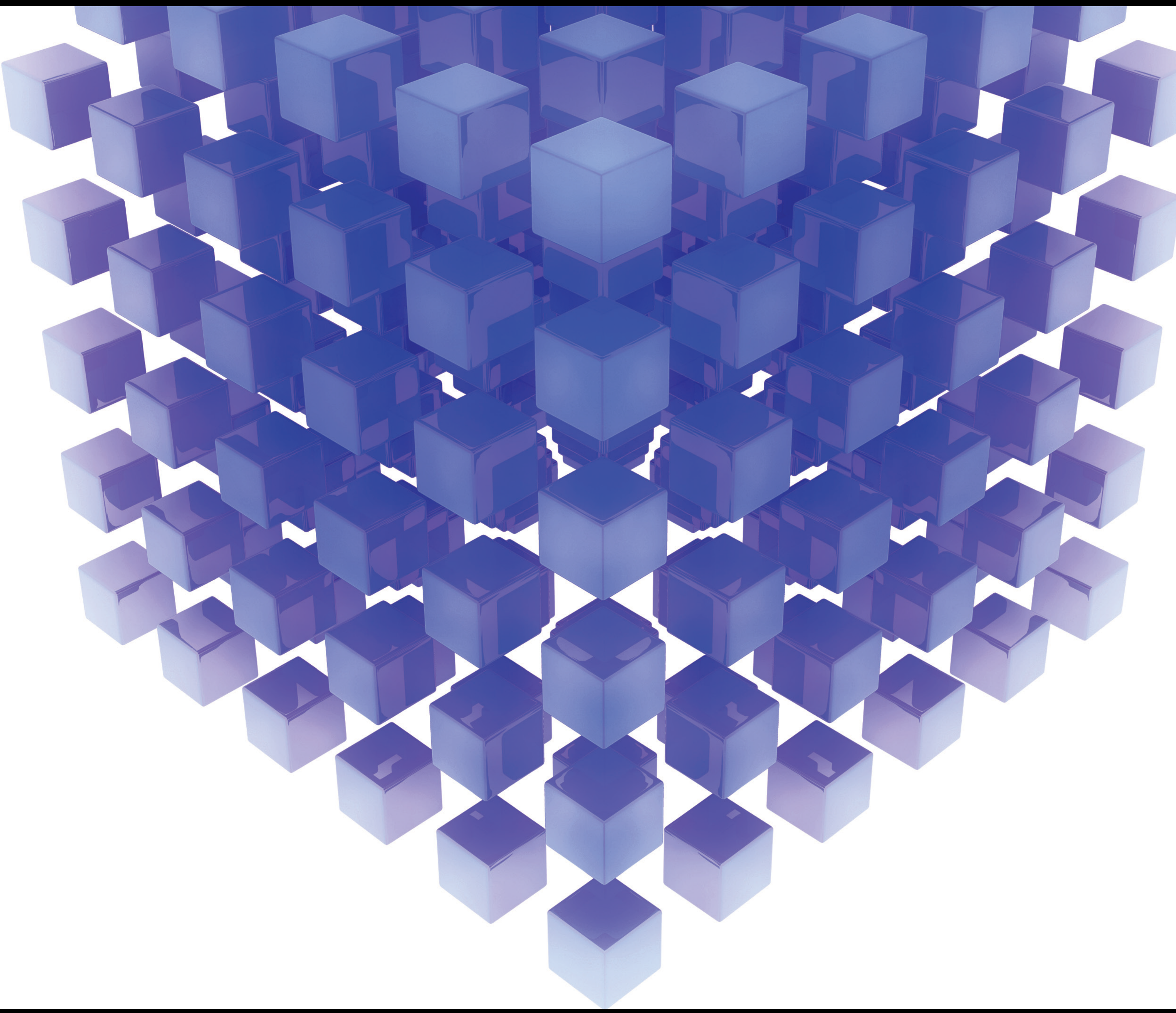
Mathematical Problems in Engineering

Advances in Observer Design and Observation of Nonlinear Systems

Lead Guest Editor: Abdellatif Ben Makhlouf
Guest Editors: Omar Naifar and Nabil Derbel



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Research Article

H_∞ Fault Estimation and Fault-Tolerant Control for T-S Fuzzy Systems with Actuator and Sensor Faults Using Sliding Mode Observer

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This paper investigates the problems of the robust fault estimation (FE) and fault-tolerant control (FTC) for the Takagi-Sugeno (T-S) fuzzy systems with unmeasurable premise variables (PVs) subject to external disturbances, actuator, and sensor faults. An adaptive fuzzy sliding mode observer (SMO) with estimated PVs is designed to reconstruct the state, actuator, and sensor faults simultaneously. Compared with the existing results, the proposed observer is with a wider application range since it does not require the knowledge of the upper bound of faults that some FE methods demand. Based on the FE information, a dynamic output-feedback fault-tolerant controller (DOFFTC) is designed to compensate the effect of faults by stabilizing the closed-loop systems. By using the H_∞ filtering method, sufficient conditions for the existence of the proposed SMO and DOFFTC are derived in terms of linear matrix inequalities (LMIs) optimization. Finally, a nonlinear inverted pendulum system is given to validate the proposed methods.

1. Introduction

In industrial applications, the increasing demand of higher performance, safety, reliability, maintainability, and survivability represent a major concern. So, it is important to support research on fault estimation (FE) and fault-tolerant control (FTC) for a class of nonlinear systems specifically Takagi-Sugeno (T-S) fuzzy models [1–5]. Many approaches have been developed in recent decades, such as sliding mode observer (SMO) [6–12], unknown input observer [13–15], adaptive observer [16, 17], and descriptor observer [18, 19].

The sliding mode (SM) scheme is a powerful tool to overcome uncertainties and external disturbances in dynamic systems, due to its good robustness, simple structure, and strong applicability. Therefore, it has a good application prospect in the field of FE and FTC and has attracted more and more attention from both academia and industry. Fruitful works can be found regarding this issue. For instance, in [8], a sliding mode observer (SMO) is designed for the estimation of actuator faults in T-S fuzzy models with digital communication channel, but the sensor faults were not considered. While in [9], the estimation of simultaneously actuator and sensor faults is realized for T + S fuzzy systems using nonquadratic Lyapunov function. However, the FTC problem was not treated. While, in [20], a FTC design for T-S fuzzy systems was established using unknown input observer approach. Moreover, a FTC scheme based on SMO for T-S fuzzy systems with local nonlinear models is proposed in [6]. Sufficient conditions are derived to calculate observer and controller gains which are solved using linear matrix inequalities (LMIs). In [21], an adaptive sliding mode FTC design is developed for a class of uncertain T-S fuzzy systems affected by multiplicative faults. Unfortunately, these previous results needed that the premise variables of the T-S fuzzy systems are measurable and the upper bounds of faults are known. Therefore, how to design a suitable SMO to overcome the above drawbacks?

Motivated by the above discussion, an adaptive fuzzy SMO is developed for the T-S fuzzy systems with unmeasurable premise variables in order to estimate the state, actuator, and sensor faults. Then, based on FE a dynamic output-feedback fault-tolerant controller (DOFFTC) is designed to compensate the fault effects by stabilizing the closed-loop system. Finally, the simulation result of a nonlinear inverted pendulum system is given to illustrate the effectiveness of the proposed method.

Compared with the existing results, the advantages of our work can be summarized as follows. First of all, we investigate the problem of fault estimation and fault-tolerant control based on the dynamic output-feedback controller for T-S fuzzy systems with external disturbances and actuator and sensor faults. Whereas, many researchers have considered only actuator faults [22, 23] or senor faults [24+26]. Then, the sliding mode observer is designed using adaptive law to avoid the hypothesis concerning knowledge of the upper bound of faults, which give less conservative results and offer more freedom in comparison with [27+29]. Moreover, authors in [6, 24] assume that the premise variables of the T-S fuzzy systems are measurable. Whereas, in this work, we consider the unmeasurable premise variables. The gains of the observer and controller are computed separately to avoid the coupling and reduce the computation complexity. Whereas, in [25], a single step solving algorithm is needed.

The rest of this paper is organized as follows. In Section 2, we describe the system and the problem studied in this paper. In Section 3, we present the design of the adaptive fuzzy sliding mode observer and the analysis of the stability of the error system. The fault estimation is studied in Section 4. In Section 5, the dynamic output-feedback fault-tolerant controller is developed. Finally, simulation example in Section 6 validates the efficiency of the proposed methods.

1.1. Notations. Throughout the paper, the following notations are used. I_n denotes an identity matrix with dimension of $n \times n$. R^n and $R^{(n\times m)}$ denote the n-dimensional Euclidean space and $n \times m$ real matrices. The pseudoinverse of a matrix A is denoted by A^+. For a real matrix A, $A > 0$ indicates that A is symmetric positive definite, and $A > 0$ indicates that A is symmetric negative definite. $\|.\|$ denotes the Euclidean norm or its induced spectral norm. The symmetric terms in a symmetric matrix are denoted by $*$. Finally, the space of square integrable functions is denoted by L_2, that is, for any $\xi(.) \in L_2[0\ \ \infty)$, $\|\xi(.)\|_2 = \sqrt{\int_0^\infty \xi^T(.)\xi(.)\mathrm{d}t}$.

2. System Description and Problem Statement

Considering the following T-S fuzzy model: Rule I: IF z_1 is M_1^i, ..., and z_g is M_g^i, THEN

$$\begin{cases} \dot{x}_p = A_{pi}x_p + B_{pi}(u + f_a) + E_{pi}d., \\ y_p = C_p x_p + N_p f_s + D_p d, \end{cases} \tag{1}$$

where $x_p \in \mathbb{R}^n$ represents the state vector; $u \in \mathbb{R}^m$ is the input; $y_p \in \mathbb{R}^p$ is the measured output; A_{pi}, B_{pi}, E_{pi}, C_p, N_p, and D_p are known matrices with compatible dimensions; f_a: $\mathbb{R}^+ \longrightarrow \mathbb{R}^q$ and f_s: $\mathbb{R}^+ \longrightarrow \mathbb{R}^h$ represent the additive faults generated by actuator and sensor, respectively; $d \in \mathbb{R}^l$ is the external disturbance which belong to $L_2[0\ \ \infty)$. It is supposed that matrices B_{pi} are of full column rank, i.e., $\operatorname{rank}(B_{pi}) = m$, matrix N_p is of full column rank, C_p is of full row rank, the pairs (A_{pi}, C_p) are observable, and the pairs (A_{pi}, B_{pi}) are controllable. Besides, $z = [z_1 \ldots z_g]$ denotes the unmeasurable premise variable, M_j^i are the fuzzy sets, k is the number of IF-THEN rules, and g is the number of the premise variable.

Using the technique [26], the overall model of system (1) is given by

$$\begin{cases} \dot{x}_p = \sum_{i=1}^{k} h_i(z)\left[A_{pi}x_p + B_{pi}(u + f_a) + E_{pi}d\right], \\ y_p = C_p x_p + N_p f_s + D_p d, \end{cases} \tag{2}$$

where $h_i(z) = w_i(z)/\sum_{i=1}^k w_i(z)$ and $w_i(z) = \prod_{j=1}^g M_j^i(z_j)$, here M_j^i is the grade of the membership function of z_j. We assume that $w_i(z) \geq 0$, $i = 1, \ldots, k$. Then, it easy to see that $\sum_{i=1}^k w_i(z) > 0$, for any z. Hence, $h_i(z)$ satisfies $h_i(z) \geq 0$ and $\sum_{i=1}^k h_i(z) = 1$.

Lemma 1 (see [27]). *For matrices A and B with appropriate dimensions, we have*

$$A^T B + B^T A \leq \varepsilon A^T A + \varepsilon^{-1} B^T B, \tag{3}$$

for any $\varepsilon > 0$.

Lemma 2 (see [28]). *If the following inequalities hold,*

$$\begin{aligned} &\Phi_{ii} < 0,\ 1 \leq i \leq k. \\ &\frac{2}{k-1}\Phi_{ii} + \Phi_{ij} + \Phi_{ji} < 0, \\ &1 \leq i \neq j \leq k, \end{aligned} \tag{4}$$

we have

$$\sum_{i=1}^{k}\sum_{j=1}^{k} h_i h_j \Phi_{ij} < 0. \tag{5}$$

Introduce the state x_f such that

$$\dot{x}_f = -A_f x_f + A_f y_p, \tag{6}$$

where $-A_f \in \mathbb{R}^{h\times h}$ is an arbitrary stable matrix. Let $x = \begin{bmatrix} x_p \\ x_f \end{bmatrix}$; then, the following augmented T-S fuzzy system is constructed:

$$\begin{cases} \dot{x} = \sum_{i=1}^{k} h_i(z)\left[A_i x + B_i\left(u + f_a\right) + N f_s + E_i d\right], \\ y = Cx, \end{cases} \tag{7}$$

where

$$\begin{aligned} A_i &= \begin{bmatrix} A_{pi} & 0 \\ A_f C_p & -A_f \end{bmatrix}, \\ B_i &= \begin{bmatrix} B_{pi} \\ 0 \end{bmatrix}, \\ N &= \begin{bmatrix} 0 \\ A_f N_p \end{bmatrix}, \\ E_i &= \begin{bmatrix} E_{pi} \\ A_f D_p \end{bmatrix}. \\ C &= \begin{bmatrix} 0 & I_h \end{bmatrix}, \\ y &= \begin{bmatrix} y_p \\ x_f \end{bmatrix}. \end{aligned} \tag{8}$$

Combining the actuator fault f_a and the sensor fault f_s in the same unknown vector $f = \begin{bmatrix} f_a & f_s \end{bmatrix}^T$ and assuming it is bounded and satisfies $\|f\| \leqslant w$, where $w > 0$ is an unknown real constant, then system (8) can be re-expressed as

$$\begin{cases} \dot{x} = \sum_{i=1}^{k} h_i(z)\left[A_i x + B_i u + M_i f + E_i d\right], \\ y = Cx, \end{cases} \tag{9}$$

where $M_i = \begin{bmatrix} B_{pi} & 0 \\ 0 & A_f N_p \end{bmatrix}$.

For designing observers, it is often assumed, in the literature, that the premise variable z is available for measurement. In this paper, it is interesting to develop a sliding mode observer for the unmeasurable premise variable T-S fuzzy system. A sliding mode observer for system (9) is in the form

$$\left\{ \dot{\hat{x}} = \sum_{i=1}^{k} h_i(\hat{z})\left[A_i \hat{x} + B_i u + G_{li} e_y + G_{ni}\vartheta\right]., \hat{y} = C\hat{x}, \right. \tag{10}$$

where $\hat{z}$ is the estimation of unmeasured premise variable, $\hat{x}$ is the state estimation of x, $\hat{y}$ is the observer output, $e_y\!: = y - \hat{y}$ is the output estimation error, G_{li} and G_{ni} are the observer gain matrices, and ϑ is the discontinuous vector to be designed.

Define the state estimation error, $e = x - \hat{x}$. Based on (9) and (10), we get the following error dynamics system:

$$\dot{e} = \sum_{i=1}^{k} h_i(\hat{z})\left[A_{oi} e + \phi + M_i f - G_{ni}\vartheta + E_i d\right], \tag{11}$$

where $A_{oi} = (A_i - G_{li} C)$ and

$$\phi = \sum_{i=1}^{k} \left(h_i(z) - h_i(\hat{z})\right)\left[A_i x + B_i u + M_i f + E_i d\right]. \tag{12}$$

Note that $\phi \longrightarrow 0$ when $e \longrightarrow 0$, that is, ϕ is treated as an unstructured vanishing perturbation which is supposed to be growth-bounded for $x, \hat{x} \in \mathbb{R}^n$ such that

$$\|\phi\| \leq \gamma \|x - \hat{x}\|, \tag{13}$$

where γ is a small Lipschitz scalar. For the simplicity, $\hat{h}_i$ denotes $h_i(\hat{z})$.

Authors in [29] have proven that the necessary and sufficient conditions for the existence of sliding mode observer when the system includes faults are

(A1) $\operatorname{rank}(CM_i) = \operatorname{rank}(M_i) = \tilde{q}$

(A2) The invariant zeros of (A_i, M_i, C) are stable

Under A1, there exists a coordinate transformation $x \longrightarrow \overline{x} = T_{1i} x$ such that system (8) is transformed into

$$\left\{ \dot{\overline{x}} = \sum_{i=1}^{k} \hat{h}_i\left[\overline{A}_i \overline{x} + \overline{\phi} + \overline{B}_i u + \overline{M}_i f_a + \overline{E}_i d\right], \overline{y} = \overline{C}\overline{x}, \right. \tag{14}$$

where

$$\begin{aligned} \overline{A}_i &= T_{1i} A_i T_{1i}^{-1} \\ &= \begin{bmatrix} \overline{A}_{1i} & \overline{A}_{2i} \\ \overline{A}_{3i} & \overline{A}_{4i} \end{bmatrix}, \\ \overline{\phi} &= T_{1i}\phi, \overline{B}_i \\ &= T_{1i} B_i \\ &= \begin{bmatrix} \overline{B}_{1i} \\ \overline{B}_{2i} \end{bmatrix}, \\ \overline{M}_i &= T_{1i} M_i \\ &= \begin{bmatrix} 0 \\ \overline{M}_{2i} \end{bmatrix}, \\ \overline{M}_{2i} &= \begin{bmatrix} 0 \\ \overline{M}_{22i} \end{bmatrix}, \\ \overline{E}_i &= T_{1i} E_i \\ &= \begin{bmatrix} \overline{E}_{1i} \\ \overline{E}_{2i} \end{bmatrix}, \\ \overline{C} &= C T_{1i}^{-1} \\ &= \begin{bmatrix} 0 & \overline{C}_2 \end{bmatrix}, \end{aligned} \tag{15}$$

where $\overline{A}_{1i} \in \mathbb{R}^{(n+h-p)\times(n+h-p)}$, $\overline{B}_{2i} \in \mathbb{R}^{p\times m}$, $\overline{M}_{22i} \in \mathbb{R}^{\overline{q}\times(q+h)}$, $\overline{E}_{2i} \in \mathbb{R}^{p\times l}$, and $\overline{C}_2 \in \mathbb{R}^{p\times p}$ is nonsingular.

Applying the linear change of coordinates T_{1i} to the error system (11), then we obtain

$$\dot{\overline{e}} = \sum_{i=1}^{k} \hat{h}_i\left[\overline{A}_{oi}\overline{e} + \overline{\phi} + \overline{M}_i f - \overline{G}_{ni}\vartheta + \overline{E}_i d\right], \tag{16}$$

where $\overline{A}_{oi} = T_{1i}A_{oi}T_{1i}^{-1}$. We suppose the observer gain $\overline{G}_{ni}$ has been the following form to facilitate the analysis:

$$\overline{G}_{ni} = \begin{bmatrix} -\overline{K}_i\overline{C}_2^{-1} \\ \overline{C}_2^{-1} \end{bmatrix}, \qquad \overline{K}_i = \begin{bmatrix} \overline{K}_{1i} & 0_{(n+h-p)\times\widetilde{q}} \end{bmatrix}, \tag{17}$$

where $\overline{K}_{1i} \in \mathbb{R}^{(n+h-p)\times(p-\widetilde{q})}$. It is noted that

$$\overline{K}_i\overline{M}_i = \begin{bmatrix} \overline{K}_{1i} & 0_{(n+h-p)\times\widetilde{q}} \end{bmatrix} \begin{bmatrix} 0_{(p-\widetilde{q})\times(q+h)} \\ \overline{M}_{22i} \end{bmatrix} = 0, \tag{18}$$

which motivates us to consider a coordinate transformation $\overline{e} \longrightarrow \widetilde{e}: = T_{2i}\overline{e}$, where

$$T_{2i} = \begin{bmatrix} I_{n+h-p} & \overline{K}_i \\ 0_{p\times(n+h-p)} & \overline{C}_2 \end{bmatrix}. \tag{19}$$

In the new set of coordinates, (16) becomes

$$\dot{\widetilde{e}} = \sum_{i=1}^{k} \widehat{h}_i \left[\widetilde{A}_{oi}\widetilde{e} + \widetilde{\phi} + \widetilde{M}_i f - \widetilde{G}_{ni}\vartheta + \widetilde{E}_i d \right], \tag{20}$$

where

$$\begin{aligned} \widetilde{A}_{oi} &= \left(T_{2i}^T\right)^{-1}\overline{A}_{oi}T_{2i}^{-1} \\ &= \widetilde{A}_i - \widetilde{G}_{li}\widetilde{C} \end{aligned} \tag{21}$$

and

$$\begin{aligned} \widetilde{A}_i &= T_{2i}\overline{A}_iT_{2i}^{-1} \\ &= \begin{bmatrix} \widetilde{A}_{1i} & \widetilde{A}_{2i} \\ \widetilde{A}_{3i} & \widetilde{A}_{4i} \end{bmatrix}, \\ \widetilde{G}_{li} &= T_{2i}\overline{G}_{li} \\ &= \begin{bmatrix} \widetilde{G}_{l1i} \\ \widetilde{G}_{l2i} \end{bmatrix}, \\ \widetilde{\phi} &= T_{2i}\overline{\phi}: \\ &= \begin{bmatrix} \widetilde{\phi}_1 \\ \widetilde{\phi}_2 \end{bmatrix}, \\ \widetilde{M}_i &= T_{2i}\overline{M}_i \\ &= \begin{bmatrix} 0 \\ \widetilde{M}_{2i} \end{bmatrix}, \\ \widetilde{E}_i &= T_{2i}\overline{E}_i \\ &= \begin{bmatrix} \widetilde{E}_{1i} \\ \widetilde{E}_{2i} \end{bmatrix}, \\ \widetilde{C} &= \overline{C}T_{2i}^{-1} \\ &= \begin{bmatrix} 0 & I_p \end{bmatrix}, \\ \widetilde{G}_{ni} &= T_{2i}\overline{G}_{ni} \\ &= \begin{bmatrix} 0 \\ I_p \end{bmatrix}, \end{aligned} \tag{22}$$

with

$$\begin{cases} \widetilde{A}_{1i} = \bar{A}_{1i} + \bar{K}_i\bar{A}_{3i}, \\ \widetilde{A}_{2i} = \left(\bar{A}_{2i} + \bar{K}_i\bar{A}_{4i}\right)C_2^{-1} - \left(\bar{A}_{1i} + \bar{K}_i\bar{A}_{3i}\right)C_2^{-1} - \bar{A}_{1i}\bar{K}_i C_2^{-1}, \\ \widetilde{A}_{3i} = \bar{C}_2\bar{A}_{3i}, \\ \widetilde{A}_{4i} = \bar{C}_2\left(\bar{A}_{4i} - \bar{A}_{3i}\bar{K}_i\right)C_2^{-1}, \\ \widetilde{M}_{2i} = \bar{C}_2\bar{M}_{2i}, \\ \widetilde{E}_{1i} = \bar{E}_{1i} + \bar{K}_i\bar{E}_{2i}, \\ \widetilde{E}_{2i} = \bar{C}_2\bar{E}_{2i}. \end{cases} \tag{23}$$

Define the matrix $\widetilde{G}_{li} = \begin{bmatrix} \widetilde{G}_{l1i} \\ \widetilde{G}_{l2i} \end{bmatrix}$. If the observer gain matrices $\widetilde{G}_{l1i}$ and $\widetilde{G}_{l2i}$ are chosen as $\widetilde{G}_{l1i} = \widetilde{A}_{2i}$ and $\widetilde{G}_{l2i} = \widetilde{A}_{4i} - \widetilde{A}_4^s$, where $\widetilde{A}_4^s$ is an arbitrary negative definite matrix, it can be concluded that

$$\begin{aligned} \widetilde{A}_{oi} &= \widetilde{A}_i - \widetilde{G}_{li}\widetilde{C} \\ &= \begin{bmatrix} \overline{A}_{1i} + \overline{K}_i\overline{A}_{3i} & 0 \\ \overline{C}_2\overline{A}_{3i} & \widetilde{A}_4^s \end{bmatrix}. \end{aligned} \tag{24}$$

According to our choice, $\widetilde{A}_4^s$ is stable. Therefore, $\widetilde{A}_{oi}$ is stable from the stability of $\overline{A}_{1i} + \overline{K}_i\overline{A}_{3i}$.

