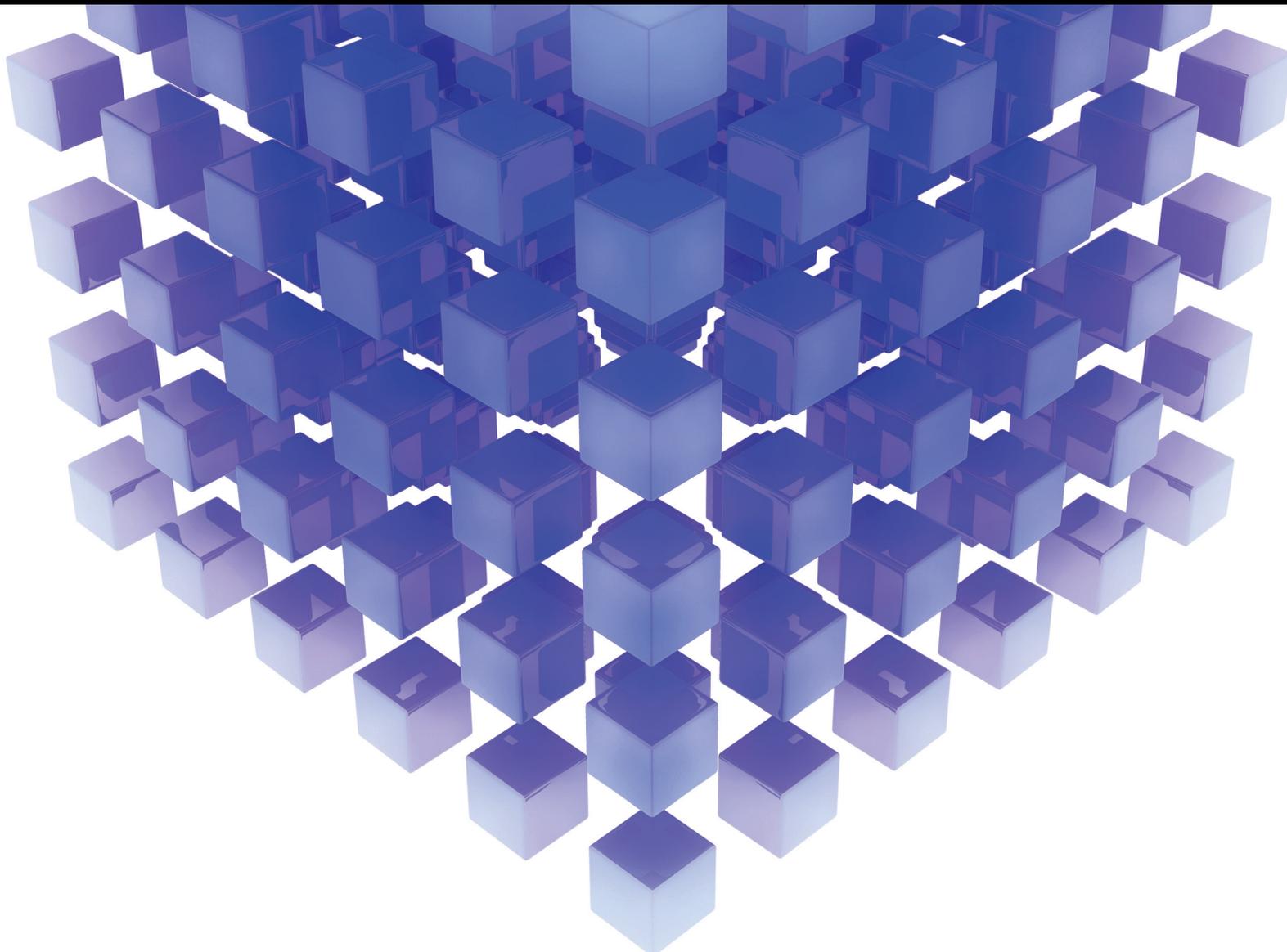


# Artificial Intelligence-Based Mathematical Modeling in Engineering: Recent Advances and Applications

Lead Guest Editor: Peiyong Zhang

Guest Editors: Muhammad Zakarya and Haotong Cao





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Mathematical Problems in Engineering

