Complexity

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*Research Article*

# Ecological Security Evaluation for Marine Ranching Based on the PLTS/ANP Method: A Case Study of Rongcheng

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The evaluation index system of ecological security of marine ranching (MRES) is based on the assumption that there is independence among evaluation indexes in the existing studies, which ignores the complex interactive paths of marine ranching as an artificial ecosystem. In this study, the MRES evaluation network model that includes interdependent relationships is established based on the Driver-Pressure-State-Impact-Response model and the analytic network process method. Then, the probabilistic linguistic term sets and analytic network process methods are used to calculate the weights of the evaluation indexes of MRES. The overall evaluation value and the contribution rate of clusters are consequently defined and analyzed to reflect the performance of MRES. Finally, a case study is carried out for evaluating the MRES of marine ranches in Rongcheng by means of the proposed method. The conclusions are summarized as follows: (1) The weights of clusters are ranked as Responses > Impact > Driver > State > Pressure, and "scientific management of fishery resources" is the most important index; (2) the MRES performance of marine ranches in the city of Rongcheng is at the medium security grade on the whole, and all 11 samples are driven by the response.

## 1. Introduction

Marine ranching is an ecological system based on the principle of ecology. It has the goals of environmental protection, resource conservation, and sustainable fishery output [1]. It follows natural productivity and uses modern engineering technology and management models to construct habitat restoration and artificial multiplication of marine resources. Research on marine ranching started later in China than elsewhere. The concept of marine ranching was first proposed by Zeng in 1979 [2]. After years of development, China's marine ranching has made great achievements in the number of ranches, construction scale, technical level, output, development mechanism, and other parameters [3]. Marine ranching is a major agricultural development strategy in China at present. It is a new fishery mode and marine economy that pursues the growth and protection of fishery resources along with the improvement of the marine ecological environment. It has ecological, economic, and social benefits [4]. In China, current research on marine ranching mainly focuses on technology, output, mechanism, and other aspects of the establishment and management of the ranching [1]. However, in terms of the ecosystem, although some studies have been conducted, such as the research on the structure and function of the food web for ecological safety, the research is still in the developing stage [5].

With the increasing ecological problems in the construction of marine ranching in China, it is of practical significance to study the ecological security of marine ranching (MRES) [6]. MRES refers to the overall balance of the resource structure and marine environment of the artificial ecosystem of marine ranching to achieve the objectives of environmental protection, resource conservation, and sustainable fishery output [7]. MRES is an important means to ensure the enhancement of fishery resources and the improvement of the ecological environment. Yang et al. [1] pointed out that it is one of the technological means for future marine ranching to develop a monitoring platform for the ecological security and environmental protection of marine ranching; the authors also suggested using multiple models to predict and evaluate the safety of marine ranching. Du and Sun [6] researched the influence paths of MRES in China and reported useful findings for managing MRES. Du and Gao [8] evaluated the ecological effects of marine ranching by comprehensively considering social and economic factors and made a systematic evaluation of MRES to provide theoretical guidance for the management practices of marine ranching. Qin et al. [9] analyzed the influencing factors of spatial variation of national marine ranching in China. Wan et al. [10] evaluated the sustainability of the supply chain of natural marine ranching and applied their novel model to determine marine ranching's sustainable performance.

In the existing literature, the research on MRES has achieved phased results, such as clarifying the concept of MRES, trying to build the evaluation index of MRES, proposing the evaluation method of MRES, and discussing the influence paths of MRES. These studies' results have important reference values for follow-up studies [11, 12]. However, the evaluation of MRES is based on the basic assumption that the indexes are independent of each other. For example, the analytic hierarchy process (AHP), which has been applied in many research about decision-making [13, 14], is a method that requires the independent relationship between elements/indexes, and it requires that the structure model is hierarchical and linear [8]. In other words, the research results of AHP will be invalid if there are interaction relationships between the indexes. The study in "influence paths of MRES" [6] indicated that marine ranching is an artificial ecosystem, and there are influence paths among its internal indexes. This means that it is questionable to simply use AHP to evaluate MRES. Based on this, how to make a more scientific evaluation of MRES on the basis of fully considering the influence relationships of indexes is an important issue to be studied. Considering the above information, this study chooses the analytic network process (ANP) to evaluate MRES.

ANP is an extended and complementary form of AHP, which was proposed and developed by Saaty [15–17]. ANP makes up for the limitations of the AHP method, which include being applicable to only linear structure [17, 18], and releases the restriction of hierarchical structure requirement because this method provides a framework that considers the interrelationships within a cluster and among different clusters between all evaluation metrics (criteria) [15]. The ANP method constructs a network system instead of the tree-shaped hierarchical structure of the AHP, thus overcoming the problem of dependence and feedback among criteria or alternatives [19]. ANP has been widely applied in various fields and has proven to be effective in solving decision-making problems, such as supply chain management [20], risk assessment [21], environmental management [22], and location selection [23]. ANP is a powerful method that has been applied to solve different decision-making problems by many researchers, and it is also applicable for constructing models with evaluation criteria and dimensions that contain complex interactions. Thus, ANP is selected as the appropriate tool to evaluate MRES performance in this study.

It should be considered that the input information of ANP is the subjective judgment information given by experts, which is often incomplete and hesitant due to the limitations of objective conditions and experts' knowledge structure. Therefore, it is particularly important to study how to express the expert judgment information and carry out decision-making scientifically. Fortunately, the probabilistic linguistic term sets (PLTS) provide a choice to express preferences by means of linguistic information. In practical applications, due to the qualitative natural of the judgment criteria, many linguistic and fuzzy methods have been developed. Rodriguez et al. [24] came up with hesitant fuzzy linguistic term sets (HFLTS) motivated by hesitant fuzzy sets [25] and linguistic term sets [26]. Based on this, Wei et al. [27] defined the operations on HFLTS. Chen et al. [28] investigated the consistency and consensus problems with the hesitant linguistic preference relations. Wang et al. [29] developed an optimization algorithm with the incomplete probabilistic linguistic term sets which could describe the qualitative pairwise judgment information in preference decision-making. These studies regarding probabilistic fuzzy or probabilistic linguistic provide fundamental theories for PLTS. As a method to model linguistic information, PLTS represents different membership degrees of all possible evaluation terms (linguistic terms) for a specific alternative, which is more likely to provide comprehensive and accurate preference information about the decision-makers.

Since the concept of PLTS was proposed while extending HFLTS, it has experienced substantial development, and it is now a hot topic in the field of multicriteria decision-making [30, 31]. In recent years, many researchers have used the probabilistic linguistic information to solve complex decision-making problems. For example, Zhang et al. [32] introduced PLTS to describe group preferences while considering fuzzy and uncertain group preferences. Song et al. [33] proposed a novel text named Word2PLTS by introducing the idea of fuzziness and uncertainty of human language. Bai et al. [34] developed a new comparison method for PLTS to overcome the shortcoming of complex computing during PLTS application. Zhou et al. [35] used the ANP method under the probabilistic linguistic environment and integrated the PLTS and FTA-ANP. The combination of ANP and PLTS (PLTS/ANP for short) as the incomplete probabilistic linguistic ANP can make up for the shortcomings of the ANP method while using the PLTS method to represent the uncertain information. Based on this previous research, this study used PLTS/ANP to obtain the weights of MRES evaluation indexes and make an overall evaluation. In the process of obtaining the weights of MRES evaluation indexes by using the ANP method, the judgment about the important value of the evaluation index from experts might be fuzzy and hesitant due to the limitation of experts' knowledge or prejudice. At the time, the PLTS as a linguistic representation model with uncertainties can represent experts' hesitant information and give the values of a linguistic variable [30, 36]. Moreover, PLTS can also reflect the associated probabilistic information about the linguistic terms, which achieves the objective of computing with expressions.

The motivation of this study is to develop an evaluation method for MRES by the integrated PLTS/ANP method and make a case study of 11 marine ranches cases in Rongcheng. MRES is a new concept, and it is very likely difficult for experts to have complete experience at this stage because of the lack of objective data and typical cases in this regard. Therefore, this study utilizes the integrated method of PLTS and ANP to evaluate MRES. This study focuses on the following three aspects:

(1) To construct an MRES evaluation network model based on the Driver-Pressure-State-Impact-Response (DPSIR) model, which includes not only evaluation indexes but also the influence relationships of indexes. The indexes for MRES involve economic, social, and ecological aspects, among which there are many complex interdependent relationships. The MRES evaluation network model shows the interdependencies that exist as inner dependence or outer dependence among clusters.

(2) To determine the prioritization of evaluation indexes by applying the PLTS/ANP method and calculate the evaluation value of MRES. This integrated PLTS/ANP method can reflect the influence mechanism between indexes and extract and integrate experts' judgment information reasonably, which is conducive for setting scientific priority weights. Combined with the weights calculated by the comprehensive method and the performance values of the marine ranching in each index, the comprehensive performances of MRES are obtained by weighted summation.

(3) To apply this proposed evaluation method to case study and conduct an effective assessment of marine ranching in Rongcheng. The evaluation value of MRES and the contribution rate of each cluster are calculated and analyzed, respectively, by using the index weight that is determined by PLTS/ANP and using the performance value of the case on each index. Finally, we give specific suggestions for improving ecosystem security for enterprises and government based on the results.

The remaining of this study is organized as follows. Section 2 constructs the evaluation index network model of the DPSIR-based MRES model and introduces the methodology of MRES evaluation using the integrated PLTS/ANP method. In Section 3, the MRES evaluation method, which integrates the PLTS and ANP, is applied to 11 marine ranches in the city of Rongcheng, and the evaluation results are analyzed as well. Based on the case study, some corresponding suggestions about MRES management improvement are given for these enterprises and government. Section 4 concludes the study with a discussion of the results in this study, the limitations of MRES evaluation by the integration method, and the future research direction.

## 2. The PLTS/ANP-Based Evaluation Method for MRES

This study utilizes the PLTS/ANP method to determine the evaluation framework for MRES and calculate the relative importance and prioritization of indexes. In other words, this study uses ANP to evaluate MRES while the PLTS provides a comprehensive way to represent complex linguistic information, so as to determine an accurate ranking of each evaluation index.

*2.1. MRES Evaluation Framework with PLTS/ANP.* The integrated PLTS/ANP used in this study to evaluate MRES includes the following steps. First, an evaluation index system is determined for MRES based on DPSIR. Second, the network model of MRES evaluation is constructed to show the influence relationships among evaluation indexes. Third, the overall weights of indexes are determined by PLTS/ANP. Fourth, the evaluation values of MRES are calculated and analyzed. The process is shown in Figure 1.

*2.2. Evaluation Index System for MRES.* The Organization for Economic Co-operation and Development (OECD) proposed the PSR model of environmental assessment in the late 1980s [37]. The United Nations (UN) adapted it for the DSR model [38], and then based on the advantages of the PSR and DSR models, the European Environment Agency (EEA) established the DPSIR framework and applied it to evaluate the relationship between environmental performance and social economy [39].

In this study, we construct the evaluation index system, which is an organic whole composed of multiple interrelated evaluation indexes from the five aspects of DPSIR for MRES. Specifically, in the evaluation system of MRES, the indexes of the driver cluster mainly come from the social system and the economic system, such as the profit margin and the enterprise's ecological awareness. The indexes of the pressure cluster mainly come from the ecological environment system and are directly affected by the driver, such as the green degree of farming methods and natural disasters. The indexes of the state cluster mainly represent the performance of the marine ranching ecological environment under pressure, such as water quality and target biomass. The indexes of the impact cluster mainly represent the effect of marine ranching on human social and economic life and the environment state, such as the improvement of fishery resources and the degree of pulling the industrial chain. The indexes of the response cluster mainly represent the positive measures taken by enterprises to improve the current situation in the process of operation and management, such as marine technology, and management and annual monitoring.

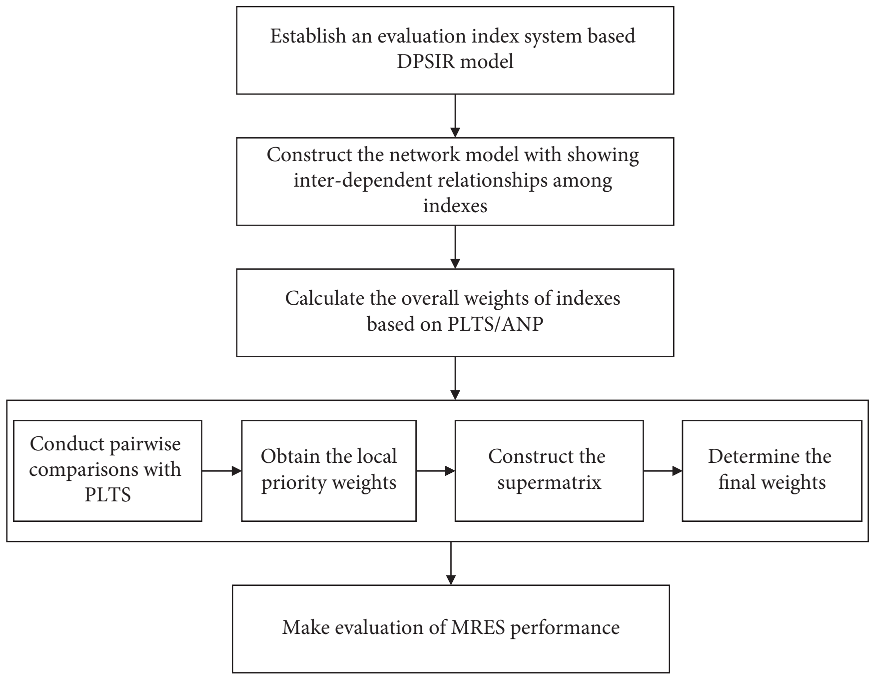


Figure 1: Framework of MRES evaluation.

The evaluation index system is established for MRES based on the DPSIR model, as shown in Table 1. From Table 1, it is obvious that the MRES index system comprises 5 clusters and 22 evaluation indexes. The evaluation indexes included in each cluster are carefully selected by referring to the relevant literature [40–42] and following the principles of being scientific, operative, forward-looking, and so on. In the attributes column of Table 1, the benefit-type index is defined as "+," and the cost-type index is defined as "−."

*2.3. Network Model Construction of MRES Evaluation.* The ANP network structure is divided into control hierarchy and network hierarchy. Inside the network hierarchy, the network structure is composed of elements that are controlled by the control hierarchy, and they may interact with each other. In this study, the control hierarchy is omitted because it only contains one target element, namely MRES. During the process of ANP application, the network structure is constructed based on the identified indexes and clusters as well as their potential interrelationships. The interdependencies can exist in the form of inner dependence or outer dependence. Thus, an accurate modeling tool of comprehensive and interdependent indexes is provided.

As mentioned in Section 1, marine ranching is an artificial ecosystem, and the indexes listed in Table 1 may be interdependent and mutually influenced. There exist both outer dependency relationships and inner dependency relationships in the MRES evaluation system. (1) The outer dependency relationships may occur between the indexes from different clusters, and these relationships are also consistent with the correlations as shown by the DPSIR model [43]. For example, the evaluation index "enterprise's ecological awareness ($e_3$)" from the "driver ($C_1$)" cluster affects the evaluation index "green degree of farming methods ($e_6$)" from the "pressure ($C_2$)" cluster. Meanwhile, the evaluation index "research support ($e_{20}$)" from the "response ($C_5$)" cluster affects the evaluation index "profit margin ($e_2$)" from the "driver ($C_1$)" cluster. The same is true for the relationships between other different clusters. (2) The inner dependency relationships may occur between the indexes included in each cluster. For example, in the "impact ($C_4$)" cluster, the index "the losses caused by natural or man-made disasters ($e_{16}$)" affects another index "the improvement of fishery resources ($e_{13}$)," and the index "the improvement of fishery resources ($e_{13}$)" has a positive effect on the index "the number of absorbed or resettled fishermen ($e_{14}$)." The same is true for the other four clusters.

Based on the above analysis, according to the DPSIR model, this study constructs the network model of MRES evaluation, as shown in Figure 2. In this network model, arrows are used to represent the influence relationships inside or outside the cluster, and arrows point one object to other objects that are affected.

*2.4. Determination of Overall Weights of Indexes.* In the existing literature, the evaluation indexes of marine ranching are based on assuming that the indexes are independent of each other. In fact, marine ranching is an ecosystem, and its MRES evaluation system exhibits the influence relationships, as shown in Figure 2. A key problem is how to reflect such influence relationships and make a reasonable evaluation. Due to the complexity of MRES, the research and understanding of some evaluation indexes are limited at present. It is difficult for experts to express preferences by means of one certain linguistic term. Therefore, this study first uses the PLTS method to effectively achieve the uncertain

Table 1: MRES evaluation index system.

| Cluster | Evaluation index | Unit | Attributes |
|---|---|---|---|
| Driver ($C_1$) | Financial fund input ($e_1$) | $10^4$yuan | + |
| | Profit margin ($e_2$) | % | + |
| | Enterprise's ecological awareness ($e_3$) | % | + |
| Pressure ($C_2$) | The number of bottom sowing and proliferation and release ($e_4$) | $10^4$ tails | + |
| | Seaweed field and seagrass bed transplant cultivation ($e_5$) | Number | + |
| | Green degree of farming methods ($e_6$) | Number | + |
| | Artificial reef construction and maintenance ($e_7$) | Number | + |
| | Natural disaster ($e_8$) | Times/year | - |
| State ($C_3$) | Water quality ($e_9$) | Number | - |
| | Marine sediment ($e_{10}$) | Number | - |
| | Target biomass ($e_{11}$) | Number | + |
| | Biodiversity index ($e_{12}$) | Number | + |
| Impact ($C_4$) | The improvement of fishery resources ($e_{13}$) | Number | + |
| | The number of absorbed or resettled fishermen ($e_{14}$) | Number | + |
| | The degree of pulling the industrial chain ($e_{15}$) | $10^4$yuan | + |
| | The losses caused by natural or man-made disasters ($e_{16}$) | $10^4$yuan | - |
| | Benefits of aquatic products ($e_{17}$) | % | + |
| Response ($C_5$) | Visualization, intelligence, information system construction ($e_{18}$) | Number | + |
| | Marine technology and management ($e_{19}$) | Number | + |
| | Research support ($e_{20}$) | Number | + |
| | Scientific management of fishery resources ($e_{21}$) | Number | + |
| | Annual monitoring ($e_{22}$) | Times/year | + |

preferences given by experts and then uses ANP to make an overall evaluation for the MRES. The associated calculation for MRES consists of the following steps.

Step 1: conduct the pairwise comparisons with PLTS.

The definition of ANP is that as a multicriteria theory, it provides the mathematics and a comprehensive structure to obtain the relative influence of one of two elements over the other in a pairwise comparison process on a third element in the system, with respect to an underlying control criterion [16]. As the ANP method has been described sufficiently in the literature, it is described very briefly in this section. For the complete process or model of constructing pairwise comparisons, see this cited article of Saaty [15].

Based on Figure 2, ANP conducts pairwise comparisons that reflect dependencies in this network model of MRES evaluation between all clusters and indexes.

For each relationship between clusters $C_i \longrightarrow C_j$ in the network structure as shown in Figure 1, the pairwise comparison matrices should be constructed. Obviously, if $i \neq j$, $C_i \longrightarrow C_j$ represents the outer dependent relationships between two different clusters; if $i = j$, it represents the inner dependent relationships in one cluster. Without loss generality, here, we suppose $C_i = \{e_{i1}, e_{i2}, \ldots, e_{iM_i}\}$ and $C_j = \{e_{j1}, e_{j2}, \ldots, e_{jN_j}\}$. Following the thought of ANP, the element/index $e_{im}$ included in cluster $C_i$ is regarded as a criterion, and then, a pairwise comparison matrix $B_j^{im}$ of cluster $C_j$ is constructed with respect to $e_{im}$.

Here, we give some concepts and definitions of PLTS and show the process for constructing $B_j^{im}$ with PLTS [44].

The most widely used concept about PLTS is the linguistic term set (LTS), which can be defined as in

$$S = \{s_0, s_1, \ldots, s_\tau, \ldots, s_{2\tau}\}, \quad (1)$$

where $2\tau + 1$ is the number of all terms in $S$; $s_\delta\,(\delta \in [0, 2\tau])$ as the linguistic term is generated by a predefined syntactic rule and restricted by a fuzzy set; $s_\tau$ means "indifference", and the remaining linguistic terms are distributed symmetrically around it.

Then, the PLTS is defined as in

$$L(p) = \left\{L^{(\ell)}\left(p^{(\ell)}\right) | L^{(\ell)} \in S, \quad p^{(\ell)} \geq 0,\, k = 1, 2, \ldots, \#L(p),\, \sum\nolimits_{\ell=1}^{\#L(p)} p^{(\ell)} \leq 1\right\}, \quad (2)$$

where $L^{(\ell)}(p^{(\ell)})$ is the $\ell$ th linguistic term of $L^{(\ell)}$ with the associated probability $p^{(\ell)}$; $\#L(p)$ is the number of all different elements in $L(p)$. Similarly, we utilize $S$ to evaluate these indexes and to present the preference degree for an index $e_j$ over $e_{j'}$ in a pairwise comparison matrix $B_j^{im}$.

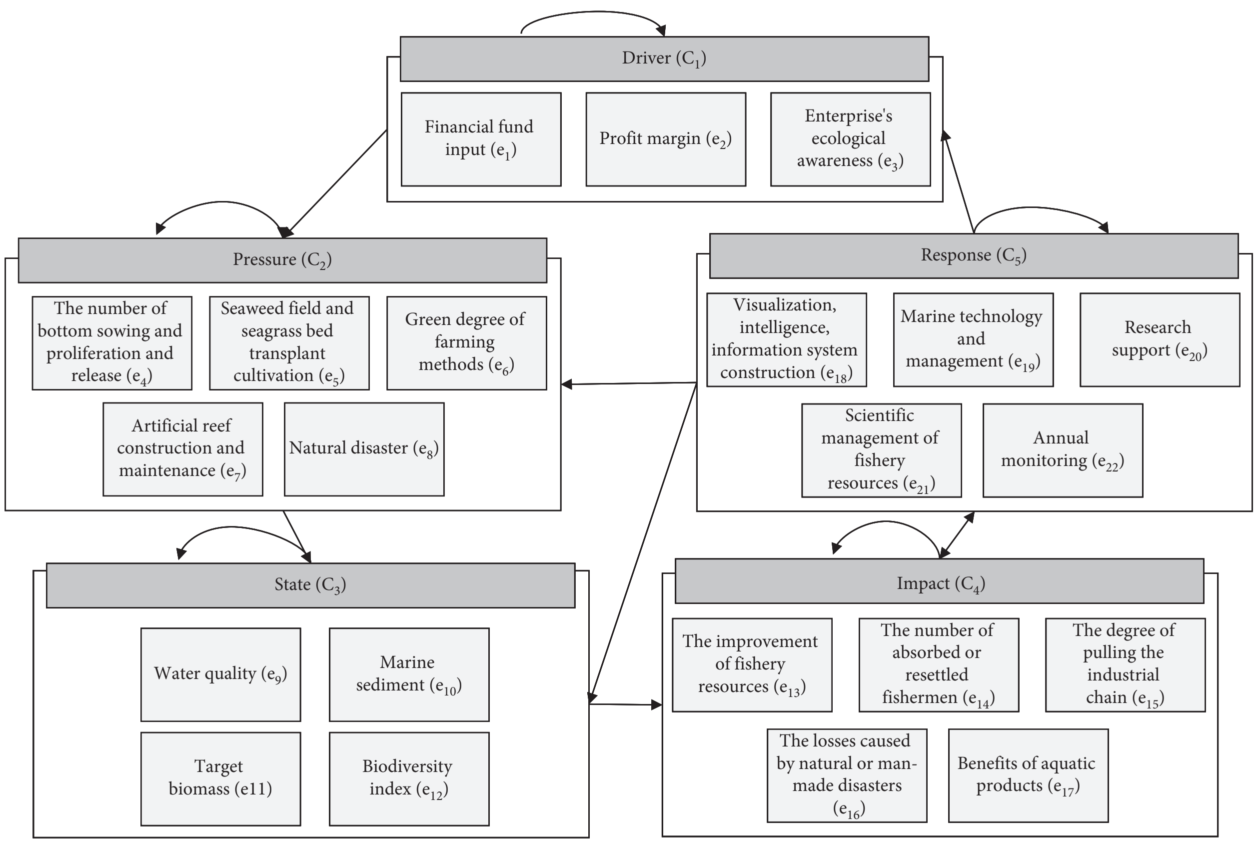


Figure 2: Network model of MRES evaluation.

$$B_j^{im} = \left(L_{kk'}(p)\right)_{N_j \times N_j},$$
$$L_{kk'}(p) = \left\{L_{kk'}^{(\ell)}\left(p_{kk'}^{(\ell)}\right) \mid \ell = 1, 2, \ldots, \#L_{kk'}(p)\right\}, \tag{3}$$

where $p_{kk'}^{(\ell)} > 0$ and $\#L_{kk'}(p)$ are the number of linguistic terms in $L_{kk'}(p)$. Moreover, there are some rules that need to be followed while constructing a matrix $B_j^{im}$, such as $p_{kk'}^{(\ell)} = p_{k'k}^{(\ell)}$, $L_{kk'}^{(\ell)} + L_{k'k}^{(\ell)} = s_{2\tau}$, $L_{kk}(p) = s_\tau(1)$, and $\#L_{kk'}(p) = \#L_{k'k}(p)$.

Step 2: obtain the local priority weights.

For a PLTS, the weighted value of the $\ell$ th element in the PLTS is defined as

$$WV^{\ell} = I\left(L^{(\ell)}\right) \times p^{(\ell)}, \tag{4}$$

where $Ind(\cdot)$ is a function that returns the subscript of a linguistic term from $S$ to $[0, 2\tau]$, for example, $Ind(s_\delta) = \delta$ for any $s_\delta \in S$.

For the pairwise matrix $B_j^{im}$ expressed by probabilistic linguistic terms, if $B_j^{im}$ is consistent, motivated by reference [45], we can get the most accurate weights of elements by the optimization model as in

$$\begin{cases} \min Z_j^{im} = \sum_{k=1}^{N_j} \sum_{k'=1}^{N_j} \sum_{\ell=1}^{\#L_{ij}} \left(\varepsilon_{kk'}^{\ell}\right)^2 = \sum_{k=1}^{N_j} \sum_{k'=1}^{N_j} \sum_{\ell=1}^{\#L_{ij}} \left(\ln \omega_k - \ln \omega_{k'} - \ln \dfrac{WV_{kk'}^{\ell}}{WV_{k'k}^{\ell}}\right)^2, \\ \text{s.t.} \sum_{k=1}^{N_j} \omega_k = 1, \quad \omega_k > 0,\ k = 1, 2, \ldots, N_j, \end{cases} \tag{5}$$

where $\varepsilon_{kk'}^{\ell} = \ln \omega_k - \ln \omega_{k'} - \ln(WV_{kk'}^{\ell}/WV_{k'k}^{\ell})$, $\ell \in 1, 2, \ldots, \#L_{ij}\}$; $\omega_k$ is the weight of the kth index corresponding to matrix $B_j^{im}$, $k = 1, 2, \ldots, N_j$.

As proved in reference [4], the optimal solution of the model as in equation (5) can be calculated by

$$\omega_k = \begin{cases} \dfrac{\exp(q_k)}{\sum_{k'=1}^{N_j-1}\exp(q_{k'})+1}, & k = 1,2,\ldots,N_j-1, \\ \\ \dfrac{1}{\sum_{k=1}^{N_j-1}\exp(q_k)+1}, & k = N_j. \end{cases} \tag{6}$$

In equation (6), $\exp(\cdot)$ is the exponential function based on the natural constant $e \approx 2.71828$, with $Q = (q_1, q_2, \ldots, q_{N_j-1}) = D^{-1}Y$, in which

$$D = \begin{pmatrix} \sum_{k=2}^{N_j}\ell_{1k} & -\ell_{12} & \cdots & -\ell_{1(N_j-1)} \\ -\ell_{21} & \sum_{\substack{k=1\\k\neq 2}}^{N_j}\ell_{2k} & \cdots & -\ell_{2(N_j-1)} \\ \vdots & \vdots & \ddots & \vdots \\ -\ell_{(N_j-1)1} & -\ell_{(N_j-1)2} & \cdots & \sum_{\substack{k=1\\k\neq N_j-1}}^{N_j}\ell_{(N_j-1)k} \end{pmatrix}, \tag{7}$$

$$Y = \begin{pmatrix} \sum_{k=1}^{N_j}\sum_{\ell=1}^{\#L_{1j}}\ln\dfrac{WV_{1k}^{\ell}}{WV_{k1}^{\ell}} \\ \sum_{k=1}^{N_j}\sum_{\ell=1}^{\#L_{2j}}\ln\dfrac{WV_{2k}^{\ell}}{WV_{k2}^{\ell}} \\ \vdots \\ \sum_{k=1}^{N_j}\sum_{\ell=1}^{\#L_{(N_j-1)k}}\ln\dfrac{WV_{(N_j-1)k}^{\ell}}{WV_{k(N_j-1)}^{\ell}} \end{pmatrix}. \tag{8}$$

Step 3: construct the supermatrix.

Using Steps 1 and 2, the vector of local priority weights ($\omega_j^{im} = (\omega_1, \omega_2, \ldots, \omega_{N_j})$) corresponding to the matrix $B_j^{im}$ is determined. To reflect the relationship between local priority weights and its corresponding criterion, here, the vector of local priority weights is redescribed as $\omega_j^{im} = (\omega_1^{im}, \omega_2^{im}, \ldots, \omega_{N_j}^{im})$ by adding the superscript into the symbols. Similarly, also for each relationship between clusters $C_i \longrightarrow C_j$, taking another element (index) $e_{im'}$ included in cluster $C_i$ as a criterion, the pairwise comparison matrix $B_j^{im'}$ of cluster $C_j$ is constructed with respect to $e_{im'}$, and its corresponding vector of local priority weights $\omega_j^{im'} = (\omega_1^{im'}, \omega_2^{im'}, \ldots, \omega_{N_j}^{im'})$ can also be determined, $m' \neq m \in \{1, 2, \ldots, M_i\}$. Consequently, all of the local priority weights for $C_i \longrightarrow C_j$ can be written as a block matrix $W_{ij}$, as in equation (9). The block matrix $W_{ij}$ represents the relative importance of the elements in cluster $C_j = \{e_{j1}, e_{j2}, \ldots, e_{jN_j}\}$ with regard to each element in cluster $C_i = \{e_{i1}, e_{i2}, \ldots, e_{iM_i}\}$ [46].

$$W_{ij} = \begin{bmatrix} \omega_1^{i1} & \omega_1^{i2} & \cdots & \omega_1^{iM_i} \\ \omega_2^{i1} & \omega_2^{i2} & \cdots & \omega_2^{iM_i} \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{N_j}^{i1} & \omega_{N_j}^{i2} & \cdots & \omega_{N_j}^{iM_i} \end{bmatrix}. \tag{9}$$

For each pair of clusters in Figure 1, if there exists a relationship between clusters $C_i \longrightarrow C_j$, then the block matrix $W_{ij}$ is determined by equations (4)–(9); else if there does not exist a relationship between clusters $C_i \longrightarrow C_j$, then the block matrix $W_{ij}$ is defined by zero matrix ($W_{ij} = 0$). When all of the block matrices are determined, a supermatrix in the ANP is constructed as in equation (10) to reflect all the local priority weights in the network model of MRES evaluation [47]. Note that, the number of clusters in the MRES evaluation is 5 resulting in there are $5 \times 5 = 25$ block matrices in the supermatrix $W$.

$$W = \left[W_{ij}\right]_{5\times5} = \begin{bmatrix} W_{11} & W_{12} & \cdots & W_{15} \\ W_{21} & W_{22} & \cdots & W_{25} \\ \vdots & \vdots & \ddots & \vdots \\ W_{51} & W_{52} & \cdots & W_{55} \end{bmatrix}. \tag{10}$$

Step 4: determine the final weights.

The supermatrix $W$, as in equation (10), is unweighted since the sum of elements in each column is not equal to 1. In order to normalize the supermatrix, the weight of each block matrix is determined by experts according to the ANP [8]. Assuming the determined weight of block matrix $W_{ij}$ is $a_{ij}$, $\sum_{i=1}^{5} a_{ij} = 1$, $a_{ij} \geq 0$, $j = 1, 2, \ldots, 5$, then the weighted supermatrix $\overline{W}$ is calculated by integrating the weight of block matrix into the supermatrix as in

$$\overline{W} = \left[\overline{W}_{ij}\right]_{5\times5} = \begin{bmatrix} a_{11}W_{11} & a_{12}W_{12} & \cdots & a_{15}W_{15} \\ a_{21}W_{21} & a_{22}W_{22} & \cdots & a_{25}W_{25} \\ \vdots & \vdots & \ddots & \vdots \\ a_{51}W_{51} & a_{52}W_{52} & \cdots & a_{55}W_{55} \end{bmatrix}. \tag{11}$$

The limit weighted supermatrix that reflects both direct influence relationships and indirect influence relationships between indexes is calculated by $\overline{W}^{\infty} = \lim_{\lambda \longrightarrow \infty} \overline{W}^{2\lambda+1}$, as in equation (12).

According to the ANP theory, the weighted supermatrix $\overline{W}$ is a column random matrix, that is, the sum of each column is equal to 1. Thus, the limit weighted supermatrix $\overline{W}^{\infty}$ must be stabilized, that is, the elements in each row are equal to each other such as $w_{n1} = w_{n2} = \cdots = w_{nN}$, $n = 1, 2, \ldots, N$. For convenience, here we suppose $w_n = w_{n1} = w_{n2} = \cdots = w_{nN}$, and it is the global priority weight of index $e_n$, $n = 1, 2, \ldots, N$. Obviously, the number of indexes in the MRES evaluation is 22 as shown in Table 1 resulting in $N = 22$ in the limit weighted supermatrix $\overline{W}^{\infty}$.

$$\overline{W}^{\infty} = \left[w_{nn'}\right]_{N\times N} = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1N} \\ w_{21} & w_{22} & \cdots & w_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ w_{N1} & w_{N2} & \cdots & w_{NN} \end{bmatrix}. \tag{12}$$

*2.5. Calculation of the Evaluation Value of MRES.* We suppose that there are $T$ marine ranches to be evaluated on the MRES, and the performance value of the $t$th marine ranching $MR_t$ on the nth index $e_n$ be $x_{tn}$; $t = 1, 2, \ldots, T$, $n = 1, 2, \ldots, 22$. As mentioned in Table 1, the evaluation indexes are divided as the benefit-type index marked with "+," and the cost-type index marked with "−." In order to eliminate dimensional and index-type effects, the collected index data of marine ranching $X = [x_{tn}]_{T\times22}$ need to be standardized before calculation by equations (13) and (14). Equation (13) is used to standardize the data on benefit index "+," and equation (14) is used to standardize that on the cost index "−".

$$y_{tn} = \frac{x_{tn} - \min\{x_{1n}, \ldots, x_{Tn}\}}{\max\{x_{1n}, \ldots, x_{Tn}\} - \min\{x_{1n}, \ldots, x_{Tn}\}}, \tag{13}$$

$$y_{tn} = \frac{\max\{x_{1n}, \ldots, x_{Tn}\} - x_{tn}}{\max\{x_{1n}, \ldots, x_{Tn}\} - \min\{x_{1n}, \ldots, x_{Tn}\}}. \tag{14}$$

After the collected data are all standardized, the comprehensive MRES evaluation value (MRES value for short) can be obtained by combining the index weights with the standardized performance data of the marine ranching, as in equation (15). The higher the value of $Q_t$ in the calculation result, the better the MRES of the marine ranching $MR_t$.

$$Q_t = \sum_{n=1}^{22} y_{tn} w_n,\ t = 1, 2, \ldots, T. \tag{15}$$

For better analysis, the MRES is usually divided into several grade levels according to comprehensive MRES evaluation values. Similar to references [8, 48], the grades of the MRES in this study are divided into three levels of insecurity, medium security, and security. The MRES is obtained according to the following steps:

Step 1: finding the maximum value $Q_{\max}$ and the minimum value $Q_{\min}$ of MRES from $Q_t$, $t = 1, 2, \ldots, T$.

Step 2: calculating the distance between the maximum and the minimum values $\Delta = Q_{\max} - Q_{\min}$.

Step 3: determining the grade level of the marine ranching $MR_t$ by equation (16), $t = 1, 2, \ldots, T$.

$$Q_t = \begin{cases} \left[Q_{\min}, Q_{\min} + \dfrac{\Delta}{3}\right), \text{ in security}, \\ \left[Q_{\min} + \dfrac{\Delta}{3} Q_{\max} - \dfrac{\Delta}{3}\right), \text{ medium} - \text{security}, \\ \left[Q_{\max} - \dfrac{\Delta}{3}, Q_{\max}\right], \text{ security}. \end{cases} \tag{16}$$

The comprehensive MRES evaluation value is composed of the performance in five clusters, that is, $Q_t = \sum_{i=1}^{5} Q_t^i$, where $Q_t^i = \sum_{n=1}^{M_i} w_n \cdot y_{ti}$, $t = 1, 2, \ldots, T$, $M_i$ is the number of indexes included in the cluster $C_i$. Consequently, the contribution rate is defined, as in equation (17), to reflect the contribution of a cluster to the MRES.

$$CR_t^i = Q_t^i / Q_t,\ i = 1, 2, \ldots, 5,\ t = 1, 2, \ldots, T. \tag{17}$$

The higher the value of $CR_t^i$, the greater the contribution rate of cluster $C_i$ to the MRES for $MR_t$. It means that the marine ranching $MR_t$ has the advantage on the cluster with a greater contribution rate. If marine ranching $MR_t$ wants to improve its MRES level, it can start from the cluster (indexes) with a lower contribution rate. Furthermore, the average contribution rate is easily calculated by $CR_i = \sum_{t=1}^{T} CR_t^i / T$ to reflect the average level of several marine ranches, $i = 1, 2, \ldots, 5$.

## 3. Case Study for Marine Ranching in Rongcheng

*3.1. Research Area and Date Source.* Located in Weihai, Shandong Province, Rongcheng has 500 km of coastline. Its marine ranches occupy an area of 313 km$^2$. As of January 2021, Shandong had 54 national marine ranches, accounting for 39.7% of the national marine ranching programs in China. Among them, Rongcheng has successfully established 10 national marine ranches and 14 provincial marine ranches, making fruitful achievements in the construction of marine ranching. In the sixth batch of the national marine ranching demonstration list published by the Ministry of Agriculture and Rural Affairs of the People's Republic of China, Rongcheng ranked first among county-level cities in China regarding the number of newly added ranches. To assess the MRES, this study collects data from 11 typical marine ranches in Rongcheng, which consist of eight national marine ranches ($MR_1$–$MR_8$) and three provincial marine ranches ($MR_9$–$MR_{11}$). We use the index system and evaluation method constructed above for analysis. The location of the study area is shown in Figure 3.

*3.2. Results and Analysis*

*3.2.1. Index Priority Weights.* As mentioned in Sections 2.1–2.4, the index weights of MRES evaluation are determined by the integrated PLTS/ANP method. First, the pairwise matrices are constructed, which reflect the connection relationships between elements and clusters according to Figure 2. Then, experts give all the pairwise comparison matrices, in which the probabilistic linguistic terms are used as needed to represent uncertain information. In this study, the set of linguistic term $S$ is defined with $\tau = 8$. The key linguistic terms are $s_0$ = extremely more unimportant, $s_2$ = very strongly unimportant, $s_4$ = strongly more unimportant, $s_6$ = moderately more unimportant, $s_8$ = equally important, $s_{10}$ = moderately more important, $s_{12}$ = strongly more important, $s_{14}$ = very strongly important, and $s_{16}$ = extremely more important. The linguistic terms $s_1$, $s_3$, $s_5$, $s_7$, $s_9$, $s_{11}$, $s_{13}$, and $s_{15}$ are the intermediate terms between the $s_{i-1}$ and $s_{i+1}$. Finally, we put the experts' information into equations (4)–(8) to obtain the vector of local priority weights of MRES evaluation indexes.

An example of the pairwise comparison matrices is shown in Table 2, which illustrates the vector of local priority weights corresponding to cluster $C_5$ with respect to the index $e_6$ in cluster $C_2$. In Table 2, the data "$s_4$ (0.8), $s_3$ (0.2)," which are in the second row and the third column, show that the relatively important degree of $e_{19}$ to $e_{21}$ with respect to $e_6$ is strongly more unimportant ($s_4$) with 0.8 probability and ($s_3$) with 0.2 probability. The final global priority weights of indexes (index weights for short) can be determined by calculating the supermatrix, weighted supermatrix, and limit-weighted supermatrix as shown in equations (9)–(12). The results are shown in Figure 4.

At the index level, the weight of an index reflects its importance degree relative to all the indexes from the evaluation index system. For an evaluation index, the higher the weight, the more impact it has on the MRES. According to Figure 4, "scientific management of fishery resources ($e_{21}$)" is the first priority index among all the evaluation indexes, with a weight of 0.1142. This means that the scientific management of fishery resources is the most critical index for evaluating MRES. Other indexes that have high weights (above 0.06) are profit margin (0.1017), enterprise's ecological awareness (0.0823), the degree of pulling the industrial chain (0.0781), the improvement of fishery resources (0.0639), and benefits of aquatic products (0.0609). This indicates that these indexes are of great importance to MRES. The number of absorbed or resettled fishermen (0.0050) and the number of bottom sowing and proliferation and release (0.0079) are relatively less influential indexes in order of importance with weights less than 0.01.

At the clusters level, according to Figure 4, the weights of clusters are ranked in the following order: response (0.2646) > impact (0.2350) > driver (0.21465) > state (0.1772) > pressure (0.1047). This shows that the five clusters from the evaluation model based on DPSIR have different influences on MRES, with the response cluster having the largest effect and the pressure cluster having the lowest effect. Response, impact, and state are the clusters with weights between 0.1772 and 0.2350.

For each cluster, the weights of its included indexes reflect the different influences on MRES within the same cluster. In the driver cluster, profit margin (0.1017) has the highest weight, followed by financial fund input (0.0345) and enterprise's ecological awareness (0.0823). In the pressure cluster, the weight of natural disaster (0.0494) is significantly higher than the remaining four indexes, whose weights are not greater than 0.021, reflecting the spacing of indexes in the one cluster. In the state cluster, the weight values of biodiversity index (0.0548), water quality (0.0530), and target biomass (0.0514) are very close, and marine sediment with the weight of 0.0180 is relatively small. In the impact cluster, the degree of pulling the industrial chain (0.0781) has the highest weight, which has an obvious difference to the number of absorbed or resettled fishermen (0.0050), which is the lowest weight in this cluster. In the response cluster, scientific management of fishery resources (0.1142) has the highest weight, and annual monitoring (0.0147) has the lowest. Visualization, intelligence,

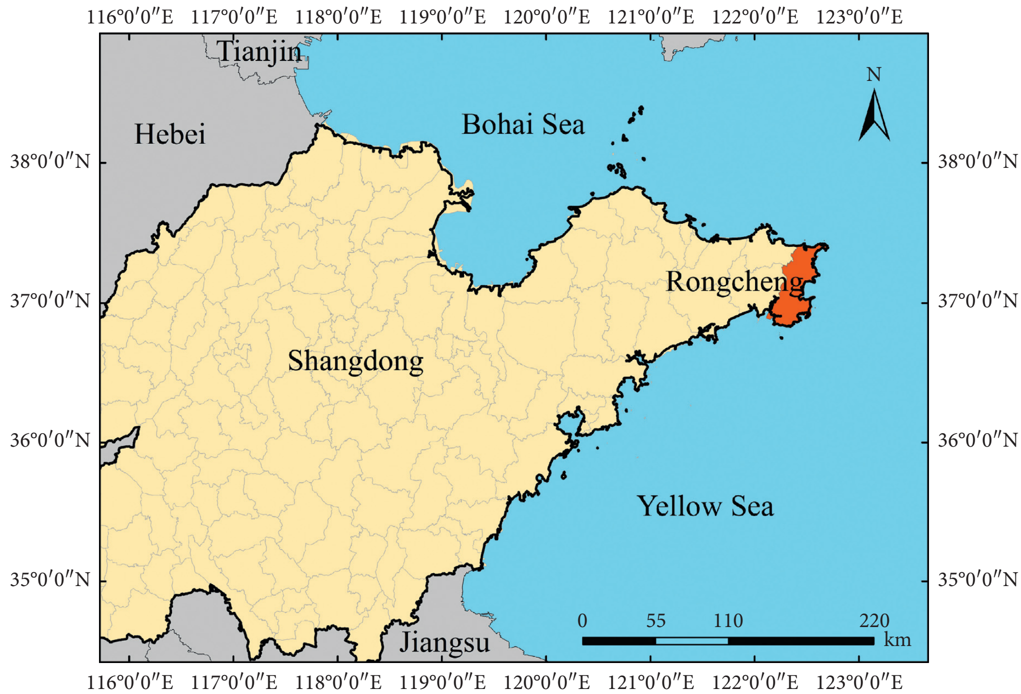


Figure 3: Geographical location of the research area of this study.

Table 2: A pairwise comparison matrix for cluster $C5$ with respect to index $e6$.

| $e_6$ | $e_{19}$ | $e_{21}$ | $e_{22}$ | Priority vector |
|---|---|---|---|---|
| $e_{19}$ | $s_8$ (1) | $s_4$ (0.8), $s_3$ (0.2) | $s_{10}$ (0.7), $s_{11}$ (0.3) | 0.1795 |
| $e_{21}$ | $s_{12}$ (0.8), $s_{13}$ (0.2) | $s_8$ (1) | $s_{14}$ (0.9), $s_{15}$ (0.1) | 0.7383 |
| $e_{22}$ | $s_6$ (0.7), $s_5$ (0.3) | $s_2$ (0.9), $s_1$ (0.1) | $s_8$ (1) | 0.0822 |

information system construction (0.0314), marine technology and management (0.0483), and research support (0.0560) have different levels of influence.

*3.2.2. Overall Evaluation Value of MRES.* As mentioned in Section 2.5, based on the index weights, the comprehensive evaluation value of MRES, which is used to reflect the final performance results of the marine ranches on ecological security, can be calculated by equations (13)–(15). Inputting the index weights calculated in Section 3.2.1 and the standardized data of the investigated 11 marine ranches in Rongcheng into equation (15), we can calculate the comprehensive evaluation values of MRES, as shown in Figure 5. The average performances of the national and provincial marine ranches with respect to MRES are obtained by calculating the mean of the MRES values of national and provincial marine ranches, respectively.

In addition, the grades of the MRES should be divided into three levels by following the three steps in Section 2.5, and each level can be determined by equation (16). If the MRES evaluation value falls within the range of [0.48, 0.58], it is defined as grade 1, namely insecurity. If the MRES evaluation value falls within the range of [0.58, 0.67], it is defined as grade 2, namely medium security. If the MRES evaluation value falls within the range of [0.67, 0.76], it is defined as grade 3, namely security. The grade levels of the MRES shown in Figure 5 reflect the security or sustainability degree of each of the 11 marine ranches operating in Rongcheng.

It is obvious that all MRES evaluation values of the 11 marine ranches range between 0.48 and 0.76. Among the evaluation values of the 11 marine ranches, four are in the insecurity grade, namely $MR_1$, $MR_2$, $ME_3$, and $MR_9$; three of them are in the medium security grade, namely $MR_{11}$, $MR_4$, and $MR_6$; four of them are in the security grade, namely $MR_8$, $MR_{10}$, $MR_7$, and $MR_5$. These evaluation results indicate that there are differences in the MRES performance of marine ranches in Rongcheng, and the development process of marine ranching is inconsistent in terms of ecological security. In other words, we can see that 36% of the marine ranches have relatively good performance in ecological security; 28% of them have average performance and have room to improve ecological security degree; and 36% of them have poor performance, so they need to be improved promptly. In particular, to find the underlying problems of the marine ranches, we conducted in-depth analysis from the national and provincial perspectives.

According to Figure 5, among the eight national marine ranches ($MR_1$–$MR_8$), 37.5% of the marine ranches are in the security grade; 25% are in the medium security grade; and 37.5% are in the insecurity grade. Overall, the average MRES value of the national marine ranches is 0.61, which is in the

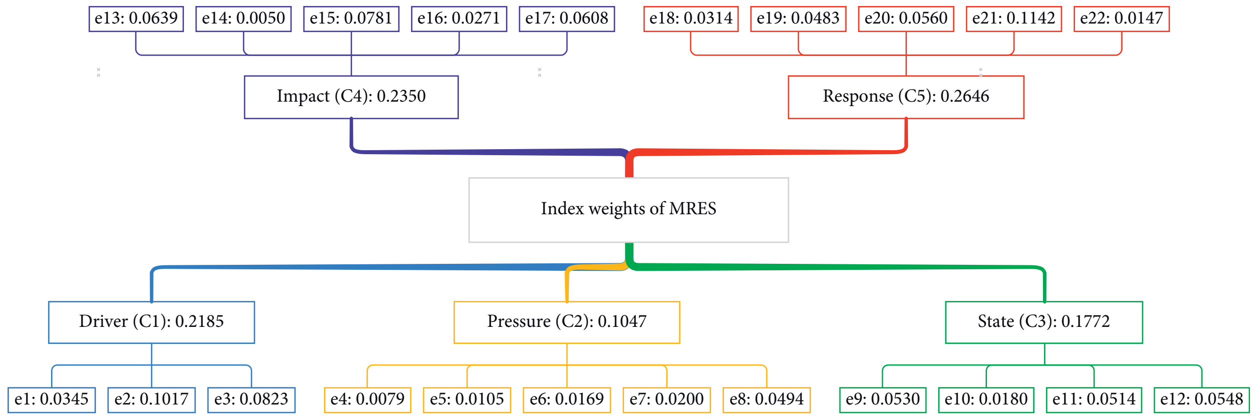


FIGURE 4: Index weights of MRES.

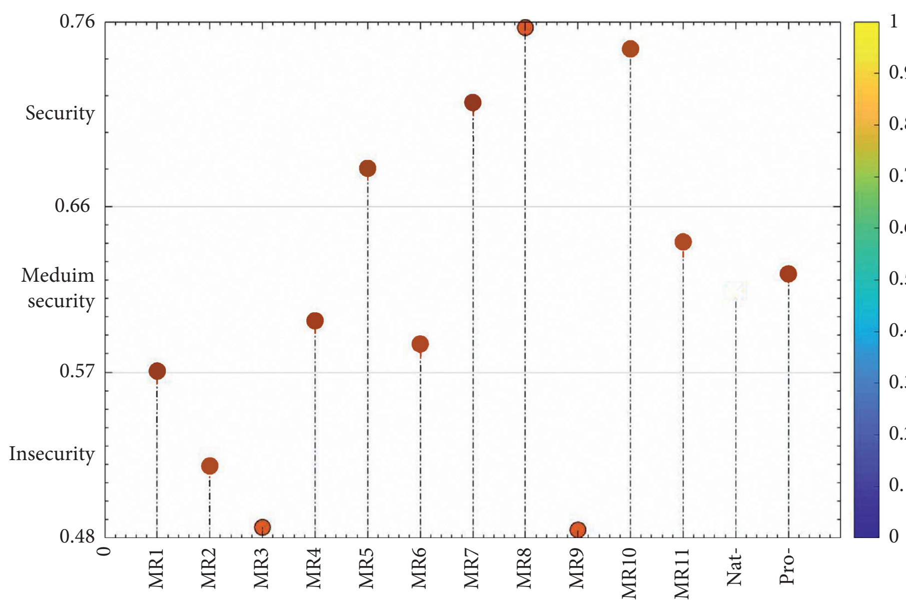


FIGURE 5: MRES evaluation values for marine ranches in Rongcheng.

medium security grade. It indicates that although a lot of efforts for pursuing the sustainable development of marine ranching and preserving the marine ecological environment have been made, for national marine ranching, there is still a gap between the current ecological security status and the ideal state.

Among the three provincial marine ranches ($MR_9$–$MR_{11}$), 33.3% are in the security grade; 33.3% are in the medium security grade; and 33.3% are in the insecurity grade. Overall, the average MRES value of the provincial marine ranches is 0.62, which is in the medium security grade. Therefore, the evaluation grade level of MRES indicates that the ecological security of provincial marine ranching also has a large room for improvement as only 33.3% of the marine ranches are in the security grade. The maximum and minimum values of MRES in provincial marine ranches are 0.75 of $MR_{10}$ and 0.48 of $MR_9$, and they are also the maximum and minimum values of MRES in the 11 marine ranches in this study, which indicates that the ecological security grade of provincial marine ranching is seriously uneven.

By comparing national and provincial marine ranching, we note that the MRES performance of national and provincial marine ranching does not show an obvious correlation with the rating level of marine ranching (i.e., national level or provincial level). First, for either national or provincial marine ranches, the distribution of marine ranching on the MRES grade level is similar, with no significant difference in percentage. For both national and provincial marine ranches, more than 60% of enterprises are not in the security grade, and they need to improve their ecological security performance. Second, the average MRES values of the national and provincial marine ranches are in the medium security grade and are very close. This means that there is no significant difference between the investigated national and provincial marine ranches in average MRES values; the provincial average MRES value (0.62) is slightly higher than the national (0.61). The above analysis shows that the performance of national marine ranches in ecological security is not necessarily higher than that of provincial marine ranching. The ecological security evaluation value of marine ranching is not related to its rating level.

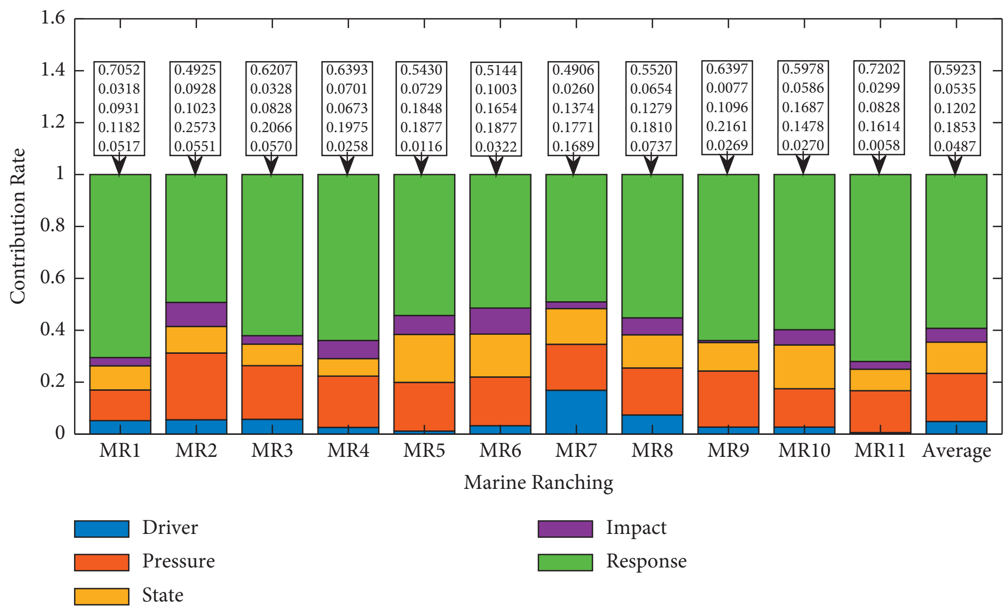


Figure 6: Contribution rates of clusters for MRES.

*3.2.3. Contribution Rate Analysis.* As mentioned in Section 2.5, the contribution rate is calculated by equation (17); it reflects the extent to which each cluster contributes to the MRES for each marine ranching. Then, the average contribution rate of each cluster, which reflects the average cluster contribution rate performance for the marine ranches, can be calculated accordingly. The calculation results are shown in Figure 6, where the data of each marine ranching from top to bottom represent the contribution rates of response, impact, state, pressure, driver.

As shown in Figure 6, the contribution rates of the five clusters to MRES in Rongcheng are different. For both national and provincial marine ranching, the contribution rate of response is significantly higher than other clusters, and the contribution rates of driver and impact are generally lower than others. The average contribution rate also reflects this feature; we can see that response is the most important cluster for MRES value with 59% contribution rate; the average contribution rates of pressure and state are 19% and 12%, and the lowest average contribution rates are driver and impact with 5%. This indicates that all the investigated marine ranches of Rongcheng are mainly driven by the response cluster. In addition, there is no distinction in national and provincial marine ranching because both of them show consistency in Figure 6. To find the underlying problems of the marine ranches of Rongcheng, we conducted an in-depth analysis of each cluster.

For the response cluster, $MR_{11}$ has the highest response contribution rate with 72%, and $MR_2$ and $MR_7$ have the lowest response contribution rates with 49%. As the highest contribution rate cluster of MRES, the response cluster is the important subsystem reflecting some positive actions of marine ranches in Rongcheng. For example, nine of 11 marine ranches have constructed "visualization, intelligence, information system;" seven have fixed marine management expenditures; and all follow scientific and sustainable marine halieutics. In addition, marine ranches have positive achievements in talent introduction, scientific cooperation, biotechnology application, and annual monitoring. In documents available for review, the government of Rongcheng established a favorable environment in terms of marine fisheries regulation and science and technology support. These demonstrate that marine ranches and the government attach great importance to science and technology construction, marine ranching's management and maintenance, and monitoring and reporting. In sum, the response is the main driving cluster of MRES in Rongcheng marine ranches, and enterprises and the government have the best performance in the response cluster.

For the driver and impact clusters with an average contribution rate of 5%, the respective maximum values for contribution rates are 17% and 9% of $MR_7$ and $MR_2$, and the corresponding minimum values for contribution rates are 1% and 1% of $MR_{11}$ ($MR_5$) and $MR_9$. As the two clusters with the lowest contribution to MRES, driver and impact have a small influence on the MRES in Rongcheng. For example, even though a series of measures are taken by the government and marine ranching in ecological environment improvement and resource protection, the performance of marine ranching on the index "enterprise's ecological awareness" is not consistent with this. In the sample period, the proportion of environmental protection expenditure is 0 for 36% marine ranches; the proportion of environmental protection expenditure is 1% for 46% marine ranches; and the proportion is 2% for 18% marine ranches. There is no obvious improvement of fishery resources, which indicates the improvement of biodiversity index, and target biomass is not successful. Moreover, as the calculated cluster weights of driver and impact are 0.22 and 0.23 in Section 3.2.1, this indicates that paying attention to these two clusters is necessary for pursuing MRES in experts' views. In sum, although driver and impact are not the main driving clusters for MRES in Rongcheng, we can improve overall MRES value to a higher level by protecting resources and establishing awareness of ecological environment protection.

For the pressure and state clusters with average contribution rates of 12% and 19%, although these two clusters are not the main driving indexes of MRES, they are significant in terms of the MRES. The contribution rates of pressure and state are between the lowest and the highest, which is determined by the specific performance of marine ranching in these clusters. For example, each marine ranching in the sample has put net cages by green framing methods and carried out the construction of artificial reefs, which have had a positive influence. However, in some indexes, the performance of different marine ranching may be very different, or there is room for further improvement. For example, 45% of the marine ranches scored zero on the index “seaweed field and seagrass bed transplant cultivation,” while other marine ranches performed better in terms of the survival status of seaweed field and seagrass bed transplant cultivation. A total of 73% of the marine ranches reached the national level II standard of seawater quality, and 27% of the marine ranches reached the highest level I standard. In sum, the contribution of pressure and state to MRES is limited. Marine ranching performs well in artificial reef construction and maintenance, but there is room for further improvement in the marine environment and the construction of seaweed field and seagrass bed.

### *3.3. Discussion and Recommendations*

*3.3.1. Discussion.* First, the study offers the prioritization of evaluation indexes determined by the PLTS/ANP method. “Scientific management of fishery resources” is the most important evaluation index among all indexes. A possible explanation is that scientific management of fishery resources is a systematic project in the MRES construction, including not only the scientific management of marine fishery organisms but also the scientific management of marine ranch staff. The scientific management of fishery resources can be realized mainly through the following three ways. The ratio of feed to breeding objects should be within a reasonable range. In fishing production, compliant fishing methods and fishing tools should be used to selectively capture fishery organisms. They should set up the harvesting range of value-added organisms to achieve sustainable utilization. This shows that the scientific management of fishery resources involves a wide range of measures, which may lead to this index being the most important index in the weight calculation dominated by expert judgment.

The response cluster has the first priority among other clusters, which shows that the response cluster and all its indexes have more effects on MRES. A possible explanation is that the response cluster is the only cluster that has relationships with the other four clusters in the DPSIR model. This in itself gives important meaning to the response cluster. The indexes under the response cluster have an internal influence on the four clusters of driver, pressure, state, and impact. This relationship is ultimately reflected in the priority weight of MRES; that is, the weight of response is the largest. Furthermore, as the cluster that reflects the positive measures taken by enterprises to improve the MRES performance, the response includes indexes that reflect the following three points: the level of information construction and human resources construction of marine ranching, the scientific nature and input intensity in fishery resources management of marine ranching, and the development of annual monitoring activities of marine ranching. All these indexes directly reflect the sustainability of marine ranching in the construction of ecological security and can effectively realize the function of marine ranching in environmental protection, resource conservation, and sustainable fishery output.

Second, the comprehensive evaluation value of MRES offers evidence supporting that the current marine ranches in Rongcheng are at the medium security grade on the whole. For all the marine ranches in the sample, only 38% are at the security grade and can be recognized as performing well in ecological security. The overall situation is at an acceptable level, but there is still great room for improvement in the construction of ecological security. A possible explanation is that Rongcheng has launched some policies and measures with the government playing the leading role and enterprises serving as the main body in the construction of marine ranching in recent years, such as advocating scientific fishing, monitoring the seawater quality, and strengthening the supervision of marine ranching, which have promoted threshold quality of MRES performance in Rongcheng. At the same time, building a good MRES is not only led by the government. Just as in the case of Rongcheng, there are differences between marine ranches in terms of ecological security; therefore, marine ranches need to further improve the performance of MRES on their own.

Moreover, the evaluation results also suggest that there is not a strong correlation between MRES performance and whether the research objects are national or provincial marine ranching. In fact, there is little difference in the performance of average MRES value and safety level distribution between the eight national marine ranches and three provincial marine ranches in this sample. A possible explanation is that as mentioned above, the government of Rongcheng has made great efforts in policy supervision, marine environmental protection, and other public services, which may make the national and provincial marine ranching consistent in the acquisition of this part of public resources, so there is no difference in the average MRES value and safety level distribution between the MRES of provincial and national enterprises. However, there are only three provincial marine ranches selected in this study, which may lead to poor representability of the data. The average MRES value and safety level distribution calculated may not really represent the ecological security situation of provincial marine ranching in Rongcheng.

Third, the contribution rates of the five clusters to MRES show that response is the most important cluster for MRES in Rongcheng. Response’s 59% average contribution rate indicates that all marine ranches of Rongcheng in this sample are mainly driven by the response cluster. A possible explanation is that Rongcheng attaches great importance to information construction, scientific research support, and talent introduction. For example, the average value of cooperation between marine ranches and universities is 2.5, and 82% of the marine ranches have built visualization, intelligence, and information systems. It can be considered that for marine ranches in Rongcheng, the impact of the response cluster of the DPSIR evaluation model is the highest, and the evaluation indexes under the cluster have great importance. Therefore, the response cluster is an important source of strong MRES performance. From another point of view, while ensuring response cluster performance, we can further enhance MRES performance through other clusters with low contribution rates but high priority weight as calculated by the PLTS/ANP method. For example, although driver and impact clusters have the lowest contribution rates, we can enhance MRES by protecting resources and establishing awareness of ecological environment protection.

*3.3.2. Recommendations.* MRES, as a hot spot in the research of marine ranching, is of great significance for marine ecological security and the construction of marine ranching. The overall MRES performance in Rongcheng is at a medium level, and there is room for further enhancement. Based on the research results and analysis of MRES evaluation index weights and overall MRES values of marine ranches in this study, the following four suggestions are put forward to help enterprises, government, and third-party institutions work together to improve the ecological security level of marine ranches in Rongcheng.

First, according to the MRES evaluation index system constructed in this study and the weight results, marine ranching should pay attention to the scientific management of fishery resources and consciously improve enterprises' awareness of ecological and environmental protection. Furthermore, for enterprises, it is necessary to emphasize profitability and to pursue profit margins. These three aspects represent the first three important indexes in the evaluation index system. In addition, the significance of the response cluster leads enterprises to pay attention to scientific and technological capabilities, talent introduction, and marine ranching monitoring, among others.

Second, the construction of MRES is inseparable from the government's policy and supervision. The government should make clear the important role of the management and guidance of MRES, establish, and improve the supervision, evaluation, and management mechanism for MRES construction. Specifically, this includes strengthening the improvement and protection of seawater quality, ensuring the scientific fishing of marine living resources, and introducing support policies for MRES. These are the reasons why some marine ranches perform well or perform poorly in the previous analysis.

Third, marine ranches in Rongcheng are driven by the response cluster. In this context, enterprises can further enhance their performance in response to improve the MRES level. However, the clusters with low contribution rates but not low priority weights like driver or impact cluster should not be ignored to improve the MRES level. To improve the performance of marine ranching in these clusters, which are not the main driving forces for MRES, it will be more challenging for enterprises. However, it is necessary for improving the MRES level.

Fourth, the participation of third-party stakeholders should be considered in the construction of MRES. Talent and technology support are of great significance for MRES, and most of them are related to the indexes in the response cluster. Therefore, marine ranching should establish cooperative relationships of knowledge sharing and joint achievements with scientific research institutes and universities and establish directional training relationships for talent. The government can also guide the collaboration between marine ranches and third-party institutions to transfer technology and knowledge to enterprises and encourage innovative academic research related to MRES.

## 4. Conclusion

In recent years, the research on marine ranching has made rich achievements in concepts, ecological evaluation, breeding carrying capacity, resources, and environmental monitoring. The ecological benefits of environmental protection and resource conservation of marine ranching have been highlighted by the government and academia, but there are still many deficiencies in the practical operation and theoretical research. Therefore, it is necessary to study the evaluation of MRES and to measure the performance of system security and ecosystem services of marine ranching. This study evaluates MRES based on the DPSIR model and PLTS/ANP method and performs a case study of 11 marine ranches in the city of Rongcheng. The main contributions of this study can be summarized as follows.

First, the network model of MRES evaluation constructed by the ANP method reflected the interdependent relationships of evaluation indexes. In the existing research on MRES, the interdependent relationships between indexes are seldom considered, while the relationship between indexes is mostly treated as a linear one. However, considering that the marine ranching ecosystem is a subsystem of the marine ecosystem, it is necessary to reflect the internal and external dependence of the evaluation indexes. Second, the weights of indexes were determined by the PLTS/ANP method. PLTS is practical for expressing experts' preferences and hesitant views, and this method reflects the probability information of each language term. We used PLTS to provide not only the linguistic values but also the corresponding probabilistic information in obtaining the pairwise matrices. Third, the proposed MRES evaluation index system and PLTS/ANP method were applied to analyze 11 marine ranches in Rongcheng, Shandong Province. Four of them were in the insecurity grade, three in the medium security grade, and four in the security grade. Moreover, according to the analysis of index weights, comprehensive MRES evaluation values, and contribution rates of 11 marine ranches, the recommendations for government, enterprises, and third-party institutions were proposed to improve MRES performance.

However, there are limitations to this study. First, in this study, PLTS and ANP, which were used to determine the weights, are two subjective methods. Further research can integrate objective methods such as the entropy weight method to obtain the weights for simultaneous subjective and objective analysis. Second, the method for order relations and operations is recently introduced on the set of PLTS, and further research may be conducted with this new method to improve the effectiveness of MRES evaluation results.

## Data Availability

The data used to support the findings of this study are currently under embargo while the research findings are commercialized. Requests for data, 12 months after publication of this article, will be considered by the corresponding author.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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*Research Article*

# Normal Wiggly Probabilistic Hesitant Fuzzy Information for Environmental Quality Evaluation

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As a valuable tool for representing uncertain information, probabilistic hesitant fuzzy sets (PHFS) have gained considerable recognition and in-depth discussion in recent years to increase the flexibility and manifest hesitant information in decision-making problems. However, decision-makers (DMs) cannot express all preferences only through a few probabilistic terms in actual decision-making. Much critical information is hidden behind the original information provided by the DMs. Keeping that in mind, we are interested in mining deeper uncertain information from the original probabilistic hesitant fuzzy evaluation data. To achieve the target, we put forward a novel representation tool called the normal wiggly probabilistic hesitant fuzzy set (NWPHFS) to extract deeper uncertain preferences from original probabilistic information. NWPHFS retains the original evaluation information and carries and assesses the potential uncertain details for increasing the rationality of decision-making outcomes. Herein, we propose some fundamental concepts of NWPHFS, for instance, some elementary operational laws, distance measures between two NWPHFSs, and score function. We also suggest two new aggregation operators, that is, the normal wiggly probabilistic hesitant fuzzy weighted averaging (NWPHFWA) and normal wiggly probabilistic hesitant fuzzy weighted geometric (NWPHFWG). A novel mechanism is proposed here to work out multiattribute decision-making (MADM) in solving normal wiggly probabilistic decision-making problems. Through a practical example of environmental quality assessment, the specific calculation steps of this method are epitomized. Finally, we have demonstrated the feasibility and advancement of the proposed approach via a comprehensive comparative study.

## 1. Introduction

MADM problems are everywhere in our daily lives, and most people frequently face uncertain decision-making in all aspects of their lives, for example, which city to travel during a short holiday, which bag is suitable for shopping today, which mobile phone brand is more suitable for my needs, which kind of fruits to buy, and which clothes to wear today. While these common decision-making issues are easy to handle, no matter how many final choices are made, no errors can be significantly highlighted in MADM applications. However, no mistake can be tolerated in significant decision-making issues. The best or optimal decisions must be made for applications, including market analysis, quality assessment, and investment strategy. To address these problems, many effective representational models have been proposed and widely implemented, especially for MADM [1, 2]. Almost all decisions take several steps to reach the final destination, and some of them can be confusing in nature. Suppose that the data is analyzed without handling the uncertainty. In that case, their decision outcomes may be extremely vague, so it is important to include the DM's preference to deal with uncertainty. Fuzzy set (FS) theory [3] and its extensions, such as intuitionistic fuzzy set (IFS) [4], type-2 fuzzy sets [5], interval-valued intuitionistic fuzzy set [6], Pythagorean fuzzy set [7], hesitant fuzzy set (HFS) [8], probabilistic HFS (PHFS) [9], and proportional HFS (PrHFS) [10], are the most effective tools to deal with the impreciseness and uncertainties in the decision-making. The purpose of the above-mentioned different types of fuzzy sets is to more effectively express information on the uncertain and complex diagnosis of DM. In recent decades, these generalizations of fuzzy sets have been widely recognized in various academic studies, and considerable achievements have been made to adapt to different application environments. In FS, DMs only evaluate the objects by stating the degree of membership (preferences), or they can only provide their assessment in a crisp value. However, for DMs, the key issue in dealing with a real complex problem is determining the crisp value based on a given standard to reflect their given uncertainty and opacity. For example, suppose that the DM cannot determine which specific number to give for object/alternative under a particular attribute. In that case, he may give several numbers instead of the specific number to represent his assessment information. Keeping this fact in mind, among these extensions, the most famous generalization of the FS is HFS [11], in which membership of a particular element is allowed to represent a set possessing many possible values between $[0, 1]$. Therefore, HFS has a wider range of applications and more practical significance and is a handy tool for expressing people's hesitation in everyday life compared to other extensions. As it leads to expressing uncertainty, it has attracted the attention of researchers. For instance, Zhang [12] proposed different aggregation operators and several useful properties and discussed the relationship between them. Li et al. [13] developed a variety of novel similarities and distance measures between HFSs, in which both the values and the number of values of HFE are taken into account. Liu et al. [14] elucidated the correlation and distance measures for hesitant fuzzy information and analyzed their properties to measure the strength of the relationship between HFSs.

Since its initiation, several researchers have proposed a lot of research to support HFS theory [15–17]. In numerous decision-making problems, information is mainly vague or ambiguous because of the inaccurate/incomplete data, lack of time, partial attention, and the information processing skills of the decision-makers. Therefore, it is difficult for DMs to express their opinions in some specific numerical values. For instance, suppose that a consumer wants to buy a car. He mainly focuses on car safety features and asks an expert for advice. If the total is 100 points, the expert is 80% sure that the car's safety could be 60, and he is 20% sure that the score could be 70. The HFS $\{0.6, 0.7\}$ cannot represents the preferences that 0.7 is more suitable than 0.6. To cope with the situation, Zhu [9] merged the probability into the HFS and proposed PHFS, which can cover the expert's hesitations and retain more information than HFS. For example, PHFS for the above case can be written as $\{0.6\,(0.8), 0.7\,(0.2)\}$, where 0.8 and 0.2 are the probabilities to the original HFE values, which can be employed to characterize DMs preferences. Zhang et al. [18] proposed the novel aggregation operators and continuous form for the improved PHFS and PHFE. Li et al. [19] proposed the MADM method with PHF information based on the dominance degree of probabilistic hesitant fuzzy elements (PHFE) and the best-worst method. Wang and Li [20] studied PHFS operations to explore MADM problems and introduced an approach based on correlation coefficients that utilize probabilistic hesitant fuzzy information.

The development of various extension forms of PHFS supports DMs to articulate their assessment information about alternatives comprehensively. For instance, the interval-valued probabilistic hesitant fuzzy set (IVPHFS) presented by Krishankumar et al. [21] permits DMs to allot the probable values in interval forms. Also, Krishankumar et al. [22] discussed the IVPHFS under context when the weights of the attributes and DMs are unknown. The weights of the attributes are calculated using the interval-valued probabilistic hesitant deviation method. In contrast, the Bayesian approximation method is used to find the weights of the DMs under the environment of IVPHFS. IVPHFS is more likely to ask DMs to provide cognitive information through probabilistic hesitant fuzzy information and then ask them to further improve uncertainty assessments in various extension forms. Recently Noor et al. [23] proposed a new MADM method (tail decision-making) to find the best alternative by using the minimal information of the probabilistic interval-valued HFS. Chen et al. [24] proposed ordered weighted averaging operators generation algorithm for MADM problems. Xiong et al. [25] presented an extended power average operators for decision-making problems. However, these extensions become more complicated, which will increase the time cost and psychological burden of DM. It becomes difficult for us to evaluate the DMs' values, which they want to elaborate. Thus, obtaining more thorough investigation information to ensure the validity of the final decision results has become a hot topic for research. Recently Ren et al. [26] presented the normal wiggly hesitant fuzzy set (NWHFS) as an extension of HFS to explore the potential information hidden behind the original data. They assume that DMs' uncertainty can be considered a general fluctuation range based on HFE diagnostic values. Liu et al. [27] developed the normal wiggly hesitant fuzzy power Muirhead means to fully exert the strength by combining power average and Muirhead mean operators on the distance measure of the NWHFE.

To more accurately describe uncertain information, Liu et al. [28] proposed the new representation mechanism with the combination of linguistic terms set and NWHFS which resulted in a useful representation tool named normal wiggly hesitant fuzzy linguistic term set (NWHFLTS). Considering the advantages of NWHFS and TODIM, Liu and Zhang [31] defined the new distance measure of two NWHFEs and put forward an extended NWHF-TODIM method to handle the MADM problems under normal wiggly information, considering that if only membership functions represent a certain degree of the attributes, the importance of uncertainty becomes ignored. Based on the idea of Pythagorean hesitant fuzzy set, Yang et al. [29] proposed a normal wiggly Pythagorean hesitant fuzzy set (NWPaHFS) that took into account both membership and nonmembership aspects. Besides, Narayanamoorthy et al. [30] presented a normal wiggly dual hesitant fuzzy set (NWDHFS) as an extension of NWHFS and defined a new score function for the new fuzzy set. It can express the profound ideas of membership and nonmembership information. Liu and Zhang [32] combined the MABAC (multiattributive border approximation area comparison) with prospect theory, which considers DMs psychological behavior and proposes a new method under a normal wiggly environment for handling the complex and uncertain decision-making problems.

To facilitate a better understanding, we summarize the features and differences discussed above, which are listed in Table 1.

However, due to the increasing complexity of the fundamental issues and the uncertainty of decision-makers perception, in many circumstances, there are some difficulties for the DMs to quantify their preferences by several possible values or the behaviors of the DMs cannot be characterized by using crisp values. As HFS has a severe deficiency of information, this loss has also converted to NWHFS, leading to extreme data loss, and it should be addressed. To overcome the issue, PHFS has been used instead of HFS to minimize information loss. Therefore, according to the analysis discussed above and for the sake of overcoming the weaknesses, we shall propose a set named normal wiggly probabilistic hesitant fuzzy set (NWPHFS). The significant excellence of NWPHFS is that it could depict different attributes of a target in a single framework: possible hesitant fuzzy set, its corresponding probability, and the extracted hidden information from the original PHFS. Moreover, we propose some elementary operational laws and aggregation operators of NWPHFS to aggregate the wiggly probabilistic data. Furthermore, we establish an efficient and authentic approach to deal with MADM problems under a probabilistic environment. Finally, we apply the proposed method to the research of the environmental quality assessment. An illustrative example shows our proposed method's implementation process and demonstrates that our approach is more reliable and logical.

Here is a summary of the main contributions of this article:

(1) Considering the uncertain preferences hidden behind the original probabilistic hesitant information, we propose the normal wiggly probabilistic hesitant fuzzy set, a new extension of HFS

(2) Two new aggregation operators, the normal wiggly probabilistic hesitant fuzzy weighted averaging and the normal wiggly hesitant fuzzy weighted geometric, are put forward to conclude the rankings results of alternatives in decision-making problems

(3) We proposed a new MADM method to streamline the normal wiggly probabilistic hesitant information based on NWPHFWA and NWPHFWG operators to obtain the best alternatives

Comprehensively, the paper framework is designed in the following way: Section 2 describes the essential concepts consisting of HFS, PHFS, and NWHFS. Section 3 elaborates the NWPHFS, a new form of PHF information, the score function, operational rules, and the comparison rule of NWPHFEs. Section 4 describes the new methodology to develop the MADM problems when attributes values are expressed in NWPHFS. Section 5 explains the application stages in comparing other theories to demonstrate the feasibility and validity of the discussed method.

## 2. Preliminaries

In this section, we mainly review some basic concepts of HFS, PHFS, and NWHFS such as the operational laws, the score function, and the comparison method. Moreover, we give some examples to explain the given theories.

*Definition 1* (see [8]). For a given nonempty set $S$, HFS $H$ on $S$ is a function of $h_S(x)$ which when applied to $S$ returns to a finite subset of $[0,1]$. Mathematically,

$$H = \{\langle x, h_S(x)\rangle | x \in S\}, \tag{1}$$

where $h_S(x)$ is the discrete set of values from $[0,1]$ representing the possible membership degrees of the element $x \in S$, also called HFE, and for simplicity, we use $h_S(x) = h$. Subsequently, the score function, deviation function, and comparison rules were proposed and investigated as the basis for their calculation and application for HFEs [33].

*Example 1.* For any set $S = \{x_1, x_2, x_3\}$, let $h(x_1) = \{0.3, 0.4, 0.5\}, h(x_2) = \{0.2, 0.3, 0.5\}$, and $h(x_3) = \{0.1, 0.2\}$ be three HFEs. Then the set $H$ is called HFS and denoted as

$$H = \{\langle x_1, (0.3, 0.4, 0.5)\rangle, \langle x_2, (0.2, 0.3, 0.5)\rangle, \langle x_3, (0.1, 0.2)\rangle\}. \tag{2}$$

To enhance the preferences in decision-making problems, Zhu [9] extended the HFS to PHFS, defined as follows.

*Definition 2.* Let $S$ be any universe of discourse; then a PHFS on $S$ can be expressed by an expression

$$H_p = \left\{\langle x, h_p(x)\rangle | x \in S\right\}, \tag{3}$$

where $h_p(x) = \gamma_i(p_i)$, representing the membership degree of the element $x \in S$ and $\gamma_i, p_i \in [0,1]$. For simplicity, we denote

$$h_p(x) = h_p = \{\gamma_i(p_i) | i = 1, 2, 3, \ldots, \#h\}, \tag{4}$$

where $\#h$ is the number of possible elements in $h_p$, $p_i$ is the hesitant probability of $\gamma_i$, and $\sum_{i=1}^{\#h} p_i = 1$.

Table 1: A summary on the normal wiggly hesitant fuzzy set and its extensions.

| Studies | Different models | Characteristic of the elements | Extract information | Probabilistic information |
|---|---|---|---|---|
| [8] | HFS | A set of possible membership values | No | No |
| [10] | PrHFS | A set of possible membership values and their associated proportion | No | No |
| [9] | PHFS | A probabilistic distribution of several membership values | No | Yes |
| [26] | NWHFS | A set of possible membership values | Yes | No |
| [28] | NWHFLTS | A set of several ordered and continuous linguistic terms | Yes | No |
| [29] | NWPaHFS | A set of membership and nonmembership values such that the sum of square of membership and nonmembership values is less than one | Yes | No |
| [30] | NWDHFS | A set of various membership and nonmembership values | Yes | No |
| Proposed | NWPHFS | A probabilistic distribution of several membership values | Yes | Yes |

*2.1. Normalization of PHFEs.* Usually, for the PHFS, we always hope that all the elements have complete probabilistic information. If given, then the calculation of the PHFEs will be more straightforward, and the outcome of this set-based decision-making will be more accurate. Unfortunately, the probabilistic information is not always complete. To overcome this issue, Zhang et al. [18] estimate the missing probabilistic details based on the following principle.

*Definition 3.* For any PHFE $h_p$, if $\sum_{i=1}^{\#h} p_i < 1$, then its associated PHFE $\widehat{h}_p$ is defined as

$$\widehat{h}_p = \left\{\gamma_i\left(\widehat{p}_i\right) \mid i = 1, 2, 3, \ldots, \#h_p\right\}, \quad (5)$$

where $\widehat{p}_i = \left(p_i / \sum_{i=1}^{\#h} p_i\right)$.

To study further deeply the probable uncertainty concealed behind the assessments of DMs, Ren et al. [26] presented the concept of NWHFS.

*Definition 4.* For a given HFE $h = \{\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_{\#h}\}$, the mean value and the deviation function of all values in $h$ can be defined as

$$\overline{h} = \frac{1}{\#h} \sum_{i=1}^{\#h} \gamma_i, \sigma_h = \sqrt{\frac{1}{\#h} \sum_{i=1}^{\#h} \left(\gamma_i - \overline{h}\right)^2}, \quad (6)$$

respectively. The mapping $\widetilde{f}$ from $h$ to $[o, \sigma_h]$, which satisfies the relation,

$$\widetilde{f}\left(\gamma_i\right) = \sigma_h \cdot e^{-\left(\left(\gamma_i - \overline{h}\right)^2 / 2\sigma_h^2\right)}, \quad (7)$$

is said to be the normal wiggly range of $\gamma_i$.

