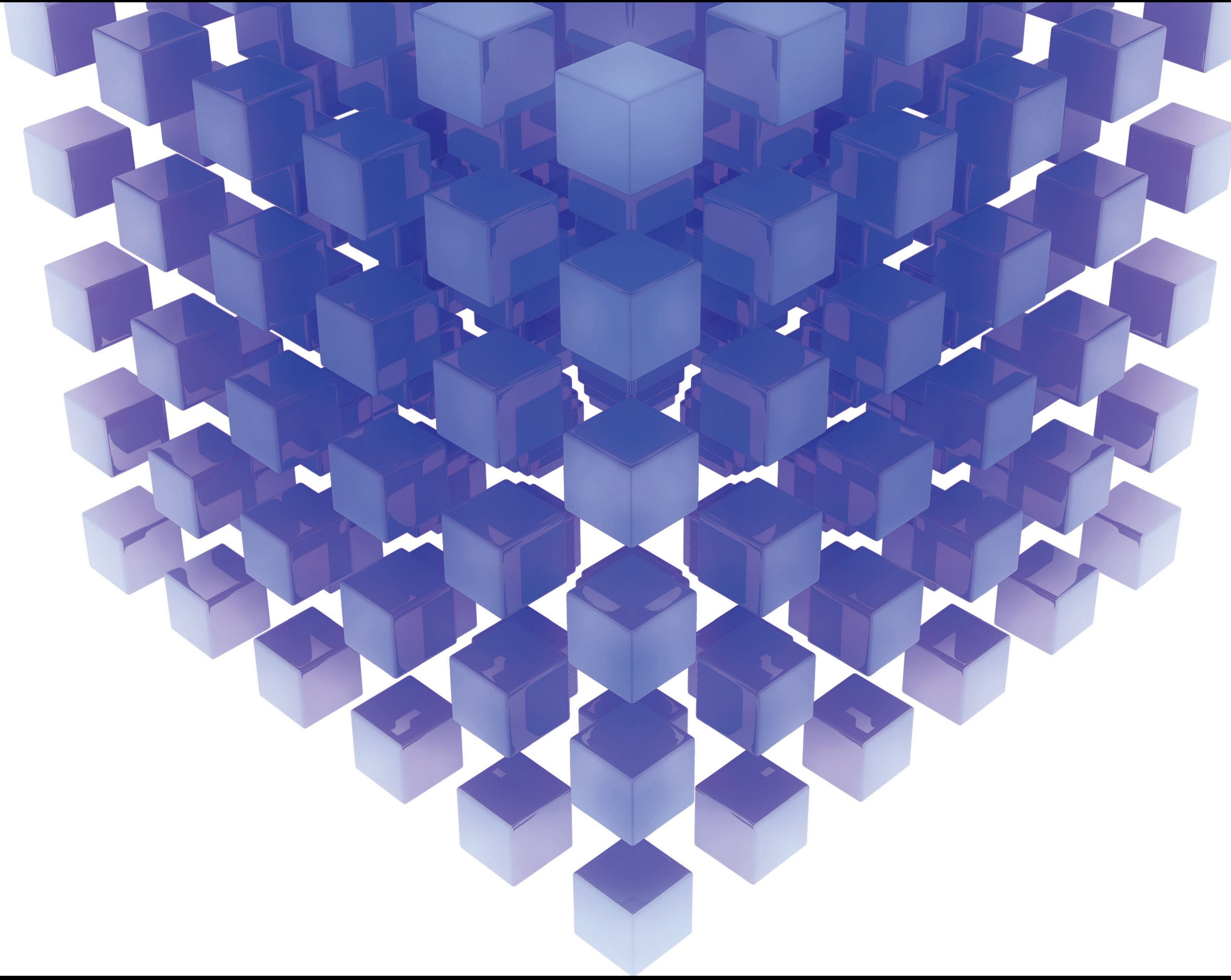
Mathematical Problems in Engineering

Green Supply Chain Network for Sustainable Inventory Management

Lead Guest Editor: Shib Sankar Sana
Guest Editors: Biswajit Sarkar and Shibaji Panda



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Research Article

Carbon Emission Reduction Decision and Revenue-Sharing Contract with Consumers' Low-Carbon Preference and CER Cost under Carbon Tax

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Since the tax of carbon emission is popular and consumers are exhibiting low-carbon preference, the green manufactures have to spend more extra cost on investing carbon emission reduction (CER) technology to decrease the carbon emission. To encourage the manufacture's CER investment efforts, this paper explores the impact of carbon tax, CER cost, and consumers' low-carbon preference on low-carbon decision-making and designs a revenue-sharing contract (RS) by constructing Stackelberg models. Based on the theoretical and numerical analysis, this paper finds that the supply chain would benefit from the increment of consumer's environmental awareness but be depressed by the increase of the CER investment cost factor. Additionally, there exists a unique optimal carbon tax to make CER degree the maximum. Furthermore, RS can effectively promote manufacturers to reduce carbon emissions and also improve the supply chain efficiency.

1. Introduction

With the rapid development of the economy, global carbon dioxide emission has sharply increased in the past few decades, which significantly resulted in serious climate and global warming [1]. It is an urgent task to reduce carbon emissions to allay global warming. And this task has drawn high attention from all social members. Governments around the world have promulgated and implemented relevant carbon policies to reduce carbon emissions in practice, such as carbon tax, cap and trade, and low-carbon offset. Among these policies, carbon tax is considered an effective environmental policy in China. In addition, more and more consumers are exhibiting low-carbon preference, that is, consumers are willing to pay higher prices for the same function but more green products [2, 3]. To comply with governments' regulatory schemes and meet the need of low-carbon preference customers, many manufacturers begin to employ the CER investment and adjust their existing production mode to produce low-carbon products. However, the manufacturer must take extra cost to employ the CER investment. The high cost of the CER investment brings the challenge to the manufacturer. Whether the increased revenue with the CER investment can balance or compensate for the additional cost is the focus of attention for the manufacturers. Therefore, it is important that government's carbon tax, customers' low-carbon preference, and CER investment cost should be considered by the manufacturer in making strategic decisions. Moreover, it is necessary to create an incentive mechanism to urge manufactures to explore CER technology investment.

The extant findings have shown that coordination contracts or carbon tax as the mechanism are used to compensate and counterbalance the CER cost, resulting in motiving the manufacture to employ CER investments [4, 5]. However, little consideration has been given to the effects of low-carbon preference, CER cost factor, and carbon tax simultaneously on the design of coordination contracts. In this context, from the perspective of encouraging manufacturers to reduce carbon emissions, this study tries to address the following questions: (1) how do the manufacturer and the retailer make decisions considering the carbon tax, customers' low-carbon preference, and CER cost factor simultaneously? (2) How to set appropriate carbon tax that encourages the manufacturer reduce carbon emissions? (3) How to design a suitable coordinate contract to improve CER degree and enhance supply chain efficiency?

To answer the above questions, this paper considers a low-carbon supply chain, where the manufacturer makes CER investment to produce low-carbon products and sell them through a single retailer. In modeling and analysis, this paper finds the equilibrium strategies of the supply chain under centralized and decentralized decision-making and discusses the impacts of low-carbon awareness, CER cost factor, and carbon tax on the supply chain in detail. Furthermore, a revenue-sharing contract (RS) is adopted to stimulate the manufacturer to improve CER degree and coordinate the supply chain. The results indicate that the supply chain is better off exhibiting low-carbon preference and will suffer a loss from the increase CER cost factor when the carbon tax is low. Moreover, the paper finds that there exists a unique optimal carbon tax to make CER degree the maximum. And RS can encourage the manufacturer's CER investments but fails to coordinate the supply chain, and the manufacturer and the retailer prefer to implement RS when the RS coefficient is in a certain threshold. This research makes three contributions to the literature. First, this paper integrates consumers' low-carbon awareness, CER cost factor, and carbon tax into supply chain models and explores how these factors affect the CER degree and firms' profit. Second, this paper finds that the government should impose different optimal carbon taxes to inspire the manufacturer to make the best efforts on CER investments in different scenarios. Finally, this paper designs RS to incentive the manufacture to improve CER degree and also enhance the supply chain efficiency.

The remainder of this paper is organized as follows. Section 2 briefly reviews the closely related literature studies. Section 3 describes model notations, assumptions, and formulations. The game models are established to obtain the equilibrium solutions in Section 4. A numerical example is conducted to compare the optimal strategies in Section 5. Section 6 provides concluding remarks. In addition, all proofs are provided in Appendix.

2. Literature Review

With the transition to the sustainable supply chain, there is a growing interest in the decision-making problems and the design of coordination/incentive contracts. Research on low-carbon supply chain decisions mainly focuses on price [6–8], low-carbon products' quantity [9, 10], manufacturing/remanufacturing [11], etc. Low-carbon supply chain coordination has been studied by different mechanisms including RS [12, 13], BB (buyback contract) [14], and CS (cost-sharing contract) [15, 16]. A brief review of the previous studies that are related to this paper is conducted with regard to low-carbon decision-making of the supply chain and RS design with CER cost and consumers' low-carbon preference and carbon tax. Table 1 compares the research scope of this paper with that of typical studies on CER.

2.1. Low-Carbon Decision-Making of the Supply Chain. Environmental issues are new hotspots in supply chain research. Currently, low-carbon decision-making of the supply chain is mainly studied from three factors: low-carbon preference, CER cost factor, and carbon tax.

In the view of carbon tax, Mishra et al. [17] developed a carbon cap and tax-regulated sustainable inventory management model. Chen and Hu [18] used the evolutionary game theory to examine the behavioral strategies of the manufacturers in response to carbon tax. Despite attracting much attention from the academia, the specific quantity or the decision of the optimal carbon tax is still not discussed totally. This paper discusses the optimal carbon tax in different scenarios to encourage the manufacturer to reduce carbon emission. In the view of CER cost, Li et al. [12] examined the impact of the CER cost factor and carbon tax on price and CER decisions. In the view of consumers' low-carbon preference, Wang et al. [19] developed game models for studying CER considering low-carbon preference in the retailer-dominant and power-balanced cases. Wu et al. [20] proposed a low-carbon decision-making model of online shopping supply chain by considering the low-carbon awareness of online shoppers. Liang and Futou [21] investigated the impact of the low-carbon preference on the optimal joint carbon reduction strategy. Wang et al. [22] found that the CER and the channel profit increase with the increase in the consumers' low-carbon preference in the dual-channel supply chain. Assuming that consumers maintain a low-carbon preference and considering CER cost and carbon tax, this paper investigates the impact of these three factors on the optimal CER and price strategy and designs RS to encourage the manufacturer to reduce carbon emission.

2.2. Revenue-Sharing Contract. Li et al. [7] investigated the impact of the RS contract offered by a retailer on CER. Yu et al. [23] deduced equilibrium emission reduction and pricing strategies in RS considering reference emission and cost learning effects. Yang and Chen [5] studied the impact of RS offered by a retailer on manufacturer's CER effort considering consumers' low-carbon preference and carbon tax simultaneously. Liu et al. [24] examined the effects of RS on product greenness and pricing strategies under different power structures. The above studies have proven that the improvement of greenness of the supply chain is strongly linked to the RS contract among supply chain members.

2.3. Implication Referred. Based on the extant literature studies, many authors analyzed the impact of various factors on the low-carbon operation decision-making of enterprises. However, the aforementioned research studies have incomprehensively considered the impact of CER cost

Table 1: Authors' contribution.

Authors	CER cost	Carbon tax	Consumers' low-carbon preference	Coordination/incentive contract
Mishra et al. [17]		✓		
Chen and Hu [18]		✓		
Wang et al. [19]			✓	
Yang and Chen [5]		✓	✓	✓
Li et al. [12]	✓		✓	✓
This paper	✓	✓	✓	✓

factor, carbon tax, and consumers' low-carbon awareness on the low-carbon decisions and RS. Our paper aims to fill the gap by introducing CER cost factor, consumers' low-carbon preference, and carbon tax into the analytical framework. Thus, the incentive effect of carbon tax and RS can be studied more accurately and realistically. The conclusion will help the firms make decisions and design incentive contract in a low-carbon supply chain, as well as provide insights for the government to formulate the optimal carbon tax policy.

3. Model Description

3.1. Notations. This paper focuses on a low-carbon supply chain comprising one manufacturer and one retailer where consumers are exhibiting low-carbon preference. The manufacturer produces low-carbon products and sells them to the retailer in the wholesale price. The manufacturer is dominant in the supply chain, who decides the CER degree and the wholesale price. The retailer decides the retail price and sells to consumers. The government imposes carbon tax on carbon emission. Due to the need of the research problem, the following variables and parameters are defined in Table 2.

3.2. Assumptions. For the purposes of discussion and without loss of generality, this paper makes the following basic assumptions.

Assumption 1. It is assumed the demand function is linearly decreasing in selling price and increasing in CER degree [25], that is, the market demand function is described as follows: $D = a - p + \beta(e_1 - e_0) = a - p + \beta e$.

Assumption 2. The manufacturer takes extra cost to invest CER technologies. It is common knowledge that the manufacturer makes initial changes in products and processes easily while the subsequent improvement being more difficult [7]. Thus, CER cost function is described as follows: $c_e = (\eta e^2/2)$, which means CER cost is a quadratic function of CER degree e, which is commonly employed in related works [26].

Assumption 3. The manufacturer must pay the carbon tax for carbon emission in the production process. Every unit carbon emission should pay t carbon tax. Therefore, the manufacturer will pay carbon tax te_1 for each low-carbon product.

Assumption 4. In order to ensure all decision variables and profits are positive, we assume $a - c - te_0 > 0, t < \beta$, and $(\beta + t)^2 < 2\eta$ throughout the paper.

3.3. Basic Model. Using the aforementioned notations and assumptions, this paper models the profits of the manufacturer, the retailer, and the total supply chain as follows. The manufacturer's profit (π_m) includes the sales revenue, production cost, CER investment cost, and carbon tax. The retailer's profit (π_r) consists of sales revenue and wholesale cost. The total profit (π_{sc}) for the whole supply chain is the sum of both firms' profit. Note that $D = a - p + \beta(e_0 - e_1) = a - p + \beta e$. So, we can get the profits (π_m, π_r) of the manufacturer and the retailer. Hence, we formulate π_m, π_r, and π_{sc} individually:

$$\pi_m = [w - c - te_1]D - c_e = [w - c - te_1](a - p + \beta e) - \frac{1}{2}\eta e^2$$
$$\pi_r = (p - w)(a - p + \beta e)$$
$$\pi_{sc} = \pi_m + \pi_r. \tag{1}$$

4. The Models and Results

4.1. Centralized Supply Chain. As a benchmark, the manufacturer and the retailer jointly determine the CER degree e^c and the retail price p^c in a centralized supply chain. This paper obtains the profit function of the centralized supply chain π_{sc}^c as follows. The superscript c is used to denote the centralized scenario.

$$\pi_{sc}^c = [p - c - t(e_0 - e)](a - p + \beta e) - \frac{\eta e^2}{2}. \tag{2}$$

Proposition 1. *In a centralized supply chain, the optimal CER degree is (e^{c*}), the retail price is (p^{c*}), and the maximum supply chain profit is (π_{sc}^{c*}):*

Table 2: Notation and parameters.

Notation	Interpretation
Decision variables	
w	Wholesale price
p	Retail price
e	Carbon emission reduction degree $e = e_1 - e_0$
Parameters	
e_1	The carbon emission of unit low-carbon product after the manufacturer's CER efforts
e_0	The initial carbon emission of unit traditional product
D	Market demand
a	The base market potential
β	Consumers' low-carbon preference coefficient
t	A carbon tax for each unit emission
η	CER cost factor
c_e	CER cost
c	Unit manufacturing cost
φ	RS coefficient/share of retailer's revenue with the manufacturer, $0 < \varphi < 1$
π_{sc}	The whole supply chain profit
π_m	The manufacturer's profit
π_r	The retailer's profit
E	The supply chain efficiency

$$e^{c*} = \frac{(a - c - te_0)(\beta + t)}{\left[2\eta - (\beta + t)^2\right]},$$

$$p^{c*} = \frac{\left[(a + c + te_0)(\eta - \beta t) - \beta^2 c - \beta^2 t e_0 - t^2 \alpha\right]}{\left[2\eta - (\beta + t)^2\right]}, \quad (3)$$

$$\pi_{sc}^{c*} = \frac{-\eta (c - \alpha + e_0 t)^2}{2\left(\beta^2 + 2\beta t + t^2 - 2\eta\right)}.$$

Proof. See Appendix.

Lemma 1. *In a centralized supply chain,*

(1) $(\partial e^{c*}/\partial \eta) < 0$, $(\partial p^{c*}/\partial \eta) < 0$, *and* $(\partial \pi_{sc}^{c*}/\partial \eta) < 0$

(2) $(\partial e^{c*}/\partial \beta) > 0$, $(\partial p^{c*}/\partial \beta) > 0$, *and* $(\partial \pi_{sc}^{c*}/\partial \beta) > 0$

(3) If $0 < t < t^{c*}$, *then* $(\partial e^{c*}/\partial t) > 0$; *if* $t^{c*} < t$, *then* $(e^{c*}/\partial t) < 0$

Proof. See Appendix.

Lemma 1 demonstrates that the optimal CER degree e^{c*}, the optimal retail price p^{c*}, and the maximum supply chain profit π_{sc}^{c*} are decreasing in the CER cost factor η and are increasing in the consumers' low-carbon preference β.

Lemma 1 also states that there is an optimal carbon tax t^{c*} in the centralized supply chain, which makes the CER degree the maximum.

4.2. Decentralized Supply Chain. In a decentralized supply chain, the relationship between the manufacturer and the retailer is modeled as a Stackelberg game, where the manufacturer is the leader and the retailer is the follower. In the first stage, the manufacturer decides the CER degree e^d and the wholesale price w^d. In the second stage, the retailer sets the retail price p^d based on the manufacturer's decisions. The profit functions of the manufacturer and the retailer are formulated as follows. The backward induction is utilized to solve this model for obtaining the optimum solution. The superscript d is used to denote the decentralized scenario.

$$\pi_m^d = \left[w - c - t(e_0 - e)\right](a - p + \beta e) - \frac{1}{2}\eta e^2, \quad (4)$$

$$\pi_r^d = (p - w)(a - p + \beta e). \quad (5)$$

Proposition 2. *In the decentralized supply chain, the optimal retail price is* (p^{d*}), *the optimal wholesale price is* (w^{d*}), *the optimal CER degree is* (e^{d*}), *and the maximum profit of the manufacturer, the retailer, and the supply chain is* $(\pi_m^{d*}, \pi_r^{d*}, \pi_{sc}^{d*})$.

$$e^{d*} = \frac{(a - c - te_0)(\beta + t)}{4\eta - (\beta + t)^2}$$

$$w^{d*} = \frac{\beta^2 c - 2c\eta - 2a\eta + at^2 + a\beta t + \beta ct - 2e_0\eta t + \beta e_0 t^2 + \beta^2 e_0 t}{\beta^2 + 2\beta t + t^2 - 4\eta},$$

$$p^{d*} = \frac{\beta^2 c - c\eta - 3a\eta + at^2 + a\beta t + \beta ct - e_0\eta t + \beta e_0 t^2 + \beta^2 e_0 t}{\beta^2 + 2\beta + t^2 - 4\eta},$$

$$\pi_m^{d*} = \frac{-\eta (c - a + e_0 t)^2}{2\left(-4\eta + 2\beta t + \beta^2 + t^2\right)},$$

$$\pi_r^{d*} = \frac{\eta^2 (c - a + e_0 t)^2}{\left(-4\eta + 2\beta t + \beta^2 + t^2\right)^2},$$

$$\pi_{sc}^{d*} = \frac{-\eta (c - a + e_0 t)^2 \left(\beta^2 + 2\beta t + t^2 - 6\eta\right)}{2\left(-4\eta + 2\beta t + \beta^2 + t^2\right)^2}. \quad (6)$$

Proof. See Appendix.

Lemma 2. *In the decentralized supply chain,*

(1) $(\partial e^{d*}/\partial\eta) < 0$, $(\partial p^{d*}/\partial\eta) < 0$, *and* $(\partial\pi_{sc}^{d*}/\partial\eta) < 0$

(2) $(\partial e^{d*}/\partial\beta) > 0$, $(\partial p^{d*}/\partial\beta) > 0$, *and* $(\partial\pi_{sc}^{d*}/\partial\beta) > 0$

(3) *If* $0 < t < t^{d*}$, *then* $(\partial e^{d*}/\partial t) > 0$; *if* $t^{d*} < t$, *then* $(e^{d*}/\partial t) < 0$

Proof. See Appendix.

Lemma 2 demonstrates that the CER degree e^{d*}, the retail price p^{d*}, and the supply chain profit π_{sc}^{d*} will increase as the CER cost factor η decreases. Moreover, the CER degree e^{d*}, the retail price p^{d*}, and the supply chain profit π_{sc}^{d*} will increase as the consumers' low-carbon preference β increases.

Lemma 2 also states that there is an optimal carbon tax t^{d*} which makes the CER degree the maximum.

According to the calculation and analysis in the above section, we compare the equilibrium outcomes in the centralized and the decentralized supply chain, which are shown in Theorem 1.

Theorem 1. *(i)* $e^{d*} < e^{c*}$ *and* $\pi_{sc}^{d*} < \pi_{sc}^{c*}$

(ii) If $\eta < t\beta + \beta^2$, $p^{c*} > p^{d*}$; *if* $\eta > t\beta + \beta^2$, $p^{c*} < p^{d*}$; *if* $\eta = t\beta + \beta^2$, $p^{c*} = p^{d*}$

Proof. See Appendix.

Theorem 1 shows that the optimal CER degree and the maximum channel profit in the centralized supply chain are both higher than those in the decentralized supply chain. This phenomenon is called "double marginal effect." The following sections design the RS contract to eliminate the double marginal effect.

4.3. Revenue-Sharing Contract for the Decentralized Supply Chain. In this section, we propose the RS contract in which the manufacturer offers the retailer a lower wholesale price, and the retailer shares revenue with the manufacturer with a fraction $\varphi\,(0 < \varphi \le 1)$.

The timeline is as follows. Firstly, the manufacturer makes CER efforts to reduce the emission of the products, which yields the CER degree e^{rs} and the wholesale price w^{rs}. Secondly, the retailer decides his selling price p^{rs}. The profit functions of the manufacturer and the retailer are as follows. They are solvable by using backward induction. The superscript rs is used to denote the RS scenario.

$$\begin{aligned}\pi_m^{rs} &= [w - c - t(e_0 - e) + 1 - \varphi p](a - p + \beta e) - \frac{1}{2}\eta e^2\\ \pi_r^{rs} &= (\varphi p - w)(a - p + \beta e).\end{aligned} \tag{7}$$

Proposition 3. *Under the RS contract, the optimal CER degree is* (e^{rs*}), *the optimal wholesale price is* (w^{rs*}), *the optimal retail price is* (p^{rs*}), *and the maximum profit of the manufacturer and the retailer is* $(\pi_m^{rs*}, \pi_r^{rs*})$:

$$\begin{aligned}
e^{rs*} &= \frac{(\beta+t)(a-c-te_0)}{2\eta(1+\varphi)-(\beta+t)^2},\\
w^{rs*} &= \frac{\varphi\left(\beta^2 c - 2c\eta + at^2 + a\beta t + \beta ct - 2a\eta\varphi - 2e_0\eta t + \beta e_0 t^2 + \beta^2 e_0 t\right)}{\beta^2 + 2t\beta + t^2 - 2\eta - 2\eta\varphi},\\
p^{rs*} &= \frac{\beta^2 c - c\eta - a\eta + at^2 + a\beta t + \beta ct - 2a\eta\varphi - e_0\eta t + \beta e_0 t^2 + \beta^2 e_0 t}{\beta^2 + 2t\beta + t^2 - 2\varphi\eta - 2\eta},\\
\pi_m^{rs*} &= \frac{-\eta(c-a+e_0 t)^2}{2\left(\beta^2 + 2\beta t + t^2 - 2\eta - 2\eta\varphi\right)}.\\
\pi_r^{rs*} &= \frac{\eta^2\varphi(c-a+e_0 t)^2}{\left(-2\eta + 2\beta t - 2\eta\varphi + \beta^2 + t^2\right)^2},
\end{aligned} \tag{8}$$

The proof is similar to that of Proposition 2 and is hence omitted here for brevity.

Lemma 3. *Under RS,*

(1) $(\partial e^{rs*}/\partial\eta) < 0$, $(\partial p^{rs*}/\partial\eta) < 0$, *and* $(\partial\pi_{sc}^{rs*}/\partial\eta) < 0$

(2) $(\partial e^{rs*}/\partial\beta) > 0$, $(\partial\pi_{sc}^{rs*}/\partial\beta) > 0$, *and* $(\partial p^{rs*}/\partial\beta) > 0$

(3) *If* $0 < t < t^{rs*}$, *then* $(\partial e^{rs*}/\partial t) > 0$; *if* $t^{rs*} < t$, *then* $(\partial e^{rs*}/\partial t) < 0$

Lemma 3 demonstrates that the CER degree e^{rs*}, the retail price p^{rs*}, and the supply chain profit π_{sc}^{rs*} will increase as the CER cost factor η decreases. Moreover, the CER degree e^{rs*}, the retail price p^{rs*}, and the supply chain profit π_{sc}^{rs*} will increase as the consumers' low-carbon preference β increases.

Lemma 3 also states that there is an optimal carbon tax t^{rs*} which makes the CER degree the maximum.

The proof is similar to that of Lemma 2 and is hence omitted here for brevity.

Theorem 2. *Under the RS contract, $e_m^{rs} > e_m^d$; if $([2\eta - (t+\beta)^4]^2/4\eta^2) < \varphi < 1$, then $\pi_m^{d*} < \pi_m^{rs*}$ and $\pi_r^{d*} < \pi_r^{rs*}$.*

Proof. See Appendix.

Theorem 2 shows that RS improves the profit of the manufacturer and the retailer in the decentralized supply chain when the RS coefficient φ is in a certain threshold, that is, the manufacturer and the retailer are willing to implement the RS contract if and only if the RS coefficient satisfies $([2\eta - (t+\beta)^4]^2/4\eta^2) < \varphi < 1$.

5. Numerical Analysis

From the aforementioned explanations, we are unable to obtain closed-form conditions for profit comparison considering various factors. So, we conduct a set of numerical experiments to study the factors how various factors, such as the CER cost factor η, the government's carbon tax t, and the consumers' low-carbon preference β influence the manufacturer's CER decision, the retailer's profit, and the manufacturer's profit under RS. The parameters in this section were assumed from a previously published paper [5]; the parameter values are $a = 20, c = 4, \beta = 1.2, e_0 = 10, t = 0.2, \eta = 6$, and $\varphi = 0.6$. And the proposed model was solved by using MATLAB software. See Figures 1–6 for the decision variables with optimum values according to the parameter values.

5.1. Effect of η on the Equilibrium Outcomes. Now, we explore how the CER cost factor η influences the CER degree. Figure 1 shows that the CER degree is the greatest in the centralized system, medium under RS, and the lowest in the decentralized system. Moreover, when the CER cost factor decreases, the CRE degree increases at different rates, and the gap of the CER degree among these three scenarios would widen further. This suggests that the benefits of vertical integration increase rapidly with lower CER cost factor. And it means that RS produces a better effect to motivate the manufacturer's CER efforts with less CER cost factor.

Figure 2 illustrates that the manufacturer's and the retailer's profit increase with the decrease in the CER cost factor because a higher CER cost would increase the manufacturer's marginal cost. The manufacturer sets a higher wholesale price, and the retailer sets a higher retail price gradually. Consequently, the demand decreases, and the marginal profit becomes lower. Thus, the total optimal profit decreases due to a higher CER cost factor.

5.2. Effect of t on the Equilibrium Outcomes. Now, we explore how the carbon tax $t$affects the CER degree and supply chain members' profit. It can be seen from Figure 3 that the CER degree initially increases and then decreases with the carbon tax t. It also says that there exists an optimal carbon tax for the government to spur the manufacturer to exert his best effort to curb carbon emission. The government should impose the highest carbon tax in the centralized system, medium under RS, and the lowest in the decentralized system.

5.3. Effect of β on the Equilibrium Outcomes. Now, we explore how the customers' low-carbon preference β affects the retail price. Figure 4 shows that the retailer should set the highest retail price in the decentralized system, medium under RS, and the lowest in the centralized system when β is lower than the threshold. An intuitive explanation for this finding is that when β is small, customers are far less likely to pay for low-carbon products; thus, it is similar to the traditional supply chain situation. The manufacturer would set a lowest wholesale price in the centralized system, medium under RS, and the highest in the decentralized system, which yields his maximum profit. So, the retailer should set higher selling price to sustain his maximum profit compared to the centralized system. When β goes beyond the certain threshold value, customers prefer to pay for low-carbon products. The retailer should set a highest retail price in the centralized system, medium under RS, and the lowest in the decentralized system. The reason is that when customers' low-carbon preference is high, the positive influence on consumers' purchase decision-making would be obvious. More customers are willing to pay premiums for low-carbon products. Thus, the product's marginal profit becomes higher. The retailer should raise the selling price to improve the profit.

Figure 5 illustrates that the manufacturer's and the retailer's profit increase with the increase in the customers' low-carbon preference. A higher customers' low-carbon preference increases the customers' willingness to pay for low-carbon products, which makes the demand to increase and improves the manufacturer's and the retailer's profit accordingly.

5.4. Effect of RS on the Equilibrium Outcomes. Now, we explore how RS affects the supply chain efficiency E. The optimal supply chain efficiency in the decentralized case ($E^{d*} = (\pi_{sc}^{d*}/\pi_{sc}^{c*})$) and under RS ($E^{rs*} = (\pi_{sc}^{rs*}/\pi_{sc}^{c*})$) is described in Figure 6.

Figure 6 shows that the supply chain efficiency under RS is higher than that in the decentralized supply chain. The supply chain efficiency in the RS and decentralized scenario decreases with β and t but increases with η. This can be explained as follows. On the one hand, for given η, the rise of

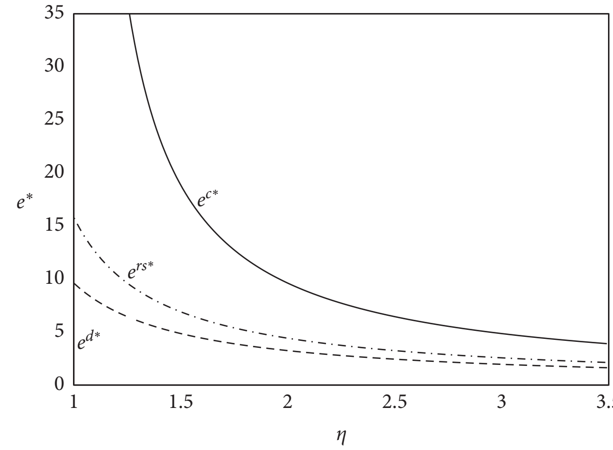

Figure 1: Effect of η on the CER degree.

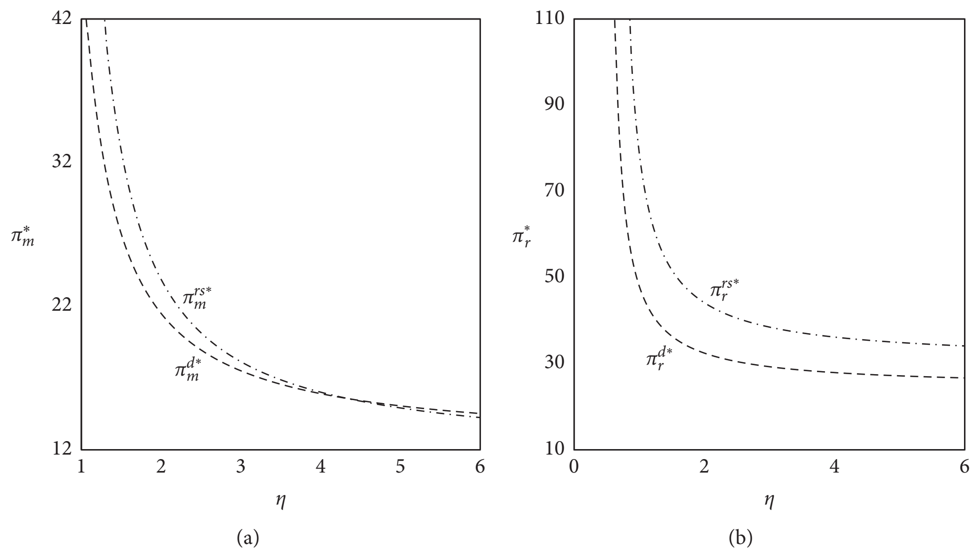

Figure 2: Effect of η on the manufacturer and retailer's profit.

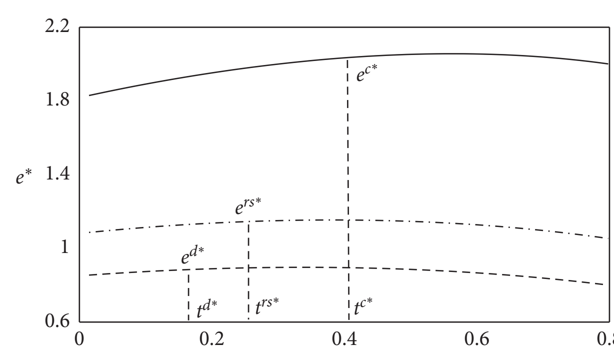

Figure 3: Effect of t on the manufacturer and retailer's profit.

the product's marginal profit enlarges as β and t increase; thus, the manufacturer exerts more effort in CER, which intensifies double marginalization and thus worsens the channel efficiency. On the other hand, for given β and t, the expansion of the product's marginal profit diminishes when η escalates; then, the manufacturer makes less effort in CER, which counteracts double marginalization and eventually improves the supply chain efficiency.

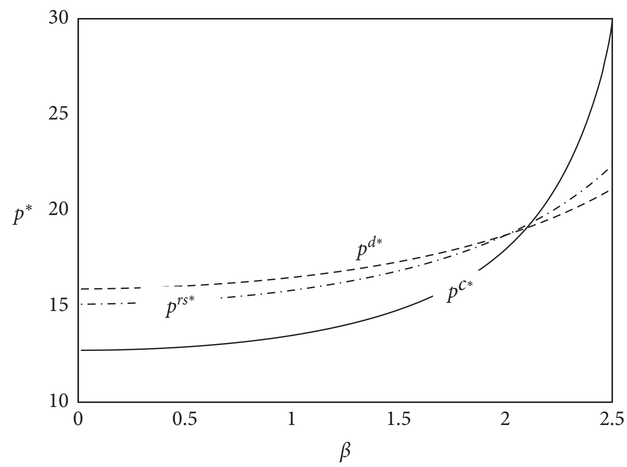

Figure 4: Effect of β on the retail price.

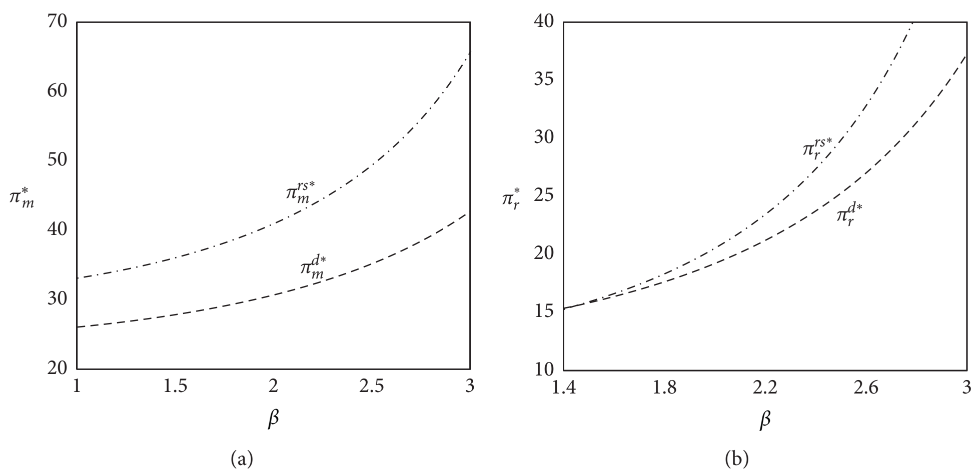

Figure 5: Effect of β on the manufacturer and retailer's profit.

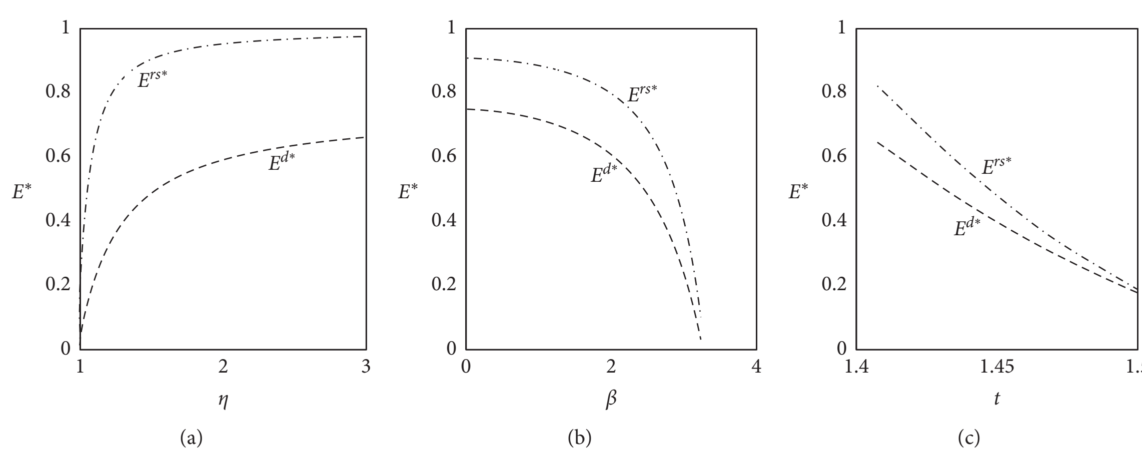

Figure 6: Impact of RS on the supply chain efficiency.

6. Managerial Implication

This paper demonstrates some key and deliberate insights for managers and policy makers who need to consider the manufacturer's CER cost factor, the government's carbon tax, and consumers' low-carbon preference. The managerial implication for managers and policy makers can be obtained as follows:

(1) With the decrease in the CER cost factor, the CER degree and the supply chain members' profits will increase. Therefore, the manufacturer should improve the CER degree with the decrease in the CER cost factor. Moreover, the manufacturer should set the highest CER degree in the centralized, medium under RS, and the lowest in the decentralized supply chain.

(2) As consumers' low-carbon preference increases, the retail price and the supply chain members' profits increase significantly. Therefore, the retailer should improve the retail price with the increase in the consumers' low-carbon preference. Moreover, the retailer should set the highest selling price in the decentralized, medium under RS, and the lowest in the centralized supply chain when β is below a certain threshold, and the retailer should set the highest selling price in the centralized, medium under RS, and the lowest in the decentralized supply chain when β is below a certain threshold.

(3) Carbon tax has a certain incentive effect on the manufacturer's CER degree. The government should impose different carbon taxes in different scenarios, i.e., the highest carbon tax in the centralized supply chain, medium under RS, and the lowest in the decentralized supply chain. Based on the optimal carbon tax imposed by the government, the incentive mechanism can stimulate the manufacturer to increase their efforts towards CER investment for environmental and economic benefits.

(4) RS cannot perfectly coordinate the supply chain but can improve the supply chain efficiency and encourage the manufacturer to increase CER investment.

7. Conclusion

In this study, some factors are taken into consideration such as the manufacturer's CER cost factor, the government's carbon tax, and consumers' low-carbon preference to establish profit models of the low-carbon supply chain. Stackelberg game models are developed to address the centralized, decentralized, and RS scenarios between the manufacturer and the retailer. The models determine the optimal CER degree and price decisions, as well as the carbon tax imposed by the government. This paper finds that the supply chain would benefit from the increment of consumer's environmental awareness but be depressed by the increase of CER cost. Additionally, there exists a unique optimal carbon tax to improve CER degree in different scenarios. Furthermore, this paper finds that RS is effective to inspire the manufacture to exert the best efforts to improve CER degree. Moreover, RS is a feasible incentive tool to the manufacturer and retailer when the RS coefficient is in a certain threshold value.

There are some limitations on this paper. The assumption is that the manufacturer is in the dominant position in the supply chain. And this paper adopts simulated data to verify the proposed model. A case study utilizing real industrial data is not inserted in this model. So, retailer-dominant or/and power-balanced scenarios should be discussed in the future. And a case study utilizing real industrial data should be extended. These extensions will help us come to a better understanding on the low-carbon supply chain operation in the future [27, 28].

Appendix

Proof of Proposition 1. The Hessian matrix of π_{sc}^{c*} $H_1 = \begin{bmatrix} (\partial^2\pi_m/\partial e^2) & (\partial^2\pi_m/\partial e\,\partial p) \\ (\partial^2\pi_m/\partial p\,\partial e) & (\partial^2\pi_m/\partial p^2) \end{bmatrix} = \begin{bmatrix} -2 & \beta - t \\ \beta - t & 2\beta t - \eta \end{bmatrix}$, which is negatively definite if $|H_1| = 2\eta - (\beta + t)^2 > 0$. π_{sc}^{C} is a strictly joint convex function of $e$and p, and there is an optimal solution.

Solving the first-order conditions $(\partial\pi_{sc}^{c}/\partial e) = 0$ and $(\partial\pi_{sc}^{c}/\partial p) = 0$ for (p, e), we obtain the optimal CER degree e^{c*} and retail price p^{c*}, respectively, in Proposition 1. To ensure $e^{c*} > 0$, we assume that $a - c - te_0 > 0$ throughout this paper.

Substituting e^{c*} and p^{c*} into (1), we obtain the maximum supply chain's profit π_{sc}^{c*} in Proposition 1. To ensure the profit is positive, we assume that $(\beta + t)^2 < 2\eta$ throughout the paper.

Proof of Lemma 1.

(1)

(i) Taking the first derivative of e^{c*} with respect toη, we obtain

$$\frac{\partial e^{c*}}{\partial \eta} = \frac{-2(\beta + t)(a - c - te_0)}{\left[2\eta - (\beta + t)^2\right]^2};\ \text{if } a - c - te_0 > 0,\ \text{then } \frac{\partial e^{c*}}{\partial \eta} < 0. \tag{A.1}$$

(ii) Taking the first derivative of p^{c*} with respect to η, we obtain

$$\frac{\partial p^{c*}}{\partial \eta} = \frac{(a - c - te_0)(t^2 - \beta^2)}{\left[2\eta - (\beta + t)^2\right]^2};\ \text{if } a - c - te_0 > 0 \text{ and } t < \beta,\ \text{then } \frac{\partial p^{c*}}{\partial \eta} < 0. \tag{A.2}$$

(iii) Similarly, $(\partial\pi_{sc}^{c*}/\partial\eta) = (2\eta(\beta + t)\ (c - a + e_0 t)^2 / 2[(\beta + t)^2 - 2\eta]^2) < 0$.

(2)

(i) Taking the first derivative of e^{c*} with respect to β, we obtain

$$\frac{\partial e^{c*}}{\partial \beta}=\frac{\left[2\eta+(\beta+t)^2\right](a-c-te_0)}{\left[2\eta-(\beta+t)^2\right]^2};\ \text{if } a-c-te_0 > 0, \text{ then } \frac{\partial e^{c*}}{\partial \beta}>0. \quad (A.3)$$

(ii) Taking the first derivative of p^{c*} with respect to β, we obtain $(\partial p^{c*}/\partial \beta) = ((a-c-te_0)[2\eta\beta - t(\beta+t)^2]/[2\eta-(\beta+t)^2]^2)$. If $a-c-te_0>0, t<\beta$, and $(\beta+t)^2<2\eta$, then $(\partial p^{c*}/\partial\beta)>0$.

(iii) Similarly, $(\partial\pi_{sc}^{c*}/\partial\beta) = (2\eta(\beta+t)(c-a+e_0t)^2/2[(\beta+t)^2-2\eta]^2)>0$.

(3) Taking the second derivative of e^{c*} with respect to t, we obtain $(\partial^2 e^{c*}/\partial t^2) = -e_0\eta-2<0$, that is, e^{c*} is a convex function of t. Solving the first-order condition $(\partial e^{c*}/\partial t) = ((c-a+\beta e_0+2e_0t)/(\beta+t)^2-2\eta) - (2(\beta+t)(\beta c - a\beta - at + ct + e_0t^2+\beta e_0t)/[(\beta+t)^2-2\eta]^2) = 0$for t, we can obtain the optimal carbon tax t^{c*}.

Proof of Proposition 2. Taking the second derivative of π_r^d with respect to p, we obtain that $(\partial^2\pi_r^d/\partial p^2) = -e_0\eta-2<0$, i.e., π_r^d is a convex function of p. Solving the first-order condition for p, we can derive the retail price reaction function:

$$p(w,e)=\frac{a+\beta e+w}{2}. \quad (A.4)$$

Substituting (A 4) into (4), we obtain that

$$\pi_m^d=\frac{1}{2}\left[w-c-t(e_0-e)\right](a+\beta e-w)-\frac{1}{2}\eta e^2. \quad (A.5)$$

According to equation (A 5), the Hessian matrix of π_m^d $H_2 = \begin{bmatrix} (\partial^2\pi_m^d/\partial e^2) & (\partial^2\pi_m^d/\partial e\,\partial w) \\ (\partial^2\pi_m^d/\partial w\partial e) & (\partial^2\pi_m^d/\partial w^2) \end{bmatrix}$ $\begin{bmatrix} t\beta-\eta & (1/2)\beta-(1/2)t \\ (1/2)\beta-(1/2)t & -1 \end{bmatrix}$, which is negatively definite if $2\eta-(t+\beta)^2>0$, $t\beta-\eta>0$. So, π_m^d is a strictly joint convex function of e and w. Solving the first-order conditions, we obtain the optimal CER degree (e^{d*}) and wholesale price (w^{d*}) in Proposition 2. Substituting e^{d*}, w^{d*} into (4)–(A 4), we obtain the optimal retail price (p^{d*}), the maximum supply chain members' profits (π_m^{d*}, π_r^{d*}), and the maximum channel's profit (π_{sc}^{d*}) in Proposition 2.

Proof of Lemma 2.

(1)

(i) Taking the first derivative of e^{d*} with respect to η, we obtain $(\partial e^{d*}/\partial\eta) = (-4(\beta+t)(a-c-te_0)/[4\eta-(\beta+t)^2]^2)$. If $a-c-te_0>0$, then $(\partial e^{d*}/\partial\eta)<0$.

(ii) Taking the first derivative of p^{d*} with respect to η, we obtain $(\partial p^{d*}/\partial\eta) = ((c-a+te_0)(3\beta-t)(\beta+t)/[4\eta-(\beta+t)^2]^2)$. If $c-a+te_0<0, \beta>t$, then $(\partial p^{d*}/\partial\eta)<0$.

(iii) Taking the first derivative of π_{sc}^{d*} with respect to η, we obtain $(\partial\pi_{sc}^{d*}/\partial\eta) = -((\beta+t)^2(c-a+e_0t)^2[(\beta+)t^2-8\eta]/2[(\beta+t)^2-4\eta]^3)$. If $(\beta+t)^2<2\eta$, then $(\partial\pi_{sc}^{d*}/\partial\eta)<0$.

(2)

(i) Taking the first derivative of e^{d*} with respect toβ, we obtain $(\partial e^{d*}/\partial\beta) = (-[4\eta+(\beta+t)^2](c-a+te_0)/[4\eta-(\beta+t)^2]^2)$. And if $c-a+te_0<0$, then $(\partial e^{d*}/\partial\beta)>0$.

(ii) Taking the first derivative of p^{d*} with respect to β, we obtain$(\partial p^{d*}/\partial\beta) = ((c-a+e_0t)[t(\beta+t)^2-2\eta(3\beta+t)]/[(\beta+t)^2-4\eta]^2)$. And if $c-a+te_0<0, t<\beta$, and $(\beta+t)^2<2\eta$, then $(\partial p^{d*}/\partial\beta)>0$.

(iii) Taking the first derivative of π_{sc}^{d*} with respect to β, we obtain $(\partial\pi_{sc}^{d*}/\partial\beta) = (\eta(\beta+t)(c-a+te_0)^2[(\beta+t)^2-8\eta]/[(\beta+t)^2-4\eta]^3)$. And if $c-a+te_0<0$ and $(\beta+t)^2<2\eta$, then $(\partial\pi_{sc}^{d*}/\partial\beta)>0$.

(3) Taking the second derivative of e^{d*} with respect to t, we obtain $(\partial^2e^{d*}/\partial t^2)<0$, i.e., e^{d*} is a convex function of t. Taking the first derivative of e^{d*} with respect to t, we obtain $(\partial e^{d*}/\partial t) = ((c-a+\beta e_0+2e_0t)/(\beta+t)^2-4\eta) - (2(\beta+t)^2(c-a+e_0t)/[(\beta+t)^2-4\eta]^2)$. Solving the first-order conditions, we can obtain the optimal carbon tax t^{d*}.

Proof of Theorem 1.

(1)

(i) If $0<\varphi<1$, it is obvious that $e^{c*}-e^{d*} = ((a-c-te_0)(\beta+t)/2\eta-(\beta+t)^2) - ((a-c-te_0)(\beta+t)/4\eta-(\beta+t)^2)>0$

(ii) If $(\beta+t)^2<2\eta$, it is obvious that $\pi_{sc}^{c*}-\pi_{sc}^{d*} = (-\eta(c-a+e_0t)^2/2[(\beta+t)^2-2\eta]) + (\eta(c-a+e_0t)^2[(\beta+t)^2-6\eta]/2[(\beta+t)^2-4\eta]^2)>0$

(iii) If $\eta<t\beta+\beta^2$, it is obvious that $p^{c*}-p^{d*} = ((a+c+te_0)(\eta-\beta t)-\beta^2c-\beta^2te_0-t^2a/2\eta-(\beta+t)^2) - ((a+c+te_0)(\eta-\beta t)-\beta^2c-\beta^2te_0-t^2a+2a\eta/4\eta-(\beta+t)^2)>0$; if $\eta>t\beta+\beta^2$, $p^{c*}-p^{d*}<0$; if $\eta=t\beta+\beta^2$, $p^{c*}=p^{d*}$

Proof of Theorem 2.

(i) If $0<\varphi<1$, it is easy to prove that $e_m^{d*}<e_m^{rs*}$.

(ii) If $\pi_m^{d*}<\pi_m^{rs*}$, then $0<\varphi<1$; if $\pi_r^{d*}<\pi_r^{rs*}$, then $([2\eta-(t+\beta)^4]^2/4\eta^2)<\varphi<1$. So, if $([2\eta-(t+\beta)^4]^2/4\eta^2)<\varphi<1$, we obtain $\pi_m^d<\pi_m^{rs}$ and $\pi_r^d<\pi_r^{rs}$.

Data Availability

The underlying data supporting the results of this study can be found in the manuscript.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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Research Article

An Inventory Model for Perishable Items with Price-, Stock-, and Time-Dependent Demand Rate considering Shelf-Life and Nonlinear Holding Costs

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Nowadays, consumers are more health conscious than before, and their demand of fresh items has intensely increased. In this context, an effective and efficient inventory management of the perishable items is needed in order to avoid the relevant losses due to their deterioration. Furthermore, the demand of products is influenced by several factors such as price, stock, and freshness state, among others. Hence, this research work develops an inventory model for perishable items, constrained by both physical and freshness condition degradations. The demand for perishable items is a multivariate function of price, current stock quantity, and freshness condition. Specific to price, six different price-dependent demand functions are used: linear, isoelastic, exponential, logit, logarithmic, and polynomial. By working with perishable items that eventually deteriorate, this inventory model also takes into consideration the expiration date, a salvage value, and the cost of deterioration. In addition, the holding cost is modelled as a quadratic function of time. The proposed inventory model jointly determines the optimal price, the replenishment cycle time, and the order quantity, which together result in maximum total profit per unit of time. The inventory model has a wide application since it can be implemented in several fields such as food goods (milk, vegetables, and meat), organisms, and ornamental flowers, among others. Some numerical examples are presented to illustrate the use of the inventory model. The results show that increasing the value of the shelf-life results in an increment in price, inventory cycle time, quantity ordered, and profits that are generated for all price demand functions. Finally, a sensitivity analysis is performed, and several managerial insights are provided.

1. Introduction

Inventory management is a valuable function for companies around the world. It seeks to control the materials from acquisition up to sales-related decision-making (how much and when to buy items) in order to avoid overstock and/or stockout. According to Yavari et al. [1], one of the challenges when managing inventories is the inherent perishability of many items, which means their freshness and quality decrease over time, and these cannot be sold after their expiration date. Tirkolaee et al. [2] noted that the inherent perishability widely occurs in food goods (e.g., milk, vegetables, and meat), organisms, and ornamental flowers. These authors also stated that the time window between preparation and sales of perishable items is very significant for producers and purchasers.

Given that the inherent perishability can occur immediately, Pal et al. [3] addressed a production-inventory model for deteriorating products when the production cost depends on both production order quantity and production rate. Later, meanwhile, Mashud et al. [4] determined the optimal replenishment policy of deteriorating goods for the classical newsboy inventory problem by considering multiple just-in-time deliveries. Additionally, Mashud et al. [5]

derived an inventory model for deteriorating products that calculates the optimal vales for replenishment time, price, and green investment cost.

There are also some items in which the inherent perishability is noninstantaneous. Mashud et al. [6] considered this by developing an inventory model for noninstantaneous deteriorating items, which jointly optimizes the cycle time, price, the spending in preservation technology, and credit financing. Alongside, Mashud et al. [7] proposed another inventory model for noninstantaneous deteriorating goods which determines the cycle length, price, and preservation cost. On the contrary, Hasan et al. [8] introduced a noninstantaneous inventory model for agricultural goods taking into account the effects of the inherent perishability. This model states the optimal pricing and timing inventory policies.

Similarly, there is currently an increasing demand for fresh items since consumers are more concerned about their health habits. In this context, consumers want to buy fresh goods far from their expiry date, so these can be stored during longer periods of time. Thus, companies should carefully manage and control their inventories of fresh items in the warehouse. One of the most common approaches to tracking the freshness and quality of perishable items in the warehouse is the radio frequency identification (RFID) smart tags, which allows to decrease the risk of selling items after their expiration date. Some examples have been exemplified by Herbon et al. [9], Herbon et al. [10], and Herbon and Ceder [11], who investigated the effect of implementing the time-temperature-indicator (TTI) in order to make information about expiration dates (for different items) available online.

Once items are available on shelves, exhibiting large quantities is one of the factors that often influence consumers to purchase more products. Then, retailers might become more profitable by increasing their goods' availability. However, this is different for cases where time plays a key role, such as perishable items, whose degradation tends to make them less attractive and unsalable at the end of shelf-life. Another factor is price, which inversely stimulates demand; lower prices result in higher demand, and vice versa. Overall, price, stock availability, time, and shelf-life are the critical factors that affect the demand of perishable products and should, consequently, be considered when developing inventory models.

The literature on inventory models with time-, price-, and stock-dependent demand is also abundant. For instance, Mashud et al. [12] constructed a price-sensitive inventory model by considering that the demand follows an exponential or isoelastic price-dependent demand.

Avinadav et al. [13] proposed two inventory models under the assumption that the demand function is dependent on both price and stock age and determined the optimal pricing and inventory policies. While the first inventory model assumes a multiplicative demand function, the second assumes an additive demand function. During the same year, Qin et al. [14] formulated an inventory model that determines the pricing and inventory policies for a perishable item considering stock-dependent demand and the effects of item's quality degradation. Later, Chen et al. [15] derived the optimal inventory policy and shelf-space size for fresh products considering the expiration time and a demand rate that depends on freshness and stock. The demand function treated by Chen et al. [15] grows in the on-hand stock level and diminishes with respect to the age of the item. Afterwards, Feng et al. [16] resolved the inventory model of Chen et al. [15] considering the demand rate jointly dependent of price, stock, and age. Meanwhile, Dobson et al. [17] built an economic order quantity (EOQ) inventory model for a perishable item with fixed shelf-life when the demand rate decreases linearly with respect to the age of the stock.

Other authors that work on a similar problem are Herbon and Khmelnitsky [18], who developed an inventory model that determines the optimal replenishment time and price when the demand rate actually depends on both time and price. Alongside, Banerjee and Agrawal [19] defined the optimal discounting and ordering policies for deteriorating products including a demand rate that depends on price and freshness. Hsieh and Dye [20] also provided the optimal pricing policy for deteriorating goods by considering the impact of reference prices under the assumption that stocks stimulate demand. Finally, Li and Teng [21] obtained the lot sizing and pricing strategies for a perishable item when the demand rate is dependent on reference price, exhibited inventory, and item freshness.

Recently, Agi and Soni [22] developed an inventory model for joint optimal pricing and inventory management for a perishable item under stock-, age-, and price-dependent demand, allowing surplus inventory at the end of cycle. These authors stated that finishing the inventory cycle period with a positive inventory level results in a benefit because the demand increases when large quantities are ordered and exhibited in the shelf space. Conversely, having inventory of perishable products at the end of their cycle is not desirable, as these cannot be stored for the next inventory cycle. Hence, these must be sold at a salvage price.

Although most inventory models consider a constant holding cost, different factors that affect inventories on the storage place make the holding cost intrinsically variable. There are common scenarios in the real world where the holding cost increases over time because longer storage periods require more sophisticated and costly warehouse facilities. For instance, a longer storage of fresh products needs refrigeration and some specific conditions in place to prevent damages. Alfares and Ghaithan [23] provided an excellent state-of-the-art review on the economic order quantity (EOQ) and economic production quantity (EPQ) inventory models with variable holding costs. Specifically, Alfares and Ghaithan [23] classified the variable holding cost into three categories: time-dependent holding cost, stock-dependent holding cost, and multiple dependence cost variability. The types of holding cost functions are constant, linear, nonlinear, step, or general. For modelling the case of variable time-dependent holding cost, there exist several functions. One is the quadratic holding cost function that is exemplified by Valliathal and Uthayakumar [24], who constructed two

EPQ inventory models for deteriorating items by including the holding cost as nonlinear time dependent. Years later, Pal et al. [25] investigated the single-period newsvendor model and obtained the optimal lot size when the customers' balking happens by taking into consideration a nonlinear holding cost that depends on both lot size and the stock level. Tripathi and Mishra [26] studied two inventory models, with time-varying holding cost, that obtain both the optimal order quantity and the optimal replenishment cycle time. While the first inventory model presents a linearly time-dependent holding cost, the second one has a quadratic time dependence, carrying inventory cost. In the same year, Sivashankari [27] also presented a comparative study of three EPQ inventory models under the assumption that the carrying inventory cost is either constant, a linear function of time, or a quadratic function of time.

Since closing the stock cycle time with zero inventory is the most appropriate strategy when dealing with perishable items, this research work builds an inventory model for perishable items with zero inventory at the end of stock cycle. On the one hand, these perishable items are subject to physical deterioration and freshness degradation over time, and on the other hand, the demand is a multivariate function of price, on-hand stock quantity, and freshness state. For the demand that is related to price, six distinct price-dependent demand functions are considered: linear, isoelastic, exponential, logit, logarithmic, and polynomial. The inventory model also considers the expiration date, a salvage value, and a deterioration cost of the perishable item. Moreover, the holding cost is considered as nonlinear with a quadratic function of time. The proposed inventory model conjointly derives the optimal policy for the price, the replenishment cycle time, and the order quantity, which together maximize the total profit per unit of time.

The rest of this research work is comprised of several sections as follows. Section 2 introduces the notation and assumptions. Section 3 develops the inventory model with price-, stock-, and time-dependent demand with nonlinear holding cost. Section 4 develops the solution procedure to determine the optimal solution. Section 5 presents the solution to six different price-dependent demand functions and solves some numerical examples. Section 6 performs a sensitivity analysis and proposes some managerial insights. Finally, Section 7 provides the conclusions and outlines several areas for further research.

2. Notation and Assumptions

2.1. Notation. The notation used to develop the inventory model with price-, stock-, and age-dependent demand, with zero inventory at the end of cycle, is given as follows. The symbols of Agi and Soni [22] are used, and some more additional symbols are defined here, in order to have a standard notation (Table 1).

2.2. Assumptions. The inventory model is based on the following assumptions:

(1) The item in the storage is subject to two types of degradations over time: physical degradation at a constant rate and freshness degradation.

(2) The item has a definite and limited shelf-life after which it is not salable.

(3) The demand of the item depends on price, the current stock amount, and its freshness. For the demand that is price-dependent, six different functions of the price-dependent demand are used: linear, isoelastic, exponential, logit, logarithmic, and polynomial.

(4) The number of remaining items at the end of the cycle is zero.

(5) The holding cost is nonlinear and modelled with a quadratic function that depends on time (i.e., $h + h_1 t + h_2 t^2$).

(6) The salvage value and deterioration cost are taken into account for the items deteriorated throughout the inventory cycle.

(7) The time horizon planning is infinite. The lead time is zero, and therefore, the replenishment rate is instantaneous.

(8) At the beginning of the stock period $t = 0$, the item is fresh and has no age effect on demand. Thus, the item loses its freshness over time, and its demand consequently decreases.

(9) The length of the stock period (T) must not surpass the item's shelf-life (n) since the item is unsalable after its expiration date is over $(T \leq n)$.

3. Inventory Model with Price-, Stock-, and Age-Dependent Demand, with Zero Inventory at the End of the Cycle

The problem under study is described as follows. A company manages a product that has an inherent perishability, and this item is subject to both physical and freshness degradation. Moreover, it is known that the item has a limited shelf-life, after which it is not marketable. Thus, the length of the stock cycle must not be greater than the item's shelf-life. Due to the nature of the item, the demand depends on price, the current stock, and its freshness. On the contrary, the target is to have zero inventory at the end of the inventory cycle. The salvage value and deterioration cost are also considered for the deteriorated items throughout the inventory period. Figure 1 shows the behavior of the inventory level over time. At the beginning of the stock cycle $t = 0$, the retailer receives a lot size of Q units, and the inventory level immediately starts to decrease due to both demand and deterioration, until the stock level reaches zero units at $t = T$.

Table 1: Notation.

Parameters	Description
s	Salvage cost of deteriorated item (currency symbol/unit)
c	Purchase cost (currency symbol/unit)
c_d	Deterioration cost (currency symbol/unit)
h	Holding cost (currency symbol/unit/unit of time)
h_1	Holding cost (currency symbol/unit/unit of time^2)
h_2	Holding cost (currency symbol/unit/unit of time^3)
K	Ordering cost (currency symbol/per cycle)
n	Shelf-life of the item upon which the remaining amount, if any, is withdrawn immediately from the storage (unit of time)
W	Maximum shelf space (units)
η	Salvage coefficient $(0 \leq \eta \leq 1)$
θ	Inventory deterioration rate $(0 \leq \theta \leq 1)$
ω	Sensitivity parameter to the current level of stock
a	Scale parameter for the part of price-dependent demand
b	Sensitivity parameter for the part of price-dependent demand
Functions	
$d(p)$	Price-dependent demand for the item, which can be linear, isoelastic, exponential, logit, logarithmic, or polynomial function (units/unit of time)
$I(t)$	Stock level at time t (units)
$D(p, I(t), t)$	Price-, stock-, and age-dependent demand for the item at time t (units/unit of time)
$H(t)$	Quadratic holding cost function
$\pi_c(p, T)$	Total profit per cycle (currency symbol/unit)
$\pi(p, T)$	Total profit per unit of time (currency symbol/unit of time)
t	Age of the stock, which is the time passed since the last replenishment (unit of time)
Decision variables	
p	Product selling price (currency symbol/unit)
T	Replenishment cycle length (unit of time)
Dependent decision variable	
Q	Order quantity (units)

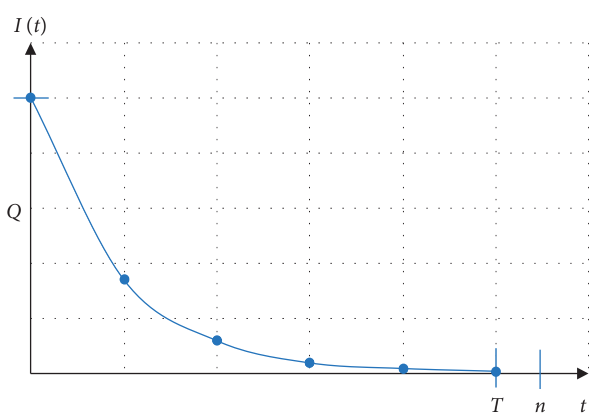

Figure 1: Graphical representation of the inventory level over time.

For the duration of the stock cycle $[0, T]$, the on-hand stock deteriorates at a constant rate (θ), while it also loses its freshness over time. The demand depends on price, stock, and age, according to the following function:

$$D(p, I(t), t) = \frac{n-t}{n} d(p) + \omega I(t), \quad n > 0, \omega \geq 0, t \leq n, \tag{1}$$

where the part of demand that is dependent on price $(d(p))$ takes one of the following expressions:

$$\begin{aligned}
d(p) &= a - bp, \quad 0 \leq p \leq \frac{a}{b} \text{ (linear demand)},\\
d(p) &= ap^{-b}, \quad 0 \leq p \leq \infty \text{ (iso} - \text{elastic demand)},\\
d(p) &= ae^{-bp}, \quad 0 \leq p \leq \infty \text{ (exponential demand)},\\
d(p) &= \frac{a}{1+e^{bp}}, \quad 0 \leq p \leq \infty \text{ (logit demand)},\\
d(p) &= a - b \ln p, \quad 0 \leq p \leq e^{(a/b)} \text{ (logarithmic demand)},\\
d(p) &= a - bp^m, \quad 0 \leq p \leq \left(\frac{a}{b}\right)^{(1/m)} \text{ (polynomial demand)}.
\end{aligned} \tag{2}$$

The six demand functions used in this study, which depend on price, adequately model the scenario where the demand increases as the price decreases, and vice versa.

By considering the aforementioned assumptions, the behavior of the on-hand inventory level $I(t)$ is modelled by the following differential equation:

$$\frac{dI(t)}{dt} = -\frac{n-t}{n} d(p) - \omega I(t) - \theta I(t), \quad 0 \leq t \leq T, \tag{3}$$

with the boundary condition:

$$I(T) = 0. \tag{4}$$

By solving the differential equation (3), the inventory level $I(t)$ is expressed as follows:

$$I(t)=\frac{d(p)}{(\omega+\theta)}\left(1+\frac{1}{n(\omega+\theta)}\right)\left(e^{(\omega+\theta)(T-t)}-1\right)-\frac{d(p)}{n(\omega+\theta)}\left(Te^{(\omega+\theta)(T-t)}-t\right),\quad 0\le t\le T. \tag{5}$$

The order quantity Q, which occurs when $I(0)$, is expressed as follows:

$$Q=\frac{d(p)}{(\omega+\theta)}\left(-1-\frac{1}{n(\omega+\theta)}\right)+\frac{d(p)e^{(\omega+\theta)T}}{(\omega+\theta)}\left(1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right)\le W. \tag{6}$$

The profit is calculated as the difference between the sales revenue of those perfect and deteriorated items and the total costs resulting from sum of the ordering cost, holding cost, purchase cost, and deterioration cost. A brief explanation about the calculation of sales revenue and costs follows. The sales revenue is the product of selling price p and the total demand occurred within the replenishment time T, where the total demand is obtained with the definite integral from zero to T of the demand function $D(p,I(t),t)$. The salvage value is computed as the product of salvage cost, salvage coefficient, and the number of deteriorated units per cycle. The ordering cost represents the cost of placing an order. The holding cost is calculated by the definite integral from zero to T, of the product of the quadratic holding cost function $H(t)$, and the stock level function $I(t)$. The purchase cost is computed by multiplying the unit purchase cost c by the order quantity Q. Finally, the deterioration cost is the product of unit deterioration cost c_d by the number of deteriorated units per cycle.

The detailed calculation of all components of the profit function is mathematically presented below.

(1) Sales revenue (SR) per cycle:

$$\begin{aligned}\mathrm{SR}&=p\int_0^T\left[d(p)\left(1-\frac{t}{n}\right)+\omega I(t)\right]\mathrm{d}t=d(p)T\left[1-\frac{T}{2n}\right]-\frac{\omega\, d(p)T}{(\omega+\theta)}\left[1-\frac{T}{2n}+\frac{1}{n(\omega+\theta)}\right]\\&\quad+\frac{\omega\, d(p)}{(\omega+\theta)}\frac{\left(e^{(\omega+\theta)T}-1\right)}{(\omega+\theta)}\left[1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right].\end{aligned} \tag{7}$$

(2) Salvage value (SV) of deteriorated items per cycle:

$$\begin{aligned}&\mathrm{SV}=s\eta\,(\text{Deteriorated units per cycle}),\\&\mathrm{SV}=s\eta\left(Q-\int_0^T\left(d(p)\left(1-\frac{t}{n}\right)+\omega I(t)\right)\mathrm{d}t\right),\\&\mathrm{SV}=s\eta\left\{\frac{d(p)}{(\omega+\theta)}\left[-1-\frac{1}{n(\omega+\theta)}\right]+\frac{d(p)e^{(\omega+\theta)T}}{(\omega+\theta)}\left[1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right]\right.\\&\left.\quad-\left[d(p)T\left(1-\frac{T}{2n}\right)-\frac{\omega\, d(p)T}{(\omega+\theta)}\left(1-\frac{T}{2n}+\frac{1}{n(\omega+\theta)}\right)+\frac{e^{(\omega+\theta)T}-1}{(\omega+\theta)}\left(\frac{\omega\, d(p)}{(\omega+\theta)}\right)\left(1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right)\right]\right\}.\end{aligned} \tag{8}$$

(3) Ordering cost (OC) per cycle:

$$\mathrm{OC} = K. \quad (9)$$

(4) The holding cost (HC) per cycle:

$$\begin{aligned}
\mathrm{HC} &= \int_0^T H(t)I(t)\mathrm{d}t = \int_0^T \left(h + h_1 t + h_2 t^2\right) I(t),\\
\mathrm{HC} &= \int_0^T H(t)I(t)\mathrm{d}t = h\left\{\frac{d(p)T}{(\omega+\theta)}\left[-1 + \frac{T}{2n} - \frac{1}{n(\omega+\theta)}\right] + \left[\frac{e^{(\omega+\theta)T} - 1}{(\omega+\theta)}\right]\left[\frac{d(p)}{(\omega+\theta)}\left(1 + \frac{1}{n(\omega+\theta)} - \frac{T}{n}\right)\right]\right\}\\
&\quad + h_1\left\{\frac{d(p)T^2}{(\omega+\theta)}\left[-\frac{1}{2} + \frac{T}{3n} - \frac{1}{2n(\omega+\theta)}\right] + \left[\frac{e^{(\omega+\theta)T} - (\omega+\theta)T - 1}{(\omega+\theta)^2}\right]\left[\frac{d(p)}{(\omega+\theta)}\left(1 + \frac{1}{n(\omega+\theta)} - \frac{T}{n}\right)\right]\right\}\\
&\quad + h_2\left\{\frac{d(p)T^3}{(\omega+\theta)}\left[-\frac{1}{3} + \frac{T}{4n} - \frac{1}{3n(\omega+\theta)}\right] + \left[\frac{2e^{(\omega+\theta)T} - [T(\omega+\theta)(T(\omega+\theta)+2)] - 2}{(\omega+\theta)^3}\right]\right.\\
&\quad \left.\cdot\left[\frac{d(p)}{(\omega+\theta)}\left(1 + \frac{1}{n(\omega+\theta)} - \frac{T}{n}\right)\right]\right\}.
\end{aligned} \quad (10)$$

(5) The purchase cost (PC) per cycle:

$$\mathrm{PC} = c\left[\frac{d(p)}{(\omega+\theta)}\left(-1 - \frac{1}{n(\omega+\theta)}\right) + \frac{d(p)e^{(\omega+\theta)T}}{(\omega+\theta)}\left(1 + \frac{1}{n(\omega+\theta)} - \frac{T}{n}\right)\right]. \quad (11)$$

(6) The deterioration cost (DC) per cycle:

$$\begin{aligned}
\mathrm{DC} &= c_d\left(Q - \int_0^T d(p)\left(1 - \frac{t}{n}\right) + \omega I(t)\right),\\
\mathrm{DC} &= c_d\left\{\frac{d(p)}{(\omega+\theta)}\left[-1 - \frac{1}{n(\omega+\theta)}\right] + \frac{d(p)e^{(\omega+\theta)T}}{(\omega+\theta)}\left[1 + \frac{1}{n(\omega+\theta)} - \frac{T}{n}\right]\right.\\
&\quad \left.-\left[d(p)T\left(1 - \frac{T}{2n}\right) - \frac{\omega\, d(p)T}{(\omega+\theta)}\left(1 - \frac{T}{2n} + \frac{1}{n(\omega+\theta)}\right) + \frac{e^{(\omega+\theta)T} - 1}{(\omega+\theta)}\left(\frac{\omega\, d(p)}{(\omega+\theta)}\right)\left(1 + \frac{1}{n(\omega+\theta)} - \frac{T}{n}\right)\right]\right\}.
\end{aligned} \quad (12)$$

Therefore, the total profit per cycle is

$$\begin{aligned}
\pi_c &= \text{sales revenue} + \text{salvage value} - \text{ordering cost} - \text{holding cost} - \text{purchase cost} - \text{deterioration cost},\\
\pi_c &= \mathrm{SR} + \mathrm{SV} - \mathrm{OC} - \mathrm{HC} - \mathrm{PC} - \mathrm{DC}.
\end{aligned} \quad (13)$$

And, it is expressed as

$$\begin{aligned}
\pi_c(p,T) = {}& d(p)T\left[1-\frac{T}{2n}\right]-\frac{\omega\, d(p)T}{(\omega+\theta)}\left[1-\frac{T}{2n}+\frac{1}{n(\omega+\theta)}\right]+\frac{\omega\, d(p)}{(\omega+\theta)}\frac{\left(e^{(\omega+\theta)T}-1\right)}{(\omega+\theta)}\left[1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right] \\
& + s\eta\left\{\frac{d(p)}{(\omega+\theta)}\left[-1-\frac{1}{n(\omega+\theta)}\right]+\frac{d(p)e^{(\omega+\theta)T}}{(\omega+\theta)}\left[1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right]\right. \\
& \left.-\left[d(p)T\left(1-\frac{T}{2n}\right)-\frac{\omega\, d(p)T}{(\omega+\theta)}\left(1-\frac{T}{2n}+\frac{1}{n(\omega+\theta)}\right)+\frac{e^{(\omega+\theta)T}-1}{(\omega+\theta)}\left(\frac{\omega\, d(p)}{(\omega+\theta)}\right)\left(1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right)\right]\right\} \\
& -\left\{h\left\{\frac{d(p)T}{(\omega+\theta)}\left[-1+\frac{T}{2n}-\frac{1}{n(\omega+\theta)}\right]+\left[\frac{e^{(\omega+\theta)T}-1}{(\omega+\theta)}\right]\left[\frac{d(p)}{(\omega+\theta)}\left(1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right)\right]\right\}\right. \\
& + h_1\left\{\frac{d(p)T^2}{(\omega+\theta)}\left[-\frac{1}{2}+\frac{T}{3n}-\frac{1}{2n(\omega+\theta)}\right]+\left[\frac{e^{(\omega+\theta)T}-(\omega+\theta)T-1}{(\omega+\theta)^2}\right]\left[\frac{d(p)}{(\omega+\theta)}\left(1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right)\right]\right\} \\
& + h_2\left\{\frac{d(p)T^3}{(\omega+\theta)}\left[-\frac{1}{3}+\frac{T}{4n}-\frac{1}{3n(\omega+\theta)}\right]+\left[\frac{2e^{(\omega+\theta)T}-[T(\omega+\theta)(T(\omega+\theta)+2)]-2}{(\omega+\theta)^3}\right]\left[\frac{d(p)}{(\omega+\theta)}\left(1\right.\right.\right. \\
& \left.\left.\left.\left.+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right)\right]\right\}\right\}-c\left[\frac{d(p)}{(\omega+\theta)}\left(-1-\frac{1}{n(\omega+\theta)}\right)+\frac{d(p)e^{(\omega+\theta)T}}{(\omega+\theta)}\left(1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right)\right] \\
& -\left\{c_d\left\{\frac{d(p)}{(\omega+\theta)}\left[-1-\frac{1}{n(\omega+\theta)}\right]+\frac{d(p)e^{(\omega+\theta)T}}{(\omega+\theta)}\left[1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right]-\left[d(p)T\left(1-\frac{T}{2n}\right)\right.\right.\right. \\
& \left.\left.\left.-\frac{\omega\, d(p)T}{(\omega+\theta)}\left(1-\frac{T}{2n}+\frac{1}{n(\omega+\theta)}\right)+\frac{e^{(\omega+\theta)T}-1}{(\omega+\theta)}\left(\frac{\omega\, d(p)}{(\omega+\theta)}\right)\left(1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right)\right]\right\}\right\}.
\end{aligned} \tag{14}$$

Hence, the total profit per unit of time is expressed as follows:

$$\begin{aligned}
\pi(p,T) = {}& \frac{1}{T}\left\{d(p)T\left[1-\frac{T}{2n}\right]-\frac{\omega\, d(p)T}{(\omega+\theta)}\left[1-\frac{T}{2n}+\frac{1}{n(\omega+\theta)}\right]+\frac{\omega\, d(p)}{(\omega+\theta)}\frac{\left(e^{(\omega+\theta)T}-1\right)}{(\omega+\theta)}\left[1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right]\right. \\
& + s\eta\left\{\frac{d(p)}{(\omega+\theta)}\left[-1-\frac{1}{n(\omega+\theta)}\right]+\frac{d(p)e^{(\omega+\theta)T}}{(\omega+\theta)}\left[1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right]\right. \\
& \left.-\left[d(p)T\left(1-\frac{T}{2n}\right)-\frac{\omega\, d(p)T}{(\omega+\theta)}\left(1-\frac{T}{2n}+\frac{1}{n(\omega+\theta)}\right)+\frac{e^{(\omega+\theta)T}-1}{(\omega+\theta)}\left(\frac{\omega\, d(p)}{(\omega+\theta)}\right)\left(1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right)\right]\right\} \\
& -\left\{h\left\{\frac{d(p)T}{(\omega+\theta)}\left[-1+\frac{T}{2n}-\frac{1}{n(\omega+\theta)}\right]+\left[\frac{e^{(\omega+\theta)T}-1}{(\omega+\theta)}\right]\left[\frac{d(p)}{(\omega+\theta)}\left(1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right)\right]\right\}\right. \\
& + h_1\left\{\frac{d(p)T^2}{(\omega+\theta)}\left[-\frac{1}{2}+\frac{T}{3n}-\frac{1}{2n(\omega+\theta)}\right]+\left[\frac{e^{(\omega+\theta)T}-(\omega+\theta)T-1}{(\omega+\theta)^2}\right]\left[\frac{d(p)}{(\omega+\theta)}\left(1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right)\right]\right\} \\
& \left.+ h_2\left\{\frac{d(p)T^3}{(\omega+\theta)}\left[-\frac{1}{3}+\frac{T}{4n}-\frac{1}{3n(\omega+\theta)}\right]+\left[\frac{2e^{(\omega+\theta)T}-[T(\omega+\theta)(T(\omega+\theta)+2)]}{(\omega+\theta)^3}\right]\left[\frac{d(p)}{(\omega+\theta)}\left(1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right)\right]\right\}\right\} \\
& - c\left[\frac{d(p)}{(\omega+\theta)}\left(-1-\frac{1}{n(\omega+\theta)}\right)+\frac{d(p)e^{(\omega+\theta)T}}{(\omega+\theta)}\left(1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right)\right] \\
& -\left\{c_d\left\{\frac{d(p)}{(\omega+\theta)}\left[-1-\frac{1}{n(\omega+\theta)}\right]+\frac{d(p)e^{(\omega+\theta)T}}{(\omega+\theta)}\left[1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right]-\left[d(p)T\left(1-\frac{T}{2n}\right)\right.\right.\right. \\
& \left.\left.\left.\left.-\frac{\omega\, d(p)T}{(\omega+\theta)}\left(1-\frac{T}{2n}+\frac{1}{n(\omega+\theta)}\right)+\frac{e^{(\omega+\theta)T}-1}{(\omega+\theta)}\left(\frac{\omega\, d(p)}{(\omega+\theta)}\right)\left(1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right)\right]\right\}\right\}\right\}.
\end{aligned} \tag{15}$$

In general, the optimization problem is formulated as follows:

$$\begin{aligned} &\text{Max}_{p,T}\pi(p,T) \\ &\text{Subject to } c \le p \le p_{\max} \text{ and } T \le n. \end{aligned} \tag{16}$$

The total profit per unit of time function given in equation (15) shows that this is highly nonlinear in both selling price (p) and replenishment cycle time (T). Therefore, it is not possible to determine a close form for these decision variables. Nonetheless, the optimal solution to p and T are found through an optimization procedure that is based on the traditional conditions for optimality. The solution procedure is explained in the following section.

4. Solution Procedure to Obtain the Optimal Solution

4.1. Theoretical Results. The goal is to determine the optimal selling price p and optimal replenishment cycle time T that maximize the total profit. Since the total profit per unit of time function $\pi(p,T)$ is continuous and twice differentiable with respect to both variables (p,T) on the interval $[0, \infty]$, there exists, then, a global maximum on that interval.

The necessary conditions for the total profit per unit of time function $\pi(p,T)$ to be maximized are as follows:

$$\frac{\partial \pi(p,T)}{\partial p} = 0, \tag{17}$$

$$\frac{\partial \pi(p,T)}{\partial T} = 0. \tag{18}$$

Moreover, for the expected total profit per unit of time function $\pi(p,T)$ to be concave, the sufficient conditions are given as follows:

$$\frac{\partial^2 \pi(p,T)}{\partial p^2} < 0, \tag{19}$$

$$\frac{\partial^2 \pi(p,T)}{\partial T^2} < 0, \tag{20}$$

$$\frac{\partial^2 \pi(p,T)}{\partial p\, \partial T} = \frac{\partial^2 \pi(p,T)}{\partial T\, \partial p}, \tag{21}$$

$$\left(\frac{\partial^2 \pi(p,T)}{\partial p^2}\right)\left(\frac{\partial^2 \pi(p,T)}{\partial T^2}\right) - \left(\frac{\partial^2 \pi(p,T)}{\partial p\, \partial T}\right)\left(\frac{\partial^2 \pi(p,T)}{\partial T\, \partial p}\right) > 0. \tag{22}$$

The optimal solution is determined by simultaneously solving the first partial derivatives of the total profit per unit of time function given in equation (15), with respect to p and T equalizing to zero.

If the solution (p,T) satisfies the conditions given by equations (19)–(22), it demonstrates that the function $\pi(p,T)$ is strictly concave in both decision variables with a negative-definite Hessian matrix. If so, the solution (p,T) is optimal.

In the optimization problem given in equation (16), it is stated that the selling price has an upper bound of $p_{\max}$ and the replenishment cycle time has an upper bound of n. The component of the demand that corresponds to price $(d(p))$ is modelled with six different functions. For isoelastic, exponential, and logit functions, the interval for the price p is $[0, \infty]$. On the contrary, for linear, logarithmic, and polynomial, there exists a maximum permissible value $p_{\max}$. The upper bound $p_{\max}$ is (a/b), $e^{(a/b)}$ and $(a/b)^{(1/m)}$ for the linear, logarithmic, and polynomial, respectively. For these demand functions, the solution for selling price is $p = p_{\max}$ when the solution for the selling price p is greater than $p_{\max}$. This allows avoiding a negative value for the demand that is dependent on price. This is mathematically right, but it does not make sense in any business of the real world.

Notice that the replenishment cycle time T is on the interval $[0, \infty]$. When the solution for the replenishment cycle time T is greater than n, the solution for the replenishment cycle time, then, is $T = n$ due to the shelf-life constrain.

4.2. Algorithm for Finding the Optimal Solution. Considering the theoretical results presented in the previous section, the following algorithm is proposed (Algorithm 1).

5. Optimal Inventory Policy for Six Different Price-Dependent Demands

5.1. Optimal Inventory Policy Using the Price-Dependent Linear Demand Function $d(p) = a - bp$. The first partial derivative of $\pi(p,T)$ with respect to p is

Step 1. Input the inventory parameters.
Step 2. Calculate $p_{\max}$.
Step 3. Solve simultaneously equations (17) and (18) to obtain the values for p and T.
Step 4. If the conditions (19)–(22) are satisfied, then the solution is optimal, and go to step 5. Otherwise, the solutions are infeasible, and go to step 14.
Step 5. If both $c \le p \le p_{\max}$ and $T \le n$ are satisfied, then set $p^* = p$ and $T^* = T$, and go to step 11.
Else, go to step 6.
Step 6. If both $p \le c$ and $T \le n$ are satisfied, then set $p^* = c$ and $T^* = T$, and go to step 11.
Else, go to step 7.
Step 7. If both $p \le c$ and $T > n$ are satisfied, then set $p^* = c$ and $T^* = n$, and go to step 11.
Else, go to step 8.
Step 8. If both $p > p_{\max}$ and $T \le n$ are satisfied, then set $p^* = p_{\max}$ and $T^* = T$, and go to step 11.
Else, go to step 9.
Step 9. If both $p \le p_{\max}$ and $T > n$ are satisfied, then set $p^* = p$ and $T^* = n$, and go to step 11.
Else, go to step 10.
Step 10. Set $p = p_{\max}$ and $T^* = n$.
Step 11. Calculate the lot size Q^* with equation (6).
Step 12. Compute the total profit per unit of time $\pi^*(p^*, T^*)$ with equation (15).
Step 13. Report the optimal solution $\pi^*(p^*, T^*)$, p^*, T^*, and Q^*.
Step 14. Stop.

Algorithm 1: Algorithm for finding the optimal solution.

$$
\begin{aligned}
\frac{\partial \pi(p,T)}{\partial p} = \Bigg\{ & [-bp + d(p)] \Bigg[1 - \frac{T}{2n} + \frac{\omega}{(\omega+\theta)} \left(-1 + \frac{T}{2n} - \frac{1}{n(\omega+\theta)} \right) \\
& + \left(\frac{\omega}{T(\omega+\theta)} \right) \left(\frac{e^{(\omega+\theta)T} - 1}{(\omega+\theta)} \right) \left(1 + \frac{1}{n(\omega+\theta)} \right) - \left(\frac{\omega}{Tn(\omega+\theta)} \right) \left(\frac{T\left(e^{(\omega+\theta)T} - 1\right)}{(\omega+\theta)} \right) \Bigg] \\
& + [s\eta(-b)] \Bigg[\frac{1}{T(\omega+\theta)} \left(-1 - \frac{1}{n(\omega+\theta)} \right) + \frac{e^{(\omega+\theta)T}}{T(\omega+\theta)} \left(1 + \frac{1}{n(\omega+\theta)} - \frac{T}{n} \right) - 1 \\
& + \frac{T}{2n} + \frac{\omega}{(\omega+\theta)} \left(1 - \frac{T}{2n} + \frac{1}{n(\omega+\theta)} \right) + \left(\frac{\omega}{T(\omega+\theta)} \right) \left(\frac{e^{(\omega+\theta)T} - 1}{(\omega+\theta)} \right) \left(-1 - \frac{1}{n(\omega+\theta)} + \frac{T}{n} \right) \Bigg] \\
& - \Bigg[(-hb) \left(\frac{1}{(\omega+\theta)} \left(-1 + \frac{T}{2n} - \frac{1}{n(\omega+\theta)} \right) + \frac{1}{T(\omega+\theta)} \left(\frac{e^{(\omega+\theta)T} - 1}{(\omega+\theta)} \right) \left(1 + \frac{1}{n(\omega+\theta)} - \frac{T}{n} \right) \right) \\
& + (-h_1 b) \left(\frac{T}{(\omega+\theta)} \left(\frac{T}{3n} - \frac{1}{2} - \frac{1}{2n(\omega+\theta)} \right) + \frac{1}{T(\omega+\theta)} \left(\frac{e^{(\omega+\theta)T} - T(\omega+\theta) - 1}{(\omega+\theta)^2} \right) \left(1 + \frac{1}{n(\omega+\theta)} - \frac{T}{n} \right) \right) \\
& + (-h_2 b) \left(\frac{T^2}{(\omega+\theta)} \left(\frac{T}{4n} - \frac{1}{3} - \frac{1}{3n(\omega+\theta)} \right) + \frac{1}{T(\omega+\theta)} \left(\frac{2e^{(\omega+\theta)T} - T(\omega+\theta)(T(\omega+\theta)+2) - 2}{(\omega+\theta)^3} \right) \right. \\
& \left. \left(1 + \frac{1}{n(\omega+\theta)} - \frac{T}{n} \right) \right) \Bigg] + \Bigg[(-cb) \left(\frac{1}{T(\omega+\theta)} \left(1 + \frac{1}{n(\omega+\theta)} \right) + \frac{e^{(\omega+\theta)T}}{T(\omega+\theta)} \left(-1 - \frac{1}{n(\omega+\theta)} + \frac{T}{n} \right) \right) \Bigg] \\
& - [c_d(-b)] \Bigg[\frac{1}{T(\omega+\theta)} \left(-1 - \frac{1}{n(\omega+\theta)} \right) + \frac{e^{(\omega+\theta)T}}{T(\omega+\theta)} \left(1 + \frac{1}{n(\omega+\theta)} - \frac{T}{n} \right) - 1 + \frac{T}{2n} \\
& + \frac{\omega}{(\omega+\theta)} \left(1 - \frac{T}{2n} + \frac{1}{n(\omega+\theta)} \right) + \left(\frac{\omega}{T(\omega+\theta)} \right) \left(\frac{e^{(\omega+\theta)T} - 1}{(\omega+\theta)} \right) \left(-1 - \frac{1}{n(\omega+\theta)} + \frac{T}{n} \right) \Bigg] \Bigg\} = 0.
\end{aligned}
\tag{23}
$$

The first partial derivative of $\pi(p,T)$ with respect to T is

$$\begin{aligned}\frac{\partial \pi(p,T)}{\partial T} = \Bigg\{ & p\left[\frac{d(p)}{2n}\left(\frac{\omega}{\omega+\theta}-1\right)+\frac{\omega\, d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{(\omega+\theta)T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)-\frac{\omega\, d(p)e^{(\omega+\theta)T}}{n(\omega+\theta)}\right] \\
& +\Bigg[s\eta\left(\frac{d(p)}{(\omega+\theta)T^2}\left(1+\frac{1}{n(\omega+\theta)}\right)+\frac{d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\right. \\
& -\frac{d(p)}{n(\omega+\theta)}(\omega+\theta)e^{(\omega+\theta)T}+\frac{d(p)}{2n}\left(1-\frac{\omega}{(\omega+\theta)}\right)-\frac{\omega\, d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{(\omega+\theta)T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right) \\
& \left.+\frac{\omega\, d(p)}{n(\omega+\theta)}e^{(\omega+\theta)T}\right)\Bigg]+\left[\frac{K}{T^2}\right]-\Bigg[\Bigg(h\left(\frac{d(p)}{2n(\omega+\theta)}+\frac{d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{(\omega+\theta)T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\right. \\
& \left.-\frac{d(p)}{n(\omega+\theta)}e^{(\omega+\theta)T}\right)\Bigg)+\Bigg(h_1\Bigg(-\frac{d(p)}{2(\omega+\theta)}+\frac{2\,d(p)T}{3n(\omega+\theta)}-\frac{d(p)}{2n(\omega+\theta)^2} \\
& +\frac{d(p)}{(\omega+\theta)}\left(\frac{\omega Te^{(\omega+\theta)T}+\theta Te^{(\omega+\theta)T}-e^{(\omega+\theta)T}+1}{(\omega+\theta)^2T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)-\frac{d(p)}{n(\omega+\theta)}\left(\frac{e^{(\omega+\theta)T}-1}{(\omega+\theta)}\right)\Bigg)\Bigg) \\
& +\Bigg(h_2\Bigg(-\frac{2\,d(p)T}{3(\omega+\theta)}+\frac{3\,d(p)T^2}{4n(\omega+\theta)}-\frac{2\,d(p)T}{3n(\omega+\theta)^2} \\
& +\frac{d(p)}{(\omega+\theta)}\left(-\frac{\theta^2T^2+2\omega\theta T^2-2\theta Te^{(\omega+\theta)T}-2\omega Te^{(\omega+\theta)T}+2e^{(\omega+\theta)T}+\omega^2T^2-2}{(\omega+\theta)^3T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right) \\
& -\frac{d(p)}{n(\omega+\theta)}\left(\frac{2e^{(\omega+\theta)T}-2T(\omega+\theta)-2}{(\omega+\theta)^2}\right)\Bigg)\Bigg)\Bigg]-\Bigg[c\Bigg(\frac{d(p)}{(\omega+\theta)T^2}\left(1+\frac{1}{n(\omega+\theta)}\right) \\
& +\frac{d(p)}{(\omega+\theta)}\left(\frac{e^{(\omega+\theta)T}(\omega T+\theta T-1)}{T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)-\frac{d(p)}{n(\omega+\theta)}(\omega+\theta)e^{(\omega+\theta)T}\Bigg)\Bigg] \\
& -\Bigg[c_d\Bigg(\frac{d(p)}{(\omega+\theta)T^2}\left(1+\frac{1}{n(\omega+\theta)}\right)+\frac{d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right) \\
& -\frac{d(p)}{n(\omega+\theta)}(\omega+\theta)e^{(\omega+\theta)T}+\frac{d(p)}{2n}\left(1-\frac{\omega}{(\omega+\theta)}\right)-\frac{\omega\, d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{(\omega+\theta)T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right) \\
& +\frac{\omega\, d(p)}{n(\omega+\theta)}e^{(\omega+\theta)T}\Bigg)\Bigg]\Bigg\}=0.\end{aligned} \tag{24}$$

Example 1. In order to represent a real scenario, let us consider a place that sells a fresh item. Assume that the price-dependent component of the demand for the fresh item is linear as follows: $d(p) = 600 - 20p$ (i.e., $a = 600$ and $b = 20$). The shelf-life of the fresh item is $n = 1$ week. There is a maximum shelf space of $W = 500$ units. The cost for placing an order to the supplier is $K = 250$ euros per order, the purchase cost is $c = 5$ euros per unit, and the salvage value of the deteriorated item is $s = 4$ euros per unit. The sensitivity coefficient to the level of stock is $\omega = 0.5$, and the stock deterioration rate is $\theta = 0.05$. The aforementioned data were taken from Agi and Soni [22]. Additional data are still needed to solve the numerical examples. The values of the holding cost are $h = 1.75$ euros per unit per week, $h_1 = 0.15$ euros per unit per week^2, and $h_2 = 0.25$ euros per unit per week^3. The deterioration cost of the item is $c_d = 2$ euros per

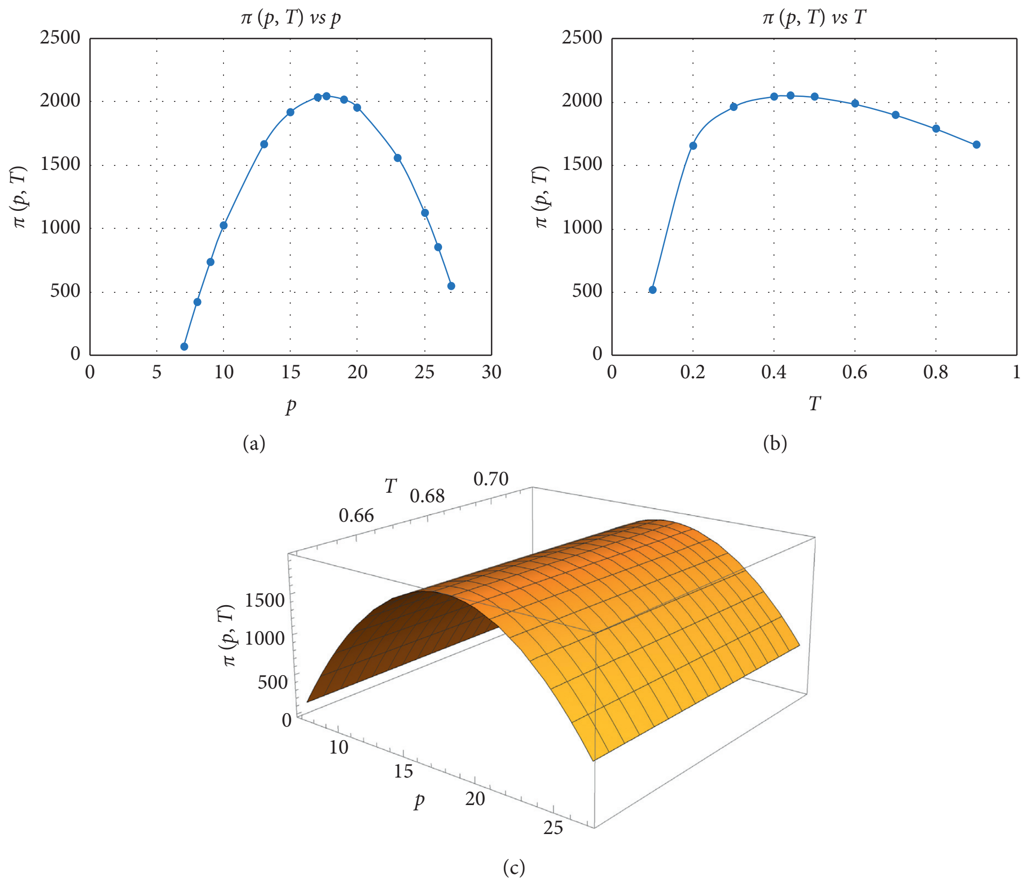

Figure 2: Concave property of the profit function $\pi(p, T)$ when the price-dependent demand function is linear.

unit, and the salvage coefficient is $\eta = 0.8$. Since $n = 1$ week, the replenishment cycle time must be $T \leq 1$ week. For the price linear demand, the upper bound for price is $p_{\max} = (a/b) = (600/20) = 30$. This means that the price must satisfy $5 \leq p \leq 30$.

