attraction with respect to p_2 and p_3, when the price game system is in a stable state with $\alpha_1 = \alpha_2 = \alpha_3 = 0.5$. The domain of attraction in system (6) is the set of initial price decision variables which can converge to the same attractor. In Figure 5 point A is the attractor for the prices of two Internet channels when the price of offline

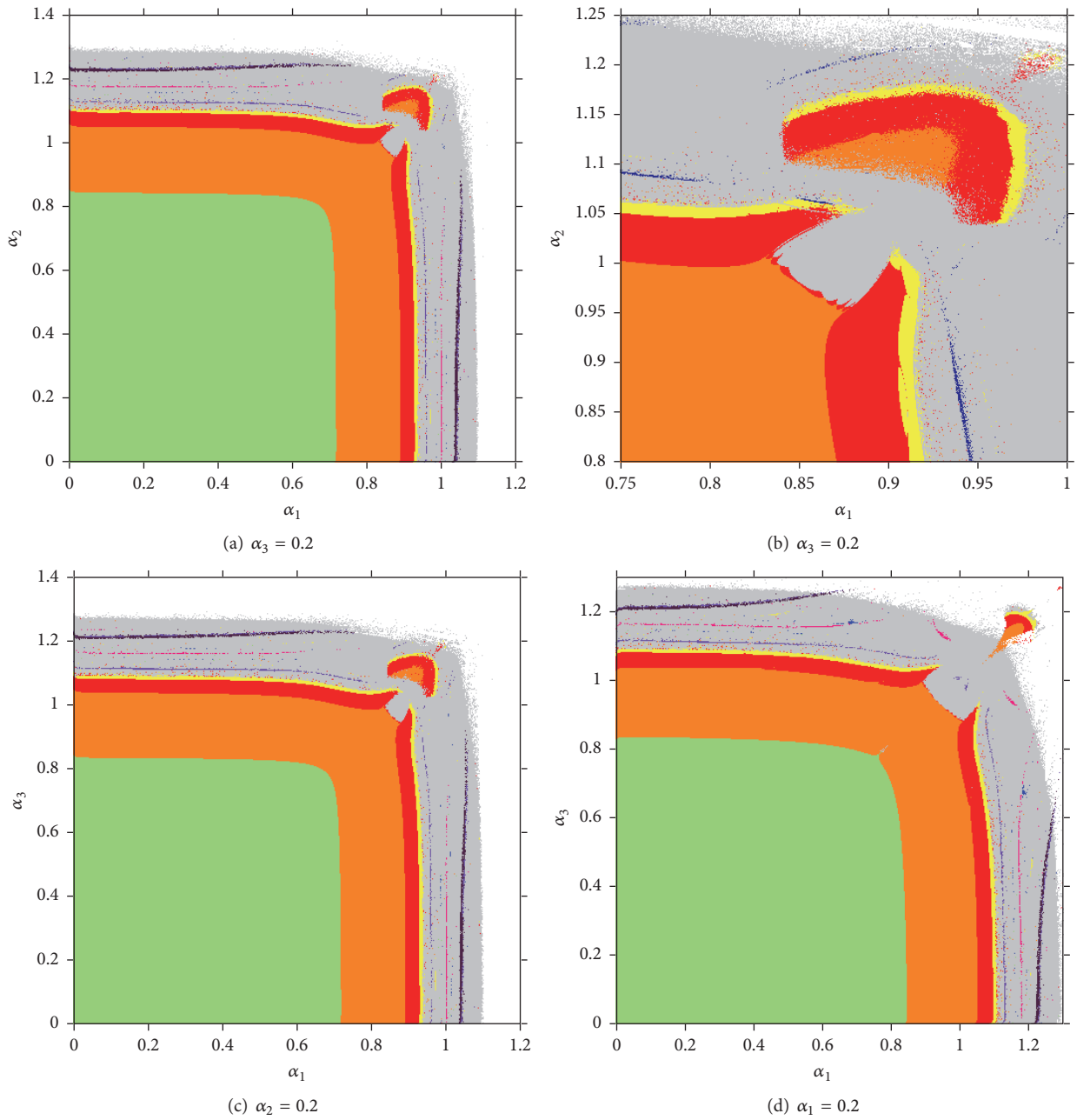

Figure 3: 2D parameter basin for periodic cycles output.