*Definition 5* (see [26]). Let $h = \{\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_{\#h}\}$ be an HFE. The normalized HFE can be calculated by the expression

$$\begin{aligned} \widetilde{h} &= \left\{\frac{\gamma_1}{\operatorname{sum}(\gamma_i)}, \frac{\gamma_2}{\operatorname{sum}(\gamma_i)}, \frac{\gamma_3}{\operatorname{sum}(\gamma_i)}, \ldots, \frac{\gamma_{\#h}}{\operatorname{sum}(\gamma_i)}\right\}, \\ \widetilde{h} &= \{\widetilde{\gamma}_1, \widetilde{\gamma}_2, \widetilde{\gamma}_3, \ldots, \widetilde{\gamma}_{\#h}\}, \end{aligned} \quad (8)$$

where $\operatorname{sum}(\gamma_i) = \sum_{i=1}^{\#h} \gamma_i$. From the normalized set $\widetilde{h}$, the real preference degree of DMs can be computed as follows:

$$\operatorname{rpd}(\widetilde{h}) = \begin{cases} \sum_{i=1}^{\#h} \widetilde{\gamma}_i \left(\frac{\#\widetilde{h} - i}{\#\widetilde{h} - 1}\right), & \text{if } \overline{h} < 0.5, \\ 1 - \sum_{i=1}^{\#h} \widetilde{\gamma}_i \left(\frac{\#\widetilde{h} - i}{\#\widetilde{h} - 1}\right), & \text{if } \overline{h} > 0.5, \\ 0.5, & \text{if } \overline{h} = 0.5, \end{cases} \quad (9)$$

which is measured based on the orness, proposed by Yager [34].

*Definition 6* (see [26]). For any nonempty set $S$, consider an HFE, $h(x) = \{\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_{\#h}\}$, in an HFS, $H = \{\langle x, h_S(x)\rangle \mid x \in S\}$. The NWHFS can be stated as

$$\mathrm{NW} = \left\{\langle x, h_S(x), \psi\left(h_S(x)\right)\rangle \mid x \in S\right\}, \quad (10)$$

where $\psi\left(h_S(x)\right) = \left\{\widehat{\gamma}_1, \widehat{\gamma}_2, \widehat{\gamma}_2, \ldots, \widehat{\gamma}_{\#h(x)}\right\}$ and

$$\widehat{\gamma}_i = \left\{\tau_i^L, \tau_i^M, \tau_i^U\right\}, \quad (11)$$

where $\tau_i^L = \max(\gamma_i - \widetilde{f}(\gamma_i), 0)$, $\tau_i^M = (2 \cdot \operatorname{rpd}(\widetilde{h}(x)) - 1)\widetilde{f}(\gamma_i) + \gamma_i$, and $\tau_i^U = \min \gamma_i + \widetilde{f}(\gamma_i), 1$. $\gamma_i$ is one of the elements in HFE, $\widetilde{f}(\gamma_i)$ is the wiggly parameter, and $\operatorname{rpd}(\widetilde{h}(x))$ is the real preference degree which can be found using (9). Furthermore, $\psi(h(x))$ is an NWE. For simplicity, $(h(x), \psi(h(x)))$ can be labeled as $(h, \psi(h))$, which is NWHFE.

For a better understanding, we give a simple numerical example of NWHFS.

*Example 2.* Let $S = \{x_1, x_{2,} x_3\}$ and consider the HFS set $H$ to be

$$H = \{\langle x_1, (0.3, 0.4, 0.5)\rangle, \langle x_2, (0.2, 0.3, 0.5)\rangle, \langle x_3, (0.1, 0.2)\rangle\}. \tag{12}$$

NWHFS can be obtained according to Definitions 5 and 6:

$$\mathrm{NW}_H = \left\{ \begin{array}{llll} x_1 & (0.2614, 0.2936, 0.3386) & (0.3184, 0.3864, 0.4816) & (0.4614, 0.4936, 0.5386) \\ x_2 & (0.1296, 0.1789, 0.2704) & (0.1797, 0.2639, 0.4203) & (0.4489, 0.4847, 0.5511) \\ x_3 & (0.0184, 0.1272, 0.1816) & (0.1614, 0.2129, 0.2386) & \end{array} \right\}. \tag{13}$$

Some properties and operational laws for the comparison of two NWHFEs are defined as follows.

*Definition 7* (see [26]). For the two NWHFEs $(h_1, \psi(h_1))$ and $(h_2, \psi(h_2))$, the score values can be calculated by the expression

$$S_{\mathrm{NW}}(h_1, \psi(h_1)) = \lambda\left(\overline{h} - \sigma_h\right) + (1-\lambda)\left(\frac{1}{\#h}\sum_{i=1}^{\#h}\left(\overline{\widehat{\gamma}}_i - \tau_{\widehat{\gamma}_i}\right)\right), \tag{14}$$

where $\overline{\widehat{\gamma}}_i = ((\tau_i^L + \tau_i^M + \tau_i^U)/3)$ and $\tau_{\widehat{\gamma}_i} = \sqrt{(\tau_i^L)^2 + (\tau_i^M)^2 + (\tau_i^U)^2 - \tau_i^L\tau_i^M - \tau_i^M\tau_i^U - \tau_i^L\tau_i^U}$ and parameter $\lambda \in (0,1)$ is the confidence level of the DMs which can be selected according to the information given by them.

The assessment of any two NWHFEs $(h_1, \psi(h_1))$ and $(h_2, \psi(h_2))$ based on Definition 7 can be defined as follows:

(1) If $S_{\mathrm{NW}}(h_1, \psi(h_1)) > S_{\mathrm{NW}}(h_2, \psi(h_2))$, then $(h_1, \psi(h_1))$ is preferable to $(h_2, \psi(h_2))$ and is described as $(h_1, \psi(h_1)) \succ (h_2, \psi(h_2))$

(2) If $S_{\mathrm{NW}}(h_1, \psi(h_1)) < S_{\mathrm{NW}}(h_2, \psi(h_2))$, then $(h_2, \psi(h_2))$ is preferable to $(h_1, \psi(h_1))$ and is described as $(h_1, \psi(h_1)) \prec (h_2, \psi(h_2))$

(3) If $S_{\mathrm{NW}}(h_1, \psi(h_1)) = S_{\mathrm{NW}}(h_2, \psi(h_2))$, then both $(h_1, \psi(h_1))$ and $(h_2, \psi(h_2))$ are equivalent and are described as $(h_1, \psi(h_1)) \sim (h_2, \psi(h_2))$

*Definition 8* (see [26]). For any two NWHFEs $(h_1, \psi(h_1))$ and $(h_2, \psi(h_2))$ and $\kappa > 0$, we have

(1) $(h_1, \psi(h_1))\widehat{\oplus}(h_2, \psi(h_2)) = (\cup_{\gamma_1 \in h_1, \gamma_2 \in h_2}\gamma_1 + \gamma_2 - \gamma_1\gamma_2, \cup_{\widehat{\gamma}_1 \in \psi(h_1), \widehat{\gamma}_2 \in \psi(h_2)}\widehat{\gamma}_1 \oplus \widehat{\gamma}_2)$

(2) $(h_1, \psi(h_1))\widehat{\otimes}(h_2, \psi(h_2)) = (\cup_{\gamma_1 \in h_1, \gamma_2 \in h_2}\gamma_1\gamma_2, \cup_{\widehat{\gamma}_1 \in \psi(h_1), \widehat{\gamma}_2 \in \psi(h_2)}\widehat{\gamma}_1 \otimes \widehat{\gamma}_2)$

(3) $(h_1, \psi(h_1))^\kappa = (\cup_{\gamma_1 \in h_1}\gamma_1^\kappa, \cup_{\widehat{\gamma}_1 \in \psi(h_1)}\widehat{\gamma}_1^\kappa)$

(4) $\kappa(h_1, \psi(h_1)) = (\cup_{\gamma_1 \in h_1} 1 - (1-\gamma_1)^\kappa, \cup_{\widehat{\gamma}_1 \in \psi(h_1)}\kappa\widehat{\gamma}_1)$

## 3. Normal Wiggly Probabilistic Hesitant Fuzzy Set

Recently, Ren et al. [26] proposed the NWHFS to ensure the validity of evaluation results, which developed a methodology to dig the secret consent of DMs from the actual evaluation information. In this part, we put forward the concepts of normal wiggly parameter and real preference degree to mine the uncertain preferences hidden behind the original PHFS. Based on this, we present the NWPHFS with its operational laws and comparison method.

*Definition 9.* Let $h_p = \{\gamma_1(p_1), \gamma_2(p_2), \gamma_3(p_3), \ldots, \gamma_{\#h}(p_{\#h})\}$ be a PHFE. Then mean and deviation of all values in $h_p$ are defined as

$$\begin{aligned} \overline{h}_p &= \frac{\left(\sum_{i=1}^{\#h}\gamma_i p_i\right)}{\sum_{i=1}^{\#h} p_i}, \\ \sigma_{h_p} &= \frac{\sum_{i=1}^{\#h} p_i\left(\gamma_i - \overline{h}_p\right)^2}{\sum_{i=1}^{\#h} p_i}, \end{aligned} \tag{15}$$

and the mapping $\widetilde{g}$: $h_p \longrightarrow [0, \sigma_{h_p}]$ is defined as

$$\widetilde{g}(\gamma_i(p_i)) = \sigma_{h_p} e^{-0.5\left(p_i\left(\gamma_i - \overline{h}_p\right)^2 / \left(\sigma_{h_p}\right)^2\right)}. \tag{16}$$

Then, the interval $[\gamma_i - \widetilde{g}(\gamma_i(p_i)), \gamma_i + \widetilde{g}(\gamma_i(p_i))]$ with the associated probabilistic element is called normal wiggly range of the element $\gamma_i(p_i)$.

For further clarity, an example is given in the following.

*Example 3.* Consider $h_p = \{0.2(0.1), 0.3(0.2), 0.6(0.3)\}$. The mean, deviation, and wiggle range value using Definition 9 can be calculated as

$$\begin{aligned}\overline{h}_p &= \frac{0.2(0.1)+0.3(0.2)+0.6(0.3)}{0.1+0.2+0.3} = 0.4333,\\ \sigma_{h_p} &= \frac{0.1(0.2-0.4333)^2+0.2(0.3-0.4333)^2+0.3(0.6-0.4333)^2}{0.1+0.2+0.3} = 0.02889,\end{aligned} \tag{17}$$

and wiggle range corresponding to the probabilistic elements using (16) is

$$\begin{cases} 0.2(0.1) \longrightarrow [0.01889, 0.02111],\\ 0.3(0.2) \longrightarrow [0.05657, 0.06343],\\ 0.6(0.3) \longrightarrow [0.1798, 0.1802]. \end{cases} \tag{18}$$

Real preference degree measured and defined by Ren et al. [26] is applicable for the HFE. For the PHFE, we extend the preference degree as follows.

*Definition 10.* The real preference degree of the DM in a PHFE can be calculated based on the degree of orness [34] which can be defined as

$$\operatorname{rpd}(\widehat{h}_p) = \begin{cases} \sum_{i=1}^{\#h} \gamma_i \left(\dfrac{\#\widehat{h}_p - i}{\#\widehat{h}_p - 1}\right)\widehat{p}_i, & \text{if } \overline{h}_p < 0.5,\\ 1 - \sum_{i=1}^{\#h} \gamma_i \left(\dfrac{\#\widehat{h}_p - i}{\#\widehat{h}_p - 1}\right)\widehat{p}_i, & \text{if } \overline{h}_p > 0.5,\\ 0.5, & \text{if } \overline{h}_p = 0.5, \end{cases} \tag{19}$$

where $\widehat{h}_p$ is the normalized set of $h_p$ calculated as

$$\widehat{h}_p = \left\{\gamma_1(\widehat{p}_1), \gamma_2(\widehat{p}_2), \widehat{\gamma}_3(p_3), \ldots, \widehat{\gamma}_{\#h_p}\left(p_{\#h_p}\right)\right\}, \tag{20}$$

and $\widehat{p}_i = (p_i / \sum_{i=1}^{\#h} p_i)$, $(i = 1, 2, 3, \ldots, \#h_p)$.

*Example 4.* Consider a PHFE $h_p = \{0.2(0.1), 0.3(0.2), 0.6(0.3)\}$. First the normalized set for the determination of real preference degree can be calculated. After the normalization, real preference degree can be find using equation (19) as

$$\widehat{h}_p = \{0.2(0.1667), 0.3(0.3333), 0.6(0.5)\}. \tag{21}$$

Then,

$$\operatorname{rpd}(\widehat{h}_p) = 0.08333. \tag{22}$$

*Definition 11.* Let $S = \left\{\langle x, h_p(x)\rangle \mid x \in S\right\}$ be a PHFS. The NWPHFS on $S$ is denoted as

$$\mathrm{NWP} = \left\{\langle x, h_p(x), \xi\left(h_p(x)\right) \mid \widehat{p}\rangle\right\}, \tag{23}$$

where $h_p(x)$ is the PHFE, and

$$\begin{aligned} \xi\left(h_p(x)\right) \mid \widehat{p} &= \left\{\widehat{\gamma}_1(\widehat{p}_1), \widehat{\gamma}_2(\widehat{p}_2), \widehat{\gamma}_3, \ldots, \widehat{\gamma}_{\#h_p}(\widehat{p}_{\#h})\right\},\\ \widehat{\gamma}_i &= \left\{\delta_i^L, \delta_i^M, \delta_i^U\right\},\\ \delta_i^L &= \max\left(\gamma_i - \overline{g}(\gamma_i(p_i)), 0\right),\\ \delta_i^M &= \left(2\operatorname{rpd}\left(\widehat{h}_p(x)\right) - 1\right)\overline{g}(\gamma_i(p_i)) + \gamma_i \cdot p_i,\\ \delta_i^U &= \min\left(\gamma_i + \overline{g}(\gamma_i(p_i)), 1\right), \end{aligned} \tag{24}$$

where $\gamma_i = 1, 2, 3, \ldots, \#h_p$. The pair $\langle h_p(x), \xi(h_p(x)) \mid \widehat{p}\rangle$ is called NWHPHFE; for simplicity, we symbolize it as $\langle h_p, \xi(h_p)\rangle$.

For further understanding, an example is given below:

$$\mathrm{NWP}_{H_p} = \begin{cases} x_1, & \begin{pmatrix}0.01889,\\0.01908,\\0.02111\end{pmatrix}\Bigg|0.1667, \begin{pmatrix}0.05657,\\0.05714,\\0.06343\end{pmatrix}\Bigg|0.3333, \begin{pmatrix}0.17980,\\0.17984,\\0.18020\end{pmatrix}\Bigg|0.5,\\ x_2, & \begin{pmatrix}0.05672,\\0.06000,\\0.06328\end{pmatrix}\Bigg|0.2, \begin{pmatrix}0.19162,\\0.20000,\\0.20838\end{pmatrix}\Bigg|0.5, \begin{pmatrix}0.23999,\\0.24000,\\0.24001\end{pmatrix}\Bigg|0.3,\\ x_3, & \begin{pmatrix}0.12000,\\0.12000,\\0.12000\end{pmatrix}\Bigg|0.3333, \begin{pmatrix}0.23841,\\0.24074,\\0.24159\end{pmatrix}\Bigg|0.4444, \begin{pmatrix}0.14000,\\0.14000,\\0.14000\end{pmatrix}\Bigg|0.2222. \end{cases} \tag{25}$$

*Example 5.* For a PHFS $H_p = \{\langle x_1, (0.2(0.1), 0.3(0.2), 0.6(0.3))\rangle, \langle x_2, (0.3(0.2), 0.4(0.5), 0.8(0.3))\rangle, \langle x_3, (0.4(0.3), 0.6(0.4), 0.7(0.2))\rangle\}$. Then, using Definition 11, we get an NWPHFS:

For the evaluation of NWPHF information, the score function defined below can simplify the probabilistic information into crisp values that can rationalize real-time information. As NWPHFS is in the form of the triangular fuzzy number [35, 36], some operations of the triangular fuzzy number are involved in finding the score of the NWPHFE.

*Definition 12.* Let $\langle h_p, \xi(h_p)\rangle$ be an NWPHFE; $\overline{h}_p$ and $\sigma_{h_p}$ are the mean value and deviation values of $h_p$. Then the score function of $\langle h_p, \xi(h_p)\rangle$ is calculated as follows:

$$S_{\mathrm{NWP}}(\langle h_p, \xi(h_p)\rangle) = \lambda\left(\overline{h}_p - \sigma_{h_p}\right) + (1-\lambda)\left(\sum_{i=1}^{\#h}\left(\overline{\widehat{\gamma}}_i - \tau_{\widehat{\gamma}_i}\right)^2 \widehat{p}_i\right), \tag{26}$$

where

$$\begin{aligned}\overline{\widehat{\gamma}}_i &= \frac{\left(\delta_i^L + \delta_i^M + \delta_i^U\right)}{3}\cdot \widehat{p}_i,\\ \tau_{\widehat{\gamma}_i} &= \sqrt{\left(\left(\delta_i^L\right)^2 + \left(\delta_i^M\right)^2 + \left(\delta_i^U\right)^2 - \delta_i^L\cdot\delta_i^M - \delta_i^M\cdot\delta_i^U - \delta_i^U\cdot\delta_i^L\right)}\cdot \widehat{p}_i.\end{aligned} \tag{27}$$

Here, $\lambda$ indicates the attitude and risk-bearing of the decision-makers.

(1) If the value of $\lambda$ is greater than 0.5, then he is a risk averter
(2) If the value of $\lambda$ is less than 0.5, then he is risk-averse
(3) If the value of $\lambda$ is 0.5, then he is not taking any risk

The following conclusions can easily be obtained based on Definition 12.

For any two different NWPHFEs $\langle h_p^1, \xi(h_p^1)\rangle$ and $\langle h_p^2, \xi(h_p^2)\rangle$, their corresponding score values are $S_{\mathrm{NWP}}\langle h_p^1, \xi(h_p^1)\rangle$ and $S_{\mathrm{NWP}}\langle h_p^2, \xi(h_p^2)\rangle$, respectively:

(1) If $S_{\mathrm{NWP}}\langle h_p^1, \xi(h_p^1)\rangle > S_{\mathrm{NWP}}\langle h_p^2, \xi(h_p^2)\rangle$, then $\langle h_p^1, \xi(h_p^1)\rangle$ is preferable to $\langle h_p^2, \xi(h_p^2)\rangle$ and is described as $\langle h_p^1, \xi(h_p^1)\rangle \succ \langle h_p^2, \xi(h_p^2)\rangle$
(2) If $S_{\mathrm{NWP}}\langle h_p^1, \xi(h_p^1)\rangle < S_{\mathrm{NWP}}\langle h_p^2, \xi(h_p^2)\rangle$, then $\langle h_p^2, \xi(h_p^2)\rangle$ is preferable to $\langle h_p^1, \xi(h_p^1)\rangle$ and is described as $\langle h_p^1, \xi(h_p^1)\rangle \prec \langle h_p^2, \xi(h_p^2)\rangle$
(3) If $S_{\mathrm{NWP}}\langle h_p^1, \xi(h_p^1)\rangle = S_{\mathrm{NWP}}\langle h_p^2, \xi(h_p^2)\rangle$, then both $\langle h_p^2, \xi(h_p^2)\rangle$ and $\langle h_p^1, \xi(h_p^1)\rangle$ are equivalent and are described as $\langle h_p^1, \xi(h_p^1)\rangle \sim \langle h_p^2, \xi(h_p^2)\rangle$

For better understanding, an example is given in the following.

*Example 6.* The NWPHFS is taken from Example 5. Then the score value and the other values which are used to find the score are shown in Table 2. The parameter $(\lambda = (1/2))$ is taken just for the simplicity which means that the DMs are neutral.

According to Definition 12, the ranking is

$$\mathrm{NWPHFE}_{x_3} \succ \mathrm{NWPHFE}_{x_2} \succ \mathrm{NWPHFE}_{x_1}. \tag{28}$$

*3.1. Basic Operations for the NWPHFEs.* Like the operational rule of HFEs [33] and NWHFEs, the following are basic rules for the operation of NWPHFEs.

(1) $(\langle h_p^1, \xi(h_p^1)\rangle)^\lambda = [\cup_{\gamma_{1_i}\in h_p^1}\{\gamma_{1_i}^\lambda(\widehat{p}_{1_i})\}, \cup_{\widehat{\gamma}_{1_i}\in h_p^1}\{\widehat{\gamma}_{1_i}^\lambda(\widehat{p}_{1_i})\}]$
(2) $\lambda(\langle h_p^1, \xi(h_p^1)\rangle) = [\cup_{\gamma_{1_i}\in h_p^1}\{[1-(1-\gamma_{1i})^l](\widehat{p}_{1_i})\}, \cup_{\widehat{\gamma}_{1_i}\in h_p^1}\{\lambda\widehat{\gamma}_{1_i}(\widehat{p}_{1_i})\}]$
(3) $\langle h_p^1, \xi(h_p^1)\rangle\widehat{\oplus}\langle h_p^2, \xi(h_p^2)\rangle = [\cup_{\gamma_{1_i}\in h_p^1,\gamma_{2_j}\in h_p^2}\{[\gamma_{1_i}+\gamma_{2_j} - \gamma_{1_i}\gamma_{2_j}](\widehat{p}_{1_i}\widehat{p}_{2_j})\}, \cup_{\widehat{\gamma}_{1_i}\in h_p^1,\widehat{\gamma}_{2_j}\in h_p^2}\{[\widehat{\gamma}_1\oplus\widehat{\gamma}_2](\widehat{p}_{1_i}\widehat{p}_{2_j})\}]$
(4) $\langle h_p^1, \xi(h_p^1)\rangle\widehat{\oplus}\langle h_p^2, \xi(h_p^2)\rangle\widehat{\otimes} = [\cup_{\gamma_{1_i}\in h_p^1,\gamma_{2_j}\in h_p^2}\{[\gamma_{1_i}\gamma_{2_j}](\widehat{p}_{1_i}\widehat{p}_{2_j})\}, \cup_{\widehat{\gamma}_{1_i}\in h_p^1,\widehat{\gamma}_{2_j}\in h_p^2}\{[\widehat{\gamma}_{1_i}\otimes\widehat{\gamma}_{2_j}](\widehat{p}_{1_i}\widehat{p}_{2_j})\}]$

*Definition 13.* Let $\langle h_p^1, \xi(h_p^1)\rangle$ and $\langle h_p^2, \xi(h_p^2)\rangle$ be two NWPHFEs and $\lambda > 0$; then

From the operational rules proposed above, we can see that the results are also NWPHFEs.

**Theorem 1.** *Let $\langle h_p^1, \xi(h_p^1)\rangle$, $\langle h_p^2, \xi(h_p^2)\rangle$, and $\langle h_p^3, \xi(h_p^3)\rangle$ be three NWPHFEs; $\lambda > 0, \lambda_1 > 0, \lambda_2 > 0$, and then*

*(1)* $\langle h_p^1, \xi(h_p^1)\rangle\widehat{\oplus}\langle h_p^2, \xi(h_p^2)\rangle = (\langle h_p^2, \xi(h_p^2)\rangle)\widehat{\oplus}(\langle h_p^1, \xi(h_p^1)\rangle)$
*(2)* $\langle h_p^1, \xi(h_p^1)\rangle\widehat{\oplus}(\langle h_p^2, \xi(h_p^2)\rangle\widehat{\oplus}\langle h_p^3, \xi(h_p^3)\rangle) = (\langle h_p^1, \xi(h_p^1)\rangle\widehat{\oplus}\langle h_p^2, \xi(h_p^2)\rangle)\widehat{\oplus}\langle h_p^3, \xi(h_p^3)\rangle$
*(3)* $\lambda(\langle h_p^1, \xi(h_p^1)\rangle\widehat{\oplus}\langle h_p^2, \xi(h_p^2)\rangle) = \lambda\langle h_p^1, \xi(h_p^1)\rangle\widehat{\oplus}\lambda\langle h_p^2, \xi(h_p^2)\rangle$
*(4)* $(\langle h_p^1, \xi(h_p^1)\rangle\widehat{\oplus}\langle h_p^2, \xi(h_p^2)\rangle)^\lambda = (\langle h_p^1, \xi(h_p^1)\rangle)^\lambda\widehat{\otimes}(\langle h_p^2, \xi(h_p^2)\rangle)^\lambda$
*(5)* $((\langle h_p^1, \xi(h_p^1)\rangle)^{\lambda_1})^{\lambda_2} = (\langle h_p^1, \xi(h_p^1)\rangle)^{\lambda_1\lambda_2}$

Table 2: Mean value and deviation value of PHFEs.

| | $\overline{h}_p$ | $\sigma_{h_p}$ | | $\overline{\overline{\gamma}}_i$ | | | $\tau_{\widehat{\gamma}_i}$ | | $S_{\mathrm{NWP}}$ |
|---|---|---|---|---|---|---|---|---|---|
| $x_1$ | 0.4333 | 0.02889 | 0.00328 | 0.01968 | 0.0900 | 0.0004 | 0.0022 | 0.0002 | 0.2043 |
| $x_2$ | 0.5 | 0.04 | 0.01200 | 0.10000 | 0.0720 | 0.0011 | 0.0073 | 0.0000 | 0.2329 |
| $x_3$ | 0.5556 | 0.01358 | 0.04 | 0.10678 | 0.0311 | 0.0000 | 0.0013 | 0.0000 | 0.2738 |

*Proof*

(1) $\langle h_p^1, \xi(h_p^1)\rangle \widehat{\oplus} \langle h_p^2, \xi(h_p^2)\rangle = [\cup_{\gamma_{1_i}\in h_p^1, \gamma_{2_j}\in h_p^2}\{[\gamma_{1_i} + \gamma_{2_j} - \gamma_{1_i}\gamma_{2_j}](p_{1_i}p_{2_j}/\sum_{i=1}^n p_{1_i}\sum_{i=1}^n p_{2_j})\}, \cup_{\widehat{\gamma}_{1_i}\in h_p^1, \widehat{\gamma}_{2_j}\in h_p^2}\{[\widehat{\gamma}_1 \oplus \widehat{\gamma}_2](p_{1_i}p_{2_j}/\sum_{i=1}^n p_{1_i}\sum_{i=1}^n p_{2_j})\}] = [\cup_{\gamma_{1_i}\in h_p^1, \gamma_{2_j}\in h_p^2}\{[\gamma_{2_i} + \gamma_{1_j} - \gamma_{2_i}\gamma_{1_j}](\widehat{p}_{1_i}\widehat{p}_{2_j})\}, \cup_{\widehat{\gamma}_{1_i}\in h_p^1, \widehat{\gamma}_{2_j}\in h_p^2}\{[\widehat{\gamma}_2 \oplus \widehat{\gamma}_1](\widehat{p}_{1_i}\widehat{p}_{2_j})\}] = \langle h_p^2, \xi(h_p^2)\rangle \widehat{\oplus} \langle h_p^1, \xi(h_p^1)\rangle$

(2) Obvious as (1)

(3) $\lambda(\langle h_p^1, \xi(h_p^1)\rangle \widehat{\oplus} \langle h_p^2, \xi(h_p^2)\rangle) = \lambda[\cup_{\gamma_{1_i}\in h_p^1, \gamma_{2_j}\in h_p^2}\{[\gamma_{1_i} + \gamma_{2_j} - \gamma_{1_i}\gamma_{2_j}](\widehat{p}_{1_i}\widehat{p}_{2_j})\}, \cup_{\widehat{\gamma}_{1_i}\in h_p^1, \widehat{\gamma}_{2_j}\in h_p^2}\{[\widehat{\gamma}_1 \oplus \widehat{\gamma}_2](\widehat{p}_{1_i}\widehat{p}_{2_j})\}] = [\cup_{\gamma_{1_i}\in h_p^1, \gamma_{2_j}\in h_p^2}\{(1-(1-(\gamma_{1_i} + \gamma_{2_j} - \gamma_{1_i}\gamma_{2_j})^{\lambda}))(\widehat{p}_{1_i}\widehat{p}_{2_j})\}, \cup_{\widehat{\gamma}_{1_i}\in h_p^1, \widehat{\gamma}_{2_j}\in h_p^2}\{\lambda(\widehat{\gamma}_1 \oplus \widehat{\gamma}_2)(\widehat{p}_{1_i}\widehat{p}_{2_j})\}] = [\cup_{\gamma_{1_i}\in h_p^1, \gamma_{2_j}\in h_p^2}\{[1-(1-(1-(1-\gamma_{1_i})(1-\gamma_{2_j}))^{\lambda})](\widehat{p}_{1_i}\widehat{p}_{2_j})\}, \cup_{\widehat{\gamma}_{1_i}\in h_p^1, \widehat{\gamma}_{2_j}\in h_p^2}\{(\lambda\widehat{\gamma}_1 \oplus \lambda\widehat{\gamma}_2)(\widehat{p}_{1_i}\widehat{p}_{2_j})\}] = [\cup_{\gamma_{1_i}\in h_p^1, \gamma_{2_j}\in h_p^2}\{[1-(1-\gamma_{1_i})^{\lambda} + 1-(1-\gamma_{2_j})^{\lambda} - (1-(1-\gamma_{1_i})^{\lambda})(1-(1-\gamma_{2_j})^{\lambda})](\widehat{p}_{1_i}\widehat{p}_{2_j})\}, \cup_{\widehat{\gamma}_{1_i}\in h_p^1, \widehat{\gamma}_{2_j}\in h_p^2}\{(\lambda\widehat{\gamma}_1 \oplus \lambda\widehat{\gamma}_2)(\widehat{p}_{1_i}\widehat{p}_{2_j})\}] = \lambda\langle h_p^1, \xi(h_p^1)\rangle \widehat{\oplus} \lambda\langle h_p^2, \xi(h_p^2)\rangle$

(4) $(\langle h_p^1, \xi(h_p^1)\rangle \widehat{\otimes} \langle h_p^2, \xi(h_p^2)\rangle)^{\lambda} = [\cup_{\gamma_{1_i}\in h_p^1, \gamma_{2_j}\in h_p^2}\{\gamma_{1_i}\gamma_{2_j}(\widehat{p}_{1_i}\widehat{p}_{2_j})\}, \cup_{\widehat{\gamma}_{1_i}\in h_p^1, \widehat{\gamma}_{2_j}\in h_p^2}\{\widehat{\gamma}_{1_i}^{\lambda}\widehat{\gamma}_{2_j}^{\lambda}(\widehat{p}_{1_i}\widehat{p}_{2_j})\}] = [\cup_{\gamma_{1_i}\in h_p^1, \gamma_{2_j}\in h_p^2}\{\gamma_{1_i}^{\lambda}\gamma_{2_j}^{\lambda}(\widehat{p}_{1_i}\widehat{p}_{2_j})\}, \cup_{\widehat{\gamma}_{1_i}\in h_p^1, \widehat{\gamma}_{2_j}\in h_p^2}\{\widehat{\gamma}_{1_i}^{\lambda}\widehat{\gamma}_{2_j}^{\lambda}(\widehat{p}_{1_i}\widehat{p}_{2_j})\}] = (\langle h_p^1, \xi(h_p^1)\rangle)^{\lambda} \widehat{\otimes} (\langle h_p^2, \xi(h_p^2)\rangle)^{\lambda}$

(5) $((\langle h_p^1, \xi(h_p^1)\rangle)^{\lambda_1})^{\lambda_2} = [\cup_{\gamma_{1_i}\in h_p^1}\{\gamma_{1_i}^{\lambda_1}(\widehat{p}_{1_i})\}, \cup_{\widehat{\gamma}_{1_i}\in h_p^1}\{\widehat{\gamma}_{1_i}^{\lambda_1}(\widehat{p}_{1_i})\}]^{\lambda_2} = [\cup_{\gamma_{1_i}\in h_p^1}\{\gamma_{1_i}^{\lambda_1\lambda_2}(\widehat{p}_{1_i})\}, \cup_{\widehat{\gamma}_{1_i}\in h_p^1}\{\widehat{\gamma}_{1_i}^{\lambda_1\lambda_2}(\widehat{p}_{1_i})\}] = (\langle h_p^1, \xi(h_p^1)\rangle)^{\lambda_1\lambda_2}$ □

### 3.2. Aggregation Operators for the NWPHFEs

Aggregation operators for the NWPHFS depend upon the properties given in Section 3.1. These operators are very suitable and significant to handle the MADM problems with NWPHF information.

*Definition 14.* Consider any NWPHFS, $\mathrm{NWP} = \{\langle h_p^i, \xi(h_p^i)\rangle | i = 1, 2, 3, \ldots, n\}$, a collection of NWPHFEs and let $\omega = (\omega_1, \omega_2, \omega_3, \ldots, \omega_n)$ be the weight vectors of $\langle h_p^i, \xi(h_p^i)\rangle$ with $\omega_i \in [0, 1]$ and $\sum_{i=1}^n \omega_i = 1$. Then, an NWPHF weighted averaging (NWPHFWA) operator is defined as follows:

$$\begin{aligned}
&\mathrm{NWPHFWA}\left(\left\{\langle h_p^i, \xi\left(h_p^i\right)\rangle | i = 1, 2, 3, \ldots, n\right\}\right) = \widehat{\oplus}_{i=1}^n \omega_i\left(\langle h_p^i, \xi\left(h_p^i\right)\rangle\right) \\
&= \left\{\bigcup_{\gamma_i \in h_p^i}\left(1 - \prod_{i=1}^n (1-\gamma_i)^{\omega_i}\right)\left(\frac{\prod_{i=1}^n p_i}{\prod_{i=1}^n \left(\sum_{i=1}^n p_i\right)}\right), \bigcup_{\widehat{\gamma}_i \in h_p^i}\left(\bigoplus_{i=1}^n \omega_i(\widehat{\gamma}_i)\right)\left(\frac{\prod_{i=1}^n p_i}{\prod_{i=1}^n \left(\sum_{i=1}^n p_i\right)}\right)\right\}.
\end{aligned} \tag{29}$$

If $\omega = ((1/n), (1/n), (1/n), \ldots, (1/n))$, then (29) reduces to NWPHFA operator. The NWPHFWA operator helps solve MADM problems, and its practical application is shown in Section 5.

$$\begin{aligned}
&\mathrm{NWPHFWG}\left(\left\{\langle h_p^i, \xi\left(h_p^i\right)\rangle | i = 1, 2, 3, \ldots, n\right\}\right) = \widehat{\otimes}_{i=1}^n \left(\langle h_p^i, \xi\left(h_p^i\right)\rangle\right)^{\omega_i} \\
&= \left\{\cup_{\gamma_i \in h_p^i}\left(\prod_{i=1}^n \gamma_i^{\omega_i}\right)\left(\frac{\prod_{i=1}^n p_i}{\prod_{i=1}^n \left(\sum_{i=1}^n p_i\right)}\right), \cup_{\widehat{\gamma}_i \in h_p^i}\left(\bigotimes_{i=1}^n (\widehat{\gamma}_i)^{\omega_i}\right)\left(\frac{\prod_{i=1}^n p_i}{\prod_{i=1}^n \left(\sum_{i=1}^n p_i\right)}\right)\right\}.
\end{aligned} \tag{30}$$

*Definition 15.* Consider any NWPHFS, $\text{NWP} = \left\{\langle h_p^i, \xi\ (h_p^i)\rangle | i = 1, 2, 3, \ldots, n\right\}$, a collection of NWPHFEs and let $\omega = (\omega_1, \omega_2, \omega_3, \ldots, \omega_n)$ be the weight vectors of $\langle h_p^i, \xi\ (h_p^i)\rangle$ with $\omega_i \in [0, 1]$ and $\sum_{i=1}^{n} \omega_i = 1$. Then, an NWPHF weighted geometric (NWPHFWG) operator is defined as follows:

If $\omega = ((1/n), (1/n), (1/n), \ldots, (1/n))$, then (30) reduces to NWPHFG operator.

Note that, in the definition above, the weight vectors must satisfy the condition $\sum_{i=1}^{n} \omega_i = 1$. This is per our habits and makes it easy for aggregation operators to be used. But this does not happen in most practical applications; most of the time, the situation is not in our favour, and $\sum_{i=1}^{n} \omega_i < 1$, which is not reasonable. The issue needs to be resolved, but, fortunately, it is not a major issue. We can normalize the weight vector, and then the new weight vector satisfies the property in which it holds most of the original information.

## 4. MADM Process with the NWPHFS

In this section, we shall propose a novel approach to MADM problems with the normal wiggly probabilistic hesitant fuzzy numbers based on the NWPHFWA and NWPHFWG operators.

*4.1. Proposed Methodology under the Normal Wiggly Probabilistic Hesitant Fuzzy Environment.* Consider a problem having $m$ alternatives, denoted by $A = \{A_1, A_2, A_3, \ldots, A_m\}$. Each alternative is assessed based on $n$ attribute, shown by $C = \{C_1, C_2, C_3, \ldots, C_n\}$, which are weighted according to the attribute weight vector $w = \left\{w_1, w_2, w_3, \ldots, w_n\right\}$ and weights should satisfy the conditions $w_i \in [0, 1]$ and $\sum_{i=1}^{n} \omega_i = 1$. Finally, for the evaluation of alternatives, some experts/DMs are invited to provide the data in the form of PHFEs.

In the following, the proposed method under wiggly probabilistic environment is applied to solve such MADM problems. The steps are as follows:

*Step 1.* Construct a decision matrix by using PHF information given by the DMs as shown in Table 3

*Step 2.* According to Definition 11, another decision matrix is obtained based on NWPHFS given in Table 4

*Step 3.* Utilize (29) or (30) to calculate the unified assessment value of each alternative

*Step 4.* For the collective results, use Definition 12 to find the scores of the alternatives by simple calculation and arrange the alternatives according to assessment values with the given comparison method

To demonstrate the process of the proposed method based on NWPHF information, a flowchart is drawn as shown in Figure 1.

We have developed the concept of NWPHFS to link probabilistic information to NWHFS to minimize information loss. NWPHFS can better deal with practical problems when DMs provide their preference values based on a random variable. To address the real issues of MADM, we have come up with an appropriate approach based on NWPHF weighted aggregation operators. It can improve the diagnostic results and handle the complex information under the wiggly probabilistic environment. In addition, this technique will be applied to environmental quality testing in the next section.

## 5. Application to the Environmental Quality Evaluation

The quality of the environment plays a significant role in human life and directly impacts human health. So people are always worried about environmental degradation and make efforts to alleviate the quality of the environment. Numerous firms plan ecological projects, specifically for the chemical industry. Therefore, environmental quality assessment has a direct impact on economic and social development. It is unbearable to disregard all businesses that can contaminate the atmosphere. For sustainable development, we must find a stable point. One possible way is to assess the ecological superiority of some diverse locations and develop environmental standards for the worst spots. After that, we can have an overall ecological standard. Therefore, the real problem is to identify an area that has a bad atmosphere between different places. A comprehensive approach is proposed for the decision-making process as follows.

The quality of the environment depends upon the region according to certain standards and assessment procedures. On the contrary, suppose that the department of environmental protection's survey shows the four areas that need to be amended. Keeping in mind the time and cost, it is beneficial to focus all the resources in a single area. The main problem is to select one of the four areas that need to be considered first. These four areas can be described as $A_1, A_2, A_3$, and $A_4$. There are many characteristics in environmental structure, but, for illustration, we consider only four of them in this article: atmospheric environment $(C_1)$, water environment $(C_2)$, noise $(C_3)$, and waste material $(C_4)$. We provide a detailed explanation of these four attributes in Table 5. According to many environmentalists, the weight of the four attribute is given as $W = (0.3, 0.25, 0.2, 0.25)$ and consists of several attributes. But, in this article, DM holistically considers each criterion to demonstrate the preferred information for each alternative in the form of PHFS. The combined information of the DMs based on PHFEs is shown in Table 6. As stated in Definition 9, we use the NWPHFS to drive all PHF information; Tables [6–9] can then be created for the NWPHF decision matrix.

Below is a summary of the concrete decision-making process:

*Step 1.* Identify the problem, a combination of each alternative $(A_1, A_2, A_3, A_4)$, set of attributes $(C_1, C_2, C_3, C_4)$, and their weight vectors $w = (0.3, 0.25, 0.2, 0.25)$.

*Step 2.* Unite experts to evaluate the alternatives under attribute, and build a PHF decision matrix as shown in Table 6.

Table 3: Probabilistic hesitant fuzzy decision matrix given by the experts.

| | $C_1$ | $C_2$ | $C_3$ | $\cdots$ | $C_n$ |
|---|---|---|---|---|---|
| $A_1$ | $h_{11}(p_{11})$ | $h_{12}(p_{12})$ | $h_{13}(p_{13})$ | $\cdots$ | $h_{1n}(p_{1n})$ |
| $A_2$ | $h_{21}(p_{21})$ | $h_{22}(p_{22})$ | $h_{23}(p_{23})$ | $\cdots$ | $h_{2n}(p_{2n})$ |
| $A_3$ | $h_{31}(p_{31})$ | $h_{32}(p_{32})$ | $h_{33}(p_{33})$ | $\cdots$ | $h_{3n}(p_{3n})$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $A_m$ | $h_{m1}(p_{m1})$ | $h_{m2}(p_{m2})$ | $h_{m3}(p_{m3})$ | $\cdots$ | $h_{mn}(p_{mn})$ |

Table 4: Normal probabilistic hesitant fuzzy decision matrix.

| | $C_1$ | $C_2$ | $C_3$ | $\cdots$ | $C_n$ |
|---|---|---|---|---|---|
| $A_1$ | $\langle h_p^{11}, \xi(h_p^{11})\rangle$ | $\langle h_p^{12}, \xi(h_p^{12})\rangle$ | $\langle h_p^{13}, \xi(h_p^{13})\rangle$ | $\cdots$ | $\langle h_p^{n1}, \xi(h_p^{n1})\rangle$ |
| $A_2$ | $\langle h_p^{21}, \xi(h_p^{21})\rangle$ | $\langle h_p^{22}, \xi(h_p^{22})\rangle$ | $\langle h_p^{23}, \xi(h_p^{23})\rangle$ | $\cdots$ | $\langle h_p^{n2}, \xi(h_p^{n2})\rangle$ |
| $A_3$ | $\langle h_p^{31}, \xi(h_p^{31})\rangle$ | $\langle h_p^{32}, \xi(h_p^{32})\rangle$ | $\langle h_p^{33}, \xi(h_p^{33})\rangle$ | $\cdots$ | $\langle h_p^{n3}, \xi(h_p^{n3})\rangle$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $A_m$ | $\langle h_p^{m1}, \xi(h_p^{m1})\rangle$ | $\langle h_p^{m2}, \xi(h_p^{m2})\rangle$ | $\langle h_p^{m3}, \xi(h_p^{m3})\rangle$ | $\cdots$ | $\langle h_p^{mn}, \xi(h_p^{mn})\rangle$ |

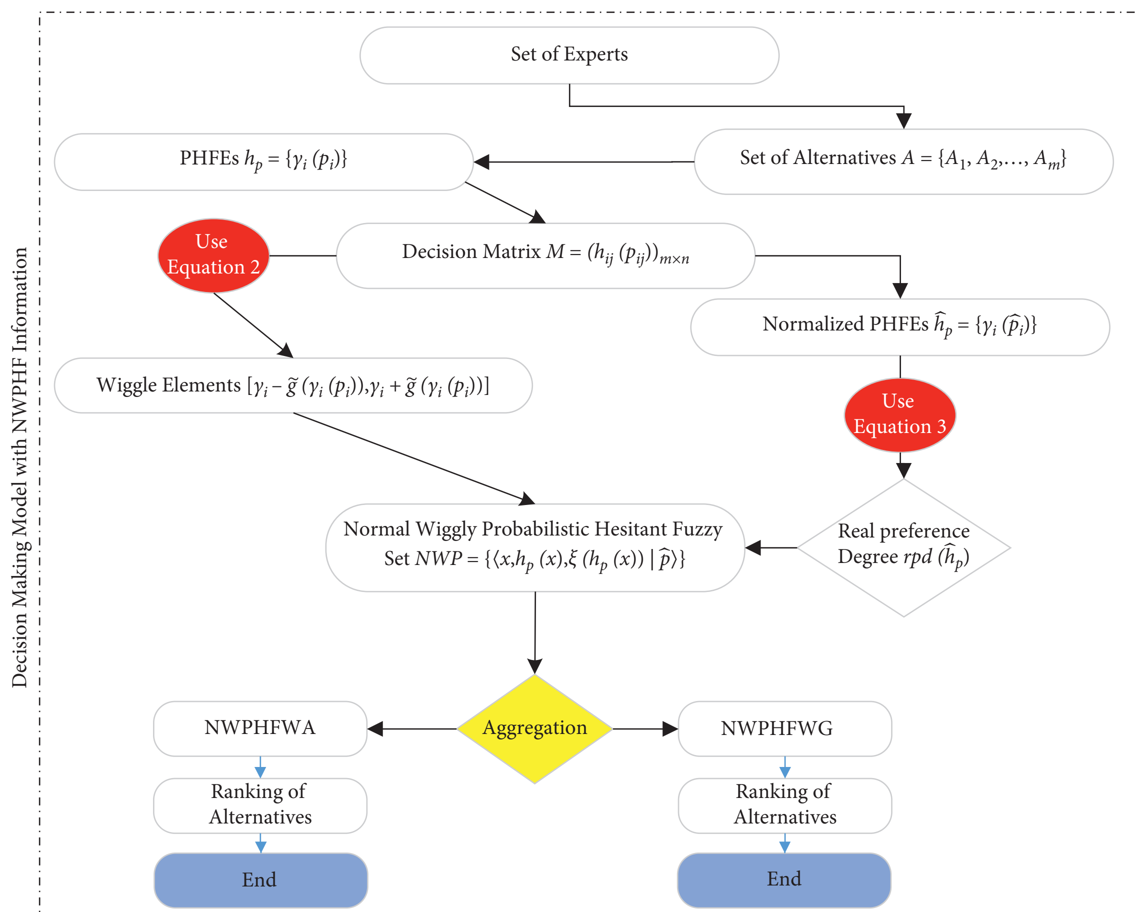


Figure 1: The schema of the whole decision-making steps using the normal wiggly probabilistic hesitant fuzzy information.

*Step 3.* NWPHF decision matrix according to Definition 9 shown in Table 7.

*Step 4.* Compute the combined evaluation values of each alternative by using the operators NWPHFWG and NWPHFWA, given in Definitions 14 and 15. Calculate the score values according to Definition 12, and rate all the alternatives according to their scores. The scores for the alternatives and final ranking are shown in Table 8.

However, if we seize the probabilities, the PHFEs in Table 6 will convert to HFEs. Then, using the NWHFS

Table 5: The description of the attributes under consideration.

| Attribute | Explanation |
|---|---|
| $C_1$: atmospheric environment | Controlling air pollution and limiting greenhouse gas emissions |
| $C_2$: water environment | Controlling regional irrigation pollution, guiding the effective use of aquatic resources, maintaining and improving water quality and the aquatic environment, ensuring the availability of adequate water resources |
| $C_3$: noise | To control noise and noise level of enclosed enclosures on urban traffic arterials, ensure the sound quality in sensitive locations, such as residential areas |
| $C_4$: waste material | To improve the construction and nature of solid waste |

Table 6: Probabilistic hesitant fuzzy decision matrix given by the experts.

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $A_1$ | $\{0.15\,(0.3), 0.35\,(0.5), 0.65\,(0.2)\}$ | $\{0.25\,(0.4), 0.65\,(0.6)\}$ | $\{0.4\,(0.3), 0.8\,(0.7)\}$ | $\{0.1\,(0.3), 0.4\,(0.3), 0.6\,(0.4)\}$ |
| $A_2$ | $\{0.35\,(0.5), 0.8\,(0.5)\}$ | $\{0.05\,(0.3), 0.35\,(0.6), 0.75\,(0.1)\}$ | $\{0.25\,(0.4), 0.7\,(0.6)\}$ | $\{0.3\,(0.4), 0.6\,(0.3), 0.1\,(0.3)\}$ |
| $A_3$ | $\{0.25\,(0.35), 0.75\,(0.65)\}$ | $\{0.25\,(0.3), 0.65\,(0.5), 0.8\,(0.2)\}$ | $\{0.3\,(0.55), 0.85\,(0.45)\}$ | $\{0.2\,(0.25), 0.4\,(0.4), 0.7\,(0.35)\}$ |
| $A_4$ | $\{0.75\,(0.4), 0.55\,(0.3), 0.1\,(0.3)\}$ | $\{0.25\,(0.4), 0.7\,(0.6)\}$ | $\{0.15\,(0.2), 0.4\,(0.4), 0.85\,(0.4)\}$ | $\{0.65\,(0.4), 0.75\,(0.2), 0.85\,(0.4)\}$ |

aggregation operators to calculate the score values of the alternatives, the score and raking can be found in Table 9.

The final ranking can be seen in Tables 8 and 9 which are different; these operators present different results, which will be discussed in the following subsection. Table 8 provides the results of NWPHFWG and NWPHFWA operators, and the result obtained by the method is given in [26] and can be obtained in Table 9. The results in Table 9 are not consistent because there is a severe loss of information. Uncertain information is dug to get more analytical results, but exploring the PHF information will provide more accurate and consistent results.

*5.1. Comparative Analysis.* Ren at al. [26] presented a MADM method based on two operators, namely, the NWHFWA and the NWHFWG, and applied them to evaluate alternatives under the normal wiggly hesitant fuzzy environment. In this section, we compare the proposed method with this approach to categorize the alternatives by calculating the final scores. NWPHFS can be seen as an extension of PHFS, which develops a technique for digging up potential information. In PHFSs, the DMs provide their assessment by a finite set of values along with their respective probabilities, which can better articulate their hesitation. Therefore, it is essential to compare the results of their classification. The original PHF information values are presented in Table 6. As NWHFS has a special case of NWPHFS, where their probabilities of each membership degrees are equal to one, the form NWPHFS is more general, which can help DMs express evaluation information. Since no such other procedure has been developed for NWPHFS, we compare our proposed procedure with the special case of NWPHFS, which is NWHFS. HFS can be obtained by seizing the probability in PHFS from Table 6. By using the method based on NWHFWA and NWHFWG operators, we calculate the ranking outcomes in Table 9. Table 9 indicates that the results of Ren et al. [26] based on NWHFWA and NWHFWG are not consistent, when compared to the results of our method given in Table 8. From Table 9, it can easily be seen that $A_3$ is the best choice and $A_2$ is the worst choice; however, $A_4$ is the more appropriate alternative, and $A_2$ is the worst by NWPHFWG operator, and the ranking order of the remaining options is also different. Moreover, by utilizing the NWHFWA operator, we see that $A_4$ is the best choice, and $A_1$ is the worst choice; but $A_4$ is the more appropriate alternative, and $A_2$ is the worst by NWPHFWA operator. The main reason for the differences is that our method takes both the original hesitant information and probabilistic information into account. Furthermore, under the NWHFWA and NWHFWG operators, the rankings are different from that of our method. NWPHFSs allow DMs to express their values in membership values along with their respective probabilities more flexibly. Finally, these basically consistent ranking results demonstrate the feasibility and effectiveness of our method. Also, we easily see that if we cease the probabilities, then the proposed method and the method defined by Ren et al. [26] are the same. This also guarantees that the proposed method can handle more complex information and more space in decision-making.

*5.2. Advantages of the Proposed Approach.* Some advantages have been pointed out from the proposed studies concerning the present:

(1) Because the PHF set is an extension of HFS and contains more information than HFS, the proposed aggregation operators (NWPHFWA and NWPHFWG) generalize the NWHFWA and NWHFWG operators. Hence, these operators can address the decision-making difficulties more efficiently.

(2) Tables 8 and 9 show that the final results of our proposed procedure do not conform to current practices under the hesitant fuzzy environment. It is also shown that conventional HFS has a severe loss of information. Thus, a comparative study reveals that the proposed measure is more appropriate and practically workable and provides a better way under the PHF environment.

Table 7: Normal probabilistic hesitant fuzzy decision matrix.

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $A_1$ | $\left\{\begin{pmatrix}0.04496,\\0.04497,\\0.04504\end{pmatrix}\mid 0.3,\begin{pmatrix}0.14500,\\0.15295,\\0.20500\end{pmatrix}\mid 0.5\begin{pmatrix}0.13000,\\0.13000,\\0.13000\end{pmatrix}\mid 0.2,\right\}$ | $\left\{\begin{pmatrix}0.09998,\\0.09999,\\0.10002\end{pmatrix}\mid 0.4,\begin{pmatrix}0.38979,\\0.38983,\\0.39021\end{pmatrix}\mid 0.6\right\}$ | $\left\{\begin{pmatrix}0.12000,\\0.12000,\\0.12000\end{pmatrix}\mid 0.3,\begin{pmatrix}0.55961,\\0.56029,\\0.56039\end{pmatrix}\mid 0.7\right\}$ | $\left\{\begin{pmatrix}0.02995,\\0.02996,\\0.03005\end{pmatrix}\mid 0.3,\begin{pmatrix}0.07745,\\0.08511,\\0.16255\end{pmatrix}\mid 0.3,\begin{pmatrix}0.23964,\\0.23971,\\0.24036\end{pmatrix}\mid 0.4\right\}$ |
| $A_2$ | $\left\{\begin{pmatrix}0.17464,\\0.17524,\\0.17536\end{pmatrix}\mid 0.5,\begin{pmatrix}0.39964,\\0.40024,\\0.40036\end{pmatrix}\mid 0.5\right\}$ | $\left\{\begin{pmatrix}0.01487,\\0.01490,\\0.01513\end{pmatrix}\mid 0.3,\begin{pmatrix}0.18436,\\0.19052,\\0.23564\end{pmatrix}\mid 0.6,\begin{pmatrix}0.07492,\\0.07494,\\0.07508\end{pmatrix}\mid 0.1\right\}$ | $\left\{\begin{pmatrix}0.09990,\\0.10008,\\0.10010\end{pmatrix}\mid 0.4,\begin{pmatrix}0.41921,\\0.42063,\\0.42079\end{pmatrix}\mid 0.6\right\}$ | $\left\{\begin{pmatrix}0.08634,\\0.10048,\\0.15366\end{pmatrix}\mid 0.4,\begin{pmatrix}0.17998,\\0.17999,\\0.18002\end{pmatrix}\mid 0.3,\begin{pmatrix}0.02984,\\0.02991,\\0.03016\end{pmatrix}\mid 0.3\right\}$ |
| $A_3$ | $\left\{\begin{pmatrix}0.08731,\\0.08765,\\0.08769\end{pmatrix}\mid 0.35,\begin{pmatrix}0.48488,\\0.48966,\\0.49012\end{pmatrix}\mid 0.65\right\}$ | $\left\{\begin{pmatrix}0.07497,\\0.07502,\\0.07503\end{pmatrix}\mid 0.3,\begin{pmatrix}0.30910,\\0.33335,\\0.34090\end{pmatrix}\mid 0.5,\begin{pmatrix}0.15761,\\0.16125,\\0.16239\end{pmatrix}\mid 0.2\right\}$ | $\left\{\begin{pmatrix}0.16129,\\0.16748,\\0.16871\end{pmatrix}\mid 0.55,\begin{pmatrix}0.38060,\\0.38377,\\0.38440\end{pmatrix}\mid 0.45\right\}$ | $\left\{\begin{pmatrix}0.04984,\\0.04988,\\0.05016\end{pmatrix}\mid 0.25,\begin{pmatrix}0.13443,\\0.14108,\\0.18557\end{pmatrix}\mid 0.4,\begin{pmatrix}0.24497,\\0.24498,\\0.24503\end{pmatrix}\mid 0.35\right\}$ |
| $A_4$ | $\left\{\begin{pmatrix}0.29326,\\0.29680,\\0.30674\end{pmatrix}\mid 0.4,\begin{pmatrix}0.09718,\\0.13279,\\0.23282\end{pmatrix}\mid 0.3,\begin{pmatrix}0.02901,\\0.02953,\\0.03099\end{pmatrix}\mid 0.3\right\}$ | $\left\{\begin{pmatrix}0.09990,\\0.10008,\\0.10010\end{pmatrix}\mid 0.4,\begin{pmatrix}0.41921,\\0.42063,\\0.42079\end{pmatrix}\mid 0.6\right\}$ | $\left\{\begin{pmatrix}0.02671,\\0.03250,\\0.03329\end{pmatrix}\mid 0.2,\begin{pmatrix}0.13105,\\0.21720,\\0.22895\end{pmatrix}\mid 0.4,\begin{pmatrix}0.33776,\\0.34170,\\0.34224\end{pmatrix}\mid 0.4\right\}$ | $\left\{\begin{pmatrix}0.26000,\\0.26000,\\0.26000\end{pmatrix}\mid 0.4,\begin{pmatrix}0.14200,\\0.15264,\\0.15800\end{pmatrix}\mid 0.2,\begin{pmatrix}0.34000,\\0.34000,\\0.34000\end{pmatrix}\mid 0.4\right\}$ |

Table 8: The collective evaluation values of the alternatives and ranking.

| | $A_1$ | $A_2$ | $A_3$ | $A_4$ | Ranking |
|---|---|---|---|---|---|
| NWPHFWG | 0.1977 | 0.1741 | 0.2350 | 0.2398 | $A_4 > A_3 > A_1 > A_2$ |
| NWPHFWA | 0.2438 | 0.2321 | 0.2701 | 0.2853 | $A_4 > A_3 > A_1 > A_2$ |

Table 9: The decision results of the different methods.

| | $A_1$ | $A_2$ | $A_3$ | $A_4$ | Ranking |
|---|---|---|---|---|---|
| NWHFWG | 0.1185 | 0.0613 | 0.1622 | 0.1581 | $A_3 > A_4 > A_1 > A_2$ |
| NWHFWA | 0.1904 | 0.2140 | 0.3046 | 0.4573 | $A_4 > A_3 > A_2 > A_1$ |

(3) Also, the operators given by [26] (NWHFWA and NWHFWG) are used in the above example, which provides different ranking on the same data, which is not practical. On the other side, the proposed operators (NWPHFWA and NWPHFWG) offer the same ranking, indicating that the proposed operators provide us more reliable and consistent results.

## 6. Conclusions and Future Prospects

In the decision-making process, the representation of uncertain information is proposed to enable the DMs to disclose their cognitive preferences fully. However, the limited knowledge of DMs leads to the fact that no complex information representation form can help DMs express all the preferred information about an alternative. Instead, it significantly increases the DMs psychological burden and time cost. Therefore, the purpose of this article is to obtain more accurate assessments from simple information. Therefore, to facilitate the DMs, we leave the dilemma of complex representation and try to find the hidden uncertain information from the original data provided by the DMs. To accomplish this objective, we propose a new representation tool, NWPHFS, to automatically find the hidden uncertain information of the original PHF information. The proposed NWPHFS is based on the assumption that human cognitive uncertainty can be considered a general fluctuation in a specific range that focuses on a value, the DM's uncertain feelings can appear objectively and realistically. In this paper, the essential theoretical knowledge of NWPHFS has been explained in detail:

(1) We propose some basic operational rules, score function, and distance measure between two NWPHFSs

(2) To aggregate the information, two aggregation operators are proposed, namely, normal wiggly probabilistic hesitant fuzzy weighted averaging and normal wiggly probabilistic hesitant fuzzy weighted geometric

(3) Based on NWPHFWA and NWPHFWG, a new MADM method is proposed to deal with MADM problems in a normal wiggly probabilistic context

(4) The effectiveness and feasibility of the proposed method are tested through an example of environmental quality assessment, and the comparative analysis revealed that the proposed method could offer more accurate and precise conclusions than the existing method

Future research can combine NWPHFS with some MADM methods considering preference relations, such as the TDM method and PROMETHEE method. Moreover, the Maclaurin Symmetric Mean and dual Maclaurin Symmetric Mean operators can be extended for the NWPHF environment. Simultaneously, we can further develop the MADM to the multiattribute group decision-making method and use this for different applications such as green supplier selection, robot selection, and environmental quality assessment.

## Data Availability

No data were used to support this study.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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*Research Article*

# A Single-Valued Extended Hesitant Fuzzy Score-Based Technique for Probabilistic Hesitant Fuzzy Multiple Criteria Decision-Making

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The probabilistic hesitant fuzzy set (PHFS) is a worthwhile extension of the hesitant fuzzy set (HFS) which allows people to improve their quantitative assessment with the corresponding probability. Recently, in order to address the issue of difficulty in aggregating decision makers' opinions, a probability splitting algorithm has been developed that drives an efficient probabilistic-unification process of PHFSs. Adopting such a unification process allows decision makers to disregard the probability part in developing fruitful theories of comparison of PHFSs. By keeping this feature in mind, we try to introduce a class of score functions for the notion of the single-valued extended hesitant fuzzy set (SVEHFS) as a novel deformation of PHFS. Interestingly, a SVEHFS not only belongs to a less dimensional space compared to that of PHFSs but also the proposed SVEHFS-based score functions satisfy a number of interesting properties. Eventually, some case studies of multiple criteria decision-making (MCDM) techniques under the PHFS environment are provided to demonstrate the effectiveness of proposed SVEHFS-based score functions.

## 1. Introduction

Hesitant fuzzy set (HFS) as an extension of the fuzzy set [1] was introduced for reflecting the hesitancy of decision makers in providing their preferences over alternatives such that the membership degree of an element in HFS is represented by a set of values in $[0, 1]$. The concept of HFS is a field that still keeps attracting a significant amount of attention from researchers, and by owing to this concept, the other extensions of HFSs have been proposed in the literature [2–6] to overcome a number of corresponding challenges.

From diverse extensions of HFS, the concept of the extended hesitant fuzzy set (EHFS) is introduced first by Zhu and Xu [7] in terms of a function that returns a finite set of membership value-groups. Then, Farhadinia and Herrera-Viedma [8] re-visited and revised the notion of EHFS as the Cartesian product of "$n$" HFSs in which each "$n$"-tuple-formed element of EHFS is referred to as the opinion of some decision makers simultaneously.

Another interesting generalization of HFSs occurs when we are required to provide experts' evaluations based on two cases: whether experts have the same weight or whether each value in a hesitant fuzzy element (HFE) gets the same probability distribution? These cases are covered by defining the concept of the probabilistic hesitant fuzzy set (PHFS) which was first developed by Zhu and Xu [9] to incorporate distribution information with the membership degrees included in hesitant fuzzy elements (HFEs). Furthermore, the PHFS concept has a great potential for handling multiple criteria decision-making (MCDM) processes in which both qualitative and quantitative criteria are to be considered [10–14].

Nowadays, among a large number of studies of PHFS notion, we may refer to the contribution of Zhang and Wu [15] in which two PHFS aggregation operators are developed by taking Archimedean $t$-norm and $t$-conorm into account. Following that work, Li and Wang [16] proposed the Hausdorff distance measure of PHFSs to extend a QUALItative FLEXible multiple (QUALIFLEX) technique for evaluating green suppliers. Yue et al. [17] developed the application of probabilistic hesitant fuzzy elements (PHFEs) in MCDM problems by proposing a set of probabilistic hesitant fuzzy aggregation operators. Following that, Zeng et al. [18] introduced the uncertain probabilistic-ordered weighted averaging distance operator in order to unify the framework between the probability and the ordered weighted averaging operator. Ding et al. [19] dealt with the situation in which the weight information is incomplete, and then, they concentrated on the class of PHFE-based multiple attribute group decision-making.

In a completely updated study, Farhadinia [20] pointed out that there exist two kinds of normalization processes in dealing with PHFS decision-making problems, namely, the probabilistic normalization and cardinal normalization. We need to mention that, among the contributions considering different types of probabilistic-unification processes, the most eminent works are those of Zhang et al. [21], Farhadinia and Xu [22], Farhadinia and Herrera-Viedma [23], Li and Wang [24], Wu et al. [25], and Lin et al. [26]. Except Lin et al.'s [26] probabilistic-unification process, Farhadinia [20] demonstrated that the other probabilistic-unification processes considered in the later-mentioned contributions are not reasonable from a mathematical point of view. It can be seen that the probabilistic-unification processes of Lin et al. [26] and Farhadinia [20] give rise to the same result with this difference that the process of Lin et al. compromises the unification of probabilities and HFE parts simultaneously, and that of Farhadinia unifies firstly the probabilities part, and then, it does the corresponding HFE part.

Keeping the latter-mentioned applications of PHFS notion in mind, the subject of PHFS ranking technique has received significant attention in the recent years. Up to now, a variety of PHFE comparison techniques have been proposed as the combination of hesitancy degree and its corresponding probability. Taking these two notions into account, there have been considerable contributions done in the past on the PHFE comparison techniques which were developed by employing the score and deviation values of each PHFE [14, 19, 21]. For instance, Lin et al. [26] put forward two types of probabilistic hesitant fuzzy aggregation operators for specifying the ranking results of alternatives in decision-making problems. Jiang and Ma [27] proposed a PHFE comparison technique using the arithmetic- and geometric-mean scores. Song et al. [28] presented a possibility degree formula for ranking PHFEs in the case where different PHFEs have common or intersecting values. This comparison technique is able to realize the optimal sorting under the hesitant fuzzy environment, and of course, it can reduce effectively the complexity of computation. Krishankumar et al. [29] suggested a ranking technique which extends a well-known VIKOR approach to the PHFS context. Wu et al. [30] supplied an enhanced satisfaction degree function on the basis of probabilistic hesitant fuzzy cumulative residual entropy for ranking the alternatives involved in a MCDM. Last but not least, Farhadinia and Xu [31] developed a thorough review of PHFS comparison techniques in MCDM and introduced a kind of PHFE ranking technique which is based on the multiplying and exponential deformation formulas of each element of a PHFE. They classified the PHFE measuring techniques in brief into the three classes which were called the element-based processes for comparing PHFEs, the one step-based processes for comparing PHFEs, and the two step-based processes for comparing PHFEs.

However, the main objective of this study is to develop a class of score functions for capturing dependencies between PHFSs. Although the ranking of PHFSs has been discussed thoroughly before, the novelty presented here lies in the fact that the comparison is done inside the less dimensional space, referred here to as the single-valued EHFSs (SVEHFSs), and it has not yet been fully exploited. The notable characteristic of proposed SVEHFS-score functions is that not only they are projected from a highly dimensional space (i.e., the PHFS space) into a less dimensional space (i.e, the SVEHFS space) but also they offer a wide variety of interesting properties. Moreover, the proposed SVEHFS-based score functions proceed in less steps, and it relieves the laborious duty of using complex rules. Besides the latter advantages, we will demonstrate that the proposed SVEHFS-score functions can be more generalized to a wider class.

The organization of this contribution is as follows. We firstly review the process of unification of PHFSs in Section 2. Then, we demonstrate that how a unified PHFS is deduced to a SVEHFS in Section 3. Section 4 is devoted to introducing a new class of SVEHFS-based score functions for the unified PHFSs which provides the decision makers with more choices and flexibility. Subsequently, by re-encountering a number of MCDM problems, we indicate that the superiority of the proposed SVEHFS-score functions compare to the existing ones for PHFSs in Section 5. Section 6 concludes this contribution and provides some perspectives.

## 2. The Probabilistic-Unification Process of PHFSs

In the following part, we are going to review a number of basic notions which will be used frequently throughout this contribution.

By taking the reference set of $X$ into consideration, Torra [1] introduced the notion of hesitant fuzzy set (HFS) in terms of a function returning a finite subset of $[0, 1]$ which is generally denoted by

$$H = \{\langle x, h(x)\rangle : x \in X\}, \tag{1}$$

where $h(x) \in [0, 1]$ is known as the hesitant fuzzy element (HFE) and denotes the possible membership degree of $x \in X$ to the set $H$.

There is another way of representing HFS already described in the form of

$$H = \left\{\langle x, \bigcup_{\hbar \in h(x)} \{\hbar\}\rangle, x \in X\right\}. \tag{2}$$

In order to emphasis on the probability occurrence of each possible value of HFE, Zhu [32] associated any element of HFE with its probability value as follows:

$$^{\wp}H = \{\langle x, {}^{\wp}h(x)\rangle : x \in X\} = \left\{\langle x, \bigcup_{\langle \hbar(x), \wp(x)\rangle \in {}^{\wp}h(x)} \{\langle \hbar(x), \wp(x)\rangle\}\rangle x \in X\right\} \tag{3}$$

where $^{\wp}h(x)$ stands for a probabilistic hesitant fuzzy element (PHFE).

As can be observed, any PHFE $^{\wp}h(x)$ is a pair of possible membership degree $\hbar(x)$ and its probability distribution in the form of $\wp(x) \in [0,1]$ such that $\sum_{^{\wp}h(x)} (\wp(x)) = 1$ for any $x \in X$.

It is obvious that if all the values of $\wp(x)$ are equal for any $x \in X$, then the PHFS $^{\wp}H$ is reduced to a typical HFS.

Keeping the probabilistic-normalization and the cardinal-normalization procedures of PHFSs in mind, Farhadinia [20] represented a probabilistic-unification type of PHFSs. Anyway, it has been presented two PHFE probabilistic-unification processes in earlier contributions, the one proposed by Farhadinia [20] and the other given by Lin et al. [26]. The main difference between these two processes is that Lin et al. [26] performed the unification process simultaneously for both the HFE part and its corresponding probability part, while Farhadinia [20] applied the unification process to probability part at the beginning and then partitioned the HFE part correspondingly. Regarding the same outcome of both Farhadinia's [20] and Lin et al.'s [26] procedures, we only consider the former one in the following.

By the use of Farhadinia's [20] algorithm which is seperated here as Algorithms 1 and 2, the initial partition of each PHFE probabilities is to be refined such that all the involved PHFEs have the same probability parts, while their corresponding HFE part remains unchanged. To explain Algorithm 1 and 2 briefly, we assume that $^{\wp}h_1 = \cup_{\langle \hbar_1, \wp_1\rangle \in {}^{\wp}h_1} \{\langle \hbar_1, \wp_1\rangle\} = \{\langle \hbar_1^1, \wp_1^1\rangle, \ldots, \langle \hbar_1^{l_1}, \wp_1^{l_1}\rangle\}$, $^{\wp}h_2 = \cup_{\langle \hbar_2, \wp_2\rangle \in {}^{\wp}h_2} \{\langle \hbar_2, \wp_2\rangle\} = \{\langle \hbar_2^1, \wp_2^1\rangle, \ldots, \langle \hbar_2^{l_2}, \wp_2^{l_2}\rangle\}$, ..., and $^{\wp}h_m = \cup_{\langle \hbar_m, \wp_m\rangle \in {}^{\wp}h_m} \{\langle \hbar_m, \wp_m\rangle\} = \{\langle \hbar_m^1, \wp_m^1\rangle, \ldots, \langle \hbar_m^{l_m}, \wp_m^{l_m}\rangle\}$ indicate $m$ arbitrary PHFEs whose probabilities can be separated in the forms of $\{\wp_1^1, \wp_1^2, \ldots, \wp_1^{l_1}\}$ $\{\wp_2^1, \wp_2^2, \ldots, \wp_2^{l_2}\}$,..., and $\{\wp_m^1, \wp_m^2, \ldots, \wp_m^{l_m}\}$, respectively. In this regard, the first phase of the PHFE probabilistic-unification process takes the following steps:

By setting $i := i + 1$, we return to Step 1.

Farhadinia [20] indicated that the following results are the inherent advantages of the above-described algorithm.

**Lemma 1** (see [20]). *If* $\prod^*: (\{\wp_1^1, \wp_1^2, \ldots, \wp_1^{l_1}\}, \{\wp_2^1, \wp_2^2, \ldots, \wp_2^{l_2}\}, \ldots, \{\wp_m^1, \wp_m^2, \ldots, \wp_m^{l_m}\}) \longrightarrow \{\wp_*^1, \wp_*^2, \ldots, \wp_*^{l_*}\}$ *where* $l_* \geq \max\{l_1, l_2, \ldots, l_m\}$, *then the aggregation operator* $\prod^*$ *is idempotent, commutative, and associative.*

**Corollary 1** (see [20]). *Let* $^{\wp}\dot{h}_1 = \cup_{\langle \dot{\hbar}_1, \dot{\wp}_1\rangle \in {}^{\wp}h_1} \{\langle \dot{\hbar}_1, \dot{\wp}_1\rangle\}$ *and* $^{\wp}\dot{h}_2 = \cup_{\langle \dot{\hbar}_2, \dot{\wp}_2\rangle \in {}^{\wp}h_2} \{\langle \dot{\hbar}_2, \dot{\wp}_2\rangle\}$ *be two unified PHFEs. Then, their corresponding probabilities sets are compatible (isomorphic).*

To gain a more clear understanding of Farhadinia's [20] PHFE probabilistic-unification algorithm, we take the following three arbitrary PHFEs:

$$\begin{aligned} ^{\wp}h_1 &= \{\langle 0.3, 0.2\rangle, \langle 0.6, 0.5\rangle, \langle 0.8, 0.3\rangle\}, \\ ^{\wp}h_2 &= \{\langle 0.4, 0.5\rangle, \langle 0.7, 0.5\rangle\}, \\ ^{\wp}h_3 &= \{\langle 0.2, 0.1\rangle, \langle 0.5, 0.7\rangle, \langle 0.9, 0.2\rangle\}. \end{aligned} \tag{4}$$

It is apparent from illustrative Figures 1–3 that the unified forms are obtained in the forms of

$$\begin{aligned} ^{\wp}\dot{h}_1 &= \{\langle 0.3, 0.1\rangle, \langle 0.3, 0.1\rangle, \langle 0.6, 0.3\rangle, \langle 0.6, 0.2\rangle, \langle 0.8, 0.1\rangle, \langle 0.8, 0.2\rangle\}, \\ ^{\wp}\dot{h}_2 &= \{\langle 0.4, 0.1\rangle, \langle 0.4, 0.1\rangle, \langle 0.4, 0.3\rangle, \langle 0.7, 0.2\rangle, \langle 0.7, 0.1\rangle, \langle 0.7, 0.2\rangle\}, \\ ^{\wp}\dot{h}_3 &= \{\langle 0.2, 0.1\rangle, \langle 0.5, 0.1\rangle, \langle 0.5, 0.3\rangle, \langle 0.5, 0.2\rangle, \langle 0.5, 0.1\rangle, \langle 0.9, 0.2\rangle\}. \end{aligned} \tag{5}$$

By the use of the above illustrative example, we find that the PHFE probabilistic-unification algorithm enables us to gain a set of PHFEs whose probabilities are in the form of a fixed vector.

## 3. Reducing Unified PHFEs to SVEHFEs

We can summarise the outcome of the previous section as follows: the PHFE probabilistic-unification algorithm leads to the set of HFE and probability pairs whose second part is a fixed vector.

As mentioned before, the purpose of this contribution is to propose a class of score functions for PHFSs with less involved factors. This fact would help us greatly reduce the model construction effort without losing the generality for different PHFSs; meanwhile, their probability part is common. Such an effort will result in defining a fundamental concept, called here as the single-valued extended hesitant fuzzy set (SVEHFS).

In the sequel, we shall present some preliminaries which will be useful for the establishment of the desired results.

Initial step: consider the input probability sets as $\begin{cases} \{\wp_1^1, \wp_1^2, \ldots, \wp_1^{l_1}\}; \\ \{\wp_2^1, \wp_2^2, \ldots, \wp_2^{l_2}\}; \\ \vdots \\ \{\wp_m^1, \wp_m^2, \ldots, \wp_m^{l_m}\}. \end{cases}$

We now let $i = 1$.

Step 1: compute $\wp_*^i = \min\{\wp_1^i, \wp_2^i, \ldots, \wp_m^i\}$.

Step 2: calculate the new probabilities: $\begin{cases} \wp_1^i := \max\{\wp_1^i - \wp_*^i, 0\}; \\ \wp_2^i := \max\{\wp_2^i - \wp_*^i, 0\}; \\ \vdots \\ \wp_m^i := \max\{\wp_m^i - \wp_*^i, 0\}. \end{cases}$

Now, if $\wp_1^i = \wp_2^i = \cdots = \wp_m^i = 0$, then STOP, and return $\wp_* = \{\wp_*^1, \wp_*^2, \ldots, \wp_*^{l_*}\}$ in which $l_* \geq \max\{l_1, l_2, \ldots, l_m\}$. Else, go to the next step.

Step 3: Define $\wp_j^{i+1} := \begin{cases} \wp_j^{i+1}, & \text{if } \wp_j^i = 0; \\ \wp_j^i, & \text{if } \wp_j^i = 0; \end{cases}$, for $j = 1, 2, \ldots, m$.

Algorithm 1: Phase 1 of Farhadinia's [20] algorithm.

Initial step: we assume that $\wp_* = \{\wp_*^1, \wp_*^2, \ldots, \wp_*^{l_*}\}$ is to be the output of Phase 1 of Farhadinia's [20] algorithm

Step 1: calculate the re-formatted probabilities as follows:

$$\wp_1^1 = \sum_{k=1}^{k_1} \wp_*^k,$$
$$\wp_1^2 = \sum_{k=k_1+1}^{k_2} \wp_*^k,$$
$$\vdots$$
$$\wp_1^{l_1} = \sum_{k=k_{l_1}+1}^{l_1} \wp_*^k.$$

Step 2: we re-arrange the HFE part of the first PHFE in the form of $\begin{array}{l} \langle \hbar_1^1, \wp_*^1 \rangle, \ldots, \langle \hbar_1^1, \wp_*^{k_1} \rangle, \\ \langle \hbar_1^2, \wp_*^{k_1+1} \rangle, \ldots, \langle \hbar_1^2, \wp_*^{k_2} \rangle, \\ \vdots \\ \langle \hbar_1^{l_1}, \wp_*^{k_{l_1}+1} \rangle, \ldots, \langle \hbar_1^{l_1}, \wp_*^{k_{l^*}} \rangle. \end{array}$

In summary, the unified form of the PHFE ${}^{\wp}h_1$ will be ${}^{\wp}\dot{h}_1 = \{\langle \dot{\hbar}_1^1, \dot{\wp}_1^1 \rangle, \ldots, \langle \dot{\hbar}_1^{k_1}, \dot{\wp}_1^{k_1} \rangle, \langle \dot{\hbar}_1^{k_1+1}, \dot{\wp}_1^{k_1+1} \rangle, \ldots, \langle \dot{\hbar}_1^{k_2}, \dot{\wp}_1^{k_2} \rangle, \vdots \langle \dot{\hbar}_1^{k_{l_1}+1}, \dot{\wp}_1^{k_{l_1}+1} \rangle, \ldots, \langle \dot{\hbar}_1^{l^*}, \dot{\wp}_1^{l^*} \rangle$.

Step 3: in a similar manner as described above, we re-format the other PHFEs ${}^{\wp}h_2$,..., and ${}^{\wp}h_m$ to ${}^{\wp}\dot{h}_2$,..., and ${}^{\wp}\dot{h}_m$.

Algorithm 2: Phase 2 of Farhadinia's [20] algorithm.

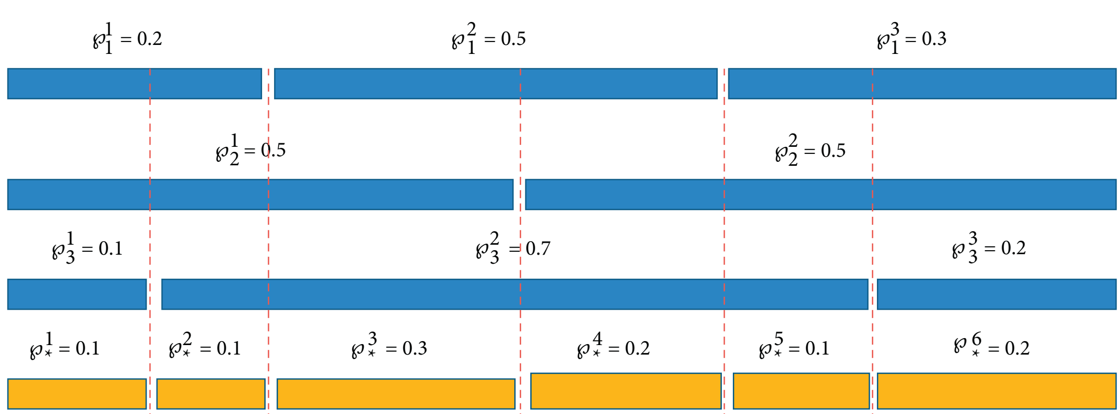


Figure 1: Stage 1 of the unification process.

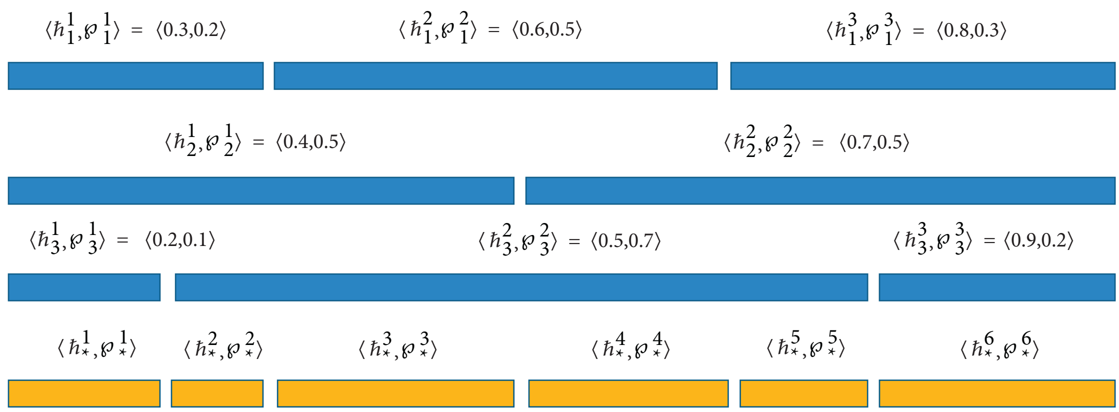


FIGURE 2: Stage 2 of the unification process.

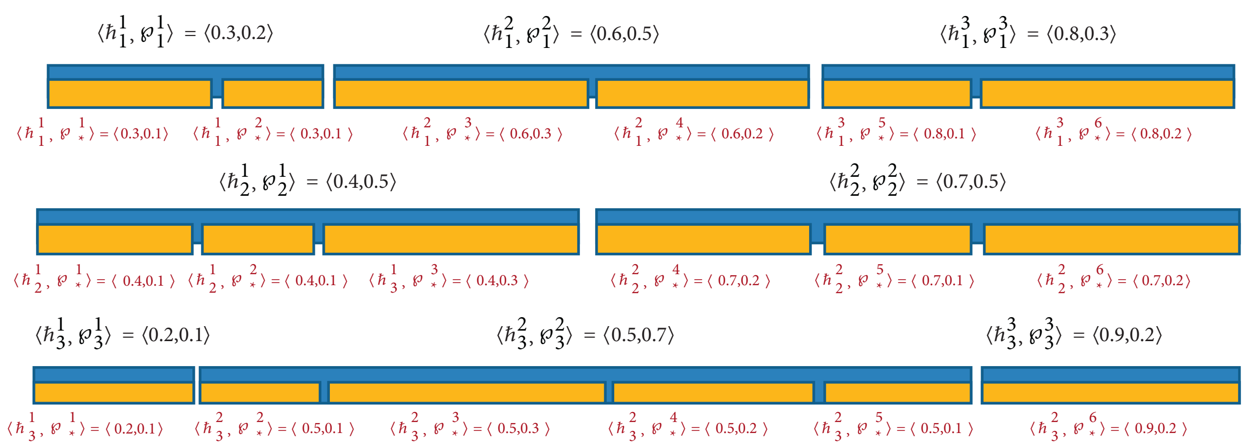


FIGURE 3: Combining both Stages 1 and 2 of the unification process.