By applying the proposed algorithm, the following optimal solution for the inventory system is calculated: $p^* = 17.69124$ euros per unit, $T^* = 0.4395923$ weeks, $Q^* = 94.42941$ units, and $\pi^*(p^*, T^*) = 2049.903$ euros per week. This solution satisfies all conditions for the optimality: $(\partial^2 \pi(p, T)/\partial p^2) = -34.56784 < 0$, $(\partial^2 \pi(p, T)/\partial T^2) = -49710.46 < 0$, $(\partial^2 \pi(p, T)/\partial p\, \partial T) = (\partial^2 \pi(p, T)/\partial T\, \partial p) = 13.01441$, and $(\partial^2 \pi(p, T)/\partial p^2)(\partial^2 \pi(p, T)/\partial T^2) - (\partial^2 \pi(p, T)/\partial p\, \partial T)(\partial^2 \pi(p, T)/\partial T\, \partial p) = 0.1718214 \times 10^7 > 0$. As the Hessian determinant is greater than zero, the total profit function then is strictly concave with a negative-definite Hessian matrix. Consequently, the solution is optimal.

By plotting the total profit per unit of time function $\pi(p, T)$ with distinct values of p (taking values between 7 and 27) given T and distinct values of T (taking values between 0 and 1) given p, it is observed that $\pi(p, T)$ is strictly concave with respect to p when T is fixed (see Figure 2(a)) and with respect to T when p is given (see Figure 2(b)). Additionally, $\pi(p, T)$ is also strictly concave with respect to both p and T (see Figure 2(c)). Thus, it is ensured that the solution corresponds a global maximum.

5.2. Optimal Inventory Policy Using the Price-Dependent Isoelastic Demand Function $d(p) = ap^{-b}$. The first partial derivative of $\pi(p,T)$ with respect to p is

$$
\begin{aligned}
\frac{\partial \pi(p,T)}{\partial p} = {}& \Bigg\{ \left[-\frac{ab}{p^{b+1}} p + d(p) \right] \Bigg[1 - \frac{T}{2n} + \frac{\omega}{(\omega+\theta)} \left(-1 + \frac{T}{2n} - \frac{1}{n(\omega+\theta)} \right) \\
& + \left(\frac{\omega}{T(\omega+\theta)} \right) \left(\frac{e^{(\omega+\theta)T} - 1}{(\omega+\theta)} \right) \left(1 + \frac{1}{n(\omega+\theta)} \right) - \left(\frac{\omega}{Tn(\omega+\theta)} \right) \left(\frac{T\left(e^{(\omega+\theta)T} - 1\right)}{(\omega+\theta)} \right) \Bigg] \\
& + \left[s\eta \left(-\frac{ab}{p^{b+1}} \right) \right] \Bigg[\frac{1}{T(\omega+\theta)} \left(-1 - \frac{1}{n(\omega+\theta)} \right) + \frac{e^{(\omega+\theta)T}}{T(\omega+\theta)} \left(1 + \frac{1}{n(\omega+\theta)} - \frac{T}{n} \right) - 1 \\
& + \frac{T}{2n} + \frac{\omega}{(\omega+\theta)} \left(1 - \frac{T}{2n} + \frac{1}{n(\omega+\theta)} \right) + \left(\frac{\omega}{T(\omega+\theta)} \right) \left(\frac{e^{(\omega+\theta)T} - 1}{(\omega+\theta)} \right) \left(-1 - \frac{1}{n(\omega+\theta)} + \frac{T}{n} \right) \Bigg] \\
& - \Bigg[\left(h \left(-\frac{ab}{p^{b+1}} \right) \right) \left(\frac{1}{(\omega+\theta)} \left(-1 + \frac{T}{2n} - \frac{1}{n(\omega+\theta)} \right) + \frac{1}{T(\omega+\theta)} \left(\frac{e^{(\omega+\theta)T} - 1}{(\omega+\theta)} \right) \left(1 + \frac{1}{n(\omega+\theta)} - \frac{T}{n} \right) \right) \\
& + \left(h_1 \left(-\frac{ab}{p^{b+1}} \right) \right) \left(\frac{T}{(\omega+\theta)} \left(\frac{T}{3n} - \frac{1}{2} - \frac{1}{2n(\omega+\theta)} \right) + \frac{1}{T(\omega+\theta)} \left(\frac{e^{(\omega+\theta)T} - T(\omega+\theta) - 1}{(\omega+\theta)^2} \right) \left(1 + \frac{1}{n(\omega+\theta)} - \frac{T}{n} \right) \right) \\
& + \left(h_2 \left(-\frac{ab}{p^{b+1}} \right) \right) \left(\frac{T^2}{(\omega+\theta)} \left(\frac{T}{4n} - \frac{1}{3} - \frac{1}{3n(\omega+\theta)} \right) + \frac{1}{T(\omega+\theta)} \left(\frac{2e^{(\omega+\theta)T} - T(\omega+\theta)(T(\omega+\theta)+2) - 2}{(\omega+\theta)^3} \right) \right. \\
& \left. \left(1 + \frac{1}{n(\omega+\theta)} - \frac{T}{n} \right) \right) \Bigg] + \left[\left(c \left(-\frac{ab}{p^{b+1}} \right) \right) \left(\frac{1}{T(\omega+\theta)} \left(1 + \frac{1}{n(\omega+\theta)} \right) + \frac{e^{(\omega+\theta)T}}{T(\omega+\theta)} \left(-1 - \frac{1}{n(\omega+\theta)} + \frac{T}{n} \right) \right) \right] \\
& - \left[c_d \left(-\frac{ab}{p^{b+1}} \right) \right] \Bigg[\frac{1}{T(\omega+\theta)} \left(-1 - \frac{1}{n(\omega+\theta)} \right) + \frac{e^{(\omega+\theta)T}}{T(\omega+\theta)} \left(1 + \frac{1}{n(\omega+\theta)} - \frac{T}{n} \right) - 1 \\
& + \frac{T}{2n} + \frac{\omega}{(\omega+\theta)} \left(1 - \frac{T}{2n} + \frac{1}{n(\omega+\theta)} \right) + \left(\frac{\omega}{T(\omega+\theta)} \right) \left(\frac{e^{(\omega+\theta)T} - 1}{(\omega+\theta)} \right) \left(-1 - \frac{1}{n(\omega+\theta)} + \frac{T}{n} \right) \Bigg] \Bigg\} = 0.
\end{aligned}
\tag{25}
$$

The first partial derivative of $\pi(p,T)$ with respect to T is

$$\begin{aligned}
\frac{\partial \pi(p,T)}{\partial T} = {} & \Bigg\{ p\Bigg[\frac{d(p)}{2n}\left(\frac{\omega}{\omega+\theta}-1\right)+\frac{\omega\, d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{(\omega+\theta)T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\\
& -\frac{\omega\, d(p)e^{(\omega+\theta)T}}{n(\omega+\theta)}\Bigg]+\Bigg[s\eta\Bigg(\frac{d(p)}{(\omega+\theta)T^2}\left(1+\frac{1}{n(\omega+\theta)}\right)+\frac{d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\\
& -\frac{d(p)}{n(\omega+\theta)}(\omega+\theta)e^{(\omega+\theta)T}+\frac{d(p)}{2n}\left(1-\frac{\omega}{(\omega+\theta)}\right)-\frac{\omega\, d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{(\omega+\theta)T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\\
& +\frac{\omega\, d(p)}{n(\omega+\theta)}e^{(\omega+\theta)T}\Bigg)\Bigg]+\left[\frac{K}{T^2}\right]-\Bigg[\Bigg(h\Bigg(\frac{d(p)}{2n(\omega+\theta)}+\frac{d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{(\omega+\theta)T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\\
& -\frac{d(p)}{n(\omega+\theta)}e^{(\omega+\theta)T}\Bigg)\Bigg)+\Bigg(h_1\Bigg(-\frac{d(p)}{2(\omega+\theta)}+\frac{2\,d(p)T}{3n(\omega+\theta)}-\frac{d(p)}{2n(\omega+\theta)^2}\\
& +\frac{d(p)}{(\omega+\theta)}\left(\frac{\omega Te^{(\omega+\theta)T}+\theta Te^{(\omega+\theta)T}-e^{(\omega+\theta)T}+1}{(\omega+\theta)^2T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)-\frac{d(p)}{n(\omega+\theta)}\left(\frac{e^{(\omega+\theta)T}-1}{(\omega+\theta)}\right)\Bigg)\Bigg)\\
& +\Bigg(h_2\Bigg(\frac{2\,d(p)T}{3(\omega+\theta)}+\frac{3\,d(p)T^2}{4n(\omega+\theta)}-\frac{2\,d(p)T}{3n(\omega+\theta)^2}\\
& +\frac{d(p)}{(\omega+\theta)}\left(-\frac{\theta^2T^2+2\omega\theta T^2-2\theta Te^{(\omega+\theta)T}-2\omega Te^{(\omega+\theta)T}+2e^{(\omega+\theta)T}+\omega^2T^2-2}{(\omega+\theta)^3T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\\
& -\frac{d(p)}{n(\omega+\theta)}\left(\frac{2e^{(\omega+\theta)T}-2T(\omega+\theta)-2}{(\omega+\theta)^2}\right)\Bigg)\Bigg)\Bigg]-\Bigg[c\Bigg(\frac{d(p)}{(\omega+\theta)T^2}\left(1+\frac{1}{n(\omega+\theta)}\right)\\
& +\frac{d(p)}{(\omega+\theta)}\left(\frac{e^{(\omega+\theta)T}(\omega T+\theta T-1)}{T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)-\frac{d(p)}{n(\omega+\theta)}(\omega+\theta)e^{(\omega+\theta)T}\Bigg)\Bigg]\\
& -\Bigg[c_d\Bigg(\frac{d(p)}{(\omega+\theta)T^2}\left(1+\frac{1}{n(\omega+\theta)}\right)+\frac{d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\\
& -\frac{d(p)}{n(\omega+\theta)}(\omega+\theta)e^{(\omega+\theta)T}+\frac{d(p)}{2n}\left(1-\frac{\omega}{(\omega+\theta)}\right)-\frac{\omega\, d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{(\omega+\theta)T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\\
& +\frac{\omega\, d(p)}{n(\omega+\theta)}e^{(\omega+\theta)T}\Bigg)\Bigg]\Bigg\}=0.
\end{aligned}\tag{26}$$

Example 2. Let us consider the same data as in Example 1. Suppose now that the price-dependent element of demand for the perishable good has an isoelastic function as follows: $d(p)=30,000p^{-1.4}$ (i.e., $a=30,000$ and $b=1.4$).

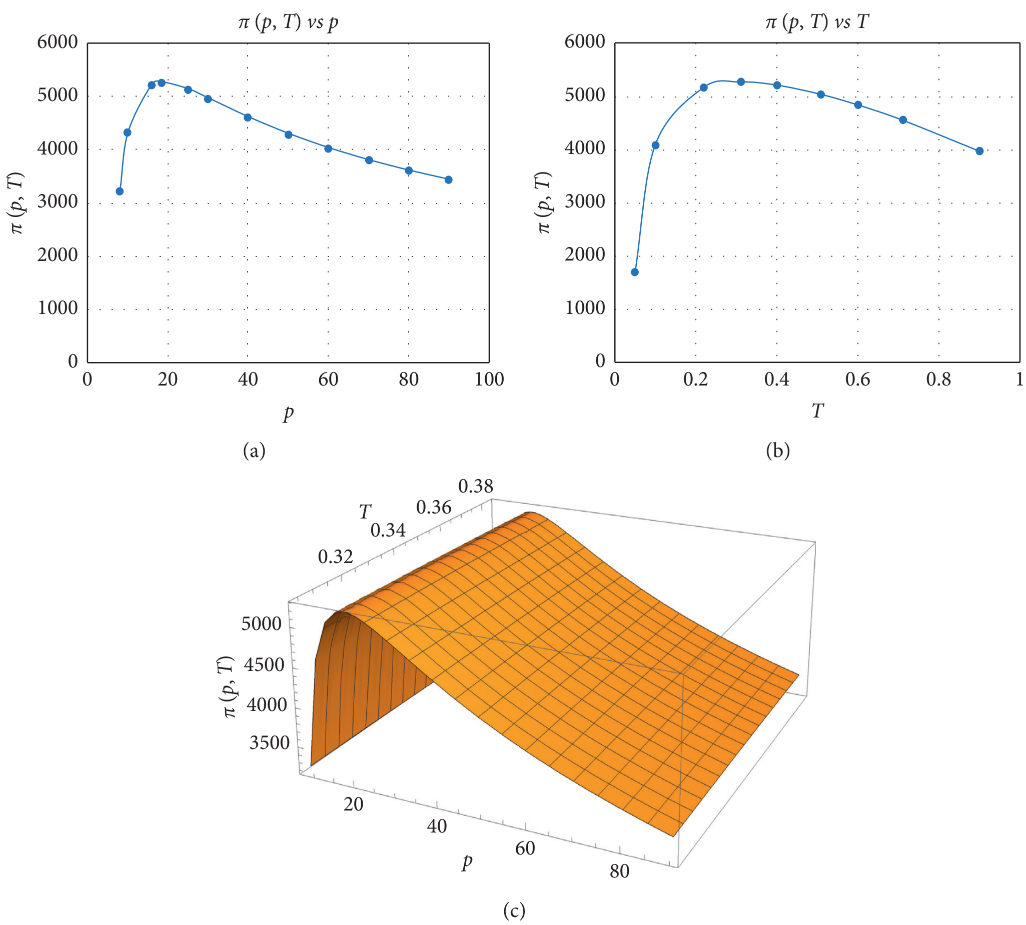

FIGURE 3: Concave property of the profit function $\pi(p,T)$ when the price-dependent demand function is isoelastic.

Since $n = 1$ week, the replenishment cycle time must be $T \leq 1$ week. For the price isoelastic demand, the upper bound for price is $p_{\max} = \infty$. Therefore, the price must be in the interval $5 \leq p \leq \infty$. By using the algorithm proposed, the following optimal solution for the inventory model is determined: $p^* = 18.47849$ euros per unit, $T^* = 0.3096932$ weeks, $Q^* = 143.5169$ units, and $\pi^*(p^*,T^*) = 5266.004$ euros per week. This solution satisfies all conditions for the optimality: $\partial^2\pi(p,T)/\partial p^2 = -9.960414 < 0$, $(\partial^2\pi(p,T)/\partial T^2) = -271515.0 < 0$, $(\partial^2\pi(p,T)/\partial p\ \partial T) = (\partial^2\pi(p,T)/\partial T\ \partial p) = 28.89635$, and $(\partial^2\pi(p,T)/\partial p^2)(\partial^2\pi(p,T)/\partial T^2) - (\partial^2\pi(p,T)/\partial p\ \partial T)(\partial^2\pi(p,T)/\partial T\ \partial p) = 0.2703567\times 10^7 > 0$ Given that the Hessian determinant is greater than zero, the total profit function is strictly concave with a negative-definite Hessian matrix. So, the solution is optimal.

By drawing the total profit per unit of time function $\pi(p,T)$, with some values of p between 8 and 90, and considering T as given and some values of T between 0 and 1 when p is fixed, it is clear that $\pi(p,T)$ is strictly concave with respect to p when T is given (see Figure 3(a)) and with respect to T when p is fixed (see Figure 3(b)). Moreover, $\pi(p,T)$ is also strictly concave with respect to both decision variables: p and T (see Figure 3(c)). This confirms the concavity property in the total profit $\pi(p,T)$, and the solution corresponds to a global maximum.

5.3. Optimal Inventory Policy Using the Price-Dependent Exponential Demand Function $d(p)=ae^{-bp}$. The first partial derivative of $\pi(p,T)$ with respect to p is

$$
\begin{aligned}
\frac{\partial\pi(p,T)}{\partial p} = \Bigg\{ & \left[-\frac{ab}{e^{bp}}p + d(p)\right]\Bigg[1-\frac{T}{2n}+\frac{\omega}{(\omega+\theta)}\left(-1+\frac{T}{2n}-\frac{1}{n(\omega+\theta)}\right) \\
& +\left(\frac{\omega}{T(\omega+\theta)}\right)\left(\frac{e^{(\omega+\theta)T}-1}{(\omega+\theta)}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)-\left(\frac{\omega}{Tn(\omega+\theta)}\right)\left(\frac{T\left(e^{(\omega+\theta)T}-1\right)}{(\omega+\theta)}\right)\Bigg] \\
& +\left[s\eta\left(-\frac{ab}{e^{bp}}\right)\right]\Bigg[\frac{1}{T(\omega+\theta)}\left(-1-\frac{1}{n(\omega+\theta)}\right)+\frac{e^{(\omega+\theta)T}}{T(\omega+\theta)}\left(1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right)-1 \\
& +\frac{T}{2n}+\frac{\omega}{(\omega+\theta)}\left(1-\frac{T}{2n}+\frac{1}{n(\omega+\theta)}\right)+\left(\frac{\omega}{T(\omega+\theta)}\right)\left(\frac{e^{(\omega+\theta)T}-1}{(\omega+\theta)}\right)\left(-1-\frac{1}{n(\omega+\theta)}+\frac{T}{n}\right)\Bigg] \\
& -\Bigg[\left(h\left(-\frac{ab}{e^{bp}}\right)\right)\left(\frac{1}{(\omega+\theta)}\left(-1+\frac{T}{2n}-\frac{1}{n(\omega+\theta)}\right)+\frac{1}{T(\omega+\theta)}\left(\frac{e^{(\omega+\theta)T}-1}{(\omega+\theta)}\right)\left(1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right)\right) \\
& +\left(h_1\left(-\frac{ab}{e^{bp}}\right)\right)\left(\frac{T}{(\omega+\theta)}\left(\frac{T}{3n}-\frac{1}{2}-\frac{1}{2n(\omega+\theta)}\right)+\frac{1}{T(\omega+\theta)}\left(\frac{e^{(\omega+\theta)T}-T(\omega+\theta)-1}{(\omega+\theta)^2}\right)\left(1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right)\right) \\
& +\left(h_2\left(-\frac{ab}{e^{bp}}\right)\right)\left(\frac{T^2}{(\omega+\theta)}\left(\frac{T}{4n}-\frac{1}{3}-\frac{1}{3n(\omega+\theta)}\right)+\frac{1}{T(\omega+\theta)}\left(\frac{2e^{(\omega+\theta)T}-T(\omega+\theta)(T(\omega+\theta)+2)-2}{(\omega+\theta)^3}\right)\right. \\
& \left.\cdot\left(1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right)\right]+\left[\left(c\left(-\frac{ab}{e^{bp}}\right)\right)\left(\frac{1}{T(\omega+\theta)}\left(1+\frac{1}{n(\omega+\theta)}\right)+\frac{e^{(\omega+\theta)T}}{T(\omega+\theta)}\left(-1-\frac{1}{n(\omega+\theta)}+\frac{T}{n}\right)\right)\right] \\
& -\left[c_d\left(-\frac{ab}{e^{bp}}\right)\right]\Bigg[\frac{1}{T(\omega+\theta)}\left(-1-\frac{1}{n(\omega+\theta)}\right)+\frac{e^{(\omega+\theta)T}}{T(\omega+\theta)}\left(1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right)-1 \\
& +\frac{T}{2n}+\frac{\omega}{(\omega+\theta)}\left(1-\frac{T}{2n}+\frac{1}{n(\omega+\theta)}\right)+\left(\frac{\omega}{T(\omega+\theta)}\right)\left(\frac{e^{(\omega+\theta)T}-1}{(\omega+\theta)}\right)\left(-1-\frac{1}{n(\omega+\theta)}+\frac{T}{n}\right)\Bigg]\Bigg\} = 0.
\end{aligned} \tag{27}
$$

The first partial derivative of $\pi(p,T)$ with respect to T is

$$\frac{\partial\pi(p,T)}{\partial T}=\Bigg\{p\Bigg[\frac{d(p)}{2n}\left(\frac{\omega}{\omega+\theta}-1\right)+\frac{\omega\,d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{(\omega+\theta)T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)$$
$$-\frac{\omega\,d(p)e^{(\omega+\theta)T}}{n(\omega+\theta)}\Bigg]+\Bigg[s\eta\Bigg(\frac{d(p)}{(\omega+\theta)T^2}\left(1+\frac{1}{n(\omega+\theta)}\right)+\frac{d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)$$
$$-\frac{d(p)}{n(\omega+\theta)}(\omega+\theta)e^{(\omega+\theta)T}+\frac{d(p)}{2n}\left(1-\frac{\omega}{(\omega+\theta)}\right)-\frac{\omega\,d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{(\omega+\theta)T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)$$
$$+\frac{\omega\,d(p)}{n(\omega+\theta)}e^{(\omega+\theta)T}\Bigg)\Bigg]+\left[\frac{K}{T^2}\right]-\Bigg[\Bigg(h\Bigg(\frac{d(p)}{2n(\omega+\theta)}+\frac{d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{(\omega+\theta)T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)$$
$$-\frac{d(p)}{n(\omega+\theta)}e^{(\omega+\theta)T}\Bigg)\Bigg)+\Bigg(h_1\Bigg(-\frac{d(p)}{2(\omega+\theta)}+\frac{2\,d(p)T}{3n(\omega+\theta)}-\frac{d(p)}{2n(\omega+\theta)^2}$$
$$+\frac{d(p)}{(\omega+\theta)}\left(\frac{\omega Te^{(\omega+\theta)T}+\theta Te^{(\omega+\theta)T}-e^{(\omega+\theta)T}+1}{(\omega+\theta)^2T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)-\frac{d(p)}{n(\omega+\theta)}\left(\frac{e^{(\omega+\theta)T}-1}{(\omega+\theta)}\right)\Bigg)\Bigg)$$
$$+\Bigg(h_2\Bigg(-\frac{2\,d(p)T}{3(\omega+\theta)}+\frac{3\,d(p)T^2}{4n(\omega+\theta)}-\frac{2\,d(p)T}{3n(\omega+\theta)^2}$$
$$+\frac{d(p)}{(\omega+\theta)}\left(-\frac{\theta^2T^2+2\omega\theta T^2-2\theta Te^{(\omega+\theta)T}-2\omega Te^{(\omega+\theta)T}+2e^{(\omega+\theta)T}+\omega^2T^2-2}{(\omega+\theta)^3T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)$$
$$-\frac{d(p)}{n(\omega+\theta)}\left(\frac{2e^{(\omega+\theta)T}-2T(\omega+\theta)-2}{(\omega+\theta)^2}\right)\Bigg)\Bigg)\Bigg]-\Bigg[c\Bigg(\frac{d(p)}{(\omega+\theta)T^2}\left(1+\frac{1}{n(\omega+\theta)}\right)$$
$$+\frac{d(p)}{(\omega+\theta)}\left(\frac{e^{(\omega+\theta)T}(\omega T+\theta T-1)}{T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)-\frac{d(p)}{n(\omega+\theta)}(\omega+\theta)e^{(\omega+\theta)T}\Bigg)\Bigg]$$
$$-\Bigg[c_d\Bigg(\frac{d(p)}{(\omega+\theta)T^2}\left(1+\frac{1}{n(\omega+\theta)}\right)+\frac{d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)$$
$$-\frac{d(p)}{n(\omega+\theta)}(\omega+\theta)e^{(\omega+\theta)T}+\frac{d(p)}{2n}\left(1-\frac{\omega}{(\omega+\theta)}\right)-\frac{\omega\,d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{(\omega+\theta)T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)$$
$$-\frac{d(p)}{n(\omega+\theta)}(\omega+\theta)e^{(\omega+\theta)T}+\frac{d(p)}{2n}\left(1-\frac{\omega}{(\omega+\theta)}\right)-\frac{\omega\,d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{(\omega+\theta)T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)$$
$$+\frac{\omega\,d(p)}{n(\omega+\theta)}e^{(\omega+\theta)T}\Bigg)\Bigg]\Bigg\}=0. \tag{28}$$

Example 3. Using the data of Example 1 with the price-dependent part of the demand modelled with an exponential function, it follows $d(p)=2000e^{-0.2p}$ (i.e., $a=2000$ and $b=0.2$). The replenishment cycle time must satisfy $T\leq 1$

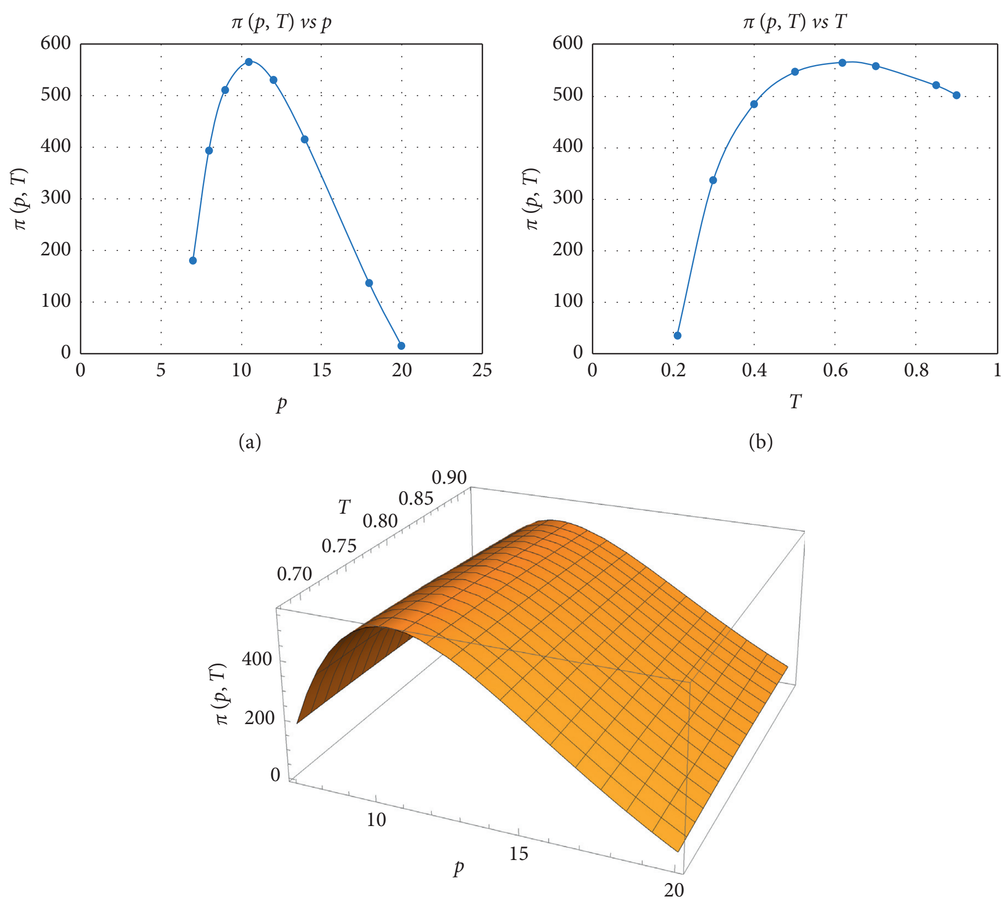

Figure 4: Concave property of the profit function $\pi(p, T)$ when the price-dependent demand function is exponential.

week because $n = 1$ week. For the price exponential demand, the upper bound for price is $p_{\max} = \infty$. As a result, the price must be into the interval $5 \le p \le \infty$.

By employing the algorithm, the optimal solution is found: $p^* = 10.50583$ euros per unit, $T^* = 0.6187657$ weeks, $Q^* = 121.3688$ units, and $\pi^*(p^*, T^*) = 564.3379$ euros per week. This solution satisfies all conditions for the optimality: $(\partial^2\pi(p,T)/\partial p^2) = -38.73461 < 0$, $(\partial^2\pi(p,T)/\partial T^2) = -17613.81 < 0$, $(\partial^2\pi(p,T)/\partial p\ \partial T) = (\partial^2\pi(p,T)/\partial T\ \partial p) = 23.85985$, and $(\partial^2\pi(p,T)/\partial p^2)(\partial^2\pi(p,T)/\partial T^2) - (\partial^2\pi(p,T)/\partial p\ \partial T)(\partial^2\pi(p,T)/\partial T\ \partial p) = 681694.7 > 0$. The Hessian determinant is greater than zero, with which the total profit function is strictly concave with a negative-definite Hessian matrix. Consequently, the solution is optimal.

By drawing the total profit per unit of time function $\pi(p,T)$ with different values of p between 7 and 20, and considering T as fixed and different values of T between 0 and 1 when p is given, $\pi(p,T)$ is strictly concave with respect to p when T is fixed (see Figure 4(a)) and with respect to T when p is given (see Figure 4(b)). Besides, $\pi(p,T)$ is also strictly concave with respect to both decision variables: p and T (see Figure 4(c)). This ratifies that total profit $\pi(p,T)$ has the concavity property, and the solution is a global maximum.

5.4. Optimal Inventory Policy Using the Price-Dependent Logit Demand Function $d(p) = (a/(1+e^{bp}))$. The first partial derivative of $\pi(p,T)$ w.r.t. p is

$$
\begin{aligned}
\frac{\partial \pi(p,T)}{\partial p} = \Bigg\{ & \left[\left[-\frac{abe^{bp}}{(1+e^{bp})^2}p + d(p)\right]\left[1 - \frac{T}{2n} + \frac{\omega}{(\omega+\theta)}\left(-1 + \frac{T}{2n} - \frac{1}{n(\omega+\theta)}\right)\right.\right. \\
& + \left(\frac{\omega}{T(\omega+\theta)}\right)\left(\frac{e^{(\omega+\theta)T}-1}{(\omega+\theta)}\right)\left(1 + \frac{1}{n(\omega+\theta)}\right) - \left(\frac{\omega}{Tn(\omega+\theta)}\right)\left(\frac{T\left(e^{(\omega+\theta)T}-1\right)}{(\omega+\theta)}\right)\Bigg] \\
& + \left[s\eta\left(-\frac{abe^{bp}}{(1+e^{bp})^2}\right)\right]\left[\frac{1}{T(\omega+\theta)}\left(-1 - \frac{1}{n(\omega+\theta)}\right) + \frac{e^{(\omega+\theta)T}}{T(\omega+\theta)}\left(1 + \frac{1}{n(\omega+\theta)} - \frac{T}{n}\right) - 1\right. \\
& \left. + \frac{T}{2n} + \frac{\omega}{(\omega+\theta)}\left(1 - \frac{T}{2n} + \frac{1}{n(\omega+\theta)}\right) + \left(\frac{\omega}{T(\omega+\theta)}\right)\left(\frac{e^{(\omega+\theta)T}-1}{(\omega+\theta)}\right)\left(-1 - \frac{1}{n(\omega+\theta)} + \frac{T}{n}\right)\right] \\
& - \left[\left(h\left(-\frac{abe^{bp}}{(1+e^{bp})^2}\right)\right)\left(\frac{1}{(\omega+\theta)}\left(-1 + \frac{T}{2n} - \frac{1}{n(\omega+\theta)}\right) + \frac{1}{T(\omega+\theta)}\left(\frac{e^{(\omega+\theta)T}-1}{(\omega+\theta)}\right)\left(1 + \frac{1}{n(\omega+\theta)} - \frac{T}{n}\right)\right)\right. \\
& + \left(h_1\left(-\frac{abe^{bp}}{(1+e^{bp})^2}\right)\right)\left(\frac{T}{(\omega+\theta)}\left(\frac{T}{3n} - \frac{1}{2} - \frac{1}{2n(\omega+\theta)}\right) + \frac{1}{T(\omega+\theta)}\left(\frac{e^{(\omega+\theta)T} - T(\omega+\theta) - 1}{(\omega+\theta)^2}\right)\left(1 + \frac{1}{n(\omega+\theta)} - \frac{T}{n}\right)\right) \\
& + \left(h_2\left(-\frac{abe^{bp}}{(1+e^{bp})^2}\right)\right)\left(\frac{T^2}{(\omega+\theta)}\left(\frac{T}{4n} - \frac{1}{3} - \frac{1}{3n(\omega+\theta)}\right) + \frac{1}{T(\omega+\theta)}\left(\frac{2e^{(\omega+\theta)T} - T(\omega+\theta)(T(\omega+\theta)+2) - 2}{(\omega+\theta)^3}\right)\right. \\
& \left.\left. \cdot\left(1 + \frac{1}{n(\omega+\theta)} - \frac{T}{n}\right)\right]\right] + \left[\left(c\left(-\frac{abe^{bp}}{(1+e^{bp})^2}\right)\right)\left(\frac{1}{T(\omega+\theta)}\left(1 + \frac{1}{n(\omega+\theta)}\right) + \frac{e^{(\omega+\theta)T}}{T(\omega+\theta)}\left(-1 - \frac{1}{n(\omega+\theta)} + \frac{T}{n}\right)\right)\right] \\
& - \left[c_d\left(-\frac{abe^{bp}}{(1+e^{bp})^2}\right)\right]\left[\frac{1}{T(\omega+\theta)}\left(-1 - \frac{1}{n(\omega+\theta)}\right) + \frac{e^{(\omega+\theta)T}}{T(\omega+\theta)}\left(1 + \frac{1}{n(\omega+\theta)} - \frac{T}{n}\right) - 1\right. \\
& \left.\left. + \frac{T}{2n} + \frac{\omega}{(\omega+\theta)}\left(1 - \frac{T}{2n} + \frac{1}{n(\omega+\theta)}\right) + \left(\frac{\omega}{T(\omega+\theta)}\right)\left(\frac{e^{(\omega+\theta)T}-1}{(\omega+\theta)}\right)\left(-1 - \frac{1}{n(\omega+\theta)} + \frac{T}{n}\right)\right]\right\} = 0.
\end{aligned} \tag{29}
$$

The first partial derivative of $\pi(p,T)$ w.r.t. T is

$$\begin{aligned}
\frac{\partial \pi(p,T)}{\partial T} = \Bigg\{ p\Bigg[&\frac{d(p)}{2n}\left(\frac{\omega}{\omega+\theta}-1\right)+\frac{\omega\, d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{(\omega+\theta)T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\\
&-\frac{\omega\, d(p) e^{(\omega+\theta)T}}{n(\omega+\theta)}\Bigg]+\Bigg[s\eta\Bigg(\frac{d(p)}{(\omega+\theta)T^2}\left(1+\frac{1}{n(\omega+\theta)}\right)+\frac{d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\\
&-\frac{d(p)}{n(\omega+\theta)}(\omega+\theta)e^{(\omega+\theta)T}+\frac{d(p)}{2n}\left(1-\frac{\omega}{(\omega+\theta)}\right)-\frac{\omega\, d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{(\omega+\theta)T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\\
&+\frac{\omega\, d(p)}{n(\omega+\theta)}e^{(\omega+\theta)T}\Bigg)\Bigg]+\left[\frac{K}{T^2}\right]-\Bigg[\Bigg(h\Bigg(\frac{d(p)}{2n(\omega+\theta)}+\frac{d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{(\omega+\theta)T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\\
&-\frac{d(p)}{n(\omega+\theta)}e^{(\omega+\theta)T}\Bigg)\Bigg)+\Bigg(h_1\Bigg(-\frac{d(p)}{2(\omega+\theta)}+\frac{2\, d(p)T}{3n(\omega+\theta)}-\frac{d(p)}{2n(\omega+\theta)^2}\\
&+\frac{d(p)}{(\omega+\theta)}\left(\frac{\omega T e^{(\omega+\theta)T}+\theta T e^{(\omega+\theta)T}-e^{(\omega+\theta)T}+1}{(\omega+\theta)^2T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)-\frac{d(p)}{n(\omega+\theta)}\left(\frac{e^{(\omega+\theta)T}-1}{(\omega+\theta)}\right)\Bigg)\Bigg)\\
&+\Bigg(h_2\Bigg(-\frac{2\, d(p)T}{3(\omega+\theta)}+\frac{3\, d(p)T^2}{4n(\omega+\theta)}-\frac{2\, d(p)T}{3n(\omega+\theta)^2}\\
&+\frac{d(p)}{(\omega+\theta)}\left(-\frac{\theta^2T^2+2\omega\theta T^2-2\theta T e^{(\omega+\theta)T}-2\omega T e^{(\omega+\theta)T}+2e^{(\omega+\theta)T}+\omega^2T^2-2}{(\omega+\theta)^3T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\\
&-\frac{d(p)}{n(\omega+\theta)}\left(\frac{2e^{(\omega+\theta)T}-2T(\omega+\theta)-2}{(\omega+\theta)^2}\right)\Bigg)\Bigg)\Bigg]-\Bigg[c\Bigg(\frac{d(p)}{(\omega+\theta)T^2}\left(1+\frac{1}{n(\omega+\theta)}\right)\\
&+\frac{d(p)}{(\omega+\theta)}\left(\frac{e^{(\omega+\theta)T}(\omega T+\theta T-1)}{T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)-\frac{d(p)}{n(\omega+\theta)}(\omega+\theta)e^{(\omega+\theta)T}\Bigg)\Bigg]\\
&-\Bigg[c_d\Bigg(\frac{d(p)}{(\omega+\theta)T^2}\left(1+\frac{1}{n(\omega+\theta)}\right)+\frac{d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\\
&-\frac{d(p)}{n(\omega+\theta)}(\omega+\theta)e^{(\omega+\theta)T}+\frac{d(p)}{2n}\left(1-\frac{\omega}{(\omega+\theta)}\right)-\frac{\omega\, d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{(\omega+\theta)T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\\
&+\frac{\omega\, d(p)}{n(\omega+\theta)}e^{(\omega+\theta)T}\Bigg)\Bigg]\Bigg\}=0.
\end{aligned} \tag{30}$$

Example 4. Consider the input parameters of Example 1 with the price-dependent term of the demand for the fresh article following a logit demand function: $d(p) = (9000/(1+e^{0.3p}))$ (i.e., $a = 9000$ and $b = 0.3$).

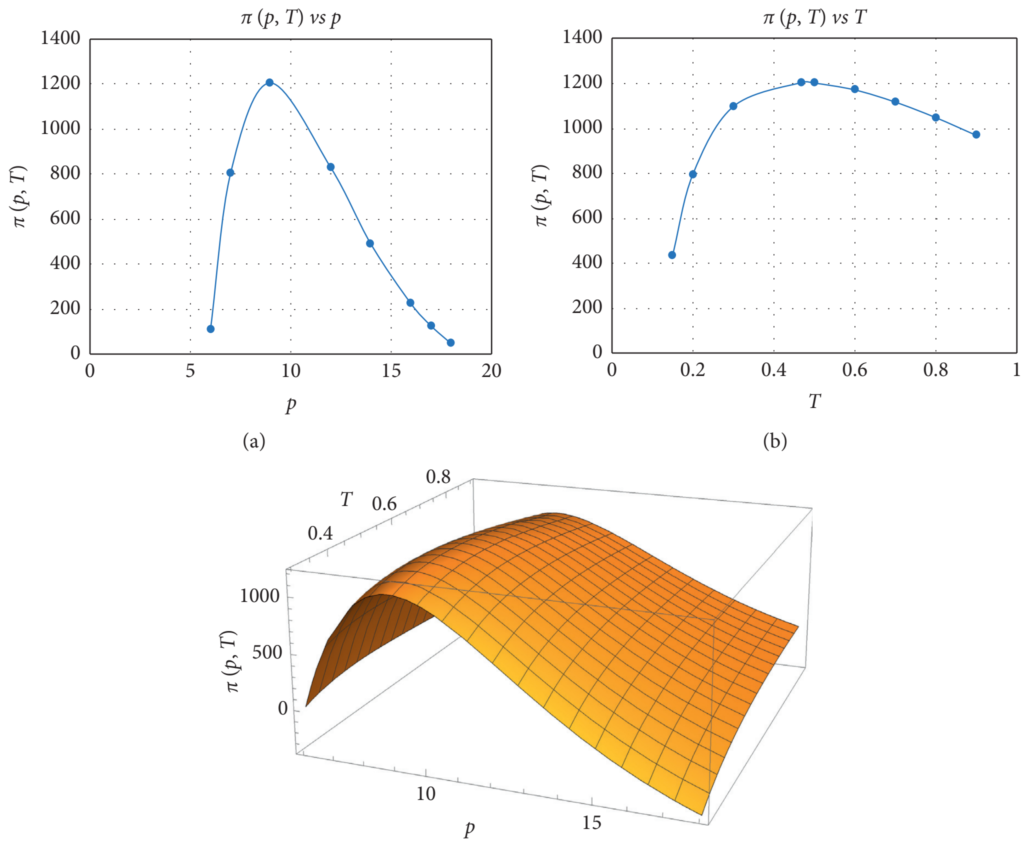

FIGURE 5: Concave property of the profit function $\pi(p,T)$ when the price-dependent demand function is logit.

The replenishment cycle time is constrained to $T \le 1$ week given that the shelf-life is $n = 1$ week. For the price logit demand, the upper bound for price is $p_{max} = \infty$. Therefore, the price is constrained by the interval $5 \le p \le \infty$.

By running the algorithm, the optimal solution is obtained: $p^* = 8.963560$ euros per unit, $T^* = 0.4682024$ weeks, $Q^* = 231.1214$ units, and $\pi^*(p^*, T^*) = 1205.467$ euros per week. This solution satisfies all conditions for the optimality: $(\partial^2\pi(p,T)/\partial p^2) = -146.5883 < 0$, $(\partial^2\pi(p,T)/\partial T^2) = -87888.64 < 0$, $(\partial^2\pi(p,T)/\partial p\,\partial T) = (\partial^2\pi(p,T)/\partial T\,\partial p) = 100.7604$, and $(\partial^2\pi(p,T)/\partial p^2)(\partial^2\pi(p,T)/\partial T^2) - (\partial^2\pi(p,T)/\partial p\,\partial T)(\partial^2\pi(p,T)/\partial T\,\partial p) = 0.128733 \times 10^8 > 0$. The Hessian determinant is greater than zero, and the total profit function is strictly concave with a negative-definite Hessian matrix. Thus, the solution is optimal.

By doing some graphs for the total profit per unit of time function $\pi(p,T)$, with different values of p between 6 and 18, considering also T as fixed, and different values of T

between 0 and 1 when p is given, $\pi(p,T)$ is strictly concave with respect to p when T is fixed (see Figure 5(a)) and with respect to T when p is given (see Figure 5(b)). Besides, $\pi(p,T)$ is strictly concave with respect to both decision variables p and T (see Figure 5(c)). This proves that total profit $\pi(p,T)$ has the concavity property, so the solution is a global maximum.

5.5. Optimal Inventory Policy Using the Price-Dependent Logarithmic Demand Function $d(p) = a - b\ln p$. The first partial derivative of $\pi(p,T)$ w.r.t. p is

$$\begin{aligned}
\frac{\partial\pi(p,T)}{\partial p} = {} & \left\{\left[-\frac{b}{p}p + d(p)\right]\left[1 - \frac{T}{2n} + \frac{\omega}{(\omega+\theta)}\left(-1 + \frac{T}{2n} - \frac{1}{n(\omega+\theta)}\right)\right.\right. \\
& \left. + \left(\frac{\omega}{T(\omega+\theta)}\right)\left(\frac{e^{(\omega+\theta)T} - 1}{(\omega+\theta)}\right)\left(1 + \frac{1}{n(\omega+\theta)}\right) - \left(\frac{\omega}{Tn(\omega+\theta)}\right)\left(\frac{T\left(e^{(\omega+\theta)T} - 1\right)}{(\omega+\theta)}\right)\right] \\
& + \left[s\eta\left(-\frac{b}{p}\right)\right]\left[\frac{1}{T(\omega+\theta)}\left(-1 - \frac{1}{n(\omega+\theta)}\right) + \frac{e^{(\omega+\theta)T}}{T(\omega+\theta)}\left(1 + \frac{1}{n(\omega+\theta)} - \frac{T}{n}\right) - 1\right. \\
& \left. + \frac{T}{2n} + \frac{\omega}{(\omega+\theta)}\left(1 - \frac{T}{2n} + \frac{1}{n(\omega+\theta)}\right) + \left(\frac{\omega}{T(\omega+\theta)}\right)\left(\frac{e^{(\omega+\theta)T} - 1}{(\omega+\theta)}\right)\left(-1 - \frac{1}{n(\omega+\theta)} + \frac{T}{n}\right)\right] \\
& - \left[\left(h\left(-\frac{b}{p}\right)\right)\left(\frac{1}{(\omega+\theta)}\left(-1 + \frac{T}{2n} - \frac{1}{n(\omega+\theta)}\right) + \frac{1}{T(\omega+\theta)}\left(\frac{e^{(\omega+\theta)T} - 1}{(\omega+\theta)}\right)\left(1 + \frac{1}{n(\omega+\theta)} - \frac{T}{n}\right)\right)\right. \\
& + \left(h_1\left(-\frac{b}{p}\right)\right)\left(\frac{T}{(\omega+\theta)}\left(\frac{T}{3n} - \frac{1}{2} - \frac{1}{2n(\omega+\theta)}\right) + \frac{1}{T(\omega+\theta)}\left(\frac{e^{(\omega+\theta)T} - T(\omega+\theta) - 1}{(\omega+\theta)^2}\right)\left(1 + \frac{1}{n(\omega+\theta)} - \frac{T}{n}\right)\right) \\
& + \left(h_2\left(-\frac{b}{p}\right)\right)\left(\frac{T^2}{(\omega+\theta)}\left(\frac{T}{4n} - \frac{1}{3} - \frac{1}{3n(\omega+\theta)}\right) + \frac{1}{T(\omega+\theta)}\left(\frac{2e^{(\omega+\theta)T} - T(\omega+\theta)(T(\omega+\theta)+2) - 2}{(\omega+\theta)^3}\right)\right. \\
& \left.\left. \cdot\left(1 + \frac{1}{n(\omega+\theta)} - \frac{T}{n}\right)\right)\right] + \left[\left(c\left(-\frac{b}{p}\right)\right)\left(\frac{1}{T(\omega+\theta)}\left(1 + \frac{1}{n(\omega+\theta)}\right) + \frac{e^{(\omega+\theta)T}}{T(\omega+\theta)}\left(-1 - \frac{1}{n(\omega+\theta)} + \frac{T}{n}\right)\right)\right] \\
& - \left[c_d\left(-\frac{b}{p}\right)\right]\left[\frac{1}{T(\omega+\theta)}\left(-1 - \frac{1}{n(\omega+\theta)}\right) + \frac{e^{(\omega+\theta)T}}{T(\omega+\theta)}\left(1 + \frac{1}{n(\omega+\theta)} - \frac{T}{n}\right) - 1\right. \\
& \left.\left. + \frac{T}{2n} + \frac{\omega}{(\omega+\theta)}\left(1 - \frac{T}{2n} + \frac{1}{n(\omega+\theta)}\right) + \left(\frac{\omega}{T(\omega+\theta)}\right)\left(\frac{e^{(\omega+\theta)T} - 1}{(\omega+\theta)}\right)\left(-1 - \frac{1}{n(\omega+\theta)} + \frac{T}{n}\right)\right]\right\} = 0.
\end{aligned} \tag{31}$$

The first partial derivative of $\pi(p,T)$ w.r.t. T is

$$\begin{aligned}
\frac{\partial \pi(p,T)}{\partial T} = &\left\{ p\left[\frac{d(p)}{2n}\left(\frac{\omega}{\omega+\theta}-1\right)+\frac{\omega\, d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{(\omega+\theta)T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\right.\right.\\
&\left.-\frac{\omega\, d(p)e^{(\omega+\theta)T}}{n(\omega+\theta)}\right]+\left[s\eta\left(\frac{d(p)}{(\omega+\theta)T^2}\left(1+\frac{1}{n(\omega+\theta)}\right)+\frac{d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\right.\right.\\
&-\frac{d(p)}{n(\omega+\theta)}(\omega+\theta)e^{(\omega+\theta)T}+\frac{d(p)}{2n}\left(1-\frac{\omega}{(\omega+\theta)}\right)-\frac{\omega\, d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{(\omega+\theta)T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\\
&\left.\left.+\frac{\omega\, d(p)}{n(\omega+\theta)}e^{(\omega+\theta)T}\right)\right]+\left[\frac{K}{T^2}\right]-\left[\left(h\left(\frac{d(p)}{2n(\omega+\theta)}+\frac{d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{(\omega+\theta)T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\right.\right.\right.\\
&\left.\left.-\frac{d(p)}{n(\omega+\theta)}e^{(\omega+\theta)T}\right)\right)+\left(h_1\left(-\frac{d(p)}{2(\omega+\theta)}+\frac{2\, d(p)T}{3n(\omega+\theta)}-\frac{d(p)}{2n(\omega+\theta)^2}\right.\right.\\
&\left.\left.+\frac{d(p)}{(\omega+\theta)}\left(\frac{\omega Te^{(\omega+\theta)T}+\theta Te^{(\omega+\theta)T}-e^{(\omega+\theta)T}+1}{(\omega+\theta)^2T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)-\frac{d(p)}{n(\omega+\theta)}\left(\frac{e^{(\omega+\theta)T}-1}{(\omega+\theta)}\right)\right)\right)\\
&+\left(h_2\left(-\frac{2\, d(p)T}{3(\omega+\theta)}+\frac{3\, d(p)T^2}{4n(\omega+\theta)}-\frac{2\, d(p)T}{3n(\omega+\theta)^2}\right.\right.\\
&+\frac{d(p)}{(\omega+\theta)}\left(-\frac{\theta^2T^2+2\omega\theta T^2-2\theta Te^{(\omega+\theta)T}-2\omega Te^{(\omega+\theta)T}+2e^{(\omega+\theta)T}+\omega^2T^2-2}{(\omega+\theta)^3T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\\
&\left.\left.\left.-\frac{d(p)}{n(\omega+\theta)}\left(\frac{2e^{(\omega+\theta)T}-2T(\omega+\theta)-2}{(\omega+\theta)^2}\right)\right)\right)\right]-\left[c\left(\frac{d(p)}{(\omega+\theta)T^2}\left(1+\frac{1}{n(\omega+\theta)}\right)\right.\right.\\
&\left.\left.+\frac{d(p)}{(\omega+\theta)}\left(\frac{e^{(\omega+\theta)T}(\omega T+\theta T-1)}{T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)-\frac{d(p)}{n(\omega+\theta)}(\omega+\theta)e^{(\omega+\theta)T}\right)\right]\\
&-\left[c_d\left(\frac{d(p)}{(\omega+\theta)T^2}\left(1+\frac{1}{n(\omega+\theta)}\right)+\frac{d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\right.\right.\\
&-\frac{d(p)}{n(\omega+\theta)}(\omega+\theta)e^{(\omega+\theta)T}+\frac{d(p)}{2n}\left(1-\frac{\omega}{(\omega+\theta)}\right)-\frac{\omega\, d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{(\omega+\theta)T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\\
&\left.\left.\left.+\frac{\omega\, d(p)}{n(\omega+\theta)}e^{(\omega+\theta)T}\right)\right]\right\}=0.
\end{aligned} \tag{32}$$

Example 5. Using the information from Example 1, now the price-dependent factor of the demand for the fresh produce follows a logarithmic demand function: $d(p) = 95 - 21 \ln p$ (i.e., $a = 95$ and $b = 21$).

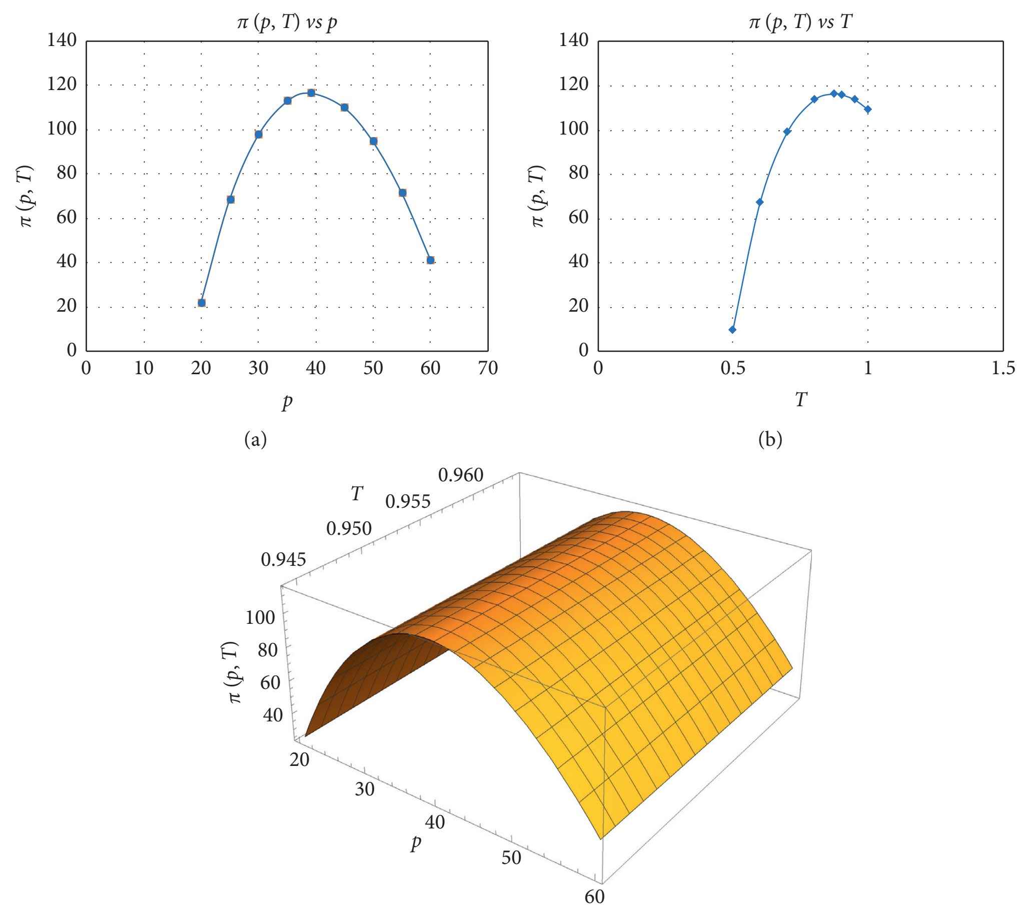

Figure 6: Concave property of the profit function $\pi(p, T)$ when the price-dependent demand function is logarithmic.

The replenishment cycle time is subjected to $T \leq 1$ week, as the shelf-life is $n = 1$ week. For the price logarithmic demand, the upper bound for price is $p_{\max} = e^{(a/b)} = e^{(95/21)} = 92.18612$. Therefore, the price is subjected to be into the interval $5 \leq p \leq 92.18612$.

By executing the algorithm, the optimal solution is computed: $p^* = 39.15353$ euros per unit, $T^* = 0.8729460$ weeks, $Q^* = 10.65368$ units, and $\pi^*(p^*, T^*) = 116.4864$ euros per week. This solution satisfies all conditions for the optimality: $(\partial^2\pi(p,T))/\partial p^2) = -0.4098830 < 0$, $(\partial^2\pi(p,T)/\partial T^2) = -1389.405 < 0$, $(\partial^2\pi(p,T)/\partial p\,\partial T) = (\partial^2\pi(p,T)/\partial T\,\partial p) = 0.1036674$, and $(\partial^2\pi(p,T)/\partial p^2)(\partial^2\pi(p,T)/\partial T^2) - (\partial^2\pi(p,T)/\partial p\,\partial T)(\partial^2\pi\ (p,T)/\partial T\,\partial p) = 569.4828 > 0$. The Hessian determinant is greater than zero, and the total profit function is strictly concave with a negative-definite Hessian matrix. Thus, the solution is optimal.

By making some diagrams for the total profit per unit of time function $\pi(p,T)$, with different values of p between 20 and 60, considering also T as fixed, and different values of T between 0 and 1 when p is given, $\pi(p,T)$ is strictly concave with respect to p when T is fixed (see Figure 6(a)) and with respect to T when p is given (see Figure 6(b)). Besides, $\pi(p,T)$ is strictly concave w.r.t. both decision variables: p and T (see Figure 6(c)). This demonstrates that total profit $\pi(p,T)$ has the concavity property. So, the solution is a global maximum.

5.6. Optimal Inventory Policy Using the Price-Dependent Polynomial Demand Function $d(p)=a-bp^m$. The partial derivative of $\pi(p,T)$ with respect to p is

$$\begin{aligned}\frac{\partial\pi(p,T)}{\partial p} &= \Bigg\{\left[\left(-mbp^{m-1}\right)p+d(p)\right]\Bigg[1-\frac{T}{2n}+\frac{\omega}{(\omega+\theta)}\left(-1+\frac{T}{2n}-\frac{1}{n(\omega+\theta)}\right)\\
&\quad+\left(\frac{\omega}{T(\omega+\theta)}\right)\left(\frac{e^{(\omega+\theta)T}-1}{(\omega+\theta)}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)-\left(\frac{\omega}{Tn(\omega+\theta)}\right)\left(\frac{T\left(e^{(\omega+\theta)T}-1\right)}{(\omega+\theta)}\right)\Bigg]\\
&\quad+\left[s\eta\left(-mbp^{m-1}\right)\right]\Bigg[\frac{1}{T(\omega+\theta)}\left(-1-\frac{1}{n(\omega+\theta)}\right)+\frac{e^{(\omega+\theta)T}}{T(\omega+\theta)}\left(1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right)-1\\
&\quad+\frac{T}{2n}+\frac{\omega}{(\omega+\theta)}\left(1-\frac{T}{2n}+\frac{1}{n(\omega+\theta)}\right)+\left(\frac{\omega}{T(\omega+\theta)}\right)\left(\frac{e^{(\omega+\theta)T}-1}{(\omega+\theta)}\right)\left(-1-\frac{1}{n(\omega+\theta)}+\frac{T}{n}\right)\Bigg]\\
&\quad-\Bigg[\left(h\left(-mbp^{m-1}\right)\right)\left(\frac{1}{(\omega+\theta)}\left(-1+\frac{T}{2n}-\frac{1}{n(\omega+\theta)}\right)+\frac{1}{T(\omega+\theta)}\left(\frac{e^{(\omega+\theta)T}-1}{(\omega+\theta)}\right)\left(1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right)\right)\\
&\quad+\left(h_1\left(-mbp^{m-1}\right)\right)\left(\frac{T}{(\omega+\theta)}\left(\frac{T}{3n}-\frac{1}{2}-\frac{1}{2n(\omega+\theta)}\right)+\frac{1}{T(\omega+\theta)}\left(\frac{e^{(\omega+\theta)T}-T(\omega+\theta)-1}{(\omega+\theta)^2}\right)\left(1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right)\right)\\
&\quad+\left(h_2\left(-mbp^{m-1}\right)\right)\left(\frac{T^2}{(\omega+\theta)}\left(\frac{T}{4n}-\frac{1}{3}-\frac{1}{3n(\omega+\theta)}\right)+\frac{1}{T(\omega+\theta)}\left(\frac{2e^{(\omega+\theta)T}-T(\omega+\theta)(T(\omega+\theta)+2)-2}{(\omega+\theta)^3}\right)\right.\\
&\quad\left.\cdot\left(1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right)\right)\Bigg]+\left[\left(c\left(-mbp^{m-1}\right)\right)\left(\frac{1}{T(\omega+\theta)}\left(1+\frac{1}{n(\omega+\theta)}\right)+\frac{e^{(\omega+\theta)T}}{T(\omega+\theta)}\left(-1-\frac{1}{n(\omega+\theta)}+\frac{T}{n}\right)\right)\right]\\
&\quad-\left[c_d\left(-mbp^{m-1}\right)\right]\Bigg[\frac{1}{T(\omega+\theta)}\left(-1-\frac{1}{n(\omega+\theta)}\right)+\frac{e^{(\omega+\theta)T}}{T(\omega+\theta)}\left(1+\frac{1}{n(\omega+\theta)}-\frac{T}{n}\right)-1\\
&\quad+\frac{T}{2n}+\frac{\omega}{(\omega+\theta)}\left(1-\frac{T}{2n}+\frac{1}{n(\omega+\theta)}\right)+\left(\frac{\omega}{T(\omega+\theta)}\right)\left(\frac{e^{(\omega+\theta)T}-1}{(\omega+\theta)}\right)\left(-1-\frac{1}{n(\omega+\theta)}+\frac{T}{n}\right)\Bigg]\Bigg\}=0.\end{aligned} \tag{33}$$

The partial derivative of $\pi(p,T)$ with respect to T is

$$
\begin{aligned}
\frac{\partial\pi(p,T)}{\partial T} = \Bigg\{ & p\Bigg[\frac{d(p)}{2n}\left(\frac{\omega}{\omega+\theta}-1\right)+\frac{\omega\, d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{(\omega+\theta)T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\\
& -\frac{\omega\, d(p)e^{(\omega+\theta)T}}{n(\omega+\theta)}\Bigg]+\Bigg[s\eta\Bigg(\frac{d(p)}{(\omega+\theta)T^2}\left(1+\frac{1}{n(\omega+\theta)}\right)+\frac{d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\\
& -\frac{d(p)}{n(\omega+\theta)}(\omega+\theta)e^{(\omega+\theta)T}+\frac{d(p)}{2n}\left(1-\frac{\omega}{(\omega+\theta)}\right)-\frac{\omega\, d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{(\omega+\theta)T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\\
& +\frac{\omega\, d(p)}{n(\omega+\theta)}e^{(\omega+\theta)T}\Bigg)\Bigg]+\left[\frac{K}{T^2}\right]-\Bigg[\Bigg(h\Bigg(\frac{d(p)}{2n(\omega+\theta)}+\frac{d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{(\omega+\theta)T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\\
& -\frac{d(p)}{n(\omega+\theta)}e^{(\omega+\theta)T}\Bigg)\Bigg)+\Bigg(h_1\Bigg(-\frac{d(p)}{2(\omega+\theta)}+\frac{2\, d(p)T}{3n(\omega+\theta)}-\frac{d(p)}{2n(\omega+\theta)^2}\\
& +\frac{d(p)}{(\omega+\theta)}\left(\frac{\omega Te^{(\omega+\theta)T}+\theta Te^{(\omega+\theta)T}-e^{(\omega+\theta)T}+1}{(\omega+\theta)^2T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)-\frac{d(p)}{n(\omega+\theta)}\left(\frac{e^{(\omega+\theta)T}-1}{(\omega+\theta)}\right)\Bigg)\Bigg)\\
& +\Bigg(h_2\Bigg(-\frac{2\, d(p)T}{3(\omega+\theta)}+\frac{3\, d(p)T^2}{4n(\omega+\theta)}-\frac{2\, d(p)T}{3n(\omega+\theta)^2}\\
& +\frac{d(p)}{(\omega+\theta)}\left(-\frac{\theta^2T^2+2\omega\theta T^2-2\theta Te^{(\omega+\theta)T}-2\omega Te^{(\omega+\theta)T}+2e^{(\omega+\theta)T}+\omega^2T^2-2}{(\omega+\theta)^3T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\\
& -\frac{d(p)}{n(\omega+\theta)}\left(\frac{2e^{(\omega+\theta)T}-2T(\omega+\theta)-2}{(\omega+\theta)^2}\right)\Bigg)\Bigg)\Bigg]-\Bigg[c\Bigg(\frac{d(p)}{(\omega+\theta)T^2}\left(1+\frac{1}{n(\omega+\theta)}\right)\\
& +\frac{d(p)}{(\omega+\theta)}\left(\frac{e^{(\omega+\theta)T}(\omega T+\theta T-1)}{T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)-\frac{d(p)}{n(\omega+\theta)}(\omega+\theta)e^{(\omega+\theta)T}\Bigg)\Bigg]\\
& -\Bigg[c_d\Bigg(\frac{d(p)}{(\omega+\theta)T^2}\left(1+\frac{1}{n(\omega+\theta)}\right)+\frac{d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\\
& -\frac{d(p)}{n(\omega+\theta)}(\omega+\theta)e^{(\omega+\theta)T}+\frac{d(p)}{2n}\left(1-\frac{\omega}{(\omega+\theta)}\right)-\frac{\omega\, d(p)}{(\omega+\theta)}\left(\frac{\left(e^{(\omega+\theta)T}(\omega T+\theta T-1)\right)+1}{(\omega+\theta)T^2}\right)\left(1+\frac{1}{n(\omega+\theta)}\right)\\
& +\frac{\omega\, d(p)}{n(\omega+\theta)}e^{(\omega+\theta)T}\Bigg)\Bigg]\Bigg\}=0.
\end{aligned}
\tag{34}
$$

Example 6. Utilizing the information given in Example 1 with the price-dependent portion of the demand for the fresh produce following a polynomial demand function: $d(p)=4000-2p^3$ (i.e., $a=4000$, $b=2$, and $m=3$).

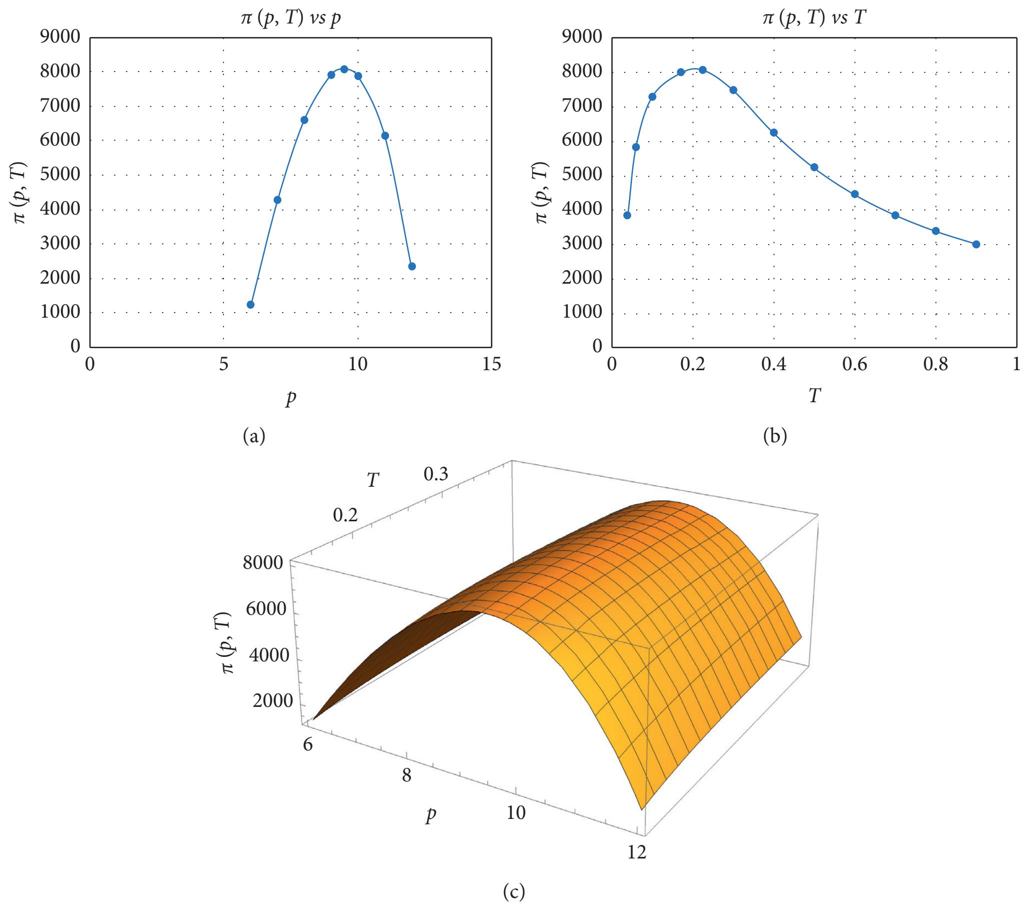

Figure 7: Concave property of the profit function $\pi(p, T)$ when the price-dependent demand function is polynomial.

The replenishment cycle time is subjected to $T \leq 1$ week because $n = 1$ week. For the price polynomial demand, the upper bound for price is $p_{\max} = (a/b)^{(1/m)} = (4000/2)^{1/3} = 12.59921$. Therefore, the price is limited to the interval $5 \leq p \leq 12.59921$.

By carrying out the algorithm, the optimal solution is $p^* = 9.475246$ euros per unit, $T^* = 0.2256880$ weeks, $Q^* = 488.7249$ units, and $\pi^*(p^*, T^*) = 8083.700$ euros per week. This solution satisfies all conditions for the optimality: $(\partial^2 \pi(p, T)/\partial p^2) = -1464.246 < 0$, $(\partial^2 \pi(p, T)/\partial T^2) = -3018587 < 0$, $(\partial^2 \pi(p, T)/\partial p\, \partial T) = (\partial^2 \pi(p, T)/\partial T\, \partial p) = 439.8788$, and $(\partial^2 \pi(p, T)/\partial p^2)(\partial^2 \pi(p, T)/\partial T^2) - (\partial^2 \pi(p, T)/\partial p\, \partial T)(\partial^2 \pi(p, T)/\partial T\, \partial p) = 0.4419761 \times 10^{10} > 0$. The Hessian determinant is greater than zero, and the total profit function is strictly concave with a negative-definite Hessian matrix, which means the solution is optimal.

By carrying out some graphs for the total profit per unit of time function $\pi(p, T)$, with different values of p between 6 and 12, considering T as fixed, and different values of T between 0 and 1, when p is given, $\pi(p, T)$ is strictly concave with respect to p when T is fixed (see Figure 7(a)) and with respect to T when p is given (see Figure 7(b)). Besides, $\pi(p, T)$ is strictly concave w.r.t. both decision variables: p and T (see Figure 7(c)). This validates both total profit $\pi(p, T)$ has the concavity property, and the solution is a global maximum.

6. Sensitivity Analysis

This section presents a sensitivity analysis that was carried out in order to examine the influence of the inventory model's input parameters, over the total profit $\pi(p, T)$ and decision variables (p, T, Q). The sensitivity analysis is performed for each of the six price-dependent demands. Distinct values for each input parameter are used, while the other input data are fixed. The results are shown in Tables 2–7. Some observations and managerial insights are also provided below.

Tables 2–7 present the impact of the input parameters on the optimal solution of p, T, Q, and the total profit for the price-dependent linear, isoelastic, exponential, logit, logarithmic, and polynomial demand functions, respectively.

Table 8 is built from the numerical experimentation shown in Tables 2–7. Table 8 shows the behavior of the variables and the total profit when the inventory model parameters are increased. The following observations and managerial insights are derived from Tables 2–8.

(i) The fresh item's maximum shelf-life n and the ordering cost K have the exact same impact over the inventory policy and total profit in all price demand functions. A longer product's maximum shelf-life n results in a higher optimal price, a

Table 2: Sensitivity analysis of the optimal solution for the price-dependent linear demand function.

Parameter		p	T	Q	Profit
n	0.5	17.62065	0.2895503	54.61395	1457.227
	0.6	17.63586	0.3211456	62.92978	1627.649
	0.7	17.65028	0.3515362	70.96242	1762.836
	0.8	17.66419	0.3811902	78.8329	1874.133
	0.9	17.6778	0.4104517	86.63123	1968.361
	1	17.69124	0.4395922	94.42941	2049.903
	1.1	17.70463	0.4688389	102.2887	2121.728
	1.2	17.71807	0.4983923	110.2643	2185.928
	1.3	17.73165	0.5284379	118.4082	2244.032
	1.4	17.74546	0.5591541	126.7715	2297.187
	1.5	17.75956	0.5907188	135.4061	2346.277
K	250	17.69124	0.4395922	94.42941	2049.903
	300	17.70497	0.4766712	100.6798	1940.786
	350	17.71696	0.5101394	106.0448	1839.465
	400	17.72758	0.5407686	110.7159	1744.321
	450	17.73707	0.5690932	114.8253	1654.228
	500	17.74562	0.5955007	118.4688	1568.368
ω	0.25	17.68249	0.4095219	84.91483	1920.088
	0.35	17.6858	0.4207697	88.45035	1970.198
	0.45	17.68936	0.4330415	92.33854	2022.689
	0.5	17.69124	0.4395922	94.42941	2049.903
	0.55	17.69317	0.4464355	96.6265	2077.807
θ	0.03	17.68352	0.4399946	94.1671	2052.515
	0.04	17.68737	0.4397943	94.29837	2051.211
	0.05	17.69124	0.4395922	94.42941	2049.903
	0.06	17.6951	0.4393881	94.5602	2048.593
	0.07	17.69898	0.4391822	94.69074	2047.279
c	4	17.18122	0.4256198	95.76367	2269.809
	4.5	17.43615	0.4324792	95.10516	2158.583
	5	17.69124	0.4395922	94.42941	2049.903
	5.5	17.94647	0.4469741	93.73557	1943.772
	6	18.20186	0.4546421	93.02275	1840.193
s	3.6	17.69273	0.439444	94.39203	2049.242
	3.8	17.69199	0.439518	94.41071	2049.572
	4	17.69124	0.439592	94.42941	2049.903
	4.2	17.69049	0.439665	94.4481	2050.234
	4.4	17.68974	0.439739	94.4668	2050.565
h, h_1, h_2	1.65, 0.05, 0.15	17.68038	0.440920	94.74612	2054.729
	1.70, 0.10, 0.20	17.68582	0.440255	94.58753	2052.314
	1.75, 0.15, 0.25	17.69124	0.439592	94.42941	2049.903
	1.80, 0.20, 0.30	17.69664	0.438930	94.27176	2047.496
	1.85, 0.25, 0.35	17.70203	0.438271	94.11458	2045.093
c_d	1	17.68655	0.440052	94.54633	2051.972
	1.5	17.68889	0.439822	94.48785	2050.937
	2	17.69124	0.439592	94.42941	2049.903
	2.5	17.69358	0.439362	94.37101	2048.87
	3	17.69592	0.439132	94.31266	2047.837
η	0.6	17.69498	0.439224	94.33599	2048.25
	0.7	17.69311	0.439408	94.38269	2049.076
	0.8	17.69124	0.439592	94.42941	2049.903
	0.9	17.68936	0.439776	94.47616	2050.731
	1	17.68748	0.439960	94.52293	2051.558
a, b fixed	360	11.79361	0.770094	69.96747	231.0563
	480	14.73238	0.554978	85.21757	956.0452
	600	17.69124	0.439592	94.42941	2049.903
	720	20.66264	0.365883	100.8173	3508.115
	840	23.64165	0.314190	105.5754	5328.862
b, a fixed	5	62.60045	0.217419	58.93376	14329.26
	10	32.63771	0.304692	76.55008	6005.921
	20	17.69124	0.439592	94.42941	2049.903
	25	14.71425	0.502487	99.05132	1313.228
	30	12.73633	0.566856	101.7333	847.1779

Table 3: Sensitivity analysis of the optimal solution for the price-dependent isoelastic demand function.

Parameter		p	T	Q	Profit
n	0.5	18.1207	0.2006021	87.68895	4386.067
	0.6	18.19569	0.2231854	99.39292	4638.565
	0.7	18.2679	0.2450845	110.6674	4839.074
	0.8	18.33853	0.2666374	121.6934	5004.362
	0.9	18.40849	0.2881024	132.6066	5144.51
	1	18.4785	0.3096932	143.5169	5266.004
	1.1	18.5492	0.3315992	154.5197	5373.237
	1.2	18.62116	0.3539997	165.7028	5469.315
	1.3	18.69494	0.3770726	177.1515	5556.509
	1.4	18.77111	0.4010032	188.9524	5636.533
	1.5	18.85028	0.4259907	201.197	5710.711
K	250	18.4785	0.3096932	143.5169	5266.004
	300	18.55645	0.3368009	153.6876	5111.347
	350	18.62591	0.3613822	162.5787	4968.135
	400	18.68867	0.3839697	170.47	4833.983
	450	18.74598	0.4049329	177.5547	4707.234
	500	18.79873	0.4245407	183.9726	4586.683
ω	0.25	18.40296	0.2805749	127.6708	5051.189
	0.35	18.43083	0.2912172	133.4547	5133.826
	0.45	18.46176	0.303162	139.956	5220.756
	0.5	18.4785	0.3096932	143.5169	5266.004
	0.55	18.49619	0.3166393	147.3102	5312.546
θ	0.03	18.43995	0.3101287	143.6865	5270.35
	0.04	18.45922	0.309911	143.6018	5268.177
	0.05	18.4785	0.3096932	143.5169	5266.004
	0.06	18.49779	0.3094752	143.4319	5263.831
	0.07	18.51709	0.3092571	143.3467	5261.658
c	4	14.90778	0.2935993	184.7868	5805.186
	4.5	16.69423	0.3021076	161.8104	5515.016
	5	18.4785	0.3096932	143.5169	5266.004
	5.5	20.26106	0.3165394	128.6391	5049.084
	6	22.04224	0.3227805	116.3246	4857.783
s	3.6	18.4861	0.3095581	143.3785	5264.953
	3.8	18.4823	0.3096256	143.4477	5265.478
	4	18.4785	0.3096932	143.5169	5266.004
	4.2	18.4747	0.3097608	143.5863	5266.529
	4.4	18.4709	0.3098285	143.6557	5267.055
h, h_1, h_2	1.65, 0.05, 0.15	18.42584	0.3107838	144.5444	5273.343
	1.70, 0.10, 0.20	18.45222	0.3102363	144.0284	5269.667
	1.75, 0.15, 0.25	18.4785	0.3096932	143.5169	5266.004
	1.80, 0.20, 0.30	18.5047	0.3091542	143.0099	5262.354
	1.85, 0.25, 0.35	18.53082	0.3086195	142.5073	5258.717
c_d	1	18.45472	0.3101169	143.9515	5269.293
	1.5	18.46662	0.3099047	143.7339	5267.647
	2	18.4785	0.3096932	143.5169	5266.004
	2.5	18.49037	0.3094822	143.3007	5264.363
	3	18.50223	0.3092719	143.0853	5262.724
η	0.6	18.49749	0.309356	143.1714	5263.379
	0.7	18.488	0.3095244	143.3439	5264.691
	0.8	18.4785	0.3096932	143.5169	5266.004
	0.9	18.469	0.3098624	143.6904	5267.318
	1	18.45948	0.3100319	143.8644	5268.634
a, b fixed	360	19.01343	0.5079969	69.67333	1350.356
	480	18.65804	0.3728992	111.0905	3266.695
	600	18.4785	0.3096932	143.5169	5266.004
	720	18.36477	0.2709628	171.0939	7308.271
	840	18.2842	0.244065	195.5106	9377.999
b, a fixed	5	31.32621	0.2405638	108.4781	10666.25
	10	22.74959	0.2742978	131.4156	7419.431
	20	18.4785	0.3096932	143.5169	5266.004
	25	15.93154	0.3474883	148.2634	3776.648
	30	14.24802	0.3882774	147.9029	2718.869

Table 4: Sensitivity analysis of the optimal solution for the price-dependent exponential demand function.

Parameter		p	T	Q	Profit
n	0.5	10.31047	0.4281761	68.1642	205.8662
	0.6	10.35651	0.4706555	80.00251	311.0982
	0.7	10.39787	0.510368	91.08083	393.5449
	0.8	10.43602	0.5479941	101.5864	460.5593
	0.9	10.47181	0.5840098	111.6502	516.5487
	1	10.50582	0.6187656	121.3688	564.3379
	1.1	10.53846	0.6525299	130.8162	605.8324
	1.2	10.57	0.6855155	140.0512	642.3708
	1.3	10.6007	0.7178965	149.1221	674.9252
	1.4	10.63073	0.7498184	158.069	704.2209
	1.5	10.66023	0.7814064	166.9269	730.8121
K	250	10.50582	0.6187656	121.3688	564.3379
	300	10.53844	0.6740886	127.3607	487
	350	10.56532	0.7243232	132.0419	415.4977
	400	10.58741	0.7705352	135.6953	348.6073
	450	10.60539	0.8134684	138.5174	285.4805
	500	10.61971	0.8536671	140.6506	225.5008
ω	0.25	10.50397	0.5998278	111.1458	498.051
	0.35	10.50459	0.6069651	115.0211	523.7045
	0.45	10.50537	0.6146843	119.1772	550.4925
	0.5	10.50582	0.6187656	121.3688	564.3379
	0.55	10.5063	0.622996	123.6407	578.4992
θ	0.03	10.48514	0.6194522	121.2816	567.9372
	0.04	10.49547	0.6191109	121.3257	566.1389
	0.05	10.50582	0.6187656	121.3688	564.3379
	0.06	10.5162	0.6184164	121.4107	562.5343
	0.07	10.5266	0.6180633	121.4516	560.7281
c	4	9.458612	0.5636089	140.2348	785.8765
	4.5	9.981914	0.5905406	130.5493	668.5296
	5	10.50582	0.6187656	121.3688	564.3379
	5.5	11.03024	0.6483456	112.6667	471.9582
	6	11.55502	0.6793453	104.418	390.1792
s	3.6	10.5098	0.6185865	121.2488	563.547
	3.8	10.50781	0.618676	121.3088	563.9424
	4	10.50582	0.6187656	121.3688	564.3379
	4.2	10.50383	0.6188552	121.4288	564.7337
	4.4	10.50185	0.6189449	121.4888	565.1296
h, h_1, h_2	1.65, 0.05, 0.15	10.47553	0.6210162	122.4024	570.4779
	1.70, 0.10, 0.20	10.4907	0.6198884	121.8838	567.4016
	1.75, 0.15, 0.25	10.50582	0.6187656	121.3688	564.3379
	1.80, 0.20, 0.30	10.52089	0.6176479	120.8572	561.2867
	1.85, 0.25, 0.35	10.53591	0.6165353	120.3493	558.2478
c_d	1	10.49339	0.6193265	121.7445	566.8144
	1.5	10.49961	0.6190458	121.5564	565.5753
	2	10.50582	0.6187656	121.3688	564.3379
	2.5	10.51203	0.6184858	121.1815	563.1023
	3	10.51824	0.6182064	120.9946	561.8685
η	0.6	10.51576	0.6183181	121.0693	562.3618
	0.7	10.51079	0.6185417	121.2189	563.3493
	0.8	10.50582	0.6187656	121.3688	564.3379
	0.9	10.50085	0.6189897	121.5189	565.3277
	1	10.49588	0.6192141	121.6693	566.3185
a, b fixed	360	10.61971	0.8536671	70.32532	112.7504
	480	10.55693	0.7080697	97.95558	329.0764
	600	10.50582	0.6187656	121.3688	564.3379
	720	10.46593	0.5567266	142.0386	811.742
	840	10.43408	0.5103661	160.7423	1067.793
b, a fixed	5	15.32865	0.3701014	143.3381	3165.293
	10	12.0851	0.4884015	136.2242	1328.054
	20	10.50582	0.6187656	121.3688	564.3379
	25	9.588089	0.7720412	102.6994	200.5329
	30	8.976881	0.9633883	81.83712	19.17279

TABLE 5: Sensitivity analysis of the optimal solution for the price-dependent logit demand function.

Parameter		p	T	Q	Profit
	0.5	8.83536	0.331024	142.0055	765.3444
	0.6	8.865511	0.3625493	162.432	896.2742
	0.7	8.892812	0.3915668	181.2635	998.0164
	0.8	8.917971	0.4186151	198.8412	1080.029
	0.9	8.941448	0.4440691	215.4026	1147.972
n	1	8.96356	0.4682024	231.1214	1205.467
	1.1	8.98454	0.4912223	246.1292	1254.953
	1.2	9.004563	0.5132901	260.5286	1298.141
	1.3	9.023767	0.5345347	274.4013	1336.271
	1.4	9.04226	0.5550609	287.8142	1370.265
	1.5	9.060133	0.5749555	300.8225	1400.829
	250	8.96356	0.4682024	231.1214	1205.467
	300	8.993896	0.5132866	246.2305	1103.581
K	350	9.020562	0.554725	258.9851	1009.949
	400	9.044224	0.5932662	269.8674	922.8403
	450	9.065344	0.6294328	279.2142	841.0543
	500	9.084255	0.663607	287.2716	763.7174
	0.25	8.952328	0.4436449	210.7649	1111.362
	0.35	8.956561	0.4528506	218.3899	1147.806
ω	0.45	8.961138	0.4628661	226.6907	1185.823
	0.5	8.96356	0.4682024	231.1214	1205.467
	0.55	8.966073	0.4737715	235.7549	1225.56
	0.03	8.94914	0.4699697	231.7329	1213.075
	0.04	8.956353	0.4690874	231.4284	1209.27
θ	0.05	8.96356	0.4682024	231.1214	1205.467
	0.06	8.97076	0.4673149	230.8119	1201.665
	0.07	8.977954	0.4664249	230.5	1197.863
	4	7.98732	0.4095817	271.7555	1780.799
	4.5	8.471951	0.4376222	251.07	1471.913
c	5	8.96356	0.4682024	231.1214	1205.467
	5.5	9.46164	0.501547	211.9723	976.7625
	6	9.965678	0.5379028	193.6657	781.43
	3.6	8.966369	0.4678248	230.7904	1203.866
	3.8	8.964965	0.4680135	230.9558	1204.666
s	4	8.96356	0.4682024	231.1214	1205.467
	4.2	8.962154	0.4683915	231.2872	1206.269
	4.4	8.960747	0.4685807	231.4532	1207.071
	1.65, 0.05, 0.15	8.943314	0.4718563	233.8835	1217.321
	1.70, 0.10, 0.20	8.953474	0.4700191	232.494	1211.376
h, h_1, h_2	1.75, 0.15, 0.25	8.96356	0.4682024	231.1214	1205.467
	1.80, 0.20, 0.30	8.973573	0.466406	229.7656	1199.592
	1.85, 0.25, 0.35	8.983515	0.4646297	228.4263	1193.751
	1	8.954758	0.469387	232.161	1210.485
	1.5	8.959163	0.4687938	231.6402	1207.974
c_d	2	8.96356	0.4682024	231.1214	1205.467
	2.5	8.967948	0.4676128	230.6046	1202.966
	3	8.972328	0.4670249	230.0898	1200.469
	0.6	8.970577	0.4672598	230.2955	1201.467
	0.7	8.967071	0.4677306	230.7078	1203.465
η	0.8	8.96356	0.4682024	231.1214	1205.467
	0.9	8.960043	0.4686754	231.5363	1207.472
	1	8.956521	0.4691495	231.9525	1209.48
	360	8.982996	0.4968576	214.1093	1013.959
	480	8.972915	0.4818983	222.7441	1109.26
a, b fixed	600	8.96356	0.4682024	231.1214	1205.467
	720	8.95485	0.455602	239.263	1302.506
	840	8.946715	0.4439588	247.1874	1400.313
	5	10.85935	0.3217571	270.4118	3869.684
	10	9.698103	0.3895725	253.8281	2170.566
b, a fixed	20	8.96356	0.4682024	231.1214	1205.467
	25	8.474152	0.5625861	204.7163	636.0077
	30	8.138176	0.680003	176.082	295.6301

Table 6: Sensitivity analysis of the optimal solution for the price-dependent logarithmic demand function.

Parameter		p	T	Q	Profit
n	0.5	38.88763	0.5	4.977036	−168.6088
	0.6	38.94428	0.6	6.078415	−80.01093
	0.7	39.0009	0.7	7.218579	−15.07501
	0.8	39.05594	0.7671033	8.380817	35.86579
	0.9	39.10635	0.8204821	9.523202	79.0227
	1	39.15353	0.872946	10.65368	116.4864
	1.1	39.19849	0.924891	11.78091	149.584
	1.2	39.24193	0.976641	12.91222	179.2519
	1.3	39.28435	1.02847	14.05402	206.1735
	1.4	39.32611	1.080619	15.21214	230.8619
	1.5	39.36751	1.133302	16.39207	253.7117
K	250	39.15353	0.872946	10.65368	116.4864
	300	39.16597	0.9351851	10.82588	61.19978
	350	39.17068	0.9904247	10.88684	9.281298
	400	39.17079	1	10.88822	−40.76307
	450	39.17079	1	10.88822	−90.76307
	500	39.17079	1	10.88822	−140.7631
ω	0.25	39.17	0.8868251	9.805374	82.89589
	0.35	39.16328	0.8805343	10.13195	95.80591
	0.45	39.15673	0.8752448	10.47529	109.4122
	0.5	39.15353	0.872946	10.65368	116.4864
	0.55	39.15036	0.870864	10.83679	123.7472
θ	0.03	39.13038	0.8727587	10.58556	116.4772
	0.04	39.14193	0.8728522	10.61954	116.4818
	0.05	39.15353	0.872946	10.65368	116.4864
	0.06	39.16517	0.8730402	10.68797	116.4909
	0.07	39.17686	0.8731347	10.72242	116.4954
c	4	38.25383	0.8630457	10.90399	128.9043
	4.5	38.70599	0.8679787	10.77778	122.6416
	5	39.15353	0.872946	10.65368	116.4864
	5.5	39.59664	0.8779487	10.53156	110.4366
	6	40.03549	0.8829876	10.41132	104.4901
s	3.6	39.15791	0.8729689	10.65237	116.4262
	3.8	39.15572	0.8729575	10.65302	116.4563
	4	39.15353	0.872946	10.65368	116.4864
	4.2	39.15133	0.8729346	10.65433	116.5165
	4.4	39.14914	0.8729232	10.65499	116.5467
h, h_1, h_2	1.65, 0.05, 0.15	39.11731	0.8728486	10.66483	116.984
	1.70, 0.10, 0.20	39.13542	0.8728972	10.65925	116.7351
	1.75, 0.15, 0.25	39.15353	0.872946	10.65368	116.4864
	1.80, 0.20, 0.30	39.17163	0.8729951	10.64811	116.2379
	1.85, 0.25, 0.35	39.18972	0.8730443	10.64254	115.9895
c_d	1	39.13981	0.8728748	10.65777	116.6747
	1.5	39.14667	0.8729104	10.65572	116.5806
	2	39.15353	0.872946	10.65368	116.4864
	2.5	39.16038	0.8729817	10.65163	116.3923
	3	39.16724	0.8730174	10.64958	116.2982
η	0.6	39.1645	0.8730031	10.6504	116.3359
	0.7	39.15901	0.8729746	10.65204	116.4111
	0.8	39.15353	0.872946	10.65368	116.4864
	0.9	39.14804	0.8729175	10.65532	116.5617
	1	39.14255	0.872889	10.65695	116.637
a, b fixed	360	33.22058	0.950225	10.52434	37.52752
	480	36.04936	0.9105406	10.61415	74.42949
	600	39.15353	0.872946	10.65368	116.4864
	720	42.56169	0.8372483	10.6493	164.2291
	840	46.30511	0.8032825	10.60662	218.2455
b, a fixed	5	212.4593	0.4632178	5.850405	2049.142
	10	103.6734	0.602643	7.845838	847.5321
	20	59.91941	0.7388322	9.513484	351.4908
	25	39.15353	0.872946	10.65368	116.4864
	30	32.79227	0.9401426	10.97968	46.17976

Table 7: Sensitivity analysis of the optimal solution for the price-dependent polynomial demand function.