retail channel is fixed, and the green region is the feasible basin of attraction for p_2 and p_3. Every initial price in the green region can converge to the fixed point A. That is to say that, although the system is stable, not every initial price decision can converge to the attractor. Once the initial prices are out of the green region area, system (6) will fall into divergence in the end. So the decisions of the initial prices are very important for every channel of the supply chain. The manufacturer and the retailer should both ensure their prices are not out of the domain of attraction, to keep the system in a long-term stable state.

In Figure 6, the abscissa axis is wholesale price between the manufacture and the retailer, and the axis of ordinates is the price of channel i or the product demand of the supply chain. The black line is a datum line determined by $p = w$. The pink line is the whole product demand of the supply chain. The red line is price of channel 3, the green line is the channel 2, and the blue line is the channel 1. As the wholesale price increases, prices of all channels will increase; however, the whole demand will decrease. Point B, which is the intersection of the black and green line, is the critical point for the change of supply chain structure. The retailer will

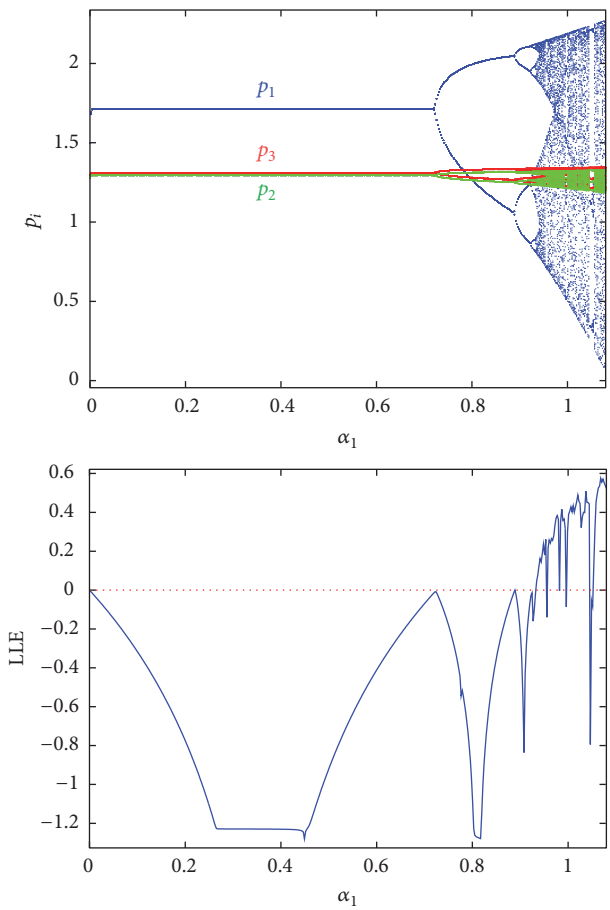

Figure 4: Bifurcation diagram and the largest Lyapunov exponent when $\alpha_2 = 0.5$ and $\alpha_3 = 0.5$.

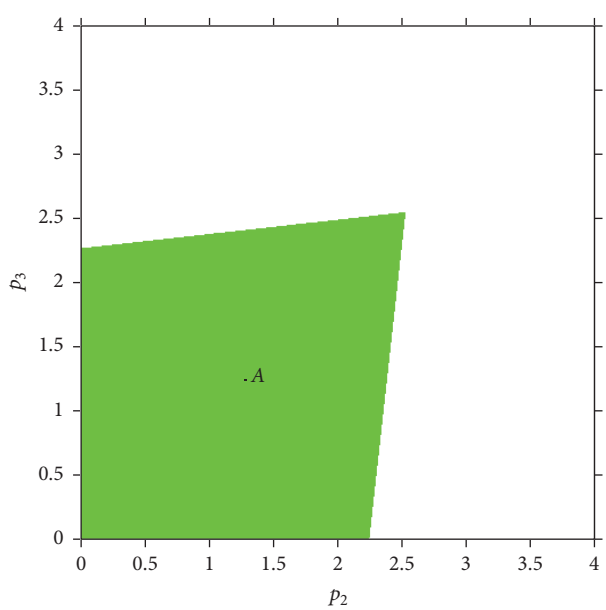

Figure 5: Basin of attraction with respect to p_2 and p_3 when $p_1 = 1.5$.