Partitioning the error system conformably to $\widetilde{e} := \begin{bmatrix} e_1^T & e_y^T \end{bmatrix}^T$ with $e_1 \in \mathbb{R}^{n+h-p}$ and $e_y \in \mathbb{R}^p$ yields

$$\begin{cases} \dot{e}_1 = \sum_{i=1}^{k} \widehat{h}_i \left[\widetilde{A}_{1i}e_1 + \widetilde{\phi}_1 + \widetilde{E}_{1i}d \right], \\ \dot{e}_y = \sum_{i=1}^{k} \widehat{h}_i \left[\widetilde{A}_4^s e_y + \widetilde{A}_{3i}e_1 + \widetilde{\phi}_2 + \widetilde{M}_{2i}f - \vartheta + \widetilde{E}_{2,i}d \right]. \end{cases} \tag{25}$$

Considering the following sliding mode surface as

$$\mathcal{S} = \left\{ \left(e_1, e_y\right) \mid e_y = 0 \right\}, \tag{26}$$

we now design the discontinuous error injection ϑ as follows:

$$\vartheta = \begin{cases} \sum_{i=1}^{k} \widehat{h}_i\rho_i \dfrac{P_{2i}e_y}{\left\| P_{2i}e_y \right\|}, & \text{if } e_y \neq 0, \\ 0, & \text{otherwise}, \end{cases} \tag{27}$$

where $P_{2i} > 0 \in \mathbb{R}^{p\times p}$ is the unique solution to the Lyapunov equation for $\widetilde{A}_4^s$ with the design matrix $Q_0 > 0 \in \mathbb{R}^{p\times p}$:

$$P_{2i}\widetilde{A}_4^s + \left(P_{2i}\widetilde{A}_4^s\right)^T = -Q_0 \tag{28}$$

and

$$\rho_i = \left\| \overline{C}_2\overline{M}_{2i} \right\| \widehat{w} + \rho_0, \tag{29}$$

with the adaptive law,

$$\dot{\widehat{w}} = -q \sum_{i=1}^{k} \widehat{h}_i \left[\left\| \overline{C}_2\overline{M}_{2i} \right\| \right] \left\| P_{2i}e_y \right\|. \tag{30}$$

ρ_0 is a small positive constant and q is designed constant.

Remark 1. The presence of both f and d in (18) poses the main challenge to the SMO design problem in this paper. To tackle this difficulty, in addition to the design of ϑ for the estimation of f in the SMO, we use the H_∞ approach for the stability analysis of the error system.

Define the controlled output of the error system m as

$$m = H\begin{bmatrix} e_1 \\ e_y \end{bmatrix}, \tag{31}$$

where $H = \operatorname{diag}(H_1, H_2)$ is a full rank design matrix.

We will utilize the H_∞ approach to analyse the stability of the error system (18) so that

(i) $\lim_{t\to\infty} \tilde{e} = 0$ if $d = 0$, $\forall t \geq 0$

(ii) $\int_0^\infty m^T m\, dt \leq \mu^2 \int_0^\infty d^T d dt$, otherwise

$\mu > 0$ is the attenuation level to be minimized.

3. Sliding Mode Observer Design

In this section, we will discuss the design method of SMO. The main results are presented as follows.

3.1. Stability Analysis

Theorem 1. *Consider system (9) with conditions* ***A1*** *and* ***A2****. The error system (18) is asymptotically stable with a minimal μ if there exist matrices $P_{1i} > 0$, $P_{2i} > 0$, $\overline{K}_i$, and small positive scalar ε such that the following convex optimization problem is solved:*

$\min(\overline{\mu})$ *subject to*

$$\begin{bmatrix} \overline{\Lambda}_{1i} & \Lambda_{2i} & P_{1i}\overline{E}_{1i} + W_i\overline{E}_{2i} & P_{1i} & 0 \\ * & \overline{\Lambda}_{4i} & P_{2i}\overline{C}_2\overline{E}_{2i} & 0 & P_{2i} \\ * & * & -\overline{\mu}I & 0 & 0 \\ * & * & * & -\varepsilon I & 0 \\ * & * & * & 0 & -\varepsilon I \end{bmatrix} < 0, \tag{32}$$

where

$$\begin{aligned} \overline{\Lambda}_{1i} &= \overline{A}_{1i}^T P_{1i} + P_{1i}\overline{A}_{1i} + W_i\overline{A}_{3i} + \overline{A}_{3i}^T W_i^T + \varepsilon\gamma^2 I + H_1^T H_1, \\ \Lambda_{2i} &= \overline{A}_{3i}^T \overline{C}_2^T P_{2i}, \\ \overline{\Lambda}_{4i} &= \widetilde{A}_4^{sT} P_{2i} + P_{2i}\widetilde{A}_4^s + \varepsilon\gamma^2 I_p + H_2^T H_2. \end{aligned} \tag{33}$$

$\overline{K}_i$ and μ are obtained as $\overline{K}_i = P_{1i}^{-1} W_i$ and $\mu = \sqrt{\overline{\mu}}$.

Proof. Consider the Lyapunov functional candidate:

$$V(\tilde{e}) = \tilde{e}^T P_i \tilde{e} + \frac{1}{q}\tilde{w}^2, \tag{34}$$

where $P_i = \operatorname{diag}\{P_{1i}, P_{2i}\}$, $P_{1i} \in \mathbb{R}^{(n+h-p)\times(n+h-p)}$, and $\tilde{w} = w - \hat{w}$. By deviating $V(\tilde{e})$, we obtain

$$\begin{aligned} \dot{V}(\tilde{e}) = \sum_{i=1}^{k} \hat{h}_i \Big[& \tilde{e}^T \left(\widetilde{A}_{oi}^T P_i + P_i\widetilde{A}_{oi} \right)\tilde{e} + 2\tilde{e}^T P_i\tilde{\phi} + 2\tilde{e}^T P_i\widetilde{E}_i d \\ & + 2\tilde{e}^T P_i\widetilde{M}_i f - 2\tilde{e}^T P_i\widetilde{G}_{ni}\vartheta + \frac{2}{q}(-\tilde{w}\dot{\hat{w}}) \Big]. \end{aligned} \tag{35}$$

According to Lemma 1, we obtain

$$2\tilde{e}^T P_i\tilde{\phi} \leq \frac{1}{\varepsilon}\tilde{e}^T P_i^2\tilde{e} + \varepsilon\|\tilde{\phi}\|^2 \leq \frac{1}{\varepsilon}\tilde{e}^T P_i^2\tilde{e} + \varepsilon\gamma^2\|\tilde{e}\|^2. \tag{36}$$

Hence, from (33), we have

$$\begin{aligned} \dot{V}(\tilde{e}) \leq \sum_{i=1}^{k} \hat{h}_i \Big[& \tilde{e}^T \left(\widetilde{A}_{oi}^T P_i + P_i\widetilde{A}_{oi} + \frac{1}{\varepsilon}P_i^2\tilde{e}^T\tilde{e} + \varepsilon\gamma^2 I \right)\tilde{e} + 2\tilde{e}^T P_i\tilde{\phi} \\ & + 2\tilde{e}^T P_i\widetilde{E}_i d + \underbrace{2\tilde{e}^T P_i\widetilde{M}_i f - 2\tilde{e}^T P_i\widetilde{G}_{ni}\vartheta + \frac{2}{q}(-\tilde{w}\dot{\hat{w}})}_{term\,(i)} \Big]. \end{aligned} \tag{37}$$

Substituting the adaptive law $\dot{\hat{w}}$ and the discontinuous vector ϑ into the term (i), we have

$$\begin{aligned} \sum_{i=1}^{k} \hat{h}_i \left[\tilde{e}^T P_i\widetilde{M}_i f - \tilde{e}^T P_i\widetilde{G}_{ni}\vartheta + \frac{1}{q}\dot{\hat{w}}\tilde{w} \right] &= \sum_{i=1}^{k} \hat{h}_i \left[e_y^T P_{2i}\overline{C}_2\overline{M}_{2i} f - e_y^T P_{2i}\vartheta + \frac{1}{q}\dot{\hat{w}}\tilde{w} \right] \\ &= \sum_{i=1}^{k} \hat{h}_i \left[e_y^T P_{2i}\overline{C}_2\overline{M}_{2i} f - e_y^T P_{2i}\left(\|\overline{C}_2\overline{M}_{2i}\|\hat{w} + \rho_0 \right) \times \frac{P_{2i}e_y}{\|P_{2i}e_y\|} + \frac{1}{q}\dot{\hat{w}}\tilde{w} \right] \\ &= \sum_{i=1}^{k} \hat{h}_i \left[e_y^T P_{2i}\overline{C}_2\overline{M}_{2i} f - \left(\|\overline{C}_2\overline{M}_{2i}\|\hat{w} + \rho_0 \right)\|P_{2i}e_y\| \right] + \frac{1}{q}(w - \hat{w})\left(-q\sum_{i=1}^{k}\hat{h}_i\left[\|\overline{C}_2\overline{M}_{2i}\| \right]\|P_{2i}e_y\| \right) \\ &\leqslant \sum_{i=1}^{k} \hat{h}_i \|P_{2i}e_y\| \left[\|\overline{C}_2\overline{M}_{2i}\| w - \rho_0 + \|\overline{C}_2\overline{M}_{2i}\| w \right] \\ &= -\rho_0 \sum_{i=1}^{k} \hat{h}_i \|P_{2i}e_y\| < 0. \end{aligned} \tag{38}$$

When substituting obtained expressions (35) into (34), we obtain

$$\dot{V}(\tilde{e}) \leq \sum_{i=1}^{k} \hat{h}_i \left[\tilde{e}^T \left(\tilde{A}_{oi}^T P_i + P_i \tilde{A}_{oi} + \frac{1}{\varepsilon} P_i^2 + \varepsilon \gamma^2 I \right) \tilde{e} + 2\tilde{e}^T P_i \tilde{E}_i d \right]. \quad (39)$$

For the case $d = 0$, then

$$\dot{V}(\tilde{e}) \leq \sum_{i=1}^{k} \hat{h}_i \left(\begin{bmatrix} e_1 \\ e_y \end{bmatrix}^T \Lambda_i \begin{bmatrix} e_1 \\ e_y \end{bmatrix} \right), \quad (40)$$

where

$$\Lambda_i = \begin{bmatrix} \Lambda_{1i} & \Lambda_{2i} \\ * & \Lambda_{4i} \end{bmatrix}, \quad (41)$$

with

$$\begin{cases} \Lambda_{1i} = \bar{A}_{1i}^T P_{1i} + P_{1i} \bar{A}_{1i} + W_i \bar{A}_{3i} + \bar{A}_{3i}^T W_i^T + \frac{1}{\varepsilon} P_{1i}^2 + \varepsilon \gamma^2 I, \\ \Lambda_{2i} = \bar{A}_{3i}^T \bar{C}_2^T P_{2i}, \\ \Lambda_{4i} = \tilde{A}_4^{sT} P_{2i} + P_{2i} \tilde{A}_4^s + \frac{1}{\varepsilon} P_{2i}^2 + \varepsilon \gamma^2 I. \end{cases} \quad (42)$$

Denote

$$\begin{aligned} W_i &= P_{1i} \bar{K}_i, \\ \bar{\mu} &= \mu^2. \end{aligned} \quad (43)$$

Using the Schur complement, if (32) holds, we have

$$\Lambda_i = \begin{bmatrix} \Lambda_{1i} & \Lambda_{2i} \\ * & \Lambda_{4i} \end{bmatrix} < 0, \quad (44)$$

which implies that $\dot{V}(\tilde{e}) < 0$, i.e., $\lim_{t \longrightarrow \infty} \tilde{e} = 0$.

On the contrary, for the case that $d \neq 0$, let

$$J_1 = \dot{V} + m^T m - \mu^2 d^T d. \quad (45)$$

Substituting (31) and (39) into (45) yields

$$\begin{aligned} J_1 &= \dot{V}(\tilde{e}) + \tilde{e}^T H^T H \tilde{e} - \mu^2 d^T d \\ &\leq \sum_{i=1}^{k} \hat{h}_i \left(\tilde{e}^T \left(\Lambda_i + H^T H \right) \tilde{e} + 2\tilde{e}^T P_i \tilde{E}_i d - \mu^2 d^T d \right) \\ &\leq \sum_{i=1}^{k} \hat{h}_i \begin{bmatrix} e_1 \\ e_y \\ d \end{bmatrix}^T \begin{bmatrix} \bar{\Lambda}_{1i} & \Lambda_{2i} & P_{1i}\bar{E}_{1i} + W_i \bar{E}_{2i} \\ * & \bar{\Lambda}_{4i} & P_{2i}\bar{C}_2 \bar{E}_{2i} \\ * & * & -\bar{\mu} I \end{bmatrix} \begin{bmatrix} e_1 \\ e_y \\ d \end{bmatrix}, \end{aligned} \quad (46)$$

where $\bar{\Lambda}_{1i} = \Lambda_{1i} + H_1^T H_1$ and $\bar{\Lambda}_{4i} = \Lambda_{4i} + H_2^T H_2$.

Using the Schur complement, we have that if (30) holds

$$\upsilon_i = \begin{bmatrix} \bar{\Lambda}_{1i} & \Lambda_{2i} & P_{1i}\bar{E}_{1i} + W_i \bar{E}_{2i} \\ * & \bar{\Lambda}_{4i} & P_{2i}\bar{C}_2 \bar{E}_{2i} \\ * & * & -\bar{\mu} I \end{bmatrix} < 0, \quad (47)$$

which means that $J_1 < 0$; i.e., the error system (23) is asymptotically stable with the H_∞ performance μ.

From (43), $\bar{K}_i$ and μ are computed as $\bar{K}_i = P_{1i}^{-1} W_i$ and $\mu = \sqrt{\bar{\mu}}$. □

Remark 2. According to Theorem 1, the error system is asymptotically stable with H_∞ performance. Thus, for some small $\varpi > 0$, we have $\|\tilde{e}\| \leq \varpi$. In addition, one has $\|e_1\| \leq \|\tilde{e}\|$.

3.1.1. Sliding Motion Analysis. In this section, the gain parameter ρ_0 in (29) will be determined to demonstrate a sliding motion occurs on $\mathcal{S}$ in finite time.

Theorem 2. *If ρ_0 in (27) is chosen to satisfy*

$$\rho_0 - \bar{\rho} \geq (\varsigma_{\max} + \gamma)\varpi + \nu_{\max}\|d\|, \quad (48)$$

where $\bar{\rho}$ is positive scalar, then a sliding motion occurs on the surface $\mathcal{S}$, for all $t \geq t_s$, where t_s is the finite time at which sliding is established.

Proof. Define

$$V_s = \frac{1}{2} e_y^T P_{2i} e_y + \frac{1}{2q} \tilde{\varpi}^2, \quad (49)$$

where the matrix P_{2i} is proposed in Theorem 1; then, the derivative of V_s satisfies

$$\dot{V}_s = \sum_{i=1}^{k} \hat{h}_i \left[e_y^T P_{2i} \left(\tilde{A}_4^s e_y + \tilde{A}_{3i} e_1 + \tilde{\phi} + \tilde{E}_{2i} d + \tilde{M}_{2i} f - \vartheta \right) + \frac{1}{q} \dot{\tilde{\varpi}} \tilde{\varpi} \right]. \quad (50)$$

According to (35), we have

$$\dot{V}_s \leq \sum_{i=1}^{k} \hat{h}_i \left[e_y^T P_{2i} \left(\tilde{A}_4^s e_y + \tilde{A}_{3i} e_1 + \tilde{\phi} + \tilde{E}_{2i} d \right) - \rho_0 \left\| P_{2i} e_y \right\| \right]. \quad (51)$$

Since $\tilde{A}_4^s$ is a stable design matrix and P_{2i} is solution to Lyapunov (28), we have

$$\begin{aligned} e_y^T P_{2i} \tilde{A}_{22}^s e_y &= \frac{1}{2} e_y^T \Bigg(\underbrace{P_{2i} \tilde{A}_4^s + \left(P_{2i} \tilde{A}_4^s \right)^T}_{-Q_{0i}} \Bigg) e_y \\ &= -\frac{1}{2} e_y^T Q_{0i} e_y \leqslant 0. \end{aligned} \quad (52)$$

Using relation (46), we have

$$\begin{aligned}\dot{V}_s &\le \sum_{i=1}^{k} \hat{h}_i \left[e_y^T P_{2i} \left(\tilde{A}_{3i} e_1 + \tilde{\phi} + \tilde{E}_{2i} d \right) - \rho_0 \left\| P_{2i} e_y \right\| \right] \\ &\le \sum_{i=1}^{k} \hat{h}_i \left[\left\| P_{2i} e_y \right\| \left(\left(\left\| \tilde{A}_{3i} \right\| + \gamma \right) \|\tilde{e}\| + \left\| \tilde{E}_{2i} \right\| \|d\| \right) - \rho_0 \right] \\ &\le \left\| P_{2i} e_y \right\| \left[(\varsigma_{\max} + \gamma) \varpi + \nu_{\max} \|d\| - \rho_0 \right],\end{aligned} \tag{53}$$

where

$$\begin{aligned}\varsigma_{\max} &= \max\left\{ \lambda_{\max}\left(\tilde{A}_{31}\right), \lambda_{\max}\left(\tilde{A}_{32}\right), \ldots, \lambda_{\max}\left(\tilde{A}_{3k}\right) \right\}, \\ \nu_{\max} &= \max\left\{ \lambda_{\max}\left(\tilde{E}_{21}\right), \lambda_{\max}\left(\tilde{E}_{22}\right), \ldots, \lambda_{\max}\left(\tilde{E}_{2k}\right) \right\},\end{aligned} \tag{54}$$

where $\lambda_{\max}(A)$ is the maximum eigenvalue of A.