Parameter		p	T	Q	Profit
	0.5	9.452645	0.1573383	319.1532	7122.271
	0.6	9.457781	0.1728178	357.2209	7406.72
	0.7	9.462536	0.1871819	392.721	7628.544
	0.8	9.466996	0.2006759	426.2251	7807.964
	0.9	9.471218	0.21347	458.1302	7957.096
n	1	9.475246	0.225688	488.7249	8083.7
	1.1	9.489529	0.2291596	500	8191.552
	1.2	9.506626	0.2277661	500	8280.137
	1.3	9.520488	0.226608	500	8353.867
	1.4	9.531955	0.2256302	500	8416.206
	1.5	9.541598	0.224794	500	8469.614
	250	9.475246	0.225688	488.7249	8083.7
	300	9.503019	0.2329333	500	7867.451
K	350	9.538332	0.2350648	500	7653.768
	400	9.573849	0.2372654	500	7442.045
	450	9.609569	0.2395383	500	7232.308
	500	9.645493	0.2418868	500	7024.585
	0.25	9.469876	0.2061207	438.4096	7840.159
	0.35	9.471851	0.2132807	456.7826	7934.293
ω	0.45	9.47405	0.2213019	477.4169	8032.715
	0.5	9.475246	0.225688	488.7249	8083.7
	0.55	9.477004	0.2299997	500	8135.968
	0.03	9.472783	0.2271689	490.8997	8101.075
	0.04	9.474017	0.2264268	489.8116	8092.38
θ	0.05	9.475246	0.225688	488.7249	8083.7
	0.06	9.476469	0.2249528	487.6396	8075.034
	0.07	9.477685	0.224221	486.5559	8066.381
	4	9.134191	0.2089559	489.8587	10340.61
	4.5	9.301964	0.2168661	489.6647	9189.811
c	5	9.475246	0.225688	488.7249	8083.7
	5.5	9.65422	0.2356095	486.9041	7025.404
	6	9.839088	0.2468774	484.0393	6018.295
	3.6	9.475739	0.2253742	488.0361	8080.03
	3.8	9.475492	0.225531	488.3802	8081.864
s	4	9.475246	0.225688	488.7249	8083.7
	4.2	9.474999	0.2258454	489.0702	8085.537
	4.4	9.474752	0.2260029	489.4161	8087.375
	1.65, 0.05, 0.15	9.471941	0.2280448	493.8519	8108.643
	1.70, 0.10, 0.20	9.473601	0.2268579	491.2705	8096.139
h, h_1, h_2	1.75, 0.15, 0.25	9.475246	0.225688	488.7249	8083.7
	1.80, 0.20, 0.30	9.476875	0.2245349	486.2145	8071.326
	1.85, 0.25, 0.35	9.478489	0.2233981	483.7386	8059.016
	1	9.473698	0.2266757	490.8924	8095.202
	1.5	9.474473	0.2261806	489.8058	8089.445
c_d	2	9.475246	0.225688	488.7249	8083.7
	2.5	9.476015	0.2251981	487.6496	8077.967
	3	9.476782	0.2247108	486.5798	8072.246
	0.6	9.476476	0.2249054	487.007	8074.533
	0.7	9.475862	0.2252959	487.8642	8079.113
η	0.8	9.475246	0.225688	488.7249	8083.7
	0.9	9.474628	0.2260818	489.5892	8088.295
	1	9.474009	0.2264773	490.4571	8092.897
	360	8.792554	0.2813326	427.128	4446.023
	480	9.148722	0.2496727	459.558	6171.702
a, b fixed	600	9.475246	0.225688	488.7249	8083.7
	720	9.786595	0.2006345	500	10164.28
	840	10.08129	0.1772878	500	12395.47
	5	11.47639	0.1919163	454.5103	13544.77
	10	10.24619	0.2103775	475.1908	10165.04
b, a fixed	20	9.475246	0.225688	488.7249	8083.7
	25	8.930435	0.2392571	498.156	6637.54
	30	8.520637	0.2493667	500	5559.827

Table 8: Behavior of the variables and the total profit when the parameters are increased.

Price-dependent demand function	Parameter	p	T	Q	Profit
Linear	n	↑	↑	↑	↑
	K	↑	↑	↑	↓
	ω	↑	↑	↑	↑
	θ	↑	↓	↑	↓
	c	↑	↑	↓	↓
	s	↓	↑	↑	↑
	h, h_1, h_2	↑	↓	↓	↓
	c_d	↑	↓	↓	↓
	η	↓	↑	↑	↑
	a, b fixed	↑	↓	↑	↑
	b, a fixed	↓	↑	↑	↓
Isoelastic	n	↑	↑	↑	↑
	K	↑	↑	↑	↓
	ω	↑	↑	↑	↑
	θ	↑	↓	↓	↓
	c	↑	↑	↓	↓
	s	↓	↑	↑	↑
	h, h_1, h_2	↑	↓	↓	↓
	c_d	↑	↓	↓	↓
	η	↓	↑	↑	↑
	a, b fixed	↓	↓	↑	↑
	b, a fixed	↓	↑	↑	↓
Exponential	n	↑	↑	↑	↑
	K	↑	↑	↑	↓
	ω	↑	↑	↑	↑
	θ	↑	↓	↑	↓
	c	↑	↑	↓	↓
	s	↓	↑	↑	↑
	h, h_1, h_2	↑	↓	↓	↓
	c_d	↑	↓	↓	↓
	η	↓	↑	↑	↑
	a, b fixed	↓	↓	↑	↑
	b, a fixed	↓	↑	↓	↓
Logit	n	↑	↑	↑	↑
	K	↑	↑	↑	↓
	ω	↑	↑	↑	↑
	θ	↑	↓	↓	↓
	c	↑	↑	↓	↓
	s	↓	↑	↑	↑
	h, h_1, h_2	↑	↓	↓	↓
	c_d	↑	↓	↓	↓
	η	↓	↑	↑	↑
	a, b fixed	↓	↓	↑	↑
	b, a fixed	↓	↑	↓	↓

Table 8: Continued.

Price-dependent demand function	Parameter	p	T	Q	Profit
Logarithmic	n	↑	↑	↑	↑
	K	↑	↑	↑	↓
	ω	↓	↓	↑	↑
	θ	↑	↑	↑	↑
	c	↑	↑	↓	↓
	s	↓	↓	↑	↑
	h, h_1, h_2	↑	↑	↓	↓
	c_d	↑	↑	↓	↓
	η	↓	↓	↑	↑
	a, b fixed	↑	↓	↑	↑
	b, a fixed	↓	↑	↑	↓
Polynomial	n	↑	↑	↑	↑
	K	↑	↑	↑	↓
	ω	↑	↑	↑	↑
	θ	↑	↓	↓	↓
	c	↑	↑	↓	↓
	s	↓	↑	↑	↑
	h, h_1, h_2	↑	↓	↓	↓
	c_d	↑	↓	↓	↓
	η	↓	↑	↑	↑
	a, b fixed	↑	↓	↑	↑
	b, a fixed	↓	↑	↑	↓

longer optimal cycle time, and a larger order quantity. This means that a fresh product with extended life span (it takes longer to lose its attractiveness) allows the marketer to offer it at a higher price. As the ordering cost K increases, the price and inventory cycle time increase as well, but the overall profit tends to be lower. Then, managers should implement actions to lower the ordering costs, so higher profits can be generated.

(ii) Higher values for the parameter of demand sensitivity to the stock level (ω), result in longer inventory cycle time, higher prices, and higher profits. This behavior is observed in five price demand functions except in the logarithmic function, where the optimal price and inventory cycle time tend to decrease as this parameter increases. For all price demand functions, it is found that higher values of the parameter of demand sensitivity to the stock level (ω) result in a larger order quantity. In this case, since ω parameter does not directly depend on the decision maker, the suggestion would be to take high values of this sensitivity parameter to generate high values of profits. This means that managers are more interested in exhibiting larger quantities of stock on shelf space.

(iii) Increasing values for the inventory deterioration rate (θ) result in significant variations in price demand functions. On the one hand, in linear and exponential price demand functions, slightly lower inventory cycle time, slightly higher prices, and lower profits are observed. On the other hand, isoelastic, logit, and polynomial price demand functions present lower inventory cycles and profits, with slightly higher prices. Therefore, if the fresh item has one of these previous five price-dependent demands, the suggestion for the buyer would be to buy products whose rate of deterioration is small so that a higher profit is generated. When using the logarithmic price demand function, the inventory deterioration rate and the optimal price are higher, and the inventory cycle time and profit are both optimal.

(iv) As the purchase cost (c) grows, it results in slightly higher price and inventory cycle time. Conversely, the profit is lower as this cost increases. This behavior is presented in all price demand functions. Therefore, the indication would be to develop low-cost purchasing policies, so profits are higher regardless of the function of demand depending on price that is used. One example would be to look for alternative suppliers that offer products at a lower cost without neglecting their quality.

(v) The increment in the salvage value (s) always results in slightly lower optimal price and slightly higher inventory cycle time and profits, particularly in the isoelastic, exponential, logit, and polynomial price demand functions. In linear price demand function, slightly higher price and inventory cycle time are found, and the profit turns out to be higher as this parameter increases. In logarithmic price demand function, a larger value results in a lower optimal price and inventory cycles, but higher profits. If linear price demand function is followed, managers should implement actions to assign higher salvage values in order to increase the profits that are generated.

(vi) Higher coefficients for the holding cost function (h, h_1, h_2) and deterioration cost (c_d) get a higher result in slightly higher selling price, but a lower profit. This behavior is observed for all price demand functions. On the contrary, the inventory cycle time increases for the logarithmic function and decreases for the rest price demand functions. It is generally advised to decision makers to implement policies to have low deterioration costs in the products in order to increase the overall profits. Likewise, lower values for the components of the holding cost function must be considered.

(vii) As the salvage coefficient (η) increases, a slightly higher inventory cycle time is observed, but slightly lower selling prices and profits are obtained in linear, isoelastic, exponential, logit, and polynomial price demand functions. In the logarithmic function, a slightly lower values in the inventory cycle time can be observed. Here, the person in charge should consider high salvage coefficients to guarantee higher profits for all cases.

(viii) By increasing the values of the scale parameter of the demand (a) and having fixed the parameter of the demand sensitivity to the price (b), higher prices and lower inventory cycle times are obtained in the linear, logarithmic, and polynomial functions. On the contrary, lower prices and lower inventory cycles are observed as scale parameter increases in the isoelastic, exponential, and logit price demand functions. In all price demand functions, increasing the value of a while keeping the value of b fixed results in larger benefits. The suggestion is, then, that this parameter is maintained with higher values.

(ix) Higher values of the demand sensitivity to price (b) while keeping the scale parameter of the demand (a) fixed together result in lower selling prices and profits, but higher inventory cycle time in all price demand functions. Hence, low values of this parameter b should be used, so the overall profits are always higher regardless of the price demand used.

7. Conclusions

This research work develops an inventory model with price-, stock-, and time-dependent demand. The physical deterioration and condition of freshness degradation over time are both considered, and zero-ending inventory is assumed. Six different types of price-dependent demand functions are studied: linear, isoelastic, exponential, logit, logarithmic, and polynomial. When working with perishable products, a salvaged value and a deterioration cost are considered in the entire cycle. A nonlinear time-dependent holding cost is included, specifically with a quadratic-type function.

Through an algorithm, the inventory model determines the optimal values for price, the inventory cycle time, and the order quantity. Some numerical examples are provided, and a sensitivity analysis is presented for all the input parameters. By observing the behavior of the decision variables and total profits, it was found that an increase in the ordering cost (K), purchasing cost (c), and shelf-life (n) results in a similar pattern in the selling price, the inventory cycle time, the quantity to order, and the total profit. Furthermore, an increase in the value of the shelf-life (n) results in an increment in price, inventory cycle time, quantity ordered, and profits generated for all functions. In addition, as the ordering cost increases (K), price, the inventory cycle time, and quantity ordered also increase for all functions. Nonetheless, the profits show a decreasing trend. Finally, by escalating the purchasing cost for all functions, there is an increase in both the price and the inventory cycle time; however, the quantity to order and total profits tend to decrease.

This research work extends and widely contributes to the state-of-the-art on the inventory field, with focus on perishable items with price-stock-time-dependent demand. The inventory model studied here has some limitations from where several directions for extension and further research are highlighted. First, an inventory model can be built with the same characteristics and demand pattern, but including the sustainability elements, so the effects of the carbon-tax and cap-and-trade mechanisms can be assessed. Second, a model that allows shortages with full or partial backlogging should be explored. Third, the trade-off and benefits of investing on preservation technology should be also studied. Fourth, the noninstantaneous item's freshness degradation can be integrated into the proposed inventory model. Finally, other components such as incorporating discount policies or advertising efforts can also be investigated.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Research Article

An Inventory Model for Growing Items with Imperfect Quality When the Demand Is Price Sensitive under Carbon Emissions and Shortages

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Nowadays, it is well known that global warming is a great hazard to the planet, and the carbon emissions are a principal source of global warming. For this reason, the customers have become more environment and quality conscious than before, and as a result, they request the firms to be ecofriendly. In this context, it is desirable that companies develop and implement inventory models which consider sustainability issues. Furthermore, the companies face problems of shortages and setting prices in order to persist in a competitive and challenging business. Besides, there exists a kind of items different than the traditional products that it is necessary to feed them until a target weight is reached in order to slaughter and sell to customers. These are named as growing items. In this sense, this research work proposes an inventory model for growing items with imperfect quality when the demand is price sensitive under carbon emissions and shortages. The shortages are fully backordered. The demand is price sensitive according to a polynomial function. The proposed inventory model determines jointly the optimal policy for the selling price of perfect-quality growing items, the order quantity, and the backordering quantity which maximize the expected total profit per unit of time. Some numerical examples are resolved in order to illustrate the use and the applicability of the inventory model. Finally, a sensitivity analysis is conducted and some managerial insights are given.

1. Introduction

Inventory management is a valued function for all firms. The main purpose of the inventory management department is to control the materials from the acquisition to sale, taking decisions of how much and when to buy the items avoiding excess or unplanned stockout.

Since the introduction of the classical economic order quantity (EOQ) inventory model proposed by Harris [1], industries have shown great interest in the development of inventory control models with new features. For example, Ghosh et al. [2] built an inventory model for a deteriorating production system when there are random machine breakdowns considering that there exists a selling price discount. Manna et al. [3] studied the impacts of inspection errors on an imperfect production-inventory model when the demand rate depends on both selling-price discount and warranty time. Khan et al. [4] and Shaikh et al. [5] introduced a two-warehouse inventory model for instantaneously deteriorating goods considering an advance payment agreement. In the same research direction, Khan et al. [6] investigated the effects of noninstantaneous deterioration on the ordering decisions for a two-warehouse inventory system taking into consideration an advance payment scheme. Shaikh et al. [7] developed an EOQ inventory model for deteriorating goods with stock-dependent demand when there exists a price discount facility.

It is well known that the EOQ determines the optimal order quantity to minimize total inventory costs. Although this inventory model has the groundwork for inventory systems, it has unrealistic assumptions. For that reason, many researchers have incorporated different characteristics in order to model real world applications such as imperfect quality, carbon emissions reduction, growing items, shortages, and pricing, among others.

Harris's [1] inventory model assumes that all items are of perfect quality. However, this assumption not always occurs because the majority of the manufacturing processes generate a portion of imperfect quality items. So, Salameh and Jaber [8] formulated an EOQ inventory model considering that the ordered lot contains imperfect-quality items and these items are sold as a single batch at a lower price. Cárdenas-Barrón [9] identified and corrected an error in the inventory model presented by Salameh and Jaber [8]. Wee et al. [10] extended Salameh and Jaber's [8] inventory model by incorporating shortages with full backordering. Afterwards, Maddah and Jaber [11] detected and rectified one flaw in Salameh and Jaber's [8] inventory model. The flaw is in the manner of determining the expected total profit per unit of time ($E[\mathrm{TPU}]$). Salameh and Jaber [8] utilized the equation $E[\mathrm{TPU}] = E[\mathrm{TP}/T]$ which is not exact. The research work of Wee et al. [10] contains the same flaw. It is important to remark that the process of producing the profit is a renewal process with renewal points at order placement periods. Therefore, in order to calculate the expected total profit per unit of time, it is needed to apply the well-known elementary renewal reward theorem given by Ross [12]. For this reason, the expected total profit per unit of time must be computed with the following equation: $E[\mathrm{TPU}] = E[\mathrm{TP}]/E[T]$. Chang and Ho [13] revisited and rescued Wee et al.'s [10] inventory model by applying the well-known elementary renewal reward theorem to get the expected total profit per unit time. Cárdenas-Barrón [14] derived closed-form expressions to determining the optimal solution to an EOQ inventory model considering items with imperfect quality. Ghiami and Beullens [15] developed a production-inventory model for a deteriorating product with partial backordering using a cash-flow net present value analysis. They generated managerial insights related if it is convenient financially to have planned shortages with partial backordering and lost sales. Zhou et al. [16] developed an economic-order quantity model considering shortages, imperfect quality, and inspection errors. Taleizadeh et al. [17] studied an inventory model when the lot contains imperfect-quality items, and these cannot be replaced with perfect-quality ones immediately, but these are repaired and then they are sold at lower price. In this line of research, Rezaei [18] introduced an inventory model considering imperfect-quality items. Sampling inspection plans are designed so the buyer would be able to decide regarding the next step.

At the present time, as we know, global warming is a big threat to the world and carbon emissions are a leading cause of global warming. For this reason, the sustainability is an extremely significant concern for all people that desire to maintain the planet healthy. It is well known that industries generate large amounts of carbon emissions. Furthermore, the transportation of goods through the supply chains produces carbon emissions too. Consequently, these damage the environment and cause global warming at the same time. In this context, the governments of the countries have imposed strict policies that limit the amounts of carbon emissions with the aim of having a sustainable environment free of pollution. The regulations related to avoid pollution impose additional costs to firms; thus, it is convenient for these to adapt and obey the regulations. In this direction, several companies have adopted inventory models that consider reducing carbon emissions. For example, Hua et al. [19] investigated how companies manage carbon emissions in an inventory management. Arslan and Turkay [20] integrated sustainability features into the design of an inventory model and proposed different policies that include environmental and social criteria. Battini et al. [21] introduced a sustainable EOQ inventory model integrating factors that have a great impact on the environment. Lin and Sarker [22] presented an inventory model that considers carbon tax policy and imperfect-quality items avoiding shortages and incorporated and evaluated some carbon tax systems. Tiwari et al. [23] investigated an integrated vendor-buyer inventory model for deteriorating goods with imperfect quality, taking into account that all processes in the supply chain cause carbon emissions. The target of their inventory model is to decrease the ecological effects. Kazemi et al. [24] proposed an inventory model with imperfect quality from a sustainable point of view, and the aim is to know the impact of emission costs into the total profit. Modak et al. [25] addressed a manufacturer-retailer supply chain inventory model by considering that there is a cost of GHG emission of the production system and determined the optimal pricing policies. Afterwards, Sinha and Modak [26] presented an economic production quantity (EPQ) inventory model taking into account aspects of carbon emissions and carbon trading. Li and Hai [27] studied an inventory system for a warehouse with multiple retailers considering carbon emissions. Manna et al. [28] constructed a production-inventory model for controlling the GHG emissions when the pollution parameters are fuzzy. Later, Manna et al. [29] studied the effects of carbon emissions on an imperfect production-inventory model. Huang et al. [30] proposed an inventory model for a two-echelon supply chain considering that all operations within the chain produce carbon emissions. Their inventory model helps companies to determine the optimal production quantity and delivery quantity which minimize the total costs under different carbon emission policies. Recently, Mishra et al. [31] revised the standard EOQ inventory model to integrate sustainability matters and developed three inventory models with and without shortage situations. Medina-Santana and Cárdenas-Barrón [32] formulated an inventory model considering a discontinuous transportation cost function and carbon emissions function. The most recent investigation about carbon emissions belongs to the authors Modak and Kelle [33]. They proposed to apply the social work donation as a scheme of corporate social responsibility into a closed-loop supply chain, taking into consideration the carbon emission tax and that demand is uncertain.

In recent years, a new topic, that is, inventory management for growing items, has also been included in the derivation of inventory models. A growing item is a kind of product which is capable of growth during time like farmyard animals, unlike inert products that do not increase in weight during storage. Growing items are incorporated for first time in an EOQ inventory model by Rezaei [34]. Here, the weight increment of items is the main difference between the inventory models proposed by Rezaei [34] and Harris [1]. Rezaei [34] developed an EOQ for growing items for different types of animals, specifically for a kind of poultry. Zhang et al. [35] revisited and extended the research work of Rezaei [34] by creating an inventory model for growing items reducing the carbon emissions in operations. Sebatjane [36] presented three inventory models for growing items. Afterwards, Khalilpourazari and Pasandideh [37] worked with growing items too. They formulated a mathematical model for multi-item economic order quantity subjected to some operational constrains. Nobil et al. [38] derived an EOQ inventory model for growing items when the shortages are allowed, and these are fully backordered. They argued that their inventory model helps poultry industries. Under an environment of a two-echelon supply chain composed by one supplier and one farmer, Malekitabar et al. [39] developed an inventory model for growing mortal items considering both the growth function and the mortality rate for the items. Recently, Sebatjane and Adetunji [40] presented an EOQ inventory model for growing items with imperfect quality. It is assumed that a certain fraction of the growing items has lower quality than desired. They mentioned that it is necessary to define the growth function of the items in order to calculate the feeding cost. In this context, Sebatjane and Adetunji [40] considered three growth functions: logistic, linear, and split linear. In the same year, Sebatjane and Adetunji [41] introduced an economic-order quantity inventory model for growing items with incremental quantity discounts. They proposed an optimal inventory policy to minimize the total inventory cost in both the owned and rented facilities. Sebatjane and Adetunji [42] continued working with growing items, but in this case, they formulated an inventory control model more realistically, considering that the items need to be transformed and packaged into a consumable form before customer demand is met. The next year, Sebatjane and Adetunji [43] built a model for managing inventory in a perishable food products supply chain that begins with farming operations where growing items live and finishes with the consumption of processed inventory. Sebatjane and Adetunji [44] derived an inventory model for a four-echelon supply chain with farming, processing, screening, and retail operations. Gharaei and Almehdawe [45] provided an economic growing quantity (EGQ) inventory model to determine the optimal economic growth and slaughter period and the economic growing quantity to minimize the total cost of the inventory system. Hidayat et al. [46] presented an EOQ inventory model with a capacitated warehouse facility and limited budget for growing products when the seller provides an incremental quantity discounts scheme. Mokhtari et al. [47] addressed a production-inventory model for growing goods which deteriorate through time. Nishandhi [48] studied an EOQ inventory model with budget-capacity constraint for growing items when a portion of the items are of imperfect quality. Pourmohammad-Zia and Karimi [49] determined the optimal replenishment and breeding policies for growing products. Afzal and Alfares [50] and Alfares and Afzal [51] developed EOQ inventory models for growing items considering shortages with full backordering. Recently, Sebatjane and Adetunji [52] created an inventory model for growing items under a three-echelon supply chain environment taking into account farming, processing, and retail echelons. Table 1 shows the inventory models related with growing items.

The organizations need to determine the optimal selling prices for the products in order to encourage that the customers buy more and more products, and therefore, these organizations can survive in the competitive business environment. In this context, the academicians and researchers are modelling the demand rate as dependent of price. For example, Khan et al. [53] analyzed two supply chain inventory systems when the demand rate depends on price. Khan et al. [54] proposed an inventory model for deteriorating merchandises when the demand is price sensitive, and there exists a discount policy according all-units arrangement. Khan et al. [55] developed inventory models for perishable products when the demand rate is dependent on both price and advertisement. On the one hand, Khan et al. [56] examined the effects of an advance payment with discount facility on ordering decisions for perishable items taking into account that the demand rate is both price and stock dependent. On the other hand, Panda et al. [57] dealt with a two-warehouse system for deteriorating items when the demand rate is dependent on both price and stock; however, the price is not optimized. Sinha et al. [58] formulated an entropic-order quantity inventory model when the demand of the product is dependent on selling price and there is an inspection process to split the imperfect-quality products from the perfect ones. Modak [59] and Modak and Kelle [60] introduced omni- and dual-channel supply chain models, respectively, by considering that there exists a price- and delivery-time-sensitive stochastic demand.

Nowadays, the firms face problems of shortages and setting the selling price in order to survive in a competitive business, which is becoming more challenging day by day. Moreover, the customers are more quality and environmental conscious than before, and therefore, they demand products of perfect quality and request firms to minimize the carbon emissions to the environment. However, the firms always face problems of process quality and contaminate the planet. Besides, there exists a type of goods different than the conventional products which requires feeding until a target weight is reached in order to slaughter and sell to customers. These are named as growing items (chickens, cows, pigs, goats, fish, shrimps, etc.). In this direction, this research work develops an inventory model for growing items with imperfect quality when the demand is price sensitive under carbon emissions and shortages. The shortages are fully backordered. The demand is price sensitive according to a polynomial function. The proposed inventory model determines jointly the optimal policy for the selling price of perfect-quality growing items, the order quantity, and

Table 1: Inventory models with growing items.

Authors	Price-dependent demand	Type of price-dependent demand	Allowed shortages	Type of backordering	Imperfect quality	Carbon tax	Type of objective function	Optimize
Rezaei [34]	No	—	No	—	No	No	Max. profit	Order quantity and slaughter time
Zhang et al. [35]	No	—	No	—	No	Yes	Min. cost	Order quantity and slaughter time
	No	—	No	—	Yes	No	Max. profit	Order quantity and cycle time
Sebatjane [36]	No	—	No	—	No	No	Min. cost	Order quantity and cycle time
	No	—	No	—	No	No	Min. cost	Order quantity and cycle time
Khalilpourazari and Pasandideh [37]	No	—	No	—	No	No	Max. profit	Order quantity and slaughter time
Nobil et al. [38]	No	—	Yes	Full	No	No	Min. cost	Order quantity, backordering quantity, and cycle time
Malekitabar et al. [39]	Yes	Linear	No	—	No	No	Max. profit	Selling price and cycle time
Sebatjane and Adetunji [40]	No	—	No	—	Yes	No	Max. profit	Order quantity and cycle time
Sebatjane and Adetunji [41]	No	—	No	—	No	No	Min. cost	Order quantity and cycle time
Sebatjane and Adetunji [42]	No	—	No	—	No	No	Min. cost	Order quantity, cycle time, and number of shipments
Sebatjane and Adetunji [43]	Yes	Exponential	No	—	No	No	Max. profit	Selling price, order quantity, cycle time, and number of shipments
Sebatjane and Adetunji [44]	No	—	No	—	Yes	No	Max. profit	Order quantity, cycle time, and number of shipments
Gharaei and Almehdawe [45]	No	—	No	—	No	No	Min. cost	Order quantity and cycle time
Hidayat et al. [46]	No	—	No	—	No	No	Min. cost	Order quantity and cycle time
Mokhtari et al. [47]	No	—	No	—	No	No	Max. profit	Order quantity and slaughter time
Nishandhi [48]	No	—	Yes	Full	Yes	No	Min. cost	Order quantity, backordering quantity, and cycle time
Pourmohammad-Zia and Karimi [49]	No	—	No	—	Yes	No	Min. cost	Order quantity and cycle time
Afzal and Alfares [50]	No	—	Yes	Full	Yes	No	Min. cost	Order quantity, backordering quantity, and cycle time
Alfares and Afzal [51]	No	—	Yes	Full	Yes	No	Min. cost	Order quantity, backordering quantity, and cycle time
Sebatjane and Adetunji [52]	No	—	No	—	No	No	Max. profit	Order quantity, cycle time, and number of shipments
This paper	Yes	Polynomial	Yes	Full	Yes	Yes	Max. profit	Selling price, order quantity, backordering quantity, and cycle time

backordering quantity which maximize the total profit per unit of time.

The remaining parts of this research work are described as follows: Section 2 introduces the notation defining symbols for the parameters, decision variables, decision-dependent variables, and functions. Section 3 states the assumptions under which the inventory model is built. Section 4 formulates the inventory model with shortages for growing items with imperfect quality when the demand is price sensitive considering carbon emissions. Section 5 determines some theoretical results, develops the solution procedure to determine the optimal solution to the inventory problem, and identifies some inventory models as special cases. Section 6 solves some numerical examples. Section 7 provides a sensitivity analysis and some managerial insights. Section 8 gives some conclusions and summaries several research points that can be addressed in the near future.

2. Notations

With the aim of having a standard notation for the inventory models with growing items, the nomenclature of Sebatjane and Adetunji [40] is used and extra symbols are defined too. Therefore, the following notations are adopted in order to develop the inventory model for growing items with imperfect quality, carbon emissions, and planned shortages.

2.1. Parameters. π: Scale parameter for the price-dependent demand

ρ: Sensitivity parameter for the price-dependent demand

n: Demand power index

v: Selling price of imperfect items (currency symbol/unit of weight)

K: Setup cost (currency symbol/cycle)

h: Holding cost (currency symbol/unit of weight/unit of time)

b: Shortage cost (currency symbol/unit of weight/unit of time)

c: Feeding cost (currency symbol/unit of weight)

p: Purchasing cost (currency symbol/unit of weight)

z: Inspection cost (currency symbol/unit of weight)

θ: Carbon tax rate (currency symbol/amount of carbon emissions)

Ec: Carbon emissions cost (currency symbol)

$\widehat{K}$: Amount of carbon emissions produced during the setup process (unit of weight/unit of time)

$\widehat{h}$: Amount of carbon emissions caused by holding items in the warehouse (unit of weight/unit of time)

$\widehat{c}$: Amount of carbon emissions generated during the feeding period (unit of weight/unit of time)

$\widehat{p}$: Amount of carbon emissions made during the purchasing activity (unit of weight/unit of time)

$\widehat{z}$: Amount of carbon emissions created during the inspection process (unit of weight/unit of time)

r: Inspection rate (unit of weight/unit of time)

α: Asymptotic weight of each item (unit of weight)

β: Integration constant (numeric value)

λ: Growth rate (numeric value/unit of time)

x: Percentage of slaughtered items that are of imperfect quality $(0 \le x \le 1)$

$E[x]$: Expected value of the percentage of imperfect items $(0 \le E[x] \le 1)$

$1 - E[x]$: Expected value of the percentage of perfect items $(0 \le 1 - E[x] \le 1)$

w_0: Weight of a newborn item (unit of weight)

w_1: Target weight of a grown item (unit of weight)

w_t: Weight of an item at time t (unit of weight)

t_1: Growing period (unit of time)

t_2: Inspection period for the backordering quantity (B) (unit of time)

t_3: Inspection period for $yw_1 - B$ units of weight (unit of time)

t_4: Consumption period of perfect items after inspection time (unit of time)

t_5: Shortages accumulation period (unit of time)

Decision variables:

y: Order quantity of newborn items (units)

B: Backordering quantity (unit of weight)

s: Selling price of perfect items (currency symbol/unit of weight)

Decision-dependent variables:

T: Cycle time (unit of time)

Q_0: Total weight at the beginning of the growing period, $Q_0 = yw_0$ (unit of weight)

Q_{t_1}: Total weight at the end of the growing period t_1, $Q_{t_1} = yw_1$ (unit of weight)

Functions:

$D(s)$: Price-dependent demand function (unit of weight/unit of time)

$w_t(t)$: Growth function

$g(x)$: Probability density function of the percentage of imperfect items

$\mathrm{TPU}(y, B, s)$: Total profit (currency symbol/unit of time)

3. Assumptions

The inventory model is based on the following assumptions:

(1) The planning horizon is infinite and a single kind of items is purchased, and these are capable of growing before the slaughter process.

(2) The shortages are permitted, and these are completely backordered.

(3) The items are slaughtered and are immediately inspected in order to sell them to consumers. Firstly, the backordering quantity is inspected in order to cover the shortages of the previous cycle.

(4) There exists an inspection process that is 100% effective.

(5) A random percentage of the slaughtered items is of imperfect quality.

(6) Imperfect-quality items are not reworked or replaced.

(7) All imperfect-quality items are salvaged and sold as a single lot at the end of the inspection process.

(8) The feeding cost for growing the items is directly related to weight gained by these.

(9) The holding cost to keep a weight unit of the slaughtered item in storage is incurred during both the inspection process and the consumption period.

(10) The demand rate $D(s)$ is a polynomial function of selling price of the perfect-quality items. It is as follows: $D(s) = \pi - \rho s^n$.

(11) The selling price of perfect-quality items is optimized, and it must be greater than that of the imperfect-quality items.

(12) Carbon emissions are taken into account, and these occur in all operations of the inventory system, except in the shortage period.

4. Inventory Model Development

The inventory model for growing items with imperfect quality, carbon emissions, and shortages is depicted in Figure 1. Consider a situation where a company orders y newborn growing items from an outside supplier at the beginning of the growing period t_1. Each newborn growing item has an initial weight of w_0. In this moment, the total initial weight of the inventory is $Q_0 = yw_0$. The growing items are fed, and eventually, they grow through time until an objective weight of w_1 is attained. Then, the growing items are slaughtered at the end of the growing time t_1. At this point, the final weight of the inventory is $Q_{t_1} = yw_1$, and this total weight contains a percentage x of imperfect items. The portion of imperfect items is a random variable with a known probability density distribution $f(x)$, and its expected value is $E[x]$. The shortages are permitted, and these are fully backordered. Therefore, immediately, the inspection process starts to screen the items to complete the backordering quantity (B) at a rate of r units of weight per unit of time during the period t_2 in order to satisfy immediate shortages from the previous cycle. So, at the end of the inspection period t_2, the inventory model diminishes vertically by B units of weight. It is important to remark that the items continue to be inspected at the same rate r during the period t_3 till the total weight is screened. The length of the inspection time is $t_2 + t_3$. It is worth mentioning that, for the duration of t_3, the on-hand inventory declines by both removing the imperfect items and current demand rate. At the end of period t_3, the imperfect items are salvaged and sold as a single lot with a less price. Consequently, the on-hand inventory drops vertically by xyw_1 units of weight. On the other hand, in the course of t_4, the on-hand inventory decreases due to the current demand rate. The inventory level continues gradually consuming until it reaches zero at the end of period t_4 and the shortages period starts. As a final point, during t_5, the shortages are accumulated at the current demand rate which are eventually satisfied in the next cycle. Without loss of generality and with the purpose to make the mathematical expressions more tractable, the cycle time is determined as follows: $T = t_3 + t_4 + t_5$.

Throughout t_1, the items are growing according to a logistic growth function which relates the weight of items with time using three input parameters. These input parameters are the asymptotic weight of the items, the integration constant, and the growth rate, which are represented by α, β, and λ, respectively. Thus, the logistic growth function of the items is mathematically expressed by

$$w_t(t) = \frac{\alpha}{1 + \beta e^{-\lambda t}}. \tag{1}$$

As it was mentioned above, the growing items are slaughtered when their weight attains the objective weight of w_1 which occurs at the end of the growth period t_1. Hence,

$$w_1 = w_t(t = t_1) = \frac{\alpha}{1 + \beta e^{-\lambda t_1}}. \tag{2}$$

The duration of the growth period (t_1) is calculated by solving equation (2) for t_1. Thus,

$$t_1 = -\frac{\ln\left[(1/\beta)\left((\alpha/w_1) - 1\right)\right]}{\lambda}. \tag{3}$$

The inventory must be inspected before being sold in order to avoid to vend imperfect quality as good ones; therefore, firstly, the backordering quantity (B) of the previous cycle must be screened at an inspection rate r. Therefore, the duration of the inspection period (t_2) is computed as follows:

$$t_2 = \frac{B}{r}. \tag{4}$$

After the inspection of the backordering quantity is conducted, the screening process continues until the total weight is screened due to the fact that there are pending $yw_1 - B$ units of weight to be inspected. This is performed during the inspection time t_3 which is given as follows:

$$t_3 = \frac{yw_1 - B}{r}. \tag{5}$$

After the inspection time t_3, the imperfect-quality items are withdrawn from the storage and sold. Now, the on-hand inventory contains only perfect-quality items, and these are consumed during t_4. The time t_4 is determined as follows:

$$E[t_4] = \frac{yw_1 - B - Dt_3 - E[x]yw_1}{D(s)}. \tag{6}$$

The shortages accumulation period (t_5) is obtained with

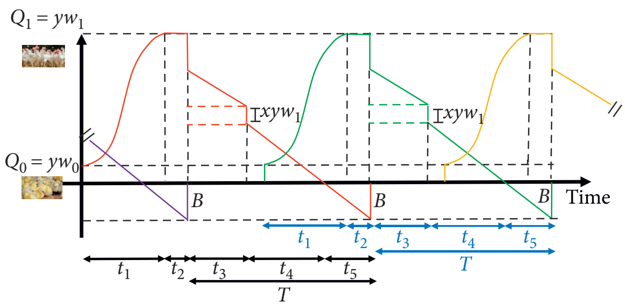

Figure 1: Inventory behavior for growing items with imperfect quality and shortages.

$$t_5 = \frac{B}{D(s)}. \quad (7)$$

With the aim to have more manageable equations and without loss of generality, it is defined that the expected cycle time T is computed by the sum of t_3, t_4, and t_5. From equations (5)–(7), the expected cycle time $E[T]$ is calculated as

$$E[T] = t_3 + E[t_4] + t_5 = \frac{yw_1(1 - E[x])}{D(s)}. \quad (8)$$

The main goal of the inventory model for growing items with imperfect quality, carbon emissions, and shortages is to determine the optimal values for the selling price, order quantity, and backordering quantity that maximize the total profit of the company. The total profit is obtained by the difference between the total revenue and the total cost of the inventory system. The details of how to calculate each component of the total revenue and the total cost are given below.

4.1. Expected Revenue per Period. Due to the fact that the company vends both perfect- and imperfect-quality items, for that reason, the expected total revenue $E[\mathrm{TR}]$ is calculated by the sum of total sales of imperfect and perfect items. The perfect-quality items are sold at a price of s per unit of weight, while at the end of the inspection period t_3, the imperfect-quality items are vended as a single batch with a less price of v per unit of weight. As a result, the expected total revenue $E[\mathrm{TR}]$ per period is given by

$$E[\mathrm{TR}] = syw_1(1 - E[x]) + vyw_1E[x]. \quad (9)$$

4.2. Total Cost per Period. The expected total cost $E[\mathrm{TC}]$ per period includes the purchasing cost, setup cost, feeding cost, expected holding cost, inspection cost, backordering cost, and expected carbon emissions cost. One has

$$E[\mathrm{TC}] = Pc + Sc + Fc + E[Hc] + Zc + Bc + E[Ec]. \quad (10)$$

The objective of this inventory model is to maximize the expected total profit $E[\mathrm{TP}]$ which is obtained by subtracting the expected total cost per period $E[\mathrm{TC}]$ from the expected total revenue $E[\mathrm{TR}]$ per period. Then,

$$E[\mathrm{TP}] = E[\mathrm{TR}] - Pc - Sc - Fc - E[Hc] - Zc - Bc - E[Ec]. \quad (11)$$

The following section provides a detailed discussion of the calculations of the aforesaid costs.

4.3. Purchasing Cost per Period. At the beginning of each period, the company buys y newborn items at a cost of p per unit of weight; each one with a weight of w_0. Thus, the purchasing cost per period is obtained with

$$Pc = pyw_0. \quad (12)$$

4.4. Setup Cost per Period. At the commencement of each period, a setup cost of K is carried out. So, the setup cost per period is determined as

$$Sc = K. \quad (13)$$

4.5. Feeding Cost per Period. The growing items are fed during t_1, and a feeding cost is incurred by the company at c per unit of weight. As the growing items become older and bigger, they need more food. For that reason, the quantity of food consumed by the items depends on the age (weight) of the items according to the growth function $w_t(t)$. The feeding cost per period is calculated with

$$\begin{aligned} Fc &= cy\int_0^{t_1} w_{t_1}(t)\mathrm{d}t = cy\int_0^{t_1}\left(\frac{\alpha}{1+\beta e^{-\lambda t}}\right)\mathrm{d}t \\ &= cy\left(\alpha t_1 + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda t_1}\right) - \ln(1+\beta)\right]\right). \end{aligned} \quad (14)$$

4.6. Expected Holding Cost per Period. The company incurs in a holding cost for maintaining in good conditions the items in storage during the cycle time T. The expected holding cost is determined by multiplying the holding cost (h) per unit of weight per unit of time by the inventory carried out during $t_2 + t_3 + t_4$. The whole inventory held is calculated as the sum of the areas $A_1 + A_2 + A_3 + A_4 + A_5$ (see Figure 5 in Appendix A). So, the expected holding cost is given by

$$\begin{aligned} E[Hc] = h\Bigg[&\frac{y^2w_1^2E\left[(1-x)^2\right]}{2D(s)} - \frac{yw_1(1-E[x])B}{D(s)} \\ &+ \frac{B^2}{2D(s)} + \frac{y^2w_1^2E[x]}{r} - \frac{yw_1E[x]B}{r} + \frac{yw_1B}{r}\Bigg]. \end{aligned} \quad (15)$$

The detailed derivation of the expected holding cost ($E[Hc]$) is given in Appendix A.

4.7. Inspection Cost per Period. Throughout $t_2 + t_3$, a 100% inspection process is performed at a rate of r with the aim of splitting the perfect-quality items from imperfect-quality items. The company incurs in an inspection cost of z per unit of weight. In consequence, the inspection cost per period is expressed as

$$Zc = zyw_1. \tag{16}$$

4.8. Shortage Cost per Period. During the course of t_5, the shortages are accumulated till these reach the backordering quantity of B units of weight. The company has a backordering cost for the management of the accumulation of shortages. The backordering cost is computed by multiplying shortage cost (b) per unit of weight per unit of time by area A_6 (see Figure 5 in Appendix A). Therefore, the backordering cost is calculated as

$$Bc = \frac{bB^2}{2D(s)}. \tag{17}$$

The detailed derivation of the backordering cost (Bc) is specified in Appendix B.

There are several research works that suppose that the carbon emissions generated by the companies are due to the transportation and warehousing activities. But, in fact, there are a lot of operations that emit carbon emissions. For example, the operations involved in the following process also cause carbon emissions: purchasing, setup, feeding, holding inventory, and inspection, among others. It is important to mention that some growing items per se generate carbon emissions (cows, goats, pigs, etc.). The carbon emissions caused by these processes are determined as follows:

4.9. Carbon Emissions Caused by the Purchasing Action.

$$\widehat{P}c = \widehat{p}yw_0. \tag{18}$$

4.10. Carbon Emissions Produced by the Setup Activity.

$$\widehat{S}c = \widehat{K}. \tag{19}$$

4.11. Carbon Emissions Generated during the Feeding Process.

$$\begin{aligned}\widehat{F}c &= \widehat{c}y\int_0^{t_1} w_{t_1}(t)\mathrm{d}t = \widehat{c}y\int_0^{t_1}\left(\frac{\alpha}{1+\beta e^{-\lambda t}}\right)\mathrm{d}t \\ &= \widehat{c}y\left(\alpha t_1 + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda t_1}\right) - \ln(1+\beta)\right]\right).\end{aligned} \tag{20}$$

4.12. Carbon Emissions Created in Holding Inventory's Operations.

$$\begin{aligned}E[\widehat{H}c] = \widehat{h}\Bigg[&\frac{y^2w_1^2E\left[(1-x)^2\right]}{2D(s)} - \frac{yw_1(1-E[x])B}{D(s)} \\ &+ \frac{B^2}{2D(s)} + \frac{y^2w_1^2E[x]}{r} - \frac{yw_1E[x]B}{r} + \frac{yw_1B}{r}\Bigg].\end{aligned} \tag{21}$$

4.13. Carbon Emissions Made by the Inspection Process.

$$\widehat{Z}c = \widehat{z}yw_1. \tag{22}$$

Carbon tax (θ) is one of the well-known mechanisms imposed by government regulations as a penalty. This means that the companies need to pay a tax on the amount of carbon emissions. As a result, the carbon emissions cost per period that the company must pay is

$$\begin{aligned}Ec = \theta\Bigg[&\widehat{p}yw_0 + \widehat{K} + \widehat{z}yw_1 + \widehat{c}y\left(\alpha t_1 + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda t_1}\right) - \ln(1+\beta)\right]\right) \\ &+\widehat{h}\left[\frac{y^2w_1^2E\left[(1-x)^2\right]}{2D(s)} - \frac{yw_1(1-E[x])B}{D(s)} + \frac{B^2}{2D(s)} + \frac{y^2w_1^2E[x]}{r} - \frac{yw_1E[x]B}{r} + \frac{yw_1B}{r}\right]\Bigg].\end{aligned} \tag{23}$$

4.14. Expected Total Profit Function. The expected total profit $E[\mathrm{TP}]$ per period is calculated in the following manner: the expected total revenue $E[\mathrm{TR}]$ per period minus the expected total cost $E[\mathrm{TC}]$ per period. Basically, the expected total profit $E[\mathrm{TP}]$ is formulated by substituting equations (9) and (12) to (17) and equation (23) into equation (11). Then, the expected total profit $E[\mathrm{TPU}]$ per unit time per period is determined as $E[\mathrm{TPU}] = (E[\mathrm{TP}]/E[T])$. Thus,

$$
\begin{aligned}
E[\mathrm{TPU}(y,B,s)] = {} & sD(s) + \frac{vD(s)E[x]}{(1-E[x])} - \frac{pD(s)w_0}{w_1(1-E[x])} - \frac{KD(s)}{yw_1(1-E[x])} - \frac{zD(s)}{(1-E[x])} \\
& - \frac{cD(s)}{w_1(1-E[x])}\left[\alpha t_1 + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda t_1}\right) - \ln(1+\beta)\right]\right] \\
& - h\left[\frac{yw_1E\left[(1-x)^2\right]}{2(1-E[x])} - B + \frac{B^2}{2yw_1(1-E[x])} + \frac{D(s)yw_1E[x]}{r(1-E[x])} + \frac{D(s)B}{r}\right] \\
& - \frac{bB^2}{2yw_1(1-E[x])} - \theta\left[\frac{\widehat{p}D(s)w_0}{w_1(1-E[x])} + \frac{\widehat{K}D(s)}{yw_1(1-E[x])} + \frac{\widehat{z}D(s)}{(1-E[x])}\right. \\
& + \frac{\widehat{c}D(s)}{w_1(1-E[x])}\left[\alpha t_1 + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda t_1}\right) - \ln(1+\beta)\right]\right] \\
& \left. + \widehat{h}\left[\frac{yw_1E\left[(1-x)^2\right]}{2(1-E[x])} - B + \frac{B^2}{2yw_1(1-E[x])} + \frac{D(s)yw_1E[x]}{r(1-E[x])} + \frac{D(s)B}{r}\right]\right].
\end{aligned}
\tag{24}
$$

Considering that the demand rate depends on selling price with a polynomial function $D(s) = \pi - \rho s^n$, the expected total profit $E[\mathrm{TPU}(y,B,s)]$ per unit of time becomes

$$
\begin{aligned}
E[\mathrm{TPU}(y,B,s)] = {} & s(\pi-\rho s^n) + \frac{v(\pi-\rho s^n)E[x]}{(1-E[x])} - \frac{p(\pi-\rho s^n)w_0}{w_1(1-E[x])} - \frac{K(\pi-\rho s^n)}{yw_1(1-E[x])} \\
& - \frac{z(\pi-\rho s^n)}{(1-E[x])} - \frac{c(\pi-\rho s^n)}{w_1(1-E[x])}\left[\alpha t_1 + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda t_1}\right) - \ln(1+\beta)\right]\right] \\
& - h\left[\frac{yw_1E\left[(1-x)^2\right]}{2(1-E[x])} - B + \frac{B^2}{2yw_1(1-E[x])} + \frac{(\pi-\rho s^n)yw_1E[x]}{r(1-E[x])} + \frac{(\pi-\rho s^n)B}{r}\right] \\
& - \frac{bB^2}{2yw_1(1-E[x])} - \theta\left[\frac{\widehat{p}(\pi-\rho s^n)w_0}{w_1(1-E[x])} + \frac{\widehat{K}(\pi-\rho s^n)}{yw_1(1-E[x])} + \frac{\widehat{z}(\pi-\rho s^n)}{(1-E[x])}\right. \\
& + \frac{\widehat{c}(\pi-\rho s^n)}{w_1(1-E[x])}\left[\alpha t_1 + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda t_1}\right) - \ln(1+\beta)\right]\right] \\
& \left. + \widehat{h}\left[\frac{yw_1E\left[(1-x)^2\right]}{2(1-E[x])} - B + \frac{B^2}{2yw_1(1-E[x])} + \frac{(\pi-\rho s^n)yw_1E[x]}{r(1-E[x])} + \frac{(\pi-\rho s^n)B}{r}\right]\right].
\end{aligned}
\tag{25}
$$

Then, the objective is to maximize the expected total profit $E[TPU(y,B,s)]$ per unit of time. Therefore, the optimization problem is formulated as follows:

$$
\underset{(y,B,s)\in\Omega}{\mathrm{Max}} \; E[\mathrm{TPU}(y,B,s)]
$$

$$
\text{where } \Omega = \left\{(y,B,s)\colon\ y>0, 0\le B\le yw_1 \text{ and } p\le s\le\left(\frac{\pi}{\rho}\right)^{(1/n)}\right\}. \tag{26}
$$

It is important to remark that the abovementioned maximization formulation is a nonlinear optimization problem.

5. Solution Procedure

Firstly, the unconstrained optimization problem is considered in order to obtain some theoretical results. Secondly, an algorithm for finding the optimal solution to the constrained problem given by equation (26) is developed.

5.1. Theoretical Results. The aim is to find the optimal values for order quantity (y), backordering quantity (B), and selling price (s) that maximize the expected total profit per unit of time. As the expected total profit per unit of time function $E[\mathrm{TPU}(y,B,s)]$ is continuous and twice differentiable with respect to the three decision variables (y,B,s) on the interval $[0,\infty]$, there exists a global maximum on that interval.

The necessary conditions which must be satisfied for the optimality of solution that maximizes the expected total profit per unit of time function are $((\partial E[\mathrm{TPU}(y,B,s)])/\partial y)=0$, $(\partial E[\mathrm{TPU}(y,B,s)]/\partial B)=0$, and $((\partial E[\mathrm{TPU}(y,B,s)])/\partial s)=0$.

The first partial derivative of $E[\mathrm{TPU}(y,B,s)]$ given in equation (25) with respect to y is expressed as follows:

$$\begin{aligned}\frac{\partial E[\mathrm{TPU}(y,B,s)]}{\partial y} &= \frac{K(\pi-\rho s^{n})}{y^{2}w_1(1-E[x])}\\ &\quad - h\left[\frac{w_1E\left[(1-x)^2\right]}{2(1-E[x])}-\frac{B^2}{2y^2w_1(1-E[x])}+\frac{(\pi-\rho s^{n})w_1E[x]}{r(1-E[x])}\right]+\frac{bB^2}{2y^2w_1(1-E[x])}\\ &\quad -\theta\left[-\frac{\widehat{K}(\pi-\rho s^{n})}{y^2w_1(1-E[x])}+\widehat{h}\left[\frac{w_1E\left[(1-x)^2\right]}{2(1-E[x])}-\frac{B^2}{2y^2w_1(1-E[x])}+\frac{(\pi-\rho s^{n})w_1E[x]}{r(1-E[x])}\right]\right]=0.\end{aligned} \tag{27}$$

The first partial derivative of $E[\mathrm{TPU}(y,B,s)]$ with respect to B is given by

$$\begin{aligned}\frac{\partial[\mathrm{TPU}(y,B,s)]}{\partial B} &= -h\left[-1+\frac{B}{yw_1(1-E[x])}+\frac{(\pi-\rho s^{n})}{r}\right]-\frac{bB}{yw_1(1-E[x])}\\ &\quad -\theta\widehat{h}\left[-1+\frac{B}{yw_1(1-E[x])}+\frac{(\pi-\rho s^{n})}{r}\right]=0.\end{aligned} \tag{28}$$

The first partial derivative of $E[\mathrm{TPU}(y,B,s)]$ with respect to s is

$$\begin{aligned}\frac{\partial E[\mathrm{TPU}(y,B,s)]}{\partial s} &= \pi-(n+1)\rho s^{n}-\frac{\rho n s^{n-1}}{w_1(1-E[x])}\left[vE[x]w_1-(p+\theta\widehat{p})w_0-\frac{(K+\theta\widehat{K})}{y}\right.\\ &\quad -(z+\theta\widehat{z})w_1-(c+\theta\widehat{c})\left[\alpha t_1+\frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda t_1}\right)-\ln(1+\beta)\right]\right]\\ &\quad \left.-\frac{(h+\theta\widehat{h})}{r}\left[yw_1^2E[x]+Bw_1(1-E[x])\right]\right]=0.\end{aligned} \tag{29}$$

By solving equations (27) and (28) simultaneously for the decision variables y and B, one has

$$y=\sqrt{\frac{2(\pi-\rho s^{n})r(K+\theta\widehat{K})}{w_1^2(h+\theta\widehat{h})\left[rE\left[(1-x)^2\right]+2(\pi-\rho s^{n})E[x]\right]-\left(\left[(h+\theta\widehat{h})w_1(1-E[x])[r-(\pi-\rho s^{n})]\right]^2/((h+b+\theta\widehat{h})r)\right)}}, \tag{30}$$

$$B=\frac{(h+\theta\widehat{h})yw_1(1-E[x])[r-(\pi-\rho s^{n})]}{(h+b+\theta\widehat{h})r}. \tag{31}$$

The sufficient conditions for the optimality of the solution that maximize $E[\mathrm{TPU}(y, B, s)]$ are given in Appendix C.

5.2. Algorithm for Finding the Optimal Solution. The following Algorithm 1 is proposed taking into account the theoretical results and constrains.

5.3. Special Cases. The proposed inventory model developed in this research work is a general inventory model because it contains the following inventory models as special cases: Sebatjane and Adetunji's [40] improved inventory model, Maddah and Jaber [11], Chang and Ho [13], Shih [61], Silver [62], and the traditional EOQ inventory models with and without shortages.

(i) When $n = 1$, it means that the product has a linear price-dependent demand $(\pi - \rho s)$. It is expressed as follows:

$$y = \sqrt{\frac{2(\pi - \rho s) r (K + \theta \widehat{K})}{w_1^2 (h + \theta \widehat{h})\left[rE\left[(1 - x)^2\right] + 2(\pi - \rho s)E[x]\right] - \left(\left[(h + \theta \widehat{h}) w_1 (1 - E[x])[r - (\pi - \rho s)]\right]^2 / ((h + b + \theta \widehat{h}) r)\right)}}, \tag{32}$$

$$B = \frac{(h + \theta \widehat{h}) y w_1 (1 - E[x])[r - (\pi - \rho s)]}{(h + b + \theta \widehat{h}) r}, \tag{33}$$

$$\begin{aligned} &\pi - 2\rho s - \frac{\rho}{w_1 (1 - E[x])}\left[vE[x]w_1 - (p + \theta \widehat{p}) w_0 - \frac{(K + \theta \widehat{K})}{y} - (z + \theta \widehat{z}) w_1\right. \\ &\left. - (c + \theta \widehat{c})\left[\alpha t_1 + \frac{\alpha}{\lambda}\left[\ln\left(1 + \beta e^{-\lambda t_1}\right) - \ln(1 + \beta)\right]\right] - \frac{(h + \theta \widehat{h})}{r}\left[y w_1^2 E[x] + B w_1 (1 - E[x])\right]\right] = 0. \end{aligned} \tag{34}$$

(ii) When shortages and carbon tax rate are not considered and the demand is not dependent on price (i.e., $b \longrightarrow \infty, \theta \longrightarrow 0$ and $\rho \longrightarrow 0$), then an improved version of Sebatjane and Adetunji's [40] inventory model is obtained. It is shown as follows:

$$y = \sqrt{\frac{2\pi K}{h w_1^2 \left[E\left[(1 - x)^2\right] + ((2\pi E[x])/r)\right]}}. \tag{35}$$

(iii) When shortages are permitted, carbon tax rate is not involved, the demand is not dependent on price, and the products are not of the type of growing items, and thus, the feeding process is not incorporated (i.e., $b > 0, \theta \longrightarrow 0, \rho \longrightarrow 0$, and $c \longrightarrow 0$), and then, the inventory model of Chang and Ho [13] is determined. It is given by

$$y = \sqrt{\frac{2\pi K}{h\left[E\left[(1 - x)^2\right] + ((2\pi E[x])/r)\right] - \left([h(1 - E[x])]^2 / (h + b)\right)}}, \tag{36}$$

$$B = \frac{h y (1 - E[x])}{(h + b)}. \tag{37}$$

(iv) When shortages are not allowed, carbon tax rate is not taken account, the demand is not dependent on price, and the products are not of the type of growing items ($b \longrightarrow \infty, \theta \longrightarrow 0, \rho \longrightarrow 0$, and $c \longrightarrow 0$); then, Maddah and Jaber's [11] inventory model is derived. It is presented as follows:

$$y = \sqrt{\frac{2\pi K}{h\left[E\left[(1 - x)^2\right] + ((2\pi E[x])/r)\right]}}. \tag{38}$$

(v) When shortages are not tolerable, carbon tax rate is not considered, the demand does not depend on price, the goods are not of the type of growing

Step 1. Provide the input parameters of the inventory system.
Step 2. Compute the selling price (s'), the order quantity (y'), and the backordering quantity (B') by solving simultaneously equations (29)–(31).
Step 3. If the optimality conditions are satisfied, then go to Step 4. Else, go to Step 8.
Step 4. If $p \le s \le (\pi/\rho)^{(1/n)}$, then go to Step 7. Else, go to Step 5.
Step 5. If $s > (\pi/\rho)^{(1/n)}$, then set $s' = (\pi/\rho)^{(1/n)}$, determine the order quantity (y') with equation (30) and the backordering quantity (B') with equation (31), and go to Step 6. Else, set $s' = p$ and calculate the order quantity (y') with equation (30) and the backordering quantity (B') with equation (31) and go to Step 6.
Step 6. Calculate the expected total profit per unit of time $E[\mathrm{TPU}(y', B', s')]$ with equation (25).
Step 7. Report the solution: $(y^*, B^*, s^*) = (y', B', s')$ and $E[\mathrm{TPU}^*(y^*, B^*, s^*)] = E[\mathrm{TPU}(y', B', s')]$.
Step 8. Stop.

ALGORITHM 1: Algorithm to find the optimal solution.

items, and inspection rate is sufficiently large (i.e., $b \longrightarrow \infty, \theta \longrightarrow 0, \rho \longrightarrow 0, c \longrightarrow 0$ and $r \longrightarrow \infty$); then, it is converted to the inventory model of Shih [61] and Silver [62]. It is expressed as

$$y = \sqrt{\frac{2\pi K}{h\left[E\left[(1-x)^2\right]\right]}}. \tag{39}$$

(vi) When shortages are permissible, carbon tax rate is not incorporated, the demand is not dependent on price, the products are not of the type of growing items, and the products are of perfect quality (i.e., $b > 0, \theta \longrightarrow 0, \rho \longrightarrow 0, c \longrightarrow 0$, and $x = 0$); then, the traditional inventory model with shortages and with full backordering is found. It is written as follows:

$$y = \sqrt{\frac{2\pi K}{h}}\sqrt{\frac{h+b}{b}}, \tag{40}$$

$$B = \frac{hy}{(h+b)}. \tag{41}$$

(vii) When shortages are not allowable, carbon tax rate is not taken into consideration, the demand does not depend on price, the goods are not of type of growing items, and these are of good quality (i.e., $b \longrightarrow \infty, \theta \longrightarrow 0, \rho \longrightarrow 0, c \longrightarrow 0$, and $x = 0$); then, it is transformed to the traditional inventory model without shortages proposed by Harris [1]. It is

$$y = \sqrt{\frac{2\pi K}{h}}. \tag{42}$$

It is important to remark that the inventory model developed in this research does not reduce to Salameh and Jaber's [8] and Wee et al.'s [10] inventory models because those inventory models contain a flaw that was identified and corrected by Maddah and Jaber [11] and Chang and Ho [13], respectively. Basically, Maddah and Jaber [11] and Chang and Ho [13] proposed improved versions for Salameh and Jaber's [8] and Wee et al.'s [10] inventory models, respectively. For this reason, the proposed inventory model developed in this research work, in fact, converges to Maddah and Jaber's [11] and Chang and Ho's [13] inventory models as it was mentioned above.

6. Numerical Examples

This section presents and solves some numerical examples in order to illustrate the applicability of the proposed inventory model.

Example 1. This example considers the input parameters of Sebatjane and Adetunji [40]. To solve this inventory problem, it is stated $b \longrightarrow \infty, \theta \longrightarrow 0$, and $\rho \longrightarrow 0$. The values for the data are $\pi = 1000000$, $\rho = 0$, $\theta = 0$, $s = 0.05$ ZAR/g, $v = 0.02$ZAR/g, $K = 1000$ ZAR/cycle, $h = 0.04$ ZAR/g/year, $c = 0.2$ ZAR/g, $p = 0.025$ ZAR/g, $z = 0.00025$ ZAR/g, $r = 5256000$ g/year, $\alpha = 6870$ g, $\beta = 120\lambda = 40$/year, $w_0 = 57$ g, and $w_1 = 1500$ g, and the percentage of imperfect growing items follows a uniform distribution $(x \sim U[\gamma, \delta])$ with the probability density function $g(x)$ which is given as follows:

$$x \sim g(x) = \begin{cases} \dfrac{1}{\delta - \gamma}, & \gamma \le x \le \delta, \\ 0, & \text{otherwise.} \end{cases} \tag{43}$$

Considering $x \sim U[0, 0.04]$,

$$x \sim g(x) = \begin{cases} 25, & 0 \le x \le 0.04, \\ 0, & \text{otherwise.} \end{cases} \tag{44}$$

Then, $E[x]$ and $E[(1-x)^2]$ are computed as follows:

$$E[x] = \int_{\gamma}^{\delta} x f(x)\mathrm{d}x = \frac{\gamma+\delta}{2} = \frac{0+0.04}{2} = 0.02,$$
$$E\left[(1-x)^2\right] = \int_{\gamma}^{\delta} (1-x)^2 f(x)\mathrm{d}x = \frac{\gamma^2+\gamma\delta+\delta^2}{3} + 1 - \gamma - \delta, \tag{45}$$
$$E\left[(1-x)^2\right] = \frac{0^2+0(0.04)+(0.04)^2}{3} + 1 - 0 - 0.04 = 0.960533333.$$

As it was mentioned before, the proposed inventory model generates an improved version of Sebatjane and Adetunji's [40] inventory model. Additionally, the proposed inventory model also can be used for optimizing the order quantity (y) and backordering quantity (B) when the selling price (s) is given. Obviously, it also optimizes the three decision variables (y, B, s). Therefore, the numerical example is solved for different values of δ for the four inventory models: (I) Sebatjane and Adetunji [40] (original version), (II) Sebatjane and Adetunji [40] (improved version), (III) the proposed inventory model when selling price (s) is given, and (IV) the proposed inventory model when the selling price (s) is optimized. Table 2 shows the comparison of the optimal solutions among the four inventory models.

From Table 2, it is observed that the expected total profit of both the proposed inventory models (see columns III and IV) is greater than that of Sebatjane and Adetunji's [40] inventory models (see columns I and II).

Example 2. This example uses the input parameters of Wee et al. [10] and Chang and Ho [13]. To solve this inventory system, the following is established: $b > 0, \theta \longrightarrow 0, \rho \longrightarrow 0$, and $c \longrightarrow 0$. The data are as follows: $\pi = 50000$, $\rho = 0$, $\theta = 0$, $c = 0$, $s = 50$ \$/unit, $v = 20$\$/unit, $K = 100$\$/cycle, $h = 5$\$/unit/year, $b = 10$\$/unit/year $p = 25$\$/unit, $z = 0.5$\$/unit, $r = 175200$units/year, and $x \sim U[0, 0.04]$. The optimal solution is as follows: $E[\mathrm{TPU}^*(y^*, B^*)] = 1213562$\$/year, $y^* = 1751.671$ units, and $B^* = 572.2127$ units. This solution is the same as in the work of Chang and Ho [13].

Example 3. Now, let us consider some dataset of Sebatjane and Adetunji [40] which is $v = 0.02$ ZAR/g, $K = 1000$ZAR/cycle, $c = 0.2$ZAR/g, $p = 0.025$ZAR/g, $z = 0.00025$ZAR/g, $r = 5256000$g/year, $\alpha = 6870$g, $\beta = 120$, $\lambda = 40$/year, $w_0 = 57$g, $w_1 = 1500$g, and $x \sim U[0, 0.04]$. Here, the holding cost is $h = 0.2$ZAR/g/year. For the implementation of the proposed inventory model, additional information is required. These data are related to the type of the demand function dependent on the price, the backordering cost, and carbon emissions. Therefore, it is necessary to state the following data: Assume that the demand rate of the growing items follows a polynomial function given by $D(s) = \pi - \rho s^n$ with values of $\pi = 135000$, $\rho = 1050$, and $n = 2$. The backordering cost is $b = 0.1$ ZAR/g/year. The relevant input parameters related to carbon emissions are $\theta = 0.0045$ ZAR/tons, $\widehat{K} = 2000$ tons/year, $\widehat{h} = 0.2$ tons/year, $\widehat{c} = 0.65$ tons/year, $\widehat{p} = 0.375$ tons/year, and $\widehat{z} = 0.005$ tons/year. By applying the proposed algorithm, the optimal solution is obtained: $E[TPU^*(y^*, B^*, s^*)] = 584997.4$ ZAR/year, $y^* = 34.26474$ units of newborn growing items, $B^* = 33054.63$ g, and $s^* = 6.555838$ ZAR/g. Figures 2–4 demonstrate graphically the concavity property of the expected total profit with respect to pairs of decision variables y and B; y and s; and B and s, respectively.

7. Sensitivity Analysis

This section provides a sensitivity analysis in order to investigate the effects of changing the input parameters on the expected total profit per unit of time $(E[\mathrm{TPU}^*])$, order quantity (y^*), backordering quantity (B^*), and selling price (s^*) of the growing items. The sensitivity analysis is performed taking into consideration the data of Example 3 when only one parameter changes at a time and other parameters are kept at their original values. Specifically, the sensitivity analysis is performed to study the effect of scale, sensitivity and power index demand parameters, selling price and expected value of a percent of poorer-quality growing items, setup cost, holding cost, backordering cost, feeding cost, purchasing cost, inspection cost, carbon tax rate, and the amount of carbon emissions caused by the operations of the processes in ordering, holding, feeding, purchasing, and screening. The results of the sensitivity analysis are given in Tables 3–5. Based on the behavioral changes as reflected in Tables 3–5, the following observations are made:

(1) The value of $E[\mathrm{TPU}^*]$ is highly sensitive to the demand parameters π, ρ, n and less sensitive to other parameters. On the one hand, the higher the value of the scale parameter of demand π, the higher the value of $E[\mathrm{TPU}^*]$ due to the fact that demand increases; therefore, the sales increase, and this leads to high profits. For this reason, it is suggested for the manager to apply advertising actions in order to boost the demand. On the other hand, when the sensitivity parameter of the demand ρ grows, the $E[\mathrm{TPU}^*]$ declines because this parameter has a negative impact on the demand, making it drop. For this reason, it is advisable for the decision maker to

Table 2: A comparison of the optimal solutions of the four inventory models.

			I		II		III			IV			
			Sebatjane and Adetunji [40] (original), $b \longrightarrow \infty, \theta = 0, \rho = 0$		Sebatjane and Adetunji [40] (improved inventory model), $b \longrightarrow \infty, \theta = 0, \rho = 0$		The proposed inventory model, $\rho = 0, s = 0.05$			The proposed inventory model, $\rho = 700000$			
γ	δ	$E[x]$	y^*	$E[TPU^*]$	y^*	$E[TPU^*]$	y^*	B^*	$E[TPU^*]$	y^*	B^*	s^*	$E[TPU^*]$
0	0.04	0.02	151.5143	34641.73	151.5039	34641.11	167.9070	57103.70	35518.41	138.9623	51818.67	0.6940517	449132.6
0	0.10	0.05	155.2887	35013.46	155.2186	35009.38	171.7566	56624.78	35880.02	142.5000	51509.93	0.6939366	449374.6
0	0.20	0.10	161.8760	35680.21	161.5588	35662.24	178.2198	55663.16	36519.49	148.5423	50865.87	0.6937328	449801.7
0	0.30	0.15	168.8356	36413.78	168.0293	36369.20	184.6550	54469.01	37209.73	154.7091	50032.07	0.6935122	450261.3
0	0.40	0.20	176.1594	37225.05	174.5420	37137.38	190.9380	53009.26	37957.34	160.9085	48973.44	0.6932723	450757.0
0	0.50	0.25	183.8244	38127.53	180.9801	37975.53	196.9174	51252.47	38770.55	167.0178	47653.14	0.6930098	451293.2

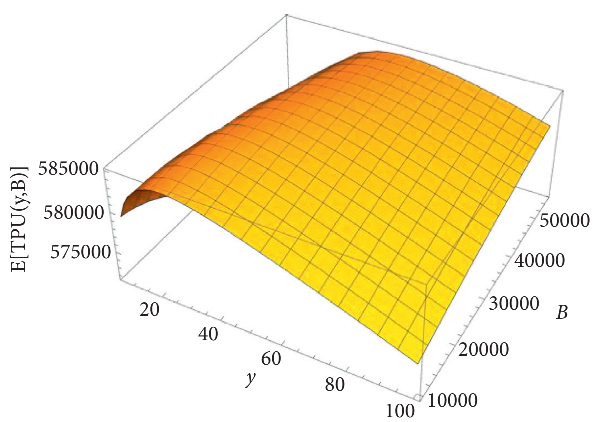

Figure 2: Concavity of $E[TPU(y, B)]$ w.r.t. y, B

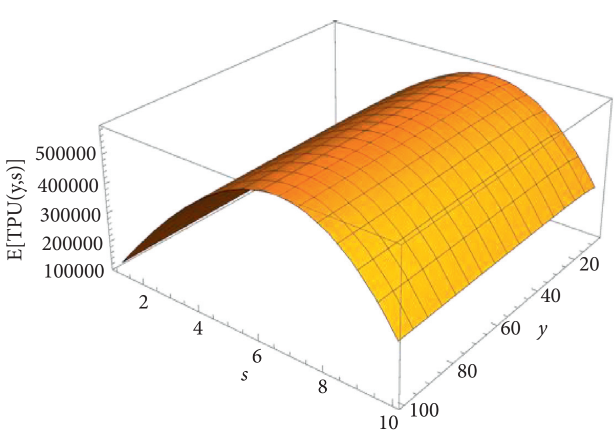

Figure 3: Concavity of $E[TPU(y, s)]$ w.r.t. y, s

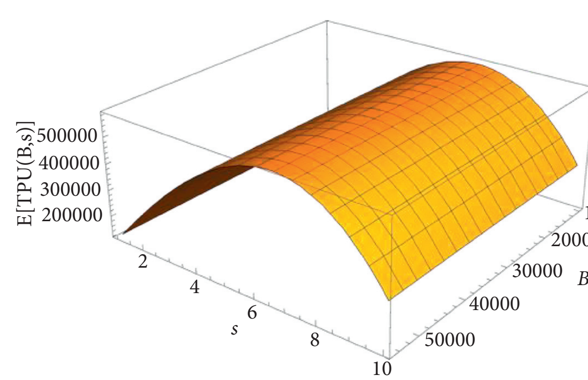

Figure 4: Concavity of $E[TPU(B, s)]$ w.r.t. B, s

reduce the selling price. However, a higher value of K induces higher values of s but a lower value for $E[TPU^*]$. This means that as the setup cost increases, the company must raise selling price s.

(2) The value of y is more sensitive to the parameters π, n, K, h, b and less sensitive to other parameters. The higher the value of the parameter b, the smaller the value of y.

(3) The value of B is more sensitive to the parameters π, n, K, h, b and less sensitive to other parameters. The higher the value of b, the smaller the value of B.

Table 3: Effects of the demand parameters (π, ρ, n), the price of imperfect growing items (v), and the percent of the imperfect growing items $(E[x])$ on the optimal solution.

Parameter	Value	y^*	B^*	s^*	$E[\mathrm{TPU}^*]$
	85000	27.51468	26714.18	5.205486	291168.5
	135000	34.26474	33054.63	6.555838	584997.4
π	185000	39.64369	37996.92	7.672019	940097.9
	235000	44.17254	42062.76	8.645232	1347288
	285000	48.10619	45509.16	9.519438	1800598
	650	34.26961	33059.16	8.329802	744653.9
	750	34.26828	33057.92	7.755266	692945.8
ρ	850	34.26703	33056.76	7.285369	650655.1
	950	34.26586	33055.67	6.891773	615231.4
	1050	34.26474	33054.63	6.555838	584997.4
	2	34.26474	33054.63	6.555838	584997.4
	5	37.98523	36485.12	1.850223	202864.7
n	7	38.84940	37274.30	1.490172	170704.7
	10	39.54134	37903.96	1.281047	151875.7
	14	40.02588	38343.67	1.167557	141757.1
	0.02	34.26474	33054.63	6.555838	584997.4
	0.06	34.26541	33055.26	6.555565	585070.7
v	0.10	34.26608	33055.88	6.555293	585144.1
	0.14	34.26675	33056.50	6.555020	585217.5
	0.18	34.26742	33057.12	6.554748	585290.8
	0.01	33.94088	33076.32	6.555872	584987.5
	0.02	34.26474	33054.63	6.555838	584997.4
$E[x]$	0.03	34.59095	33028.81	6.555804	585007.0
	0.04	34.91934	32998.64	6.555770	585016.4
	0.05	35.24975	32963.88	6.555736	585025.6

Table 4: Impacts of the costs of the inventory system $(K, h, b, c, p, z, \theta)$ on the optimal solution.

Parameter	Value	y^*	B^*	s^*	$E[\mathrm{TPU}^*]$
	1000	34.26474	33054.63	6.555838	584997.4
	1200	37.50542	36180.93	6.556512	584656.6
K	1400	40.48714	39057.41	6.557133	584343.1
	1600	43.26350	41735.80	6.557711	584051.2
	1800	45.87182	44252.08	6.558254	583776.9
	0.12	38.34897	30325.67	6.554933	585380.6
	0.16	35.88691	31977.51	6.555439	585160.0
h	0.2	34.26474	33054.63	6.555838	584997.4
	0.24	33.09099	33786.80	6.556172	584869.7
	0.28	32.18620	34295.59	6.556464	584765.0
	0.1	34.26474	33054.63	6.555838	584997.4
	0.15	30.54445	25267.19	6.556552	584559.0
b	0.2	28.43147	20585.99	6.557049	584259.0
	0.25	27.05971	17420.14	6.557416	584039.2
	0.3	26.09415	15121.73	6.557700	583870.6
	0.05	34.26816	33057.81	6.554445	585372.3
	0.1	34.26702	33056.75	6.554909	585247.3
c	0.15	34.26588	33055.69	6.555374	585122.3
	0.2	34.26474	33054.63	6.555838	584997.4
	0.25	34.26360	33053.58	6.556302	584872.4
	0.025	34.26474	33054.63	6.555838	584997.4
	0.05	34.26395	33053.90	6.556162	584910.2
p	0.075	34.26315	33053.16	6.556485	584823.1
	0.1	34.26236	33052.42	6.556809	584736.0
	0.125	34.26156	33051.68	6.557133	584648.9
	0.00025	34.26474	33054.63	6.555838	584997.4
	0.0005	34.26453	33054.44	6.555923	584974.4
z	0.00075	34.26432	33054.25	6.556008	584951.5
	0.001	34.26412	33054.05	6.556094	584928.6
	0.00125	34.26391	33053.86	6.556179	584905.6

Table 4: Continued.

Parameter	Value	y^*	B^*	s^*	$E[\text{TPU}^*]$
	0.0045	34.26474	33054.63	6.555838	584997.4
	0.01	34.41339	33258.23	6.555956	584955.2
θ	0.0155	34.56096	33460.86	6.556074	584913.2
	0.021	34.70746	33662.55	6.556192	584871.2
	0.0265	34.85293	33863.29	6.556309	584829.3

Table 5: Impacts of the carbon emission parameters $(\widehat{K}, \widehat{h}, \widehat{c}, \widehat{p}, \widehat{z})$ on the optimal solution.

Parameter	Value	y^*	B^*	s^*	$E[\text{TPU}^*]$
	2000	34.26474	33054.63	6.555838	584997.4
	2500	34.30291	33091.45	6.555846	584993.3
$\widehat{K}$	3000	34.34103	33128.23	6.555854	584989.3
	3500	34.37910	33164.96	6.555862	584985.3
	4000	34.41714	33201.65	6.555870	584981.3
	0.2	34.26474	33054.63	6.555838	584997.4
	0.4	34.23432	33074.31	6.555846	584994.2
$\widehat{h}$	0.6	34.20411	33093.81	6.555854	584991.0
	0.8	34.17412	33113.15	6.555862	584987.8
	1	34.14434	33132.32	6.555870	584984.7
	0.65	34.26474	33054.63	6.555838	584997.4
	1.3	34.26468	33054.57	6.555865	584990.0
$\widehat{c}$	1.95	34.26461	33054.51	6.555892	584982.7
	2.6	34.26454	33054.45	6.555919	584975.4
	3.25	34.26448	33054.39	6.555947	584968.1
	0.375	34.26474	33054.63	6.555838	584997.4
	0.75	34.26469	33054.58	6.555860	584991.5
$\widehat{p}$	1.125	34.26464	33054.53	6.555882	584985.6
	1.5	34.26458	33054.48	6.555904	584979.7
	1.875	34.26453	33054.44	6.555925	584973.8
	0.005	34.26474	33054.63	6.555838	584997.4
	0.015	34.26470	33054.60	6.555853	584993.2
$\widehat{z}$	0.025	34.26467	33054.56	6.555869	584989.1
	0.035	34.26463	33054.53	6.555884	584985.0
	0.045	34.26459	33054.49	6.555899	584980.8

Table 6: Sensitivity analysis of the key parameters of the inventory system.

Parameter	y^*	B^*	s^*	$E[\text{TPU}^*]$
$\pi\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$
$\rho\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$n\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$
$v\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$
$K\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$
$h\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$
$b\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$
$c\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$
$p\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$
$z\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$
$\theta\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$
$\widehat{K}\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$
$\widehat{h}\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$
$\widehat{c}\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$
$\widehat{p}\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$
$\widehat{z}\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$
$E[x]\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$

(4) The value of s is more sensitive to the parameters π, ρ, n and less sensitive to other parameters. The higher the value of the parameter π, the higher the value of s. This means that as demand scale parameter π increases, the company must raise selling price which will also directly impact positively the expected total profit. The higher the value of the parameters ρ, n, the smaller the value of s.

(5) When the expected value of the defective growing items $E[x]$ decreases, the expected total profit also decreases $E[\text{TPU}^*]$.

(6) Changes on carbon emission parameters have a regular influence on the expected total profit $E[\text{TPU}^*]$.

With the information of Tables 3–5 is constructed Table 6. Table 6 summarizes the sensitivity analysis study in a visual manner which is more helpful for the decision makers due to that these can more easily observe how changes in parameters affect to the decisions variables and the expected total profit per unit of time in order to take the best decisions based on numerical facts.

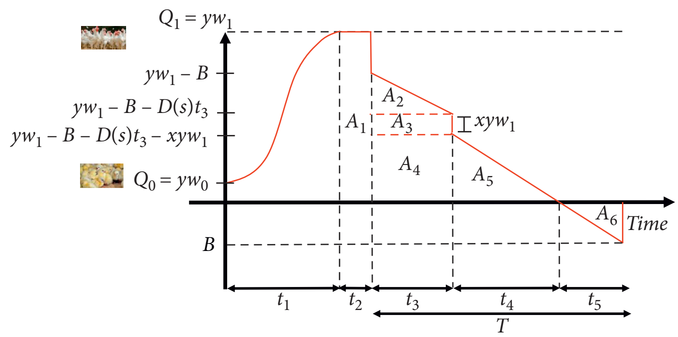

Figure 5: Inventory behavior through time.

8. Conclusions and Future Research

This paper develops an inventory model for growing items with imperfect quality and carbon emissions in which the demand rate is price sensitive according to a polynomial function. Furthermore, the shortages are permissible and these are fully backordered. To optimize the expected total profit per unit of time, some theoretical results are derived, and with these results, an effective and efficient solution procedure is developed. It is important to state that the algorithm generates the optimal solution to order quantity, backordering quantity, and selling price which maximizes the expected total profit per unit of time. The effects of carbon tax and the amount of carbon emissions are studied. Some important managerial insights are obtained from numerical examples and sensitivity analysis. The results show that carbon tax has a regular role on the reduction of carbon emissions. It is important to pay attention to quality and ensure that the percentage of defective items is kept to a minimum. Furthermore, if the setup cost increases, the company must raise selling price s, and when the demand elasticity parameter increases, the company must raise selling price which also directly impacts the total profit. Furthermore, the proposed inventory model is a generalized inventory model due the fact that several previously published inventory models are particular cases. It was found that the policy that permits shortages with full backordering is more economical that the one that avoids shortages. Moreover, it was also found that the proposed inventory model outperforms the inventory model of Sebatjane and Adetunji [40].

There are several possible extensions of the proposed inventory model that can be explored. These extensions constitute future research endeavors in the inventory management of imperfect growing items. For instance, consider to investigate the effect that only a percentage of defective items can be reworked, and the others must be eliminated immediately. Other research studies that can be conducted are to include new aspects such as stock-dependent demand, nonlinear holding cost, vendor-managed inventory (VMI) with consignment stock (CS), inflation, volume discounts, deterioration, trade credit, supply chain environment, and a vendor-buyer inventory model with multiple shipments, partial backordering, advertising, and multiple products subject to constrains such as space, budget, and time. These are, among others, some interesting and challenging subjects of ongoing future investigation that academicians and researchers would like to study in the future.

Appendix

A. Determination of the Expected Holding Cost $(E[Hc])$

The holding cost is calculated by the multiplication of the unit holding cost (h) and the total inventory accumulated during $t_2 + t_3 + t_4$, and this is the area above level zero in Figure 5. This area is computed as the sum of the areas $A_1 + A_2 + A_3 + A_4 + A_5$.

Considering Figure 5, the five areas are defined as follows.

The area A_1 is obtained as follows:

$$A_1 = (t_2)(yw_1) = \left(\frac{B}{r}\right)(yw_1) = \frac{yw_1 B}{r}. \tag{A.1}$$

The area A_2 is found as follows:

$$A_2 = \frac{(t_3)(D(s))(t_3)}{2} = \frac{(D(s))\left(t_3^2\right)}{2} = \frac{(D(s))\left((yw_1 - B)/r\right)^2}{2} = \frac{(D(s))(yw_1 - B)^2}{2r^2}. \tag{A.2}$$

The area A_3 is determined in the following manner:

$$A_3 = (t_3)(xyw_1) = \left(\frac{yw_1 - B}{r}\right)(xyw_1) = \frac{y^2 w_1^2 x}{r} - \frac{yw_1 xB}{r}. \tag{A.3}$$

The area A_4 is defined as follows:

$$\begin{aligned} A_4 &= (t_3)(yw_1 - B - D(s)t_3 - xyw_1) \\ &= (t_3)(yw_1(1-x) - B - D(s)t_3), \end{aligned} \tag{A.4}$$
$$A_4 = (yw_1(1-x) - B)t_3 - D(s)t_3^2.$$

The area A_5 is given by

$$A_5 = \frac{(t_4)(yw_1 - B - D(s)t_3 - xyw_1)}{2} = \left(\frac{yw_1 - B - D(s)t_3 - xyw_1}{D(s)}\right)\left(\frac{yw_1(1-x) - B - D(s)t_3}{2}\right),$$
$$A_5 = \left(\frac{yw_1(1-x) - B - D(s)t_3}{D(s)}\right)\left(\frac{yw_1(1-x) - B - D(s)t_3}{2}\right) = \frac{(yw_1(1-x) - B - D(s)t_3)^2}{2D(s)},$$
$$A_5 = \frac{(yw_1(1-x) - B)^2 - 2(yw_1(1-x) - B)D(s)t_3 + (D(s)t_3)^2}{2D(s)},$$
$$A_5 = \frac{(yw_1(1-x) - B)^2}{2D(s)} - (yw_1(1-x) - B)t_3 + \frac{D(s)t_3^2}{2}. \quad \text{(A.5)}$$

The total inventory accumulated (TIA) during $t_2 + t_3 + t_4$ is computed as follows:

$$\text{TIA} = A_1 + A_2 + A_3 + A_4 + A_5. \quad \text{(A.6)}$$

Substituting the corresponding areas,

$$TIA = \frac{yw_1B}{r} + \frac{(D(s))(t_3^2)}{2} + \frac{y^2w_1^2x}{r} - \frac{yw_1xB}{r} + (yw_1(1-x) - B)t_3 - D(s)t_3^2$$
$$+ \frac{(yw_1(1-x) - B)^2}{2D(s)} - (yw_1(1-x) - B)t_3 + \frac{D(s)t_3^2}{2}. \quad \text{(A.7)}$$

Simplifying,

$$\text{TIA} = \frac{yw_1B(1-x)}{r} + \frac{y^2w_1^2x}{r} + \frac{(yw_1(1-x) - B)^2}{2D(s)},$$
$$\text{TIA} = \frac{yw_1B(1-x)}{r} + \frac{y^2w_1^2x}{r} + \frac{y^2w_1^2(1-x)^2 - 2yw_1(1-x)B + B^2}{2D(s)},$$
$$\text{TIA} = \frac{yw_1B(1-x)}{r} + \frac{y^2w_1^2x}{r} + \frac{y^2w_1^2(1-x)^2 - 2yw_1(1-x)B + B^2}{2D(s)}. \quad \text{(A.8)}$$

Taking the expected value of the total inventory accumulated (TIA), the following expression is obtained:

$$E[\text{TIA}] = \frac{yw_1B(1-E[x])}{r} + \frac{y^2w_1^2E[x]}{r} + \frac{y^2w_1^2E\left[(1-x)^2\right] - 2yw_1(1-E[x])B + B^2}{2D(s)}. \quad \text{(A.9)}$$

Rearranging terms,

$$E[TIA] = \frac{y^2w_1^2E\left[(1-x)^2\right]}{2D(s)} - \frac{yw_1(1-E[x])B}{D(s)} + \frac{B^2}{2D(s)} + \frac{y^2w_1^2E[x]}{r} - \frac{yw_1E[x]B}{r} + \frac{yw_1B}{r}. \quad \text{(A.10)}$$

Finally, the expected holding cost $(E[Hc])$ is given by

$$E[Hc] = h\left[\frac{y^2 w_1^2 E\left[(1-x)^2\right]}{2D(s)} - \frac{y w_1 (1-E[x])B}{D(s)} + \frac{B^2}{2D(s)} + \frac{y^2 w_1^2 E[x]}{r} - \frac{y w_1 E[x] B}{r} + \frac{y w_1 B}{r}\right]. \quad (A.11)$$

B. Determination of the Backordering Cost (Bc)

The backordering cost is determined by the multiplication of the unit backordering cost (b) and the total shortages accumulated during t_5, and this is the area A_6 shown below level zero in Figure 5. This area (A_6) is obtained as follows:

$$A_6 = \frac{(t_5)(B)}{2} = \frac{(B/D(s))(B)}{2} = \frac{B^2}{2D(s)}. \quad (B.1)$$

Thus, the backordering cost (Bc) is given by

$$Bc = \frac{bB^2}{2D(s)}. \quad (B.2)$$

C. Sufficient Conditions for the Optimality

For the sake of brevity, only the final equations are provided for the direct and cross second-order partial derivatives.

The direct second-order partial derivatives are given as follows.

The second-order partial derivative of equation (25) with respect to y is as follows:

$$\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial y^2} = -\frac{B^2[h+b+\theta\widehat{h}] + 2(\pi - \rho s^n)[K+\theta\widehat{K}]}{y^3 w_1 (1-E[x])} < 0. \quad (C.1)$$

The second-order partial derivative of equation (25) with respect to B is expressed as

$$\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial B^2} = -\frac{h+b+\theta\widehat{h}}{y w_1 (1-E[x])} < 0. \quad (C.2)$$

The second-order partial derivative of equation (25) with respect to s is given as follows:

$$\begin{aligned}\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial s^2} = &-(n+1)\rho n s^{n-1} - \frac{\rho n(n-1)s^{n-2}}{w_1(1-E[x])}\Bigg[vE[x]w_1 - (p+\theta\widehat{p})w_0 - \frac{(K+\theta\widehat{K})}{y} \\ &- (z+\theta\widehat{z})w_1 - (c+\theta\widehat{c})\left[\alpha t_1 + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda t_1}\right) - \ln(1+\beta)\right]\right] \\ &- \frac{(h+\theta\widehat{h})}{r}\left[y w_1^2 E[x] + B w_1 (1-E[x])\right] < 0.\end{aligned} \quad (C.3)$$

The second-order cross partial derivatives are given as follows.

The second-order cross partial derivative $(\partial^2/\partial y\partial B)$ of equation (25) is presented as follows:

$$\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial y \partial B} = \frac{B[h+b+\theta\widehat{h}]}{y^2 w_1 (1-E[x])}. \quad (C.4)$$

The second-order cross partial derivative $(\partial^2/\partial y\partial s)$ of equation (25) is written in the following manner:

$$\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial y \partial s} = -\frac{\rho n s^{n-1}}{y^2 w_1 (1-E[x])}(K+\theta\widehat{K}) + \frac{w_1 E[x]\rho n s^{n-1}}{r(1-E[x])}(h+\theta\widehat{h}). \quad (C.5)$$

The second-order cross partial derivative $(\partial^2/\partial B\partial y)$ of equation (25) is shown in the following way:

$$\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial B \partial y} = \frac{B[h+b+\theta\widehat{h}]}{y^2 w_1 (1-E[x])}. \quad (C.6)$$

The second-order cross partial derivative $(\partial^2/\partial B\partial s)$ of equation (25) is given as follows:

$$\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial B \partial s} = \frac{\rho n s^{n-1}[h+\theta\widehat{h}]}{r}. \quad (C.7)$$

The second-order cross partial derivative $(\partial^2/\partial s\partial y)$ of equation (25) is given as follows:

$$\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial s \partial y} = -\frac{\rho n s^{n-1}}{y^2 w_1 (1-E[x])}(K+\theta\widehat{K}) + \frac{w_1 E[x]\rho n s^{n-1}}{r(1-E[x])}(h+\theta\widehat{h}). \quad (C.8)$$

The second-order cross partial derivative $(\partial^2/\partial s\partial B)$ of equation (25) is

$$\frac{\partial^2 E[\text{TPU}(y,B,s)]}{\partial s\partial B} = \frac{\rho n s^{n-1}[h+\theta\widehat{h}]}{r}. \quad (\text{C.9})$$

Optimality is given as follows.

To prove the optimality of three decision variables, it is required to construct the Hessian matrix H, which is given as follows:

$$H = \begin{bmatrix} \frac{\partial^2 E[\text{TPU}(y,B,s)]}{\partial y^2} & \frac{\partial^2 E[\text{TPU}(y,B,s)]}{\partial y\partial B} & \frac{\partial^2 E[\text{TPU}(y,B,s)]}{\partial y\partial s} \\ \frac{\partial^2 E[\text{TPU}(y,B,s)]}{\partial B\partial y} & \frac{\partial^2 E[\text{TPU}(y,B,s)]}{\partial B^2} & \frac{\partial^2 E[\text{TPU}(y,B,s)]}{\partial B\partial s} \\ \frac{\partial^2 E[\text{TPU}(y,B,s)]}{\partial s\partial y} & \frac{\partial^2 E[\text{TPU}(y,B,s)]}{\partial s\partial B} & \frac{\partial^2 E[\text{TPU}(y,B,s)]}{\partial s^2} \end{bmatrix}. \quad (\text{C.10})$$

The sufficient conditions for optimality through the Hessian matrix H are as follows:

$\text{Det}(H_1) < 0, \text{Det}(H_2) > 0$, and $\text{Det}(H_3) < 0$, where H_1, H_2, and H_3 are the following matrixes:

$$H_1 = \left[\frac{\partial^2 E[\text{TPU}(y,B,s)]}{\partial y^2}\right],$$

$$H_2 = \begin{bmatrix} \frac{\partial^2 E[\text{TPU}(y,B,s)]}{\partial y^2} & \frac{\partial^2 E[\text{TPU}(y,B,s)]}{\partial y\partial B} \\ \frac{\partial^2 E[\text{TPU}(y,B,s)]}{\partial B\partial y} & \frac{\partial^2 E[\text{TPU}(y,B,s)]}{\partial B^2} \end{bmatrix}, \quad (\text{C.11})$$

$$H_3 = \begin{bmatrix} \frac{\partial^2 E[TPU(y,B,s)]}{\partial y^2} & \frac{\partial^2 E[\text{TPU}(y,B,s)]}{\partial y\partial B} & \frac{\partial^2 E[\text{TPU}(y,B,s)]}{\partial y\partial s} \\ \frac{\partial^2 E[\text{TPU}(y,B,s)]}{\partial B\partial y} & \frac{\partial^2 E[\text{TPU}(y,B,s)]}{\partial B^2} & \frac{\partial^2 E[\text{TPU}(y,B,s)]}{\partial B\partial s} \\ \frac{\partial^2 E[\text{TPU}(y,B,s)]}{\partial s\partial y} & \frac{\partial^2 E[\text{TPU}(y,B,s)]}{\partial s\partial B} & \frac{\partial^2 E[\text{TPU}(y,B,s)]}{\partial s^2} \end{bmatrix}.$$

Thus,

$$\begin{aligned}
\operatorname{Det}(H_1) &= \frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial y^2} = -\frac{B^2[h+b+\theta\widehat{h}] + 2(\pi-\rho s^n)[K+\theta\widehat{K}]}{y^3 w_1(1-E[x])} < 0,\\
\operatorname{Det}(H_2) &= \left(\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial y^2}\right)\left(\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial B^2}\right) - \left(\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial y\partial B}\right)^2 > 0,\\
\operatorname{Det}(H_2) &= \left(-\frac{B^2[h+b+\theta\widehat{h}] + 2(\pi-\rho s^n)[K+\theta\widehat{K}]}{y^3 w_1(1-E[x])}\right)\left(-\frac{h+b+\theta\widehat{h}}{y w_1(1-E[x])}\right)\\
&\quad - \left(\frac{B[h+b+\theta\widehat{h}]}{y^2 w_1(1-E[x])}\right)^2 > 0,\\
\operatorname{Det}(H_2) &= \left(\frac{2(\pi-\rho s^n)[K+\theta\widehat{K}]}{y^4 w_1^2(1-E[x])^2}\right)(h+b+\theta\widehat{h}) > 0,\\
\operatorname{Det}(H_3) &= \left(\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial y^2}\right)\left(\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial B^2}\right)\left(\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial s^2}\right)\\
&\quad + \left(\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial B\partial y}\right)\left(\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial s\partial B}\right)\left(\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial y\partial s}\right)\\
&\quad + \left(\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial s\partial y}\right)\left(\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial y\partial B}\right)\left(\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial B\partial s}\right)\\
&\quad - \left(\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial y\partial s}\right)\left(\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial B^2}\right)\left(\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial s\partial y}\right)\\
&\quad - \left(\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial B\partial s}\right)\left(\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial s\partial B}\right)\left(\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial y^2}\right)\\
&\quad - \left(\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial s^2}\right)\left(\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial y\partial B}\right)\left(\frac{\partial^2 E[\mathrm{TPU}(y,B,s)]}{\partial B\partial y}\right) < 0.
\end{aligned} \tag{C.12}$$

The abovementioned analysis proves that the function $E[TPU(y,B,s)]$ is strictly concave and shows that the Hessian is negative-definite. Therefore, the optimal solution for the decision variables (y^*, B^*, s^*) exists and maximizes the expected total profit.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Research Article

Effect of Optimal Subsidy Rate and Strategic Behaviour of Supply Chain Members under Competition on Green Product Retailing

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This paper investigates the impact of the subsidy and horizontal strategic cooperation on a green supply chain where two competing manufacturers distribute substitutable green products through exclusive retailers. Models are formulated in three-stage game structures in five different scenarios, where the government organization determines optimal subsidy by pursuing social welfare maximization. Both manufacturers invest in improving green quality levels of products. The study aims to explore the advantage of vertical integration and strategic collusion from the perspective of green supply chain practice in the presence of subsidy. The key contributions from the present study indicate that under competition, members of both supply chains are able to receive higher profits through horizontal collusion, but green quality levels of the product remain suboptimal. If upstream manufacturers cooperate, government subsidy does not necessarily improve product quality level, and the amount of government expenditure increased substantially. By comparing outcomes where members are vertically integrated with scenarios where members make strategic collusion, we found that the former might outperform by later. Cross-price sensitivity appears as a significant parameter affecting supply chain members' performance and the amount of government expenditure. Cooperation between members at the horizontal level is a more robust strategic measure than vertical integration if consumers are highly price-sensitive.