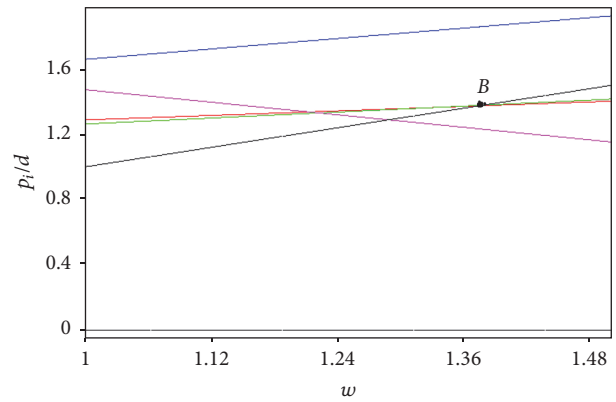

Figure 6: The impact of wholesale price on the price equilibrium points and the demands.

keep the wholesale channel although the wholesale price w increases, when the wholesale price is less than w_B. However, once the wholesale price is more than w_B, the retailer will not wholesale products from the manufacturer; it will purchase products from the manufacturer's Internet channel directly with a lower price than the wholesale price w.

4. The Bullwhip Effect of Supply Chains

4.1. The Demand Model. The demand of channel is determined by the its retail price, so we can get the demands according to (1). It is obvious that the price competition will lead to the demand fluctuation. Let the initial prices of all channels be as follows: $p_{1,0} = 1.5$, $p_{2,0} = 1.5$, $p_{3,0} = 1.5$.

The demand process of stable and chaotic state is shown in Figure 7. The red line represents the demand of channel i in a stable state ($\alpha_2 = 0.8$), and the blue line is the demand of channel i in a chaotic state ($\alpha_2 = 1.2$). We can find that the amplitude of demand in a chaotic state is more than that in a stable state. In particular, the demand amplitude of channel 2 is much more than the other two channels in a chaotic state. The demand fluctuation is the source of bullwhip effect [5]. So we investigate the bullwhip effect of the multichannel supply chain in the following sections.

4.2. Replenishment Policy. In this paper we assume that the order-up-to-inventory policy is employed by every channel. The manufacturer and retailer have known their own demand of current period $d_{i,t}$ at the end of period t and estimate their own inventory of period $t+1$ ($S_{i,t+1}$) and send the order of period t ($q_{i,t+1}$) to the manufacturer. After the lead time L_i, the retailer receives the products from the manufacturer at the period $t + L_i$. The expected inventory of the period $t+1$ is determined by the demand prediction of the lead time.

$$S_{i,t+1} = \widehat{D}_{i,t+1}^{L_i} + z\widehat{\sigma}_{i,t+1}^{L_i}. \tag{12}$$

Here, $\widehat{\sigma}_{i,t+1}^{L_i}$ represents the standard deviation between the actual demand and demand prediction of the lead time. The parameter z is a safety factor, on behalf of the desired service level. In order to deduce a more simple expression