If condition (42) holds, then the sliding mode reaching condition

$$\dot{V}_s \leqslant -\rho_0 \left\| P_{2i} e_y(t) \right\| \leqslant -\rho_0 \sqrt{\lambda_{\min}(P_{2i})} V_s^{1/2} \tag{55}$$

is guaranteed. Applying the chain rule $(\mathrm{d}/\mathrm{d}t)\sqrt{V_s} = (1/2\sqrt{V_s})\dot{V}_s$, the reaching condition (47) can be integrated and rearranged to obtain an estimate for t_s:

$$\begin{aligned}&\frac{1}{2\sqrt{V_s}} \dot{V}_s \le -\rho_0 \sqrt{\lambda_{\min}(P_{2i})} \\ &\Rightarrow \int_0^{t_s} \frac{\mathrm{d}}{\mathrm{d}t} \left(\sqrt{V_s} \right) \mathrm{d}t \\ &\le -\rho_0 \sqrt{\lambda_{\min}(P_{2i})} \int_0^{t_s} \mathrm{d}t \\ &\Rightarrow \left[\sqrt{V_s(t_s)} - \sqrt{V_s(0)} \right] \\ &\le \rho_0 \sqrt{\lambda_{\min}(P_{2i})} t_s \\ &\Rightarrow t_s \le \frac{1}{\rho_0} \frac{\sqrt{V_s(0)}}{\sqrt{\lambda_{\min}(P_{2i})}}.\end{aligned} \tag{56}$$

This proves that the sliding surface $\mathcal{S}$ is thus reached in finite time t_s. □

4. Robust Reconstruction of Actuator and Sensor Faults

In Theorems 1 and 2, one has proven that the error system is asymptotically stable and can thus be driven onto sliding surface $\mathcal{S}$ at some time instant t_s; the error system (23) is reduced to

$$0 = \sum_{i=1}^{k} \hat{h}_i \left[e_y^T P_{2i} \left(\tilde{A}_{3i} e_1 + \tilde{\phi}_2 + \tilde{M}_{2i} f + \tilde{E}_{2i} d - \vartheta_{eq} \right) \right], \forall t \ge t_s, \tag{57}$$

where

$$\vartheta_{eq} = \sum_{i=1}^{k} \hat{h}_i \rho_i \frac{P_{2i} e_y}{\left\| P_{2i} e_y \right\| + \delta}, \tag{58}$$

with $\delta > 0$ is small scalar. Define

$$g(e_1, x, \hat{x}, u) = \tilde{A}_{3i} e_1 + \tilde{\phi}_2 + \tilde{E}_{2i} d. \tag{59}$$

Computing the norm of (59) yields

$$\begin{aligned}\|g(e_1, x, \hat{x}, u)\| &\le \left(\left\| \tilde{A}_{21i} \right\| + \gamma_2 \right) \|e_1\| \\ &+ \left\| \tilde{E}_{2i} \right\| \|d\| \le (\varsigma_{\max} + \gamma) \|\tilde{e}\| + \nu_{\max} \|d\| \\ &\le \left[\bar{\mu} (\varsigma_{\max} + \gamma) \sigma_{\max}\left(H^{-1} \right) + \nu_{\max} \right] \|d\|,\end{aligned} \tag{60}$$

where $\sigma_{\max}(A)$ for a matrix A denotes the maximum singular value of the matrix. The result (52) follows by keeping in mind that $\|\tilde{e}\| \le \sigma_{\max}(H^{-1}) \bar{\mu} \|d\|$. Therefore, it follows that

$$\begin{aligned}\sup_{\|d\| \ne 0} &= \frac{\|g(e_1, x, \hat{x}, u)\|}{\|d\|} \\ &= \beta_1 \sqrt{\bar{\mu}} + \beta_2,\end{aligned} \tag{61}$$

where $\beta_1 = (\varsigma_{\max} + \gamma)\sigma_{\max}(H^{-1})$ and $\beta_2 = \nu_{\max}$. Thus, for a small $\beta_1 \sqrt{\bar{\mu}} + \beta_2$, both actuator and sensor faults are estimated by

$$\begin{aligned}\hat{f} &= \begin{bmatrix} \hat{f}_a \\ \hat{f}_s \end{bmatrix} \\ &= \sum_{i=1}^{k} \hat{h}_i \tilde{M}_{2i}^{+} \rho_i \frac{P_{2i} e_y}{\left\| P_{2i} e_y \right\| + \delta}.\end{aligned} \tag{62}$$

5. Dynamic Output-Feedback Fault-Tolerant Control Design

In this section, a fuzzy dynamic output-feedback FTC (DOFFTC) will be constructed to guarantee the stability of closed-loop system (2). The following corrected output will be given which is obtained by subtracting the reconstructed sensor faults from the (faulty) outputs:

$$y_c = C_p x_p + N_p \left(f_s - \hat{f}_s \right) + D_p d, \tag{63}$$

where $\hat{f}_s$ is estimation of f_s obtained from the FE scheme. System (2) becomes

$$\begin{cases} \dot{x}_p = \sum_{i=1}^{k} h_i \left[A_{pi} x_p + B_{pi} (u + f_a) + E_{pi} d \right], \\ y_c = C_p x_p + N_p e_{fs} + D_p d, \end{cases} \tag{64}$$

where $e_{f_s} = f_s - \hat{f}_s$ is the sensor fault estimation error. The fuzzy DOFFTC for system (56) is constructed as follows:

$$\begin{cases} \dot{x}_c = \sum_{i=1}^{k}\sum_{j=1}^{k} h_i h_j \left[A_{cij}x_c + B_{ci}y_c\right], \\ u = \sum_{i=1}^{k} h_i \left[C_{ci}x_c + D_{ci}y_c - \widehat{f}_a\right], \end{cases} \tag{65}$$

where $x_c \in \mathbb{R}^{n\times n}$ is the controller state, $\widehat{f}_a$ is the estimation of f_a, and $A_{cij} \in \mathbb{R}^{n\times n}$, $B_{ci} \in \mathbb{R}^{n\times p}$, $C_{ci} \in \mathbb{R}^{m\times n}$, and $D_{ci} \in \mathbb{R}^{m\times p}$ denote controller matrices to be obtained later, respectively.

Substituting (63) into (65), we obtain

$$\begin{cases} \dot{x}_c = \sum_{i=1}^{k}\sum_{j=1}^{k} h_i h_j \left[A_{cij}x_c + B_{ci}C_p x_p + B_{ci}N_p e_{fs} + B_{ci}D_p d\right], \\ u = \sum_{i=1}^{k} h_i \left[C_{ci}x_c + D_{ci}C_p x_p + D_{ci}N_p e_{fs} + D_{ci}D_p d - \widehat{f}_a\right. \end{cases} \tag{66}$$

Then, substituting (64) into (66), we further obtain

$$\begin{aligned} \dot{x}_p = \sum_{i=1}^{k} h_i \big[& A_{pi}x_p + B_{pi}C_{ci}x_c + B_{pi}D_{ci}C_p x_p + B_{pi}D_{ci}N_p e_{fs} \\ & + B_{pi}D_{ci}D_p d + B_{pi}e_{f_a} + E_{pi}d\big], \end{aligned} \tag{67}$$

where $e_{f_a} = f_a - \widehat{f}_a$ is the actuator fault estimation error.

The dynamic equation of the closed-loop system is obtained as follows:

$$\left\{ \dot{\tilde{x}} = \sum_{i=1}^{k}\sum_{j=1}^{k} h_i h_j \left[\mathscr{A}_{ij}\tilde{x} + \mathscr{E}_{ij}\varpi\right],\ y_c = \mathscr{C}\tilde{x} + \mathscr{D}\varpi, \right. \tag{68}$$

where $\tilde{x} = \begin{bmatrix} x_p \\ x_c \end{bmatrix}$, $\varpi = \begin{bmatrix} e_{f_a}^T & e_{fs}^T & d^T \end{bmatrix}^T$, and

$$\begin{aligned} \mathscr{A}_{ij} &= \begin{bmatrix} A_{pi} + B_{pi}D_{ci}C_p & B_{pi}C_{ci} \\ B_{ci}C_p & A_{cij} \end{bmatrix}, \\ \mathscr{E}_{ij} &= \begin{bmatrix} B_{pi} & B_{pi}D_{ci}N_p & \left(B_{pi}D_{ci}D_p + E_{pi}\right) \\ 0 & B_{ci}N_p & B_{ci}D_p \end{bmatrix}, \\ \mathscr{C} &= \begin{bmatrix} C_p 0 \end{bmatrix}, \\ \mathscr{D} &= \begin{bmatrix} 0 N_p D_p \end{bmatrix}. \end{aligned} \tag{69}$$

So far, the control purpose in this paper for the closed-loop system (60) is to design the controller gain matrices of (57) such that the corrected output satisfies the H_∞ performance as follows:

$$\int_0^\infty y_c^T y_c \mathrm{d}t \le \mu_c \int_0^\infty \varpi^T \varpi \mathrm{d}t, \tag{70}$$

for $\varpi \in L_2[0\ \ \infty)$ and attenuation level $\mu_c > 0$.

Theorem 3. *The closed-loop system (60) is asymptotically stable with a minimal μ_c in (61) if there exist matrices $X > 0$, $Y > 0$, and $\widehat{A}_{cij}$, $\widehat{B}_{ci}$, $\widehat{C}_{ci}$, and $\widehat{D}_{ci}$, $i, j = 1, \ldots, k$, such that the following convex optimization problem is solved:*

$$\begin{aligned} &\min(\bar{\mu}_c) \text{ subject to} \\ &\quad \Phi_{ii} < 0,\ 1 \le i \le k, \\ &\quad \frac{2}{k-1}\Phi_{ii} + \Phi_{ij} + \Phi_{ji} < 0,\ 1 \le i < j \le k, \end{aligned} \tag{71}$$

where

$$\Phi_{ij} = \begin{bmatrix} \Upsilon_{1ij} & \Upsilon_{2ij} & B_{pi} & \Upsilon_{3ij} & \Upsilon_{4ij} & E_{pi} & XC_p^T \\ * & \Upsilon_{5ij} & YB_{pi} & \Upsilon_{6ij} & \Upsilon_{7ij} & YE_{pi} & C_p^T \\ * & * & -\bar{\mu}_c I & 0 & 0 & 0 & 0 \\ * & * & * & -\bar{\mu}_c I & 0 & 0 & 0 \\ * & * & * & * & -\bar{\mu}_c I & 0 & N_p^T \\ * & * & * & * & * & -\bar{\mu}_c I & D_p^T \\ * & * & * & * & * & * & -\bar{\mu}_c I \end{bmatrix}, \tag{72}$$

with

$$\begin{aligned} \Upsilon_{1ij} &= A_{pi}X + XA_{pi}^T + B_{pi}\widehat{C}_{ci} + \widehat{C}_{ci}^T B_{pi}^T, \\ \Upsilon_{2ij} &= \widehat{A}_{cij}^T + A_{pi} + B_{pi}\widehat{D}_{ci}C_p, \\ \Upsilon_{3ij} &= B_{pi}\widehat{D}_{ci}N_p, \\ \Upsilon_{4ij} &= B_{pi}\widehat{D}_{ci}D_p, \\ \Upsilon_{5ij} &= YA_{pi} + A_{pi}^T Y^T + \widehat{B}_{ci}C_p + C_p^T \widehat{B}_{ci}^T, \\ \Upsilon_{6ij} &= \widehat{B}_{ci}N_p, \\ \Upsilon_{7ij} &= \widehat{B}_{ci}D_p. \end{aligned} \tag{73}$$

The gain matrices of the DOFFTC are as follows:

$$\begin{aligned} D_{ci} &= \widehat{D}_{ci}, \\ A_{cij} &= S^{-1}\left(\widehat{A}_{cij} - Y\left(A_{pi} + B_{pi}\widehat{D}_{ci}C_p\right)X\right)Q^{-T} \\ &\quad - S^{-1}YB_{pi}C_{cj} - B_{ci}C_pXQ^{-T}, \\ C_{ci} &= \left(\widehat{C}_{ci} - D_{ci}C_pX\right)Q^{-T}, \\ B_{ci} &= S^{-1}\left(\widehat{B}_{ci} - YB_{pi}D_{ci}\right). \end{aligned} \tag{74}$$

μ_c is obtained as $\mu_c = \sqrt{\bar{\mu}_c}$.

Proof. Consider the Lyapunov function $V_x = \tilde{x}^T P_x \tilde{x}$, where $P_x > 0$; then, the derivative of V_x can be obtained as

$$\dot{V}_x = \sum_{i=1}^{k}\sum_{j=1}^{k} h_i h_j \left[\tilde{x}^T\left(\mathscr{A}_{ij}^T P_x + P_x\mathscr{A}_{ij}\right)\tilde{x}^T + 2\tilde{x}^T P_x \mathscr{E}_{ij}\varpi\right]. \tag{75}$$

Let

$$J_2 = \dot{V}_x + y_c^T y_c - \mu_c^2 \varpi^T \varpi. \tag{76}$$

Substituting (76) into (75), we obtain

$$
\begin{aligned}
J_2 &= \sum_{i=1}^{k}\sum_{j=1}^{k} h_i h_j \left[\tilde{x}^T\left(\mathscr{A}_{ij}^T P_x + P_x \mathscr{A}_{ij}\right)\tilde{x}^T + 2\tilde{x}^T P_x \mathscr{E}_{ij}\varpi + y_c^T y_c - \mu_c^2 \varpi^T \varpi\right] \\
&= \sum_{i=1}^{k}\sum_{j=1}^{k} h_i h_j \left[\tilde{x}^T\left(\mathscr{A}_{ij}^T P_x + P_x \mathscr{A}_{ij}\right)\tilde{x}^T + 2\tilde{x}^T P_x \mathscr{E}_{ij}\varpi + \tilde{x}^T \mathscr{C}^T \mathscr{C}\tilde{x} + 2\tilde{x}^T \mathscr{C}^T \mathscr{D}\varpi + \varpi^T \mathscr{D}^T \mathscr{D}\varpi - \mu_c^2 \varpi^T \varpi\right] \\
&= \sum_{i=1}^{k}\sum_{j=1}^{k} h_i h_j \left[\tilde{x}^T\left(\mathscr{A}_{ij}^T P_x + P_x \mathscr{A}_{ij} + \mathscr{C}^T \mathscr{C}\right)\tilde{x}^T + 2\tilde{x}^T P_x \mathscr{E}_{ij}\varpi + 2\tilde{x}^T \mathscr{C}^T \mathscr{D}\varpi + \varpi^T \mathscr{D}^T \mathscr{D}\varpi - \mu_c^2 \varpi^T \varpi\right] \\
&= \begin{bmatrix} \tilde{x} \\ \varpi \end{bmatrix}^T \Theta \begin{bmatrix} \tilde{x} \\ \varpi \end{bmatrix},
\end{aligned}
\tag{77}
$$

where

$$
\Theta = \sum_{i=1}^{k}\sum_{j=1}^{k} h_i h_j \begin{bmatrix} \mathscr{A}_{ij}^T P_x + P_x \mathscr{A}_{ij} + \mathscr{C}^T \mathscr{C} & P_x \mathscr{E}_{ij} + \mathscr{C}^T \mathscr{D} \\ * & \mathscr{D}^T \mathscr{D} - \mu_c^2 I \end{bmatrix}. \tag{78}
$$

It is easy to find that $J_2 < 0$ if $\Theta < 0$. Using the Schur complement, $\Theta < 0$ is equivalent to

$$
\sum_{i=1}^{k}\sum_{j=1}^{k} h_i h_j \begin{bmatrix} \mathscr{A}_{ij}^T P_x + P_x \mathscr{A}_{ij} & P_x \mathscr{E}_{ij} & \mathscr{C}^T \\ * & -\mu_c^2 I & \mathscr{D}^T \\ * & * & -I \end{bmatrix} < 0. \tag{79}
$$

Let us define the matrix P_x and its inverse P_x^{-1}:

$$
\begin{aligned}
P_x &= \begin{bmatrix} Y & S \\ S^T & W \end{bmatrix}, \\
P_x^{-1} &= \begin{bmatrix} X & Q \\ Q^T & Z \end{bmatrix}.
\end{aligned}
\tag{80}
$$

Due to $P_x P_x^{-1} = I_{2n}$, we have

$$
\begin{aligned}
P_x \begin{bmatrix} X \\ Q^T \end{bmatrix} &= \begin{bmatrix} I_n \\ 0 \end{bmatrix}, \\
P_x \begin{bmatrix} X & I_n \\ Q^T & 0 \end{bmatrix} &= \begin{bmatrix} I_n & Y \\ 0 & S^T \end{bmatrix}.
\end{aligned}
\tag{81}
$$

We will also define the matrices:

$$
\begin{aligned}
\Pi_1 &= \begin{bmatrix} X & I \\ Q^T & 0 \end{bmatrix}, \\
\Pi_2 &= P_x \Pi_1 \\
&= \begin{bmatrix} I & Y \\ 0 & S^T \end{bmatrix}.
\end{aligned}
\tag{82}
$$

Pre- and post-multiplying (79) by $\operatorname{diag}(\Pi_1^T, I, I)$ and its transpose and by using the variable change:

$$
\begin{aligned}
\widehat{A}_{ij} &= Y\left(A_{pi} + B_{pi} D_{ci} C_p\right) X + S B_{ci} C_p X + Y B_{pi} C_{cj} Q^T \\
&\quad + S A_{cij} Q^T, \\
\widehat{B}_{ci} &= Y B_{pi} D_{ci} + S B_{ci}, \\
\widehat{C}_{ci} &= D_{ci} C X + C_{ci} Q^T, \\
\widehat{D}_{ci} &= D_{ci}.
\end{aligned}
\tag{83}
$$

Inequality (72) can thus be easily obtained. □

6. Inverted Pendulum Example

Consider the nonlinear inverted pendulum system from [30]:

$$
\begin{cases}
\dot{x}_1 = x_2, \\
\dot{x}_2 = \dfrac{g\sin(x_1) - mlax_2^2 \sin(2x_1)/2 - ba\cos(x_1)x_4 - a\cos(x_1)(F - f_c)}{4l/3 - mla\cos(x_1)^2}, \\
\dot{x}_3 = x_4, \\
\dot{x}_4 = \dfrac{-mga\sin(2x_1)/2 + 4mla/3x_2^2\sin(x_1) - bax_4 + 4a/3(F - f_c)}{4/3 - ma\cos(x_1)^2},
\end{cases}
\tag{84}
$$

where $m = 2kg$, $M = 8kg$, $l = 0.5m$, $b = 0.06Ns/\text{rad}$, $\rho = 0.05$, $g = 9.81ms^{-2}$, $L = 2m$, $a = 1/(m + M)$, and $f_c = \rho\,\text{sign}(x_4)$. In [30], the above nonlinear system is expressed by two-rule T + S fuzzy model (2) with

$$A_{p1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \dfrac{g}{4l/3 - mla} & 0 & 0 & \dfrac{ba}{4l/3 - mla} \\ 0 & 0 & 0 & 1 \\ \dfrac{-mga}{4/3 - ma} & 0 & 0 & \dfrac{-ba}{4/3 - ma} \end{bmatrix},$$

$$B_{p1} = \begin{bmatrix} 0 \\ \dfrac{-a}{4l/3 - mla} \\ 0 \\ \dfrac{4a/3}{4/3 - ma} \end{bmatrix},$$

$$E_{p1} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix},$$

$$A_{p2} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \dfrac{g2\sqrt{2}/\pi}{4l/3 - mla/2} & 0 & 0 & \dfrac{ba\sqrt{2}/2}{4l/3 - mla/2} \\ 0 & 0 & 0 & 1 \\ \dfrac{-mga2/\pi}{4/3 - ma/2} & 0 & 0 & -\dfrac{ba}{4/3 - ma/2} \end{bmatrix}, \quad (85)$$

$$B_{p2} = \begin{bmatrix} 0 \\ \dfrac{-a\sqrt{2}/2}{4l/3 - mla/2} \\ 0 \\ \dfrac{4a/3}{4/3 - ma/2} \end{bmatrix},$$

$$E_{p2} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix},$$

$$C_p = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$N_p = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},$$

$$D_p = \begin{bmatrix} 0 \\ 0.5 \\ 0 \end{bmatrix},$$

With membership functions are $h_1(x_1) = 1 - (1/1 + \exp(-14(x_1 - (\pi/8))))/1 + \exp(-14(x_1 + (\pi/8)))$ and $h_2(x_1) = 1 - h_1(x_1)$. The membership functions are chosen based on the method of sector nonlinearity [30].

Choosing $A_f = 1$, $\widetilde{A}_4^s = \mathrm{diag}(-3, -5, -7)$, $H_1 = I_{2\times 2}$, and $H_2 = I_{3\times 3}$ and solving LMI optimization problem given in Theorem 1, we can calculate the H_∞ performance level $\mu = 0.6128$ and the following observer gains:

$$\overline{K}_{11} = \begin{bmatrix} 1.5283 \\ 3.8106 \end{bmatrix},$$

$$\overline{K}_{12} = \begin{bmatrix} 0.4747 \\ 0.7659 \end{bmatrix},$$

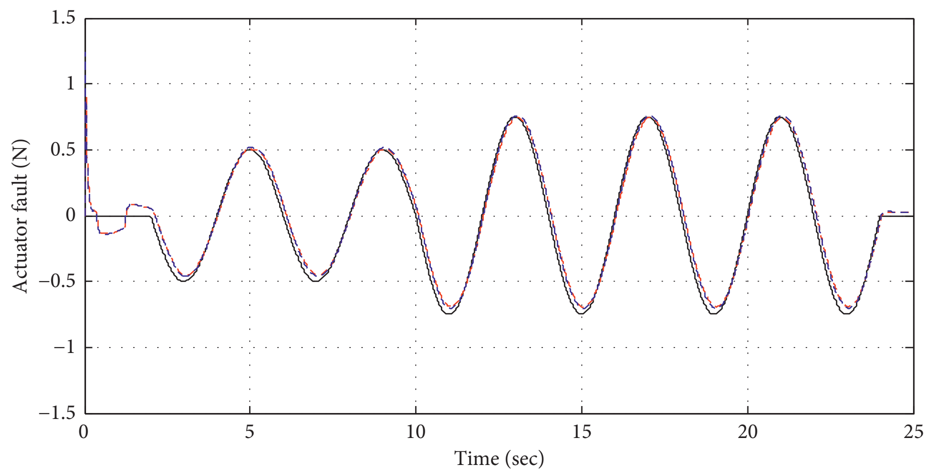
$$G_{l1} = \begin{bmatrix} -1.500 & -8.8106 & 0 \\ -4.3941 & -36.4793 & 0 \\ 1 & 4.3226 & 0 \\ 2.9471 & -3.6149 & 0 \\ 0 & -1.0807 & 6 \end{bmatrix},$$

$$G_{l2} = \begin{bmatrix} -1.0607 & -5.7659 & 0 \\ -3.1132 & -18.1749 & 0 \\ 1 & 1.3427 & 0 \\ 2.9514 & -0.6493 & 0 \\ 0 & -0.3357 & 6 \end{bmatrix}, \quad (86)$$

$$Gn1 = \begin{bmatrix} 0 & -1 & 0 \\ -1.5000 & -3.8106 & 0 \\ 0 & 1.0807 & 0 \\ 1 & -1.0807 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

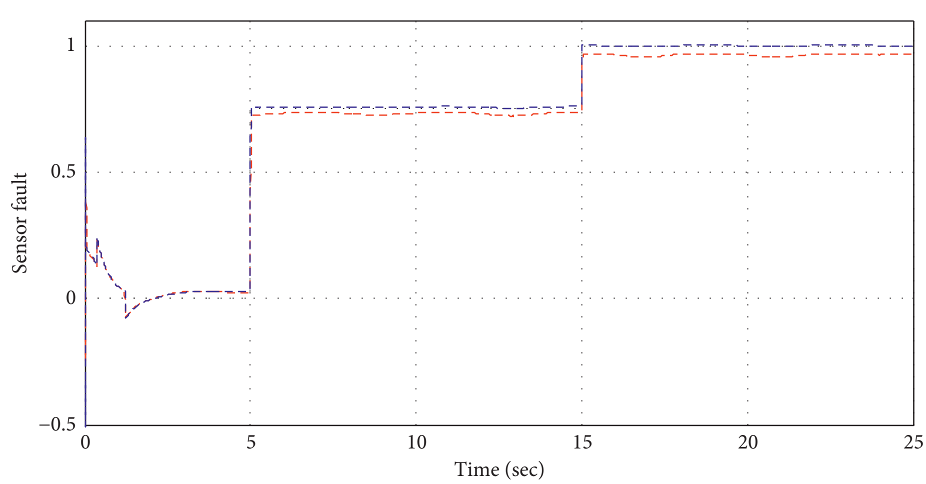
$$Gn2 = \begin{bmatrix} 0 & -1 & 0 \\ -1.0607 & -0.7659 & 0 \\ 0 & 0.3357 & 0 \\ 1 & -0.3357 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

According to Theorem 3, we obtain the H_∞ performance level, $\mu_c = 1.0746$, and the following controller gain matrices:



Real actuator fault
Estimated actuator fault
Estimated actuator fault with adaptive law

FIGURE 1: The actuator fault f_a and its estimation.