1. Introduction

In the past few decades, with increasing environmental awareness, issues on investment in green quality improvement have been regarded as one of the significant solutions to sustainability issues [1, 2]. A recent global survey by Accenture reported that 83% of consumers consider product greenness when making purchasing decisions [3]. Spurred by this market force, an increasing number of manufacturers make green technology investments in their production process to fulfill the demands of environmentally concerned consumers, gain competitiveness, and strengthen their reputations. However, the cost of investment in green technology is usually substantial, which is viewed as one of the main barriers to green production. Manufacturers need to consider an explicit trade-off between the pros and cons of several issues associated with an investment in green technology. Particularly, investment decisions become more complicated while trading with substitutable products, and in such a situation, a manufacture's decision will be further affected by the rival manufacturer's decisions and even the strategic decision of downstream retailers. In this study, we analyze the equilibrium of two competing supply chains (SCs), each of which consists of a single manufacturer, selling its products exclusively through a single retailer. Upstream manufacturers determine wholesale prices and investment in improving green quality levels, and downstream retailers determine market prices. The proposed SC structures fit numerous industries such as gasoline, soft drink, garments, footwear, cars, and electronics accessories, where a manufacturer trades with an exclusive retailer. The demand for substitute products has a negative correlation and creates a rivalry between two competing SCs. Therefore, the first research question addressed by the study is as follows: how does the investment efficiency under competition affect the downstream retailers and consumers?

In pragmatic environment, "collusion" between two competing manufacturers is not rare [4]. For example, stable collaborative relationships between Apple and Samsung [5, 6] justify this possibility. Upstream manufacturers cooperate for many reasons, such as to increase joint market size, to develop products with new features to protect the present and future share of the market, and to conquer a larger share of the market [7]. In the existing literature, researchers have explored empirically identified scenarios where manufacturers may collude [8, 9]. Nocke and White [10] examined the condition when the collusive effect between upstream manufacturers improves the utility of the overall distribution channel. Piccolo and Reisinger [11] compared the equilibrium price of two competing SCs where two colluding manufacturers maximize joint profits and found that exclusive territories sometime favor collusion. Huang [12] studied the effect of downstream collusion and reported that it can cause a detrimental effect on the performance of the upstream manufacturers. In some recent studies, the strategic aspect of collusion between upstream manufacturers [13] or downstream collusion between retailers [14] is also documented. Therefore, the second research question addressed by the study is as follows: can strategic collusion between manufacturer-manufacturer and retailer-retailer enhance the overall performance of each SC and sustainability? And if so, is it beneficial between upstream members or downstream members?

In green supply chain management, government organizations play an important role by providing subsidy to supply chain members [15–26]. However, the impact of government subsidy on manufacturers' green technology investment decisions and strategic collusion between horizontal members under chain-to-chain competition have not been well understood in the literature. Therefore, the third intriguing research question arises in that consequence is as follows: does the government expenditure increase in the presence of strategic collusion?

Our work complements the literature on subsidizing manufacturers selling substitutable products under competition in different strategic settings. We consider five scenarios to investigate the characteristics of optimal decisions under horizontal collusion and vertical integration on each firm's profit, green quality levels, and social welfare. In Scenario UDLD, two upstream manufacturers make wholesale pricing and green quality decisions and two retailers set their respective retail prices by maximizing their respective profit functions [26]. We consider the scenario as a benchmark. In Scenario UCLD, two upstream manufacturers make wholesale pricing and green quality by maximizing total upstream profits, not individual profits, and then two retailers set their respective retail prices by maximizing their individual profits. This game structure is similar to the "collusion" game as discussed by Bian et al. [4]. In Scenario UDLC, two upstream manufacturers make their decision by maximizing their respective profits, but two retailers set retail prices by maximizing the sum of downstream profits. Finally, in Scenario UCLC, both two upstream manufactures and downstream retailers make their decision by maximizing the sum of upstream and downstream profits, respectively. Therefore, Scenarios UCLD, UDLC, and UCLC represent all possible options of horizontal collusion under competition. Finally, Scenario CC is considered where the manufacturer and retailer in each SC are vertically integrated [27]. This will assist us in finding the answer to our third research question: do the outcomes under horizontal collusion outperform the decision attained under vertical integration?

The main insights of our research are summarized as follows: first, to some extent, a dominant equilibrium strategy is for both manufacturers to make upstream collusion; however, they can encounter a prisoner's dilemma. Increasing consumer cross-price elasticity can intensify the competition to a point where eventually both manufacturers are worse off. While both manufacturers invest to improve green quality levels and make collusion, they try to upsurge the wholesale prices to compensate investment costs, which in turn raises retail prices and worsens double marginalization. Second, higher government subsidy does not always ensure a higher green quality level. Due to higher price-setting power, profits for each firm and government expenditure increase under collusion, but social welfare and green quality levels will be always less. Finally, under SC competition, researchers largely highlighted the benefits of vertical integration between SC members under competition, but we prove that horizontal collusion can be a useful strategic option for competing SC members to improve their respective profits.

1.1. Literature Review. Our study is closely related to three different streams of research such as (i) decision under supply chain competition, (ii) supply chain decision under price and green quality level-sensitive demand, and (iii) government subsidy on green technology investment.

Early seminal work in decisions under SC competition is done by McGuire and Staelin [28], Choi [29], and Moorthy [30], where pricing behaviours of two competing SCs are explored where there are two manufacturers, each sells substitutable products through an independent retailer. However, in the last decades, this research stream is gaining priority form the research community. In this direction, one can categorize the number of publishing articles into two groups. In the first groups, researchers mainly focused on the effect the information asymmetry in SC competition [31], Ai et al. [32], Bian et al. [33], Lee [34], and others. The authors explored optimal decisions mostly in a single game structure under price-dependent demand and explored the effect of information asymmetry in decentralized and centralized settings. In contrast, a group of researchers studied optimal decisions in various game models under symmetric information, and our work is closely related to this stream of research [35]. For example, Wu and Mallik [36] discussed the optimal decisions of competing SCs where members separately maximize their respective profits non-cooperatively, members of one SC imply decentralized decision, and others imply centralized decision, and compared optimal decision by benchmarking the centralized decision. Zhou and Cao [27] studied the optimal decision of two competing SCs under the price and display quantity dependent demand. The authors derived optimal decisions in three decision-making scenarios, (1) two members in each decentralized SC set decision under the "manufacturer-Stackelberg" game, (ii) manufacturer and corresponding exclusive retailer in each SC are agreeing to form "an integrated firm" by negotiation, and (iii) one of the SCs implies integrated decision and others strict with the decentralized decision. By comparing profits, the authors conclude that the relatively fierce price competition between two SCs may eliminate the negative effects of double marginalization on the overall SC profit. Similar to Zhou and Cao [27], Li and Li [37] explored optimal decisions for a sustainable SC in three different game structures. By comparing equilibrium, the authors found that the vertical integration of two competing SCs can be beneficial when the product quality competition degree is low and decentralized SC decision can be more preferable if competition degree is more fierce. In this direction, Wang et al. [38] also compared the equilibriums of two competing SCs in three game structures, where the manufacturer acts as a Stackelberg leader, the retailer act as a Stackelberg leader, and both the manufacturer and the retailer act as Bertrand–Nash competitors. However, the work by Zhu and He [39] is different, and instead of comparing optimal decisions under different games, the authors derive optimal decisions in single retailer-single manufacturer, single manufacturer-two retailers, and two manufacturers-two retailers settings. The authors reported that price competition between two retailers could increase the equilibrium product qualities. Seyedhosseini et al. [40] analyzed optimal decision under four game structures, namely, Stackelberg-Cournot, Stackelberg-Collusion, Nash-Cournot, and Nash-Collusion. The authors found that the Nash-Collusion game structure can yield maximum supply chain profits. Bian et al. [4] also studied a model with a different context, where each manufacturer can distribute their products through either a single retailer or both retailers. The authors found that SC members prefer a single distribution channel if the products are substitutable to a sufficient extent. However, the aforementioned studies do not take into consideration of SC members' option collusion formation and government subsidy. Our study contributes to this stream of research as we investigate horizontal and vertical cooperation in competing SCs.

Another stream of research related to the study is the study where the authors studied the characteristics of supply chain equilibrium where the demand function is influenced by the green degree of the products and retail price [15, 41–48]. This group of researchers mainly focused on the equilibrium decision under the various game structures and on the way to improve the performance of supply chain members through various coordination mechanisms under a single manufacturer-single retailer setting. However, we study the optimal decision under SC competition. In this direction, our work is closely related to Li and Li [37] and Yang et al. [49], where the authors explored the characteristics of optimal decisions under competition. He et al. [50] studied the impact of subsidy in a dual-channel closed-loop supply chain and found that higher subsidy can benefit consumers but not necessarily improve environmental performance. Li and He [51] investigated the pricing and information disclosure strategies in a green supply chain and found that a manufacturer can receive higher benefit by disclosing information. Sana [52] pointed out the strategic advantage of selling green products in the presence of subsidy and reported subsidy is important to encourage manufacturers to trade with green products. However, the authors ignored the combined effect of retail prices, collusion behaviour of SC members, and government subsidy in a two manufacturers-two retailers supply chain model. Therefore, our perspective differs from the existing literature.

Finally, we study the effect of government subsidy on green supply chain practices. To support green product manufacturing and promote consumption, government organizations in different countries design various subsidy and tax policies [45, 46, 53, 54]. In the existing literature, researchers studied the impact of several forms of subsidy, such as direct subsidy to manufacturers [15–17, 55], to retailer [55, 56], to consumers [18, 48, 55, 57, 58], to both retailers and manufacturers [20], to manufacturers and consumers [59], and in others way to improve environmental sustainability. However, the literature on the influence of subsidy in SC competition is sparse. In this study, we consider two competitive SCs, each of which consists of one manufacturer and one retailer and discuss the scenarios where two upstream manufacturers receive subsidies based on green technology investment. The optimal subsidy rate is determined by maximizing social welfare function in each of the five scenarios [56]. Comparative analysis among optimal decision in five scenarios can help policymakers to understand how strategic cooperation affects the green quality of the product and explore the trade-off between expenditure and social welfare optimization goal.

2. Model Settings

We consider two ex-ante symmetric supply chains, indexed by $i = 1, 2$, and each consists of one manufacturer M_i and corresponding one exclusive retailer R_i. The two manufacturers sell substitutable products and compete in the end-customer market under price and green quality level-sensitive demand. We consider five different scenarios as presented in Figure 1.

First, Scenario UDLD is considered as benchmark, where members in both SCs take decentralized decision [27, 37, 60]. The next three Scenarios UCLD, UDLC, and UCLC are considered to analyze the effect of horizontal collusion. In Scenario UCLD, two manufacturers optimize the sum of upstream profits, but two downstream retailers optimize their respective profits [4]. In Scenario UDLC, two manufacturers optimize their respective profits, but downstream retailers set retail prices by optimizing total downstream profits. In Scenario UCLC, horizontal and vertical members make their respective decision by maximizing the sum of upstream and downstream profits.

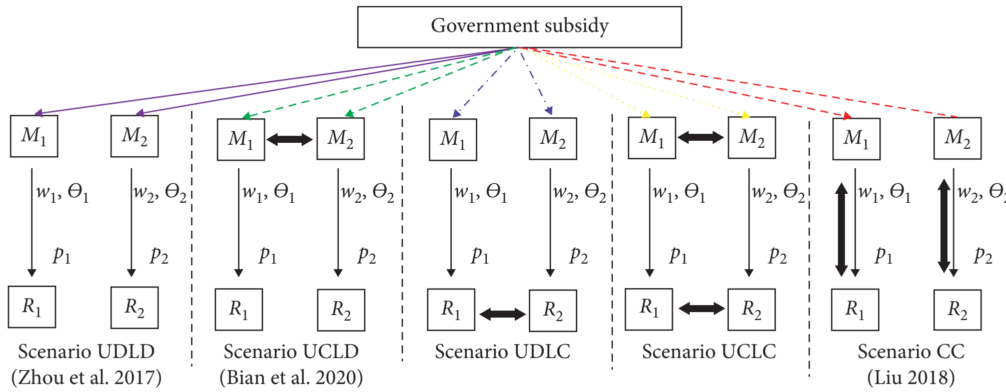

Figure 1: Decision scenarios considered in this study.

Finally, we consider Scenario CC where each SC is vertically integrated [27].

To characterize the demand (D_i^s) for product i, we adopt the following functional forms:

$$D_i^s\left(p_i^s, \theta_i^s\right) = a - p_i^s + \beta p_j^s + \gamma \theta_i^s - \delta \theta_j^s, \quad i = 1, 2, j = 3 - i, \tag{1}$$

where p_i^s and θ_i^s represent market price and green quality level, respectively. Therefore, D_i^s has positive correlations with θ_i^s and p_j^s and negative correlations with θ_j^s and p_i^s. Note that without the influence of prices, the demand function is similar to Li and Li [37] or without the impact of green quality level, it is similar to Bian et al. [4].

Similar to [45], we adopt a quadratic form function $\eta_i \theta_i^{s2}$ to represent the green investment cost, where η_i is the green investment efficiency forM_i. To enable fair comparison among the optimal decisions for different scenarios and for analytical simplicity, we assume $\eta_1 = \eta_2 = \eta$ [56, 61]. For parsimony, we further assume that operational costs for each SC are constant and normalized to zero. We assume that all the parameters are deterministic and evaluate the equilibrium of the five scenarios under symmetric information [16].

To encourage green product manufacturing, the government provides a subsidy on total investment in green technology adaptation, and the government decides the subsidy rate by maximizing social welfare. Therefore, we employ a three-stage game structure. Under competition, the decision about retail prices is taken by retailers, and decisions about wholesale prices and investment to improve product green quality levels are taken by manufacturers. Finally, the government determines the subsidy rate by optimizing the social welfare (SW) function.

For the rest of the paper, we assume that

$$\eta > \max\left\{\frac{\left(28 - 24\beta^2 + 5\beta^4\right)(\gamma - \delta)^2}{2(2-\beta)^2\left(4 - \beta + 2\beta^2\right)^2}, \frac{(7 - 5\beta)(\gamma - \delta)^2}{8(2-\beta)^2(1-\beta)}, \frac{(7-\beta)(\gamma - \delta)^2}{32(1-\beta)}\right\}. \tag{2}$$

Not only does the condition ensure that there must be a threshold for the efficiency limit of the manufacturers' green technology investment, but also the condition is sufficient for many subsequent analytical results related with the existence of optimal decision in five scenarios. Moreover, under the above assumptions, equilibrium wholesale prices, green quality levels, retail prices, and profits are always positive.

Based on the demand function in equation (1) and assumptions, the profit functions for manufacturers, retailers, and SW function for government organization are given as

$$\Pi_{ri}^s = \left(p_i^s - w_i^s\right) D_i^s, \quad i = 1, 2, \tag{3}$$

$$\Pi_{mi}^s = w_i^s D_i^s - \eta\left(1 - \alpha^s\right)\left(\theta_i^s\right)^2, \quad i = 1, 2, \tag{4}$$

$$\mathrm{SW}^s = \sum_{i=1}^{2} \Pi_{ri}^s + \sum_{i=1}^{2} \Pi_{mi}^s + \mathrm{CS} - \sum_{i=1}^{2} \eta \alpha^s \left(\theta_i^s\right)^2. \tag{5}$$

Similar to the existing study, we consider the impact of profits of both SCs, the members' and consumer surplus (CS), and total government expenditure on the social welfare function [62]. For each scenario, we determined subsidy rate by optimizing SW. We summarize notations used to distinguish optimal decisions under five scenarios in Table 1.

3. Model and Decision-Making

In this section, we first characterize optimal decisions in four scenarios and then explore the impact of collusion and subsidy.

3.1. Optimal Decisions in Scenarios UDLD and UCLD. In these two scenarios, two downstream retailers take their decision independently, but two upstream manufacturers take their decision independently in Scenario UDLD, and

Table 1: Notation and descriptions.

	Descriptions
Indices	
Subscripts i	Index for ith SC, $i \in \{1, 2\}$
Subscript r, m	r represents retailers, and m represents manufacturers
Superscript s	Different scenarios in the presence of subsidy, $s \in$ {udld, udlc, ucld, uclc, cc}
Subscripts n	n is used additionally to represent decision in the absence of subsidy
Parameters	
a	Intrinsic market demand for each SC
β	The cross-price sensitivity of consumers between two products, $\beta \in [0, 1)$
γ	The green quality level sensitivity of consumers with that SC, $\gamma \geq 0$
δ	The green quality level sensitivity of consumers with that of rival SC, $0 \leq \delta < \gamma$
η	The investment efficiency of each manufacturer, $\eta > 0$
Variables	
p_i^s	Market price per unit of ith product
w_i^s	Wholesale price per unit of ith product
θ_i^s	Green quality level of ith product
α^s	Subsidy rate
Π_{ri}^s	Profit for the ith retailer
Π_{mi}^s	Profit for the ith manufacturer
Π_c^s	$\Pi_c^s = \sum_i^2 \Pi_{ri}^s + \sum_i^2 \Pi_{mi}^s$, i.e., total profit for two SCs in scenario s
SW^s	Social welfare
TS^s	Total government subsidy
Q_i^s	Sales volume of ith product

jointly in Scenario UCLD, respectively. Scenario UDLD is discussed commonly in the literature [4, 27, 63], and Scenario UCLD represents "collusion" as stated by Bian et al. [4]. After exploring the equilibrium of two scenarios, we will try to find how the strategic behaviour of upstream members affects the product green quality and profits for two retailers. The sequence of the decision in Scenarios UDLD and UCLD is as presented follows.

Stage 1. The government organization decides the subsidy rate $(\alpha^{\mathrm{udld}}/\alpha^{\mathrm{ucld}})$ to maximize $(\mathrm{SW}^{\mathrm{udld}}/\mathrm{SW}^{\mathrm{ucld}})$.

Stage 2. Two upstream manufacturers quote wholesale prices (w_i^{udld}) and green quality levels $(\theta_i^{\mathrm{udld}})$ by maximizing their respective profits in Scenario UDLD. However, two manufacturers quote wholesale prices (w_i^{ucld}) and green qualities $(\theta_i^{\mathrm{ucld}})$ by maximizing sum of upstream profits in Scenario UCLD.

Stage 3. Two downstream retailers choose market prices (p_i^{udld}) and (p_i^{ucld}) in Scenario UDLD and UCLD, respectively.

Therefore, the optimization problem in Scenario UDLD is as follows:

$$\begin{cases} \max_{\alpha^{\mathrm{udld}}} \mathrm{SW}^{\mathrm{udld}}, \\ \begin{cases} \max_{\left(w_1^{\mathrm{udld}},\theta_1^{\mathrm{udld}}\right)} \Pi_{m_1}^{\mathrm{udld}} + \max_{\left(w_2^{\mathrm{udld}},\theta_2^{\mathrm{udld}}\right)} \Pi_{m_2}^{\mathrm{udld}}, \\ \left\{ \max_{p_1^{\mathrm{udld}}} \Pi_{r_1}^{\mathrm{udld}} + \max_{p_2^{\mathrm{udld}}} \Pi_{r_2}^{\mathrm{udld}}. \right. \end{cases} \end{cases} \tag{6}$$

First, two retailers' responses for retail prices are determined by assuming decision variables of upstream members are given. Solving the first-order conditions $(\mathrm{d}\Pi_{ri}^{\mathrm{udld}}/\mathrm{d}p_i^{\mathrm{udld}}) = 0, i = 1, 2,$ simultaneously, optimal responses on retail prices are obtained as follows:

$$p_i^{\mathrm{udld}} = \frac{a(2+\beta) + 2w_i^{\mathrm{udld}} + w_j^{\mathrm{udld}}\beta + (2\gamma - \beta\delta)\theta_i^{\mathrm{udld}} + (\beta\gamma - 2\delta)\theta_j^{\mathrm{udld}}}{4-\beta^2}, \quad i = 1, 2, j = 3 - i. \tag{7}$$

From the above expression, we observe that the wholesale prices of both products have a positive correlation with retail prices. The retail price of any product would increase with the increase of wholesale prices of both products. Because $2\gamma > \beta\delta$, we observe that retail price increased with the green quality level of that product. Moreover, if $\beta\gamma > 2\delta$, then retail price increases with the green quality levels of both products. Because $(\mathrm{d}^2\Pi_{\mathrm{ri}}^{\mathrm{udld}}/\mathrm{d}p_i^{\mathrm{udld}2}) = -2 < 0$, i.e., the optimality is ensured. Plugging response for two retailers in equation (4), profit functions for two manufacturers are obtained as follows:

$$\Pi_{\mathrm{mi}}^{\mathrm{udld}} = \frac{w_i^{\mathrm{udld}}\left(a(2+\beta) + w_j^{\mathrm{udld}}\beta - w_i^{\mathrm{udld}}\left(2-\beta^2\right) + (2\gamma-\beta\delta)\theta_i^{\mathrm{udld}} + (\beta\gamma - 2\delta)\theta_j^{\mathrm{udld}}\right)}{4-\beta^2} - \left(1-\alpha^{\mathrm{udld}}\right)\eta\theta_i^{\mathrm{udld}2}, \quad i = 1,2, j = 3-i. \tag{8}$$

To determine optimal response for two manufacturers, we solve the first-order conditions $(\partial\Pi_{\mathrm{mi}}^{\mathrm{udld}}/\partial w_i^{\mathrm{udld}}) = 0$ and $(\partial\Pi_{\mathrm{mi}}^{\mathrm{udld}}/\partial\theta_i^{\mathrm{udld}}) = 0$, simultaneously. On simplification, wholesale prices and green quality levels are obtained as follows:

$$\begin{aligned} w_i^{\mathrm{udld}} &= \frac{2a\left(1-\alpha^{\mathrm{udld}}\right)\left(4-\beta^2\right)\eta}{2\left(1-\alpha^{\mathrm{udld}}\right)(2-\beta)\left(4-\beta-2\beta^2\right)\eta - (\gamma-\delta)(2\gamma-\beta\delta)}, \\ \theta_i^{\mathrm{udld}} &= \frac{a(2\gamma-\beta\delta)}{2\left(1-\alpha^{\mathrm{udld}}\right)(2-\beta)\left(4-\beta-2\beta^2\right)\eta - (\gamma-\delta)(2\gamma-\beta\delta)}, \quad i = 1,2. \end{aligned} \tag{9}$$

From the expressions of green qualities, it is observed that both are decreased as the investment efficiency of two manufacturers η decreased and reverse trend is observed with the subsidy rate α^{udld} (please find Appendix A). Therefore, the green quality levels will be least if $\alpha^{\mathrm{udld}} = 0$ and consumers get benefited from the presence of subsidy. Note that in the absence of a subsidy, there is no need to execute the first stage. We represent the optimal solution in the absence of subsidy in this scenario with the other three scenarios in Table 2, and we use the results in the absence of subsidy as a benchmark. The value of the determinant of the Hessian matrix $(H_{\mathrm{mi}}^{\mathrm{udld}})$ for the profit function of each manufacturer is obtained as follows:

$$H_{\mathrm{mi}}^{\mathrm{udld}} = \begin{vmatrix} \dfrac{\partial^2\Pi_{mi}^{\mathrm{udld}}}{\partial w_i^{\mathrm{udld}2}} & \dfrac{\partial^2\Pi_{\mathrm{mi}}^{\mathrm{udld}}}{\partial w_i^{\mathrm{udld}}\partial\theta_i^{\mathrm{udld}}} \\ \dfrac{\partial^2\Pi_{\mathrm{mi}}^{\mathrm{udld}}}{\partial w_i^{\mathrm{udld}}\partial\theta_i^{\mathrm{udld}}} & \dfrac{\partial^2\Pi_{mi}^{\mathrm{udld}}}{\partial w_i^{\mathrm{udld}}} \end{vmatrix} = \begin{vmatrix} -\dfrac{2\left(2-\beta^2\right)}{\left(4-\beta^2\right)} & \dfrac{2\gamma-\beta\delta}{4-\beta^2} \\ \dfrac{2\gamma-\beta\delta}{4-\beta^2} & 2\left(1-\alpha^{\mathrm{udld}}\right)\eta \end{vmatrix} = \frac{\Theta_1}{\left(4-\beta^2\right)^2}.a. \tag{10}$$

We can see that all the diagonal elements of the above the Hessian matrix $(H_{\mathrm{mi}}^{\mathrm{udld}})$ are negative; therefore, profit function for each manufacturer is concave if $\Theta_1 = 4(1-\alpha^{\mathrm{udld}})(8-6\beta^2+\beta^4)\eta - (2\gamma-\beta\delta)^2 > 0$.

Using the response for the manufacturers and retailers in equation (5), the social welfare function $(\mathrm{SW}^{\mathrm{udld}})$ is obtained as follows:

$$\mathrm{SW}^{\mathrm{udld}} = \frac{2a^2\eta\left[2\left(1-\alpha^{\mathrm{udld}}\right)^2\Phi_1\eta - (2\gamma-\beta\delta)^2\right]}{\left(2\left(1-\alpha^{\mathrm{udld}}\right)(2-\beta)\left(4-\beta-2\beta^2\right)\eta - (\gamma-\delta)(2\gamma-\beta\delta)\right)^2}, \tag{11}$$

where $\Phi_1 = 28 - 24\beta^2 + 5\beta^4$. Therefore, the optimal subsidy rate is obtained by solving $(d\mathrm{SW}^{\mathrm{udld}}/d\alpha^{\mathrm{udld}}) = 0$. On simplification,

$$\alpha^{\mathrm{udld}} = \frac{\left(12+\beta(2-\beta)\left(6-6\beta-5\beta^2\right)\right)\gamma - \left(28-\beta\left(8+3\beta\left(6-\beta-\beta^2\right)\right)\right)\delta}{\Phi_1(\gamma-\delta)}. \tag{12}$$

Table 2: Optimal decisions in the absence of subsidy.

	Scenario UDLDN	Scenario UCLDN	Scenario UDLCN	Scenario UCLCN
w_{in}^j	$(2a(4-\beta^2)\eta/\Delta_{1n})$	$(2a(2-\beta)\eta/\Delta_{2n})$	$(4a\eta/\Delta_{3n})$	$(4a\eta/\Delta_{4n})$
θ_{in}^j	$(a(2\gamma-\beta\delta)/\Delta_{1n})$	$(a(\gamma-\delta)/\Delta_{2n})$	$(a\gamma/\Delta_{3n})$	$(a(\gamma-\delta)/\Delta_{4n})$
p_{in}^j	$(4a\eta(3-\beta^2)/\Delta_{1n})$	$(2a(3-2\beta)\eta/\Delta_{2n})$	$(2a(3-2\beta)\eta/(1-\beta)\Delta_{3n})$	$(6a\eta/\Delta_{4n})$
q_{in}^j	$(2a\eta(2-\beta^2)/\Delta_{1n})$	$(2a(1-\beta)\eta/\Delta_{2n})$	$(2a\eta/\Delta_{3n})$	$(2a(1-\beta)\eta/\Delta_{4n})$
Π_{rin}^j	$(4a^2\eta^2(2-\beta^2)^2/\Delta_{1n}^2)$	$(4a^2(1-\beta)^2\eta^2/\Delta_{2n})$	$(4a^2\eta^2/(1-\beta)\Delta_{3n}^2)$	$(4a^2(1-\beta)\eta^2/\Delta_{4n})$
Π_{min}^j	$(a^2\eta(4(8-6\beta^2+\beta^4)\eta-(2\gamma-\beta\delta)^2)/\Delta_{1n}^2)$	$(a^2\eta/\Delta_{2n})$	$(a^2\eta(8\eta-\gamma^2)/\Delta_{3n}^2)$	$(a^2\eta/\Delta_{4n})$

$\Delta_{1n}=2(2-\beta)(4-\beta-2\beta^2)\eta-(\gamma-\delta)(2\gamma-\beta\delta)$; $\Delta_{2n}=4(2-\beta)\eta-\gamma(\gamma-\delta)$; $\Delta_{3n}=4(2-\beta)(1-\beta)\eta-(\gamma-\delta)^2$; and $\Delta_{4n}=8(1-\beta)\eta-(\gamma-\delta)^2$.

From the above expression of the subsidy rate, we can observe that the subsidy rate is independent η and intrinsic market demand. Note that the SW^{udld} function is also concave with respect to α^{udld}, if $\Delta_1>0$ because

$$\frac{d^2SW^{udld}}{d\alpha^{udld2}}\Big|_{\alpha^{udld}}=-\frac{8a^2\Phi_1^4(\gamma-\delta)^4\eta^2}{(2\gamma-\beta\delta)^2\Delta_1^3}<0, \tag{13}$$

where$\Delta_1=2(2-\beta)^2(4-\beta+2\beta^2)^2\eta-\Phi_1(\gamma-\delta)^2$. In addition,

$$\Theta_1|_{\alpha^{udld}}=\frac{(2\gamma-\beta\delta)\Delta_{11}}{\left(14-5\beta^2\right)(\gamma-\delta)}, \tag{14}$$

where $\Delta_{11}=4(2-\beta)^2(2+\beta)(4-\beta-2\beta^2)\eta-\ (14-5\beta^2)(\gamma-\delta)(2\gamma-\beta\delta)$. Therefore, a unique equilibrium always exists in Scenario UDLD if $\Delta_{11}>0$ and $\Delta_1>0$. By using backward substitution, we summarized the simplified optimal decision in Scenario UDLD in the following proposition.

Proposition 1. *Optimal decision in Scenario UDLD is obtained as follows:*

$$\begin{aligned}
\alpha^{udld}&=\frac{\left(12+\beta(2-\beta)\left(6-6\beta-5\beta^2\right)\right)\gamma-\left(28-\beta\left(8+3\beta\left(6-\beta-\beta^2\right)\right)\right)\delta}{\Phi_1(\gamma-\delta)},\\
w_i^{udld}&=\frac{2a(2-\beta)^2(2+\beta)\left(4-\beta-2\beta^2\right)\eta}{\Delta_1},\\
\theta_i^{udld}&=\frac{a\Phi_1(\gamma-\delta)}{\Delta_1},\\
p_i^{udld}&=\frac{4a(2-\beta)\left(3-\beta^2\right)\left(4-\beta-2\beta^2\right)\eta}{\Delta_1},\\
\Pi_{mi}^{udld}&=\frac{a^2(2-\beta)\left(2-\beta^2\right)\left(4-\beta-2\beta^2\right)\eta\left(4(2-\beta)^2(2+\beta)\left(4-\beta-2\beta^2\right)\eta-\left(14-5\beta^2\right)(\gamma-\delta)(2\gamma-\beta\delta)\right)}{\Delta_1^2},\\
Q_i^{udld}&=\frac{2a(2-\beta)\left(2-\beta^2\right)\left(4-\beta-2\beta^2\right)\eta}{\Delta_1},\\
SW^{udld}&=\frac{2a^2\Phi_1\eta}{\Delta_1},\\
TS^{udld}&=\frac{2a^2\Phi_1(\gamma-\delta)\left((12+(2-\beta)\beta\left(6-6\beta-5\beta^2\right))\gamma-\left(28-\beta\left(8+3\beta\left(6-\beta-\beta^2\right)\right)\right)\delta\right)\eta}{\Delta_1},\\
\Pi_c^{udld}&=\frac{2a^2(2-\beta)\left(2-\beta^2\right)\left(4-\beta-2\beta^2\right)\eta\left(8(2-\beta)\left(3-\beta^2\right)\left(4-\beta-2\beta^2\right)\eta-\left(14-5\beta^2\right)(\gamma-\delta)(2\gamma-\beta\delta)\right)}{\Delta_1^2}.
\end{aligned} \tag{15}$$

Next, we present optimization problem in Scenario UDLC as follows:

$$\begin{cases} \max_{\alpha^{\text{ucld}}} SW^{\text{ucld}}, \\ \begin{cases} \max_{\left(w_i^{\text{ucld}},\theta_i^{\text{ucld}}\right)} \Pi_{m1}^{\text{ucld}} + \Pi_{m2}^{\text{ucld}}, \\ \left\{ \max_{p_1^{\text{ucld}}} \Pi_{r1}^{\text{ucld}} + \max_{p_2^{\text{ucld}}} \Pi_{r2}^{\text{ucld}}. \right. \end{cases} \end{cases} \quad (16)$$

Therefore, upstream manufacturers optimize their sum of profits, instead of individual profits. We present the detailed derivations in Appendix A for the simplicity of the presentation. The equilibrium decision in Scenario is summarized in Proposition 2.

Proposition 2. *Optimal decision in Scenario UCLD is obtained as follows:*

$$\begin{aligned}
\alpha^{\text{ucld}} &= \frac{3(1-\beta)}{7-5\beta}, \\
w_i^{\text{ucld}} &= \frac{4a(2-\beta)^2\eta}{\Delta_2}, \\
\theta_i^{\text{ucld}} &= \frac{a(\gamma-\delta)(7-5\beta)}{\Delta_2}, \\
p_i^{\text{ucld}} &= \frac{4a(2-\beta)(3-2\beta)\eta}{\Delta_2}, \\
\Pi_{ri}^{\text{ucld}} &= \frac{16a^2\left(2-3\beta+\beta^2\right)^2\eta^2}{\Delta_2^2}, \\
\Pi_{\text{mi}}^{\text{ucld}} &= \frac{2a^2(2-\beta)\eta}{\Delta_2}, \\
Q_i^{\text{ucld}} &= \frac{4a(2-\beta)(1-\beta)\eta}{\Delta_2}, \\
SW^{\text{ucld}} &= \frac{2a^2(7-5\beta)\eta}{\Delta_2}, \\
TSG^{\text{ucld}} &= \frac{6a^2(1-\beta)(7-5\beta)(\gamma-\delta)^2\eta}{\Delta_2}, \\
\Pi_c^{\text{ucld}} &= \frac{4a^2(2-\beta)\eta\left(8(2-\beta)(1-\beta)(3-2\beta)\eta-(7-5\beta)(\gamma-\delta)^2\right)}{\Delta_2^2}.
\end{aligned} \quad (17)$$

By comparing results in Propositions 1 and 2, we identify one of the key results of the study which is presented in Proposition 3.

Proposition 3. *In between Scenarios UDLD and UCLD,*

(1) wholesale and retail prices are higher in Scenario UCLD

(2) product green quality levels and SW are higher in Scenario UDLD if $\beta < 0.8477$

(3) sales volume is always higher in Scenario UDLD

We refer to Appendix C for the proof. According to Proposition 3, in the presence of upstream collusion, competing manufacturer members reduce the quality of products. Predictably, the upstream collusion may harm the downstream members because manufacturers have higher price-setting power. The results also reflect that fact. Noticeably, cross-price sensitivity (β) between two SCs is the major parameter affecting the variation. We present comparative analysis, indicating how the optimal decisions behave to changes in the four key parameter values, one at a time. We offer the results formally below in Table 3.

The graphical proof against results in Table 3 is presented in a supplementary document (8) (available here). For graphical validation, we use the following parameters: $a = 300$, $\beta = 0.3$, $\eta = 0.5$, $\gamma = 0.5$, and $\delta = .2$. According to Table 3, one can find that green quality levels, sales volumes, SW, and subsidy rate increase with γ and β and decrease with δ and η. It is sensible that the green quality levels decrease if

Table 3: Characteristics of optimal decision in Scenario UDLD and UCLD.

	α^{udld}	w_i^{udld}	p_i^{udld}	θ_i^{udld}	Q_i^{udld}	Π_{ri}^{ucld}	Π_{mi}^{udld}	SW^{udld}
γ	↑	↑	↑	$\uparrow_g$	↑	↑	$\uparrow_g$	↑
η	0	↓	↓	↓	↓	↓	$\downarrow_n$	↓
δ	↓	↓	↓	↓	↓	↓	$\downarrow_n$	↓
β	$\uparrow_g$	$\uparrow_g$	$\uparrow_g$	$\uparrow_g$	$\uparrow_g$	$\uparrow_g$	$\uparrow_g$	$\uparrow_g$
	α^{ucld}	w_i^{ucld}	p_i^{ucld}	θ_i^{ucld}	Q_i^{ucld}	Π_{ri}^{ucld}	Π_{mi}^{ucld}	SW^{ucld}
γ	0	↑	↑	↑	↑	↑	↑	↑
η	0	↓	↓	↓	↓	↓	↓	↓
δ	0	↓	↓	↓	↓	↓	↓	↓
β	↓	$\uparrow_g$	$\uparrow_g$	↑	↑	$\uparrow_g$	$\uparrow_g$	↑

↓↑ indicates that column variable decrease (increases) with row parameters, and 0 indicates that column variables remain independent. Here, a subscript g is used to indicate a relationship determined graphically.

the manufacturer is not efficient in investment, and consequently, overall performance decreased. On the other hand, in a green-sensitive market, if γ increases, the demand for the product increases. A notable result is that the consumers' cross-price elasticity or sensitivity with the green quality level of other available products is higher than the manufacturers' need to produce products with higher quality. Therefore, competition eventually enforces manufacturers toward sustainability. Because the price of the product also increases, consequently all the members in both SCs have the opportunity to receive higher profits. The results also demonstrate that fact.

3.2. Optimal Decisions in Scenarios UDLC and UCLC. In this subsection, we derive the optimal decisions to pinpoint the influence of downstream collusion under competition. In contrast to the previous section, here both retailers set retail prices that maximize the sum of downstream profits. The sequence of the decision in Scenarios UDLC and UCLC is as follows.

Stage 4. The government organization decides the subsidy rate ($\alpha^{udlc}/\alpha^{uclc}$) to maximize ($SW^{udlc}/SW^{uclc}$).

Stage 5. Two upstream manufacturers quote wholesale prices w_i^{udlc} and green quality levels θ_i^{udlc} by maximizing their respective profits in Scenario UDLC. However, two manufacturers quote wholesale prices w_i^{uclc} and green quality levels θ_i^{uclc} by maximizing sum of upstream profits in Scenario UCLC.

Stage 6. Two retailers choose market prices (p_i^{udlc}) and (p_i^{uclc}) in Scenario UDLC and UCLC, respectively, by maximizing sum of downstream profits.

Now, the optimization problem in Scenario UDLC is presented as follows:

$$\begin{cases} \max_{\alpha^{udlc}} SW^{udlc}, \\ \begin{cases} \max_{\left(w_1^{udlc},\theta_1^{udlc}\right)} \Pi_{m_1}^{udlc} + \max_{\left(w_2^{udlc},\theta_2^{udlc}\right)} \Pi_{m_2}^{udlc}, \\ \left\{ \max_{\left(p_1^{udlc},p_2^{udlc}\right)} \Pi_{r_1}^{udlc} + \Pi_{r_2}^{udlc}. \right. \end{cases} \end{cases} \quad (18)$$

In this scenario, both retailers decide their respective prices that optimize sum of profits for two retailers, i.e., by maximizing $\pi_r^{udlc} = \Pi_{r1}^{udlc} + \Pi_{r2}^{udlc}$. Therefore, the optimal response for two retailers is obtained by solving $(\partial \pi_r^{udlc}/\partial p_i^{udlc}) = 0, i = 1, 2$, simultaneously. After simplification, responses for two retailers on market prices are obtained as follows:

$$p_i^{udlc} = \frac{a(1+\beta) + w_i^{udlc}\left(1-\beta^2\right) + (\gamma - \beta\delta)\theta_i^{udlc} + (\beta\gamma - \delta)\theta_j^{udlc}}{2 - 2\beta^2}, \quad i = 1, 2, j = 3 - i. \quad (19)$$

In contrast with Scenario UDLD, we can observe that retail prices of products increase with the wholesale price of that product only, not other products. Consequently, the downstream collusion reduces the wholesale price differentiation effect of two manufacturers. If $\gamma > \beta\delta$, retail prices of the product increase with green quality levels of that product, and it will increase with the green quality level of other products also if $\beta\gamma > \delta$. In particular, if consumers' cross elasticity with green quality level becomes negligible ($\delta = 0$), then only retail prices of both products will increase with green quality levels of both products. The sum of downstream profit functions is concave because the value of the determinant of the Hessian matrix (H_r^{udlc}) is obtained as follows:

$$H_r^{udlc} = \begin{vmatrix} \dfrac{\partial^2 \pi_r^{uclc}}{\partial p_1^{udlc2}} & \dfrac{\partial^2 \pi_r^{udlc}}{\partial p_1^{udlc} \partial p_2^{udlc}} \\ \dfrac{\partial^2 \pi_r^{udlc}}{\partial p_1^{udlc} \partial p_2^{udlc}} & \dfrac{\partial^2 \pi_r^{udlc}}{\partial p_2^{udlc}} \end{vmatrix} = \begin{vmatrix} -2 & 2\beta \\ 2\beta & -2 \end{vmatrix} = 4\left(1-\beta^2\right) > 0. \quad (20)$$

Moreover, diagonal elements are also negative. Substituting optimal responses for two retailers, profit functions for two manufacturers are obtained as follows:

$$\Pi_{mi}^{\text{udlc}} = \frac{w_i^{\text{udlc}}\left(a - w_i^{\text{udlc}} + w_j^{\text{udlc}}\beta + \gamma\theta_i^{\text{udlc}} - \delta\theta_j^{\text{udlc}}\right)}{2} - \left(1 - \alpha^{\text{udlc}}\right)\eta\theta_{\text{i}}^{\text{udlc2}}, \quad i = 1, 2, j = 3 - i. \tag{21}$$

Therefore, the optimal response for two manufacturers is obtained by solving $(\partial\Pi_{\text{mi}}^{\text{udlc}}/\partial w_i^{\text{udlc}}) = 0$ and $(\partial\Pi_{\text{mi}}^{\text{udlc}}/\partial\theta_i^{\text{udlc}}) = 0$, respectively. After simplification, the optimal responses for two manufacturers are obtained as follows:

$$\begin{aligned} w_i^{\text{udlc}} &= \frac{4a\left(1 - \alpha^{\text{udlc}}\right)\eta}{4\left(1 - \alpha^{\text{udlc}}\right)(2 - \beta)\eta - \gamma(\gamma - \delta)}, \\ \theta_i^{\text{udlc}} &= \frac{a(\gamma - \delta)}{4\left(1 - \alpha^{\text{udlc}}\right)(2 - \beta)\eta - \gamma(\gamma - \delta)}. \end{aligned} \tag{22}$$

To verify concavity, we determine the value of determinant of the Hessian matrix $(H_{\text{mi}}^{\text{udlc}})$ for the profit function of each manufacturer as follows:

$$H_{\text{mi}}^{\text{udlc}} = \begin{vmatrix} \dfrac{\partial^2\Pi_{mi}^{\text{udlc}}}{\partial w_i^{\text{udlc2}}} & \dfrac{\partial^2\Pi_{\text{mi}}^{\text{udlc}}}{\partial w_i^{\text{udlc}}\partial\theta_i^{\text{udlc}}} \\ \dfrac{\partial^2\Pi_{\text{mi}}^{\text{udlc}}}{\partial w_i^{\text{udlc}}\partial\theta_i^{\text{udlc}}} & \dfrac{\partial^2\Pi_{\text{mi}}^{\text{udlc}}}{\partial\theta_i^{\text{udlc2}}} \end{vmatrix} = \begin{vmatrix} -1 & \dfrac{\gamma}{2} \\ \dfrac{\gamma}{2} & -2\left(1 - \alpha^{\text{udlc}}\right)\eta \end{vmatrix} = \frac{\Theta_4}{4}. \tag{23}$$

Because the values of diagonal elements are negative, therefore, the profit function for the manufacturer is also concave if $\Theta_4 = 8(1 - \alpha^{\text{udlc}})\eta - \gamma^2$. Substituting optimal responses for both manufacturers and retailers, the SW^{udlc} function is obtained as

$$\text{SW}^{\text{udlc}} = \frac{2a^2\eta\left(2\left(1 - \alpha^{\text{udlc}}\right)^2(7 - 5\beta)\eta - (1 - \beta)\gamma^2\right)}{(1 - \beta)\left(4\left(1 - \alpha^{\text{udlc}}\right)(2 - \beta)\eta - \gamma^2 + \gamma\delta\right)^2}. \tag{24}$$

Therefore, the optimal subsidy rate is obtained by solving $(d\text{SW}^{\text{udlc}}/d\alpha^{\text{udlc}}) = 0$. On simplification, the subsidy rate is obtained as $\alpha^{\text{udlc}} = 1 - (2(2 - \beta)(1 - \beta)\gamma/(7 - 5\beta)(\gamma - \delta))$. Note that the SW^{ucld} function is concave with respect to α^{ucld}, because

$$\frac{d^2\text{SW}^{\text{udlc}}}{d\alpha^{\text{ucld}_2}}\Big|_{\alpha^{\text{ucld}}} = -\frac{8a^2(7 - 5\beta)^4(\gamma - \delta)^4\eta^2}{(1 - \beta)\gamma^2\Delta_2^3} < 0. \tag{25}$$

Moreover, $\Theta_4|_{\alpha^{\text{udlc}}} = (\gamma\Delta_{31}/4(7 - 5\beta)(\gamma - \delta)) > 0$, if $\Delta_{31} = 16(2 - \beta)(1 - \beta)\eta - \gamma(7 - 5\beta)(\gamma - \delta) > 0$. Therefore, optimal solution always exists if $\Delta_2 > 0$ and $\Delta_{31} > 0$. By using back substitution, we obtain simplified values of optimal decision as presented in Proposition 4.

Proposition 4. *Optimal decision in Scenario UDLC is obtained as follows:*

$$\begin{aligned} \alpha^{\text{udlc}} &= 1 - \frac{2(2 - \beta)(1 - \beta)\gamma}{(7 - 5\beta)(\gamma - \delta)}, \\ \theta_i^{\text{udlc}} &= \frac{a(7 - 5\beta)(\gamma - \delta)}{\Delta_2}, \\ w_i^{\text{udlc}} &= \frac{8a(2 - \beta)(1 - \beta)\eta}{\Delta_2}, \\ p_i^{\text{udlc}} &= \frac{4a(2 - \beta)(3 - 2\beta)\eta}{\Delta_2}, \\ \Pi_{ri}^{\text{udlc}} &= \frac{16a^2(2 - \beta)^2(1 - \beta)\eta^2}{\Delta_2^2}, \\ \Pi_{\text{mi}}^{\text{udlc}} &= \frac{2a^2(2 - \beta)(1 - \beta)\eta(16(2 - \beta)(1 - \beta)\eta - (7 - 5\beta)\gamma(\gamma - \delta))}{\Delta_2^2}, \\ Q_i^{\text{udlc}} &= \frac{4a(2 - \beta)(3 - 2\beta)\eta}{\Delta_2}, \\ \text{SW}^{\text{udlc}} &= \frac{2a^2(7 - 5\beta)\eta}{\Delta_2}, \\ \text{TS}^{\text{udlc}} &= \frac{2a^2(7 - 5\beta)(\gamma - \delta)((1 + \beta)(3 - 2\beta)\gamma - (7 - 5\beta)\delta)\eta}{\Delta_2^3}, \\ \Pi_c^{\text{udlc}} &= \frac{4a^2(2 - \beta)(1 - \beta)\eta(8(2 - \beta)(3 - 2\beta)\eta - (7 - 5\beta)\gamma(\gamma - \delta))}{\Delta_2^2}. \end{aligned} \tag{26}$$

Next, we present optimization problem in Scenario UCLC as follows:

$$\begin{cases} \max_{\alpha^{\text{uclc}}} \text{SW}^{\text{uclc}}, \\ \begin{cases} \max_{\left(w_i^{\text{uclc}}, \theta_i^{\text{uclc}}\right)} \Pi_{m_1}^{\text{uclc}} + \Pi_{m_2}^{\text{uclc}}, \\ \left\{ \max_{p_i^{\text{uclc}}} \Pi_{r_1}^{\text{uclc}} + \Pi_{r_2}^{\text{uclc}}. \right. \end{cases} \end{cases} \tag{27}$$

Therefore, upstream manufacturers optimize their sum of profits, instead of individual profits. We presented the detailed derivations in Appendix B for the simplicity of the presentation. The equilibrium decision in Scenario UCLC is summarized in 5.

Proposition 5. *Optimal decision in Scenario UCLC is obtained as follows:*

$$\alpha^{uclc} = \frac{3-\beta}{7-\beta},$$
$$w_i^{uclc} = \frac{16a\eta}{\Delta_2},$$
$$\theta_i^{uclc} = \frac{a(\gamma-\delta)}{\Delta_3},$$
$$p_i^{uclc} = \frac{24a\eta}{\Delta_3},$$
$$\Pi_{ri}^{uclc} = \frac{64a^2(1-\beta)\eta^2}{\Delta_3},$$
$$\Pi_{mi}^{uclc} = \frac{4a^2\eta}{\Delta_3}, \tag{28}$$
$$Q_i^{uclc} = \frac{8a(1-\beta)\eta}{\Delta_3},$$
$$SW^{uclc} = \frac{2a^2(7-\beta)\eta}{\Delta_3},$$
$$TG^{uclc} = \frac{2a^2(7-\beta)(3-\beta)(\gamma-\delta)^2\eta}{\Delta_3},$$
$$\Pi_c^{uclc} = \frac{8a^2\eta\left(48(1-\beta)\eta-(7-\beta)(\gamma-\delta)^2\right)}{\Delta_3^2}.$$

Now, comparing results in Propositions 4 and 5, we obtain another key result of the study as presented in Proposition 6.

Proposition 6. *In between Scenarios UDLC and UCLC,*

(1) wholesale and retail prices are higher in Scenario UCLC

(2) product green quality levels, SW, and sales volumes are always higher in Scenario UDLC

We refer to Appendix D for the proof of Proposition 6, and we observe that results are similar to Proposition 3. Combining results in the above six propositions, we conclude that the strategic collusion can reduce the green quality levels of products under competition. We present a comparative analysis of decision variables to four key parameters in Scenarios UDLC and UCLC in Table 4.

Except for the subsidy rate, the trends of the optimal decisions remain similar in four scenarios. The cross-price elasticity is the major parameter mostly responsible for the variation of government subsidy rate. In all four scenarios, the subsidy rate in independent from investment efficiency for the manufacturers. Note that an increase in β or δ increases the competitive gap between two manufacturers. Because an increase in a manufacturers' capability compared to the competitor usually prompts a relative variation in the wholesale price or green quality level in a reverse way, the result also reflects that fact.

As noted earlier, the Table 2 below summarizes optimal decision in the absence of subsidy.

In the next section, we use the results to evaluate the effect of subsidy.

4. Model Analysis

4.1. Nature of Retail Prices and Green Quality Levels. In the section, we focus mainly on market prices and green quality levels of the products; that is, we analyze optimal decisions to pinpoint the scenario which is favorable from the perspective of consumers. Previously, we prove that $\theta_i^{udld} > \theta_i^{ucld}$ and $\theta_i^{udlc} > \theta_i^{uclc}$. From Propositions 2 and 4, we observe $\theta_i^{ucld} = \theta_i^{udlc}$ and Propositions 1 and 5,

$$\theta_i^{udld} - \theta_i^{uclc} = \frac{2a\beta(2+\beta)(3-2\beta)\left(48-58\beta-11\beta^2+21\beta^3-2\beta^4\right)(\gamma-\delta)\eta}{\Delta_3\Delta_1} > 0. \tag{29}$$

if $\beta < 0.9282$. Similarly, comparing market prices, we obtain

$$p_i^{uclc} - p_i^{udld} = \frac{4a\beta(2+\beta)(3-2\beta)\eta * * * 4(2-\beta)\left(4-\beta-2\beta^2\right)\eta - \left(25-\beta\left(3+9\beta-\beta^2\right)(\gamma-\delta)^2\right)}{\Delta_1\Delta_3} > 0, \tag{30}$$

$$p_i^{ucld} - p_i^{udlc} = 0. \tag{31}$$

TABLE 4: Characteristics of optimal decision in Scenarios UDLC and UCLC.

	α^{udlc}	w_i^{udlc}	p_i^{udlc}	θ_i^{udlc}	Q_i^{udlc}	Π_{ri}^{udlc}	Π_{mi}^{udlc}	SW^{udlc}
γ	↑	↑	↑	↑	↑	↑	$\uparrow_g$	↑
η	0	↓	↓	↓	↓	↓	$\downarrow_g$	↓
δ	↓	↓	↓	↓	↓	↓	$\downarrow_g$	↓
β	↑	↑	$\uparrow_g$	↑	↑	$\uparrow_g$	$\uparrow_g$	↑
	α^{udlc}	w_i^{udlc}	p_i^{udlc}	θ_i^{udlc}	Q_i^{udlc}	Π_{ri}^{udlc}	Π_{mi}^{udlc}	SW_i^{udlc}
γ	0	↑	↑	↑	↑	↑	↑	↑
η	0	↓	↓	↓	↓	↓	↓	↓
δ	0	↓	↓	↓	↓	↓	↓	↓
β	↓	↑	↑	↑	↑	↑	↑	↑

Therefore, using the results in Propositions 3 and 6, we propose the following proposition.

Proposition 7

(1) Optimal GLs satisfy $\theta_i^{uclc} < \theta_i^{ucld} = \theta_i^{udlc} < \theta_i^{udld}$ *if* $\beta < 0.8477$

(2) Optimal retail prices satisfy $p_i^{udld} < p_i^{ucld} = p_i^{udlc} < p_i^{uclc}$

From Proposition 7, we can observe that the quality level of the products is always higher in Scenario UDLD and least in Scenario UCLC. However, the reverse trend is observed for retail prices. Therefore, if members form collusion, then consumers may need to pay more for lower quality products. Note that members form collusion to achieve greater control for their decision, consequently, they have more price-setting power. The downstream retailers are able to set higher market prices, and upstream manufacturers reduce the investment to gain higher profits, subsequently, consumers suffer, and the results reflect that fact. To obtain more detailed insights, we draw the following figure that represents prices in different scenarios and the ratios (θ_i^s/p_i^s)*, which reflects a comparative view about how much consumers need to pay for green quality.*

Some notable insights from Figure 2 are as follows: (i) although the quality of product is always higher in the presence of subsidy, consumers also need to pay more. Therefore, under SC competition, government subsidy might not keep market prices. (ii) The ratio of (θ_i^s/p_i^s) is higher in Scenario UDLD, consequently, without collusion always favorable for consumers. (iii) If upstream and downstream members form collusion, then government subsidy may have the least impact. As we have seen, the ratio in Scenario UCLC may be lower compared to the Scenarios UDLDN or UCLDN, when the members do not receive any subsidy. Overall, a government subsidy to the manufacturers becomes less effective in the perspective of consumers if members form collusion.

4.2. Nature of Profits for Manufacturers and Retailers. Before we look into individual profits for members in each SC, first we compare the sales volumes. From Propositions 2 and 4, we can observe that $Q_i^{ucld} - Q_i^{udlc} = 0$. Moreover,

$$Q_i^{udld} - Q_i^{uclc} = \frac{2a\beta(2+\beta)(3-2\beta)\eta\left(8(2-\beta)(1-\beta)\left(4-\beta-2\beta^2\right)\eta-(1+\beta)\left(2-\beta^2\right)(\gamma-\delta)^2\right)}{\Delta_1\Delta_2} > 0. \quad (32)$$

Therefore, using the results in Propositions 3 and 6, we propose the following proposition.

Proposition 8. *Optimal sales volume satisfies* $Q_i^{uclc} < Q_i^{ucld} = Q_i^{udlc} < Q_i^{udld}$.

The outcome of Proposition 8 is consistent with the previous results. If the consumers need to pay more with lower quality products, then the demand decreases. Therefore, collusion among upstream or downstream members in the presence of subsidy reduces the consumption of products. Note that it is difficult to identify a straightforward relationship among profits for both SC members, till one can observe that the profits for manufacturers are always greater in Scenario UCLD compared to UDLC or UCLC because

$$\begin{aligned}\Pi_{mi}^{ucld} - \Pi_{mi}^{udlc} &= \frac{2a^2(2-\beta)\eta(8(2-\beta)(1-\beta)\beta\eta-(7-5\beta)(\gamma-\delta)(\beta\gamma-\delta))}{\Delta_2^2} > 0,\\ \Pi_{mi}^{ucld} - \Pi_{mi}^{uclc} &= \frac{2a^2\eta***16(2-\beta)(1-\beta)\beta\eta-\beta(1+\beta)(\gamma-\delta)^2}{\Delta_2\Delta_3} > 0,\end{aligned} \quad (33)$$

respectively. Results make sense because both retailers have more power price-setting power under downstream collusion. Therefore, both manufacturers can face a challenge. On the contrary, in the perspective for the retailers,

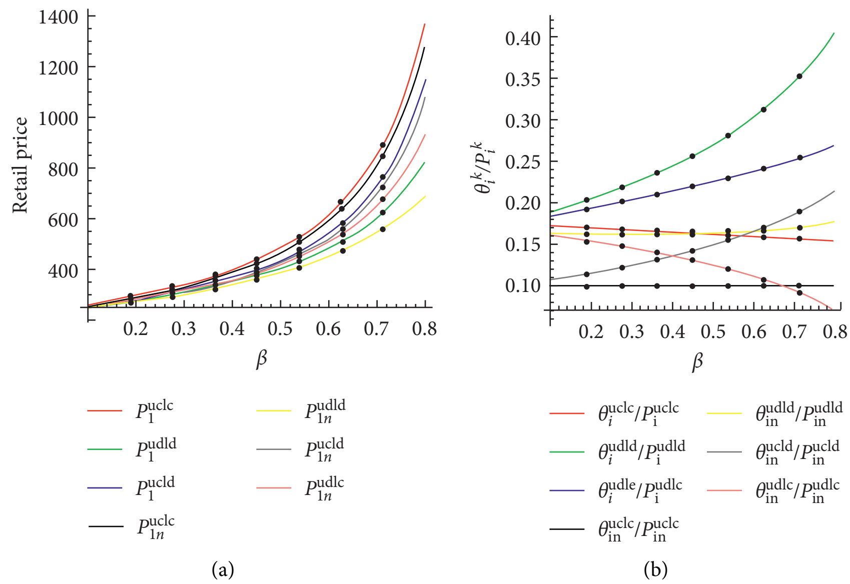

Figure 2: (a) Retail prices and (b) the ratios of green quality level and retail price, in seven scenarios.

$$\Pi_{ri}^{udlc} - \Pi_{ri}^{ucld} = \frac{16a^2\beta\eta^2(2-\beta)^2(1-\beta)}{\Delta_2^2} > 0,$$

$$\Pi_{ri}^{udlc} - \Pi_{ri}^{uclc} = \frac{16a^2\beta(1-\beta)\eta^2\left(16(2-\beta)(1-\beta)\eta - \beta(1+\beta)(\gamma-\delta)^2\right)\left(16(4-\beta)(2-\beta)(1-\beta)\eta + (28-(19-\beta)\beta)(\gamma-\delta)^2\right)}{\Delta_2^2\Delta_3^2} > 0. \tag{34}$$

Consequently, both retailers receive lower profits if upstream manufacturers maximize their joint profits. The graphical representation of profits for retailers and manufacturers and the total amount of government expenditure in four different scenarios are presented in the figure below:

Figure 3 demonstrates the following key outcomes: (i) as expected, both manufacturers prefer competition at the downstream level and gain higher benefit in Scenario UCLD or UDLD. The above figure reflects that fact. (ii) Noticeably, both retailers prefer competition at upstream level and receive higher profits in Scenario UDLC or UDLD. (iii) Scenario UCLC remains dominated by other three, which is in line with Propositions 7 and 8. (iv) Most importantly, higher government expenditure might not yield a higher green quality product. It can be observed that manufacturers receive a higher amount of subsidy in Scenario UCLD, but product quality always remains at the highest level in Scenario UDLD.

Therefore, members in the competing SCs face prisoner's dilemma. Through collusion, they can achieve profit maximization objective, but they need to trade lower quality product. Till, a region exists that represents unique preference, i.e., Scenario UDLD, where all the members can receive higher profits if cross-price elasticity and green level sensitivity are higher. However, this occurs due to the presence of this subsidy. Note that under the value of the same parameter, two manufacturers always receive higher profits in Scenario UCLDN, and retailers receive higher profits in Scenario UDLCN or UCLDN as presented in Figure 4.

From the above Figure 4, we can observe that competing SC members need to change their strategic collaboration decision with their competitors in the presence of subsidy. Downstream retailers can prefer strategic collusion between upstream members in the absence of subsidy, which is not true previously. Therefore, whether to make collusion with rivals at the horizontal level is influenced by government intervention. Next, we examine the optimal decision from the perspective of government organizations.

4.3. *Nature of Social Welfare and Government Subsidy.*

First, we identify the scenario where SW reaches at higher level. From Propositions 2 and 4, we can observe that $SW^{ucld} - SW^{udlc} = 0$. Moreover,

$$SW^{udld} - SW^{uclc} = \frac{4a^2\beta\eta^2(2+\beta)(3-2\beta)\left(48-58\beta-11\beta^2+21\beta^3-2\beta^4\right)}{\Delta_1\Delta_3} > 0. \tag{35}$$

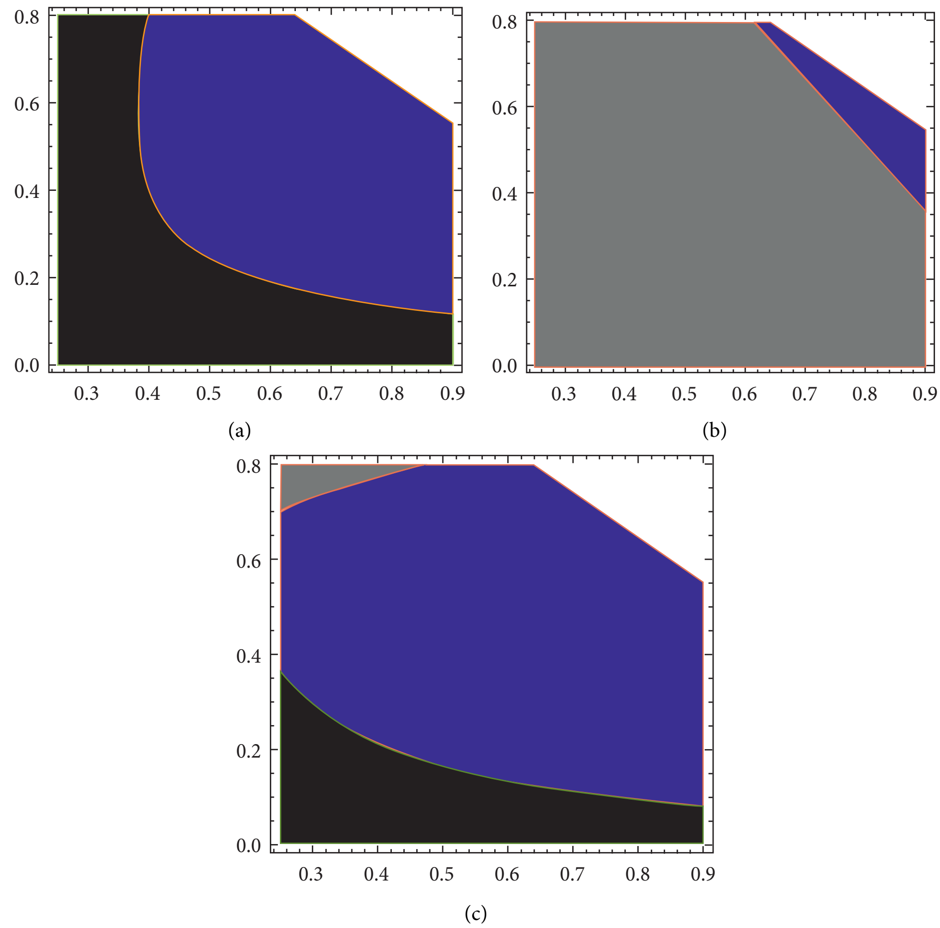

FIGURE 3: (a) Profits for manufacturers, (b) profits for retailers, and (c) total government subsidies (blue: UDLD $\geq$ max{UCLD, UDLC, UCLC}; red: UCLC $\geq$ max{UCLD, UDLC, UDLD}; black: UCLD $\geq$ max{UDLD, UDLC, UCLC}; gray: UDLC $\geq$ max{UCLC, UCLD, UDLD}; white: no feasible region for $\beta \in (0, 0.8)$ and $\gamma \in (0, 0.9)$.

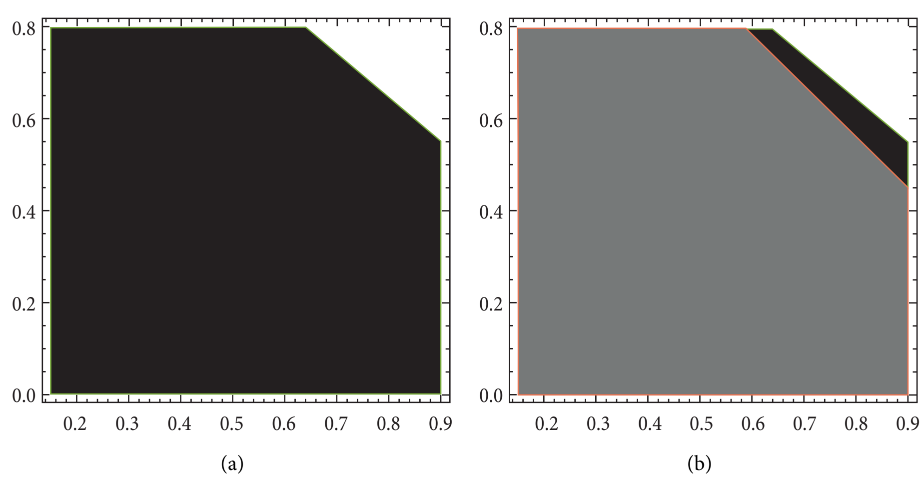

FIGURE 4: (a) Profits for manufacturers and (b) profits for retailers (blue: UDLDN $\geq$ max{UCLDN, UDLCN, UCLCN}; red: UCLCN $\geq$ max {UCLDN, UDLCN, UDLDN}; black: UCLDN $\geq$ max{UDLDN, UDLCN, UCLCN}; gray: UDLCN $\geq$ max{UCLCN, UCLDN, UDLDN}; white: no feasible region for $\beta \in (0, 0.8)$ and $\gamma \in (0, 0.9)$.

If $\beta < 0.9281$, therefore, using the results in Propositions 3 and 6, we propose the following proposition.

Proposition 9. *Optimal SWs satisfy* $SW^{uclc} < SW^{ucld} = SW^{udlc} < SW^{udld}$ *if* $\beta < 0.8477$.

The result makes sense, and green quality level, sales volume, and total subsidy are maximum in Scenario UDLD. Consequently, if members make collusion, then the impact of subsidy reduces. Figure 5 demonstrates the increment of sales volume and green quality level and total subsidy in different scenarios.

From Figure 5, one can find that government subsidy might improve both product consumption and quality, which is anticipated. Noticeably, all three figures demonstrate a clear difference of four scenarios based on the nature of strategic collusion. In Scenarios UCLC and UCLD, product consumption and quality are higher for the lower value of β; however, as β increases, those are higher in UDLD and UDLC. From the perspective of the total expenditure, the figure exhibits a similar trend.

5. An Extension: Two Integrated Supply Chains (CC)

So far, we have analyzed four scenarios to study the impact of horizontal cooperation. To analyze the consequence of vertical cooperation, we derive the optimal decision where members of both SCs implement the vertically integrated decision. In this scenario, both the manufacturers and retailers in each SC are willing to form “an integrated firm” and jointly determine the retail price and green quality level which maximizes the total profit for each SC, and they belong to [27, 37]. In this scenario, profit functions for two competing SCs (Π_{ti}^{cc}, $i = 1, 2$) are obtained as follows:

$$\Pi_{ti}^{cc} = p_i^{cc} D_i^{cc} - \eta\left(1 - \alpha^{cc}\right)\left(\theta_i^{cc}\right)^2, \quad i = 1, 2. \tag{36}$$

Note that if both SCs are integrated, wholesale prices become irrelevant. The derivation of the optimal decision in this scenario remains similar; hence, we omitted the proof. The simplified expressions of decision variables are presented below.

Proposition 10. *Optimal decision in Scenario CC is obtained as follows:*

$$\alpha^{cc} = \frac{\gamma + \beta\gamma - 3\delta}{3\gamma - 3\delta},$$

$$p_i^{cc} = \frac{2a(2-\beta)\eta}{\Delta_{cc}},$$

$$\theta_i^{cc} = \frac{3a(\gamma - \delta)}{\Delta_{cc}},$$

$$Q_i^{cc} = \frac{2a(2-\beta)\eta}{\Delta_{cc}} \Pi_{ti}^{cc} = \frac{a^2(2-\beta)\eta(4(2-\beta)\eta - 3\gamma(\gamma-\delta))}{\Delta_{cc}^2},$$

$$TS^{cc} = \frac{6a^2(\gamma + \beta\gamma - 3\delta)(\gamma - \delta)\eta}{\Delta_{cc}^2},$$

$$SW^{cc} = \frac{6a^2\eta}{\Delta_{cc}},$$

$$\Pi_c^{cc} = \frac{2a^2(2-\beta)\eta(4(2-\beta)\eta - 3\gamma(\gamma-\delta))}{\Delta_{cc}^2}, \tag{37}$$

where $\Delta_{cc} = 2(2-\beta)^2\eta - 3(\gamma-\delta)^2$.

There is a long debate on the issue of whether the members in two competing SCs should cooperate with the vertical members or with their rival, i.e., horizontal members. As mentioned earlier in the work by Zhao and Cao [27], Li and Li [37], and Fang and Shou [64], the authors emphasized on the issue of vertical cooperation by comparing the outcomes in three different models: (i) members make a decentralized decision, similar to Scenario UDLD, (ii) one integrated SC, and one decentralized SC, and (iii) both SCs are decentralized. However, they ignore the impact of government subsidy. Therefore, we find the answer for the following question: is it profitable for SC members to cooperate with their rival instead of members belong to the same SC? First, we compare the green quality levels in Scenarios UDLD and CC, and we obtain

$$\theta_i^{ucld} - \theta_i^{cc} = \frac{2(2-\beta)^2(2+\beta)\left(17\beta + 2\beta^2 - 7\beta^3 - 10\right)\eta}{\Delta_1\Delta_{cc}}. \tag{38}$$

Therefore, one can find that consumers’ cross-price elasticity is the only parameter responsible for the difference, and if $\beta > 0.6525$, then product quality is always less in Scenario CC. Subsequently, we conclude that horizontal cooperation can enhance quality level. Note that the quality level is always higher in UDLD compared to UCLD, ULLC, and UDLC. Similarly, the difference between green quality levels in between Scenarios CC and UCLC is

$$\theta_i^{uclc} - \theta_i^{cc} = \frac{2a\eta(\gamma-\delta)(2+\beta)\left(10 - 13\beta + \beta^2\right)}{\Delta_3\Delta_{cc}} > 0, \tag{39}$$

if $\beta > 0.8211$. Based on the discussion, we proposed the following proposition.

Proposition 11. *(1) Optimal green quality level is higher in Scenario UDLD compared to Scenario CC if* $\beta > 0.6525$.

The graphical representation for the total profits for two competing SCs in five scenarios is presented in Figure 6.

From the above, we can note another important contribution of the study is that members have the option on strategic agreements with their rivals, which can not only ensure higher profits but also can build consumers’ resilience with higher product quality. Figure 6 shows that total profits and green quality level in Scenario CC are always higher compared to Scenario UCLC, but it is not true with if

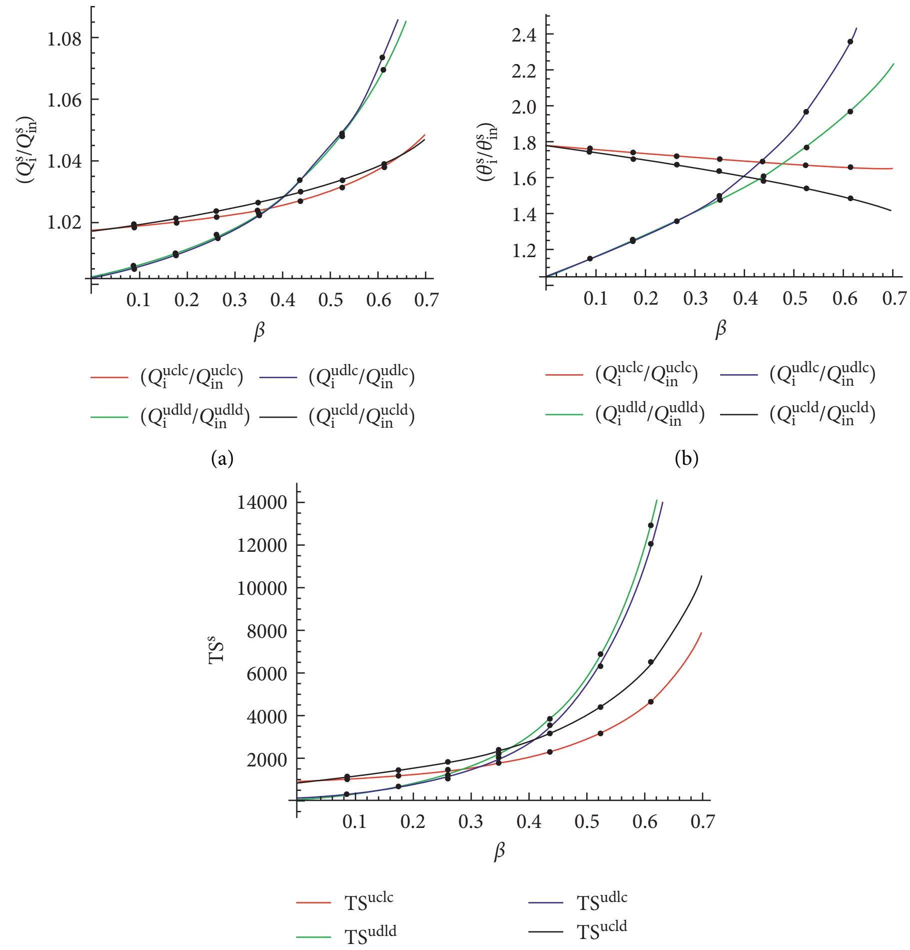

FIGURE 5: (a) Increment in sales volume and (b) product green quality level in the presence of subsidy and (c) total amount of subsidy in Scenarios UDLD, UCLD, UDLC, and UCLC.

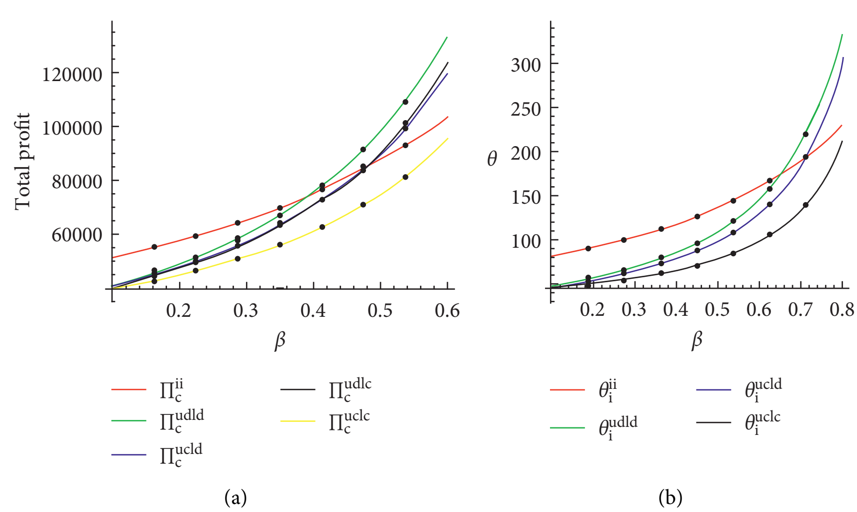

FIGURE 6: (a) Total supply chain profits and (b) green quality level in Scenarios CC, UDLD, UDLC, UCLD, and UCLC.

we see the results with other three scenarios where the competing SC members collaborate with horizontal competitors. From the managerial perspective, in today's business world, we can find many instances where competition increasingly transformed from confrontation to cooperation in achieving economies of scale and range [65].

Continuing in this direction, this study made an effort to analyze such a possibility of collusion and compared the corresponding equilibrium. Our analysis reveals that such cooperation has the potential to improve the overall SC performance which will be an interesting insight to the managers in handling such a business environment.

6. Discussion

This work is motivated by governments' push for sustainability via various subsidy programs. For instance, government organizations subsidize firms that produce energy-efficient appliances in countries such as the USA and Canada, China, Germany, India, and others in different modes such as green credit mode, manufacture subsidy mode, and sales subsidy mode. In this context, supply chain members aim to maximize their respective profits, while the government emphases on measuring its impact on social welfare, consumer surplus, environmental benefit, and other goals. In the existing literature, researchers also highlighted this issue form various perspectives; however, the literature on strategic cooperation under government subsidy is sparse. Our findings can provide some guidance for the government and supply chain members to comprehend regarding their action. It highlights that the strategic collusion can reduce the green quality levels of products although the government needs to allocate more funding. Essentially, collusion is a strategic measure for competing members to accomplish their pricing decision to increase profits. Consequently, it hurts the consumers, increases government expenditure, and reduces the overall social welfare. The results also demonstrate that the competing manufacturers can set prices that are higher than the competitive prices. As a result, regulators need to monitor the situation that conditions facilitate the formation of cartels and then implement subsidy program to accomplish sustainability goal.

7. Conclusion

In conclusion, we have modeled interaction among government organizations, manufacturers, retailers, and consumers where two manufacturers distribute products through two exclusive retailers. Optimal decisions are derived under five different scenarios to pinpoint government subsidy's impact to improve the green quality levels under competition. Comparative studies are conducted analytically and numerically to highlight managerial implications for SC members and policymakers on how strategic collusion or vertical integration can affect government expenditure to stimulate environmental performance.

Our study's key findings are as follows: first, regarding performance for manufacturers and retailers in the perspective of their respective profits, we find that they are benefited more from the collusion. However, product green quality level is less. Both upstream manufacturers permanently welcome more downstream competition whereas downstream retailers welcome upstream competition. Second, in the perspective of green product quality, strategic collusion always leads to suboptimal product quality in the presence of subsidy. Product consumption also reduces. Although the significance of vertical cooperation is studied in the literature, we pointed out that the horizontal collusion can serve as a strategic tool for the competing supply chain members to receive higher profits. Third, in the presence of collusion, government expenditure increased but not product quality. We find that if SC members optimize their respective profits, then government subsidy is higher, and the green quality level is also increased. Fourth, the study indicates that there is an optimal subsidy rate for all five scenarios, more significant levels of those may increase expenditure without bringing potential outcomes. Towards another step ahead, we prove a potential correlation between both strategic pacts among competitors both in upstream and downstream levels and total expenditure. A careful examination is warranted from a government organization's perspective, and they must take care to identify the possibility of such deals before subsidizing manufacturers.

In terms of future research, the present study can be extended in several directions. We ignore the effect of cross-channel selling. Consequently, it will be interesting to explore the characteristics of the optimal decisions in the presence of another degree of competition. We explore the scenarios where the manufacturers receive the subsidy; therefore, it could be fruitful to examine the characteristics where the subsidy to be received by retailers or customers, or both. We ignored the effect of cost-sharing agreement between the manufacturer and retailer at the vertical level or between two manufacturers at a horizontal level. Therefore, one can study the effect of contract mechanisms such as cost-sharing contract, trade-credit policy, cost-tariff contract [66], and revenue-sharing at vertical level or bargaining contract mechanism horizontal level. Next, in the proposed supply chain strategic structure, one can introduce market uncertainty or limits on government expenditure to assess how it might affect product green quality levels [68, 69].

Appendix

(A). Optimal Decision in Scenario UCLD

Because in Scenarios UDLD and UCLD, both downstream retailers take their respective decisions by optimizing their respective profits. Therefore, in both scenarios, the optimal response for the retailers remains the same, and with their response, profits for two manufacturers are obtained as

$$\Pi_{mi}^{\text{ucld}} = \frac{w_i^{\text{ucld}}\left(a(2+\beta) + w_j^{\text{ucld}}\beta - w_i^{\text{ucld}}\left(2-\beta^2\right) + (2\gamma-\beta\delta)\theta_i^{\text{ucld}} + (\beta\gamma-2\delta)\theta_j^{\text{ucld}}\right)}{4-\beta^2} - \left(1-\alpha^{\text{ucld}}\right)\eta\theta_i^{\text{ucld}2}, \tag{A.1}$$

where $i = 1,2$, $j = 3 - i$. In contrast to Scenario UDLD, two manufacturers optimize the sum of total profits as $\pi_m^{\text{ucld}} = \Pi_{m1}^{\text{ucld}} + \Pi_{m2}^{\text{ucld}}$. Therefore, the optimal response for two manufacturers is obtained by solving $(\partial \pi_m^{\text{ucld}}/\partial w_i^{\text{ucld}}) = 0$ and $(\partial \pi_m^{\text{ucld}}/\partial \theta_i^{\text{ucld}}) = 0$, respectively. After simplification, the optimal responses are obtained as follows:

$$w_i^{\text{ucld}} = \frac{2a(1-\alpha^{\text{ucld}})(2-\beta)\eta}{4(1-\alpha^{\text{ucld}})(2-\beta)(1-\beta)\eta - (\gamma-\delta)^2},$$
$$\theta_i^{\text{ucld}} = \frac{a(\gamma-\delta)}{4(1-\alpha^{ucld})(2-\beta)(1-\beta)\eta - (\gamma-\delta)^2}. \tag{A.2}$$

From the above expressions, we can see that wholesale prices decrease and product quality increases with α^{ucld}, but the reverse trend follows for η. The Hessian matrix (H_m^{ucld}) for the sum of the profit function two manufactures is obtained as

$$H_m^{\text{ucld}} = \begin{bmatrix} \frac{\partial^2 \pi_m^{\text{ucld}}}{\partial w_1^{\text{ucld}2}} & \frac{\partial^2 \pi_m^{\text{ucld}}}{\partial w_1^{\text{ucld}} \partial w_2^{ucld}} & \frac{\partial^2 \pi_m^{\text{ucld}}}{\partial w_1^{\text{ucld}} \partial \theta_1^{\text{ucld}}} & \frac{\partial^2 \pi_m^{\text{ucld}}}{\partial w_1^{\text{ucld}} \partial \theta_2^{\text{ucld}}} \\ \frac{\partial^2 \pi_m^{\text{ucld}}}{\partial w_1^{\text{ucld}} \partial w_2^{\text{ucld}}} & \frac{\partial^2 \pi_m^{\text{ucld}}}{\partial w_2^{\text{ucld}2}} & \frac{\partial^2 \pi_m^{\text{ucld}}}{\partial w_2^{\text{ucld}} \partial \theta_1^{\text{ucld}}} & \frac{\partial^2 \pi_m^{\text{ucld}}}{\partial w_2^{\text{ucld}} \partial \theta_2^{\text{ucld}}} \\ \frac{\partial^2 \pi_m^{\text{ucld}}}{\partial w_1^{\text{ucld}} \partial \theta_1^{\text{ucld}}} & \frac{\partial^2 \pi_m^{\text{ucld}}}{\partial w_2^{\text{ucld}} \partial \theta_1^{\text{ucld}}} & \frac{\partial^2 \pi_m^{\text{ucld}}}{\partial \theta_1^{\text{ucld}2}} & \frac{\partial^2 \pi_m^{\text{ucld}}}{\partial \theta_1^{\text{ucld}} \theta_2^{\text{ucld}}} \\ \frac{\partial^2 \pi_m^{\text{ucld}}}{\partial w_1^{\text{ucld}} \partial \theta_2^{\text{ucld}}} & \frac{\partial^2 \pi_m^{\text{ucld}}}{\partial w_2^{\text{ucld}} \partial \theta_2^{\text{ucld}}} & \frac{\partial^2 \pi_m^{\text{ucld}}}{\partial \theta_1^{\text{ucld}} \theta_2^{\text{ucld}}} & \frac{\partial^2 \pi_m^{\text{ucld}}}{\partial \theta_2^{\text{ucld}2}} \end{bmatrix}$$
$$= \begin{bmatrix} \frac{-2(2-\beta^2)}{4-\beta^2} & \frac{2\beta}{4-\beta^2} & \frac{2\gamma-\beta\delta}{4-\beta^2} & \frac{\beta\gamma-2\delta}{4-\beta^2} \\ \frac{2\beta}{4-\beta^2} & \frac{-2(2-\beta^2)}{4-\beta^2} & \frac{\beta\gamma-2\delta}{4-\beta^2} & \frac{2\gamma-\beta\delta}{4-\beta^2} \\ \frac{2\gamma-\beta\delta}{4-\beta^2} & \frac{\beta\gamma-2\delta}{4-\beta^2} & -2(1-\alpha^{ucld})\eta & 0 \\ \frac{\beta\gamma-2\delta}{4-\beta^2} & \frac{2\gamma-\beta\delta}{4-\beta^2} & 0 & -2(1-\alpha^{\text{ucld}})\eta \end{bmatrix}. \tag{A.3}$$

The values of principal minors of above Hessian matrix (H_m^{ucld}) are

$$H_{m_1}^{\text{ucld}} = \frac{-2(2-\beta^2)}{4-\beta^2} < 0,$$
$$H_{m_2}^{\text{ucld}} = \frac{4(1-\beta^2)}{4-\beta^2} > 0,$$
$$H_{m_3}^{\text{ucld}} = \frac{-2\Theta_2}{(4-\beta^2)^2},$$
$$H_{m_4}^{\text{ucld}} = \frac{\Theta_3}{(4-\beta^2)^2}, \tag{A.4}$$

respectively. Therefore, the joint profit function for two manufacturers is concave if

$$\Theta_2 = 4(1-\alpha^{\text{ucld}})(4-5\beta^2+\beta^4)\eta - (2+\beta^2)(\gamma^2+\delta^2) + 6\beta\gamma\delta > 0.$$
$$\Theta_3 = (\gamma^2-\delta^2)^2 - 8(1-\alpha^{\text{ucld}}) \cdot ((2+\beta^2)(\gamma^2+\delta^2) - 6\beta\gamma\delta)\eta + 16(1-\alpha^{\text{ucld}})^2(4-5\beta^2+\beta^4)\eta^2 > 0,$$
$$\text{for } \alpha^{\text{ucld}} \in (0,1). \tag{A.5}$$

Similar to previous Scenario UDLD, if we substitute $\alpha^{\text{ucld}} = 0$, we can obtain an optimal decision as presented in Table 2.

Substituting optimal responses for both manufacturers and retailers, the SW^{ucld} function is obtained as

$$\text{SW}^{\text{ucld}} = \frac{2a^2\eta\left[2(1-\alpha^{\text{ucld}})^2(1-\beta)(7-5\beta)\eta - (\gamma-\delta)^2\right]}{\left(4(1-\alpha^{\text{ucld}})(2-\beta)(1-\beta)\eta - (\gamma-\delta)^2\right)^2}. \tag{A.6}$$

Therefore, the optimal subsidy rate is obtained by solving $(d\text{SW}^{\text{ucld}}/d\alpha^{\text{ucld}}) = 0$. On simplification, the subsidy rate is obtained as $\alpha^{\text{ucld}} = (3(1-\beta)/7-5\beta)$. Note that the SW^{ucld} function is concave with respect to α^{ucld} if $\Delta_2 = 8(2-\beta)^2(1-\beta)\eta - (7-5\beta)(\gamma-\delta)^2 > 0$, because

$$\frac{d^2\text{SW}^{\text{ucld}}}{d\alpha^{\text{ucld}2}}\Big|_{\alpha^{\text{ucld}}} = -\frac{8a^2(7-5\beta)^4(1-\beta)(\gamma-\delta)^2\eta^2}{\Delta^{(3/2)}}. \tag{A.7}$$

Moreover,

$$\Theta_2|_{\alpha=\alpha^{\text{ucld}}} = \frac{2\Delta_{21}}{7-5\beta},$$
$$\Theta_3|_{\alpha=\alpha^{\text{ucld}}} = \frac{\Delta_2\Delta_{22}}{(7-5\beta)^2}, \tag{A.8}$$

where $\Delta_{21} = 8(2-\beta)(4-5\beta^2+\beta^4)\eta - (7-5\beta)((2+\beta^2)\gamma^2 - 6\beta\gamma\delta + (2+\beta^2)\delta^2)$ and $\Delta_{22} = 8(2-\beta)(1+\beta)(2+\beta)\eta - (7-5\beta)(\gamma+\delta)^2$; that is, optimal solution exists in Scenario UCLD if $\Delta_2 > 0$, $\Delta_{21} > 0$, and $\Delta_{22} > 0$.

The following inequalities ensure that the wholesale price and green quality levels in Scenario UDLD decreased with η and increased with α^{udld},

$$\begin{aligned}
\frac{\partial w_i^{\text{udld}}}{\partial \eta} &= \frac{-2a\left(1-\alpha^{\text{udld}}\right)\left(4-\beta^2\right)(\gamma-\delta)(2\gamma-\beta\delta)}{\left(2\left(1-\alpha^{\text{udld}}\right)(2-\beta)\left(4-\beta-2\beta^2\right)\eta-(\gamma-\delta)(2\gamma-\beta\delta)\right)^2}<0,\\
\frac{\partial \theta_i^{\text{udld}}}{\partial \eta} &= \frac{-2a\left(1-\alpha^{\text{udld}}\right)(2-\beta)\left(4-\beta-2\beta^2\right)(2\gamma-\beta\delta)}{\left(2\left(1-\alpha^{\text{udld}}\right)(2-\beta)\left(4-\beta-2\beta^2\right)\eta-(\gamma-\delta)(2\gamma-\beta\delta)\right)^2}<0,\\
\frac{\partial w_i^{\text{udld}}}{\partial \alpha^{\text{udld}}} &= \frac{\left[2a\left(4-\beta^2\right)-(\gamma-\delta)(2\gamma+\beta\delta)\eta\right]}{\left(2\left(1-\alpha^{\text{udld}}\right)(2-\beta)\left(4-\beta-2\beta^2\right)\eta-(\gamma-\delta)(2\gamma-\beta\delta)\right)^2}>0,\\
\frac{\partial \theta_i^{\text{udld}}}{\partial \alpha^{\text{udld}}} &= \frac{2a(2-\beta)\left(4-\beta-2\beta^2\right)(2\gamma+\beta\delta)\eta}{\left(2\left(1-\alpha^{\text{udld}}\right)(2-\beta)\left(4-\beta-2\beta^2\right)\eta-(\gamma-\delta)(2\gamma-\beta\delta)\right)^2}>0.
\end{aligned} \tag{A.9}$$

Therefore, optimal is ensured if η satisfies the condition in model formulation.

The following inequalities ensure that the wholesale price and green qualities decreased with η and increased with α^{udlc},

$$\begin{aligned}
\frac{\partial w_i^{\text{udlc}}}{\partial \eta} &= \frac{4a\left(1-\alpha^{\text{udlc}}\right)\gamma(\gamma-\delta)}{\left(4\left(1-\alpha^{\text{udlc}}\right)(2-\beta)\eta-\gamma(\gamma-\delta)\right)^2}<0,\\
\frac{\partial \theta_i^{\text{udlc}}}{\partial \eta} &= \frac{4a\left(1-\alpha^{\text{udlc}}\right)(2-\beta)\gamma}{\left(4\left(1-\alpha^{\text{udlc}}\right)(2-\beta)\eta-\gamma(\gamma-\delta)\right)^2}<0,\\
\frac{\partial w_i^{\text{udlc}}}{\partial \alpha^{\text{udlc}}} &= \frac{4a\gamma(\gamma-\delta)\eta}{\left(4\left(1-\alpha^{\text{udlc}}\right)(2-\beta)\eta-\gamma(\gamma-\delta)\right)^2}>0,\\
\frac{\partial theta_i^{\text{udlc}}}{\partial \alpha^{\text{udlc}}} &= \frac{4a(2-\beta)\gamma\eta}{\left(4\left(1-\alpha^{\text{udlc}}\right)(2-\beta)\eta-\gamma(\gamma-\delta)\right)^2}>0.
\end{aligned} \tag{A.10}$$

(B). Optimal Decision in Scenario UCLC

In Scenarios UDLC and UCLC, both downstream retailers take their respective decisions by optimizing the sum of downstream profits. Therefore, in both scenarios, the optimal response for the retailers remains the same. Similar to Scenario UDLC, the optimal response for two retailers on their respective retail price will be the same, and with their response, profits for two manufacturers are obtained as

$$\begin{aligned}
\Pi_{mi}^{\text{uclc}} &= \frac{w_i^{\text{uclc}}\left(a-w_i^{\text{uclc}}+w_j^{\text{uclc}}\beta+\gamma\theta_i^{\text{uclc}}-\delta\theta_j^{\text{uclc}}\right)}{2}\\
&\quad-\left(1-\alpha^{\text{uclc}}\right)\eta\theta_i^{\text{uclc}_2}.
\end{aligned} \tag{B.1}$$

However, in contrast with the Scenario UDLC, two manufacturers optimize the sum of upstream profits as $\pi_m^{\text{uclc}} = \Pi_{m_1}^{\text{uclc}} + \Pi_{m_2}^{\text{uclc}}$. Therefore, the optimal response for two manufacturers is obtained by solving $(\partial\pi_m^{\text{uclc}}/\partial w_1^{\text{uclc}}) = 0$, $(\partial\pi_m^{\text{uclc}}/\partial\theta_1^{\text{uclc}}) = 0$, $(\partial\pi_m^{\text{uclc}}/\partial w_2^{\text{uclc}}) = 0$, and $(\partial\pi_m^{\text{uclc}}/\partial\theta_2^{\text{uclc}}) = 0$, respectively. After simplification, the optimal responses are obtained as follows:

$$\begin{aligned}
w_i^{\text{uclc}} &= \frac{4a\left(1-\alpha^{\text{uclc}}\right)\eta}{8\left(1-\alpha^{\text{uclc}}\right)(1-\beta)\eta-(\gamma-\delta)^2},\\
\theta_i^{\text{uclc}} &= \frac{a(\gamma-\delta)}{8\left(1-\alpha^{\text{uclc}}\right)(1-\beta)\eta-(\gamma-\delta)^2}.
\end{aligned} \tag{B.2}$$

To verify concavity, we compute the Hessian matrix (H_m^{uclc}) for the joint profit function for two manufacturers as follows:

$$H_m^{ucld} = \begin{bmatrix} \frac{\partial^2 \pi_m^{uclc}}{\partial w_1^{uclc}} & \frac{\partial^2 \pi_m^{uclc}}{\partial w_1^{uclc}\partial w_2^{uclc}} & \frac{\partial^2 \pi_m^{uclc}}{\partial w_1^{uclc}\partial \theta_1^{uclc}} & \frac{\partial^2 \pi_m^{uclc}}{\partial w_1^{uclc}\partial \theta_2^{uclc}} \\ \frac{\partial^2 \pi_m^{uclc}}{\partial w_1^{uclc}\partial w_2^{uclc}} & \frac{\partial^2 \pi_m^{uclc}}{\partial w_2^{uclc2}} & \frac{\partial^2 \pi_m^{uclc}}{\partial w_2^{uclc}\partial \theta_1^{uclc}} & \frac{\partial^2 \pi_m^{uclc}}{\partial w_2^{uclc}\partial \theta_2^{uclc}} \\ \frac{\partial^2 \pi_m^{uclc}}{\partial w_1^{uclc}\partial \theta_1^{uclc}} & \frac{\partial^2 \pi_m^{uclc}}{\partial w_2^{uclc}\partial \theta_1^{uclc}} & \frac{\partial^2 \pi_m^{uclc}}{\partial \theta_1^{uclc2}} & \frac{\partial^2 \pi_m^{uclc}}{\partial \theta_1^{uclc}\theta_2^{uclc}} \\ \frac{\partial^2 \pi_m^{uclc}}{\partial w_1^{uclc}\partial \theta_2^{uclc}} & \frac{\partial^2 \pi_m^{uclc}}{\partial w_2^{uclc}\partial \theta_2^{uclc}} & \frac{\partial^2 \pi_m^{uclc}}{\partial \theta_1^{uclc}\theta_2^{uclc}} & \frac{\partial^2 \pi_m^{uclc}}{\partial \theta_2^{uclc}} \end{bmatrix}$$

$$= \begin{bmatrix} -1 & \beta & \frac{\gamma}{2} & \frac{-\delta}{2} \\ \beta & -1 & \frac{-\delta}{2} & \frac{\gamma}{2} \\ \frac{\gamma}{2} & \frac{-\delta}{2} & -2\left(1-\alpha^{uclc}\right)\eta & 0 \\ \frac{-\delta}{2} & \frac{\gamma}{2} & 0 & -2\left(1-\alpha^{uclc}\right)\eta \end{bmatrix}. \tag{B.3}$$

The values of principal minors of the above Hessian matrix are $H_{m_1}^{uclc} = -1 < 0$; $H_{m_2}^{uclc} = 1-\beta^2 > 0$; $H_{m_3}^{uclc} = (-\Theta_5/4)$; and $H_{m_4}^{uclc} = (\Theta_6/16)$, respectively, where $\Theta_5 = (8(1-\alpha^{uclc})(1-\beta^2)\eta - \gamma^2 + 2\beta\gamma\delta - \delta^2)$ and $\Theta_6 = (\gamma^2-\delta^2)^2 - 16(1-\alpha^{uclc})(\gamma^2 - 2\beta\gamma\delta + \delta^2)\eta + 64(1-\alpha^{uclc})^2(1-\beta^2)\eta^2$. Therefore, joint profit function for two manufacturer is also concave if $\Theta_5 > 0$ and $\Theta_6 > 0$.