In a recent work, Zhu and Xu [7] introduced the notion of extended HFS (EHFS) in terms of a function which returns a finite set of membership value-groups. Then, Farhadinia and Herrera-Viedma [8] indicated that each element of an EHFS, known as the extended hesitant fuzzy element (EHFE), is indeed a set of $n$-tuples which demonstrates the opinion of $n$ number of decision makers. They introduced an extended hesitant fuzzy set (EHFS) on the reference set $X$ in the form of

$$\mathbf{H} = \{\langle x, \mathbf{h}(x)\rangle | x \in X\} = \left\{ \langle x, \bigcup_{(\gamma_1(x),\ldots,\gamma_m(x))\in \mathbf{h}(x)} \{(\gamma_1(x), \ldots, \gamma_m(x))\}\rangle | x \in X \right\}, \tag{6}$$

where

$$\mathbf{h} = \bigcup_{(\gamma_1,\ldots,\gamma_m)\in \mathbf{h}} \{(\gamma_1, \ldots, \gamma_m)\}, \tag{7}$$

which stands for an extended HFE (EHFE).

For instance, if we suppose that $X = \{x_1, x_2\}$ is the reference set and $\mathbf{h}_1(x) = \{(0.6, 0.3, 0.3), (0.5, 0.2, 0.2)\}$ and $\mathbf{h}_2(x) = \{(0.3, 0.2, 0.1)\}$ are two EHFEs on $X$, then the EHFS $\mathbf{H}$ is characterized by

$$\mathbf{H} = \{\langle x_1, \mathbf{h}_1(x)\rangle, \langle x_2, \mathbf{h}_2(x)\rangle\} = \{\langle x_1, \{(0.6, 0.3, 0.3), (0.5, 0.2, 0.2)\}\rangle, \langle x_2, \{(0.3, 0.2, 0.1)\}\rangle\}. \tag{8}$$

Keeping the concept of EHFS in mind, we are now able to derive the concept of the single-valued extended hesitant fuzzy set (SVEHFS) as follows:

*Definition 1.* Let $\mathbf{H} = \{\langle x, \cup_{(\gamma_1(x),\ldots,\gamma_m(x))\in\mathbf{h}(x)}\{(\gamma_1(x),\ldots,\gamma_m(x))\}\rangle \mid x \in X\}$ be an extended hesitant fuzzy set (EHFS) on the reference set $X$. A single-valued extended hesitant fuzzy set (SVEHFS) is interpreted as the reduced form of $\mathbf{H}$ being characterized by

$$\dot{H} = \{\langle x, \{(\gamma_1(x),\ldots,\gamma_m(x))\}\rangle \mid x \in X\}, \quad (9)$$

where, for a fixed $x \in X$,

$$\mathbf{h}(x) = \{(\gamma_1(x),\ldots,\gamma_m(x))\}, \quad (10)$$

which stands for a single-valued extended hesitant fuzzy element (SVEHFE).

To give a more specific example, let us consider again the above example of EHFS, but in the form of SVEHFS, suppose that $X = \{x_1, x_2\}$ is the reference set and $\mathbf{h}_1(x) = \{(0.6, 0.3, 0.3)\}$ and $\mathbf{h}_2(x) = \{(0.3, 0.2, 0.1)\}$ are two SVEHFEs on $X$. Then, the SVEHFS $\dot{H}$ is characterized by

$$\dot{H} = \{\langle x_1, \mathbf{h}_1(x)\rangle, \langle x_2, \mathbf{h}_2(x)\rangle\} = \{\langle x_1, \{(0.6, 0.3, 0.3)\}\rangle, \langle x_2, \{(0.3, 0.2, 0.1)\}\rangle\}. \quad (11)$$

Now, we turn back to the beginning expression in this section where it was stated that all of the pairs involved in a unified PHFE correspond to a fixed vector as their probability part.

If we put aside the probability part of the unified PHFEs, then it gives rise to forming the corresponding SVEHFEs. For more explanation, we suppose that ${}^{\wp}h_1 = \langle \hbar_1^1, \wp_1^1\rangle, \langle \hbar_1^2, \wp_1^2\rangle, \langle \hbar_1^3, \wp_1^3\rangle, {}^{\wp}h_2 = \{\langle \hbar_2^1, \wp_2^1\rangle, \langle \hbar_2^2, \wp_2^2\rangle\}$, and ${}^{\wp}h_3 = \{\langle \hbar_3^1, \wp_3^1\rangle, \langle \hbar_3^2, \wp_3^2\rangle, \langle \hbar_3^3, \wp_3^3\rangle\}$ are three arbitrary PHFEs. Then, their unified forms can be derived as follows:

$$\begin{aligned}
{}^{\wp}\dot{h}_1 &= \{\langle \hbar_1^1, \wp_*^1\rangle, \langle \hbar_1^1, \wp_*^2\rangle, \langle \hbar_1^2, \wp_*^3\rangle, \langle \hbar_1^2, \wp_*^4\rangle, \langle \hbar_1^3, \wp_*^5\rangle, \langle \hbar_1^3, \wp_*^6\rangle\},\\
{}^{\wp}\dot{h}_2 &= \{\langle \hbar_2^1, \wp_*^1\rangle, \langle \hbar_2^1, \wp_*^2\rangle, \langle \hbar_2^1, \wp_*^3\rangle, \langle \hbar_2^2, \wp_*^4\rangle, \langle \hbar_2^2, \wp_*^5\rangle, \langle \hbar_2^2, \wp_*^6\rangle\},\\
{}^{\wp}\dot{h}_3 &= \{\langle \hbar_3^1, \wp_*^1\rangle, \langle \hbar_3^2, \wp_*^2\rangle, \langle \hbar_3^2, \wp_*^3\rangle, \langle \hbar_3^2, \wp_*^4\rangle, \langle \hbar_3^2, \wp_*^5\rangle, \langle \hbar_3^3, \wp_*^6\rangle\}.
\end{aligned} \quad (12)$$

If we put all the same probability part $(\wp_*^1, \wp_*^2, \ldots, \wp_*^6)$ of the later unified PHFEs ${}^{\wp}\dot{h}_1$, ${}^{\wp}\dot{h}_2$, and ${}^{\wp}\dot{h}_3$ aside, then the corresponding SVEHFEs are given as follows:

$$\begin{aligned}
\dot{h}_1 &:= \mathbf{h}_1 = \{(\hbar_1^1, \hbar_1^1, \hbar_1^2, \hbar_1^2, \hbar_1^3, \hbar_1^3)\},\\
\dot{h}_2 &:= \mathbf{h}_2 = \{(\hbar_2^1, \hbar_2^1, \hbar_2^1, \hbar_2^2, \hbar_2^2, \hbar_2^2)\},\\
\dot{h}_3 &:= \mathbf{h}_3 = \{(\hbar_3^1, \hbar_3^2, \hbar_3^2, \hbar_3^2, \hbar_3^2, \hbar_3^3)\}.
\end{aligned} \quad (13)$$

Before ending this section, we are required to discuss about the issue of distance measures for SVEHFEs. Generally, an unified-PHFE distance measure is constructed using the different part of hesitancy and probability parts. This is while the probability part of PHFEs is released in defining the concept of SVEHFEs. Therefore, the probability difference part may not make sense in developing distance measures for SVEHFEs, and only the hesitancy difference part is kept instead.

Now, if we assume that the weight of element $x_i \in X$ is to be denoted by $w_i$, satisfying $w_i \in [0, 1]$ and $\sum_{i=1}^N w_i = 1$, then a series of weighted distance measures for SVEHFSs $\dot{H}_1 = \{\langle x, \dot{h}_1(x) = \{(\gamma_1^1(x),\ldots,\gamma_m^1(x))\}\rangle : x \in X\}$ and $\dot{H}_2 = \{\langle x, \dot{h}_2(x) = \{(\gamma_1^2(x),\ldots,\gamma_m^2(x))\}\rangle : x \in X\}$ will be developed as the following:

(1) The single-valued extended hesitant weighted distance measure:

$$d_1(\dot{H}_1, \dot{H}_2) = \left[\sum_{i=1}^N w_i \left(\frac{1}{m}\sum_{j=1}^m \left|\gamma_j^1(x_i) - \gamma_j^2(x_i)^\lambda\right|\right)\right]^{1/\lambda}, \quad \lambda > 0. \quad (14)$$

(2) The single-valued extended hesitant weighted Hausdorff distance measure:

$$d_2(\dot{H}_1, \dot{H}_2) = \left[\sum_{i=1}^N w_i \max_{1\le j\le m}\left\{\left|\gamma_j^1(x_i) - \gamma_j^2(x_i)\right|^\lambda\right\}\right]^{1/\lambda}, \quad \lambda > 0. \quad (15)$$

(3) The single-valued extended hesitant weighted hybrid distance measure:

$$d_3(\dot{H}_1, \dot{H}_2) = \left[\sum_{i=1}^N w_i \times \frac{1}{2}\left(\frac{1}{m}\sum_{j=1}^m \left|\gamma_j^1(x_i) - \gamma_j^2(x_i)\right|^\lambda + \max_{1\le j\le m}\left\{\left|\gamma_j^1(x_i) - \gamma_j^2(x_i)^\lambda\right|\right\}\right)\right]^{1/\lambda}, \quad \lambda > 0. \quad (16)$$

(4) The generalized single-valued extended hesitant weighted hybrid distance measure:

$$\mathrm{d}_g\left(\dot{H}_1, \dot{H}_2\right) = \left[\sum_{i=1}^{N} w_i \times \alpha\left(\frac{1}{m}\sum_{j=1}^{m}\left|\gamma_j^1(x_i) - \gamma_j^2(x_i)\right|^{\lambda}\right) + \beta\left(\max_{1\le j\le m}\left\{\left|\gamma_j^1(x_i) - \gamma_j^2(x_i)\right|^{\lambda}\right\}\right)\right]^{1/\lambda}, \quad \lambda > 0, \tag{17}$$

where $0 \le \alpha, \beta \le 1$, and $\alpha + \beta = 1$.

Needless to say that all the abovementioned distance measures $d_1(.,.)$, $d_2(.,.)$, and $d_3(.,.)$ can be derived from the generalized form of $d_g(.,.)$.

**Theorem 1.** *Let* $\dot{H}_1 = \left\{\langle x, \dot{h}_1(x) = \{(\gamma_1^1(x), \ldots, \gamma_m^1(x))\}\rangle : x \in X\right\}$ *and* $\dot{H}_2 = \left\{\langle x, \dot{h}_2(x) = \{(\gamma_1^2(x), \ldots, \gamma_m^2(x))\}\rangle : x \in X\right\}$ *be two SVEHFSs. Then, the weighted distance measures for SVEHFSs given by (21)–(24) satisfy the following properties:*

*(1)* $0 \le d(\dot{H}_1, \dot{H}_2) \le 1$

*(2)* $d(\dot{H}_1, \dot{H}_2) = 0$ *if and only if* $\dot{H}_1 = \dot{H}_2$

*(3)* $d(\dot{H}_1, \dot{H}_2) = d(\dot{H}_2, \dot{H}_1)$

*(4)* $d(\dot{H}_1, \dot{H}_2) \le d(\dot{H}_1, \dot{H}_3)$ *if* $\dot{H}_1 \le_H \dot{H}_2 \le_H \dot{H}_3$ *implying that* $\dot{h}_1(x) \le_h \dot{h}_2(x) \le_h \dot{h}_3(x)$ *for any* $x \in X$, *that is,* $\gamma_j^1(x) \le \gamma_j^2(x) \le \gamma_j^3(x)$ *for any* $j = 1, 2, \ldots, m$

*Proof.* We only prove the above assertions for the distance measure $\mathrm{d}_g(.,.)$ given by (17), and the others can be deduced easily. □

*Axiom 1.* Keeping equation (17) in mind, we easily deduce that $0 \le |\gamma_j^1(x_i) - \gamma_j^2(x_i)| \le 1$ for any $0 \le \gamma_j^1(x_i) \le 1$ and $0 \le \gamma_j^2(x_i) \le 1$ in which $i = 1, 2, \ldots, N$ and $j = 1, 2, \ldots, m$.

These easily give rise to

$$0 \le \frac{1}{m}\sum_{j=1}^{m}\left|\gamma_j^1(x_i) - \gamma_j^2(x_i)\right|^{\lambda} \le 1, \quad \text{and } 0 \le \max_{1\le j\le m}\left\{\left|\gamma_j^1(x_i) - \gamma_j^2(x_i)\right|^{\lambda}\right\} \le 1, \tag{18}$$

for any $\lambda > 0$. Now, by taking $w_i \in [0,1]$ and $\sum_{i=1}^{N} w_i = 1$, and moreover, $0 \le \alpha, \beta \le 1$ and $\alpha + \beta = 1$, we result in $0 \le \mathrm{d}_g(\dot{H}_1, \dot{H}_2) \le 1$.

*Axiom 2.* Let

$$\mathrm{d}_g\left(\dot{H}_1, \dot{H}_2\right) = \left[\sum_{i=1}^{N} w_i \times \alpha\left(\frac{1}{m}\sum_{j=1}^{m}\left|\gamma_j^1(x_i) - \gamma_j^2(x_i)\right|^{\lambda}\right) + \beta\left(\max_{1\le j\le m}\left\{\left|\gamma_j^1(x_i) - \gamma_j^2(x_i)\right|^{\lambda}\right\}\right)\right]^{1/\lambda} = 0. \tag{19}$$

This implies that

$$\frac{1}{m}\sum_{j=1}^{m}\left|\gamma_j^1(x_i) - \gamma_j^2(x_i)\right|^{\lambda} = 0, \quad \text{and } \max_{1\le j\le m}\left\{\left|\gamma_j^1(x_i) - \gamma_j^2(x_i)\right|^{\lambda}\right\} = 0, \tag{20}$$

in which both of them lead to $|\gamma_j^1(x_i) - \gamma_j^2(x_i)| = 0$, that is, $\gamma_j^1(x_i) = \gamma_j^2(x_i)$ for any $i = 1, 2, \ldots, N$ and $j = 1, 2, \ldots, m$. Thus, we conclude that $\dot{H}_1 = \dot{H}_2$.

The inverse axiom will be easily proved in the same manner.

*Axiom 3.* The proof is immediate from definition of distance measure $\mathrm{d}_g(.,.)$ given by (17).

*Axiom 4.* If $\dot{H}_1 \le_H \dot{H}_2 \le_H \dot{H}_3$, then it implies that $\dot{h}_1(x_i) \le_h \dot{h}_2(x_i) \le_h \dot{h}_3(x_i)$ for any $x_i \in X$, that is,

$\gamma_j^1(x_i) \le \gamma_j^2(x_i) \le \gamma_j^3(x_i)$ for any $j = (1, 2, \ldots, m)$ and $x_i \in X$. The latter inequalities give rise to $|\gamma_j^1(x_i) - \gamma_j^2(x_i)| \le |\gamma_j^1(x_i) - \gamma_j^3(x_i)|$ for any $k = (1, 2, \ldots, m)$ and $x_i \in X$, and therefore,

$$\begin{aligned} \mathrm{d}_g\left(\dot{H}_1, \dot{H}_2\right) &= \left[\sum_{i=1}^{N} w_i \times \alpha\left(\frac{1}{m}\sum_{j=1}^{m}\left|\gamma_j^1(x_i) - \gamma_j^2(x_i)\right|^{\lambda}\right) + \beta\left(\max_{1\le j\le m}\left\{\left|\gamma_j^1(x_i) - \gamma_j^2(x_i)\right|^{\lambda}\right\}\right)\right]^{1/\lambda} \\ &\le \left[\sum_{i=1}^{N} w_i \times \alpha\left(\frac{1}{m}\sum_{j=1}^{m}\left|\gamma_j^1(x_i) - \gamma_j^3(x_i)\right|^{\lambda}\right) + \beta\left(\max_{1\le j\le m}\left\{\left|\gamma_j^1(x_i) - \gamma_j^3(x_i)\right|^{\lambda}\right\}\right)\right]^{1/\lambda} = \mathrm{d}_g\left(\dot{H}_1, \dot{H}_3\right). \end{aligned} \tag{21}$$

## 4. SVEHFS-Based Score Function for PHFSs

As will be shown later, the score function of SVEHFS is fundamentally defined in accordance with the score function of its SVEHFEs, and therefore, we only discuss the score functions for SVEHFEs.

Now, we are in a position to introduce a class of SVEHFE-score functions by the help of SVEHFE distance measures given by (14)–(17).

*Definition 2.* Let $\dot{h} = \{(\gamma_1, \ldots, \gamma_m)\}$ be a SVEHFE. The score function $Sc(.)$ is defined as

$$Sc(\dot{h}) := Sc((\gamma_1, \ldots, \gamma_m)) = 1 - \mathrm{d}((\gamma_1, \ldots, \gamma_m), 1), \tag{22}$$

where $\mathrm{d}(.,.)$ is a distant measure for SVEHFE and 1 stands for the SVEHFE $(1, \ldots, 1)$.

As will be shown below, score function (22) satisfies the fundamental properties, known as monotonicity, boundary conditions, idempotency, and duality.

*Property 1* (monotonicity). Let $\dot{h}_1 = \{(\gamma_1^1, \ldots, \gamma_m^1)\}$ and $\dot{h}_2 = \{(\gamma_1^2, \ldots, \gamma_m^2)\}$ be two SVEHFEs such that $\dot{h}_1 \le_h \dot{h}_2$, that is, $\gamma_j^1 \le \gamma_j^2$ for any $j = 1, 2, \ldots, m$. Then, the score function $Sc(.)$ given by (22) satisfies

$$Sc\left(\dot{h}_1\right) \le Sc\left(\dot{h}_2\right). \tag{23}$$

*Proof.* From the fact that $\dot{h}_1 \le_h \dot{h}_2 \le_h 1$ and the monotonicity property of any distance $\mathrm{d}(.,.)$, we find that $\mathrm{d}((\gamma_1^1, \ldots, \gamma_m^1), 1) \ge \mathrm{d}((\gamma_1^2, \ldots, \gamma_m^2), 1)$ which easily implies that

$$\mathrm{Sc}\left(\dot{h}_1\right) = 1 - \mathrm{d}\left(\left(\gamma_1^1, \ldots, \gamma_m^1\right), 1\right) \le 1 - \mathrm{d}\left(\left(\gamma_1^2, \ldots, \gamma_m^2\right), 1\right) = Sc\left(\dot{h}_2\right). \tag{24}$$

□

*Property 2* (boundary conditions). Let $1 = (1, 1, \ldots, 1)$ and $0 = (0, 0, \ldots, 0)$ be One-SVEHFE and Zero-SVEHFE, respectively. Then, we conclude that the score function $Sc(.)$ given by (22) satisfies

$$Sc(1) = 1, \quad \text{and } Sc(0) = 0. \tag{25}$$

*Proof.* By keeping the axiom $0 \le \mathrm{d}(\dot{h}_1, \dot{h}_2) \le 1$ (for any SVEHFEs $\dot{h}_1$ and $\dot{h}_2$) in mind, the proof is evident. □

*Property 3* (idempotency). Let $\dot{h} = \{(\gamma_1, \ldots, \gamma_m)\} = \{(\gamma, \ldots, \gamma)\}$. If $\mathrm{d}((\gamma, \ldots, \gamma), 1) = 1 - \gamma$, then the score function $Sc(.)$ given by (22) satisfies

$$Sc(\dot{h}) = \gamma. \tag{26}$$

*Proof.* The proof is obvious. □

*Definition 3.* If $Sc(.)$ stands for a score function of SVEHFEs, then

$$D(Sc(\dot{h})) = D(Sc(\{(\gamma_1, \ldots, \gamma_m)\})) := 1 - Sc(\{(1 - \gamma_1, \ldots, 1 - \gamma_m)\}), \tag{27}$$

which defines the dual form of $Sc(.)$.

*Property 4* (duality). The score function $Sc(.)$ given by (22) satisfies

$$D(D(Sc(\dot{h}))) = Sc(\dot{h}). \tag{28}$$

*Proof.* Following from Definition 3, we get that

$$D(D(Sc(\dot{h}))) = D(1 - Sc(\{(1-\gamma_1,\ldots,1-\gamma_m)\})) \\ := 1 - (1 - Sc(\{(1-(1-\gamma_1),\ldots,1-(1-\gamma_m))\})) = Sc(\{(\gamma_1,\ldots,\gamma_m)\}). \tag{29}$$

□

*Property 5* (generalization). Let $\Theta$: $[0,1] \longrightarrow [0,1]$ be a strictly monotone decreasing real function and $\mathrm{d}(.,.)$ be a distance measure between SVEHFEs. Then,

$$Sc_\Theta(\dot{h}) = \Theta(\mathrm{d}((\gamma_1,\ldots,\gamma_m),1)), \tag{30}$$

which defines a score function for SVEHFEs.

*Proof.* We show that $Sc_\Theta(.)$ satisfies two fundamental properties, called above as monotonicity and boundary conditions.

Monotonicity property: from the fact that $\dot{h}_1 \le_h \dot{h}_2 \le_h 1$ and the monotonicity property of any distance $\mathrm{d}(.,.)$, we find that $\mathrm{d}((\gamma_1^1,\ldots,\gamma_m^1),1) \ge \mathrm{d}((\gamma_1^2,\ldots,\gamma_m^2),1)$. On the contrary, the latter inequality and the strictly monotone decreasing property of $\Theta$ give rise to

$$\Theta\left(\mathrm{d}\left(\left(\gamma_1^1,\ldots,\gamma_m^1\right),1\right)\right) \le \Theta\left(\mathrm{d}\left(\left(\gamma_1^2,\ldots,\gamma_m^2\right),1\right)\right), \tag{31}$$

which implies that

$$Sc_\Theta\left(\dot{h}_1\right) \le Sc_\Theta\left(\dot{h}_2\right). \tag{32}$$

Boundary conditions' property: consider the One-SVEHFE $1 = (1,1,\ldots,1)$ and the Zero-SVEHFE $0 = (0,0,\ldots,0)$. Then, by keeping the axiom $0 \le \mathrm{d}(\dot{h}_1,\dot{h}_2) \le 1$ (for any SVEHFEs $\dot{h}_1$ and $\dot{h}_2$) in mind, we conclude easily that the score function $Sc_\Theta(.)$ given by (30) satisfies

$$Sc_\Theta(1) = 1, \quad \text{and } Sc_\Theta(0) = 0. \tag{33}$$

By the help of Property 5, we will be able to develop different formulas of score functions for SVEHFEs by taking into account different strictly monotone decreasing functions $\Theta$: $[0,1] \longrightarrow [0,1]$, for instance, (1) $\Theta_1(x) = 1 - x$; (2) $\Theta_2(x) = 1 - x/1 + x$; (3) $\Theta_3(x) = 1 - xe^{x-1}$; (4) $\Theta_4(x) = 1 - x^2$.

From this property, the following SVEHFE-score functions can be established:

$$\begin{aligned} Sc_{\Theta_1}(\dot{h}) &= 1 - \mathrm{d}((\gamma_1,\ldots,\gamma_m),1); \\ Sc_{\Theta_2}(\dot{h}) &= \frac{1 - \mathrm{d}((\gamma_1,\ldots,\gamma_m),1)}{1 + \mathrm{d}((\gamma_1,\ldots,\gamma_m),1)}; \\ Sc_{\Theta_3}(\dot{h}) &= 1 - \mathrm{d}((\gamma_1,\ldots,\gamma_m),1)e^{\mathrm{d}((\gamma_1,\ldots,\gamma_m),1)-1}; \\ Sc_{\Theta_4}(\dot{h}) &= 1 - \mathrm{d}^2((\gamma_1,\ldots,\gamma_m),1). \end{aligned} \tag{34}$$

□

## 5. SVEHFS Score-Based Multiple Criteria Decision-Making

This section provides three practical case studies to demonstrate that the proposed concept of SVEHFS is effective enough in the field of score-based optimization methods.

Briefly speaking, the proposed SVEHFS score-based decision-making procedure is composed of the following three stages: the unification process of PHFSs, the reduction process of PHFSs to SVEHFSs, and the selection procedure. The first two stages have been served in Sections 2 and 3. The last stage given in Section 4 describes the process of ranking alternatives in accordance with their values of score function and selecting the best one with the greatest value.

Now, in order for more systematically being understood, we give following Algorithm 3 (see Figure 4).

*5.1. Case Study I.* In this portion, we adopt an optimization problem which was originally solved in [33] by the use of a probabilistic linguistic term set- (PLTS-) based algorithm. Here, in order to have a better understanding of how the proposed SVEHFS- (or PHFS-) based algorithm behaves over the later-mentioned multiple criteria decision-making problem, we transform PLTS information to PHFS (or SVEHFS) data. This is done by the help of Theorem 1 in [34] in which the bijective transformation between PLTSs and PHFSs is explained.

A company needs to plan the development of large projects (strategy initiatives) for the next five years. To do this end, the company invites five experts to form the board of directors. Moreover, the company takes three possible projects $A_i$ ($i = 1, 2$, and 3) into consideration which should be evaluated based on their importance. These projects should be ranked in accordance with four criteria of the benefit type which are suggested by the balanced scorecard methodology as follows:

$C_1$: financial perspective

$C_2$: the customer satisfaction

$C_3$: internal business process perspective

$C_4$: learning and growth perspective

Now, by adopting Algorithm 3 and the assumption that five experts apply the linguistic term set $S = \{s_0 = \text{none}, s_1 = \text{verylow}, s_2 = \text{low}, s_3 = \text{medium}, \quad s_4 = \text{high}, s_5 = \text{veryhigh}, s_6 = \text{perfect}\}$, we are able to evaluate the projects $A_i$ ($i = 1, 2$, and 3) by means of PLTSs in Step 1. The corresponding data is presented in Table 1.

To save more space, we only present the transformation form of the probabilistic linguistic decision matrix into that of PHFSs as explained above. Consequently, the result will be that given in Table 2.

Now, by the help of Step 2 of Algorithm 3, we are in a position to use the proposed unification process for the data of Table 2 and draw those results being summarized in Table 3.

In what follows, by the use of Step 3 of Algorithm 3, we will derive the corresponding SVEHFEs, as shown in Table 4.

If we now consider the weight vector of criteria $C_i$ ($i = 1, 2, 3, 4$) in the form of $w = (0.138, 0.304, 0.416, 0.142)$

**Input**: the probabilistic hesitant fuzzy decision matrix
**Output**: the ranking of alternatives and the best one
**Step 1**: build the probabilistic hesitant fuzzy decision matrix
**Step 2**: extract the unified form of PHFSs from the decision matrix
**Step 3**: reduced the probabilistic-unified PHFSs to so-called SVEHFSs
**Step 4**: find the best alternative(s) in accordance with their SVEHFS score values

Algorithm 3: Proposed SVEHFS score-based decision-making algorithm.

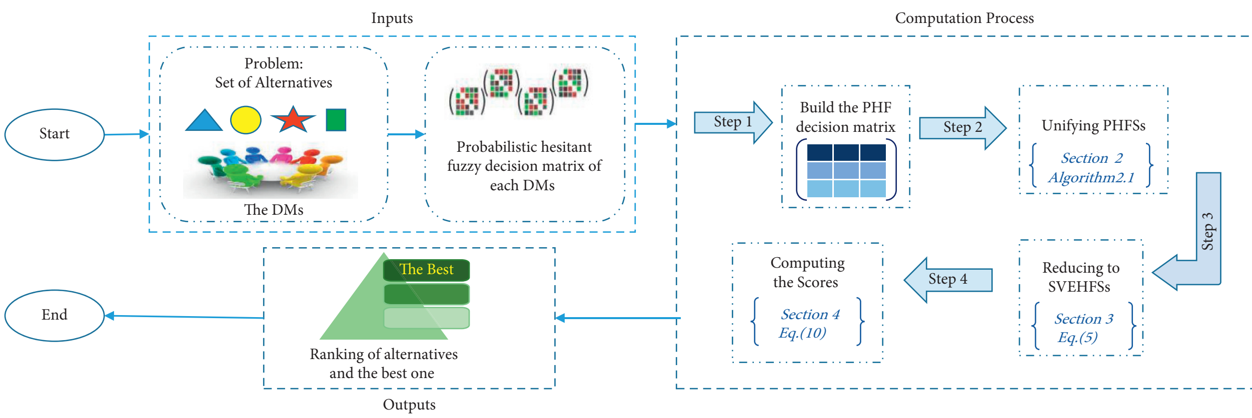


Figure 4: Proposed SVEHFS score-based decision-making algorithm.

Table 1: The probabilistic linguistic decision matrix.

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $A_1$ | $\{s_3(0.4), s_4(0.6)\}$ | $\{s_2(0.2), s_4(0.8)\}$ | $\{s_3(0.2), s_4(0.8)\}$ | $\{s_3(0.4), s_5(0.6)\}$ |
| $A_2$ | $\{s_3(0.8), s_5(0.2)\}$ | $\{s_2(0.3), s_3(0.4), s_4(0.3)\}$ | $\{s_1(0.3), s_2(0.4), s_3(0.3)\}$ | $\{s_3(0.8), s_4(0.2)\}$ |
| $A_3$ | $\{s_3(0.6), s_4(0.4)\}$ | $\{s_3(0.6), s_4(0.2), s_5(0.2)\}$ | $\{s_3(0.4), s_4(0.2), s_5(0.4)\}$ | $\{s_4(0.7), s_6(0.3)\}$ |

Table 2: The probabilistic hesitant fuzzy decision matrix.

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $A_1$ | $\{0.5(0.4), 0.67(0.6)\}$ | $\{0.33(0.2), 0.67(0.8)\}$ | $\{0.5(0.2), 0.67(0.8)\}$ | $\{0.5(0.4), 0.83(0.6)\}$ |
| $A_2$ | $\{0.5(0.8), 0.83(0.2)\}$ | $\{0.33(0.3), 0.5(0.4), 0.67(0.3)\}$ | $\{0.17(0.3), 0.33(0.4), 0.5(0.3)\}$ | $\{0.5(0.8), 0.67(0.2)\}$ |
| $A_3$ | $\{0.5(0.6), 0.67(0.4)\}$ | $\{0.5(0.6), 0.67(0.2), 0.83(0.2)\}$ | $\{0.5(0.4), 0.67(0.2), 0.83(0.4)\}$ | $\{0.67(0.7), 1(0.3)\}$ |

together with $\lambda = 1$ for distance measures $d_1(.,.)$, $d_2(.,.)$, and $d_3(.,.)$ given, respectively, by (14)–(16); then, following Step 4 of Algorithm 3, the proposed SVEHFS-score function $Sc(.)$ gives rise to the priorities of projects listed in Table 5. In addition to these results, the output of Pang et al.'s TOPSIS-based and aggregation-based techniques [33] has been presented in Table 5.

Generally, the TOPSIS-based and aggregation-based techniques are chosen in accordance with the decision makers' need on one side, and on the other side, Pang et al.'s TOPSIS- and aggregation-based techniques [33] impose the extracondition of normalization by adding a number of artificial linguistic terms with "zero" probability. By imposing such artificial PLTS normalization process, the underlying optimization procedure will cause the computational process with more complexity. In contrast, the SVEHFS-score-based technique maintains the integrity and authenticity of decision information as far as possible, which results in much more reasonable decisions.

*5.2. Case Study II.* In this part of contribution, we implement the proposed SVEHFS-score function for specifying the best Chinese hospital from a collection of considered hospitals. Such a problem was originally discussed by Song et al. [28], and then, it was more investigated by some other researchers

Table 3: The unified probabilistic hesitant fuzzy decision matrix.

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $A_1$ | {0.5 (0.2), 0.5 (0.1), 0.5 (0.1), 0.67 (0.2), 0.67 (0.1), 0.67 (0.1), 0.67 (0.2) | {0.33 (0.2), 0.67 (0.1), 0.67 (0.1), 0.67 (0.2), 0.67 (0.1), 0.67 (0.1), 0.67 (0.2) | {0.5 (0.2), 0.67 (0.1), 0.67 (0.1), 0.67 (0.2), 0.67 (0.1), 0.67 (0.1), 0.67 (0.2) | {0.5 (0.2), 0.5 (0.1), 0.5 (0.1), 0.83 (0.2), 0.83 (0.1), 0.83 (0.1), 0.83 (0.2) |
| $A_2$ | {0.5 (0.2), 0.5 (0.1), 0.5 (0.1), 0.5 (0.2), 0.5 (0.1), 0.5 (0.1), 0.83 (0.2) | {0.33 (0.2), 0.33 (0.1), 0.5 (0.1), 0.5 (0.2), 0.5 (0.1), 0.67 (0.1), 0.67 (0.2) | {0.17 (0.2), 0.17 (0.1), 0.33 (0.1), 0.33 (0.2), 0.33 (0.1), 0.5 (0.1), 0.5 (0.2) | {0.5 (0.2), 0.5 (0.1), 0.5 (0.1), 0.5 (0.2), 0.5 (0.1), 0.5 (0.1), 0.67 (0.2) |
| $A_3$ | {0.5 (0.2), 0.5 (0.1), 0.5 (0.1), 0.5 (0.2), 0.67 (0.1), 0.67 (0.1), 0.67 (0.2) | {0.5 (0.2), 0.5 (0.1), 0.5 (0.1), 0.5 (0.2), 0.67 (0.1), 0.67 (0.1), 0.83 (0.2) | {0.5 (0.2), 0.5 (0.1), 0.5 (0.1), 0.67 (0.2), 0.83 (0.1), 0.83 (0.1), 0.83 (0.2) | {0.67 (0.2), 0.67 (0.1), 0.67 (0.1), 0.67 (0.2), 0.67 (0.1), 1 (0.1), 1 (0.2) |

Table 4: The SVEHLFS decision matrix.

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $A_1$ | $\{(0.5, 0.5, 0.5, 0.67, 0.67, 0.67, 0.67)\}$ | $\{(0.33, 0.67, 0.67, 0.67, 0.67, 0.67, 0.67)\}$ | $\{(0.5, 0.67, 0.67, 0.67, 0.67, 0.67, 0.67)\}$ | $\{(0.5, 0.5, 0.5, 0.83, 0.83, 0.83, 0.83)\}$ |
| $A_2$ | $\{(0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.83)\}$ | $\{(0.33, 0.33, 0.5, 0.5, 0.5, 0.67, 0.67)\}$ | $\{(0.17, 0.17, 0.33, 0.33, 0.33, 0.5, 0.5)\}$ | $\{(0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.67)\}$ |
| $A_3$ | $\{(0.5, 0.5, 0.5, 0.5, 0.67, 0.67, 0.67)\}$ | $\{(0.5, 0.5, 0.5, 0.5, 0.67, 0.67, 0.83)\}$ | $\{(0.5, 0.5, 0.5, 0.67, 0.83, 0.83, 0.83)\}$ | $\{(0.67, 0.67, 0.67, 0.67, 0.67, 1, 1)\}$ |

Table 5: Ranking results of the different techniques.

| | Ranking order | Optimal alternative |
|---|---|---|
| Pang et al.'s TOPSIS-based technique [33] | $A_1 > A_3 > A_2$ | $A_1$ |
| Pang et al.'s aggregation-based technique [33] | $A_1 > A_3 > A_2$ | $A_1$ |
| The proposed SVEHFS-score function $Sc_{d_1}$ | $A_3 > A_1 > A_2$ | $A_3$ |
| The proposed SVEHFS-score function $Sc_{d_2}$ | $A_3 > A_1 > A_2$ | $A_3$ |
| The proposed SVEHFS-score function $Sc_{d_3}$ | $A_3 > A_1 > A_2$ | $A_3$ |

Table 6: The probabilistic hesitant fuzzy decision matrix.

| | Environment of health service $(C_1)$ | Treatment optimization $(C_2)$ | Social resource allocation $(C_3)$ |
|---|---|---|---|
| West China Hospital $(A_1)$ | $\{0.5(0.4), 0.7(0.6)\}$ | $\{0.9(1)\}$ | $\{0.3(0.2), 0.5(0.8)\}$ |
| Huashan Hospital $(A_2)$ | $\{0.8(0.3), 0.9(0.7)\}$ | $\{0.5(1)\}$ | $\{0.8(0.4), 0.9(0.6)\}$ |
| Union Medical College Hospital $(A_3)$ | $\{0.5(1)\}$ | $\{0.7(0.5), 0.9(0.5)\}$ | $\{0.8(0.6), 0.9(0.4)\}$ |
| PLA General Hospital $(A_4)$ | $\{0.8(0.5), 0.9(0.5)\}$ | $\{0.3(0.5), 0.6(0.5)\}$ | $\{0.7(1)\}$ |

including Zhang et al. [21] and Farhadinia and Herrera-Viedma [23].

The problem here is that we are seeking the best Chinese hospital with regards to the medical resource restriction and the old-age limitation of target population. In this regard, three criteria are mainly considered as $C_1$: environment of health service, $C_2$: treatment optimization, and $C_3$: social resource allocation. The corresponding weight vector of criteria is supposed to be $w = (0.2, 0.1, 0.7)$. For this optimization problem, we evaluate four candidate hospitals including $A_1$: West China Hospital of Sichuan University, $A_2$: Huashan Hospital of Fudan University, $A_3$: Union Medical College Hospital, and $A_4$: Chinese PLA General Hospital. Since one option is not able to describe the influence factor, a number of experts are asked to express their preferences related to the abovementioned hospitals based on available criteria in the form of PHFSs.

Now, performing Step 1 of Algorithm 3 leads to constructing the following probabilistic hesitant fuzzy decision matrix (see Table 6).

Similar to what is discussed in Section 3 and applying Steps 2 and 3 of Algorithm 3, we will derive the corresponding unified PHFEs' and SVEHFEs' matrices, as shown in Tables 7 and 8, respectively.

Following Step 4 of Algorithm 3, if we now consider the weight vector of criteria $C_i\,(i = 1, 2, 3)$ in the form of $w = (0.2, 0.1, 0.7)$ together with $\lambda = 1$ for distance measures $d_1(.,.)$, $d_2(.,.)$, and $d_3(.,.)$ given, respectively, by (14)–(16), then the proposed SVEHFS-score function $Sc(.)$ gives rise to the priority of Chinese hospitals listed in Tables 9 and 10. In addition to these results, the output of Zhang et al.'s [21], Song et al.'s [28], and Farhadinia and Xu [22] techniques have been also presented in Tables 9 and 10.

As what can be observed from Tables 9 and 10, the most repeated alternative is $A_2$ which implies that the most appropriate hospital is the Huashan Hospital of Fudan University. This is exactly what we observe from the last three rows of Table 10 dedicated to the results of proposed SVEHFS-score functions.

Now, let us conclude the part of this section with some discussions on the pros and cons of proposed SVEHFS-score functions. The techniques of Zhang et al. [21] and Song et al. [28] are restricted directly to the normalization process of PHFSs, and Farhadinia and Xu [22] techniques are related to the multiplying and exponential deformation formulas of each pair of possible membership degree and its associated probability. This is while the proposed SVEHFS-score functions do not change the original form of PHFSs, and this can be seen as a pro. The other significant advantage of SVEHFS-based score functions over the other above-mentioned techniques is its ease of use.

*5.3. Case Study III.* Because of competition and limitation of research funding in universities of China, a few outstanding research topics are annually supported. In order to select the best research topic several aspects including practicality, innovativeness, and feasibility are taken into consideration.

In March 2018, the business school of university A in China asked three instructors to submit their research topics for evaluating which one is more suitable for granting the university research funding. In this project, three professors $DM_k$ $(k = 1, 2, \text{ and } 3)$ are invited for evaluating the quality of the three research topics $A_i$ $(i = 1, 2, \text{ and } 3)$ in accordance with three criteria: $C_{j=1}$: innovativeness, $C_{j=2}$: practicality, and $C_{j=3}$: feasibility. All the criteria are benefit types, and all the corresponding evaluations of three professors $DM_k$ $(k = 1, 2, \text{ and } 3)$ are represented in the form of PHFE-based decision matrices (see Tables 11–13).

By applying Steps 2 and 3 of Algorithm 3, the individual unified PHFEs are computed as the data given in Tables 14–16.

Following the process discussed by Li et al. [35], the decision makers' weights are obtained as

$$\varpi_k = \frac{\sum_{j=1}^{n}\sum_{i=1}^{m-1}\sum_{g=i+1}^{m} d\left(\gamma_{ij}^k, \gamma_{gj}^k\right)}{\sum_{k=1}^{z}\sum_{j=1}^{n}\sum_{i=1}^{m-1}\sum_{g=i+1}^{m} d\left(\gamma_{ij}^k, \gamma_{gj}^k\right)}, \quad k = 1, 2, \text{ and } 3, \tag{35}$$

Table 7: The unified probabilistic hesitant fuzzy decision matrix.

| | $C_1$ | $C_2$ | $C_3$ |
|---|---|---|---|
| $A_1$ | {0.5 (0.2), 0.5 (0.1), 0.5 (0.1), 0.7 (0.1), 0.7 (0.1), 0.7 (0.4) | {0.9 (0.2), 0.9 (0.1), 0.9 (0.1), 0.9 (0.1), 0.9 (0.1), 0.9 (0.4) | {0.3 (0.2), 0.5 (0.1), 0.5 (0.1), 0.5 (0.1), 0.5 (0.1), 0.5 (0.4) |
| $A_2$ | {0.8 (0.2), 0.8 (0.1), 0.9 (0.1), 0.9 (0.1), 0.9 (0.1), 0.9 (0.4) | {0.5 (0.2), 0.5 (0.1), 0.5 (0.1), 0.5 (0.1), 0.5 (0.1), 0.5 (0.4) | {0.8 (0.2), 0.8 (0.1), 0.8 (0.1), 0.9 (0.1), 0.9 (0.1), 0.9 (0.4) |
| $A_3$ | {0.5 (0.2), 0.5 (0.1), 0.5 (0.1), 0.5 (0.1), 0.5 (0.1), 0.5 (0.4) | {0.7 (0.2), 0.7 (0.1), 0.7 (0.1), 0.7 (0.1), 0.9 (0.1), 0.9 (0.4) | {0.8 (0.2), 0.8 (0.1), 0.8 (0.1), 0.8 (0.1), 0.8 (0.1), 0.9 (0.4) |
| $A_4$ | {0.8 (0.2), 0.8 (0.1), 0.8 (0.1), 0.8 (0.1), 0.9 (0.1), 0.9 (0.4) | {0.3 (0.2), 0.3 (0.1), 0.3 (0.1), 0.3 (0.1), 0.7 (0.1), 0.7 (0.4) | {0.7 (0.2), 0.7 (0.1), 0.7 (0.1), 0.7 (0.1), 0.7 (0.1), 0.7 (0.4) |

Table 8: The SVEHFS decision matrix.

| | $C_1$ | $C_2$ | $C_3$ |
|---|---|---|---|
| $A_1$ | $\{(0.5, 0.5, 0.5, 0.7, 0.7, 0.7)\}$ | $\{(0.9, 0.9, 0.9, 0.9, 0.9, 0.9)\}$ | $\{(0.3, 0.5, 0.5, 0.5, 0.5, 0.5)\}$ |
| $A_2$ | $\{(0.8, 0.8, 0.8, 0.9, 0.9, 0.9)\}$ | $\{(0.5, 0.5, 0.5, 0.5, 0.5, 0.5)\}$ | $\{(0.8, 0.8, 0.8, 0.9, 0.9, 0.9)\}$ |
| $A_3$ | $\{(0.5, 0.5, 0.5, 0.5, 0.5, 0.5)\}$ | $\{(0.7, 0.7, 0.7, 0.7, 0.9, 0.9)\}$ | $\{(0.8, 0.8, 0.8, 0.8, 0.8, 0.9)\}$ |
| $A_4$ | $\{(0.8, 0.8, 0.8, 0.8, 0.9, 0.9)\}$ | $\{(0.3, 0.3, 0.3, 0.3, 0.7, 0.7)\}$ | $\{(0.7, 0.7, 0.7, 0.7, 0.7, 0.7)\}$ |

Table 9: Ranking results of Chinese hospitals.

| Score function | Score of hospitals | Ranking order | Optimal alternative |
|---|---|---|---|
| $\overline{S}^{d_1}$ | 0.8555 0.7884 0.8002 0.8186 | $A_2 > A_3 > A_4 > A_1$ | $A_2$ |
| $\underline{S}^{d_1}$ | 0.1268 0.0410 0.0546 0.0771 | $A_2 > A_3 > A_4 > A_1$ | $A_2$ |
| $\overline{S}^{d_2}$ | 0.8618 0.7951 0.8007 0.8186 | $A_2 > A_3 > A_4 > A_1$ | $A_2$ |
| $\underline{S}^{d_2}$ | 0.1406 0.0432 0.0595 0.0774 | $A_2 > A_3 > A_4 > A_1$ | $A_2$ |
| $\overline{S}^{d_3}$ | 0.9631 0.9063 0.8310 0.8255 | $A_4 > A_3 > A_2 > A_1$ | $A_4$ |
| $\underline{S}^{d_3}$ | 0.2075 0.0576 0.0827 0.0860 | $A_2 > A_3 > A_4 > A_1$ | $A_2$ |
| $\overline{S}^{d_4}$ | 0.9093 0.8473 0.8156 0.8220 | $A_3 > A_4 > A_2 > A_1$ | $A_3$ |
| $\underline{S}^{d_4}$ | 0.1671 0.0493 0.0686 0.0815 | $A_2 > A_3 > A_4 > A_1$ | $A_2$ |
| $\overline{S}_{AM}$ | 0.1445 0.2116 0.1998 0.1814 | $A_2 > A_3 > A_4 > A_1$ | $A_2$ |
| $\underline{S}_{AM}$ | 0.8732 0.9590 0.9454 0.9229 | $A_2 > A_3 > A_4 > A_1$ | $A_2$ |
| $\overline{S}_{GM}$ | 0.1054 0.1871 0.1977 0.1814 | $A_3 > A_2 > A_4 > A_1$ | $A_3$ |
| $\underline{S}_{GM}$ | 0.8711 0.9589 0.9451 0.9229 | $A_2 > A_3 > A_4 > A_1$ | $A_2$ |
| $\overline{S}_{Min}$ | 0.0369 0.0937 0.1690 0.1745 | $A_4 > A_3 > A_2 > A_1$ | $A_4$ |
| $\underline{S}_{Min}$ | 0.7925 0.9424 0.9173 0.9140 | $A_2 > A_3 > A_4 > A_1$ | $A_2$ |
| $\overline{S}_{Max}$ | 0.2957 0.3704 0.2328 0.1880 | $A_2 > A_1 > A_3 > A_4$ | $A_2$ |
| $\underline{S}_{Max}$ | 0.9399 0.9742 0.9707 0.9312 | $A_2 > A_3 > A_1 > A_4$ | $A_2$ |
| $\overline{S}_{P}$ | 0.0001 0.0012 0.0015 0.0011 | $A_3 > A_2 > A_4 > A_1$ | $A_3$ |
| $\underline{S}_{P}$ | 0.5758 0.8455 0.7979 0.7255 | $A_2 > A_3 > A_1 > A_4$ | $A_2$ |
| $\overline{S}_{BS}$ | 0.5779 0.8464 0.7992 0.7257 | $A_2 > A_3 > A_1 > A_4$ | $A_2$ |
| $\underline{S}_{BS}$ | 1 1 1 1 | $A_1 > A_2 > A_3 > A_4$ | $A_1$ |
| $\overline{S}_{F}$ | 0.0007 0.0112 0.0112 0.0069 | $A_3 > A_2 > A_4 > A_1$ | $A_3$ |
| $\underline{S}_{F}$ | 0.9993 1.0000 1.0000 0.9998 | $A_2 > A_3 > A_1 > A_4$ | $A_2$ |

Table 10: Continues from Table 9.

| Technique | Ranking order | Optimal alternative |
|---|---|---|
| Zhang et al.'s [21] | $A_2 > A_3 > A_4 > A_1$ | $A_2$ |
| Song et al.'s [28] | $A_2 >^{0.838} A_3 >^{0.819} A_4 >^{1} A_1$ | $A_2$ |
| Farhadinia and Xu's [22] first two step-based process multiplying deformation formula: $\overline{S}_1(A_1) = 0.144$, $\overline{S}_1(A_2) = 0.2116$, $\overline{S}_1(A_3) = 0.199$, and $\overline{S}_1(A_4) = 0.1814$ | $A_2 > A_3 > A_4 > A_1$ | $A_2$ |
| Exponential deformation formula: $\underline{S}_1(A_1) = 0.873$, $\underline{S}_1(A_2) = 0.9590$, $\underline{S}_1(A_3) = 0.945$, and $\underline{S}_1(A_4) = 0.9229$ | $A_2 > A_3 > A_4 > A_1$ | $A_2$ |
| Farhadinia and Xu's [22] second two step-based process multiplying deformation formula: $\overline{S}_2(A_1) = 0.1445$, $\overline{S}_2(A_2) = 0.2116$, $\overline{S}_2(A_3) = 0.1998$, and $\overline{S}_2(A_4) = 0.1814$ | $A_2 > A_3 > A_4 > A_1$ | $A_2$ |
| Exponential deformation formula: $\underline{S}_2(A_1) = 0.8732$, $\underline{S}_2(A_2) = 0.9590$, $\underline{S}_2(A_3) = 0.9454$, and $\underline{S}_2(A_4) = 0.9229$ | $A_2 > A_3 > A_4 > A_1$ | $A_2$ |
| Farhadinia and Xu's [22] third two step-based process multiplying deformation formula: $\overline{R}_3(A_1) = (0.1445, 0.1376)$, $\overline{R}_3(A_2) = (0.2116, 0.1171)$, $\overline{R}_3(A_3) = (0.1998, 0.0428)$, $\overline{R}_3(A_4) = (0.1814, 0.1824)$ | $\overline{R}_3(A_2) \geq_{lex} \overline{R}_3(A_3) \geq_{lex} \overline{R}_3(A_4) \geq_{\text{lex}} \overline{R}_3(A_1)$ $A_2 > A_3 > A_4 > A_1$ | $A_2$ |
| Exponential deformation formula: $\underline{R}_3(A_1) = (0.8732, 0.1499)$, $\underline{R}_3(A_2) = (0.9590, 0.1114)$, $\underline{R}_3(A_3) = (0.9454, 0.0475)$, and $\underline{R}_3(A_4) = (0.9229, 0.0430)$ | $\underline{R}_3(A_2) \geq_{lex} \underline{R}_3(A_3) \geq_{lex} \underline{R}_3(A_4) \geq_{\text{lex}} \underline{R}_3(A_1)$ $A_2 > A_3 > A_4 > A_1$ | $A_2$ |
| The proposed SVEHFS-score function $Sc_{d_1}$: $Sc_{d_1}(A_1) = 0.5471$, $Sc_{d_1}(A_2) = 0.8735$, $Sc_{d_1}(A_3) = 0.7483$, and $Sc_{d_1}(A_4) = 0.7000$ | $A_2 > A_3 > A_4 > A_1$ | $A_2$ |
| The proposed SVEHFS-score function $Sc_{d_2}$ $Sc_{d_2}(A_1) = 0.4310$, $Sc_{d_2}(A_2) = 0.7970$, $Sc_{d_2}(A_3) = 0.7300$, and $Sc_{d_2}(A_4) = 0.6800$ | $A_2 > A_3 > A_4 > A_1$ | $A_2$ |
| The proposed SVEHFS-score function $Sc_{d_2}$ $Sc_{d_3}(A_1) = 0.4890$, $Sc_{d_3}(A_2) = 0.8352$, $Sc_{d_3}(A_3) = 0.7391$, and $Sc_{d_3}(A_4) = 0.6900$ | $A_2 > A_3 > A_4 > A_1$ | $A_2$ |

Table 11: Evaluation information provided by $DM_1$.

| | $C_1$ | $C_2$ | $C_3$ |
|---|---|---|---|
| $A_1$ | $\{0.3\,(0.3), 0.4\,(0.4), 0.5\,(0.3)\}$ | $\{0.4\,(0.3), 0.5\,(0.4), 0.6\,(0.3)\}$ | $\{0.2\,(1)\}$ |
| $A_2$ | $\{0.7\,(1)\}$ | $\{0.3\,(0.5), 0.4\,(0.5)\}$ | $\{0.8\,(0.5), 0.9\,(0.5)\}$ |
| $A_3$ | $\{0.6\,(0.5), 0.8\,(0.5)\}$ | $\{0.7\,(0.5), 0.9\,(0.5)\}$ | $\{0.3\,(0.5), 0.4\,(0.5)\}$ |

Table 12: Evaluation information provided by $DM_2$.

| | $C_1$ | $C_2$ | $C_3$ |
|---|---|---|---|
| $A_1$ | $\{0.4\,(0.5), 0.5\,(0.5)\}$ | $\{0.6\,(1)\}$ | $\{0.5\,(0.3), 0.7\,(0.4), 0.8\,(0.3)\}$ |
| $A_2$ | $\{0.3\,(0.5), 0.4\,(0.5)\}$ | $\{0.4\,(0.3), 0.5\,(0.4), 0.6\,(0.3)\}$ | $\{0.6\,(0.5), 0.7\,(0.5)\}$ |
| $A_3$ | $\{0.5\,(0.5), 0.6\,(0.5)\}$ | $\{0.8\,(0.5), 0.9\,(0.5)\}$ | $\{0.6\,(1)\}$ |

Table 13: Evaluation information provided by $DM_3$.

| | $C_1$ | $C_2$ | $C_3$ |
|---|---|---|---|
| $A_1$ | $\{0.1\,(0.5), 0.3\,(0.5)\}$ | $\{0.3\,(0.3), 0.4\,(0.4), 0.5\,(0.3)\}$ | $\{0.6\,(0.5), 0.7\,(0.5)\}$ |
| $A_2$ | $\{0.7\,(0.3), 0.8\,(0.4), 0.9\,(0.3)\}$ | $\{0.5\,(0.3), 0.6\,(0.4), 0.8\,(0.3)\}$ | $\{0.3\,(1)\}$ |
| $A_3$ | $\{0.4\,(0.5), 0.5\,(0.5)\}$ | $\{0.9\,(1)\}$ | $\{0.7\,(0.5), 0.8\,(0.5)\}$ |

Table 14: The unified probabilistic hesitant fuzzy decision matrix for $DM_1$.

| | $C_1$ | $C_2$ | $C_3$ |
|---|---|---|---|
| $A_1$ | $\{0.3\,(0.3), 0.4\,(0.2), 0.4\,(0.2), 0.5\,(0.3)\}$ | $\{0.4\,(0.3), 0.5\,(0.2), 0.5\,(0.2), 0.6\,(0.3)\}$ | $\{0.2\,(0.3), 0.2\,(0.2), 0.2\,(0.2), 0.2\,(0.3)\}$ |
| $A_2$ | $\{0.7\,(0.3), 0.7\,(0.2), 0.7\,(0.2), 0.7\,(0.3)\}$ | $\{0.3\,(0.3), 0.3\,(0.2), 0.4\,(0.2), 0.4\,(0.3)\}$ | $\{0.8\,(0.3), 0.8\,(0.2), 0.9\,(0.2), 0.9\,(0.3)\}$ |
| $A_3$ | $\{0.6\,(0.3), 0.6\,(0.2), 0.8\,(0.2), 0.8\,(0.3)\}$ | $\{0.7\,(0.3), 0.7\,(0.2), 0.9\,(0.2), 0.9\,(0.3)\}$ | $\{0.3\,(0.3), 0.3\,(0.2), 0.4\,(0.3), 0.4\,(0.3)\}$ |

Table 15: The unified probabilistic hesitant fuzzy decision matrix for $DM_2$.

| | $C_1$ | $C_2$ | $C_3$ |
|---|---|---|---|
| $A_1$ | $\{0.4\,(0.3), 0.4\,(0.2), 0.5\,(0.2), 0.5\,(0.3)\}$ | $\{0.6\,(0.3), 0.6\,(0.2), 0.6\,(0.2), 0.6\,(0.3)\}$ | $\{0.5\,(0.3), 0.7\,(0.2), 0.7\,(0.2), 0.8\,(0.3)\}$ |
| $A_2$ | $\{0.3\,(0.3), 0.3\,(0.2), 0.4\,(0.2), 0.4\,(0.3)\}$ | $\{0.4\,(0.3), 0.5\,(0.2), 0.5\,(0.2), 0.6\,(0.3)\}$ | $\{0.6\,(0.3), 0.6\,(0.2), 0.7\,(0.2), 0.7\,(0.3)\}$ |
| $A_3$ | $\{0.5\,(0.3), 0.5\,(0.2), 0.6\,(0.2), 0.6\,(0.3)\}$ | $\{0.8\,(0.3), 0.8\,(0.2), 0.9\,(0.2), 0.9\,(0.3)\}$ | $\{0.6\,(0.3), 0.6\,(0.2), 0.6\,(0.2), 0.6\,(0.3)\}$ |

Table 16: The unified probabilistic hesitant fuzzy decision matrix for $DM_3$.

| | $C_1$ | $C_2$ | $C_3$ |
|---|---|---|---|
| $A_1$ | $\{0.1\,(0.3), 0.1\,(0.2), 0.3\,(0.2), 0.3\,(0.3)\}$ | $\{0.3\,(0.3), 0.4\,(0.2), 0.4\,(0.2), 0.5\,(0.3)\}$ | $\{0.6\,(0.3), 0.6\,(0.2), 0.7\,(0.2), 0.7\,(0.3)\}$ |
| $A_2$ | $\{0.7\,(0.3), 0.8\,(0.2), 0.8\,(0.2), 0.9\,(0.3)\}$ | $\{0.5\,(0.3), 0.6\,(0.2), 0.6\,(0.2), 0.8\,(0.3)\}$ | $\{0.3\,(0.3), 0.3\,(0.2), 0.3\,(0.2), 0.3\,(0.3)\}$ |
| $A_3$ | $\{0.4\,(0.3), 0.4\,(0.2), 0.5\,(0.2), 0.5\,(0.3)\}$ | $\{0.9\,(0.3), 0.9\,(0.2), 0.9\,(0.2), 0.9\,(0.3)\}$ | $\{0.7\,(0.3), 0.7\,(0.2), 0.8\,(0.2), 0.8\,(0.3)\}$ |

Table 17: The SVEHFS decision matrix for $DM_1$.

| | $C_1$ | $C_2$ | $C_3$ |
|---|---|---|---|
| $A_1$ | $\{\text{blue}\,(0.3, 0.4, 0.4, 0.5)\}$ | $\{(0.4, 0.5, 0.5, 0.6)\}$ | $\{(0.2, 0.2, 0.2, 0.2)\}$ |
| $A_2$ | $\{0.7, 0.7, 0.7, 0.7\}$ | $\{0.3, 0.3, 0.4, 0.4\}$ | $\{0.8, 0.8, 0.9, 0.9\}$ |
| $A_3$ | $\{0.6, 0.6, 0.8, 0.8\}$ | $\{0.7, 0.7, 0.9, 0.9\}$ | $\{0.3, 0.3, 0.4, 0.4\}$ |

Table 18: The SVEHFS decision matrix for $DM_2$.

| | $C_1$ | $C_2$ | $C_3$ |
|---|---|---|---|
| $A_1$ | $\{0.4, 0.4, 0.5, 0.5\}$ | $\{0.6, 0.6, 0.6, 0.6\}$ | $\{0.5, 0.7, 0.7, 0.8\}$ |
| $A_2$ | $\{0.3, 0.3, 0.4, 0.4\}$ | $\{0.4, 0.5, 0.5, 0.6\}$ | $\{0.6, 0.6, 0.7, 0.7\}$ |
| $A_3$ | $\{0.5, 0.5, 0.6, 0.6\}$ | $\{0.8, 0.8, 0.9, 0.9\}$ | $\{0.6, 0.6, 0.6, 0.6\}$ |

Table 19: The SVEHFS decision matrix for $DM_3$.

| | $C_1$ | $C_2$ | $C_3$ |
|---|---|---|---|
| $A_1$ | $\{0.1, 0.1, 0.3, 0.3\}$ | $\{0.3, 0.4, 0.4, 0.5\}$ | $\{0.6, 0.6, 0.7, 0.7\}$ |
| $A_2$ | $\{0.7, 0.8, 0.8, 0.9\}$ | $\{0.5, 0.6, 0.6, 0.8\}$ | $\{0.3, 0.3, 0.3, 0.3\}$ |
| $A_3$ | $\{0.4, 0.4, 0.5, 0.5\}$ | $\{0.9, 0.9, 0.9, 0.9\}$ | $\{0.7, 0.7, 0.8, 0.8\}$ |

Table 20: The collective SVEHFS decision matrix.

| | $C_1$ | $C_2$ | $C_3$ |
|---|---|---|---|
| $A_1$ | $\{(0.2537, 0.2934, 0.3891, 0.4273)\}$ | $\{0.4212, 0.4901, 0.4901, 0.5623\}$ | $\{0.4602, 0.5230, 0.5753, 0.6150\}$ |
| $A_2$ | $\{0.6317, 0.6873, 0.6987, 0.7722\}$ | $\{0.4112, 0.4851, 0.5125, 0.6508\}$ | $\{0.6079, 0.6079, 0.7140, 0.7140\}$ |
| $A_3$ | $\{0.5028, 0.5028, 0.6577, 0.6577\}$ | $\{0.8254, 0.8254, 0.9000, 0.9000\}$ | $\{0.5657, 0.5657, 0.6508, 0.6508\}$ |

in which $\mathrm{d}(.,.)$ is a distance measure.

If we implement distance measures $\mathrm{d}_1(.,.)$, $\mathrm{d}_2(.,.)$, and $\mathrm{d}_3(.,.)$ provided, respectively, by (14)–(16) with $\lambda = 1$, then the weight vectors will be

$$\begin{aligned} \varpi_{d_1} &= (\varpi_1, \varpi_2, \varpi_3) = (0.3545, 0.2421, 0.4034), \\ \varpi_{d_2} &= (\varpi_1, \varpi_2, \varpi_3) = (0.3864, 0.1932, 0.4204), \\ \varpi_{d_3} &= (\varpi_1, \varpi_2, \varpi_3) = (0.3927, 0.1859, 0.4215). \end{aligned} \tag{36}$$

To save more space for convenient storage, we only list the subsequent results for $\varpi_{d_1}$.

Now, if we aggregate the individual SVEHFS matrices given in Tables 17–19 by the help of the following rule

$$\dot{h} = \oplus_{k=1}^{3}\left(\varpi_k \times \dot{h}_k\right) = \left\{ \left(1 - \prod_{k=1}^{3}\left(1 - \dot{h}_k\right)^{\varpi_k}\right)^{(1)}, \ldots, \left(1 - \prod_{k=1}^{3}\left(1 - \dot{h}_k\right)^{\varpi_k}\right)^{(m)} \right\}, \tag{37}$$

in which $\varpi_k$ stands for the weight of the decision makers $DM_k$ ($k = 1, 2,$ and $3$) and the notation $(j)$ (for $j = 1, \ldots, m$) denotes the $j$th element of collective SVEHFS, then the collective SVEHFS matrices can be derived in the form of Table 20.

Table 21 shows the comparison outcomes of different techniques. The ranking results obtained by the techniques of Xu et al. [36, 37] and Li et al. [35] are identical to those of proposed SVEHFS-score techniques. Such identical ranking results are possibly related to the same steps of processing which are performed using the latter-mentioned techniques. Briefly speaking, the common steps of these techniques are the integration of evaluation information given by the decision makers, the calculation of score value of the collective evaluation information, and the comparison of alternatives by the help of their score values. The outcomes of such identical steps are seen in identical ranking results.

But, the result of classical ORESTE technique [38] is quite different from that of other abovementioned techniques. Such a different ranking result arises from the two-stage integrating ranking process. The initial stage calculates utility values for determining the weak ranking of alternatives. Then, the subsequent stage will derive the preference, indifference, and incomparability relations with conflict analysis. Finally, the strong ranking of alternatives is extracted. However, due to such complicated two-stage procedure, the best alternative obtained from classical ORESTE technique does not appear convincing enough.

In summary, the comparison with other considered techniques suggests that the proposed SVEHFS-score techniques have superior performance and also less computational complexity.

Table 21: Ranking results of the different techniques.

| Technique | Score of alternatives | | | Ranking order | Optimal alternative |
|---|---|---|---|---|---|
| PHFWA-based technique of Xu et al. [36] | 0.4584 | 0.5990 | 0.7245 | $A_1 < A_2 < A_3$ | $A_3$ |
| PHFWG-based technique of Xu et al. [36] | 0.4117 | 0.5284 | 0.6909 | $A_1 < A_2 < A_3$ | $A_3$ |
| Xu et al. [37] | 0.2804 | 0.3653 | 0.7974 | $A_1 < A_2 < A_3$ | $A_3$ |
| Classical ORESTE [38] | 4.8172 | 7.1306 | 6.7856 | $A_2 < A_3 < A_1$ | $A_1$ |
| Li et al. [35] | 0.2563 | 0.1933 | 0.1212 | $A_1 < A_2 < A_3$ | $A_3$ |
| The proposed SVEHFS-score function $Sc_{d_1}$ | 0.4583 | 0.6244 | 0.6837 | $A_1 < A_2 < A_3$ | $A_3$ |
| The proposed SVEHFS-score function $Sc_{d_2}$ | 0.3784 | 0.5503 | 0.6313 | $A_1 < A_2 < A_3$ | $A_3$ |
| The proposed SVEHFS-score function $Sc_{d_3}$ | 0.4183 | 0.5873 | 0.6575 | $A_1 < A_2 < A_3$ | $A_3$ |

## 6. Conclusion

Adopting a probability splitting algorithm for deriving an efficient probabilistic-unification process of PHFSs, we developed a class of score functions for SVEHFSs which are novel deformation of PHFSs. As we demonstrated here, the concept of SVEHFS belongs to a less dimensional space compared to that of PHFSs. Furthermore, we indicated that the proposed SVEHFS-based score functions satisfy a number of interesting properties. It may be of interest to mention that the proposed SVEHFS-based score functions are able to be more generalized to a wider class. Lastly, three case studies were prepared to illustrate the applicability and efficiency of proposed SVEHFS-based score functions compared to other existing PHFS-based techniques. In contrast to the other existing techniques for PHFSs, the SVEHFS-based score functions are associated with less complexity and computation requirements.

In future work, we will work towards opportunities to investigate the meaning and essence of SVEHFS-based score functions in the MCDM under probabilistic hesitant fuzzy setting.

## Data Availability

No data were used to support this study.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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*Research Article*

# Probabilistic Linguistic-Based Group DEMATEL Method with Both Positive and Negative Influences

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Decision-making trial and evaluation laboratory (DEMATEL) is a widely accepted factor analysis algorithm for complex systems. The rationality of the evaluation scale is the basis of sound DEMATEL decision-making. Unfortunately, the existing evaluation scales of DEMATEL failed to reasonably distinguish and describe the positive and negative influences between factors. Generally, the positive and negative influences between factors should be considered at the same time. In other words, negative influence between factors should not be directly ignored, which is improper and unrealistic. To better address this issue, we extend the evaluation scale of DEMATEL. We also integrate the scale-based group DEMATEL method with probabilistic linguistic term sets (PLTSs) to increase its effectiveness, which allows experts to express incomplete and uncertain linguistic preferences in DEMATEL decision-making. An experts' subjective weight adjustment method based on the similarity degree between PLTSs is introduced to determine experts' weights. Finally, an algorithm of probabilistic linguistic-based group DEMATEL method with both positive and negative influences is summarized, and an example is used to illustrate the proposed method and demonstrate its superiority. Our results demonstrate that the method proposed in this paper deals reasonably with realistic problems.

## 1. Introduction

As a factor analysis algorithm for complex socioeconomic system problems, by making full use of expertise and prior experience, decision-making trial and evaluation laboratory (DEMATEL) uses the form of an evaluation scale to judge the influence relationships between system factors and forms a direct influence matrix between factors. Then, the judgment of the relative importance and the direct and indirect causal relationships between the factors can be estimated through matrix operations.

The rationality of the evaluation scale of DEMATEL is the key to accurate decision-making. The evaluation scale of 0, 1, 2, and 3 was initially used to indicate the degree of direct influence between factors, representing "no influence," "low influence," "medium influence," and "high influence," respectively [1–3]. To effectively differentiate the intensity of influence between factors within a limited scale, Chiu et al. [4], Liou et al. [5], Tseng [6], Chen et al. [7], Lin et al. [8], and Uygun et al. [9] extended the DEMATEL scale to five levels, and the evaluation scale of 0, 1, 2, 3, and 4 was employed for complex problem analysis, where the scale value 0 still indicates "no influence," and 1, 2, 3, and 4, respectively, indicate "low influence," "medium influence," "high influence," and "very high influence." Huang et al. [10] and Tseng and Huang [11] proposed 11 evaluation levels, from 0 to 10, ranging from "no influence" to "very high influence," for influence relationship analysis. Dytczak and Ginda [12] further extended the DEMATEL evaluation scale to a more general form and reckoned that a scale reflecting the intensity of influences between factors could be expressed as 0 to $N$, where 0 means "no influence," and $N$ is any assumed positive integer indicating the maximum degree of influence. Wu et al. [13] came up with a 1–5 scale, using 1 for "no influence," and 2, 3, 4, and 5 for "very low influence," "low influence," "high influence," and "very high influence," respectively. The 0–4 scale is the most widely used. Regrettably, although many scholars have studied and extended the DEMATEL scale, no existing scale can distinguish the positive and negative influences between factors, i.e., negative influences between factors are regarded and treated the same as positive influences resulting in those negative influences not being effectively reflected. The effects of positive and negative influences are obviously different, and it is unreasonable and inconsistent with practice to equate negative influences with positive ones. Therefore, in this paper, we define a new DEMATEL scale that considers and reflects both positive and negative influences between factors and propose operating rules and processing methods for matrices in the DEMATEL method using the new scale.

With regard to the judgment expression form, experts tend to make linguistic judgments when judging the influence relationships between factors due to the complex decision-making environment and the vagueness inherent in human thinking. Considering that experts may hesitate among several possible linguistic terms when expressing preferences by means of linguistic information, Rodríguez et al. [14] proposed hesitant fuzzy linguistic term sets (HFLTSs) on the basis of linguistic term sets [15] and hesitant fuzzy sets [16] to enable an expert to propose several possible values for a linguistic variable. In most studies on HFLTSs, all possible values given by experts have equal importance, which is unrealistic. To solve this problem, Pang et al. [17] developed the concept of probabilistic linguistic term sets (PLTSs) as an extension of HFLTSs. The linguistic term is associated with a probability that can be interpreted as a probabilistic distribution or degree of belief. Moreover, considering the limitations of experts' prior experience, such as knowledge width and professional background, partial ignorance is accepted.

Owing to its usefulness and efficiency, the PLTS has attracted a lot of researchers' attention, and fruitful research achievements regarding it has been published since it was introduced in 2016. Lin et al. [18] suggested a novel score-entropy-based ELECTRE II method to process the edge node selection problem with the evaluation information of PLTSs. Their main contributions in PLTSs are as follows: first, a novel distance measure for PLTSs was defined. Second, a novel comparison method based on the score function and information entropy of PLTSs was proposed. Then, the concordance and discordance values of alternatives were compared with the determined concordance and discordance levels according to the established rules. Lin et al. [19] evaluated ten regions' general higher education in China by probabilistic linguistic clustering algorithm based on the scale of higher education institutions, the number of higher education institutions, the number of students in higher education institutions, and the staff situation of the faculty. Jin et al. [20] proposed the concept of uncertain probabilistic linguistic term set (UPLTS) to serve as an extension of the existing tools, and we developed an aggregation-based method and presented the application of the UPLTSs in multiple attribute group decision-making. Liu et al. [21] developed the probabilistic linguistic Archimedean MM (PLAMM) operator, probabilistic linguistic Archimedean weighted MM (PLAWMM) operator, probabilistic linguistic Archimedean dual MM (PLADMM) operator, and probabilistic linguistic Archimedean dual weighted MM (PLADWMM) operator, and provided two multiple attribute decision-making (MADM) methods built on the proposed operators. Gu et al. [22] proposed a decision-making framework based on prospect theory. In this framework, the outcomes are characterized by probabilistic linguistic term sets (PLTSs), which furnishes a paradigm to extend prospect theory to accommodate other forms of fuzzy and linguistic input. Since then, PLTSs have been widely used in cloud decisions [23], investment decisions [24, 25], water security evaluation [26], and site selection of solar power plants [27]. Scholars have combined the WASPAS method, the Dempster–Shafer (D-S) evidence theory, the ELECTRE III method, the MULTIMOORA method, the ORESTE method, and the ANP method with PLTSs [28–33].

Much research has also been done on methodological improvements to PLTSs. Gou and Xu [34] pointed out that the operations of PLTSs proposed by Pang et al. [17] might cause the result to exceed the boundary of the linguistic term set and also to lose probabilistic information. Hence, they proposed new operation laws of PLTSs. Bai et al. [35] indicated that either comparison methods of fuzzy numbers did not fully consider fuzzy information, or the comparison process was too complicated. Hence, they proposed a possibility degree-based ranking method using the graphical method to analyze the structure of PLTSs. By analyzing some illustrative examples, Mao et al. [36] demonstrated two main drawbacks relating to PLTS ranking methods. On the one hand, their robustness was so poor that a small change in the probability might cause the reversal of a PLTS ranking. On the other hand, they might result in the unreasonable judgment that two different PLTSs were identical. To overcome these defects, they proposed a possibility algorithm for ranking PLTSs. In addition, they defined the Euclidean distance between PLTSs and presented a judgment similarity-based correction method for experts' subjective weights. However, they failed to fully consider the structure of PLTSs given by various experts, giving rise to poor reliability of similarity evaluations. So, this paper defines the similarity degree between PLTSs by combining the possibility algorithm of PLTS ranking and the Euclidean distance between PLTSs of Mao et al. [36], and devises a method to determine experts' weights.

Although the DEMATEL method has been widely applied in complicated socioeconomic system issues analysis, experts' judgments must be certain and precise, so the traditional method is infeasible when experts hesitate among several possible linguistic terms. As an uncertain linguistic preference expression method, PLTS has unique advantages in dealing with multi-attribute decision-making problems. Like HFLTS, it allows experts to express their views using several linguistic terms, and it extends HFLS by adding probability information to prevent the loss of original linguistic information provided by experts. In addition, it permits experts to give incomplete judgment information. The contributions of this paper are summarized as follows.

Therefore, based on the advantages of PLTSs in terms of information processing as well as accurate description of uncertainty, in this paper we combine the DEMATEL method with PLTSs and adopt PLTSs as the form of information collection to overcome the shortcomings of the traditional DEMATEL method. In addition, based on the unique data structure of PLTSs, our study redefines the similarity measure between PLTSs and designs a subjective expert weight method to facilitate a more scientific judgment of the importance of experts and achieve accurate aggregation of multigroup expert information. At the same time, in order to extend the traditional DEMATEL method to increasingly complex socioeconomic systems, we propose the concept of negative scaling. This will enable experts to express their multidimensional and multidirectional decision information more clearly, make comprehensive judgments on the causal relationships between influencing factors from multiple perspectives, and provide more accurate decision results. The motivation of this paper is to propose a probabilistic linguistic-based group DEMATEL method considering both the positive and negative influences between factors, where all information provided by experts is characterized by probabilistic linguistic terms, and the evaluation information can be partially ignored.

The remainder of this paper is organized as follows. Section 2 introduces the preliminary details of DEMATEL method and the PLTSs. In Section 3, we develop a new DEMATEL scale that considers both the positive and negative influences between factors, and we also give an improved method for adjustment of experts' subjective weights under probabilistic linguistic environment. Then, the procedures of the new group DEMATEL method are presented, and the algorithm corresponding to the new method is also summarized. In Section 4, an illustrative example is given, and our method is compared to other DEMATEL methods to illustrate its feasibility and effectiveness. Section 5 provides conclusions and suggests future work.

## 2. Preliminaries

In this section, we mainly recall the detailed procedures of traditional DEMATEL method and describe some concepts and operations related to PLTSs.

*2.1. Decision-Making Trial and Evaluation Laboratory.* DEMATEL is an effective method for system factors analysis, which can deal with complex socioeconomic problems by making full use of experts' knowledge and experience. In DEMATEL decision-making, experts are invited to judge the direct influence relationships between factors by using an evaluation scale of DEMATEL and form the direct influence matrices, and critical factors of the system will be identified by matrix operations. The procedures of traditional DEMATEL can be summarized as follows [37]:

Step 1: construct the direct influence matrix. let $E = \{e^1, e^2, \ldots, e^m\}$ be the set of experts, let $F = \{f_1, f_2, \ldots, f_n\}$ be a finite set of influencing factors. The experts are asked to judge the direct influence degree that factor $f_i$ has on factor $f_j$ by using the evaluation scale of DEMATEL, i.e. "no influence (0)," "low influence (1)," "medium influence (2)," "high influence (3)," and "very high influence (4)." Then, the direct influence matrix provided by expert $e^\lambda\,(\lambda = 1, \ldots, m)$ can be gotten as $A^\lambda = [a_{ij}^\lambda]_{n\times n}\,(i, j = 1, 2, \ldots, n)$, where $a_{ij}^\lambda$ indicates the direct influence degree that factor $f_i$ has on $f_j$ given by the $\lambda$th expert. By combining the judgments of all experts, the group direct influence matrix can be formed as $A = [a_{ij}]_{n\times n}$, where:

$$a_{ij} = \frac{\sum_{\lambda=1}^m a_{ij}^\lambda}{m}, i, j = 1, 2, \ldots, n. \tag{1}$$

Step 2: normalize the direct influence matrix. By using equation (2), the direct influence matrix $A = [a_{ij}]_{n\times n}$ can be normalized and the normalized direct influence matrix $G$ will be obtained.

$$G = \frac{A}{\max\limits_{1\le i\le n} \sum_{j=1}^n a_{ij}}. \tag{2}$$

Step 3: calculate the total influence matrix. Let $T$ be the total influence matrix, then its calculation formula is

$$T = \left[t_{ij}\right]_{n\times n} = G(I - G)^{-1}. \tag{3}$$

Step 4: determine the centrality of the factors and calculate the cause and effect groups. Let $d_i$ and $r_i$, which can be calculated by equation (4) and (5), be the sums of the $i$th row and column of matrix $T$, respectively. Then, a causal diagram of system factors can be drawn, where $d_i$ and $r_i$ are located in the horizontal and vertical axes, respectively.

$$d_i = \sum_{j=1}^n t_{ij}, \quad i = 1, 2, \ldots, n, \tag{4}$$

$$r_i = \sum_{j=1}^n t_{ji}, \quad i = 1, 2, \ldots, n, \tag{5}$$

where $d_i$ indicates the centrality of factor $f_i$ in the entire system, and $r_i$ indicates whether factor $f_i$ belongs to the cause group or the effect group. Factors having positive values of $r_i$ are in the cause group and dispatch influence to other factors, and factors having negative values of $r_i$ are in the effect group and receive influence from other factors.

*2.2. Probabilistic Linguistic Term Sets.* As an extension of HFLTSs, PLTSs allow experts to hesitate among several possible linguistic terms when expressing their judgments under the linguistic environment. The probabilistic distribution of these linguistic terms is also collected in PLTSs, and partial ignorance is allowed. All these properties are desirable in expressing preferences in decision-making.

*Definition 1* (see [17]). Let $S = \{s_0, s_1, \ldots, s_\tau\}$ be a linguistic term set (LTS). A PLTS can be defined as

$$L(p)=\left\{L^{(k)}\left(p^{(k)}\right)|L^{(k)}\in S,p^{(k)}\geq 0,k=1,2,\ldots,\#L(p),\sum_{k=1}^{\#L(p)}p^{(k)}\leq 1\right\},\tag{6}$$

where $L^{(k)}(p^{(k)})$ is the linguistic term $L^{(k)}$ associated with its probability $p^{(k)}$, and $\#L(p)$ is the number of all different linguistic terms in $L(p)$.

*Definition 2* (see [17]). The score $E(L(p))$ and the deviation degree $\sigma(L(p))$ of PLTS $L(p)=\{L^{(k)}(p^{(k)})|k=1,2,\ldots,\#L(p)\}$ are denoted by

$$E(L(p))=s_{\overline{\alpha}},$$
$$\sigma(L(p))=\frac{\left(\sum_{k=1}^{\#L(p)}\left(p^{(k)}\left(r^{(k)}-\overline{\alpha}\right)\right)^2\right)^{1/2}}{\sum_{k=1}^{\#L(p)}p^{(k)}},\tag{7}$$

where $\overline{\alpha}=\sum_{k=1}^{\#L(p)}r^{(k)}p^{(k)}/\sum_{k=1}^{\#L(p)}p^{(k)}$, and $r^{(k)}$ is the subscript of linguistic term $L^{(k)}$.

It can be noted from equation (6) that $\sum_{k=1}^{\#L(p)}p^{(k)}\leq 1$, that is to say, partial ignorance is acceptable. So, estimating the ignorance of probabilistic information is a crucial work for the use of PLTSs. To handle this issue, Pang et al. [17] assigned the ignorance $(1-\sum_{k=1}^{\#L(p)}p^{(k)})$ to the linguistic terms in $L(p)$ averagely as follows to get the associated complete PLTS $L(p)$ for $L(p)$.

*Definition 3* (see [17]). Let $L(p)$ be a PLTS with $\sum_{k=1}^{\#L(p)}p^{(k)}\leq 1$. Then, the associated complete PLTS $L(p)$ can be defined by

$$\dot{L}(p)=\left\{L^{(k)}\left(\dot{p}^{(k)}\right)|k=1,2,\ldots,\#L(p)\right\},\tag{8}$$

where $\dot{p}^{(k)}=p^{(k)}/\sum_{k=1}^{\#L(p)}p^{(k)}$, and $k=1,2,\ldots,\#L(p)$.

*Definition 4* (see [17]). Let $L_1(p)=\left\{L_1^{(k)}(p_1^{(k)})|k=1,2,\ldots,\#L_1(p)\right\}$ and $L_2(p)=\left\{L_2^{(k)}(p_2^{(k)})|k=1,2,\ldots,\#L_2(p)\right\}$ be two complete PLTSs, where $\#L_1(p)$ and $\#L_2(p)$ are, respectively, the numbers of linguistic terms in $L_1(p)$ and $L_2(p)$. If $\#L_1(p)>\#L_2(p)$, then $(\#L_1(p)-\#L_2(p))$ linguistic terms will be added to $L_2(p)$. The added linguistic terms are the smallest ones in $L_2(p)$ and the probabilities related to the added linguistic terms are zero.

So far, the PLTSs $L_1(p)$ and $L_2(p)$ have been normalized. For convenience, we still denote the normalized PLTSs by $L_1(p)$ and $L_2(p)$.