Real sensor fault
Estimated sensor fault
Estimated sensor fault with adaptive law

FIGURE 2: The sensor fault f_s and its estimation.

$$X = \begin{bmatrix} 6.5946 & -22.8284 & -1.4260 & -0.3703 \\ -22.8284 & 89.0575 & 6.3415 & -10.5593 \\ -1.4260 & 6.3415 & 7.6086 & -7.8653 \\ -0.3703 & -10.5593 & -7.8653 & 21.3122 \end{bmatrix},$$

$$Y = \begin{bmatrix} 106.4288 & -22.7293 & -12.3762 & -22.2923 \\ -22.7293 & 8.6729 & 4.1400 & 7.7688 \\ -12.3762 & 4.1400 & 83.7854 & 6.0534 \\ -22.2923 & 7.7688 & 6.0534 & 20.5865 \end{bmatrix},$$

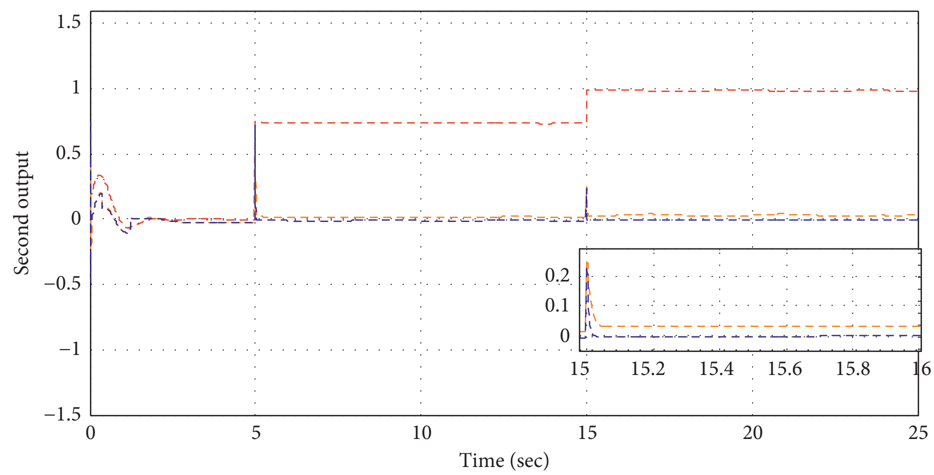


FIGURE 3: Second closed-loop system output response: output response with free faults (black line), output response without FTC (red line), output response with FTC without adaptive law (orange line), and output response with the proposed FTC (blue line).

$$
\begin{gathered}
A_{c11}=\begin{bmatrix} -4.8054 & -4.6433 & -29.6951 & 186.8313 \\ 0.1944 & -1.9736 & -6.0099 & 13.6829 \\ -1.0255 & -3.2484 & -24.8435 & 194.3215 \\ -0.2002 & 0.3665 & 2.2852 & -20.0596 \end{bmatrix}, \\
A_{c12}=\begin{bmatrix} -4.8921 & -4.8143 & -31.5827 & 397.3524 \\ 0.1805 & -1.9966 & -6.2623 & 44.6510 \\ -1.1243 & -3.4524 & -27.0566 & 463.3002 \\ -0.1881 & 0.3910 & 2.5087 & -56.4758 \end{bmatrix}, \\
A_{c21}=\begin{bmatrix} -3.7054 & -1.8250 & -9.8471 & 15.8856 \\ 0.1188 & -1.8469 & -6.2064 & 28.5907 \\ -0.7277 & -1.1952 & -15.1229 & 198.4365 \\ -0.3068 & -0.0084 & -0.1040 & -1.6435 \end{bmatrix}, \\
A_{c22}=\begin{bmatrix} -3.6976 & -1.8083 & -9.6846 & -4.7204 \\ 0.1008 & -1.8815 & -6.5937 & 79.3446 \\ -0.8225 & -1.3877 & -17.2258 & 451.0355 \\ -0.3028 & -0.0007 & -0.0702 & -11.9604 \end{bmatrix}, \\
B_{c1}=\begin{bmatrix} -152.1400 & 0.0680 & -193.0902 \\ -14.6685 & 7.4849 & -16.7177 \\ -180.1739 & -8.0082 & -296.1359 \\ -46.7432 & 0.7054 & 30.6986 \end{bmatrix}, \\
B_{c2}=\begin{bmatrix} 13.0136 & 6.1728 & 18.4474 \\ -30.5119 & -3.3066 & -14.3595 \\ -153.5522 & -52.0403 & -169.8636 \\ -61.5354 & 1.8005 & 3.2691 \end{bmatrix},
\end{gathered}
\tag{87}
$$

$C_{c1} = [\,0.0656\ \ 0.2233\ \ 1.5563\ \ -14.3353\,]$, $C_{c2} = [0.0547\ \ 0.0863\ 1.0121\ -32.8360]$, $D_{c1} = [12.9881\ 0.3879\ 16.8240]$, and $D_{c2} = [\,11.4862\ \ 3.7856\ \ 7.6871\,]$. In the corresponding simulation, the parameters associated with the equivalent output error injection ϑ_{eq} have been chosen to be $\rho_1 = 10$, $\rho_2 = 15$, $\rho_0 = 5$, $\overline{\rho} = 0.1$, and $\delta = 0.01$ and initial conditions $x_{10} = \pi/20$, $x_{20} = 0$, $x_{30} = 2$, and $x_{40} = 0$. The considered actuator and sensor faults have, respectively,

$$f_a = \begin{cases} 0, & t < 2, \\ 0.5\ \sin(0.25t), & 2 \le t < 10, \\ 0.5\ \sin(0.75t), & t \ge 10, \end{cases}$$
$$f_s = \begin{cases} 0, & t < 5, \\ 0.75, & 5 \le t < 15, \\ 1, & t \ge 15. \end{cases} \tag{88}$$

And the external disturbances d are supposed to be random noises from −0.1 and 0.1. The simulation results are provided with online simultaneous actuator and sensor faults' injection. Figures 1 and 2 indicate that the adaptive SMO can estimate existing faults simultaneously with satisfactory precision by rejecting the effects of disturbances.

Simulation result for the output response y_2 (considered faulty) is provided in Figure 3. It is observed that the output without FTC does not converge to the output of the fault-free model (i.e., without any fault). However, the output trajectory of y_2 with FTC reaches the output of the nominal model. Therefore, the proposed fuzzy DOFFTC design achieves the performance under faults and disturbances, and the stability of the closed-loop system is guaranteed while satisfying the prescribed $\mathscr{H}_\infty$ performance.

7. Conclusion

This paper focuses on the problems of FE and FTC for T-S fuzzy systems with unmeasurable PVs and having external disturbances, actuator, and sensor faults. An adaptive fuzzy SMO is designed for estimating the state, actuator, and sensor faults, simultaneously. Using the FE scheme, a DOFFTC is designed to compensate the faults and to stabilize the closed-loop system. Finally, simulation results show the effectiveness of the proposed methods.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Research Article

Adaptive Proportional Integral Observer Design for Interval Type 2 Takagi–Sugeno Fuzzy Systems

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In this paper, the problem of fault estimation in systems described by Takagi–Sugeno fuzzy systems is studied. A proportional integral observer is conceived in order to reconstruct state and faults which can affect the studied system. Proportional integral observer can easily estimate actuator faults which are assimilated to be as unknown inputs. In order to estimate actuator and sensor faults, a mathematical transformation is used to conceive an augmented system, in which the initial sensor fault appears as an unknown input. Considering the augmented state, it is possible to conceive an adaptive observer which is able to estimate the whole state and faults. The noise effect on the state and fault estimation is also minimized in this study, which provides some robustness properties to the proposed observer. The proportional integral observer is conceived for nonlinear systems described by Takagi–Sugeno fuzzy models.

1. Introduction

State estimation can have numerous applications in control and diagnosis. In often cases, the system state is globally or partially unknown, so its estimation can be a solution.

Generally, the process is affected by disturbances, measurement uncertainties, and sensor and/or actuator faults. Disturbances and faults are usually considered as unknown inputs which can have a random behavior in time, and they can have harmful effects on the process. Observers with unknown inputs were the subject of many works [1–3]. Indeed, methods of simultaneous estimation of the unknown inputs and the system state were proposed in [3]. In [1], authors present a method of simultaneous estimations of system state and unknown inputs and outputs, and it is considered that some outputs are not accessible to measure. In [2], a comparison study is proposed between sliding mode observers and unknown inputs' observers in the context of fault estimation, but only the actuator fault is considered.

Takagi–Sugeno fuzzy systems, named also multiple models [4], are an efficient approach to handle complex nonlinear systems [5, 6]. They are composed of a set of linear models weighted by nonlinear activation functions verifying the convex-sum property [6, 7]. Using the same activation functions, nonlinear observers can be designed to make the state estimation. These observers are called multiple observers [5, 8]. Indeed, works presented in [6, 7] can be considered as the first works regarding this kind of models. It is proved in these works that these models can approximate well the nonlinear system behavior. Works presented in [5, 8] are interested in some application of state and fault estimation using this kind of models.

Approaches using Takagi–Sugeno fuzzy models are the subject of numerous works [9–12], dealing with state estimation in the presence of unknown inputs or parameter uncertainties. In [9], authors propose to consider singulary perturbed Takagi–Sugeno models where the activation function is depended on unmeasurable variables such as the system state. In [10], a case of the switching system is considered as a particular form of Takagi–Sugeno models. In this kind of models, the activation function can be 0 or 1. In [11], authors are interested in Takagi–Sugeno–Kang models for online identification with application to crane systems. Sensor networks are considered in [12] more precisely in the case of nonfragile distributed filters with Takagi–Sugeno models. In this context, the problem of state and fault estimation is studied in [2, 4, 5, 8, 13]. In [4], a sliding mode observer with unknown input for the case of uncertain Takagi–Sugeno models is proposed. In [5], a method of state and unknown input estimation is presented for multiple models. In [8], a method of sensor faults' estimation is proposed for systems described by Takagi–Sugeno models. For linear systems, Edwards proposes, in [2], to use a mathematical transformation in order to conceive a new system where the sensor fault appears as an unknown input. This transformation is used next in [14] for the fault estimation in the context of linear systems.

It is possible to estimate simultaneously the system state and the fault affecting the system using the proportional integral observer. This kind of observer is composed of two estimators (proportional and integral) [4, 5, 8, 13]. In practice, the design of the proportional integral observer is reduced to the computation of the two gains (proportional and integral) where the proportional term lets to estimate the system state and the integral term permits to estimate the fault [4, 5, 8, 13]. Works presented in [4, 5, 8, 13] are interested in state and fault estimation in the context of Takagi–Sugeno systems using proportional integral observers which are composed of two estimators; the first one called proportional terms is used to estimate the system state and the second one called integral term allows estimating the fault affecting the system. Some academic and real applications are given like the application to the model of turbo-reactor presented in [4].

Takagi–Sugeno fuzzy models can be of type 1 or 2 [15–18]. A type 2 fuzzy set uses upper and lower primary membership functions and a secondary membership function [15–18]. Contrary to a type 1 fuzzy set which has only one primary membership function, by consequent, a type 2 fuzzy set is more able to handle uncertainties and ambiguities. Type 2 fuzzy sets were proposed by Zadeh in 1975, but there were not many researchers interested in them until these last years. Some researchers started to consider type 2 fuzzy systems in the past several years due to their relative novelty [15–18]. Works presented in [15–18] are interested in type 2 Takagi–Sugeno fuzzy models. Indeed, it is shown that this type can be used to reduce the system complexity and the number of local models comparing with type 1. The main difference between type 1 and type 2 is in the form of activation functions. Indeed, activation functions for type 1 are characterized by a real term for each time. For type 2, in each time, the activation functions are characterized by fuzzy sets which are defined often in cases by their upper and lower bounds which is why they are called type 2 interval Takagi–Sugeno fuzzy systems. For each time, activation function is varying between upper and lower bounds.

Many other works focus on the state and fault estimation in several contexts. Let us cite briefly some of them and give the difference between them and the present work. In [19, 20], authors are interested on multiagent systems which are modeled by several agents where each agent presents a nonlinearity which is different from the principle of the Takagi–Sugeno models where the local models are linear and the nonlinearity is given by the activation functions. The obtained results in these works are important but the main difference is in the used model. Multiagent systems are also considered for fault estimation in [21] by considering distributed ℓ_1 state and fault estimation. Asymptotic fault and state estimation is proposed in [22] in the context of nonlinear systems which is different from this work where systems are modeled by Takagi–Sugeno models. The same work is extended to conceive fault tolerant control. In [23], Lipschitz condition is assumed for the state and fault estimation in the context of Takagi–Sugeno models, and these conditions are not considered in this work. In [24], only actuator fault is estimated simultaneously with the state estimate in the context of interval Takagi–Sugeno systems contrary with this work where both actuators and sensors' faults are considered. The Takagi–Sugeno discrete model is considered in [25–27], but in this work, we focus on continuous models. In [28], Takagi–Sugeno models with delay assumptions are considered; in this work, we do not consider delay. The application of fault estimation to the fault tolerant control is given in [29, 30]. Ellipsoidal bounding conditions are assumed in [31].

The main contribution in this work is to extend the method of simultaneous estimation of the system state and actuator and sensor faults developed in the context of type 1 Takagi–Sugeno fussy systems for the case of type 2 Takagi–Sugeno fuzzy models. Indeed, the structure of type 2 Takagi–Sugeno fuzzy systems based on varying activation functions lets the extension of obtained results in the case of type 1 Takagi–Sugeno systems not evident since type 2 models are based on double fuzzy sets which are the model and the activation functions, which make the presented work more important. An adaptive proportional integral observer is proposed and used to assure this estimation. Classically, this observer is used to estimate system state and unknown inputs. This paper shows that it is possible to adapt this observer to estimate, simultaneously, the system state, the actuator, and the sensor faults. Indeed, the mathematical transformation proposed in [2], for the case of linear systems and extended to the case of systems described by Takagi–Sugeno models in [4, 8, 13], is adapted to interval type 2 Takagi–Sugeno models in this paper. Based on the adapted form of this transformation, an augmented system state is obtained. This augmented system presents a generalized unknown input which contains the initial sensor and actuator faults. At this level, a proportional integral observer able to estimate the augmented state and the generalized unknown input is proposed. The estimation of the initial sensor and actuator faults is reduced to the estimation of the generalized unknown input. This work presents the three possible cases of faults' estimation:

(i) State and actuator fault estimation

(ii) State and sensor fault estimation

(iii) Simultaneous estimation of state, actuator, and sensor faults

The paper is organized as follows. Section 2 recalls the principle of Takagi–Sugeno multiple models type 1 and type 2. Section 3 describes the design of the proportional integral observer for the state and actuator faults' estimation. This observer is adapted to sensor faults' estimation in Section 4. Section 5 proposes a method to estimate simultaneously system state, sensor, and actuator faults. An example of simulation, showing the quality of estimation, is given in Section 7.

2. Takagi–Sugeno Fuzzy Systems

2.1. Elementary Background on Type 1 Takagi–Sugeno Fuzzy Systems. Takagi–Sugeno fuzzy systems are an appropriate tool which permits to model large class of complex and nonlinear systems with a mathematical model which can be used for analysis [32, 33], control [34, 35], and observer design [1, 8, 13, 36]. This approach is based on a decomposition of the system operating space into a finite number of operating zones. Hence, a simple linear model describes the system dynamic behavior inside each operating zone. The contribution of each submodel in the global model is quantified using a nonlinear weighting function which can have various structures. The submodels are associated in the state equation using a common state vector. This model has been proposed, in a fuzzy modeling framework, by Takagi and Sugeno [7].

Takagi–Sugeno fuzzy systems are based on the assumption that each nonlinear dynamic system can be simply described as the fuzzy fusion of many linear models, where each linear model represents the local system behavior around an operating point. A Takagi–Sugeno model is described by fuzzy IF-THEN rules which represent local linear inputs/outputs' relations of the nonlinear system. It has a rule base of M rules, each having p antecedents, where the i^{th} rule is expressed as follows:

$$R^i\text{: IF } \xi_1 \text{ is } F_1^i \text{ and} \dots \text{and } \xi_p \text{ is } F_p^i, \tag{1}$$

where $i \in \{1, \dots, M\}, F_j^i \, (j \in \{1, \dots, p\})$ are fuzzy sets and $\xi = [\xi_1, \xi_2, \dots, \xi_p]$ is a known vector of premise variables [5] which may depend on the state, the input, or the output. Variable ξ is called the decision variable.

The global Takagi–Sugeno fuzzy model is given by the aggregation of the submodels using the weighting functions as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t)) \left(A_i x(t) + B_i u(t)\right), \\ y(t) = \sum_{i=1}^{M} \mu_i(\xi(t)) C_i x(t), \end{cases} \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^r$ is the control vector, $y(t) \in \mathbb{R}^m$ is the vector of measures, and A_i, B_i, and C_i are known constant matrices with appropriate dimensions.

The weighting functions $\mu_i(\xi(t))$ assure a progressive passage between the local models and verifies the property of the convex sum: $\sum_{i=1}^{M} \mu_i(\xi(t)) = 1, \forall t$ and $0 \le \mu_i(\xi(t)) \le 1, \forall i = 1, \dots, M, \forall t$.

If, in the equation of the output, it is supposed that $C_1 = C_2 = \cdots = C_M = C$, the output of the multiple model (2) is reduced to $y(t) = Cx(t)$, and the multiple model state equation becomes

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t)) \left(A_i x(t) + B_i u(t)\right), \\ y(t) = Cx(t). \end{cases} \tag{3}$$

2.2. Type 2 Takagi–Sugeno Fuzzy Systems. Interval type 2 Takagi–Sugeno fuzzy models are nonlinear systems with M rules, where the Rule R^i is as follows.

IF $f_1(\xi(t))$ is $\widetilde{F}_1^i$ AND ...AND $f_p(\xi(t))$ is $\widetilde{F}_p^i$ THEN

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t), \\ y = C_i x(t), \end{cases} \tag{4}$$

where $f_j(\xi(t))$ is the premise variable depending on a known decision variable ξ and $\widetilde{F}_i^j$ is an interval type 2 fuzzy set, for $i \in \{1, 2, \dots, M\}$ and $j \in \{1, 2, \dots, p\}$, p is a positive integer, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^q$ in the system input, $y(t) \in \mathbb{R}^l$ is the system output, and A_i, B_i, and C_i are known matrices with appropriate dimensions.

The i^{th} fuzzy rule can be described by the interval sets: $\mu_i(\xi(t)) = [\underline{\mu}_i(\xi(t)), \overline{\mu}_i(\xi(t))]$, $i \in \{1, 2, \dots, M\}$, where $\underline{\mu}_i(\xi(t)) = \prod_{j=1}^{M} \underline{\mu}_{\widetilde{F}_j^i}(f_i(\xi(t))) \ge 0$ and $\overline{\mu}_i(\xi(t)) = \prod_{i=1}^{M} \overline{\mu}_{\widetilde{F}_j^i}(f_i(\xi(t))) \ge 0$ are the lower and upper grades of membership, respectively. $\overline{\mu}_{\widetilde{F}_j^i}(f_i(\xi(t))) \ge 0$ and $\underline{\mu}_{\widetilde{F}_j^i}(f_i(\xi(t))) \ge 0$ denote the lower and upper weighting functions, respectively. Therefore, it is assumed that $\overline{\mu}_{\widetilde{F}_j^i}(f_i(\xi(t))) \ge \underline{\mu}_{\widetilde{F}_j^i}(f_i(\xi(t))) \ge 0$ and $\underline{\mu}_i(\xi(t)) \ge \overline{\mu}_i(\xi(t))$ for all i.