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## Research Article

# An Experimental Testing of Optimized Fuzzy Logic-Based MPPT for a Standalone PV System Using Genetic Algorithms

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The choice and the dimensioning of the controller for the maximum power point tracking (MPPT) are determined for the ideal energy efficiency of the photovoltaic (PV) systems. Many works have been developed in the field of MPPT methods, especially fuzzy logic controllers. However, these are robust if the parameters of the membership functions have been well designed. In this paper, the performances of an intelligent fuzzy logic controller (FLC)-based MPPT method have been optimized by an evolutionary genetic algorithm (GA). The works presented in the literature have shown the efficiency of the proposed method compared to classical methods. In our paper, the validation of the experimental results obtained is given with respect to a reference signal. The control of the simulated PV source and the proposed method are built using the Simulink/Matlab environment and implemented on the dSPACE DS1104 controller to validate the practical execution of the suggested method. The standalone PV system has been tested in an emulated test bench experimentation. Experimental results confirm the efficiency of the proposed method and its high accuracy in handling fast varying load conditions.

## 1. Introduction

Renewable energy is critical to sustainable development around the globe. Photovoltaic (PV) energy is one of the most abundantly available and cost-effective renewable energy sources. PV power is utilized in grid-integrated, standalone, and hybrid energy systems [1]. Standalone PV systems are utilized in distant locations where power from the main AC grids is not available. PV systems that include an energy storage system. PV power is also integrated into

a grid using power converters, and in order to meet the standards of grid codes, the integration of such power source needs sophisticated control techniques. Hybrid power systems can be produced by combining solar photovoltaic systems with wind, tidal, and thermal energy [2]. However, due to the stochastic nature of solar/PV energy, the output power produced by PV systems is also fluctuating.

The solar panel can give the maximum amount of power to the load at its ideal operating point. Maximum power point (MPP) is the common name for the specific operating

point. Because solar irradiation and cell temperature have a significant impact on the current-voltage characteristic of PV modules, the locus of MPP exhibits nonlinear change. The MPPT is to be created for the PV system due to the nonlinearity of PV modules. At any solar irradiation and temperature, MPPT is able to determine the PV panel MPP operating voltage. The PV system controls the PV module's voltage to the MPP operating voltage. It is possible to draw as much power as you can. As a result, the PV system's effectiveness can be increased [2, 3].

Several research algorithms have been reported to extract the maximum power from solar panels under stochastic environmental conditions. A number of techniques have been presented in recent years as examples, including perturb and observe (P&O) [4], incremental conductance (INC) [5], and fuzzy logic controller (FLC) [6].

The simplest algorithms are P&O and INC. However, due to the fact that the disturbance continues even when the system is functioning at the MPP, these techniques lead to power oscillations around the MPP. The duty cycle step might be decreased to address this problem. However, in this scenario, as atmospheric circumstances change, the system will track the MPP slowly. The PV system's total efficiency is decreased as a result of the power losses. A slight change in the step size in the P&O algorithm disrupts algorithm control. The measurement of PV system output power causes a slight modification to the direction of the step size specified by the P&O technique. Common problems could occur if a PV panel's output power is increased or decreased abruptly [7].

Alternatively, the FLC is used to track an MPP with more accuracy due to its inherent advantages in handling nonlinearity and the lack of a mathematical model, but this algorithm is greatly related to user experience with the PV panel characteristics. As reported in [8], an MPPT algorithm based on the FLC has been applied successfully for a photovoltaic generator system with a variable resistive. From the presented results, the FLC-based MPPT algorithm ensured enhanced performance as compared to the P&O technique. Similarly, in [9], an FLC-based MPPT controller is tested under variable atmospheric conditions. The FLC MPPT controller proposed in [9] guaranteed the optimal operation and performance of the autonomous PV system. In [10, 11], authors reported fuzzy controllers to regulate the power generated in a hybrid system containing PV and wind turbine systems. The study of interval type-3 fuzzy logic controllers (IT3 FLCs) has gotten a lot of attention recently. Numerous studies have demonstrated that IT3 FLCs can manage uncertainties better than their type-1 (T1) and type-2 (T2) counterparts [12].