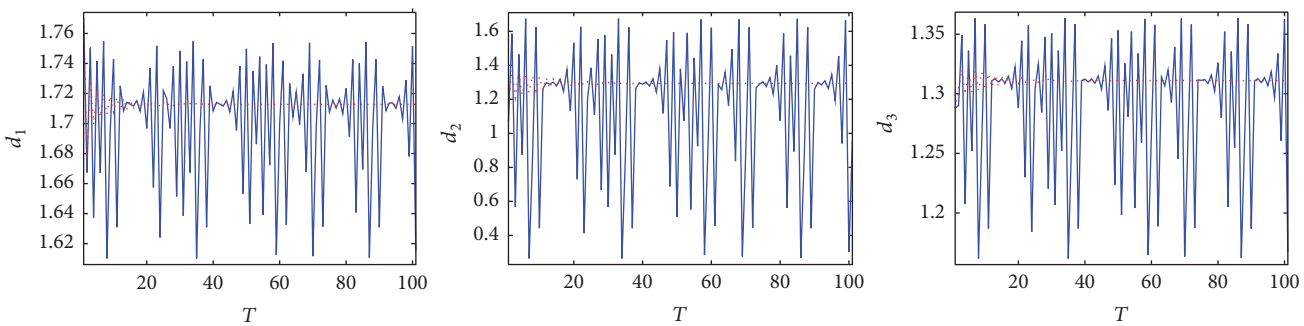

Figure 7: The demand process of stable and chaotic state.

for the variance of the orders placed by the retailer to the manufacturer, we assume that the safety factor z is equal to 0, as this inventory policy is quite common in practice. Taking $z = 0$ does not necessarily imply a low service level, because the retailer will often make the lead time parameter L plus one to achieve the desired service level when they take this policy [16].

In order to achieve the desired inventory goal, in the beginning of the period $t+1$, the retailer must send the order $q_{i,t}$ to the manufacturer. After the lead time L, the retailer receives the goods at the period $t+L$. Therefore

$$q_{i,t+1} = S_{i,t+1} - S_{i,t} + d_{i,t} = \widehat{D}_{i,t+1}^{L_i} - \widehat{D}_{i,t}^{L_i} + d_{i,t}. \quad (13)$$

The sum of demand prediction of the lead time is as follows:

$$\widehat{D}_{i,t+1}^{L_i} = \widehat{d}_{i,t+1} + \widehat{d}_{i,t+2} + \cdots + \widehat{d}_{i,t+L_i}, \quad (14)$$

where $\widehat{d}_{i,t+1}, \ldots, \widehat{d}_{i,t+L-1}, \widehat{d}_{i,t+L}$ denote the corresponding predicted values of $d_{i,t+1}, \ldots, d_{i,t+L-1}, d_{i,t+L}$. In practice, the assumption that $\widehat{d}_{i,t+2} \cdots \widehat{d}_{i,t+L}$ are equal to $\widehat{d}_{i,t+1}$ is very common. Therefore, we can get the order of channel i on the basis of (14):

$$q_{i,t+1} = \widehat{D}_{i,t+1}^{L_i} - \widehat{D}_{i,t}^{L_i} + d_{i,t} = L_i \widehat{d}_{i,t+1} - L_i \widehat{d}_{i,t} + d_{i,t}. \quad (15)$$

4.3. The Demand Forecast. At the end of period t, all channels have known the current price of each channel and can make their price forecast of the next period according to (6), but they cannot know the rival prices of next period; therefore, they just think their opponents take the same price in the next period as the current period. So each channel can give its demand forecast of the next period via (1):

$$\widehat{d}_{i,t+1} = d_{i,t+1}\left(\widehat{p}_{i,t+1}, p_{j,t}, p_{6-i-j,t}\right), \quad i, j = 1, 2, 3, \ \ j \neq i. \quad (16)$$

Here,

$$\widehat{p}_{i,t+1} = p_{i,t} + \alpha_{i,t} p_{i,t} \frac{\partial u_{i,t}}{\partial p_{i,t}}. \quad (17)$$

Therefore, using (6), (15), (16), and (17), we can get the order of each channel on the basis of the forecasting price.

The total demand is determined by $d_t = d_{1,t} + d_{2,t} + d_{3,t}$, and the total order is determined by $q_t = q_{1,t} + q_{2,t} + q_{3,t}$. According to the work of Chen et al. [16], the measure of the bullwhip effect in the multichannel supply chain system can be expressed as

$$\mathrm{BWE} = \frac{\sigma_q^2 / u_q}{\sigma_d^2 / u_d}. \quad (18)$$

4.4. The Bullwhip Effect in Stable and Chaotic State. From the above analysis we can see that the demands of the two retailers experience stable, period doubling, and chaotic state, with the continuous growth of the price adjustment speed of retailers. Considering the demands are price-sensitive and would impact the bullwhip effect, we design a numerical experiment to investigate how the bullwhip effect is affected by the channels' price strategy in different states.