Substituting optimal responses for both manufacturers and retailers, the SW^{uclc} function is obtained as

$$SW^{uclc} = \frac{2a^2\eta\left(2\left(1-\alpha^{uclc}\right)^2(7-\beta)(1-\beta)\eta - (\gamma-\delta)^2\right)}{\left(8\left(1-\alpha^{uclc}\right)(1-\beta)\eta - (\gamma-\delta)^2\right)^2}. \tag{B.4}$$

Therefore, the optimal subsidy rate is obtained by solving $(dSW^{uclc}/d\alpha^{uclc}) = 0$. On simplification, the subsidy rate is obtained as $\alpha^{uclc} = (3-\beta/7-\beta)$. Note that the SW^{uclc} function is concave with respect to α^{uclc} if

$$\frac{d^2SW^{uclc}}{d\alpha^{uclc2}}\Big|_{\alpha^{udlc}} = -\frac{8a^2(7-\beta)^4(1-\beta)(\gamma-\delta)^2\eta^2}{(32(1-\beta)\eta - (7-\beta)(\gamma-\delta))^3} < 0. \tag{B.5}$$

In addition,

$$\Theta_5\big|_{\alpha^{udlc}} = \frac{\left(32(1-\beta)\eta - (7-\beta)(\gamma-\delta)^2\right)^2}{16(7-\beta)^2},$$
$$\Theta_6\big|_{\alpha=\alpha^{udlc}} = \frac{32\left(1-\beta^2\right)\eta - (7-\beta)\left(\gamma^2 - 2\beta\gamma\delta + \delta^2\right)}{4(7-\beta)}. \tag{B.6}$$

Therefore, optimal decision exists if $\Delta_3 > 0$.

(C). Proof of Proposition 3

By comparing and simplifying optimal decision in Scenarios UCLD and UDLD, we obtain the following relations:

$$w_i^{ucld} - w_i^{udld} = \frac{2a(2-\beta)^2\eta\beta\left(4(2-\beta)^2\beta\left(4-\beta-2\beta^2\right)\eta - (26-\beta(3+11\beta))(\gamma-\delta)^2\right)}{\Delta_1\Delta_2} > 0,$$

$$p_i^{ucld} - p_i^{udld} = \frac{4a\eta\beta(2-\beta)\left(2(2-\beta)^2\left(4-\beta-2\beta^2\right)\eta - \beta\left(25-\beta\left(17+9\beta-6\beta^2\right)\right)(\gamma-\delta)^2\right)}{\Delta_1\Delta_2},$$

$$\theta_i^{udld} - \theta_i^{ucld} = \frac{2a\eta(\gamma-\delta)(2-\beta)^2\beta\left(24-31\beta-7\beta^2+12\beta^3\right)}{\Delta_2\Delta_1} > 0, \quad \text{if } \beta < 0.8477, \tag{C.1}$$

$$Q_i^{ucld} - Q_i^{udld} = \frac{-2a(2-\beta)\eta * * * 4(2-\beta)^2(1-\beta)\beta\left(4-\beta-2\beta^2\right)\eta - \beta(1+\beta)\left(2-\beta^2\right)(\gamma-\delta)^2}{\Delta_1\Delta_2} < 0,$$

$$SW^{udld} - SW^{uclc} = \frac{4a^2\eta^2(2-\beta)^2\beta\left(24-31\beta-7\beta^2+12\beta^3\right)}{\Delta_2\Delta_1} > 0, \quad \text{if } \beta < 0.8477.$$

The above relations ensure the proof.

(D). Proof of Proposition 6

The following inequalities ensure the proof of Proposition 6:

$$w_i^{\mathrm{uclc}} - w_i^{\mathrm{udlc}} = \frac{8a\eta\beta\left(16(2-\beta)(1-\beta)\beta\eta - (13-(10-\beta)\beta)(\gamma-\delta)^2\right)}{\Delta_2\Delta_3} > 0,$$

$$p_i^{\mathrm{ucld}} - p_i^{\mathrm{udlc}} = \frac{4a\eta\beta\left(16(2-\beta)(1-\beta)\eta - \left(25-21\beta+2\beta^2\right)(\gamma-\delta)^2\right)}{\Delta_2\Delta_3} > 0,$$

$$\theta_i^{\mathrm{udlc}} - \theta_i^{\mathrm{uclc}} = \frac{8a(1-\beta)\beta\left(12-11\beta+\beta^2\right)(\gamma-\delta)\eta}{\Delta_2\Delta_3} > 0, \tag{D.1}$$

$$Q_i^{\mathrm{udlc}} - Q_i^{\mathrm{uclc}} = \frac{4a(1-\beta)\eta\left(16(2-\beta)(1-\beta)\beta\eta - \beta(1+\beta)(\gamma-\delta)^2\right)}{\Delta_2\Delta_3} > 0,$$

$$\mathrm{SW}^{\mathrm{ucld}} - \mathrm{SW}^{\mathrm{uclc}} = \frac{16a^2\eta^2(1-\beta)\beta\left(12-11\beta+\beta^2\right)}{\Delta_2\Delta_3} > 0.$$

The proposition is proved.

Data Availability

The data used to support the findings of this study are included within the manuscript.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Supplementary Materials

We presented the graphical representation of sensitivity analysis of wholesale prices, retail prices, green quality levels, sales volumes, profit for the retailers, profits for the manufacturers, total subsidies, and social welfare functions in four scenarios with respect to green quality sensitivity, investment efficiency of manufacturers, cross-green quality sensitivity, and cross-price elasticity in Figures S1, S2, S3, and S4, respectively. (*Supplementary Materials*)

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Research Article

A Bilevel Programming Location Approach to Regional Waste Electric and Electronic Equipment Collection Centers: A Study in China

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As one of the largest markets of electronic and electric equipment, China has not completely established the formal recycling system of WEEE compared with the developed countries. As a result, China is facing the huge challenge of resource waste and water/soil environmental pollution. In this paper, according to the current regulations on WEEE recycling and disposal issued by Chinese government, the business model of the Chinese WEEE recycling system was designed, and a bilevel programming-based model was proposed to help the disposal factories to establish the regional efficient and economical WEEE recycling network. This model addressed the optimization of bilateral benefits of disposal factories and the third-party recycling agencies/stations. An experiment based on a regional WEEE recycling business data was solved by the NSGA algorithm to validate the proposed model. With the result, the proposed model was compared with the related studies on Chinese WEEE recycling network planning. With the comparison and the analysis on the experiment result, it was found that the proposed model had considerably stable convergence and optimization performance, which proved that this model can be regarded as a useful tool to the planning of the Chinese regional WEEE recycling network. In the last part, the future improvement of the proposed model was also discussed.

1. Introduction

The demand for electric and electronic equipment is growing very rapidly. Moreover, the life cycles of these products get shorter. It results in a growing amount of waste electric and electronic equipment (WEEE) which needs to be reused or disposed for the economic and environmental purpose. In WEEE, there are much precious renewable materials. For example, there are 35 percent of components made of metal in computers and the proportion of Cu is about 20 percent (Luo et al., 2006) [1]. In addition, the top 1 component, plastic, can be used as a kind of fuel, which produces the energy 1.3 times of that of coal. The treatment of WEEE not only protects the environment but also makes commercial benefits. Therefore, the disadvantage of maltreatment and the advantage of right treatment have drawn great attention from the public.

Many countries, especially the developed ones, have promulgated acts to recycle WEEE. The USA issued the compulsory act on WEEE treatment in the early 1990s and more acts on the recycle of WEEE in 2002 (Lu et al., 2014) [2]. In the USA, most states enforce the producer responsibility with which the producers are responsible for the recycle of WEEE and pay for the recycle cost. The customer responsibility which forces the customers to pay for the recycle is also enforced in other states. At present, more than 96 percent of the WEEE is reused while the rest is disposed as rubbish (Yang et al., 2019) [3]. The European Union passed WEEE (2002/96/EC) and RoHS (2002/95/EC) to guide member countries to regulate the recycle of WEEE. The two directives state that the producers pay the cost of the recycle and disposal of WEEE and the nonprofit organizations dispose of the WEEE (Robinson, 2009) [4]. In Japan, producers are responsible for the disposal of WEEE and the dealers recycle and deliver the WEEE to producers. Different from other countries, Japanese customers are obligated to pay the cost of recycle according to the kind of WEEE and it is illegal to abandon WEEE anywhere (Menikpura et al., 2014; Xu and Zhou, 2019) [5, 6].

Many developing countries also start making an effort on building infrastructures for WEEE reverse logistic network design processes (Temur and Bolat, 2017) [7]. According to the report of Chinese electronic and electrical equipment (EEE) market in 2019, the overall turnover of EEE in Chinese market reached 126 billion dollars in 2019 (CCID, 2020) [8]. The People's Daily reported that according to the white paper on the recycling, processing, and comprehensive utilization of China's WEEE released by the China household electrical appliance research institute (CHEARI), in 2018, about 150 million TV sets, refrigerators, air conditioners, washing machines, and computers were scrapped. The weight of the WEEE theoretically exceeded 4.06 million tons. It is optimistically estimated that amount all WEEE, only 50 to 60 percent of all has been formally disposed of (Kou, 2020) [9]. As the largest developing country, China is facing the huge challenge of WEEE treatment.

Contrary to the developed countries, China has not completely established the regular recycle system of WEEE. The Chinese recycling business model (Figure 1) showed that in China, the WEEE was mainly collected by recycle individual agencies/stations rather than dealers and producers, and the customers sold WEEE to individual recycling agencies/stations and then the agencies sold WEEE to WEEE disposal factories. It is a kind of commercial business. Since the customers are not compulsory to hand in WEEE in right way, some WEEE is even abandoned as rubbish, which may cause the serious environmental pollution. According to the investigation, nearly 90 percent of WEEE is not processed in a formal way in China (Intelligence Research Group, 2019) [10].

According to official reports, there are about 90,000 registered recycling agencies/stations and more than 300,000 unregistered ones in China. More than 90 percent of the WEEE is transferred to disposal factories by these recycling agencies/stations. The recycling agencies/stations play the important role in the recycling system. For examples, there are more than 5000 ones in Beijing, which is only an ordinary epitome of the distribution of the Chinese recycling network. In general, these recycling agencies/stations are in small size and equipped with poor facilities. They are generally scattered in city districts, self-owned, or run by unemployed persons. They collect all kinds of renewable materials including WEEE manually and sort out the materials to transfer them to different disposal factories.

In order to improve the normal disposal of WEEE, Chinese government has been enforcing the regulations and licensing on the WEEE recycling and disposal industry from 2011. Unlicensed organizations are banned to recycle and dispose of WEEE. With these measures, Chinese WEEE business has been developed to some extent in nearly 10 years. Until 2018, there were 109 disposal companies added into the licensed list. In order to support the sustainable running of disposal companies, Chinese government also enacted WEEE disposal fund regulation (WDFR) to establish the disposal fund to provide subsidies to disposal factories from 2012. According to the regulation, the EEE producers and importers pay for the fund according to the product category and production volume (The State Council of People's Republic of China, 2012) [11]. However, there still exist many problems faced by all disposal factories (Ministry of Commerce of People's Republic of China, 2018). (a) Low income: the main income of Chinese WEEE disposal companies is derived from the sales of decomposed components from WEEE and the financial support by the government. According to the investigation by the Intelligence Research Group, by the end of 2018, the recycling fund had collected 2.36 billion dollars and allocated 2.28 billion dollars in subsidies, which barely maintained a balance [12]. (b) High recycling cost: in China, since the recycle logistics chain is much longer and composed of individual recycling agencies with diverse scale and capability, the efficiency of the recycle logistics systems is considerably poor. In addition, the money paid to the citizens to collect the WEEE further increased the cost of recycling. That meant the total cost was eventually afforded by the disposal factories. With the low operation efficiency and the high cost of the recycling, the WEEE disposal factories made too little profit to survive. It was reported that even with the financial support which was about 60 percent of the disposal factories' total income, nearly 66 percent of them were about to pause running or go bankrupt (Intelligence Research Group, 2019) [10].

From 2014, Chinese government upgraded the subsidies' requirements. The disposal companies cannot get the subsidies from the government any more if the amount of the disposal of WEEE is lower than 20 percent of the licensed disposal capacity. This measure helped regulate the WEEE disposal industry. Qualified companies would get more support to maintain the sustainable business while those who failed to meet the requirements would be excluded from the market. However, WEEE is still recycled from the public with money and directly transferred to the disposal plants by the recycling agencies/stations. In addition, the producers and retailers are still not enforced but only encouraged to collect WEEE as possible as they can. In fact, few of them are willing to participate in the recycling business considering the extra workload.

As was described above, in the special business model in China, the WEEE disposal factories had to establish their own collection centers to pack all small batches of WEEE from recycling agencies/stations into big batches in order to decrease the collection cost. Meanwhile, they also take the benefits of the recycling agencies/stations into account to collect more WEEE.

In this paper, the bilevel programming approach was used to assist disposal factories to establish an efficient and economical recycling network. The work addressed the special WEEE business situation in China. On the one hand, by satisfying the benefit interest of the public in WEEE recycling, disposal factories will collect as much WEEE as possible to meet the basic conditions for obtaining government subsidies. On the other hand, the approach will help factories to decrease the cost of recycling logistics so as to make more profit to survive.

2. Related Study

Many related studies on WEEE recycling network planning have been launched recent years. Optimization and simulation are the prevailing methods used in the related studies.

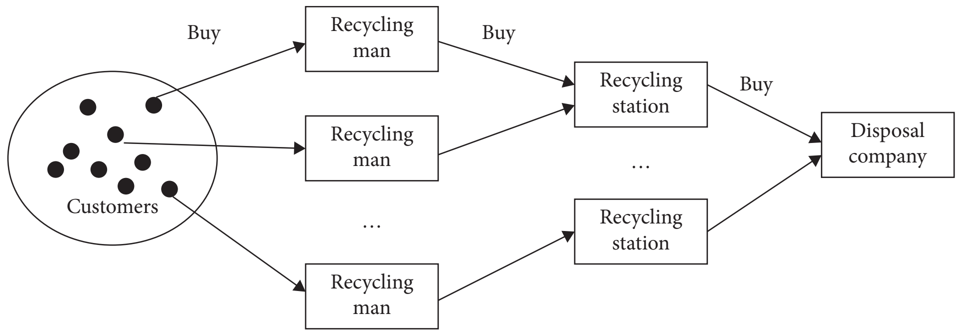

Figure 1: The WEEE recycling business process in China.

2.1. Simulation-Based Study. Golinska and Kawa (2012) proposed a model based on graph theory and agent technology that provided dynamic configuration of the recovery network among a pool of cooperating companies in dynamically changing conditions [13]. Shokohyar and Mansour (2013) developed a simulation optimization model to determine the best locations for the collection centers and also recycling plants for WEEE. In their work, economic, environmental, and social issues were considered to help the government to simultaneously perform the trade-off between environmental issues and economical and social impacts. The proposed model was examined through an illustrative case study from a developing country's WEEE situation [14]. Ravi et al. (2017) measured the performance of the reverse logistic enterprise with the agent-based simulation model. They designed all kinds of agents in the reverse logistic business: collector agent, sorting-cum-reuse agent, remanufacturing agent, recycler agent, supplier agent, and distributor agent. The model implemented the whole recycling process from collection to disposal to simulate the activities in a cost-effective manner. Eventually, the recommendations were given to improve the total performance of the reverse logistic enterprise [15]. Popa and Cotet (2017) proposed a material flow management optimizing algorithm based on a virtual model of the selecting and processing architecture of the waste collection integrated system. The material flow management of this system was based on its virtual model in order to identify and eliminate material flow concentrators and increase productivity. Simulation was used to diagnose the initial performance of the system structural elements as well as to validate the optimized system performances after eliminating the bottlenecks [16]. Llamas et al. (2020) designed a very large simulation model linking up to 223 detailed modeled unit operations, over 860 flows and 30 elements, and all associated compounds to analyze the resource efficiency limits and evaluate the material recovery, resource consumption, and environmental impacts of different processing routes of the circular economy system [17]. Suyabatmaza et al. (2014) considered manufacturers that strategically decided to outsource the company specific reverse logistics activities to a third-party logistics (3PL) service provider. They proposed a hybrid simulation-analytical modeling approach which iteratively used mixed integer programming models and simulation to create the framework for handling the uncertainties in the stochastic reverse logistics network design problem [18].

2.2. Optimization-Based Study. Mar-Ortiz et al. (2011) studied the optimization of the design of the reverse logistic network for the collection of WEEE in the Spanish region of Galicia. They proposed a three-phase hierarchical approach. The first phase is for the facility location by means of a mixed integer linear programming, and the second phase is for the corresponding heterogeneous fleet vehicle routing with a new integer programming formulation and a savings-based heuristic algorithm. The third phase is a simulation study on the collection routes in order to assess the overall performance of the recovery system. They claimed that the performance of the proposed procedure was good and the configuration of the recovery network was improved, compared to the one currently in use whose transportation costs were reduced by nearly 30 percent [19]. Tari and Alumur (2014) studied the collection center location problem with equity considerations within reverse logistics network design. The aim of the problem is for three objectives. The first one is to minimize total cost, the second one is to ensure equity among different firms, and the third one is to provide steady flow of products to each firm. The multiobjective mixed integer programming formulation was used considering the changes in the fixed costs and container capacities, changes in the amount of supply, and changes in the growth rate. An implementation of the problem in Turkey within the context of WEEE collection was presented to illustrate the potential of the model and value of using a multiperiod model as opposed to using a static one [20]. Baxter and Bø (2017) launched a detailed analysis of the logistical and cost-effectiveness of the collection and transport of WEEE in Norway. The study revealed regional geography to be a particularly important factor which varied significantly across the country and heavily influences the cost of collection. The study also explored the influence of other factors relating to operational effectiveness and customer service in the WEEE collection and transport business [21]. Di et al. (2013) proposed a bilevel programming model to collection station location choice. The top level aimed at the minimum total cost and the maximum volume of the WEEE collection. The bottom level aimed at the maximum satisfaction of the customers considering the collection price paid to customers and the distance between collections and customers. The model was proved to be beneficial to determine the number of establishing collection stations, location, and the served customers and more beneficial to the waste household appliance recycling management [22]. Huang and Zhang (2018) constructed the improved maximum coverage location model of waster mobile phone recycling business points and used Lagrangian relaxation algorithm with subgradient optimization to solve the model. It was proved that the model and the designed algorithm which had good practicality can guide the government or companies to deal with the waste mobile phone recycling problem [23]. Yu and Solvang (2016) suggested a general reverse logistics network and formulated it through multiobjective mixed integer programming. The reverse logistics system was an independent network and comprised of three echelons for collection, recycling, and disposal of waste. Their work explicitly showed the trade-off between the cost-effectiveness for improving environmental performance, and influences from resource utilization had great practical implication on decision-making of network configurations and transportation planning of a reverse logistics system [24].

The above related studies have proved that simulation methods and optimization methods contribute most to the WEEE recycling business in practical and academic domains. However, the models in the related studies did not exactly and completely address the current situation of WEEE recycling business in China. In most models, the cost for the collection of WEEE from the customers (WEEE holders) was not taken into account and the recycling stations were assumed established by disposal companies. The bilevel programming model of Di et al. (2013) considered the collection cost of WEEE and took it as an important factor in the model. However, their study aimed at establishing the recycling stations especially for WEEE, which was not feasible because most recycling stations, and recycling vendors worked for all kinds of renewable resources and their business is independent of disposal factories.

3. Problem Definition

According to the Chinese WEEE recycling business model in Section 1, it was hypothesized as follows:

(1) Since the third-party recycling agencies/station network had been established, the location, capacity, and number of recycling agencies/stations were predetermined.

(2) Since the concerns about the environment pollution, the number and the location of disposal factories was usually predetermined and authorized by the local government before the planning of collection centers. Therefore, in this business model, there was only one disposal factory in a certain area, whose location and disposal capacity was predetermined.

(3) All WEEE must be transferred from recycling agencies/stations to collection centers established by disposal factory, and the links between recycling agencies/stations and collection centers were fixed.

(4) The collection price of WEEE was determined in WEEE categories by the disposal factory.

(5) Since most recycling agencies/stations were owned by individuals, the recycling price paid to the customers by different recycling agencies/stations varied to some extent.

(6) The time windows of collection centers were deployed with different values by the disposal factory in order to schedule the WEEE delivery tasks of collection centers in good order without confusion.

Considering the nature of the location problem, the bilevel programming algorithm was used to design the location models (Figure 2). The top-level model addressed the concern of the disposal companies about the collection volume and cost. The bottom-level model addressed the benefits of the third-party recycling agencies/stations.

3.1. The Multiobjective Optimal Model in Top Level. In top level, the disposal factories aimed at the maximum collection volume and the minimum collection cost. The collection volume was the total collection volume of all collection centers. The collection cost included construction cost, regular operational cost, and the transportation cost of all collection centers. The optimal model in top level was defined in the following equations. It was a typical multiobjective programming problem. In equation (1), the total cost of the WEEE recycling network was defined as the minimum sum of construction cost of collection centers (kA_j), the operational cost of collection centers (t_jO_j), and the transportation cost of collection centers ($V_jR_jd_j$), while in equation (2), the maximum total WEEE collection volume from collection centers was defined. The two objectives were in opposite optimal directions, which hinted that they can hardly be integrated into one single objective:

$$\min T_c = \sum_{j=1}^{N} y_j\left(kA_j + t_jO_j + Vc_jR_jd_j\right), \tag{1}$$

$$\max V_c = \sum_{j=1}^{N} y_jVc_j, \tag{2}$$

s.t.

$$\begin{cases} \sum_{j=1}^{N} y_jkA_j \le T_I \\ \sum_{j=1}^{N} y_j > 0, \\ Vc_j \le \overline{Vc_j}, \\ 0.2V_T \le \sum_{j=1}^{N} y_jVc_j \le V_T, \\ y_j \in \{0,1\}, j = 1,\cdots N, \end{cases} \tag{3}$$

where t_j was the time window of collection center j. In this model, the time window was a kind of relative value, where

$$t_j = \frac{\text{work hours}}{8}. \tag{4}$$

T_I was the budget of total construction cost of collection centers and A_j was the construction area size of collection

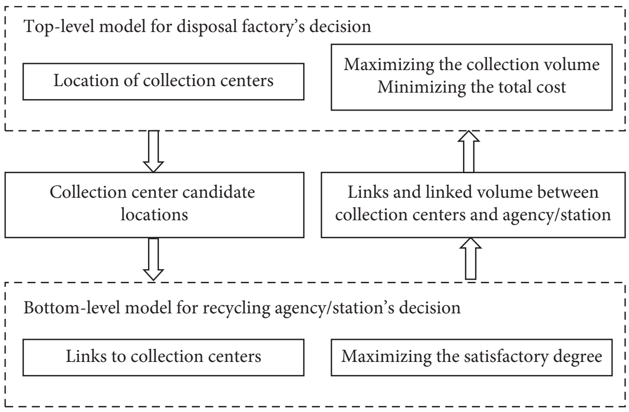

Figure 2: Bilevel programming model for location of WEEE collection centers.

center j. K was the construction cost per m^2. As was defined in equation (3), the construction cost of all collection centers cannot exceed the budget T_I. O_j was the unit operational cost, and y_j was the variable to decide whether to set up the collection center j $(y_j = 1)$ or not $(y_j = 0)$ (as defined in equation (3)). As was defined in equation (3), there was at least 1 collection center set up in the planning area. $\mathrm{Vc_j}$ was stock volume of collection center j, and $\overline{Vc_j}$ was the max stock volume of collection j. As was defined in equation (3), Vc_j should be less than $\overline{Vc_j}$. V_T was the annual licensed disposal capacity divided by the average times of the transportation from collection centers to disposal factory. The average times were estimated by the disposal factories according to their recycling business experience. Equation (3) meant that the disposal factories cannot work if they did not live up to the requirement of getting the fund from the government. R_j was the unit transportation cost from collection center j to disposal factory, and d_j was the distance between collection center j and disposal factory. N was the maximum number of candidate collection centers.

3.2. The Optimal Model in Bottom Level. In bottom level, the third-party recycling agencies/stations ran for higher income with lower WEEE recycling cost. The transportation distance to the collection centers, the recycling price, and the time window of collection centers were key factors to influence their business. The optimal model in bottom level was defined in the following equations:

$$\max U = \sum_{i=1}^{M} \left(p_{x_i} - p_i - R_{ix_i} d_{ix_i}\right) Vr'_i t_{x_i}, \tag{5}$$

s.t.

$$\sum Vr'_i \le Vc_{x_i}, \quad i = 1, \cdots M, \ j = 1, \cdots N, \tag{6}$$

$$Vr'_i \le Vr_i, \quad i = 1, \cdots M, \tag{7}$$

where x_i was the variable to decide whether recycling agency/station i delivered WEEE to collection center j $(x_\mathrm{i} = j)$, p_i was the price recycling agency/station i paid to the customers when collecting WEEE from customers, p_{x_i} was the price paid by collection center x_i to the recycling agency/station i, R_{ix_i} was the transportation fee rate from recycling agency/station i to collection center x_i, d_{ix_i} was the distance from recycling agency/station i to collection center x_i, Vr_i was the maximum capacity of recycling agency/station i, Vr'_i was the WEEE capacity of recycling agency/station i transferred to collection center x_i, and M was the number of recycling agencies/stations.

As was defined in equation (4), U was the utility function of recycling agencies/stations, which was taken as the weighted profit of all recycling agencies/stations. The weights were the time window of collection centers. The larger the time window was, the more convenient for recycling agencies/stations to transfer WEEE. In equation (5), the total volume of recycling agencies/stations cannot exceed the volume of the collection center that was linked with the recycling agencies/stations.

4. Method to Solve the Problem

4.1. Method to Solve the Bilevel Programming Location Model. Bilevel programming was proved to be a typical NP-hard problem. The models based on bilevel programming were usually solved by heuristic algorithms such as genetic algorithm (GA), ant colony algorithm (ACO), and simulated annealing (SA). In this study, GA was used to solve the bilevel-based location model of WEEE collection centers.

The GA family has many branches. Since the optimal model in top level was multiobjective-oriented with nonweighted objectives, the nondominated sorting genetic algorithm (NSGA) (Srinivas, N. and Deb, K., 1994) was considered to solve the optimal model. As one of the most well-known multiobjective optimal algorithms, NSGA has three versions, NSGA-I (1994), NSGA-II (2002), and NSGA-III (2014) [25, 26]. The basic framework of NSGA-III remains similar to the original NSGA-II with significant changes in its selection mechanism. Unlike in NSGA-II, the maintenance of diversity among population members in NSGA-III is aided by supplying and adaptively updating a number of well-spread reference points (Deb, K. and Jain, H., 2014) [26]. As compared with NSGA-II and other multiobjective optimal algorithms, NSGA-III significantly reduces the computation cost, especially for high-dimension multiobjective problems. For low-dimension multiobjective problems such as the proposed model in this paper, NSGA-II and NSGA-III are all suitable. However, considering NSGA-III is the latest improved version of NSGA-II, we chose it as the preferred algorithm.

According to the bilevel programming process in Section 3, the NSGA-III algorithm was realized as shown in Figure 3. The feasible location schemes of collection centers were generated and passed to the GA algorithm in the bottom level to get the optimal business collaboration between collection centers and recycling agencies/stations. Then, the optimal location scheme of collection centers was got.

In the above NSGA algorithm, the encoding scheme of the feasible location scheme of collection centers was defined as the binary-like chromosome shown in

$$cc = \{y_1, y_2, \cdots y_N\}, \quad (8)$$

where $y_j = 1$ meant a collection center was set up at position j and $y_j = 0$ meant a collection center was not set up at this position.

The optimal model in bottom level was single objective-oriented, so the classical genetic algorithm was used to get the optimal solution in bottom level (see Figure 4). The algorithm performed genetic operations (selection, crossover, and mutation) and repeated the operations driven by the algorithm in Figure 2 until the optimal solution of the top-level model was got.

In this algorithm, the encoding scheme rc and the feasible solution of business collaboration between recycling agencies/stations and collection centers was defined as the integer vector in

$$rc = (x_1, x_2, \cdots x_m). \quad (9)$$

The value of x_i was the collection center to which recycling agency/station i transferred WEEE. For example, one feasible location solution of collection centers in the top-level model was cc = (0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1) (see equation (8)). In cc, $y_3 = y_7 = y_{12} = 1$, it meant that only collection center 3, 7, and 12 were set up to collect WEEE from all recycling agencies/stations. Thus, one feasible solution in the bottom-level model can be defined as rc = (3, 3, 7, 3, 12, 12, 7, 3, 12, 7, 12, 7, 3, 12, 12, 7, 7) (see equation (9)). In rc, $x_1 = x_2 = x_4 = x_8 = x_{13} = 3$, it meant that recycling agency/station 1, 2, 4, 8, and 13 transferred WEEE to collection center 3.

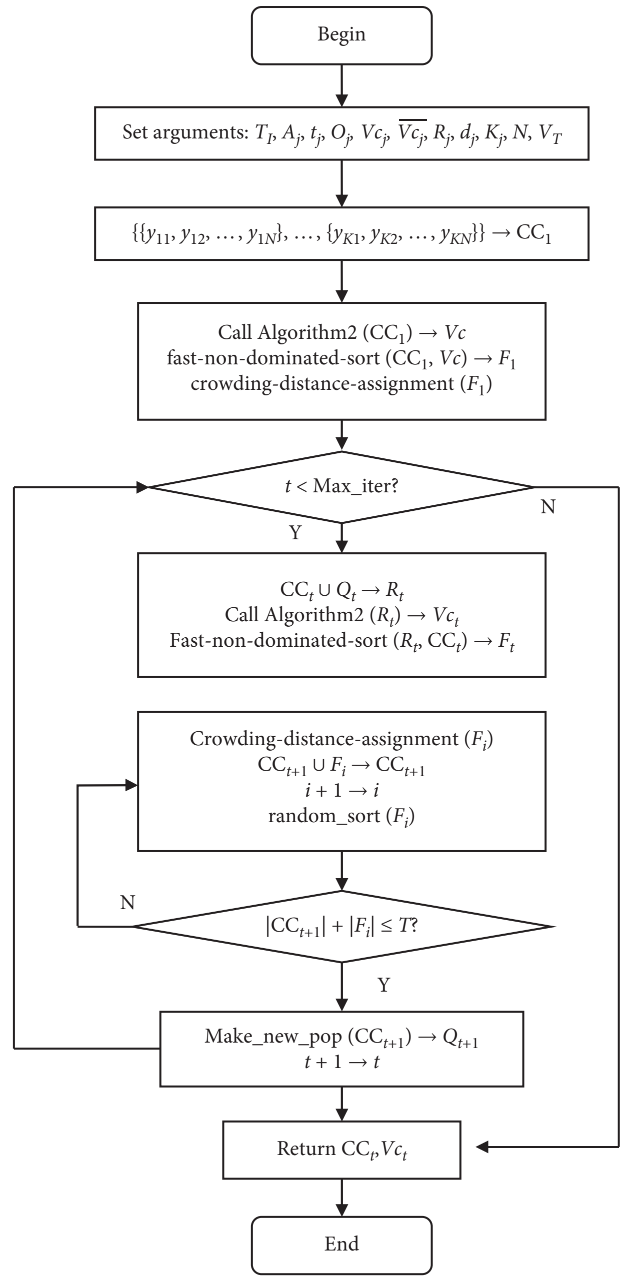

Figure 3: Algorithm to solve the top-level model.

4.2. Validation of the Model. The validation dataset was about the business of a city in China. There were 44 third-party recycling agencies/stations and 18 candidate collection centers scattered in the city. The total investment (T_I) was up to 10 million yuan. The annual disposal licensed capacity was 1.5 million, and the average time of transportation from collection centers to disposal factory was 100. Therefore, $V_T = 15000$.

The raw data about recycling stations and candidate collection centers were collected by the authors in 2014. The distance between collection centers and disposal factory was measured by ArcGIS with longitude and latitude data, so was the distance between collection centers and recycling agencies/stations. The final dataset for validation is shown in Tables 1–3.

The parameters of NSGA and GA are shown in Table 4.

Geatpy (Jazzbin, 2020), an evolutionary algorithm toolbox and framework with high performance in Python [27], was used to implement the bilevel programming algorithms due to the following features:

(1) As open source software written in pure Python, Geatpy helps researchers easily implement complex models with templates in programming mode such as MatLab
(2) Be capable of solving single-objective, multi-objective, many-objective, and combinatorial optimization problems
(3) A huge number of operators with high performance of evolutionary algorithms (selection, recombination, mutation, and migration)
(4) Many evolutionary algorithm templates, including GA, DE, and ES for single-/multiobjective evolution
(5) Support parallelization and distribution of evaluations
(6) Support tracking analysis of the evolution iteration

The algorithms were repeatedly performed in 20 turns to validate the stability. The values of objective function $f1$ and $f2$ are shown in Table 5.

In this experiment, there was only one nondominated solution in each run, which demonstrated a seemingly

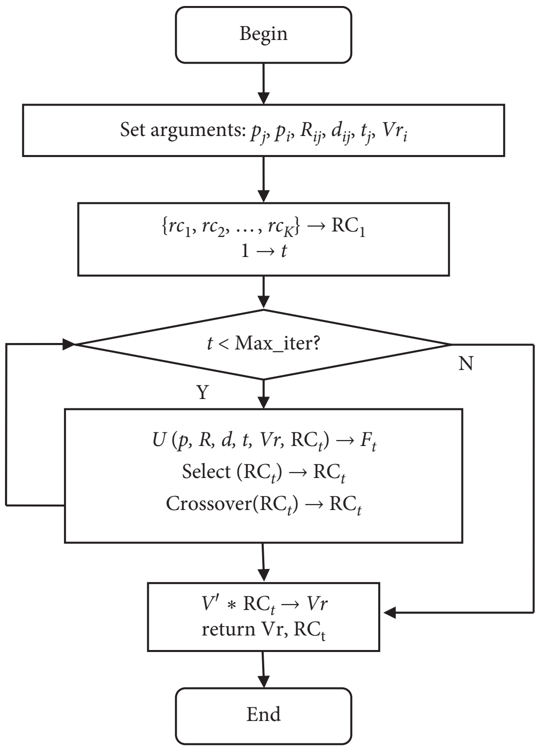

Figure 4: Algorithm to solve the bottom-level model.

confusing result. Through the tracking analysis of the evolution process, it was found that in the bottom-level model, all recycling agencies/stations tried to transfer WEEE to collection centers as more as possible (see equation (6)), which made $f2$ (the total volume of collected WEEE) at a stably highest level in each run. The value of $f2$ would be the total of the maximum capacity of all recycling agencies/stations (if the total was less than V_T) or V_T (if the total was larger than V_T). The result hinted that the third-party recycling agencies/stations had the overwhelming impacts on Chinese WEEE recycling business.

With the fixed value of $f2$, $f1$ would theoretically converge to a stable level. The convergence level of $f1$ was analyzed in the hypothesis test. The normal distribution test was used to test the distribution of the sampling error of f1. The p value was 0.278 ($\alpha = 0.05$), which meant that there was no significant difference in the values of f1 in all runs.

One of the feasible solutions was selected as the final solution of the location problem. Figure 5 shows the evolution process of $f1$. The optimal solution was got at iteration 78. The values of $f1$ and $f2$ were 5219652 and 4394. The collaboration between recycling agencies/stations and collection centers is shown in Table 6.

5. Discussion

There were some representative related studies on Chinese WEEE recycling business. For example, Di et al. (2013) considered the collection cost of WEEE and took it as an important factor in the model. Their study aimed at establishing the recycling stations especially for WEEE, which was not feasible because most recycling agencies/stations and recycling vendors worked for all kinds of renewable resources and their business was independent of disposal factories. Huang and Zhang (2018) constructed an improved maximum coverage location model of waste mobile phone recycling point, using the waste mobile phone recycling spatiotemporal demand retrieved by mobile signaling data mining, and the position and quantity of demand were all changing [23]. They considered the cluster feature of mobile users to plan the location of collection centers, and it was not suitable for all kinds of WEEE. Furthermore, the third-party recycling agencies/stations as the major WEEE recycling institutions were not considered in their model, and it was not factual in the Chinese WEEE recycling market. Di and Wang (2012) proposed a 2-objective optimal model to plan the Chinese WEEE recycling network. The definition of 2 objective functions was similar to that of the model in this paper. To solve the multiobjective model, the values of 2 objective functions were separately calculated and normalized, and then weighted sum method was used to transform the multiobjective model into a single-objective model [28]. However, there were some aspects that could be argued: (1) the possible conflicts of 2 objectives were taken as negative problems. In fact, in multiobjective problems, conflicts of objectives were common and acceptable. (2) The validity of the transformation method was uncertain because it was difficult to determine the reasonable values of the weights of objective functions.

Compared with the related studies, the proposed model addressed the location problem of the Chinese WEEE recycling network and adequately reflected the demand of disposal companies in the Chinese WEEE industry. The features of this study are summarized in the following aspects:

(1) Since the third-party recycling agencies/stations were independent of disposal factories and running with different capacity, the benefits of third-party recycling agencies/stations were also taken into account other than related studies.

(2) Compared with the multiobjective and single-objective algorithms used in related studies, the non-dominated solution tactic was more reasonable and universal than those of other algorithms.

(3) In the implementation part of the proposed model, in order to simplify the algorithm, the encoding scheme of feasible solutions was also elaborately designed. It was noted that in the bottom-level model, the collaborations among recycling agencies/stations and collection centers were defined by a M-dimension vector (see equation (9)) other than by the traditional M ∗ N matrix shown in equation (10). With the M-dimension vector, a feasible solution containing M numbers was evaluated by GA, which significantly reduced the complexity of the bottom-level model and saved more computation time:

Table 1: Information about candidate collection centers.

ID	Distance	Collection price	Area	Volume	Trans. fee rate	Operation fee	Time window	K
1	14.1	17	1000	800	1.8	160	0.95	1000
2	10.2	15	1000	700	1.7	180	1	1000
3	15.6	16	850	700	1.75	180	1	900
...	...	...	...	...	...	...	...	...
15	9.2	15	850	800	1.7	180	0.9	1100
16	14.1	16	1200	800	1.8	180	1	1000
17	16	17	900	900	1.7	180	0.95	900
18	13	15	1000	850	1.5	150	1.1	1000

Table 2: Information about recycling agencies/stations.

ID	Volume	Collection price
1	80	12
2	120	13
3	80	13
...	...	...
42	72	13
43	89	13
44	60	12

Table 3: Information about transportation between collection centers and recycling agencies/stations.

Collection centers	Recycling agencies/stations	Distance	Trans. fee rate
1	1	2	1.8
1	2	5	1.7
1	3	1	1.75
...	...	...	...
2	1	4.5	1.6
2	2	6.4	1.7
2	3	3	1.7
...	...	...	...
18	44	2.2	1.7

Table 4: Parameters of NSGA and GA algorithms.

Algorithms	Number of individuals	Max. iterations
NSGA-III for top-level model	40	500
GA for bottom-level model	40	500

Table 5: The values of $f1$ and $f2$ in 20-turn random experiment.

Turn	$f1$	Sampling error of $f1$	$f2$	Time (sec.)
1	5264756	−80683	4394	1320
2	5438353	67043	4394	1428
...	...	...	...	
18	5268428	−54543	4394	1365
19	5238616	119054	4394	1518
20	5264756	−50871	4394	986

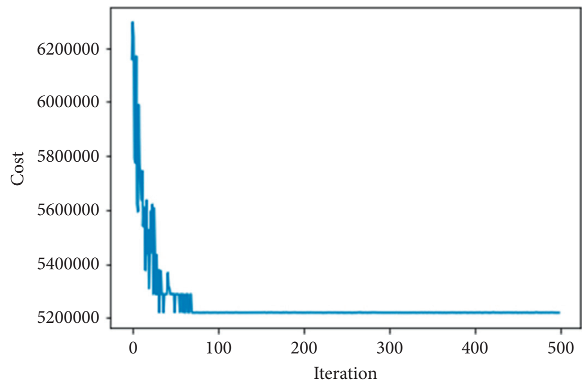

Figure 5: The value of objective function $f1$ in all iterations.

Table 6: The business links between collection centers and recycling agencies/stations.

Collection centers	Recycling stations	Group volume
2	0, 5, 6, 7,13, 32, 33	677
7	2, 4, 8, 10, 14, 18, 31	723
8	19, 22, 28, 29, 34, 37	548
10	1, 9, 12, 17, 23, 30	749
14	11, 16, 21, 24, 25, 27, 38, 39	799
16	3, 15, 20, 26, 35, 36, 40, 41, 42, 43	898

$$\text{rc} = \left\{ \begin{matrix} 1 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \\ \cdots & & & \\ 0 & 1 & \cdots & 0 \end{matrix} \right\}. \tag{10}$$

6. Conclusions

As was illustrated in the introduction and related study sections, at present, considering the regulations enforced by Chinese government and the structure of the reverse supply chain, the disposal factories can hardly establish their own recycling stations to directly collect WEEE from the customers. They had to cooperate with the third-party recycling agencies/stations to set up their own local collection centers to collect more WEEE with lower logistic cost. Moreover, the WEEE disposal volume must meet with the 20 percent of the licensed volume to get the subsidies from the government. Through the experiment in this paper, the bilevel programming-based location model was proved to work with considerably stable convergence and optimization performance. It met the requirements of disposal factories and the recycling agencies/stations and considered the regulations issued by the Chinese government. It can be a new useful approach to the treatment of WEEE in China.

However, the study introduced in this paper was based on the current situation of China WEEE policies which was still in practice now; therefore, the model should be improved according to the change of related policies. Meanwhile, even the model proposed in this paper was Chinese market-oriented, and if the business mode of other countries was similar to that of China, the model was also available through minor revision in variables and objective function definition, especially in developing countries. However, since the WEEE business was heavily affected by the regulations or policies, the proposed model was not completely guaranteed to universally work well.

In addition, more factors such as competition among WEEE disposal companies and the income from the disposal of WEEE were not considered. The cost and volume were not calculated elaborately with category of WEEE, either. As a result, the objective functions and constraints were not perfectly defined in the proposed model, which led to few nondominated solutions for business managers. In future study, these unconsidered factors should also be considered to improve the optimal model for better planning result.

Data Availability

The data used to support the findings of this study are available from the first author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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Research Article

Exploiting GWmZd Model by Exploring Knowledge-Based Grey-Holistic Technique for Sustainable Vendor Evaluation

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It is investigated that the thoughts of sustainability gained the momentum among researchers in the present era. The industrial empirical research survey proved that green operations, waste minimization strategies, and zero defect planning are three potential pillars in mapping the sustainability of supplier/vendor units. The authors built and proposed a grey knowledge-based green-waste minimizing and zero defect (GWmZd) sustainability appraisement hierarchical structural evaluation model/framework to production companies for assessing the sustainability ratio of their candidate suppliers/partners. Due to the uncertainty associated with the measures and metrics of the proposed model, the incomplete information is procured from a cluster of professionals' vs GWmZd measures and metrics in the terms of the grey (except fuzzy) set. It is sensed by the prior literature survey that a few grey knowledge-based sustainability model are framed, but they were constrained to the individual first level layer without weight evaluation cum noncomparative analysis-based modern technique, and it is respected as a major research gap and challenge. To compensate the major research gap, the authors elected AHP and enabled AHP (analytic hierarchy process) to materialize and aggregate the assigned rating of each expert for evaluating weights of 2nd level metrics (overcoming the drawback of combined/group ratings). Later, the authors structured and proposed a new mathematical equation, assisted authors to evaluate the global weight of the first layer-three pillars-measures of the proposed model. Eventually, the authors constructed and fruitfully implemented a grey-holistic technique (merger of grey-MOORA-FMF fused with the dominance theory) on the model to compute sustainability index and score of suppliers. A production company is investigated to exhibit the application of the research work practically. The sustainability of supplier A_1 is found the best than the residue of suppliers. The research forum can be explored by production companies to opt the feasible supplier under the proposed model. The conducted research has a value across the global production companies. The research forum can be explored by managers of production companies for benchmarking the performance of global suppliers under GWmZd and future advancing models along with grey-holistic technique fused with the dominance theory.

1. Introduction of Sustainability

The application of technological and performance measurement tools towards attaining the green-lean-economic architectures in vendor units is called as vendor sustainability. The vendor firms can gain the sustainability if firms preserve the cost-effective and best practices/processes, i.e., green, waste minimizing, and zero defects planning across production measures practices, as these practices holistically enable the vendor organizations sustainable at the global platform. It is sensed that numerous vendor organizations attain the sustainability by minimizing miscellaneous wastes, preserving the best green performance, and maintaining economic tradeoffs in their production units in the present era. It is ascertained by the literature survey that advancement in metrics-practices of the green-waste minimizing and zero defect (GW_MZ_D) model advise the production companies to materialize the sustainability of vendor organizations, where the green measure stimulate the vendor companies to preserve the green manufacturing via ramping up the renewable energies utilization, abasing hazard materials byproducts, and recycling of waste water [1–3]. Next, minimization of waste leads to lean manufacturing and eradicate the vendor companies for reducing multiple wastes, i.e., idle time, over processing, unwanted production, and unnecessary movement. Zero defects planning instructs the vendor companies to trim down the defective products, salvaging, and reworks and leads in reducing the cost of the quality. Figure 1 shows the GWmZd model whose base pillars were taken from the manuscript published by [4].

1.1. Green Practice, Waste Minimization, and Zero Defects Concept. Green practice/measure aids the industrialists to eliminate the causes of carbon attacks, minimization of ill-biological particles, fossil fuels, hazard particles, ill-particles, and toxic gasses. Green practice/measure is utilized in the present era as it is considered as a leading competitive strategy to overcome the trust of product consumers. Green practice/measure can be attained by vendor firms by pursuing, such as advanced practices-metrics, i.e., renewable energy process, recycling of waste and hazard materials, and recycling of waste water. A green practice is a procedure to eliminate the amount of many wastes, which are produced during the production. In vendor firms, green practice is used as an effective process for butchery the waste production. Waste can be minimized if vendor firms pursue, such as advanced green metrics-practices, i.e., over processing, most excellent production, and effective movement.

Waste minimization refers to the lean manufacturing strategy, whose objective is the use of economic sources and recycling methods prior to disposal of the wastes. As per the United Nations Green Programme (UNGP), waste minimization refers a strategy that has the aim to prevent waste via upstream interventions. In case of production in vendor firms, these strategies are focusing the on optimizing resource and energy use and lowering toxicity levels during the manufacturing time. Strategies are considered to minimize waste and thus improve resource efficiency before the manufacturing process, i.e., product design, cleaner production, reuse of scrap material, improved quality control, and waste exchanges. Recently, it is observed that minimization of waste does not only eliminate the waste such as idle time, over processing, unwanted production, and unnecessary movement (aids to economic production) but also preserve the healthy green environment, resulting in higher productivity. Minimization of waste helps the vendor firms to achieve sustainability.

Zero defects motivate the vendor firms for abolishing the wastes at first (that leads to cost reduction). Thus, zero defects leads to waste reduction along with cost cutting. All these processes improve the services and therefore, make customer pleasure. Zero defects can be attained by stimulating the vendor firm for pursuing, such as advanced metrics-practices, i.e., elimination of manufacturing of defective products, meager rejection of goods, and minimization of reworking and salvaging.

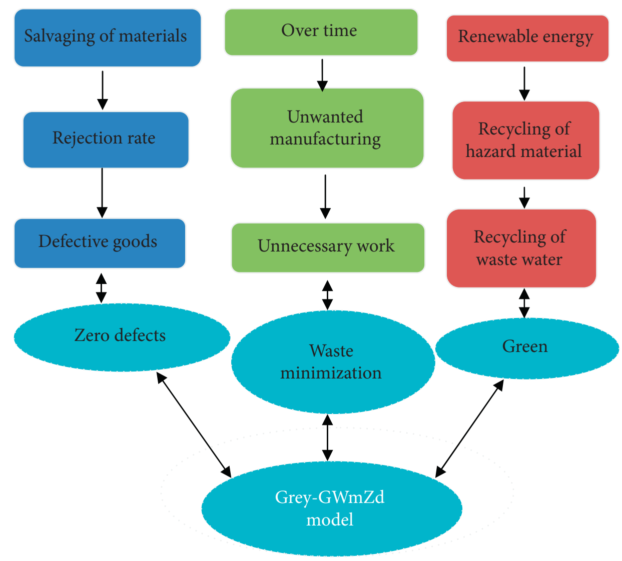

Figure 1: Grey-GWmZd sustainably model.

2. Literature Survey

Literature survey is conducted into two parts: in the first part, the literature survey is conducted in the context of green manufacturing, waste minimization, and zero defects philosophies for structure and framing the model.

Later, the literature survey is conducted in extant of the multiobjective optimization MOORA methods and grey set theory and their application towards measuring the sustainability of production and vendor firms.

2.1. Part: I

2.1.1. Literature Survey is Conducted in the Context of Green Manufacturing. In [5], the author said that the green stressors may be responsible for protecting the quality and quantity of freshwater resources. Reference [6] articulated an innovative green regulation, which was integrated with licensed Ireland's pharmaceutical manufacturing sector as a part of the case study. In [7], the authors stated that green audit is a management tool, which provides a structure and comprehensive mechanism for ensuring that the goods of an enterprise do not cause unacceptable effects on the atmosphere. Reference [8] proposed a model for appraising the worth of sustainable material providers. The expert's method was applied to distinguish the criterion for assessing the performance of traditional material providers and sustainable material providers. Reference [9] stated that green is a major source of greenhouse gas emissions (55%).

Reference [10] defined that an idea of sustainability can be built by optimizing the link among global society and its natural atmosphere, taking into account society's social, economic, and green chains. In [11], the authors stated that sustainability is an important goal such as promoting economic development, decreasing poverty, and improving quality. The green agenda is a necessary part of holistic, city-led strategies for economic, social, and green sustainability. In [12], the authors stated that green sustainability added the green value in the growing proportion of the world's population, lives in cities. Reference [13] proposed the sustainability tool to aware about the status of sustainability development in organizations by dimensions such as green, social, and economic. In [14], the authors proposed a double layers green supply chain efficient appraisement model for benchmarking the green alternative suppliers. A triangular fuzzy sets theory is used to handle vagueness of the supplier's model and select the most significant supplier. In [15], the authors determined during the case study of coal enterprises of China that various driving mechanisms, i.e., government regulations, enterprise resource capability, and supply chain, aid the global industries to attain the green innovation. Reference [16] developed a multiobjective decision-making hierarchical model, which integrated the forward and reverse logistics in objective to reduce the recycling and manufacturing costs in industries. Reference [17] investigated the united green innovation policy and pricing strategies in a remanufacturing system to attract the huge customers. In [18], the authors investigated the GSC as retailer's strategy. It is ascertained that GSC aids the retailer to improve their retail profit with poor promotional efforts.

2.1.2. Literature Survey is Conducted in the Context of Waste Minimization Manufacturing. Reference [19] explained the principles and conceptions of reuse aspects in case studies of Ecosan in developing countries. Reference [20] outlined the nature of the wastes, waste generating industries, waste characterization, health and green implications of wastes management practices, steps towards planning, design and development of models for effective hazardous waste management, treatment, approaches, and regulations for disposal of hazardous waste. Evaluation of the entire situation with reference to Indian scenario has attempted in order that a better cost-effective strategies for waste management be evolved in future. Reference [21] developed a green vendor selection model, which is solved by application of the artificial neural network (ANN) with two more multiattribute judgment analyses (MAJA) techniques such as data envelopment analysis (DEA) and the analytic network process (ANP) [22]. Industry is a chief consumer of natural resources and a major donor to the overall pollution load. As per organization for economic cooperation and development, it accounts for about one-third of global energy consumption of their member states and for about 10% of the total water withdrawal. The relative contribution to the total pollution load is obviously higher for industry-related pollutants, i.e., organic substances, sulfur dioxide, particulates, and nutrients. In [23], the authors said that lean and agile indicators can be jointed within supply chain. The authors also said that combining agility and leanness in one supply chain via the strategic utility of a decoupling point is called as legality. The legality model can be constructed in future. Reference [24] applied a fuzzy-based quality function deployment (QFD) on lean SC. A case study was conducted out in an Indian electronics switches manufacturing organization. The applied techniques were found so effective in the recognition of lean indicators, lean decision domains, lean attributes, and lean enablers for the business. Reference [25] proposed an agility lean model to evaluate the agility and leanness of an individual firm. The model consisted of a set of agile and lean supply chain practices integrated in an assessment model. Reference [26] estimated the leanness of a manufacturing firm in a fuzzy context. Various leanness indicators have been considered in order to measure the performance of the manufacturing firm by using the concept of the trapezoidal fuzzy number set [27]. Waste minimization is one of the strategies, adopted for minimizing the industrial pollution. The objective of the scheme is to assist the small and medium scale industry in adoption of cleaner production practices.

2.1.3. Literature Survey is Conducted in the Context of Zero Defect (Six Sigma) Philosophy. In [28], zero defects philosophy is a thought that gained its focus from 1960s. It is a programme to take away defects from the industrial production and was primarily intended for automobile production [29]. Zero defects (ZDs) philosophy is a management tool, which has the aim to reduce the defects through preventions. Vendor firm's employees are directed to prevent mistakes by developing a constant, conscious desire to do their job right the first time. In [30], the Six Sigma tools have aims to process the best product quality. Six Sigma tools is the need of organization to build customized products. Reference [31] conducted a systematic review on Lean Six Sigma and found that the environmental (green) is the best method for improving the quality in manufacturing operations. Reference [32] investigated the application of Lean Six Sigma (LSS) tools in food processing industries. It is evaluated that LSS impact on environmental sustainability. Reference [33] proposed an integrated DEA technique with the Six Sigma projects evaluation model for selecting Six Sigma projects. The findings demonstrated that selected projects confirmed expert opinions. Reference [34] evaluated the university leadership performance by using the Lean Six Sigma (LSS) framework. Reference [35] constructed a structural measurement model by creating a link between Lean Six Sigma (LSS) and sustainable manufacturing strategies. Next, structural equation modeling technique is applied to validate the existing links.

2.2. Part: II

2.2.1. Literature Survey is Conducted in Extant of the Multiobjective Optimization MOORA-FMF Methods. Reference [36] explored the multiobjective optimization by ratio analysis (MOORA) technique for project management in a transaction economy. Reference [37] explored the MOORA technique to solve the inner climate problems. Reference [38] proposed the MOORA method to solve many economic, managerial, and construction problems. Reference [39] employed the MOORA method to define the economic policy for balanced regional development in

Lithuania. Reference [40] applied the MOORA method in the construction field to solve the problems related to energy loss in heating buildings. Reference [41] explored the MOORA method to solve different decision-making problems in the real-time manufacturing green. Six decision-making problems are solved. Reference [42] presented the extended MOORA method for solving decision-making problems with interval data to determine the most preferable alternative among possible alternatives. Reference [43] extended the MULTIMOORA method with type-2 grey sets for solving the personnel selection problem under uncertain assessments.

2.2.2. Grey System Application towards Achieving the Sustainability. Reference [44] utilized the grey set and rough set theory towards integrating the sustainability with the vendor selection procedure. Reference [45] compared conventional statistical tools with the grey system theory and declared the three superiorities of the grey system theory, i.e., (a) provides easy calculation, (b) requires few sample size, and (c) has an exact accuracy for prediction. Reference [46] proposed an efficient grey TOPSIS and grey COPRAS methodology with the vendor appraisement platform for vendor evaluation under green concerns. Reference [47] utilized the grey sets theory with integrated MULTIMOORA to benchmark the CNC machine tool under CNC machine tool evaluation criteria. Reference [48] suggested the outline of the grey set so that an upcoming researcher might use the concept for decision-making. Reference [49] extended the application of the grey theory in many decision-making problems. In [50], the authors applied the grey system theory with the expert panel method to set up evaluation index for the material provider. The performance of mass production by Commercial Aircraft Corporation of China Ltd (COMAC) was measured. Reference [51] proposed a fractional reverse accumulative grey Verhulst model to enhance the model stability and improve the prediction accuracy in responding to the characteristics of information on the test. In [52], the authors proposed a fractional order reverse accumulation generation gm model and revealed its applications in solving industrial problems. Reference [53] presented a new ranking method to determine the ranking order of the professionals. The authors built a novel graph model with grey information to solve equilibrium states and decision paths problem. In [27], the authors proposed a QFD network for the early design of a complex product, which demonstrated the top-down decomposition design process. Moreover, an uncertain multilevel programming model and its algorithm are proposed to aid the designers to get an optimal solution. Reference [54] described the way to control and utilize the grey system theory under lack of information or incomplete information. Reference [55] utilized grey concepts and the corresponding theories to develop a multiobjective grey wolf optimizer for handling multiple objectives optimization problems. Reference [56] presented the grey DEMATEL approach to identify and evaluate criterions and alternatives under incomplete information.

3. Motivation to Conduct Research Work, Especially for Sustainability

Sustainability is a thought, which mainly focuses to minimize the industrial wastes by ramping up the quality across processes with green and zero defects planning schemes and concerns. Sustainability can be attained via increasing the renewable energy processes, recycling of waste and hazard materials, recycling of waste water, over processing, most excellent production, effective movement, elimination of manufacturing of defective products, meager rejection of goods, and minimization of reworking. The sustainability assessment of supplier organizations must be carried out by production companies if production companies desire to sustain at global market for a long period of time [6, 9, 12, 22].

(i) Recently, it is virtually investigated that previous authors proposed a single and a few double layers sustainability assessment hierarchical structural model in addressing fuzzy ratings-based simulation techniques such as TOPSIS, PROMATHEE, SAW, VIKOR, and MLMCDM for assessing sustainability of vendor firm alternatives. A few authors attempted for grey ratings evaluation techniques, but they were capable to solve only the single layer sustainability assessment hierarchical structural model. Aforesaid gaps are probed as a first research gap.

(ii) It is also observed and probed by authors, especially focusing over the model structure and framing embedded with measures/metrics that previous researchers introduced ordinary measures practices and its interrelated metrics such as economic and employee retention in framing sustainably-based models (except focusing over the green, minimization of waste, and zero defect measures practices with their interrelated metrics in inducing into the model), and it is noticed as a second research gap.

(iii) Furthermore, the authors observed via the same literature survey that a few proposed sustainability-based hierarchical structural models are facilitated with crisp AHP technique, but AHP was capable to tackle only fused or combined ratings (except individual rating of each member). Next, the crisp AHP weight evaluation method application is observed accompanied with single TOPSIS, PROMATHEE, SAW, VIKOR, and MLMCDM ratings evaluation techniques under only the fuzzy set (except grey set), for assessing sustainability of vendor firm alternatives is found as the third research gap.

(iv) There are no weight evaluation mathematical formulas, which can be used to calculate the weight of measures from metrics data (calculated by the AHP), and it is respected as a forth research gap.

(v) The authors had no research evidence pertaining to grey-based holistic-robust technique that can deliver accurate results as reliability of decision is a big concern, and it is respected as the fifth research gap.

The aforesaid grounds motivated authors to develop the multilevel knowledge-based GWmZd sustainability appraisement hierarchical structural evaluation model, with introducing AHP with new weights, and global weight mathematical formula, with grey set-based holistic technique embedded with the dominance approach to compensate all research gaps.

4. Grey-Holistic Approach with AHP

4.1. Analytic Hierarchy Process (AHP). Analytic hierarchy process (AHP) is one of the multipractices decision-making techniques. In short, it is a technique to derive ratio scales from paired comparisons [57, 58]. The input can be obtained from the actual measurement such as price and weight or from the subjective opinion such as satisfaction feelings and preference. AHP allows some small inconsistency in judgment because human is not always consistent. The ratio scales are derived from the principal eigenvectors, and the consistency index is derived from the principal eigen value.

It is a tool used for solving complex decision problems to evaluate many dilemma in different areas of human requirements, such as political, financial, and various others different interests. The AHP provides a comprehensive and rational framework to help managers set priorities and make the best decision when both qualitative and quantitative aspects of a decision need to be considered. In conventional AHP, the pairwise comparison is established using a scale which converts the human preferences between available alternatives. Even though the discrete scale of AHP has the advantages of simplicity and ease of use, it is not sufficient to take into account the uncertainty associated with mapping of one's perception to a number. However, due to vagueness and uncertainty in the decision maker's judgment, a crisp, pairwise comparison with a conventional AHP may be unable to accurately capture the decision maker's judgment.

Definition 1. Consistency of the pairwise comparison matrix [57].

In the classical AHP, we consider an $n \times n$ a pairwise comparison matrix A with positive elements, such that

$$A = \begin{bmatrix} 1 & a_{12} & \dots & a_{1n} \\ a_{21} & 1 & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & 1 \end{bmatrix}. \quad (1)$$

This matrix is reciprocal if $a_{ij} = 1/a_{ji}$, for each $1 \le i, j \le n$.We say that A is consistent if

$$a_{ij} * a_{jk} = a_{ik}, \quad \text{for each } 1 \le i, j, k \le n. \quad (2)$$

From the geometrical means, the relative normalized weights of each attributes/criterions can be calculated by normalizing the geometrical means of raw in the comparison matrix. This can be presented in equations (1) and (2); the geometric means method of the AHP is explored to find out the relative normalized weights of the criterion due to its simplicity and easiness to find out the maximum eigen value and to reduce the inconsistency in judgment.

$$A_1 = \left[b_{ij}\right], \quad (3)$$

$$\text{GM} = \left[\prod_{j=1}^{n} b_{ij}\right]^{1/M}, \quad (4)$$

$$A_2 = W_j = \frac{\text{GM}}{\sum_{j=1}^{n} \text{GM}_j}.$$

$$\text{Calculation of matrices } A_3 \text{ and } A_4 \text{ such that } A_3 = A_1 \times A_2 \text{ and } A_4 = \frac{A_3}{A_2}, \quad (5)$$

$$\text{where } A_2 = \left[w_1, w_2, w_3, \dots, w_j\right]^T, \text{ and } A_i \text{ is a decision matrix.}$$

Determine the maximum eigen value ($\lambda_{\max}$), i.e., the average of matrix A_4.

Consistency index is evaluated by the following equation:

$$\text{Consistency index (CI)} = \frac{\text{Principle eigen value} - \text{size of the matrix}}{\text{Size of the matrix} - 1} = \frac{\lambda_{\max} - \text{n}}{\text{n} - 1}. \quad (6)$$

For the index of consistency for random judgments, Saaty [57] defined the consistency ratio (CR) as

$$\mathrm{CR} = \frac{\mathrm{CI}}{\mathrm{RI}}, \tag{7}$$

where RI is chosen by the matrix size using the Saaty [58] (Table 1)

4.2. Global Weights Equation [46].

$$\sum_{i=1}^{n} w_i = 1, \quad w_i = w_1, w_2, w_3, \ldots, j = n. \tag{8}$$

4.3. Theory of Grey Numbers: Mathematical Basis. Grey theory has become a very effective method of solving uncertainty problems under discrete data and incomplete information. Grey theory has now been applied to various areas such as forecasting, system control, and decision-making and computer graphics. Here, we give some basic definitions regarding the relevant mathematical background of the grey system, grey set, and grey number in the grey theory [27, 44, 46, 47, 59, 60].

Definition 2. A grey system is defined as a system containing uncertain information presented by grey numbers and grey variables

Definition 3. Let X be the universal set. Then, a grey set G of X is defined by its two mappings:

$$\begin{cases} \overline{\mu}_G(x)\colon x \longrightarrow [0,1], \\ \underline{\mu}_G(x)\colon x \longrightarrow [0,1]. \end{cases} \tag{9}$$

$\overline{\mu}_G(x) \geq \underline{\mu}_G(x)$, $x \in X$, $X = R$, $\overline{\mu}_G(x)$, and $\underline{\mu}_G(x)$ are the upper and lower membership functions in G, respectively. When $\overline{\mu}_G(x) = \underline{\mu}_G(x)$, the grey number G becomes a grey set. It shows that the grey theory considers the condition of fuzziness and can flexibly deal with the fuzziness situation

Definition 3. Definition 4A grey number is one of which the exact value is unknown, while the upper and/or the lower limits can be estimated. Generally grey number is written as $(\otimes G = G|_{\underline{\mu}}^{\overline{\mu}})$

Definition 5. If only the lower limit of G can be possibly estimated and G is defined as the lower limit grey number,

$$\otimes G = [\underline{G}, \infty]. \tag{10}$$

Definition 6. If only the upper limit of G can be possibly estimated and G is defined as the upper limit grey number,

$$\otimes G = [\infty, \overline{G}]. \tag{11}$$

Definition 7. If the lower and upper limits of G can be estimated and G is defined as the interval grey number,

$$\otimes G = [\underline{G}, \overline{G}]. \tag{12}$$

Definition 8. The basic operations of grey numbers $\otimes x_1 = [\underline{x}_1, \overline{x}_1]$ and $\otimes x_2 = [\underline{x}_2, \overline{x}_2]$ can be expressed as follows:

$$\left.\begin{aligned} &\otimes x_1 + \otimes x_2 = [\underline{x}_1 + \underline{x}_2, \overline{x}_1 + \overline{x}_2] \\ &\otimes x_1 - \otimes x_2 = [\underline{x}_1 - \overline{x}_2, \overline{x}_1 - \underline{x}_2] \\ &\otimes x_1 \times \otimes x_2 = \mathrm{Min}[\underline{x}_1\underline{x}_2, \underline{x}_1\overline{x}_2, \overline{x}_1\underline{x}_2, \overline{x}_1\overline{x}_2], \mathrm{Max}[\underline{x}_1\underline{x}_2\underline{x}_1\overline{x}_2, \overline{x}_1\underline{x}_2, \overline{x}_1\overline{x}_2] \\ &\frac{\otimes x_1}{\otimes x_2} = [\underline{x}_1, \overline{x}_1] \times \left[\frac{1}{\underline{x}_2}, \frac{1}{\overline{x}_2}\right] \end{aligned}\right\}. \tag{13}$$

Whitened value: The whitened value of an interval grey number, $\otimes x$, is a deterministic number with its value lying between the upper and lower bounds of interval $\otimes x$. For a given interval grey number $\otimes x = [\underline{x}, \overline{x}]$, the whitened value $x_{(\lambda)}$ can be determined as follows [44, 46, 47, 50].

$$x_{(\lambda)} = \lambda\, \underline{x} + (1-\lambda)\overline{x}, \tag{14}$$

where λ is the whitening coefficient, and $\lambda \in [0,1]$. Because of its similarity with a popular λ function, formula (15) is often shown in the following form:

$$x_{(\lambda)} = (1-\lambda)\, \underline{x} + \lambda\overline{x}. \tag{15}$$

For $\lambda = 0.5$, formula (16) gets the following form:

Table 1: The value of random consistency index.

M	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48	1.56	1.57	1.59

$$x_{(\lambda=0.5)} = \frac{1}{2}\left(\underline{x} + \overline{x}\right). \tag{16}$$

Signed distance: let $\otimes x_1 = [\underline{x}_1, \overline{x}_1]$ and $\otimes x_2 = [\underline{x}_2, \overline{x}_2]$ be two positive interval grey numbers. Then, the distance between $\otimes x_1$ and $\otimes x_2$ can be calculated as a signed difference between its centers as shown in the following equation:

$$\begin{aligned} d\left(\otimes x_1, \otimes x_2\right) &= \frac{\underline{x}_1 + \overline{x}_1}{2} - \frac{\underline{x}_2 + \overline{x}_2}{2} \\ &= \frac{1}{2}\left[\left(\underline{x}_1 - \underline{x}_2\right) + \left(\overline{x}_1 - \overline{x}_2\right)\right]. \end{aligned} \tag{17}$$

4.4. Evaluation of Rating from 2^nd^ to 1^st^ Level [46].

$$\begin{aligned} \mathbf{R} &= \left(\otimes r_i\right)_{m\times n} \\ &= \frac{\otimes r_{ik1} + \otimes r_{ik2} + \otimes r_{ik3} + \otimes r_{ik4} + \otimes r_{ik5} + \otimes r_{ik6}, \ldots, \otimes r_{ikn}}{C_{ikn}}. \end{aligned} \tag{18}$$

By using equation (18), denominator $\mathbf{R} = \left(\otimes r_j\right)_{m\times n}$, the computed i^{th} rating of 1st level measures vs alternatives j^{th}, can be computed on availing assigned ratings data of 2nd level metrics $(\otimes r_{ik})$. C_{ikn} is the number of metrics that are aligned with its father measure.

4.5. The MOORA Technique. Multiobjective optimization by ratio analysis (MOORA) method is introduced by [37–39, 41, 42] on the basis of previous research studies. The method starts with a matrix of responses of different alternatives on different objectives:

$$X = \left[x_{ij}\right]_{m\times n}, \tag{19}$$

wherex_{ij} is the response of alternative j on objective or attribute$i$; $j = 1, 2, \ldots, m$ is the alternative; and $i = 1, 2, \ldots, n$ is the attribute.

The MOORA method consists of two parts: the ratio system and the reference point approach [38].

4.5.1. The Ratio System Approach of the MOORA Method. Reference [42] proved that the most robust choice for the denominator is the square root of the sum of squares of each alternative per objective, and therefore, the use of the vector normalization method is recommended in order to normalize responses of alternatives. As a result, the following formula is obtained:

$$x_{ij}^* = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{m} x_{ij}^2}}, \tag{20}$$

where x_{ij} is the response of alternative j on objective or attribute i; $j = 1, 2, \ldots, m$ is the number of alternatives; $i = 1, 2, \ldots, n$, where n is the number of objectives; x_{ij}^* is the normalized response of alternative i on objective j; and $x_{ij}^* \in [0, 1]$.

Let $W = (w_1, w_2, \ldots, w_n)$ be the relative weight vector about the practices, evaluated by grey AHP satisfying $\sum_{i=1}^{n} w_i = 1$.

$$\tilde{R} = \left(\tilde{r}_{ij}\right)_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccccc} c_1 & c_2 & \cdots & \ldots\ldots & c_n \end{array} \\ \begin{bmatrix} x_{11}^* & x_{12}^* & \cdots & x_{1n}^* \\ x_{21}^* & x_{22}^* & \ldots & x_{2n}^* \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1}^* & x_{m2}^* & \ldots & x_{mn}^* \end{bmatrix} \end{array}. \tag{21}$$

Calculate the weighted normalized decision matrix.

$$v = (v_{ij})_{mn},$$

$$\begin{aligned} \tilde{R} = \left(\tilde{r}_{ij}\right)_{m\times n} &= \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccccc} c_1 & c_2 & \cdots & \ldots\ldots & c_n \end{array} \\ \begin{bmatrix} x_{11}^* & x_{12}^* & \cdots & x_{1n}^* \\ x_{21}^* & x_{22}^* & \ldots & x_{2n}^* \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1}^* & x_{m2}^* & \cdots & x_{mn}^* \end{bmatrix} \end{array} \times \begin{bmatrix} w_j \\ w_j \\ \cdot \\ w_j \end{bmatrix}, \\ \tilde{V} = \left(\tilde{v}_{ij}\right)_{m\times n} &= \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \begin{bmatrix} \tilde{v}_{11} & \tilde{v}_{12} & \cdots & \tilde{v}_{1n} \\ \tilde{v}_{21} & \tilde{v}_{22} & \ldots & \tilde{v}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{v}_{m1} & \tilde{v}_{m2} & \ldots & \tilde{v}_{mn} \end{bmatrix} \end{array}. \end{aligned} \tag{22}$$

For optimization based on the ratio system approach of the MOORA method, normalized responses are added in case of maximization and subtracted in case of minimization, which can be expressed by the following formula:

$$y_j^* = \sum_{i=1}^{g} x_{ij}^* - \sum_{i=g+1}^{i=n} x_{ij}^*, \tag{23}$$

where x_{ij}^* is the normalized response of alternative j on objectives i; $i = 1, 2, \ldots, g$ is the objective to be maximized; $i = g + 1, g + 2, \ldots, n$ is the objective to be minimized; $j = 1, 2, \ldots, m$ is the alternatives; and y_j^* is the overall ranking index of alternative$j$. $y_j^* \in [-1, 1]$ provided and proved by [42].

4.5.2. The Importance Given to Objectives. When solving real-world problems using MCDM methods, objectives do not always have the same importance, i.e., some objectives are more important than the others. In order to give more importance to an objective, it could be multiplied with a significance coefficient [42]. Importance given to objectives has influence on the ratio system and reference point approach of the MOORA method. In the ratio system approach, importance given to objectives is included by modifying formula (23), which gets the following form:

$$\ddot{y}_j^* = \sum_{i=1}^{g} s_i x_{ij}^* - \sum_{i=g+1}^{i=n} s_i x_{ij}^*, \tag{24}$$

where s_i is the significance coefficient of objective$i$; $i = 1, 2, \ldots, g$ is the objective to be maximized; $i = g+1, g+2, \ldots, n$ is the objective to be minimized; $j = 1, 2, \ldots, m$ is the alternative; and $\ddot{y}_j^*$ is the overall ranking index of alternative j with respect to all objectives with significance coefficients, $\ddot{y}_j^* \in [-1, 1]$.

After that, formula (24) still remains to determine the most appropriate alternative based on the ratio system approach of the MOORA method.

4.5.3. The Grey-MOORA. The procedure of selecting the most appropriate alternative using the MOORA method involves several important stages that should be considered before an extension of the MOORA method with interval grey numbers, and these are [43]

Stage 1: transforming responses of alternatives into dimensionless values

Stage 2: determining overall ranking indexes for considered alternatives based on the ratio system part of the MOORA method and

Stage 3: determining distances between considered alternatives and the reference point based on the reference point part of the MOORA method

Stage 1: transformation into dimensionless values

For the normalization of responses of alternatives expressed in the form of interval numbers, suggested the use of the following formula:

$$\otimes x_{ij}^* = \frac{\otimes x_{ij}}{\sqrt{\sum_{j=1}^{m}\left(\underline{x}_{ij}^2 + \overline{x}_{ij}^2\right)}}. \tag{25}$$

Formula (25) provides the appropriate form for normalizing responses of alternatives expressed by interval grey numbers. However, in cases of multipractices optimizations, which require simultaneously the use of crisp and interval grey numbers, the previously mentioned formula gives unsatisfactory results.

Stage 2: determining overall ranking index based on the ratio system approach of the MOORA method for optimization based on the ratio system part of the MOORA method, we start from the formula

$$\begin{aligned}
y_j^* &= y_j^{\vee} - y_j^{\wedge}, \\
y_j^* &= \sum_{i\in\Omega_G^+} \otimes s_i x_{ij}^* - \sum_{i\in\Omega_G^-} \otimes s_i x_{ij}^*, \\
y_j^{\vee} &= \sum_{i\in\Omega_G^+} \otimes s_i x_{ij}^*, \\
y_j^{\wedge} &= \sum_{i\in\Omega_G^-} \otimes s_i x_{ij}^*,
\end{aligned} \tag{26}$$

where y_j^* is the overall ranking index of alternative j; $y_j^{\vee}$ and $y_j^{\wedge}$ are the total sums of maximizing and minimizing responses of alternative j to objectives i, respectively; s_i is the significance coefficient of objective i; x_{ij}^* or $\otimes x_{ij}^*$ as the normalized responses of alternative j on different objectives i, which are expressed in the form on crisp or interval grey numbers; Ω_G^+ are the assets of objectives to be maximized and expressed in the form on crisp or interval grey numbers, and Ω_G^- are the sets of objectives to be minimized and expressed in the form on crisp or interval grey numbers:

(i) When decision makers have the same significance ($\lambda = 0$),

$$\begin{aligned}
y_j^* = {} & (1-\lambda)\left(\sum_{i\in\Omega_G^+} \underline{s_i x}_{ij}^* - \sum_{i\in\Omega_G^-} \underline{s_i x}_{ij}^*\right) \\
& + \lambda\left(\sum_{i\in\Omega_G^+} \overline{s_i x}_{ij}^* - \sum_{i\in\Omega_G^-} \overline{s_i x}_{ij}^*\right).
\end{aligned} \tag{27}$$

(ii) When the decision maker has no preferences ($\lambda = 0.5$),

$$\begin{aligned}
y_j^* = {} & \frac{1}{2}\left(\sum_{i\in\Omega_G^+} \underline{s_i x}_{ij}^* - \sum_{i\in\Omega_G^-} \underline{s_i x}_{ij}^*,\right) \\
& + \frac{1}{2}\left(\sum_{i\in\Omega_G^+} \overline{s_i x}_{ij}^* - \sum_{i\in\Omega_G^-} \overline{s_i x}_{ij}^*\right).
\end{aligned} \tag{28}$$

(iii) When the decision maker has no preference and objectives have the same significance ($\lambda = 1$),

$$\begin{aligned}
y_j^* = {} & \lambda\left(\sum_{i\in\Omega_G^+} \overline{s_i x}_{ij}^* - \sum_{i\in\Omega_G^-} \overline{s_i x}_{ij}^*,\right) \\
& + (1-\lambda)\left(\sum_{i\in\Omega_G^+} \underline{s_i x}_{ij}^* - \sum_{i\in\Omega_G^-} \underline{s_i x}_{ij}^*\right).
\end{aligned} \tag{29}$$

During problem solution, i.e., ranking of alternatives, the attitude of the professionals can lie between pessimistic and optimistic, and the whitening coefficientλ allows the expression of professionals' degree of optimism or pessimism.

In the cases of particularly expressed optimism, the whitening coefficient λ, in accordance with the aforesaid formula, takes

Table 2: Grey knowledge-based GWmZd sustainability appraisement hierarchical structural evaluation model.

Goal (C)	Measures (C_i)	Metrics (C_{ik})
		Renewable energy ($C_{1,1}$)
	Green (C_1)	Recycling of hazard material ($C_{1,2}$)
		Recycling of waste water ($C_{1,3}$)
		Over time, ($C_{2,1}$)
Sustainability measurement	Waste minimization (C_2)	Unwanted manufacturing ($C_{2,2}$)
		Unnecessary work ($C_{2,3}$)
		Defective goods ($C_{3,1}$)
	Zero defect (C_3)	Rejection rate ($C_{3,2}$)
		Salvaging of materials ($C_{3,3}$)

Table 3: Attitude of the measures and their interrelated metrics against supplier organizations A_1, A_2, and A_3.

Goal (C)	Measures (C_i)	Attitudes	Metrics (C_{ik})	Attitudes
			Renewable energy ($C_{1,1}$)	(+)
	Green (C_1)	(+)	Recycling of hazard material ($C_{1,2}$)	(+)
			Recycling of waste water ($C_{1,3}$)	(+)
			Over time ($C_{2,1}$)	(−)
Sustainability measurement	Waste minimization (C_2)	(−)	Unwanted manufacturing ($C_{2,2}$)	(−)
			Unnecessary work ($C_{2,3}$)	(−)
			Defective goods ($C_{3,1}$)	(−)
	Zero defect (C_3)	(−)	Rejection rate ($C_{3,2}$)	(−)
			Salvaging of materials ($C_{3,3}$)	(−)

Table 4: Definition of measures.

C_j	Definition of measures
(C_1)	This focus on maximizing the renewable energy, recycling of waste materials, hazard materials, and recycling of water
(C_2)	The aim is to reduce over processing, unwanted production, and unnecessary movements
(C_3)	The aim is to eliminate the defective product, rejection, and rework

Table 5: The verbal scale of importance by AHP.

Intensity of importance	Verbal scale	Description
1	Equal importance	Two activities contribute equally to the objective
3	Weak importance of one over another	Experience and judgment slightly favor one activity over another
5	Essential or strong importance	Experience and judgment strongly favor one activity over another
7	Demonstrated importance	An activity is strongly favored and its dominance demonstrated in practice
9	Absolute importance	The evidence favoring one activity over another is of the highest possible order of affirmation

higher values ($\lambda \longrightarrow 1$), and the ranking order of alternatives is mainly based on the upper bounds of intervals with which the overall response of each alternative is expressed, $y_{j(\lambda=1)} = y_j^*$. On the other hand, in the cases of particularly expressed pessimistic, the whitening coefficient λ takes lower values ($\lambda \longrightarrow 0$), and the ranking order of alternatives is mainly based on lower bounds of the intervals, $y_{j(\lambda=0)} = y_j^*$. On the other hand, in the cases of particularly expressed moderate, the whitening coefficient λ takes half of lower and upper values ($\lambda \longrightarrow 0.5$), and the ranking order of alternatives is mainly based on lower bounds of the intervals, $y_{j(\lambda=0.5)} = y_j^*$.

4.5.4. The Grey-FMF. Determining overall ranking index is based on multiobjective optimization on the full multification form decision-making evaluation technique; it was the extensive part of MOORA formula [37, 39–41]:

$$y_j^+ = \frac{\prod_{i\in\Omega_G^+} s_i x_{ij}^*}{\prod_{i\in\Omega_G^-} s_i x_{ij}^*}. \tag{30}$$

(i) When objectives have the same significance ($\lambda = 0$),

$$y_j^+ = (1-\lambda)\left(\frac{\prod_{i\in\Omega_G^+} \underline{s_i x}_{ij}^*}{\prod_{i\in\Omega_G^-} \underline{s_i x}_{ij}^*}\right) + \lambda\left(\frac{\prod_{i\in\Omega_G^+} \overline{s_i x}_{ij}^*}{\prod_{i\in\Omega_G^-} \overline{s_i x}_{ij}^*}\right). \tag{31}$$

(ii) When the decision maker has no preferences ($\lambda = 0.5$),

TABLE 6: Importance against (C_1–C_9), assigned by P_1.

C_i	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
C_1	1	1/3	1/5	1/3	1/5	1/7	1/5	1/3	1/5
C_2	3	1	1/3	1/5	1/3	1/5	1/7	1/5	1/7
C_3	5	3	1	1/5	1/7	1/5	1/3	1/5	1/3
C_4	3	5	5	1	1/9	1/5	1/5	1/5	1/3
C_5	5	3	7	9	1	1/5	1/9	1/9	1/5
C_6	7	5	5	5	5	1	1/5	1/7	1/5
C_7	5	7	3	5	9	5	1	1/5	1/7
C_8	3	5	5	5	9	7	5	1	1/9
C_9	5	7	3	3	5	5	7	9	1
Crisp representation									
C_1	1.000	0.333	0.200	0.333	0.200	0.143	0.200	0.333	0.200
C_2	3.000	1.000	0.333	0.200	0.333	0.200	0.143	0.200	0.143
C_3	5.000	3.000	1.000	0.200	0.143	0.200	0.333	0.200	0.333
C_4	3.000	5.000	5.000	1.000	0.111	0.200	0.200	0.200	0.333
C_5	5.000	3.000	7.000	9.000	1.000	0.200	0.111	0.111	0.200
C_6	7.000	5.000	5.000	5.000	5.000	1.000	0.200	0.143	0.200
C_7	5.000	7.000	3.000	5.000	9.000	5.000	1.000	0.200	0.143
C_8	3.000	5.000	5.000	5.000	9.000	7.000	5.000	1.000	0.111
C_9	5.000	7.000	3.000	3.000	5.000	5.000	7.000	9.000	1.000

TABLE 7: Importance against (C_1–C_9), assigned by P_2.

C_i	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
C_1	1	1/5	1/5	1/5	1/5	1/7	1/9	1/3	1/5
C_2	5	1	1/5	1/5	1/3	1/5	1/7	1/5	1/7
C_3	5	5	1	1/5	1/7	1/5	1/3	1/5	1/3
C_4	5	5	5	1	1/9	1/5	1/5	1/5	1/3
C_5	5	3	7	9	1	1/5	1/9	1/9	1/5
C_6	7	5	5	5	5	1	1/5	1/7	1/5
C_7	9	7	3	5	9	5	1	1/5	1/7
C_8	3	5	5	5	9	7	5	1	1/9
C_9	5	7	3	3	5	5	7	9	1
Crisp representation									
C_1	1.000	0.200	0.200	0.200	0.200	0.143	0.111	0.333	0.200
C_2	5.000	1.000	0.200	0.200	0.333	0.200	0.143	0.200	0.143
C_3	5.000	5.000	1.000	0.200	0.143	0.200	0.333	0.200	0.333
C_4	5.000	5.000	5.000	1.000	0.111	0.200	0.200	0.200	0.333
C_5	5.000	3.000	7.000	9.000	1.000	0.200	0.111	0.111	0.200
C_6	7.000	5.000	5.000	5.000	5.000	1.000	0.200	0.143	0.200
C_7	9.000	7.000	3.000	5.000	9.000	5.000	1.000	0.200	0.143
C_8	3.000	5.000	5.000	5.000	9.000	7.000	5.000	1.000	0.111
C_9	5.000	7.000	3.000	3.000	5.000	5.000	7.000	9.000	1.000

TABLE 8: Importance against (C_1–C_9), assigned by P_3.

C_i	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
C_1	1	1/9	1/9	1/9	1/9	1/9	1/9	1/9	1/9
C_2	9	1	1/5	1/5	1/5	1/5	1/5	1/5	1/5
C_3	9	5	1	1/5	1/7	1/5	1/3	1/5	1/3
C_4	9	5	5	1	1/9	1/5	1/5	1/5	1/3
C_5	9	5	7	9	1	1/5	1/9	1/9	1/5
C_6	9	5	5	5	5	1	1/5	1/7	1/5
C_7	9	5	3	5	9	5	1	1/5	1/7
C_8	9	5	5	5	9	7	5	1	1/9
C_9	9	5	3	3	5	5	7	9	1
Crisp representation									
C_1	1.000	0.111	0.111	0.111	0.111	0.111	0.111	0.111	0.111
C_2	9.000	1.000	0.200	0.200	0.200	0.200	0.200	0.200	0.200

Table 8: Continued.

C_i	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
C_3	9.000	5.000	1.000	0.200	0.143	0.200	0.333	0.200	0.333
C_4	9.000	5.000	5.000	1.000	0.111	0.200	0.200	0.200	0.333
C_5	9.000	5.000	7.000	9.000	1.000	0.200	0.111	0.111	0.200
C_6	9.000	5.000	5.000	5.000	5.000	1.000	0.200	0.143	0.200
C_7	9.000	5.000	3.000	5.000	9.000	5.000	1.000	0.200	0.143
C_8	9.000	5.000	5.000	5.000	9.000	7.000	5.000	1.000	0.111
C_9	9.000	5.000	3.000	3.000	5.000	5.000	7.000	9.000	1.000

Table 9: Importance against (C_1–C_9), assigned by P_4.

C_i	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
C_1	1	1/3	1/3	1/3	1/3	1/3	1/3	1/3	1/3
C_2	3	1	1/7	1/7	1/7	1/7	1/7	1/7	1/7
C_3	3	7	1	1/5	1/7	1/5	1/3	1/5	1/3
C_4	3	7	5	1	1/9	1/5	1/5	1/5	1/3
C_5	3	7	7	9	1	1/5	1/9	1/9	1/5
C_6	3	7	5	5	5	1	1/5	1/7	1/5
C_7	3	7	3	5	9	5	1	1/5	1/7
C_8	3	7	5	5	9	7	5	1	1/9
C_9	3	7	3	3	5	5	7	9	1
Crisp representation									
C_1	1.000	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333
C_2	3.000	1.000	0.143	0.143	0.143	0.143	0.143	0.143	0.143
C_3	3.000	7.000	1.000	0.200	0.143	0.200	0.333	0.200	0.333
C_4	3.000	7.000	5.000	1.000	0.111	0.200	0.200	0.200	0.333
C_5	3.000	7.000	7.000	9.000	1.000	0.200	0.111	0.111	0.200
C_6	3.000	7.000	5.000	5.000	5.000	1.000	0.200	0.143	0.200
C_7	3.000	7.000	3.000	5.000	9.000	5.000	1.000	0.200	0.143
C_8	3.000	7.000	5.000	5.000	9.000	7.000	5.000	1.000	0.111
C_9	3.000	7.000	3.000	3.000	5.000	5.000	7.000	9.000	1.000

Table 10: Importance against (C_1–C_9), assigned by P_5.

C_i	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
C_1	1	1/5	1/5	1/5	1/5	1/5	1/5	1/5	1/5
C_2	5	1	1/7	1/7	1/7	1/7	1/7	1/7	1/7
C_3	5	7	1	1/5	1/7	1/5	1/3	1/5	1/3
C_4	5	7	5	1	1/9	1/5	1/5	1/5	1/3
C_5	5	7	7	9	1	1/5	1/9	1/9	1/5
C_6	5	7	5	5	5	1	1/5	1/7	1/5
C_7	5	7	3	5	9	5	1	1/5	1/7
C_8	5	7	5	5	9	7	5	1	1/9
C_9	5	7	3	3	5	5	7	9	1
Crisp representation									
C_1	1.000	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200
C_2	5.000	1.000	0.143	0.143	0.143	0.143	0.143	0.143	0.143
C_3	5.000	7.000	1.000	0.200	0.143	0.200	0.333	0.200	0.333
C_4	5.000	7.000	5.000	1.000	0.111	0.200	0.200	0.200	0.333
C_5	5.000	7.000	7.000	9.000	1.000	0.200	0.111	0.111	0.200
C_6	5.000	7.000	5.000	5.000	5.000	1.000	0.200	0.143	0.200
C_7	5.000	7.000	3.000	5.000	9.000	5.000	1.000	0.200	0.143
C_8	5.000	7.000	5.000	5.000	9.000	7.000	5.000	1.000	0.111
C_9	5.000	7.000	3.000	3.000	5.000	5.000	7.000	9.000	1.000

Table 11: Aggregated importance against (C_1–C_9), assigned by $P_{1,2,3,4,5}$.

C_i	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
C_1	1.000	0.235	0.209	0.235	0.209	0.186	0.191	0.262	0.209
C_2	5.000	1.000	0.204	0.177	0.230	0.177	0.154	0.177	0.154
C_3	5.400	5.400	1.000	0.200	0.143	0.200	0.333	0.200	0.333
C_4	5.000	5.800	5.000	1.000	0.111	0.200	0.200	0.200	0.333
C_5	5.400	5.000	7.000	9.000	1.000	0.200	0.111	0.111	0.200
C_6	6.200	5.800	5.000	5.000	5.000	1.000	0.200	0.143	0.200
C_7	6.200	6.600	3.000	5.000	9.000	5.000	1.000	0.200	0.143
C_8	4.600	5.800	5.000	5.000	9.000	7.000	5.000	1.000	0.111
C_9	5.400	6.600	3.000	3.000	5.000	5.000	7.000	9.000	1.000

Table 12: Evaluated eigenvector by AHP.

C_i	M^{th} root of products of values	Eigenvectors
C_1	0.256	0.019
C_2	0.316	0.023
C_3	0.537	0.039
C_4	0.705	0.051
C_5	0.981	0.072
C_6	1.434	0.105
C_7	2.099	0.153
C_8	3.057	0.223
C_9	4.302	0.314

Table 13: Evaluated global weights.

Goal (C)	Measures (C_i)	Global weights	Metrics (C_{ik})	Eigenvectors
			Renewable energy ($C_{1,1}$)	0.019
	Green (C_1)	0.081	Recycling of hazard material ($C_{1,2}$)	0.023
			Recycling of waste water ($C_{1,3}$)	0.039
			Over time ($C_{2,1}$)	0.051
Sustainability measurement	Waste minimization (C_2)	0.228	Unwanted manufacturing ($C_{2,2}$)	0.072
			Unnecessary work ($C_{2,3}$)	0.105
			Defective goods ($C_{3,1}$)	0.153
	Zero defect (C_3)	0.691	Rejection rate ($C_{3,2}$)	0.223
			Salvaging of materials ($C_{3,3}$)	0.314

$$y_j^+ = \lambda\left(\frac{\prod_{i\in\Omega_G^+} s_i \underline{x}_{ij}^*}{\prod_{i\in\Omega_G^-} s_i \underline{x}_{ij}^*},\right) + \lambda\left(\frac{\prod_{i\in\Omega_G^+} \overline{s_i x}_{ij}^*}{\prod_{i\in\Omega_G^-} \overline{s_i x}_{ij}^*}\right). \tag{32}$$

(iii) When the decision makers have no preference and objectives have the same significance ($\lambda = 1$),

$$y_j^+ = \lambda\left(\frac{\prod_{i\in\Omega_G^+} \overline{s_i x}_{ij}^*}{\prod_{i\in\Omega_G^-} \overline{s_i x}_{ij}^*},\right) + (1-\lambda)\left(\frac{\prod_{i\in\Omega_G^+} s_i \underline{x}_{ij}^*}{\prod_{i\in\Omega_G^-} s_i \underline{x}_{ij}^*}\right). \tag{33}$$

During the problem solution, i.e., ranking of alternatives, the attitude of the professionals can lie between pessimistic and optimistic, and the whitening coefficientλ allows the expression of professionals' degree of optimism or pessimism.