The interval type 2 Takagi–Sugeno fuzzy system is given by the following state equations:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(x(t)) \left(A_i x(t) + B_i u(t)\right), \\ y(t) = \sum_{i=1}^{M} \mu_i(x(t)) C_i x(t), \end{cases} \tag{5}$$

with

$$\mu_i(\xi(t)) = \frac{\omega_i(\xi(t))}{\sum_{i=1}^{M} \omega_i(\xi(t))}, \qquad \sum_{i=1}^{M} \mu_i(\xi(t)) = 1, \tag{6}$$

where

$$\omega_i(\xi(t)) = \underline{\vartheta}_i(\xi(t)) \underline{\mu}_i(\xi(t)) + \overline{\vartheta}_i(\xi(t)) \overline{\mu}_i(\xi(t)) \ge 0, \quad \forall i. \tag{7}$$

The nonlinear functions $\underline{\vartheta}_i(\xi(t))$ et $\overline{\vartheta}_i(\xi(t))$ must verify the following conditions:

$$0 \le \overline{\vartheta}_i(\xi(t)) \le 1, \tag{8}$$

$$0 \le \underline{\vartheta}_i(\xi(t)) \le 1, \tag{9}$$

$$\overline{\vartheta}_i(\xi(t)) + \underline{\vartheta}_i(\xi(t)) = 1. \tag{10}$$

To summarize, it is possible to write an interval type 2 Takagi–Sugeno system in the following form:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(x(t))\left(A_i x(t) + B_i u(t)\right), \\ y(t) = \sum_{i=1}^{M} \mu_i(x(t)) C_i x(t), \end{cases} \tag{11}$$

where $\sum_{i=1}^{M} \mu_i(\xi(t)) = 1$, $\forall t$ and $0 \le \mu_i(\xi(t)) \le 1$, $\forall t, \forall i = 1, \ldots, M$ and

$$\mu_i(\xi(t)) = \frac{\overline{\mu}_i(\xi(t))\overline{\vartheta}_i(\xi(t)) + \underline{\mu}_i(\xi(t))\underline{\vartheta}_i(\xi(t))}{\sum_{i=1}^{M}\left(\overline{\mu}_i(\xi(t))\overline{\vartheta}_i(\xi(t)) + \underline{\mu}_i(\xi(t))\underline{\vartheta}_i(\xi(t))\right)}. \tag{12}$$

3. Actuator Faults' Estimation

The objective of this part is to conceive a proportional integral observer able to estimate actuator faults affecting nonlinear systems represented by interval type 2 Takagi–Sugeno models.

Consider the following interval type 2 Takagi–Sugeno fuzzy system affected by an actuator fault and a measurement noise:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))\left(A_i x(t) + B_i u(t) + E_i f_a(t)\right), \\ y(t) = Cx(t) + Dw(t), \end{cases} \tag{13}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $y(t) \in \mathbb{R}^m$ is the measured output, $u(t) \in \mathbb{R}^r$ is the input, $f_a(t)$ is the actuator fault which is assumed to be bounded, and $w(t)$ is the measurement noise. A_i, B_i, and C are known constant matrices with appropriate dimensions. E_i and D are, respectively, the fault and noise distribution matrices which are assumed to be known. The scalar M represents the number of the local models. $\mu_i(\xi(t))$ are the activation functions verifying equation (12).

The structure of the proportional integral observer is chosen as follows:

$$\begin{cases} \dot{\widehat{x}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))\left(A_i\widehat{x}(t) + B_i u(t) + E_i\widehat{f}_a(t) + K_i\widetilde{y}(t)\right), \\ \dot{\widehat{f}}_a(t) = \sum_{i=1}^{M} \mu_i(\xi(t)) L_i \widetilde{y}(t), \\ \widehat{y}(t) = C\widehat{x}(t), \end{cases} \tag{14}$$

where $\widehat{x}(t) \in \mathbb{R}^n$ is the estimated system state, $\widehat{f}_a(t)$ represents the estimated fault, $\widehat{y}(t) \in \mathbb{R}^m$ is the estimated output, $\widetilde{y}(t) = y(t) - \widehat{y}(t)$, K_i are the local proportional observer gains, and L_i are the local integral gains to be computed.

Let us define the state estimation error $\widetilde{x}(t)$ and the fault estimation error $\widetilde{f}_a(t)$. They are given by the following equalities:

$$\widetilde{x}(t) = x(t) - \widehat{x}(t), \tag{15}$$

$$\widetilde{f}_a(t) = f_a(t) - \widehat{f}_a(t). \tag{16}$$

The dynamics of the state estimation error is given by the computation of $\dot{\widetilde{x}}(t)$ which is written as follows:

$$\dot{\widetilde{x}}(t) = \dot{x}(t) - \dot{\widehat{x}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))\left(A_i - K_i C\widetilde{x}(t) + E_i\widetilde{f}_a(t) + K_i Dw(t)\right). \tag{17}$$

The dynamics of the fault estimation error is given by the expression of $\dot{\widetilde{f_a}}(t)$ written below:

$$\dot{\widetilde{f_a}}(t) = \dot{f_a}(t) - \dot{\widehat{f_a}}(t) = \dot{f_a}(t) - \sum_{i=1}^{M} \mu_i(\xi(t))\left(L_i C\widetilde{x}(t) - L_i Dw(t)\right). \tag{18}$$

The following matrices are introduced:

$$\varphi_a = \begin{bmatrix} \tilde{x}(t) \\ \tilde{f}_a(t) \end{bmatrix},$$
$$\varepsilon_a = \begin{bmatrix} w(t) \\ \dot{f}_a(t) \end{bmatrix}. \tag{19}$$

Equations (17) and (18) can be rewritten as follows:

$$\dot{\varphi}_a = A_{\mathrm{ma}}\varphi_a + B_{\mathrm{ma}}\varepsilon_a, \tag{20}$$

with

$$A_{\mathrm{ma}} = \sum_{i=1}^{M} \mu_i(\xi(t))A_{ai},$$
$$B_{\mathrm{ma}} = \sum_{i=1}^{M} \mu_i(\xi(t))B_{ai}, \tag{21}$$

where

$$A_{ai} = \begin{bmatrix} A_i - K_iC & E_i \\ -L_iC & 0 \end{bmatrix},$$
$$B_{ai} = \begin{bmatrix} -K_iD & 0 \\ -L_iD & I \end{bmatrix}. \tag{22}$$

The matrix I is the identity matrix with appropriate dimensions.

In order to analyse the convergence of the generalized estimation error $\varphi_a(t)$, the quadratic Lyapunov candidate function $V_a(t) = \varphi_a(t)^T P\varphi_a(t)$ is considered, where P denotes a symmetric definite positive matrix.

The problem of robust state and fault estimation is reduced to find the gains K_i and L_i of the observer to ensure an asymptotic convergence of $\varphi_a(t)$ toward zero if $\varepsilon_a(t) = 0$ and to ensure a bounded error in the case, where $\varepsilon_a(t) \neq 0$, i.e.,

$$\lim_{t\longrightarrow\infty} \varphi_a(t) = 0, \qquad \text{for } \varepsilon_a(t) = 0,$$
$$\left\|\varphi_a(t)\right\|_{Q_\varphi} \leq \lambda\left\|\varepsilon_a(t)\right\|_{Q_\varepsilon}, \qquad \text{for } \varepsilon_a(t) \neq 0 \text{ and } e(0) = 0, \tag{23}$$

where $\lambda > 0$ is the attenuation level.

To satisfy constraints (23), it is sufficient to find a Lyapunov function $V_a(t)$ such that

$$\dot{V}_a(t) + \varphi_a^T Q_\varphi \varphi_a - \lambda^2 \varepsilon_a^T Q_\varepsilon \varepsilon_a < 0, \tag{24}$$

where Q_φ and Q_ε are two positive definite matrices.

In order to simplify the notations, the time index (t) will be omitted henceforth.

Inequality (24) can also be written as follows:

$$\psi_a^T \Omega_a \psi_a < 0, \tag{25}$$

with

$$\psi_a = \begin{bmatrix} \varphi_a \\ \varepsilon_a \end{bmatrix},$$
$$\Omega_a = \begin{bmatrix} A_{\mathrm{ma}}^T P + PA_{\mathrm{ma}} + Q_\varphi & PB_{\mathrm{ma}} \\ B_{\mathrm{ma}}^T P & -\lambda^2 Q_\varepsilon \end{bmatrix}. \tag{26}$$

Inequality (25) has a quadratic form, and it holds iff $\Omega_a < 0$.

The matrices A_{ma} and B_{ma} can be written as

$$A_{\mathrm{ma}} = \widetilde{A}_{\mathrm{ma}} - \widetilde{K}_{\mathrm{ma}}\widetilde{C},$$
$$B_{\mathrm{ma}} = \widetilde{I} - \widetilde{K}_{\mathrm{ma}}\widetilde{D}, \tag{27}$$

where

$$\widetilde{A}_{\mathrm{ma}} = \sum_{i=1}^{M} \mu_i(\xi(t))\widetilde{A}_{\mathrm{ma}},$$
$$\widetilde{K}_{\mathrm{ma}} = \sum_{i=1}^{M} \mu_i(\xi(t))\widetilde{K}_{\mathrm{ma}}, \tag{28}$$

with

$$\widetilde{K}_{\mathrm{mai}} = \begin{bmatrix} K_i \\ L_i \end{bmatrix},$$
$$\widetilde{A}_{\mathrm{mai}} = \begin{bmatrix} A_i & E_i \\ 0 & 0 \end{bmatrix},$$
$$\widetilde{I} = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix},$$
$$\widetilde{C} = \begin{bmatrix} C & 0 \end{bmatrix},$$
$$\widetilde{D} = \begin{bmatrix} D & 0 \end{bmatrix}. \tag{29}$$

With the changes of variables $G_{\mathrm{ma}} = P\widetilde{K}_{\mathrm{ma}}$ and $\overline{\lambda} = \lambda^2$, the matrix Ω_a can be put as follows:

$$\Omega_a = \begin{bmatrix} \vartheta_a & -G_{\mathrm{ma}}\widetilde{D} + P\widetilde{I} \\ \widetilde{I}^T P - \widetilde{D}^T G_{\mathrm{ma}}^T & -\overline{\lambda}Q_\varepsilon \end{bmatrix}, \tag{30}$$

where $\vartheta_a = P\widetilde{A}_{\mathrm{ma}} + \widetilde{A}_{\mathrm{ma}}^T P - G_{\mathrm{ma}}\widetilde{C} - \widetilde{C}^T G_{\mathrm{ma}}^T + Q_\varphi$.

As $\Omega_a = \sum_{i=1}^{M} \mu_i(\xi(t))\Omega_{ai}$, the negativity of Ω is assured if, for $i = 1, \ldots, M$,

$$\Omega_{\mathrm{ai}} < 0, \tag{31}$$

with

$$\Omega_{\mathrm{ai}} = \begin{bmatrix} \vartheta_{\mathrm{ai}} & -G_{\mathrm{ai}}\widetilde{D} + P\widetilde{I} \\ \widetilde{I}^T P - \widetilde{D}^T G_{\mathrm{ai}}^T & -\overline{\lambda}Q_\varepsilon \end{bmatrix}, \tag{32}$$

where $\vartheta_{\mathrm{ai}} = P\widetilde{A}_{\mathrm{mai}} + \widetilde{A}_{\mathrm{mai}}^T P - G_{\mathrm{ai}}\widetilde{C} - \widetilde{C}^T G_{\mathrm{ai}}^T + Q_\varphi$ and $G_{\mathrm{ai}} = P\widetilde{K}_{\mathrm{mai}}$.

The resolution of LMI (31) leads to the determination of the matrices P and G_{ai} and the scalar $\overline{\lambda}$. The gain matrices are then deduced by the equation $\widetilde{K}_{\mathrm{mai}} = P^{-1}G_{\mathrm{ai}}$.

The observer design is summarized by the following theorem.

Theorem 1. *System (20) describing the time evolution of the state estimation error $\tilde{x}$ and the fault estimation error $\widetilde{f_a}$ is stable and the $\mathscr{L}_2$-gain of the transfer from $\varepsilon_a(t)$ to $\varphi_a(t)$ is bounded if there exists a symmetric, positive definite matrix P, gain matrices G_{ai}, $i \in \{1, \ldots, M\}$, and a positive scalar $\overline{\lambda}$ such that the following LMI are verified:*

$$\begin{bmatrix} \vartheta_{\mathrm{ai}} & -G_{\mathrm{ai}}\widetilde{D} + P\widetilde{I} \\ \widetilde{I}^T P - \widetilde{D}^T G_{\mathrm{ai}}^T & -\overline{\lambda} Q_\varepsilon \end{bmatrix} < 0, \quad i \in \{1, \ldots, M\}, \tag{33}$$

where $\vartheta_{ai} = P\widetilde{A}_{mai} + \widetilde{A}_{mai}^T P - G_{ai}\widetilde{C} - \widetilde{C}^T G_{ai}^T + Q_\varphi$. *The observer gains (proportional and integral gains) are computed using* $\widetilde{K}_{mai} = P^{-1}G_{ai}$ *and the attenuation level is given by* $\lambda = \sqrt{\overline{\lambda}}$.

4. Sensor Faults' Estimation

The objective of this part is to adapt the proportional integral observer proposed in Section 3 to estimate sensor faults affecting the nonlinear system described by interval type 2 Takagi–Sugeno fuzzy model.

Let us consider the following interval type 2 Takagi–Sugeno system affected by sensor fault $f_s(t)$ and measurement noise $w(t)$:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))\left(A_i x(t) + B_i u(t)\right), \\ y(t) = Cx(t) + Ff_s(t) + Dw(t), \end{cases} \tag{34}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $y(t) \in \mathbb{R}^m$ is the measured output, $u(t) \in \mathbb{R}^r$ is the system input, A_i, B_i, and C are known constant matrices with appropriate dimensions, F and D are, respectively, the fault and noise distribution matrices which are assumed to be known, the scalar M is the number of local models, and $\mu_i(\xi(t))$ are the activation functions verifying equation (12).

Consider the state $z(t) \in \mathbb{R}^p$ [8, 13] given by

$$\dot{z}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))\left(-\overline{A}_i z(t) + \overline{A}_i C x(t) + \overline{A}_i F f_s(t)\right), \tag{35}$$

where $-\overline{A}_i$, $i \in \{1, \ldots, M\}$ are stables matrices.

Remark 1. The introduced new state $z(t)$ has the form of a particular filter for the output of the system; it was initially extended to the context of Takagi–Sugeno models in [8], and it was used in [13]. The main advantage of this new state is to conceive an augmented system where all the faults affecting the initial system (actuator and sensor faults) appear as unknown inputs which let possible to use an augmented proportional integral observer to estimate this unknown input considered as actuator faults. The use of this state is important because the classic proportional integral observer allows only estimating actuator faults which let the impossible to estimate the sensor fault based on the classical proportional observer.

The augmented state $\mathbf{x}(t) = \left[\, x^T(t) \;\; z^T(t) \,\right]^T$ is introduced. It is given by equation (36):

$$\begin{cases} \dot{\mathbf{x}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))\left(A_{gi}\mathbf{x}(t) + B_{gi}u(t) + E_{gi}f_s(t)\right), \\ \mathbf{y}(t) = C_g\mathbf{x}(t) + D_g w(t), \end{cases} \tag{36}$$

with

$$\begin{aligned} A_{gi} &= \begin{bmatrix} A_i & 0 \\ \overline{A}_i C & -\overline{A}_i \end{bmatrix}, \\ B_{gi} &= \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \\ E_{gi} &= \begin{bmatrix} 0 \\ \overline{A}_i F \end{bmatrix}, \\ C_g &= \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix}, \\ D_g &= \begin{bmatrix} D \\ 0 \end{bmatrix}. \end{aligned} \tag{37}$$

A proportional integral observer is able to estimate simultaneously the augmented state $\mathbf{x}(t)$ and the sensor fault $f_s(t)$ is chosen as follows:

$$\begin{cases} \dot{\widehat{\mathbf{x}}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))\left(A_{gi}\widehat{\mathbf{x}}(t) + B_{gi}u(t) + E_{gi}\widehat{f}_s(t) + K_i\widetilde{\mathbf{y}}(t)\right), \\ \dot{\widehat{f}}_s(t) = \sum_{i=1}^{M} \mu_i(\xi(t))\left(L_i\widetilde{\mathbf{y}}(t)\right), \\ \widehat{\mathbf{y}}(t) = C_g\widehat{\mathbf{x}}(t), \end{cases} \tag{38}$$

where $\widehat{\mathbf{x}}(t)$ is the estimated system state, $\widehat{f}_s(t)$ is the estimated sensor fault, $\widehat{\mathbf{y}}(t)$ is the estimated output, $\widetilde{\mathbf{y}}(t) = \mathbf{y}(t) - \widehat{\mathbf{y}}(t)$, K_i are the local proportional observer gains, and L_i are the local integral observer gains to be computed. It is assumed that $f_s(t)$ is bounded.

The augmented state estimation error $\widetilde{\mathbf{x}}(t)$ and the fault estimation error $\widetilde{f}_s(t)$ are defined as follows:

$$\widetilde{\mathbf{x}}(t) = \mathbf{x}(t) - \widehat{\mathbf{x}}(t), \tag{39}$$

$$\widetilde{f}_s(t) = f_s(t) - \widehat{f}_s(t). \tag{40}$$

The dynamics of the augmented state reconstruction error is given by the computation of $\dot{\widetilde{\mathbf{x}}}(t) = \dot{\mathbf{x}}(t) - \dot{\widehat{\mathbf{x}}}(t)$ which is written as follows:

$$\dot{\widetilde{\mathbf{x}}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))\left(\left(A_{ai} - K_i C_a\right)\widetilde{\mathbf{x}}(t) + E_{ai}\widetilde{f}_s(t) + K_i D_a w(t)\right). \tag{41}$$

The dynamic of the sensor fault estimation error can be computed as follows:

$$\dot{\widetilde{f}}_s(t) = \dot{f}_s(t) - \sum_{i=1}^{M} \mu_i(\xi(t))\left(L_i C_a\widetilde{\mathbf{x}}(t) - L_i D_a w(t)\right). \tag{42}$$

The following matrices are introduced:

$$\varphi_s = \begin{bmatrix} \mathbf{x}(t) \\ \tilde{f}_s(t) \end{bmatrix}, \quad \varepsilon_s = \begin{bmatrix} w(t) \\ \dot{f}_s(t) \end{bmatrix}. \tag{43}$$

By omitting to denote the dependence with regard to the time t, equations (41) and (42) can be rewritten as follows:

$$\dot{\varphi}_s = A_{\text{ms}}\varphi_s + B_{\text{ms}}\varepsilon_s. \tag{44}$$

The matrices A_{ms} and B_{ms} have the following expressions:

$$A_{\text{ms}} = \sum_{i=1}^{M} \mu_i(\xi(t))\widetilde{A_{si}}, \quad B_{\text{ms}} = \sum_{i=1}^{M} \mu_i(\xi(t))\widetilde{B_{si}}, \tag{45}$$

where

$$\widetilde{A_{si}} = \begin{bmatrix} A_{ai} - K_iC_a & E_{ai} \\ -L_iC_a & 0 \end{bmatrix}, \quad \widetilde{B_{si}} = \begin{bmatrix} -K_iD_a & 0 \\ -L_iD_a & I \end{bmatrix}. \tag{46}$$

Using the Lyapunov function $V_s(t)$ given by $V_s(t) = \varphi_s(t)^T P\varphi_s(t)$ and following the same reasoning as for actuator faults' estimation, the convergence of state and fault estimation errors as well as the attenuation level is guaranteed if $\psi_s^T\Omega_s\psi_s < 0$ with

$$\psi_s = \begin{bmatrix} \varphi_s \\ \varepsilon_s \end{bmatrix}, \quad \Omega_s = \begin{bmatrix} A_{\text{mg}}^T P + \text{PA}_{\text{mg}} + Q_\varphi & \text{PB}_{\text{mg}} \\ B_{\text{mg}}^T P & -\lambda^2 Q_\varepsilon \end{bmatrix}. \tag{47}$$

The matrices A_{ms} and B_{ms} are written as $A_{\text{ms}} = \widetilde{A}_{\text{ms}} - \widetilde{K}_{\text{ms}}\widetilde{C}_g$ and $B_{\text{ms}} = \widetilde{I} - \widetilde{K}_{\text{ms}}\widetilde{D}_g$ with

$$\widetilde{A}_{\text{ms}} = \sum_{i=1}^{M} \mu_i(\xi(t))\widetilde{A}_{\text{msi}}, \quad \widetilde{K}_{\text{ms}} = \sum_{i=1}^{M} \mu_i(\xi(t))\widetilde{K}_{\text{msi}}, \tag{48}$$

where

$$\widetilde{A}_{\text{msi}} = \begin{bmatrix} A_{\text{gi}} & E_{\text{gi}} \\ 0 & 0 \end{bmatrix}, \quad \widetilde{K}_{\text{msi}} = \begin{bmatrix} K_i \\ L_i \end{bmatrix}, \quad \widetilde{I} = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}, \quad \widetilde{C}_g = \begin{bmatrix} C_g & 0 \end{bmatrix}, \quad \widetilde{D}_g = \begin{bmatrix} D_g & 0 \end{bmatrix}. \tag{49}$$

Using the changes of variables $G_{\text{ms}} = P\widetilde{K}_{\text{ms}}$ and $\bar{\lambda} = \lambda^2$ and choosing $Q_\varphi = Q_\varepsilon = I$, the matrix Ω_s can be put in the following form:

$$\Omega_s = \begin{bmatrix} \vartheta_s & P\widetilde{I} - G_{\text{ms}}\widetilde{D}_a \\ \widetilde{I}^T P - \widetilde{D}_a^T G_{\text{ms}}^T & -\bar{\lambda} I \end{bmatrix}, \tag{50}$$

with $\vartheta_s = P\widetilde{A}_{\text{ms}} + \widetilde{A}_{\text{ms}}^T P - G_{\text{ms}}\widetilde{C}_a - \widetilde{C}_a^T G_{\text{ms}}^T + I$.