Gou and Xu [34] extended LTS for PLTSs to $S=\{s_t|t=-\tau,\ldots,-1,0,1,\ldots,\tau\}$, and they defined some basic operational laws of PLTSs as

$$wL(p)=g^{-1}\left(\cup_{\eta^{(k)}\in g(L)}\left\{\left(1-\left(1-\eta^{(k)}\right)^w\right)\left(p^{(k)}\right)\right\}\right),\tag{9}$$

$$L_1(p)\oplus L_2(p)=g^{-1}\left(\cup_{\eta_1^{(i)}\in g(L_1),\eta_2^{(j)}\in g(L_2)}\left\{\left(\eta_1^{(i)}+\eta_2^{(j)}-\eta_1^{(i)}\eta_2^{(j)}\right)\left(p_1^{(i)}p_2^{(j)}\right)\right\}\right),\tag{10}$$

$$\begin{aligned}L^w(p)&=g^{-1}\left(\cup_{\eta^{(k)}\in g(L)}\left\{\left(\eta^{(k)}\right)^w\left(p^{(k)}\right)\right\}\right),\\ L_1(p)\otimes L_2(p)&=g^{-1}\left(\cup_{\eta_1^{(i)}\in g(L_1),\eta_2^{(j)}\in g(L_2)}\left\{\left(\eta_1^{(i)}\eta_2^{(j)}\right)\left(p_1^{(i)}p_2^{(j)}\right)\right\}\right),\end{aligned}\tag{11}$$

where $L(p)$, $L_1(p)$, and $L_2(p)$ are three PLTSs, and $w$ is a positive real number; $\eta^{(k)}\in g(L)$, $\eta_1^{(i)}\in g(L_1)$, $\eta_2^{(j)}\in g(L_2)$, where $k=1,2,\ldots,\#L$, $i=1,2,\ldots,\#L_1$, $j=1,2,\ldots,\#L_2$; $g$ and $g^{-1}$ are the equivalent functions proposed by Gou et al. [38]:

$$g\colon[-\tau,\tau]\longrightarrow[0,1],g(L(p))=\left\{\left(\left(r^{(k)}/2\tau\right)+(1/2)\right)\left(p^{(k)}\right)\right\}=L_\gamma(p),\gamma\in[0,1],\tag{12}$$

$$g^{-1}\colon[0,1]\longrightarrow[-\tau,\tau],g^{-1}\left(L_\gamma(p)\right)=\left\{s_{(2\gamma-1)\tau}\left(p^{(\gamma)}\right)|\gamma\in[0,1]\right\}=L(p).\tag{13}$$

*Definition 5* (see [31]). Let $S = \{s_\alpha | \alpha = -\tau, \ldots, -1, 0, 1, \ldots, \tau\}$ be an LTS. $L_1(p) = \{L_1^{(k_1)}(p_1^{(k_1)}) | \ k_1 = 1, 2, \ldots, \#L_1(p)\}$ and $L_2(p) = \{L_2^{(k_2)}(p_2^{(k_2)}) | k_2 = 1, 2, \ldots, \#L_2(p)\}$ are two complete PLTSs. Then, the possibility degree that $L_1(p)$ is not less than $L_2(p)$ can be defined by

$$P(L_1(p) \geq L_2(p)) = \sum_{k_1=1}^{\#L_1(p)} \sum_{k_2=1}^{\#L_2(p)} R\left(L_1^{(k_1)}, L_2^{(k_2)}\right), \quad (14)$$

where $R(L_1^{(k_1)}, L_2^{(k_2)}) = \begin{cases} p_1^{(k_1)} p_2^{(k_2)}, L_1^{(k_1)} > L_2^{(k_2)} \\ 1/2 p_1^{(k_1)} p_2^{(k_2)}, L_1^{(k_1)} = L_2^{(k_2)} \\ 0, L_1^{(k_1)} < L_2^{(k_2)} \end{cases}$, indicating the possibility degree that $L_1^{(k_1)}$ in $L_1(p)$ is not smaller than $L_2^{(k_2)}$ in $L_2(p)$.

The possibility degree has the following properties:

(1) $0 \leq P(L_1(p) \geq L_2(p)) \leq 1$

(2) $P(L_1(p) \geq L_1(p)) = 0.5$

(3) $P(L_1(p) \geq L_2(p)) + P(L_2(p) \geq L_1(p)) = 1$

(4) If $P(L_1(p) \geq L_2(p)) = P(L_2(p) \geq L_1(p))$, then $P(L_1(p) \geq L_2(p)) = P(L_2(p) \geq L_1(p)) = 0.5$

In addition, Mao et al. [36] also gave the equation for Euclidean distance calculation between two PLTSs.

*Definition 6* (see [36]). Let $L_1(p)$ and $L_2(p)$ be two normalized PLTSs. The Euclidean distance between $L_1(p)$ and $L_2(p)$ is defined as

$$d(L_1(p), L_2(p)) = \sqrt{\sum_{k=1}^{\#L_1(p)} \left(p_1^{(k)} g\left(L_1^{(k)}\right) - p_2^{(k)} g\left(L_2^{(k)}\right)\right)^2 / \#L_1(p)}, \quad (15)$$

where $d(L_1(p), L_2(p)) \in [0, 1]$, and $g$ is the equivalent function in equation (12).

## 3. New Group DEMATEL Method

*3.1. Proposed New Scale for the DEMATEL Method.* Four kinds of affecting decision-making factors, benefit, opportunity, cost, and risk could be fully taken into account to make optimal decisions. Similarly, positive and negative influences in DEMATEL are the two kinds of influence relationships between factors. When analyzing the factors in a system, they should be properly considered to reasonably determine the relationships between factors and their positions in the system. However, evaluation scales in exiting DEMATEL cannot distinguish these influences, and negative influences are treated as positive, which is not consistent with reality. To overcome the above drawbacks, we extend a DEMATEL scale to a more reasonable form by the following definition.

*Definition 7.* he evaluation scale for pairwise comparison in DEMATEL can be presented in nine levels, where −4, −3, −2, −1, 0, 1, 2, 3, and 4, respectively, represent "very high negative influence," "high negative influence," "medium negative influence," "low negative influence," "no influence," "low positive influence," "medium positive influence," "high positive influence," and "very high positive influence."

To facilitate the combination with PLTSs, the definition of LTS corresponding to the above scale is as follows.

*Definition 8.* Let $s_\alpha$ $(\alpha = -4, \ldots, 0, \ldots, 4)$ be possible values for the influence degree expressed by a linguistic form. Then, the LTS corresponding to the new scale of DEMATEL can be defined as: $S_D = \{s_{-4}$ = very high negative influence, $s_{-3}$ = high negative influence, $s_{-2}$ = medium negative influence, $s_{-1}$ = low negative influence, $s_0$ = no influence, $s_1$ = low positive influence, $s_2$ = medium positive influence, $s_3$ = high positive influence, and $s_4$ = very high positive influence$\}$.

Based on the above scale, the direct influence matrix of influencing factors in DEMATEL can be expressed as follows.

*Definition 9.* For a group DEMATEL decision-making problem, let $E = \{e^1, e^2, \ldots, e^m\}$ be the set of experts, and let $F = \{f_1, f_2, \ldots, f_n\}$ be a finite set of influencing factors. The direct influence matrix provided by expert $e^\lambda$ $(\lambda = 1, \ldots, m)$ using the new scale can be defined as\scale90%

$$A^\lambda = \left[a_{ij}^\lambda\right]_{n\times n} = A^{\lambda_+} + A^{\lambda_-} = \left[a_{ij}^{\lambda_+}\right]_{n\times n} + \left[a_{ij}^{\lambda_-}\right]_{n\times n}, \quad i, j = 1, 2, \ldots, n, \quad (16)$$

where $a_{ij}^\lambda \in \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$ indicates the direct influence direction and influence degree of factor $f_i$ on factor $f_j$, $a_{ij}^{\lambda_+} \in \{0, 1, 2, 3, 4\}$ indicates the degree of positive direct influence of factor $f_i$ on factor $f_j$, and $a_{ij}^{\lambda_-} \in \{-4, -3, -2, -1, 0\}$ indicates the degree of negative direct influence of factor $f_i$ on factor $f_j$. $A^{\lambda_+}$ and $A^{\lambda_-}$ are the positive and negative direct influence matrices, respectively, which can be characterized as follows:

$$A^{\lambda_+} = \left[a_{ij}^{\lambda_+}\right]_{n\times n} = \left[\frac{\left(a_{ij}^\lambda + \left|a_{ij}^\lambda\right|\right)}{2}\right]_{n\times n}, \quad i, j = 1, 2, \ldots, n, \quad (17)$$

$$A^{\lambda_-} = \left[a_{ij}^{\lambda_-}\right]_{n\times n} = \left[\frac{\left(a_{ij}^\lambda - \left|a_{ij}^\lambda\right|\right)}{2}\right]_{n\times n}, \quad i, j = 1, 2, \ldots, n. \quad (18)$$

*3.2. Determination of Experts' Weights.* Determining experts' weights is an important part of integrating their judgment information. Subjective weights are often given in advance, which may be biased. Mao et al. [36] proposed a similarity-based adjustment coefficient to adjust them. However, this coefficient only considers the Euclidean distance between probabilistic linguistic matrices, and fails to fully consider the structural differences between PLTSs, resulting in poor reliability of similarity evaluation results. To overcome this, we define the similarity degree between two normalized PLTSs, which is based on the possibility degree in Definition 5 and the Euclidean distance in Definition 6.

*Definition 10.* Let $L_1(p)=\{L_1^{(k)}(p_1^{(k)})|k=1,2,\ \ldots,\#L_1(p)\}$ and $L_2(p)=\{L_2^{(k)}(p_2^{(k)})|k=1,2,\ldots,\#L_2(p)\}$ be two normalized PLTSs. $d(L_1(p),L_2(p))\in[0,1]$ is the Euclidean distance between $L_1(p)$ and $L_2(p)$, and $P(L_1(p)\ge L_2(p))$ is the possibility degree that $L_1(p)$ is not less than $L_2(p)$. Then, the similarity degree between $L_1(p)$ and $L_2(p)$ is defined as

$$
\begin{aligned}
C(L_1(p),L_2(p)) &= 2\left(0.5-\left(0.5-\left(\left(1-\sqrt{d(L_1(p),L_2(p))}\right)/2\right)+\left|P(L_1(p)\ge L_2(p))-0.5\right|\right)/2\right),\\
&=\begin{cases} 2\left(0.5-\dfrac{\left(0.5-\left(1-\sqrt{d(L_1(p),L_2(p))}\right)/2+0.5-P(L_1(p)\ge L_2(p))\right)}{2}\right), & P(L_1(p)\ge L_2(p)\in[0,0.5],\\[2ex] 2\left(0.5-\dfrac{\left(0.5-\left(1-\sqrt{d(L_1(p),L_2(p))}\right)/2+P(L_1(p)\ge L_2(p))-0.5\right)}{2}\right), & P(L_1(p)\ge L_2(p)\in(0.5,1],\end{cases}\\
&=\begin{cases} 0.5-0.5\sqrt{d(L_1(p),L_2(p))}+P(L_1(p)\ge L_2(p)), P(L_1(p)\ge L_2(p)\in[0,0.5],\\ 1.5-0.5\sqrt{d(L_1(p),L_2(p))}-P(L_1(p)\ge L_2(p)), P(L_1(p)\ge L_2(p)\in(0.5,1].\end{cases}
\end{aligned}
\tag{19}
$$

**Proposition 1.** *For two normalized PLTSs $L_1(p)$ and $L_2(p)$, the similarity degree has the following desirable properties:*

*(1)* $C(L_1(p),L_2(p))\in[0,1]$

*(2)* $C(L_1(p),L_2(p))=C(L_2(p),L_1(p))$

*(3)* $C(L_i(p),L_i(p))=1$

*Proof*

(1) Since $\sqrt{d(L_1(p),L_2(p))}, P(L_1(p)\ge L_2(p))\in[0,1]$, $0.5-(1-\sqrt{d(L_1(p),L_2(p))})/2$ and $|P(L_1(p)\ge L_2(p))-0.5|\in[0,0.5]$, and we have $(0.5-(1-\sqrt{d(L_1(p),L_2(p))})/2+|P(L_1(p)\ge L_2(p)-0.5|/2)\in[0,0.5]$. So, $C(L_1(p),L_2(p))=2(0.5-0.5-(1-\sqrt{d(L_1(p),L_2(p))})/2+|P(L_1(p)\ge P(L_2(p))|-0.5/2\in[0,1])$.

(2) $C(L_2(p),L_1(p))=2(0.5-(0.5-(1-\sqrt{d(L_1(p),L_2(p))})/2+|P(L_2(p)\ge L_1(p))-0.5|)/2)2\ (0.5-(0.5-(1-\sqrt{d(L_1(p),L_2(p))}\ /2+|1-P(L_1(p)\ge L_2(p))-0.5\ /2|)=2(0.5-\ (0.5-(1-\sqrt{d(L_1(p),L_2(p))}\ /2)+|P(L_1(p)\ge L_2(p))-0.5|)/2)=C(L_1(p),L_2(p))$.

(3) Since $\sqrt{d(L_i(p),L_i(p))}=0$, $P(L_i(p)\ge L_i(p))=0.5$, $0.5-(1-\sqrt{d(L_i(p),L_i(p))}/2)+|P(L_i(p)\ge L_i(p))-0.5|=0$.
$C(L_i(p),L_i(p))=2(0.5-(0.5-(1-\sqrt{d(L_i(p),L_i(p))})/2+(|P(L_i(p))\ge L_i(p)-0.5|)/2)=1$.

The similarity degree between two probabilistic linguistic decision matrices (PLDMs) is given as follows. □

*Definition 11.* Let $L^1=[L_{ij}^1(p)]_{n\times n}$ and $L^2=[L_{ij}^2(p)]_{n\times n}$ be two matrices with PLTSs. The similarity degree between them is

$$C(L^1,L^2)=\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{C(L_{ij}^1(p),L_{ij}^2(p))}{n^2}, \tag{20}$$

where $C(L_{ij}^1(p),L_{ij}^2(p))$ is the similarity degree between $L_{ij}^1(p)$ and $L_{ij}^2(p)$ by equation (19).

The relative consistent degree $\rho^{\lambda r}(\lambda,r=1,\ldots,m,\lambda\ne r)$ between experts $e^\lambda$ and $e^r$ can be calculated as

$$\rho^{\lambda r}=\frac{C(L^\lambda,L^r)}{\sum_{k=1,k\ne r}^{m}C(L^k,L^r)}, \tag{21}$$

where $C(L^k,L^r)(k=1,\ldots,m,k\ne r)$ is obtained by equation (20), and $m$ is the number of experts.

Then, an adjustment coefficient $\rho^\lambda$ of $e^\lambda$ is defined as

$$\rho^\lambda=\frac{\sum_{r=1,r\ne\lambda}^{m}\rho^{\lambda r}}{(m-1)}. \tag{22}$$

According to the given experts' subjective weights $\eta^\lambda$ and the adjustment coefficients $\rho^\lambda$, the final weights are

$$w^\lambda = \frac{\rho^\lambda \eta^\lambda}{\sum_{k=1}^{m} \rho^k \eta^k}, \quad \lambda = 1, \ldots, m. \tag{23}$$

*3.3. Procedures for the New Group DEMATEL Method.* Based on the above analysis, the probabilistic linguistic-based group DEMATEL method with both positive and negative influences is determined as follows, and its process can be illustrated in Figure 1.

Step 1: construct the original probabilistic linguistic decision matrix. The experts in the expert group $E$ utilize the LTS $S_D$ Definition deff8, to evaluate the direct influence relationships between factors in $F = \{f_1, f_2, \ldots, f_n\}$ by means of PLTSs to derive the original PLDMs. It should be noted that partial ignorance for the evaluation is acceptable. The PLDM $L^\lambda$ provided by expert $e^\lambda$ $(\lambda = 1, 2, \ldots, m)$ can be described as

$$L^\lambda = \left[L_{ij}^\lambda(p)\right]_{n\times n}, \quad i, j = 1, 2, \ldots, n, \tag{24}$$

where $L_{ij}^\lambda(p) = \{L_{ij}^{\lambda,k}(p^{\lambda,k}) | L_{ij}^{\lambda,k} \in S_D, p^{\lambda,k} \geq 0, k = 1, 2, \ldots, \#L_{ij}^\lambda(p), \sum_{k=1}^{\#L_{ij}^\lambda(p)} p^{\lambda,k} \leq 1\}$, $\lambda = 1, 2, \ldots, m$, indicating the evaluation result of the influence of factor $f_i$ on factor $f_j$ given by expert $e^\lambda$ under the probabilistic linguistic environment.

Step 2: determine the experts' weights. First, we can get the complete and normalized form for each PLTS in $L^\lambda$ $(\lambda = 1, 2, \ldots, m)$ according to Definitions deff3 and deff4. For convenience, the complete and normalized PLDMs are still denoted as $L^\lambda$ $(\lambda = 1, 2, \ldots, m)$. Let $\eta = (\eta^1, \ldots, \eta^m)$ be the subjective weight vector of experts which is determined by a preliminary discussion of the expert group. Based on equations (19)–(23), the final weight vector $w = (w^1, \ldots, w^m)$ of experts can be obtained.

Step 3: determine the aggregated probabilistic linguistic decision matrix. Based on equations (9), (10), (12), and (13), the group's aggregated PLDM can be derived as

$$L = \left[L_{ij}(p)\right]_{n\times n}, \quad i, j = 1, 2, \ldots, n, \tag{25}$$

where $L_{ij}(p) = w^1 L_{ij}^1(p) \oplus w^2 L_{ij}^2(p) \oplus \cdots \oplus w^m L_{ij}^m(p)$.

Step 4: transform $L$ to the direct influence matrix under the new scale. Based on Definition deff2, the score of each PLTS $L_{ij}(p)$ in $L$ can be calculated as

$$E\left(L_{ij}(p)\right) = s_{\overline{\alpha}_{ij}}, \quad i, j = 1, 2, \ldots, n, \tag{26}$$

where $\overline{\alpha}_{ij} = \sum_{k=1}^{\#L_{ij}(p)} r_{ij}^k p_{ij}^k$, $r_{ij}^k$ is the subscript of linguistic term $L_{ij}^k$, and $\sum_{k=1}^{\#L_{ij}(p)} p_{ij}^k = 1$.

Then, the aggregated direct influence matrix can be formed as $A = [a_{ij}]_{n\times n}$, where$a_{ij} = \text{round}(\overline{\alpha}_{ij})$, where round$(\cdot)$ is the usual rounding operation. The aggregated positive direct influence matrix and aggregated negative direct influence matrix, denoted as $A^+ = [a_{ij}^+]_{n\times n}$ and $A^- = [a_{ij}^-]_{n\times n}$, respectively, can be derived as per equation (17) and (18).

Step 5: calculate the normalized direct influence matrices. The normalized positive direct influence matrix $G^+$ and normalized negative direct influence matrix $G^-$ can be calculated as

$$G^+ = \frac{A^+}{\max_{1\leq i\leq n} \sum_{j=1}^{n} a_{ij}^+}, \tag{27}$$

$$G^- = \frac{A^-}{\min_{1\leq i\leq n} \sum_{j=1}^{n} a_{ij}^-}. \tag{28}$$

Step 6: determine the total influence matrices. The total positive influence matrix $T^+$ and total negative influence matrix $T^-$ can be determined as

$$T^+ = \left[t_{ij}^+\right]_{n\times n} = G^+\left(I - G^+\right)^{-1}, \tag{29}$$

$$T^- = \left[t_{ij}^-\right]_{n\times n} = G^-\left(I - G^-\right)^{-1}, \tag{30}$$

where $i, j = 1, 2 \ldots, n$, and $I$ is the unit matrix.

Step 7: determine the centrality of the factors and calculate the cause and effect groups. Let the vectors $D^+ = (d_1^+, \ldots, d_n^+)^T$ and $D^- = (d_1^-, \ldots, d_n^-)^T$ be the sums of rows of matrices $T^+$ and $T^-$, respectively, and let the vectors $R^+ = (r_1^+, \ldots, r_n^+)$ and $R^- = (r_1^-, \ldots, r_n^-)$ be the sums of columns of matrices $T^+$ and $T^-$, respectively. The $i$th elements in $D^+$, $D^-$, $R^+$, and $R^-$ can be calculated as

$$d_i^+ = \sum_{j=1}^{n} t_{ij}^+, \quad i = 1, 2, \ldots, n, \tag{31}$$

$$d_i^- = \sum_{j=1}^{n} t_{ij}^-, \quad i = 1, 2, \ldots, n, \tag{32}$$

$$r_i^+ = \sum_{j=1}^{n} t_{ji}^+, \quad i = 1, 2, \ldots, n, \tag{33}$$

$$r_i^- = \sum_{j=1}^{n} t_{ji}^-, \quad i = 1, 2, \ldots, n. \tag{34}$$

Let $\vartheta_i = \vartheta_i^+ - \vartheta_i^- = (d_i^+ + r_i^+) - (d_i^- + r_i^-)$ and $\psi_i = \psi_i^+ - \psi_i^- = (d_i^+ - r_i^+) - (d_i^- - r_i^-)$.

Drawing on the thoughts of Saaty and Ozdemir [39], we propose that $\vartheta_i$ indicates the degree of importance that factor $f_i$ plays in the entire system. The factor having greater absolute value of $\vartheta_i$ is more important. What is more, the factors that have positive values of $\vartheta_i$ have a positive influence on the whole system, and factors with negative values of $\vartheta_i$ have a negative influence on the whole system. We use $\psi_i$ to calculate the cause and effect groups. Factors having positive values of $\psi_i$ are in the cause group and dispatch

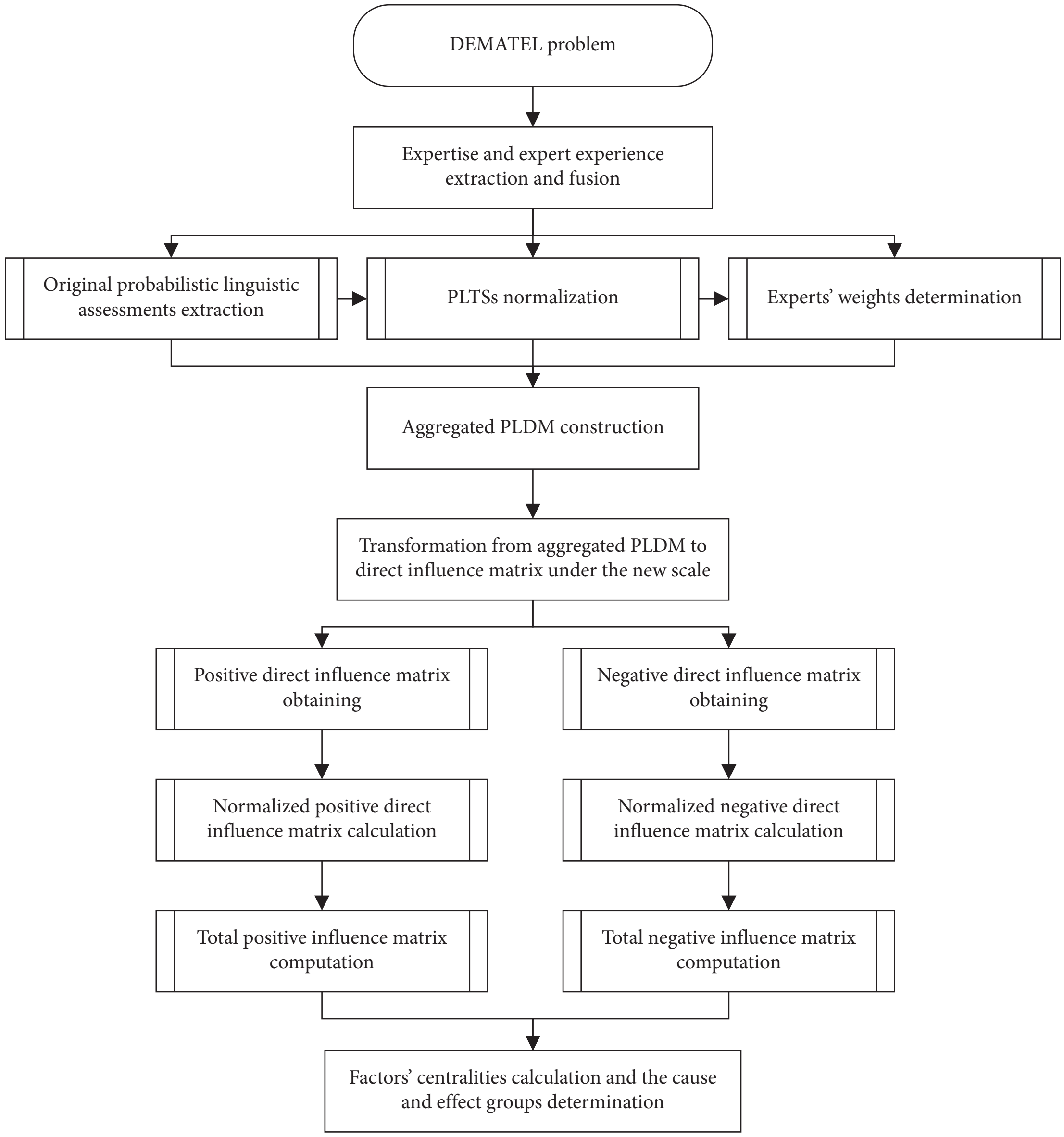



Figure 1: The process of the new group DEMATEL.

influence to other factors, and factors having negative values of $\psi_i$ are in the effect group and receive influence from other factors.

The algorithm for the new group DEMATEL can be summarized in Algorithm 1.

As can be seen through the methodological process described above, the new group DEMATEL method improves several aspects of decision-making from a systems perspective. Firstly, this method changes the traditional form of information input by describing expert preferences through PLTSs, which not only retains more complete input information but also ameliorates the impact of incomplete information on the decision outcome. Secondly, this model revises the processing flow of input information and introduces the concept of negative scaling, allowing decision-makers to make two-way decisions. The addition of a negative influence matrix allows experts to more intuitively understand the degree and direction of influence of system factors in the decision-making process, making the results more interpretable. Finally, this study defines similarity measures based on the structure of PLTSs, designs an expert subjective weight method, outputs a more reliable aggregated initial influence matrix, and improves the accuracy of factor analysis. The new group DEMATEL method optimizes the decision-making process from a system perspective in terms of the input, processing, and output of information, respectively, making the efficient DEMATEL more applicable to complex decision-making systems based on the original one.

## 4. Illustrative Example

We provide an example of a pharmaceutical enterprise called $M$ to illustrate the application of the proposed method. Wu

Inputs: the set of factors $F = \{f_1, \ldots, f_n\}$, linguistic term set $S_D = \{s_{-4}, \ldots, s_4\}$, set of evaluation grade levels $\Theta = \{\theta_{-4}, \ldots, \theta_4\}$, set of experts $E = \{e^1, \ldots, e^m\}$, subjective weight vector of experts $\eta = (\eta^1, \ldots, \eta^m)$.
Outputs: the $\vartheta_i$ and $\psi_i\,(i = 1, \ldots, n)$ values of factors.
Begin
% Original probabilistic linguistic decision matrices (PLDMs) construction
For $i = 1$ to $n$
  For $j = 1$ to $n$
    If $i \neq j$
      For $\lambda = 1$ to $m$
        Expert $e^\lambda$ assesses the influence degree of factor $f_i$ on $f_j$ by $L_{ij}^\lambda(p) = \left\{L_{ij}^{\lambda,k}(p^{\lambda,k}) | L_{ij}^{\lambda,k} \in S_D, p^{\lambda,k} \geq 0, k = 1, 2, \ldots, \#L_{ij}^\lambda(p), \sum_{k=1}^{\#L_{ij}^\lambda(p)} p^{\lambda,k} \leq 1\right\}$
        EndFor
        Assign the assessment result to the element of PLDM $L^\lambda$ by $L_{ij}^\lambda = L_{ij}^\lambda(p)$
      Else
        Assign linguistic term $s_0$ to the element of PLDM $L^\lambda$ by $L_{ij}^\lambda = \{s_0(1)\}$
      EndIf
    EndFor
EndFor
Get the PLDMs $L^\lambda\,(\lambda = 1, \ldots m)$
% Determine aggregated PLDM
Adjust subjective weight vector $\eta = (\eta^1, \ldots, \eta^m)$ of experts to the final weight vector $W = (w^1, \ldots, w^m)$
  For $i = 1$ to $n$
    For $j = 1$ to $n$
      Initialize the aggregation result by $L_{ij}(p) = \{s_{-4}(1)\}$
      If $i \neq j$
        For $\lambda = 1$ to $m$
          Make aggregation for the first $\lambda$ experts by $L_{ij}(p) = L_{ij}(p) \oplus w^\lambda L_{ij}^\lambda(p)$
        EndFor
        Assign the aggregation result to the element of aggregated PLDM by $L_{ij} = L_{ij}(p)$
        Else
        Assign linguistic term $s_0$ to the element of aggregated PLDM by $L_{ij} = \{s_0(1)\}$
      EndIf
    EndFor
EndFor
Get the aggregated PLDM $L = [L_{ij}(p)]_{n\times n}$
% Transform $L$ to the direct influence matrix under the new scale
For $i = 1$ to $n$
    For $j = 1$ to $n$
      Obtain the direct influence degree of factor $f_i$ on $f_j$ by $a_{ij} = \mathrm{round}(\sum_{k=1}^{\#L_{ij}(p)} r_{ij}^k p_{ij}^k)$
      Obtain the positive and negative direct influence degrees of factor $f_i$ on $f_j$ by $a_{ij}^+ = (a_{ij} + |a_{ij}|)/2$ and $a_{ij}^- = (a_{ij} - |a_{ij}|)/2$
      Construct direct influence matrix, positive and negative direct influence matrices by $A = [a_{ij}]_{n\times n}$, $A^+ = [a_{ij}^+]_{n\times n}$, and $A^- = [a_{ij}^-]_{n\times n}$
    EndFor
EndFor
Calculate the normalized positive and negative direct influence matrices by $G^+ = A^+ / \max_{1\leq i\leq n} \sum_{j=1}^n a_{ij}^+$ and $G^- = A^- / \max_{1\leq i\leq n} \sum_{j=1}^n a_{ij}^-$
Calculate the total positive and negative influence matrices by $T^+ = [t_{ij}^+]_{n\times n} = G^+(I - G^+)^{-1}$ and $T^- = [t_{ij}^-]_{n\times n} = G^-(I - G^-)^{-1}$
Calculate the importance degree, cause and effect degree of factors by $\vartheta_i = (\sum_{j=1}^n t_{ij}^+ + \sum_{j=1}^n t_{ji}^+) - (\sum_{j=1}^n t_{ij}^- + \sum_{j=1}^n t_{ji}^-)$ and $\psi_i = (\sum_{j=1}^n t_{ij}^+ - \sum_{j=1}^n t_{ji}^+) - (\sum_{j=1}^n t_{ij}^- - \sum_{j=1}^n t_{ji}^-)$
End

Algorithm 1: The algorithm for the new group DEMATEL method.

and Shanley [40] pointed out that companies develop new knowledge based on their existing knowledge stock so as to keep pace with new developments and maintain innovation competence. Jiménez and Valle [41] recommended that innovation requires the transformation and development of existing knowledge by asking employees to share information and knowledge. As Nonaka [42] suggested, knowledge-sharing among employees plays a fundamental role in innovation.

Enterprise $M$ has always taken pharmaceutical research and development (R&D) as the focus of innovation and the weapon to enhance the core competitiveness. So, it is important for enterprise $M$ to analyze the relationships between influencing factors of knowledge-sharing among pharmaceutical R&D employees. In this example, five influencing factors of knowledge-sharing among pharmaceutical R&D employees in enterprise $M$ are selected: the richness of knowledge transfer channels ($f_1$), knowledge-sharing ability of employees ($f_2$), implication and dispersion of knowledge ($f_3$), employees' hierarchy bias ($f_4$), and employees' willingness to share knowledge ($f_5$). Three experts, $e^1$, $e^2$, and $e^3$, are invited to analyze the direct influence relationships between influencing factors by using the new linguistic term set $S_D$, and the PLDMs $L^{\lambda}$ $(\lambda = 1, 2, 3)$ provided by the experts are as follows:

$$L^1 = \begin{bmatrix} \{s_0(1)\} & \{s_2(0.6), s_3(0.4)\} & \underline{\{s_{-3}(0.5), s_{-2}(0.4)\}} & \{s_{-2}(0.3), s_{-1}(0.5), s_0(0.2)\} & \{s_0(0.6), s_1(0.4)\} \\ \{s_0(0.1), s_1(0.3), s_2(0.6)\} & \{s_0(1)\} & \{s_{-3}(0.6), s_{-1}(0.4)\} & \{s_0(1)\} & \{s_1(0.3), s_2(0.7)\} \\ \{s_0(0.2), s_1(0.8)\} & \{s_{-4}(0.2), s_{-3}(0.8)\} & \{s_0(1)\} & \{s_1(0.6), s_2(0.4)\} & \{s_{-2}(0.8), s_{-1}(0.2)\} \\ \{s_0(1)\} & \{s_{-1}(0.7), s_0(0.3)\} & \{s_0(0.1), s_1(0.9)\} & \{s_0(1)\} & \{s_{-4}(0.8), s_{-3}(0.2)\} \\ \{s_0(0.2), s_1(0.8)\} & \{s_1(0.3), s_2(0.7)\} & \{s_{-2}(0.2), s_{-1}(0.8)\} & \{s_{-2}(0.7), s_0(0.3)\} & \{s_0(1)\} \end{bmatrix},$$

$$L^2 = \begin{bmatrix} \{s_0(1)\} & \{s_3(0.7), s_4(0.3)\} & \{s_{-3}(0.1), s_{-2}(0.6), s_{-1}(0.3)\} & \underline{\{s_{-1}(0.7), s_0(0.2)\}} & \{s_1(0.7), s_2(0.3)\} \\ \{s_1(0.3), s_2(0.7)\} & \{s_0(1)\} & \{s_{-2}(0.7), s_{-1}(0.3)\} & \{s_0(1)\} & \{s_1(0.2), s_2(0.8)\} \\ \{s_1(0.7), s_2(0.3)\} & \{s_{-3}(0.7), s_{-2}(0.3)\} & \{s_0(1)\} & \{s_0(0.3), s_1(0.7)\} & \{s_{-2}(0.7), s_{-1}(0.3)\} \\ \{s_{-1}(0.2), s_0(0.8)\} & \{s_{-2}(0.5), s_{-1}(0.5)\} & \{s_1(0.2), s_2(0.8)\} & \{s_0(1)\} & \{s_{-4}(1)\} \\ \{s_1(0.7), s_2(0.3)\} & \{s_0(0.2), s_1(0.8)\} & \{s_{-3}(0.6), s_{-2}(0.4)\} & \{s_{-3}(0.2), s_{-2}(0.8)\} & \{s_0(1)\} \end{bmatrix},$$

$$L^3 = \begin{bmatrix} \{s_0(1)\} & \begin{matrix}\{s_2(0.7), s_3(0.2), \\ s_4(0.1)\}\end{matrix} & \{s_{-3}(0.5), s_{-2}(0.5)\} & \underline{\{s_{-1}(0.6), s_0(0.3)\}} & \begin{matrix}\{s_0(0.7), s_1(0.2), \\ s_2(0.1)\}\end{matrix} \\ \{s_2(0.4), s_3(0.6)\} & \{s_0(1)\} & \{s_{-2}(0.5), s_{-1}(0.5)\} & \{s_0(1)\} & \{s_2(0.6), s_3(0.4)\} \\ \{s_{-1}(0.2), s_0(0.8)\} & \begin{matrix}\{s_{-4}(0.2), s_{-3}(0.7), \\ s_{-1}(0.1)\}\end{matrix} & \{s_0(1)\} & \{s_0(0.8), s_1(0.2)\} & \{s_{-3}(0.4), s_{-2}(0.6)\} \\ \{s_0(1)\} & \{s_{-1}(0.9), s_0(0.1)\}, & \{s_2(0.3), s_3(0.7)\} & \{s_0(1)\} & \{s_{-4}(1)\} \\ \{s_0(0.1), s_1(0.9)\} & \{s_2(0.3), s_3(0.7)\}, & \{s_{-1}(0.6), s_0(0.4)\} & \{s_{-2}(0.8), s_{-1}(0.2)\} & \{s_0(1)\} \end{bmatrix}. \tag{35}$$

It can be noticed that $L^1_{13}$, $L^2_{14}$, and $L^3_{14}$ are incomplete. Based on Definition deff3, the complete forms for these three PLTSs can be expressed as $\{s_{-3}(0.56), s_{-2}(0.44)\}$, $\{s_{-1}(0.78), s_0(0.22)\}$, and $\{s_{-1}(0.67), s_0(0.33)\}$, respectively. After preliminary discussion by the expert group, the subjective weight vector of experts is determined as $\eta = (0.32, 0.35, 0.33)$. According to the adjustment method of experts' weights proposed in Section 3.2, the final weight vector of experts is obtained as $w = (0.335, 0.337, 0.328)$.

*4.1. Analysis of Influencing Factors by the Traditional DEMATEL Method.* In the traditional DEMATEL method, experts usually use a scale of 0–4 to reflect the influence relationships between factors, which are expressed through a single linguistic term. That is to say, the corresponding linguistic value for the influence degree must be nonnegative, certain, and precise. Furthermore, partial ignorance is not allowed in the traditional DEMATEL method. So, the scores $E(L^{\lambda}_{ij}(p))$ of $L^{\lambda}_{ij}$ $(i, j = 1, 2, \ldots, 5, \lambda = 1, 2, 3)$ are calculated, and only those $E(L^{\lambda}_{ij}(p))$ which corresponding PLTSs have complete probability information will be used. Then, the absolute values of $\mathrm{round}(E(L^{\lambda}_{ij}(p)))$ are used to characterize the experts' judgments, and the direct influence matrices $A^{\lambda}_t$ $(\lambda = 1, 2, 3)$ can be expressed as follows, where "–" represents that an expert gave no exact judgment, so this judgment is not considered:

$$A^1_t = \begin{bmatrix} 0 & 2 & - & 1 & 0 \\ 2 & 0 & 2 & 0 & 2 \\ 1 & 3 & 0 & 1 & 2 \\ 0 & 1 & 1 & 0 & 4 \\ 1 & 2 & 1 & 1 & 0 \end{bmatrix},$$

$$A^2_t = \begin{bmatrix} 0 & 3 & 2 & - & 1 \\ 2 & 0 & 2 & 0 & 2 \\ 1 & 3 & 0 & 1 & 2 \\ 0 & 2 & 2 & 0 & 4 \\ 1 & 1 & 3 & 2 & 0 \end{bmatrix}, \tag{36}$$

$$A^3_t = \begin{bmatrix} 0 & 2 & 3 & - & 0 \\ 3 & 0 & 2 & 0 & 2 \\ 0 & 3 & 0 & 0 & 2 \\ 0 & 1 & 3 & 0 & 4 \\ 1 & 3 & 1 & 2 & 0 \end{bmatrix}.$$

Then, the aggregated direct influence matrix $A_t$ can be obtained as follows, as per equation $A_t = \sum_{\lambda=1}^{m} \eta^{\lambda} A_t^{\lambda}$:

$$A_t = \begin{bmatrix} 0 & 2 & 2 & 1 & 0 \\ 2 & 0 & 2 & 0 & 2 \\ 1 & 3 & 0 & 1 & 2 \\ 0 & 1 & 2 & 0 & 4 \\ 1 & 2 & 2 & 2 & 0 \end{bmatrix}. \quad (37)$$

The total influence matrix $T_t$ calculated per equations (27) and (29) is

$$T_t = \begin{bmatrix} 1.302 & 2.396 & 2.28 & 1.239 & 2.044 \\ 1.738 & 2.523 & 2.605 & 1.339 & 2.516 \\ 1.884 & 3.233 & 2.775 & 1.65 & 2.945 \\ 1.871 & 3.258 & 3.201 & 1.691 & 3.383 \\ 1.898 & 3.203 & 3.063 & 1.8 & 2.819 \end{bmatrix}. \quad (38)$$

Based on matrix $T_t$, equations (31) and (33) are used to calculate $d_i^+$ and $r_i^+$. The corresponding $\vartheta_i^+$ and $\psi_i^+$ are shown in Table 1.

*4.2. Analysis of Influencing Factors by the New Scale-Based DEMATEL Method.* The new scale defined in this paper extends the scale to −4, −3, −2, −1, 0, 1, 2, 3, and 4, which can reflect both positive and negative influences between factors. Unlike the traditional DEMATEL method, the new scale-based DEMATEL method does not require the corresponding linguistic value for influence degree to be nonnegative. Therefore, the values of $\text{round}(E(L_{ij}^{\lambda}(p)))(i, j = 1, 2, \ldots, 5, \lambda = 1, 2, 3)$ are directly used to characterize the experts' judgments, and similarly, we use only those $\text{round}(E(L_{ij}^{\lambda}(p)))$ values which corresponding PLTSs have complete probability information. The direct influence matrices $A_s^{\lambda}(\lambda = 1, 2, 3)$ are

$$A_s^1 = \begin{bmatrix} 0 & 2 & - & -1 & 0 \\ 2 & 0 & -2 & 0 & 2 \\ 1 & -3 & 0 & 1 & -2 \\ 0 & -1 & 1 & 0 & -4 \\ 1 & 2 & -1 & -1 & 0 \end{bmatrix},$$
$$A_s^2 = \begin{bmatrix} 0 & 3 & -2 & - & 1 \\ 2 & 0 & -2 & 0 & 2 \\ 1 & -3 & 0 & 1 & -2 \\ 0 & -2 & 2 & 0 & -4 \\ 1 & 1 & -3 & -2 & 0 \end{bmatrix}, \quad (39)$$
$$A_s^3 = \begin{bmatrix} 0 & 2 & -3 & - & 0 \\ 3 & 0 & -2 & 0 & 2 \\ 0 & -3 & 0 & 0 & -2 \\ 0 & -1 & 3 & 0 & -4 \\ 1 & 3 & -1 & -2 & 0 \end{bmatrix}.$$

Using the equation $A_s = \sum_{\lambda=1}^{m} \eta^{\lambda} A_s^{\lambda}$, the aggregated direct influence matrix is

$$A_s = \begin{bmatrix} 0 & 2 & -2 & -1 & 0 \\ 2 & 0 & -2 & 0 & 2 \\ 1 & -3 & 0 & 1 & -2 \\ 0 & -1 & 2 & 0 & -4 \\ 1 & 2 & -2 & -2 & 0 \end{bmatrix}. \quad (40)$$

The positive and negative direct influence matrices calculated by equations (17) and (18) are

$$A_s^+ = \begin{bmatrix} 0 & 2 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 2 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 \end{bmatrix},$$
$$A_s^- = \begin{bmatrix} 0 & 0 & -2 & -1 & 0 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & -3 & 0 & 0 & -2 \\ 0 & -1 & 0 & 0 & -4 \\ 0 & 0 & -2 & -2 & 0 \end{bmatrix}. \quad (41)$$

The total positive and negative influence matrices calculated by equations (27)–(30) are

$$T_s^+ = \begin{bmatrix} 0.714 & 1.143 & 0 & 0 & 0.571 \\ 1.429 & 1.286 & 0 & 0 & 1.143 \\ 0.49 & 0.327 & 0.143 & 0.286 & 0.163 \\ 0.245 & 0.163 & 0.571 & 0.143 & 0.082 \\ 1.143 & 1.429 & 0 & 0 & 0.714 \end{bmatrix},$$
$$T_s^- = \begin{bmatrix} 0 & 0.721 & 1.023 & 0.535 & 0.837 \\ 0 & 0.512 & 0.791 & 0.186 & 0.465 \\ 0 & 1.279 & 0.977 & 0.465 & 1.163 \\ 0 & 1.047 & 1.163 & 0.744 & 1.861 \\ 0 & 0.93 & 1.256 & 0.884 & 1.209 \end{bmatrix}. \quad (42)$$

Based on matrices $T_s^+$ and $T_s^-$, $d_i^+$, $r_i^+$, $d_i^-$, and $r_i^-$ are calculated by equations (31)–(34), and the values of $\vartheta_i$ and $\psi_i$ are shown in Table 2.

*4.3. Analysis of Influencing Factors by the DEMATEL Method Proposed in This Paper.* According to Step 3 in Section 3.3, based on the PLDMs $L^{\lambda}$ $(\lambda = 1, 2, 3)$, the elements in the aggregated PLDM $L$ can be derived as shown in Table 3.

Then, following Step 4 in Section 3.3, the aggregated direct influence matrix, aggregated positive direct influence matrix, and aggregated negative direct influence matrix are given, respectively.

Table 1: $\vartheta_i^+$ and $\psi_i^+$ values of factors by the traditional DEMATEL method.

| | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ | Sorting |
|---|---|---|---|---|---|---|
| $\vartheta_i$ | 17.954 | 25.335 | 26.409 | 21.122 | 26.490 | $f_5 \succ f_3 \succ f_2 \succ f_4 \succ f_1$ |
| $\psi_i$ | 0.566 | −3.890 | −1.438 | 5.685 | −0.923 | $f_4 \succ f_1 \succ f_5 \succ f_3 \succ f_2$ |

Table 2: $\vartheta_i$ and $\psi_i$ values by the new scale-based DEMATEL method.

| | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ | Sorting |
|---|---|---|---|---|---|---|
| $\vartheta_i$ | 3.333 | 1.762 | −6.971 | −5.995 | −3.855 | $f_1 \succ f_2 \succ f_5 \succ f_4 \succ f_3$ |
| $\psi_i$ | −4.708 | 2.045 | 2.020 | −1.225 | 1.868 | $f_2 \succ f_3 \succ f_5 \succ f_4 \succ f_1$ |

$$A = \begin{bmatrix} 0 & 3 & -2 & -1 & 1 \\ 2 & 0 & -2 & 0 & 2 \\ 1 & -3 & 0 & 1 & -2 \\ 0 & -1 & 2 & 0 & -4 \\ 1 & 2 & -1 & -2 & 0 \end{bmatrix},$$

$$A^+ = \begin{bmatrix} 0 & 3 & 0 & 0 & 1 \\ 2 & 0 & 0 & 0 & 2 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 \end{bmatrix}, \tag{43}$$

$$A^- = \begin{bmatrix} 0 & 0 & -2 & -1 & 0 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & -3 & 0 & 0 & -2 \\ 0 & -1 & 0 & 0 & -4 \\ 0 & 0 & -1 & -2 & 0 \end{bmatrix}.$$

The total positive influence matrix $T^+$ and total negative influence matrix $T^-$ can be determined by equations (27)–(30) as

$$T^+ = \begin{bmatrix} 3.8 & 5.6 & 0 & 0 & 4 \\ 4 & 5 & 0 & 0 & 4 \\ 1.371 & 1.6 & 0.143 & 0.286 & 1.143 \\ 0.686 & 0.8 & 0.571 & 0.143 & 0.571 \\ 3.2 & 4.4 & 0 & 0 & 3 \end{bmatrix},$$

$$T^- = \begin{bmatrix} 0 & 0.547 & 0.755 & 0.472 & 0.679 \\ 0 & 0.415 & 0.642 & 0.151 & 0.377 \\ 0 & 1.038 & 0.604 & 0.377 & 0.943 \\ 0 & 0.66 & 0.566 & 0.604 & 1.509 \\ 0 & 0.472 & 0.547 & 0.717 & 0.793 \end{bmatrix}. \tag{44}$$

Based on matrices $T^+$ and $T^-$, $d_i^+$, $r_i^+$, $d_i^-$, and $r_i^-$ are calculated by equations (31)–(34), and the corresponding $\vartheta_i$ and $\psi_i$ are shown in Table 4.

According to the results in Table 4, $f_2$ (knowledge-sharing ability of employees) and $f_1$ (richness of knowledge transfer channels) are the two most important influencing factors. Factors $f_2$, $f_1$, and $f_5$ (employees' willingness to share knowledge) have a positive impact on the whole knowledge-sharing system, while $f_4$ (employees' hierarchy bias) and $f_3$ (the implication and dispersion of knowledge) influence the whole system negatively. Factors $f_3$ and $f_4$ are on the cause group, and $f_5$, $f_1$, and $f_2$ are the factors to be influenced by other factors.

*4.4. Comparisons and Discussion.* To demonstrate that the proposed DEMATEL method is more reasonable and practical, an illustrative example was analyzed using traditional DEMATEL, the new scale-based DEMATEL, and the proposed DEMATEL method. The results of these methods and the rankings of the influencing factors are shown in Tables 1, 2, and 4. The main differences in some special attributes between these three methods are shown in Table 5. From the comparison, it can be seen that the proposed DEMATEL method has some merits.

(1) Positive and negative influence relationships between influencing factors are fully considered. In traditional DEMATEL, the evaluation scale values are all nonnegative integers. Thus, any negative influence will be analyzed and processed as a positive influence. In general, positive influences have positive effects and need to be strengthened, while negative influences have negative effects and need to be suppressed. It is unreasonable to process these two kinds of influences equally. On the contrary, the proposed new scale of DEMATEL extends the traditional one, and makes it possible to rationally distinguish and express positive and negative influences between factors. Therefore, by using the proposed new evaluation scale of DEMATEL, experts will no longer be confused when expressing the positive and negative influences between factors, and the actual influence relationships between factors will be better described and analyzed to make more scientific decisions.

(2) Partial ignorance is permitted. The corresponding linguistic values for influence relationships between factors must be complete in traditional DEMATEL and the new scale-based DEMATEL. In practical decision-making, however, experts may give incomplete judgment information because of the limitation of their professional background and previous experience. So, the experts are often forced to offer complete judgment information when a judgment oversteps an expert's expertise and experience, which may lead to judgmental distortion. When determining the influence relationship between two factors, once an expert gives an incomplete judgment, it must be deleted, and only complete judgments provided by other experts will be considered. On the contrary, the proposed

Table 3: Elements in group's aggregated PLDM $L$.

| | Aggregated PLTSs |
|---|---|
| $L_{11}(p)$ | $\{s_0(1)\}$ |
| $L_{12}(p)$ | $\{s_{2.416}(0.294), s_{2.736}(0.084), s_{2.744}(0.196), s_3(0.056), s_4(0.37)\}$ |
| $L_{13}(p)$ | $\{s_{-3}(0.028), s_{-2.656}(0.028), s_{-2.648}(0.19), s_{-2.32}(0.19), s_{-2.312}(0.132), s_{-2.248}(0.084), s_{-2}(0.132), s_{-1.944}(0.084), s_{-1.936}(0.066), s_{-1.64}(0.066)\}$ |
| $L_{14}(p)$ | $\{s_{-1.312}(0.15678), s_{-1}(0.2613), s_{-0.944}(0.07722), s_{-0.928}(0.04422), s_{-0.648}(0.1287), s_{-0.64}(0.10452), s_{-0.632}(0.0737), s_{-0.584}(0.02178), s_{-0.32}(0.05148), s_{-0.312}(0.0363), s_{-0.304}(0.02948), s_{-0.008}(0.01452)\}$ |
| $L_{15}(p)$ | $\{s_{0.368}(0.294), s_{0.696}(0.28), s_{0.832}(0.126), s_{0.992}(0.056), s_{1.104}(0.042), s_{1.12}(0.036), s_{1.128}(0.084), s_{1.368}(0.028), s_{1.384}(0.024), s_{1.472}(0.018), s_{1.712}(0.012)\}$ |
| $L_{21}(p)$ | $\{s_{1.104}(0.012), s_{1.368}(0.036), s_{1.472}(0.028), s_{1.696}(0.018), s_{1.704}(0.072), s_{1.712}0.084, s_{1.904}(0.054), s_{1.992}(0.042), s_2(0.168), s_{2.168}(0.108), s_{2.176}(0.126), s_{2.408}(0.252)\}$ |
| $L_{22}(p)$ | $\{s_0(1)\}$ |
| $L_{23}(p)$ | $\{s_{-2.32}(0.21), s_{-1.952}(0.21), s_{-1.944}(0.09), s_{-1.64}(0.14), s_{-1.592}(0.09), s_{-1.312}(0.14), s_{-1.304}(0.06), s_{-1}(0.06)\}$ |
| $L_{24}(p)$ | $\{s_0(1)\}$ |
| $L_{25}(p)$ | $\{s_{1.368}(0.036), s_{1.704}(0.084), s_{1.712}(0.144), s_{1.904}(0.024), s_2(0.336), s_{2.168}(0.056), s_{2.176}(0.096), s_{2.408}(0.224)\}$ |
| $L_{31}(p)$ | $\{s_{0.096}(0.028), s_{0.368}(0.112), s_{0.448}(0.112), s_{0.592}(0.012), s_{0.696}(0.448), s_{0.832}(0.048), s_{0.904}(0.048), s_{1.128}(0.192)\}$ |
| $L_{32}(p)$ | $\{s_{-3.648}(0.028), s_{-3.32}(0.098), s_{-3.312}(0.112), s_{-3.264}(0.012), s_{-3}(0.392), s_{-2.952}(0.042), s_{-2.944}(0.048), s_{-2.648}(0.168), s_{-2.552}(0.014), s_{-2.264}(0.056), s_{-2.224}(0.006), s_{-1.952}(0.024)\}$. |
| $L_{33}(p)$ | $\{s_0(1)\}$ |
| $L_{34}(p)$ | $\{s_{0.368}(0.144), s_{0.696}(0.372), s_{0.824}(0.096), s_{0.992}(0.084), s_{1.112}(0.024), s_{1.12}(0.224), s_{1.376}(0.056)\}$ |
| $L_{35}(p)$ | $\{s_{-2.312}(0.224), s_{-2}(0.336), s_{-1.936}(0.152), s_{-1.64}(0.228), s_{-1.584}(0.024), s_{-1.304}(0.036)\}$ |
| $L_{41}(p)$ | $\{s_{-0.32}(0.2), s_{-0.008}(0.8)\}$ |
| $L_{42}(p)$ | $\{s_{-1.312}(0.315), s_{-1}(0.315), s_{-0.944}(0.035), s_{-0.936}(0.135), s_{-0.648}(0.035), s_{-0.64}(0.135), s_{-0.592}(0.015), s_{-0.32}(0.015)\}$ |
| $L_{43}(p)$ | $\{s_{1.104}(0.006), s_{1.368}(0.054), s_{1.472}(0.024), s_{1.696}(0.014), s_{1.712}(0.216), s_{1.904}(0.126), s_{1.992}(0.056), s_{2.176}(0.504)\}$ |
| $L_{44}(p)$ | $\{s_0(1)\}$ |
| $L_{45}(p)$ | $\{s_{-4}(0.8), s_{-3.648}(0.2)\}$ |
| $L_{51}(p)$ | $\{s_{0.368}(0.014), s_{0.696}(0.182), s_{0.832}(0.006), s_{0.992}(0.504), s_{1.12}(0.054), s_{1.128}(0.024), s_{1.384}(0.216)\}$ |
| $L_{52}(p)$ | $\{s_{1.104}(0.018), s_{1.368}(0.072), s_{1.472}(0.042), s_{1.696}(0.042), s_{1.704}(0.168), s_{1.904}(0.168), s_{1.984}(0.098), s_{2.168}(0.392)\}$ |
| $L_{53}(p)$ | $\{s_{-1.952}(0.072), s_{-1.648}(0.048), s_{-1.592}(0.288), s_{-1.536}(0.048), s_{-1.312}(0.192), s_{-1.256}(0.032), s_{-1.2}(0.192), s_{-0.944}(0.128)\}$. |
| $L_{54}(p)$ | $\{s_{-2.32}(0.112), s_{-2}(0.448), s_{-1.952}(0.028), s_{-1.648}(0.112), s_{-1.52}(0.048), s_{-1.24}(0.192), s_{-1.2}(0.012), s_{-0.936}(0.048)\}$ |
| $L_{55}(p)$ | $\{s_0(1)\}$ |

Table 4: $\vartheta_i$ and $\psi_i$ values of factors by the proposed DEMATEL method.

| | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ | Sorting |
|---|---|---|---|---|---|---|
| $\vartheta_i$ | 24.004 | 25.683 | −2.460 | −0.818 | 16.484 | $f_2 \succ f_1 \succ f_5 \succ f_4 \succ f_3$ |
| $\psi_i$ | −2.110 | −2.853 | 3.980 | 1.324 | −0.341 | $f_3 \succ f_4 \succ f_5 \succ f_1 \succ f_2$ |

Table 5: Principal differences in some special attributes between three kinds of DEMATEL methods.

| | Negative influences between factors | Incomplete information | Experts' hesitant information | Adjustment of experts' weights |
|---|---|---|---|---|
| Traditional DEMATEL method | × | × | × | × |
| New scale-based DEMATEL method | √ | × | × | × |
| Proposed DEMATEL method | √ | √ | √ | √ |

DEMATEL method allows experts to give incomplete judgment information, so that all judgment information will be adopted without losing valuable data. If the judgment information is incomplete, the probabilities in these incomplete PLTSs will be normalized to make them complete, as shown in Definition deff3.

(3) Hesitation among several possible values is allowed. In traditional DEMATEL and the new scale-based DEMATEL, the linguistic value of the influence degree must be certain and precise. However, due to the complexity of practical problems, experts may hesitate among several possible linguistic terms when judging the influence relationships between factors. In this case, if the experts are required to give precise judgment information, the judgment may be distorted. On the contrary, the proposed DEMATEL method allows experts to hesitate among several possible linguistic terms, and their preferences for these possible linguistic terms will be expressed in the form of a probability distribution. In this way, the expression of judgment information will be more flexible, and all valuable information will be collected and considered.

(4) An adjustment method of experts' subjective weights is introduced. In traditional DEMATEL and the new scale-based DEMATEL, it is often assumed that the experts are equally important, which is arbitrary, or the experts' weights are given directly by the expert group, which only considers the experts' confidence in their judgments and does not consider the reliability of experts' judgments. To overcome these defects, the proposed DEMATEL method introduces an experts' subjective weights adjustment method to determine the final experts' weights, which is based on the similarity degree between PLTSs defined in this paper. First, the subjective weights of experts are determined by the expert group through a preliminary discussion. Then, the final weights will be gotten by adjusting the subjective weights based on the similarity degree of experts' judgments which are expressed in PLTSs. On the one hand, the final weights can reflect experts' confidence in their judgments. On the other hand, the final weights which are determined by the similarity degree of judgments can reflect the reliability of experts' judgments to some extent. Therefore, the experts' weights determination method proposed in this paper is more reasonable.

Table 6: Initial subjective weights of three experts and adjusted weights.

| Initial subjective weights | Adjusted weights |
|---|---|
| $\omega_1$ $\omega_1^t = \{0.1, 0.3, 0.6\}$ | $\omega_1^a = \{0.081, 0.224, 0.695\}$ |
| $\omega_2$ $\omega_2^t = \{0.2, 0.1, 0.7\}$ | $\omega_2^a = \{0.155, 0.071, 0.774\}$ |
| $\omega_3$ $\omega_3^t = \{0.4, 0.3, 0.3\}$ | $\omega_3^a = \{0.363, 0.250, 0.388\}$ |
| $\omega_4$ $\omega_4^t = \{0.5, 0.1, 0.4\}$ | $\omega_4^a = \{0.430, 0.079, 0.491\}$ |
| $\omega_5$ $\omega_5^t = \{0.6, 0.2, 0.2\}$ | $\omega_5^a = \{0.561, 0.172, 0.267\}$ |
| $\omega_6$ $\omega_6^t = \{0.8, 0.1, 0.1\}$ | $\omega_6^a = \{0.773, 0.089, 0.138\}$ |

*4.5. Sensitivity Analysis.* In order to further validate the effectiveness and superiority of the new group method, the sensitivity of each of the three models will be tested and analyzed in this study using multiple sets of weights for three experts. The data for the six sets of weight changes required in the analysis are shown in Table 6, where the initial subjective weights of three experts given by the decision-maker is denoted as $\omega_t$, according to the adjustment method of experts' weights proposed in Section 3.2, and the final weight vector of experts is denoted as $\omega_a$.

Then, following steps in Sections 4.1–4.3, the results by the three algorithms with six sets of weight variations were obtained and are shown in Tables 7–9, respectively.

The results of the above calculations show that the line graphs of the DEMATEL method proposed in this paper change more significantly compared to the other two methods that have not been improved. The change in the new scale-based DEMATEL method is not obvious and the direction is basically flat, indicating that if the information is collected in a form other than PLTSs, the loss of information will affect the sensitivity of the ranking results and will also have a greater impact on the decision results. In contrast, the

Table 7: Results of the traditional DEMATEL method.

| | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ | Sorting | Line chart |
|---|---|---|---|---|---|---|---|
| $\vartheta^1$ | 5.758 | 8.572 | 7.856 | 6.485 | 8.419 | $f_2 \succ f_5 \succ f_3 \succ f_4 \succ f_1$ | |
| $\psi^1$ | 0.077 | −1.042 | −1.723 | 2.874 | −0.185 | $f_4 \succ f_1 \succ f_5 \succ f_2 \succ f_3$ | |
| $\vartheta^2$ | 6.034 | 9.230 | 7.486 | 6.642 | 8.636 | $f_2 \succ f_5 \succ f_3 \succ f_4 \succ f_1$ | |
| $\psi^2$ | −0.098 | −1.548 | −1.217 | 2.946 | −0.083 | $f_4 \succ f_5 \succ f_1 \succ f_3 \succ f_2$ | |
| $\vartheta^3$ | 17.954 | 25.335 | 26.409 | 21.122 | 26.490 | $f_5 \succ f_3 \succ f_2 \succ f_4 \succ f_1$ | |
| $\psi^3$ | 0.566 | −3.890 | −1.438 | 5.685 | −0.923 | $f_4 \succ f_1 \succ f_5 \succ f_3 \succ f_2$ | |
| $\vartheta^4$ | 12.397 | 17.142 | 16.834 | 14.095 | 16.957 | $f_2 \succ f_5 \succ f_3 \succ f_4 \succ f_1$ | |
| $\psi^4$ | 0.581 | −2.522 | 0.151 | 3.537 | −1.747 | $f_4 \succ f_1 \succ f_3 \succ f_5 \succ f_2$ | |
| $\vartheta^5$ | 9.534 | 12.908 | 12.550 | 9.253 | 11.516 | $f_2 \succ f_3 \succ f_5 \succ f_1 \succ f_4$ | |
| $\psi^5$ | 0.433 | −1.994 | 0.097 | 3.396 | −1.932 | $f_4 \succ f_1 \succ f_3 \succ f_5 \succ f_2$ | |
| $\vartheta^6$ | 8.450 | 11.494 | 10.751 | 7.387 | 10.197 | $f_2 \succ f_3 \succ f_5 \succ f_1 \succ f_4$ | |
| $\psi^6$ | 0.426 | −1.570 | 0.580 | 2.235 | −1.671 | $f_4 \succ f_3 \succ f_1 \succ f_2 \succ f_5$ | |

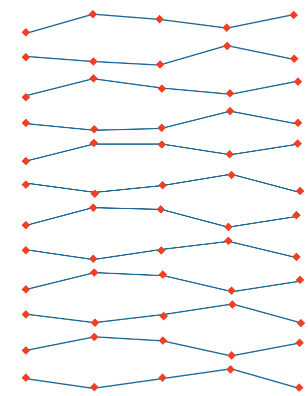


Table 8: Results of the new scale-based DEMATEL method.

| | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ | Sorting | Line chart |
|---|---|---|---|---|---|---|---|
| $\vartheta^1$ | 0.799 | −4.551 | −8.493 | −7.028 | −6.532 | $f_1 \succ f_2 \succ f_5 \succ f_4 \succ f_3$ | |
| $\psi^1$ | −4.102 | 2.887 | 0.726 | −0.727 | −1.400 | $f_2 \succ f_3 \succ f_4 \succ f_5 \succ f_1$ | |
| $\vartheta^2$ | 2.350 | −0.599 | −5.475 | −5.060 | −2.027 | $f_1 \succ f_2 \succ f_5 \succ f_4 \succ f_3$ | |
| $\psi^2$ | −3.846 | 1.547 | −0.449 | −0.419 | 3.167 | $f_5 \succ f_2 \succ f_4 \succ f_3 \succ f_1$ | |
| $\vartheta^3$ | 3.333 | −0.599 | −5.475 | −5.060 | −2.027 | $f_1 \succ f_2 \succ f_5 \succ f_4 \succ f_3$ | |
| $\psi^3$ | −3.846 | 1.547 | −0.449 | −0.419 | 3.167 | $f_5 \succ f_2 \succ f_4 \succ f_3 \succ f_1$ | |
| $\vartheta^4$ | 3.996 | 1.209 | −3.953 | −4.028 | −0.871 | $f_1 \succ f_2 \succ f_5 \succ f_4 \succ f_3$ | |
| $\psi^4$ | −4.045 | 1.057 | 0.845 | −0.243 | 2.386 | $f_5 \succ f_2 \succ f_3 \succ f_4 \succ f_1$ | |
| $\vartheta^5$ | 4.449 | 2.123 | −2.863 | −1.715 | 1.553 | $f_1 \succ f_2 \succ f_5 \succ f_4 \succ f_3$ | |
| $\psi^5$ | −3.592 | 0.771 | 1.013 | −0.543 | 2.351 | $f_5 \succ f_3 \succ f_2 \succ f_4 \succ f_1$ | |
| $\vartheta^6$ | 4.286 | 2.014 | −3.405 | −2.453 | 1.499 | $f_1 \succ f_2 \succ f_5 \succ f_4 \succ f_3$ | |
| $\psi^6$ | −3.429 | 0.880 | 1.233 | −1.090 | 2.406 | $f_5 \succ f_3 \succ f_2 \succ f_4 \succ f_1$ | |

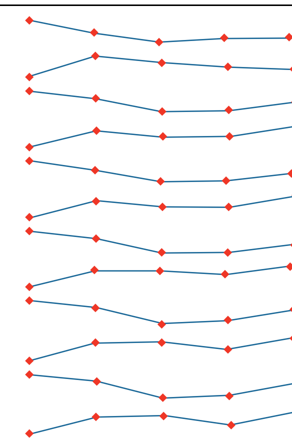
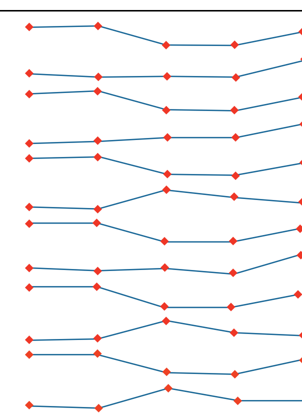


Table 9: Results of the DEMATEL method proposed in this paper.

| | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ | Sorting | Line chart |
|---|---|---|---|---|---|---|---|
| $\vartheta^1$ | 21.947 | 23.283 | −5.576 | −5.164 | 14.770 | $f_2 \succ f_1 \succ f_5 \succ f_4 \succ f_3$ | |
| $\psi^1$ | −0.053 | −0.453 | −0.349 | −0.519 | 1.374 | $f_5 \succ f_1 \succ f_3 \succ f_2 \succ f_4$ | |
| $\vartheta^2$ | 18.747 | 24.083 | −5.576 | −5.160 | 14.770 | $f_2 \succ f_1 \succ f_5 \succ f_4 \succ f_3$ | |
| $\psi^2$ | −3.353 | −2.053 | −0.349 | −0.519 | 6.174 | $f_5 \succ f_3 \succ f_4 \succ f_2 \succ f_1$ | |
| $\vartheta^3$ | 24.004 | 25.683 | −0.818 | −2.460 | 16.484 | $f_2 \succ f_1 \succ f_5 \succ f_3 \succ f_4$ | |
| $\psi^3$ | −2.110 | −2.853 | 3.980 | 1.324 | −0.341 | $f_3 \succ f_4 \succ f_5 \succ f_1 \succ f_2$ | |
| $\vartheta^4$ | 21.947 | 23.283 | −4.933 | −4.518 | 14.770 | $f_2 \succ f_1 \succ f_5 \succ f_4 \succ f_3$ | |
| $\psi^4$ | −0.053 | −0.453 | −0.135 | −0.733 | 1.374 | $f_5 \succ f_1 \succ f_3 \succ f_2 \succ f_4$ | |
| $\vartheta^5$ | 23.341 | 23.958 | −3.836 | −4.428 | 13.500 | $f_2 \succ f_1 \succ f_5 \succ f_3 \succ f_4$ | |
| $\psi^5$ | −2.773 | −1.865 | 5.154 | 0.343 | −0.586 | $f_3 \succ f_4 \succ f_5 \succ f_2 \succ f_1$ | |
| $\vartheta^6$ | 23.547 | 25.150 | −1.569 | −4.034 | 16.103 | $f_2 \succ f_1 \succ f_5 \succ f_3 \succ f_4$ | |
| $\psi^6$ | −1.653 | −2.320 | 3.910 | −0.059 | 0.040 | $f_3 \succ f_5 \succ f_4 \succ f_1 \succ f_2$ | |



DEMATEL method proposed in this paper, which considers both negative scaling and PLTSs, is more superior.

Then, to intuitively observe the changes in the position of the factors in the system, this research analyses the sensitivity of the three models from the perspective of the factors, using the change in the position of the cause degree ($\psi_i$) of the factors as an example. The results of the change in the location of the factors are shown in Figures 2–6.

It is evident from Figures 2 to 6 that the DEMATEL method proposed in this paper is sensitive to the weight

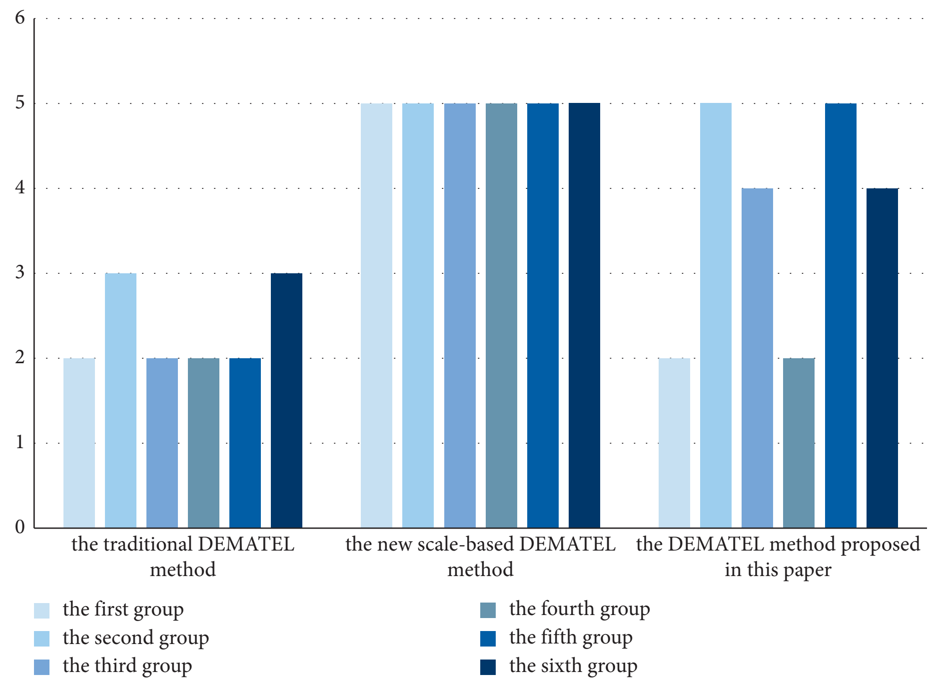



FIGURE 2: Location change of system element $f_1$.

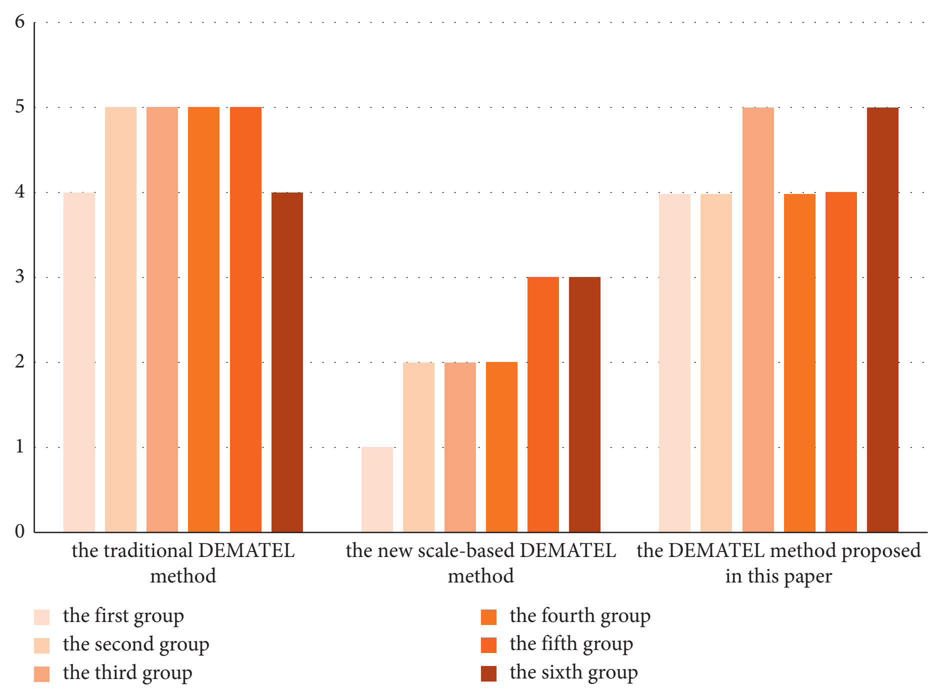



FIGURE 3: Location change of system element $f_2$.

change of experts, with more significant differences in the position of the factors in the system as the varying expert weights. By comparison, the new scale-based DEMATEL method is less sensitive and the weight change of experts has no greater impact on the decision outcome, which is judged to be inconsistent with the real world.

In addition, the traditional DEMATEL method has a relatively high sensitivity as does the DEMATEL method proposed in this paper, but the new group DEMATEL in this paper contains a more diverse amount of information and also adds a negative scale as a basis for evaluation. It can be seen that although the new group DEMATEL method is computationally complex, the sensitivity of this method does not decrease due to the increased amount of information and computational complexity; so, the new group DEMATEL model is more advantageous in solving more complex system issues.

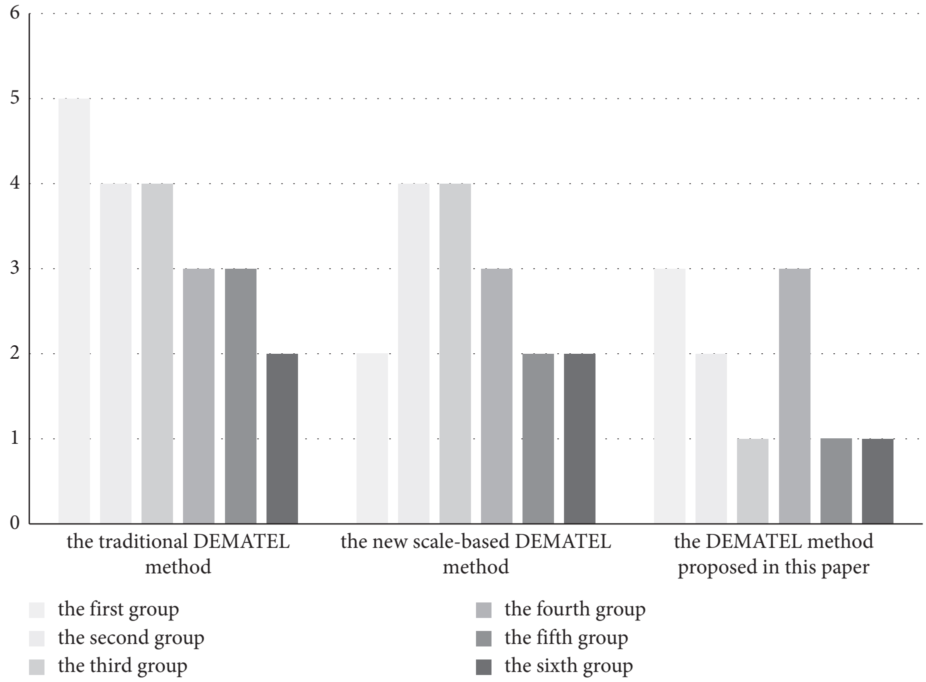



Figure 4: Location change of system element $f_3$.

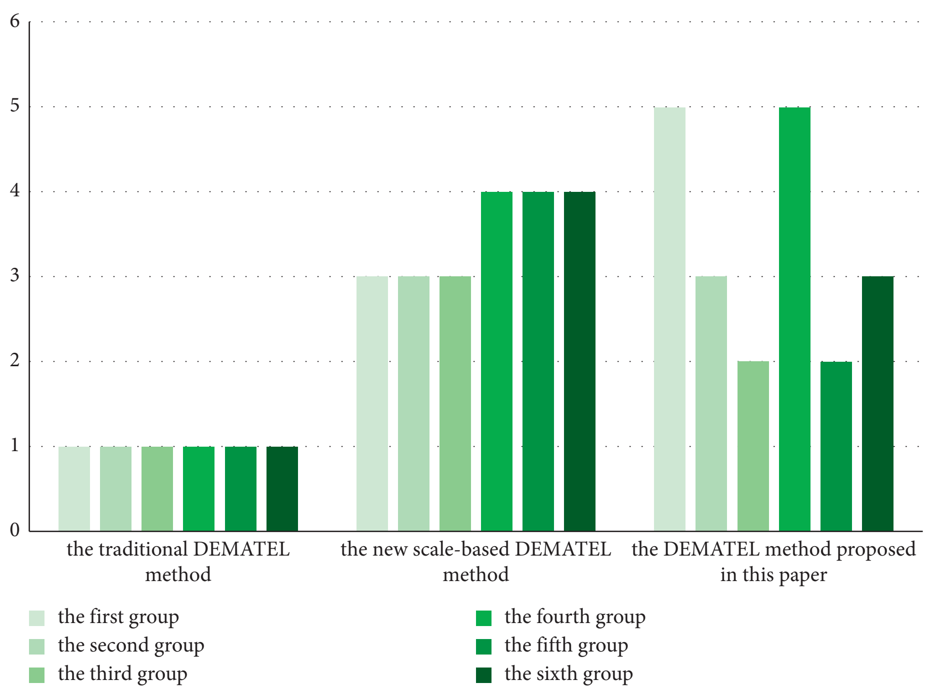



Figure 5: Location change of system element $f_4$.

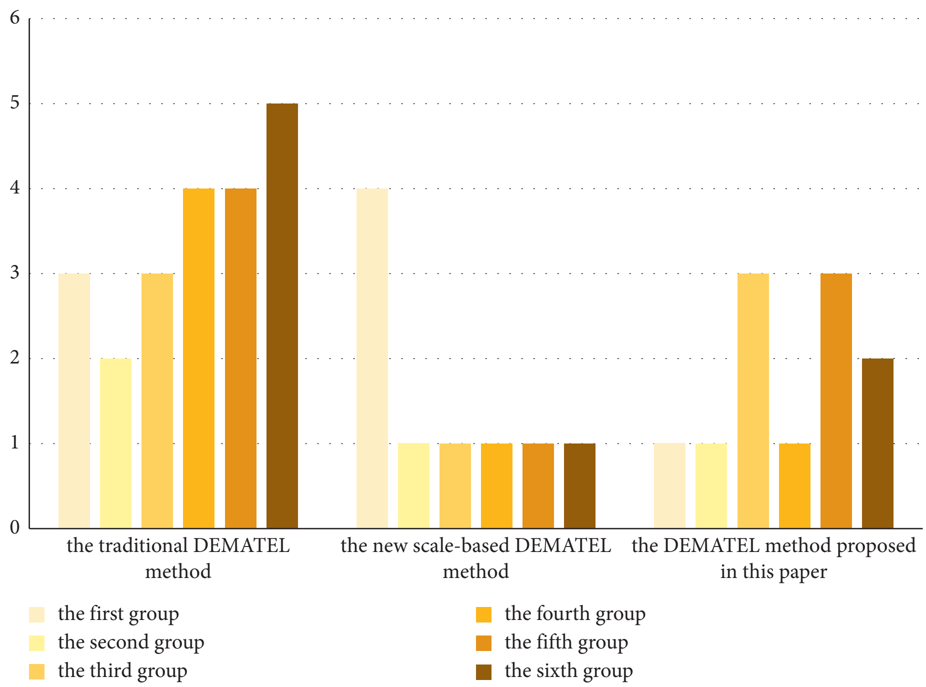



Figure 6: Location change of system element $f_5$.

## 5. Conclusions and Future Directions

The values of the existing evaluation scales of DEMATEL are all nonnegative integers without exception, and the negative influences between factors are expressed and treated as positive ones. However, positive and negative influences have different effects and need to be managed differently. In addition, the experts are assumed to be able to give precise and complete judgment information, which may lead to judgmental distortion when a judgment oversteps an expert's expertise and experience. In order to solve the mentioned problems, the new probabilistic linguistic-based group DEMATEL method is developed in this paper to analyze both positive and negative influence relationships between factors. To demonstrate the proposed DEMATEL method, an illustrative example of an innovative pharmaceutical enterprise is adopted. The proposed group DEMATEL method is compared to traditional DEMATEL and the new scale-based DEMATEL to prove the feasibility and advantages of the proposed new method. Our results show that the proposed DEMATEL method is more reasonable and practical.

The main contributions of this study can be summarized into four aspects. Firstly, by extending the traditional evaluation scale of DEMATEL, a new scale is defined to distinguish and describe both positive and negative influences between factors. The positive and negative direct influence matrices under the new scale are also defined, and the corresponding matrix operations of DEMATEL are introduced. This is the first time that negative influences between factors are considered in the method of DEMATEL; therefore, it is more in line with the practical situation of complex system factor analysis. Secondly, PLTS theory is introduced to integrate with the new scale-based group DEMATEL method, which allows experts to express incomplete and uncertain linguistic preference. Thirdly, to better determine the final experts' weights, an experts' subjective weights adjustment method based on the similarity degree of experts' judgments under the probabilistic linguistic environment is introduced. Fourthly, an algorithm of probabilistic linguistic-based group DEMATEL method with both positive and negative influences is summarized to make the idea and process of the proposed method clear.

In future research, we expect to employ the proposed DEMATEL method to solve practical system factor analysis problems, such as analysis of factors influencing the location selection of freight villages, factors influencing the implementation of innovation strategies, and analysis of influencing factors of green supplier selection.

## Data Availability

The data used to support the findings of this study are included within the article.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## Acknowledgments

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*Research Article*

# Integrating an Extended Outranking-TOPSIS Method with Probabilistic Linguistic Term Sets for Multiattribute Group Decision-Making

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Group decision-making is a common activity in organizational management and economic conditions. In practice, the opinions of experts may be fuzzy. This paper proposes integrating an extended outranking-TOPSIS method with probabilistic linguistic term sets for multiattribute group decision-making, which is used to solve the real-world public-private partnership (PPP) project selection problem. First, an extended outranking method based on probabilistic linguistic term sets is proposed, and each expert's ranking of alternatives is obtained according to this method. After the individual ranking is completed, the large-scale expert group is clustered by the K-means clustering method, and then the improved consensus mechanism is used to study the degree of consensus of the expert group. If the consensus of the group is not up to the standard, then, for clusters with a lower degree of consensus with the group, the feedback mechanism is used to adjust the weight between different clusters so that the group consensus can be improved. After achieving the target group consensus, an improved technique for order preference by similarity to an ideal solution (TOPSIS) method is used to synthesize expert opinions, and the ranking results are obtained. Finally, there are cases used to demonstrate the feasibility and rationality of the method.