From the figure of bifurcation and the largest Lyapunov exponent, we can find the system is in stable or chaotic state, when the parameters takes values $\alpha_2 = 0.8$ or $\alpha_2 = 1.2$, respectively. In this section, we mainly compare the bullwhip effect of the supply chain on the basis of the demand processes used in Figure 7.

As shown in Figure 8, the red line is the order variance ratio of the whole supply chain in a stable state. The blue line is the order variance ratio of the whole supply chain in the chaotic state. No matter what state the system is, there is bullwhip effect in the multichannel supply chain. After the initial little values, the bullwhip effects in different states decrease gradually and tend to be stable. It is easy to find that the whole supply chain in a chaotic state suffers a larger bullwhip effect than that in a stable state.

4.5. The Impact of the Price Adjustment Speed on Bullwhip Effect. From the above analysis, we may find that the speed of the price adjustment has an important impact on the bullwhip effect of the supply chain. In this section, we intend to investigate the differences of the long-term bullwhip effect when the parameters of the speed of the price adjustment take different values. In particular, we are interested in the interaction of three channel's price strategy.

In Figure 9, we plot 4D map of the bullwhip effect of the whole supply chain using the RGB color map. The drawing standard is that the colors from green to red are

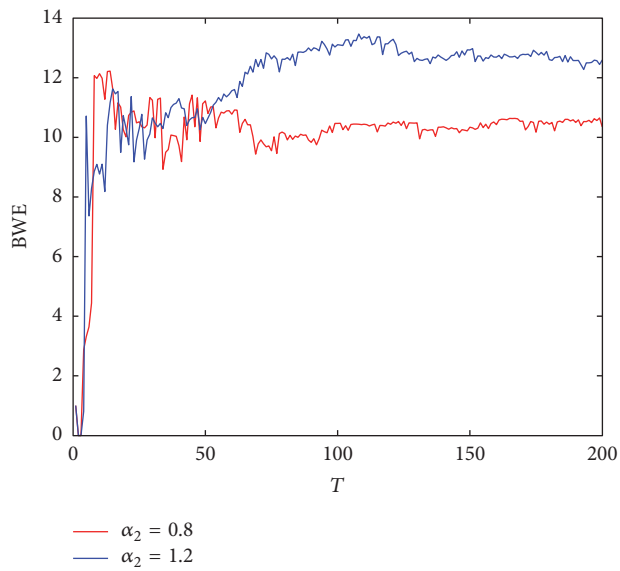

Figure 8: The timing diagram of bullwhip effect in stable and chaotic state.

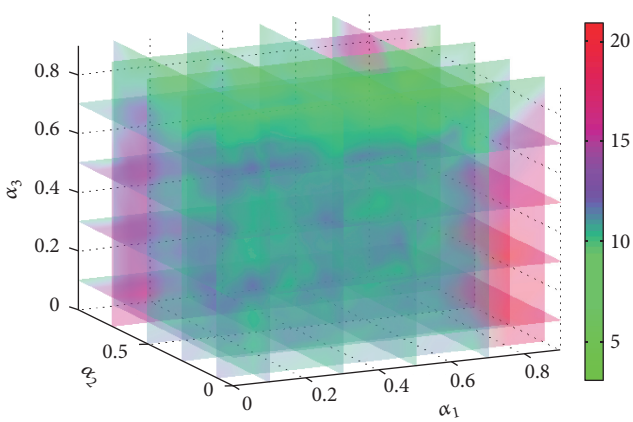

Figure 9: The impact of α_i on the bullwhip effect.

corresponding to the bullwhip effect from 4 to 20, as shown in the color bar. As shown in Figures 2 and 9, the color changes tell us that the bullwhip effect of supply chain in the stable region is usually less than that out of the stable region. We can find that the bullwhip effect increases as the growth of the retailer's price adjustment speeds (α_1 and α_2); however, the growth of the manufacturer's price adjustment speeds (α_3) can mitigate the bullwhip effect of the whole supply chain. This result provides a useful managerial insight for the household appliances manufacturer that they can improve their own speed of the price adjustment to reduce the bullwhip effect.