As $\Omega_s = \sum_{i=1}^{M} \mu_i(\xi(t))\Omega_{\text{si}}$, the negativity of Ω_s is assured if, for $i \in \{1, \ldots, M\}$,

$$\Omega_{\text{si}} < 0, \tag{51}$$

with

$$\Omega_{\text{si}} = \begin{bmatrix} \vartheta_{\text{si}} & P\widetilde{I} - G_{\text{si}}\widetilde{D}_a \\ \widetilde{I}^T P - \widetilde{D}_a^T G_{\text{si}}^T & -\bar{\lambda} I \end{bmatrix}, \tag{52}$$

where $\vartheta_{\text{si}} = P\widetilde{A}_{\text{msi}} + \widetilde{A}_{\text{msi}}^T P - G_{\text{si}}\widetilde{C}_a - \widetilde{C}_a^T G_{\text{si}}^T + I$ and $G_{\text{si}} = P\widetilde{K}_{\text{msi}}$.

The resolution of LMI (51) leads to the determination of the matrices P and G_{si} and the scalar $\bar{\lambda}$. The gain matrices are then deduced from the equation $\widetilde{K}_{\text{msi}} = P^{-1}G_{\text{si}}$. The observer design is summarized by the following theorem:

Theorem 2. *System (44) describing the time evolution of the state estimation error $\tilde{x}$ and the fault estimation error $\tilde{f}$ is stable and the $\mathscr{L}_2$-gain of the transfer from $\varepsilon_g(t)$ to $\varphi_g(t)$ is bounded if there exists a symmetric, positive definite matrix P, gain matrices G_{gi}, $i \in \{1, \ldots, M\}$, and a positive scalar m such that the following LMI is verified:*

$$\begin{bmatrix} \vartheta_{\text{si}} & P\widetilde{I} - G_{\text{gi}}\widetilde{D}_a \\ \widetilde{I}^T P - \widetilde{D}_a^T G_{\text{gi}}^T & -\bar{\lambda} I \end{bmatrix} < 0, \quad i \in \{1, \ldots, M\}, \tag{53}$$

with $\vartheta_{si} = P\widetilde{A}_{mgi} + \widetilde{A}_{mgi}^T P - G_{gi}\widetilde{C}_a - \widetilde{C}_a^T G_{gi}^T + I$. The observer gains (proportional and integral gains) are computed using the equation $\widetilde{K}_{mgi} = P^{-1}G_{gi}$, and the attenuation level is given by $\lambda = \sqrt{\bar{\lambda}}$.

The main advantage of the proposed method is to estimate the sensor fault using a proportional integral observer. The use of the mathematical transformation (35) lets to estimate it since it appears as an actuator fault in the augmented system.

5. Actuator and Sensor Faults' Estimation

The objective of this part is to conceive a proportional integral observer which is able to estimate simultaneously actuator and sensor faults affecting the nonlinear system represented by interval type 2 Takagi–Sugeno model.

Let us consider the following nonlinear interval type 2 Takagi–Sugeno system affected by a sensor fault $f_s(t)$, actuator fault $f_a(t)$, and a measurement noise $w(t)$:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))\left(A_i x(t) + B_i u(t) + E_i f_s(t)\right), \\ y(t) = Cx(t) + Ff_s(t) + Dw(t), \end{cases} \tag{54}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $y(t) \in \mathbb{R}^m$ is the measured output, $u(t) \in \mathbb{R}^r$ is the system input, A_i, B_i, and C are known constant matrices with appropriate dimensions, E_i, F, and D are, respectively, the fault and noise distribution matrices which are assumed to be known, M is the number of local models, and $\mu_i(\xi(t))$ are the activation functions verifying equation (12).

By considering the state $z(t) \in \mathbb{R}^p$ given in (35), the augmented state $\mathbf{x}(t) = \begin{bmatrix} x^T(t) & z^T(t) \end{bmatrix}^T$ is given by

$$\begin{cases} \dot{\mathbf{x}}(t) = \sum_{i=1}^{M} \mu_i \xi(t)\left(A_{\mathrm{gi}}\mathbf{x}(t) + B_{\mathrm{gi}}u(t) + W_{\mathrm{gi}}f(t)\right), \\ \mathbf{y}(t) = C_g\mathbf{x}(t) + D_g w(t), \end{cases} \tag{55}$$

with

$$\begin{aligned} A_{\mathrm{gi}} &= \begin{bmatrix} A_i & 0 \\ \overline{A}_i C & -\overline{A}_i \end{bmatrix}, \\ B_{\mathrm{gi}} &= \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \\ W_{\mathrm{gi}} &= \begin{bmatrix} E & 0 \\ 0 & \overline{A}_i F \end{bmatrix}, \\ C_g &= \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix}, \\ D_g &= \begin{bmatrix} D \\ 0 \end{bmatrix}, \\ f &= \begin{bmatrix} f_a \\ f_s \end{bmatrix}. \end{aligned} \tag{56}$$

An adaptive structure of the proportional integral observer which is able to estimate simultaneously the system augmented state and the generalized fault $f(t)$ is chosen as follows:

$$\begin{cases} \dot{\hat{\mathbf{x}}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))\left(A_{\mathrm{gi}}\hat{\mathbf{x}}(t) + B_{\mathrm{gi}}u(t) + W_{\mathrm{gi}}\hat{f}(t) + K_i\tilde{\mathbf{y}}\right), \\ \hat{f}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))\left(L_i\tilde{\mathbf{y}}(t)\right), \\ \hat{\mathbf{y}}(t) = C_g\hat{\mathbf{x}}(t), \end{cases} \tag{57}$$

where $\hat{\mathbf{x}}(t)$ is the estimated augmented system state, $\hat{f}(t)$ is the estimated fault, $\hat{\mathbf{y}}(t)$ is the estimated output, $\tilde{\mathbf{y}}(t) = \mathbf{y}(t) - \hat{\mathbf{y}}(t)$, K_i are the local proportional observer gains, and L_i are the local integral observer gains to be computed. It is assumed that $f(t)$ is bounded.

Remark 2. The designed observer is called adaptive observer because it is based on the adaptive global model, and it lets the estimation of the actuator and the sensor fault after the application of the mathematical transformation. So, it has the same structure of a proportional integral observer, but this structure is adapted to the augmented system.

Let us define the augmented state estimation error $\tilde{\mathbf{x}}(t) = \mathbf{x}(t) - \hat{\mathbf{x}}(t)$ and the fault estimation error $\tilde{f}(t) = f(t) - \hat{f}(t)$.

Their dynamics are given as follows:

$$\dot{\tilde{\mathbf{x}}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))\left(\left(A_{\mathrm{gi}} - K_iC_g\right)\tilde{\mathbf{x}}(t) + E_{gi}\tilde{f}(t) + K_iD_g w(t)\right), \tag{58}$$

$$\dot{\tilde{f}}(t) = \dot{f}(t) - \sum_{i=1}^{M} \mu_i(\xi(t))\left(L_iC_g\tilde{\mathbf{x}}(t) - L_iD_g w(t)\right). \tag{59}$$

The following matrices are introduced:

$$\begin{aligned} \varphi_f &= \begin{bmatrix} \tilde{\mathbf{x}}(t) \\ \tilde{f}(t) \end{bmatrix}, \\ \varepsilon_f &= \begin{bmatrix} w(t) \\ \dot{f}(t) \end{bmatrix}. \end{aligned} \tag{60}$$

By omitting to denote the dependence with regard to the time t, equations (58) and (59) become

$$\dot{\varphi}_f = A_{\mathrm{mf}}\varphi_f + B_{\mathrm{mf}}\varepsilon_f. \tag{61}$$

The matrices A_{mf} and B_{mf} have the expressions: $A_{\mathrm{mf}} = \sum_{i=1}^{M} \mu_i(\xi(t))\widetilde{A_{fi}}$ and $B_{\mathrm{mf}} = \sum_{i=1}^{m} \mu_i(\xi(t))\widetilde{B_{fi}}$. with

$$\begin{aligned} \widetilde{A_{\mathrm{fi}}} &= \begin{bmatrix} A_{\mathrm{gi}} - K_iC_g & E_{\mathrm{gi}} \\ -L_iC_g & 0 \end{bmatrix}, \\ \widetilde{B_{\mathrm{fi}}} &= \begin{bmatrix} -K_iD_g & 0 \\ -L_iD_g & I \end{bmatrix}. \end{aligned} \tag{62}$$

The Lyapunov function $V_f(t) = \varphi_f(t)^T P\varphi_f(t)$ is considered. By following the same reasoning as for actuator faults' estimation, the convergence of state and fault estimation errors as well as attenuation level are guaranteed if $\psi_f^T\Omega_f\psi_f < 0$ with

$$\begin{aligned} \psi_f &= \begin{bmatrix} \varphi_f \\ \varepsilon_f \end{bmatrix}, \\ \Omega_f &= \begin{bmatrix} A_{\mathrm{mf}}^T P + \mathrm{PA}_{\mathrm{mf}} + \mathrm{Q}_\varphi & \mathrm{PB}_{\mathrm{mf}} \\ B_{\mathrm{mf}}^T P & -\lambda^2 Q_\varepsilon \end{bmatrix}. \end{aligned} \tag{63}$$

The matrices A_{mf} and B_{mf} can be written as $A_{\mathrm{mf}} = \widetilde{A}_{\mathrm{mf}} - \widetilde{K}_{\mathrm{mf}}\widetilde{C}_g$ and $B_{\mathrm{mf}} = \widetilde{I} - \widetilde{K}_{\mathrm{mf}}\widetilde{D}_g$ with

$$\begin{aligned} \widetilde{A}_{\mathrm{mf}} &= \sum_{i=1}^{M} \mu_i(\xi(t))\widetilde{A}_{\mathrm{mfi}}, \\ \widetilde{K}_{\mathrm{mf}} &= \sum_{i=1}^{M} \mu_i(\xi(t))\widetilde{K}_{\mathrm{mfi}}, \end{aligned} \tag{64}$$

where

$$\begin{aligned}\widetilde{A}_{\text{mfi}} &= \begin{bmatrix} A_{\text{gi}} & E_{\text{gi}} \\ 0 & 0 \end{bmatrix},\\ \widetilde{K}_{\text{mfi}} &= \begin{bmatrix} K_i \\ L_i \end{bmatrix},\\ \widetilde{I} &= \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix},\\ \widetilde{C}_g &= \begin{bmatrix} C_g & 0 \end{bmatrix},\\ \widetilde{D}_g &= \begin{bmatrix} D_g & 0 \end{bmatrix}.\end{aligned} \tag{65}$$

Using the changes of variables $G_{\text{mf}} = P\widetilde{K}_{\text{mf}}$ and $\overline{\lambda} = \lambda^2$ and choosing $Q_{\varphi} = Q_{\varepsilon} = I$, the matrix Ω_f can be put in the following form:

$$\Omega_f = \begin{bmatrix} \vartheta_f & P\widetilde{I} - G_{mf}\widetilde{D}_a \\ \widetilde{I}^T P - \widetilde{D}_a^T G_{mf}^T & -mI \end{bmatrix}, \tag{66}$$

with $\vartheta_f = P\widetilde{A}_{\text{mf}} + \widetilde{A}_{\text{mf}}^T P - G_{\text{mf}}\widetilde{C}_a - \widetilde{C}_a^T G_{\text{mf}}^T + I$.

As $\Omega_f = \sum_{i=1}^{M} \mu_i(\xi(t))\Omega_{\text{fi}}$, the negativity of Ω_f is assured if, for $i \in \{1, \ldots, M\}$,

$$\Omega_{\text{fi}} < 0, \tag{67}$$

with

$$\Omega_{\text{fi}} = \begin{bmatrix} \vartheta_{\text{fi}} & P\widetilde{I} - G_{\text{fi}}\widetilde{D}_a \\ \widetilde{I}^T P - \widetilde{D}_a^T G_{\text{fi}}^T & -mI \end{bmatrix}, \tag{68}$$

where $\vartheta_{\text{fi}} = P\widetilde{A}_{\text{mfi}} + \widetilde{A}_{\text{mfi}}^T P - G_{\text{fi}}\widetilde{C}_a - \widetilde{C}_a^T G_{\text{fi}}^T + I$ and $G_{\text{fi}} = P\widetilde{K}_{\text{mfi}}$.

The resolution of LMI (67) leads to the determination of the matrices P and G_{fi} and the scalar m. The gain matrices are then deduced using the equation: $\widetilde{K}_{\text{mfi}} = P^{-1}G_{\text{fi}}$.

The observer design is summarized by the following theorem.

Theorem 3. *System (61) describing the time evolution of the state estimation error $\widetilde{x}$ and the fault estimation error $\widetilde{f}$ is stable and the $\mathscr{L}_2$-gain of the transfer from $\varepsilon_f(t)$ to $\varphi_f(t)$ is bounded if there exists a symmetric, positive definite matrix P, gain matrices G_{fi}, $i \in \{1, \ldots, M\}$, and a positive scalar m such that the following LMI is verified:*

$$\begin{bmatrix} \vartheta_{\text{fi}} & P\widetilde{I} - G_{\text{fi}}\widetilde{D}_a \\ \widetilde{I}^T P - \widetilde{D}_a^T G_{\text{fi}}^T & -mI \end{bmatrix} < 0, \quad i \in \{1, \ldots, M\}, \tag{69}$$

where $\vartheta_{fi} = P\widetilde{A}_{mfi} + \widetilde{A}_{mfi}^T P - G_{fi}\widetilde{C}_a - \widetilde{C}_a^T G_{fi}^T + I$. The observer gains (proportional and integral gains) are computed using the equation $\widetilde{K}_{mfi} = P^{-1}G_{fi}$ and the attenuation level is given by $\lambda = \sqrt{\overline{\lambda}}$.

The main advantage of this method is to estimate simultaneously the sensor and the actuator faults using a proportional integral observer. The use of the mathematical transformation 34 lets to obtain a generalized fault which combines the actuator and the sensor faults. This generalized fault appears as an actuator fault in the obtained augmented system. Its estimation leads to estimate the sensor and the actuator fault.

6. Example of Simulation

The objective of this section is to apply the proposed method to a hydraulic process made up of three tanks [37]. The system is supposed affected simultaneously by sensor and actuator faults. The three tanks T_1, T_2, and T_3 with identical sections ρ are connected to each other by cylindrical pipes of identical sections S_n. The output valve is located at the output of tank T_2; it ensures to empty the tank filled by the flow of pumps 1 and 2 with, respectively, flow rates $Q_1(t)$ and $a_2(t)$. Combinations of the three water levels are measured. The pipes of communication between the tanks are manually closed or open. The three levels x_1, x_2, and x_3 are governed by the constraint $x_1 > x_3 > x_2$; the process described by Figure 1 is modeled by equation (70). Taking into account the fundamental laws of conservation of the fluid, a nonlinear model is expressed by the following state equations [37]:

$$\begin{cases} \rho \dfrac{\mathrm{d}x_1(t)}{\mathrm{d}t} = -\alpha_1 S_n\left(2g\left(x_1(t) - x_3(t)\right)\right)^{1/2} + Q_1(t) + Qf_1 f_a(t), \\ \rho \dfrac{\mathrm{d}x_2(t)}{\mathrm{d}t} = -\alpha_3 S_n\left(2g\left(x_3(t) - x_2(t)\right)\right)^{1/2} - \alpha_2 S_n\left(2g x_2(t)\right)^{1/2} + Q_2(t) + Qf_2 f_a(t), \\ \rho \dfrac{\mathrm{d}x_3(t)}{\mathrm{d}t} = -\alpha_1 S_n\left(2g\left(x_1(t) - x_3(t)\right)\right)^{1/2} - \alpha_3 S_n\left(2g\left(x_3(t) - x_2(t)\right)\right)^{1/2} + Qf_3 f_a(t), \end{cases} \tag{70}$$

where α_1, α_2, and α_3 are constants, $f_a(t)$ is regarded as an unknown input, and $f_s(t)$ is a sensor fault affecting the process. $Qf/fi(t)$, $i \in \{1, \ldots, 3\}$, denote the additional mass flows into the tanks caused by leaks and g is the gravity constant.

The multiple model, with $\xi(t) = u(t)$, which approximates the nonlinear system (70), is given by the following state equation:

$$\begin{cases} \dot{x}(t) = \displaystyle\sum_{i=1}^{M} \mu_i(u(t))\left(A_i x(t) + B_i u(t) + E_i f_s(t)\right), \\ y(t) = Cx(t) + Ff_s(t) + Dw(t). \end{cases} \tag{71}$$

The matrices A_i, Bi, and d_i are calculated by linearizing the initial system (70) around four points chosen in the operation range of the system. Four local models have been selected in a heuristic way. That number guarantees a good

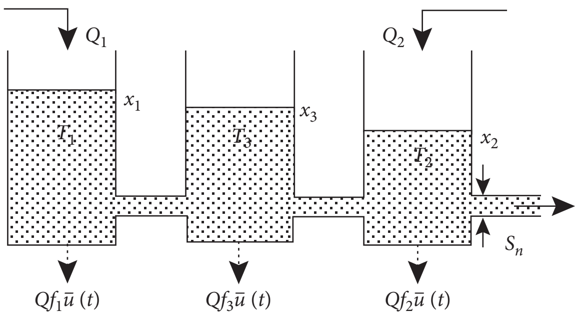

Figure 1: Three tanks' system.

approximation of the state of the real system by the multiple model [?]. The following numerical values were obtained:

$$A_1 = \begin{bmatrix} -0.0109 & 0 & 0.0109 \\ 0 & -0.0206 & 0.0106 \\ 0.0109 & 0.0106 & -0.0215 \end{bmatrix},$$
$$d_1 = 10^{-3} * \begin{bmatrix} -2.86 \\ -0.38 \\ 0.11 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -0.0110 & 0 & 0.0110 \\ 0 & -0.0205 & 0.0104 \\ 0.0110 & 0.0104 & -0.0215 \end{bmatrix},$$
$$d_2 = 10^{-3} * \begin{bmatrix} -2.86 \\ -0.34 \\ 0.038 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} -0.0084 & 0 & 0.0084 \\ 0 & -0.0206 & 0.0095 \\ 0.0084 & 0.0095 & -0.0180 \end{bmatrix},$$
$$d_3 = 10^{-3} * \begin{bmatrix} -3.7 \\ -0.14 \\ 0.69 \end{bmatrix}, \quad (72)$$
$$A_4 = \begin{bmatrix} -0.0085 & 0 & 0.0085 \\ 0 & -0.0205 & 0.0095 \\ 0.0085 & 0.0095 & -0.0180 \end{bmatrix},$$
$$d_4 = 10^{-3} * \begin{bmatrix} -3.67 \\ -0.18 \\ 0.62 \end{bmatrix},$$
$$B_i = E_i$$
$$D = \begin{bmatrix} 0.1 & 0.5 \\ 0.2 & 0.1 \\ 0.1 & 0.3 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

The functions Qf_1, Qf_2, and Qf_3 are constant and the numerical application is made with

$$Qf_i = 10^{-4},\ \forall i \in \{1,\dots,4\},\ \text{and. } t \in [0, x, [$$
$$\alpha_1 = 0.78,\ \alpha_2 = 0.78,\ \text{and } \alpha_3 = 0.75$$
$$g = 9.8,\ S_n = 5 * 10^{-5},\ \text{and } \rho = 0.0154$$

The type 2 activation functions are shown in Figure 2.

Figure 2 shows the evolution of activation functions $\mu_i(\xi(t))$ in time. For each activation function $\mu_i(\xi(t))$, $i \in 1,\dots,4$, the upper bound $\overline{\mu}_i(\xi(t))$ and the under bound $\underline{\mu}_i(\xi(t))$ are represented. The evolution of the activation function in time is between these two bounds.

The actuator fault $f_a(t) = [\, f_{a1}(t) \;\; f_{a2}(t) \,]^T$ is defined as

$$f_{a1} = \begin{cases} 0.4, & 100\,\text{s} < t < 700\,\text{s}, \\ 0, & \text{otherwise}, \end{cases}$$
$$f_{a1} = \begin{cases} 0.5, & 60\,\text{s} < t < 360\,\text{s}, \\ 0.8 * \sin(0.2\pi t), & 360\,\text{s} < t < 800\,\text{s}, \\ 0, & \text{otherwise}. \end{cases} \quad (73)$$

The sensor fault $f_s(t)$ is $f_s(t) = [\, f_{s1}(t) \;\; f_{s2}(t) \,]^T$ with

$$f_{s1} = \begin{cases} 0.7 * \sin(0.4\pi t), & 200\,\text{s} < t < 700\,\text{s}, \\ 0.6, & 700\,\text{s} < t < 1000\,\text{s}, \\ 0, & \text{otherwise}, \end{cases}$$
$$f_{s2} = \begin{cases} 0.8 * \sin(0.1\pi t), & 350 < t < 550\,\text{s}, \\ 0.6, & 600\,\text{s} < t < 800\,\text{s}, \\ 0, & \text{otherwise}. \end{cases} \quad (74)$$

Matrices $\overline{A}_i$ are chosen as $\overline{A}_1 = 5 * I$, $\overline{A}_2 = 10 * I$, $\overline{A}_3 = 15 * I$, and $\overline{A}_4 = 20 * I$.