## 1. Introduction

Research on decision-making has been conducted for many years, and it has long been recognized by enterprises and other entities. In the decision-making problem, the language of the decision-maker and its expression are worthy of being studied. The decision-maker, similar to the decision-making problem, faces many constraints. As specific events cannot be accurately quantified and each decision-maker's personal educational background, living environment, abilities, and so on are different, ambiguities are present in the decision-making process. Therefore, it is necessary to convert qualitative expressions into quantitative expressions. Zadeh [1] proposed fuzzy linguistics and the use of linguistic variables to represent decision-making. However, linguistic variables are not sufficient to accurately reflect opinions. Torra [2, 3] proposed the concept of hesitant fuzzy sets and pointed out that its membership degree exists in a subset of [0, 1]. Rodriguez et al. [4] proposed a hesitant fuzzy linguistic term set to obtain a contiguous subset of a set of linguistic terms to describe linguistic variables. The hesitant fuzzy linguistic term set assigns the same weight to each linguistic variable, which is not sufficient to reflect the probability difference between different linguistic variables assigned by the decision-maker. In this context, Pang et al. [5] proposed a new probabilistic linguistic term set (PLTS) that can fully quantify the linguistic scores of decision-makers and reflect the quantitative differences between the linguistic variables. In this manner, comprehensive and accurate preference information of decision-makers can be obtained.

There have been many studies on PLTS in recent years. Zhang and She [6] used it for multicriteria decision-making (MCDM) problems. Liao et al. [7] also proposed a method based on PLTS to solve the multicriteria decision-making problems. Lin et al. [8] constructed an IoT evaluation system and introduced the concept of probabilistic linguistic term set to express the group preference information of the IoT platform with respect to the criteria. Yao et al. [9] proposed a probabilistic linguistic term envelopment analysis and studied the optimization method of the allocation efficiency of PM2.5 emission rights. Bai et al. [10] proposed an interval-valued probabilistic linguistic term set and studied the related operations and comparison laws of the theory to solve the multicriteria group decision-making (MCGDM) problem. Yu et al. [11] proposed probabilistic linguistic weight average (PLWA) and probabilistic linguistic order weight average (PLOWA) operators and studied their properties, and they proposed a multicriteria decision-making method based on the proposed operators. Gou and Xu [12] proposed some new operation laws for hesitant fuzzy linguistic elements and probabilistic linguistic term sets based on two equivalent transformation functions. By using PLTS for calculations, the probability information can be kept complete.

With the development of the economy and technology, the importance of decision science is increasing [13, 14]. Multiattribute group decision-making is an important part of modern decision science, and its theories and methods are widely used in the fields of economy, management, and military strategy. Different experts or decision-makers are needed to evaluate alternatives with multiple attributes and finally give a ranking of the options approved by the decision-making group. Liu et al. [15] first determined the percentage distribution of the assessment by each group of each alternative and then aggregated the subjective weights provided by the organizer and the objective weights determined by the level of consensus between the participant evaluations to obtain the decision weights for each group of each option and then rank by comparing the advantages between the programs. Wu et al. [16] used linguistic principal component analysis to reduce the attribute dimension. Shen et al. [17] used a new intuitionistic fuzzy ordering method to solve related multiattribute group decision problems. Rodríguez et al. [18] combined hesitant fuzzy linguistic and group decision-making to expand the scope of hesitant fuzzy linguistic method. The Score-HeDLiSF proposed by Liao et al. [19] has advantages in dealing with balanced and unbalanced language information with hesitant and linguistic scale functions. Lin et al. [20] proposed an aggregation-based technology to sort alternatives to solve the problem of multiattribute group decision-making. Gou et al. [21] proposed a similarity-based clustering method and a double hierarchy information entropy-based weighting method and consensus metric. Lin et al. [22] proposed a new PDOWA operator to address multiattribute group decision-making problems and gave an example to illustrate. Yu et al. [23] extended the classic TODIM method to develop a new MCGDM method based on unbalanced hesitant fuzzy linguistic term sets (HFLTS).

In many situations, a large number of experts participate in the group decision-making effort. Due to factors such as the observation angle of the experts and personal abilities, the degree of decision-making consensus is often not high enough. At the same time, the increase in the number of group decision-making participants may make some models no longer applicable. This is the problem of large-scale group decision-making. The problem of large-scale group decision-making requires increasing the degree of consensus among experts and unifying expert opinions. Wu et al. [24] combined the interval type-2 fuzzy method with the TOPSIS method to solve the large-scale multiattribute group decision problem. Xu et al. [25] proposed some new concepts, including collective adjustment proposals and rationality to solve large-scale group decision-making problems. Du et al. [26] proposed a new large-scale group decision-making method considering the expert knowledge structure and proposed an information extraction mechanism that provides three kinds of reasoning methods: single-attribute reasoning, local integral reasoning, and global integral reasoning. Liu et al. [27] proposed a dynamic weight penalty mechanism to increase the degree of consensus for overconfident decision-makers in large-scale group decision-making problems. Liu et al. [28] aimed at large-scale group decision-making in the social network environment and detected and reduced conflicts among decision-makers in the three processes of trust propagation, conflict detection and elimination, and selection.

Clustering is a common method to solve group decision-making problems and improve the degree of consensus. It groups experts with similar opinions, so as to effectively adjust the opinions between experts or adjust the weights attributed by experts. Ma et al. [29] applied a fuzzy clustering approach to create expert clusters based on expert similarity and phase and attribute weights. Kamis et al. [30] suggested three steps, namely, the identification of experts who contribute less to consensus, identification of leaders in the network, and generation of recommendations to achieve clustering mechanisms. Yoon et al. [31] proposed a mediation group decision-making method based on preference clustering to minimize subjectivity issues. Xu et al. [32] also constructed a group membership clustering algorithm to cluster large groups and then obtained the best alternative algorithm by comparing the exact functions of the score function and interval intuitionistic fuzzy numbers. Wu and Liu [33] used interval type-2 fuzzy equivalence clustering to classify decision-makers. In addition to this, there are other methods to optimize the consensus of group decision problems. For example, Wu et al. [34] used equivalent integer linear programming to optimize the size of the change, the number of modifications, and the individuals who need to modify their preferences. To improve the acceptability of the proposed preferences, an interactive consistency process and an interactive consensus process based on the multistage model were also designed to illustrate the developed method. Wu and Xu [35] proposed a process of direct consensus to solve the HFLPR consensus problem. The consensus arrival process has a salient feature that the feedback system is directly based on the degree of consensus, thereby effectively reducing the proximity measure calculations. Some existing consensus improvement mechanisms have practices that completely ignore the opinions of marginal expert groups, such as transferring all of their opinions directly to other expert groups, which is not in line with the idea of group decision-making. At present, it is necessary to propose a new consensus improvement mechanism to increase consensus on the basis of respecting the opinions of marginal expert groups.

This article has the following innovations and contributions:

(1) This work proposes a new extended outranking method based on probabilistic linguistic term sets that can effectively rank the alternatives. Then, clustering is based on the proposed extended outranking relation, and the matrix used is an asymmetric matrix. Compared with the traditional symmetric matrix based on probabilistic linguistic term sets, such as the Euclidean distance matrix between ordinary probabilistic linguistic term sets, the information is more abundant and complete, and the clustered expert group opinions are closer.

(2) This work uses an improved consensus improvement mechanism. The feedback mechanism respects the opinions of marginal expert groups and respects their weight adjustments. This helps to improve the persuasiveness and acceptability of decision-making, and it can also effectively improve the overall consensus degree. By setting the threshold of the consensus mechanism, a good improvement effect can be obtained, thereby making the conclusion more accurate and reliable.

(3) This work proposes an improved TOPSIS ranking method based on net credibility and weight adjustment. This method constructs the initial matrix by using net credibility as relative closeness and introduces the adjusted weights of decision-makers to obtain the group closeness matrix, which can effectively solve the problem of large-scale multi-attribute group decision-making.

The rest of the article is organized as follows: Section 2introduces the basic concepts related to the probabilistic linguistic term set, and Section 3introduces the extended outranking method based on probabilistic linguistic term sets, clustering mechanism, consensus mechanism and feedback mechanism, and the improved TOPSIS method. Section 4introduces the specific examples to illustrate the method of this work. Section 5compares and discusses related literature. Section 6gives the conclusions. The Appendix section provides the original data of this article.

## 2. Basic Concepts and Theories and Their Relationship

In this section, we introduce the basic concepts related to the probabilistic linguistic term set (PLTS).

*Definition 1* (see [5]). Consider $S = \{S_i | i = 0, 1, 2, \ldots, g\}$as a set of linguistic terms, then the PLTS can be defined as follows:

$$L(p) = \left\{ L^{(k)}\left(p^{(k)}\right) | L^{(k)} \in S, p^{(k)} \geq 0, k = 1, 2, \ldots, \#L(p), \sum_{k=1}^{\#L(p)} p^{(k)} \leq 1 \right\}, \quad (1)$$

where $L^{(k)}(p^{(k)})$is the linguistic term $L^{(k)}$associated with probability $p^{(k)}$and $\#L(p)$is the number of all different linguistic terms in $L^{(p)}$.

The advantage of the probabilistic linguistic term set is that it can reflect the complete probabilistic distribution in linguistic terms. Thus, in a complex decision-making environment, decision-makers can selectively assign several linguistic terms and their probabilities, which is convenient for decision-makers to use linguistic information to effectively articulate decision-making views and to express linguistic information flexibly. Therefore, it is more in line with the inner thinking of decision-makers.

To effectively compare the two probabilistic linguistic term sets, it is necessary to introduce the concept of score function, using the score function based on concentration degree for probabilistic linguistic term sets score function proposed by Lin et al.

*Definition 2* (see [36]). Let $L(p) = \{L^{(k)}(p^{(k)}) | k = 1, 2, \ldots, \#L(p)\}$be a PLTS, and let $r^{(k)}$be the subscript for the linguistic term $L^{(k)}$. Thus, the score function $F(L(p))$of $L(p)$can be expressed as follows:

$$F(L(p)) = s_{\overline{\alpha} \times cd(L(P))}. \quad (2)$$

In (2), $\overline{\alpha} = \sum_{l=1}^{L} I(s^{(l)}) p^{(l)} / \sum_{l=1}^{L} p^{(l)}$. The concentration degree of$L(p)$is $c\,d(L(P)) = 1 + \sum_{l=1}^{L} p^{(l)} \log_2 (1 - (|I(s^{(l)}) - I(E(L(P)))| / I(d_{lts})))$. The expected value of $L(P)$is $E(L(P))$, $I(s^{(l)})$is the subscript of the linguistic term $s^{(l)}, I(d_{\mathrm{lts}})$is the subscript of the linguistic term which is the difference value between the maximum linguistic term and the minimum linguistic term in the LTS $S$, and $I(E(L(P)))$is the subscript of the expectation value of $L(P)$. It considers hesitance and uncertainty degree in the concentration degree.

The score function is composed of expectation value and concentration degree, effectively processing the probability information contained in probabilistic linguistic term sets and achieving a comparison of probabilistic linguistic term sets: for two probabilistic linguistic term sets $L_{P_1}$and $L_{P_2}$, if $F(L_{P_1}) < F(L_{P_2})$, then $L_{P_1} < L_{P_2}$. If $F(L_{P_1}) = F(L_{P_2})$, then $L_{P_1} = L_{P_2}$.

## 3. A Multiattribute Group Decision-Making Method with Probabilistic Linguistic Term Sets

*3.1. An Extended Outranking Method Based on Probabilistic Linguistic Term Sets.* This article proposes introducing the score function $F$of the probabilistic linguistic term sets as the attribute score of the scheme into the ELECTRE_IIIalgorithm as a new extended outranking method based on probabilistic linguistic term sets. The superior and inferior relationships between the schemes are obtained according to this method. To effectively study the multiattribute group decision problem in this article, the following symbols are adopted: the attribute set $G = \{G_1, G_2, \ldots, G_m\}$, scheme set $A = \{A_1, A_2, \ldots, A_n\}$, expert set $X = \left\{x_1, x_2, \ldots, x_q\right\}$, expert weight set $\omega_x = \left\{\omega_{x_1}, \omega_{x_2}, \ldots, \omega_{x_q}\right\}$, and the score $F_j(A_i)$, where $F$is the function defined in Definition 2above, $A_i$is the $i$-th scheme in the scheme set $A$, and $j$is the $j$-th attribute in the attribute set $G$.

In the extended outranking relation based on probabilistic linguistic term sets, the attribute threshold is used to determine the scheme level difference and the attribute threshold is divided into three thresholds: indifference threshold $q$, preference threshold $p$, and veto threshold $v$. The indifference threshold $q$occurs when schemes $A_i$and $A_k$are compared on a certain attribute $G_j$; if the score difference is less than $q$, that is, $A_i - A_k < q$, it can be considered that there is no difference between the two schemes on this attribute. The preference threshold $p$occurs when schemes $A_i$and $A_k$are compared on a certain attribute $G_j$; if the score difference is greater than $p$, that is, $A_i - A_k > p$, it can be considered that $A_i$strictly takes precedence over $A_k$on this attribute. The veto threshold $v$occurs when schemes $A_i$and $A_k$are compared on a certain attribute $G_j$; if the score difference is greater than or equal to $v$, that is, $A_i - A_k > v$, it can be considered that the attribute is higher than $A_k$on the $A_i$level. At the same time, multiattribute decision-making research needs to analyze the level relationship of the program and introduce the harmony degree, rejection degree, and credibility index to determine the merits and demerits of each attribute.

*Definition 3.* The harmony degree based on probabilistic linguistic term sets indicates the degree to which "scheme $A_i$is higher than $A_k$," and the calculation method is given by the following equation:

$$R(A_i, A_k) = \frac{1}{\sum_{j=1}^{n} w_j} \sum_{j=1}^{n} w_j r_j(A_i, A_k), \tag{3}$$

where for the degree that scheme $A_i$is better than $A_k$on attribute $G_j$, it is represented by $r_j(A_i, A_k)$, and the calculation method is given by the following equation:

$$r_j(A_i, A_k) = \begin{cases} 0, & \text{if } F_j(A_i) + p \le F_j(A_k), \\ 1, & \text{if } F_j(A_i) + q \ge F_j(A_k), \\ \dfrac{F_j(A_i) + p - F_j(A_k)}{p - q}, & \text{others.} \end{cases} \tag{4}$$

*Definition 4.* For the rejection degree of scheme $A_i$is better than that of $A_k$on attribute $G_j$, it is represented by $t_j(A_i, A_k)$, which is calculated by the following equation:

$$t_j(A_i, A_k) = \begin{cases} 0, & \text{if } F_j(A_i) + p \ge F_j(A_k), \\ 1, & \text{if } F_j(A_i) + v \le F_j(A_k), \\ \dfrac{F_j(A_k) - F_j(A_i) - p}{v - p}, & \text{others.} \end{cases} \tag{5}$$

*Definition 5.* The credibility index based on probabilistic linguistic term sets is expressed as the degree of trust that scheme $A_i$is higher than $A_k$in all attributes. It is necessary to comprehensively consider the degree that scheme $A_i$is better than $A_k$on attribute $G_j$and the rejection degree of scheme $A_i$is better than that of $A_k$on attribute $G_j$, and its size is defined by the following equation:

$$U(A_i, A_k) = \begin{cases} R(A_i, A_k), & \text{if } H = \varnothing, \\ R(A_i, A_k) * \prod_{j \in H} \dfrac{1 - t_j(A_i, A_k)}{1 - R(A_i, A_k)}, & \text{if } H \ne \varnothing, \end{cases} \tag{6}$$

where$H = \left\{ j \mid t_j(A_i, A_k) > R(A_i, A_k) \right\}$and $t_j(A_i, A_k)$is obtained by (5).

### 3.2. Expert Consensus Improvement Mechanism Based on Clustering Improvement

*3.2.1. Clustering Mechanism.* The K-means clustering method was introduced to effectively adjust the weight assigned by each expert and improve the consensus of the program. Given that some experts may be too vague and the overall consensus level could be too low, the K-means clustering method is used to optimize the whole group decision-making to increase the group decision-making consensus. Some previous works have used this clustering method [37–39]. This work clusters the credibility given by the extended outranking relation based on probabilistic linguistic term sets. According to the reliability given by this method, $U(A_i, A_k)$and $U(A_k, A_i)$are not necessarily equal, which can effectively reflect the extended outranking relation between the two schemes. General clustering based on linguistic term sets uses a symmetric matrix. Symmetry means that there is only a unique absolute mathematical relationship between the two schemes, which cannot effectively reflect the fuzziness of each scheme on a certain attribute. Using the asymmetric matrix of credibility given by the extended outranking relation based on probabilistic linguistic term sets to cluster, the information contained in it is richer, and the opinions of the clustered expert group have more reference value.

First, the overall degree of ambiguity and consensus of the opinions given by experts are measured and Euclidean is used to measure the distance between the credibility matrices of each expert. As the elements of the matrix obtained by the extended outranking relation based on probabilistic linguistic term sets are located between 0 and 1, it is not necessary to standardize and can be directly substituted into the K-means clustering method.

In the K-means clustering method, the standardized Euclidean distance between the two is given by the following equation:

$$\mathrm{d}E(x, y) = \sqrt{\sum_{k=1}^{n} (x_k - y_k)^2}. \tag{7}$$

For fuzzy preference relationships, consensus measures can be given at three different levels: pairs of alternatives, alternatives, and relations [40–43].

For each expert's credibility index matrix $(U(A_i, A_k))_{n\times n}$, calculate the Euclidean distance between each other, and cluster the credibility index matrices as follows:

Step 1: as the $n$ elements $U(A_i, A_i)$on the main diagonal of the obtained matrix are necessarily 1, it has no meaning for the group decision consensus calculation and is eliminated. To facilitate the operation of the K-means clustering method, for the matrix of the $\delta$ thexpert, the remaining matrix elements are successively listed as a row vector as shown in the following equation:

$$U_\delta = \{U_\delta(A_1, A_2), U_\delta(A_1, A_3), \ldots, U_\delta(A_1, A_n), U_\delta(A_2, A_1), \ldots, U_\delta(A_n, A_{n-1})\}. \tag{8}$$

The row vector has a total of $n \times (n-1)$elements. Let $\varphi = n \times (n-1)$, let $\xi$denote the $\xi$ thelement of the vector, and let $U_{\delta_1} = U_\delta(A_1, A_2), U_{\delta_2} = U_\delta(A_1, A_3), \ldots, U_{\delta_\varphi} = U_\delta(A_n, A_{n-1})$.

Step 2: set the value of $K$in the K-means clustering method based on experience. Then, $K$cluster centroid points are randomly generated, and the $t$-th cluster centroid points are expressed as shown in the following equation:

$$\Omega_t^0 = \left\{\Omega_{t_1}^0, \Omega_{t_2}^0, \ldots, \Omega_{t_\varphi}^0\right\}. \tag{9}$$

Step 3: calculate the distance from each point in the data set to the centroid of the cluster in which it is located and assign the data points to the closest cluster. For the $t$-th cluster, the normalized Euclidean distance formula for the $\delta$th expert to the centroid in the cluster (where $\eta$is the number of updates) is given by the following equation:

$$\mathrm{d}E(U_\delta, \Omega_t^\eta) = \sqrt{\sum_{l=1}^{\varphi} \left(U_{\delta_l} - \Omega_{t_l}^\eta\right)^2}. \tag{10}$$

Step 4: for each cluster, calculate the mean value of all points in the cluster. According to the calculation result, the mean value is taken as the new cluster centroid $\Omega_t^\eta = \left\{\Omega_{t_1}^\eta, \Omega_{t_2}^\eta, \ldots, \Omega_{t_\varphi}^\eta\right\}$, update $\eta$, and repeat step 3 until a stable cluster is generated.

Step 5: output of the final clustered result is given by the following equation:

$$\theta = \{\theta_1, \theta_2, \ldots, \theta_K\} \tag{11}$$

Step 6: end of process.

*3.2.2. Consensus Mechanism.* The consensus mechanism is to determine whether the overall consensus of the expert group is up to the standard, so as to determine whether the expert opinion or expert weights need to be adjusted. Through the linkage with the feedback mechanism, the overall consensus of the expert group can be improved, and the opinions of the group decision-making are more consistent so that the final program can be ranked. Set the cluster set as $\{\theta_1, \theta_2, \ldots, \theta_K\}$.

Step 1: for the clustering results obtained, a consensus mechanism is needed to obtain the overall consensus degree, and then it is determined whether the overall consensus degree meets the requirements. First, the expert weight vector is standardized:

For a finally obtained cluster $\theta_1$, there are a total of $\rho$experts. The weight of the entire cluster is $\omega_{\theta_1} = \sum_{i=1}^{\rho} \omega_{x_i}^{\theta_1}$; the normalized result of the weight vector of expert $i$is given by $\omega_{x_i}^{\theta_1} = (\omega_{x_i}^{\theta_1}/\omega_{\theta_1})$.

*Example 1.* Suppose that there are 10 experts $\{x_1, x_2, \ldots, x_{10}\}$, and the weights are $\{0.10.0.05, 0.2, 0.06, 0.03, 0.04, 0.17, 0.15, 0.09, 0.11\}$. Now cluster $\{x_2, x_4, x_6, x_8\}$into a cluster $\theta_1$. Then, within the cluster, the weight of $x_2$is $\omega_{x_2}^{\theta_1} = (0.05/0.05 + 0.06 + 0.04 + 0.15) = (1/6)$.

Step 2: the research consensus is divided into three stages to obtain the overall consensus [44, 45]:

Step 2.1: obtain the intracluster consensus matrix.

For each cluster, find a weighted average of the credibility of the two schemes within its cluster as shown in the following equation:

$$\overline{cd_{\theta_1}^l} = \frac{\sum_{i=1}^{\rho} \omega_{x_i}^{\theta_1} U_{x_{il}}}{\rho}. \tag{12}$$

From the above, the cluster consensus degree matrix can be obtained as follows:

$$cd_{\theta_1} = \begin{pmatrix} - & \overline{cd_{\theta_1}^1} & & \overline{cd_{\theta_1}^{n-2}} & \overline{cd_{\theta_1}^{n-1}} \\ \overline{cd_{\theta_1}^n} & - & \cdots & \overline{cd_{\theta_1}^{2n-3}} & \overline{cd_{\theta_1}^{2n-2}} \\ & \vdots & \ddots & \vdots & \\ \overline{cd_{\theta_1}^{n^2-3n+3}} & \overline{cd_{\theta_1}^{n^2-3n+4}} & & - & \overline{cd_{\theta_1}^{(n-1)(n-1)}} \\ \overline{cd_{\theta_1}^{n^2-2n+2}} & \overline{cd_{\theta_1}^{n^2-2n+3}} & \cdots & \overline{cd_{\theta_1}^{n(n-1)}} & - \end{pmatrix}. \tag{13}$$

Step 2.2: obtain the intercluster similarity matrix and aggregation.

For the two clusters $\theta_\alpha$and $\theta_\beta$, the intercluster similarity matrix element is calculated as given in [45] and shown in the following equation:

$$sm_{\alpha\beta}^{ik} = 1 - \left| cd_{\theta_\alpha}^{ik} - cd_{\theta_\beta}^{ik} \right|. \tag{14}$$

As $\mathrm{d}E(cd_{\theta_\alpha}^{ik}, cd_{\theta_\beta}^{ik}) = \mathrm{d}E(cd_{\theta_\beta}^{ik}, cd_{\theta_\alpha}^{ik})$, there are $(K(K-1)/2)$intercluster similarity matrices. For the convenience of identification, take the matrices of $\alpha < \beta$.

For the obtained $(K(K-1)/2)$intercluster similarity matrix, a specific aggregation function $\Gamma$is used to aggregate it as given in [45] and shown in the following equation:

$$cm_{ik} = \Gamma\left(sm_{\alpha\beta}^{ik}\right). \tag{15}$$

In general, the aggregation method is a weighted average, and the obtained elements $cm_{ik}$are arranged in a matrix to obtain an overall consensus degree matrix, that is, $cm$.

Step 2.3: obtain a general consensus degree matrix.

Level 1: the consensus degree between any two schemes: for the consensus degree relationship between any two schemes $(A_i, A_k)$, $cp_{ik}$, directly take the corresponding positional element $cm_{ik}$from matrix $cm$[44] as shown in the following equation:

$$cp_{ik} = cm_{ik}. \tag{16}$$

Level 2: the consensus level for a scheme: for a scheme $A_i$, its consensus degree is expressed by $ca_i$, which is defined as in [44] and shown by the following equation:

$$ca_i = \frac{\sum_{k=1, k\neq i}^{n} cp_{ik}}{n-1}. \tag{17}$$

Level 3: the overall consensus degree: it is expressed in terms of ocd, which is used to measure the degree of consensus of the entire group. The calculation method is shown in [44] and according to the following equation:

$$\mathrm{ocd} = \frac{\sum_{i=1}^{n} ca_i}{n}. \tag{18}$$

### 3.2.3. Feedback Mechanism

Step 1: the use of the aforementioned consensus mechanism can effectively obtain the consensus within the cluster and the overall consensus. Assume that the overall consensus degree of the presupposition is $\overline{\mathrm{ocd}}$. If the obtained $\mathrm{ocd} \geq \overline{\mathrm{ocd}}$, then it is the situation where the overall consensus degree meets the expected requirements; then go directly to the next step of the method. For $\mathrm{ocd} < \overline{\mathrm{ocd}}$, as a situation where the expected requirements are not met, a feedback mechanism is required. This feedback mechanism has been inspired by the literature [38] and has been improved.

Through the definition of the overall consensus degree $\overline{cd^l} = \sum_{t=1}^{K} \omega_{\theta_t} \overline{cd_{\theta_t}^l}$, it can be seen that, in all clusters for positions unchanged, lowering the expert weight of the clusters with farther distances can effectively improve the overall consensus.

Step 2: obtain the overall average consensus matrix.

Obtain the weighted average value of the elements of the cluster consensus matrix as shown in the following equation:

$$\overline{cd^l} = \sum_{t=1}^{K} \omega_{\theta_t} \overline{cd_{\theta_t}^l}. \tag{19}$$

Rank them in a matrix to obtain the overall average consensus matrix as given by the following equation:

$$cd = \begin{pmatrix} - & \overline{cd^{1}} & \cdots & \overline{cd^{n-2}} & \overline{cd^{n-1}} \\ \overline{cd^{n}} & - & \cdots & \overline{cd^{2n-3}} & \overline{cd^{2n-2}} \\ \vdots & & \ddots & & \vdots \\ \overline{cd^{n^2-3n+3}} & \overline{cd^{n^2-3n+4}} & & - & \overline{cd^{n^2-3n+4}} \\ \overline{cd^{n^2-2n+2}} & \overline{cd^{n^2-2n+3}} & \cdots & \overline{cd^{n(n-1)}} & - \end{pmatrix}. \tag{20}$$

As the overall average consensus matrix is the centroid of the entire cluster, for a series of clusters $\{\theta_1, \theta_2, \ldots, \theta_K\}$, a way can be adopted, similar to the consensus mechanism, to identify the distance from the centroid of the cluster, thus providing a theoretical basis to adjust the weight of each cluster.

Level 1: the degree of consensus between any two schemes obtained by the consensus relationship between any two schemes $(A_i, A_k)$between the group centroid and the cluster $\theta_t$centroid $gp_{ik}^t$is given in [45] and shown in the following equation:

$$gp_{ik}^t = 1 - \left|cd^{ik}, cd_{\theta_t}^{ik}\right|. \tag{21}$$

Level 2: the consensus degree for a scheme: for a scheme $A_i$, the degree of consensus between the group centroid and the cluster $\theta_t$centroid is expressed by $pa_i^t$, which is defined in [45] and shown in the following equation:

$$pa_i^t = \frac{\sum_{k=1,k\neq i}^{n} gp_{ik}^t}{n-1}. \tag{22}$$

Level 3: the overall consensus degree: it is expressed in $g$, and $g^{\theta_t}$is used to measure the degree of consensus between the entire group and the entire cluster $\theta_t$. The calculation method is given in [45] and shown in the following equation:

$$g^{\theta_t} = \frac{\sum_{i=1}^{n} pa_i^t}{n}. \tag{23}$$

Step 3: after obtaining the degree of consensus between each cluster and the group, the mechanism for adjusting the weight can be enabled for the clusters with too low consensus level, and part of the weight $\omega_{\theta_t}$of the experts is distributed to other cluster experts.

The weight adjustment system should be based on the opinion of the adjusted expert group that chooses which one or several expert clusters to obtain the weight of the adjustment part, $\Delta\omega_{\theta_t}$ $(0 < \Delta\omega_{\theta_t} < \omega_{\theta_t})$. The existing weight of the expert cluster $p$that obtained the new weight is given in [45] and shown in the following equation:

$$\omega_{\theta_p}^{r+1} = \omega_{\theta_p}^{r} + \mu \times \Delta\omega_{\theta_t}, \tag{24}$$

where $\mu$is the ratio of the weighted portion $\Delta\omega_{\theta_t}$to the expert cluster and $r$is the number of times the weight is adjusted.

At the same time, although there are some differences between these experts and the group opinions, there may be merits in their opinions. The expert group cannot completely disperse the weight of this part of the experts. The expert group should coordinate and set a weighted upper limit, such as 80%; that is, $\Delta\omega_{\theta_t} \leq 0.8\omega_{\theta_t}$, so that some opinions of the cluster experts can be retained. After the first adjustment, the existing weights of the clusters with the worst consensus are obtained as in the following equation:

$$\omega_{\theta_t}^{1} = \omega_{\theta_t}^{0} - \Delta\omega_{\theta_t}^{0}. \tag{25}$$

The cluster expert weight will not be affected by the subsequent weight adjustments.

After the first weight adjustment is completed, the degree of consensus is recalculated. If the standard is still not met, the weight of the expert group with the second lowest degree of consensus is adjusted, and it is assigned to other expert clusters (except for the clusters whose degree of consensus is worse). The above steps are repeated until the target consensus degree is reached.

After the $r$-th adjustment reaches the target consensus degree, for the expert $\delta$belonging to the cluster $\theta_t$, his expert weight at this time is given by the following equation:

$$\omega_{\delta}^{r} = \omega_{\theta_t}^{r}. \tag{26}$$

The resulting expert cluster weight vector is obtained from the following equation:

$$\omega = \left\{\omega_{\theta_1}^{r}, \omega_{\theta_2}^{r}, \ldots, \omega_{\theta_K}^{r}\right\}. \tag{27}$$

*3.3. Improved TOPSIS Ranking Method Based on the Net Credibility and Weight Adjustment.* After solving the expert consensus degree problem through clustering and adjusting the expert weight, the credibility index based on probabilistic linguistic term sets should be compared. This requires the introduction of the concepts of consistent credibility, inconsistent credibility, and net credibility to achieve a comparison of the merits and demerits of a scheme as compared with all other schemes.

*Definition 6.* The consistent credibility $\Phi^{+}(A_i)$is used to describe the total extent of scheme $A_i$over other schemes. The calculation formula is given by the following equation:

$$\Phi^{+}(A_i) = \sum_{k \neq i} U(A_i, A_k). \tag{28}$$

*Definition 7.* Inconsistent credibility $\Phi^{-}(A_i)$is used to describe the total degree of other schemes better than scheme $A_i$. The calculation formula is given by the following equation:

$$\Phi^{-}(A_i) = \sum_{k \neq i} U(A_k, A_i). \tag{29}$$

*Definition 8.* The net credibility $\Phi(A_i)$represents the difference between the scheme and other schemes under this attribute. The calculation formula is given by the following equation:

$$\Phi(A_i) = \Phi^{+}(A_i) - \Phi^{-}(A_i). \tag{30}$$

The larger $\Phi(A_i)$, the better scheme $A_i$as compared with other schemes, and the higher the ranking.

TOPSIS is a method that uses virtual "positive ideal target points" and "negative ideal target points" to achieve program ordering. There are many works in the literature which use TOPSIS and its improved methods to solve group decision problems [46–49]. The vector formed by all positive ideal target points is the positive ideal solution, and the vector formed by all negative ideal target points is the negative ideal solution. Take the deviation squared form to measure the square of the distance. After the square root is obtained, calculate the relative closeness degree to rank the schemes. The core principle of TOPSIS improvement in this work is to construct the initial matrix with net credibility as the relative closeness degree and introduce the adjusted decision-maker weights to obtain the group closeness matrix.

Step 1: First, use the net credibility as the relative closeness degree to build the initial matrix as given in the following equation:

$$X = \begin{bmatrix} \Phi_1(A_1) & \Phi_2(A_1) & \dots & \Phi_q(A_1) \\ \Phi_1(A_2) & \Phi_2(A_2) & \dots & \Phi_q(A_2) \\ \dots & \dots & \dots & \dots \\ \Phi_1(A_n) & \Phi_2(A_n) & \dots & \Phi_q(A_n) \end{bmatrix}. \tag{31}$$

Step 2: only normalized values can be guaranteed to lie in $[-1,1]$and thus are comparable. If the initial matrix does not lie in $[-1,1]$, the initial matrix values are normalized to obtain a group relative closeness normalization matrix. The standardization method is to mark $\Phi_s(A_i)$with $x_{is}$, $y_{is} = (x_{is}/\sqrt{\sum_{i=1}^{n} x_{is}^2})$as given by the following equation:

$$Y = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1q} \\ y_{21} & y_{22} & \cdots & y_{2q} \\ \cdots & \cdots & \cdots & \cdots \\ y_{n1} & y_{n2} & \cdots & y_{nq} \end{bmatrix}. \tag{32}$$

Step 3: in decision-making, in general, the experience, level, and status of different decision-makers are different, so the weight of their decision-making is generally different. This is to introduce the final decision-maker's decision weight vector $\omega = \{\omega_{x_1}^r, \omega_{x_2}^r, \dots, \omega_{x_q}^r\}$to obtain the group closeness degree matrix as shown in the following equation:

$$Z = \begin{bmatrix} \omega_{x_1}^r y_{11} & \omega_{x_2}^r y_{12} & \dots & \omega_{x_q}^r y_{1q} \\ \omega_{x_1}^r y_{21} & \omega_{x_2}^r y_{22} & \dots & \omega_{x_q}^r y_{2q} \\ \dots & \dots & \dots & \dots \\ \omega_{x_1}^r y_{n1} & \omega_{x_2}^r y_{n2} & \dots & \omega_{x_q}^r y_{nq} \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & \dots & z_{1q} \\ z_{21} & z_{22} & \dots & z_{2q} \\ \dots & \dots & \dots & \dots \\ z_{n1} & z_{n2} & \dots & z_{nq} \end{bmatrix}. \tag{33}$$

Step 4: according to the obtained group closeness matrix, the values of the positive and negative ideal solutions can be determined. Positive ideal solution is given by $z^{+} = \{z_1^{+}, z_2^{+}, \dots, z_n^{+}\}$, where, for each specific $s$, $z_s^{+} = \max\{z_{1s}, z_{2s}, \dots, z_{ns}\}$. Negative ideal solution is given by $z^{-} = \{z_1^{-}, z_2^{-}, \dots, z_n^{-}\}$, and, for each specific $s$, $z_s^{-} = \min\{z_{1s}, z_{2s}, \dots, z_{ns}\}$. According to the obtained positive and negative ideal solutions, the positive and negative ideal solution distances can be determined: $d^{+} = \sqrt{\sum_{j=1}^{m} (z_{is} - z_s^{+})^2}$and $d^{-} = \sqrt{\sum_{j=1}^{m} (z_{is} - z_s^{-})^2}$. Finally, the relative closeness degree can be calculated: $D = (d^{-}/d^{-} + d^{+})$. The larger $D$is, the better the scheme is and the higher the ranking is.

Through the foregoing method, the problem of multiattribute group decision-making is realized.

*3.4. Algorithm.* This section proposes a group decision-making method that combines extended outranking relation based on probabilistic linguistic term sets, clustering method, consensus mechanism, feedback mechanism, and improved TOPSIS ranking method based on the net credibility and weight adjustment. First, according to the probabilistic linguistic term set matrix, the credibility index based on probabilistic linguistic term sets is obtained through the extended outranking method based on probabilistic linguistic term sets. Using the clustering algorithm to cluster, calculating the overall consensus degree through the consensus mechanism, and using the feedback mechanism to improve the overall consensus degree, and finally substituting the weight vector to obtain the corresponding plan ranking result are the steps in the improved TOPSIS method.

Large-scale multiattribute group decision-making methods can effectively solve complex decision problems.

For this type of question, this article takes the following symbols:

There are $n$schemes, denoted as $A = \{A_1, A_2, \ldots, A_n\}$

There are $m$attributes, denoted as $G = \{G_1, G_2, \ldots, G_m\}$

There are $q$experts, recorded as $X = \{x_1, x_2, \ldots, x_q\}$

The initial expert weight vector is $\omega_x = \{\omega_{x_1}, \omega_{x_2}, \ldots, \omega_{x_q}\}$

A probabilistic linguistic term set $L(p) = \{L^{(k)}(p^{(k)}) | k = 1, 2, \ldots, \#L(p)\}$

The degree to which $A_i$is better than $A_k$on attribute $G_j$, denoted as $r_j(A_i, A_k)$

The degree of rejection scheme $A_i$is better than that of $A_k$on attribute $G_j$, denoted as $t_j(A_i, A_k)$

The credibility index of schemes $A_i$and $A_k$is denoted as $U(A_i, A_k)$

To effectively rank the schemes, the following steps are adopted:

Step 1: a probabilistic linguistic term set matrix (PLTSM) is given, denoted as $\mathrm{PLTSM}_{x_q} = [L(p)_{x_q}]_{n\times m}$

The score function is used to calculate the scoring values of the expert scores one by one, and the score function matrix $\mathrm{FM}_{x_q} = [F_j(L_{P_i})_{x_q}]_{n\times m}$is listed

Step 2: for each attribute, the indifference threshold $q$, preference threshold $p$, and veto threshold $v$are given by expert group discussion and related calculations, respectively.

In general, the threshold of each attribute is given by the function of the relevant historical research or the experience of the expert or related regulations and is empirical. There is generally a correlation between $q$, $p$, and $v$.

Step 3: the expert's score function matrix is given as $[F_j(L_{P_i})_{x_q}]_{n\times m}$. According to the score function, the degree of merits and demerits between the two schemes under each attribute is obtained by the extended outranking method based on probabilistic linguistic term sets, and the credibility index between the two schemes under each attribute is calculated according to the degree of the advantages and disadvantages.

Step 4: using the clustering method such as K-means, cluster the expert groups to obtain the final clustering $\theta = \{\theta_1, \theta_2, \ldots, \theta_K\}$.

Step 5: use the consensus mechanism to obtain the overall consensus degree ocd. If the obtained $\mathrm{ocd} > \overline{\mathrm{ocd}}$, then the overall consensus degree meets the expected requirements, and go directly to the improved TOPSIS method of Step 7 for group decision analysis; otherwise, go to Step 6.

Step 6: for the case of $\mathrm{ocd} < \overline{\mathrm{ocd}}$, as a situation where the expected requirements are not met, a feedback mechanism is required. The expert weights are adjusted according to the feedback mechanism described in Section 3.2.

Step 7: the ranking results are obtained using the improved TOPSIS ranking method based on the net credibility and weight adjustment in Section 3.3.

## 4. An Illustrative Example

PPP is a partnership between the government and private capital owners. As a way of building a public foundation project, it effectively reduces government financial pressures and allows private capital to participate in projects that were previously impossible or difficult to initiate. The public and private sectors share the risks and can also effectively reduce the risks faced by a single party.

PPP projects often have a pool of experts. Experts conduct a comprehensive evaluation of a project to effectively avoid project risks. However, experts often give highly subjective suggestions. By combining with the multiattribute group decision-making method based on probabilistic linguistic term sets proposed in this work, it can effectively avoid the subjective problem of the original expert evaluation.

There are five PPP projects $(A1, A2, A3, A4, A5)$that can be selected by a city investment company, depending on the company's funds and other factors. Under the existing priority given to the projects with a higher ranking, this requires a comprehensive ranking of the five projects. The following factors need to be considered comprehensively: the government support dimension ($G1$), the project risk dimension ($G2$), the project sustainability dimension ($G3$), the project benefit dimension ($G4$), and the macroeconomic dimension ($G5$). For convenience, the weights of the attributes are equal. The existing 20 PPP project experts $X = \{x_1, x_2, \ldots, x_{20}\}$have equal initial weights, and the weight vector is $\omega_x = ((1/20), (1/20), \ldots, (1/20))$. Twenty experts scored five attributes of the five projects, which were divided into $VL$, $L$, $M$, $H$, and $VH$(five-scale linguistic evaluation sets: $VL$—S0, L—S1, M—S2, H—S3, VH—S4), and can give a continuous score, such as $(VL, L)$.

Step 1: expert evaluation matrices are given, where the elements of the matrix row represent the scores of the attributes of the scheme, and the elements of the matrix column represent the scores of the schemes of the attribute. Due to the large number of elements, please refer to Appendix for details.

Due to space limitations, this article only shows the process of obtaining the credibility matrix of expert 1 and so on for the rest of the experts:

The score function matrix for all scores is obtained from equation (2):

$$\mathrm{FM}_{x_1} = \begin{pmatrix} 1.8835 & 1.0484 & 0.8084 & 0.8340 & 1.6200 \\ 0.5340 & 1.8430 & 1.3869 & 0.9148 & 0.6456 \\ 0.7085 & 1.3684 & 1.5978 & 1.0490 & 1.0407 \\ 1.1680 & 1.4758 & 0.6851 & 0.5967 & 1.4681 \\ 1.1897 & 1.2006 & 1.2705 & 0.7717 & 1.2264 \end{pmatrix}. \tag{34}$$

Step 2: the expert group gives the indifference threshold, the preference threshold, and the veto threshold for each attribute, which is common to all experts (see Table 1).

Step 3: from equations (4) and (5), the matrix of the pros and cons of the experts under different attributes is calculated according to the given threshold:

$(r_1(A_i, A_k))_{n\times n}$and $(t_1(A_i, A_k))_{n\times n}$of expert 1 under the $G1$attribute:

$$\begin{aligned} (r_1(A_i, A_k))_{n\times n} &= \begin{pmatrix} 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0 & 1.0000 & 1.0000 & 0 & 0 \\ 0 & 1.0000 & 1.0000 & 0.1621 & 0.0753 \\ 0 & 1.0000 & 1.0000 & 1.0000 & 0 \\ 0 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \end{pmatrix}, \\ (t_1(A_i, A_k))_{n\times n} &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1.0000 & 0 & 0 & 0.2680 & 0.3114 \\ 1.0000 & 0 & 0 & 0 & 0 \\ 0.4311 & 0 & 0 & 0 & 1.0000 \\ 0.3877 & 0 & 0 & 0 & 0 \end{pmatrix}. \end{aligned} \tag{35}$$

$(r_2(A_i, A_k))_{n\times n}$and $(t_2(A_i, A_k))_{n\times n}$of expert 1 under the $G2$attribute:

$$\begin{aligned} (r_2(A_i, A_k))_{n\times n} &= \begin{pmatrix} 1.0000 & 0 & 0.4002 & 0 & 1.0000 \\ 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 1.0000 & 0 & 1.0000 & 1.0000 & 1.0000 \\ 1.0000 & 0.1641 & 1.0000 & 1.0000 & 1.0000 \\ 1.0000 & 0 & 1.0000 & 0.6240 & 1.0000 \end{pmatrix}, \\ (t_2(A_i, A_k))_{n\times n} &= \begin{pmatrix} 0 & 0.9865 & 0 & 0.0685 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1866 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0.6060 & 0 & 0 & 0 \end{pmatrix}. \end{aligned} \tag{36}$$

$(r_3(A_i, A_k))_{n\times n}$and $(t_3(A_i, A_k))_{n\times n}$of expert 1 under the $G3$attribute:

Table 1: The indifference threshold, preference threshold, and veto threshold of the five attributes.

| Threshold | G1 | G2 | G3 | G4 | G5 |
|---|---|---|---|---|---|
| Indifference threshold | 0.25 | 0.2 | 0.3 | 0.25 | 0.25 |
| Preference threshold | 0.5 | 0.4 | 0.6 | 0.5 | 0.5 |
| Veto threshold | 1 | 0.8 | 1.2 | 1 | 1 |

$$(r_3(A_i, A_k))_{n\times n} = \begin{pmatrix} 1.0000 & 0.0716 & 0 & 1.0000 & 0.4598 \\ 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 1.0000 & 0 & 0 & 1.0000 & 0.0487 \\ 1.0000 & 1.0000 & 0.9087 & 1.0000 & 1.0000 \end{pmatrix},$$
$$(t_3(A_i, A_k))_{n\times n} = \begin{pmatrix} 0 & 0 & 0.3157 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1697 & 0.5213 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{37}$$

$(r_4(A_i, A_k))_{n\times n}$and $(t_4(A_i, A_k))_{n\times n}$of expert 1 under the $G4$attribute:

$$(r_4(A_i, A_k))_{n\times n} = \begin{pmatrix} 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 1.0000 & 0.7273 & 0.1906 & 1.0000 & 1.0000 \\ 1.0000 & 1.0000 & 0.8908 & 1.0000 & 1.0000 \end{pmatrix},$$
$$(t_4(A_i, A_k))_{n\times n} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{38}$$

$(r_5(A_i, A_k))_{n\times n}$and $(t_5(A_i, A_k))_{n\times n}$of expert 1 under the $G5$attribute:

$$
\left(r_5\left(A_i, A_k\right)\right)_{n\times n} = \begin{pmatrix} 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0 & 1.0000 & 0.4195 & 0 & 0 \\ 0 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 1.0000 & 0.7273 & 0.1906 & 1.0000 & 1.0000 \\ 0.4257 & 1.0000 & 0.8908 & 1.0000 & 1.0000 \end{pmatrix},
$$
$$
\left(t_5\left(A_i, A_k\right)\right)_{n\times n} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0.9488 & 0 & 0 & 0.6451 & 0.1616 \\ 0.1586 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{39}
$$

From equation (3), $\left(R_1\left(A_i, A_k\right)\right)_{n\times n}$of expert 1 can be obtained:

$$
\left(R_1\left(A_i, A_k\right)\right)_{n\times n} = \begin{pmatrix} 1.0000 & 0.6143 & 0.6800 & 0.8000 & 0.8920 \\ 0.6000 & 1.0000 & 0.8839 & 0.6000 & 0.6000 \\ 0.6000 & 0.8000 & 1.0000 & 0.6905 & 0.8151 \\ 0.8000 & 0.5783 & 0.6381 & 1.0000 & 0.8097 \\ 0.6851 & 0.8000 & 0.9599 & 0.9248 & 1.0000 \end{pmatrix}. \tag{40}
$$

From equation (6), $\left(U_1\left(A_i, A_k\right)\right)_{n\times n}$of expert 1 can be obtained:

$$
U_1 = \left(U_1\left(A_i, A_k\right)\right)_{n\times n} = \begin{pmatrix} 1.0000 & 0.0215 & 0.6800 & 0.8000 & 0.8920 \\ 0 & 1.0000 & 0.8839 & 0.5323 & 0.6000 \\ 0 & 0.8000 & 1.0000 & 0.6905 & 0.8151 \\ 0.8000 & 0.5783 & 0.6381 & 1.0000 & 0.8097 \\ 0.6851 & 0.8000 & 0.9599 & 0.9248 & 1.0000 \end{pmatrix}. \tag{41}
$$

The same method is used to obtain the credibility matrices given by other experts under the five attributes.

Step 4: after obtaining the credibility matrix of the 20 experts, remove the main diagonal as described in Step 1 of the classification mechanism, retain other elements and group them into credibility vectors one by one, and cluster. For example, for $U_1 = \left(U_1\left(A_i, A_k\right)\right)_{n\times n}$, $U_1 =$ {0.0215,0.6800,0.8000,0.8920,0.0000,0.8839,0.5323,0.6000,0.0000,0.8000,0.6905,0.8151,0.8000,0.5783,0.6381, 0.8907,0.6851,0.8000,0.9599,0.9248} (see Table 2).

Clustering is performed by equations (7)–(11), and $K$in the K-means cluster is set to 5, and the result is clustered:

$$
\begin{aligned}
\theta_1 &= \{U_1, U_2, U_4, U_6, U_7, U_{11}, U_{16}, U_{17}\}, \\
\theta_2 &= \{U_3, U_{13}, U_{14}\}, \\
\theta_3 &= \{U_9, U_{10}, U_{15}, U_{19}\}, \\
\theta_4 &= \{U_5, U_{12}, U_{20}\}, \\
\theta_5 &= \{U_8, U_{18}\}.
\end{aligned} \tag{42}
$$

From equations (12)–(14) for clusters 1 and 2, the intercluster similarity matrix is as follows:

Table 2: The element value of the credibility vector.

| $\xi$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $U_1$ | 0.0215 | 0.6800 | 0.8000 | 0.8920 | 0.0000 | 0.8839 | 0.5323 | 0.6000 | 0.0000 | 0.8000 | 0.6905 | 0.8151 | 0.8000 | 0.5783 | 0.6381 | 0.8097 | 0.6851 | 0.8000 | 0.9599 | 0.9248 |
| $U_2$ | 0.4962 | 0.9278 | 0.9293 | 0.8708 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9137 | 0.5484 | 0.9613 | 1.0000 | 0.8000 | 0.2264 | 1.0000 | 1.0000 | 0.9451 | 0.5911 | 1.0000 | 0.9175 |
| $U_3$ | 0.8000 | 0.8679 | 0.8163 | 0.6000 | 0.6000 | 0.7446 | 0.4000 | 0.0000 | 0.9834 | 0.8862 | 0.7715 | 0.3371 | 1.0000 | 0.8000 | 0.9035 | 0.6000 | 0.8989 | 0.8000 | 0.8000 | 0.6857 |
| $U_4$ | 0.8000 | 1.0000 | 1.0000 | 1.0000 | 0.8919 | 1.0000 | 0.8545 | 0.8201 | 0.8178 | 0.7608 | 0.6451 | 0.7963 | 1.0000 | 0.8000 | 1.0000 | 1.0000 | 0.7462 | 0.4591 | 0.8799 | 0.6957 |
| $U_5$ | 0.0000 | 0.8407 | 0.8128 | 0.4101 | 0.6135 | 1.0000 | 0.0000 | 1.0000 | 0.7365 | 0.0000 | 0.4244 | 0.8004 | 0.9815 | 0.0000 | 0.8360 | 0.3636 | 0.4467 | 0.0000 | 1.0000 | 0.0000 |
| $U_6$ | 1.0000 | 1.0000 | 0.8000 | 1.0000 | 0.5485 | 0.6580 | 0.5377 | 0.9054 | 0.2878 | 0.8000 | 0.7390 | 0.8167 | 0.8000 | 0.8000 | 0.9739 | 0.8000 | 0.6966 | 1.0000 | 0.8292 | 0.7156 |
| $U_7$ | 0.8000 | 0.8595 | 0.1608 | 0.8000 | 0.7925 | 0.7856 | 0.7959 | 0.8000 | 0.8000 | 0.8000 | 0.8000 | 0.7449 | 0.8256 | 0.8272 | 1.0000 | 0.7901 | 1.0000 | 0.7924 | 0.8000 | 0.0000 |
| $U_8$ | 0.6000 | 0.6622 | 0.8000 | 0.9334 | 0.0000 | 0.6396 | 0.0000 | 0.7734 | 0.7893 | 0.9073 | 0.7299 | 0.8251 | 0.8605 | 0.7747 | 0.7900 | 0.8000 | 0.0000 | 0.3499 | 0.1959 | 0.0000 |
| $U_9$ | 0.6333 | 0.0000 | 0.6569 | 0.8000 | 0.8000 | 0.0000 | 0.8000 | 0.8000 | 1.0000 | 0.8000 | 0.7723 | 1.0000 | 0.7233 | 0.8000 | 0.0000 | 0.7475 | 1.0000 | 0.7000 | 0.0000 | 0.4184 |
| $U_{10}$ | 1.0000 | 0.0000 | 0.8000 | 0.8429 | 0.7528 | 0.0650 | 0.0000 | 0.8000 | 0.7654 | 0.8405 | 0.1960 | 1.0000 | 1.0000 | 1.0000 | 0.1398 | 0.9160 | 0.8156 | 1.0000 | 0.1981 | 0.2982 |
| $U_{11}$ | 0.9901 | 1.0000 | 0.7315 | 0.8000 | 0.8707 | 1.0000 | 0.8344 | 0.8000 | 0.7047 | 0.7201 | 0.8208 | 0.6092 | 0.5782 | 0.7457 | 0.9998 | 0.6564 | 0.9663 | 1.0000 | 1.0000 | 0.8000 |
| $U_{12}$ | 0.6688 | 1.0000 | 0.8206 | 0.7371 | 0.0000 | 0.9040 | 0.7341 | 0.6067 | 0.7002 | 0.5157 | 0.6000 | 0.7509 | 0.6873 | 0.0000 | 0.8735 | 0.2061 | 0.0000 | 0.1697 | 0.8006 | 0.7585 |
| $U_{13}$ | 0.8590 | 0.9498 | 0.6000 | 0.0000 | 0.7461 | 0.8550 | 0.4226 | 0.0000 | 0.8000 | 0.9882 | 0.8000 | 0.0000 | 0.7732 | 0.6122 | 0.9544 | 0.0040 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| $U_{14}$ | 0.0534 | 0.8081 | 0.0000 | 0.0000 | 0.8000 | 1.0000 | 0.9210 | 0.6000 | 0.8000 | 0.8000 | 0.8000 | 0.6000 | 0.7321 | 0.9902 | 0.8036 | 0.7674 | 0.8526 | 0.5417 | 0.7013 | 0.7479 |
| $U_{15}$ | 0.9816 | 0.8000 | 0.8040 | 1.0000 | 0.7505 | 0.0000 | 0.5869 | 1.0000 | 0.8000 | 0.9995 | 0.6815 | 1.0000 | 0.8000 | 0.8656 | 0.7520 | 0.9179 | 0.2947 | 0.8205 | 0.0000 | 0.0000 |
| $U_{16}$ | 0.8000 | 1.0000 | 0.7418 | 0.8000 | 0.6718 | 0.9448 | 0.2423 | 0.8000 | 0.6192 | 0.8000 | 0.4629 | 0.6467 | 0.8000 | 0.7083 | 1.0000 | 0.8000 | 0.7598 | 1.0000 | 1.0000 | 0.5300 |
| $U_{17}$ | 1.0000 | 0.7797 | 1.0000 | 0.8276 | 0.7962 | 0.3290 | 0.9078 | 0.4158 | 0.5937 | 0.6000 | 0.8000 | 0.6000 | 0.4711 | 0.8000 | 0.5707 | 0.6363 | 0.8313 | 0.8000 | 0.8801 | 1.0000 |
| $U_{18}$ | 0.9452 | 0.3841 | 0.0000 | 0.0000 | 0.0000 | 0.5165 | 0.0000 | 0.4139 | 0.8000 | 1.0000 | 0.4980 | 0.8000 | 0.8000 | 0.9623 | 0.6000 | 0.0000 | 0.0000 | 1.0000 | 0.5316 | 0.0000 |
| $U_{19}$ | 0.8000 | 0.0000 | 0.6000 | 0.6819 | 0.6634 | 0.2854 | 0.9656 | 0.9498 | 1.0000 | 0.8622 | 0.8000 | 0.8000 | 0.0000 | 0.9389 | 0.3175 | 0.8000 | 0.8748 | 0.9285 | 0.5471 | 1.0000 |
| $U_{20}$ | 0.7232 | 0.7341 | 1.0000 | 0.9381 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9822 | 0.7387 | 1.0000 | 1.0000 | 0.3060 | 0.0387 | 0.1813 | 0.9740 | 0.0000 | 0.0380 | 0.5338 | 1.0000 |

$$sm_{12} = \begin{pmatrix} - & 0.8323 & 0.9694 & 0.7017 & 0.3262 \\ 0.9811 & - & 0.9586 & 0.8681 & 0.4323 \\ 0.7310 & 0.8372 & - & 0.9494 & 0.5587 \\ 0.9243 & 0.8849 & 0.9893 & - & 0.6456 \\ 0.9116 & 0.9752 & 0.9151 & 0.8868 & - \end{pmatrix}. \quad (43)$$

By analogy, other intercluster similarity matrices are obtained. The weight of each cluster is as follows:

$$\begin{aligned} \omega_{\theta_1}^0 &= 0.4, \\ \omega_{\theta_2}^0 &= 0.15, \\ \omega_{\theta_3}^0 &= 0.20, \\ \omega_{\theta_4}^0 &= 0.15, \\ \omega_{\theta_5}^0 &= 0.10. \end{aligned} \quad (44)$$

The following are available from equations (15)–(18): $ca_1 = 0.7304$, $ca_2 = 0.7102$, $ca_3 = 0.8092$, $ca_4 = 0.7417$, $ca_5 = 0.6427$, and ocd = 0.7268.

Let $\overline{\text{ocd}} = 0.75$; then ocd < 0.75, triggering the feedback mechanism.

The following are available from equations (19)–(23), the degrees of consensus for all clusters and groups: $g^{\theta_1} = 0.8920$, $g^{\theta_2} = 0.8020$, $g^{\theta_3} = 0.7931$, $g^{\theta_4} = 0.8088$, and $g^{\theta_5} = 0.7631$, where $g^{\theta_1} > g^{\theta_4} > g^{\theta_2} > g^{\theta_3} > g^{\theta_5}$.

The weights of the lowest cluster $\theta_5$are adjusted by formulas (24)–(25). After consultation with the expert group, $\mu_{\max} = 0.8$is set. After the adjusted $\theta_5$experts negotiate, it is decided to give $\theta_1$ $50\%\omega_{\theta_5}^0$weight and give $\theta_4$ $30\%\omega_{\theta_5}^0$weight; then the weight of each cluster is as follows:

$$\begin{aligned} \omega_{\theta_1}^0 &= 0.45, \\ \omega_{\theta_2}^0 &= 0.15, \\ \omega_{\theta_3}^0 &= 0.2, \\ \omega_{\theta_4}^0 &= 0.18, \\ \omega_{\theta_5}^0 &= 0.02. \end{aligned} \quad (45)$$

Reaggregate the calculations to arrive at a new consensus: $ca_1 = 0.7403$, $ca_2 = 0.7612$, $ca_3 = 0.7993$, $ca_4 = 0.7291$, $ca_5 = 0.6717$, and ocd = 0.7403.

Now, ocd < 0.75, and it triggers the feedback mechanism again.

Consensus of each cluster and group is as follows: $g^{\theta_1} = 0.8889$, $g^{\theta_2} = 0.7886$, $g^{\theta_3} = 0.7877$, $g^{\theta_4} = 0.8169$, and $g^{\theta_5} = 0.7449$, where $g^{\theta_1} > g^{\theta_4} > g^{\theta_2} > g^{\theta_3} > g^{\theta_5}$.

As $\theta_5$has been adjusted, adjust the weight of the second smallest cluster $\theta_4$. After the adjusted $\theta_4$experts had negotiated, it was decided to give $\theta_1$ $40\%\omega_{\theta_4}^1$weight and give $\theta_3$ $40\%\omega_{\theta_4}^1$weight, at which time the cluster weights are as follows:

$$\begin{aligned} \omega_{\theta_1}^0 &= 0.53, \\ \omega_{\theta_2}^0 &= 0.15, \\ \omega_{\theta_3}^0 &= 0.04, \\ \omega_{\theta_4}^0 &= 0.26, \\ \omega_{\theta_5}^0 &= 0.02. \end{aligned} \quad (46)$$

Reaggregate calculations to arrive at a new consensus: $ca_1 = 0.7731$, $ca_2 = 0.8004$, $ca_3 = 0.7963$, $ca_4 = 0.7285$, $ca_5 = 0.6666$, and ocd = 0.7530.

Now, ocd > 0.75does not trigger the feedback mechanism. The weights of the experts obtained by formulas (26) and (27) are as follows:

$$\begin{aligned} \omega_{x_1} &= 0.06625, \\ \omega_{x_2} &= 0.6625, \\ \omega_{x_3} &= 0.05, \\ \omega_{x_4} &= 0.06625, \\ \omega_{x_5} &= 0.08667, \\ \omega_{x_6} &= 0.06625, \\ \omega_{x_7} &= 0.01, \\ \omega_{x_8} &= 0.01, \\ \omega_{x_9} &= 0.01, \\ \omega_{x_{10}} &= 0.06625, \\ \omega_{x_{11}} &= 0.08667, \\ \omega_{x_{12}} &= 0.05, \\ \omega_{x_{13}} &= 0.05, \\ \omega_{x_{14}} &= 0.05, \\ \omega_{x_{15}} &= 0.01, \\ \omega_{x_{16}} &= 0.06625, \\ \omega_{x_{17}} &= 0.06625, \\ \omega_{x_{18}} &= 0.01, \\ \omega_{x_{19}} &= 0.01, \\ \omega_{x_{20}} &= 0.08667. \end{aligned} \quad (47)$$

The net credibility of each scheme is calculated from equations (28)–(30). Finally, the constructed relative closeness degree matrix is used to find the distance and relative closeness degree from equations (31)–(33) as summarized in Table 3.

TABLE 3: The distance from each scheme to the ideal solution.

| Distance | $d^+$ | $d^-$ | $D = (d^- / d^- + d^+)$ |
|---|---|---|---|
| $A1$ | 0.0681 | 0.1152 | 0.6284 |
| $A2$ | 0.0878 | 0.0873 | 0.4987 |
| $A3$ | 0.1349 | 0.0335 | 0.1989 |
| $A4$ | 0.0874 | 0.0910 | 0.5103 |
| $A5$ | 0.0887 | 0.0945 | 0.5157 |

In summary, the program ranks $A1 > A5 > A4 > A2 > A3$.

## 5. Comparison and Discussion

The probabilistic linguistic term set effectively explains the degree of hesitation and probability distribution of expert opinions in the nonideal case, which can make the linguistic information flexibly expressed. Through combination with TOPSIS, VIKOR [50], and other methods, the method proposed in this article can effectively solve the problem of multiattribute group decision-making based on the probabilistic linguistic term sets.

In group decision-making, there may be situations where the opinions of experts are extremely conflicted, which requires an increase in consensus and consistency. There are currently several ways to increase consensus, including adjusting expert weights or adjusting expert preferences. How to adjust the relationship of expert preferences and the distribution of expert weights is a question worth exploring. In [44], the authors used the clustering algorithm to divide the expert group into three clusters. After clustering, the preference of the adjustment expert was selected. After the expert preference was adjusted cluster by cluster, clustering was rerun. The consensus degree increased from 0.6925 to 0.7861 and then increased to 0.7970; thus, the consensus degree basically met the requirements. In [44], choosing to adjust the preferences of experts and improving the consensus degree through consultation and exchange methods may have problems in group decision-making operations that require multiple consultations. Repeated consultations will inevitably take time and effort, and it is not as convenient to adjust the weight as the method given in this article.

In [45], after the authors used K-means clustering, the expert group was divided into six clusters. In contrast to the method proposed in this article, the method given in [45] directly withdraws the clusters with poor consensus and calculates the weights of these experts to other expert clusters, thus improving the group consensus. If the cluster of experts is directly withdrawn, the opinions of these experts are actually meaningless and have problems. In practice, there may be some opinions of this group of experts which cannot be completely ignored. However, the weight adjustment and improvement made in this paper have a maximum adjustment threshold, which effectively avoids the problem of total loss of the opinions of some experts as seen in [45].

## 6. Conclusions

This work proposes integrating a new extended outranking-TOPSIS method with probabilistic linguistic term sets for multiattribute group decision-making. First, for the application of probabilistic linguistic term sets in multiattribute group decision-making problems, a new extended outranking relation based on probabilistic linguistic term sets is proposed to determine the superior and inferior relationships between the schemes. Second, according to the expert opinions obtained, the expert consensus improvement mechanism based on clustering improvement is used to determine and improve the consensus degree. Finally, an improved TOPSIS ranking method based on the net credibility and weight adjustment is proposed to rank the schemes. This article also provides an application case of PPP to illustrate the method proposed in this article.

The theory and calculation of the extended outranking relation based on probabilistic linguistic term sets proposed in this work are not complicated, which is convenient for practical application. Furthermore, it solves the problem of ignoring the differences in the degree of hesitation which exists in similar outranking methods. The proposed expert consensus improvement mechanism based on clustering improvement can also effectively respect the opinions of marginal expert groups, respect the concept of group decision-making, and facilitate the development of group decision-making, and it can be effectively applied to various group decision-making application problems. By improving the TOPSIS method, it can be effectively applied to the ranking of schemes based on probabilistic linguistic term sets.

This study requires some improvements. Keeping expert opinions from too much influence of the model is a point that needs to be paid attention to in the consensus study of multiattribute group decision-making models. The K-means clustering method is used in this article, and $K$ in K-means clustering method is set manually as a hyperparameter. Due to the relative subjectivity of manual settings, some expert opinions that should be maintained may be affected. In the future, methods such as grid search will be used to traverse each $K$ to find the situation with the largest initial group decision consensus degree, so as to minimize the extent of expert weight adjustment transfer. It helps to maintain the opinions of the expert group, making the analysis results closer to the initial opinion of the expert group.

At present, there are few studies regarding the application of PLTS in large-scale multiattribute group decision-making problems. In future work, for the combination of PLTS and large-scale multiattribute group decision-making, the following research can be carried out. First, we can study the use of different operators and distance measurement methods to reduce information loss in the large-scale multiattribute group decision-making process. For the study of consensus mechanisms, in addition to clustering, there are other algorithms that can be utilized to improve expert weights and expert preference relationships. In addition, more optimization algorithms or a combination of multiple optimization algorithms, such as other machine learning methods, can be introduced to make large-scale multiattribute group decision-making more comprehensive and more accurate.

## Appendix

All the probabilistic linguistic term sets in the illustrative example in this article are listed as follows:

$\mathrm{PLTSM}_{x_1}$

$$\mathrm{PLTSM}_{x_2}=\begin{pmatrix} \{S_0(0.05),S_1(0.35),S_2(0.15),S_3(0.05),S_4(0.15)\} & \{S_0(0.25),S_1(0.1),S_2(0.05),S_3(0.05),S_4(0.35)\} & \{S_0(0.15),S_1(0.05),S_2(0.35),S_3(0.2),S_4(0.2)\} & \{S_0(0.2),S_1(0.2),S_2(0.05),S_3(0.05),S_4(0.2)\} & \{S_0(0.05),S_1(0.3),S_2(0.3),S_3(0.15),S_4(0.2)\} \\ \{S_0(0.05),S_1(0.1),S_2(0.25),S_3(0.35),S_4(0.1)\} & \{S_0(0.2),S_1(0.25),S_2(0.2),S_3(0.2),S_4(0.05)\} & \{S_0(0.1),S_1(0.1),S_2(0.45),S_3(0.15),S_4(0.1)\} & \{S_0(0.05),S_1(0.15),S_2(0.25),S_3(0.2),S_4(0.1)\} & \{S_0(0.05),S_1(0.05),S_2(0.3),S_3(0.2),S_4(0.35)\} \\ \{S_0(0.1),S_1(0.15),S_2(0.25),S_3(0.1),S_4(0.05)\} & \{S_0(0.2),S_1(0.35),S_2(0.2),S_3(0.2),S_4(0.05)\} & \{S_0(0.2),S_1(0.25),S_2(0.1),S_3(0.3),S_4(0.05)\} & \{S_0(0.05),S_1(0.3),S_2(0.2),S_3(0.35),S_4(0.1)\} & \{S_0(0.25),S_1(0.05),S_2(0.2),S_3(0.25),S_4(0.25)\} \\ \{S_0(0.1),S_1(0.1),S_2(0.1),S_3(0.2),S_4(0.15)\} & \{S_0(0.25),S_1(0.05),S_2(0.15),S_3(0.1),S_4(0.1)\} & \{S_0(0.05),S_1(0.3),S_2(0.25),S_3(0.25),S_4(0.1)\} & \{S_0(0.15),S_1(0.1),S_2(0.3),S_3(0.1),S_4(0.05)\} & \{S_0(0.3),S_1(0.15),S_2(0.05),S_3(0.2),S_4(0.2)\} \\ \{S_0(0.1),S_1(0.3),S_2(0.35),S_3(0.1),S_4(0.1)\} & \{S_0(0.3),S_1(0.2),S_2(0.2),S_3(0.05),S_4(0)\} & \{S_0(0.1),S_1(0.25),S_2(0.1),S_3(0.25),S_4(0.2)\} & \{S_0(0.15),S_1(0.05),S_2(0.15),S_3(0.3),S_4(0.15)\} & \{S_0(0.25),S_1(0.15),S_2(0.05),S_3(0.15),S_4(0.05)\} \end{pmatrix},$$

$$\mathrm{PLTSM}_{x_3}=\begin{pmatrix} \{S_0(0.2),S_1(0.25),S_2(0.1),S_3(0.25),S_4(0.15)\} & \{S_0(0.3),S_1(0.05),S_2(0.1),S_3(0.05),S_4(0.15)\} & \{S_0(0.1),S_1(0.15),S_2(0.2),S_3(0.3),S_4(0.05)\} & \{S_0(0.2),S_1(0.15),S_2(0.05),S_3(0.3),S_4(0.25)\} & \{S_0(0.2),S_1(0.05),S_2(0.2),S_3(0.25),S_4(0.15)\} \\ \{S_0(0.3),S_1(0.1),S_2(0.25),S_3(0.25),S_4(0.05)\} & \{S_0(0.1),S_1(0.35),S_2(0.1),S_3(0.2),S_4(0.2)\} & \{S_0(0.35),S_1(0.35),S_2(0.1),S_3(0.15),S_4(0.05)\} & \{S_0(0.05),S_1(0.1),S_2(0.2),S_3(0.35),S_4(0.2)\} & \{S_0(0.2),S_1(0.3),S_2(0.05),S_3(0.1),S_4(0.35)\} \\ \{S_0(0.35),S_1(0.25),S_2(0.15),S_3(0.2),S_4(0)\} & \{S_0(0.2),S_1(0.25),S_2(0.1),S_3(0.15),S_4(0.15)\} & \{S_0(0.2),S_1(0.05),S_2(0.1),S_3(0.3),S_4(0.25)\} & \{S_0(0.05),S_1(0.25),S_2(0.15),S_3(0.25),S_4(0)\} & \{S_0(0.05),S_1(0.25),S_2(0.2),S_3(0.25),S_4(0.05)\} \\ \{S_0(0.1),S_1(0.15),S_2(0.1),S_3(0.15),S_4(0.05)\} & \{S_0(0.25),S_1(0.3),S_2(0.05),S_3(0.1),S_4(0.05)\} & \{S_0(0.1),S_1(0.05),S_2(0.1),S_3(0.05),S_4(0.05)\} & \{S_0(0.25),S_1(0.2),S_2(0.15),S_3(0.25),S_4(0)\} & \{S_0(0.15),S_1(0.25),S_2(0.05),S_3(0.05),S_4(0.3)\} \\ \{S_0(0.3),S_1(0.2),S_2(0.15),S_3(0.15),S_4(0.1)\} & \{S_0(0),S_1(0.35),S_2(0.35),S_3(0.05),S_4(0)\} & \{S_0(0),S_1(0.05),S_2(0.25),S_3(0.2),S_4(0.35)\} & \{S_0(0.35),S_1(0.3),S_2(0.05),S_3(0),S_4(0)\} & \{S_0(0.15),S_1(0.1),S_2(0.15),S_3(0.1),S_4(0.2)\} \end{pmatrix},$$

$$\mathrm{PLTSM}_{x_4}=\begin{pmatrix} \{S_0(0.2),S_1(0),S_2(0),S_3(0.3),S_4(0.25)\} & \{S_0(0.2),S_1(0.25),S_2(0.2),S_3(0.1),S_4(0.1)\} & \{S_0(0.15),S_1(0.3),S_2(0.35),S_3(0.2),S_4(0)\} & \{S_0(0.1),S_1(0.15),S_2(0.35),S_3(0.35),S_4(0.05)\} & \{S_0(0.25),S_1(0),S_2(0),S_3(0.2),S_4(0.35)\} \\ \{S_0(0.15),S_1(0.3),S_2(0.1),S_3(0.05),S_4(0.2)\} & \{S_0(0.15),S_1(0.3),S_2(0.1),S_3(0.05),S_4(0.35)\} & \{S_0(0.05),S_1(0.45),S_2(0.15),S_3(0.1),S_4(0.15)\} & \{S_0(0.05),S_1(0.35),S_2(0.3),S_3(0.1),S_4(0.1)\} & \{S_0(0.35),S_1(0.25),S_2(0.15),S_3(0.05),S_4(0.35)\} \\ \{S_0(0.15),S_1(0.3),S_2(0.1),S_3(0.05),S_4(0.2)\} & \{S_0(0.3),S_1(0.15),S_2(0.1),S_3(0.05),S_4(0.3)\} & \{S_0(0.3),S_1(0.25),S_2(0.05),S_3(0.1),S_4(0)\} & \{S_0(0.05),S_1(0.25),S_2(0.15),S_3(0.2),S_4(0.35)\} & \{S_0(0.25),S_1(0.25),S_2(0.25),S_3(0.1),S_4(0.1)\} \\ \{S_0(0.05),S_1(0.35),S_2(0.1),S_3(0.3),S_4(0.1)\} & \{S_0(0.15),S_1(0.25),S_2(0.25),S_3(0.1),S_4(0.05)\} & \{S_0(0.15),S_1(0.1),S_2(0.15),S_3(0.05),S_4(0.2)\} & \{S_0(0.05),S_1(0.15),S_2(0.35),S_3(0.1),S_4(0.25)\} & \{S_0(0.2),S_1(0.15),S_2(0.05),S_3(0.05),S_4(0.25)\} \\ \{S_0(0.15),S_1(0.05),S_2(0.35),S_3(0.3),S_4(0.05)\} & \{S_0(0.25),S_1(0.25),S_2(0.1),S_3(0.15),S_4(0.1)\} & \{S_0(0.2),S_1(0.05),S_2(0.2),S_3(0.05),S_4(0.15)\} & \{S_0(0.3),S_1(0.15),S_2(0.25),S_3(0),S_4(0.1)\} & \{S_0(0.25),S_1(0.35),S_2(0.05),S_3(0.05),S_4(0.25)\} \end{pmatrix},$$

$$\mathrm{PLTSM}_{x_5}=\begin{pmatrix} \{S_0(0.35),S_1(0.1),S_2(0.3),S_3(0.05),S_4(0.15)\} & \{S_0(0.15),S_1(0.3),S_2(0.3),S_3(0.05),S_4(0.05)\} & \{S_0(0),S_1(0.35),S_2(0.3),S_3(0.25),S_4(0)\} & \{S_0(0.1),S_1(0.15),S_2(0.05),S_3(0.15),S_4(0.1)\} & \{S_0(0.05),S_1(0.2),S_2(0.3),S_3(0.2),S_4(0.1)\} \\ \{S_0(0),S_1(0.15),S_2(0.3),S_3(0.15),S_4(0.2)\} & \{S_0(0.05),S_1(0.05),S_2(0.35),S_3(0.2),S_4(0.05)\} & \{S_0(0.3),S_1(0.05),S_2(0.2),S_3(0.25),S_4(0.2)\} & \{S_0(0.05),S_1(0.35),S_2(0.1),S_3(0.2),S_4(0.15)\} & \{S_0(0.1),S_1(0.25),S_2(0.4),S_3(0.05),S_4(0.15)\} \\ \{S_0(0.2),S_1(0),S_2(0.3),S_3(0.05),S_4(0.05)\} & \{S_0(0.25),S_1(0.25),S_2(0.25),S_3(0.1),S_4(0.1)\} & \{S_0(0.15),S_1(0.2),S_2(0.1),S_3(0.15),S_4(0.3)\} & \{S_0(0.2),S_1(0.2),S_2(0.15),S_3(0.15),S_4(0.25)\} & \{S_0(0.1),S_1(0.25),S_2(0.25),S_3(0.1),S_4(0.25)\} \\ \{S_0(0.3),S_1(0.15),S_2(0.25),S_3(0),S_4(0.25)\} & \{S_0(0.05),S_1(0.3),S_2(0.05),S_3(0.35),S_4(0.25)\} & \{S_0(0.05),S_1(0.05),S_2(0.1),S_3(0.35),S_4(0.2)\} & \{S_0(0.15),S_1(0.5),S_2(0.15),S_3(0),S_4(0)\} & \{S_0(0.1),S_1(0.2),S_2(0.3),S_3(0.1),S_4(0.1)\} \\ \{S_0(0),S_1(0.2),S_2(0.2),S_3(0.3),S_4(0.2)\} & \{S_0(0.35),S_1(0.2),S_2(0.2),S_3(0.15),S_4(0.1)\} & \{S_0(0.3),S_1(0.1),S_2(0.1),S_3(0.25),S_4(0.1)\} & \{S_0(0.2),S_1(0.25),S_2(0.15),S_3(0.1),S_4(0.2)\} & \{S_0(0.2),S_1(0.15),S_2(0.05),S_3(0.25),S_4(0.1)\} \end{pmatrix},$$

$$\mathrm{PLTSM}_{x_6}=\begin{pmatrix} \{S_0(0.2),S_1(0.25),S_2(0.15),S_3(0.35),S_4(0)\} & \{S_0(0.05),S_1(0.3),S_2(0.05),S_3(0.2),S_4(0.35)\} & \{S_0(0.15),S_1(0.3),S_2(0.05),S_3(0.2),S_4(0.35)\} & \{S_0(0),S_1(0.05),S_2(0.35),S_3(0.3),S_4(0.2)\} & \{S_0(0.1),S_1(0.05),S_2(0.15),S_3(0.35),S_4(0.25)\} \\ \{S_0(0.35),S_1(0.1),S_2(0.05),S_3(0.25),S_4(0.15)\} & \{S_0(0.05),S_1(0.3),S_2(0.05),S_3(0.15),S_4(0.2)\} & \{S_0(0.1),S_1(0.35),S_2(0.05),S_3(0.25),S_4(0.15)\} & \{S_0(0.25),S_1(0.2),S_2(0.3),S_3(0.05),S_4(0.05)\} & \{S_0(0.15),S_1(0.25),S_2(0.3),S_3(0.1),S_4(0)\} \\ \{S_0(0.15),S_1(0.2),S_2(0.35),S_3(0.3),S_4(0.3)\} & \{S_0(0.25),S_1(0.35),S_2(0.25),S_3(0.1),S_4(0.05)\} & \{S_0(0.3),S_1(0.2),S_2(0.15),S_3(0.05),S_4(0.35)\} & \{S_0(0.15),S_1(0.05),S_2(0.3),S_3(0.35),S_4(0.1)\} & \{S_0(0.15),S_1(0.15),S_2(0.3),S_3(0.15),S_4(0.15)\} \\ \{S_0(0.25),S_1(0.2),S_2(0.25),S_3(0.25),S_4(0.05)\} & \{S_0(0.3),S_1(0.2),S_2(0.3),S_3(0.05),S_4(0.15)\} & \{S_0(0.05),S_1(0.1),S_2(0.3),S_3(0.35),S_4(0)\} & \{S_0(0.05),S_1(0.1),S_2(0.3),S_3(0.3),S_4(0.1)\} & \{S_0(0.1),S_1(0.2),S_2(0.35),S_3(0.3),S_4(0)\} \\ \{S_0(0.35),S_1(0.3),S_2(0.1),S_3(0.05),S_4(0.05)\} & \{S_0(0.15),S_1(0.1),S_2(0.2),S_3(0.35),S_4(0.15)\} & \{S_0(0.2),S_1(0.05),S_2(0.25),S_3(0.1),S_4(0.05)\} & \{S_0(0.1),S_1(0.2),S_2(0.05),S_3(0.25),S_4(0.2)\} & \{S_0(0.15),S_1(0.1),S_2(0.05),S_3(0.2),S_4(0.3)\} \end{pmatrix},$$

$$\mathrm{PLTSM}_{x_7}=\begin{pmatrix} \{S_0(0.2),S_1(0.15),S_2(0.15),S_3(0.35),S_4(0.1)\} & \{S_0(0.15),S_1(0.2),S_2(0.25),S_3(0.15),S_4(0.1)\} & \{S_0(0.25),S_1(0.15),S_2(0.05),S_3(0.5),S_4(0.05)\} & \{S_0(0.15),S_1(0.05),S_2(0.3),S_3(0.25),S_4(0.05)\} & \{S_0(0.3),S_1(0.15),S_2(0.35),S_3(0.1),S_4(0.1)\} \\ \{S_0(0.1),S_1(0.05),S_2(0.05),S_3(0.35),S_4(0.3)\} & \{S_0(0.25),S_1(0.2),S_2(0.2),S_3(0.3),S_4(0.05)\} & \{S_0(0.2),S_1(0.15),S_2(0.2),S_3(0.2),S_4(0.15)\} & \{S_0(0.25),S_1(0.05),S_2(0.25),S_3(0.2),S_4(0.2)\} & \{S_0(0.2),S_1(0.2),S_2(0.2),S_3(0.05),S_4(0.2)\} \\ \{S_0(0.15),S_1(0.2),S_2(0.25),S_3(0.25),S_4(0.1)\} & \{S_0(0.05),S_1(0.35),S_2(0.45),S_3(0.05),S_4(0.05)\} & \{S_0(0.35),S_1(0.25),S_2(0.2),S_3(0.15),S_4(0)\} & \{S_0(0.35),S_1(0.05),S_2(0.2),S_3(0.05),S_4(0.15)\} & \{S_0(0.15),S_1(0.1),S_2(0.2),S_3(0.05),S_4(0.1)\} \\ \{S_0(0.05),S_1(0.35),S_2(0.1),S_3(0.2),S_4(0.1)\} & \{S_0(0.2),S_1(0.1),S_2(0.2),S_3(0.05),S_4(0.15)\} & \{S_0(0.35),S_1(0.2),S_2(0.2),S_3(0.05),S_4(0.1)\} & \{S_0(0.25),S_1(0.15),S_2(0.1),S_3(0.2),S_4(0.05)\} & \{S_0(0.05),S_1(0.1),S_2(0.1),S_3(0.3),S_4(0.05)\} \\ \{S_0(0.2),S_1(0.1),S_2(0.05),S_3(0.3),S_4(0.25)\} & \{S_0(0.05),S_1(0.15),S_2(0.2),S_3(0.05),S_4(0.2)\} & \{S_0(0.2),S_1(0.25),S_2(0.25),S_3(0.1),S_4(0.2)\} & \{S_0(0.1),S_1(0.2),S_2(0.15),S_3(0.2),S_4(0.25)\} & \{S_0(0.35),S_1(0.1),S_2(0.1),S_3(0.1),S_4(0.15)\} \end{pmatrix},$$