In the cases of particularly expressed optimism, the whitening coefficient λ, in accordance with formula (29), takes higher values ($\lambda \longrightarrow 1$), and the ranking order of alternatives is mainly based on the upper bounds of intervals with which the overall response of each alternative is expressed, $y_{j(\lambda=1)} = y_j^+$. On the other hand, in the cases of particularly expressed pessimistic, the whitening coefficient λ takes lower values ($\lambda \longrightarrow 0$), and the ranking order of alternatives is mainly based on lower bounds of the intervals, $y_{j(\lambda=0)} = y_j^+$. On the other hand, in the cases of particularly expressed moderate, the whitening coefficient λ takes half of lower and upper values ($\lambda \longrightarrow 0.5$), and the ranking order of alternatives is mainly based on lower bounds of the intervals, $y_{j(\lambda=0.5)} = y_j^+$.

Table 14: The scale of attribute ratings $\otimes G$.

Scale	$\otimes r$
Very poor (VP)	(0, 1)
Poor (P)	(1, 3)
Medium poor (MP)	(3, 4)
Fair (F)	(4, 5)
Medium good (MG)	(5, 6)
Good (G)	(6, 9)
Very good (VG)	(9, 10)

Table 15: Appropriateness grey rating against metrics for A_1.

Metrics (C_{ik})	P1	P2	P3	P4	P5
Renewable energy ($C_{1,1}$)	VG	G	G	G	VG
Recycling of hazard material ($C_{1,2}$)	VG	VG	VG	VG	MP
Recycling of waste water ($C_{1,3}$)	MG	F	F	MP	MP
Over time ($C_{2,1}$)	G	F	F	MP	VG
Unwanted manufacturing ($C_{2,2}$)	VG	F	F	MP	F
Unnecessary work ($C_{2,3}$)	MG	F	P	MP	F
Defective goods ($C_{3,1}$)	MG	MP	VG	VG	F
Rejection rate ($C_{3,2}$)	MG	MP	F	MG	G
Salvaging of materials ($C_{3,3}$)	F	MP	F	MG	G

Table 16: Appropriateness grey rating against metrics for A_2.

Metrics (C_{ik})	P1	P2	P3	P4	P5
Renewable energy ($C_{1,1}$)	F	G	MG	MG	G
Recycling of hazard material ($C_{1,2}$)	G	MG	MG	G	G
Recycling of waste water ($C_{1,3}$)	MG	MP	MG	G	G
Over time ($C_{2,1}$)	MG	MP	F	G	G
Unwanted manufacturing ($C_{2,2}$)	MG	F	F	G	MG
Unnecessary work ($C_{2,3}$)	VG	F	VG	MG	MG
Defective goods ($C_{3,1}$)	F	VG	F	MG	MG
Rejection rate ($C_{3,2}$)	MG	MP	F	G	F
Salvaging of materials ($C_{3,3}$)	MG	MP	VG	G	MG

Table 17: Appropriateness grey rating against metrics for A_3.

Metrics (C_{ik})	P1	P2	P3	P4	P5
Renewable energy ($C_{1,1}$)	MG	MP	F	G	MG
Recycling of hazard material ($C_{1,2}$)	VG	MP	F	G	MG
Recycling of waste water ($C_{1,3}$)	F	VG	F	G	MG
Over time ($C_{2,1}$)	F	F	G	VG	MG
Unwanted manufacturing ($C_{2,2}$)	MG	F	P	VG	VG
Unnecessary work ($C_{2,3}$)	F	F	VP	MG	G
Defective goods ($C_{3,1}$)	F	VG	VP	MG	G
Rejection rate ($C_{3,2}$)	G	F	G	MG	G
Salvaging of materials ($C_{3,3}$)	MG	VG	G	MG	VG

Table 18: Addition of grey global rating for A_1.

Goal (C)	Measures (C_i)	Global ratings	Attitudes	Metrics (C_{ik})	Metrics ratings	Attitudes
Sustainability measurement	(C_1)	(6.267, 7.667)	(+)	($C_{1,1}$)	(7.2000, 9.4000)	(+)
				($C_{1,2}$)	(7.8000, 8.8000)	(+)
				($C_{1,2}$)	(3.8000, 4.8000)	(+)
	(C_2)	(4.667, 5.800)	(–)	($C_{2,1}$)	(5.2000, 6.6000)	(–)
				($C_{2,2}$)	(4.8000, 5.8000)	(–)
				($C_{2,3}$)	(4.0000, 5.0000)	(–)
	(C_3)	(5.000, 6.267)	(–)	($C_{3,1}$)	(6.0000, 7.0000)	(–)
				($C_{3,2}$)	(4.6000, 6.0000)	(–)
				($C_{3,3}$)	(4.4000, 5.8000)	(–)

5. Empirical Research: Sustainability Evaluation of Alternative Organizations

The grey knowledge-based GWmZd sustainability appraisement hierarchical structural evaluation model is constructed by scrutinizing 3 (three) momentous measures and 9 (nine) interrelated metrics via the literature review of [4–6, 8–16, 18–22, 27, 28, 30, 34, 35, 61, 62]. The model is found valid towards assessing the sustainability of supplier organizations. The model's attitude and measures' definitions are shown in Tables 2–4, respectively. In the presented research work, 3 significant practices/measures, i.e., green, (C_1), waste minimization, (C_2), and zero defect, (C_3), are considered at the 1st level, whilst identified and shortlisted 9

Table 19: Addition of grey global rating for A_2.

Goal (C)	Measures (C_i)	Global ratings	Attitudes	Metrics (C_{ik})	Metrics ratings	Attitudes
				($C_{1,1}$)	(5.2000, 7.0000)	(+)
	(C_1)	(5.267, 7.200)	(+)	($C_{1,2}$)	(5.6000, 7.8000)	(+)
				($C_{1,2}$)	(5.0000, 6.8000)	(+)
				($C_{2,1}$)	(4.8000, 6.6000)	(−)
Sustainability measurement	(C_2)	(5.333, 6.733)	(−)	($C_{2,2}$)	(4.8000, 6.2000)	(−)
				($C_{2,3}$)	(6.4000, 7.4000)	(−)
				($C_{3,1}$)	(5.4000, 6.4000)	(−)
	(C_3)	(5.133, 6.400)	(−)	($C_{3,2}$)	(4.4000, 5.8000)	(−)
				($C_{3,3}$)	(5.6000, 7.0000)	(−)

Table 20: Addition of grey global rating for A_3.

Goal (C)	Measures (C_i)	Global ratings	Attitudes	Metrics (C_{ik})	Metrics ratings	Attitudes
				($C_{1,1}$)	(4.6000, 6.0000)	(+)
	(C_1)	(5.200, 6.600)	(+)	($C_{1,2}$)	(5.4000, 6.8000)	(+)
				($C_{1,2}$)	(5.6000, 7.0000)	(+)
				($C_{2,1}$)	(5.6000, 7.0000)	(−)
Sustainability measurement	(C_2)	(5.000, 6.333)	(−)	($C_{2,2}$)	(5.6000, 6.8000)	(−)
				($C_{2,3}$)	(3.8000, 5.2000)	(−)
				($C_{3,1}$)	(4.8000, 6.2000)	(−)
	(C_3)	(5.667, 7.333)	(−)	($C_{3,2}$)	(5.4000, 7.6000)	(−)
				($C_{3,3}$)	(6.8000, 8.2000)	(−)

Table 21: Grey global rating matrix for sustainability measurement.

Goal (C)	Alternatives A_j	(C_1)	(C_2)	(C_3)
	A_1	(6.267, 7.667)	(4.667, 5.800)	(5.000,6.267)
Sustainability measurement	A_2	(5.267, 7.200)	(5.333, 6.733)	(5.133, 6.400)
	A_3	(5.200, 6.600)	(5.000, 6.333)	(5.667, 7.333)

Table 22: Grey global rating normalized matrix.

Goal (C)	Alternatives A_j	(C_1)	(C_2)	(C_3)
	A_1	(0.362, 0.442)	(0.260, 0.323)	(0.324, 0.406)
Sustainability measurement	A_2	(0.304, 0.415)	(0.297, 0.375)	(0.332, 0.414)
	A_3	(0.300, 0.381)	(0.278, 0.353)	(0.367, 0.475)

Table 23: Weighted normalized matrix.

Goal (C)	Alternatives A_j	(C_1)	(C_2)	(C_3)
	A_1	(0.02929, 0.0358)	(0.0748, 0.093)	(0.2236, 0.2803)
Sustainability measurement	A_2	(0.02462, 0.0337)	(0.0855, 0.108)	(0.2295, 0.2862)
	A_3	(0.0243, 0.0308)	(0.0800, 0.1016)	(0.2534, 0.3258)

metrics (measures' interrelated factors), i.e., renewable energy, ($C_{1,1}$), recycling of hazard material, ($C_{1,2}$), recycling of waste water, ($C_{1,3}$), over time, ($C_{2,1}$), unwanted manufacturing, ($C_{2,2}$), unnecessary work, ($C_{2,3}$), defective goods, ($C_{3,1}$), rejection rate, ($C_{3,2}$), and salvaging of materials, ($C_{3,3}$), are set up over the 2nd level. The model has an objective to evaluate the sustainability scores of supplier organizations in application of dominance comparative analysis. To evaluate results, a committee of five highly experience professionals are formed from units of the case study manufacturing industry and requested to overview and judge the supplier partners/organizations. The procedure steps are following:

Step 1: Verbal information has been collected by a committee of five highly experienced professionals (P), and a team is formed from the cross functional units of the material purchaser (production) industry. Information is collected from each member of a group of professionals (P) via the crisp AHP linguistic scale as shown in Table 5 in relation to 9 metrics. The assigned pairwise weights against 9 metrics are shown in

Table 24: Ranking results obtained using MOORA technique for $\lambda = 0, 0.5, 1$.

λ	$\lambda = 0$		$\lambda = 0.5$		$\lambda = 1$	
Alternatives A_j	y_j^*	Ranking	y_j^*	Ranking	y_j^*	Ranking
A_1	−0.269	1.000	−0.303	1.000	−0.338	1.000
A_2	−0.290	2.000	−0.326	2.000	−0.361	2.000
A_3	−0.309	3.000	−0.353	3.000	−0.397	3.000

Table 25: Ranking results obtained using full multification form technique for $\lambda = 0, 0.5, 1$.

λ	$\lambda = 0$		$\lambda = 0.5$		$\lambda = 1$	
Alternatives, A_j	y_j^+	Ranking	y_j^+	Ranking	y_j^+	Ranking
A_1	1.746	1.000	1.560	1.000	1.373	1.000
A_2	1.254	2.000	1.170	2.000	1.087	2.000
A_3	1.201	3.000	1.066	3.000	0.930	3.000

Table 26: Preferences of supplier organizations at $\lambda = 0, 0.5, 1$.

λ	$\lambda = 0, 0.5, 1$ by MOORA			$\lambda = 0, 0.5, 1$ by FMF			Final rank
Alternatives, A_j	Ranking	Ranking	Ranking	Ranking	Ranking	Ranking	
A_1	1.000	1.000	1.000	1.000	1.000	1.000	Alternative vendor organization A1 sustainability is the best
A_2	2.000	2.000	2.000	2.000	2.000	2.000	
A_3	3.000	3.000	3.000	3.000	3.000	3.000	

Table 27: The conducted research work's applications, limitations, economic values, and commercial values.

Applications	The proposed grey knowledge-based GWmZd sustainability appraisement hierarchical structural evaluation model can be explored for measuring the sustainability of alternative vendor organizations under the proposed grey set-based approach with the dominance theory. Benchmarking and several election problems of industries, i.e., trucks, cranks, and hand trucks, can also be solved by using same research forum.
Limitations	The model is found versatile in nature. It can solve the many decision-making problems, i.e., the evaluation problem of facility locations and routes for new manufacturing firm by replacing the chain of sustainability assessment measures and their interrelated metrics corresponding to defining alternatives. However, single and multivariables linear programming problem under the boundary constraint value cannot be solved.
Economic values	Proposed model neither requires a specific software nor high skill professional. It can be solved by excel sheet.
Commercial values	Proposed model is sharable with other manufacturing firm by e-mail, fax, and other electronic media. The proposed research work is suitable for solving many problems of the manufacturing firm on extension/exchange of the chain of sustainability appraisement measures and their interrelated metrics against defining or facing alternatives.

Tables 6–10 and aggregated by average rule, and data are shown in Table 11. Equations (1)–(5) are applied to compute weights against 9 metrics of the 2nd level as shown in Table 12. The weights are (0.019, 0.023, 0.039, 0.051, 0.072, 0.105, 0.153, 0.223, and 0.314).

In order to check consistency, equation (4) is applied on calculated $\lambda_{\max} = 9.1$ (considered $M = 9$). Then, the consistency (for CI = 1.45) is checked by using equation (7), depicted $0.0862 < 0.1$. The proposing new equation (8) is applied to compute global weights of measures. Table 13 dealt with the global weights.

Step 2: Later, using the concept of the grey set theory, the grey variables, shown in Table 14, are used by the team of same five professionals (P) to assign grey ratings against supplier organizations, i.e., A_1, A_2, and A_3. The assigned grey ratings are aggregated by equation (13) as shown in Tables 15–17 and Tables 18–20 against suppliers, i.e., A_1, A_2, and A_3.

Step 3: Then, to compute grey global rating of the 1st level measures from the 2nd level metrics, equation (18) is utilized to jump from the 2nd level to the 1st level as shown in Tables 18–20. Grey global rating matrix computed for organizations A_1, A_2, and A_3 is depicted in Table 21.

Step 4: Then, normalization is carried out by using equation (25) for bringing grey set values in the interval of 0-1 excluding transforming the nonbeneficial criterion into beneficial measures as shown in Table 22. Then, global weights are multiplied by its measures and constructed the weighted normalized matrix by using equation (22), shown in Table 23.

Step 5: Grey-MOORA is applied on the data available in Table 23. Ranking results are obtained using MOORA technique by equation (26), and ranks computed for $\lambda = 0, 0.5, 1$ by using equations (27)–(29) shown in Table 24. Grey-Full multification form is applied on the data available in Table 23. Ranking results obtained using grey-FMF technique for $\lambda = 0, 0.5, 1$ is calculated by equations (30)–(33), shown in Table 25.

Step 6: The preference's order vs sustainability of supplier organizations under GWmZd measures is obtained by exploring the comparative analysis at $\lambda = 0, 0.5, 1$ as shown in Table 26. It is found that A_1 is the more sustainable supplier than others. It must be the elected material purchasing company for placing orders (Table 27).

6. Applications, Practical Implications, Economic Values, Commercial Values, and Limitations

Applications, practical implications, economic values, commercial values, and limitations are provided in Table 27

7. Conclusions

In the presented research work, the grey knowledge-based GWmZd sustainability appraisement hierarchical structural evaluation model is constructed by the literature survey, consisted of 3 measures, i.e., green, (C_1), waste minimization, (C_2), and zero defect, (C_3) and 9 measures' interrelated metrics. The authors applied the AHP in addressing individual rating of each expert and aggregated all the expert opinion vs 9 metrics as weights. Later, global weights are computed vs 3 measures. Eventually, the grey-holistic approach amalgamated with the dominance theory [37, 39, 41, 42] is applied in order to get consistent results with respect to attitude of experts, i.e., $\lambda = 0, 0.5, 1$. Eventually, it is found that sustainability of 1st vendor candidate alternative is the best. It must be elected by production firm for placing order.

The others conclusions are given in below section:

(i) The presented grey knowledge-based GWmZd model can be powered by Microsoft Windows XP, and manual computation is also possible

(ii) Proposed GWmZd models can be extended with the advance chain of sustainability appraisement measures and their interrelated metrics against defining alternatives with the grey-holistic approach merged with dominance theory to compute robust results

(iii) Proposed work does not require a special support of the software. It could be solved by excel sheet.

(iv) The authors developed and proposed new mathematical equations, assisted authors to evaluate the global weight of first layer-three pillars from second level metrics

The future scope of the presented research work is that the novel GWmZd model can be constructed with advanced sustainability measures and their interrelated metrics in future. The decision can be simulated by using the same grey-holistic approach fused with the dominance theory.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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Research Article

The Impact of Green Credit Guidelines on the Technological Innovation of Heavily Polluting Enterprises: A Quasi-Natural Experiment from China

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This study investigates the impact of China's Green Credit Guidelines on the technological innovations of heavily polluting enterprises. This study uses data obtained from the CSMAR database (2007–2018) and China Marketization Index Report by Province 2018 and uses the Green Credit Guidelines as a quasi-natural experiment. The sample was divided into an experimental group and a control group; the experimental group disclosed environmental and sustainable development information, while the control group did not. This study's primary finding is that the Green Credit Guidelines can improve the level of technological innovation of heavily polluting enterprises and have a greater impact in areas with high levels of marketization, indicating that the Green Credit Guidelines have a positive effect on the technological innovation of heavily polluting enterprises. This provides China with an experience constructing relevant policies and regulations and provides empirical evidence regarding the technological innovations of heavily polluting enterprises from the perspective of factor market distortions and the Porter hypothesis.

1. Introduction

Increasingly more ecological problems have emerged with the rapid development of urbanization; consequently, environmental problems have become a topic of common concern in both the practical and academic communities. The Chinese government also attaches a great importance to ecological and environmental issues; to strengthen environmental and resource protection, the government has formulated more policies and regulations to restrict the behavior of market players. The construction of an ecological civilization has become a strategic issue at the national level and a core task for governments and people [1, 2]. There is currently an urgent need to solve environmental problems at the institutional level and achieve a win-win situation between the public and the country.

Green credit refers to the concept of using environmental leverage to guide environmental protection and achieve coordinated development of the economy, society, and environment through control and coordination of resources, the environment, and pollution. Its original purposes were to control the expansion and development of the "three high" enterprises, guide funds to environmentally friendly enterprises, optimize the credit structure, serve the real economy, and ultimately reduce environmental pollution. First of all, green credit can promote the optimization and upgrading of the industrial structure through preferential loans to environmental protection companies and fines for polluting companies [3], and it plays a vital role in improving investment efficiency [3, 4] and reducing the pollution [5]. And green credit regulates the flow of social capital to strengthen environmental governance and promote social green production, which plays an increasingly important role in promoting environment friendly enterprises and limiting polluting enterprises [6]. This marks the beginning of a war in China on energy conservation and emission reduction to promote development of an ecological economy and a green economy. In the context of rapid economic development, the task of energy conservation and emission reduction is even more urgent, and the government's environmental protection requirements for enterprises are also more stringent. Inadequate control methods and efforts have an increased credit risk. Therefore, on February 24, 2012, the CBRC issued the Green Credit Guidelines to effectively prevent risks that may be encountered during the development of green credit and to play a greater restrictive role. In the context of rapid economic development, the industrial production model of "polluting first and then treating" is no longer consistent with the main mode of pollution control. Enterprise technological innovation has become an effective strategy for pollution control during the production process [7].

In terms of research on technological innovation, some researchers studied the factors that affected China's energy consumption from 1981 to 1987 and found that technological progress was the main force behind energy conservation [8]. Also some research studies studied the changes in China's energy intensity from 1987 to 1992, and the results showed that technological progress was the main factor in reducing energy consumption [9]. The key reason is that technological innovation can reduce energy consumption by upgrading and adjusting the industrial structure. To a certain extent, traditional product and process innovation can stimulate the production activities of companies in heavily polluting industries (pharmaceutical and steel), thereby improving their energy consumption [10]. Technological innovation has become one of the main means of saving energy, reducing emissions, and decreasing pollution [11, 12]; it has effectively reduced unit energy consumption and changed its structure, thereby reducing energy waste and significantly impacting the sustainable development of enterprises. Therefore, technological innovation has become an important driving force for the high-quality development of the regional economy, which is the key to improving the regional technological innovation performance and production efficiency [13].

However, China's current green industry level is still relatively low, and effective implementation of green credit depends not only on banks' financial incentives but also on enterprises' technical upgrades [1]. In particular, heavily polluting companies have suffered high social pressure, investment risks, environmental litigation risks, and reputational risks. At the same time, when financial development has an important impact on financial constraints, green credit has a crucial impact on corporate financing [14], especially for the financing costs and maturity of heavily polluting companies [15, 16]; this will also indirectly affect the innovation and development of enterprises [17], which has an important impact on the development of green economy [3]. So credit policies and financial constraints have important impacts on the technology investments of enterprises [18]. Will they strengthen technology innovation to reduce emissions, obtain funds, and continue to qualify for credit? By contrast, after qualifying for financing, will they only focus on their interests and abandon the pursuit of environmental benefits? Thus, the study of the green credit guidelines has an important impact on and significant policy implications for core element technology innovation in the research and development of heavily polluting enterprises. Based on the above research, this study takes heavily polluting companies listed on Shanghai and Shenzhen A share markets from 2007 to 2018 as the research sample and classifies them according to whether they disclose environmental and sustainable development information. Using a difference in the differences method, empirical tests are performed to examine the effect of the green credit guidelines on corporate technological innovation, with further analysis of the impact under different environmental regulations and levels of marketization.

The innovations and contributions of this study are mainly as follows. First, the difference in the differences method is used to analyze the quasi-natural experiment of the green credit guidelines and examine the impact of macropolicies and regulations on the innovation of microenterprises, thereby enriching the literature on macropolicies and corporate innovation and opening new research perspectives. Second, there is very little literature on the impact of green credit and corporate technological innovation in the context of microcosmic enterprises; this study helps fill this gap in the technological innovation research. Third, while testing the role of policies and regulations, the study also tests the operating conditions of the market mechanism and provides suggestions for adjusting macropolicies and market mechanisms to better serve the development of the real economy. The results can be used as a reference for the relevant regulatory and policy-making departments and contribute to the economy's sustainable development. Fourth, the results of this paper provide experience for the country to formulate relevant policies and regulations in different regions and provide empirical evidence for technological innovation of heavily polluting companies from the perspective of factor market distortion and Porter's hypothesis.

The remainder of this article is organized as follows. The second section reviews the theory and develops the hypotheses. The third section describes the research design, and the fourth section discusses the empirical analysis. The fifth section presents the concluding remarks.

2. Theory and Hypotheses

The literature includes many studies on green credit policies and technical innovation; some scholars believe there is a positive relationship between the two [19]. Green credit policy plays an important role in resource allocation [15], especially the unconventional monetary policy and credit policy will affect the investment and financing behavior of enterprises [18]. When China implemented the green credit policy, its aim was to achieve sustainable economic development and achieve the dual goal of saving energy and reducing emissions and optimizing and upgrading the industrial structure [19]. The policy plays a restrictive role for heavily polluting enterprises through credit constraints, which are stricter for high-pollution companies with a poor environmental performance [20, 21]. Because the availability of green credit loans is closely related to a company's R&D

investment and technical achievements, to qualify for green credit financing, companies must make technological progress in reducing environmental pollution, provide environmental protection through technological progress, and reduce policy ambiguity and the impact of the lack of information on corporate credit. Therefore, under environmental and public pressure, more enterprises have begun to be motivated to become green. They are actively responding to the ecology and adopting a series of energy saving and consumption reduction measures to promote sustainable economic development [22]. Environmental management capabilities are positively related to company performance, and the stronger the environmental management, the more significant the positive returns [23]; these in turn make the company more enthusiastic about engaging in technological innovation. Therefore, the green credit policy plays an important role in guiding green resources and improving resource utilization efficiency [24]. A company's lean production may generate more public benefit spillovers, thereby playing a role in improving environmental benefits [25]; thus, the positive circular effect is obvious. At the same time, a company is also affected by its social responsibility for the environment. From the perspective of the environment and resources, the higher a company's environmental and social responsibility, the better its stock price performance [26] and the more positive the impact on obtaining credit rights. When financial development has an important impact on financial constraints, green credit also has a vital impact on corporate financing [14], especially for the financing costs and maturity of heavily polluting companies [15, 16], plays an important role in the innovation and development of enterprises [17], and contributes to the green development of the economy [3]. And financial development will alleviate the financing constraints of enterprises and affect their financing behavior [3, 4, 27]. Liu et al. [15] used the double difference (DID) model to conduct a quasi-natural experiment on the "Green Credit Guidelines" issued by China and found that after the introduction of the green credit policy, the proportion and maturity of the debt financing of Chinese companies with serious pollution will drop significantly. In addition, environmental regulations have an important impact on the production efficiency of technological innovation [28], and the impact of green credit on financing is stable and continuous [15, 24]. It is believed that heavily polluting enterprises will definitely increase their emphasis on environmental pollution to qualify for financing and thus increase their investment in technologies that are closely related to pollution discharge. The Porter hypothesis proposes that proper environmental regulation will speed up technological innovation, and the productivity improvements brought about by these innovations will offset the costs incurred to respond to environmental protection, ultimately increasing enterprise profitability.

However, promulgation of the Green Credit Guidelines policy is an important factor that affects creditors' risk perception. Environmental risks affect bank lending behaviors [29]. Financing is essential for companies to conduct production and operating activities, and this is closely related to technical decisions and the environmental performance [30]. Credit policies and financial constraints have a significant impact on corporate investment; more liquid assets will promote the company's R&D investment, and more long-term debt and commercial bank credit may reduce its research and development investment [18]. Moreover, the debt financing capacity of heavily polluting companies has decreased significantly. Liu et al. [24] also indicate that the goal guidance in the green credit policy has greatly reduced the total financing of energy-intensive industries and had a significant inhibitory effect on investment [24]. Thus, the Green Credit Guidelines may limit financing for heavily polluting enterprises, which is not conducive to developing enterprise technological innovation. That is to say, although environmental laws and regulations can promote technological innovation and are one of the important means to achieve green transformation, due to the high cost of energy conservation and emission reduction, it is ultimately not conducive to the development of green innovation [31].

Signaling theory suggests that a company's green credit financing announcement will affect the company by indicating that the company has obtained a loan, which will increase stock price expectations in the market [26]. At this time, on the one hand, the company has a fluke mentality, thinking that it is operating well, and it will not strengthen its technological innovation, leading to loan constraints in a later period. An improvement in technological innovation forms a positive cycle. However, the impact of the green credit policy on heavily polluting and energy-consuming enterprises is still unclear [20]. Figure 1 shows the impact mechanism of the green credit guidelines on technological innovation.

What is the mechanism through which it influences corporate technological innovation? This leads to competitive hypothesis 1:

> H1a: green credit guidelines have a positive effect on the technological innovation of heavily polluting enterprises.
>
> H1b: green credit guidelines have a negative effect on the technological innovation of heavily polluting enterprises.

The distortion in the factor market will cause the factor's market price to deviate from its opportunity cost, leading to the problem of insufficient efficiency in the allocation of market resources. Prior to China's reform and opening up, to support the development of the heavy industry, the low prices resulting from various factors in the planned economy created significant price distortions in the Chinese market. After the reform and opening up, the heavy industry's development strategy has changed, and local governments have become champions of their local economy's GDP. Under this driving mechanism, Chinese officials have been forced to intervene and control land, capital, labor, and other factors to control the market [20], which has an important impact on the level of resource allocation in the entire market. There is a close relationship between regional economic innovation and regional economic development

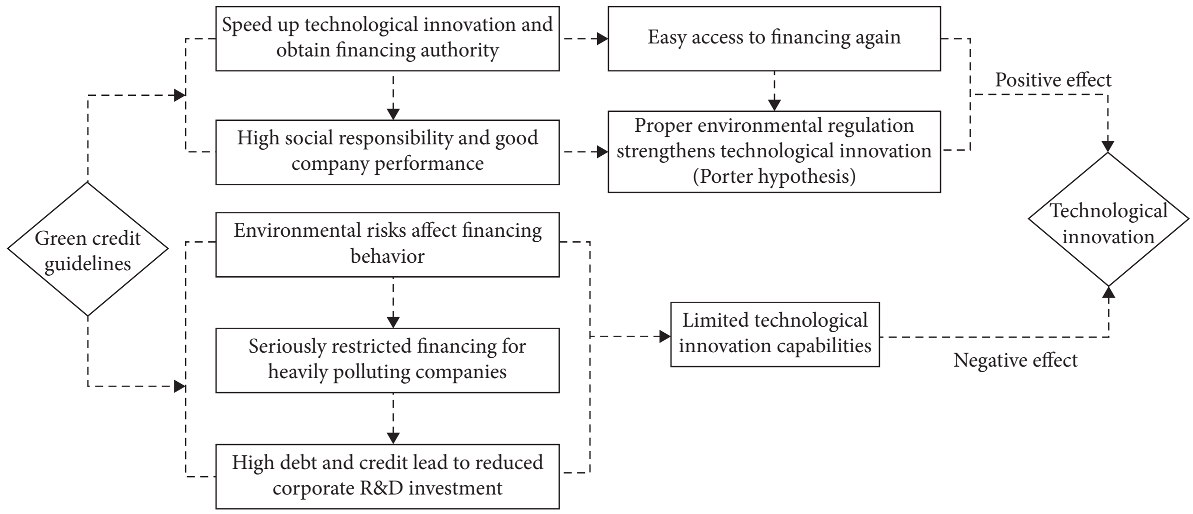

Figure 1: Impact mechanism of the green credit guidelines on technological innovation. Source: created by the author.

[2], so the invisible hand of the marketization level in different regions has an important effect on market innovation. As technological innovation is an important driving force for the high-quality development of the regional economy, improving factor endowment conditions and distribution efficiency is the key to regional technological innovation [13].

The factor endowment hypothesis suggests that when companies can benefit from factor inputs, that is, when the cost of factor inputs is lower than their benefit, they will comply with the corresponding environmental regulations. When the cost of obtaining green credit through technological innovation is lower than the cost of obtaining green credit without technological innovation, they will follow the rules to strengthen technical innovation; otherwise, they will abandon regulation. However, if the factor market is distorted, it will seriously inhibit improvement in China's green economy production efficiency, which is not conducive to overall economic development [32]. The Porter hypothesis suggests that proper environmental regulation will accelerate technological innovation, and the productivity improvements brought about by these innovations will offset the costs of responding to environmental protection, ultimately increasing the profitability of enterprises in the market. Therefore, it is believed that the stronger the environmental regulations, the higher the technological innovation of enterprises. However, China has always had disproportionate development between the east, central, and west. There are great differences in the level of economic development and marketization among regions [33]. Environmental supervision has a significantly positive impact on the efficiency of regional capital allocation [34]. Omri [35] found that technological innovation can contribute to the three pillars of sustainable development at the same time only in rich countries, and it only affects the economic and environmental aspects of middle-income countries, whereas it has no effect on low-income countries. Xu and Li [6] further verified that the economically developed regions are more affected by green credit than the economically undeveloped regions. Where the degree of marketization is high, the level of technological development is high, the institutional environment is good, and the cost of technological innovation is relatively low. In contrast, where the degree of marketization is low, the market environment is relatively poor, and there are many government interventions. The problem is that local governments use their authority to deliberately increase the burden of enterprise approvals and licenses. As a result, an enterprise's technical costs increase accordingly, making it more difficult for it to innovate.

Therefore, the following hypothesis is proposed:

> H2: in areas with a high level of marketization, the stronger the environmental regulations, the more obvious the effect of green credit policies in guiding heavily polluting enterprises to increase their technological innovation capabilities; otherwise, the opposite is true

3. Research Design

3.1. Sample and Data Sources. This study selects heavily polluting listed companies in Shanghai and Shenzhen A share markets from 2007 to 2018 as the research object; 2007 was chosen as the starting year because it is the year the new accounting standard was implemented. Excluded from the sample were (1) current year companies marked ST or ∗ ST and (2) companies with missing data. To eliminate the effects of extreme values, all continuous variables were winsorized at the 5% level, leaving a final sample of 2,337 observations. The financial data were obtained from the CSMAR database and the China Marketization Index Report by Province 2018 and were cross-checked manually.

3.2. Model Setting and Variable Definitions

3.2.1. Model Setting. The Green Credit Guidelines implemented in 2012 present a natural experiment. This study uses a difference in the differences method to evaluate the impact of the green credit guidelines on the technological innovation of heavily polluting enterprises. Based on controlling other variables, the difference in the differences method can test whether there is a significant difference in the processing group's technological innovation development status and that of the control group before and after the green credit guidelines were implemented. The model is set as in the following equation [15, 36]:

$$\mathrm{TI}_{i,t} = \beta_0 + \beta_1 \,\mathrm{DID}_{i,t} + \beta_2 \,\mathrm{treat}_{i,t} + \beta_3 \,\mathrm{post}_{i,t} + \mathrm{controls} + \mathrm{year} + \mathrm{indu} + \mathrm{region} + \varepsilon_{i,t}, \otimes \tag{1}$$

where $\mathrm{TI}_{i,t}$ is the dependent variable used to measure the listed company's degree of technological innovation. $\mathrm{DID}_{i,t}$ is the core explanatory variable, and $\mathrm{DID}_{i,t} = \mathrm{treat}_i \times \mathrm{post}_t$. During the sample period, if a listed company discloses environmental and sustainable development information, $\mathrm{treat}_i = 1$; otherwise, it equals 0. When $t \geq 2012$, $\mathrm{post}_t = 1$; otherwise, it equals 0. Controls are the control variables, year represents the annual effects, indu represents the industry effects, region represents the regional effects, and $\varepsilon_{i,t}$ is the error term. At the same time, the clustered file standard error reported in this study can solve potential serial correlation and heteroscedasticity problems. The processing group in this article includes listed companies that disclose environmental and sustainable development information, while the control group is composed of listed companies that do not disclose environmental and sustainable development. The estimated coefficient β_1 is the policy effect that is the focus of this study. If the policy is effective, the coefficient will be significantly positive.

3.2.2. Variables

(1) Technical innovation. Since the number of patents is an important indicator of a company's technological level, technological innovation is measured in this study as the cumulative number of patents applied for, obtained, authorized, or accepted as of the end of the reporting period

(2) The difference in differences is the cross product of the experimental variable and the time variable

(3) Control variables. According to De Jonghe et al. [33], it also controls for relevant company-level variables that can affect corporate technology innovation, including the sales ratio (SALES), asset-liability ratio (LEV), growth in sales (GROWTH), return on assets (ROA), ratio of independent directors (INDR), shareholding ratio of the largest shareholder (TOPHLD), nature of listed company (SOE), whether the company's chairman and CEO are the same individual (DUAL), asset size (SIZE), and annual, industry, and regional effects. The definitions of the main variables are shown in Table 1 [33].

4. Empirical Analysis

4.1. Descriptive Statistics. According to the descriptive statistics in Table 2, the standard error of the technical innovation of heavily polluting companies is 1,125.579, which indicates technological innovation among heavily polluting companies is heterogeneous, and the distribution of innovation results is very uneven. The average value of the disclosure of sustainable development information is 0.389. This indicates most companies still pay relatively little attention to environmental governance, have poor environmental awareness, and need improvement.

4.2. Trends of the Treatment and Control Groups before Policy Implementation. Figures 2 and 3 show that whether the dependent variable is a technological innovation or its residual mean, the treatment and control groups maintained the same basic trend from 2007 to 2012. However, a significant difference begins to appear after 2012, and the condition of the treatment group is better than that of the control group. The reason for the difference may be that, after the 2012 Green Credit Guidelines policy was issued, heavily polluting companies that focus on disclosing environmental and sustainable development information will be able to obtain credit from financing institutions because of their good environmental management. With fewer financing constraints, more funds can be used to develop the enterprise itself, forming a positive economic cycle and helping the enterprise upgrade, innovate, and develop technology.

4.3. Empirical Analysis

4.3.1. Results for the Benchmark Model. To test hypothesis 1, it uses equation (1) to estimate the impact of the green credit policy on the technological innovation of heavily polluting enterprises. The regression results are shown in Table 3. Column (1) shows the regression that includes the core variables treat and post and their interaction terms. Column (2) through column (4) controls for annual effects, industry effects, and regional effects, respectively, by adding control variables. Column (5) controls for annual, industry, and regional effects by adding control variables. The results show that there is a positive relationship between the green credit policy and technological innovation. The estimated value of DID is positive and significant at the 10% level. That is, implementation of the green credit policy significantly promotes development of technological innovation, and the effect of technological innovation is significant. The results remain consistent regardless of the control variables included or excluded. This may be because, to prepare to qualify for credit financing, increasingly more heavily polluting enterprises have begun to pay attention to their own technical problems and improve environmental protection standards under the guidance of the green credit policy. In this process, the technological innovation ability of heavily polluting enterprises is significantly improved. After obtaining credit funds, they continue to strengthen their technological development to improve their economic development efficiency rate and environmental efficiency. Therefore, the benchmark regression results show that implementing the green credit policy has a significantly

Table 1: Variable definitions.

Variable	Variable definition
TI	Number of patents applied for, obtained, authorized, or accepted as of the end of the reporting period
Treat	Equals 1 if the company discloses environmental and sustainable development information, otherwise 0
Post	Indicates whether the observation is before or after promulgation of the Green Credit Guidelines policy; equals 0 from 2007 to 2011 and 1 from 2012 to 2016
DID	Difference in differences term; the product of the experimental variables and time variables
SALES	Sales ratio = sales revenue/total assets at the beginning of the year
LEV	Liabilities to assets ratio = total liabilities at the end of the period/total assets at the end of the period
GROWTH	Sales revenue of the current year-sales revenue of the prior year/sales revenue of the prior year
ROA	Return on assets = EBIT/total assets at the end of the period
INDR	Number of independent directors/total number of board members
TOPHLD	Shareholding of the largest shareholder
SOE	The nature of the listed company; equals 1 for a state-owned enterprise, otherwise 0
DUAL	Equals 1 if the chairman and CEO are the same individual, otherwise 0
SIZE	Asset size, measured as the natural logarithm of total assets at the end of the period
MKTIDX	Fan Gang marketization index
Year	Year dummy variable
Indu	Industry dummy variable
Region	Regional dummy variable

Source: created by the author.

Table 2: Descriptive statistics.

Variable	Mean	Std. deviation	Minimum	Maximum
TI	202.697	1125.579	0	34279
Treat	0.389	0.488	0	1
Post	0.655	0.475	0	1
MKTIDX	2.55	3.692	0	9.85
SIZE	21.915	1.332	20.07	24.492
ROA	0.056	0.048	−0.034	0.16
INDR	0.37	0.044	0.333	0.455
DUAL	0.256	0.437	0	1
SOE	0.42	0.494	0	1
SALES	0.615	0.356	0.142	1.497
GROWTH	0.248	0.547	−0.359	2.155

Source: all numbers are calculated based on the CSMAR database and China Marketization Index Report by Province (2018).

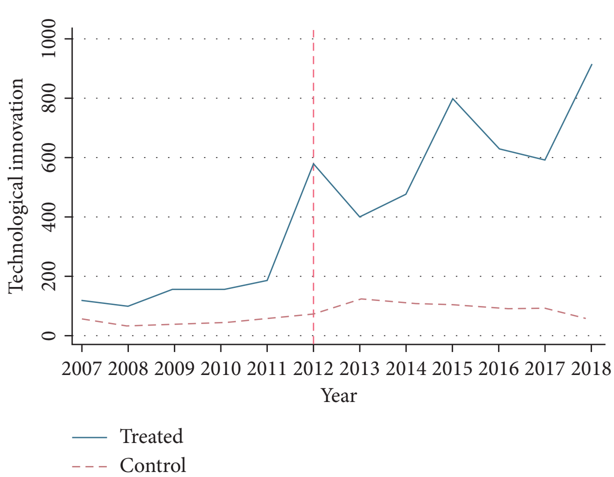

Figure 2: Pretreatment trends for the control and treatment groups based on TI. Source: author prepared using stata based on data from the CSMAR database and China Marketization Index Report by Province 2018.

Figure 3: Pretreatment trends for the control and treatment groups based on TI residuals. Source: author prepared using stata based on data from the CSMAR database and China Marketization Index Report by Province 2018.

Table 3: Result for the benchmark model.

	TI				
	(1)	(2)	(3)	(4)	(5)
Treat	99.0021*	41.0373*	41.8444*	31.9381*	25.7140*
	(56.57)	(24.6200)	(24.8961)	(17.71)	(14.29)
Post	46.9430*	31.2174*	121.6668*	33.2540*	58.8774*
	(26.86)	(17.71)	(69.14)	(18.86)	(33.71)
DID	380.2850***	267.0631*	271.8076*	294.5685*	309.9278*
	(98.3472)	(153.3355)	(163.7389)	(168.2958)	(176.8331)
SALES		219.6296	221.1714	258.4925	264.2739
		(279.9041)	(280.2297)	(176.0542)	(178.0389)
LEV		$-5.1e+02$	$-5.0e+02$	$-1.6e+02$	$-1.5e+02$
		(331.1338)	(329.8066)	(124.2742)	(124.2957)
GROWTH		15.6963	15.2347*	−2.8583	−7.3051
		(30.2491)	(8.71)	(22.4815)	(22.6065)
ROA		$-1.1e+03$	$-1.1e+03$	$-1.8e+03$***	$-1.8e+03$***
		(750.0931)	(791.8576)	(851.6609)	(896.4834)
TOPHLD		509.9625*	515.7855	296.2690	313.4170
		(290.855)	(385.4141)	(219.3283)	(224.1709)
SOE		$-1.5e+02$	$-1.5e+02$	$-2.2e+02$	$-2.2e+02$
		(134.7405)	(134.4034)	(153.3731)	(153.4348)
DUAL		64.1087	63.0375	58.6877	58.2025*
		(54.7293)	(53.9711)	(52.1979)	(33.1409)
SIZE		152.8322***	151.1647***	105.0601*	101.0064*
		(57.9052)	(56.9995)	(60.9728)	(57.7105)
MKTIDX		27.7000***	27.7010**	33.6743	30.5579
		(13.7978)	(13.7642)	(29.9241)	(28.9877)
INDR		$-7.2e+02$	$-7.2e+02$	$-7.9e+02$	$-7.8e+02$
		(739.6559)	(742.3059)	(680.9940)	(677.1396)
_cons	48.4940	$-3.0e+03$***	$-3.0e+03$***	$-2.0e+03$*	$-1.9e+03$
	(54.1332)	$(1.1e+03)$	$(1.1e+03)$	$(1.1e+03)$	$(1.2e+03)$
Year		Yes			Yes
Indu			Yes		Yes
Region				Yes	Yes
R^2	0.0321	0.0717	0.0723	0.2758	0.2769
N	2337	2168	2168	2168	2168

Source: all numbers are calculated based on equation (1). *Note.* The following brackets are standard errors below the coefficients, which represent significant values at the levels of 10%, 5%, and 1%, respectively.

positive impact on the technological innovation of heavily polluting enterprises. Hypothesis 1a is supported.

4.3.2. Heterogeneous Treatment Effects. Due to the existence of heterogeneous effects such as economic basis, environmental supervision, resource endowment, and geographical location, policy implementation effects will differ between regions. Therefore, it is necessary to analyze the heterogeneity of the benchmark regression results. This study examines the intensity of regional marketization and environmental regulation. Table 4 shows the results of the heterogeneity test; column (1) through column (4) reflects the group with a higher degree of marketization, while column (5) through column (8) reflects the group with a lower degree of marketization. The coefficient of DID is positive and significant at the 10% level for the group with a higher degree of marketization, while it is not significant for the group with a lower degree of marketization, indicating that the green credit guidance policy has no significant effect on the marketization process. The significant influence in the group with a higher degree of marketization supports hypothesis 2. This may be because environmental regulations are relatively strict when there is a high degree of marketization, and the market development environment is relatively good. For heavily polluting enterprises, the cost of technological innovation will be lower than the cost of responding to environmental protection. Therefore, enterprises will increase technological innovation and improve the production performance to offset technological innovation and environmental regulations.

4.3.3. Identification Tests. These research results show that implementation of the green credit policy is conducive to strengthening the technological innovation capacity of heavily polluting enterprises. However, the conclusion may be affected by omitted variables bias. The following identification tests are performed to verify the reliability of applying DID to policy identification.

4.3.4. Pretreatment Trends Test for the Control and Treatment Groups. To further test the pretreatment trends and verify whether the policy has a time lag effect, it uses the event

Table 4: Heterogeneous treatment effects test.

	TI				TI			
	MKTIDX > 2.55				MKTIDX < 2.55			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Treat	53.258**	1.943**	16.5864*	32.5401*	36.5737	37.1431	5.157	6.0322
	(26.0430)	(0.9501)	(9.0636)	(17.7815)	(35.2489)	(35.7226)	(45.4045)	(45.9361)
Post	27.9042*	71.1915*	34.9341**	87.1666**	25.2016*	−6.2051	27.7566**	4.9212
	(15.2482)	(38.9024)	(17.0817)	(42.6242)	(14.7689)	(31.5871)	(13.5758)	(34.239)
DID	485.7164*	550.8624*	373.8458*	408.8617*	10.3176	11.9261	4.9124	6.6593
	(265.3575)	(300.8521)	(204.2841)	(222.9501)	(56.1344)	(61.9884)	(56.4754)	(62.3552)
SALES	524.9007	531.0286	876.7075*	904.1218*	−16.5762	−15.5392	−36.3065	−35.4515
	(−643.1779	(−648.4518)	(−501.6764)	(−509.9591)	(−37.9322)	(−37.8976)	(−29.0135)	(−28.9442)
LEV	−2.0e + 03**	−2.0e + 03**	−8.9e + 02*	−8.8e + 02*	44.9515	47.2881	50.1226	52.901
	(−999.1107)	(−996.8959)	(−476.9471)	(−473.4563)	(−59.7938)	(−61.2012)	(−55.5566)	(−56.9065)
GROWTH	10.9057	28.1169	−79.3031	−84.6772	6.8464	6.2795	8.7677	7.9987
	(−91.4759)	(−104.0257)	(−82.1888)	(−92.0356)	(−9.9956)	(−10.5696)	(−10.0929)	(−10.6262)
ROA	−3.8e + 03*	−3.6e + 03*	−5.6e + 03**	−5.8e + 03**	236.287	262.305	244.3799	262.9136
	(−2.00E + 03)	(−2.00E + 03)	(−2.40E + 03)	(−2.50E + 03)	(−252.1742)	(−258.999)	(−266.3702)	(−268.962)
TOPHLD	1.7e + 03*	1.7e + 03*	1.7e + 03**	1.7e + 03**	−1.30E + 02	−1.30E + 02	−51.9833	−52.9766
	(−867.9094)	(−886.8309)	(−850.0596)	(−869.6945)	(−109.2968)	(−107.4172)	(−77.9857)	(−77.4566)
SOE	−4.10E + 02	−4.10E + 02	−6.20E + 02	−6.30E + 02	−42.7409*	−41.1779*	−37.9613*	−36.1360*
	(−341.1923)	(−342.0849)	(−419.5122)	(−422.7072)	(−24.6236)	(−24.5478)	(−21.6326)	(−21.3102)
DUAL	295.1416	300.2588	121.5137	133.4344	31.2318	31.5271	15.1674	15.2547
	(−252.0513)	(−261.6512)	(−232.8186)	(−242.1271)	(−23.5903)	(−23.6486)	(−21.0245)	(−21.0239)
SIZE	396.5692**	396.9989**	340.8621*	337.4935*	37.7347*	37.1001*	25.6333**	24.6832*
	(−158.0985)	(−157.3843)	(−191.2962)	(−192.8438)	(−20.559)	(−20.4612)	(−12.6146)	(−12.6999)
MKTIDX	199.1509**	216.4785**	92.6105	80.3357	−86.5086***	−86.6099**	−5.7e + 02***	−5.7e + 02***
	(−89.1296)	(−97.7303)	(−57.7957)	(−55.3344)	(−33.276)	(−33.4966)	(−199.4349)	(−205.1018)
INDR	−3.60E + 03	−3.60E + 03	−2.60E + 03	−2.60E + 03	110.7845	120.3381	108.1091	115.9929
	(−2.50E + 03)	(−2.50E + 03)	(−1.90E + 03)	(−1.90E + 03)	(−213.747)	(−216.6523)	(−211.5991)	(−214.5007)
_cons	−8.5e + 03**	−8.8e + 03**	−7.5e + 03*	−7.2e + 03*	−7.6e + 02*	−7.5e + 02*	−5.1e + 02**	−4.9e + 02**
	(−3.30E + 03)	(−3.40E + 03)	(−4.00E + 03)	(−4.00E + 03)	(−416.2372)	(−413.004)	(−242.9753)	(−246.062)
Year	Yes			Yes	Yes			Yes
Indu		Yes		Yes		Yes		Yes
Region			Yes	Yes			Yes	Yes
R^2	0.1243	0.1276	0.31	0.3124	0.056	0.0594	0.1038	0.1069
N	764	764	764	764	1404	1404	1404	1404

Source: all numbers are calculated based on equation (1). *Note*. The following brackets are standard errors below the coefficients, which represent significant values at the levels of 10%, 5%, and 1%, respectively.

study method to study the dynamic effect of the green credit guidance policy [37, 38]. Specifically, it replaces DID in formula (1) with a dummy variable indicating several years before and after the green credit policy implementation; the dependent variable remains unchanged, as shown in the following equation:

$$TI_{i,t} = \beta_0 + \prod_{s\geq -5}^{4} \beta_s D_s + \beta_4\, \text{control}_{i,t} + \delta_i + \gamma_t + \varepsilon_{i,t}, \qquad (2)$$

where D_0 is the dummy variable for the year the green credit policy was implemented. A negative number S indicates S years before implementation of the green credit policy, while a positive number indicates S years after green credit policy implementation. Figure 4 shows the parameter estimates for $\{\beta_{-5}, \beta_{-4}, \beta_{-3}, \ldots, \beta_3, \beta_4\}$. The figure illustrates that the coefficients before policy implementation are generally not significant, while the coefficients after policy implementation are generally significant at the lowest confidence interval level. The test results further verify the parallel trend hypothesis and show that the policy effect shows a gradual upward trend and has continuity after implementation occurs.

Figure 4 provides further evidence on the parallel trend hypothesis. The coefficient curves and maximum confidence intervals are all above 0; some coefficients of the minimum confidence interval are also greater than 0, which satisfies the pretreatment trends assumption of the DID model.

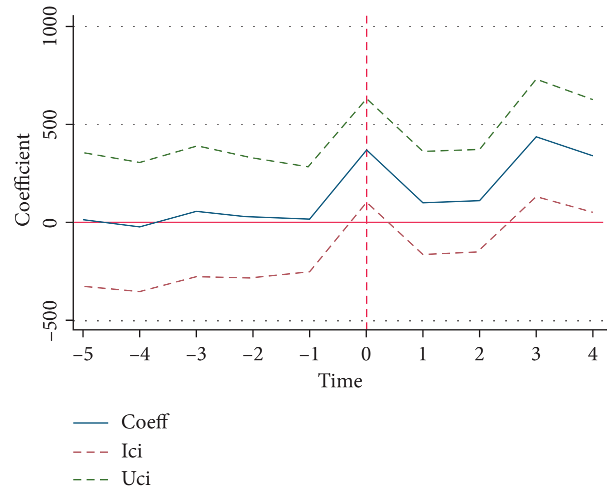

Figure 4: Dynamic impact of the green credit guidance policy. Source: author calculated using stata based on data from the CSMAR database and China Marketization Index Report by Province 2018.

5. Placebo Test

To eliminate interference of other factors or unobserved missing variables in the study's basic conclusions, it performs a placebo test [30, 39]. It draws 1,500 random samples; 10 samples are randomly selected each time from the whole sample as the treatment group for the indirect test. According to equation (1), using the regression results of column (5) in Table 3 as the benchmark results, the coefficient estimate for $DID_{i,t}$ is as follows:

$$\widehat{\beta_1} = \beta_1 + \mu \frac{\text{cov}\,(DID_{ct}, \varepsilon_{ct}|\text{control})}{\text{var}\,(DID_{ct}|\text{control})}. \qquad (3)$$

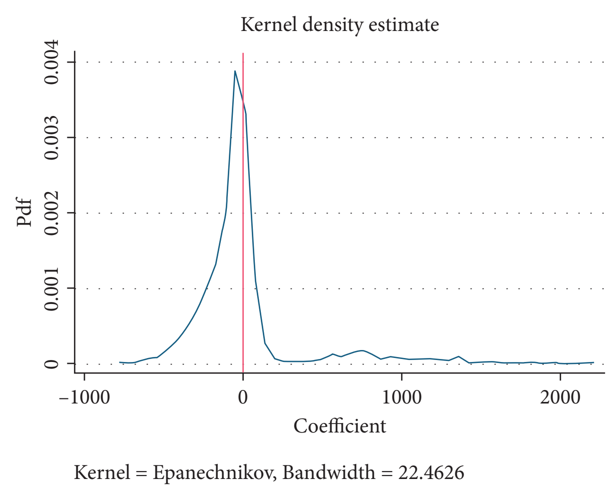

Figure 5: Kernel density estimate for the placebo test. Source: author calculated using stata based on data from the CSMAR database and China Marketization Index Report by Province 2018.

In equation (3), the control variables reflect all variables that cannot be observed. If the estimate of β_1 is unbiased, then μ must be 0. However, it is not known whether μ is 0 or whether the unobserved factors affect the test results. According to the relevant economic theory, $DID_{i,t}$ does not impact the interpreted variables in random samples. If $\beta_1 = 0$, then, it can also be deduced that μ is 0. Figure 5 reports the kernel density estimate of the estimated coefficient. Because the estimates are concentrated around 0, it can deduce that μ is 0, which proves that our basic conclusion is not affected by other random factors. That is, the randomly established green credit policy has no effect on the technological innovation of heavily polluting enterprises, so implementing the green credit guidance policy in 2012 has a significant promoting effect on the treatment group. To sum up, the positive impact of green credit policy implementation on the technological innovation of the treatment team is not affected by other unobserved factors.

The occurrence of exogenous events may not be unique, and the impact of the green credit guidelines on environmental information disclosure of enterprises may be a "false fact," that is, there is no special time point that will lead to improvement in the environmental information disclosure quality. It sets the green credit guidance time as 2013 and 2015, or 2013–2016 and 2015–2016 as the years after the establishment of green credit, which equals 1 for 2013 and 2015, and 0 for the rest of the years. The test results are shown in Table 5; columns (1) and (2) are the regression results of replacing the policy implementation point with 2013, and columns (3) and (4) are the results of replacing the policy implementation point with 2015. The item (treat × post) DID2 and DID3 are no longer significant,

Table 5: Transformation of policy implementation points.

	(1) TI	(2) TI	(3) TI	(4) TI
Treat	60.5914 (73.6671)	101.4942 (68.8888)	104.5407 (65.4154)	109.8326** (53.5740)
Post2	69.6015 (59.6959)	91.2790 (163.3056)		
DID2	279.0212 (215.5341)	165.8258 (140.2437)		
Post3			25.7314 (57.2225)	46.2126 (129.1283)
DID3			372.4849 (290.4272)	276.7606 (210.1416)
SALES		315.5692 (297.4583)		322.0162 (299.5797)
LEV		$-3.7e+02$ (246.6655)		$-3.7e+02$ (243.2950)
GROWTH		8.9980 (32.4159)		11.8972 (32.8058)
ROA		$-1.5e+03$ ($1.1e+03$)		$-1.5e+03$ ($1.1e+03$)
TOPHLD		388.7778 (294.4725)		392.9291 (295.9822)
SOE		$-2.5e+02$* (134.0068)		$-2.5e+02$* (133.1534)
DUAL		52.6587 (51.5991)		51.1028 (50.1108)
SIZE		174.7900*** (61.1703)		175.1140*** (61.0578)
MKTIDX		225.0020 (137.9179)		198.7667 (123.3613)
INDR		$-7.3e+02$ (747.6475)		$-7.4e+02$ (750.0367)
_cons	88.8955* (47.9528)	$-4.2e+03$*** ($1.5e+03$)	116.1809*** (41.1425)	$-4.1e+03$*** ($1.5e+03$)
Year		Yes		Yes
Indu		Yes		Yes
Region		Yes		Yes
R^2	0.1047	0.1485	0.1039	0.1497
N	2337	2168	2337	2168

Source: all numbers are calculated based on equation (1). *Note*. The following brackets are standard errors below the coefficients, which represent significant values at the levels of 10%, 5%, and 1%, respectively.

reinforcing that it is an exogenous shock, and the conclusion is reliable.

6. Robustness Test

The PSM-DID method is used as a robustness test to further analyze the policy effect of the green credit guidance.

First, to facilitate comparison, the control variables in the previous tests are used in a logit regression to predict the probability of each enterprise's disclosure of environmental and sustainable development information. The nearest neighbor, radius, and kernel matching methods are then used to match the control group to the sample (treatment group) that actively discloses environmental and sustainable development information, so the processing group and control group are in the green credit guidance as far as possible. There is no significant difference before the policy impact, reducing the endogeneity problems caused by the self-selection bias of the choice to disclose environmental and sustainable development information.

Second, the DID method is used to identify the net impact of the green credit guidance on the technology innovation of heavily polluting enterprises. Because the tendency score can best solve the deviation problem of observable covariates and the double difference method can eliminate the influence of unobservable variables such as time-varying variables, the combination of these two methods can better identify the policy effect. No matter which matching method is used, the t-test of the observations before and after matching is not significant, and the difference is small as given in Appendix.

The regression results are shown in Table 6. The estimated results of radius, kernel, and nearest neighbor matching are shown, respectively, in columns (1)–(3). In principle, the estimation results of any matching method will not be very different [40]. From the estimation results of the

Table 6: Robustness test results.

	TI		
	Radius matching (1)	Kernel matching (2)	Nearest neighbor matching (3)
Treat	19.6033* (11.6686)	19.6030* (11.6683)	19.6032* (11.6685)
Post	38.5234* (22.9306)	38.5236* (22.9308)	38.5233* (22.9304)
DID	289.5674* (172.6691)	289.5674* (172.669)	289.5674* (172.6691)
SALES	317.0722 (−300.8824)	317.0722 (−300.882)	317.0722 (−300.8824)
LEV	$-3.80E+02$ (−249.9302)	$-3.80E+02$ (−249.93)	$-3.80E+02$ (−249.9302)
GROWTH	10.2342 (−33.4457)	10.2342 (−33.4457)	10.2342 (−33.4457)
ROA	$-1.50E+03$ ($-1.10E+03$)	$-1.50E+03$ ($-1.10E+03$)	$-1.50E+03$ ($-1.10E+03$)
INDR	$-7.40E+02$ (−749.5888)	$-7.40E+02$ (−749.589)	$-7.40E+02$ (−749.5888)
TOPHLD	398.3817 (−300.6895)	398.3817 (−300.69)	398.3817 (−300.6895)
SOE	$-2.5e+02$* (−134.2982)	$-2.5e+02$* (−134.298)	$-2.5e+02$* (−134.2982)
DUAL	53.9892 (−49.533)	53.9892 (−49.533)	53.9892 (−49.533)
SIZE	173.3524*** (−60.6272)	173.3524*** (−60.6272)	173.3524*** (−60.6272)
MKTIDX	205.7158 (−136.6827)	205.7158 (−136.683)	205.7158 (−136.6827)
_cons	$-4.0e+03$*** ($-1.50E+03$)	$-4.0e+03$*** ($-1.50E+03$)	$-4.0e+03$*** ($-1.50E+03$)
Year	YES	YES	YES
Indu	YES	YES	YES
Region	YES	YES	YES
R^2	0.1501	0.1501	0.1501
N	2168	2168	2168

Source: all numbers are calculated based on equation (1). *Note.* The following brackets are standard errors below the coefficients, which represent significant values at the levels of 10%, 5%, and 1%, respectively.

three matching methods in Table 6, the estimation coefficients, signs, and significance levels of the matching methods are basically consistent with the results of the benchmark regression in Table 3. Therefore, the estimate of the impact of the green credit guidance on technology innovation of heavily polluting enterprises is stable.

7. Concluding Remarks

The purpose of this study was to investigate the influence of the green credit guidelines on technological innovation of heavily polluting enterprises. After estimating several specifications of the DID model using samples obtained from the CSMAR database (2007–2018) and China Marketization Index Report by Province (2018), the research results show that the green credit guidance can improve the technological innovation level of heavily polluting enterprises. Further analysis shows that due to differences in the factor endowment structure, economic basis, environmental supervision intensity, and geographical factors, which lead to heterogeneous policy effects between different regions, the impact is greater in areas with high levels of market orientation. The conclusion is consistent throughout several recognition and robustness tests and shows that the green credit guidance has a positive effect on the technological innovation of enterprises. It not only plays an important role in developing market environment mechanisms but also provides an experience constructing relevant policies and regulations in China. Furthermore, the study provides empirical evidence by analyzing the technological innovation of heavily polluting enterprises from the perspective of factor market distortion and the Porter hypothesis.

In view of the availability of data and the limitations of research methods, this research still has some shortcomings, which are not enough to fully control the operation of the entire macroeconomic and microeconomics; so, it is necessary to pay attention to the universality and particularity of contradictions. In the future, we need to focus on introducing more complex models to measure the degree of technological innovation of heavily polluting companies, so as to put forward more meaningful solutions for different types of polluting companies, and put forward substantive suggestions for the development of China's economy.

Based on this study's conclusion and the current state of domestic and international development, it offers the following suggestions. First, it should strengthen the willingness of heavily polluting enterprises to disclose environmental information, improve their environmental awareness, and tap their endogenous development power from inside to outside. At the same time, the government should further strengthen the relevant laws and regulations, strengthen supervision, and guide heavily polluting enterprises. Second, it should focus on the differences in the enterprise development under different marketization levels and conduct targeted environmental management for heavily polluting enterprises. It should give full play to the three-dimensional coordination mechanism of a "government market society" in constructing an ecological civilization and provide support for the innovation and development of heavily polluting enterprises.

7.1. Notes

(1) This study takes listed companies that actively disclose environmental and sustainable development information as the treatment group affected by the green credit guidance policy. To implement macrocontrol policies such as the comprehensive work plan for energy conservation and emission reduction of the 12$^{\text{th}}$ Five Year Plan of the State Council (GF (2011) No. 26), the opinions of the State Council on strengthening the keywork of environmental protection (GF (2011) No. 35), and the requirements of the combination of regulatory policies and industrial policies, banking financial institutions should be encouraged to take green credit as the starting point and actively adjust the credit structure. The CBRC has formulated the green credit guidelines to effectively prevent environmental and social risks, better serve the real economy, and promote the transformation of the economic development mode and economic restructuring.

(2) This study uses the industry codes of the industry classification guidelines for listed companies as revised by the China Securities Regulatory Commission in 2012 and selects companies in the heavily polluting industries listed in Shanghai and Shenzhen A share markets from 2007 to 2018 as the research sample for analysis. The sorted heavily polluting industry codes are B01, B03, B05, B07, C01, C03, C05, C11, C14, C31, C35, C41, C43, C61, C65, C67, C81, D01, H01, and H03.

Data Availability

The data used to support this study are included within this article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Supplementary Materials

Table 1: t-test for radius matching; Table 2: t-test for kernel matching; Table 3: t-test for nearest neighbor matching; Table 4: sample comparison; Figure 1: difference between unmatched sample and matched sample; and Figure 2: propensity score for unmatched sample and matched sample. (*Supplementary Materials*)

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Research Article

A New Equilibrium Strategy of Supply and Demand for the Supply Chain of Pig Cycle

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The pig market had experienced a cycle of price rise and fall, also known as the "pig cycle." This paper analyzes the fluctuation relationship between pig price, pig supply, and pork demand, constructs a system dynamics model of the pig industry by decomposing the structure of the pig supply chain, and then discusses the causes of "pig cycle," as well as the supply chain management strategy and industrial policy, to stabilize the pig industry market. Research shows that reducing the cost of pig breeding, countercyclical adjustment, and government macrocontrol can effectively reduce the fluctuation of pig prices. Among them, reducing the pig breeding cost is the most effective long-term strategy to stabilize the pig price.

1. Introduction

China is the world's largest producer and consumer of pork, consuming approximately 700 million live pigs each year. During this process, the price of live pigs in China had experienced huge fluctuations. In 2018, scholars pointed out that the profit-seeking behavior of live pig farmers caused fluctuations in live pig prices and caused environmental pollution. The government immediately introduced a "decapacity" policy for individual farmers to withdraw from pig breeding. The African swine fever epidemic caused a sharp drop in the number of live pigs in China in 2019, and then the price of live pigs reached a record high of 6 USD/kg. Due to the high price of live pigs and rising of CPI, consumers suffered losses. Once the production capacity was restored, the price of live pigs fell sharply again, which caused losses to the pig breeding companies. Therefore, in order to maximize the benefits of the supply chain, it is necessary to study the reasons of the pig cycle and the strategies of stabilizing the price of pigs.

Regular cyclical changes are often observed in the prices and production of agricultural products. In the 1920s, scholars observed the vibration law of pig price, which is called the "pig cycle" (Coase and Fowler [1, 2]). Larry and Limothy [3] investigated the development of the pig industry in the United States from 1940 to 1990 and found that the average period of the "pig cycle" was 4.08 years, and its fluctuation range was 2 to 6 years. The traditional pig cycle is 4 years (Dawson [4]; Sterman [5]). There are also cycles existing in other breeding industries, such as the "cow cycle," which is generally 10–12 years (Huang [6]).

The classic economic theory about the commodity cycle is "cobweb theory" (Ezekiel [7]; Harlow [8]; Talpaz [9]). Cobweb theory is a theoretical tool, which is used to explain the cycle caused by production lag response. Harlow perfected the simple cobweb model from the four supply factors of sow number, slaughtered pig number, the effect of beef, and other substitutes on the pork price. Waugh [10] solved the influence of nonlinear and multi-input factors on functions. By introducing the concept of the multidimensional cobweb model, Waugh solved the problem of the influence of other factors pointed out by the pig price on production decisions.

The traditional cobweb model has two assumptions: the supply and demand functions are linear; the supply and demand of each phase are balanced. Therefore, scholars also pointed out that the "cobweb model" is not suitable for serious simulation of market dynamics (Sterman [5]).

Because the cobweb model does not consider the stock flow structure and assumes that the commodity cycle is twice the production delay, the pig cycle is much more than twice the production delay (McClements [11]). For example, the breeding cycle of live pigs is about 11 months, while the pig cycle is about 4 years. Besides, spectral analysis (Slade [12]) and harmonic motion models (McClements [11]) are also commonly used.

Scholars have also analyzed the causes of the pig cycle. In economics, it is believed that cycles result from delayed responses to changes in prices and other variables. Haas and Ezekiel [13] described the relationship between pig price and corn price with a linear model. Harlow [8] speculated that the pig cycle may be internal to the industry rather than the result of delayed response to external influences. According to the classification of internal and external influencing factors, it can be roughly divided into internal transmission mechanisms (such as supply and demand laws and pig breeding cycle) and external impact mechanisms (such as epidemic, macrocontrol, and consumption habits).

In the market economy, price is at the core of adjusting the balance between supply and demand. The quantity and value of goods in past periods determine the quantity and value of goods in subsequent periods. Due to the long delay in the response of supply to price changes, the effect of competition game and bounded rationality (Simon [14]) and supply and demand is out of balance, and then a business cycle occurs. Accordingly, this paper constructs a system dynamics model of pig supply chain from three aspects: pig price, pork demand, and pork supply.

System dynamics was found by Professor Jay Forrester of MIT in the 1950s. John Sterman, a professor at MIT and currently director of the MIT Institute of System Dynamics, expanded it into a business dynamic analysis method and had widely used in commodity market analysis (Berg and Huffaker [15]). The application of dynamics in biology and mathematics has resulted in population dynamics and evolutionary dynamics.

This paper tries to answer the following two questions:

(1) Is the pig cycle caused by the interaction of pig prices, pig supply, and pork demand? How do the three influence each other?

(2) Which supply chain management strategies can stabilize the pig cycle? What is its role?

The novel contributions of this paper are as follows:

(1) Most scholars study the pig cycle coping strategies of a certain company or part of the pig supply chain participants from the microlevel, while this article through the macrolevel, to construct a more comprehensive pig supply chain system dynamics model, which can be viewed from a macroperspective and observe the reactions of participants in the pig supply chain

(2) This paper studies the impact of pig supply chain management strategies such as reducing pig breeding cost, countercyclical adjustments, and government macrocontrol on stabilizing the pig cycle and then constructs a model for simulation as well as quantitative analysis

2. Literature Review

There are three streams of literature that are closely related to our work: (1) live pig price; (2) pork supply and demand; and (3) supply chain coordination.

2.1. Live Pig Price. Tversky and Kahneman [16] found that the price of live pigs is constrained by historical prices with strong inertia. Bessler and Brandt [17] believed that the expectation of live pig prices is greatly affected by recent prices, while the impact on historical prices more than one year is small. Behavioral economics shows that people will make the linear speculative analysis of the recent situation and then use it to imagine future trends (Rostow [18]).

Kahneman and Tversky [19] created the anchoring and adjustment method. This method estimates the adjustment by reviewing known reference points (anchor points); then, decision makers use mental simulations to estimate significant or vague influence factors. Nerlove [20] and Marcet and Sargent [21] showed that the expected value of the explanatory variable will satisfy the adaptive adjustment process. Gerlow et al. [22] believed that the balance of supply and demand and unit breeding cost are the decisive factors that affect the price of live pigs. Tan and Zapata [23] believed that the pig breeding cost in Europe and America is much lower than China, which is the main reason for the relatively stable pig prices in Europe and the United States. Breimyer [24] believed that live pigs are the largest and most adjustable consumer of corn, so the price of live pigs is affected by corn supply and price changes. Shun et al. [25] conducted an empirical analysis of the influencing factors of pork price fluctuation in China. The analysis shows that pork price is greatly influenced by itself. The above research shows that the price of live pigs is affected by historical pig price, as well as the cost of pig breeding and the balance of supply and demand. These studies laid the foundation for the construction of a pig price model in this article.

2.2. Pork Demand and Supply. Norris [26] believed that consumption was mainly influenced by income, price, and consumption preferences. According to economic principles, substitutes such as mutton, beef, poultry, and fish will also affect pig consumption. Qi [27] calculated that the impact coefficient of pork and the substitutes is only 0.15, indicating that substitutes have little impact on pork consumption. When the residents' income reaches a higher level, the increase in income does not significantly change the consumption of pork. He et al. [28] calculated the cross-price elasticity of pork and other five substitutes; the research found that the cross-price elasticity was almost zero. According to the prospect theory, the Heiner model, and the Simon model, the transaction parties do not expect to maximize their interests, but are "satisfied" (Heiner [29]). Pennings and Smidts's research [30] on the Dutch pig market also showed that when making risk decisions, using a measurement method based on the expected utility model has a better behavioral prediction effect. The complexity of the details causes people to pay too much attention to their own business and ignore the plans of others, which leads to the phenomenon of "competitive neglect," which makes the behavior of farmers consistent (Kahneman [31]). Through the regression analysis of the agricultural output and price changes, Bean [32] found that equilibrium prices usually lead to stable planting areas and livestock numbers. The above studies have conducted a detailed analysis of the factors affecting the supply and demand of live pigs.

2.3. Supply Chain Coordination. Bailey [33] believed that countercyclical adjustment can effectively increase profits and reduce losses. Hayes and Schmitz [34] pointed out that if the business cycle exists, the shrewd producers will take countercyclical adjustment to obtain high profits and avoid losses, while the poorly run producers will be eliminated. When more people take countercyclical adjustments, the business cycle will disappear. He and Zhao [35, 36] showed that, in the face of supply chain uncertainty, contracts can achieve flexible benefit distribution and effectively stabilize supply chain fluctuations. Modak and Kelle [37] also believed that shared revenue contracts can effectively resolve channel conflicts and then achieve higher profit margins. Modak et al. [38] analyzed the profit distribution and channel conflicts of the two-level supply chain. The analysis reveals that the subgame perfect equilibrium retail price cuts out the channel conflict and ensure the stability of the supply chain. In order to solve the coping problems of supply chain participants when demand is uncertain, Modak and Kelle [39] studied the Nash game problem between retailers and manufacturers, and they solved the equilibrium point of cooperation between the two parties. He et al. [40] demonstrated the role of government subsidies in the supply chain performance. Research has shown that government subsidies are always beneficial to consumers and then improve the performance of the entire supply chain. The government can also release meat reserves to subsidize consumers, but this policy also has drawbacks. Long-term storage of pork may result in quality degradation and high inventory holding costs (Sana et al. [41]). The storage, distribution, and market issues in the food supply chain have also caused extensive discussions among scholars (He et al [42]). We should also emphasize the importance of corporate social responsibility to prevent some companies from hoarding goods and driving up prices (Modak and Kelle [37]). The above research lays the foundation for the coordination research of the supply chain in this article.

3. Problem Description and Modeling

The basic assumptions of the system dynamics model of the pig supply chain are as follows:

(1) Political and economic stability: the pig breeding environment is stable, free from major diseases and disasters

(2) Stable behavioral preferences, including the stability of supply chain participants and the stability of consumer consumption

(3) Every link in the pig supply chain is considered as a whole

3.1. Pig Price Influencing Factors' Analysis. Price modeling is the most difficult part of economic modeling. In some papers, it is unreasonable to adjust the balance of supply and demand around the balanced price of live pigs. The market does not know the true supply curve and demand curve, so the equilibrium price cannot be assumed by calculating the average price of live pigs. Market decision makers can only adjust prices through trial and error, gradually adjusting prices to equilibrium prices; the process is called mountain climbing. According to the anchor and adjustment principle, this paper constructed a pig breeding model, and the price function is constructed as

$$P = P^{*} \times f_1(\text{signal}_1) \times f_2(\text{signal}_2), \ldots, f_n(\text{signal}_n). \tag{1}$$

As the anchor point, the expected price P^* will be adjusted to the historical price. We use adaptive expectations, such as exponential smoothing, to construct the expected price function. The signal in the formula represents various factors that affect the price of live pigs, such as supply and demand balance, unit cost, and other factors, to realize the price adjustment process.

As can be seen from Figure 1, since 2003, China's pig price has gone through four cycles: 2003.5–2006.5, 2006.5–2009.5, 2009.5–2014.4, and 2014.4–2018.5. The cycle length is 3 years, 3 years, 5 years, and 4 years, respectively, with an average cycle length of 3.75 years, which conforms to the traditional pig cycle.

Research on the pig cycle in the United States also usually relies on the price ratio of pigs-to-corn (Holt and Craig [43]). China's pig cycle and the pig-to-corn ratio cycle also show considerable consistency (Zhou and Koemle [44]), as shown in Figure 2.

Other factors may also affect pig prices, such as new technologies, economic cycles, inflation, and import and export of pork. In the initial model, we will not consider these factors for the time being.

Based on the above analysis, the price function of live pigs is constructed as follows:

Pig price = trader's expected price * effect of stock supply ratio on price * effect of cost on price

Among them,

The dealer's expected price = INTEGRAL (the dealer's expected price change, pig price)

Trader's expected price change = (indicator price − -trader's expected price)/trader's time to adjust expected price

Indicator price = MAX (pig price, lowest price)

Lowest price = expected variable cost of breeding

The expected variable cost of breeding is characterized by the grain ratio of profit and loss of pigs. According to the

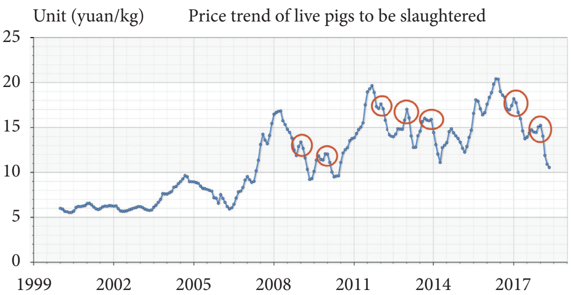

Figure 1: Price trend of live pigs to be slaughtered in China. The dates of the red circles are December 2008-January 2009; December 2009-January 2010; December 2011-January 2012; December 2012-January 2013; December 2013 -January 2014; December 2016-January 2017; December 2017-January 2018. The Spring Festival effect of pork consumption is obvious when the pig price falls. The data are from the China Animal Husbandry Information Network.

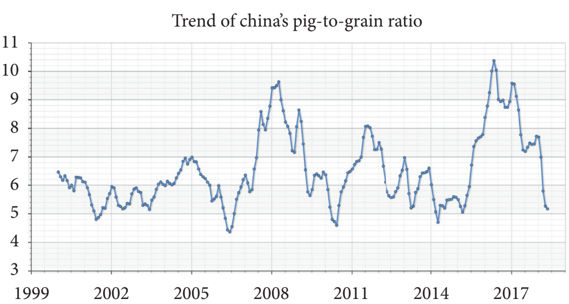

Figure 2: Trend of China's pig-to-grain ratio. The data are from the China Animal Husbandry Information Network.

China Animal Husbandry and Veterinary Yearbook, China's profit-loss pig-to-grain ratio is 5.5. The price of corn is shown in Figure 3.

Assuming the price of corn is 2 yuan/kg, the expected variable pig breeding cost is 11 yuan/kg.

Impact of stock supply ratio on price = POWER (relative stock supply ratio, elasticity of price to stock supply ratio)

Relative stock supply ratio = perceived stock supply ratio/reference stock supply ratio

The reference stock supply ratio refers to the average ratio of pork to consumption in the market. Reference inventory includes the sum of pork inventory held by slaughterhouses, distributors, wholesalers, retailers, and consumers. According to industry experience, it is estimated to be 1.5 months.

Perceived inventory supply ratio = SMOOTH (inventory supply ratio, inventory perceived time)

Stock supply ratio = supply-side pork stock/pork supply rate

Cost-to-price impact = 1 + price-to-cost elasticity * (expected breeding cost/trader's expected price – 1)

The price-to-cost elasticity reflects the influence of cost to the price. When price-to-cost elasticity is close to 1, it indicates that cost has a greater impact on price, such as some agricultural products; when price-to-cost elasticity is close to zero, it indicates that cost has little effect on the price, such as some luxury goods.

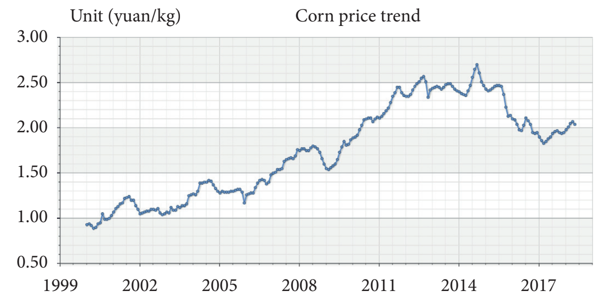

Figure 3: China 2000–2017 corn price trend chart. The data are from the China Animal Husbandry Information Network.

The price construction method has universal significance and is supported by many research results. The adaptive expected price model has shown good prediction ability (Nerlove [20]; Gerlow et al. [22]). Econometric studies also showed that prices will respond to the inventory supply ratio and unit cost (Sterman [5]).

3.2. Pork Demand Influencing Factors' Analysis. Similarly, the pork demand function is set based on the anchor and adjustment process. The pork demand function is constructed as follows:

$$D = D^{*} \times f_1(\text{signal}_1) \times f_2(\text{signal}_2), \ldots, f_n(\text{signal}_n). \tag{2}$$

Among them, the anchor value D^{*} is assumed to be the average pork consumption from 2010 to 2017. Studies have shown that factors affecting pork demand include income, price, consumption preferences, population size, per capita pork consumption, substitutes, seasonal changes, pig epidemics, and holidays.

In recent years, the growth rate of China's pork consumption has begun to decline. From 2001 to 2005, the average annual growth rate of total pork consumption was 3.37%, from 2005 to 2010, it fell to 2.37%, and from 2010 to 2017, it further fell to 1.03%. Economic principles show that an increase in residents' income will lead to an increase in pork consumption. According to the China Animal Husbandry and Veterinary Yearbook, China's per capita pork consumption reached a maximum of 41.5 kg in 2014. The income of Chinese residents increased, but pork consumption decreased from 2015 to 2017, indicating that the increase in income of residents has no effective impact on pork consumption at this stage. The decrease of pork consumption in China in recent years is also related to the diversified consumer preferences of the meat diet.

The trend of pork consumption and pig slaughter in China is the same and showed obvious seasonal fluctuation: November to January of the following year is the peak period of consumption in the whole year, while March to May after Spring Festival is the weakest period. Generally speaking, pork consumption started to pick up from August until Spring Festival. After Spring Festival, the sales volume dropped rapidly, and then the consumption entered a downturn period until August when it returned to positive growth. Statistics on pig prices in China from 2010 to 2018 are shown in Table 1.

Considering the principle of matching supply and demand, this paper depicts the fluctuation of demand through slaughter quantity data of slaughter enterprises, as shown in Table 2.

The pig demand function is constructed as follows:

Indicating pork demand rate = population quantity ∗ per capita consumption ∗ influence of seasonal fluctuation on consumption ∗ influence of price on consumption

Given the irregularity of seasonal fluctuation and the incompleteness of data, this paper makes a simplified treatment and constructs a table function of seasonal fluctuation on consumption based on Table 2, as shown in Figure 4.

Impact of price on consumption = 1 + impact coefficient of price on consumption ∗ (breeding cost − pig price)/pig price

The influence coefficient of price on consumption is calculated by the price corresponding to the annual output. The statistical results are shown in Table 3.

From Table 3, it can be seen that pig prices have an impact on consumption. The prices in 2012, 2013, 2015, and 2017 are close, with an average price of 15.215 yuan/kg and a relative change in average consumption of 1.0071. When the price of live pigs was lower in 2014, the consumption was higher. When pig prices were higher in 2016, consumption was lower. Consumers will form historical prices to judge whether the current prices are high or low and adjust consumption accordingly, which economists call "reference price effect." According to the data in the table, the influence coefficient of the pig price on consumption is calculated to be 0.25.

Pork demand is adjusted with delay around demand indicated by commodity prices, so consumption will not have an immediate response to prices. According to this, the pig demand function is constructed as follows:

Pork demand rate = SMOOTH (indicating pork demand rate, demand adjustment time)

3.3. Pork Supply Influencing Factors' Analysis. For industrial products, the key to the demand cycle is the core variable that determines the price. For agricultural products, the supply cycle is the core variable that determines the price. This is because the production of industrial products is relatively stable and can be planned and increased or decreased at any time. Agricultural products, on the contrary, are rigid in demand and relatively stable in consumption, but highly uncertain in supply. They are the main sources of price fluctuations. Besides, the supply cycle of agricultural products is long while the production cannot be resumed as planned. Therefore, it is difficult to resume production in a short period in case of insufficient supply.

According to the principles of behavioral economics, decision makers often make predictions based on convincing evidence to make decisions. The expected pig price function is constructed according to the extrapolated expectation.

Expected price = perceived price ∗ impact of trend on expected price

Perceived price = SMOOTH (pig price, the time required to perceive current pig price)

Impact of trend on expected price = EXP (expected growth rate ratio ∗ expected price time span)

Meadows [45] measured that the expected price adjustment time was about 6 months and then constructed the expected growth rate proportional function as follows:

Expected growth rate ratio = TREND (pig price, 6, 0)

According to the expected price, the expected profit rate function is constructed as follows:

Expected profit rate = (expected price − average breeding cost)/average breeding cost

Philip Green Wright said: "Business and price cycles are cyclical recurrences caused by large-scale psychology through capitalist production." Economic research showed that when profits increase, existing enterprises tend to expand their scale as soon as possible, and new enterprises will enter the market. When losses occur, existing enterprises tend to reduce production scale, and enterprises with poor profits will withdraw from the market.

According to the anchor adjustment rule as well as the expected profit rate, the expected sow number function is constructed as follows:

Expected sows = number of industry reference sows ∗ effect of expected profit rate on expected sows

Effect of expected profit rate on expected sow number = IF THEN ELSE (expected profit rate > 0, 1 + profit impact coefficient ∗ expected profit rate, 1 + loss impact coefficient ∗ expected profit rate)

Based on the average price of live pigs and the number of live pigs sold in China from 2010 to 2017, this paper determines the profit impact coefficient and loss impact coefficient (Table 3) and then calculates the profit impact coefficient to be 0.2 and the loss impact coefficient to be 0.13.

The breeder adjusts the number of sows in stock according to the expected number of sows, thereby constructing a sow increase rate function as follows:

Sow increase rate = DELAY1 (MAX (sow stock adjustment rate + sow elimination rate, 0), 1)

Sow stock adjustment rate = (expected sow number − number of sows able to reproduce)/sow stock adjustment time

Meadows [45] measured that the adjustment time of sow stock was about 5 months. When the market is prosperous, the breeder will prolong the breeding time of the sow, even if the number of farrowing of the sow is lower than the average

Table 1: Statistics of pig prices in China from 2010 to 2018.

Month/year	2010	2011	2012	2013	2014	2015	2016	2017	2018	Average
Jan.	12.05	**13.88**	17.65	17.04	14.46	13.38	17.62	18.22	15.25	15.51
Feb.	11.14	14.35	17.10	16.09	13.11	12.71	18.37	17.78	14.06	14.97
Mar.	10.06	14.78	15.83	14.07	12.05	**12.27**	18.90	16.71	11.91	14.06
Apr.	**9.53**	15.05	14.80	**12.80**	**11.12**	12.91	19.84	16.00	10.93	**13.66**
May	9.62	15.53	14.25	12.82	12.79	13.92	20.45	14.63	**10.57**	13.84
Jun.	9.64	17.54	14.05	14.10	13.06	14.72	20.41	**13.78**	11.32	14.29
Jul.	11.14	18.98	**14.00**	14.61	13.34	16.59	19.03	13.96	12.02	14.85
Aug.	12.19	19.33	14.28	15.70	14.56	18.12	18.62	14.41	13.36	15.62
Sept.	12.55	19.68	14.89	16.04	*14.88*	17.94	18.36	14.75	14.14	15.91
Oct.	12.78	18.93	14.85	15.86	14.42	17.10	17.06	14.52	14.10	15.51
Nov.	13.55	17.35	14.83	15.77	14.09	16.43	**16.98**	14.47	13.87	15.26
Dec.	**13.79**	17.15	15.83	15.92	13.81	16.68	17.46	15.07	13.95	15.52
Average	11.50	16.88	15.20	15.07	13.47	15.23	18.59	15.36	12.96	14.92

The data are from China's Animal Husbandry Information Network. The italic font in the table is the highest price for the current year, the bold font is the lowest price for the current year, and 14.92 yuan/kg in the lower right corner is the average pig price from 2010 to 2018.

Table 2: Slaughter quantity of pig slaughtering enterprises above the national scale.

Month	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	Average
Slaughter quantity	2198	1809	1658	1735	1695	1589	1491	1500	1703	1817	1891	2293	1782
Relative rate of change	1.23	1.02	0.93	0.97	0.95	0.89	0.84	0.84	0.96	1.02	1.06	1.29	1

The relative change rate is the slaughter quantity of the current month divided by the average value. The data are from the China Animal Husbandry and Veterinary Yearbook.

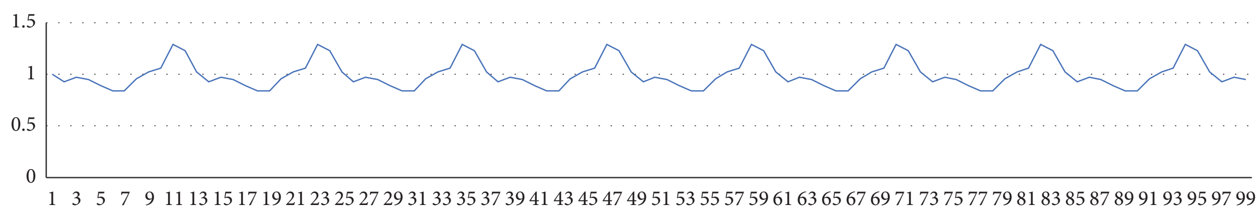

Figure 4: Influence of seasonal fluctuation on pig consumption. The data are calculated by the author.

Table 3: Statistics of China's live pig output, consumption, and annual average price from 2010 to 2017.

Year	Output (unit: 10,000 pigs)	Population (unit: 10,000 people)	Annual per capita consumption (unit: head/ person/year)	Relative changes in per capita consumption	Average price (unit: yuan/ kg)	Per capita consumption (unit: head/ person/year)
2017	68,800	139,008	0.4949	0.9726	15.36	0.5089
2016	68,500	138,300	0.4953	0.9733	18.59	
2015	70,825	137,500	0.5151	1.0122	15.23	
2014	73,500	136,800	0.5373	1.0558	13.47	
2013	71,557	136,100	0.5258	1.0332	15.07	
2012	69,628	135,400	0.5142	1.0105	15.2	
2011	66,170	134,700	0.4912	0.9653	16.88	
2010	66,686	134,100	0.4973	0.9772	11.5	

The output data are from China's Animal Husbandry and Veterinary Yearbook, and population data are from the China statistical yearbook. Other data are calculated by the author.

level. When the industry suffers serious losses, many farmers slaughter sows to speed up the elimination of sows. For example, the slaughter of sows occurred in 1999 and 2006. Judging from experience, the number of slaughtered sows should be in the range of 1%–5%. According to this, the elimination adjustment time function is constructed as follows:

Elimination adjustment time = elimination adjustment coefficient ∗ expected profit margin

Due to the lack of sufficient data, simplified processing is done here. After calculation, the elimination adjustment coefficient is equal to 1.

Sow elimination rate = number of sows capable of reproduction/(average cycle of sow reproduction + elimination adjustment time)

The process of pig fattening can be divided into pregnancy delay and fattening delay. Among them, 90% of piglets were born 111 to 119 days after conception, which is a high-order delay (Meadows [45]). The difference in fattening delay is larger than pregnancy delay, which can be expressed by third-order delay. According to the China Animal Husbandry and Veterinary Yearbook, the fattening time of live pigs is about 170 days, and the number of sows giving birth is 2.2 times a year. The average number of piglets per litter provided by sows is also increasing. In 2015, the average number of piglets per litter is 7.48. In 2017, it reached about 9. According to this, the formula is constructed as follows:

Pregnancy rate = number of sows capable of reproduction ∗ monthly pregnancy frequency ∗ average number of piglets per litter

Birth rate = DELAY FIXED (pregnancy rate, pregnancy cycle)

Supply rate = DELAY3 (birth rate, average fattening time)

Because pig diseases are quite unpredictable, this paper does not consider the impact of diseases on the pig supply chain. Most pig diseases have not significantly affected the stability of the pig supply chain.

4. Model Construction and Initialization

According to the above analysis, the system dynamics model of the pig industry is built by using system dynamics software Vensim PLE, as shown in Figure 5.

According to price theory, there is general equilibrium. Schumpeter [46] thought that, as the pig market is close to a completely competitive market, new enterprises will enter the market as long as there is profit, and then the final equilibrium is zero profit price. However, our common sense is that profits exist. Schumpeter explained that entrepreneurs have produced value surplus in the recombination of production factors. The surplus will gradually disappear as production continues. Profit is a special case of value surplus. That is, under the condition of balance between supply and demand, the system will move to the equilibrium gradually, making "Pig price" = "Dealer expected price" = "Expected breeding cost." The time required to achieve equilibrium depends in part on the speed with which farms learn from experience. According to "Little's rule" (Stock = Rate ∗ Time), the initial sow pregnancy preparation delay is 1,119,911, the sow population is 35,727,335, the pregnancy delay is 24,894,004, the fattening delay is 327,762,575, the pork stock on the supply side is 88,425,155, and the monthly pork demand is 58,950,103. Other parameters are shown in Table 4.

The simulation time is 100 months, and the simulation step length is 1 month. The simulation results are shown in Figure 6.

The simulation reveals two characteristics of the pig supply chain:

(1) Oscillation: the price of live pigs oscillates around the breeding cost. The price of live pigs reached the highest value in the 8th, 53rd, and 99th months, 20.49, 20.76, and 19.70 yuan/kg, respectively. The average pig price for 100 months is 14.61 yuan/kg, with a variance of 10.26. The highest price is 20.76 yuan/kg, the lowest price is 9.98 yuan/kg, and the price difference is 10.78 yuan/kg. The pig cycle is 45 months and 46 months, respectively, with an average pig cycle of 3.79 years, which is very close to the average pig cycle of 3.75 years in China since 2003.

(2) Phase lag: there is a significant phase lag relationship between pig price, number of sows capable of reproduction, and supply rate, i.e., its peak and trough appear in turn, and there is a lag delay.

Oscillation and phase lag are common phenomena in the supply chain (Sterman [47]). Note that the pig price is not a simple mirror image of the pig supply rate.

5. Numerical Experiments and Analysis

The purpose of this section is to measure the degree of mitigation of different supply chain management strategies on the pig cycle, especially on the degree of fluctuation in the pig price. This section verifies the effectiveness of the supply chain management strategy from the perspective of enterprises (reducing the pig breeding cost and countercyclical regulation) and from the perspective of government (government macroregulation).

5.1. Reducing the Pig Breeding Cost. The pig breeding cost greatly affects the pig price. Assuming that the cost of pig breeding in China is reduced to 2/3 and 1/2 of the original cost, i.e., 9.81 yuan/kg and 7.36 yuan/kg, the other parameters remain unchanged. The simulation results of pig prices are shown in Figure 7.

By reducing the pig breeding cost, the fluctuation of pig price had greatly stabilized.

When pig breeding cost = 9.81 yuan/kg, the average price of live pigs dropped to 9.71 yuan/kg. The difference between the highest price of 12.72 yuan/kg and the lowest price of 7.52 yuan/kg is 5.2 yuan/kg. The price fluctuation of live pigs decreased by 51.76%. When pig breeding cost = 7.36 yuan/kg, its 100-month variance of pig price is only 0.53. The average price of live pigs dropped to 7 yuan/kg. The difference between the highest price of 8.75 yuan/kg and the lowest price of 5.67 yuan/kg is 3.08 yuan/kg. The price fluctuation of live pigs decreased by 71.37%. The simulation shows that the reduction of pig breeding costs can bring stability to the pig market.