Figure 3 visualizes the two actuator faults and their estimations. The actuator faults' error estimation is shown in Figure 4. In Figure 5, the two sensor faults and their estimations are represented, and the sensor faults' error estimation is shown in Figure 6. The state error estimation is visualized in Figure 7.

Figures 3 and 4 show that the proposed proportional observer allows estimating the actuator fault well even in the case of time-varying faults.

Figures 5 and 6 show that the proposed proportional observer allows estimating the sensor fault well even in the case of time-varying faults. Figure 7 shows also that this proposed observer allows estimating the system state well. The effect of the measurement noise is minimized using the $\mathscr{L}_2$ approach.

The obtained results show the effectiveness of the proposed proportional integral observer.

Figures 3 to 7 show that the proposed method gives a state and fault estimation with high performances. The proposed adaptive proportional observer lets to estimate the system state and the actuator and/or the sensor fault well. The proposed observer gives good results even in the case of time-varying faults. It is shown also that the proposed

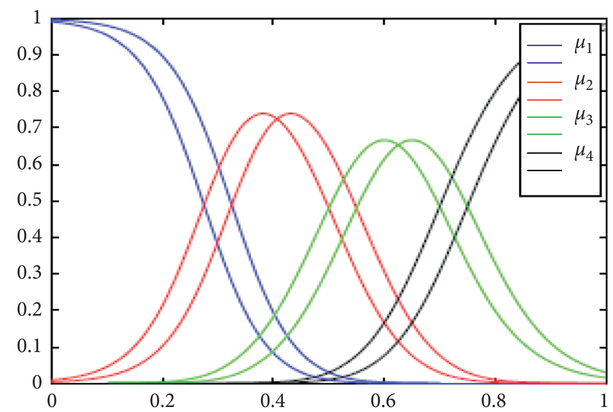

Figure 2: Actuator faults' estimation error.

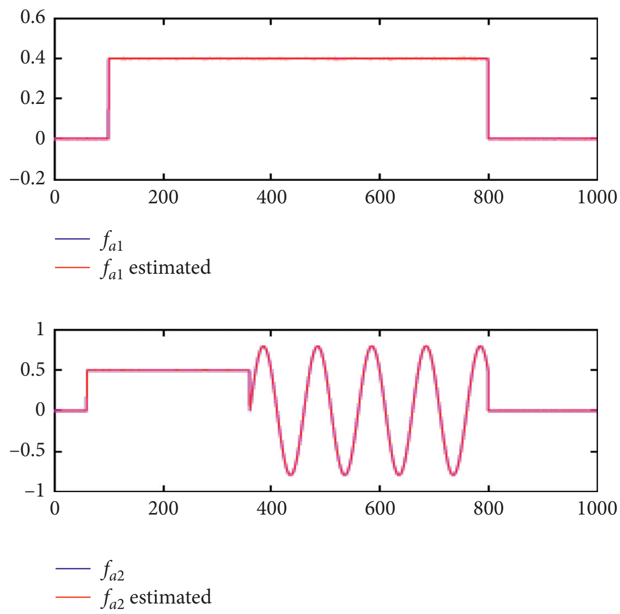

Figure 3: Actuator faults and their estimation.

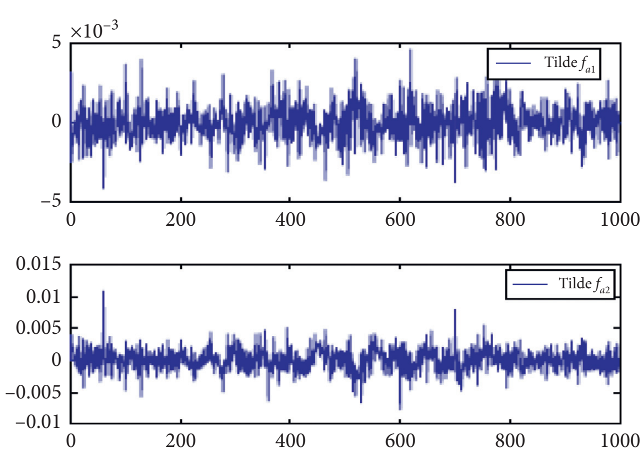

Figure 4: Actuator faults' estimation error.

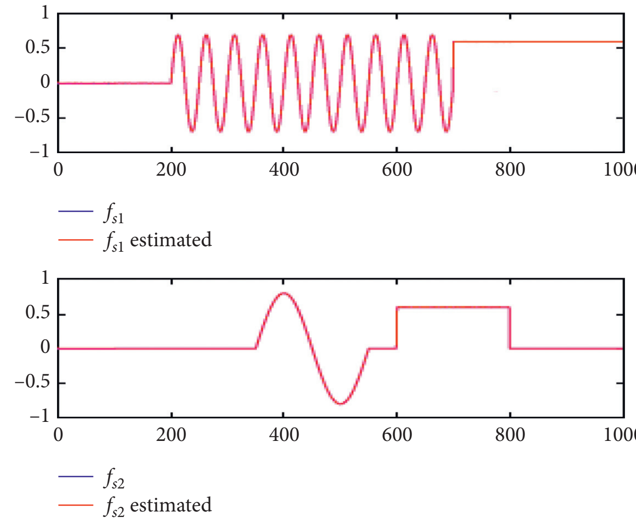

Figure 5: Sensor faults and their estimation.

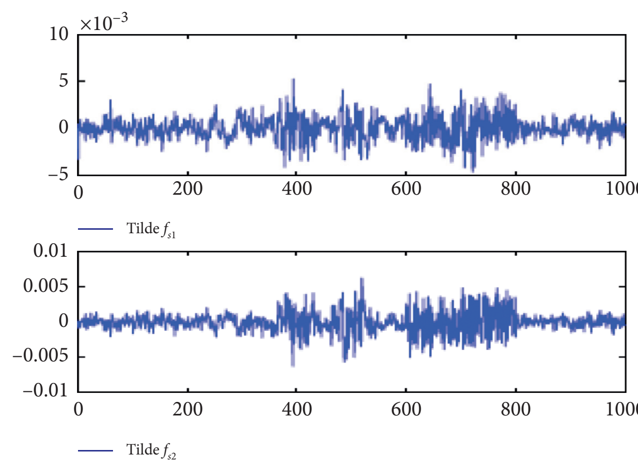

Figure 6: Sensor faults' estimation error.

observer is rapid and lets to estimate the system state and the fault in a very short time.

Simulations results present the robustness of the designed observer for state and fault estimation. Indeed, it is shown that the actuator fault and its estimation are nearly superposed. The same situation is obtained for sensor faults. The estimation error is less than 1% which is supposed acceptable estimation error for fault or state estimation. The analysis of simulation result lets to conclude that the conceived observer allows an acceptable state and fault estimation by its application to the three tanks system modeled by a type 2 interval Takagi–Sugeno model. The considered fault for the simulation is time varying which lets to obtain a general result without the assumption of null fault derivative.

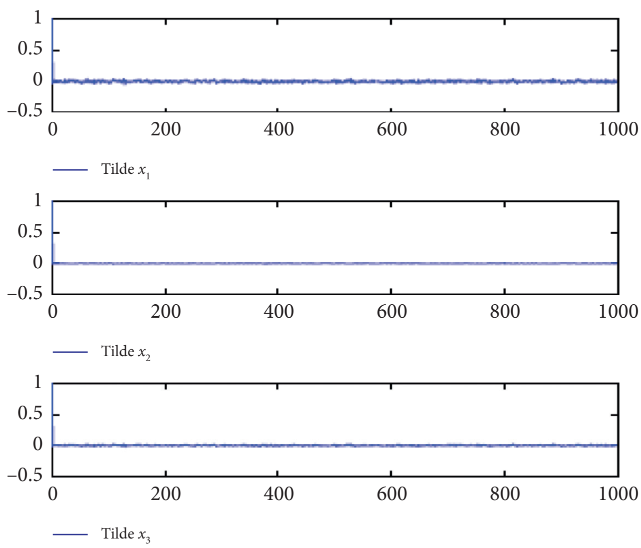

Figure 7: State estimation error.

Remark 3. The particularity of this work compared with others in the same context is that this work proposes an observer able to estimate actuator and sensor fault using a mathematical transformation which can be considered as a filter of the system output. Many works in this context suppose that the fault derivate is null, and this assumption is not considered in this work which makes it more general. It is shown that time-varying fault is well estimated.

7. Conclusion

This paper presents a method of a proportional integral observer design based on the principle of interval type 2 Takagi–Sugeno fuzzy systems. The proposed observer is able to estimate simultaneously the system state and the faults' affecting system. In this work, actuator faults are considered as unknown inputs. To estimate sensor faults, a mathematical transformation is used to conceive an augmented system in which the initial sensor fault appears as an unknown input. In this work, the system affected by actuator fault is considered firstly; then, the system affected by the sensor fault is treated. Moreover, the case, where the studied system is affected simultaneously by sensor and actuator faults, is considered. The computation of the global observer gains is reduced to the computation of the gains of the local observers.

This method allows estimating the faults well even in the case of time-varying faults. The noise effect on the state and fault estimation is also minimized using the $\mathscr{L}_2$ approach. The direct application of this observer could be the base for the design of a detection procedure and localization of faults. It is possible also to use the sensor and/or actuator fault estimation to conceive a fault tolerant control which can remain the evolution of the faulty system state to the state of the system where no faults are affecting it. In the same context of Takagi–Sugeno approach, more works will be developed taking into account more parameters such as system delays and fractional order systems. Other type of observers will be also developed, in particular, proportional multiple integral observers and observers with unknown input. These works will be extended in the second step to treat the problem of the design of fault tolerant control using the active approach.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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Research Article

Adaptive Fault Estimation and Fault Tolerant Control for Polynomial Systems: Application to Electronic and Mechanical Systems

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This paper presents a sum of squares (SOS) approach for active fault tolerant control (AFTC) of nonlinear polynomial systems. A polynomial adaptive fault estimation algorithm for polynomial systems is firstly proposed. Then, sufficient conditions for the existence of the fault estimator are given in terms of SOS which can be numerically (partially symbolically) solved via the recently developed SOSTOOLS. Based on the obtained online fault estimation information, a fault-tolerant control strategy is designed for both compensating the effect of actuator faults in real time and stabilizing the closed-loop system. Finally, tunnel diode circuit and mass-spring-damper systems are used to demonstrate the applicability of the proposed approach.

1. Introduction

The growing complexity of modern industrial processes gives rise to increasing demands regarding fault estimation (FE) and fault-tolerant control (FTC). Various FE strategies have been proposed in the literature. For example, proportional integral sliding mode observer has been considered in Elleuch et al. [1]; learning algorithm has been proposed in Jia et al. [2]. In Zhang et al.'s study [3], a fast adaptive actuator fault estimation technique is proposed for linear models. In Tayari et al.'s study [4], two adaptive sliding mode observers are adopted for a class of uncertain linear parameter varying systems.

Since the most practical systems are nonlinear, the adaptive observer approach has been extended to deal with nonlinear models. The well-known Takagi Sugeno (TS) fuzzy model has attracted much attention in the past two decades due to its powerful capability to approximate complex nonlinear systems. In Ichalal et al.'s study [5], the state and the actuator faults are well estimated by using the fast adaptive observer proposed in Zhang et al. [3] for TS fuzzy systems. An adaptive observer-based FTC has been designed for a class of TS fuzzy systems with both actuator and sensor faults in Kharrat et al. [6]. Moreover, the H_∞ performance is used in this work to attenuate the external disturbance effect.

Polynomial system is a class of nonlinear systems in the form of $\dot{x} = A(x)x + B(x)u$, where $A(x)$ and $B(x)$ are polynomial matrices. This class of systems can describe many engineering systems such as electronic circuits, mechanical systems, and communications systems. For instance, the electronic circuit with tunnel and the mass-spring-damper are described by polynomial models in Zhao et al. [7] and Li et al. [8], respectively. In Zhao et al.'s study [7], a functional observer is designed in order to estimate the system state and the unknown input, in which the observer design scheme has been applied to electronic circuit with tunnel described by the polynomial model. When this original model is considered, the LMI-based analysis approach cannot be applied directly. For this reason, the control of this electronic circuit is based on the transformation of the original polynomial model into TS fuzzy form. In this case, the controller is designed using LMI-based approach. Similarly, a mass-spring-damper system is used by Li et al. [8] to show the applicability of the proposed controller design.

The latest developments in sum of squares (SOS) programming techniques make it possible to deal directly with polynomial systems. So far, extensive results have been presented for investigating different classes of polynomial-based systems such as polynomial systems, positive polynomial systems, polynomial fuzzy systems, and polynomial fuzzy systems with time delay. Topics on delay-free case cover a wide range including stability analysis by Han et al. [9], stabilization by Zhao et al. [10], fuzzy observer design by Liu et al. [11], passive fault tolerant control by Ye et al. [12], and fault detection filter design by Chibani et al. [13]. Recently, in the direction of investigating several classes of time delay polynomial systems, some results have been proposed in the literature, e.g., stabilization by Gassara et al. [14]; control under actuator saturation by Gassara et al. [15], and observer-based control for positive polynomial systems with time delay by Iben Ammar et al. [16]. These various results are presented in terms of sum of squares (SOS), in which conditions are numerically (partially symbolically) solved via the recently developed SOSTOOLS by Prajna et al. [17]. These results clearly demonstrate that the SOS approach can be used as an effective alternative technique to the LMI-based approaches for nonlinear systems with polynomial matrices. However, to our knowledge, there are no results for adaptive observer-based FTC for polynomial-based systems. Motivated by the aforementioned observation, in this work, the adaptive fault tolerant control problem for a class of polynomial model with actuator faults is investigated, in which polynomial terms depend only on the measurable variables.

The main contributions of this paper can be summarized as follows:

(i) A novel polynomial adaptive observer is proposed. Despite, standard adaptive observer has been extensively studied in literature for fault estimation, polynomial adaptive observer is not yet investigated for the class of polynomial model. The main advantage of the proposed polynomial adaptive observer-based fault estimation compared with the standard one is that the observer gain $L(y(t))$ is not constant but polynomial.

(ii) Various practical engineering systems can be modeled by the proposed polynomial model. For design purpose, the dynamics of these systems are generally approximated in literature by TS fuzzy models. In this case, the polynomial model can reduce the computational load especially when the number of fuzzy rules is high.

(iii) The proposed polynomial model can also increase the modelling accuracy. In fact, we can deal with the original model without using the sector nonlinearity concept to transform the original model into the TS fuzzy model. This allows to avoid setting the variation bounds of some system states.

It becomes increasingly apparent that the SOS approach can be extended to deal with large research topics, e.g., adaptive tracking control by Chen et al. [18] and Wang et al. [19]; event-triggered control by Xie et al. [20]; and finite-time adaptive fault-tolerant control by Wang et al. [21].

This paper is organized as follows. In Section 2, we present a description of a class of polynomial models with actuator faults. Sufficient conditions for the existence of the actuator fault estimator are given in Section 3. These conditions are given in terms of SOS. Meanwhile, based on the online fault estimation, the controller law is then designed to compensate the effect of actuator faults. In Section 4, a tunnel diode circuit and mass-spring-damper systems are presented to demonstrate the applicability of the proposed result. Finally, Section 5 concludes the paper.

2. Problem Formulation

Consider the following polynomial model with additive actuator faults:

$$\begin{cases} \dot{x}(t) = A(\zeta(t))x + B(\zeta(t))(u(t) + f(t)), \\ y(t) = Cx(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state vector, $u(t) \in \mathbb{R}^{n_u}$ is the input vector, $f(t) \in \mathbb{R}^{n_f}$ is the additive actuator fault vector, $\zeta(t)$ is available, such as the partial system state variables and the system outputs as in Pang and Zhang's study [22], $y(t) \in \mathbb{R}^{n_y}$ is the measurement output vector, C is constant real matrix, and $A(\zeta(t))$ and $B(\zeta(t))$ are polynomial matrices in $\zeta(t)$.

The derivative of $f(t)$ with respect to time is norm-bounded:

$$\|\dot{f}(t)\| \le f_{1\max}; \quad 0 \le f_{1\max} < \infty. \tag{2}$$

To estimate actuator faults, the following polynomial adaptive fault diagnosis observer is considered:

$$\left\{ \dot{\hat{x}}(t) = A(\zeta(t))\hat{x}(t) + B(\zeta(t))(u(t) + \hat{f}(t)) + L(\zeta(t))e_y(t), e_y(t) = y(t) - \hat{y}(t), \hat{y}(t) = C\hat{x}(t), \right. \tag{3}$$

where $\hat{x}(t) \in \mathbb{R}^{n_x}$ is the observer state vector, $\hat{y}(t) \in \mathbb{R}^{n_y}$ is the observer output vector, and $\hat{f}(t) \in \mathbb{R}^{n_f}$ is an estimate of actuator fault $f(t)$.

Denote the state and fault estimation errors as follows:

$$\begin{aligned} e_x(t) &= x(t) - \hat{x}(t), \\ e_f(t) &= f(t) - \hat{f}(t). \end{aligned} \tag{4}$$

State estimation error $e_x(t)$ is written as

$$\dot{e}_x(t) = (A(\zeta(t)) - L(\zeta(t))C)e_x(t) + B(\zeta(t))e_f(t). \quad (5)$$

The conventional adaptive fault estimation algorithm is given by

$$\dot{\hat{f}}(t) = \Gamma F(\zeta(t))\left(\dot{e}_y(t) + \sigma e_y(t)\right), \quad (6)$$

where $\Gamma \in \mathbb{R}^{n_f \times n_f}$ is the learning rate.

In Figure 1, the block diagram illustrates the proposed polynomial FTC strategy.

From now, to lighten the notation, we will drop the notation with respect to time t. For instance, we will employ x, $\hat{x}$, and ζ instead of $x(t)$, $\hat{x}(t)$, and $\zeta(t)$, respectively.

3. Main Results

3.1. Fault Estimation Based on Polynomial Adaptive Algorithm. In this section, the stability of the error dynamics is guaranteed by the following theorem.

Theorem 1. *If there exists positive definite matrix P_1 and polynomial matrices $W_L(\zeta)$ and $F(\zeta)$ such that the following SOS optimization problem is feasible.*

Minimize η subject to

$$v_1^T(P_1 - \varepsilon_1 I)v_1 \text{ is SOS}, \quad (7)$$

$$v_2^T(\Lambda(\zeta) - \varepsilon_2(\zeta)I)v_2 \text{ is SOS}, \quad (8)$$

$$-v_3^T(\Xi(\zeta) + \varepsilon_3(\zeta)I)v_3 \text{ is SOS}, \quad (9)$$

where v_1, v_2, and v_3 denote vectors that are independent of x, $\hat{x}$, and ζ.

$$\Lambda(\zeta) = \begin{bmatrix} \eta I & B^T(\zeta)P_1 - F(\zeta)C \\ * & I \end{bmatrix},$$
$$\Xi(\zeta) = \begin{bmatrix} \xi_{11}(\zeta) & \xi_{12}(\zeta) \\ * & \xi_{22}(\zeta) \end{bmatrix}, \quad (10)$$

in which

$$\xi_{11}(\zeta) = P_1A(\zeta) - W_L(\zeta)C + A^T(\zeta)P_1 - C^TW_L^T(\zeta),$$
$$\xi_{12}(\zeta) = -\frac{1}{\sigma}A^T(\zeta)P_1B(\zeta) + \frac{1}{\sigma}C^TW_L(\zeta)B(\zeta),$$
$$\xi_{22}(\zeta) = -2\frac{1}{\sigma}B^T(\zeta)P_1B(\zeta) + \frac{1}{\sigma}M, \quad (11)$$

then the state estimation error e_x and the fault estimation error e_f are bounded. Furthermore, if the bound of the first time derivative of f is zero, these variables converge asymptotically to zero. In this case, the gain of the polynomial adaptive observer-based is given by $L(\zeta) = P_1^{-1}W_L(\zeta)$.