$$\mathrm{PLTSM}_{x_8}=\begin{pmatrix} \{S_0(0.15),S_1(0.15),S_2(0.1),S_3(0.1),S_4(0.1)\} & \{S_0(0.05),S_1(0.1),S_2(0.1),S_3(0.3),S_4(0.35)\} & \{S_0(0.05),S_1(0.3),S_2(0.05),S_3(0.15),S_4(0.05)\} & \{S_0(0.3),S_1(0.35),S_2(0.1),S_3(0.05),S_4(0.1)\} & \{S_0(0.35),S_1(0.1),S_2(0.15),S_3(0.15),S_4(0.15)\} \\ \{S_0(0.3),S_1(0.4),S_2(0),S_3(0),S_4(0.2)\} & \{S_0(0.1),S_1(0.35),S_2(0.05),S_3(0.05),S_4(0.3)\} & \{S_0(0.25),S_1(0.05),S_2(0.3),S_3(0.15),S_4(0.15)\} & \{S_0(0.1),S_1(0.05),S_2(0.1),S_3(0.3),S_4(0.25)\} & \{S_0(0.15),S_1(0.2),S_2(0.25),S_3(0.05),S_4(0.25)\} \\ \{S_0(0.2),S_1(0.05),S_2(0.1),S_3(0.2),S_4(0.25)\} & \{S_0(0.1),S_1(0.05),S_2(0.35),S_3(0.25),S_4(0.1)\} & \{S_0(0.3),S_1(0.05),S_2(0.2),S_3(0.15),S_4(0.25)\} & \{S_0(0.1),S_1(0.2),S_2(0.05),S_3(0.25),S_4(0.15)\} & \{S_0(0.15),S_1(0.2),S_2(0.45),S_3(0.05),S_4(0.15)\} \\ \{S_0(0.25),S_1(0.25),S_2(0.05),S_3(0.05),S_4(0.35)\} & \{S_0(0.05),S_1(0.1),S_2(0.05),S_3(0.2),S_4(0.15)\} & \{S_0(0.1),S_1(0.15),S_2(0.1),S_3(0.05),S_4(0.35)\} & \{S_0(0.15),S_1(0.05),S_2(0.1),S_3(0.3),S_4(0.25)\} & \{S_0(0.2),S_1(0.35),S_2(0.05),S_3(0.35),S_4(0.05)\} \\ \{S_0(0.1),S_1(0.15),S_2(0.05),S_3(0.15),S_4(0.05)\} & \{S_0(0.15),S_1(0.25),S_2(0.05),S_3(0.15),S_4(0.35)\} & \{S_0(0.15),S_1(0.05),S_2(0.1),S_3(0.3),S_4(0.2)\} & \{S_0(0.3),S_1(0.2),S_2(0.35),S_3(0.05),S_4(0.1)\} & \{S_0(0.15),S_1(0.35),S_2(0.05),S_3(0.05),S_4(0.25)\} \end{pmatrix},$$

$$\mathrm{PLTSM}_{x_9}=\begin{pmatrix} \{S_0(0.15),S_1(0.1),S_2(0.25),S_3(0.1),S_4(0.2)\} & \{S_0(0.15),S_1(0.1),S_2(0.2),S_3(0.2),S_4(0.35)\} & \{S_0(0.25),S_1(0.05),S_2(0.2),S_3(0.2),S_4(0.2)\} & \{S_0(0.2),S_1(0.15),S_2(0.3),S_3(0.35),S_4(0)\} & \{S_0(0.25),S_1(0.1),S_2(0.05),S_3(0.25),S_4(0.35)\} \\ \{S_0(0.3),S_1(0.05),S_2(0.15),S_3(0.25),S_4(0.25)\} & \{S_0(0.1),S_1(0.15),S_2(0.15),S_3(0.1),S_4(0.1)\} & \{S_0(0.05),S_1(0.25),S_2(0.05),S_3(0.3),S_4(0.05)\} & \{S_0(0),S_1(0.3),S_2(0.3),S_3(0.15),S_4(0.2)\} & \{S_0(0.35),S_1(0.2),S_2(0.15),S_3(0.2),S_4(0.05)\} \\ \{S_0(0.05),S_1(0.2),S_2(0.3),S_3(0.2),S_4(0.1)\} & \{S_0(0),S_1(0),S_2(0.05),S_3(0.5),S_4(0.25)\} & \{S_0(0.3),S_1(0.2),S_2(0.05),S_3(0.15),S_4(0.2)\} & \{S_0(0.15),S_1(0.15),S_2(0.25),S_3(0.25),S_4(0.15)\} & \{S_0(0.2),S_1(0.05),S_2(0.35),S_3(0.1),S_4(0.2)\} \\ \{S_0(0.3),S_1(0.1),S_2(0.2),S_3(0.15),S_4(0.25)\} & \{S_0(0.15),S_1(0.05),S_2(0.15),S_3(0.35),S_4(0.15)\} & \{S_0(0.1),S_1(0.2),S_2(0.35),S_3(0.1),S_4(0.25)\} & \{S_0(0.25),S_1(0.35),S_2(0.05),S_3(0.2),S_4(0.15)\} & \{S_0(0.05),S_1(0.2),S_2(0.1),S_3(0.35),S_4(0.1)\} \\ \{S_0(0),S_1(0.35),S_2(0.35),S_3(0.1),S_4(0)\} & \{S_0(0.05),S_1(0.15),S_2(0.2),S_3(0.25),S_4(0.15)\} & \{S_0(0.2),S_1(0.25),S_2(0.15),S_3(0.05),S_4(0.3)\} & \{S_0(0.15),S_1(0.3),S_2(0.2),S_3(0.05),S_4(0.1)\} & \{S_0(0.35),S_1(0.1),S_2(0.15),S_3(0.15),S_4(0.25)\} \end{pmatrix},$$

$$\mathrm{PLTSM}_{x_{10}}=\begin{pmatrix} \{S_0(0.1),S_1(0.3),S_2(0.2),S_3(0.25),S_4(0.15)\} & \{S_0(0.05),S_1(0.35),S_2(0.15),S_3(0.2),S_4(0.2)\} & \{S_0(0.05),S_1(0.15),S_2(0.25),S_3(0.1),S_4(0.2)\} & \{S_0(0.15),S_1(0.2),S_2(0.25),S_3(0.1),S_4(0,05)\} & \{S_0(0.35),S_1(0.25),S_2(0.05),S_3(0.3),S_4(0)\} \\ \{S_0(0.1),S_1(0.2),S_2(0.35),S_3(0.05),S_4(0.25)\} & \{S_0(0.15),S_1(0.35),S_2(0.05),S_3(0.05),S_4(0.35)\} & \{S_0(0.05),S_1(0.25),S_2(0.05),S_3(0.35),S_4(0.05)\} & \{S_0(0.3),S_1(0.25),S_2(0.25),S_3(0.1),S_4(0)\} & \{S_0(0.3),S_1(0.1),S_2(0.2),S_3(0.35),S_4(0.05)\} \\ \{S_0(0.05),S_1(0.25),S_2(0.15),S_3(0.15),S_4(0.2)\} & \{S_0(0.25),S_1(0.2),S_2(0.1),S_3(0.35),S_4(0.1)\} & \{S_0(0),S_1(0.05),S_2(0.2),S_3(0.35),S_4(0.15)\} & \{S_0(0.1),S_1(0.1),S_2(0.15),S_3(0.05),S_4(0.25)\} & \{S_0(0.05),S_1(0.05),S_2(0.3),S_3(0.3),S_4(0.1)\} \\ \{S_0(0.2),S_1(0.1),S_2(0.2),S_3(0.1),S_4(0.3)\} & \{S_0(0.05),S_1(0.15),S_2(0.1),S_3(0.15),S_4(0.3)\} & \{S_0(0.1),S_1(0.1),S_2(0.15),S_3(0.4),S_4(0.1)\} & \{S_0(0.15),S_1(0.2),S_2(0.05),S_3(0.5),S_4(0.05)\} & \{S_0(0.25),S_1(0.15),S_2(0.25),S_3(0.15),S_4(0.2)\} \\ \{S_0(0.2),S_1(0.2),S_2(0.15),S_3(0.1),S_4(0.15)\} & \{S_0(0.25),S_1(0.2),S_2(0),S_3(0.15),S_4(0.15)\} & \{S_0(0.15),S_1(0.2),S_2(0.3),S_3(0.15),S_4(0.1)\} & \{S_0(0.1),S_1(0.05),S_2(0.25),S_3(0.2),S_4(0.25)\} & \{S_0(0.1),S_1(0.35),S_2(0.15),S_3(0.15),S_4(0.15)\} \end{pmatrix},$$

$$\mathrm{PLTSM}_{x_{11}}=\begin{pmatrix} \{S_0(0.1),S_1(0.05),S_2(0.25),S_3(0.3),S_4(0.25)\} & \{S_0(0.15),S_1(0.15),S_2(0.05),S_3(0.25),S_4(0.3)\} & \{S_0(0.2),S_1(0.1),S_2(0.05),S_3(0.3),S_4(0.15)\} & \{S_0(0.35),S_1(0.05),S_2(0.05),S_3(0.25),S_4(0.05)\} & \{S_0(0.15),S_1(0.1),S_2(0.2),S_3(0.25),S_4(0.15)\} \\ \{S_0(0.1),S_1(0.2),S_2(0.3),S_3(0.35),S_4(0.05)\} & \{S_0(0.3),S_1(0.1),S_2(0.05),S_3(0.05),S_4(0.25)\} & \{S_0(0.15),S_1(0.35),S_2(0.1),S_3(0.05),S_4(0.05)\} & \{S_0(0.2),S_1(0.15),S_2(0.35),S_3(0.1),S_4(0.1)\} & \{S_0(0.1),S_1(0.15),S_2(0.1),S_3(0.1),S_4(0.1)\} \\ \{S_0(0.3),S_1(0.05),S_2(0.35),S_3(0.15),S_4(0.1)\} & \{S_0(0.3),S_1(0.1),S_2(0.35),S_3(0.05),S_4(0.15)\} & \{S_0(0.1),S_1(0.25),S_2(0.3),S_3(0.1),S_4(0.25)\} & \{S_0(0.3),S_1(0.05),S_2(0.3),S_3(0.2),S_4(0.1)\} & \{S_0(0.2),S_1(0.25),S_2(0.1),S_3(0.1),S_4(0.05)\} \\ \{S_0(0.2),S_1(0.3),S_2(0.05),S_3(0.2),S_4(0.1)\} & \{S_0(0.3),S_1(0.45),S_2(0.05),S_3(0),S_4(0)\} & \{S_0(0.1),S_1(0.1),S_2(0.1),S_3(0.35),S_4(0.05)\} & \{S_0(0.05),S_1(0.3),S_2(0.35),S_3(0.1),S_4(0.2)\} & \{S_0(0.15),S_1(0.15),S_2(0.2),S_3(0.15),S_4(0.35)\} \\ \{S_0(0.15),S_1(0.05),S_2(0.3),S_3(0.2),S_4(0.1)\} & \{S_0(0.25),S_1(0.25),S_2(0.3),S_3(0.1),S_4(0.1)\} & \{S_0(0.25),S_1(0.2),S_2(0.3),S_3(0.1),S_4(0)\} & \{S_0(0.1),S_1(0.05),S_2(0.05),S_3(0.2),S_4(0.1)\} & \{S_0(0.05),S_1(0.3),S_2(0.1),S_3(0.25),S_4(0.25)\} \end{pmatrix},$$

$$\mathrm{PLTSM}_{x_{12}}=\begin{pmatrix} \{S_0(0),S_1(0.05),S_2(0.35),S_3(0.35),S_4(0)\} & \{S_0(0.2),S_1(0.25),S_2(0.1),S_3(0.1),S_4(0.1)\} & \{S_0(0.2),S_1(0.1),S_2(0.3),S_3(0.3),S_4(0.05)\} & \{S_0(0.2),S_1(0.1),S_2(0.35),S_3(0.2),S_4(0.1)\} & \{S_0(0.15),S_1(0.15),S_2(0.3),S_3(0.15),S_4(0.2)\} \\ \{S_0(0.25),S_1(0.25),S_2(0.3),S_3(0.15),S_4(0.05)\} & \{S_0(0.05),S_1(0.15),S_2(0.15),S_3(0.1),S_4(0.1)\} & \{S_0(0.2),S_1(0.35),S_2(0.15),S_3(0.2),S_4(0)\} & \{S_0(0.3),S_1(0),S_2(0.05),S_3(0.35),S_4(0.15)\} & \{S_0(0),S_1(0.3),S_2(0.3),S_3(0.3),S_4(0)\} \\ \{S_0(0.1),S_1(0.2),S_2(0.15),S_3(0.35),S_4(0.15)\} & \{S_0(0.2),S_1(0.3),S_2(0.35),S_3(0),S_4(0.1)\} & \{S_0(0.1),S_1(0.3),S_2(0.1),S_3(0.1),S_4(0.1)\} & \{S_0(0.35),S_1(0.35),S_2(0.15),S_3(0.15),S_4(0)\} & \{S_0(0.05),S_1(0.35),S_2(0.2),S_3(0.1),S_4(0.3)\} \\ \{S_0(0.1),S_1(0.2),S_2(0.05),S_3(0.35),S_4(0.1)\} & \{S_0(0.25),S_1(0.2),S_2(0.05),S_3(0.3),S_4(0.15)\} & \{S_0(0.35),S_1(0.2),S_2(0.1),S_3(0.1),S_4(0.15)\} & \{S_0(0.1),S_1(0.2),S_2(0.3),S_3(0.15),S_4(0.15)\} & \{S_0(0),S_1(0.25),S_2(0.35),S_3(0.35),S_4(0)\} \\ \{S_0(0.35),S_1(0.2),S_2(0),S_3(0.3),S_4(0)\} & \{S_0(0.25),S_1(0.25),S_2(0.05),S_3(0.25),S_4(0.1)\} & \{S_0(0.05),S_1(0.15),S_2(0.15),S_3(0.35),S_4(0.3)\} & \{S_0(0.15),S_1(0.05),S_2(0.35),S_3(0.15),S_4(0)\} & \{S_0(0.05),S_1(0.3),S_2(0.35),S_3(0.05),S_4(0.05)\} \end{pmatrix},$$

$$\mathrm{PLTSM}_{x_{13}}=\begin{pmatrix} \{S_0(0.1),S_1(0.05),S_2(0.35),S_3(0.2),S_4(0)\} & \{S_0(0.15),S_1(0.35),S_2(0.2),S_3(0),S_4(0.15)\} & \{S_0(0.35),S_1(0.05),S_2(0.05),S_3(0.15),S_4(0.35)\} & \{S_0(0.3),S_1(0.5),S_2(0.15),S_3(0.2),S_4(0.1)\} & \{S_0(0.1),S_1(0.35),S_2(0.25),S_3(0.05),S_4(0.1)\} \\ \{S_0(0.05),S_1(0.35),S_2(0.3),S_3(0.05),S_4(0.05)\} & \{S_0(0.25),S_1(0.15),S_2(0.1),S_3(0.1),S_4(0.2)\} & \{S_0(0.25),S_1(0.35),S_2(0.05),S_3(0.05),S_4(0.05)\} & \{S_0(0.1),S_1(0.25),S_2(0.5),S_3(0),S_4(0.5)\} & \{S_0(0.35),S_1(0.05),S_2(0.05),S_3(0.2),S_4(0.35)\} \\ \{S_0(0.15),S_1(0.45),S_2(0.2),S_3(0.15),S_4(0)\} & \{S_0(0.15),S_1(0.15),S_2(0.2),S_3(0.1),S_4(0.35)\} & \{S_0(0.3),S_1(0.05),S_2(0.35),S_3(0.05),S_4(0.1)\} & \{S_0(0.25),S_1(0.05),S_2(0),S_3(0.3),S_4(0.1)\} & \{S_0(0.2),S_1(0.2),S_2(0.15),S_3(0.05),S_4(0.25)\} \\ \{S_0(0.3),S_1(0.2),S_2(0.1),S_3(0.35),S_4(0.05)\} & \{S_0(0.05),S_1(0.1),S_2(0.25),S_3(0.2),S_4(0.3)\} & \{S_0(0.1),S_1(0.35),S_2(0.35),S_3(0.2),S_4(0)\} & \{S_0(0.35),S_1(0.1),S_2(0.2),S_3(0.1),S_4(0.15)\} & \{S_0(0.3),S_1(0.15),S_2(0.2),S_3(0.25),S_4(0.05)\} \\ \{S_0(0.1),S_1(0.2),S_2(0.35),S_3(0.2),S_4(0.1)\} & \{S_0(0),S_1(0),S_2(0.35),S_3(0.5),S_4(0.05)\} & \{S_0(0.05),S_1(0.15),S_2(0.35),S_3(3),S_4(0.05)\} & \{S_0(0.15),S_1(0.1),S_2(0.15),S_3(0.2),S_4(0.25)\} & \{S_0(0.1),S_1(0.05),S_2(0.2),S_3(0.05),S_4(0.35)\} \end{pmatrix},$$

$$\mathrm{PLTSM}_{x_{14}}=\begin{pmatrix}\{S_0(0.05),S_1(0.25),S_2(0.2),S_3(0.1),S_4(0.3)\} & \{S_0(0.25),S_1(0.05),S_2(0.15),S_3(0.25),S_4(0.3)\} & \{S_0(0.15),S_1(0),S_2(0),S_3(0.35),S_4(0.25)\} & \{S_0(0.15),S_1(0),S_2(0),S_3(0.35),S_4(0.2)\} & \{S_0(0.3),S_1(0.2),S_2(0.15),S_3(0.1),S_4(0.25)\}\\ \{S_0(0.1),S_1(0.3),S_2(0.2),S_3(0.2),S_4(0.05)\} & \{S_0(0.1),S_1(0.05),S_2(0.15),S_3(0.3),S_4(0.15)\} & \{S_0(0.2),S_1(0.3),S_2(0.1),S_3(0.05),S_4(0.1)\} & \{S_0(0.2),S_1(0.3),S_2(0.1),S_3(0.05),S_4(0.1)\} & \{S_0(0.1),S_1(0.3),S_2(0.15),S_3(0.1),S_4(0.3)\}\\ \{S_0(0.05),S_1(0.25),S_2(0),S_3(0.45),S_4(0.2)\} & \{S_0(0.15),S_1(0.35),S_2(0.25),S_3(0.1),S_4(0.05)\} & \{S_0(0.1),S_1(0.3),S_2(0.05),S_3(0.2),S_4(0.35)\} & \{S_0(0.1),S_1(0.3),S_2(0.05),S_3(0.2),S_4(0.35)\} & \{S_0(0.05),S_1(0.35),S_2(0.25),S_3(0.1),S_4(0.25)\}\\ \{S_0(0.2),S_1(0.15),S_2(0.15),S_3(0.2),S_4(0.25)\} & \{S_0(0.05),S_1(0.05),S_2(0.45),S_3(0.15),S_4(0.1)\} & \{S_0(0.15),S_1(0.25),S_2(0.05),S_3(0.05),S_4(0.1)\} & \{S_0(0.15),S_1(0.25),S_2(0.05),S_3(0.05),S_4(0.1)\} & \{S_0(0.15),S_1(0.05),S_2(0.2),S_3(0.05),S_4(0.2)\}\\ \{S_0(0.3),S_1(0.1),S_2(0.05),S_3(0.25),S_4(0.05)\} & \{S_0(0.2),S_1(0.05),S_2(0.25),S_3(0),S_4(0.1)\} & \{S_0(0.1),S_1(0.1),S_2(0.2),S_3(0.25),S_4(0.1)\} & \{S_0(0.1),S_1(0.1),S_2(0.2),S_3(0.25),S_4(0.1)\} & \{S_0(0.1),S_1(0),S_2(0.2),S_3(0.35),S_4(0)\}\end{pmatrix},$$

$$\mathrm{PLTSM}_{x_{15}}=\begin{pmatrix}\{S_0(0.15),S_1(0.35),S_2(0.05),S_3(0.15),S_4(0.1)\} & \{S_0(0.1),S_1(0.1),S_2(0.05),S_3(0.15),S_4(0.2)\} & \{S_0(0.25),S_1(0.1),S_2(0.1),S_3(0.35),S_4(0)\} & \{S_0(0.1),S_1(0.05),S_2(0.15),S_3(0.15),S_4(0.2)\} & \{S_0(0.05),S_1(0.2),S_2(0.15),S_3(0.05),S_4(0.05)\}\\ \{S_0(0.2),S_1(0.1),S_2(0.25),S_3(0.15),S_4(0.15)\} & \{S_0(0.15),S_1(0.1),S_2(0.25),S_3(0.15),S_4(0.1)\} & \{S_0(0.05),S_1(0.25),S_2(0.3),S_3(0.15),S_4(0.1)\} & \{S_0(0.25),S_1(0.2),S_2(0),S_3(0.15),S_4(0.35)\} & \{S_0(0.2),S_1(0.05),S_2(0.1),S_3(0.15),S_4(0.2)\}\\ \{S_0(0.25),S_1(0.25),S_2(0.1),S_3(0.1),S_4(0.1)\} & \{S_0(0.05),S_1(0.05),S_2(0.05),S_3(0.15),S_4(0.35)\} & \{S_0(0.05),S_1(0.2),S_2(0.25),S_3(0.1),S_4(0.35)\} & \{S_0(0.3),S_1(0.05),S_2(0.15),S_3(0.2),S_4(0.3)\} & \{S_0(0.1),S_1(0.2),S_2(0.2),S_3(0.05),S_4(0.25)\}\\ \{S_0(0.05),S_1(0.25),S_2(0.25),S_3(0.25),S_4(0.1)\} & \{S_0(0.05),S_1(0.1),S_2(0.1),S_3(0.15),S_4(0.2)\} & \{S_0(0.15),S_1(0.05),S_2(0.5),S_3(0.1),S_4(0)\} & \{S_0(0.15),S_1(0.25),S_2(0.45),S_3(0.05),S_4(0.1)\} & \{S_0(0.15),S_1(0.15),S_2(0.1),S_3(0.05),S_4(0.15)\}\\ \{S_0(0.2),S_1(0.2),S_2(0.2),S_3(0.25),S_4(0.1)\} & \{S_0(0.25),S_1(0.2),S_2(0.35),S_3(0.05),S_4(0)\} & \{S_0(0.15),S_1(0.1),S_2(0.3),S_3(0.35),S_4(0.1)\} & \{S_0(0.25),S_1(0.35),S_2(0.25),S_3(0.05),S_4(0.1)\} & \{S_0(0.2),S_1(0.25),S_2(0.15),S_3(0.2),S_4(0.15)\}\end{pmatrix},$$

$$\mathrm{PLTSM}_{x_{16}}=\begin{pmatrix}\{S_0(0.1),S_1(0.1),S_2(0.35),S_3(0.25),S_4(0.1)\} & \{S_0(0.15),S_1(0.3),S_2(0.35),S_3(0.05),S_4(0)\} & \{S_0(0.3),S_1(0.1),S_2(0.1),S_3(0.2),S_4(0.1)\} & \{S_0(0),S_1(0.35),S_2(0.3),S_3(0.1),S_4(0.25)\} & \{S_0(0.3),S_1(0.05),S_2(0.05),S_3(0.35),S_4(0.15)\}\\ \{S_0(0.15),S_1(0.25),S_2(0.25),S_3(0.05),S_4(0.2)\} & \{S_0(0.25),S_1(0.15),S_2(0.1),S_3(0.1),S_4(0.2)\} & \{S_0(0.35),S_1(0.2),S_2(0.05),S_3(0.35),S_4(0)\} & \{S_0(0.35),S_1(0.05),S_2(0.05),S_3(0.2),S_4(0.35)\} & \{S_0(0.1),S_1(0.05),S_2(0.1),S_3(0.25),S_4(0.05)\}\\ \{S_0(0.3),S_1(0.05),S_2(0.05),S_3(0.35),S_4(0.1)\} & \{S_0(0.1),S_1(0.3),S_2(0.05),S_3(0.25),S_4(0.3)\} & \{S_0(0.2),S_1(0.15),S_2(0.15),S_3(0.2),S_4(0.25)\} & \{S_0(0.3),S_1(0.35),S_2(0.2),S_3(0.15),S_4(0)\} & \{S_0(0.25),S_1(0.15),S_2(0.2),S_3(0.2),S_4(0.15)\}\\ \{S_0(0.3),S_1(0.2),S_2(0.15),S_3(0.15),S_4(0.05)\} & \{S_0(0.05),S_1(0.25),S_2(0.2),S_3(0.25),S_4(0.2)\} & \{S_0(0.05),S_1(0.1),S_2(0.1),S_3(0.05),S_4(0.1)\} & \{S_0(0.1),S_1(0.1),S_2(0.05),S_3(0.3),S_4(0.25)\} & \{S_0(0.2),S_1(0.25),S_2(0.1),S_3(0.05),S_4(0.35)\}\\ \{S_0(0.25),S_1(0.1),S_2(0.05),S_3(0.3),S_4(0.2)\} & \{S_0(0.3),S_1(0.15),S_2(0.35),S_3(0.1),S_4(0.1)\} & \{S_0(0.35),S_1(0.05),S_2(0.05),S_3(0.1),S_4(0.1)\} & \{S_0(0.1),S_1(0.25),S_2(0.35),S_3(0.15),S_4(0.05)\} & \{S_0(0.05),S_1(0.25),S_2(0.3),S_3(0.3),S_4(0.05)\}\end{pmatrix},$$

$$\mathrm{PLTSM}_{x_{17}}=\begin{pmatrix}\{S_0(0),S_1(0.25),S_2(0.15),S_3(0.35),S_4(0.1)\} & \{S_0(0.1),S_1(0.25),S_2(0.15),S_3(0.2),S_4(0.15)\} & \{S_0(0.25),S_1(0.15),S_2(0.05),S_3(0.05),S_4(0.2)\} & \{S_0(0.05),S_1(0.05),S_2(0.3),S_3(0.1),S_4(0.35)\} & \{S_0(0.25),S_1(0.15),S_2(0.25),S_3(0.2),S_4(0)\}\\ \{S_0(0.05),S_1(0.1),S_2(0.35),S_3(0.2),S_4(0)\} & \{S_0(0.2),S_1(0.05),S_2(0.15),S_3(0.15),S_4(0.35)\} & \{S_0(0.3),S_1(0.05),S_2(0.2),S_3(0.1),S_4(0.25)\} & \{S_0(0.35),S_1(0.1),S_2(0.15),S_3(0.15),S_4(0)\} & \{S_0(0.15),S_1(0.1),S_2(0.15),S_3(0.1),S_4(0)\}\\ \{S_0(0.15),S_1(0.1),S_2(0.35),S_3(0.05),S_4(0.25)\} & \{S_0(0.05),S_1(0.2),S_2(0.25),S_3(0.3),S_4(0.15)\} & \{S_0(0.05),S_1(0.05),S_2(0.25),S_3(0.3),S_4(0.35)\} & \{S_0(0.1),S_1(0.25),S_2(0.2),S_3(0.25),S_4(0.1)\} & \{S_0(0.35),S_1(0.3),S_2(0.05),S_3(0.1),S_4(0.2)\}\\ \{S_0(0.25),S_1(0.15),S_2(0.25),S_3(0.05),S_4(0)\} & \{S_0(0.25),S_1(0.15),S_2(0.25),S_3(0.2),S_4(0.15)\} & \{S_0(0.1),S_1(0.3),S_2(0.05),S_3(0.15),S_4(0.35)\} & \{S_0(0.15),S_1(0.2),S_2(0.25),S_3(0.25),S_4(0.1)\} & \{S_0(0.2),S_1(0.15),S_2(0.3),S_3(0.25),S_4(0.05)\}\\ \{S_0(0.15),S_1(0.2),S_2(0.05),S_3(0.3),S_4(0.05)\} & \{S_0(0.1),S_1(0.3),S_2(0.3),S_3(0.25),S_4(0)\} & \{S_0(0.15),S_1(0.05),S_2(0.2),S_3(0.15),S_4(0.15)\} & \{S_0(0),S_1(0.1),S_2(0.35),S_3(0.35),S_4(0.1)\} & \{S_0(0.15),S_1(0.05),S_2(0.1),S_3(0.15),S_4(0.3)\}\end{pmatrix},$$

$$\mathrm{PLTSM}_{x_{18}}=\begin{pmatrix}\{S_0(0.05),S_1(0.05),S_2(0.1),S_3(0.35),S_4(0.3)\} & \{S_0(0.35),S_1(0.05),S_2(0.35),S_3(0.05),S_4(0)\} & \{S_0(0.2),S_1(0.35),S_2(0.3),S_3(0),S_4(0)\} & \{S_0(0.2),S_1(0.15),S_2(0.15),S_3(0.05),S_4(0.1)\} & \{S_0(0.35),S_1(0.05),S_2(0.25),S_3(0.1),S_4(0.2)\}\\ \{S_0(0.25),S_1(0.25),S_2(0.1),S_3(0.3),S_4(0)\} & \{S_0(0.2),S_1(0.1),S_2(0.05),S_3(0.3),S_4(0.2)\} & \{S_0(0.15),S_1(0.35),S_2(0.15),S_3(0.3),S_4(0.05)\} & \{S_0(0.2),S_1(0.05),S_2(0.1),S_3(0.3),S_4(0.1)\} & \{S_0(0.25),S_1(0.05),S_2(0.1),S_3(0.05),S_4(0.3)\}\\ \{S_0(0.1),S_1(0.2),S_2(0.25),S_3(0.2),S_4(0.15)\} & \{S_0(0.1),S_1(0.2),S_2(0.1),S_3(0.1),S_4(0.05)\} & \{S_0(0.05),S_1(0.2),S_2(0.05),S_3(0.3),S_4(0.05)\} & \{S_0(0.25),S_1(0.05),S_2(0.15),S_3(0.1),S_4(0.2)\} & \{S_0(0.05),S_1(0.25),S_2(0.3),S_3(0.15),S_4(0.1)\}\\ \{S_0(0.1),S_1(0.1),S_2(0.25),S_3(0.2),S_4(0.1)\} & \{S_0(0.2),S_1(0.35),S_2(0.15),S_3(0.05),S_4(0.2)\} & \{S_0(0.3),S_1(0.15),S_2(0.2),S_3(0.1),S_4(0.1)\} & \{S_0(0.05),S_1(0.05),S_2(0.3),S_3(0.05),S_4(0.35)\} & \{S_0(0),S_1(0.1),S_2(0.1),S_3(0.35),S_4(0.3)\}\\ \{S_0(0.25),S_1(0.3),S_2(0.2),S_3(0.25),S_4(0)\} & \{S_0(0.05),S_1(0.1),S_2(0.1),S_3(0.15),S_4(0.25)\} & \{S_0(0.25),S_1(0.25),S_2(0.25),S_3(0.1),S_4(0.1)\} & \{S_0(0.1),S_1(0.35),S_2(0.2),S_3(0.2),S_4(0.05)\} & \{S_0(0.2),S_1(0.3),S_2(0.45),S_3(0.05),S_4(0)\}\end{pmatrix},$$

$$\mathrm{PLTSM}_{x_{19}}=\begin{pmatrix}\{S_0(0.35),S_1(0.05),S_2(0.05),S_3(0.1),S_4(0.3)\} & \{S_0(0.35),S_1(0.35),S_2(0.05),S_3(0.15),S_4(0.05)\} & \{S_0(0.15),S_1(0.05),S_2(0.3),S_3(0.25),S_4(0.15)\} & \{S_0(0.15),S_1(0.3),S_2(0.15),S_3(0.3),S_4(0.2)\} & \{S_0(0.1),S_1(0),S_2(0.5),S_3(0.3),S_4(0.05)\}\\ \{S_0(0.15),S_1(0.15),S_2(0.3),S_3(0.3),S_4(0)\} & \{S_0(0.2),S_1(0.3),S_2(0.25),S_3(0.1),S_4(0.15)\} & \{S_0(0.1),S_1(0.1),S_2(0.05),S_3(0.2),S_4(0.3)\} & \{S_0(0.3),S_1(0.25),S_2(0.05),S_3(0.15),S_4(0.2)\} & \{S_0(0.25),S_1(0.15),S_2(0.15),S_3(0.15),S_4(0.05)\}\\ \{S_0(0.3),S_1(0.1),S_2(0.1),S_3(0.1),S_4(0.15)\} & \{S_0(0),S_1(0.3),S_2(0.35),S_3(0.25),S_4(0)\} & \{S_0(0.05),S_1(0.2),S_2(0.35),S_3(0.3),S_4(0.05)\} & \{S_0(0.25),S_1(0.2),S_2(0.3),S_3(0.05),S_4(0.25)\} & \{S_0(0.05),S_1(0.2),S_2(0.3),S_3(0.25),S_4(0.2)\}\\ \{S_0(0.15),S_1(0.05),S_2(0.3),S_3(0.25),S_4(0.1)\} & \{S_0(0.2),S_1(0.05),S_2(0.25),S_3(0.2),S_4(0.2)\} & \{S_0(0.15),S_1(0.05),S_2(0.25),S_3(0.05),S_4(0.3)\} & \{S_0(0.05),S_1(0.2),S_2(0.15),S_3(0.15),S_4(0.1)\} & \{S_0(0.35),S_1(0.35),S_2(0.1),S_3(0),S_4(0)\}\\ \{S_0(0.1),S_1(0.1),S_2(0.3),S_3(0.05),S_4(0.05)\} & \{S_0(0.2),S_1(0.15),S_2(0.15),S_3(0.05),S_4(0.15)\} & \{S_0(0.1),S_1(0.35),S_2(0.25),S_3(0.1),S_4(0.1)\} & \{S_0(0.2),S_1(0.3),S_2(0.1),S_3(0.1),S_4(0.25)\} & \{S_0(0.1),S_1(0.25),S_2(0.2),S_3(0.2),S_4(0.05)\}\end{pmatrix},$$

$$\mathrm{PLTSM}_{x_{20}}=\begin{pmatrix}\{S_0(0.1),S_1(0.1),S_2(0.3),S_3(0.05),S_4(0.25)\} & \{S_0(0.1),S_1(0.05),S_2(0.3),S_3(0.05),S_4(0.35)\} & \{S_0(0.1),S_1(0.3),S_2(0.15),S_3(0.25),S_4(0.15)\} & \{S_0(0.25),S_1(0.25),S_2(0.25),S_3(0.15),S_4(0)\} & \{S_0(0.2),S_1(0.1),S_2(0.15),S_3(0.25),S_4(0)\}\\ \{S_0(0),S_1(0.2),S_2(0.5),S_3(0.2),S_4(0)\} & \{S_0(0.05),S_1(0.25),S_2(0.15),S_3(0.05),S_4(0.35)\} & \{S_0(0.15),S_1(0.25),S_2(0.25),S_3(0.05),S_4(0.1)\} & \{S_0(0.05),S_1(0.05),S_2(0.15),S_3(0.2),S_4(0.05)\} & \{S_0(0.15),S_1(0.05),S_2(0.15),S_3(0.1),S_4(0.15)\}\\ \{S_0(0.1),S_1(0.35),S_2(0.15),S_3(0.1),S_4(0.05)\} & \{S_0(0.1),S_1(0.2),S_2(0.2),S_3(0.2),S_4(0.25)\} & \{S_0(0.1),S_1(0.35),S_2(0.05),S_3(0.1),S_4(0.35)\} & \{S_0(0.1),S_1(0.05),S_2(0.15),S_3(0.3),S_4(0.3)\} & \{S_0(0.1),S_1(0.1),S_2(0.1),S_3(0.2),S_4(0.25)\}\\ \{S_0(0.3),S_1(0.1),S_2(0.25),S_3(0.25),S_4(0.05)\} & \{S_0(0.25),S_1(0.35),S_2(0.15),S_3(0.05),S_4(0.15)\} & \{S_0(0.1),S_1(0.25),S_2(0.2),S_3(0.15),S_4(0.25)\} & \{S_0(0.15),S_1(0.25),S_2(0.1),S_3(0.15),S_4(0.2)\} & \{S_0(0.2),S_1(0.35),S_2(0.05),S_3(0.05),S_4(0.35)\}\\ \{S_0(0.25),S_1(0.25),S_2(0.35),S_3(0.1),S_4(0)\} & \{S_0(0.05),S_1(0.1),S_2(0.15),S_3(0.05),S_4(0.3)\} & \{S_0(0.15),S_1(0.15),S_2(0.15),S_3(0.25),S_4(0.25)\} & \{S_0(0.15),S_1(0.15),S_2(0.2),S_3(0.25),S_4(0.05)\} & \{S_0(0.2),S_1(0.3),S_2(0.1),S_3(0.35),S_4(0.05)\}\end{pmatrix}. \tag{A.1}$$

## Data Availability

All of the data used to support the study have been included within the article.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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*Research Article*

# The Defects and Improvements of the Internal Control Audit in Chinese Universities with respect to the Probabilistic Hesitant Fuzzy Environment

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This paper analyzes the survey data and qualitative evaluation information of the internal control audit of some universities directly under the Ministry of Education of China (MEC) by using the principal component analysis. The qualitative evaluation information is given by experts and presented by the probabilistic hesitant fuzzy elements. The analysis results show that the internal control audit system and process are not perfect, the professional knowledge and skills of internal auditors in universities are not very adaptable to internal control audit, and the effectiveness of internal control audit management conducted by external firms is not good. These are the three most serious defects of the current internal control audit in universities. The most critical factors influencing the quality of internal control audit in universities are, in order, the difficulty of rational use of the results of internal control audit by universities, the poor support of the leadership for internal control audit, and the insufficient professional knowledge and skills of internal auditors to conduct internal control audit. Thus, this paper proposes that the optimization suggestions for the internal control audit of universities are to establish and improve their internal control audit system and process to continually promote the application of internal control audit results, to increase the degree of attention of the leadership, and to strengthen the construction of the internal audit teams.

## 1. Introduction

At the end of 2012, the Ministry of Finance issued the "Regulation for Internal Control of Administrative Institutions (for Trial Implementation)" (Finance and Accounting (2012) No. 21, China), requiring that the regulation must be implemented nationwide by January 1, 2014, which triggered a research boom on the internal control construction of domestic administrative institutions. With the continuous increase of this research boom and China's investment in education, the Ministry of Education of China (MEC) has continually emphasized and promoted the construction of internal control of universities directly under MEC in recent years. During this period, the internal control audit has become a recontrol of the internal control of universities and a research hotpot of all social circles. In practice, at the end of 2016, MEC required that universities directly under MEC must fill out and report internal audit work, including the development of internal control audit. This was the first time that the statistics on the internal audit circumstances of universities directly under MEC listed the "internal control audit" as an independent audit item, which is enough to reflect the importance of MEC.

This article mainly investigates some universities directly under MEC and adopts a questionnaire survey and the experts' qualitative evaluation to research the defects of the current internal audit of universities in China and the influencing factors that affect the quality of the internal control audit of universities. The paper attempts to conduct profound analysis using the principal component analysis in order to find the most serious defects and the most critical factors. At last, the article puts forward more targeted and reasonable recommendations for the optimization of the internal control audit of universities, hoping that such optimization proposals will further promote the sound development of the internal control audit of universities.

## 2. Literature Review

Academic research on the internal control of enterprises has a long history, but research on the internal control of administrative institutions is more recent and on the rise. At present, the research on the internal control audit of universities has mainly occurred in three aspects, which are the defects of the internal control audit of universities, the causes of those defects, and suggestions for the optimization of the internal control audit of universities.

The defects of internal control audit in universities mainly include seven aspects. First, the audit coverage of internal control audit is not comprehensive enough, and the content is relatively narrow [1–4]. Second, the use of internal control audit results is not ideal [2, 5, 6]. Third, the internal control audit mainly focuses on postevaluation and lacks prior control and in-process supervision, and the audit process is not sound [2, 7–9]. Fourth, internal control audit lacks mature risk assessment mechanism and effective internal control environment [7, 10, 11]. Fifth, the system of internal control self-assessment in universities is not perfect and lacks a complete index system [4, 5, 7, 12–16]. Sixth, the internal control audit implementation procedure is not standardized, and the focus is comparatively limited [4, 17]. Seventh, internal control audit is too arbitrary and tend to be more formal [12, 16].

The causes of defects in internal control audit in universities mainly include seven aspects. First, the concept of internal audit in universities is weak [4, 7, 13, 15, 18, 19]. Second, the process and system of internal control audit in universities are not perfect [4, 5, 18–20]. Third, internal auditors in universities have poor professional qualifications and skills, which cannot adapt to the current new environment [16, 18–20]. Fourth, independence of internal audit institutions is insufficient in China universities [18, 19]. Fifth, there is a big obstacle to the implementation of risk management audit in universities [13, 15]. Sixth, risk of internal control audit in universities is high [5, 12, 14]. Seventh, the internal audit of universities does not focus on cost-effectiveness, resulting in the fact that funds cannot meet audit needs [18].

There are six main suggestions for the optimization of internal control audit in universities. First, a good internal control environment and risk assessment mechanism should be established [10, 12, 18]. Second, universities should change the concept of thinking and implement an internal control audit based on the overall framework of internal control in universities [2, 7, 13]. Third, both the construction of the audit team and the quality of personnel should be strengthened [2, 14, 16, 18, 21]. Fourth, responsibility of the leadership and self-assessment should be intensified [5, 22]. Fifth, universities ought to raise the awareness of internal control of personnel and improve the internal control audit system and evaluation index system [2, 7, 14, 16, 17]. Sixth, universities are supposed to strengthen the supervision function of audit and improve the independence of internal audit institutions in universities [18, 19, 21].

The above research results have established a good foundation for later studies. At the same time, this paper finds that the current research on "internal control audit in universities" is generally less theoretical than empirical. The empirical research mainly adopts analytic hierarchy process and fuzzy evaluation. Due to the lack of empirical support in some documents, the description of "deficiencies and causes of internal control audit in colleges and universities" is basically subjective, and objectivity and universality are doubtful. In the empirical research, the factors affecting the quality of internal control audit in universities have not been clearly distinguished, which may lead to the corresponding optimization suggestions, which may reduce the optimization effect. Therefore, the goal of this paper is to promote empirical research, combined with the investigation and principal component analysis, and fuse the qualitative evaluation information presented by the probabilistic hesitant fuzzy set [23–25] of the internal control audit of some universities directly under MEC, to find the most serious flaws in the current internal control audit of universities in China and the most critical factor affecting the quality of universities' internal control audit influencing factors, to further improve the objectivity, universality, and rationality of the internal control audit optimization recommendations for universities.

## 3. The Questionnaire Survey and Analysis

### 3.1. Sample Selection and Data Sources

*3.1.1. Sample Selection.* This article takes the auditing office of the universities directly under MEC as the survey object. Considering the representativeness of the sample and the availability of data, the selected universities cover the eastern, southern, western, northern, and central regions of China. The main representatives of universities in the eastern region include Donghua University, Nanjing University, Southeast University, China Pharmaceutical University, Hohai University, Nanjing Agricultural University, China University of Mining and Technology, and Jiangnan University. The main representatives of universities in the southern region include South China University of Technology, Chongqing University, Sichuan University, and Southwest University; the main representative of universities in the western region is Xinjiang University (double-first-class university); the main representatives of universities in the northern region are Peking University, Tsinghua University, University of Science and Technology Beijing, and Jilin University; the main representatives of universities in Central Region are Central South University and Wuhan University.

*3.1.2. Data Sources.* Data collection methods are as follows: universities in Jiangsu Province mainly focus on on-site interviews; universities outside Jiangsu Province filled out questionnaires (star questionnaire online version) through online platforms such as the WeChat group (college audit sister group) and the QQ group (college internal audit group and educational audit work exchange group). Note that some alternatives and attributes are judged and then presented by the real numbers and the probabilistic hesitant elements, namely, PHFEs [6, 25], in the questionnaire results.

*3.2. Content of the Investigation.* The questionnaire survey was divided into three parts. The first part explores the basic situation of the internal control audit of universities, including ten aspects: which school the respondent works in; the position of the investigated person in the audit department (director, deputy chief, chief/deputy chief/member of the financial audit section, and chief/deputy chief/ member of the engineering audit section); whether the university under investigation has carried out internal control audit and the starting time and frequency; the object of the internal control audit of the university where the subject is located; the object of the internal control audit of the college where the subject is located (the whole school, two or more colleges/departments/units of the school, and individual colleges/departments/units of the school); subject of the internal control audit of the university in which the respondents serve (school as a whole, two or more colleges/ departments/units of the school, and individual colleges/ departments/units of the school); the scope of the internal control audit of the university where the subject is located (the overall internal control of the school, the comprehensive internal control of two or more aspects of the school, and the internal control of a single aspect of the school); the content and focus of the internal control audit of the universities where the respondents are located (internal control at the school unit level: control environment, risk assessment, control activities, information and communication, and internal supervision; internal control at school business level: internal control of budget operations, revenue and expenditure internal control of business, internal control of government procurement, internal control of asset management, internal control of construction project management, and internal control of contract management); types of audit projects for internal control audits carried out by the universities under investigation (independently carry out internal control audits, integrate budget execution and final accounts audit projects, integrate economic responsibility audit projects, integrate scientific research funding audit projects, integrate financial revenue and expenditure audit projects, and integrate other audit projects).

The second part of the investigation conducted by the university where the respondent is located is to investigate the defects of the internal control audit of the university, mainly including whether the internal control audit method and process are complete, whether the internal audit institution is independent, whether the internal auditor's business knowledge can adapt to the internal control audit, what is the effectiveness of the hired firm's internal control audit management, and whether the internal control audit finds problems and corrects them in a timely manner.

The third part is to investigate the influencing factors of the internal control audit quality of colleges and universities, including the leadership's support for internal control audit, the degree of cooperation of various departments in internal control audit, the lack or imperfection of internal control audit system and process, the degree of restriction on internal control auditing, whether the knowledge and skills of internal auditors meet the needs of internal control auditing, whether the funds for internal control auditing are sufficient, and the use of internal control audit results.

*3.3. Survey Results and Analysis.* The survey took back 110 questionnaires, eliminated 27 invalid questionnaires, and obtained 83 valid questionnaires. The effective response rate was 75.45%.

*3.3.1. Descriptive Statistics of Survey Results*

*(1) Investigation Results on the Basic Situation of Internal Control Audit in Universities.* In this survey, the universities that received the most valid questionnaires were mainly in the eastern region, accounting for 67.74%; the most positive responses were financial auditors, accounting for 63.3% of the questionnaires returned.

During the investigation, we found that although the 2009 "Internal Auditing Practice Guide No. 4-University Internal Auditing" (referred to as the "Guide," the same below) pointed out that the auditors can choose to audit all or part of the unit's internal control, some universities' auditors only accept the overall internal control audit of the school, denying that the audit of the internal control of parts of the school or individual colleges/departments/units is an internal control audit; individual colleges and universities have not independently carried out internal control audit projects but incorporate it into other audit projects. The survey results show that the publication of the "Guide" has only caused the initial concern of the Auditing Offices of various universities, and the implementation has not been popularized. The projects of university's internal control auditing projects have generally commenced throughout the country starting in 2016. That is, MEC clearly requires all universities to report internally. The frequency is usually once a year.

In addition, 60.98% of colleges and universities selected the audit department as the subject of internal control audit; 40.96% selected internal control auditing objects as schools as a whole; 39.76% selected internal control auditing objects as colleges, universities, departments, or units; and nearly 75% of universities chose the overall internal control of the school and two or more situations of comprehensive internal control of schools as the scope of internal control audit; only 30.12% of universities selected content and focus of internal control audit as all aspects of the school unit level and business level. 25.3% of universities selected the type of conducting independent internal control audits. Most universities integrate internal control auditing in other audit projects.

*(2) Investigation Results on the Defects of Internal Control Audit in Universities.* According to the statistics of the survey, nearly 40% of the universities have not established internal control audit system and process; 96.34% of universities have independent internal audit structure; and 14.63% of the universities believe that the internal audit personnel's knowledge and skills can well adapt to the internal control audit; 28% of the universities believe that the management effect of internal control audit conducted by external firms is normal, not very good, or very bad; only 15.85% of the universities can rectify the problem of internal control audits timely.

*(3) Survey Results of Factors Affecting the Quality of Internal Control Audits in Universities.* The survey results show that the leadership of the university lacks sufficient understanding and support for conducting internal control audits, and the degree of cooperation among various departments of the school is not satisfactory. The proportion of "very good" selection is only 1.83%; 3.67% of universities considered that the shortage of internal control audit system and process would hamper the implementation of internal control audits. It is no wonder that universities as a whole have insufficient emphasis on the establishment and perfection of internal control auditing systems; only 11.93% of colleges and universities believe that internal auditors fully possess the knowledge and skills of internal control auditing; 72.48% of colleges and universities think that the necessary funds for internal control audit are adequate or sufficient; 55% of colleges and universities thought that the situation of using internal control results is normal, not very good, or very bad.

*3.4. Survey Results' Analysis Based on Principal Component Analysis.* The survey used SPSS19.0 software to process the surveyed information. According to the severity of internal control auditing defects and the degree of influence of various factors restricting the quality of internal control auditing, the paper evaluates the value from low to high, that is, from the A to D or E option of the questionnaire to the value of 1 to 4 or 5 in turn. Moreover, the surveyed information presented by the PHFEs is transformed based on their score functions. Thus, we can fully show the uncertainty of the qualitative evaluation information given by the experts and directly calculate and model the qualitative information simultaneously.

*3.4.1. Principal Component Analysis of Internal Control Audit Defects.* There are five indicators of internal control audit defects, and this paper uses $Y1$–$Y5$ to indicate as shown in Table 1.

Firstly, the paper used the principal component analysis method; then it extracted three principal components from the five defects of the internal control audit. The cumulative variance contribution rate of the three principal components reaches 94.207% as shown in Table 2, which can well explain the information contained in the original five indicators. Note that there are two indicators, namely, $Y1$ and $Y3$, which are evaluated by the PHFE information and then transformed into real numbers based on their score functions.

Secondly, the paper extracted the eigenvectors corresponding to the three principal components; that is, the coefficients of the three principal components corresponding to the original five indicators are shown in Table 3.

Thirdly, the paper took the proportion of the eigenvalues as the weight; the above three principal components can be weighted and averaged in order to obtain a comprehensive principal component index $F_B$, which can represent the synthesis of the listed defects. The expression is as follows:

$$F_B = 0.639628 \times F_{B1} + 0.221545 \times F_{B2} + 0.138827 \times F_{B3}. \tag{1}$$

Furthermore, the expressions of $F_{B1}$, $F_{B2}$, and $F_{B3}$ are brought into the above formula, and the expression of $F_B$ on the five indexes is

$$\begin{aligned} F_B = {} & 0.488705 \times Y_1 - 0.02888 \times Y_2 + 0.299625 \times Y_3 \\ & + 0.280475 \times Y_4 + 0.270253 \times Y_5. \end{aligned} \tag{2}$$

By comparing the weights of the above five indicators, we can see the importance of the five indicators. According to the weights of the five indicators, the comparison results obtained are as follows.

According to Table 4, it can be seen that the most serious defects in the internal control audit of universities at this stage are as follows: the defects represented by $Y_1$, $Y_3$, and $Y_4$, that is, the internal control audit system and process, are not perfect, and the knowledge and skills of internal auditors in colleges and universities cannot adapt to internal control audits, and the management effects of internal control audit conducted by external firms are not good.

*3.4.2. Principal Component Analysis of Influencing Factors on Internal Control Audit Quality.* There are six main influencing factors that affect the quality of internal control audit in universities. This paper uses $Z1$–$Z6$ to indicate, and the corresponding explanations are given in Table 5. Note that there are three main influencing factors, namely, $Z2$, $Z3$, and $Z4$, which are evaluated by the PHFE information and then transformed into the real numbers based on their score functions.

Firstly, the paper used the principal component analysis method; then, it extracted three principal components from six influencing factors. The cumulative variance contribution rate of the three principal components is 87.8% as shown in Table 6, which can well explain the information contained in the original six indicators.

Secondly, the paper extracted the Characteristic Vectors corresponding to the three principal components; that is, the coefficients of the three principal components corresponding to the original six indicators are shown in Table 7.

Thirdly, the paper took the proportion of the eigenvalues as the weight; the above three principal components can be weighted and averaged in order to obtain a comprehensive principal component index $F_C$, which can represent the synthesis of the listed defects. The expression is as follows:

Table 1: Meaning of indicators of internal control audit defects.

| Index | Meaning | Defect severity values (from low to high) |
|---|---|---|
| $Y_1$ | The degree of imperfection in internal control audit methods and procedures | 1–5 |
| $Y_2$ | Internal audit agency does not have independence | 1–5 |
| $Y_3$ | Internal auditor's business knowledge cannot adapt to the internal control audit's defect severity | 1–5 |
| $Y_4$ | External offices defective in the effectiveness of internal control audit management | 1–5 |
| $Y_5$ | The problem that internal control audit found was not resolved in a timely manner | 1–5 |

Table 2: Eigenvalues and cumulative contribution rates for internal control auditing defects.

| No. | Eigenvalues | Variance contribution rate (%) | Cumulative variance contribution rate (%) |
|---|---|---|---|
| 1 | 1.995 | 60.253 | 60.253 |
| 2 | 0.691 | 20.868 | 81.121 |
| 3 | 0.433 | 13.086 | 94.207 |
| 4 | 0.165 | 4.984 | 99.191 |
| 5 | 0.027 | 0.809 | 100.000 |

Table 3: Characteristic vectors of internal control audit defects.

| Index | The first principal component $F_{B1}$ | The second principal component $F_{B2}$ | The third principal component $F_{B3}$ |
|---|---|---|---|
| $Y_1$ | 0.9211 | −0.3525 | −0.1611 |
| $Y_2$ | −0.0290 | −0.0457 | 0.0106 |
| $Y_3$ | 0.2457 | 0.3838 | 0.3997 |
| $Y_4$ | 0.2365 | 0.8349 | −0.4149 |
| $Y_5$ | 0.1862 | 0.1708 | 0.8024 |

Table 4: Ranking results of internal control audit defect indicators.

| $Y_1$ | $Y_3$ | $Y_4$ | $Y_5$ | $Y_2$ |
|---|---|---|---|---|
| 0.488705 | 0.299625 | 0.280475 | 0.270253 | −0.02888 |

Table 5: Meaning of indicators of internal control auditing factors.

| Index | Meaning | Influence level (from low to high) |
|---|---|---|
| $Z_1$ | Leadership does not support internal control audit | 1–5 |
| $Z_2$ | Noncooperation degree of internal control audit among various departments | 1–5 |
| $Z_3$ | Restriction degree due to the imperfections of the internal control audit system and process | 1–5 |
| $Z_4$ | Incompleteness of knowledge and skills of internal auditors | 1–5 |
| $Z_5$ | Insufficient funding for internal control audit | 1–5 |
| $Z_6$ | Poor use of audit results | 1–5 |

Table 6: Eigenvalues, cumulative variance, and contribution rate of internal control audit influencing factors.

| No. | Eigenvalues | Variance contribution rate (%) | Cumulative variance contribution rate (%) |
|---|---|---|---|
| 1 | 2.199 | 56.900 | 56.900 |
| 2 | 0.824 | 21.322 | 78.222 |
| 3 | 0.370 | 9.578 | 87.800 |
| 4 | 0.224 | 5.794 | 93.594 |
| 5 | 0.153 | 3.959 | 97.553 |
| 6 | 0.095 | 2.447 | 100.000 |

Table 7: Characteristic vectors of internal control audit influencing factors.

| Index | The first principal component $F_{C1}$ | The second principal component $F_{C2}$ | The third principal component $F_{C3}$ |
|---|---|---|---|
| $Z_1$ | 0.4120 | 0.2269 | 0.2893 |
| $Z_2$ | 0.2691 | 0.1774 | 0.5984 |
| $Z_3$ | 0.3756 | −0.9210 | 0.0822 |
| $Z_4$ | 0.4046 | 0.1575 | −0.0838 |
| $Z_5$ | 0.3203 | 0.0936 | −0.7349 |
| $Z_6$ | 0.5921 | 0.1873 | −0.0707 |

$$F_C = 0.648099 \times F_{C1} + 0.242853 \times F_{C2} + 0.109048 \times F_{C3}. \tag{3}$$

Furthermore, the expressions of $F_{C1}$, $F_{C2}$, and $F_{C3}$ are brought into the above formula, and the expression of $F_C$ on the six indexes is

$$\begin{aligned} F_C = {} & 0.353668 \times Z_1 + 0.282740 \times Z_2 + 0.028722 \times Z_3 \\ & + 0.291332 \times Z_4 + 0.150178 \times Z_5 + 0.421516 \times Z_6. \end{aligned} \tag{4}$$

By comparing the weights of the above six indicators, we can see the importance of the six indicators. According to the weights of the six indicators, the comparison results are as follows.

According to Table 8, it can be seen that the most critical influencing factors in the quality of internal control audits in universities at this stage are $Z_6$ (the results of internal audit of colleges and universities are difficult to use rationally), $Z_1$ (the leadership's support for internal control audits is poor), and $Z_4$ (insufficient business knowledge and skills of internal auditors in conducting internal control audits).

*3.4.3. Robustness Test.* Based on regression analysis, the paper verified the credibility of the relationship between internal control audit defects in universities and the factors affecting quality of internal control audit. Then, a multiple regression analysis model was established. Independent variables were $Z_6$, $Z_1$, and $Z_4$; the dependent variable was $F_B$. The influence of these three factors on the comprehensive evaluation index $F_B$ of internal control auditing defects was examined. The parameter estimation results of the regression model are shown in Table 9.

The multiple regression model expression is

$$F_B = 0.400 \times Z_1 + 0.424 \times Z_4 + 0.483 \times Z_6. \tag{5}$$

Among them, the regression coefficient of the regression model is $R^2 = 0.972$, the whole equation passes the significance test, and the $P$ value of each influencing factor is less than 0.1 or even less than 0.05, which indicate that the confidence degree of the regression result is high. Therefore, the regression result clearly shows that $Z_6$ (the results of the auditing of internal control of universities are difficult to use rationally), $Z_1$ (the leadership's support for internal control auditing is poor), and $Z_4$ (the business knowledge and skills of internal auditors conducting internal control auditing are insufficient) are indeed the most critical influencing factors for the comprehensive principal component $F_B$ of internal control audit defects.

Table 8: Ranking results of internal control auditing influencing factors.

| $Z_6$ | $Z_1$ | $Z_4$ | $Z_2$ | $Z_5$ | $Z_3$ |
|---|---|---|---|---|---|
| 0.421516 | 0.353668 | 0.291332 | 0.28274 | 0.150178 | 0.028722 |

Table 9: Multiple regression results' table.

| Index | Coefficient | Standard error | $T$-test statistic | $P$ value |
|---|---|---|---|---|
| $Z_1$ | 0.400 | 0.234 | 1.714 | 0.091 |
| $Z_4$ | 0.424 | 0.193 | 2.194 | 0.031 |
| $Z_6$ | 0.483 | 0.158 | 3.054 | 0.003 |

## 4. Conclusions and Suggestions

The defects of internal control audit in universities are closely related to the quality of internal control audit. To some extent, the fewer the defects of internal control audit, the higher the quality of internal control audit, and vice versa. Based on the above investigation and analysis results with respect to the qualitative evaluation given by experts and presented by the real numbers and PHFEs, this paper is based on the critical degree of the internal control audit defect severity and the influencing factors of internal control audit quality. It is suggested that, according to the principle of importance, the internal audit of universities must be gradually optimized as follows.

*4.1. Establish and Improve the Internal Control Audit System and Process.* A perfect audit system and a standardized audit process are the institutional guarantees for the successful implementation of the audit project. Establishing and improving the internal control audit system and process are the primary way to optimize internal audit in universities. Universities should continue to advance the integration of systems, processes, and informatization based on the school's internal control objectives, information-based service processes, process-oriented service systems, and institutionalized service management and constantly improve internal control audit systems and processes.

*4.2. Promote the Application of the Results of Internal Control Audit Continuously.* Internal control audit and result application are the relationship between process and end point. In order to reflect the value of internal control audit, it is necessary to strengthen the application of the results of internal control audit. Universities should promote the follow-up review system of disclosure of internal control audit results and feedback on the implementation of reforms. The evaluation of the use of internal control audit results should be used as the annual assessment indicator of the relevant person in charge of the relevant departments of the school. This index linked to personal interests is directly related to its removal, promotion, punishment, reward, etc.

*4.3. Raise the Attention of the Leadership.* The construction of internal control is a leading project, as is the internal control audit. Only when university presidents have paid enough attention can the development of internal control audits truly go smoothly. Colleges and universities need to increase publicity, emphasize the importance of internal control audit, and make them deeply rooted in people's minds, so as to mobilize the enthusiasm of management, teachers, and students to cooperate; universities should increase information transparency and strengthen information among leaders, management, and students, so as to improve the effectiveness of communication and grasp the core business of internal control timely and systematically. The concerted efforts of all levels of the university can help to purify the internal control audit environment and improve the efficiency and quality of the internal control audit.

*4.4. Strengthen the Construction of the Internal Audit Team.* The second most serious flaw in the defect of internal control audit in universities and the third most important factor in the factors affecting the internal audit quality of universities are "the adaptability of the knowledge and skills of internal auditors is not powerful. The degree of weakness is that the internal auditors have insufficient business knowledge and skills to carry out internal control audits." In fact, the third-ranked defect, "the poor management effect of internal control audit conducted by external offices," is a deficient derivative of internal auditing power. Therefore, it is crucial to strengthen the construction of an internal audit team. It is necessary to establish a long-term auditing personnel training mechanism and motivate internal auditors to learn constantly. While improving their own auditing knowledge and skills, they must also pay attention to improving the management capabilities and level of external auditors of internal auditors.

*4.5. Future Works.* In this paper, we use the real numbers and PHFEs to show the experts and DMs' qualitative evaluation information in the given questionnaires. Then, the principal component analysis is applied to deal with these data and derive the above conclusions. It is pointed out that the PHFEs are used to fully show the subjective information and then transformed into the score values. After that, the principal component analysis can be effectively used. As the experts and DMs concluded, the PHFE is a convenient tool to describe their subjective evaluation. Then, how to further reasonably and effectively model the given PHFE information could be an interesting research direction in future. Moreover, we can find that the PHFE information is transformed firstly and then the principal component analysis is used to calculate them. In our opinion, this process is imperfect and then the whole model should be studied to analyze the principal components and calculate the PHFEs. Therefore, these directions are two important study issues to improve our methods which could be used to judge the subject defects and object defects of the current internal audit of universities in China, as well as the influencing factors that affect the quality of the internal control audit of universities. This will be our next works.

## Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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*Research Article*

# A Novel Group Decision-Making Method Based on Generalized Distance Measures of PLTSs on E-Commerce Shopping

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In multiattribute group decision-making (MAGDM), due to quantity, fuzziness, and complexity of evaluation linguistic information on commodities, traditional distance measures need to be extended to the integration of evaluation information under a multigranular probabilistic linguistic environment. A more reasonable method is proposed to deal with the missing value in the evaluation information. On the basis of the generalized distance measures and filling in the missing evaluation information, some novel distance measures between two multigranular probabilistic linguistic term sets (PLTSs) are presented in this paper. Based on these distance measures, three extended decision-making (DM) algorithms based on TOPSIS, the extended TOPSIS, and VIKOR are proposed, which are MGPL-TOPSIS, MGPL-ETOPSIS, and MGPL-VIKOR, respectively. The case analyses on purchasing a car are provided to illustrate the application of the extended multiattribute group decision-making (MAGDM) algorithms. Then, sensitivity analyses based on PT are proposed as well. In particular, the extended TOPSIS method is presented. These results demonstrate the novelty, feasibility, and rationality of the distance measures between two multigranular PLTSs proposed in this paper.

## 1. Introduction

With the popularity of the Internet, online shopping has become an important way of daily shopping. Shopping on an e-commerce platform, one alternative can be evaluated on different platforms, or one alternative may be described on the same platform by different granular fuzzy linguistic information; for example, we suppose that a consumer wants to buy a new energy car from USD 20,000 to USD 30,000. In this case, they can visit the auto home website to learn about comments of these cars from other consumers and make a reasonable decision to buy a car. Consumers who have purchased these cars can comment on them through many channels. They can score cars on the same platform, post word-of-mouth comments, post through the community, or evaluate through different media. The granularity of evaluation information is different in these evaluation channels. How can we more effectively make purchase decisions of products with different granularity evaluation?

In DM, due to the complexity of the real world, Zadeh proposed fuzzy set [1]. Furthermore, Zadeh proposed linguistic variables to represent uncertain and imprecise information intuitively, expressing human thoughts better [2]. In practice, the DMs are always hesitant among some evaluation values, and then, Torra proposed hesitant fuzzy set (HFS) first [3]. The hesitant fuzzy linguistic term set (HFLTS) was proposed to tackle the flexibility of the membership degree of HFS [4]. However, in some cases, the linguistic variables may be uncertain because of the complexity of DM problems, and the DMs may have different preferences with different belief degrees [5], possibility distributions [6], and importance degrees [7]. Then, using several linguistic terms to express evaluation information is more scientific. In this case, the hesitant fuzzy set based on probability was proposed as the probability-based hesitant fuzzy set (PHFS) [8, 9], and probabilistic linguistic term set (PLTS) by Qi Pang et al. is proposed to describe the object more effectively [10]. The PLTSs allow the DMs to give several linguistic terms, serving as the value of a linguistic variable, which enriches the flexibility of the expression of linguistic information. The DMs can express their linguistic evaluations or preference information better. Meanwhile, the PLTSs can provide different importance degrees or weights of all the possible evaluation preferences of one object.

The traditional method for group decision-making (GDM) under the same granular linguistic information cannot integrate hybrid evaluation information. Therefore, multigranular linguistic term sets need to be described efficiently. Then, how can two PLTSs with multigranular probabilistic linguistic information be measured? Considerable research has been conducted about the distance measures; for example, Zhai et al. presented probabilistic interval-valued intuitionistic hesitant fuzzy sets [11]. Wu et al. proposed a probabilistic linguistic MULTIMOORA method in multicriteria group decision-making (MCGDM) based on the probabilistic linguistic expectation function [12]. However, a few research types on distance measures of PLTSs with multigranular linguistic information remain. Then, distance measures for PLTSs with multigranular linguistic details need to be extended. Some DM models have been used to deal with probabilistic linguistic information. Gou and Xu proposed a new score function of linguistic terms and defined the operations of PLTSs [13]. Pang et al. proposed a probabilistic linguistic representation model based on TOPSIS [10]. Liu and Li presented the PROMTHEE II method [14]. Liao et al. gave the PL-LINMAP method for multiple criteria decision-making (MCDM) with PLTSs [15]. Li and Wang presented an extended QUALIFLEX plan for selecting green suppliers [16]. Wu and Liao proposed the ORESTE method with probabilistic linguistic information [17]. Liao et al. proposed the PL-ELECTRE III method with PLTSs [18]. Abdolhamid et al. extended the VIKOR method for GDM with extended hesitant fuzzy linguistic information [19]. Zhang et al. proposed a probabilistic linguistic method based on VIKOR to evaluate green supply chain initiatives [20]. Zhang and Xu et al. used the probabilistic linguistic VIKOR method to tackle water-human harmony evaluation [21]. Gou et al. improved the VIKOR method for the application of smart healthcare with probabilistic double hierarchy linguistic term set [22].

Some researchers have applied the PLTS to solve some practical problems; for example, Hao et al. presented a probabilistic dual-hesitant fuzzy set and its application in risk evaluation [23]. Gao et al. proposed a dynamic reference point method for emergency response [24]. Sharaf extended TOPSIS to similarity measures for MADM and applied it to network selection [25]. Muhammad Sajjad Ali Khan et al. extended TOPSIS for MCDM [26]. Asif Ali and Tabasam Rashid presented a generalized interval-valued trapezoidal fuzzy best-worst MCDM method [27]. Rajkumar Verma presented MAGDM based on aggregation operators for linguistic trapezoidal fuzzy intuitionistic fuzzy sets [28].

However, the traditional linguistic information missing is usually filled with the minimum value or ignored. This method is flawed. Linguistic evaluation information is closely related with the psychological activities of decision-makers. On the basis of the discussion above, we present a novel method to deal with the missing value in the evaluation information and generalized distance measures for the PLTSs with multigranular linguistic information. Then, we apply it to solve the problems of MAGDM on the decision-making of purchasing a car.

Based on the discussion above, this paper proposes distance measures for the PLTSs with multigranular linguistic information and then applies them in MAGDM.

## 2. Preliminaries

*2.1. Linguistic Term Sets.* The DMs can use LTSs to describe their preferences on the considered alternatives. The additive LTS is used most widely, which is defined as follows [28]:

$$S = \{S_\alpha | \alpha = 0, 1, \ldots, g-1\}, \tag{1}$$

where $S$ is a $g$-granular fuzzy linguistic set; $S_\alpha$ is a linguistic variable with $S_0$ and $S_g$, namely, the lower and upper limits of the linguistic terms; and $g$ is a positive integer.

Considering the situations where the DMs may hesitate among several possible values in DM, which is similar to the hesitant fuzzy set, the concept of HFLTSs is as follows.

*Definition 1* (see [29]). We let $S = \{S_0, S_1, \ldots S_{g-1}\}$ be a LTS, and then, HFLTs $b_s$ is an ordered finite subset of consecutive linguistic terms $S$.

*2.2. Probabilistic Linguistic Term Sets*

*Definition 2* (see [10]). We let $S = \{S_0, S_1, \ldots S_{g-1}\}$ be an LTS. A PLTS is defined as

$$L(P) = \left\{ \left(L^{(k)} P^{(k)}\right) | L^{(k)} \in S, P^{(k)} \geq 0, k = 1, 2, \ldots, \#L(P), \sum_{k=1}^{\#L(P)} P^{(k)} \leq 1 \right\}, \tag{2}$$

where $L^{(k)}P^{(k)}$ is the linguistic term $L^{(k)}$ associated with the probability $P^{(k)}$ and $\#L(P)$ is the number of all different linguistic terms in $L(P)$.

If $\sum_{k=1}^{\#L(P)} P(k) = 1$, then we obtain the complete information on the probabilistic distribution with all the possible linguistic terms. If $\sum_{k=1}^{\#L(P)} P(k) < 1$, then partial ignorance exists because of current insufficient evaluation information. Especially, $\sum_{k=1}^{\#L(P)} P(k) = 0$ means complete ignorance. Therefore, handling ignorance $L(P)$ is crucial research for the application of PLTSs.

*Definition 3* (see [10]). Given a PLTS $L(P)$ with $\sum_{k=1}^{\#L(P)} P(k) < 1$, then the associated PLTS $\dot{L}(P)$ is defined by

$$\dot{L}(P) = \left\{ L^{(k)}\left(\dot{P}^{(k)}\right) | k = 1, 2, \ldots, \#L(P) \right\}, \tag{3}$$

where $\dot{P}^{(k)} = P^{(k)} / \sum_{k=1}^{\#L(P)} P^{(k)}$ for all $k = 1, 2, \ldots, \#L(P)$.

*2.3. Multigranular Probabilistic Linguistic Term Sets*

*Definition 4* (see [30]). We let $S = \left\{S_0, S_1, \ldots S_{g-1}\right\}$ be $g$-granular LTS and $S' = \left\{S_0, S_1, \ldots S_{g\prime-1}\right\}$ be $g'$-granular LTS. $L_1(P)$ and $L_2(P)$ are two different granular PLTSs on the attribute set $x = \{x_1, x_2, \ldots x_n\}$. Multigranular PLTSs can be defined as

$$\begin{aligned} L_1(P) &= \left\{ L_1^{(k_1)}\left(P_1^{(k_1)}\right) | L_1^{(k_1)} \in S, P_1^{(k_1)} \geq 0, \quad k_1 = 1, 2, \ldots, \#L_1(P_1), \sum_{k_1=1}^{\#L_1(P)} P_1^{(k_1)} \leq 1 \right\}, \\ L_2(P) &= \left\{ L_2^{(k_2)}\left(P_2^{(k_2)}\right) | L_2^{(k_2)} \in S', P_2^{(k_2)} \geq 0, \quad k_2 = 1, 2, \ldots, \#L_2(P_2), \sum_{k_2=1}^{\#L_2(P)} P_2^{(k_2)} \leq 1 \right\}, \end{aligned} \tag{4}$$

where $L_1^{(k_1)}(P_1^{(k_1)})$ is the linguistic term $L_1^{(k_1)}$ associated with the probability $P_1^{(k_1)}$ and $L_2^{(k_2)}(P_2^{(k_2)})$ is the linguistic term $L_2^{(k_2)}$ related to the probability $P_2^{(k_2)}$.

The numbers of linguistic terms in PLTSs are usually different for a DM. Therefore, the numbers of linguistic terms need to be added, in which numbers are relatively small. Then, the numbers of linguistic terms are the same.

The numbers of $L_1(P)$ and $L_2(P)$ are denoted as $\#L_1(P)$ and$\#L_2(P)$, respectively. If $\#L_1(P) < \#L_2(P)$, then$\#L_1(P) - \#L_2(P)$ linguistic terms are added to $L_2(P)$, leading to the numbers of $L_1(P)$ and $L_2(P)$ to be equal. The added linguistic terms are the smallest ones,$L_2(P)$, and all the linguistic probabilities are zero.

*Definition 5* (see [30]). We let $L_1(P)$ and $L_2(P)$ be two multigranular PLTSs. Then, the normalization processes are as follows:

(1) If $\sum_{k_i=1}^{\#L_i(P)} P_i^{(k_i)} < 1$ by Definition 3, we calculate $\dot{L}_i(P), i = 1, 2$.

(2) If $\#L_1(P) \neq \#L_2(P)$, then by Definition 4, we add some elements to the one with the smaller number of elements.

The PLTSs obtained by Definition 5 are denoted the normalized PLTSs. Conveniently, the normalized PLTSs are marked by $L_1(P)$ and $L_2(P)$ as well.

Given the positions of elements in a PLTS are arbitrary, we need to obtain the ordered PLTSs first, leading to the operational results in PLTSs being determined directly.

*Definition 6* (see [30]). We let $S = \left\{S_0, S_1, \ldots S_{g-1}\right\}$ be $g$-granular LTS. Given a PLTS, $L(P) = \{L^{(k)}(P^{(k)}) | L^{(k)} \in S, P^{(k)} \geq 0, k = 1, 2, \ldots \#L(P), \sum_{k=1}^{\#L(P)} P^{(k)} \leq 1\}$, and $r^{(k)}(L^{(k)})$ is the subscript of the linguistic term $L^{(k)}L(P)$. It is named an ordered multigranular PLTS if the $L^{(k)}(P^{(k)})(k = 1, 2, \ldots, \#L(P))$ descending order's values arrange$\alpha(L^{(k)}) = (r^{(k)}(L^{(k)})/g) \times P^{(k)}(k = 1, 2, \ldots, \#L(P))$ linguistic terms.

## 3. Main Results in Discrete Case

*3.1. Generalized Distance Measures between Multigranular PLTSs.* The traditional method of handling ignorance is not very scientific. Then, we extend the method and present the novel method to calculate the missing values. Inspired by [31], we present Definition 7 as follows.

*Definition 7.* We let $L_1(P)$ and $L_2(P)$ be two multigranular PLTSs. Then, the extended normalization processes are as follows:

(1) If $\sum_{k_i=1}^{\#L_i(P)} P_i^{(k_i)} < 1$, then by Definition 3, we calculate $\dot{L}_1(P), i = 1, 2$.

(2) If $\#L_1(P) \neq \#L_2(P)$, then we add some elements to the one with

$$\alpha\left(L^{(k)}\right) = t \times \max\left\{\frac{r^{(k)}\left(L^{(k)}\right)}{g}\right\} + (1-t)\min\left\{\frac{r^{(k)}\left(L^{(k)}\right)}{g}\right\}, 0 \leq t \leq 1, k = 1, 2, \ldots, \#L(P), \tag{5}$$

where $t$ represents the risk preferences of the DMs. If $t > 0.5$, it means the DMs are optimistic. If $t > 0.5$, it means they are pessimistic. The value $t$ should be given by the DMs previously.

Conveniently, we suppose that the PLTEs are the extended normalized and ordered multigranular PLTEs as Definitions 5 and 7 in all the following sections in this paper.

The normalized distance measures are extended, and the generalized distance measures between two multigranular PLTSs in discrete cases are presented as follows.

*Example 1.* Let $S = \{S_0, S_1, \ldots, S_6\}$, $L_1(P) = \{S_3(0.4), S_2(0.2), S_1(0.2)\}$ and $L_2(P) = \{S_3(0.1), S_2(0.3)\}$ be two PLTSs, then (1) according to Definition 7, $\dot{L}_1(P) = \{S_3(0.5), S_2(0.25), S_1(0.25)\}$, $\dot{L}_2(P) = \{S_2(0.75), S_3(0.25)\}$. (2) Since $\#L_2(P) < \#L_1(P)$, then we add the linguistic term $t \times (1.5/5) + (1-t) \times (0.75/5) = 0.15t + 0.15$. When $t = 0.2$, then after normalization, $\alpha(\dot{L}_1^{(k)}) = \{0.3, 0.1, 0.05\}$, $\alpha(\dot{L}_2^{(k)}) = \{0.3, 0.18, 0.15\}$.

Conveniently, suppose the PLTEs are the extended normalized and ordered multigranular PLTEs as Definition 5 and Definition 7 in all the following sections in this paper.

The normalized distance measures are extended, and the generalized distance measures between two multigranular PLTSs in discrete cases are proposed as follows.

*Definition 8.* We let $L_1^{(k)}(P_1^{(k)}) \in L_1(P)$ and $L_2^{(k)}(P_2^{(k)}) \in L_2(P)$ be two PLTEs as in Definition 4. Then, the distance measured between them is defined as

$$d\left(L_1^{(k)}\left(P_1^{(k)}\right), L_2^{(k)}\left(P_2^{(k)}\right)\right) = \left|\alpha\left(L_1^{(k)}\left(P_1^{(k)}\right)\right) - \alpha\left(L_2^{(k)}\left(P_2^{(k)}\right)\right)\right|. \tag{6}$$

*Example 2.* Let $\alpha(\dot{L}_1^{(k)}) = \{0.1\}$, $\alpha(\dot{L}_2^{(k)}) = \{0.18\}$, then $d(L_1^{(k)}(P_1^{(k)}), L_2^{(k)}(P_2^{(k)})) = |0.1 - 0.18| = 0.08$.

*Definition 9.* We let $L_1^{(k)}(P_1^{(k)}) \in L_1(P)$ and $L_2^{(k)}(P_2^{(k)}) \in L_2(P)$ be two PLTEs on the attribute set, denoted by $x = \{x_1, x_2, \ldots, x_n\}$, where $x_j$ is the $j$th attribute of the alternatives and $j = 1, 2, \ldots, n$. Then, the generalized Hamming distance between $L_1(P)$ and $L_2(P)$ is defined as follows:

$$d_{hd}(L_1(P), L_2(P)) = \frac{1}{L}\sum_{k=1}^{L} d\left(L_1^{(k)}\left(P_1^{(k)}\right) L_2^{(k)}\left(P_2^{(k)}\right)\right), \tag{7}$$

where $\#L_1(P) = \#L_2(P) = L$.

*Example 3.* Let $\alpha(\dot{L}_1^{(k)}) = \{0.3, 0.1, 0.05\}$ and $\alpha(\dot{L}_2^{(k)}) = \{0.3, 0.18, 0.15\}$, then $d_{hd}(L_1(P), L_2(P)) = (1/3)(|0.3 - 0.3| + |0.1 - 0.18| + |0.05 - 0.15|) = 0.06$.

The generalized Euclidean distance between $L_1(P)$ and $L_2(P)$ is as follows:

$$d_{ed}(L_1(P), L_2(P)) = \left[\frac{1}{L}\sum_{k=1}^{L}\left(d\left(L_1^{(k)}\left(P_1^{(k)}\right), L_2^{(k)}\left(P_2^{(k)}\right)\right)\right)^2\right]^{(1/2)}. \tag{8}$$

*Example 4.* Let $\alpha(\dot{L}_1^{(k)}) = \{0.3, 0.1, 0.05\}$ and $\alpha(\dot{L}_2^{(k)}) = \{0.3, 0.18, 0.15\}$, then $d_{ed}(L_1(P), L_2(P)) = \sqrt{(1/3)[(0.3 - 0.3)^2 + (0.1 - 0.18)^2 + (0.05 - 0.15)^2]} = 0.0739$.

The generalized distance between $L_1(P)$ and $L_2(P)$ is as follows:

$$d_{gd}(L_1(P), L_2(P)) = \left[\frac{1}{L}\sum_{k=1}^{L}\left(d\left(L_1^{(k)}\left(P_1^{(k)}\right), L_2^{(k)}\left(P_2^{(k)}\right)\right)\right)^{\lambda}\right]^{(1/\lambda)}, \quad \lambda > 0 \tag{9}$$

Significantly, if $g = g'$, the generalized distance reduces to the generalized Hamming distance. If $n = 1, \lambda = 1$, the generalized distance reduces to the normalized Hamming distance. If $n = 1, \lambda = 2$, it reduces to the normalized Euclidean distance. Definition 9 extends the normalized Hamming distance and Euclidean distance.

### 3.2. Generalized Weighted Distance Measures between Multigranular PLTSs.

We let $S = \{S_0, S_1, \ldots S_{g-1}\}$ be $g$-granular LTS and $S' = \{S_0, S_1, \ldots S_{g'-1}\}$ be $g'$-granular LTS. $L_1(P)$ and $L_2(P)$ are two different granular PLTSs on the attribute set $x = \{x_1, x_2, \ldots, x_n\}$ with the weight vector $w = \{w_1, w_2, \ldots, w_n\}^T$, where $x_j$ is the $j$th attribute of the alternatives, $j = 1, 2, \ldots, n, 0 \le w_j \le 1, \sum_{j=1}^{n} w_j = 1$. Then, the normalized weighted distance measures are extended similar to Section 3.1. The generalized weighted distance measures between $L_1(P)$ and $L_2(P)$ are defined as follows.

*Definition 10.* A generalized weighted distance between $L_1(P)$ and $L_2(P)$ is defined as

$$d_{gwd}(L_1(P), L_2(P)) = \left[\sum_{j=1}^{n}\frac{w_j}{L}\sum_{k=1}^{L}\left(d\left(L_1^{(k)}\left(P_1^{(k)}\right), L_2^{(k)}\left(P_2^{(k)}\right)\right)\right)^{\lambda}\right]^{1/\lambda}, \quad \lambda > 0. \tag{10}$$

Primarily, two exceptional cases of the generalized weighted distance are as follows:

(1) If $\lambda = 1$, then generalized weighted distance reduces to the generalized weighted Hamming distance as follows:

$$d_{gwhd}(L_1(P), L_2(P)) = \sum_{j=1}^{n}\frac{w_j}{L}\sum_{k=1}^{L}\left(d\left(L_1^{(k)}\left(P_1^{(k)}\right), L_2^{(k)}\left(P_2^{(k)}\right)\right)\right). \tag{11}$$

(2) If $\lambda = 2$, then generalized weighted distance reduces to the generalized weighted Euclidean distance as follows:

$$d_{gwed}(L_1(P), L_2(P)) = \left[\sum_{j=1}^{n}\frac{w_j}{L}\sum_{k=1}^{L}\left(d\left(L_1^{(k)}\left(P_1^{(k)}\right), L_2^{(k)}\left(P_2^{(k)}\right)\right)\right)^2\right]^{(1/2)}. \tag{12}$$

## 4. Applications of Generalized Distance Measures in MAGDM

*4.1. Description of the Problem.* A set of alternatives $A = (A_1, A_2, \ldots, A_m)$ is presented, the attribute vector is $x = (x_1, x_2, \ldots, x_n)^T$, $w = (w_1, w_2, \ldots, w_n)^T$ is the weight vector, and $x_j$ is the $j$th attribute of the alternatives,

$j = 1, 2, \ldots, n$, $0 \le w_j \le 1$, $\sum_{j=1}^{n} w_j = 1$. The DMs assess $m$ alternatives on $n$ attributes by utilizing a linguistic term set to get a set of linguistic decision matrices.