A greater adjustment speed can help retail channels get more quickly the maximum effect and obtain greater long-term profits. But this will also bring greater bullwhip effect; therefore, there must be a suitable adjustment speed for whole supply chain to obtain greater profits and maintain a low bullwhip effect. This suitable adjustment speed must be in a stable state. That is to say, the appliance retail channels cannot adopt a fast and wide range of retail price adjustment strategy or a too cautious price adjustment strategy. They should take a suitable price adjustment speed within their own range to adjust the retail prices in a timely manner according to the price changes of the competitive channels.

A typical example is Midea's home appliance multichannel retail supply chain. Established in 1968, Midea is a publicly listed as (and since July 2016) Fortune 500 company that offers one of the most comprehensive ranges in the home appliance industry. Midea Group sells one kind of rice cooker whose product type is MB-FS406C through Midea Mall which is Midea's only official Internet mall and GOME which is one of mainland China's largest appliance retail chain enterprises. The latter sells this rice cooker via their retail stores and gome.com.cn which is an online shopping mall operated by GOME. The retail price data overserved from three channels shows that the retail price in gome.com.cn changes more frequently with larger amplitude and is lower than the other two channels. This is consistent with the results of our model.

5. Chaos Control and Mitigation of Bullwhip

The simulation of bullwhip effect subject to the adjustment speed of price suggests that the bullwhip effect would be mitigated in the equilibrium state compared to the period doubling and chaotic state. From the perspective of supply chain management, firms of supply chain all hope to find some methods to control the chaos and period doubling of the system and lighten their bullwhip effect. In the ways of chaos control, the delayed feedback control method has been widely used in the chaos of supply chain system [9, 17]. In this paper, we propose the delayed feedback control method to control the system. We make the retailer control its Internet channel's adjustment speed when making price decisions with the help of the control parameter K. With the controlling, the price decision process of the retailer's Internet channel can be rewritten as

$$\begin{aligned} p_{2,t+1} = {} & p_{2,t} + \alpha_2 p_{2,t} \left(a\delta_2 + p_{1,t}\beta + p_{3,t}\eta \right. \\ & \left. - 2p_{2,t}\left(\beta + \eta + \theta_2\right) + vw\left(\beta + \eta + \theta_2\right)\right) - K\left(p_{2,t+T} \right. \\ & \left. - p_{2,t}\right), \end{aligned} \tag{19}$$

where T is the length of the lag time and K is the control parameter. We consider the control of chaos in the system with $T = 1$; then we can obtain a new price game system for the two retailers of the supply chain, under the control of channel 2:

$$\begin{aligned} p_{1,t+1} = {} & p_{1,t} + \alpha_1 p_{1,t} \left(a\delta_1 + p_{2,t}\beta + p_{3,t}\gamma \right. \\ & \left. - 2p_{1,t}\left(\beta + \gamma + \theta_1\right) + w\left(\beta + \gamma + \theta_1\right)\right), \end{aligned}$$

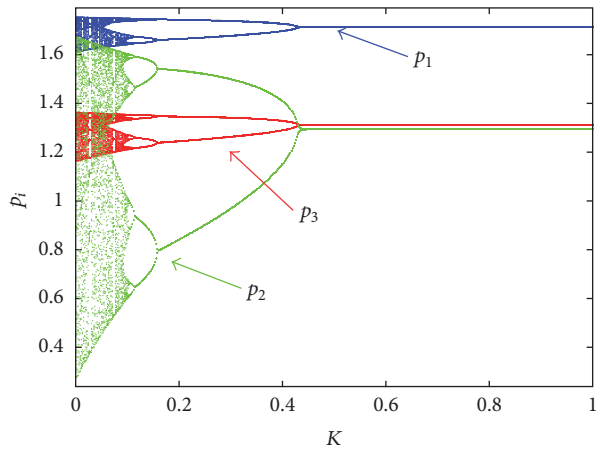

FIGURE 10: Bifurcation diagram with $\alpha_1 = \alpha_3 = 0.5$ and $\alpha_2 = 1.2$, K varying from 0 to 1.