The price of live pigs in Europe and the United States is more stable. The reasons may include the following: lower pig breeding cost; the pig market opens longer, and pig farmers have more experience in the pig cycle; and Europe

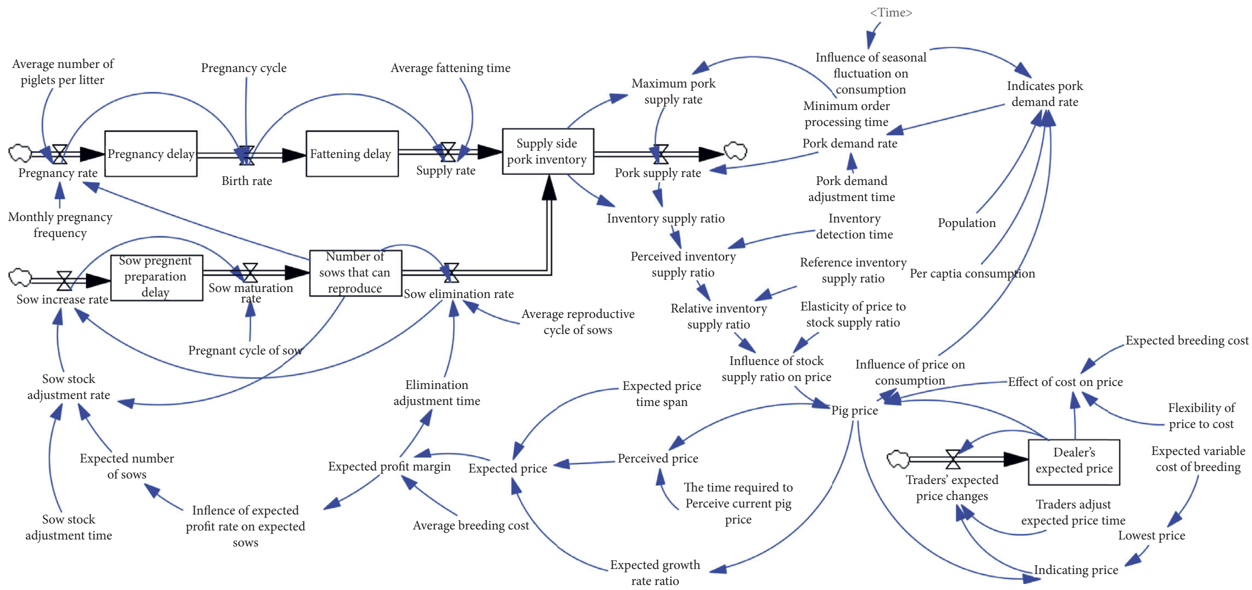

Figure 5: Pig supply chain stock flowchart.

Table 4: Some parameters of pig industry models.

Parameter	Number	Unit
Time for sows to prepare for pregnancy	1	Month
Average reproductive cycle of sows	30	Month
The average number of piglets per litter	9	Head/nest
Monthly pregnancy frequency	0.1833	Nest/head/month
Pregnancy cycle	3.8	Month
Average fattening time	5.56	Month
Sow stock adjustment time	5	Month
Reference inventory supply ratio	1.5	Month
Minimum order processing time	1	Month
Expected price period	3	Month
Adjustment time of pork demand	6	Month
Elasticity of price to stock supply ratio	−0.75	Dimensionless
Inventory detection time	0.5	Month
Flexibility of price to cost	0.5	Dimensionless
Traders adjust expected price time	12	Month
Population	139,008	Ten thousand people
Per capita consumption	0.0424	Head/ person/month
Expected breeding cost	14.72	yuan/kg

Population data are from the China Statistical Yearbook. Other parameters are calculated by the author.

and the United States have a higher degree of scale and more stable production. Figure 8 shows the comparison of live pig prices between China, the United States, and Europe from 2000 to 2013.

With the reduction of pig breeding costs, the price curve that eliminates periodic fluctuations will tilt downward, resulting in a decrease in pig market profits. The ideal equilibrium state in the economic system had never reached, but it is constantly "pursued." The competition between enterprises broke the balance of the economic system, making the economic system struggle around the new equilibrium position (new pig breeding cost).

5.2. Countercyclical Adjustment. The countercyclical strategy refers to the fact that producers increase production when the overall production capacity is low, thereby limiting the high prices caused by short-term supply shortages. In the case of high overall production capacity, production is reduced to minimize losses. T. S. Adams also pointed out that "predict the cycle is to make the cycle invalid." If the prediction accuracy is improved, the pig cycle can be suppressed or even eliminated. However, the unpredictability of the cycle (i.e., imperfect information) will continue because the current prediction model cannot fully calculate all the dynamics of the price cycle time path. A common mistake made by forecasters is to treat seemingly random disturbances as white noise. Disturbances may be part of the nonlinear dynamics of the system.

However, the long-standing business cycle shows that the regularity of the business cycle is very strong and cannot be changed through countercyclical adjustment and speculative strategies. The profit of countercyclical adjustment must exceed the cost of adjustment; the high adjustment cost made the profit of countercyclical production decline in the past century. The behavior of "sticky" producers explains the persistence of the cycle. Increasing anticipation of the cycle may weaken the normal periodic movement, but it cannot prevent the cycle from happening.

Assuming that the countercyclical adjustment is well implemented, the profit impact coefficient is reduced to 0.15, and the loss impact coefficient is reduced to 0.1, the simulation results are shown in Figure 9(a). When the profit impact coefficient is reduced to 0.12 and the loss impact coefficient is reduced to 0.8, the simulation results are shown

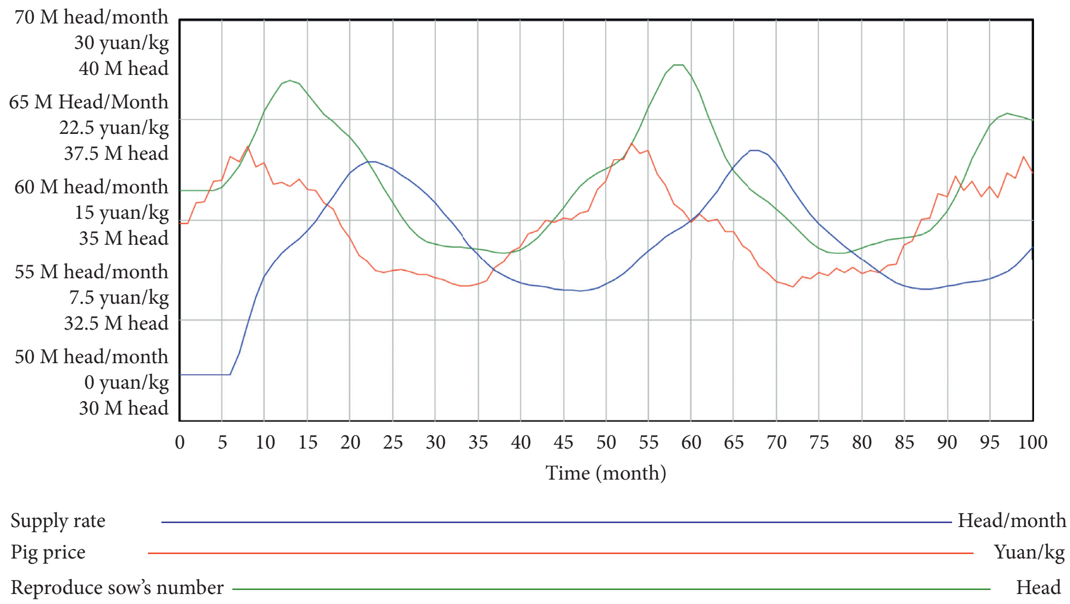

Figure 6: Simulation results of the pig supply chain.

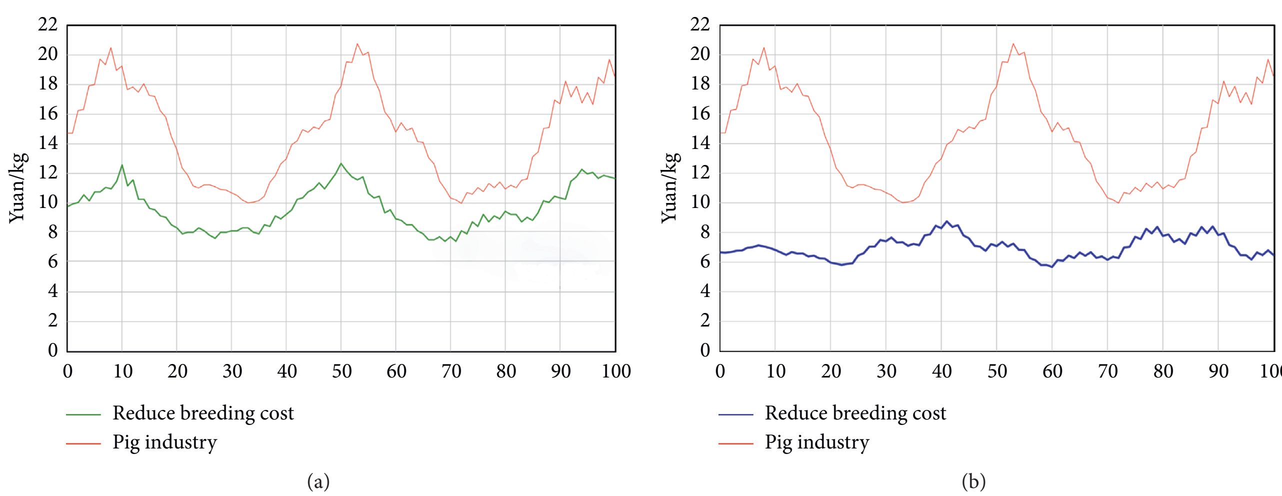

Figure 7: Effect of reducing pig breeding cost on the pig price. (a) Pig breeding cost = 9.81 yuan/kg. (b) Pig breeding cost = 7.36 yuan/kg.

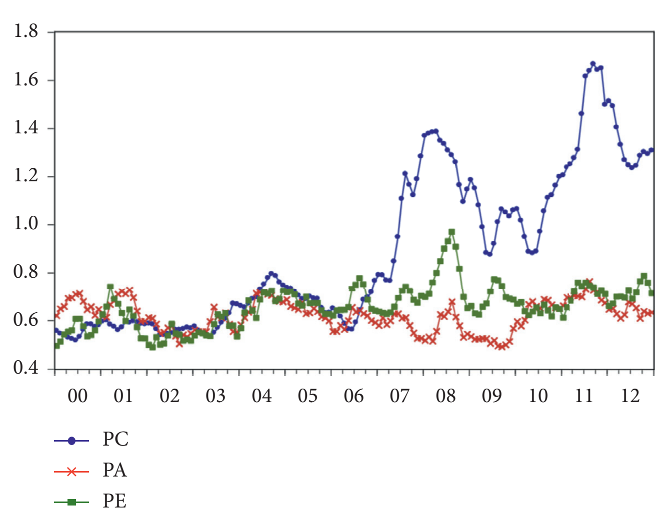

Figure 8: Comparison of Chinese pig price, American pig price, and European pig price. The abscissa is the year, and the ordinate is the pig price (unit: USD/kg). The picture is taken from Hog Price Transmission in Global Markets: China, EU, and U.S., 2014.

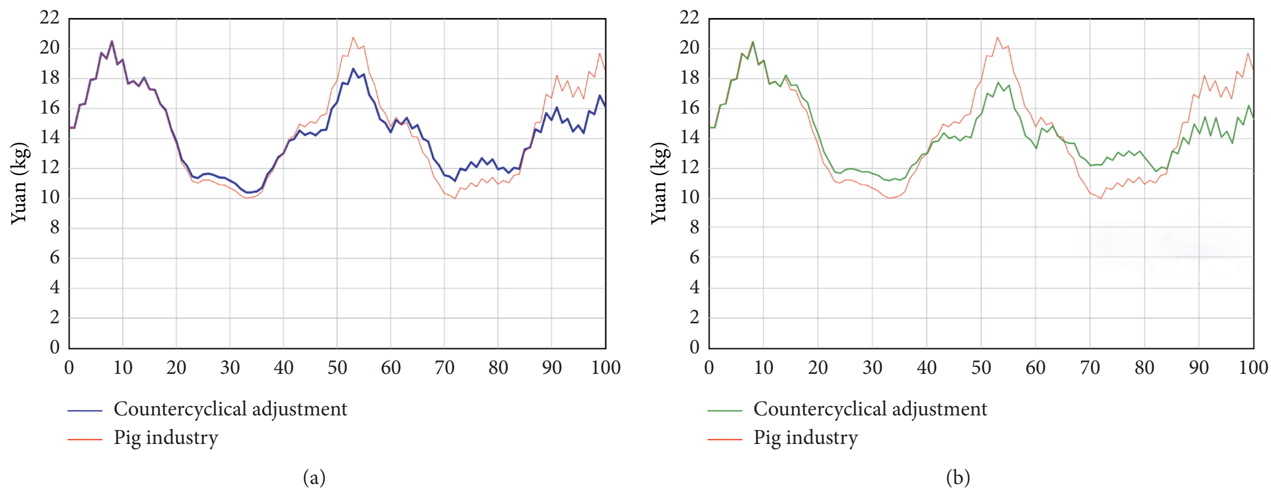

Figure 9: Effect of countercyclical adjustment on the pig price.

in Figure 9(b). Pay attention to the lag of countercyclical adjustment.

5.3. Government's Macrocontrol Strategy. Frank Knight pointed out that we should be alert to the free market because the free market creates wealth by stimulating human greed and other dark sides. There are many evil sides in the market economy. Based on this, the government should play its macrocontrol role to ease the fluctuation of pig prices.

The government's macrocontrol measures for the pig market include the meat reserve policy and import and export policies, as well as financial subsidy policy.

Import and export policies are also conducive to the stability of the pig market. For example, Britain is no longer self-sufficient in pork: pork imports have grown steadily over the past 20 years and now exceed domestic production. As a result, the UK pork prices have become more stable since 2000. Meanwhile, pork imports have problems such as food safety, then the government has adopted strict control policies on pig imports, so China's pig supply chain is a "self-sufficient" model. Imports of pork averaged 940,000 tons from 2013 to 2017, with about 11.575 million pigs, accounting for 1.64% of the annual consumption. When the price of live pigs is high, the government will increase imports.

Financial subsidies can also effectively guarantee the supply of a live pig market. When losses occur, financial subsidies prevent farmers from significantly reducing the scale of farming and can promote the restoration of farming. When the price of live pigs rose and then the market was optimistic, many enterprises often expanded their scale or entered pig farming across industries, increasing the supply of live pigs and then bringing down the price of live pigs again. So, when the price of pigs fell, the government needed corresponding subsidies for farmers to operate stably.

According to this, the government regulation function is constructed as follows:

Government regulation = IF THEN ELSE (government-perceived price > pig breeding cost, import and delivery coefficient ∗ government-perceived price, export and storage coefficient ∗ (government-perceived price – pig breeding cost)/government-perceived price)

Government-perceived price = SMOOTH (pig price, government-perceived price time)

Pork imports = import and release coefficient ∗ government-perceived price

To simplify processing, the delay in importing pork was ignored. The government perceives the price set at one month. As the government implements financial subsidies, it is assumed that the loss reduction impact coefficient is 0.12. Since the initial model did not consider the macrocontrol of the government, the import and delivery coefficient was set at 10,000, and the export and storage coefficient was set at 50,000; the stock flowchart is shown in Figure 10, and the simulation result is shown in Figure 11.

Under the macrocontrol of the government, the pig market is more stable. The average price of live pigs was 14.39 yuan/kg, and the variance was reduced to 7.07. The difference between the highest price of 19.94 yuan/kg and the lowest price of 10.32 yuan/kg was 9.62 yuan/kg, lower than the previous difference of 10.78 yuan/kg.

Despite the high cost of the meat reserve policy and no significant effect on market stability, it has two major roles. First, during the period of high prices, panic will spread, and people will be exhausted. The government should not only avoid the loss of high prices to the public but also try to protect the public from the impact of panic. The policy of reserve meat is a "heart-strengthening needle" to restore market stability during the period of low pig prices and a "sedative" to cool the overheated

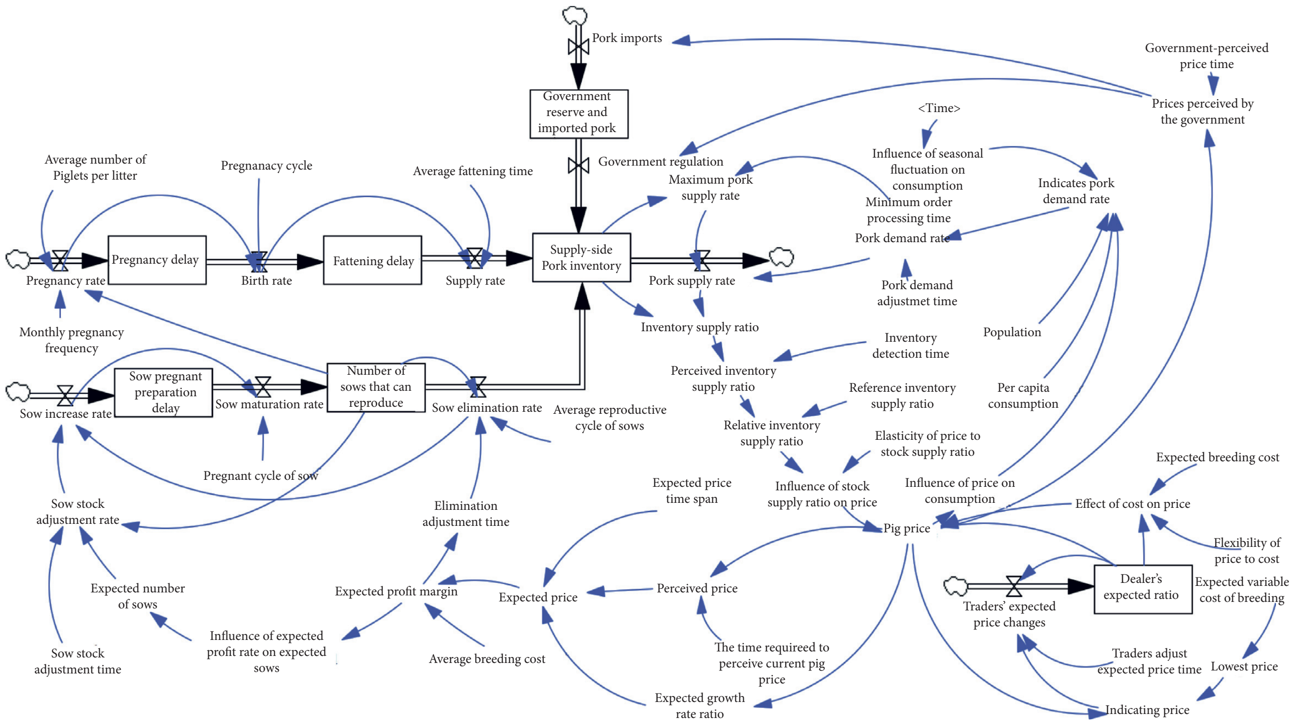

Figure 10: The stock flowchart of China's pig supply chain regulated by the government.

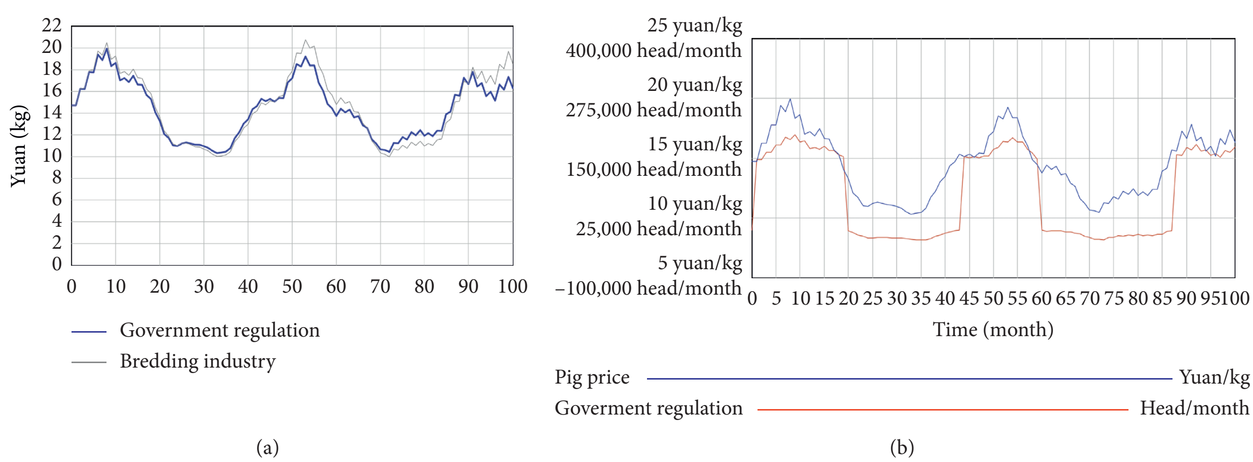

Figure 11: (a) Stabilizing effect of the government regulation on the pig market; (b) the relationship between the pig price and the number of pigs imported and released by the government.

market during the period of high pig prices. Second, when the government buys at a low price, it can increase its fiscal revenue by putting in at a high price. At the same time, we should also realize that this policy has encroached on a certain market share and compressed the profit space of the pig supply chain.

Import and export policies are convenient and flexible, but it should be noted that early regulation is better than late regulation. In particular, when the price of live pigs is depressed due to the increase in the supply of live pigs, the government should ensure an increase in pork exports. Besides, when there is overcapacity, the government should actively seek ways to expand exports. China's difficulties in increasing pork exports lie in the following: first, the high pig breeding cost leads to the lack of price competitiveness; second, China accounts for half of the world's pig consumption, and the international market is relatively limited; and third is food safety.

Financial subsidy policy is an effective policy to stabilize the market, but more subsidies should be given to small farmers; this also reflects the fairness concern and the fairness of income distribution. Besides, subsidies are only needed to help enterprises solve infrastructure or negative externalities. Therefore, the government should stop subsidizing large enterprises to avoid the deterioration of income distribution and the spread of corruption.

Note that the market will also generate compensatory feedback to the direct regulation of the government, and then the compensatory feedback will weaken the direct regulation effect of the government. Industrial policies will also mislead entrepreneurs and make them invest resources in areas or projects that they should not invest, thus causing overcapacity and harming the healthy development of the industry. While the government intervenes, it creates rent and causes rent-seeking behavior, which in turn leads to corruption and unfair income distribution.

The existence of a large number of small-scale farmers may be one of the important reasons for the price fluctuation of pork in China to be larger than Europe and America. The "company + farmer" model can effectively stabilize the scale of small- and medium-sized farmers, which may be the development direction of small- and medium-sized farmers in the future.

6. Conclusion

The "pig cycle" originates from the interaction between the physical delay in pig breeding and the capacity of decisions made by individual producers/investors under the guidance of bounded rationality; in addition, pig cycle is interfered by pig diseases, industrial policies, natural environment, etc.. The "pig cycle" takes the pig endogenous cycle as the expected value and fluctuates up and down around the expected value.

This paper constructs a system dynamics model of China's pig supply chain and then simulates the strategy of stabilizing the pig market. Simulation results show that reducing the pig breeding cost and countercyclical adjustment, as well as government macrocontrol, can effectively stabilize the pig market. Among them, reducing the pig breeding cost is a long-term strategy to stabilize the pig market. The government's direct regulation occupies a large number of resources that causes compensation feedback. Therefore, the government should weaken the direct intervention strategy, starting with reducing the pig breeding cost and countercyclical adjustment, and play the regulatory role of "invisible hand" to guide the market. The social significance of the government's macrocontrol is greater than its economic significance, and it should continue to be implemented.

As the paper focuses on the fluctuation of the pig cycle, some model parameters and formulas have not been accurately calibrated, and some feedback are not added to the model. Since the impact of random shocks is not considered, the simulated pig price is smoother than the actual pig price data. Econometrics can be used to estimate parameters more accurately. Besides, further decomposition of the structure of the pig supply chain can make the model results closer to actual data.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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Research Article

Integrated Optimization of Sustainable Transportation and Inventory with Multiplayer Dynamic Game under Carbon Tax Policy

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Growth in environmental sustainability has prompted the logistics industry to seek sustainable development, and carbon tax policies are considered an effective approach to reducing carbon emissions. This study investigates the optimization of sustainable transportation and inventory under a carbon tax policy and explores effective methods for coordinating the interests of governments and enterprises. The results can provide insights into sustainable logistics for decision-making by enterprises and policy-making by governments. We first examine a Stackelberg game model and design an iterative solution to optimize sustainable transportation and inventory under the carbon tax policy. We then establish a three-stage dynamic game model to optimize the wholesale price, carbon tax rate, and proportion of sustainable investment shared by the government. Furthermore, we perform a simulation to identify the optimal solution of the three-stage game, and we compare the simulation results with a numerical example. The results indicate that a carbon tax policy can improve social welfare and the sustainability of transportation and inventory but could hinder corporate profits. An appropriate sustainable investment-sharing strategy could compensate for the shortcomings of the carbon tax policy and result in positive outcomes for governments and enterprises.

1. Introduction

With increasingly severe environmental pollution worldwide, people have gradually become aware of the importance of sustainable development. An increasing number of countries are taking measures to control greenhouse gas emissions, and organizations and individuals are undertaking environmental-protection actions to reduce carbon emissions [1]. To address pollution, developed and developing countries have implemented regulations aimed at reducing carbon emissions [2]. A carbon tax is one such emissions-reduction regulation that has been widely implemented to encourage companies to reduce their carbon emissions [3]. Carbon taxes have been adopted in Australia, Japan, Denmark, Austria, Finland, and Ireland [2]. In Australia, the carbon tax was implemented in 2011, with a rate of AU$23 per ton of carbon in 2012 [4]. A carbon tax increases the cost of operations [5], which encourages enterprises to increase their sustainable investments. Dong et al. [6] indicated that sustainable investment can reduce the carbon emissions of manufacturers, and both manufacturers and sellers can benefit from it. Sustainable investment improves not only environmental sustainability but also a firm's long-term profitability and competitiveness [7]. For example, in 2007, Marks & Spencer invested £200 million in carbon-reduction efforts; moreover, Walmart has reduced its carbon emissions by 667,000 m^3 after requiring its 60,000 suppliers to reduce their packaging by 5% [8].

Given this context, enterprises are motivated to optimize their supply chains and invest in clean-energy technologies to reduce carbon emissions [9]. Sustainable supply chain management is a growing concern because production and logistics activities lead to waste and pollution [10]. The logistics industry consumes energy and fossil fuels and is a major source of carbon emissions in the supply chain [11]. A survey of the supply chain operations of 2,500 large enterprises revealed that logistics-induced emissions exceeded 20% of total supply chain emissions [12]. This result demonstrates that logistics has a significant negative impact on both environmental sustainability and economic growth. To protect the environment, governments should implement strict environmental-protection policies and encourage sustainable investment in the logistics industry [13, 14].

Transportation and inventory management are the main sources of carbon emissions in the logistics industry [15]. Transportation is the largest source of carbon emissions. Inventory control determines logistics factors, such as inventory level, warehousing activities, and transportation frequency [12], which are crucial for decision-making in supply chain operations. In Europe, transportation is the sector with the second highest carbon emissions (the energy supply sector has the highest), accounting for 23% of total carbon emissions [5]. Transportation and inventory are the two most critical elements of logistics in terms of both economics and ecology. The efficient management of a logistics system requires an integrated approach that combines various logistics functions [16]. Furthermore, few studies have explored the optimization of sustainable investment through the integration of transportation and inventory decision-making on a global scale. Therefore, research on integrating decision-making in sustainable transportation and inventory would facilitate the reduction of carbon emissions in the logistics industry and promote sustainable development.

This study investigated the relationships between government, suppliers, and retailers (Figure 1). Governments adopt carbon tax policies to optimize social welfare. Suppliers and retailers make decisions concerning transportation and inventory management, respectively, with the aim of maximizing profits. Carbon emissions in transportation and inventory management have been analyzed, but carbon emissions due to other activities in the supply chain have not been considered. Therefore, we investigated the following research questions:

(1) Under a carbon tax, how can suppliers and retailers maintain sustainable levels of transportation and inventory?

(2) How do governments set carbon tax rates and incentives?

(3) What effect does a carbon tax have on supply chain performance, and how can government and enterprise objectives be harmonized?

This study aimed to achieve three specific goals: (1) to establish an integrated optimization model for sustainable levels of transportation and inventory under a carbon tax; (2) to construct a multiplayer dynamic game model to determine the optimal carbon tax rate and the sustainable investment-sharing strategy; and (3) to solve this multiplayer dynamic game model through a MATLAB simulation and to explore the effect of a carbon tax on sustainable transportation and inventory, enterprise profits, and social welfare.

The remainder of this paper is organized as follows. Relevant studies are discussed in Section 2. Section 3 proposes assumptions and details of the notations used. Section 4 presents the model for sustainable transportation and inventory optimization under a carbon tax, along with an analysis of and solution to the model. A three-stage game model, which was designed to optimize the carbon tax rate and sustainable investment-sharing strategies, is presented in Section 5. Section 6 provides the results of simulations performed to solve the three-stage model numerically, along with an analysis of the effect of a carbon tax on decision-making. Conclusion is presented in Section 7.

2. Literature Review

Numerous studies have explored sustainability in the supply chain. Those highly related to the present topic were divided into three categories according to the topic: carbon tax, integrated environmental optimization of transportation and inventory management, and sustainable investment in the supply chain.

2.1. Carbon Tax. A carbon tax is an effective approach to mitigate climate change and is one of the two main instruments for redesigning the supply chain (the other being cap-and-trade programs) [2]. A succession of studies have focused on the effect of a carbon tax on emission reduction. Shu et al. [17] reported that a carbon tax can place a heavier financial burden on companies to meet certain emission targets than other carbon-reduction policies can. Chen and Nie [18] demonstrated that a specific number of carbon taxes in the production process can improve social welfare, whereas consumption and redistribution reduce social welfare. Moreover, Olsen et al. [19] indicated that if a carbon tax is implemented after properly analyzing its impact, its revenue can be used to support government investment in projects that reduce emissions. Ma et al. [20] argued that economists and policymakers prefer a carbon tax because its implementation requires less management than that of other carbon-reduction policies. However, Xie et al. [21] indicated that a carbon tax harms economic growth because it inevitably entails companies incurring additional costs. Many studies have investigated the significant impact of a carbon tax on supply chain performance. Bao and Zhang [22] developed a mixed linear programming model to explore sustainable procurement relationships in a supply chain under a carbon tax scheme. Fahimnia et al. [4] proposed a tactical supply-planning model that optimizes carbon emissions and economic targets under a carbon tax. Xin et al. [23] discussed the problem of sustainably scheduling a shuttle tanker fleet with variable tanker speed under a carbon tax. Wang et al. [24] examined decisions about a carbon tax in decentralized and centralized supply chains by using the Stackelberg game model. They proved that a carbon tax in a decentralized supply chain should be higher than that in a

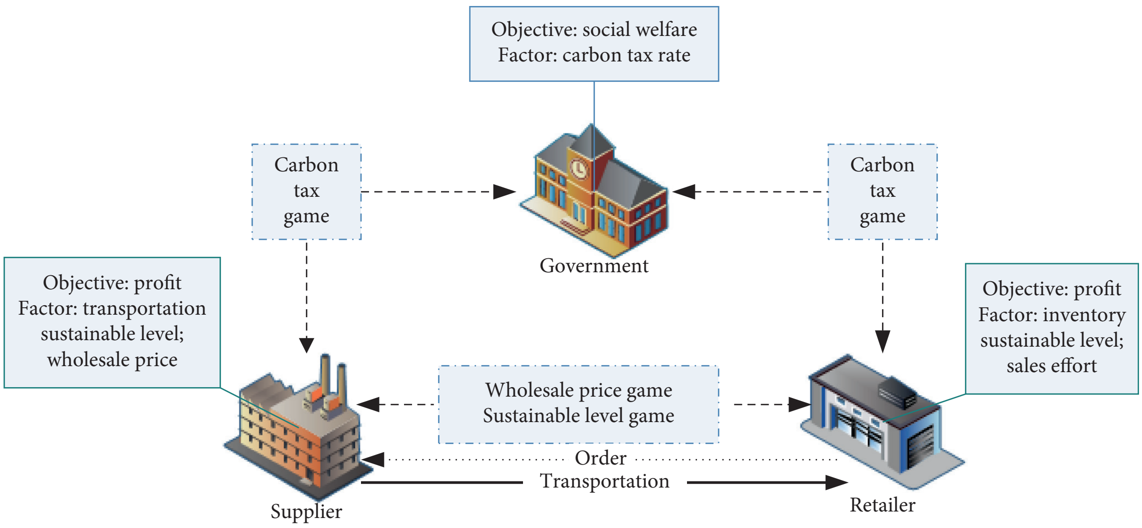

Figure 1: Relationships of the multiplayer game.

centralized supply chain. Wang et al. [25] derived the optimal level of supply chain emission reduction with stochastic demand under a carbon tax. They reported that optimal emission reductions gradually increased with an increase in the carbon tax rate. Ding et al. [26] developed a diffusion model of energy technology to explore the possible impacts of various carbon tax conditions on the diffusion of energy technologies in China. Alizadeh et al. [27] studied the impact of the uncertainty of the carbon tax rate on emissions in a supply chain. Their results demonstrated that increasing the uncertainty of the carbon tax rate from 0 to 30 reduced total network emissions by 2.8%. However, integrating sustainable investment into logistics operating decisions has not been considered in most of these studies.

2.2. Integrated Environmental Optimization of Transportation and Inventory Management. Employees usually make decisions regarding inventory management and transportation in an organization. Collaboration is the optimal supply chain strategy [28]. The interaction between inventory parameter decisions and transportation choices indicates that, to optimize transportation and inventory management, they must be integrated. Integrated optimization problems can be categorized as strategic or tactical. Gaur and Fisher [29] divided integration optimization problems into those with finite and infinite periods. Four main types of logistics networks exist, namely, one-to-one, one-to-many, many-to-one, and many-to-many [30]. This study focused on one-to-one networks. One-to-one networks have not a routing problem, whereas the other three network types do.

Recently, increasing numbers of scholars have included carbon emission factors in integrated decision models for transportation and inventory management. Economic order quantity (EOQ) models with various carbon emission regulations are the predominant tool employed in most studies on transportation and inventory management. Bonney and Jaber [31] presented an EOQ model associated with carbon emissions and waste produced by transportation and inventory operations. Tang et al. [32] integrated emissions, transportation costs, and inventory costs into the (R, Q) policy with stochastic demand. Hovelaque and Bironneau [33] considered the carbon emission problem in an EOQ model with demand dependent on price and carbon emissions. Hua et al. [34] constructed an EOQ model for a carbon cap-and-trade policy and analyzed the trends of replenishment decisions, costs, and carbon emissions with changes in the carbon trading price. Chen et al. [35] proposed a condition for achieving a carbon emission-reduction target by adjusting the order quantity based on an EOQ model. Gautam et al. [36] integrated defect management into a sustainable supply chain model to jointly optimize the number of shipments and quantities of orders and backorders. Konur and Schaefer [12] developed integration models for transportation and inventory management for four carbon policies. They compared and analyzed the impacts of these policies on the optimal order quantity. Wang et al. [39] studied the optimization of refined oil distribution under a carbon tax. Their results revealed that a carbon tax effectively reduced the carbon emissions due to refined oil distribution. Reddy et al. [38] established a multiperiod reverse logistics network design model and analyzed the impact of a carbon tax on the optimal decisions. Konur and Schaefer [12] examined the EOQ model for less-than-truckload and truckload shipping under cap-and-offset, cap-and-trade, carbon cap, and carbon tax policies. Micheli and Mantella [5] designed a comprehensive model of transportation and inventory under the cap, cap-and-offset, cap-and-trade, and carbon tax. They involved a comprehensive emission model with vehicles.

In-depth research has been conducted on optimizing the integration of transportation and inventory management. In such research, the main decision-making objectives are to optimize transportation and inventory management under certain economic and social goals. Studies on environmental factors have focused on the impact of carbon emissions and carbon-reduction policies on transportation and inventory integration decisions. The present study extends the literature by introducing sustainable investment factors into the decision-making model for transportation and inventory integration, and we focused on the optimal decisions to reach sustainable levels of emissions due to transportation and inventory.

2.3. Sustainable Investment in a Supply Chain. Supply chain emission reduction has become a critical, theoretical, and practical topic, with numerous studies discussing it and proposing investment in clean technologies to reduce carbon emissions (i.e., sustainable investment). Several studies have focused on the effectiveness of sustainable investment. Drake and Spinler [37] noted that the effectiveness of clean technologies in sustainable economic development should not be underestimated. Shi et al. [40] analyzed the comprehensive effects of power structures and sustainable investment in the supply chain on the economy and environment. Su et al. [41] developed a pricing decision model to explore the effects of government subsidies on optimal decisions for sustainable supply chain management under various government subsidies and power structures. Scholars have often studied sustainable investment decision-making under multiple carbon-reduction policies. Benjaafar et al. [42] studied the impact of clean technologies on carbon reduction. The results indicated that a carbon cap-and-trade regulation can effectively encourage companies to adopt clean technologies if the benefits of a clean technology are substantial. Krass et al. [43] believed that increasing taxes may prompt companies to switch to clean technology when opting for emission control technology. If the input cost of clean technology is compensated for, the negative environmental impact will be eliminated and the tax will have proved effective. Chen [44] explored the impact of a carbon tax on the choice of energy-saving technologies by enterprises under market competition. Chen's results demonstrated that clean and conventional technologies can coexist and that companies choose technologies according to the tax rate. Shifting to clean technology is not always the optimal choice when dealing with a rising tax rate. Drake et al. [45] compared the impacts of a carbon tax and a carbon cap-and-trade policy on the corporate sustainable technology choice and capacity decisions, and their results revealed that the expected profit under a cap-and-trade policy was higher than that under a carbon tax.

With the deepening of research into sustainable investment, integrating sustainable investment into supply chain activities such as procurement, production, and inventory has received increasing attention. Toptal et al. [46] introduced retailers' sustainable investments into an EOQ model and constructed joint decisions for sustainable investment and inventory management under carbon cap and cap-and-trade policies. The model revealed that sustainable investment can simultaneously reduce carbon emissions and costs. Dong et al. [6] studied the sustainable investment problem under a carbon cap-and-trade policy and constructed a decision-making model for decentralized coordination and centralized control of a supply chain. The conclusions indicated that sustainable investment substantially influenced the optimal supply chain strategy. Based on the results of Dong et al. [6], Cheng et al.[8] introduced big data to the sustainable investment decision-making model. Their results revealed that adopting big data was not always the optimal choice for retailers. Moreover, they showed that whether manufacturers increase sustainable investment depends on the service level set by the retailer.

With continual advancements in sustainability, consumers are increasingly concerned about the environmental impact of goods, and this is referred to as consumer environmental awareness [5]. Consumer environmental awareness has often been integrated into the supply chain optimization model along with sustainable investment. For example, Zhang and Liu [47] examined pricing strategies in a three-level supply chain where demand was relative to the environmental impact of products. Du et al. [48] proposed a carbon-sensitive demand function to investigate the impact of consumers' preferences for reducing carbon emissions on a supply chain, and they designed various supply chain coordination contracts. Consumer environmental awareness has two main impacts on the sustainable supply chain model. The first is the impact on market demand; when carbon emissions decrease, market demand increases [6, 8]. The second is the impact on prices; when consumer environmental awareness increases, low-carbon products receive more recognition, even among companies with higher prices [48]. The higher consumer environmental awareness is, the greater is the willingness of consumers to pay for sustainable products [49]. Consumer environmental awareness motivates manufacturers to produce more sustainable products, but this does not necessarily mean higher profits for manufacturers [50].

Toptal et al. [46], Dong et al. [6], and Cheng et al. [8] have conducted similar investigations into retailers' perspectives regarding inventory replenishment and sustainable investment. Dong et al. [6] and Cheng et al. [8] have investigated sustainable investments in production. By contrast, the present study investigated sustainable investments in logistics. Toptal et al. [46] analyzed sustainable investment in inventory management, whereas we considered sustainable investments in both transportation and inventory management. Moreover, Toptal et al. [46] explored deterministic demand. By contrast, the present study included the sales effort factor and sustainable investment in a random demand function to harmonize the optimization model with the real enterprise situation. Table 1 presents the studies related to the present paper.

3. Model Assumptions and Notation

In this study, we considered a supply chain consisting of one supplier and one retailer selling a single product. Over a single cycle, the supplier sells the product to the retailer at the wholesale price, and the supplier is responsible for shipping the product from its factory to the retailer's warehouse. The retailer is responsible for product ordering and inventory management, and the retailer sells the product

Table 1: The literature positioning of this paper.

Papers	Carbon emission	Sustainable investment	Carbon tax	Integrated optimization of transportation and inventory
Fahimnia et al. [4], Wang et al. [24], Wang et al. [25]	✓		✓	
Webb and Larson [51], Gaur and Fisher [29]				✓
Bonney and Jaber [31], Hua et al. [34], Tang et al. [32]	✓			✓
Konur and Schaefer [12], Micheli and Mantella [5], Wang et al. [37]	✓		✓	✓
Toptal et al. [46], Dong et al. [6], Cheng et al. [8]	✓	✓		
This paper	✓	✓	✓	✓

to consumers at the market retail price. In a carbon tax scenario, decision-making involves two consecutive steps. In the first step, the supplier determines the appropriate extent of sustainable investment for transportation given the goal of maximizing profit. In the second step, given a sustainable level of transportation, the retailer determines the sustainable inventory level and order quantity for the product. The notations used in the models are summarized in Table 2. The models are limited by the following assumptions:

(1) A supplier trades with a retailer during a single period. The retailer can only order once during the period. If the retailer's order quantity exceeds market demand, the leftover stock is disposed at the unit salvage value v. Without loss of generality, we assume that $v < C_s$, where C_s is the unit production cost. Moreover, we assume that the explicit shortage cost is zero if the market demand exceeds the retailer's order quantity [6]. When market demand exceeds the retailer's order quantity, the retailer sells out its inventory; urgent orders are not allowed.

(2) The sustainability level is a dimensionless index that measures carbon emissions. The higher the sustainability level is, the lower are the carbon emissions. Assume that the sustainable transportation level $s_p \in [0, (a_p/b_p)]$ and the sustainable inventory level $s_l \in [0, (a_l/b_l)]$. When $s_p = 0$, carbon emitted per unit of the product for transportation is a_p. When $s_l = 0$, carbon emitted per unit of the product for transportation is a_l. Carbon emitted per unit of the product for inventory is $a_p - b_p s_p$. Carbon emitted per unit of the product for inventory is $a_l - b_l s_l$.

(3) Sales effort x is a comprehensive index that reflects the retailer's efforts to promote market demand through activities such as advertising, services, and sales promotions. Let $x \geq 0$; we assume that $f'(x) > 0$, $f''(x) < 0$, $g'(x) > 0$, and $g''(x) \geq 0$, where $f(x)$ represents the demand generated by sales effort and $g(x)$ represents the costs incurred by sales effort. The assumption indicates that $f(x)$ and $g(x)$ are increasing functions. However, the rate of demand increase decreases with x, and the rate of cost increase is zero or increasing with x [52].

(4) Product demand D is uncertain; it depends not only on the retailer's sales effort x, the sustainable inventory level s_l, and the sustainable transportation level of the supplier s_p but also on the market's random demand factor ξ, as expressed in the following: $D(x, s_p, s_l) = f(x) + \beta(s_p + s_l) + \xi$. Moreover, β is the coefficient of the effect of sustainability on increasing demand, ξ is the random demand with mean μ, and the probability density function and probability distribution function are $\phi(y)$ and $F(y)$, respectively. Furthermore, $\beta(s_p + s_l)$ represents the demand generated by the sustainable transportation level s_p and sustainable inventory level s_l. If $\beta > 0$, then the sustainability level has a positive effect on the demand. When $s_p = 0$ and $s_l = 0$, the demand generated by the sustainability level is zero. The maximum values of demand generated by s_p and s_l are $\beta a_p/b_p$ and $\beta a_l/b_l$, respectively.

(5) The higher the sustainability level is, the higher is the cost of sustainable investment and the faster is the cost increase. Similar to [6, 8, 53], we assume that the sustainable investment costs for transportation and inventory are quadratic functions, specifically $\delta_l s_l^2/2$ and $\delta_p s_p^2/2$, respectively, where δ_l and δ_p are the sustainable investment coefficients for transportation and inventory, respectively. In practice, the costs of sustainable investment are usually high. Therefore, we assume that δ_l and δ_p are sufficiently high that $\delta_l \geq 2rb_l\beta$ and $\delta_p \geq 2rb_p\beta$, where r is the carbon tax rate. Otherwise, the lower bound and upper bound of the sustainability level interval are the optimal values of s_p and s_l; that is, $s_p = 0$ or (a_p/b_p) and $s_l = 0$ or (a_l/b_l). The assumption is that $\delta_l \geq 2rb_l\beta$ and $\delta_p \geq 2rb_p\beta$ can avoid these simple solutions [6].

4. Optimization Model for Sustainable Transportation and Inventory under a Carbon Tax

Based on the Stackelberg game between the supplier and the retailer, this paper uses a backward sequential decision-making approach. First, the retailer's sales effort, sustainable

Table 2: Notations.

Notation	Meaning
p	Market price
w	Wholesale price
C_s	Unit production cost
C_p	Unit transportation cost
C_l	Unit inventory cost
v	Unit salvage cost
Q	Retailer's reorder quantity
D	Demand function
L	Service level
x	Sales effort
s_p	Sustainable transportation level
s_l	Sustainable inventory level
β	Coefficient of the effect of sustainability on demand
δ_l	Coefficient of sustainable inventory investment
δ_p	Coefficient of sustainable transportation investment
a_p	Emissions when sustainable transportation level is zero
b_p	Coefficient of the effect of sustainability on transportation emissions
a_l	Emissions when sustainable inventory level is zero
b_l	Coefficient of the effect of sustainability on inventory emissions
r	Carbon tax rate
θ	Coefficient of the effect of environmental damage on environmental utility
π_R	Retailer's profit function
π_V	Supplier's profit function
SW	Social welfare
U_{ec}	Economic utility
U_{en}	Environmental utility

inventory level, and order quantity are determined on the basis of the sustainable transportation level of the supplier. Subsequently, we solve the supplier's problem according to the retailer's decision-making plan and obtain the optimal sustainable transportation level.

4.1. Decision-Making under a Carbon Tax

4.1.1. Retailer's Decision. For a given sustainable transportation level, the retailer maximizes the expected profit by optimizing the order quantity, sales effort, and inventory sustainability level. Under a carbon tax policy, enterprises are required to pay a tax related to carbon emissions generated during their operations [21]. When considering the carbon tax policy, the expected profit of retailers is calculated as follows:

$$\begin{aligned}\pi_R = E[p\min(Q,D) - (w + c_l)Q - g(x) - \frac{\delta_l s_l^2}{2} \\ + v(Q-D)^+ - r(a_l - b_l s_l)Q].\end{aligned} \tag{1}$$

In equation (1), the first term is the income of the sales product, the second term is the purchasing cost of the product from the supplier and the logistics cost, the third term is the sales cost of the retailer, the fourth term is the sustainable investment cost in the inventory, the fifth term is the remaining cost, the sixth term is the carbon tax cost, and $(a_l - b_l s_l)Q$ is the carbon emissions due to inventory. Based on equation (1), when considering the carbon tax, the retailer's sales effort x is affected by the sustainable inventory level s_l. The retailer must integrate decisions related to the sales effort x and the sustainable inventory level s_l, which add complexity to the decision process.

Given x, s_p, and s_l, the first and second derivatives of equation (1) with respect to Q yield the following proposition.

Proposition 1. *Under the carbon tax, the retailer's optimal order quantity, denoted by Q^*, for given x, s_p, and s_l, is as follows:*

$$Q^* = f(x) + \beta(s_p + s_l) + F^{-1}\left(\frac{p - w - c_l - r(a_l - b_l s_l)}{p - v}\right). \tag{2}$$

The optimal order quantity Q^* increases with an increase in the remaining cost v and decreases with increases in the market price p, the wholesale price w, and the inventory cost c_l. The first derivatives of Q^* with respect to x and s_p are shown in equation (3). These equations indicate that the optimal order quantity Q^* increases with increases in x and s_p.

$$\begin{aligned}\frac{dQ}{dx} &= f'(x), \\ \frac{dQ}{ds_p} &= \beta.\end{aligned} \tag{3}$$

By substituting Q^* into equation (1) and incorporating the first-order and second-order conditions of equation (1) with respect to x and s_l, we obtain Propositions 2 and 3, respectively.

Proposition 2. *Under the carbon tax, the retailer's optimal sales effort, denoted by x^*, for given s_l, should satisfy the following equation:*

$$\left(p - w - c_l - r\left(a_l - b_l s_l\right)\right) f'\left(x^*\right) = g'\left(x^*\right). \tag{4}$$

Proposition 3. *Under the carbon tax, the retailer's optimal sustainable inventory level, denoted by s_l^*, for given x, is as follows:*

$$s_l^* = \frac{\left(p - w - c_l - ra_l\right)\beta + rb_l\left(f(x) + \beta s_p\right)}{\delta_l - 2rb_l\beta}. \tag{5}$$

4.1.2. Supplier's Decision. The supplier supplies products according to the retailer's order quantity and determines the sustainable transportation level. When considering the carbon tax cost, the supplier's profit function is shown in equation (6), and $(a_p - b_p s_p)Q$ represents the carbon emissions due to transportation:

$$\pi_V = \left(w - c_s - c_p\right)Q - r\left(a_p - b_p s_p\right)Q - \frac{\delta_p s_p^2}{2}. \tag{6}$$

By substituting the retailer's optimal order quantity Q^* into equation (6), we can obtain Proposition 4 based on the first-order and second-order derivatives of equation (6) with respect to s_p.

Proposition 4. *Under the carbon tax, the supplier's optimal sustainable transportation level for given x and s_l is as follows:*

$$s_p^* = \frac{\left(w - c_s - c_p - ra_p\right)\beta + rb_p\left(f(x) + \beta s_l + F^{-1}\left(\left(p - w - c_l - r\left(a_l - b_l s_l\right)\right)/(p - v)\right)\right)}{\delta_p - 2rb_p\beta}. \tag{7}$$

Proposition 4 indicates that s_p^* monotonically increases with increases in w and β and monotonously decreases with increases in c_s, c_p, and δ_p.

4.2. Analysis of Decision Results. The decision model results support the following conclusions.

Corollary 1. *The carbon tax reduces the retailer's sales effort s_l.*

When $r = 0$ (i.e., no carbon tax policy), the retailer's optimal sales effort, denoted by x^{0*}, should satisfy equation (8). Comparing equations (4) and (8) reveals that $x^* < x^{0*}$. Therefore, increases in the carbon tax rate reduce the retailer's profits per product unit, thus reducing sales effort and enthusiasm [2, 18].

$$\left(p - w - c_l\right) f'\left(x^{0*}\right) = g'\left(x^{0*}\right). \tag{8}$$

Corollary 2. *Under the carbon tax, increasing the retailer's sustainable inventory level s_l can increase its sales effort x^*.*

Equation (4) illustrates that x^* increases monotonically with increase in s_l. This means that the higher the sustainable inventory level is, the lower is the carbon tax cost and the higher is the profit per unit product. Therefore, the retailer increases its sales effort to increase profit. Equation (5) indicates that s_l^* increases monotonically with increase in x. By increasing its sales effort x, a retailer can enhance the sustainable inventory level s_l^*.

Corollary 3. *The retailer's sustainable inventory level s_l and the supplier's sustainable transportation level s_p are positively related under the carbon tax.*

Equation (5) shows that if the supplier increases the sustainable transportation level, then the retailer will increase the sustainable inventory level. Moreover, equation (7) shows that if the retailer increases the sustainable inventory level, then the supplier will increase the sustainable transportation level. Therefore, when considering the carbon tax, the retailer's sustainable inventory investment decisions and the supplier's sustainable transportation investment decisions interact with and promote each other.

Corollary 4. *Under the carbon tax, the retailer's optimal sustainable inventory level is $s_l^* > (p - w_1 - c_l - ra_l)\beta/(\delta_l - 2rb_l\beta)$.*

Equation (5) indicates that when $x = 0$ and $s_p = 0$, the retailer's optimal sustainable inventory level is $s_l = (p - w_1 - c_l - ra_l)\beta/(\delta_l - 2rb_l\beta)$. According to Corollaries 2 and 3, increases in x and s_p can enhance s_l^*. Under the carbon tax, $x > 0$ and $s_p > 0$. Therefore, $s_l^* > (p - w_1 - c_l - ra_l)\beta/(\delta_l - 2rb_l\beta)$.

4.3. Solution for Optimal Sustainable Levels under a Carbon Tax. In the preceding analysis, equations (4), (5), and (7) provide analytical expressions of x^*, s_l^*, and s_p^*, respectively. However, these three decision variables are nested in each other in these three equations, and they cannot be directly obtained from the corresponding formula. An iterative method is required to solve for these three decision variables.

A careful analysis revealed three implicit mathematical results from the previous conclusions:

(1) The retailer's sales profit increases monotonically with the sustainable inventory level s_l

(2) The retailer's profit from sustainable inventory investment increases monotonically for $s_l \in [0, (p - w_1 - c_l - ra_l)\beta/(\delta_l - 2rb_l\beta)]$ and decreases

monotonically for $s_l \in [(p - w_1 - c_l - ra_l)\beta/(\delta_l - 2rb_l\beta), (a_l/b_l)]$

(3) The supplier's profit increases monotonically with the sustainable inventory level s_l

According to the aforementioned information, we can conclude that, in the interval $[(p - w_1 - c_l - ra_l)\beta/(\delta_l - 2rb_l\beta), (a_l/b_l)]$, an optimal sustainable inventory level s_l^* must maximize the expected profits of the retailer and the supplier. Therefore, the following iterative method is introduced to obtain the optimal solutions:

Step 1: let $i = 1$ and $s_l(i) = (p - w_1 - c_l - ra_l)\beta/(\delta_l - 2rb_l\beta)$; initialize $\pi_R(0)$ as zero.

Step 2: calculate $Q(i)$, $x(i)$, and $s_p(i)$ by substituting $s_l(i)$ into equations (2), (4), and (7), respectively.

Step 3: calculate $\pi_R(i)$ by substituting $s_l(i)$, $x(i)$, $s_p(i)$, and $Q(i)$ into equation (1). If $\pi_R(i) \leq \pi_R(i-1)$, the procedure terminates, and $s_l(i)$, $x(i)$, and $s_p(i)$ are the optimal values; thus, $s_l^* = s_l(i)$, $x^* = x(i)$, and $s_p^* = s_p(i)$. Otherwise, let $i = i + 1$ and go to Step 4.

Step 4: calculate $s_l(i)$ by substituting $x(i-1)$ and $s_p(i-1)$ into equation (13) and return to Step 2.

5. Multiplayer Dynamic Game Optimization Model

The optimal sustainable levels for transportation and inventory can be determined using the optimization method described in Section 4 given the wholesale price w and carbon tax rate r. However, sustainable transportation and inventory levels are related to carbon tax rates and wholesale prices, which affect retailers' and suppliers' profits. Therefore, appropriate carbon tax rates and wholesale prices are crucial for the coordinated operation of the supply chain. The dynamic game method with complete information was used to construct a three-stage game optimization model to determine the optimal wholesale prices and carbon tax rates. Table 3 displays the basic information of the game, and the optimization processes of each stage of the game simulation are illustrated in Figure 2.

5.1. Wholesale Price Game (Stage 1). In stage 1, the effect of the carbon tax is neglected (i.e., $r = 0$). By substituting $r = 0$ into equations (5) and (7), we can determine the optimal sustainable levels for inventory and transportation (denoted by s_l^{0*} and s_p^{0*}, respectively) as follows:

$$s_l^{0*} = \frac{(p - w - c_l)\beta}{\delta_l},$$
$$s_p^{0*} = \frac{(w - c_s - c_p)\beta}{\delta_p}. \tag{9}$$

We then calculate the retailer's profit and supplier's profit by substituting $r = 0$, s_l^{0*}, and s_p^{0*} into equations (1) and (6).

In the supplier-retailer relationship, the supplier is the dominant party because wholesale prices are set by the supplier. If the retailer accepts this price, both parties trade at this price, and the game is finished. If the retailer does not accept this price, then the supplier gradually reduces the price until the retailer accepts it. When considering the opportunity cost, the equilibrium condition for a retailer is that its profit rate is higher than the social average. Otherwise, the retailer would choose another investment opportunity. When a retailer's profit rate is higher than the social average, the supplier will not reduce the wholesale price because it can easily find other retailers in the market. Therefore, retailers no longer require suppliers to reduce wholesale prices, and the Nash equilibrium is achieved.

5.2. Carbon Tax Rate Game (Stage 2). The dominant party in stage 2 is the government, the goal of which is to maximize social welfare. Social welfare is expressed in equation (10). The first term in equation (10) is economic utility, and the second term is environmental utility. Economic utility is a positive utility that is determined using equation (11). In equation (11), the first term is the supplier's profit, the second term is the retailer's profit, and the third term is the carbon tax revenue. Environmental utility is a disutility and reflects the environmental damage caused by carbon emissions. A quadratic environmental damage function [54] is used to represent the environmental utility, as illustrated in equation (12).

$$SW = U_{\text{ec}} - U_{\text{en}}, \tag{10}$$

$$U_{\text{ec}} = \pi_R + \pi_V + r\left(a_l + a_p - b_l s_l - b_p s_p\right)Q, \tag{11}$$

$$U_{\text{en}} = \frac{\theta\left(a_l + a_p - b_l s_l - b_p s_p\right)^2 Q^2}{2}. \tag{12}$$

Stage 2 involves three players. Based on the Stackelberg game, a government sets a carbon tax rate, and suppliers and retailers select sustainable levels of transportation and inventory by using the optimization model presented in Section 4. If profits are lower than expected, suppliers and retailers will abandon the investments, which will result in no social welfare. In the simulation game, the government gradually reduces the carbon tax rate from a high level until it is accepted by suppliers and retailers. The game equilibrium maximizes social welfare while suppliers and retailers achieve their expected profit rates.

5.3. Sustainable Investment-Sharing Strategy Game (Stage 3). Carbon tax policy imposes a negative financial burden on companies [17]. To encourage enterprises to strengthen their efforts to reduce carbon emissions, a government can implement a sustainable investment-sharing strategy. The sharing proportions of the government and enterprises are k and $1 - k$, respectively. In this strategy, government

Table 3: Basic game information.

Stage	Players: Government	Supplier	Retailer	Optimization objective
1		✓	✓	Wholesale price
2	✓	✓	✓	Carbon tax rate
3	✓	✓	✓	Sustainable investment-sharing proportion

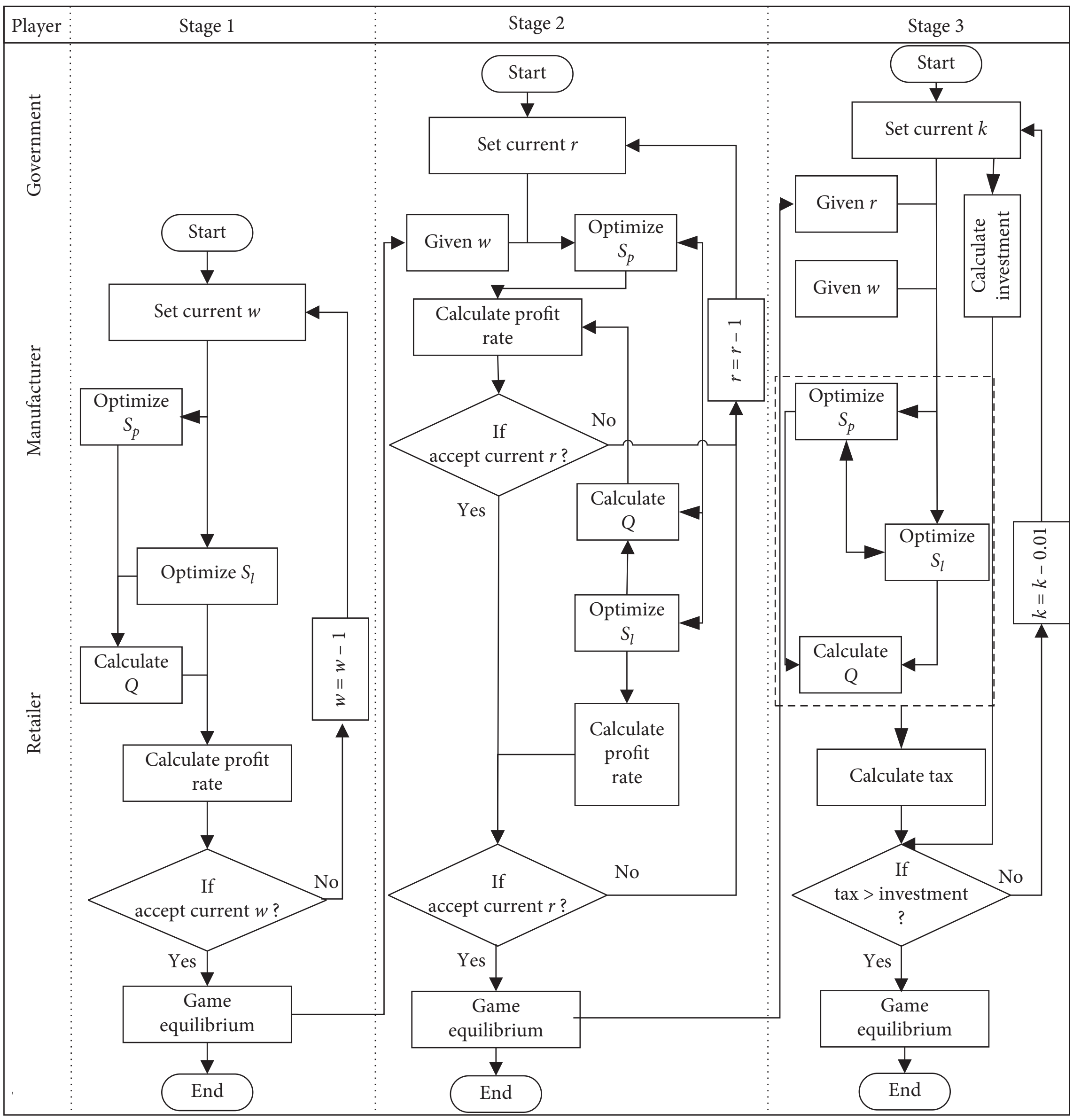

Figure 2: Three-stage game simulation of optimization process.

sustainable investment is represented as $k(\delta_l s_l^2 + \delta_p s_p^2)/2$, and sustainable investments of suppliers and retailers are $(1-k)\delta_l s_l^2/2$ and $(1-k)\delta_p s_p^2/2$, respectively. The expected profit functions of retailers and suppliers are as follows:

$$\pi_R^I = E\left[p\min(Q,D) - (w + c_l)Q - g(x) - \frac{(1-k)\delta_l s_l^2}{2} + v(Q-D)^+ - r(a_l - b_l s_l)Q\right],$$
$$\pi_V^I = \left(w - c_s - c_p - r\left(a_p - b_p s_p\right)\right)Q - \frac{(1-k)\delta_p s_p^2}{2}. \tag{13}$$

By using the same derivation as in Section 4, we can obtain the optimal sustainable levels of inventory and transportation as follows:

$$s_l^{I*} = \frac{(p - w - c_l - ra_l)\beta + rb_l\left(f(x^{II}) + \beta s_p\right)}{(1-k)\delta_l - 2rb_l\beta}, \tag{14}$$

$$s_p^{I*} = \frac{\left(w - c_s - c_p - ra_p\right)\beta + rb_p\left(f(x^{II}) + \beta s_l^{II} + F^{-1}\left(p - w - c_l - r\left(a_l - b_l s_l^{II}\right)/(p - v)\right)\right)}{(1-k)\delta_p - 2rb_p\beta}. \tag{15}$$

Equations (14) and (15) indicate that the optimal sustainable levels of inventory and transportation increase monotonically with the sharing proportion of government k.

The sustainable investment-sharing strategy is optimized based on the Stackelberg game between the government and the enterprises (i.e., suppliers and retailers). Suppliers and retailers will accept any government strategy. However, higher proportions of government investment induce higher sustainable levels of transportation and inventory, which improve social welfare but reduce carbon tax revenue. Therefore, a balance between social welfare, investment, and carbon tax revenue is required. However, the primary aim of the carbon tax policy is to improve the environment rather than raise revenue. This model assumes that governments aim to maximize social welfare when carbon tax revenue is higher than the investment. In this game, the government first sets the investment-sharing proportion to a relatively high value and then gradually reduces it. When the government achieves the decision objectives, the game reaches its Nash equilibrium.

6. Simulation Analysis and Results

6.1. Simulation Method and Accuracy Analysis. To illustrate the simulation method for game optimization and analyze the effect of a carbon tax on supply chain performance, supply chains were studied in specific situations. In the following numerical examples, let $\xi \sim N(200, 20)$, $p = 80$ CNY, $C_s = 25$ CNY, $C_l = 2$ CNY, $C_p = 3$ CNY, $v = 5$ CNY, $a_p = 50$ kg, $b_p = 5$ kg, $a_l = 30$ kg, $b_l = 3$ kg, $\beta = 140$, $\delta_p = 1500$ CNY, $\delta_l = 1200$ CNY, and $\theta = 3$ CNY. Given that $f'(x) > 0$, $f''(x) < 0$, $g'(x) > 0$, and $g''(x) \geq 0$, we set $f(x) = 10x^{1/2}$ and $g(x) = x$ [55]. The expression $f(x) = 10x^{1/2}$ shows that the regularity of demand generated by sales effort varies with sales effort, and $g(x) = x$ indicates that when the retailer increases sales effort by one unit, the cost incurred for sales effort is 1 CNY. The constant operating costs of the supplier and the retailer were set at 20,000 CNY and 15,000 CNY, respectively.

The simulation calculations for each stage of the three-stage game optimization process were programmed separately in MATLAB (Figure 2). A total of 500 simulation cycles were implemented. The relative error ε of the simulation result was calculated using equation (16), where X and σ are the mean and standard deviation, respectively, of 500 simulation results. The simulation accuracy was set to 99%, and the allowable relative error was set to 5%. Social welfare was selected as the test index, and statistical analyses of the simulation results are illustrated in Table 4. According to Table 4, the relative errors of the results for the 500 simulations were all <5%. The accuracy of the simulation results satisfied research requirements, and these results were used for decision analysis.

$$\varepsilon = \frac{t_{499,0.99}\sqrt{\sigma^2/500}}{X(500)}. \tag{16}$$

6.2. Simulation Results and Optimal Decisions

6.2.1. Simulation Results of the Wholesale Price Game (Stage 1). Let $r = 0$, and suppose that the average social profit rate for the product is 15%. Set $w = 30, 31, \ldots, 65$ CNY, and simulate the supply chain operations for each of the 36 wholesale price situations. Table 5 summarizes the results of stage 1. When $w = 49$ CNY, retailer profit was 15.34%. When $w > 49$ CNY, retailer profit was <15%. Therefore, the maximum acceptable wholesale price for retailers was 49 CNY. The relationship between the supplier and the retailer indicated that the optimal wholesale price determined by the supplier was 49 CNY.

6.2.2. Simulation Results of the Carbon Tax Rate Game (Stage 2). Let $w = 49$ CNY, and suppose that the average social profit rate for the product is 10% after the implementation of the carbon tax. Simulation experiments revealed that the sustainable transportation and inventory levels were maximized when $r = 623$ CNY/ton. For $r > 623$ CNY/ton, increases in the carbon tax rate had no effect on the operation of the supply chain. Therefore, we investigated tax rates in the range of 1–623 CNY/ton and simulated the supply chain operations for each of the 623 carbon tax rates. Table 6 summarizes the results of stage 2.

According to the simulation results, social welfare exhibits an inverted U-shaped trend with increases in the carbon tax rate. When $r = 529$ CNY/ton, social welfare was maximized at a value of 51,276 CNY. However, the profits of the supplier and the retailer were −6.86% and 9.02%, respectively. If the government sets the tax rate at 529CNY/ton, both suppliers and retailers abandon their

Table 4: Accuracy of the simulation results.

Game stage	Mean (CNY)	Standard deviation	Half-interval	Relative error (%)
Stage 1	7944	2924.6	336.79	4.24
Stage 2	23,462	1802.2	328.15	1.40
Stage 3	31,455	1681.6	306.18	0.97

Table 5: Decision information for the wholesale price game (stage 1).

Indicators	Wholesale price w (CNY)				
	30~47 (%)	48 (%)	49 (%)	50 (%)	51~63 (%)
Retailer's profit rate	35.25~17.34	16.34	15.34	14.34	13.33~0.98

businesses because profits do not satisfy expectations. Therefore, after considering the effect on the supplier and the retailer, social welfare was 0. Therefore, tax rates must be reduced to ensure that the supplier and the retailer achieve their expected profit rates. When the carbon tax rate was reduced to 163 CNY/ton, both supplier and retailer achieved their expected profits. Thus, the carbon tax rate game reached equilibrium with an optimal carbon tax rate of 163 CNY/ton.

Under a carbon tax, the supplier bears more costs for sustainable investment and the carbon tax than the retailer does, resulting in a considerable decrease in the supplier's profit. Hence, the supplier attempts to increase the wholesale price w to shift some costs to retailers. We set $r = 163$ CNY/ton; consider $w = 30, 31, \ldots, 65$ CNY; and simulate the supply chain operations for each of the 36 wholesale prices. The results are summarized in Table 7. When $w = 51$ CNY, the retailer's profit was 10.45%. For $w \geq 52$ CNY, the retailer's profit was <10%. Therefore, the supplier could adjust the wholesale price w to 51 CNY.

6.2.3. Simulation Results of the Sustainable Investment-Sharing Strategy Game (Stage 3). Let $w = 51$ CNY and $r = 163$ CNY/ton. Through simulation experiments, we determined that the sustainable transportation and inventory levels reached their maximum when the government's sharing proportion was $k = 57\%$. At this value, further increases in the shared proportion did not improve social welfare; therefore, the shared proportion set by the government should not exceed 57%. With $k = 1\%, 2\%, \ldots, 57\%$, we simulated supply chain operations for each of the 57 investment-sharing strategies. Table 8 summarizes the results of stage 3.

The simulation results indicated that carbon tax revenue decreased with an increase in the shared proportion, whereas government investment increased with an increase in the shared proportion. With $k = 44\%$, social welfare was maximized. For this value, the carbon tax revenue was 12,850 CNY, and the government investment was 31,636 CNY. Therefore, the government was required to invest additional 18,786 CNY. For $k < 44\%$, social welfare increased with the shared proportion. For $k > 44\%$, social welfare decreased with the shared proportion. For $k < 57\%$, tax revenues decreased, and the government investment increased with an increasing shared proportion. For $k = 35\%$, tax revenue was higher than government investment. Therefore, the optimal shared proportion was 35%.

6.3. Summary and Comparison of Optimal Results. The optimal results of each stage are summarized in Table 9. Comparing the results of stages 1 and 2 revealed that the effects of the carbon tax on the supply chain were principally manifested in the following four aspects:

(1) The sustainable transportation level increased by 44.90%, from 1.96 to 2.84, and the sustainable inventory level increased by 7.36%, from 3.38 to 3.64.

(2) Social welfare increased by 24.93%, from 21,664 CNY to 27,254 CNY. Environmental utility decreased by 25.80%, from 30,733 CNY to 26,061 CNY. Economic utility increased by 3.23%, from 52,397 CNY to 50,703 CNY.

(3) Supplier and retailer profits decreased by 51.92% and 31.57%, respectively. The profit structure in the supply chain changed substantially. Without the carbon tax, supplier and retailer profits were 51.87% and 48.13%, respectively. After adopting the carbon tax, supplier and retailer profits were 43.09% and 56.91%, respectively.

(4) The carbon tax revenue increased to 20,380 CNY.

These results demonstrated that the carbon tax improved the sustainable transportation and inventory levels and reduced carbon emissions in logistics, thus reducing environmental utility. When the supplier and the retailer improve their sustainability levels, their sustainable investments are increased correspondingly. However, the increase in sustainable investment does not result in an increase in profits. By contrast, supplier and retailer profits decreased after the introduction of the carbon tax. The conflict between investment and profit reflected the negative effect of the carbon tax on the supplier and the retailer. The increase in the sustainable transportation level after the introduction of the carbon tax was higher than that of the sustainable inventory level, encouraging sustainable investment by the supplier. Moreover, the carbon tax revenue from the supplier was higher than that from the retailer because emissions from transportation are higher than those from inventory. Therefore, the reduction in the supplier's profits was higher than that of the retailer's profits, which adjusted the profit structure in the supply chain. Although the carbon tax had a negative effect on profit in the supply chain, the economic utility of the supply chain did not

Table 6: Decision information for the carbon tax rate game (stage 2).

Indicators	Carbon tax rate r (CNY/ton)					
	1	2~162	163	164~528	529	530~623
Social welfare (CNY)	21,767	21,803~27,034	27,065	27,096~41,274	41,276	36,294~29,811
Supplier's profit rate (%)	30.17	30.01~10.19	10.09	9.99~−6.89	−6.86	−6.82~-0.89
Retailer's profit rate (%)	15.36	15.33~12.13	12.11	12.09~9.00	9.02	9.03~9.83

Table 7: Wholesale price adjustment in stage 2.

Indicators	Wholesale price w (CNY)				
	30~49 (%)	50 (%)	51 (%)	52 (%)	53~63 (%)
Retailer's profit rate	34.13~12.12	11.54	10.45	9.37	8.28~−10.48

Table 8: Decision information for the sustainable investment-sharing strategy game (stage 3).

Indicators (CNY)	Government's sharing proportion k					
	1%	2%~34%	35%	36%~43%	44%	45%~57%
Social welfare	26,491	26,275~39,121	39,588	40,046~42,395	42,460	43,395~27,248
Tax revenue	20,372	20,363~17,499	17,210	16,892~13,551	12,850	12,069~0
Government's investments	153.8	295.13~15,875	15,875	17,101~29,210	31,636	34,302~76,950

Table 9: Summary of optimal results.

Indicators	Stage 1	Stage 2	Stage 3
Transportation sustainable level	1.96	2.84	6.34
Inventory sustainable level	3.38	3.64	5.32
Supplier's profit (CNY)	27,176	13,065	25,132
Retailer's profit (CNY)	25,221	17,258	29,842
Profit of the whole supply chain (CNY)	52,397	30,323	54,974
Supplier's profit rate (%)	30.32	12.67	19.11
Retailer's profit rate (%)	15.34	10.45	13.89
Economical utility (CNY)	52,397	50,703	56,309
Environmental utility (CNY)	30,733	23,449	16,721
Social welfare (CNY)	21,664	27,254	39,588
Tax revenues (CNY)	0	20,380	17,210
Government's investments (CNY)	0	0	15,875

decrease because the government received carbon tax revenue. However, with the rapid decline in the environmental utility, the social welfare of the entire supply chain was significantly improved. Therefore, although the carbon tax had negative effects on enterprises [17], it induced positive effects for the society.

The optimization results of stages 2 and 3 indicated that the effects of sustainable investment-sharing strategies on the supply chain were as follows:

(1) The sustainable transportation and inventory levels increased by 123.24% and 46.15%, respectively.

(2) Social welfare increased by 45.26% to 39,588 CNY, and economic utility increased by 11.06% to 56,309 CNY, whereas environmental utility decreased by 28.69% to 16,721 CNY.

(3) Supplier and retailer profits increased by 92.36% and 72.92%, respectively. The proportions of supplier and retailer profits were 45.72% and 54.28%, respectively. The profit structure in the supply chain trended towards a balance.

(4) The carbon tax revenue decreased to 17,210 CNY. The sustainable investment of the government was 15,875 CNY, and the actual revenue (i.e., tax revenue minus investment) was 1,335 CNY.

The sustainable investment-sharing strategy improved supply chain performance for the government and enterprises. It reduced the marginal investment cost for the supplier and the retailer, thus providing an incentive to improve sustainability. A higher government share proportion induced more sustainable practices. By sharing the investment cost with the government, the supplier and the retailer obtained more revenue at a reduced cost. With the improvement in sustainability, the supplier and the retailer reduced their carbon emissions in logistics, thereby reducing the environmental utility and their carbon tax costs. The reduction in carbon tax costs increased the profits of enterprises, thereby increasing their economic utility. The dual function of economic utility and environmental utility resulted in a rapid increase in social welfare. Notably, the sustainable investment-sharing strategy had a more substantial effect on the supplier because suppliers generate higher carbon emissions than retailers do.

Compared with the results of stage 1, the optimization strategy of stage 3 increased the government revenues (1,335 CNY), social welfare (39,588 CNY), and the profit of the entire supply chain (54,974 CNY), thus coordinating the interests of the government and enterprises. These results indicated that the sustainable investment-sharing strategy not only promoted the positive role of the carbon tax for the sustainable logistics level, social welfare, and environmental utility but also effectively compensated for the negative effect of the carbon tax on enterprise profits. Therefore, the

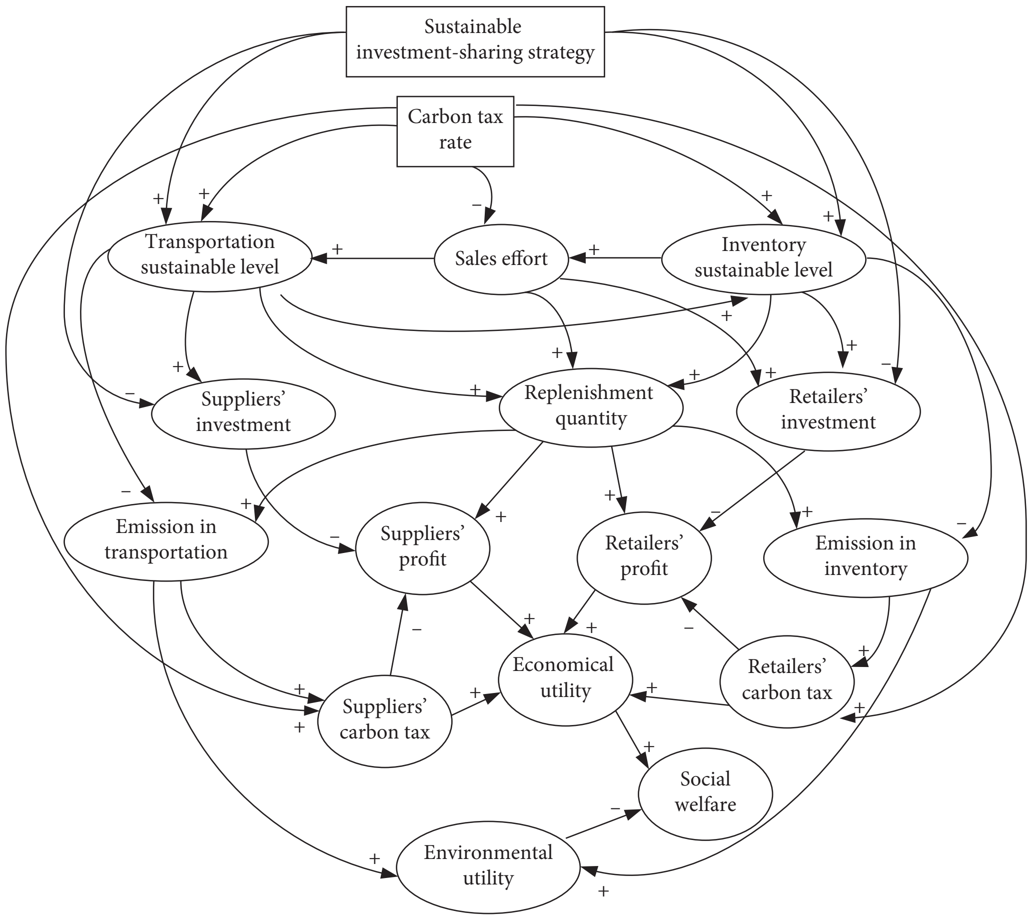

Figure 3: Causal relationships of the sustainable transportation and inventory system.

proposed combined optimization of the carbon tax and the sustainable investment-sharing strategy can increase social welfare, stimulate enthusiasm for carbon emission reduction in enterprise logistics, and facilitate supply chain coordination.

6.4. Implications for Management. Figure 3 shows the mechanisms and effects of the carbon tax and the sustainable investment-sharing strategy as well as the internal relationships between the supply chain elements.

The decisions of suppliers and retailers regarding sustainable investment are interlinked. To ensure that enterprises can make optimal investments in transportation and inventory to reduce carbon emissions, both suppliers and retailers must establish communication mechanisms to reach consensus and reduce carbon emissions [46]. Suppliers' sustainable investments in inventory benefit retailers, and retailers' sustainable investment and sales investment benefit suppliers. Suppliers and retailers should use the benefits of supply chain spillovers to reasonably share investment costs and encourage the other parties to enhance their sustainable investment [6, 45]. Therefore, incentive mechanisms should be implemented to reduce carbon emissions by suppliers and retailers. When formulating policies, governments should compromise between social welfare and corporate goals. High carbon taxes can harm government revenues and enterprises. When implementing a sustainable investment-sharing strategy, the government should prioritize enterprises with high carbon emissions to maximize the effect of emission reduction. When financial expenditure is sufficient or the task of reducing carbon emissions is challenging, governments can increase their share of sustainable investment to reduce environmental utility and improve social welfare.

7. Conclusions

7.1. Discussion. Sustainable supply chain management is a widely discussed topic in environmental sustainability, and a carbon tax is an effective method for reducing carbon emissions. Transportation and inventory management are the two core factors that affect economic and environmental utilities in logistics. However, research on joint decision-making for sustainable transportation and inventory management when a carbon tax is implemented on a global scale is lacking. We investigated a two-part supply chain in which the supplier first determines the sustainable transportation level and the retailer then places an order and assesses the sustainable inventory level relative to sales effort. We integrated the optimization of sustainable transportation and inventory levels and identified an approach to improve sustainable supply chain performance when a carbon tax is implemented and the government shares sustainable investment with enterprises.

For suppliers and retailers, they must maintain appropriate sustainable transportation and inventory levels under a carbon tax. We adopted a Stackelberg game to optimize the sustainable transportation and inventory levels. Because the three factors, namely, sales effort, inventory sustainability, and transportation sustainability, influenced each other in the decision-making process, optimal results could not be directly deduced, and thus, an iterative process was designed to solve the optimization problem. The results revealed that the carbon tax policy encouraged suppliers and retailers to increase their sustainable transportation and inventory levels [25], thus reducing carbon emissions in the logistics industry. Increasing the sustainable inventory level improved retailers' sales efforts. However, an increase in carbon tax costs reduced sales motivation.

For the government, the carbon tax rate must be set to improve environmental utility and social welfare. To solve this problem, we developed a three-stage dynamic game optimization model. The wholesale price was determined in the first stage, which included suppliers and retailers. In the second stage, which also included the government, the carbon tax rate was optimized, and the wholesale price was adjusted accordingly. The sustainable investment-sharing strategy was considered in the third stage. We implemented a MATLAB simulation to solve this three-stage game model.

A single carbon tax has a dual effect on the performance of the supply chain, and an appropriate sustainable investment-sharing strategy can harmonize government and enterprise objectives. The results of numerical studies indicated that the carbon tax played a strong role in improving environmental utility and social welfare. However, such a policy hinders enterprises [43]. Enterprises increased their sustainable investments, but their profits decreased. The sustainable investment-sharing strategy can stimulate enterprises to promote sustainability, reduce environmental utility, and increase social welfare. This may enable an increase in profits for enterprises with less investment, which would offset the negative effect of the carbon tax on enterprises. An appropriate sustainable investment-sharing strategy can promote the interests of governments and enterprises and improve sustainability in the supply chain.

The novel contributions of this paper are summarized as follows. An integrated optimization model for sustainable transportation and inventory management was developed. To the best of our knowledge, this study is the first to combine two sustainable logistics factors in the supply chain. Furthermore, a three-stage dynamic game model was proposed to optimize wholesale prices, carbon tax rates, and investment-sharing strategies. The proposed model could be used as a framework for governments that intend to implement a carbon tax and enterprises that are committed to developing sustainable logistics.

7.2. Limitations and Future Research. Although this paper makes several novel contributions, certain limitations should be noted, and future studies should address the following: (1) the carbon tax rate was set as a fixed value in the model. Uncertain carbon tax rates should be discussed in future studies. (2) Our study only considered the carbon tax. However, sustainable investment under other carbon emission-reduction policies, such as carbon cap, carbon cap-and-trade, and carbon offset, also warrants further study. (3) The supply chain network structure has certain limitations. Future research should investigate more complex supply chain networks such as one-to-many, many-to-one, and many-to-many. Routing problems in these networks complicate the optimization process. In such networks, the measurement of sustainable transportation levels in different routing schedules and the differences in consumer environmental awareness in different markets would increase the value and challenge of the problem. (4) The spillover effect of sustainable investment has a major influence on decision-making in supply chain enterprises. Future studies should focus on the spillover effect in one-to-many, many-to-one, and many-to-many supply chains. We should further analyze the horizontal and vertical benefit spillovers and the coordination mechanisms within these supply chains.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Research Article

Pricing Decisions in a Competitive Closed-Loop Supply Chain with Duopolistic Recyclers

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In this study, we consider a three-echelon closed-loop supply chain consisting of a manufacturer, a collector, and two duopolistic recyclers. In the supply chain, the collector collects end-of-life products from consumers in the market. Then, both recyclers purchase the recyclable waste from the collector, and each recycler turns them into new materials. The manufacturer has no recycling facilities; therefore, the manufacturer only purchases the recycled and new materials for its production from the two recyclers. Under this scenario, price competition between recyclers is inevitable. With two pricing structures (Nash and Stackelberg) of the leaders group and three competition behaviors (Collusion, Cournot, and Stackelberg) of the followers group, we suggest six different pricing game models. In each of them, we establish a pricing game model among the members, prove the uniqueness of the equilibrium prices of the supply chain members, and discuss the effects of competition on the overall supply chain's profitability. Our numerical experiment indicates that as the price competition between recyclers intensifies, the supply chain profitability decreases. Moreover, the greater the recyclability degree of the waste is, the higher the profits in the supply chain become.

1. Introduction

Over the past few decades, profitability improvements and cost leadership have been the main goals of supply chain management. However, more recently, the increasing rates of environmental degradation and resource depletion triggered by rapid industrialization have shifted this focus to socioenvironmental issues; in the context of supply chain research, this has led to more concern about sustainability, with the concept of supply chain sustainability emerging [1]. For many industries, supply chain sustainability is one of the most critical tasks of their operations and long-term planning. In addition to business performance of the supply chain, environmental and social effects of the supply chain have been increasingly perceived as critical aspects of supply chain performance by shareholders. As a result, sustainable supply chain management has become one of the main interests of business managers and stakeholders [2–4].

The supply chain is an important branch of operational management, and it has a significant impact on the environment through hazardous gas emission and pollution. Companies in various industries are now attempting to minimize their environmental impacts by integrating environmental issues into their supply chain operations. The integration of environmental issues into supply chain management practices is referred to as green supply chain management [5–8]. Green supply chain has become an important research topic in academia and industry. This topic includes environmental management such as eco-friendly product/service designs, green purchasing, reuse, remanufacturing, and recycling [9–11]. Among these solutions, recycling and reuse are considered more desirable because they require less of a quality-of-life compromise of the type closely linked to intensive material consumption [12]. Many countries have been promoting the policies related to the economic resource circulation. For example, Japan has been encouraging what is termed a sound material-cycle society by implementing the 3R (reduce, reuse, and recycle) strategy [13]. Material and energy flows must become part of an increasingly sustainable and circular economy, a concept introduced by the European Union as follows: In a circular economy, the value of products and materials is maintained for as long as possible. Waste and resource use are minimized, and when a product reaches the end of its life, it is used again to create further value. This can bring major economic benefits, contributing to innovation, growth, and job creation. In 2018, Apple announced that the 2018 models of its MacBook Air and Mac mini would both be manufactured with 100 percent recycled aluminum. Additionally, the Mac mini would be constructed from 60 percent recycled plastic.

Recycling is the process of collecting and processing waste that would otherwise be thrown away as trash and turning the waste into new products for environmental protection. It also includes the optimal management of waste disposal facilities. Another aim of recycling is to encourage ecofriendly management and to manage limited supplies of resources. The supply chain term refers to a type of systematic collaboration between people, processes, and information to create tangible or intangible value and deliver it to consumers. The main purpose of supply chain management is to facilitate better material and information flows among stakeholders in the supply chain. This creates a better relationship between stakeholders in the supply chain, which increases the profitability of the entire supply chain [14, 15]. With the depletion of resources around the world, waste from end-of-life products is becoming an important resource that can be managed globally. As consumers' interest in environmental issues has increased along with the amounts of waste, industrialists and researchers are now focusing on sustainable products. Reverse and closed-loop supply chains are well adapted to sustainability goals [16]. Generally, a reverse supply chain and closed-loop supply chain consist of certain operations such as collecting recyclable waste, transforming it into new materials, and transferring these materials to a manufacturer for remanufacturing. A dual channel for collection can also be implemented in the supply chain. Sometimes, it is found that dual-channel recycling outperforms single-channel recycling [17, 18].

Recyclability is a characteristic of a material that can retain useful chemical and/or physical properties after achieving their original purpose, thus allowing it to be reused or remanufactured into additional products through a recognized process. Thus, recyclability must be observed and considered as the baseline during design, production, and waste management activities. With the development of information and communication technology (ICT), the demand for electronic products has increased greatly, and this has led to more generation of waste electrical and electronic equipment (WEEE). The disruption of rare-earth metal exports to Japan, triggered by the Senkaku Islands dispute in 2010, led to confusion not only in Japan but also in the global market, paradoxically emphasizing the importance of securing resources. In 2016, the potential recovery of seven precious resources, specifically iron, copper, gold, silver, aluminum, palladium, and plastics in WEEE, amounts to approximately 12.3 million tonnes in Europe [19]. In light of this fact, the recyclability of WEEE plays a key role not only in terms of environmental protection but also with regard to the stable supply and demand for various ICT products.

Based on these observations, this paper deals with a closed-loop supply chain in which two recyclers compete with each other. More specifically, each recycler purchases recyclable waste from a collector, recycles the waste, and finally sells the recycled materials to a manufacturer. In this process, the two recyclers compete with regard to the selling prices of their recycled materials. The aim of this study is to investigate pricing and ordering decisions during the waste recycling process in a three-echelon closed-loop supply chain with duopolistic recyclers using a game-theoretic framework. Due to the competition between recyclers, the price offered by one recycler affects not only the price of the other recycler but also those of all other members in the supply chain. Therefore, demand and profit in the supply chain are sensitive in all cases to price competition. This study proposes several pricing game models based on pricing structures and competitive behavior. The main research questions for this study are as follows:

(i) How can each member in the supply chain increase the profit?
(ii) What are the profits and equilibrium variables of the supply chain members?
(iii) How strong is the effect of the price competition between recyclers in the supply chain?
(iv) Does the recyclability degree of wastes have a positive effect on the profit of each member in the supply chain?
(v) Does an imbalance in the market share between competitors affect the profit of the entire supply chain?

In order to answer the above questions, we revisit Jafari et al.'s study [20] by considering price competition between recyclers. The main contributions of this research are threefold. First, the impact of price competition between recyclers with different competitive behaviors on the sustainability of the supply chain is investigated. Second, our study concerns the economic as well as environmental aspects of the supply chain. Third, we suggest six different pricing game models and show the existence of equilibrium solutions for each game. Finally, we compare the results of the six pricing game models through a numerical example.

The rest of this study is organized as follows. In Section 2, we present a brief overview of the relevant literature, after which we introduce the six pricing game models and then review the notations used and assumptions in Section 3. In Section 4, we conduct a preliminary analysis of our main results. Section 5 deals with the equilibrium quantities for each of the six pricing game models. Various numerical experiments are conducted in Section 6 to investigate the effects of certain parameters on the equilibrium quantities. In Section 6, we give a summary of the paper and provide future research topics.

2. Literature Review

In this section, we review the relevant literature considering the main stream of research studies: recycling issues in the sustainable supply chains.

The economic and environmental benefits of sustainable supply chain management have been widely recognized over the past two decades, and the closed-loop supply chain (CLSC) has therefore attracted significant attention from both industry and academia. A CLSC consists of a forward and a reverse supply chain. The forward supply chain involves the movement of products from upstream suppliers to downstream consumers, while the reverse supply chain involves the movement of used or end-of-life products from consumers to upstream suppliers [21]. In a CLSC, it is suggested that once the products are sold to consumers, the responsibilities of producers for dealing with sustainability issues should not end. There should be some accountability with regard to the impacts of the products during their consumption and during the postconsumption phase. Accordingly, waste management programs should be adopted. As a result, the linear paradigm of the supply chain becomes circular. Input materials into the CLSC are reduced because some of the generated waste is retrieved to be reused as resources. Hence, the energy and resource dependencies are reduced without affecting economic growth [22]. The CLSC stimulates the circulation of resources by slowing, narrowing, intensifying, and closing resource loops [23]. From this point of view, recycling is one of the major avenues used to improve waste management systems [12]. Several studies have investigated this issue. Savaskan et al. [24] dealt with a CLSC capable of product collection and recycling and found that retailer collection is the most effective means of product collection activity for the manufacturer. Savaskan and Van Wassenhove [25] studied the reverse channel design and optimal pricing decisions of a CLSC in which two competing recyclers collect used products. Chen and Sheu [26] developed a differential game model established in view of sales competition and recycling dynamics as well as regulation-related profit function. Atasu et al. [27] investigated the impact of collection cost structures on optimal reverse channel decisions based on the work of Savaskan et al. [24]. Hong et al. [28] investigated three reverse hybrid collection channel structures in a manufacturer-oriented CLSC and found that the retailer's and manufacturer's hybrid collection channel is the most effective. Huang et al. [18] studied the impacts of recycling competition on pricing and recycling strategies. They showed that dual-channel recycling outperforms a single-channel recycling. A similar problem with a different end-of-life product collection structure was studied by Wang et al. [29] and Modak et al. [30, 31]. Modak et al. proposed a two-echelon duopolistic retailer supply chain model with a recycling facility by considering the Cournot and Collusion behaviors of retailers. Saha et al. [32] considered pricing strategies in a dual-channel CLSC under three systems for the collection of used products: third-party collection, manufacturer collection, and retailer collection. Liu et al. [33] studied the dual-recycling channel collection to investigate pricing and best reverse-channel choice decisions. Their work suggested that the retailer dual-collecting model was the best channel structure for a CLSC. In a dual-reverse-supply chain, consumer preference was considered by Feng et al. [17], who assumed a case in which consumers return used products via three different recycling channels. Jafari et al. [20] considered a dual-channel recycling structure through a collector or a recycler. They considered recycling while assuming a smooth waste collection flow, i.e., with no shortcomings occurring on the collector's side. They established various Stackelberg game models to determine the equilibrium prices for recyclable waste, recycled materials, and finished products. By considering a more realistic situation of recycling, Giri and Dey [34] extended the model by Jafari et al. [20] to a CLSC with a secondary or backup supplier who supplies shortfalls of materials to tackle critical situations and obstacles preventing the smooth running of the supply chain. Wei et al. [35] studied a remanufacturing supply chain with dual-collecting channels under a dynamic setting and established three two-period game models by considering both the profit discount and competition between the two collecting channels. Jian et al. [36] explored collaborative collection effort strategies involving a third-party collector and an e-tailer based on the "internet + recycling" business model. Nielsen et al. [37] examined the effects of government subsidy policies in a CLSC and suggested that government organizations must inspect carefully the product types, power structures, and investment efficiency before implementing any subsidy policies. Saha et al. [38] also dealt with a CLSC under the influence of government incentives. They found that the greening level and used product return rate in a CLSC are always higher under retailer-led Stackelberg game.