Then, the evaluation of linguistic information is used to make up a multigranular probabilistic linguistic decision matrix as follows:

$$R = \left[L_{ij}(P)\right]_{m\times n} = \begin{bmatrix} L_{11}(P) & L_{12}(P) & \cdots & L_{1n}(P) \\ L_{21}(P) & L_{22}(P) & \cdots & L_{2n}(P) \\ \vdots & \vdots & \ddots & \vdots \\ L_{m1}(P) & L_{m2}(P) & \cdots & L_{mn}(P) \end{bmatrix}, \quad (13)$$

where $L_{ij}(P) = \left\{L_{ij}^{k_{ij}}\left(P_{ij}^{k_{ij}}\right) \mid L_{ij}^{(k_{ij})} \in S_i, P_{ij}^{k_{ij}} \le 0, k_{ij} = 1, 2, \cdots, \#L_{ij}(P), \sum_{k_{ij}=1}^{\#L_{ij}(P)} P_{ij}^{(k_{ij})} \le 1\right\}$ is a multigranular PLTS denoting the degree of the alternative $A_i$ on the attribute $x_j$, $S_i = \left\{S_0, S_1, \ldots, S_{g_i-1}\right\}$ is a $g_i$-granular fuzzy linguistic set, and $r_{ij}^{(k_{ij})}$ is the subscript of the linguistic term $L_{ij}^{(k_{ij})}\left(P_{ij}^{(k_{ij})}\right)$, which is associated with the probability $P_{ij}^{(k_{ij})}$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$.

In MAGDM problems, the attributes can be classified into two types: benefits and costs. The higher the benefit attribute, the better the situation, whereas the opposite it applies to the cost attribute. In this paper, we suppose that the attributes are benefits.

On the basis of the generalized distance measures of Section 3, the extended TOPSIS is presented as follows.

*4.2. MGPL-TOPSIS Algorithm.* MGPL-TOPSIS algorithm is a MAGDM approach based on TOPSIS under multigranular probabilistic fuzzy linguistic environment proposed as follows.

*Step 1.* Individual preferences over the alternatives on different attributes provided by experts are gathered as $R = [L_{ij}(P)]_{m\times n}$.

*Step 2.* (see [30]). The weight vector $w = (w_1, w_2, \ldots, w_n)^T$ of $n$ attributes $x = (x_1, x_2, \ldots, x_n)^T$ is computed as follows:

$$\begin{aligned} E_j = \frac{1}{m}\sum_{i=1}^{m}\Bigg(1 - \frac{2}{L_j T}\sum_{i=1}^{L_j}\Bigg(\Bigg(1 + q r_{ij}^{(k_{ij})}\Bigg) \\ \cdot \ln\left(1 + q r_{ij}^{(k_{ij})}\right) + \left(1 + q\left(1 - r_{(L_j-i+1)j}^{(k_{ij})}\right)\right) \\ \cdot \ln\left(1 + q\left(1 - r_{(L_j-i+1)j}^{(k_{ij})}\right)\right)\Bigg)\Bigg/2 \\ - \left(\left(2 + q r_{ij}^{(k_{ij})} + q\left(1 - r_{(L_j-i+1)j}^{(k_{ij})}\right)\right)\Big/2\right) \\ \cdot \ln\left(\left(2 + q r_{ij}^{(k_{ij})} + q\left(1 - r_{(L_j-i+1)j}^{(k_{ij})}\right)\right)\Big/2\right)\Bigg)\Bigg), \end{aligned} \quad (14)$$

$$w_j = \frac{1 - E_j}{n - \sum_{j=1}^{n} E_j}, \quad i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n,\ L_j = \#L_{ij}(P),$$

where $T = (1+q)\mathrm{In}(1+\mathrm{q}) - (2+q)(\mathrm{In}(2+\mathrm{q}) - \mathrm{In}\,2)$, $q > 0$, $0 \le w_j \le 1$, and $\sum_{j=1}^{n} w_j = 1$. In this paper, we let $q = 2$ [30].

*Step 3.* The positive ideal solution and the negative ideal solution, respectively, are calculated.

The probabilistic linguistic positive ideal solution (PLPIS) and the probabilistic linguistic negative ideal solution (PLNIS) are defined, respectively.

The PLNIS of the alternatives is

$$L^{+} = \left(L_1(P)^{+}, L_2(P)^{+}, \ldots, L_n(P)^{+}\right). \quad (15)$$

The PLNIS of the alternatives is

$$L^{-} = \left(L_1(P)^{-}, L_2(P)^{-}, \ldots, L_n(P)^{-}\right), \quad (16)$$

where $L_j(P)^{+} = L_{\triangle}$, in which $\triangle = \max_{i,j,k}\left\{\alpha\left(L_{ij}^{(k_{ij})}\right)\right\}$, and $L_j(P)^{-} = L_{\nabla}$, in which $\nabla = \min_{i,j,k}\left\{\alpha\left(L_{ij}^{(k_{ij})}\right)\right\}$.

$$\alpha\left(L_{ij}^{(k_{ij})}\right) = \frac{r_{ij}^{(k_{ij})}\left(L_{ij}^{(k_{ij})}\right)}{g_i} \times P_{ij}^{(k_{ij})}, \quad i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n,\ k_{ij} = 1, 2, \ldots, \#L_{ij}(P). \quad (17)$$

*Step 4.* The distance between $A_i$ and $L^+$, denoted by $d(A_i, L^+)$, and the distance between $A_i$ and $L^-$, denoted by $d(A_i, L^-)$, are computed.

*Step 5.* The closeness degree of each alternative is computed as follows:

$$\mathrm{CD}_i = \frac{(1-\delta)d(A_i, L^-)}{\delta\, d(A_i, L^+) + (1-\delta)d(A_i, L^-)}, \tag{18}$$

where the parameter $\delta \in [0, 1]$ represents the risk preferences of the decision-makers. If $\delta < 0.5$, then the DMs are optimistic. If $\delta > 0.5$, then they are pessimistic. The value $\delta$ should be given by the DMs previously.

*Step 6.* The alternatives are ranked according to the values $\mathrm{CD}_i$ of $A_i$.

The larger the closeness degree, the better the alternative.

*4.3. MGPL-ETOPSIS Algorithm.* MGPL-ETOPSIS algorithm is based on the extended TOPSIS by Qi Pang et al. [10], which is a MAGDM approach under multigranular probabilistic fuzzy linguistic environment proposed as follows:

Step 1: individual preferences over the alternatives on different attributes provided by experts are gathered as $R = [L_{ij}(P)]_{m\times n}$.

Step 2: the weight vector (see [30]) $w = (w_1, w_2, \ldots, w_n)^T$ of $n$ attributes $x = (x_1, x_2, \ldots, x_n)^T$ is computed as follows, as seen in equation (14).

Step 3: the positive ideal solution and the negative ideal solution, respectively, are calculated.

Step 4: the distance between $A_i$ and $L^+$, denoted by $d(A_i, L^+)$, and the distance between $A_i$ and $L^-$, denoted by $d(A_i, L^-)$, are computed.

Step 5: compute the closeness coefficient $\mathrm{CI}_i$ of each alternative $A_i$ as follows:

$$\mathrm{CI}_i = \frac{d(A_i, L^-)}{d_{\max}(A_i, L^-)} - \frac{d(A_i, L^+)}{d_{\min}(A_i, L^+)}. \tag{19}$$

Rank the alternatives by $\mathrm{CI}_i$. Obviously, the bigger the closeness coefficient, the better the alternative.

*4.4. MGPL-VIKOR Algorithm.* MGPL-VIKOR algorithm is a MAGDM approach based on VIKOR under multigranular probabilistic fuzzy linguistic environment proposed as follows:

Step 1: individual preferences over the alternatives on different attributes provided by experts are gathered as $R = [L_{ij}(P)]_{m\times n}$.

Step 2: the weight vector (see [30]) $w = (w_1, w_2, \ldots, w_n)^T$ of $n$ attributes $x = (x_1, x_2, \ldots, x_n)^T$ is computed as follows, as seen in equation (14).

Step 3: compute the distance $d(L_{ij}(P), L_j(P)^+)$ and $d(L_j(P)^+, L_j(P)^-)$.

Step 4: compute the whole benefit $\mathrm{MU}_i$ and individual regret $\mathrm{MR}_i$, $\mathrm{MU}^+$, $\mathrm{MU}^-$, $\mathrm{MR}^+$, and $\mathrm{MR}^-$, respectively, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$.

Step 5: compute the compromise index $\mathrm{MC}_i$ of $A_i$. Rank the alternatives, according to $\mathrm{MC}_i$. Obviously, the bigger the compromise index, the better the alternative [31].

The definitions of whole benefit, $\mathrm{MU}_i$, individual regret $\mathrm{MR}_i$, and the compromise index $\mathrm{MC}_i$, are as follows:

$$\begin{aligned}
\mathrm{MU}_i &= \sum_{j=1}^{n} w_j \frac{d(L_{ij}(P), L_j(P)^+)}{d(L_j(P)^+, L_j(P)^-)},\\
\mathrm{MR}_i &= \max\left[w_j \frac{d(L_{ij}(P), L_j(P)^+)}{d(L_j(P)^+, L_j(P)^-)}\right],\\
\mathrm{MC}_i &= \nu \frac{\mathrm{MU}_i - \mathrm{MU}^+}{\mathrm{MU}^- - \mathrm{MU}^+} + (1-\nu)\frac{\mathrm{MR}_i - \mathrm{MR}^+}{\mathrm{MR}^- - \mathrm{MR}^+},
\end{aligned} \tag{20}$$

where $\mathrm{MU}^+ = \max\{\mathrm{MU}_i\}$, $\mathrm{MU}^- = \min\{\mathrm{MU}_i\}$, $\mathrm{MR}^+ = \max\{\mathrm{MR}_i\}$, and $\mathrm{MR}^- = \min\{\mathrm{MR}_i\}$, $i = 1, 2, \ldots, m$. The parameter $\nu$ denotes the weight of the strategy of the maximum whole benefits, whereas $1 - \nu$ is the weight of the individual regret strategy.

Rank the alternatives by $\mathrm{MC}_i$. The higher the $\mathrm{MC}_i$, the more preferred the alternative.

## 5. Illustrative Example

In the real world, people usually encounter the DM problems, such as healthcare management, project evaluation, education assessment, emergency management, and smart city construction, especially COVID-19 prevention and control; for example, someone will purchase one of the five new energy cars, who can find all kinds of evaluation information of these five cars through the network for the development of Internet information. The more professional and popular website about auto information is the "Auto Home" website. Some evaluation information of these cars is presented on the "Auto Home" website in three ways: scoring data, word-of-mouth data, and forum reviews on eight attributes. The eight attributes are space $(x_1)$, power $(x_2)$, manipulate $(x_3)$, power consumption $(x_4)$, comfort $(x_5)$, appearance $(x_6)$, interior decoration $(x_7)$, and cost performance $(x_8)$, respectively. The five cars are Tiggo3Xe $(A_1)$, ZhongTaiE200 $(A_2)$, Yuan New Energy $(A_3)$, Song New Energy $(A_4)$, and Qin Pro New Energy $(A_5)$. Given that scoring data online is a five-point system, and the scoring data can be mapped to 5-granular linguistic term sets. The average word-of-mouth data can be mapped to 7-granular linguistic term sets. Because of the complexity of forum reviews, this information can be mapped to 9-granular linguistic term sets.

Then, we use the generalized distance measure formula (equation (10)) as an example to apply the algorithm (Section 4.2) as follows.

*5.1. Application of MGPL-TOPSIS Algorithm.* The application of the algorithm based on MGPL-TOPSIS is shown as follows (Tables 1–13):

Step 1: the users' evaluation information on the "Auto Home" website until Feb 17 in 2019 is collected (Tables 1–3).

Here, the scoring data are the five cars' final average values on eight attributes from the scoring data (Table 1). The evaluation information is the general impression of the word-of-mouth data (Table 2). The evaluation information is from the forum review data (Table 3). These data are obtained on the "Auto Home" website.

Then, we obtain the users' overall evaluation linguistic term sets (Table 4).

Then, we obtain the users' overall evaluation probabilistic linguistic term sets by Definition 7. We suppose that the DMs are the most pessimistic $t = 0$. The probability of evaluation information is calculated by probability definition, and we obtain the probabilistic linguistic evaluation matrix as follows (Table 5).

Then, we obtain the extended normalized DM matrix by Definition 7 (Table 6).

Step 2: the weight vector is calculated on the eight attributes by equation (14) (Table 7).

Step 3: the PLPIS and PLNIS are calculated, respectively (Tables 8 and 9). To calculate conveniently, we only denote $\alpha_i$ instead of $S_{\alpha_i}$.

Step 4: $d(A_i, L^-)$ and $d(A_i, L^+), i = 1, 2, \ldots, 5$ are calculated. The results are as follows (Tables 10 and 11).

Step 5: the closeness coefficient $CD_i$ of $A_i(i = 1, 2, \ldots, 5)$ by equation (18) is calculated. The results are as follows (Table 12).

Step 6: The alternatives are ranked by $CD_i$. Here, we let $\delta = 0.5$ (Table 13 and Figure 1).

Table 13 and Figure 1 show that when$\lambda = 1$, the ranking of$A_i(i = 1, 2, \ldots, 5)$ is "$A_1 \succ A_5 \succ A_4 \succ A_2 \succ A_3$," when$\lambda = 2$, the ranking is "$A_1 \succ A_5 \succ A_4 \succ A_3 \succ A_2$," and when $\lambda = 3, 4, \ldots, 10$, the ranking is "$A_5 \succ A_1 \succ A_4 \succ A_3 \succ A_2$," illustrating that the ranking results are changed with $\lambda$. When $\lambda$ is different, the ranking result is different. When $\lambda = 3$, the ranking is stable, demonstrating that the algorithm based on TOPSIS is available.

In real life, we can use the MGPL-TOPSIS algorithm to deal with some GDM problems, such as environmental pollution management, urban traffic planning, treatment options, project evaluation, education assessment, emergency management, and smart city construction, especially COVID-19 prevention and control. The calculation method of the MGPL-TOPSIS algorithm is more convenient and effective, and the scientific method to make up for the DMs' missing information reduces the loss of effective information, so as to make the DM results more objective and effective.

*5.2. Comparative Analysis and Discussion.* To demonstrate the feasibility and efficiency of the algorithm based on generalized distance measures of the PLTSs, we calculate the other results by the two different algorithms based on MGPL-ETOPSIS and MGPL-VIKOR, respectively. Here, we let $\delta = 0.5$ (Tables 14 and 15).

In Figures 1 and 2, we can find that the results based on MGPL-TOPSIS and MGPL-ETOPSIS are the same as follows. When $\lambda = 1$, the ranking of $A_i(i = 1, 2, \ldots, 5)$ is "$A_1 \succ A_5 \succ A_4 \succ A_2 \succ A_3$." When $\lambda = 2$, the ranking is "$A_1 \succ A_5 \succ A_4 \succ A_3 \succ A_2$," and when $\lambda = 3, 4, \ldots, 10$, the ranking is "$A_5 \succ A_1 \succ A_4 \succ A_3 \succ A_2$." The results based on MGPL-VIKOR are as follows (Figure 3). When $\lambda = 1$, $\lambda = 2$, and$\lambda = 3$, the ranking is "$A_1 \succ A_4 \succ A_5 \succ A_2 \succ A_3$," and when$\lambda = 4, 5, \ldots, 10$, the ranking is "$A_1 \succ A_5 \succ A_4 \succ A_3 \succ A_2$." The results based on PT are as follows. When $\lambda = 1$ and 2, the ranking is "$A_5 \succ A_1 \succ A_2 \succ A_4 \succ A_3$," and when $\lambda = 3, 4, \ldots, 10$, the ranking is "$A_5 \succ A_1 \succ A_2 \succ A_3 \succ A_4$."

*5.3. Sensitivity Analysis.* To calculate the sensitivity parameter$\lambda$, we take the algorithm's different parameters based on PT [29] and rank $A_i(i = 1, 2, \ldots, 5)$. Then, the results are shown in Tables 16–19.

Tables 16–20 show that the results are as follows. When $\lambda = 1$, the ranking of $A_i$ $(i = 1, 2, \ldots, 5)$ is "$A_1 \succ A_5 \succ A_4 \succ A_2 \succ A_3$." When $\lambda = 2$, the ranking is "$A_1 \succ A_5 \succ A_4 \succ A_3 \succ A_2$," and when $\lambda = 3, 4, \ldots, 10$, the ranking is "$A_5 \succ A_1 \succ A_4 \succ A_3 \succ A_2$" although the parameters $\theta$, $\alpha$, and $\beta$ are changed. Therefore, the larger the parameter $\lambda$, the ranking $A_i$ tends to be stable, which is not affected by the parameters' subjective psychological factors $\theta$, $\alpha$, and $\beta$. The definitions of distance measures between two PLTSs under a multigranular linguistic environment are scientifically presented in Section 3.

## 6. The Extended TOPSIS Method

There is a set of three alternatives $A = \{A_1, A_2, A_3\}$ and the weight vector $w = (w_1, w_2, w_3, w_4)^T$ of attribute vector $x = (x_1, x_2, x_3, x_4)^T$, where $0 \le w_j \le 1$, $\sum_{j=1}^{n} w_j = 1$ [10]. The five DMs assess the three alternatives on four attributes by the multigranular linguistic set, which is $S = \{S_0 = \text{none}, S_1 = \text{very low}, S_2 = \text{low}, S_3 = \text{medium}, S_4 = \text{high}, S_5 = \text{very high}, S_6 = \text{perfect}\}$ (Tables 21–25).

Step 1: collect the five DMs' evaluation information:

Their original decision matrices are shown in Tables 21–25. The probabilistic linguistic decision matrix and the normalized probabilistic linguistic decision matrix of the group are shown in Tables 26 and 27.

Step 2: calculate the weight vector of the attributes $x_j(j = 1, 2, 3, 4)$:

$$w = (0.2396, 0.2340, 0.1332, 0.3931)^T. \tag{21}$$

Step 3: determine the PLPIS $L^+$ and the PLNIS $L^-$, respectively (Table 28).

Step 4: calculate the deviation degrees between each alternative and the PLPIS (PLNIS), respectively $(i = 1, 2, 3)$:

Table 1: Evaluation information by $s_5$ (scoring data).

| Alternative | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ |
|---|---|---|---|---|---|---|---|---|
| $A_1$ | $S^5_{4.87}$ | $S^5_5$ | $S^5_5$ | $S^5_5$ | $S^5_{4.47}$ | $S^5_{4.63}$ | $S^5_{4.3}$ | $S^5_{4.83}$ |
| $A_2$ | $S^5_{4.79}$ | $S^5_{4.81}$ | $S^5_{4.75}$ | $S^5_{4.65}$ | $S^5_{4.02}$ | $S^5_{4.75}$ | $S^5_{4.64}$ | $S^5_{4.75}$ |
| $A_3$ | $S^5_{4.33}$ | $S^5_{4.71}$ | $S^5_{4.67}$ | $S^5_{4.45}$ | $S^5_{4.39}$ | $S^5_{4.85}$ | $S^5_{4.24}$ | $S^5_{4.75}$ |
| $A_4$ | $S^5_{4.48}$ | $S^5_{4.77}$ | $S^5_{4.55}$ | $S^5_{4.30}$ | $S^5_{4.39}$ | $S^5_{4.64}$ | $S^5_{4.47}$ | $S^5_{4.55}$ |
| $A_5$ | $S^5_{4.40}$ | $S^5_{4.78}$ | $S^5_{4.61}$ | $S^5_{4.45}$ | $S^5_{4.16}$ | $S^5_{4.81}$ | $S^5_{4.27}$ | $S^5_{4.45}$ |

Table 2: Evaluation information by $s_7$ (word-of-mouth data).

| Alternative | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ |
|---|---|---|---|---|---|---|---|---|
| $A_1$ | $S^7_7$ | $S^7_7$ | $S^7_7$ | — | $S^7_7$ | $S^7_6$ | — | $S^7_7$ |
| $A_2$ | $S^7_5$ | $S^7_7$ | $S^7_6$ | — | $S^7_4$ | $S^7_7$ | $S^7_6$ | $S^7_6$ |
| $A_3$ | $S^7_3$ | $S^7_5$ | $S^7_6$ | $S^7_1$ | $S^7_1$ | $S^7_7$ | $S^7_4$ | $S^7_4$ |
| $A_4$ | $S^7_7$ | $S^7_3$ | $S^7_7$ | $S^7_1$ | $S^7_6$ | $S^7_6$ | $S^7_7$ | $S^7_5$ |
| $A_5$ | $S^7_7$ | — | $S^7_7$ | — | $S^7_7$ | $S^7_7$ | $S^7_6$ | $S^7_7$ |

Table 3: Evaluation information by $s_9$ (forum reviews data).

| Alternative | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ |
|---|---|---|---|---|---|---|---|---|
| $A_1$ | $S^9_9$ | $S^9_8$ | $S^9_7$ | $S^9_3$ | $S^9_7$ | $S^9_8$ | $S^9_5$ | $S^9_9$ |
| $A_2$ | $S^9_7$ | $S^9_8$ | $S^9_7$ | — | $S^9_5$ | $S^9_2$ | $S^9_1$ | $S^9_3$ |
| $A_3$ | $S^9_5$ | $S^9_6$ | $S^9_5$ | $S^9_2$ | $S^9_3$ | $S^9_9$ | $S^9_5$ | $S^9_4$ |
| $A_4$ | $S^9_8$ | $S^9_4$ | $S^9_8$ | $S^9_3$ | $S^9_4$ | $S^9_5$ | $S^9_9$ | $S^9_7$ |
| $A_5$ | $S^9_9$ | $S^9_1$ | $S^9_9$ | — | $S^9_9$ | $S^9_8$ | $S^9_7$ | $S^9_9$ |

Table 4: The linguistic evaluation matrix.

| Alternative | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|---|
| $A_1$ | $\{S^5_{4.87}, S^7_7, S^9_9\}$ | $\{S^5_5, S^7_7, S^9_8\}$ | $\{S^5_5, S^7_7, S^9_7\}$ | $\{S^5_5, S^9_3\}$ |
| $A_2$ | $\{S^5_{4.79}, S^7_5, S^9_7\}$ | $\{S^5_{4.81}, S^7_7, S^9_8\}$ | $\{S^5_{4.75}, S^7_6, S^9_7\}$ | $\{S^5_{4.65}\}$ |
| $A_3$ | $\{S^5_{4.33}, S^7_3, S^9_5\}$ | $\{S^5_{4.71}, S^7_5, S^9_6\}$ | $\{S^5_{4.67}, S^7_6, S^9_5\}$ | $\{S^5_{4.45}, S^7_1, S^9_2\}$ |
| $A_4$ | $\{S^5_{4.48}, S^7_7, S^9_8\}$ | $\{S^5_{4.77}, S^7_3, S^9_4\}$ | $\{S^5_{4.55}, S^7_7, S^9_8\}$ | $\{S^5_{4.3}, S^7_1, S^9_3\}$ |
| $A_5$ | $\{S^5_{4.4}, S^7_7, S^9_9\}$ | $\{S^5_{4.78}, S^9_1\}$ | $\{S^5_{4.61}, S^7_7, S^9_9\}$ | $\{S^5_{4.45}\}$ |
| Alternative | $x_5$ | $x_6$ | $x_7$ | $x_8$ |
| $A_1$ | $\{S^5_{4.47}, S^7_7, S^9_7\}$ | $\{S^5_{4.63}, S^7_6, S^9_8\}$ | $\{S^5_{4.3}, S^7_4, S^9_5\}$ | $\{S^5_{4.83}, S^7_7, S^9_9\}$ |
| $A_2$ | $\{S^5_{4.02}, S^7_4, S^9_5\}$ | $\{S^5_{4.75}, S^7_7, S^9_2\}$ | $\{S^5_{5.64}, S^7_6, S^9_1\}$ | $\{S^5_{4.75}, S^7_6, S^9_3\}$ |
| $A_3$ | $\{S^5_{4.39}, S^7_1, S^9_3\}$ | $\{S^5_{4.85}, S^7_7, S^9_9\}$ | $\{S^5_{4.24}, S^7_4, S^9_5\}$ | $\{S^5_{4.75}, S^7_4, S^9_4\}$ |
| $A_4$ | $\{S^5_{4.39}, S^7_6, S^9_4\}$ | $\{S^5_{4.64}, S^7_6, S^9_5\}$ | $\{S^5_{4.47}, S^7_7, S^9_9\}$ | $\{S^5_{4.55}, S^7_5, S^9_7\}$ |
| $A_5$ | $\{S^5_{4.16}, S^7_7, S^9_9\}$ | $\{S^5_{4.81}, S^7_7, S^9_8\}$ | $\{S^5_{4.27}, S^7_6, S^9_7\}$ | $\{S^5_{4.45}, S^7_7, S^9_9\}$ |

Table 5: The probabilistic linguistic evaluation matrix.

| Alternative | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|---|
| $A_1$ | $\{S^5_{4.87}(1/3), S^7_7(1/3), S^9_9(1/3)\}$ | $\{S^5_5(1/3), S^7_7(1/3), S^9_8(1/3)\}$ | $\{S^5_5(1/3), S^7_7(1/3), S^9_7(1/3)\}$ | $\{S^5_5(1/3), S^7_1(1/3), S^9_3(1/3)\}$ |
| $A_2$ | $\{S^5_{4.79}(1/3), S^7_5(1/3), S^9_7(1/3)\}$ | $\{S^5_{4.81}(1/3), S^7_7(1/3), S^9_8(1/3)\}$ | $\{S^5_{4.75}(1/3), S^7_6(1/3), S^9_7(1/3)\}$ | $\{S^5_{4.65}(1/3), S^7_1(1/3), S^9_2(1/3)\}$ |
| $A_3$ | $\{S^5_{4.33}(1/3), S^7_3(1/3), S^9_5(1/3)\}$ | $\{S^5_{4.71}(1/3), S^7_5(1/3), S^9_6(1/3)\}$ | $\{S^5_{4.67}(1/3), S^7_6(1/3), S^9_5(1/3)\}$ | $\{S^5_{4.45}(1/3), S^7_1(1/3), S^9_2(1/3)\}$ |
| $A_4$ | $\{S^5_{4.48}(1/3), S^7_7(1/3), S^9_8(1/3)\}$ | $\{S^5_{4.77}(1/3), S^7_3(1/3), S^9_4(1/3)\}$ | $\{S^5_{4.55}(1/3), S^7_7(1/3), S^9_8(1/3)\}$ | $\{S^5_{4.3}(1/3), S^7_1(1/3), S^9_3(1/3)\}$ |
| $A_5$ | $\{S^5_{4.4}(1/3), S^7_7(1/3), S^9_9(1/3)\}$ | $\{S^5_{4.78}(1/3), S^7_3(1/3), S^9_1(1/3)\}$ | $\{S^5_{4.61}(1/3), S^7_7(1/3), S^9_9(1/3)\}$ | $\{S^5_{4.45}(1/3), S^7_1(1/3), S^9_2(1/3)\}$ |
| Alternative | $x_5$ | $x_6$ | $x_7$ | $x_8$ |
| $A_1$ | $\{S^5_{4.47}(1/3), S^7_7(1/3), S^9_7(1/3)\}$ | $\{S^5_{4.63}(1/3), S^7_6(1/3), S^9_8(1/3)\}$ | $\{S^5_{4.3}(1/3), S^7_4(1/3), S^9_5(1/3)\}$ | $\{S^5_{4.83}(1/3), S^7_7(1/3), S^9_9(1/3)\}$ |
| $A_2$ | $\{S^5_{4.02}(1/3), S^7_4(1/3), S^9_5(1/3)\}$ | $\{S^5_{4.75}(1/3), S^7_7(1/3), S^9_2(1/3)\}$ | $\{S^5_{5.64}(1/3), S^7_6(1/3), S^9_1(1/3)\}$ | $\{S^5_{4.75}(1/3), S^7_6(1/3), S^9_3(1/3)\}$ |
| $A_3$ | $\{S^5_{4.39}(1/3), S^7_1(1/3), S^9_3(1/3)\}$ | $\{S^5_{4.85}(1/3), S^7_7(1/3), S^9_9(1/3)\}$ | $\{S^5_{4.24}(1/3), S^7_4(1/3), S^9_5(1/3)\}$ | $\{S^5_{4.75}(1/3), S^7_4(1/3), S^9_4(1/3)\}$ |
| $A_4$ | $\{S^5_{4.39}(1/3), S^7_6(1/3), S^9_4(1/3)\}$ | $\{S^5_{4.64}(1/3), S^7_6(1/3), S^9_5(1/3)\}$ | $\{S^5_{4.47}(1/3), S^7_7(1/3), S^9_9(1/3)\}$ | $\{S^5_{4.55}(1/3), S^7_5(1/3), S^9_7(1/3)\}$ |
| $A_5$ | $\{S^5_{4.16}(1/3), S^7_7(1/3), S^9_9(1/3)\}$ | $\{S^5_{4.81}(1/3), S^7_7(1/3), S^9_8(1/3)\}$ | $\{S^5_{4.27}(1/3), S^7_6(1/3), S^9_7(1/3)\}$ | $\{S^5_{4.45}(1/3), S^7_7(1/3), S^9_9(1/3)\}$ |

Table 6: The normalized DM matrix.

| Alternative | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ |
|---|---|---|---|---|---|---|---|---|
| $A_1$ | $\{S_{(2/3)}, S_{(4.87/15)}, S_0\}$ | $\{S_{(2/3)}, S_{(8/27)}, S_0\}$ | $\{S_{(2/3)}, S_{(7/27)}, S_0\}$ | $\{S_{(1/3)}, S_{(1/9)}, S_{(1/21)}\}$ | $\{S_{(1/3)}, S_{(4.47/15)}, S_{(7/27)}\}$ | $\{S_{(4.63/15)}, S_{(8/27)}, S_{(2/7)}\}$ | $\{S_{(4.3/15)}, S_{(4/21)}, S_{(5/27)}\}$ | $\{S_{2/3}, S_{4.83/15}, S_0\}$ |
| $A_2$ | $\{S_{(4.79/15)}, S_{(7/27)}, S_{(5/21)}\}$ | $\{S_{(1/3)}, S_{(4.81/15)}, S_{(8/27)}\}$ | $\{S_{(4.75/15)}, S_{(2/7)}, S_{(7/27)}\}$ | $\{S_{(4.65/15)}, S_{(2/27)}, S_{(1/21)}\}$ | $\{S_{(4.02/15)}, S_{(4/21)}, S_{(5/27)}\}$ | $\{S_{(1/3)}, S_{(4.75/15)}, S_{(2/27)}\}$ | $\{S_{4.64/15}, S_{2/7}, S_{1/27}\}$ | $\{S_{4.75/15}, S_{2/7}, S_{1/9}\}$ |
| $A_3$ | $\{S_{(4.33/15)}, S_{(5/27)}, S_{(1/7)}\}$ | $\{S_{(4.71/15)}, S_{(5/21)}, S_{(2/9)}\}$ | $\{S_{(4.67/15)}, S_{(2/7)}, S_{(5/27)}\}$ | $\{S_{(4.45/15)}, S_{(2/27)}, S_{(1/21)}\}$ | $\{S_{(4.39/15)}, S_{(1/9)}, S_{(1/21)}\}$ | $\{S_{(2/3)}, S_{(4.85/15)}, S_0\}$ | $\{S_{4.24/15}, S_{4/21}, S_{5/27}\}$ | $\{S_{4.75/15}, S_{4/21}, S_{4/27}\}$ |
| $A_4$ | $\{S_{(1/3)}, S_{(4.48/15)}, S_{(8/27)}\}$ | $\{S_{(4.77/15)}, S_{(4/27)}, S_{(1/7)}\}$ | $\{S_{(1/3)}, S_{(4.55/15)}, S_{(8/27)}\}$ | $\{S_{(4.3/15)}, S_{(1/9)}, S_{(1/21)}\}$ | $\{S_{(4.39/15)}, S_{(2/7)}, S_{(4/27)}\}$ | $\{S_{(4.64/15)}, S_{(2/7)}, S_{(5/27)}\}$ | $\{S_{2/3}, S_{4.47/15}, S_0\}$ | $\{S_{4.55/15}, S_{7/27}, S_{5/21}\}$ |
| $A_5$ | $\{S_{(2/3)}, S_{(4.4/15)}, S_0\}$ | $\{S_{(4.78/15)}, S_{(1/7)}, S_{(1/27)}\}$ | $\{S_{(2/3)}, S_{(4.61/15)}, S_0\}$ | $\{S_{(4.45/15)}, S_{(2/27)}, S_{(1/21)}\}$ | $\{S_{(2/3)}, S_{(4.16/15)}, S_0\}$ | $\{S_{(1/3)}, S_{(4.81/15)}, S_{(8/27)}\}$ | $\{S_{2/7}, S_{4.27/15}, S_{7/27}\}$ | $\{S_{2/3}, S_{4.45/15}, S_0\}$ |

TABLE 7: The weights of the attributes.

| Attribute | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ |
|---|---|---|---|---|---|---|---|---|
| Weight | 0.0921 | 0.1265 | 0.0807 | 0.2466 | 0.1350 | 0.0895 | 0.1262 | 0.1034 |

TABLE 8: The normalized PLPIS.

| Attribute | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|---|
| PLPIS | (0.6667, 0.3247, 0.2963) | (0.6667, 0.3207, 0.2963) | (0.6667, 0.3073, 0.2963) | (0.3333, 0.1111, 0.0476) |
| Attribute | $x_5$ | $x_6$ | $x_7$ | $x_8$ |
| PLPIS | (0.6667, 0.2980, 0.2593) | (0.6667, 0.3233, 0.2963) | (0.6667, 0.2980, 0.2593) | (0.6667, 0.3220, 0.2381) |

TABLE 9: The normalized PLNIS.

| Attribute | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|---|
| PLNIS | (0.2887, 0.1852, 0.2963) | (0.3140, 0.1429, 0.2963) | (0.3113, 0.2593, 0.2963) | (0.2867, 0.0741, 0.0476) |
| Attribute | $x_5$ | $x_6$ | $x_7$ | $x_8$ |
| PLNIS | (0.2680, 0.1111, 0.2593) | (0.3087, 0.2857, 0.2963) | (0.2827, 0.1905, 0.2593) | (0.3033, 0.1905, 0.2381) |

TABLE 10: The results of $d(A_i, L^-)$.

| Alternative | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ |
|---|---|---|---|---|---|
| $\lambda = 1$ | 0.1238 | 0.0549 | 0.0549 | 0.0549 | 0.0549 |
| $\lambda = 2$ | 0.1810 | 0.0846 | 0.1036 | 0.1168 | 0.1867 |
| $\lambda = 3$ | 0.2160 | 0.1071 | 0.1412 | 0.1551 | 0.2236 |
| $\lambda = 4$ | 0.2395 | 0.1241 | 0.1702 | 0.1859 | 0.2481 |
| $\lambda = 5$ | 0.2564 | 0.1372 | 0.1928 | 0.2106 | 0.2658 |
| $\lambda = 6$ | 0.2692 | 0.1473 | 0.2107 | 0.2304 | 0.2795 |
| $\lambda = 7$ | 0.2793 | 0.1554 | 0.2252 | 0.2466 | 0.2904 |
| $\lambda = 8$ | 0.2874 | 0.1619 | 0.2372 | 0.2599 | 0.2993 |
| $\lambda = 9$ | 0.2941 | 0.1673 | 0.2472 | 0.2709 | 0.3068 |
| $\lambda = 10$ | 0.2998 | 0.1719 | 0.2557 | 0.2803 | 0.3132 |

TABLE 11: The results of $d(A_i, L^+)$.

| Alternative | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ |
|---|---|---|---|---|---|
| $\lambda = 1$ | 0.0905 | 0.1299 | 0.1484 | 0.1155 | 0.1055 |
| $\lambda = 2$ | 0.1626 | 0.1926 | 0.1999 | 0.1805 | 0.1704 |
| $\lambda = 3$ | 0.2039 | 0.2309 | 0.2344 | 0.2191 | 0.2075 |
| $\lambda = 4$ | 0.2302 | 0.2557 | 0.2583 | 0.2444 | 0.2313 |
| $\lambda = 5$ | 0.2486 | 0.2729 | 0.2754 | 0.2621 | 0.2483 |
| $\lambda = 6$ | 0.2622 | 0.2856 | 0.2882 | 0.2751 | 0.2612 |
| $\lambda = 7$ | 0.2729 | 0.2954 | 0.2981 | 0.2852 | 0.2715 |
| $\lambda = 8$ | 0.2815 | 0.3033 | 0.3059 | 0.2931 | 0.2801 |
| $\lambda = 9$ | 0.2888 | 0.3098 | 0.3122 | 0.2996 | 0.2873 |
| $\lambda = 10$ | 0.2949 | 0.3153 | 0.3175 | 0.3050 | 0.2936 |

TABLE 12: The closeness coefficient $CD_i$.

| Alternative | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ |
|---|---|---|---|---|---|
| $\lambda = 1$ | 0.5776 | 0.2969 | 0.2680 | 0.3699 | 0.5366 |
| $\lambda = 2$ | 0.5268 | 0.3053 | 0.3414 | 0.3928 | 0.5228 |
| $\lambda = 3$ | 0.5144 | 0.3168 | 0.3760 | 0.4145 | 0.5186 |
| $\lambda = 4$ | 0.5099 | 0.3268 | 0.3972 | 0.4320 | 0.5175 |
| $\lambda = 5$ | 0.5078 | 0.3345 | 0.4118 | 0.4455 | 0.5171 |
| $\lambda = 6$ | 0.5066 | 0.3403 | 0.4223 | 0.4558 | 0.5169 |
| $\lambda = 7$ | 0.5058 | 0.3447 | 0.4304 | 0.4637 | 0.5168 |
| $\lambda = 8$ | 0.5052 | 0.3481 | 0.4367 | 0.4699 | 0.5166 |
| $\lambda = 9$ | 0.5046 | 0.3507 | 0.4419 | 0.4749 | 0.5164 |
| $\lambda = 10$ | 0.5041 | 0.3528 | 0.4461 | 0.4789 | 0.5162 |

Table 13: The ranking of $A_i$.

| Distance parameter | Rank |
|---|---|
| $\lambda = 1$ | $A_1 \succ A_5 \succ A_4 \succ A_2 \succ A_3$ |
| $\lambda = 2$ | $A_1 \succ A_5 \succ A_4 \succ A_3 \succ A_2$ |
| $\lambda = 3$ | $A_5 \succ A_1 \succ A_4 \succ A_3 \succ A_2$ |
| $\lambda = 4$ | $A_5 \succ A_1 \succ A_4 \succ A_3 \succ A_2$ |
| $\lambda = 5$ | $A_5 \succ A_1 \succ A_4 \succ A_3 \succ A_2$ |
| $\lambda = 6$ | $A_5 \succ A_1 \succ A_4 \succ A_3 \succ A_2$ |
| $\lambda = 7$ | $A_5 \succ A_1 \succ A_4 \succ A_3 \succ A_2$ |
| $\lambda = 8$ | $A_5 \succ A_1 \succ A_4 \succ A_3 \succ A_2$ |
| $\lambda = 9$ | $A_5 \succ A_1 \succ A_4 \succ A_3 \succ A_2$ |
| $\lambda = 10$ | $A_5 \succ A_1 \succ A_4 \succ A_3 \succ A_2$ |

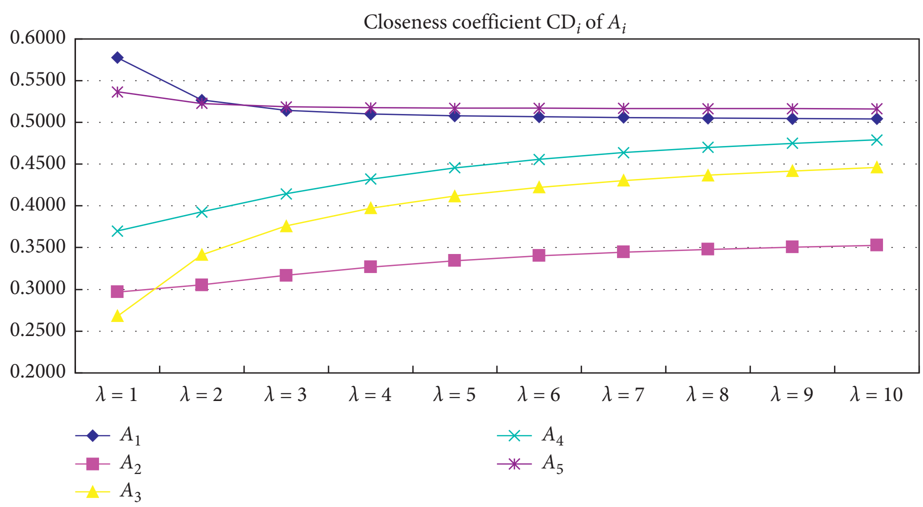


Figure 1: $CD_i$ based on MGPL-TOPSIS.

Table 14: The compromise index $CI_i$ based on MGPL-ETOPSIS.

| Alternative | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ |
|---|---|---|---|---|---|
| $\lambda = 1$ | −0.0005 | −0.9926 | −1.2005 | −0.7282 | −0.1789 |
| $\lambda = 2$ | −0.0304 | −0.7314 | −0.6742 | −0.4844 | −0.0481 |
| $\lambda = 3$ | −0.0340 | −0.6535 | −0.5179 | −0.3809 | −0.0178 |
| $\lambda = 4$ | −0.0347 | −0.6103 | −0.4359 | −0.3123 | −0.0050 |
| $\lambda = 5$ | −0.0363 | −0.5830 | −0.3839 | −0.2633 | 0.0003 |
| $\lambda = 6$ | −0.0407 | −0.5663 | −0.3495 | −0.2289 | 0.0000 |
| $\lambda = 7$ | −0.0434 | −0.5531 | −0.3223 | −0.2013 | −0.0003 |
| $\lambda = 8$ | −0.0449 | −0.5418 | −0.2996 | −0.1783 | 0.0000 |
| $\lambda = 9$ | −0.0463 | −0.5329 | −0.2810 | −0.1597 | −0.0001 |
| $\lambda = 10$ | −0.0471 | −0.5251 | −0.2649 | −0.1438 | 0.0001 |

Table 15: The compromise index $MC_i$ based on MGPL-VIKOR.

| Alternative | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ |
|---|---|---|---|---|---|
| $\lambda = 1$ | 0.9243 | 0.4534 | 0.0000 | 0.7823 | 0.5470 |
| $\lambda = 2$ | 0.8473 | 0.2973 | 0.0000 | 0.6797 | 0.4548 |
| $\lambda = 3$ | 0.9238 | 0.1604 | 0.0000 | 0.6262 | 0.5279 |
| $\lambda = 4$ | 1.0000 | 0.0459 | 0.2072 | 0.5743 | 0.6898 |
| $\lambda = 5$ | 1.0000 | 0.0427 | 0.3241 | 0.5230 | 0.7447 |
| $\lambda = 6$ | 1.0000 | 0.0407 | 0.3717 | 0.5021 | 0.7564 |
| $\lambda = 7$ | 1.0000 | 0.0390 | 0.3931 | 0.4922 | 0.7599 |
| $\lambda = 8$ | 1.0000 | 0.0374 | 0.4032 | 0.4867 | 0.7625 |
| $\lambda = 9$ | 1.0000 | 0.0360 | 0.4081 | 0.4834 | 0.7644 |
| $\lambda = 10$ | 1.0000 | 0.0347 | 0.4105 | 0.4811 | 0.7659 |

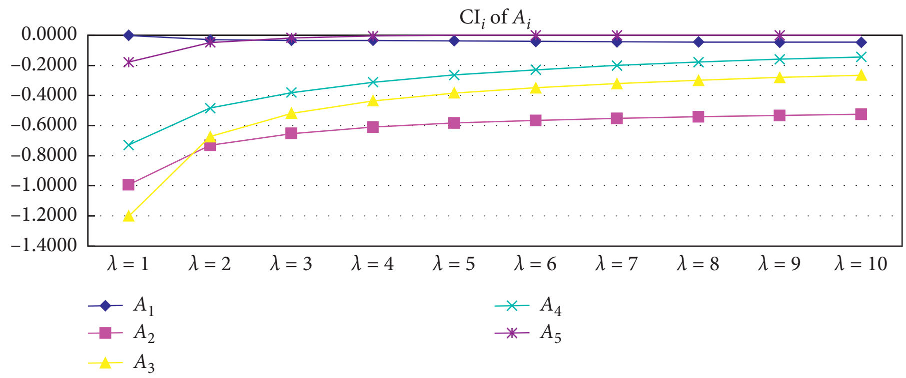


Figure 2: CI$_i$ based on MGPL-ETOPSIS.

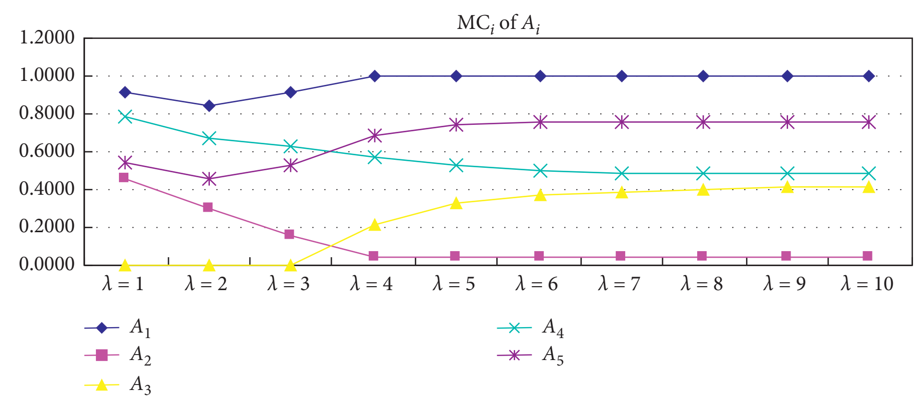


Figure 3: The MC$_i$ based on MGPL-VIKOR.

Table 16: The ration CP$_i$ based on PT ($\alpha = 0.85, \theta = 4.1$, and $\beta = 0.85$).

| Alternative | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ |
|---|---|---|---|---|---|
| $\lambda = 1$ | 0.3130 | 0.1145 | 0.1076 | 0.1382 | 0.2853 |
| $\lambda = 2$ | 0.2409 | 0.1109 | 0.1195 | 0.1283 | 0.2355 |
| $\lambda = 3$ | 0.2203 | 0.1094 | 0.1209 | 0.1247 | 0.2222 |
| $\lambda = 4$ | 0.2115 | 0.1085 | 0.1205 | 0.1230 | 0.2168 |
| $\lambda = 5$ | 0.2071 | 0.1080 | 0.1199 | 0.1221 | 0.2141 |
| $\lambda = 6$ | 0.2045 | 0.1077 | 0.1194 | 0.1217 | 0.2127 |
| $\lambda = 7$ | 0.2029 | 0.1074 | 0.1190 | 0.1214 | 0.2118 |
| $\lambda = 8$ | 0.2019 | 0.1072 | 0.1188 | 0.1213 | 0.2112 |
| $\lambda = 9$ | 0.2012 | 0.1071 | 0.1186 | 0.1212 | 0.2109 |
| $\lambda = 10$ | 0.2006 | 0.1070 | 0.1184 | 0.1211 | 0.2106 |

Table 17: The ration CP$_i$ based on PT ($\alpha = 0.88, \theta = 2.25$, and $\beta = 0.88$).

| Alternative | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ |
|---|---|---|---|---|---|
| $\lambda = 1$ | 0.5783 | 0.2036 | 0.1921 | 0.2487 | 0.5267 |
| $\lambda = 2$ | 0.4407 | 0.1970 | 0.2136 | 0.2298 | 0.4314 |
| $\lambda = 3$ | 0.4015 | 0.1943 | 0.2161 | 0.2230 | 0.4060 |
| $\lambda = 4$ | 0.3850 | 0.1928 | 0.2152 | 0.2198 | 0.3958 |
| $\lambda = 5$ | 0.3766 | 0.1918 | 0.2141 | 0.2182 | 0.3908 |
| $\lambda = 6$ | 0.3717 | 0.1912 | 0.2131 | 0.2173 | 0.3880 |
| $\lambda = 7$ | 0.3687 | 0.1907 | 0.2124 | 0.2168 | 0.3863 |
| $\lambda = 8$ | 0.3667 | 0.1904 | 0.2119 | 0.2165 | 0.3852 |
| $\lambda = 9$ | 0.3654 | 0.1901 | 0.2116 | 0.2164 | 0.3845 |
| $\lambda = 10$ | 0.3644 | 0.1899 | 0.2113 | 0.2163 | 0.3841 |

Table 18: The ration $CP_i$ based on PT ($\alpha = 0.89, \theta = 2.25$, and $\beta = 0.92$).

| Alternative | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ |
|---|---|---|---|---|---|
| $\lambda = 1$ | 0.6189 | 0.2134 | 0.2008 | 0.2625 | 0.5625 |
| $\lambda = 2$ | 0.4642 | 0.2047 | 0.2223 | 0.2400 | 0.4549 |
| $\lambda = 3$ | 0.4202 | 0.2010 | 0.2240 | 0.2316 | 0.4259 |
| $\lambda = 4$ | 0.4017 | 0.1990 | 0.2226 | 0.2277 | 0.4140 |
| $\lambda = 5$ | 0.3921 | 0.1977 | 0.2211 | 0.2257 | 0.4081 |
| $\lambda = 6$ | 0.3866 | 0.1968 | 0.2198 | 0.2245 | 0.4047 |
| $\lambda = 7$ | 0.3831 | 0.1962 | 0.2189 | 0.2238 | 0.4026 |
| $\lambda = 8$ | 0.3808 | 0.1958 | 0.2183 | 0.2234 | 0.4013 |
| $\lambda = 9$ | 0.3792 | 0.1954 | 0.2178 | 0.2231 | 0.4004 |
| $\lambda = 10$ | 0.3780 | 0.1952 | 0.2174 | 0.2229 | 0.3997 |

Table 19: The ration $C_i$ ($\alpha = 0.725, \theta = 2.04$, and $\beta = 0.717$).

| Alternative | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ |
|---|---|---|---|---|---|
| $\lambda = 1$ | 0.5861 | 0.2517 | 0.2342 | 0.2894 | 0.5353 |
| $\lambda = 2$ | 0.4721 | 0.2452 | 0.2581 | 0.2745 | 0.4575 |
| $\lambda = 3$ | 0.4385 | 0.2424 | 0.2614 | 0.2688 | 0.4365 |
| $\lambda = 4$ | 0.4242 | 0.2408 | 0.2610 | 0.2662 | 0.4281 |
| $\lambda = 5$ | 0.4169 | 0.2398 | 0.2602 | 0.2648 | 0.4240 |
| $\lambda = 6$ | 0.4127 | 0.2392 | 0.2595 | 0.2641 | 0.4217 |
| $\lambda = 7$ | 0.4101 | 0.2387 | 0.2589 | 0.2637 | 0.4204 |
| $\lambda = 8$ | 0.4083 | 0.2384 | 0.2585 | 0.2635 | 0.4742 |
| $\lambda = 9$ | 0.4072 | 0.2381 | 0.2582 | 0.2634 | 0.4190 |
| $\lambda = 10$ | 0.4063 | 0.2379 | 0.2580 | 0.2633 | 0.4187 |

Table 20: The ranking of $A_i$.

| Algorithm | Parameter | | Rank |
|---|---|---|---|
| PT | $\alpha = 0.85, \theta = 4.1, \beta = 0.85$; $\alpha = 0.88, \theta = 2.25, \beta = 0.88$; $\alpha = 0.89, \theta = 2.25, \beta = 0.92$; $\alpha = 0.725, \theta = 2.04, \beta = 0.717$ | $\lambda = 1$ | $A_1 \succ A_5 \succ A_4 \succ A_2 \succ A_3$ |
| | | $\lambda = 2$ | $A_1 \succ A_5 \succ A_4 \succ A_3 \succ A_2$ |
| | | $\lambda = 3$ | $A_5 \succ A_1 \succ A_4 \succ A_3 \succ A_2$ |
| | | $\lambda = 4$ | $A_5 \succ A_1 \succ A_4 \succ A_3 \succ A_2$ |
| | | $\lambda = 5$ | $A_5 \succ A_1 \succ A_4 \succ A_3 \succ A_2$ |
| | | $\lambda = 6$ | $A_5 \succ A_1 \succ A_4 \succ A_3 \succ A_2$ |
| | | $\lambda = 7$ | $A_5 \succ A_1 \succ A_4 \succ A_3 \succ A_2$ |
| | | $\lambda = 8$ | $A_5 \succ A_1 \succ A_4 \succ A_3 \succ A_2$ |
| | | $\lambda = 9$ | $A_5 \succ A_1 \succ A_4 \succ A_3 \succ A_2$ |
| | | $\lambda = 10$ | $A_5 \succ A_1 \succ A_4 \succ A_3 \succ A_2$ |

Table 21: The linguistic decision matrix provided by the first DM.

| Alternative | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|---|
| $A_1$ | $S_3$ | $S_4$ | $S_5$ | $S_4$ |
| $A_2$ | $S_2$ | $S_3$ | $S_1$ | $S_2$ |
| $A_3$ | $S_4$ | $S_3$ | — | $S_5$ |

Table 22: The linguistic decision matrix provided by the second DM.

| Alternative | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|---|
| $A_1$ | $S_4$ | $S_2$ | $S_4$ | $S_5$ |
| $A_2$ | $S_3$ | $S_1$ | — | $S_3$ |
| $A_3$ | $S_5$ | $S_3$ | $S_4$ | $S_5$ |

Table 23: The linguistic decision matrix provided by the third DM.

| Alternative | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|---|
| $A_1$ | $S_4$ | — | $S_4$ | $S_4$ |
| $A_2$ | $S_3$ | $S_2$ | $S_5$ | $S_3$ |
| $A_3$ | $S_4$ | $S_3$ | $S_1$ | $S_5$ |

Table 24: The linguistic decision matrix provided by the fourth DM.

| Alternative | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|---|
| $A_1$ | $S_4$ | $S_4$ | $S_5$ | $S_3$ |
| $A_2$ | $S_5$ | $S_2$ | $S_4$ | — |
| $A_3$ | $S_3$ | $S_2$ | $S_4$ | $S_4$ |

Table 25: The linguistic decision matrix provided by the fifth DM.

| Alternative | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|---|
| $A_1$ | $S_3$ | $S_4$ | $S_2$ | $S_5$ |
| $A_2$ | $S_3$ | $S_3$ | $S_4$ | $S_2$ |
| $A_3$ | $S_3$ | — | $S_5$ | $S_4$ |

Table 26: The probabilistic linguistic decision matrix of the group.

| Attributes | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|---|
| $A_1$ | $\{S_3(0.4), S_4(0.6)\}$ | $\{S_4(0.6), S_2(0.2)\}$ | $\{S_5(0.4), S_2(0.2), S_4(0.4)\}$ | $\{S_4(0.4), S_3(0.2), S_5(0.4)\}$ |
| $A_2$ | $\{S_2(0.2), S_3(0.6), S_5(0.2)\}$ | $\{S_3(0.4), S_1(0.2), S_2(0.4)\}$ | $\{S_1(0.2), S_5(0.2), S_4(0.4)\}$ | $\{S_3(0.4), S_2(0.4)\}$ |
| $A_3$ | $\{S_4(0.4), S_3(0.4), S_5(0.2)\}$ | $\{S_3(0.6), S_2(0.2)\}$ | $\{S_1(0.2), S_5(0.2), S_4(0.4)\}$ | $\{S_4(0.4), S_5(0.6)\}$ |

Table 27: The normalized probabilistic linguistic decision matrix of the group.

| Attributes | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|---|
| $A_1$ | $\{S_4(0.6), S_3(0.4), S_3(0)\}$ | $\{S_4(0.6), S_2(0.2), S_2(0)\}$ | $\{S_5(0.4), S_4(0.4), S_2(0.2)\}$ | $\{S_5(0.4), S_4(0.4), S_3(0.2)\}$ |
| $A_2$ | $\{S_3(0.6), S_5(0.2), S_2(0.2)\}$ | $\{S_3(0.4), S_2(0.4), S_1(0.2)\}$ | $\{S_4(0.4), S_5(0.2), S_1(0.2)\}$ | $\{S_3(0.4), S_2(0.4), S_2(0)\}$ |
| $A_3$ | $\{S_4(0.4), S_3(0.4), S_5(0.2)\}$ | $\{S_3(0.6), S_2(0.2), S_2(0)\}$ | $\{S_4(0.4), S_5(0.2), S_1(0.2)\}$ | $\{S_5(0.6), S_4(0.4), S_4(0)\}$ |

Table 28: The positive ideal solution and the negative ideal solution.

| Attributes | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|---|
| PLPIS | (2.4, 1.2, 1) | (2.4, 0.8, 0.2) | (2, 1.6, 0.4) | (3, 1.6, 0.6) |
| Attributes | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
| PLNIS | (1.6, 1, 0) | (1.2, 0.4, 0) | (1.6, 1, 0.2) | (1.2, 0.8, 0) |

$$
\begin{aligned}
d\left(A_1, L^+\right) &= 0.4257,\\
d\left(A_2, L^+\right) &= 0.8076,\\
d\left(A_3, L^+\right) &= 0.4055,\\
d\left(A_1, L^-\right) &= 0.6244,\\
d\left(A_2, L^-\right) &= 0.1223,\\
d\left(A_3, L^-\right) &= 0.6692,\\
d_{\min}\left(A_i, L^+\right) &= 0.4055,\\
d_{\max}\left(A_i, L^-\right) &= 0.6692.
\end{aligned} \tag{22}
$$

Step 5: calculate the closeness coefficient $\mathrm{CI}(A_i)\,(i = 1, 2, 3)$:

$$
\begin{aligned}
\mathrm{CI}\left(A_1\right) &= -0.1168,\\
\mathrm{CI}\left(A_2\right) &= -1.8089,\\
\mathrm{CI}\left(A_3\right) &= 0.
\end{aligned} \tag{23}
$$

Step 6: rank the alternatives $A_i$ according to $\mathrm{CI}(A_i)\,(i = 1, 2, 3)$:

$$A_3 > A_1 > A_2 \tag{24}$$

By comparing the proposed classical algorithm above, we can find it is only a special example of the proposed algorithms; in this paper, when $\lambda = 2$, that is, the proposed algorithms extended the classical algorithm.

## 7. Conclusions

This paper presents generalized distance measures between two PLTSs with multigranular probabilistic linguistic information and the method of filling with the missing evaluation information, which can be used to deal with MAGDM problems. The main advantages of this paper can be given as follows: (1) the generalized distance measures improve the accuracy of multigranular linguistic information in the MAGDM issues, and even some evaluation information is null, a more reasonable method is presented to address the missing value of the evaluation information; (2) the parameter $\lambda$ of the generalized distance measures is a variable that can be used to obtain different distance measures formula according to the DMs' needs; since then, the rankings of the alternatives are stable; (3) under these distance measures, the presented three algorithms MGPL-TOPSIS, MGPL-ETOPSIS, and MGPL-VIKOR are more available; (4) the proposed calculation method of PLTSs is more effective; (5) the entropy weight method can save more information and become more objective and accurate.

The generalized distance measures presented in this paper also have some limitations: (1) whether more appropriate ways exist to measure the distances between two PLTSs with multigranular linguistic information is a worthwhile question; (2) how to select the $\lambda$ proper to measure the distance between two PLTSs and the algorithm according to the practical problem have potential to be studied further. Some directions for further research are identified. First, the applications of these extended distance measures are interesting to study in cluster analysis and other MAGDM problems. Second, linguistic information can be expressed by interval fuzzy numbers.

## Data Availability

All relevant data are available within the paper. All relevant data are available from the "Auto Home" website (http://www.autohome.com.cn).

## Disclosure

This research does not involve any human or animal participation.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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*Research Article*

# Evaluation of Nursing Homes Using a Novel PROMETHEE Method for Probabilistic Linguistic Term Sets

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Aging has become a serious social problem in China. Traditional informal long-term care is hard to sustain because of the reduction in family size and elders' children migration to big cities. The institution offering services for the disabled elders has been a tendency. There exists a strange phenomenon: some nursing homes are difficult to enter for most disabled elders, while the other ones must search for elders to maintain operation. Therefore, for the evaluation of nursing homes, two problems should be considered: (1) selecting suitable nursing homes for disabled elders; (2) obtaining the key factors influencing the selection of elders and helping nursing homes improve their services based on the key factors. First, we propose a new DEMATEL (Decision-Making Trial and Evaluation Laboratory) method for PLTSs to solve the second problem. Then, we present a novel PROMETHEE (Preference Ranking Organization Methods for Enrichment Evaluations) method to rank the alternatives and make a sensitivity analysis for criteria. Finally, we illustrate our proposed methods to an evaluation problem in Zhenjiang City by a case study. Based on the case study, we can obtain that our proposed methods are effective and practicable.

## 1. Introduction

With the development of society and medical level, China is entering an aging society. By the end of 2019, there are more than 253 million old people aged 60 or over. The population of disabled elders is more than 40 million. With the reducing size of families and adult children moving to cities, many disabled elders live alone and lack long-term care [1]. Traditional informal long-term care may lead to some problems for disabled elders, such as psychological loneliness [2] and reduction in household income [3]. Therefore, it is necessary for disabled elders to seek long-term care from nursing homes [4]. There have been numerous nursing homes in every city. The service levels of different nursing homes are multifarious. On one hand, some nursing homes are very popular that most disabled elders must wait for several years to enter them. On the other hand, many nursing homes' occupancy rate is very low. To cope with the contradiction, there are two key problems that need to be solved. (1) How to help disabled elders choose suitable nursing homes? (2) What are the factors of concern for disabled elders and how to improve these factors? For the first problem, we can use a multiple criteria decision-making (MCDM) method to solve. With respect to the second one, in this paper, we use a Decision-Making Trial and Evaluation Laboratory (DEMATEL) method to obtain the key factors for evaluation system.

For MCDM problems, there have been a large number of researches. In many cases, decision-makers (DMs) usually use linguistic information [5] to express their viewpoints. There have been lots of studies for linguistic information [6, 7]. To aggregate the information of different DMs easily, Pang et al. [8] proposed the definition of probabilistic linguistic term set (PLTS). Many studies for PLTSs have emerged from theory to application. As for aggregating rules for PLTSs, Pang et al. [8] first proposed the basic rules for PLTSs. Liao et al. [9] proposed some new operational rules based on disparity degrees. Li and Wei [10] put forward a series of new rules based on evidence theory. With regard to the application of PLTSs, Lin et al. [11] put forward a novel best worst method for PLTS and applied it to evaluate IoT platforms. Li et al. [12] proposed a new case-based reasoning method for PLTS and solved the evaluation of poverty-stricken families. Lin et al. [13] proposed some clustering algorithms for PLTSs. Lin et al. [13] proposed an ELECTREE method for PLTS. Lin et al. [14] proposed a new score function for PLTS and applied it to select children English educational organization.

DEMATEL method is an effective way to obtain the key factors influencing the evaluation system. Cause-effect interactions for different criteria (factors) can be obtained by processing a comprehensive direct influencing matrix. DEMATEL method has been expanded to different uncertain information, such as fuzzy numbers [15], grey numbers [16], hesitant fuzzy linguistic term set [17], and PLTS [18]. DEMATEL method has been applied to many research areas, such as supply chain management [19], new energy [20], and business ecosystem [21].

To solve the second key problem mentioned in the first paragraph, we will use the DEMATEL method to analyze the key factors influencing the choices of disabled elders. Because the evaluation information is expressed by PLTSs, we should extend the traditional DEMATEL method to probabilistic linguistic environment. Furthermore, in order to address the first key problem, we will propose a new PROMETHEE method to help disabled elders select suitable nursing homes for them. We choose the PROMETHEE method because it is easy to make a sensitivity analysis for criteria weights.

In this paper, we will propose a new DEMATEL method for PLTS to make an analysis of key factors influencing the evaluation system. Then, we will put forward a novel PROMETHEE method to rank the alternatives and make a sensitivity analysis for criteria. The main contributions and innovation points of this paper can be concluded as follows:

(1) Propose a new DEMATEL method for PLTSs by transforming PLTSs into TFNs based on WOWA operators, which will help nursing homes obtain the concern factors of elders and can improve their services precisely

(2) Propose a novel PROMETHEE II method for PLTSs, which will help elders select the most suitable nursing homes

(3) Propose an approach to sensitivity analysis of criteria weights using a stability interval (WSI) method for PLTSs, which can help DMs find the variation range of criteria weights if the ranking results are stable

This paper is organized as follows. Section 2 reviews some basic definitions of PLTSs, TFNs, and WOWA operator. Section 3 proposes a novel DEMATEL method for PLTSs to obtain key factors for the evaluation system. Section 4 presents a PROMETHEE II method for PLTS and makes a sensitivity analysis for criteria. Section 5 applies our methods to an evaluation problem for nursing homes in Zhenjiang City. Section 6 makes a summary for this paper.

## 2. Preliminaries

In this section, we will review the basic definitions for PLTSs, TFNs, and WOWA operator.

*2.1. PLTS.* In real life, DMs may use linguistic information, such as "high" and "low," to express their opinions for evaluating some objects. A typical linguistic term set (LTS) can be described as $S = \{s_t | t = -\tau, \ldots, -1, 0, 1, \ldots, \tau\}$, where $\tau$ is a positive integer and $2\tau + 1$ is called granularity of LTS $S$.

It is easy to find that LTS can describe the subjectivity of DM. However, in many cases, there are many DMs participating in the decision process. Traditional LTS cannot express the information conveniently in this situation. To address this issue, Pang et al. [8] proposed the definition of PLTS, which can effectively describe the information of many DMs using LTSs.

*Definition 1* (see [8]). Let $S$ be an LTS; then a PLTS can be defined as

$$L(p) = \left\{ L^{(k)}\left(p^{(k)}\right) | L^{(k)} \in S, \quad p^{(k)} \geq 0, k = 1, 2, \ldots, \#L(p), \sum_{k=1}^{\#L(p)} p^{(k)} \leq 1 \right\}, \tag{1}$$

where $L^{(k)}(p^{(k)})$ is the linguistic term $L^{(k)}$ associated with probability $p^{(k)}$ and $\#L(p)$ is the number of all different linguistic terms in $L(p)$.

*Example 1.* Given an LTS $S = \{s_{-2}, s_{-1}, s_0, s_1, s_2\}$, then $L_1(p) = \{s_{-1}(0.4), s_0(0.4), s_1(0.2)\}$ and $L_1(p) = \{s_{-1}(0.4), s_1(0.4)\}$ are both PLTSs.

From Example 1, we can give some explanations: (1) for PLTS $L_1(p)$, 40 percent of the DMs give evaluations using $s_{-1}$, 40 percent give of the DMs give evaluations using $s_0$, and 20 percent of the DMs give evaluations using $s_1$; (2) for PLTS $L_2(p)$, 40 percent of the DMs give evaluations using $s_{-1}$, 40 percent of the DMs give evaluations using $s_1$, and 20 percent of the DMs give up their opinions for some reasons. We can find that PLTSs can effectively describe the linguistic information of many DMs.

*2.2. Triangular Fuzzy Number (TFN).* TFN uses three elements to describe uncertain information. It is convenient to use TFNs to express some uncertain linguistic information [12]. The definition of TFN can be seen as in Definition 2.

*Definition 2* (see [22]). A three tuple $A = (a, b, c)$ is defined as a TFN if it satisfies

$$\mu_A(x) = \begin{cases} 0, & x < a, \\ \dfrac{x-a}{b-a}, & a \le x \le b, \\ \dfrac{c-x}{c-b}, & b \le x \le c, \\ 0, & x > c. \end{cases} \quad (2)$$

Given three TFNs $A = (a, b, c)$, $A_1 = (a_1, b_1, c_1)$, and $A_2 = (a_2, b_2, c_2)$, then the following operational rules hold [22]:

(1) $A_1 \oplus A_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$

(2) $A_1 \Theta A_2 = (a_1 - c_2, b_1 - b_2, c_1 - a_2)$

(3) $\eta A = (\eta a, \eta b, \eta c)$, $\eta \ge 0$

*Definition 3* (see [18, 22]). Let $A = (a, b, c)$ be a TFN; then its defuzzified centroid can be defined as

$$\mathrm{DC}(A) = \frac{a+b+c}{3}. \quad (3)$$

*2.3. WOWA Operator.* The definition of the WOWA operator is defined as follows.

*Definition 4* (see [23]). Let $P = (p_1, p_2, \ldots, p_n)$ be a weighting vector of numbers $a_1, a_2, \ldots, a_n$ satisfying $0 \le p_i \le 1$ and $\sum_{i=1}^n p_i = 1$. Then, mapping $f_{\mathrm{WOWA}}^{P,W}: R \longrightarrow R$, which has an associated weighting vector $W = (w_1, w_2, \ldots, w_n)$ such that $0 \le w_i \le 1$ and $\sum_{i=1}^n w_i = 1$, is called a WOWA operator if

$$f_{\mathrm{WOWA}}^{P,W}(a_1, a_2, \ldots, a_n) = \sum_{i=1}^n v_i b_i, \quad (4)$$

where $b_i$ is the i-th largest element in $a_1, a_2, \ldots, a_n$ and $v_i$ is called comprehensive weight and can be obtained by

$$v_i = w^*\left(\sum_{j=1}^{i} p_{\sigma(j)}\right) - w^*\left(\sum_{j=1}^{i-1} p_{\sigma(j)}\right), \quad (5)$$

where $w^*$ is monotone increasing function and can be seen in the paper proposed by Li et al. [18].

For simplicity, we call $P = (p_1, p_2, \ldots, p_n)$ and $W = (w_1, w_2, \ldots, w_n)$ importance weighting vector and position weighting vector, respectively. We can obtain the position weighting vector $W = (w_1, w_2, \ldots, w_n)$ by the following mathematical programming [24]:

$$\min \delta = \delta^*,$$
$$(P1)\ \mathrm{s.t.} \begin{cases} \mathrm{orness}(w^*) = \displaystyle\sum_{n=1}^{k} \frac{k-n}{k-1} \cdot w_n^* = \lambda, \\ \left|w_{n-1}^* - w_n^*\right| \le \delta^*, \\ \displaystyle\sum_{n=1}^{k} w_n^* = 1, \ \forall w_n^* \ge 0, \end{cases} \quad (6)$$

where parameter $\lambda$ can be given by DMs.

## 3. Obtaining Key Factors for Evaluation System by DEMATEL Method for PLTSs

When evaluating nursing homes, ranking alternatives are important but not the only target. Obtaining the key factors for the evaluation system is another target because it can help DMs to find the reasons leading to the decision results. DEMATEL method is an effective method to seek key factors and obtain criteria weights [18]. In the traditional DEMATEL method, DMs need to make a comparison between two criteria and give a comprehensive direct influencing matrix and then a total influencing matrix. The information for PLTSs cannot be used directly in the DEMATEL method. Therefore, we need to transform PLTSs into TFNs.

Given a PLTS $L(p)$, we can use the WOWA method to transform it into a TFN. We assume that a PLTS is $L(p) = \{s_i(p_i), s_{i+1}(p_{i+1}), \ldots, s_j(p_j)\}$. It is worth noting that, in traditional PLTS, the subscripts of linguistic terms may be not continuous. We need to add the missing linguistic terms with their probabilities equal to 0. We can transform the linguistic term $s_k$ $(i \le k \le j)$ into TFN $A_k = (a_k^L, a_k^M, a_k^R)$ by the following rules [18, 25]:

(1) If $-\tau < k < \tau$, then $a_k^L = (\tau + k - 1/2\tau)$, $a_k^M = (\tau + k/2\tau)$, and $a_k^R = (\tau + k + 1/2\tau)$

(2) If $k = \tau$, then $a_k^L = (2\tau - 1/2\tau)$ and $a_k^M = a_k^R = 1$

(3) If $k = -\tau$, then $a_k^L = a_k^M = 0$ and $a_k^R = (1/2\tau)$

We can write $L(p) = \{s_i(p_i), s_{i+1}(p_{i+1}), \ldots, s_j(p_j)\}$ as a numerical set $T = \{a_i^L, a_i^M, a_{i+1}^M, \ldots, a_j^M, a_j^R\}$. Then, we can transform PLTS $L(p)$ into a TFN based on the following rules [18]:

Rule 1: if $j = \tau$, then TFN $A = (a_i^L, f_{\mathrm{WOWA}}^{P,W}(a_i^M, a_{i+1}^M, \ldots, a_\tau^M), a_\tau^M)$

Rule 2: if $i = -\tau$, then TFN $A = (a_{-\tau}^M, f_{\mathrm{WOWA}}^{P,W}(a_{-\tau}^M, a_{-\tau+1}^M, \ldots, a_j^M), a_j^R)$

Rule 3: If $i > -\tau$ and $j < \tau$, then $A = (a_i^L, f_{\mathrm{WOWA}}^{P,W}(a_i^M, a_{i+1}^M, \ldots, a_j^M), a_j^R)$

Based on Rules 1–3, PLTS $L(p)$ can be transformed into a TFN $A$. We then obtain key factors for evaluation system by the DEMATEL method as follows.

*3.1. Establish a Comprehensive Direct Influencing Matrix Q.* For the evaluation problem, experts $E_1, E_2, \ldots, E_m$ express their opinions by making pair comparisons for criteria $C_1, C_2, \ldots, C_n$ using LTS $S$. We aggregate the information from these experts and obtain a comprehensive direct influencing matrix $Q = (L_{ij}(p))_{n\times n}$.

*3.2. Transform the Matrix Q into TFN Matrix A.* Because we cannot illustrate PLTS matrix $Q$ to process the DEMATEL method, so we transform matrix $Q$ into TFN matrix $A = (a_{ij}^L, a_{ij}^M, a_{ij}^R)_{n\times n}$ based on Rules 1–3.

*3.3. Obtain Total Influencing Matrix T.* Normalize TFN matrix $A$ to $B = (b_{ij}^L, b_{ij}^M, b_{ij}^R)_{n\times n} = ((a_{ij}^L/\rho), (a_{ij}^M/\rho), (a_{ij}^R/\rho))_{n\times n}$, where $\rho = \max\{\max_i\{\sum_j a_{ij}^R\}, \max_j\{\sum_i a_{ij}^R\}\}$. Compute the total influencing matrix $T$ based on

$$T = \left(t_{ij}^L, t_{ij}^M, t_{ij}^R\right)_{n\times n} = B + B^2 + \cdots = B(I-B)^{-1}. \tag{7}$$

*3.4. Make an Analysis for Relationship of Criteria.* Based on total influencing matrix $T$, compute the defuzzified centroid matrix $Y = (y_{ij})_{n\times n} = (t_{ij}^L + t_{ij}^M + t_{ij}^R/3)_{n\times n}$. Calculate sums of rows and columns of matrix $Y$ as follows:

$$r_i = \sum_{j=1}^{n} y_{ij}, \qquad c_i = \sum_{i=1}^{n} y_{ij}. \tag{8}$$

Set a threshold $\varepsilon = (\sum_i^n \sum_j^n y_{ij}/n^2)$ [26]. In the defuzzified centroid matrix $Y$, if the element $y_{ij} > \varepsilon$, we can say criterion $C_i$ has influence on $C_j$.

Make an analysis based on the values of $r_i - c_i$, which indicates net effect of criterion $C_i$ to the evaluation system. If $r_i - c_i > 0$, then criterion $C_i$ is called net cause factor. On the contrary, if $r_i - c_i < 0$, then criterion $C_i$ is called result factor.

Furthermore, we can obtain the criteria weights by the following [27]:

$$\omega_i = \frac{\sqrt{2(r_i)^2 + 2(c_i)^2}}{\sum_{i=1}^{n}\sqrt{2(r_i)^2 + 2(c_i)^2}}. \tag{9}$$

## 4. A Novel PROMETHEE II Method for PLTS and Sensitivity Analysis for Criteria

In this section, we will propose a novel PROMETHEE II method for PLTS and make a sensitivity analysis for criteria based on the WSI method.

*4.1. A PROMETHEE II Method for PLTS.* For evaluating nursing homes problems, criterion set is $C = (C_1, C_2, \ldots, C_n)$, alternative set (nursing homes) is $X = \{X_1, X_2, \ldots, X_m\}$, and the criteria weights set is $W = (\omega_1, \omega_2, \ldots, \omega_n)$. Experts give a decision matrix $U = (u_{ij})_{m\times n}$, where $u_{ij}$ is a PLTS and indicates the value alternative $X_i$ with respect to criterion $C_j$.

*Step 1.* Transform PLTS matrix $U$ into TFN information

To make the decision process easy to compute, we firstly transform PLTS matrix $U$ into TFN matrix $H = (h_{ij}^L, h_{ij}^M, h_{ij}^R)_{m\times n}$ based on Rules 1–3 in Section 3.

*Step 2.* Obtain the defuzzified centroid matrix $Z$

Compute the defuzzified centroid matrix of TFN matrix $H$ as $Z = (z_{ij})_{m\times n}$, where

$$z_{ij} = \frac{h_{ij}^L + h_{ij}^M + h_{ij}^R}{3}. \tag{10}$$

*Step 3.* Determine the positive flow $\Phi^+$ and negative flow $\Phi^-$

For criterion $C_j$, the preference degree for alternative $X_i$ over $X_k$ can be obtained by

$$F_j(X_i, X_k) = H_j\left(z_{ij} - z_{kj}\right), \tag{11}$$

where $H_j$ is a nondecreasing preference function. There are mainly six types of preference functions to choose [28]. In this paper, in order to compute simply, we choose the following preference function:

$$H_j(x, y) = \begin{cases} 1, & x \geq y, \\ 0, & x < y. \end{cases} \tag{12}$$

Then, we can obtain the overall preference value of alternative $X_i$ over $X_k$ as

$$F(X_i, X_k) = \sum_{j=1}^{n} \omega_j F_j(X_i, X_k), \tag{13}$$

where $\omega_j$ is the criterion weight for $C_j$ and can be obtained by equation (9).

The positive flow $\Phi^+(X_i)$ and negative flow $\Phi^-(X_i)$ for alternative $X_i$ can be calculated by (14) and (15), respectively:

$$\Phi^+(X_i) = \frac{1}{m-1}\sum_{k=1,k\neq i}^{m} F(X_i, X_k), \quad (14)$$

$$\Phi^-(X_i) = \frac{1}{m-1}\sum_{k=1,k\neq i}^{m} F(X_k, X_i). \quad (15)$$

*Step 4.* Compute the net flow $\Phi$ and rank the alternatives

The net flow $\Phi(X_i)$ for alternative $X_i$ can be calculated by (16):

$$\Phi(X_i) = \Phi^+(X_i) - \Phi^-(X_i). \quad (16)$$

Rank the alternatives according to the values of net flow for all alternatives. The larger the values of net flow of the alternative, the higher the priority of the alternative.

*4.2. Sensitivity Analysis for Criteria Using WSI Method.* Accurate criteria weights are very important to make a reasonable decision. WSI method [29] is an effective method to make a sensitivity analysis for criteria. For criterion $C_j$, its weight is $\omega_j$, and we will see how the weight value can be modified without changing the ranking result. The new criteria weights are defined as follows:

$$\begin{aligned} &\omega_j' = (1+\alpha)\cdot\omega_j, \alpha \geq -1,\\ &\omega_l' = \beta\cdot\omega_l, \beta = \frac{1-(1+\alpha)\omega_j}{1-\omega_j}, 0\leq\beta\leq\frac{1}{1-\omega_j}, \forall l\neq j. \end{aligned} \quad (17)$$

Based on the PROMETHEE II method, we assume that

$$\begin{aligned} &\Delta(X_i, X_k) = \Phi(X_i) - \Phi(X_k),\\ &\Delta_j(X_i, X_k) = \Phi_j(X_i) - \Phi_j(X_k) = \frac{1}{m-1}\left(\sum_{h=1,h\neq i}^{m} F_j(X_i, X_h) - \sum_{h=1,h\neq k}^{m} F_j(X_k, X_h)\right). \end{aligned} \quad (18)$$

We give the following definition:

$$\begin{aligned} &\Omega^0 = \{(X_i, X_k)\in X\times X, \text{s.t. } \Delta_j(X_i, X_k) < 0 \text{ and } \Delta(X_i, X_k) = 0\},\\ &\Omega^- = \{(X_i, X_k)\in X\times X, \text{s.t. } \Delta(X_i, X_k)\cdot\Delta_j(X_i, X_k) < 0\},\\ &\Omega^+ = \{(X_i, X_k)\in X\times X, \text{s.t. } \Delta(X_i, X_k)\cdot\Delta_j(X_i, X_k) > \Delta^2(X_i, X_k)\},\\ &\beta_j^- = \max\frac{\Delta(X_i, X_k)\cdot\Delta_j(X_i, X_k)}{\Delta(X_i, X_k)\cdot\Delta_j(X_i, X_k) - \Delta^2(X_i, X_k)}, \quad (X_i, X_k)\in\Omega^-,\\ &\beta_j^+ = \min\frac{\Delta(X_i, X_k)\cdot\Delta_j(X_i, X_k)}{\Delta(X_i, X_k)\cdot\Delta_j(X_i, X_k) - \Delta^2(X_i, X_k)}, \quad (X_i, X_k)\in\Omega^+. \end{aligned} \quad (19)$$

Then, we can get the weight stability interval of the criterion $C_j$ as follows:

$$(\omega_j^-, \omega_j^+) = (1-(1-\omega_j)\cdot\beta_j^+, 1-(1-\omega_j)\cdot\beta_j^-), \quad (20)$$

where $\omega_j^-$ and $\omega_j^+$ are the lower and upper bounds of the weight stable interval of criterion $C_j$.

## 5. A Case Study

In this section, we will use our proposed methods to solve the evaluation of nursing homes in Zhenjiang City, Jiangsu Province. This section will include four parts: (1) decision problem description, (2) obtaining key factors for evaluation system and criteria weights based on DEMATEL, (3) ranking alternatives based on PROMETHEE II method for PLTS, and (4) further discussions and sensitivity analysis using WSI method.

*5.1. Decision Problem Description.* In recent years, due to the influence of the fertility policy, the number of the elderly populations in China began to increase continuously. China has become an aging population country. In such a population environment, the pension service industry began to develop. There are a variety of different pension models for the elderly to choose. As part of the pension pattern, institutional pensions are defined as institutions that provide

Table 1: Comprehensive direct influencing matrix $Q$.

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $C_1$ | 0 | $\{s_{-2}(0.2), s_{-1}(0.2), s_1(0.4), s_2(0.2)\}$ | $\{s_{-2}(0.2), s_{-1}(0.5), s_2(0.3)\}$ | $\{s_{-2}(0.3), s_{-1}(0.3), s_0(0.4)\}$ |
| $C_2$ | $\{s_{-2}(0.2), s_{-1}(0.3), s_0(0.5)\}$ | 0 | $\{s_{-2}(0.2), S_1(0.6), s_2(0.2)\}$ | $\{s_{-1}(0.2), S_1(0.6), s_2(0.2)\}$ |
| $C_3$ | $\{s_{-1}(0.3), s_0(0.2), s_1(0.2), s_2(0.3)\}$ | $\{s_0(0.4), s_1(0.2), S_2(0.4)\}$ | 0 | $\{s_{-2}(0.2), s_{-1}(0.6), s_2(0.2)\}$ |
| $C_4$ | $\{s_{-2}(0.3), s_{-1}(0.1), s_0(0.6)\}$ | $\{s_0(0.5), s_1(0.3), s_2(0.2)\}$ | $\{s_0(0.3), s_1(0.6), s_2(0.1)\}$ | 0 |

Table 2: TFN matrix $A$.