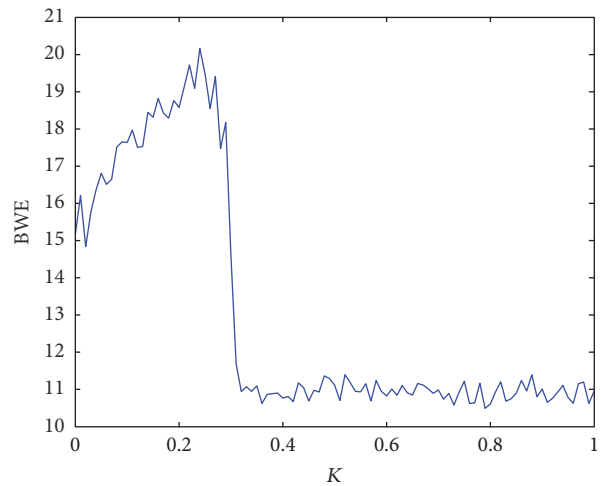

FIGURE 11: Effect of K on the bullwhip effect when $\alpha_1 = \alpha_3 = 0.5$ and $\alpha_2 = 1.2$.

$$\begin{aligned} p_{2,t+1} &= p_{2,t} + \alpha_2 p_{2,t} \left(a\delta_2 + p_{1,t}\beta + p_{3,t}\eta \right. \\ &\quad \left. - 2p_{2,t}\left(\beta + \eta + \theta_2\right) + vw\left(\beta + \eta + \theta_2\right)\right) - K\left(p_{2,t+1} \right. \\ &\quad \left. - p_{2,t}\right), \\ p_{3,t+1} &= p_{3,t} + \alpha_3 p_{3,t}\left(a\left(1 - \delta_1 - \delta_2\right) + p_{1,t}\gamma + p_{2,t}\eta \right. \\ &\quad \left. - 2p_{3,t}\left(\gamma + \eta + \theta_3\right) + w\left(\gamma + \eta\right) + c\theta_3\right). \end{aligned} \tag{20}$$

5.1. The Effect of the Control Parameter K on the Price Game System. Based on the above numerical simulation results, we can make the price adjustment parameters $\alpha_1 = \alpha_3 = 0.5$ and $\alpha_2 = 1.2$ to simulate a chaotic state of system (6). We plot a price bifurcation diagram with respect to K. From Figure 10 we can find that under the action of delayed feedback control parameter K the system gradually changes from chaos to period cycles and equilibrium state at last. When $K > 0.15$, the system is in a state of twofold period. When $K > 0.42$, the competition system enters into a stable state.

5.2. The Effect of the Control Parameter K on the Bullwhip Effect. In order to check the effect of the control parameter K on the bullwhip effect, we give a numerical simulation when $\alpha_1 = \alpha_3 = 0.5$ and $\alpha_2 = 1.2$.

Figure 11 shows the impact of the control parameter K on the bullwhip effect. As the control parameter's value increases from 0 to 0.22, the bullwhip effect has the upward growth trend. After that the value of bullwhip effect will be steady if the control parameter's value is larger than 0.32. A significant sharp drop of bullwhip effect takes place when the control parameter's value continues to grow from 0.22 to 0.32.

This shows that the delayed feedback control has achieved a good effect and can effectively alleviate the bullwhip effect the supply chain. The supply chain firms can make the chaotic system controlled by adjusting the control parameter K. The retail channel with a strong profit-seeking motive should review the price adjustment at each stage of price decision. If the price adjustment is too large, the price adjustment should be appropriately reduced, so that the system can be stabilized.

6. Conclusions

This paper focuses on the dynamic pricing game of household appliance supply chain with more than one Internet channel and investigates the nonlinear characteristics of the multichannel household appliance, and we explore how their price adjustment speeds affect the stability of the system and the bullwhip effect of the whole supply chain.