Proof. Consider the following Lyapunov function

$$V = e_x^TP_1e_x + \frac{1}{\sigma}e_f^T\Gamma^{-1}e_f. \quad (12)$$

Differentiating V with respect to time t and considering (3), (5), and (6), it leads to

$$\dot{V} = 2e_x^TP_1\left((A(\zeta) - L(\zeta)C)e_x + B(\zeta)e_f\right)$$
$$+ 2\frac{1}{\sigma}e_f^T\Gamma^{-1}\dot{f} - 2e_f^TF(\zeta)Ce_x - 2\frac{1}{\sigma}e_f^TF(\zeta)C\dot{e}_x. \quad (13)$$

One has

$$2\frac{1}{\sigma}e_f^T\Gamma^{-1}\dot{f} \le \frac{1}{\sigma}e_f^TMe_f(t) + \frac{1}{\sigma}\dot{f}(t)^T\Gamma^{-1}M^{-1}\Gamma^{-1}\dot{f}(t)$$
$$\le \frac{1}{\sigma}e_f(t)^TMe_f(t) + \delta, \quad (14)$$

where

$$\delta = \frac{1}{\sigma}f_{1\max}^2\lambda_{\max}\left(\Gamma^{-1}M^{-1}\Gamma^{-1}\right). \quad (15)$$

If (8) holds, then

$$\Lambda(\zeta) = \begin{bmatrix} \eta I & B^T(\zeta)P_1 - F(\zeta)C \\ * & I \end{bmatrix} \pm 0. \quad (16)$$

Applying Schur complement to (16) implies that

$$\left(F(\zeta)C - B^T(\zeta)P_1\right)\left(F(\zeta)C - B^T(\zeta)P_1\right)^T \prec \eta I. \quad (17)$$

The minimization of η leads to the following equality:

$$F(\zeta)C = B^T(\zeta)P_1. \quad (18)$$

Hence,

$$\dot{V} \le 2e_x^TP_1(A(\zeta) - L(\zeta)C)e_x + \frac{1}{\sigma}e_f(t)^TMe_f(t)$$
$$+ \delta - 2\frac{1}{\sigma}e_f^TB^T(\zeta)P_1\left((A(\zeta) - L(\zeta)C)e_x + B(\zeta)e_f\right). \quad (19)$$

This inequality can been rewritten as follows:

$$\dot{V} \le \tilde{x}^T\Omega(\zeta)\tilde{x} + \delta, \quad (20)$$

where $\tilde{x} = \begin{bmatrix} e_x(t) \\ e_f(t) \end{bmatrix}$, $\Omega(\zeta) = \begin{bmatrix} \omega_{11}(\zeta) & \omega_{12}(\zeta) \\ * & \xi_{22}(\zeta) \end{bmatrix}$ in which

$$\omega_{11}(\zeta) = P_1(A(\zeta) - L(\zeta)C) + (A(\zeta) - L(\zeta)C)^TP_1,$$
$$\omega_{12}(\zeta) = -\frac{1}{\sigma}(A(\zeta) - L(\zeta)C)^TP_1B(\zeta). \quad (21)$$

If (9) holds, then $\Omega(\zeta) \le 0$. Furthermore, if (9) holds with $\varepsilon_3(\zeta) > 0$ for $\zeta \ne 0$, then $\Omega(\zeta) < 0$. Therefore, there exists a scalar $\vartheta > 0$ such that

$$\dot{V} < -\vartheta\|\tilde{x}\|^2 + \delta. \quad (22)$$

It follows that $\dot{V} < 0$ if $\vartheta\|\tilde{x}\|^2 > \delta$, and according to Lyapunov stability theory $\tilde{x}$ converges to the following set:

$$S = \left\{\frac{\tilde{x}}{\|\tilde{x}\|^2} \le \frac{\delta}{\vartheta}\right\}. \quad (23)$$

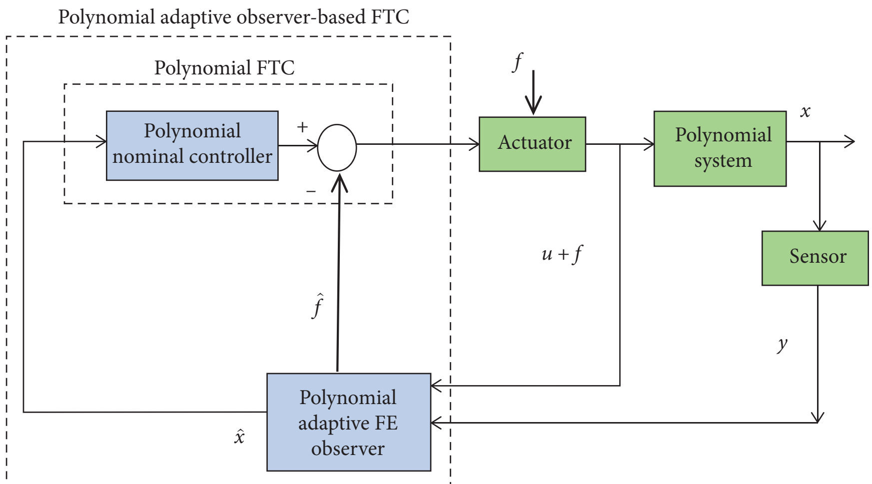

Figure 1: Polynomial fault tolerant control scheme.

Thus, estimation errors of both the state and the fault are uniformly ultimately bounded. □

Remark 1. Selection of the learning rate Γ influences the accuracy of the system state and actuator faults estimation (see Kharrat et al. [6]). This parameter should be adjusted such that δ is minimised.

Remark 2. The nonnegative polynomials $\varepsilon_2(\zeta) > 0$ and $\varepsilon_3(\zeta) > 0$ for $\zeta \neq 0$ can be accommodated by SOS optimization as in Papachristodoulou and Prajna's study. [23].

3.2. Fault Accommodation. After that, the fault information is obtained, and we will consider the fault-tolerant control design problem of system (1) to compensate the effect of actuator faults and to stabilize the resulting closed loop systems by considering the following FTC law:

$$u = -K(\zeta)\hat{x} - \hat{f}. \tag{24}$$

Substituting (24) into (1), we obtain the following dynamic of the closed-loop system:

$$\dot{x} = (A(\zeta) - B(\zeta)K(\zeta))x + \rho, \tag{25}$$

where

$$\rho = B(\zeta)K(\zeta)e_x + B(\zeta)e_f. \tag{26}$$

ρ can be considered as an external disturbance, and the boundedness of e_x and e_f can be guaranteed by Section 3.1. So, if the polynomial state feedback controller

$$u = -K(\zeta)x \tag{27}$$

can ensure that the following polynomial system is asymptotically stable:

$$\dot{x} = A(\zeta)x + B(\zeta)u, \tag{28}$$

then state vector x is uniformly ultimately bounded under observer-based fault tolerant controller (3) according to the input-to-state stability theory.

The polynomial state feedback controller (27) can be obtained by solving the SOS conditions presented in the following theorem.

Theorem 2. *Control law (27) stabilizes polynomial system (28) if there exists a symmetric matrix P_2 and a polynomial matrix $W_K(\zeta)$ such that the following SOS conditions are satisfied:*

$$\begin{aligned} &v_1^T(P_2 - \varepsilon_1 I)v_1 \text{ is SOS}, \\ -&v_2^T(\Upsilon(\zeta) + \varepsilon_2(\zeta)I)v_2 \text{ is SOS}, \end{aligned} \tag{29}$$

where v_1 and v_2 denote vectors that are independent of ζ.

$$\Upsilon(\zeta) = A(\zeta)P_2 - B(\zeta)W_K(\zeta) + P_2A^T(\zeta) - W_K^T(\zeta)B^T(\zeta). \tag{30}$$

In this case, a stabilizing feedback gain $K(\zeta)$ can be obtained from P_2 and $W_K(\zeta)$ as $K(\zeta) = W_K(\zeta)P_2^{-1}$.

4. Simulation Examples

4.1. Example 1: Tunnel Diode Circuit. A tunnel diode circuit shown in Figure 2 is adopted from the study by Zhao et al. [7] and Iben Ammar et al. [16]. This electronic circuit can be described as follows:

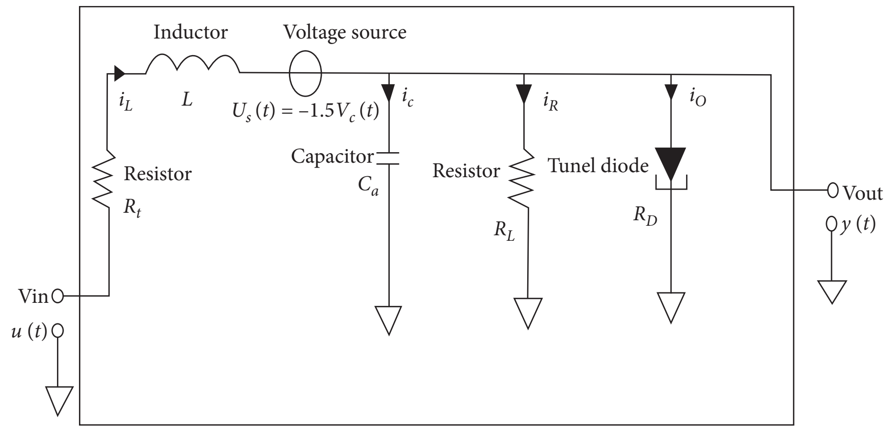

Figure 2: Tunnel diode circuit.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{R_L C_c} - \frac{1}{C_c}\left(0.002 + 0.01x_1^2\right) & \frac{1}{C_c} \\ \frac{1}{2L} & -\frac{R_E}{L} \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} u,$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \tag{31}$$

where R_L and R_E are two resistors, C_c is a capacitor, and L is an inductor.

The tunnel diode circuit parameters are taken as

$$C_c = 25\,\text{mF},$$
$$L = 20\,\text{H},$$
$$R_E = 200\Omega,$$
$$R_L = 2\,\text{k}\Omega. \tag{32}$$

Based on the concept of nonlinearity sector, a T–S fuzzy model is proposed in Zhao et al. [7] to represent the dynamics of this system under $x_1 \in [\,\overline{m}_1 \;\; \overline{m}_2\,]$. In this paper, we deal directly with polynomial model (31), without any assumption. Furthermore, x_1 is not restricted to be in $[\,\overline{m}_1 \;\; \overline{m}_2\,]$.

In order to illustrate the use of result, we assume that we have an actuator fault. In this case, the polynomial model is as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{R_L C_c} - \frac{1}{C_c}\left(0.002 + 0.01x_1^2\right) & \frac{1}{C_c} \\ \frac{1}{2L} & -\frac{R_E}{L} \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} (u + f),$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \tag{33}$$

where f is an additive actuator fault defined by

$$f = \begin{cases} 0, & 0 \le t < 5, \\ 0.5e^{0.05(t-5)} - 0.5, & 5 \le t < 80, \\ 0.5e^{0.05*75} - 0.5, & 80 \le t < 150. \end{cases} \quad (34)$$

We choose $\varepsilon_1 = \varepsilon_2(\zeta) = \varepsilon_3(\zeta) = 10^{-3}$, $\sigma = 1$, degrees of $P_1, W_L(\zeta)$, and $F(\zeta)$ are 0, 2, and 0, respectively. Solving the SOS conditions in Section 3.1, one can obtain that

$$\begin{aligned} \gamma &= 0.0004, \\ F &= 0.0050057, \\ L &= \begin{bmatrix} 1253.442\zeta^2 + 7084.977 \\ -313.3588\zeta^2 - 1770.6725 \end{bmatrix}. \end{aligned} \quad (35)$$

The polynomial state feedback gain matrix $K(y)$ is calculated by solving SOS conditions in Section 3.2. By choosing $\varepsilon_1 = \varepsilon_2(\zeta) = 10^{-3}$, the degrees of P_2, $W_K(\zeta)$ are 0 and 2, respectively. We get

$$K = \begin{bmatrix} 2.8013\zeta^2 + 3.6931 & 140.3681\zeta^2 + 151.395 \end{bmatrix}. \quad (36)$$

By taking learning rate $\Gamma = 10^6$ and $\sigma = 1$, we obtain $\delta = 25 \times 10^{-7}$. Simulation results are shown in Figures 3–5. Figure 3 shows the evolution of actuator fault and its estimated values. Figures 4 and 5 show the evolution of system states x_1 and x_2, respectively, with nominal control and fault tolerant control law.

It is noted that when actuator failures occurs, the stability of the closed-loop polynomial model with the nominal controller is not even guaranteed, whereas the closed-loop system using the fault tolerant control still operates correctly and remains maintained.

4.2. Example 2: Mass-Spring-Damper System. In this example, we consider a mass-spring-damper system (Figure 6) described by the following polynomial model given in Li et al. [8]:where M is the mass, $x_1(t)$ is the displacement of the mass, $x_2(t)$ is the velocity of the mass, and u(t) is the input force.

$$\begin{aligned} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -\dfrac{c_3}{M} & -\dfrac{c_1 + c_2 x_1^2(t)}{M} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1 + c_4 x_2^2(t)}{M} \end{bmatrix} u(t), \\ y &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \end{aligned} \quad (37)$$

Mass-spring-damper system parameters are $M = 1$, $c_1 = 0.003$, $c_2 = 0.001$, $c_3 = 0.80$, and $c_4 = 0.1$.

We notice that the mass-spring-damper system is modeled as a polynomial system, whereas in Li et al.'s study [8], it has been modeled as an uncertain system with polytopic uncertainties by restricting the state variables $x_1(t)$ and $x_2(t)$ such as $x_1(t) \in [-a \;\; a]$ and $x_2(t) \in [-b \;\; b]$, $a > 0$, $b > 0$. However, in this paper, we do not need this restriction, we take the original model as it is.

By adding the actuator fault on the system, the polynomial model can be described by

$$\begin{aligned} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -\dfrac{c_3}{M} & -\dfrac{c_1 + c_2 x_1^2(t)}{M} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1 + c_4 x_2^2(t)}{M} \end{bmatrix} (u(t) + f(t)), \\ y &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \end{aligned} \quad (38)$$

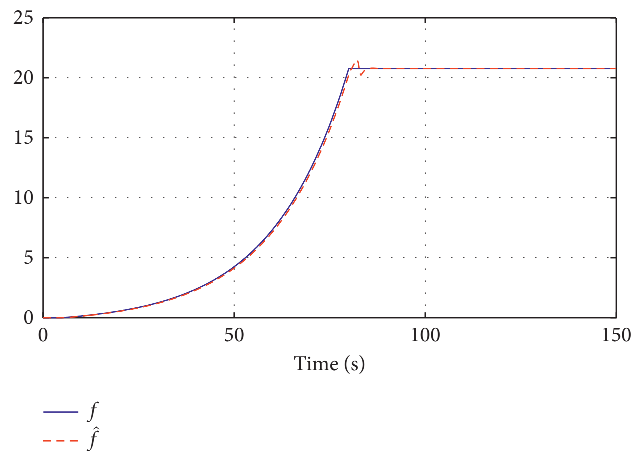

Figure 3: Fault and its estimated in example 1.

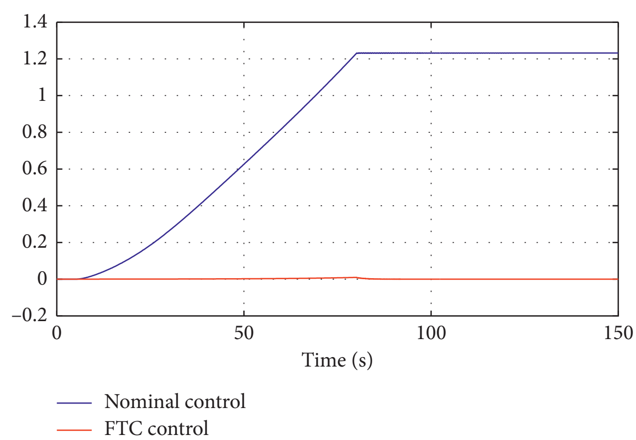

Figure 4: System state x_1 with nominal control and fault tolerant control law in example 1.

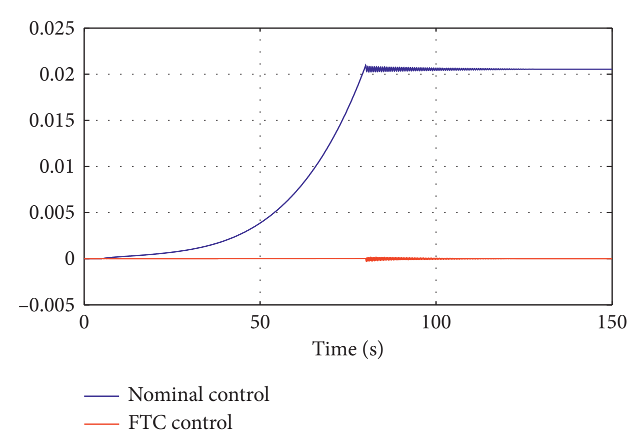

Figure 5: System state x_2 with nominal control and fault tolerant control law in example 1.

where f is an additive actuator fault defined by

$$f = \begin{cases} 0, & 0 \le t < 5, \\ \cos(\pi t) + 2, & 5 \le t \le 20. \end{cases} \tag{39}$$

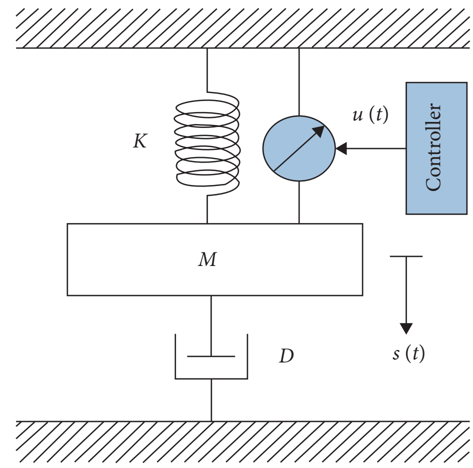

Figure 6: A mass-spring-damper system.

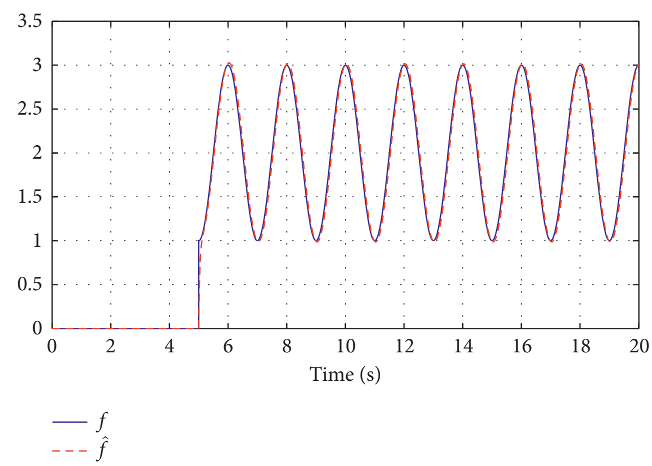

Figure 7: Fault and its estimated in example 2.

We choose $\varepsilon_1 = \varepsilon_2(\zeta) = \varepsilon_3(\zeta) = 10^{-3}$, $\sigma = 1$, $F(\zeta)$ of degree 2 in ζ_2, and $W_L(\zeta)$ of degree 2 in ζ_1. By solving SOS conditions in Section 3.1, we get

$$\begin{aligned} \gamma &= 0.1 \times 10^{-10}, \\ F(y) &= \left[0.00024962\zeta_2^2 + 0.0024962 \quad 0.034298\zeta_2^2 + 0.34298 \right], \\ L(y) &= \begin{bmatrix} 0.683066\zeta_1^2 + 1.026 & 0.70759 \\ -0.0049713\zeta_1^2 - 0.622816 & 0.4218721 - 0.001\zeta_1^2 \end{bmatrix}. \end{aligned} \tag{40}$$

Now, we choose $\varepsilon_1 = \varepsilon_2(\zeta) = 10^{-3}$, $W_K(\zeta)$ of degree 2 in ζ_1. By solving the SOS conditions in Section 3.2, the polynomial controller gain is obtained as:

$$K(\zeta) = \left[0.62\zeta_1^4 + 0.52\zeta_1^2 + 0.76 \quad 1.3\zeta_1^4 + 1.09\zeta_1^2 + 1.58 \right]. \tag{41}$$

By choosing $\Gamma = 1000$, we obtain $\delta = 0.00010$. Similar to example 1, we show the evolution of actuator fault and its estimated values in Figure 7. The evolution of system state x_1 with nominal control and fault tolerant control law is given

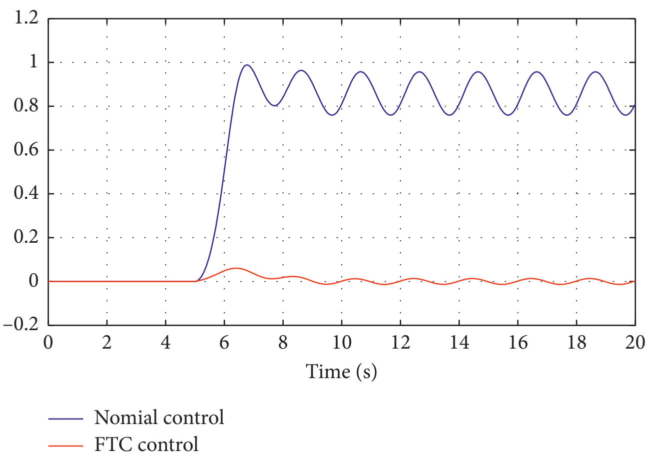

FIGURE 8: System state x_1 with nominal control and fault tolerant control law in example 2.

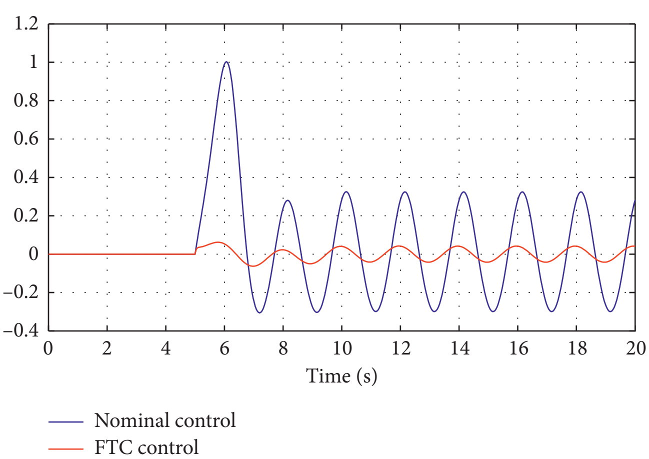

FIGURE 9: System state x_2 with nominal control and fault tolerant control law in example 2.

in Figure 8 and the evolution of system state x_2 with nominal control and fault tolerant control law in Figure 9.

5. Conclusion

In this paper, we have developed an adaptive actuator FTC strategy for a class of polynomial models, and sufficient analysis and design conditions in terms of SOS are proposed. Based on the adaptive fault estimation, a fault tolerant control is designed to guarantee the stability of the closed loop systems despite fault presence. Two simulation examples are presented to demonstrate the applicability of the proposed polynomial active fault-tolerant controller using the adaptive fault estimation algorithm. As future work, we will investigate the problem of adaptive fault estimation and fault-tolerant control for polynomial fuzzy systems with time delay considering measurable and unmeasurable premise variables.

Data Availability

The data used to support the findings of this study are available upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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