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $C_1$ | $<0.000, 0.000, 0.000>$ | $<0.000, 0.620, 1.000>$ | $<0.000, 0.775, 1.000>$ | $<0.000, 0.225, 0.750>$ |
| $C_2$ | $<0.000, 0.282, 0.750>$ | $<0.000, 0.000, 0.000>$ | $<0.000, 0.800, 1.000>$ | $<0.000, 0.805, 1.000>$ |
| $C_3$ | $<0.000, 0.606, 1.000>$ | $<0.250, 0.757, 1.000>$ | $<0.000, 0.000, 0.000>$ | $<0.000, 0.800, 1.000>$ |
| $C_4$ | $<0.000, 0.339, 0.750>$ | $<0.250, 0.782, 1.000>$ | $<0.250, 0.804, 1.000>$ | $<0.000, 0.000, 0.000>$ |

Table 3: Total influencing matrix $T$.

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $C_1$ | $<0.000, 0.191, 3.267>$ | $<0.000, 0.472, 4.000>$ | $<0.000, 0.529, 4.000>$ | $<0.000, 0.357, 3.733>$ |
| $C_2$ | $<0.000, 0.306, 3.467>$ | $<0.000, 0.351, 3.750>$ | $<0.000, 0.583, 4.000>$ | $<0.000, 0.540, 3.783>$ |
| $C_3$ | $<0.000, 0.403, 3.733>$ | $<0.083, 0.585, 4.250>$ | $<0.000, 0.411, 4.000>$ | $<0.000, 0.563, 4.017>$ |
| $C_4$ | $<0.000, 0.322, 3.467>$ | $<0.090, 0.562, 4.000>$ | $<0.083, 0.589, 4.000>$ | $<0.000, 0.332, 3.533>$ |

Table 4: Defuzzified centroid matrix $Y$.

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $C_1$ | 1.153 | **1.491** | **1.510** | 1.363 |
| $C_2$ | 1.257 | 1.367 | **1.528** | **1.441** |
| $C_3$ | 1.379 | **1.640** | 1.470 | **1.527** |
| $C_4$ | 1.263 | **1.551** | **1.558** | 1.288 |

centralized housing and care services for the elderly, such as nursing homes.

Recently, we walked into a service community for the elderly in Zhenjiang City, Jiangsu Province. In the early stage of preparation, we collected the relevant information of caregivers in four local nursing homes. In the process of investigation, we provided the relevant materials of caregivers in four nursing homes to the elderly in the community and invited them to make a comprehensive evaluation of caregivers in four nursing homes $(X_1, X_2, X_3, X_4)$. There are four criteria [30] considered in the evaluation process: price acceptability $(C_1)$, sustainability of service $(C_2)$, responsibility $(C_3)$, and service quality $(C_4)$.

*5.2. Obtaining Key Factors for Evaluation System and Criteria Weights Based on DEMATEL.*

(1) We invite ten experts to make a comparison between two criteria using LTSs. By aggregating the information, we obtain the comprehensive direct influencing matrix $Q$ as shown in Table 1.

(2) Based on Rules 1–3, we obtain the TFN matrix $A$ as shown in Table 2.

(3) Based on equation (6), we obtain the total influencing matrix $T$ as shown in Table 3.

(4) We can compute the defuzzified centroid matrix $Y$ as shown in Table 4.

We can obtain the threshold $\varepsilon = (\sum_i^n \sum_j^n y_{ij}/n^2) = 1.424$. Influence relation between criteria can be seen in Figure 1.

The values of $r_i c_i$, $r_i + c_i$, $r_i - c_i$ and $\omega_j$ can be seen in Table 5.

We can obtain the cause-effect relationship of criteria as shown in Figure 2.

It can be seen from Figure 2 that criteria price acceptability $(C_1)$ and service quality $(C_4)$ are cause factors influencing the evaluation system, while and sustainability of service $(C_2)$ and responsibility $(C_3)$ are effect ones that are affected by the evaluation system. In other words, price acceptability $(C_1)$ and service quality $(C_4)$ are the most concerned factors of elders in four ones. Nursing homes should lower the service price and improve service quality.

*5.3. Ranking Alternatives Based on PROMETHEE II Method for PLTS.* Based on the aggregation of the experts' opinions, the decision matrix $U$ can be seen in Table 6.

*Step 1.* Based on Rules 1–3, we can transform PLTS matrix $U$ into TFN decision matrix $H$ as shown in Table 7.

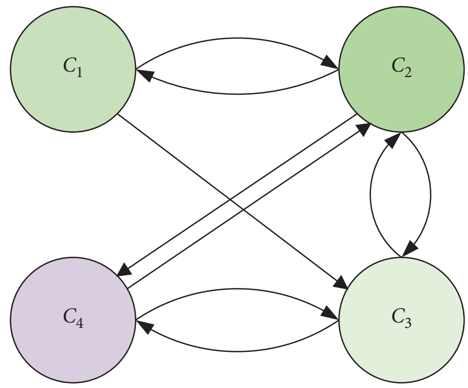


Figure 1: Influence relation diagram between criteria.

Table 5: Values of $r_i c_i$, $r_i + c_i$, $r_i - c_i$, and $\omega_j$.

| | $r_i$ | $c_i$ | $r_i + c_i$ | $r_i - c_i$ | $\omega_j$ |
|---|---|---|---|---|---|
| $C_1$ | 5.516 | 5.052 | 10.568 | 0.464 | 0.232 |
| $C_2$ | 5.593 | 6.048 | 11.641 | −0.454 | 0.255 |
| $C_3$ | 6.015 | 6.065 | 12.080 | −0.050 | 0.265 |
| $C_4$ | 5.659 | 5.620 | 11.279 | 0.040 | 0.248 |

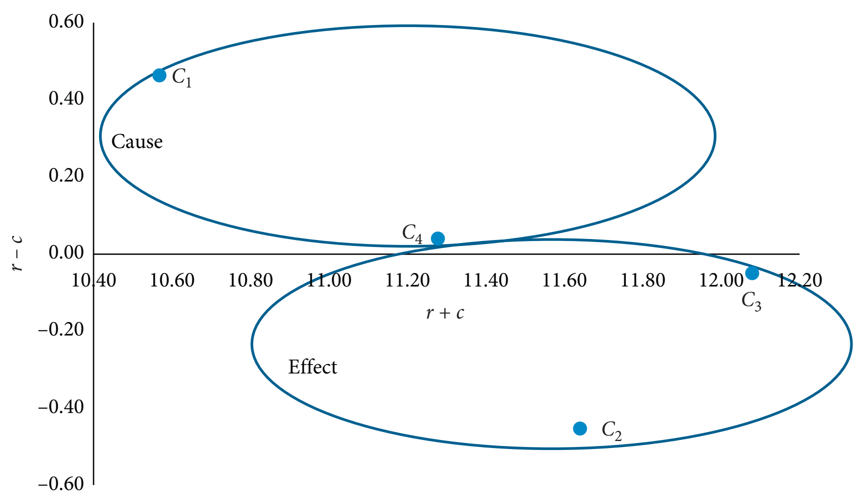


Figure 2: Cause-effect relationship of criteria.

Table 6: Decision matrix $U$.

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $X_1$ | $\{s_{-1}(0.4), s_0(0.2), S_1(0.4)\}$ | $\{s_{-1}(0.2), s_1(0.8)\}$ | $\{s_{-2}(0.6), s_1(0.4)\}$ | $\{s_{-2}(0.2), s_{-1}(0.4), s_0(0.2), s_2(0.2)\}$ |
| $X_2$ | $\{s_{-1}(0.2), s_0(0.4), s_1(0.2), s_2(0.2)\}$ | $\{s_{-1}(0.4), S_1(0.4), s_2(0.2)\}$ | $\{s_{-2}(0.2), s_{-1}(0.6), s_0(0.2)\}$ | $\{s_{-1}(0.4), s_1(0.6)\}$ |
| $X_3$ | $\{s_{-2}(0.3), s_{-1}(0.3), s_0(0.4)\}$ | $\{s_{-1}(0.4), s_0(0.2), s_1(0.2), s_2(0.2)\}$ | $\{s_{-1}(0.2), s_0(0.3), s_1(0.4), s_2(0.1)\}$ | $\{s_{-2}(0.4), s_0(0.6)\}$ |
| $X_4$ | $\{s_{-1}(0.3), s_1(0.5), s_2(0.2)\}$ | $\{s_0(0.4), s_1(0.4), s_2(0.2)\}$ | $\{s_{-1}(0.2), s_0(0.2), s_1(0.4), s_2(0.2)\}$ | $\{s_0(0.2), s_1(0.6), s_2(0.2)\}$ |

Table 7: TFN decision matrix $H$.

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $X_1$ | <0.000, 0.507, 1.000> | <0.000, 0.683, 1.000> | <0.000, 0.623, 1.000> | <0.000, 0.620, 1.000> |
| $X_2$ | <0.000, 0.628, 1.000> | <0.000, 0.755, 1.000> | <0.000, 0.307, 0.750> | <0.000, 0.624, 1.000> |
| $X_3$ | <0.000, 0.225, 0.750> | <0.000, 0.628, 1.000> | <0.000, 0.690, 1.000> | <0.000, 0.372, 0.750> |
| $X_4$ | <0.000, 0.777, 1.000> | <0.250, 0.757, 1.000> | <0.000, 0.628, 1.000> | <0.250, 0.807, 1.000> |

Table 8: Defuzzified centroid matrix $Z$.

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $X_1$ | 0.502 | 0.561 | 0.541 | 0.540 |
| $X_2$ | 0.543 | 0.585 | 0.352 | 0.541 |
| $X_3$ | 0.325 | 0.543 | 0.563 | 0.374 |
| $X_4$ | 0.592 | 0.669 | 0.543 | 0.686 |

Table 9: Values of positive flow $\Phi^+$.

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $X_1$ | 1 | 1 | 1 | 1 |
| $X_2$ | 2 | 2 | 0 | 2 |
| $X_3$ | 0 | 0 | 3 | 0 |
| $X_4$ | 3 | 3 | 2 | 3 |

Table 10: Values of negative flow $\Phi^-$.

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $X_1$ | 2 | 2 | 2 | 2 |
| $X_2$ | 1 | 1 | 3 | 1 |
| $X_3$ | 3 | 3 | 0 | 3 |
| $X_4$ | 0 | 0 | 1 | 0 |

Table 11: Comparison results with the method of Pang et al. [8].

| Decision-making method | Ranking order | Optimal alternative |
|---|---|---|
| Pang et al. [8] method | $X_4 \succ X_1 \succ X_2 \succ X_3$ | $X_4$ |
| Proposed method | $X_4 \succ X_2 \succ X_1 \succ X_3$ | $X_4$ |

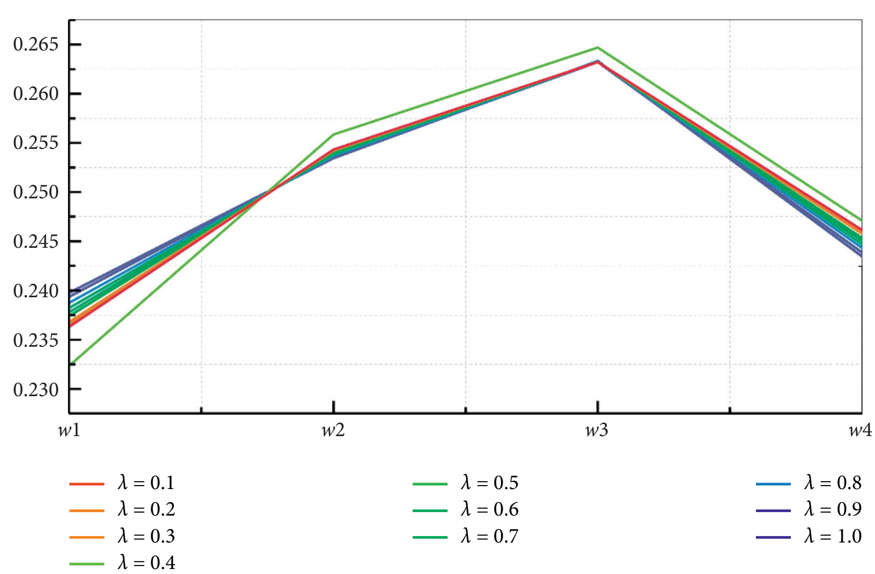


Figure 3: Values of criteria weights under different parameter $\lambda$.

Table 12: Ranking results under different parameter $\lambda$.

| $\lambda$ | Ranking order | Optimal alternative |
|---|---|---|
| 0.1 | $X_4 > X_1 > X_2 > X_3$ | $X_4$ |
| 0.2 | $X_4 > X_2 > X_1 > X_3$ | $X_4$ |
| 0.3 | $X_4 > X_2 > X_1 > X_3$ | $X_4$ |
| 0.4 | $X_4 > X_2 > X_1 > X_3$ | $X_4$ |
| 0.5 | $X_4 > X_2 > X_3 > X_1$ | $X_4$ |
| 0.6 | $X_4 > X_2 > X_3 > X_1$ | $X_4$ |
| 0.7 | $X_4 > X_2 > X_3 > X_1$ | $X_4$ |
| 0.8 | $X_4 > X_2 > X_3 > X_1$ | $X_4$ |
| 0.9 | $X_4 > X_2 > X_3 > X_1$ | $X_4$ |
| 1.0 | $X_4 > X_2 > X_3 > X_1$ | $X_4$ |

Table 13: Results of stable weight intervals using the WSI method.

| $\lambda = 0.1$ | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $(\beta_j^-, \beta_j^+)$ | (0.000, 1.308) | (0.000, 1.340) | (0.777, 1.357) | (0.556, 1.326) |
| $(\omega_j^-, \omega_j^+)$ | (0.000, 1.000) | (0.000, 1.000) | (0.000, 0.428) | (0.000, 0.581) |
| $\lambda = 0.4$ | $C_1$ | $C2$ | $C3$ | $C4$ |
| $(\beta_j^-, \beta_j^+)$ | (0.000, 1.258) | (0.000, 1.258) | (0.907, 1.361) | (0.000, 1.258) |
| $(\omega_j^-, \omega_j^+)$ | (0.034, 1.000) | (0.063, 1.000) | (0.000, 0.333) | (0.054, 1.000) |
| $\lambda = 0.7$ | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
| $(\beta_j^-, \beta_j^+)$ | (0.949, 1.312) | (0.000, 1.056) | (0.808, 1.027) | (0.974, 1.325) |
| $(\omega_j^-, \omega_j^+)$ | (0.001, 0.276) | (0.212, 1.000) | (0.243, 0.405) | (0.000, 0.264) |

*Step 2.* We can obtain the defuzzified centroid matrix $Z$ as shown in Table 8.

*Step 3.* Obtain the positive flow $\Phi^+$ and negative flow $\Phi^-$ as shown in Tables 9 and 10, respectively.

*Step 4.* Compute the net flow $\Phi$ of four alternatives as follows:

$$\begin{aligned} \Phi(X_1) &= -0.333, \\ \Phi(X_2) &= -0.020, \\ \Phi(X_3) &= -0.470, \\ \Phi(X_4) &= 0.823. \end{aligned} \tag{21}$$

The ranking result is $X_4 \succ X_2 \succ X_1 \succ X_3$.

We make a comparison between our proposed method and the method proposed by Pang et al. [8] as shown in Table 11.

As can be seen from Table 11, the ranking results of the two methods are different. The main reason may be lie in the different decision-making mechanisms for the two methods. The method proposed by Pang et al. [8] uses the traditional TOPSIS method, while our method uses a specific outranking method. The advantages of the method to the other one are simple calculation and convenience in sensitivity analysis using the WSI method.

*5.4. Further Discussions and Sensitivity Analysis Using WSI Method.* In Section 3, the values parameter $\lambda$ can make an influence on the final decision results. We will choose different values of $\lambda$ to discuss the ranking results and sensitivity analysis situations. The criteria weights and ranking results under different values of $\lambda$ can be seen in Figure 3 and Table 12, respectively.

We choose $\lambda = 0.1, \lambda = 0.4$, and $\lambda = 0.7$ to make a sensitivity analysis using the WSI method as shown in Table 13.

As can be seen from Table 13, the smaller the value of parameter $\lambda$, the larger stable the weight intervals. In fact, the parameter can reflect the confidences of DMs. If DMs have enough confidence on their judgments, the variation of weights will be very small. This is also true of reality.

## 6. Conclusions

China has been one of the most serious aging countries in the world. With the liberalization of the family planning policy, more and more families have two children. Many families have the 4-2-2 family structure, which means a couple should support four elders and raise two children. It is difficult for the couple to spend enough time to support the four elders. Furthermore, many young people migrate to big cities to bring home the bacon. Traditional informal preserving pattern for the elders based on families is not realizable for many families. The institution offering services for the elders is a new tendency. A few nursing homes' supply falls short of demand that leads to the fact that most elders cannot enjoy their care services. While most private nursing homes operate hardly and have to look for customers. In order to resolve the contradiction, it is necessary to evaluate nursing homes, which will not only help elders to select suitable nursing homes but also find key concern point for elders. Therefore, in this paper, the evaluation process has two phases: seeking key factors and ranking results. We illustrate the DEMATEL method for PLTSs to analyze the key factors influencing evaluation process and obtain criteria weights. Then, we propose a novel PROMETHEE method to rank the alternatives and make a sensitivity analysis for criteria. Finally, we applied our methods to solve the nursing homes evaluation problem in Zhenjiang City to illustrate the effectiveness and practicability of our methods.

Future research will focus on clustering disabled elders to different categories based on case-based reasoning method, which will help government to give different financial support for certain disabled elders [31].

## Data Availability

No data were used to support this study.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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*Research Article*

# Portfolio Selection with respect to the Probabilistic Preference in Variable Risk Appetites: A Double-Hierarchy Analysis Method

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Traditional portfolio selection models mainly obtain the optimized portfolio ratio by focusing on the prices of financial products. However, investors' multiple preferences and risk appetites are also significant factors that should be taken into account. In consideration of these two factors simultaneously, we propose a double-hierarchy model in this paper. Specifically, the first hierarchy quantifies investors' risk appetite based on a historical simulation method and probabilistic preference theory. This hierarchy can be utilized to describe investors' variable risk appetites and ensure the obtained investment ratios meet investors' immediate risk requirements. Then, using the cross-efficiency evaluation principle, the optimal investment ratios can be derived by fusing investors' multiple preferences and risk appetites in the second hierarchy. Lastly, an illustrative example about evaluating the 10 largest capitalized stocks on the Shenzhen Stock Exchange is given to verify the feasibility and effectiveness of our newly proposed model. We make the theoretical contribution to improve the traditional portfolio selection model, especially considering investors' subjective preferences and risk appetite. Moreover, the proposed model can be practical for assisting investors with their investment strategies in real life.

## 1. Introduction

Portfolio selection has been broadly discussed in the field of economics over years. As an effective tool to diversify risks and increase profits, how to optimize the portfolio has also become a hot issue for investors. Since optimized portfolios are capable to diversify risks, they can be differently and flexibly changed according to investors' risk appetites. Even though people would mainly concern about the expected returns, some key elements should also be taken into consideration, such as investors' risk appetites and their subjective preference. These two key elements are variable for different investors. Moreover, an investor's risk appetite and preferences can be dynamic in different periods. As a result, they can be impacted by external and internal factors. On the contrary, how to quantitate investors' dynamic risk appetite and multiple preferences is the issue that this paper addresses as well. To optimize portfolio selection from the perspectives of investors' dynamic risk appetites and subjective preferences, this study proposes a double-hierarchy model that considers investors' dynamic risk appetites with probabilistic preference and multiple preferences simultaneously. Furthermore, the feasibility and effectiveness of our newly proposed model are proved by providing an empirical analysis based on the data of China's Shenzhen A-share market.

In the previous studies in the portfolio field, most scholars focus on returns and risks [1], stockholder wealth, asset allocation, and financial product price. However, rational investors would make investment decisions merely based on a single preference such as financial product price. Instead, they tend to consider multiple relative preferences of investment objectives with dynamic risk appetites. In view of that, some scholars propose multipreference models to cope with the above issues. For example, Kwan [2] proposed an optimal algorithm for seven types of multipreference models, which does not need a clear ranking of stocks. Based on this algorithm, Jun [3] proposed that the price of financial products must be considered when considering the least risk and the greatest benefit, and the price also determines the weight of the portfolio selection. Chamberlain et al. [4] employed it to calculate the optimal portfolio, enabling investors to amplify across different forms of assets. Other scholars mainly pay attention to portfolio selection, forecast of stock return [5], and so on.

As the medium of investment, portfolio analysis, multipreference analysis, and selection are key factors that cannot be ignored in the financial product market. Therefore, considering multipreference portfolio analysis is also the key to current research. From the perspective of preference analysis, different scholars have carried out research studies on investment portfolio preference selection issues, preference evaluation, and preference effects. This article is mainly based on the cross-efficiency model and selected multiple-preference portfolio model. Previous studies have shown that both single preference and dual preference can affect venture capital to a certain extent. In the classic efficiency evaluation, the biggest difficulty at present is how to choose the aggressive and benevolent. Researchers have started more in-depth research and analysis for this. Mei [6] applied the dual-preference analysis of the portfolio to get a new strategy and obtained a higher rate of return for the first time. Penman and Sougiannis [7] used momentum preference indicators and value preference indicators to analyze market efficiency, and the results showed that they can generate higher profits. Based on their analysis, Paul and Bergin [8] found that the rates of return of the two are independent of each other, and the rates of return have increased, but they are not relevant. Li and Chuang [9]; Reikvam et al. [10]; and Asai & Mcaleer [11] carry out detailed studies from the perspectives of accuracy, feasibility, and simplicity of preference selection, respectively. Rowley and Kwon [12] analyzed current portfolio preferences from multiple perspectives of immediacy, representativeness, and risk and improved the adaptive level of preference selection. In addition, Xue and Zhou [13] and Israelov and Klein [14] classified the nature of the multipreference indicators; on this basis, Haley [15] studied the heterogeneity in multiple-preference selection and applied the research model to the portfolio in the stock market. They found that due to external effects, indicators with only profitability preferences are not applicable to the portfolio market.

In general, the current investment portfolio preference focuses on investment transaction risk, profitability, and representative indicators. Although the research results are quite abundant, the existing theories are still difficult to match the continuous development of the financial product market investment portfolio. Thus, we use variable multiple preferences to meet the growing portfolio demand.

The above investigations consider some essential elements of the portfolio. However, typical investors will adjust their portfolio selection due to the increase or decrease of their income, the changes of original investment targets, and the vicissitude of the investment macroenvironment. Therefore, this paper introduces several representative factors of investor risk appetite into the portfolio selection model.

Investor risk appetite has been a hot issue in the investment field. It is usually classified into risk-averse, risk-neutral, and risk-seeking. Scholars mainly focus on in the field of pure contagion, commodity returns, and exchange rates [16]. Being different from the traditional qualitative empirical tests, some researchers attempt to research the variable risk appetites of investors [17]. Furthermore, some papers broaden the risk appetite coefficient [18–20]. Smimoua et al. [21] believed that portfolio selection needs to refer to certain preferences. On the basis of these, Smith and Ierapetritou [22] put forward a single evaluation criterion, and Zrs and Bayoumi [23] proposed multiple evaluation criteria for portfolios; both of them are enriching the research content of the portfolio evaluation criterion. In addition, some scholars focus on the venture capital and technological performance by multiple evaluation criteria [24–26]. Investors' risk appetite is also an important research content in the field of investment portfolio. Since Friend and Blume [27] proposed the risk attitude coefficient, firstly, many scholars pay attention to the risk attitude research, and Hansen and Singleton [28] measured the risk attitude coefficient range from 0.68 to 0.97. Other scholars adopt different data to measure risk attitude coefficient and obtain different ranges; Halek and Eisenhauer [29] measured that the average risk appetite coefficient is 3.735 based on life insurance data. Nosic and Weber [30] found that investors' risk appetites are changeable in the face of diversified financial products; especially, Zhang et al. [5] believed that risk appetites will change with changes in previous returns. Dulleck et al. and Yanling and Dragon [31, 32] showed that risk appetites will change in macroeconomic indicators. As a result, we attempt to investigate investors' variable risk appetites and quantify them by using historical simulation and probabilistic preference theory.

As mentioned above, the previous studies mainly focus on a single preference of the portfolio, such as financial product price, but less attention is paid to investors' variable risk appetites. Therefore, we quantify the variable risk appetites and introduce them into a multipreference portfolio selection model to carry out further empirical investigation.

The remainder of this paper is organized as follows: the next section of this paper describes the multiple-preference portfolio selection model (MPPS). Double-hierarchy multiple-preference model (DHMP) will be elucidated in Section 3. Section 4 explains the modeling steps and the illustrated example. Section 5 conducts the empirical study. The fifth part gives conclusions and some research directions. The process is shown in Figure 1.

## 2. Multiple-Preference Portfolio Selection Model in the Background of the Probabilistic Preference

*2.1. Preferences and Multiple Preferences.* In the financial product market, investors make a choice that whether to select the product in the past mainly based on the current and historical prices of financial products. In other words, portfolio selection is affected by price preference. However, with the financial product market continuing to improve, increasing investors consider multiple preferences. The

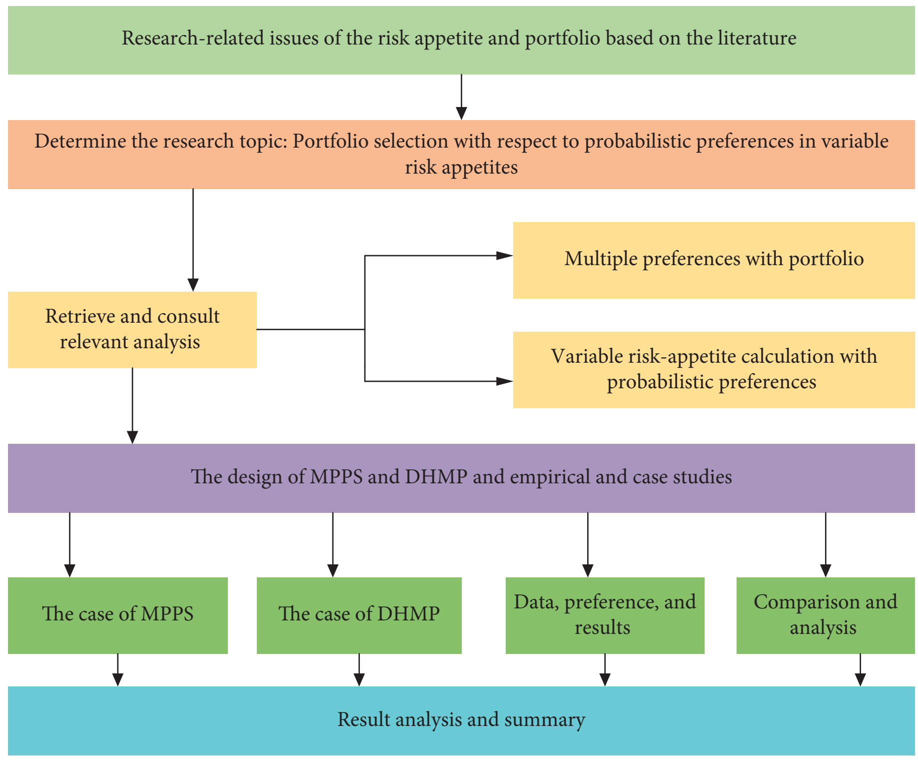


Figure 1: The research structure of this paper.

preferences of |products are very wide. From the perspective of return and risk attributes, there are high risk and high return, low risk and low return, and risk-free return. From the perspective of industry attributes, there are high energy consumption, low energy consumption, and green environmental protection. And we analyze the portfolio model from the multiple-preference attributes of returns and risk and measure the preference of financial products from the indicator characteristics of the investment object. Different investment indicators reflect different preferences. Drawing on the cross-efficiency evaluation method, we introduce the inclusion of multiple preferences to conduct the portfolio model.

With the help of efficiency evaluation ideas, we interpret preferences from the perspective of risk and return. When there is a single risk preference in the market, there is only one evaluation unit for investment products, but there are often multiple preferences and multiple decision-making units in practical investment. Through multiple preferences, multiple risks and return issues of investment products can be analyzed clearly, the decision maker does not need to give any subjective information, and there is no need to preset a certain production function.

*2.2. Multiple-Preference Portfolio Selection Model and Calculation.* Then, we build the multiple-preference portfolio selection in the background of cross-efficiency. Assume that there are $n$ evaluation units. There are $m$ different risk preferences and $s$ return preferences in evaluation units, and the risks and benefit vectors are

$$X_j = \left(x_{1j}, x_{2j}, \ldots, x_{mj}\right), Y_j = \left(y_{1j}, y_{2j}, \ldots, y_{sj}\right). \quad (1)$$

For the multiple-preference value $E_{dd}$, we can use the traditional CCR model for calculation:

$$E_{dd} = \max \sum_{r=1}^{s} \mu_{rd} y_{rd},$$
$$\text{s.t.} \begin{cases} \displaystyle\sum_{i=1}^{m} w_{id} x_{id} - \sum_{r=1}^{s} \mu_{rd} y_{rj} \geq 0, & j = 1, 2, \ldots, n, \\ \displaystyle\sum_{i=1}^{m} w_{id} x_{id} = 1, & \\ w_{id} \geq 0, & i = 1, 2, \ldots, m, \\ \mu_{rd} \geq 0, & r = 1, 2, \ldots, s, \end{cases} \quad (2)$$

where $\omega_{id}\,(i = 1, 2, \ldots, m)$ and $\omega_{id}\,(i = 1, 2, \ldots, m)$ represent the weight of risk preferences $x_{ij}$ and return preferences $y_{rj}$. We can get the optimal weight $\omega_{1d}^{*}, \ldots, \omega_{md}^{*}, \mu_{1d}^{*}, \ldots, \mu_{sd}^{*}$ of the multiple-preference unit by solving model (2); furthermore, the value of cross-evaluation can be achieved by the following equation:

$$E_{dj} = \frac{\sum_{r=1}^{s} \mu_{rd}^{*} y_{rj}}{\sum_{i=1}^{m} \omega_{id}^{*} x_{ij}}, \quad d, j = 1, 2, \ldots, n. \tag{3}$$

The cross-evaluation value of the preference unit can be obtained by comparing equations (2) and (3). For every preference unit, all cross-evaluation values $E_{dj}\,(d = 1, 2, \ldots, n)$ are averaged by the following equation:

$$\bar{E}_j = \frac{1}{n} \sum_{d=1}^{n} E_{dj}\,(j = 1, 2, \ldots, n). \tag{4}$$

$\bar{E}_j$ is the final efficiency value. In the above equation, the optimal target value of model (2) is unique, but the optimal weight is not unique possibly. Through the calculation of equation (3), we may solve multiple different cross-efficiency preference values; to solve this problem, we build the following equation to form a multiple-preference portfolio selection model:

$$\beta = opt \sum_{j=1, j\neq d}^{\delta} \mu_{rd} \left( \sum_{j=1, j\neq d}^{n} y_{rj} \right),$$

$$\text{s.t.} \begin{cases} \sum_{i=1}^{m} w_{id} x_{ij} - \sum_{r=1}^{s} \mu_{rd} y_{rj} \geq 0, \quad j = 1, 2, \ldots, n, \\ \sum_{i=1}^{m} w_{id} \left( \sum_{j=1, j\neq d}^{n} x_{ij} \right) = 1, \\ \sum_{r=1}^{m} \mu_{rd} y_{rd} - \beta_{dd} \sum_{i=1}^{m} w_{id} x_{id} = 0, \\ w_{id} \geq 0, i = 1, 2, \ldots, m, \\ \mu_{rd} \geq 0, r = 1, 2, \ldots, s, \\ d = 1, 2, \ldots, n, \end{cases} \tag{5}$$

where $m$ and $s$ represent the standard number of risk preferences and profitable preferences, which are presented as $X_j = (x_{1j}, x_{2j}, \ldots, x_{mj})$ and $Y_j = (y_{1j}, y_{2j}, \ldots, y_{sj})$. $w_{id}\,(i = 1, 2, \ldots, m)$ and $\mu_{\rho d}\,(\rho = 1, 2, \ldots, s)$ denote the proportion vector of $x_{ij}$ and $y_{ij}$, respectively. In addition, $\beta_{dd}$ denotes the self-evaluation value, and $\beta_{dj}$ is the cross-evaluation value and calculated by the model.

Then, after we calculate $\beta_{dj}$, take the mean and variance of $\beta_{dj}$. Finally, we take it into using the mean variance model for conducting the multiple-preference portfolio selection model.

$$\min \delta(R_P) = \sum \sum x_i x_j \text{cov}\left(R_i - R_j\right),$$

$$\text{s.t.} \begin{cases} R_P = \sum x_i R_i, \\ 1 = \sum x_i, \\ or 1 = \sum x_i, x_i \geq 0, \end{cases} \tag{6}$$

where $\delta(R_P)$ represents the variance of the investor's choice, that is, the risk measure of the overall selection, $\text{cov}(R_i - R_j)$ is the covariance of investment products, and $x_i$ and $x_j$ are the weights of different financial products. In addition, $R_P$ is the expected rate of return; we combined multiple preferences and portfolio theory to build a multiple-preference portfolio selection model.

## 3. Double-Hierarchy Multiple-Preference Model Based on the Probabilistic Preference

Based on the multiple preferences, we propose a double-hierarchy multiple-preference model composed of variable risk appetite coefficients based on probabilistic preference and multiple preferences; we first introduce the historical simulation method and probabilistic preference to describe the variable risk appetites, and after the second hierarchy of calculation, the weight of the optimal portfolio is obtained; the process is displayed in Figure 2.

*3.1. First Hierarchy Calculation of the Variable Risk Appetites in the Probabilistic Preference.* In this part, we introduce the interval probabilistic preference as the evidence measure, assume that the probability of occurrence of the event is an interval value, and describe the ambiguity of choice, according to the probability theory [33, 34]. The preference of investors for the program is described by four types of relationships: surpass $\succ$, cannot compare, equal $\approx$, worse than $\prec$, and expand to surpass, worse than, equal, incomparable but there is a previous $^{\vee}$, not compare with upper bound $\|_{\wedge}$, cannot compare with upper bound and lower bound $\|_{\wedge}^{\vee}$ cannot compare with unbounded $\| \|$; we take interval probability theory as a tool for uncertain decisions. Set the program selection problem; the alternatives are $X = [A_1, \ldots, A_i, \ldots, A_k, \ldots, A_m]$, the decision-making group is $E = [e_1, e_2, \ldots, e_j, \ldots, e_n]$, the weight of $e_j$ is $w_j$, $\sum_{j=1}^{n} w_j = 1$, $1 \geq w_j \geq 0$, $e_j$is the preference relationship and $A_k$, $R = \{\succ, \|^{\vee}, \|_{\wedge}, \|_{\wedge}^{\vee}, \approx,\}$ is the probability distribution on the interval, and the probability of $A_i r A_k$: $q_{j,r} = [q_{j,*r}, q_{*j,r}]$ and $q_{j,*r} \leq q_{*j,r}$, $\forall r \in R$.

Determine the objective function of maximizing the probability of the investor's preference relationship with the following formula:

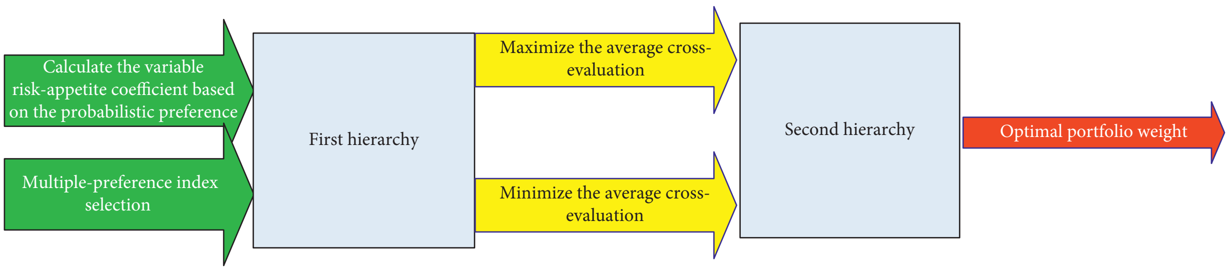


FIGURE 2: The process of the double-hierarchy model with respect to the probabilistic preference.

$$\max_{r\in R=\left\{\succ,\|,\overset{\vee}{\|},\underset{\wedge}{\|},\|^{\vee}_{\wedge},\approx,\prec\right\}} Z=\prod_{j=1}^{n}\left(w_j q_j, r\right)^{x_r},$$
$$\text{s.t.}\begin{cases} q_{j,*r}\le q_{j,r}\le q_{j,*r}, \quad \displaystyle\sum_{r\in R=\left\{\succ,\|,\overset{\vee}{\|},\underset{\wedge}{\|},\|\,\|^{\vee}_{\wedge},\approx,\prec\right\}} q_{j,r}=1, \quad \forall_j=1,2,\ldots,n, \displaystyle\sum_{j=1}^{n} w_j=1, \\ \displaystyle\sum_{r\in R=\left\{\succ,\|,\overset{\vee}{\|},\underset{\wedge}{\|},\|^{\vee}_{\wedge},\approx,\prec\right\}} x_r\ge 1, \quad x_r=0,1, \end{cases} \tag{7}$$

where $\prod_{j=1}^{n}(w_j q_j, r)^{x_r}$ represents the total probability. Determine the collective preference of all schemes in $B$, and get the collective preference by solving model $R^*$. Use the binary relationship priority rule to filter and get the preference information about the program. And we take the average of the upper bound and lower bound as the foundation of historical simulation.

Because investors prefer to construct portfolios based on their own risk appetites, after the probabilistic preference calculation, we combine with a historical simulation to describe investors' variable risk appetites. First, investors are provided with some financial products based on their interests. Then, the list of the investors' preferred financial products can be given. Thus, we calculate the variable risk-appetite coefficient of investors according to their lists using the following model:

$$\max f(k_i)=\max\left(k_1+k_2+\cdots+k_{n-1}\right)$$
$$\text{s.t.}\begin{cases} E_{\min}+\theta\left(E_{\max}-E_{\min}\right)-E^2_{\min}-\theta\left(E^2_{\max}-E^2_{\min}\right)+a_1-b_1>0 \\ E^2_{\min}+\theta\left(E^2_{\max}-E^2_{\min}\right)-E^3_{\min}-\theta\left(E^3_{\max}-E^3_{\min}\right)+a_2-b_2>0 \\ \vdots \\ E^{n-1}_{\min}+\theta\left(E^{n-1}_{\max}-E^{n-1}_{\min}\right)-E^n_{\min}-\theta\left(E^n_{\max}-E^n_{\min}\right)+a_{n-1}-b_{n-1}>0, \\ a_1b_1=0, a_2b_2=0,\ldots,a_{n-1}b_{n-1}=0, \\ b_1-a_1k_1\ge 0,\ b_2-a_2k_2\ge 0,\ \ldots, b_{n-1}-a_nk_{n-1}\ge 0, \\ a,b\ge 0, \theta\in[0,1], k_i\in\{0,1\}, i=1,\ldots,n-1, \end{cases} \tag{8}$$

where $E_{\max}$ and $E_{\min}$ are the optimization results calculated by models (9) and (10), respectively, which stand for the highest profitable risk matrix and the lowest one, $k$ is nonnegative and $k\in\{0,1\}$, $n$ is the number of portfolio choices, and $\theta$ is the variable risk-appetite coefficient and $\theta\in[0,1]$. Besides, $a$ and $b$ are the slack variables, and $a,b\ge 0$.

We can find that investors should provide a rank for $n$ objectives first, and then their variable risk appetites can be calculated by this model. For example, if there are four stocks, $A$, $B$, $C$, and $D$, an investor gives his/her rank based on his/her subjective evaluation, such as $A\ge B\ge C\ge D$. Then, we use models (8) and (9) to calculate $E_{\min}$ and $E_{\max}$, respectively, and derive the variable risk-appetite parameter $\theta$ based on model (8).

*3.2. Second Hierarchy Fusion of Probabilistic Preferences and Variable Risk Appetites.* To fuse the above variable risk appetite and calculate the optimal investment ratios, we build a multipreference portfolio selection model on the basis of the idea of cross-efficiency [35]. The model can assess financial products with multiple preferences and provide portfolio selection. Moreover, the weights of different financial products in a portfolio and the efficient frontier can be calculated by the following models:

$$E_{\max} = \max \sum_{j=1, j\neq d}^{s} \mu_{rd} \left( \sum_{j=1, j\neq d}^{n} y_{rj} \right),$$
$$\text{s.t.} \begin{cases} \sum_{i=1}^{m} \omega_{id} x_{ij} - \sum_{r=1}^{s} \mu_{rd} y_{rj} \geq 0, \quad j = 1, 2, \ldots, n, \\ \sum_{i=1}^{m} \omega_{id} \left( \sum_{j=1, j\neq d}^{n} x_{ij} \right) = 1, \\ \sum_{r=1}^{s} \mu_{rd} y_{rd} - E_{dd} \sum_{i=1}^{m} \omega_{id} x_{id} = 0, \\ \omega_{id} \geq 0, \quad i = 1, 2, \ldots, m, \\ \mu_{rd} \geq 0, \quad r = 1, 2, \ldots, s, \\ d = 1, 2, \ldots, n, \end{cases} \tag{9}$$

$$E_{\min} = \min \sum_{j=1, j\neq d}^{s} \mu_{rd} \left( \sum_{j=1, j\neq d}^{n} y_{rj} \right),$$
$$\text{s.t.} \begin{cases} \sum_{i=1}^{m} \omega_{id} x_{ij} - \sum_{r=1}^{s} \mu_{rd} y_{rj} \geq 0, \quad j = 1, 2, \ldots, n, \\ \sum_{i=1}^{m} \omega_{id} \left( \sum_{j=1, j\neq d}^{n} x_{ij} \right) = 1, \\ \sum_{r=1}^{s} \mu_{rd} y_{rd} - E_{dd} \sum_{i=1}^{m} \omega_{id} x_{id} = 0, \\ \omega_{id} \geq 0, \quad i = 1, 2, \ldots, m, \\ \mu_{rd} \geq 0, \quad r = 1, 2, \ldots, s, \\ d = 1, 2, \ldots, n, \end{cases} \tag{10}$$

where $m$ and $s$ represent the numbers of assessed risky and profitable variables, which are presented as $X_j = (x_{1j}, x_{2j}, \ldots, x_{mj})$ and $Y_j = (y_{1j}, y_{2j}, \ldots, y_{sj})$, and $\omega_{\iota d} (\iota = 1, 2, \ldots, m)$ and $\mu_{\rho d} (\rho = 1, 2, \ldots, s)$ denote the proportion vector of $x_{ij}$ and $y_{ij}$, respectively. In addition, $E_{dd}$ denotes the self-evaluation value, $E_{dj}$ is the cross-evaluation value, and it can be calculated by $E_{dj} = \sum_{r=1}^{s} \mu_{rd} y_{rj} / \sum_{i=1}^{m} \omega_{id} x_{ij}$, $E_{\max}$ is obtained based on the benevolent strategy to maximize the average cross-evaluation value of each variable, and $E_{\min}$ is obtained based on the aggressive strategy to minimize the average cross-evaluation value of each variable.

By integrating models (9) and (10) with the variable risk-appetite coefficient $\theta$, we construct the multipreference portfolio selection model, namely, model (11).

Based on model (11), we can use matrix $z$ to calculate two statistic preferences similar to the price's mean and variance preferences. We can find that $z$ is fused by multiple preferences and variable risk appetites, based on which we can obtain the optimal investment ratios and get the portfolio by using the traditional portfolio model. Clearly, the calculated optimal investment ratios are calculated on the basis of the investor's variable risk appetites and multiple relative preferences.

$$z = \theta \max \sum_{j=1, j\neq d}^{s} \mu_{rd} \left( \sum_{j=1, j\neq d}^{n} y_{rj} \right) + (1-\theta) \min \sum_{j=1, j\neq d}^{s} \mu_{rd} \left( \sum_{j=1, j\neq d}^{n} y_{rj} \right),$$
$$\text{s.t.} \begin{cases} \sum_{i=1}^{m} \omega_{id} x_{ij} - \sum_{r=1}^{s} \mu_{rd} y_{rj} \geq 0, \quad j = 1, 2, \ldots, n \\ \sum_{i=1}^{m} \omega_{id} \left( \sum_{j=1, j\neq d}^{n} x_{ij} \right) = 1 \\ \sum_{r=1}^{s} \mu_{rd} y_{rd} - E_{dd} \sum_{i=1}^{m} \omega_{id} x_{id} = 0 \\ \omega_{id} \geq 0, \quad i = 1, 2, \ldots, m \\ \mu_{rd} \geq 0, \quad r = 1, 2, \ldots, s \\ d = 1, 2, \ldots, n. \end{cases} \tag{11}$$

## 4. Portfolio Selection with respect to Probabilistic Preference Steps and Illustrated Examples

In this section, we combine the definitions to model and make a multiple-preference selection case. This case is about a program that the investment agencies can prepare for their customers, which is applied to verify the validity and practicality of the MPPS and DHMO models. Subsequently, more in-depth analyses prove the effectiveness of the method in the article.

First of all, we give the risk and profitability indicators in multiple preferences, which mainly have the following properties: the number of risk indicators and profitability indicators in multiple preferences does not need to be equal, and multiple risks can be compared to multiple profitability. It can also be more risky versus less profitable; multiple-preference indicators should fully reflect risk and profitability. Risk indicators and profitability indicators are given in the form of a matrix. If much preference information is displayed in the matrix, then the weights of the multiple-preference portfolio model can also have multiple preferences because multiple portfolio models contain all the preference information; the above criteria can be summarized in Algorithm 1.

Furthermore, the reason we normalize and correct the multiple efficiencies is that (1) normalizing the efficiency value simplifies the complexity of the calculation process; (2) eliminating outliers helps to ensure the stability and accuracy of the results of multipreference portfolios. In addition,

Step 1: give the fundamental elements $X_j = (x_{1j}, x_{2j}, \ldots, x_{mj})$ and $Y_j = (y_{1j}, y_{2j}, \ldots, y_{sj})$
Step 2: normalize them, put $X_j$ and $Y_j$ in 0 and 1, and calculate the value of CCR by $\max \sum_{r=1}^{s} \mu_{rd} y_{rd}$, which is denoted with $E_{dd}\,(i, j = 1, 2, \ldots, n)$
Step 3: correct the CCR efficiency value $E_{ii}$ and put it into the cross-efficiency value evaluation $\bar{E}_{dd}$; this process mainly eliminates outliers
Step 4: according to the equation $1/n \sum_{d=1}^{n} E_{dj}$, $\mathrm{var}(\bar{E}_{dd})$, the multiple preferences that contain the mean and variance of the efficiency value will be found
Step 5: select the Markowitz portfolio model that allows short selling or does not allow short selling, and the optimal weight will be calculated by running software (software used by the authors is MATLAB)
Step 6: if one needs to visually see the results that include the effective frontier of the portfolio, click on software to get the effective frontier map

Algorithm 1: Multiple-preference portfolio selection model.

taking the CCR value first is also to ensure the validity of the multipreference analysis; if the abnormal value of the efficiency value is not eliminated, the error of results easily increases. To comprehensively display the process of MPPS, a simple example of the algorithm is given as follows.

*Example 1.* The MPPS model is used to obtain the efficiency value with multiple preferences based on the existing information. For example, we can assume that the existing information matrix of MPPS is $\bar{Q}_i$ and $\bar{Q}_o$, and the specific form is as follows:

$$\bar{Q}_i = \begin{bmatrix} 1 & 1/8 & 1/4 & 1/5 \\ 1/8 & 1 & 1/3 & 1/6 \\ 1/4 & 1/3 & 1 & 1/5 \\ 1/5 & 1/6 & 1/5 & 1 \end{bmatrix}, \ldots \bar{Q}_o = \begin{bmatrix} 1 & 1/7 & 1/3 & 1/4 \\ 1/7 & 1 & 1/3 & 1/6 \\ 1/3 & 1/3 & 1 & 1/4 \\ 1/4 & 1/6 & 1/4 & 1 \end{bmatrix}. \tag{12}$$

Step 1: mark the MPPS multipreference risk matrix,

$$\bar{Q}_i = \begin{bmatrix} 1 & 0.13 & 0.25 & 0.20 \\ 0.13 & 1.00 & 0.33 & 0.17 \\ 0.25 & 0.33 & 1.00 & 0.20 \\ 0.20 & 0.17 & 0.20 & 1.00 \end{bmatrix}, \tag{13}$$

and multipreference profit matrix,

$$\bar{Q}_o = \begin{bmatrix} 1 & 0.13 & 0.25 & 0.20 \\ 0.13 & 1.00 & 0.33 & 0.17 \\ 0.25 & 0.33 & 1.00 & 0.20 \\ 0.20 & 0.17 & 0.20 & 1.00 \end{bmatrix}. \tag{14}$$

Step 2: solve the multiple-preference self-evaluation efficiency value and average cross-efficiency matrix:

$$\bar{Q}_E = \begin{bmatrix} 1 & 0.0182 & 0.0825 & 0.0500 \\ 0.0182 & 1 & 0.1089 & 0.0289 \\ 0.0825 & 0.1089 & 1 & 0.0289 \\ 0.0500 & 0.0289 & 0.0500 & 1 \end{bmatrix}, \bar{E} = \begin{bmatrix} 0.2822 & 0.2877 & 0.2890 & 0.3104 \end{bmatrix}. \tag{15}$$

Step 3: give the multiple-preference optimal ranking: **rank** $= \begin{bmatrix} 4 & 1 & 2 & 3 \end{bmatrix}$.

Step 4: calculate the variance and covariance matrix of $\bar{Q}_E$: var $(\bar{Q}_E)$, cov $(\bar{Q}_E)$.

$$\mathrm{var}\left(\bar{Q}_E\right) = [0.2262\ 0.2263\ 0.2119\ 0.2290], \mathrm{cov}\left(\bar{Q}_E\right) = \begin{bmatrix} 0.2262 & -0.0952 & -0.0625 & -0.0734 \\ -0.0952 & 0.2263 & -0.0460 & -0.0874 \\ -0.0625 & -0.0460 & 0.2120 & -0.0810 \\ -0.0734 & -0.0874 & -0.0810 & 0.2291 \end{bmatrix}. \tag{16}$$

Step 5: it is easy to make a portfolio by using the mean and covariance of efficiency.

According to the above equation, the MPPS multi ple-preference matrix can be obtained; however, when we select a portfolio by MPPS, if the variance and covariance matrix are negative definite, we need to change them to positive definite. To clearly demonstrate the last process of MPPS, we would like to take a case in point.

Assume we have three bonds ($A$, $B$, $C$), their expected return $\text{Exp Return} = \left[\, 0.12 \;\; 0.21 \;\; 0.16 \,\right]$, and the expected covariance:

$$\text{Cov Covariance} = \begin{bmatrix} 0.0100 & -0.0061 & 0.0042 \\ -0.0061 & 0.04000 & -0.0252 \\ 0.0042 & -0.0252 & 0.0225 \end{bmatrix}. \tag{17}$$

Select 10 security portfolios among them, draw a risk-return curve, and generate an effective boundary.

It is easy for us to get the effective frontier by running the MPPS: the set of best advantages of the portfolio forms an effective frontier curve in Figure 3. The abscissa represents the variance, and the ordinate represents the expected revenue. In addition, the algorithm of MPPS also gives the expected return under ten different security portfolio forms, and investors can choose their own preferences according to different security portfolio forms; the details are shown in Table 1.

As mentioned previously, we need to connect the investor's risk-appetite coefficient and multiple preferences, build a double-hierarchy portfolio model, and analyze portfolio selection clearly. Therefore, in the process of solving the variable risk-preference coefficient, we also give a simple case to illustrate the effectiveness of the model solution in Algorithm 2.

We can get $\lambda_1 = [0.5, 0.6, 0.7]^T$ and $\lambda_2 = [0.8, 0.9, 1.5]^T$. Then, we can establish the following equation:

$$\begin{aligned} &\max f(k_i) = \max\left(k_1 + k_2 + \cdots + k_{n-1}\right) \\ &\text{s.t.} \begin{cases} E_{\min} + \theta\left(E_{\max} - E_{\min}\right) - E_{\min}^2 - \theta\left(E_{\max}^2 - E_{\min}^2\right) + a_1 - b_1 > 0, \\ E_{\min}^2 + \theta\left(E_{\max}^2 - E_{\min}^2\right) - E_{\min}^3 - \theta\left(E_{\max}^3 - E_{\min}^3\right) + a_2 - b_2 > 0, \\ \vdots \\ E_{\min}^{n-1} + \theta\left(E_{\max}^{n-1} - E_{\min}^{n-1}\right) - E_{\min}^{n} - \theta\left(E_{\max}^{n} - E_{\min}^{n}\right) + a_{n-1} - b_{n-1} > 0, \\ a_1 b_1 = 0, \\ a_2 b_2 = 0, \\ \vdots \\ a_{n-1} b_{n-1} = 0 \\ b_1 - a_1 k_1 \geq 0, \\ b_2 - a_2 k_2 \geq 0, \\ \vdots \\ b_{n-1} - a_n k_{n-1} \geq 0, \\ a, b \geq 0,\ \theta \in [0,1], \\ k_i \in \{0,1\}, i = 1, \ldots, n-1, \end{cases} \end{aligned} \Longrightarrow \begin{aligned} &\max k = k_1 + k_2 + k_3 \\ &\begin{cases} 0.5 - 0.8 * x + a_1 - b_1 = 0, \\ 0.6 + 0.9 * x + a_2 - b_2 = 0, \\ 0.7 + 1.5 * x + a_3 - b_3 = 0, \\ a_1 * b_1 = 0, \\ a_2 * b_2 = 0, \\ a_3 * b_3 = 0, \\ a_1 - b_1 * k_1 > 0, \\ a_2 - b_2 * k_2 > 0, \\ a_3 - b_3 * k_3 > 0, \\ a_1 >= 0, \\ a_2 >= 0, \\ a_3 >= 0, \\ b_1 >= 0, \\ b_2 >= 0, \\ b_3 >= 0, \\ x >= 0, \\ x <= 1. \end{cases} \end{aligned} \tag{18}$$

We calculate the appetite coefficient $\theta = 0.65$ based on the above equation; as long as investors make a choice, we can calculate the variable risk-preference coefficient through historical simulation and probabilistic preference.

## 5. Empirical Study of the Double-Hierarchy Model with respect to the Probabilistic Preference

*5.1. Data and Results.* To prove the feasibility of the double-hierarchy model, in this section, the 10 largest capitalized stocks on the Shenzhen Stock Exchange that have just been listed for one year are selected as the research objectives. Due to the insufficient data and incomplete investment information of these stocks, the traditional portfolio selection methods cannot be utilized in this situation. Instead, the proposed double-hierarchy model that combines variable risk appetites in the probabilistic preference and multiple-preference indicators is more suitable to address this issue. This model uses data from the Shenzhen $A$-share market and eliminates stocks with incomplete information (see Table 3).

In terms of investors' preference, an example of 4 stocks, $A_1$, $A_2$, $A_3$, and $A_4$, is listed as $A_4 \geq A_3 \geq A_2 \geq A_1$ or in other

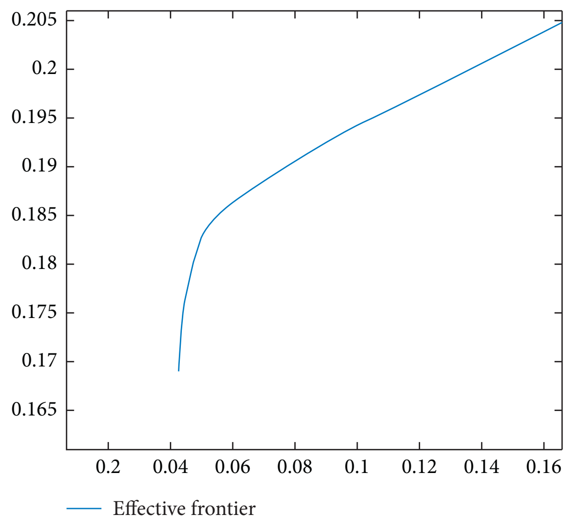


Figure 3: The effective frontier of MPPS.

Table 1: Ten security portfolio forms' expected rate of return and risk.

| Security | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| portrisk | 0.043 | 0.043 | 0.043 | 0.043 | 0.044 | 0.045 | 0.045 | 0.046 | 0.047 | 0.049 |
| portreturn | 0.169 | 0.170 | 0.172 | 0.173 | 0.175 | 0.176 | 0.178 | 0.179 | 0.180 | 0.182 |

Step 1: provide the portfolio of different types about financial products for investors, and investors will make their choice according to their own risk and return preferences. Investment program: $A_1, A_2, A_3, \ldots, A_n$, investors can choose $n$ rankings according to their preferences, and this ranking includes all financial products we provided.
Step 2: calculate $E_{\max}$ and $E_{\min}$ based on models (3) and (4) simultaneously.
Step 3: calculate the appetite coefficient. We can obtain $E_{\max}$ and $E_{\min}$ after merging similar items based on the investor's choice; then, $\lambda_1 = [a_1, a_2, \ldots, a_n]$ and $\lambda_2 = [b_1, b_2, \ldots, b_n]$.
Step 4: take $\lambda_1$ and $\lambda_2$ into model (2) based on software, and we will get the appetite coefficient conveniently.
Step 5: to clearly calculate the appetite coefficient $\theta$, like Step 3, we assume that there are four financial products to choose, allowing investors to choose between financial products' risks and returns according to model (8), and get the result in Table 2.

Algorithm 2: Risk-appetite coefficient model by historical simulation and probabilistic preference.

Table 2: The preference information of the expert on $A$.

| Investor | $\succ$ | $\parallel$ | $\parallel^{\vee}$ | $\parallel_{\wedge}$ | $\parallel^{\vee}_{\wedge}$ | $\approx$ | $\prec$ | Average |
|---|---|---|---|---|---|---|---|---|
| 1 | [0.4,0.9] | [0.6,0.7] | [0.8,0.5] | [0.2,1.1] | [0.5,0.8] | [0.7,0.6] | [0.3,1.0] | [0.5,0.8] |
| 2 | [0.1,0.9] | [1.1,0.9] | [0.7,0.8] | [0.5,1.0] | [0.4,0.6] | [0.6,0.9] | [0.8,1.2] | [0.6,0.9] |
| 3 | [0.8,2.1] | [0.6,0.9] | [0.4,1.6] | [1.0,1.4] | [0.7,1.5] | [1.1,0.8] | [0.3,0.7] | [0.7,1.5] |

Table 3: Summary of example data.

| Stock code | Name |
|---|---|
| 000038.SZ | Shenzhen Capstone Industrial Co., Ltd. |
| 000156.SZ | Wasu Media Holding Co., Ltd. |
| 000607.SZ | Holley Pharmaceuticals (Chongqing) Co., Ltd. |
| 000665.SZ | Hubei Radio & Television Information Network Co., Ltd. |
| 000673.SZ | Contemporary Eastern Investment Co., Ltd. |
| 000681.SZ | Visual China Group Co., Ltd. |
| 000719.SZ | Central China Land Media Co., Ltd. |
| 000793.SZ | Huawen Media Group |
| 000802.SZ | Miracle in June (Beijing) Culture Media Co., Ltd. |
| 000839.SZ | CITIC Guoan Information Industry Co., Ltd. |

Table 4: Multipreference attributes and indices.

| Multipreference attribute | Index |
|---|---|
| The risk preference variables | Inventory turnover<br>Current ratio<br>Quick ratio |
| The profitability preference variables | Earnings per share growth ratio<br>Net income growth rate<br>Return on assets |

Table 5: Empirical results of the illustrated example.

| Stock codes | Weight (portfolio with a risk appetite coefficient of 0.5) | | | | | | | | | | Mean | Weight (portfolio with a risk appetite coefficient of 0) | | | | | | | | | | Mean |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 000038.SZ | 0 | 0 | 0.11 | 0 | 0.04 | 0 | 0.32 | 0 | 0 | 0.53 | 0.11 | 0 | 0 | 0.1 | 0 | 0 | 0 | 0.24 | 0 | 0 | 0.66 | 0 |
| 000156.SZ | 0 | 0 | 0.13 | 0 | 0.12 | 0 | 0.32 | 0 | 0 | 0.44 | 0.04 | 0 | 0 | 0.09 | 0.02 | 0 | 0 | 0.21 | 0 | 0 | 0.68 | 0 |
| 000607.SZ | 0 | 0 | 0.14 | 0 | 0.19 | 0 | 0.32 | 0 | 0 | 0.34 | 0.05 | 0 | 0 | 0.07 | 0.05 | 0 | 0 | 0.18 | 0 | 0 | 0.69 | 0.04 |
| 000665.SZ | 0 | 0 | 0.15 | 0 | 0.27 | 0 | 0.32 | 0 | 0 | 0.25 | 0.04 | 0 | 0 | 0.06 | 0.08 | 0 | 0 | 0.15 | 0 | 0 | 0.7 | 0.21 |
| 000673.SZ | 0.09 | 0.15 | 0 | 0.04 | 0.25 | 0.08 | 0.2 | 0 | 0 | 0.18 | 0.22 | 0 | 0 | 0.05 | 0.12 | 0 | 0 | 0.12 | 0 | 0 | 0.71 | 0 |
| 000681.SZ | 0.16 | 0.28 | 0 | 0.11 | 0.12 | 0.12 | 0.02 | 0.12 | 0 | 0.07 | 0.11 | 0 | 0 | 0.04 | 0.15 | 0 | 0 | 0.09 | 0 | 0 | 0.72 | 0 |
| 000719.SZ | 0.27 | 0 | 0 | 0.21 | 0.03 | 0 | 0 | 0.16 | 0.28 | 0 | 0.15 | 0 | 0 | 0.02 | 0.18 | 0 | 0 | 0.06 | 0 | 0 | 0.73 | 0.11 |
| 000793.SZ | 0.34 | 0 | 0 | 0.02 | 0.05 | 0.31 | 0 | 0.29 | 0 | 0 | 0.06 | 0 | 0 | 0.01 | 0.21 | 0 | 0 | 0.03 | 0 | 0 | 0.74 | 0 |
| 000802.SZ | 0.25 | 0 | 0 | 0 | 0.12 | 0.57 | 0 | 0.06 | 0 | 0 | 0.03 | 0 | 0 | 0 | 0.25 | 0 | 0 | 0 | 0 | 0 | 0.75 | 0 |
| 000839.SZ | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0.18 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0.64 |
| | Weight (portfolio with a risk appetite coefficient of 1) | | | | | | | | | | Mean | Weight (portfolio with a risk appetite coefficient of 0.47) | | | | | | | | | | Mean |
| 000038.SZ | 0 | 0 | 0 | 0 | 0.15 | 0 | 0.25 | 0.06 | 0 | 0.55 | 0.1 | 0 | 0 | 0.27 | 0 | 0 | 0 | 0.34 | 0 | 0 | 0.39 | 0 |
| 000156.SZ | 0 | 0 | 0 | 0 | 0.24 | 0 | 0.31 | 0 | 0 | 0.45 | 0.05 | 0 | 0 | 0.2 | 0.01 | 0.05 | 0 | 0.32 | 0 | 0 | 0.41 | 0.08 |
| 000607.SZ | 0 | 0.14 | 0.05 | 0 | 0.24 | 0 | 0.28 | 0 | 0 | 0.3 | 0.04 | 0 | 0.08 | 0.19 | 0.1 | 0.01 | 0 | 0.24 | 0 | 0 | 0.39 | 0.11 |
| 000665.SZ | 0 | 0 | 0.15 | 0.04 | 0.23 | 0.08 | 0.27 | 0 | 0 | 0.23 | 0.03 | 0 | 0.12 | 0.16 | 0.17 | 0 | 0 | 0.17 | 0 | 0 | 0.39 | 0.18 |
| 000673.SZ | 0.02 | 0 | 0.22 | 0.16 | 0 | 0.13 | 0.08 | 0.09 | 0.21 | 0.1 | 0.24 | 0 | 0.15 | 0.13 | 0.24 | 0 | 0 | 0.1 | 0 | 0 | 0.38 | 0.01 |
| 000681.SZ | 0.22 | 0.26 | 0 | 0.09 | 0.1 | 0.1 | 0 | 0.2 | 0 | 0.02 | 0.16 | 0 | 0.18 | 0.1 | 0.31 | 0 | 0 | 0.03 | 0 | 0 | 0.38 | 0 |
| 000719.SZ | 0.23 | 0.05 | 0 | 0 | 0.04 | 0.39 | 0 | 0.26 | 0 | 0.03 | 0.12 | 0 | 0.25 | 0.04 | 0.35 | 0 | 0 | 0 | 0 | 0 | 0.36 | 0.12 |
| 000793.SZ | 0.25 | 0 | 0 | 0 | 0.13 | 0.43 | 0 | 0.19 | 0 | 0 | 0.08 | 0 | 0 | 0 | 0.45 | 0 | 0 | 0 | 0.1 | 0 | 0.44 | 0.17 |
| 000802.SZ | 0.24 | 0 | 0 | 0 | 0.29 | 0.45 | 0 | 0.02 | 0 | 0 | 0.02 | 0 | 0 | 0 | 0.16 | 0 | 0 | 0 | 0.56 | 0 | 0.27 | 0 |
| 000839.SZ | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0.17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0.34 |

relations. After that, the minimum value $E_{\mathbf{min}}$ and the maximum value $E_{\max}$ can be calculated by models (7)–(9). It is assumed that their maximum values are 0.8, 0.9, 1.5, and 0.5, and the minimum values are 0.5, 0.6, 0.7, and 0.8, respectively. Then, based on probabilistic preference theory (model (7)), we can obtain a corresponding risk-appetite coefficient of 0.47.

In Table 4, the risk preference variables are the inventory turnover, the current ratio, and the quick ratio, and the profitability preference variables are the earnings per share growth ratio, the net income growth rate, and the return on assets. In this example, we choose these preference indicators because they are broadly applied in the investment field, which can reflect the characteristics of our model.

We can calculate the double-hierarchy model and obtain the results as shown in Table 5. Several conclusions are derived according to the empirical result: (1) each row gives the optimal portfolio selection, and the corresponding proportion of capital investment can be seen in Figure 4, which gives investors a more detailed investment suggestion. (2) Investors can make suitable and dynamic decisions in consideration of their variable investment risk appetites.

*5.2. Portfolio Comparison in Different Risk Appetites Calculated by the Probabilistic Preference.* In this section, we compared the portfolio effective frontiers under four different risk appetites and multiple preferences and compared the results of the double-hierarchy portfolio with risk and return.

Figure 5 displays the portfolio effective frontiers about the appetites of 0.5, 0, 1, and 0.47. It is easy to see that the efficient frontier of investors synchronous changes with the investor's appetite; in addition, the frontier slows down as the value of risk appetite decreases.

Figures 4 and 6 are the results calculated by the proposed double-hierarchy model, which present the minimum variance set based on four different risk appetites. In addition, we calculate and compare the average weight of the

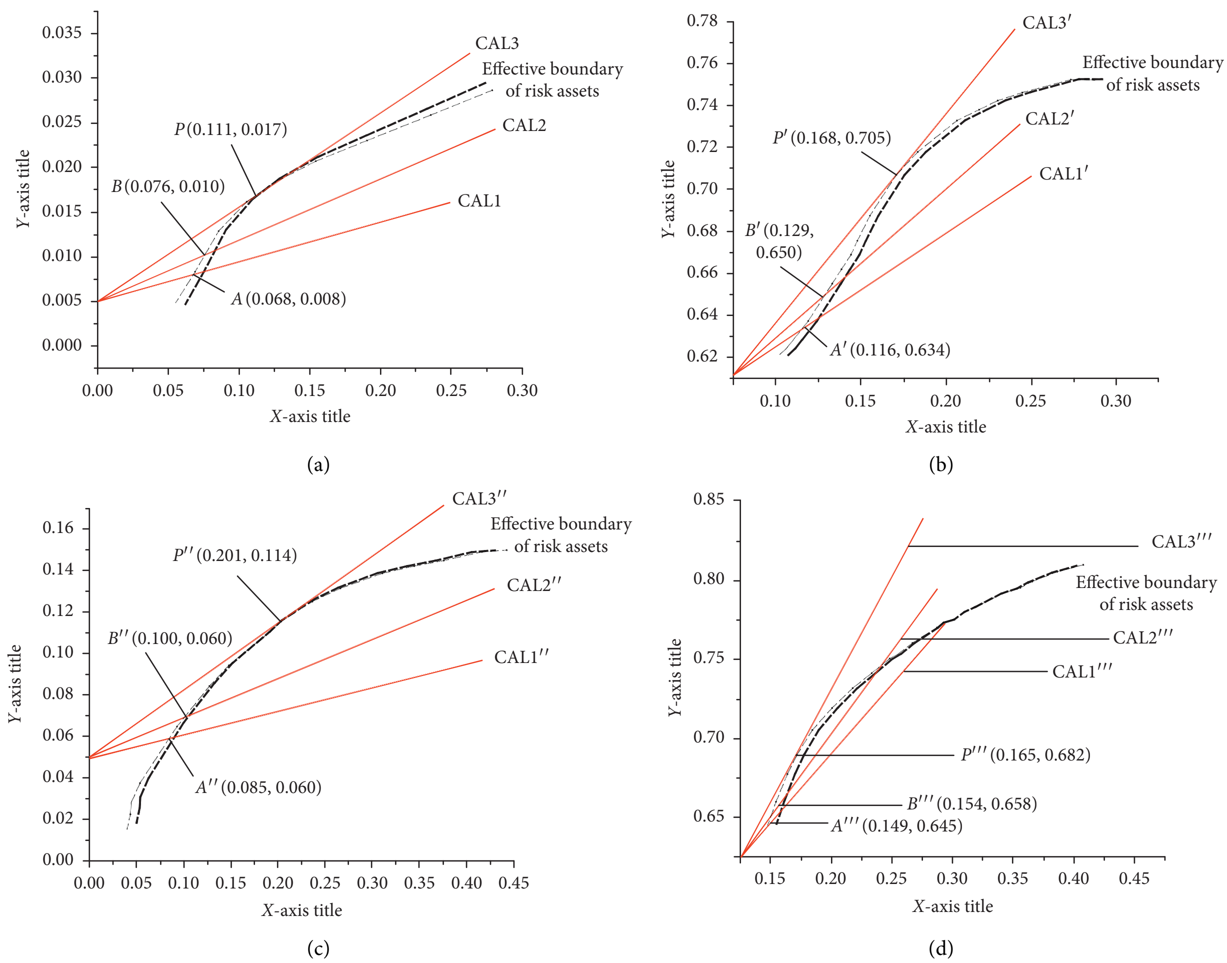


Figure 4: The optimal strategies under different risk attitudes.

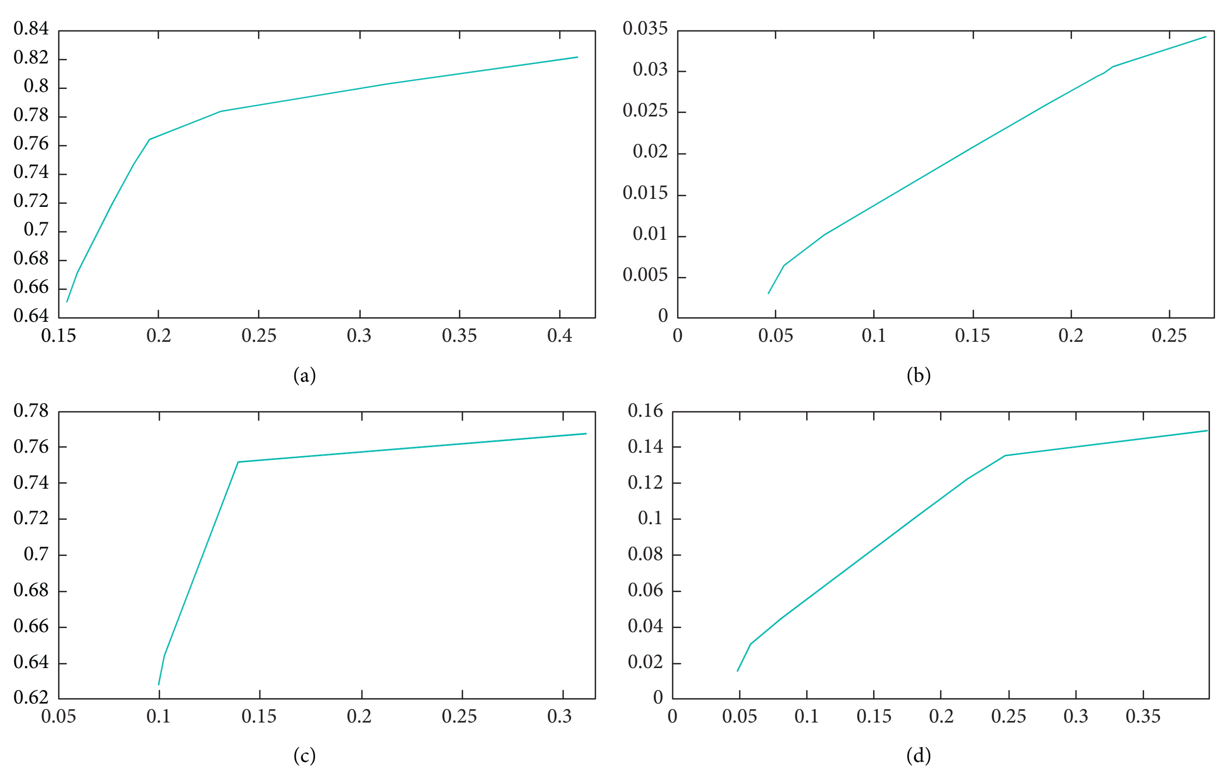


Figure 5: The value of the efficient set.

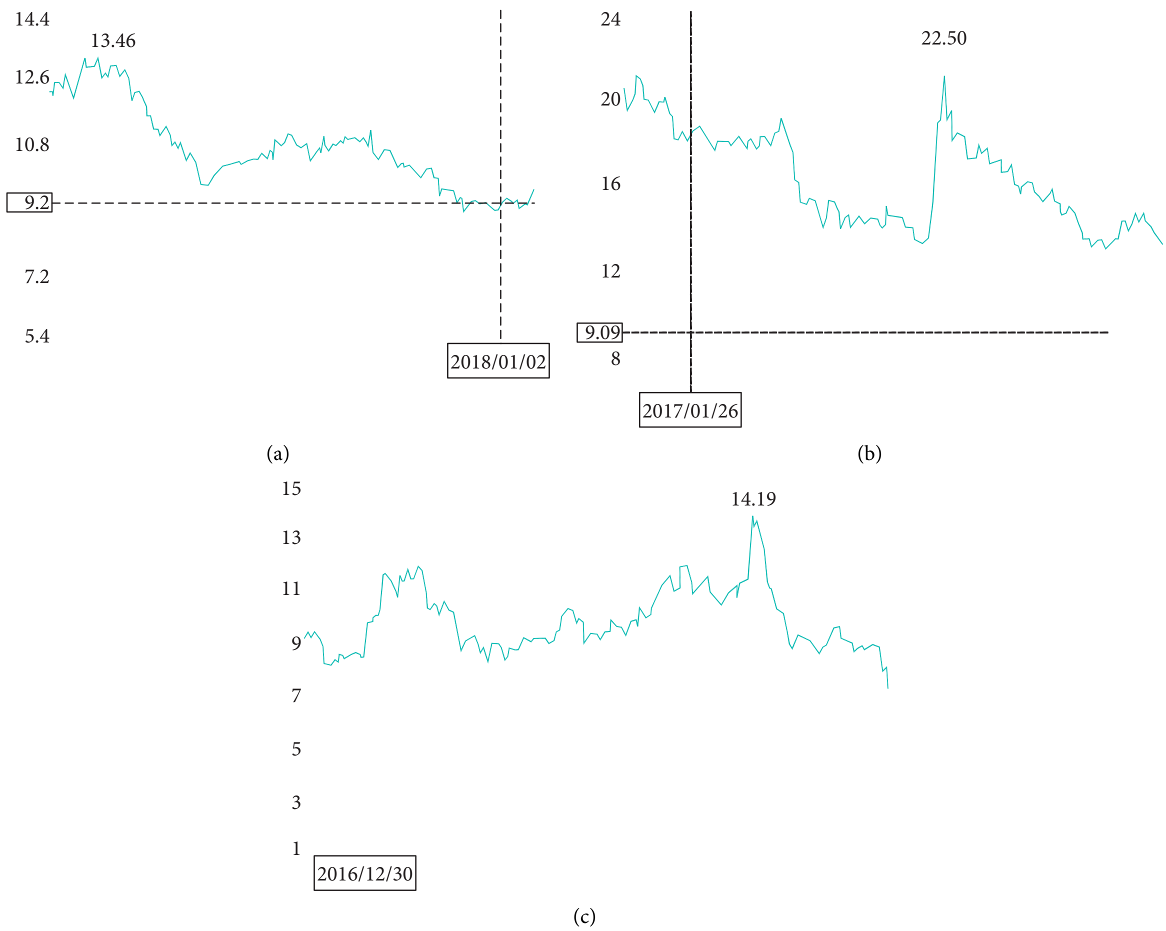


Figure 6: Actual comparison object. (a) 000719.SZ. (b) 000802.SZ. (c) 000839.SZ.

Table 6: The weight mean value under dynamic risk-appetite coefficient.

| Stock code | Mean (1) | Mean (0.75) | Mean (0.65) | Mean (0.5) | Mean (0.25) | Mean (0) | Mean (all) |
|---|---|---|---|---|---|---|---|
| 000038.SZ | 0.1 | 0 | 0 | 0.11 | 0 | 0 | 0.04 |
| 000156.SZ | 0.05 | 0 | 0 | 0.04 | 0 | 0 | 0.02 |
| 000607.SZ | 0.04 | 0.15 | 0.06 | 0.05 | 0.02 | 0.04 | 0.06 |
| 000665.SZ | 0.03 | 0.06 | 0.07 | 0.04 | 0.14 | 0.21 | 0.09 |
| 000673.SZ | 0.24 | 0.02 | 0.02 | 0.22 | 0 | 0 | 0.08 |
| 000681.SZ | 0.16 | 0 | 0 | 0.11 | 0 | 0 | 0.05 |
| 000719.SZ | 0.12 | 0.03 | 0.03 | 0.15 | 0.026 | 0.11 | 0.08 |
| 000793.SZ | 0.08 | 0.42 | 0.45 | 0.06 | 0.395 | 0 | 0.23 |
| 000802.SZ | 0.02 | 0 | 0 | 0.03 | 0 | 0 | 0.01 |
| 000839.SZ | 0.17 | 0.32 | 0.37 | 0.18 | 0.419 | 0.64 | 0.35 |

portfolios in different risk appetites. The results show that the portfolios considering investors who are risk-averse, risk-neutral, and risk-seeking are highly fitted to the outcomes of the double-hierarchy model, except for 000673.SZ under the portfolio with a risk appetite coefficient of 0.5. Actually, this stock's shareholders froze their accounts in 2018. Therefore, we take the second largest weight for further comparison, and the results also match the ones calculated by the double-hierarchy model. The higher and lower weights of risk-averse are 000839.SZ and 000802.SZ, and their advance-decline are −5.16% and −19.59%, respectively. The higher and lower weights in risk-neutral are 000839.SZ and 000802.SZ, and their advance-decline are −5.16% and −19.59%, respectively. The higher and lower weights in risk-seeking are 000719.SZ and 000802.SZ, and their advance-decline are −5.16% and −19.59%, respectively (see Figure 6).

Figure 5 describes the optimal portfolio point $P'$, $P''$, $P'''$, and $P''''$; the corresponding abscissa and ordinate are the risk and return in the field of investment portfolio, respectively. Each point on the efficient frontier represents the

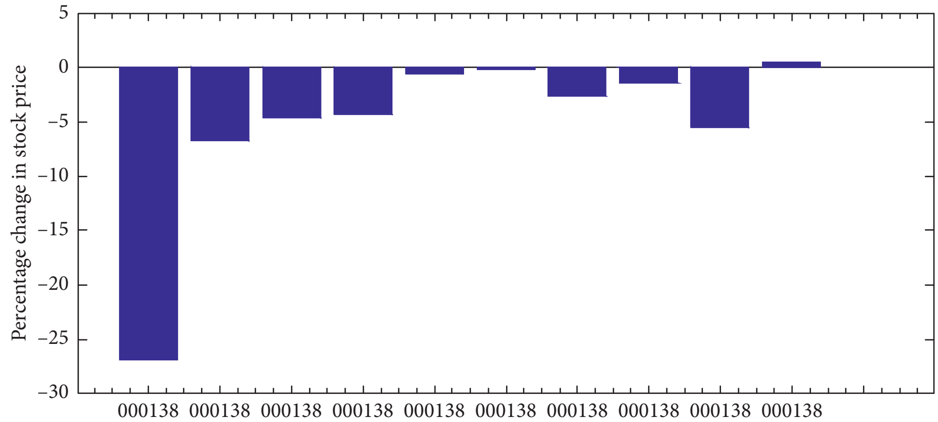


Figure 7: Changes in stock gains.

combination of returns and risks corresponding to various portfolio stocks. The portfolio securities in other points are not the best because they cannot get the maximum benefit with the least risk, and we have not drawn the corresponding frontier of inefficient investment in the lower half. The convex set is meaningless, and it is impossible for investors to reduce returns in exchange for higher risk. Table 6 shows the average optimal weights under different risk appetites.

As can be seen in Figure 7, two stocks 000038.SZ and 000156.SZ float down significantly; from the optimal weight calculated by DHMP, it can be found that the weight of these two stocks is smaller, which effectively reduces the risk of investment. The results show that this research proves the feasibility and effectiveness of the proposed double-hierarchy model with respect to the probabilistic preference.

## 6. Conclusions

Portfolio theory is playing an increasingly important role in modern economic theory. How to accurately and conveniently measure investors' risk appetite is the top priority. Because the impact factors that determine investors' subjective risk appetites are in a wide scope including time, age, and asset status, it is hard to comprehensively analyze and quantify investors' risk appetites in a portfolio calculation process. To address this issue, we take the historical simulation and probabilistic preference, MPPS, and DHMP models which take investors' risk appetites and multiple preferences into account. Therefore, optimal investment ratios can be obtained for investors to provide them with more accurate and personal investment strategies. In this paper, we have made some contributions from the perspectives of theoretical improvement and practical application. First, we provide optimal portfolio selection for different investors with diverse investment demands. Second, investors' dynamic risk appetite can be measured by probabilistic preferences. Lastly, the newly proposed method has been applied to the practical application to verify its feasibility and effectiveness.

However, there are still some limitations in the DHMP model and its application. For example, we have not given a normal form and standard for multiple-preference selection, and the time span is limited. In addition, in future research, we will attempt to provide a more reasonable preference selection with probabilistic preferences and use panel data.

## Data Availability

The stock data (which were collected from the Wind database) in the empirical research used to support the findings of this study are included within the article.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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