More recently, many studies of corporate social responsibility (CSR) in a CLSC have been carried out due to growing consumer interest in environmental protection. Modak et al. [39] investigated a socially responsible supply chain with duopolistic retailers, using Cournot and Collusion games to demonstrate that a manufacturer's CSR has a significant effect on wholesale prices because intensive CSR practice can result in negative wholesale prices. Panda et al. [40] developed a socially responsible CLSC with recycling. They insisted that the channel's nonprofit maximizing motive through corporate social responsibility practices generated a higher profit margin than the profit maximizing objective and that there must be a recycling limit for the optimal benefit of the channel. Modak et al. [41] examined the influence of a manufacturer's social responsibility on the collection activity of a third party in a CLSC and showed that product recycling is directly affected by the manufacturer's corporate social responsibility concerns and that there must be a recycling threshold for the optimal benefit. Modak and Kelle [42] suggested social work donation (SWD) as a tool of CSR in a CLSC considering carbon taxes and demand uncertainty. They revealed that SWD is beneficial when used as an investment in CSR activity if the demand has a higher SWD elasticity parameter than the price sensitivity parameter. Dual-channel CLSC coordination under SWD was also analyzed by Modak et al. [43], who asserted that if a channel recycles used products and has socially concerned consumers, then consumers have the power to accelerate SWD and recycling simultaneously. An excellent survey on reverse logistics and CLSC management studies can be found in Kazemi et al. [44] and the references therein.

Although comprehensive research has been conducted on recycling and pricing issues in various sustainable supply chains, few studies have investigated price competition between recyclers in the recycling market. In this work, we consider forward and reverse supply chains where recyclable waste, recycled materials, and finished goods flow. From our pricing decision models, we investigate the effects of price competition between recyclers on the profits of the members in the supply chain and on the total profit of the supply chain. This work also discusses how an imbalance in the market share between recyclers affects the profit of the entire supply chain.

3. Model Description and Assumptions

3.1. Notations. To model the investigated supply chain, the following notations are used throughout the paper:

Index

i: recyclers $(i = 1, 2)$

Decision variables

P_m: selling price of the finished product offered by the manufacturer
P_{ri}: selling price of the recycled material offered by the recycler i
P_c: selling price of the recyclable wastes offered by the collector

Parameters

C_p: unit production cost of the finished product to the manufacturer
C_{ri}: unit recycling cost of the recyclable wastes to the recycler i
C_c: unit collection cost of the end-of-life product to the collector
γ: quantity of the recycled materials required to produce one unit of the finished product $(\gamma > 1)$
θ: recyclability degree of the wastes $(0 < \theta < 1)$
α: potential market demand for the finished product
β_m: consumer's price sensitivity for the finished product
β_r: manufacturer's price sensitivity for the recycled material
ω: cross-price sensitivity for the recycled material $(\omega < \beta_r)$
δ_i: market share of the recycler i $(0 < \delta_i < 1$ and $\sum_i \delta_i = 1)$

Functions

D: demand faced by the manufacturer
D_{mi}: quantity ordered by the manufacturer to the recycler i
D_{ri}: quantity ordered by the recycler i to the collector
Π_m: manufacturer's profit
Π_c: collector's profit
Π_{ri}: recycler i's profit

3.2. Assumptions. In this study, pricing and ordering decisions are investigated on the waste recycling process of a three-echelon CLSC. The proposed CLSC consists of one monopolistic manufacturer, one monopolistic collector, and two duopolistic recyclers. Figure 1 depicts the overall configuration and the material and cash flows of the investigated supply chain.

The manufacturer produces finished products which are made mainly from recycled wastes. In other words, the manufacturer uses the recycled (raw) materials to produce the product. This assumption is reasonable because, in practice, 100% recycled products are now being sold in the market. For example, Apple's MacBook Air is made with 100% recycled aluminum. Seventh Generation released paper towels made from 100% recycled paper. In addition, 100% recycled products are found in many remanufacturing industries such as footwear (Allbirds), plastic bottle (Rothy's), watches (Wewood), and clothing and accessories (Cotopaxi, Recover Brands, and Looptworks) [45]. It was reported that Looptworks significantly reduces the amount of garbage in the region and minimizes carbon emissions by working with those who gather manufacturing materials from landfills.

The manufacturer purchases the recycled materials only from the recyclers. Unlike Jafari et al. [20] and Giri and Dey [34], we assume that the manufacturer is not in charge of recycling the wastes. Hence, the manufacturer can obtain the recycled materials directly from the recyclers and then produce the finished product. Under the considered supply chain, the market demand for the finished product is determined based on the final price charged by the manufacturer to consumers in the market. We assume that the market demand, D, for the finished product is a linear function of the selling price, P_m, set by the manufacturer. We take $D = \alpha - \beta_m P_m$, where α and β_m indicate the potential market demand and the price sensitivity of the finished product, respectively. We assume that $\alpha, \beta_m > 0$ and $D > 0$.

The recycler's main activities are to purchase the recyclable waste from the collector and provide the manufacturer with the recycled materials. We assume that there are two competing recyclers operating their own recycling facilities. The two recyclers sell identical recycled materials to the manufacturer. We equivalently index the two recyclers by $i = 1, 2$. Each recycler is assumed to employ a uniform pricing strategy to attract the manufacturer. Thus, the demand of the recycled materials is price-dependent and is assumed to be a decreasing linear function of the price. Let D_{mi} denote the demand function of recycler i. Then, D_{mi} has the form of

$$D_{mi} = \delta_i \gamma D - \beta_r P_{ri} + \omega P_{rj} > 0, \quad i = 1, 2\ j = 3 - i, \qquad (1)$$

where, δ_i, γ, and P_{ri} indicate recycler i's market share of recycling, the quantity of recycled materials required to produce one unit of the finished product, and the price

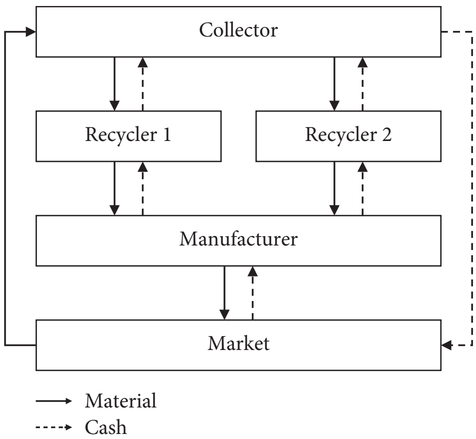


Figure 1: Material and cash flow diagram in the closed-loop supply chain with duopolistic recyclers.

offered by recycler i, respectively. In equation (1), β_r represents the manufacturer's price sensitivity with regard to the recycled materials. The term ω is the cross-price sensitivity, which reflects the degree of cannibalization between the two competing recyclers. In other words, ω represents the leakage of the demand from one recycler to the other recycler. Therefore, the term ω is a competition parameter between the two recyclers. As ω increases, the price competition between the two recyclers in the supply chain becomes more intense. Throughout the paper, it is assumed that $\beta_r > \omega$. The linear type of the demand function considering the price competition is assumed in most studies [20, 34, 39, 46, 47].

The collector is responsible for collecting waste in the form of end-of-life products from consumers. The collected waste can be classified as either recyclable or nonrecyclable, and the collector transfers the recyclable waste to the recyclers. In this study, the recyclability degree of the waste is also considered; i.e., it is assumed that only a constant share of the waste remains after the recycling process operated by the recyclers. Let D_{ri} denote the ordering quantity of recycler i for the recyclable waste to the collector. Then, D_{ri} is simply given by $D_{ri} = (1/\theta)D_{mi} > 0$ for $i = 1, 2$, where the term θ indicates the recyclability degree of the waste. Therefore, the function of the collector's total demand becomes $\sum_i D_{ri} > 0$.

Based on the demand functions of the participants in the CLSC considered here, the profit function of each participant is obtained by the following equation:

$$\begin{aligned}\Pi_m &= \left(P_m - C_p\right)D - \sum_i P_{ri}D_{mi},\\ \Pi_{ri} &= P_{ri}D_{mi} - \left(P_c + C_{ri}\right)D_{ri}, \quad i = 1, 2,\\ \Pi_c &= \sum_i \left(P_c - C_c\right)D_{ri},\end{aligned} \tag{2}$$

where Π_m, Π_{ri}, and Π_c denote the profits of the manufacturer, the recycler i, and the collector, respectively. Note that it is useless to study pricing decisions with no positive profits of participants in practice; therefore, the following assumption must be made: $\Pi_m > 0$, $\Pi_{ri} > 0$, and $\Pi_c > 0$.

3.3. Decision-Making Structure. This study utilizes game theory to model the problem of determining the equilibrium prices of the participants in the investigated CLSC. The players participating in the six pricing game models (which are described in the next section) are the manufacturer, the collector, recycler 1, and recycler 2. These four players are divided into two groups: the leaders group and the followers group. We assume that the manufacturer and the collector belong to the leaders group and that the two recyclers belong to the followers group. The basic structure of the pricing game model is as follows. The leaders group initially determines the prices devised by the collector and the manufacturer, after which the followers group determines those devised by the two recyclers. This assumption is reasonable because, in our setting, there exists price competition between the two recyclers, and they are under pressure from the collector and manufacturer with regard to the demand for recyclable waste and the supply of recycled materials, respectively. Therefore, the decision power of the leaders group is greater than that of the followers group.

In the leaders group, we deal with two pricing structure types: Stackelberg and Nash. In the Stackelberg pricing structure, the manufacturer acts as the Stackelberg game leader and the collector reacts as the follower [48]. In the Stackelberg pricing structure, the manufacturer initializes the selling price for the finished product and the collector then decides on the selling price for the recyclable waste based on the manufacturer's price. In the game theory literature, a Nash game is a simultaneous-move game in which the manufacturer and the collector make their decisions simultaneously [49]. In the followers group, we deal with three types of competition behavior: Collusion, Cournot, and Stackelberg. When engaging in Collusion behavior, the recyclers cooperatively decide on their selling prices, while their selling prices are set competitively when displaying Cournot behavior [50]. When using Stackelberg behavior, the leader of the two recyclers initially sets the price, after which the follower determines its own price based on the leader's price. Without a loss of generality, we assume that recycler 1 (recycler 2) is a leader (follower) throughout the paper. Therefore, with the two aforementioned pricing structures in the leaders group and the three types of competition behavior in the followers group, six combinations of different pricing game models can be investigated: (i) Nash-Collusion, (ii) Nash-Cournot, (iii) Nash-Stackelberg, (iv) Stackelberg-Collusion, (v) Stackelberg-Cournot, and (vi) Stackelberg-Stackelberg. Figure 2 illustrates the six different

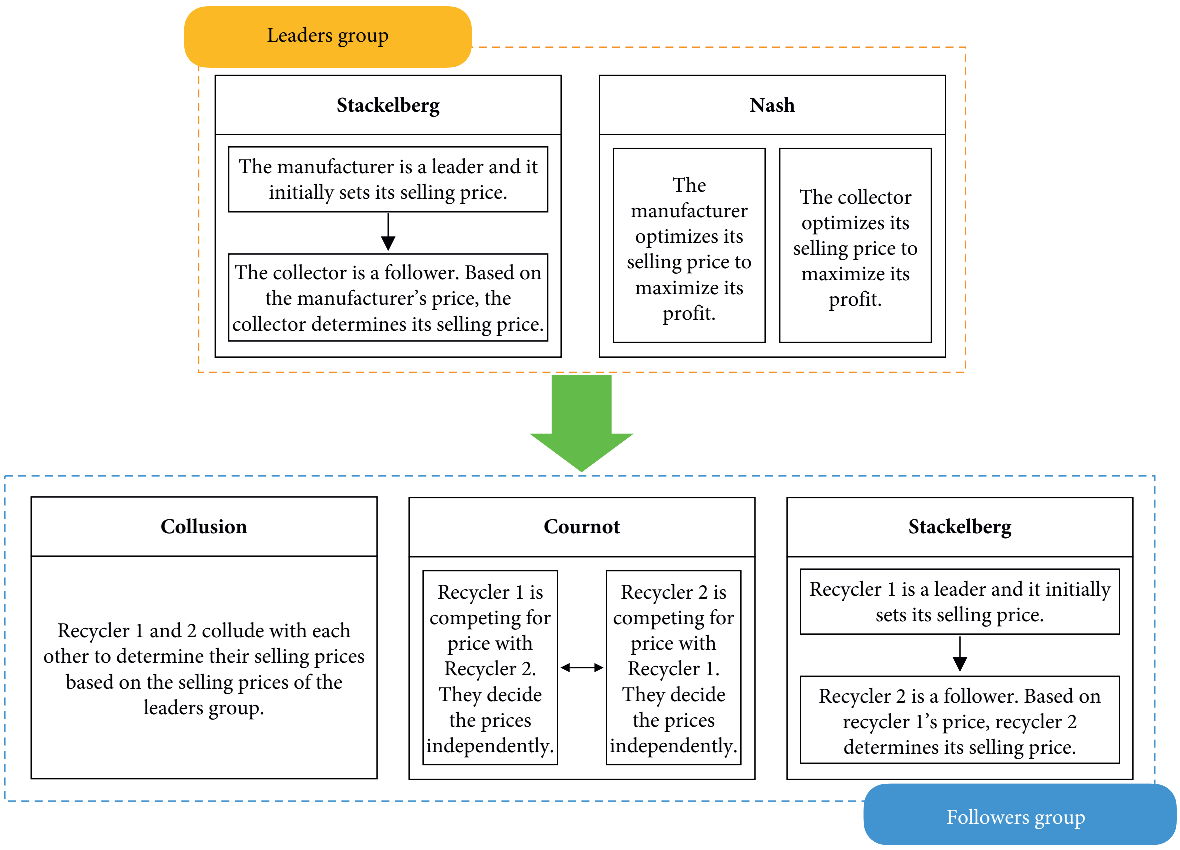


FIGURE 2: Conceptual framework of pricing game models in the dual-channel recycling supply chain.

pricing game models. In the next sections, we develop the mathematical programming and prove the uniqueness of the four players' prices for each pricing game model.

4. Preliminaries: Pricing Behaviors in the Followers Group

In this section, we discuss the three competition behaviors of the recyclers in the followers group and obtain the preliminary results for the next section.

4.1. Collusion Behavior in the Followers Group. In the Collusion behavior, the recyclers collude with each other to set their prices for recycled materials. More specifically, the Collusion behavior is similar to the case in which the two recyclers recognize their interdependence and agree to act in union in order to maximize the total profit of the recycling market. Note that the two recyclers cooperate in pricing but still compete in selling. The total profit of the recyclers, Π_{rt}, in the Collusion behavior can be formulated as follows: $\Pi_{rt} = \Pi_{r1} + \Pi_{r2}$. Hence, the recyclers' pricing problem, $\mathrm{RP}^{\mathrm{CL}}$ in the Collusion behavior can be formulated as follows:

$$\begin{aligned} \mathrm{RP}^{\mathrm{CL}}: \quad & \max_{(P_{r1},P_{r2})\in\mathbb{R}_+^2} \quad \Pi_{rt}(P_{r1},P_{r2}) = \sum_i (P_{ri}D_{mi}) - (P_c + C_{ri})D_{ri} \\ & \text{s.t.} \quad \Pi_{rt}(P_{r1},P_{r2}) > 0. \end{aligned} \tag{3}$$

For the recyclers' equilibrium prices, we have the following proposition.

Proposition 1. *Given the values of the manufacturer's price, P_m and the collector's price P_c, in the Collusion behavior, there exists a unique equilibrium under the recycler i's price, P_{ri}^{CL}:*

$$P_{ri}^{\mathrm{CL}} = \frac{P_c + C_{ri}}{2\theta} + \frac{K(\beta_r\delta_i + \omega\delta_j)}{2\theta(\beta_r^2 - \omega^2)}, \quad \text{for } i = 1,2,\ j = 3 - i, \tag{4}$$

where $K = \gamma\theta(\alpha - \beta_m P_m)$.

Proof. We consider the following Hessian matrix of the objective function in RP^{CL}:

$$\mathbf{H}_r^{\mathrm{CL}} = \begin{bmatrix} \dfrac{\partial^2 \Pi_{rt}}{\partial P_{r1}^2} & \dfrac{\partial^2 \Pi_{rt}}{\partial P_{r1}\partial P_{r2}} \\ \dfrac{\partial^2 \Pi_{rt}}{\partial P_{r2}\partial P_{r1}} & \dfrac{\partial^2 \Pi_{rt}}{\partial P_{r2}^2} \end{bmatrix} = \begin{bmatrix} -2\beta_r & \omega \\ \omega & -2\beta_r \end{bmatrix}. \tag{5}$$

We define Δ_k^{CL} as the leading principal minor of order k in $\mathbf{H}_r^{CL}$. We then find that $\Delta_1^{CL} = -2\beta_r < 0$ and $\Delta_2^{\mathrm{SCL}} = 4\beta_r^2 - \omega^2 > 0$ because we assume that $\beta_r > \omega$. Therefore, $\mathbf{H}_r^{\mathrm{CL}}$ is negatively definite, implying that Π_{rt} is strictly concave in the feasible region and that the stationary point of Π_{rt} becomes the global maximizer of $\mathrm{RP}^{\mathrm{CL}}$. Consequently, given the competitor's price, each recycler can find its own pricing strategy by setting $\partial\Pi_{rt}/\partial P_{ri} = 0$ for $i = 1, 2$:

$$P_{r1} = \frac{\beta_r (P_c + C_{r1}) - \omega (P_c + C_{r2}) + 2\theta\omega P_{r2} + K\delta_1}{2\beta_r \theta},$$
$$P_{r2} = \frac{\beta_r (P_c + C_{r2}) - \omega (P_c + C_{r1}) + 2\theta\omega P_{r1} + K\delta_2}{2\beta_r \theta}. \quad (6)$$

Solving the equations system in equation (6) leads to equation (4). This completes the proof. □

4.2. Cournot Behavior in the Followers Group. The Cournot behavior forces the recyclers to decide on their prices simultaneously. In other words, each recycler independently sets its price by assuming its competitor's selling price as a parameter. Hence, recycler *i*'s pricing problem, $\mathrm{RP}_i^{\mathrm{CT}}$, considering price competition is modeled as follows:

$$\mathrm{RP}_i^{\mathrm{CT}}: \quad \max_{P_{ri}\in\mathbb{R}_+} \quad \Pi_{ri}(P_{ri}) = P_{ri}D_{mi} - (P_c + C_{ri})D_{ri} \quad \text{s.t.} \quad \Pi_{ri}(P_{ri}) > 0. \quad (7)$$

For the recyclers' equilibrium prices, we have the following proposition.

Proposition 2. *Given the values of P_m and P_c, in the Cournot behavior, there exists a unique equilibrium under the recycler i's price, P_{ri}^{CT}:*

$$P_{ri}^{\mathrm{CT}} = \frac{K(2\beta_r\delta_i + \omega\delta_j) + \beta_r\left[2\beta_r (P_c + C_{ri}) + \omega(P_c + C_{rj})\right]}{\theta(4\beta_r^2 - \omega^2)}, \quad \text{for } i = 1, 2, \ j = 3 - i. \quad (8)$$

Proof. The second-order derivative of the objective function in $\mathrm{RP}_i^{\mathrm{CT}}$ is given by $\partial^2\Pi_{ri}/\partial P_{ri}^2 = -2\beta_r < 0$, for $i = 1, 2$. Hence, each recycler's profit function is strictly concave on its own decision and there exists a unique equilibrium price for each recycler. Consequently, given the competitor's price, each recycler can find its own pricing strategy by setting $\partial\Pi_{ri}/\partial P_{ri} = 0$ for $i = 1, 2$:

$$P_{r1} = \frac{\beta_r (P_c + C_{r1}) + \theta\omega P_{r2} + K\delta_1}{2\theta\beta_r},$$
$$P_{r2} = \frac{\beta_r (P_c + C_{r2}) + \theta\omega P_{r1} + K\delta_2}{2\theta\beta_r}. \quad (9)$$

Solving the equations system in equation (9) leads to equation (8). This completes the proof. □

4.3. Stackelberg Behavior in the Followers Group. In the Stackelberg behavior, also known as a sequential game, the leader of the game initially sets the price and the follower then determines its own price based on the leader's price. As noted in Section 3, we assume that recycler 1 acts as the Stackelberg leader while recycler 2 acts as the Stackelberg follower. With this assumption, recycler 1's decision power and market share are greater than those of recycler 2. Accordingly, it is natural to assume that $\delta_1 \geq \delta_2$. Then, recycler *i*'s pricing model, $\mathrm{RP}_i^{\mathrm{ST}}$, in the Stackelberg behavior is modeled as follows:

$$\mathrm{RP}_2^{\mathrm{ST}}: \quad \max_{P_{r2}\in\mathbb{R}_+} \quad \Pi_{r2}(P_{r2}) = P_{r2}D_{m2} - (P_c + C_{r2})D_{r2} \quad \text{s.t.} \quad \Pi_{r2}(P_{r2}) > 0,$$
$$\mathrm{RP}_1^{\mathrm{ST}}: \quad \max_{P_{r1}\in\mathbb{R}_+} \quad \Pi_{r1}(P_{r1}) = P_{r1}D_{m1} - (P_c + C_{r1})D_{r1} \quad \text{s.t.} \quad P_{r2} \in \arg\max \Pi_{r2}(P_{r2}), \ \Pi_{r1}(P_{r1}) > 0. \quad (10)$$

For the sequential game above, we have the following proposition.

Proposition 3. *Given the values of P_m and P_c, in the Stackelberg behavior, there exists a unique equilibrium under recycler i's price, P_{ri}^{ST}:*

$$P_{r1}^{ST} = \frac{P_c + C_{r1}}{2\theta} + \frac{\beta_r\omega (P_c + C_{r2}) + K(2\beta_r\delta_1 + \omega\delta_2)}{2\theta(2\beta_r^2 - \omega^2)},$$
$$P_{r2}^{ST} = \frac{\omega P_{r1}^{ST}}{2\beta_r} + \frac{P_c + C_{r2}}{2\theta} + \frac{K\delta_2}{2\theta\beta_r}. \quad (11)$$

Proof. The second-order derivative of the objective function in $\mathrm{RP}_2^{\mathrm{ST}}$ is given by $\partial^2\Pi_{r2}/\partial P_{r2}^2 = -2\beta_r < 0$. Therefore, Π_{r2} is strictly concave with respect to (w.r.t.) P_{r2} and, by solving $\partial\Pi_{r2}/\partial P_{r2} = 0$, the global maximizer of Π_{r2} is obtained as

$$P_{r2}^{\mathrm{ST}} = \frac{\omega P_{r1}}{2\beta_r} + \frac{P_c + C_{r2}}{2\theta} + \frac{K\delta_2}{2\theta\beta_r}. \quad (12)$$

By integrating P_{r2}^{ST} in equation (12) into $\mathrm{RP}_1^{\mathrm{ST}}$, it follows that $\partial^2\Pi_{r1}/\partial P_{r1}^2 = -(2\beta_r^2 - \omega^2)/\beta_r < 0$. Hence, Π_{r1} is also strictly concave w.r.t, P_{r1} and, by solving $\partial\Pi_{r1}/\partial P_{r1} = 0$, the equilibrium solution of RP_1^{ST} is given by

$$P_{r1}^{\mathrm{ST}} = \frac{P_c + C_{r1}}{2\theta} + \frac{\beta_r\omega (P_c + C_{r2}) + K(2\beta_r\delta_1 + \omega\delta_2)}{2\theta(2\beta_r^2 - \omega^2)}. \quad (13)$$

This completes the proof. □

5. Development of the Six Pricing Game Models

In this section, we present the main results of this paper. Six different pricing game models are carefully analyzed one by one.

5.1. Nash Game Structure in the Leaders Group. The Nash game structure is a simultaneous-move game in which the manufacturer and the collector make their decisions simultaneously.

5.1.1. Nash-Collusion Model. In the Nash-Collusion game model, which is denoted by NCL, a Nash game is played between the collector and the manufacturer, while the recyclers collude with each other to set the prices for recycled materials. In the first stage of the NCL model, the manufacturer and the collector announce their sales prices simultaneously. Based on this, in the second stage, the recyclers cooperate in terms of pricing with each other and set their prices to maximize the sum of their profits. According to Proposition 1, we know that the total profit of the recyclers, Π_{rt} is strictly concave with respect to the recyclers' prices, P_{r1} and P_{r2}, and the equilibrium price of recycler i in the NCL model is expressed as

$$P_{ri}^{\mathrm{NCL}} = \frac{P_c + C_{ri}}{2\theta} + \frac{K\left(\beta_r\delta_i + \omega\delta_j\right)}{2\theta\left(\beta_r^2 - \omega^2\right)}, \quad \text{for } i = 1, 2,\ j = 3 - i. \tag{14}$$

With recyclers' prices in equation (14), the collector's pricing problem, CP^{NCL}, and the manufacturer's pricing problem, MP^{NCL}, are formulated as follows:

$$\begin{aligned}
CP^{NCL}:\quad & \max_{P_c\in\mathbb{R}_+} \quad \Pi_c\left(P_c\right) = \sum_i \left(P_c - C_c\right) D_{ri} \\
& \text{s.t.} \quad \text{eq. (14)} \\
& \qquad \Pi_c\left(P_c\right) > 0, \\
MP^{NCL}:\quad & \max_{P_m\in\mathbb{R}_+} \quad \Pi_m\left(P_m\right) = \left(P_m - C_p\right) D - \sum_i P_{ri} D_{mi} \\
& \text{s.t.} \quad \text{eq. (14)} \\
& \qquad \Pi_m\left(P_m\right) > 0.
\end{aligned} \tag{15}$$

Proposition 4 states the result of the NCL pricing game model.

Proposition 4. *In the Nash-Collusion pricing game model, there exists a unique Nash equilibrium under the collector's price,* P_c^{NCL}, *and the manufacturer's price,* P_m^{NCL}:

$$\begin{aligned}
P_c^{\mathrm{NCL}} &= \frac{2C_c - C_{r1} - C_{r2}}{4} + \frac{\gamma\theta\left(\alpha - \beta_m C_p\right)\left(\beta_r + \omega\right)}{8\left(\beta_r^2 - \omega^2\right) + 2\gamma^2\beta_m\left(2\omega\delta_1\delta_2 + \beta_r\sum_i\delta_i^2\right)}, \\
P_m^{\mathrm{NCL}} &= \frac{\alpha}{\beta_m} - \frac{2\left(\alpha - \beta_m C_p\right)\left(\beta_r^2 - \omega^2\right)}{4\beta_m\left(\beta_r^2 - \omega^2\right) + \gamma^2\beta_m^2\left(2\omega\delta_1\delta_2 + \beta_r\sum_i\delta_i^2\right)}.
\end{aligned} \tag{16}$$

Proof. From the objective functions in CP^{NCL} and MP^{NCL}, their second-order derivatives are given by, respectively,

$$\begin{aligned}
\frac{\partial^2\Pi_c}{\partial P_c^2} &= -\frac{2\left(\beta_r - \omega\right)}{\theta^2} < 0, \\
\frac{\partial^2\Pi_m}{\partial P_m^2} &= -\beta_m\left[2 + \frac{\gamma^2\beta_m\left(2\omega\delta_1\delta_2 + \beta_r\sum_i\delta_i^2\right)}{2\left(\beta_r^2 - \omega^2\right)}\right] < 0.
\end{aligned} \tag{17}$$

Therefore, the collector's and the manufacturer's profit functions are both strictly concave on their own decision variables and there exist unique equilibrium prices for the collector and the manufacturer. Consequently, the collector and the manufacturer can find their own pricing strategies by solving the first-order conditions of $\partial\Pi_c/\partial P_c = 0$ and $\partial\Pi_m/\partial P_m = 0$:

$$\begin{aligned}
P_c &= \frac{2C_c - C_{r1} - C_{r2}}{4} + \frac{K}{4\left(\beta_r - \omega\right)}, \\
P_m &= \beta_m^{-1}\left[\alpha - \frac{2\left(\alpha - \beta_m C_p\right)\left(\beta_r^2 - \omega^2\right)}{4\left(\beta_r^2 - \omega^2\right) + \gamma^2\beta_m\left(2\omega\delta_1\delta_2 + \beta_r\sum_i\delta_i^2\right)}\right].
\end{aligned} \tag{18}$$

Solving equation (18) simultaneously leads to equation (16). This completes the proof. □

By the backward induction, the equilibrium prices of the recyclers in the NCL model are determined as follows:

$$P_{ri}^{\mathrm{NCL}} = \frac{P_c^{\mathrm{NCL}} + C_{ri}}{2\theta} + \frac{\gamma\theta\left(\alpha - \beta_m P_m^{\mathrm{NCL}}\right)\left(\beta_r\delta_i + \omega\delta_j\right)}{2\theta\left(\beta_r^2 - \omega^2\right)}, \quad \text{for } i = 1, 2,\ j = 3 - i. \tag{19}$$

5.1.2. Nash-Cournot Model. In the Nash-Cournot game model, which is denoted by NCT, a Nash game is played between the collector and the manufacturer, with the Cournot behavior displayed by the two recyclers. In the first stage of the NCT model, the manufacturer and the collector announce their sales prices simultaneously. Based on this, in the second stage, the equilibrium prices of the two recyclers are simultaneously and independently obtained. According to Proposition 2, the equilibrium price of recycler i in the NCT model is expressed as

$$P_{ri}^{\mathrm{NCT}} = \frac{K\left(2\beta_r\delta_i + \omega\delta_j\right) + \beta_r\left[2\beta_r\left(P_c + C_{ri}\right) + \omega\left(P_c + C_{rj}\right)\right]}{\theta\left(4\beta_r^2 - \omega^2\right)}, \quad \text{for } i = 1, 2,\ j = 3 - i. \tag{20}$$

With the recyclers' equilibrium prices in equation (20), the collector's and the manufacturer's pricing games are formulated as follows, respectively,

$$\begin{aligned}
CP^{\mathrm{NCT}}:\quad & \max_{P_c\in\mathbb{R}_+} \quad \Pi_c\left(P_c\right) = \sum_i \left(P_c - C_c\right) D_{ri} \\
& \text{s.t.} \quad \text{eq. (20)} \\
& \qquad \Pi_c\left(P_c\right) > 0, \\
MP^{\mathrm{NCT}}:\quad & \max_{P_m\in\mathbb{R}_+} \quad \Pi_m\left(P_m\right) = \left(P_m - C_p\right) D - \sum_i P_{ri} D_{mi} \\
& \text{s.t.} \quad \text{eq. (20)} \\
& \qquad \Pi_m\left(P_m\right) > 0.
\end{aligned} \tag{21}$$

The following proposition pertains to the results of the NCT model.

Proposition 5. *In the Nash-Cournot pricing game model, there exists a unique Nash equilibrium under the collector's price,* P_c^{NCT}, *and the manufacturer's price,* P_m^{NCT}.

Proof. From the objective functions in CP^{NCT} and MP^{NCT}, their second-order derivatives are given, respectively, by

$$\frac{\partial^2 \Pi_c}{\partial P_c^2} = -\frac{4\beta_r(\beta_r - \omega)}{\theta^2(2\beta_r - \omega)} < 0, \qquad \frac{\partial^2 \Pi_m}{\partial P_m^2} = -2\beta_m(1 + X_1) < 0, \tag{22}$$

where

$$X_1 = \frac{\gamma^2\beta_m\beta_r\left[8\omega\beta_r\delta_1\delta_2 + \left(4\beta_r^2 - \omega^2\right)\sum_i \delta_i^2\right]}{\left(4\beta_r^2 - \omega^2\right)^2} > 0. \tag{23}$$

Note that the collector's and the manufacturer's profit functions are both strictly concave on their own decision variables. Thus, there exist unique Nash equilibrium prices for the collector and the manufacturer. Consequently, by solving the first-order conditions, $\partial\Pi_c/\partial P_c = 0$ and $\partial\Pi_m/\partial P_m = 0$, the collector's equilibrium price, P_c^{NCT}, and the manufacturer's equilibrium price, P_m^{NCT}, are obtained. This completes the proof. □

The explicit expressions of P_c^{NCT} and P_m^{NCT} are long and complicated. Instead, we present a brief version of the solving procedure of the NCT model in Appendix A.

5.1.3. Nash-Stackelberg Model. In the Nash-Stackelberg game model, which is denoted by NST, a Nash game is played between the collector and the manufacturer, while a Stackelberg game is played between the two recyclers. In the first stage of the NST model, the manufacturer and the collector announce their sales prices simultaneously. Based on this, in the second stage, recycler 1 determines its sales price. In the third stage, following recycler 1's price, recycler 2 makes a further pricing decision. By considering the Stackelberg behavior of the recyclers, their prices are already known as

$$P_{r1}^{\mathrm{NST}} = \frac{P_c + C_{r1}}{2\theta} + \frac{\beta_r\omega(P_c + C_{r2}) + K(2\beta_r\delta_1 + \omega\delta_2)}{2\theta\left(2\beta_r^2 - \omega^2\right)}, \qquad P_{r2}^{\mathrm{NST}} = \frac{\omega P_{r1}^{\mathrm{NST}}}{2\beta_r} + \frac{P_c + C_{r2}}{2\theta} + \frac{K\delta_2}{2\theta\beta_r}. \tag{24}$$

Therefore, with (23), the collector's pricing problem, CP^{NST}, and the manufacturer's pricing problem, MP^{NST}, are formulated as follows, respectively,

$$\begin{aligned} CP^{\mathrm{NST}}: \quad & \max_{P_c \in \mathbb{R}_+} \quad \Pi_c(P_c) = \sum_i (P_c - C_c) D_{ri} \\ & \text{s.t.} \quad \text{eq. (24)} \\ & \qquad\ \ \Pi_c(P_c) > 0, \\ MP^{\mathrm{NST}}: \quad & \max_{P_m \in \mathbb{R}_+} \quad \Pi_m(P_m) = \left(P_m - C_p\right) D - \sum_i P_{ri} D_{mi} \\ & \text{s.t.} \quad \text{eq. (24)} \\ & \qquad\ \ \Pi_m(P_m) > 0. \end{aligned} \tag{25}$$

Proposition 6 states the result of the NST pricing game model.

Proposition 6. *In the Nash-Stackelberg pricing game model, there exists a unique Nash equilibrium under the collector's price, P_c^{NST}, and the manufacturer's price, P_m^{NST}.*

Proof. The second-order derivative of Π_c w.r.t. P_c is given by

$$\frac{\partial^2 \Pi_c}{\partial P_c^2} = -\frac{(\beta_r - \omega)\left[8\beta_r^3 - \omega^3 + \beta_r\omega(4\beta_r - 3\omega)\right]}{2\theta^2\beta_r\left(2\beta_r^2 - \omega^2\right)}. \tag{26}$$

From the fact that $\beta_r > \omega \Longrightarrow 8\beta_r^3 > \omega^3$ *and* $4\beta_r > 3\omega$, it is easy to show that $\partial^2\Pi_c/\partial P_c^2 < 0$. Therefore, Π_c is strictly concave w.r.t. P_c. The second-order derivative of Π_m w.r.t. P_m is also given by $\partial^2\Pi_m/\partial P_m^2 = -2\beta_m(1 + X_2)$, where

$$X_2 = \frac{\gamma^2\beta_m\left\{4\beta_r^2\left[\delta_1^2\left(2\beta_r^2 - \omega^2\right) + \delta_2^2\left(4\beta_r^2 - \omega^2\right)\right] + 4\beta_r\omega\delta_1\delta_2\left(8\beta_r^2 - \omega^2\right) + 8\beta_r^4\delta_1^2 - \omega^4\delta_2^2\right\}}{16\beta_r\left(2\beta_r^2 - \omega^2\right)^2}. \tag{27}$$

Upon the assumption that $\delta_1 \geq \delta_2$, it follows that $8\beta_r^4\delta_1^2 - \omega^4\delta_2^2 \geq \delta_2^2(8\beta_r^4 - \omega^4) > 0$. Therefore, we have the following relationship: $\delta_1 \geq \delta_2 \Longrightarrow X_2 > 0 \Longrightarrow \partial^2\Pi_m/\partial P_m^2 < 0$. Thus, Π_m is also strictly concave w.r.t. P_m and there exist unique Nash equilibrium prices for the collector and the manufacturer. Consequently, by solving the first-order conditions, $\partial\Pi_c/\partial P_c = 0$ and $\partial\Pi_m/\partial P_m = 0$, the collector's equilibrium price, P_c^{NST}, and the manufacturer's equilibrium price, P_m^{NST}, are obtained. This completes the proof. □

The explicit expressions of P_c^{NST} and P_m^{NST} are long and complicated. Instead, we present a brief version of the solving procedure of the NST model in Appendix A.

5.2. Stackelberg Game Structure in the Leaders Group. In the Stackelberg game structure, the manufacturer acts as a Stackelberg game leader and the collector reacts as the follower.

5.2.1. Stackelberg-Collusion Model. In the Stackelberg-Collusion game model, which is denoted by SCL, a Stackelberg game is played between the collector and the manufacturer, while the recyclers collude with each other to set the prices of recycled materials. In the first stage of the SCL model, the manufacturer announces the selling price for the finished product. Based on this, in the second stage of the game, the collector determines its selling price for the recyclable waste. In the last stage, following the collector's price, two recyclers make a decision on their prices cooperatively. According to Proposition 1, the recycler i's equilibrium price, P_{ri}^{SCL}, in the SCL model is given by

$$P_{ri}^{\mathrm{SCL}} = \frac{P_c + C_{ri}}{2\theta} + \frac{K\left(\beta_r\delta_i + \omega\delta_j\right)}{2\theta\left(\beta_r^2 - \omega^2\right)}, \quad \text{for } i = 1, 2,\ j = 3 - i. \tag{28}$$

By substituting equation (28) into the collector's profit function, the collector's pricing problem, CP^{SCL}, regarding the SCL model can be formulated as follows:

$$\begin{aligned} CP^{\mathrm{SCL}}: \quad & \max_{P_c \in \mathbb{R}_+} \quad \Pi_c\left(P_c\right) = \sum_i \left(P_c - C_c\right) D_{ri} \\ & \text{s.t.} \quad \text{eq. (28)} \\ & \qquad \Pi_c\left(P_c\right) > 0. \end{aligned} \tag{29}$$

Proposition 7. *Given the value of P_m, there exists a unique equilibrium solution, P_c^{SCL}, which maximizes the collector's profit:*

$$P_c^{\mathrm{SCL}} = \frac{2C_c - C_{r1} - C_{r2}}{4} + \frac{K}{4\left(\beta_r - \omega\right)}. \tag{30}$$

Proof. The second-order derivative of Π_c w.r.t. P_c can be obtained as $\partial^2\Pi_c/P_c^2 = -2\left(\beta_r - \omega\right)/\theta^2 < 0$. Therefore, the objective function Π_c in CP^{SCL} is strictly concave and there exists a unique global maximizer of CP^{SCL}. By setting $\partial\Pi_c/\partial P_c = 0$, we obtain the equilibrium solution P_c^{SCL} in equation (30). This completes the proof.

Accordingly, by substituting equations (28) and (30) into the manufacturer's profit function, the manufacturer's pricing problem, MP^{SCL}, regarding the SCL model can be formulated as follows:

$$\begin{aligned} MP^{\mathrm{SCL}}: \quad & \max_{P_m \in \mathbb{R}_+} \quad \Pi_m\left(P_m\right) = \left(P_m - C_p\right) D - \sum_i P_{ri} D_{mi} \\ & \text{s.t.} \quad \text{eqs. (28) and (30)} \\ & \qquad \Pi_m\left(P_m\right) > 0. \end{aligned} \tag{31}$$

□

Proposition 8. *In the Stackelberg-Collusion pricing game model, there exists a unique Stackelberg equilibrium under the manufacturer's price, P_m^{SCL}:*

$$P_m^{\mathrm{SCL}} = \frac{\left(\beta_r^2 - \omega^2\right)\left[\gamma\beta_m\left(2C_c + C_{r1} + C_{r2}\right) - 16\theta\left(\alpha + \beta_m C_p\right)\right] + \alpha\gamma^2\theta\beta_m\left[16\delta_1\delta_2\left(\beta_r - \omega\right) - 7\beta_r + \omega\right]}{\theta\beta_m\left\{\gamma^2\beta_m\left[16\delta_1\delta_2\left(\beta_r - \omega\right) - 7\beta_r + \omega\right] - 32\left(\beta_r^2 - \omega^2\right)\right\}}. \tag{32}$$

Proof. The second-order derivative of Π_m w.r.t. P_m can be obtained as

$$\frac{\partial^2\Pi_m}{\partial P_m^2} = \beta_m\left\{\frac{\gamma^2\beta_m\left[16\delta_1\delta_2\left(\beta_r - \omega\right) - 7\beta_r + \omega\right]}{16\left(\beta_r^2 - \omega^2\right)} - 2\right\}. \tag{33}$$

If the condition $\partial^2\Pi_m/\partial P_m^2 < 0$ is satisfied, the objective function in MP^{SCL} is strictly concave and there exists a unique global maximizer of MP^{SCL}. Because $0 \le \delta_1, \delta_2 \le 1$, and $\delta_1 + \delta_2 = 1$, the maximum value of $\delta_1\delta_2$ must be 0.25, where $\delta_1 = \delta_2 = 0.5$. The upper bound of $\partial^2\Pi_m/\partial P_m^2$ is obtained as follows:

$$\begin{aligned} \frac{\partial^2\Pi_m}{\partial P_m^2} &= \beta_m\left\{\frac{\gamma^2\beta_m\left[16\delta_1\delta_2\left(\beta_r - \omega\right) - 7\beta_r + \omega\right]}{16\left(\beta_r^2 - \omega^2\right)} - 2\right\} \\ &\le \beta_m\left\{\frac{\gamma^2\beta_m\left[4\left(\beta_r - \omega\right) - 7\beta_r + \omega\right]}{16\left(\beta_r^2 - \omega^2\right)} - 2\right\} \\ &= -\beta_m\left[\frac{3\gamma^2\beta_m}{16\left(\beta_r - \omega\right)} + 2\right]. \end{aligned} \tag{34}$$

Therefore, it is obvious that $\partial^2\Pi_m/\partial P_m^2 < 0$. From the first-order condition, $\partial\Pi_m/\partial P_m = 0$, we can obtain the manufacturer's equilibrium solution P_m^{SCL} in equation (32). This completes the proof.

Hence, considering the backward induction, the equilibrium prices of the collector and the recyclers in the SCL model are determined as follows:

$$P_c^{SCL} = \frac{2C_c - C_{r1} - C_{r2}}{4} + \frac{\gamma\theta(\alpha - \beta_m P_m^{SCL})}{4(\beta_r - \omega)},$$

$$P_{r1}^{SCL} = \frac{\gamma\theta(\alpha - \beta_m P_m^{SCL})(\beta_r\delta_1 + \omega\delta_2) + (\beta_r^2 - \omega^2)(P_c^{SCL} + C_{r1})}{2\theta(\beta_r^2 - \omega^2)},$$

$$P_{r2}^{SCL} = \frac{\gamma\theta(\alpha - \beta_m P_m^{SCL})(\beta_r\delta_2 + \omega\delta_1) + (\beta_r^2 - \omega^2)(P_c^{SCL} + C_{r2})}{2\theta(\beta_r^2 - \omega^2)}. \tag{35}$$

□

5.2.2. Stackelberg-Cournot Model. In the Stackelberg-Cournot game model, which is denoted by SCT, a Stackelberg game is played between the collector and the manufacturer, with the Cournot behavior following the two recyclers. In the first stage of the SCT model, the manufacturer announces the selling price for the finished product. Based on this, in the second stage game, the collector determines its selling price for the recyclable waste. In the last stage, following the collector's price, each recycler reaches a price decision independently. According to Proposition 2, recycler i's equilibrium price, P_{ri}^{SCT}, in the SCT model is given by

$$P_{ri}^{SCT} = \frac{K(2\beta_r\delta_i + \omega\delta_j) + \beta_r[2\beta_r(P_c + C_{ri}) + \omega(P_c + C_{rj})]}{\theta(4\beta_r^2 - \omega^2)}, \quad \text{for } i = 1, 2, j = 3 - i. \tag{36}$$

Then, by replacing equation (36) into the collector's profit function, the collector's pricing problem, CP^{SCT}, regarding the SCT model can be formulated as follows:

$$CP^{SCT}: \quad \max_{P_c \in \mathbb{R}_+} \quad \Pi_c(P_c) = \sum_i (P_c - C_c) D_{ri} \quad \text{s.t.} \quad \text{eq. (36)} \quad \Pi_c(P_c) > 0. \tag{37}$$

Proposition 9. *Given the value of P_m, there exists a unique equilibrium solution, P_c^{SCT}, which maximizes the collector's profit:*

$$P_c^{SCT} = \frac{2C_c - C_{r1} - C_{r2}}{4} + \frac{K}{4(\beta_r - \omega)}. \tag{38}$$

Proof. The second-order derivative of Π_c w.r.t. P_c can be obtained as $\partial^2\Pi_c/\partial P_c^2 = -4\beta_r(\beta_r - \omega)/\theta^2(2\beta_r - \omega) < 0$. Therefore, the objective function Π_c in CP^{SCT} is strictly concave and there exists a unique global maximizer of CP^{SCT}. By setting $\partial\Pi_c/\partial P_c = 0$, we obtain the equilibrium solution P_c^{SCT} in equation (38). This completes the proof.

Ultimately, by integrating equations (36) and (38) into the manufacturer's profit function, the manufacturer's pricing problem, MP^{SCT}, in the SCT model can be formulated as follows:

$$MP^{SCT}: \quad \max_{P_m \in \mathbb{R}_+} \quad \Pi_m(P_m) = (P_m - C_p)D - \sum_i P_{ri} D_{mi} \quad \text{s.t.} \quad \text{eqs. (36) and (38)} \quad \Pi_m(P_m) > 0. \tag{39}$$

□

Proposition 10. *In the Stackelberg-Cournot pricing game model, there exists a unique Stackelberg equilibrium under the manufacturer's price, P_m^{SCT}. By solving the first-order condition, P_m^{SCT} is obtained.*

Proof. The second-order derivative of Π_m w.r.t. P_m can be obtained as

$$\frac{\partial^2\Pi_m}{\partial P_m^2} = \beta_m\left[\frac{4\gamma^2\delta_1\delta_2\beta_m\beta_r}{(2\beta_r + \omega)^2} + \frac{\gamma^2\beta_m\beta_r X_3}{4(\beta_r - \omega)(4\beta_r^2 - \omega^2)^2} - 2\right]. \tag{40}$$

where $X_3 = 6\omega^3 - 15\omega^2\beta_r + 28\omega\beta_r^2 - 28\beta_r^3$. If the condition $\partial^2\Pi_m/\partial P_m^2 < 0$ is satisfied, the objective function in MP^{SCT} is strictly concave and there exists a unique global maximizer of MP^{SCT}. Like the proof of Proposition 8, the upper bound of $\partial^2\Pi_m/\partial P_m^2$ is obtained as follows:

$$\begin{aligned}\frac{\partial^2\Pi_m}{\partial P_m^2} &= \beta_m\left[\frac{4\gamma^2\delta_1\delta_2\beta_m\beta_r}{(2\beta_r + \omega)^2} + \frac{\gamma^2\beta_m\beta_r X_3}{4(\beta_r - \omega)(4\beta_r^2 - \omega^2)^2} - 2\right] \\ &\le \beta_m\left[\frac{\gamma^2\beta_m\beta_r}{(2\beta_r + \omega)^2} + \frac{\gamma^2\beta_m\beta_r X_3}{4(\beta_r - \omega)(4\beta_r^2 - \omega^2)^2} - 2\right] \\ &= -\beta_m\left[\frac{\gamma^2\beta_m\beta_r(3\beta_r - 2\omega)}{4(\beta_r - \omega)(2\beta_r - \omega)^2} + 2\right].\end{aligned} \tag{41}$$

Here, it is true that $3\beta_r > 2\omega$ because we assume that $\beta_r > \omega$. This implies that

$$3\beta_r > 2\omega \Longrightarrow \frac{\gamma^2\beta_m\beta_r(3\beta_r - 2\omega)}{4(\beta_r - \omega)(2\beta_r - \omega)^2} > 0 \Longrightarrow \frac{\partial^2\Pi_m}{\partial P_m^2} < 0. \tag{42}$$

Hence, from the first-order condition, $\partial\Pi_m/\partial P_m = 0$, we can obtain the manufacturer's equilibrium solution P_m^{SCT}. This completes the proof. □

The explicit expression of P_m^{SCT} is long and complicated. Instead, we present a brief version of the solving procedure in Appendix B.

5.2.3. Stackelberg-Stackelberg Model. In the Stackelberg-Stackelberg game model, which is denoted by SST, only one

Stackelberg game is played throughout the investigated supply chain. This Stackelberg game consists of four stages. In the first stage, the manufacturer announces the selling price of the finished product. Based on this, in the second stage of the game, the collector determines its selling price for the recyclable waste. In the third stage, following the collector's selling price, recycler 1 sets the selling price for the recycled materials. Finally, in the last stage, recycler 2 makes a decision on its selling price based on the information on other players' prices. By considering the Stackelberg behavior of the recyclers, their prices are already known as

$$P_{r1}^{\mathrm{SST}} = \frac{P_c + C_{r1}}{2\theta} + \frac{\beta_r \omega \left(P_c + C_{r2}\right) + K\left(2\beta_r \delta_1 + \omega \delta_2\right)}{2\theta\left(2\beta_r^2 - \omega^2\right)},$$
$$P_{r2}^{\mathrm{SST}} = \frac{\omega P_{r1}^{\mathrm{SST}}}{2\beta_r} + \frac{P_c + C_{r2}}{2\theta} + \frac{K\delta_2}{2\theta\beta_r}. \tag{43}$$

Then, by substituting equation (43) into the collector's profit function, the collector's pricing problem, CP^{SST} regarding the SST model can be formulated as follows:

$$\begin{aligned} CP^{\mathrm{SST}}:\quad & \max_{P_c \in \mathbb{R}_+} \quad \Pi_c\left(P_c\right) = \sum_i \left(P_c - C_c\right) D_{ri} \\ & \text{s.t.} \quad \text{eq. (43)} \\ & \qquad\ \Pi_c\left(P_c\right) > 0. \end{aligned} \tag{44}$$

Proposition 11. *Given the value of P_m, there exists a unique equilibrium solution, P_c^{SST}, which maximizes the collector's profit:*

$$P_c^{\mathrm{SST}} = \frac{1}{4}\left\{2C_c - C_{r1} - C_{r2} + \frac{K}{\beta_r - \omega} + \frac{\omega^2\left[K\left(\delta_2 - \delta_1\right) + \left(\beta_r + \omega\right)\left(C_{r1} - C_{r2}\right)\right]}{8\beta_r^3 - \omega^3 + \beta_r \omega\left(4\beta_r - 3\omega\right)}\right\}. \tag{45}$$

Proof. The second-order derivative of Π_c w.r.t. P_c can be obtained as

$$\frac{\partial^2 \Pi_c}{\partial P_c^2} = -\frac{\left(\beta_r - \omega\right)\left[8\beta_r^3 - \omega^3 + \beta_r \omega\left(4\beta_r - 3\omega\right)\right]}{2\theta^2 \beta_r\left(2\beta_r^2 - \omega^2\right)} < 0. \tag{46}$$

Therefore, the objective function Π_c in CP^{SCT} is strictly concave and there exists a unique global maximizer of CP^{SCS}. By setting $\partial\Pi_c/\partial P_c = 0$, we obtain the equilibrium solution P_c^{SCT} in equation (45). This completes the proof.

Ultimately, by integrating equations (43) and (45) into the manufacturer's profit function, the manufacturer's pricing problem, MP^{SST}, in the SST model can be formulated as follows:

$$\begin{aligned} MP^{\mathrm{SST}}:\quad & \max_{P_m \in \mathbb{R}_+} \quad \Pi_m\left(P_m\right) = \left(P_m - C_p\right) D - \sum_i P_{ri} D_{mi} \\ & \text{s.t.} \quad \text{eqs. (43) and (45)} \\ & \qquad\ \Pi_m\left(P_m\right) > 0. \end{aligned} \tag{47}$$

For the uniqueness of the manufacturer's equilibrium price, we introduce the following conjecture: □

Conjecture 1. *The objective function, Π_m, in MP^{SST} is strictly concave w.r.t. the manufacturer's price, P_m.*

Because the second-order derivative of the manufacturer's profit function has a highly complicated form in terms of input parameters, it is difficult to prove the concavity of the manufacturer's profit function. Instead, we present a simple numerical example to show that Conjecture 1 is reasonable. We set the parameters as follows: $\alpha = 500$, $\beta_m = 4$, $\beta_r = 10$, $\omega = 5$, $\delta_1 = 0.7$, $\gamma = 8$, $\theta = 0.5$, $C_p = 1$, $C_{r1} = C_{r2} = 0.5$, and $C_c = 0.2$. In Figure 3, we can observe that Π_m is strictly concave w.r.t. P_m, and the maximum is obtained at $P_m = 115.946$.

Proposition 12. *Assuming that Conjecture 1 is true, in the Stackelberg-Stackelberg pricing game model, there exists a unique Stackelberg equilibrium under the manufacturer's price, P_m^{SST}. By solving the first-order condition, P_m^{SST} is obtained.*

Proof. If Conjecture 1 is true, there exists a unique global maximizer of MP^{SST}. By setting $\partial\Pi_m/\partial P_m = 0$, we obtain the equilibrium solution P_m^{SST}. This completes the proof. □

The explicit expression of P_m^{SST} is long and complicated. Instead, we present a brief version of the solving procedure in Appendix B.

6. Numerical Examples

This section numerically investigates the effects of parameters on the optimal equilibrium quantities. The main dataset used for the analysis is as follows: $\alpha = 500$, $\beta_m = 4$, $\beta_r = 10$, $\omega = 5$, $\delta_1 = 0.6$, $\gamma = 8$, $\theta = 0.5$, $C_p = 1$, $C_{r1} = C_{r2} = 0.5$, and $C_c = 0.2$. For this dataset, we obtain the equilibrium price and profit for each player in the developed pricing game models in the next sections.

6.1. Effect of ω on Equilibrium Quantities. Parameter ω in the recyclers' demand functions indicates the competition intensity between the two recyclers. We are interested in an investigation of the effects of the competition intensity on equilibrium prices and profits. To do this, we consider the

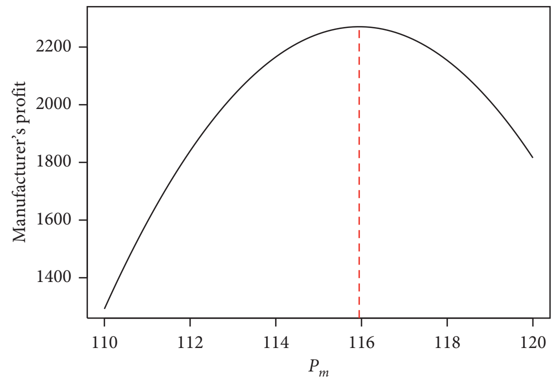


FIGURE 3: Concavity of the manufacturer's profit in the SST model.

main dataset and vary ω from 0 to 8. The equilibrium quantities of the decision variables and the obtained profits for different values of ω are plotted in Figure 4. As the value of ω increases, we observe the following:

(i) The manufacturer's and the collector's prices increase in each of the six pricing game models.

(ii) In the NCL and the SCL models, recycler 1's price increases, while in the remaining four models, recycler 1's price decreases.

(iii) In the NCL and the SCL models, recycler 2's price increases. In the remaining four models, recycler 2's price initially increases to a certain level, i.e., it has a maximum, and then decreases.

(iv) The profit of all members except recycler 2 decreases in each of the six pricing game models.

(v) Recycler 2's profit initially increases to a certain level, i.e., it has a maximum, and then decreases in each of the six pricing game models. This is an interesting phenomenon by which a recycler with smaller market share will benefit from limited competition.

From the facts observed above, we suggest the following managerial insight:

Insight 1. As the competition between the two recyclers intensifies, the profit of all members except recycler 2 decreases. This leads to lower total profit of the supply chain. In other words, competition has a negative effect on the profits of not only the supply chain but also its members. Note that the competition intensity ω is the key parameter for raising the prices of supply chain members when the recyclers collude with each other.

6.2. Effect of β_r on Equilibrium Quantities. Parameter β_r in the recyclers' demand functions indicates the manufacturer's price sensitivity with regard to the recycled materials. To investigate the effects of the manufacturer's price sensitivity on equilibrium prices and profits, we consider the main dataset and vary β_r from 5.5 to 10. Figure 5 shows the trends of the equilibrium quantities of the decision variables and the obtained profits for different values of β_r. As the value of β_r increases, we observe the following:

(i) The manufacturer's and the collector's prices decrease in each of the six pricing game models.

(ii) In the NCL and SCL models, recycler 1's and recycler 2's prices decrease, while in the remaining four models, their prices increase.

(iii) The profits of all members in the supply chain increase in each of the six pricing game models. Therefore, the total profit of the supply chain also increases.

From the facts observed above, we suggest the following managerial insight:

Insight 2. We can infer that the demand for the finished product is an increasing function of β_r. This is because that, as the value of β_r increases, the manufacturer's price decreases while the manufacturer's profit increases. In other words, increasing β_r boosts the demands of all the members in the supply chain. These boosted demands then cause the profits of all members in the supply chain to increase. Consequently, the total profit of the supply chain also increases. Note that the manufacturer's price sensitivity β_r is the key parameter for lowering the prices of supply chain members when the recyclers collude with each other.

6.3. Effect of δ_1 on Equilibrium Quantities. Parameter δ_i in the recyclers' demand functions indicates the recycler *i*'s market share. In this section, we conduct a sensitivity analysis of the market share on the equilibrium prices and profits. Varying δ_1 from 0.5 to 0.7, Figure 6 records the trends of the equilibrium quantities of the decision variables and the obtained profits for different values of δ_1. As the value of δ_1 increases (i.e., recycler 1's market share grows), we observe the following:

(i) The manufacturer's and recycler 1's prices increase in each of the six pricing game models.

(ii) The collector's and recycler 2's prices decrease in each of the six pricing game models.

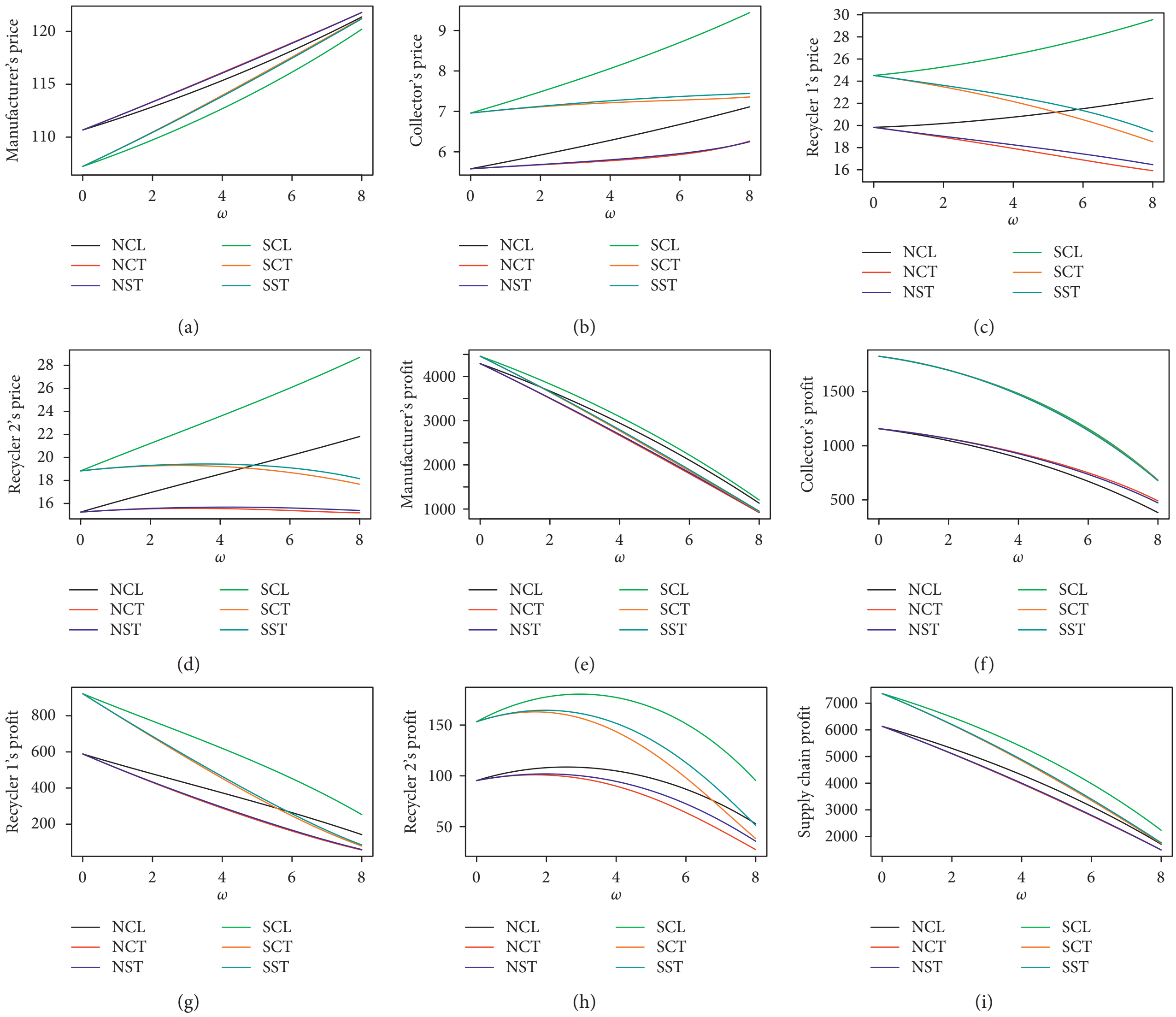


Figure 4: Effect of ω on equilibrium prices and profits.

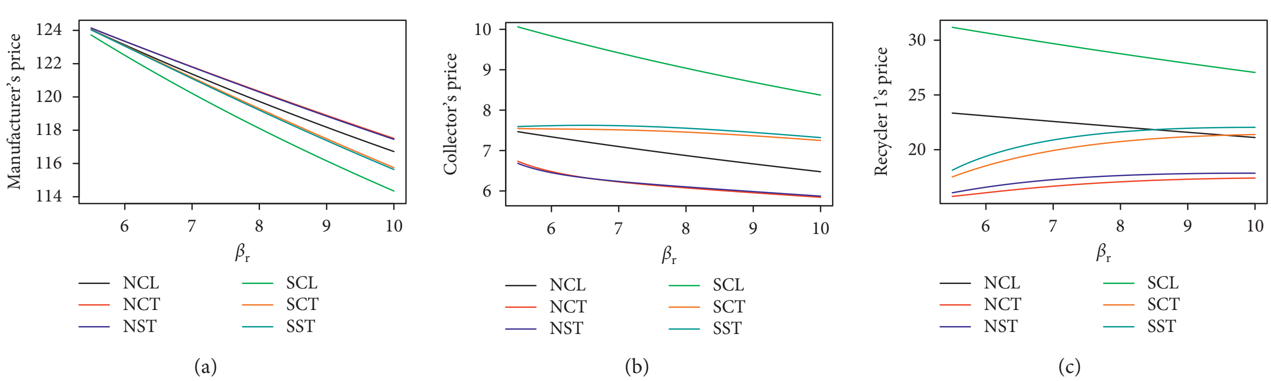


Figure 5: Continued.

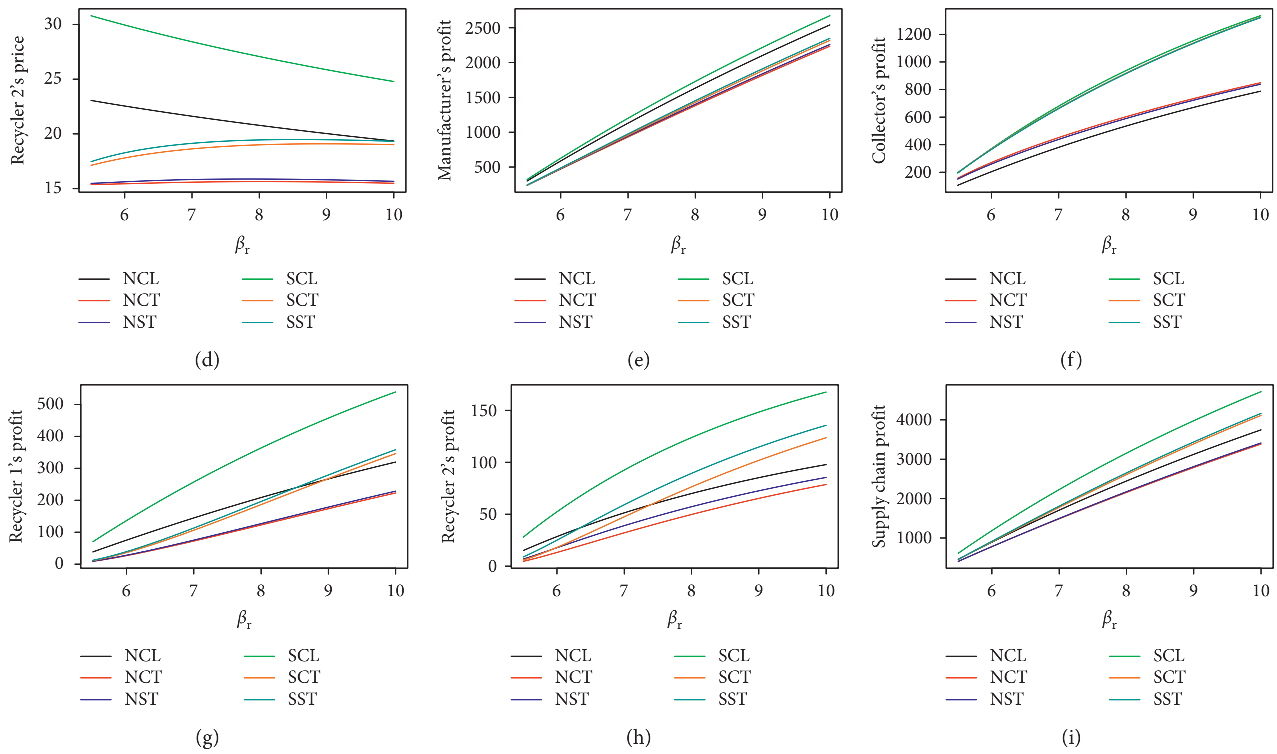
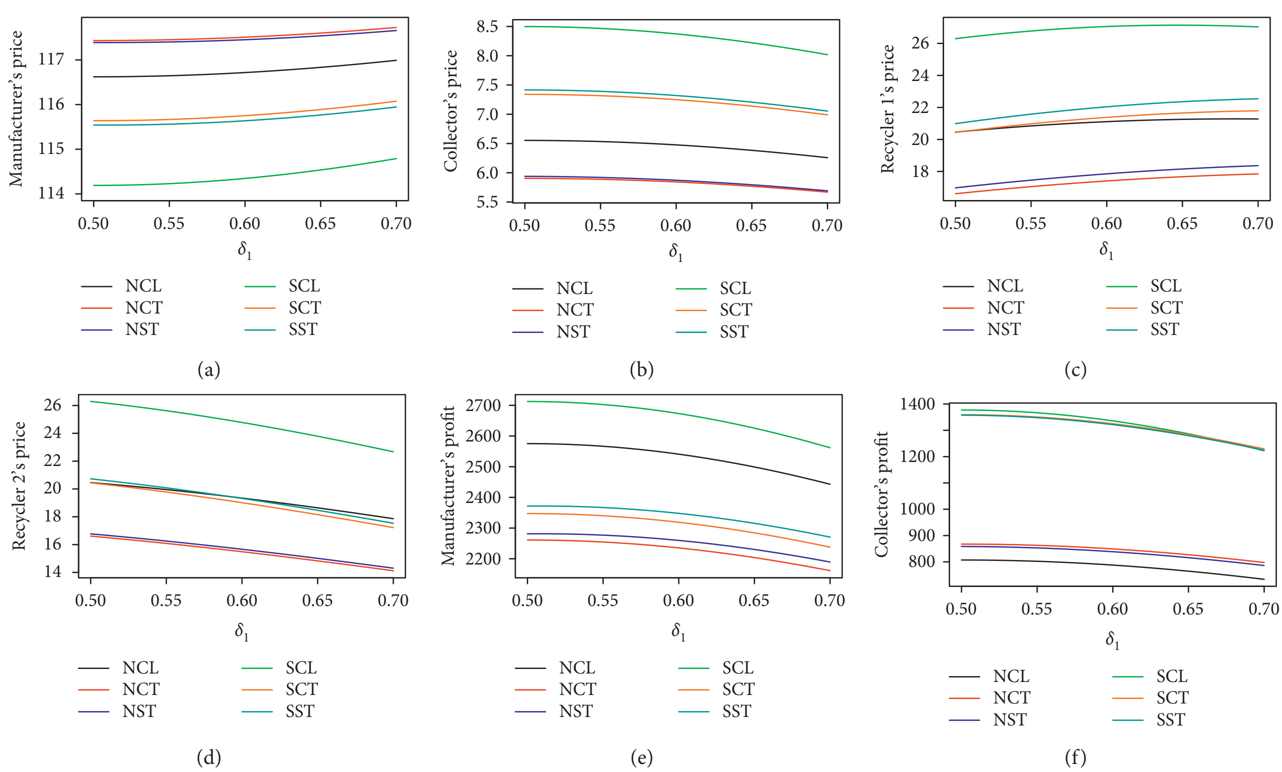


Figure 5: Effect of β_r on equilibrium prices and profits.



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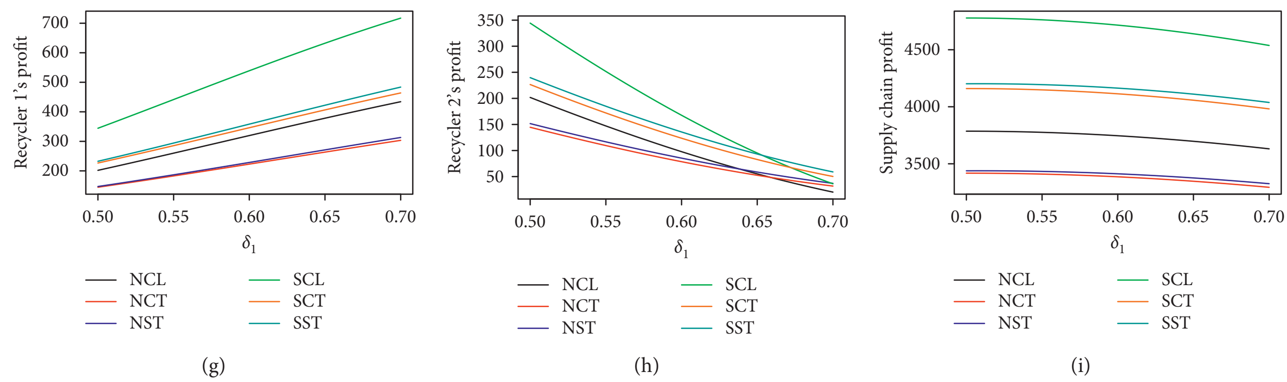

Figure 6: Effect of δ_1 on equilibrium prices and profits.

(iii) The profit decreases for all members except for recycler 1 in each of the six pricing game models. Only recycler 1's profit increases.

From the facts observed above, we suggest the following managerial insight:

Insight 3. An increase in δ_1 means that the imbalance in the market share is intensifying in the recycling market. As δ_1 increases, only the recycler 1's profit increases, whereas the profits of the other members decrease. This causes the total profit of the supply chain to decrease. Summarizing the above, the imbalance in the market share has a negative effect on the profit of the supply chain.

6.4. Effect of θ on Equilibrium Quantities. Parameter θ in the collector's demand indicates the recyclability of the waste that can be recovered and turned into raw materials. This section investigates the effects of the recyclability degree on the equilibrium prices and profits. Varying θ from 0.1 to 0.9, Figure 7 records the trends of the equilibrium quantities of the decision variables and the obtained profits for different values of θ. As the value of θ increases, we observe the following.

In the Stackelberg game structure of the leaders group, the manufacturer's price increases. However, in the NCT and the NST models, the manufacturer's price decreases. Finally, in the NCL model, the manufacturer's price is not affected by the recyclability:

(i) The collector's price increases in each of the six pricing game models.

(ii) Recycler 1's and recycler 2's prices both decrease in each of the six pricing game models.

(iii) The profit increases for all members except for the manufacturer. Only the manufacturer's profit decreases.

From the facts observed above, we suggest the following managerial insight:

Insight 4. As θ increases, the profit increases for all members except for the manufacturer, causing the total profit of the supply chain to increase. The greater the recyclability degree of the waste is, the higher the profits for all the members involved in the recycling process become. In other words, recyclability has a positive impact on the total profit of the supply chain. Note that θ raises the price of the collector while it lowers the price of both retailers in all game models.

6.5. Effect of γ on Equilibrium Quantities. Parameter γ in the recyclers' demand functions indicates the quantity of the recycled materials required to produce one unit of the finished product. This section investigates the effects of the γ on the equilibrium prices and profits. Varying γ from 4 to 12, Figure 8 records the trends of the equilibrium quantities of the decision variables and the obtained profits for different values of γ. As the value of γ increases, we observe the following:

(i) The price decreases for all members except for the manufacturer in each of the six pricing game models. Only the manufacturer's price increases.

(ii) The profit decreases for all members in each of the six pricing game models.

From the facts observed above, we suggest the following managerial insight:

Insight 5. It is observed that a higher value of γ leads to lowering the profits of all members. As γ increases, the quantity of the recycled materials needed to meet the customer's demand increases. Moreover, the collector must collect more waste, leading to higher costs. As a result, the profit of the supply chain gradually decreases.

6.6. Effect of β_m on Equilibrium Quantities. Parameter β_m in the manufacturer's demand functions indicates the consumer's price sensitivity with regard to the finished product. We are interested in an investigation of the effects of β_m on equilibrium of prices and profits. To do this, we consider the main dataset and vary β_m from 1 to 7. Figure 9 records the trends of the equilibrium quantities of the decision variables

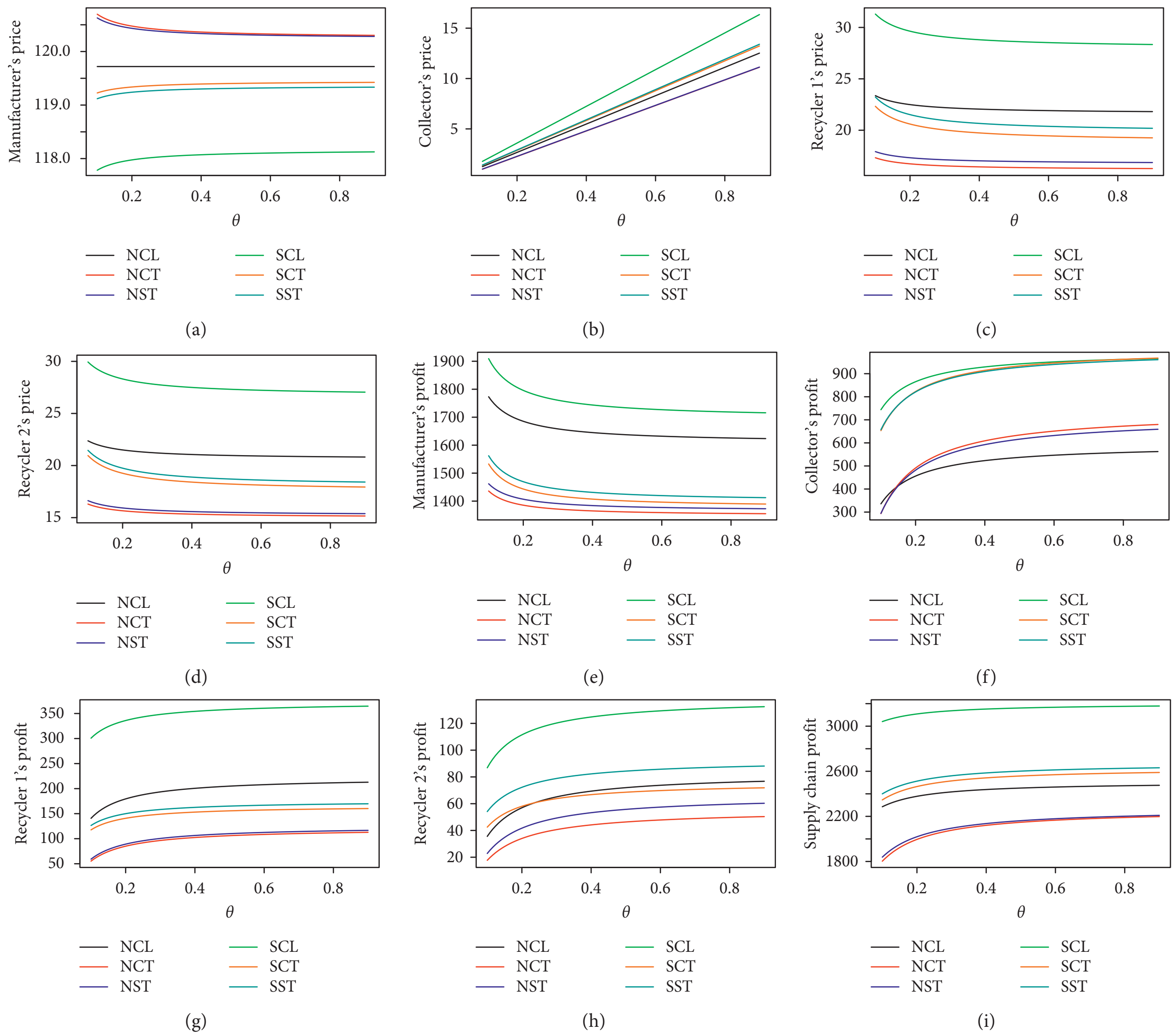

Figure 7: Effect of θ on equilibrium prices and profits.

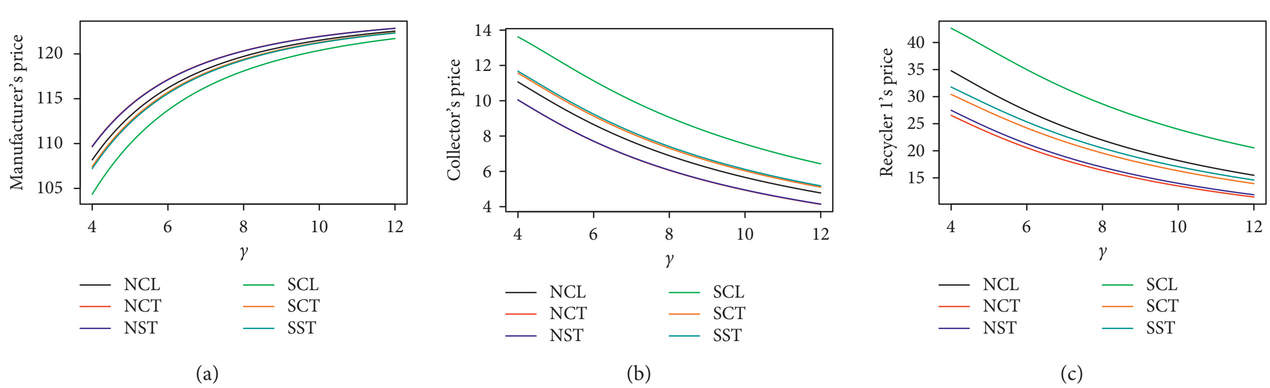

Figure 8: Continued.

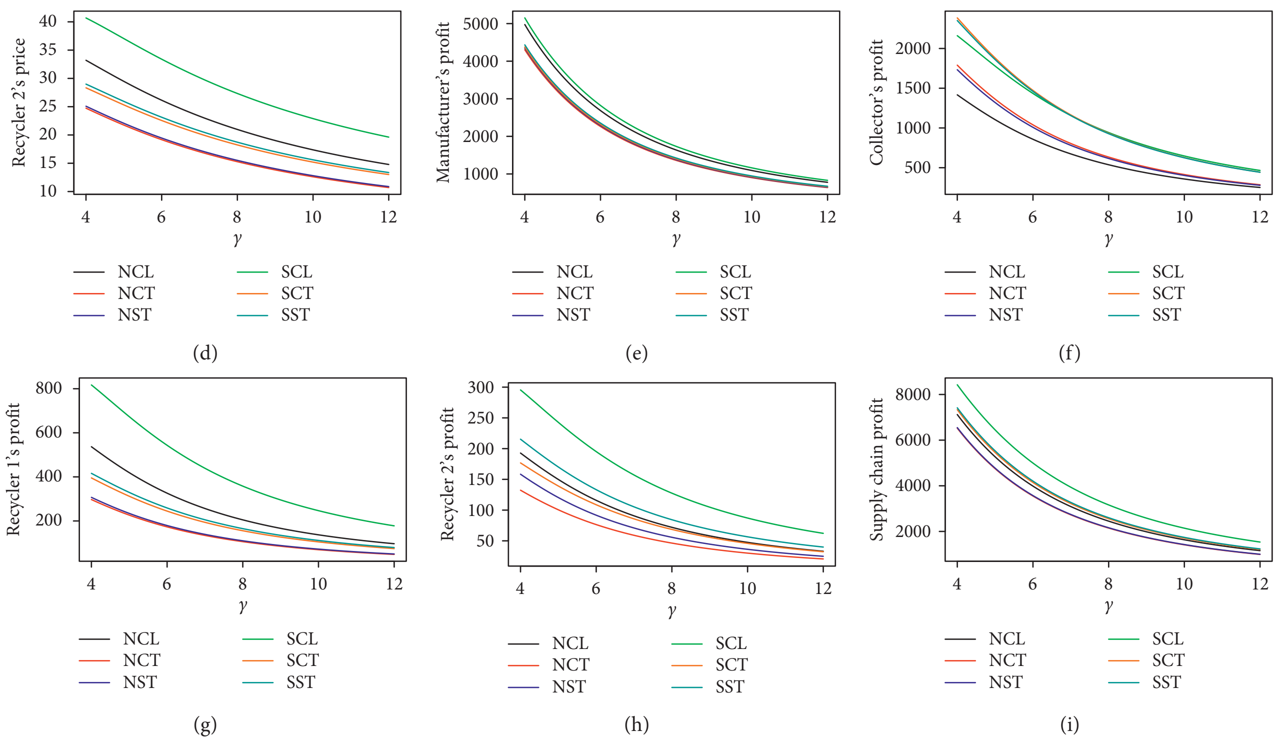

Figure 8: Effect of γ on equilibrium prices and profits.

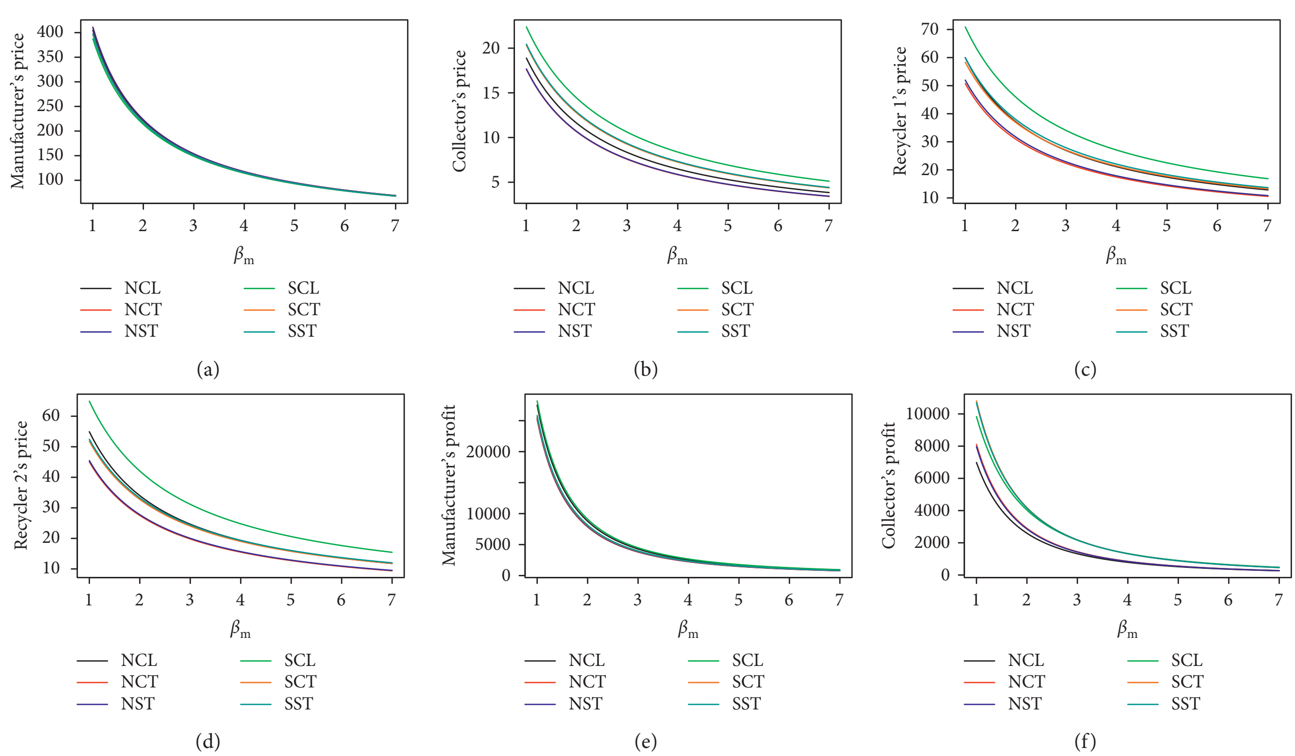

Figure 9: Continued.

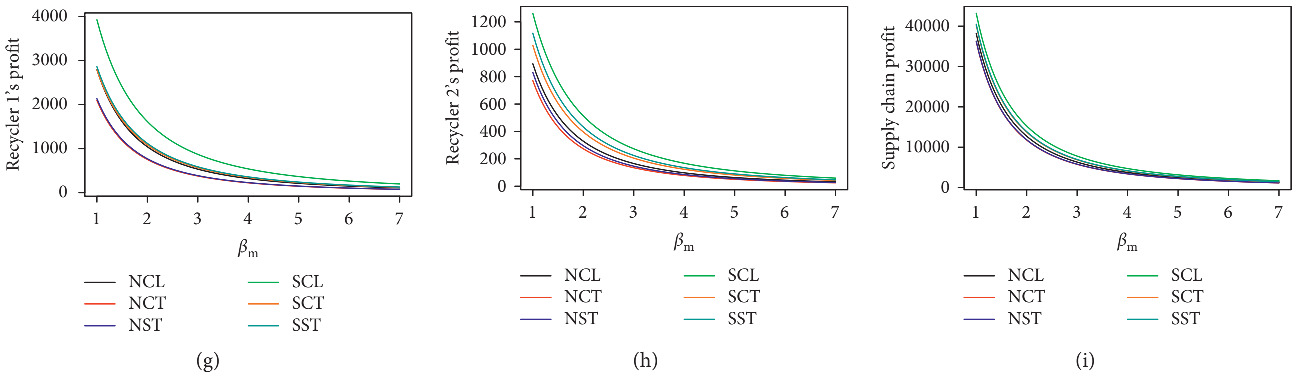

Figure 9: Effect of β_m on equilibrium prices and profits.

and the obtained profits for different values of β_m. As the value of β_m increases, we observe the following:

(i) Prices decrease for all members in each of the six pricing game models

(ii) Profits decrease for all members in each of the six pricing game models

From the facts observed above, we suggest the following managerial insight:

Insight 6. It is obvious that as β_m increases, the demand for the finished product decreases resulting in lowering the demands for the recyclable waste and the recycled materials, respectively. As a result, the profit for each member decreases. Note that, in all game models, β_m lowers the prices of all supply chain members as well as their profits.

6.7. Comparison among Six Pricing Game Models. From Figures 4–9, we can find the following:

(i) In the case of the Nash game structure of the leaders group, the manufacturer's price is higher than that of the Stackelberg game structure: $P_m^{NCL} > P_m^{SCL}$, $P_m^{NCT} > P_m^{SCT}$, and $P_m^{NST} > P_m^{SST}$.

(ii) In the case of the Stackelberg game structure of the leaders group, the collector's price is higher than that of the Nash game structure: $P_c^{NCL} < P_c^{SCL}$, $P_c^{NCT} < P_c^{SCT}$, and $P_c^{NST} < P_c^{SST}$.

(iii) In the case of the Stackelberg game structure of the leaders group, recycler 1's price is higher than that of the Nash game structure: $P_{r1}^{NCL} < P_{r1}^{SCL}$, $P_{r1}^{NCT} < P_{r1}^{SCT}$, and $P_{r1}^{NST} < P_{r1}^{SST}$.

(iv) In the case of the Stackelberg game structure of the leaders group, recycler 2's price is higher than that of the Nash game structure: $P_{r2}^{NCL} < P_{r2}^{SCL}$, $P_{r2}^{NCT} < P_{r2}^{SCT}$, and $P_{r2}^{NST} < P_{r2}^{SST}$.

(v) In the case of the Stackelberg game structure of the leaders group, the manufacturer's profit is higher than that of the Nash game structure: $\Pi_m^{NCL} < \Pi_m^{SCL}$, $\Pi_m^{NCT} < \Pi_m^{SCT}$, and $\Pi_m^{NST} < \Pi_m^{SST}$. In terms of profitability, the SCL (NCT) pricing game model is the most advantageous (disadvantageous) for the manufacturer.

(vi) In the case of the Stackelberg game structure of the leaders group, the collector's profit is higher than that of the Nash game structure: $\Pi_c^{NCL} < \Pi_c^{SCL}$, $\Pi_c^{NCT} < \Pi_c^{SCT}$, and $\Pi_c^{NST} < \Pi_c^{SST}$. In terms of profitability, the SCL (NCL) pricing game model is the most advantageous (disadvantageous) for the collector.

(vii) In the case of the Stackelberg game structure of the leaders group, the recycler 1's profit is higher than that of the Nash game structure: $\Pi_{r1}^{NCL} < \Pi_{r1}^{SCL}$, $\Pi_{r1}^{NCT} < \Pi_{r1}^{SCT}$, and $\Pi_{r1}^{NST} < \Pi_{r1}^{SST}$. In terms of profitability, the SCL (NCT) pricing game model is the most advantageous (disadvantageous) for recycler 1.

(viii) In the case of the Stackelberg game structure of the leaders group, the recycler 1's profit is higher than that of the Nash game structure: $\Pi_{r2}^{NCL} < \Pi_{r2}^{SCL}$, $\Pi_{r2}^{NCT} < \Pi_{r2}^{SCT}$, and $\Pi_{r2}^{NST} < \Pi_{r2}^{SST}$. In terms of profitability, the SCL (NCT) pricing game model is the most advantageous (disadvantageous) for recycler 2.

(ix) In terms of the total profit of the supply chain, the Stackelberg game structure of the leaders group outperforms the Nash game structure. In addition, the recyclers' Collusion (Cournot) behavior performs the best (the worst): $\Pi_{sc}^{NCT} < \Pi_{sc}^{NST} < \Pi_{sc}^{NCL} < \Pi_{sc}^{SCT} < \Pi_{sc}^{SST} < \Pi_{sc}^{SCL}$, where Π_{sc} is the total profit of the supply chain.

7. Conclusion

In this work, we discussed a topic related to environmental sustainability through an investigation of the collecting and recycling processes of recyclable waste in a three-echelon CLSC consisting of one manufacturer, one collector, and two recyclers. This study utilized game theory to model the problem of determining the equilibrium prices of participants in the investigated supply chain considered. We assumed that the manufacturer and the collector belong to the leaders group and that the two recyclers belong to the followers group. In the leaders group, we dealt with two pricing structure types: the Stackelberg and Nash types. In the followers group, price competition was assumed to exist between the two recyclers, and we dealt with three types of competition behavior for the recyclers: Collusion, Cournot, and Stackelberg. With the two pricing structure types in the leaders group and the three types of competition behavior in the followers group, six different pricing game models were developed. For each of the pricing game models, the uniqueness of equilibrium and optimal pricing was analytically proved. Finally, various numerical experiments were conducted to investigate the effects of the experimental parameters on the equilibrium prices and profits of the supply chain members. To the best of our knowledge, the current paper is the first to consider the concept of selling price competition between recyclers in a three-echelon CLSC.

In terms of the total profit of this supply chain, our main findings are as follows:

(i) As the price competition intensifies between the recyclers, the total profit of the supply chain decreases
(ii) The higher the manufacturer's price sensitivity for the recycled materials is, the higher the total profit of the supply chain becomes
(iii) As the imbalance in the market share intensifies in the recycling market, the total profit of the supply chain decreases
(iv) The more likely the waste is to be recycled, the higher the total profit of the supply chain
(v) As the quantity of the recycled materials required to produce a finished product increases, the total profit of the supply chain decreases
(vi) The Stackelberg game structure of the leaders group outperforms the Nash game structure
(vii) The recyclers' Collusion (Cournot) behavior performs the best (the worst)

Several future research studies related to this topic are possible. One can modify the model to consider different coordination contracts between the players to obtain higher profits. Moreover, different demand patterns, especially of the nonlinear demand of the consumer, can be assumed when studying similar types of problems. Another possibility is to consider the price competition between manufacturers and collectors and to analyze their outcomes.

Appendix

A. Solving Procedure for the Equilibrium Prices

For all. $g \in \{\mathrm{NCT}, \mathrm{NST}\}$

Step 1: set $P_{ri} = P_{ri}^g$, $i = 1, 2$

Step 2: solve the equations system $\partial \Pi_c / \partial P_c = 0$ and $\partial \Pi_m / \partial P_m = 0$ w.r.t. P_c and P_m, and set the roots to the collector's price, P_c^g, and the manufacturer's price, P_m^g, respectively

Step 3: calculate $P_{ri}^g = P_{ri}(P_m^g, P_c^g)$, $i = 1, 2$.

B. Solving Procedure for the Equilibrium Prices

For all. $g \in \{\mathrm{SCT}, \mathrm{SST}\}$

Step 1: set $P_c = P_c^g$ and $P_{ri} = P_{ri}^g$, $i = 1, 2$

Step 2: solve the equation $\partial \Pi_m / \partial P_m = 0$ w.r.t. P_m, and set the root to the manufacturer's price, P_m^g

Step 3: calculate $P_c^g = P_c(P_m^g)$ and $P_{ri}^g = P_{ri}(P_m^g, P_c^g)$, $i = 1, 2$.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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