This paper constructs a model of multichannel household appliance supply chains including a retailer with and a manufacturer with an Internet channel. All channels take price as the competitive variable and make bounded rational decision in order to obtain the optimal profit or much more market share. This paper studies the complex characteristics caused by the price competition in multichannel household appliance supply chains. Considering price competition often leads to the demand and order fluctuation, we also investigate the bullwhip effect of the multichannel supply chains on the basis of the order-up-to-inventory policy. From the numerical simulation, we find a system in a chaotic state will suffer larger bullwhip effect than a stable system, and a large price adjustment speed in the manufacturer's Internet channel is helpful to mitigate the bullwhip effect. Our results provide useful managerial inspirations for the household manufacturer who operates an Internet retail channel and retailers who have adopted a multichannel strategy. Firstly, both the household appliance manufacturer and its retailer should manage their retail channel with a suitable price adjustment speed in the stable region, and each of their pricing decisions cannot exceed the domain of attraction. Secondly, if the manufacturer wants to mitigate the bullwhip effect, they can adopt a more radical pricing strategy in their Internet channel. Thirdly, the feedback control on the retail channel with a strong profit-seeking motive is an effective

method to control the chaos of price game system and mitigate the bullwhip effect of the supply chain. The price adjustment should be reviewed and be appropriately reduced if the price adjustment is too large.

Appendix

The profit functions of three channels are as follows:

$$\begin{aligned}\pi_1 &= d_1(p_1 - w) = (\theta_1 a - s_1 p_1 + \beta p_2 + \gamma p_3)(p_1 - w),\\ \pi_2 &= (\theta_2 a + \beta p_1 - s_2 p_2 + \eta p_3)(p_2 - w),\\ \pi_3 &= ((1-\theta_1-\theta_2)a + \gamma p_1 + \eta p_2 - s_3 p_3)(p_3 - c)\\ &\quad + (\theta_1 a - s_1 p_1 + \beta p_2 + \gamma p_3 + \theta_2 a + \beta p_1 - s_2 p_2\\ &\quad + \eta p_3)(w - c).\end{aligned} \tag{A.1}$$

According the assumption about the utility of three channels, their functions can be expressed as follows:

$$\begin{aligned}u_1 &= \pi_1 = d_1(p_1 - w),\\ u_2 &= v\pi_2 + (1-v)e_2 = vd_2(p_2 - w) + (1-v)d_2 p_2,\\ u_3 &= \pi_3 = d_3(p_3 - c) + (d_1 + d_2)(w - c).\end{aligned} \tag{A.2}$$

So, their marginal utility can be written in the following form:

$$\begin{aligned}\frac{\partial u_1}{\partial p_1} &= a\delta_1 + p_2\beta + p_3\gamma - 2p_1(\beta + \gamma + \theta_1)\\ &\quad + w(\beta + \gamma + \theta_1),\\ \frac{\partial u_2}{\partial p_2} &= a\delta_2 + p_1\beta + p_3\eta - 2p_2(\beta + \eta + \theta_2)\\ &\quad + vw(\beta + \eta + \theta_2),\\ \frac{\partial u_3}{\partial p_3} &= a(1 - \delta_1 - \delta_2) + p_1\gamma + p_2\eta\\ &\quad - 2p_3(\gamma + \eta + \theta_3) + w(\gamma + \eta) + c\theta_3.\end{aligned} \tag{A.3}$$

Key Notations

Variables

$d_{i,t}$: The market demand of channel i $(i = 1, 2, 3)$ at time t
$p_{i,t}$: The price of channel i $(i = 1, 2, 3)$ at time t
q_t: The order of the whole channels
$q_{i,t}$: The order quantity of channel i
$S_{i,t}$: The order-up-to point of channel i
$\widehat{D}_t^L$: The estimate vector of the lead time demand
π_i: The expected profit of the ith channel.

Parameters

c: The unit cost of the product
w: The wholesale price
α_i: The price adjustment speed of channel i
a: The possible largest market demand
δ_i: The initial proportion of the market share of channel i
b_i: The price sensitivity of consumers for channel i
β: The price-gap sensitivity of consumers between channel 1 and channel 2
γ: The price-gap sensitivity of consumers between channel 1 and channel 3
η: The price-gap sensitivity of consumers between channel 2 and channel 3
E: The stable set of the price decision-making coefficients
L_i: The lead time of channel i
z: The safety factor.

Competing Interests

The authors declare no conflict of interests regarding the publication of this paper.

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