Even though FLC controllers have many advantages, building them is still challenging since there is no standardized method for locating fuzzy control rules and fine-tuning the membership function parameters of the controllers. To get the better of this limitation, the design procedures are formulated as optimization problems that are typically solved using evolutionary algorithms [13–15], such as genetic algorithms (GAs). GAs are widely regarded as one of the most effective optimization techniques. The GA is

capable of solving a wide range of complex optimization problems, including those with nonderivable cost functions [16]. Genetic algorithms attempt to simulate the evolutionary process of species in their natural environment: an artificial transposition of basic concepts of genetics and the laws of survival stated by Darwin.

A GA is constructed in quite analogous way. In the solution set of an optimization problem, a population of size  $N$  consists of  $N$  solutions (the individuals in the population) suitably marked by coding that identifies them completely. An evaluation procedure is necessary to determine the strength of each individual in the population. Then, there is a selection phase (in which individuals are chosen in proportion to their strength) and a recombination phase (in which artificial operators of crossing and mutation are used to generate a new population of individuals with a good chance of being stronger than those in the previous generation). From generation to generation, the strength of the individuals in the population increases, and after a certain number of iterations, the population is entirely composed of strong individuals, that is to say, of quasi-optimal solutions to the problem.

Theoretical work on the optimization problem of fuzzy controllers has been widely reported in the literature. A GA-based optimization problem is solved in [17] to find the optimal scaling parameters of a fuzzy logic-based MPPT controller that maximizes the efficiency of a PV pumping system. In [18], the algorithm GA is used to optimize the following two controllers: the fuzzy PI (proportional and integrator) and the P&O PI. The simulation results presented in this article have demonstrated that the fuzzy PI-GA is better than the P&O PI-GA in terms of response time. Similarly, in [19], theoretical simulations have been carried out. The authors have used a hierarchical genetic algorithm to design a fuzzy controller for the command of a photovoltaic conversion system.

The implementation aspect of optimized fuzzy controllers has been reported in [20–25]. An experimental study on the MPPT controller using the FPGA (field programmable gate array) has been presented in [21]. Based on the obtained results, the authors concluded that the optimized FLC outperforms the conventional P&O method in terms of response time and steady-state fluctuations. In [23], FPGA implementation of MPPT-based fuzzy logic is investigated for standalone PV conversion systems. In [24], FPGA implementation of a fuzzy controller is reported for a PV system. The authors in [25] discuss the MPPT technique based on GAs for a standalone PV conversion system. The results have been compared with the conventional techniques. The results have been validated experimentally using a dSPACE control board. Another two hardware implementations of fuzzy controllers for MPPT of PV systems have been described in recent works [26, 27]. The experimental results obtained have been compared to a conventional P&O method. The fuzzy controller design has been better than the P&O. However, it shows significant oscillations at the optimal point.

This paper discusses the hardware implementation of a fuzzy controller optimized by the GA algorithm for MPPT tracking using the dSPACE 1104 control board. The

proposed MPPT method is tested for maximum harvesting power under real-time environmental conditions. The following is a summary of the primary differences between the current work and the related literature:

- (i) The fuzzy controller is optimized to improve the convergence speed for maximum power harvesting of the PV system. The suggested technique can also track the global MPP (GMPP) efficiently, which is very beneficial in variable atmospheric conditions.
- (ii) The proposed algorithm is tested for fast-changing solar irradiation and a variable resistance load.
- (iii) In the cited literature [20–25], the design of a fuzzy MPPT controller using the GA optimization algorithm is accomplished for a learning profile of constant conditions ( $T = 25^\circ\text{C}$  and  $S = 1000 \text{ W/m}^2$ ). However, atmospheric conditions are constantly changing. In this work, different variations of the irradiance ( $600 \text{ W/m}^2$ ,  $1000 \text{ W/m}^2$ , and  $800 \text{ W/m}^2$ ) are adopted to test the robustness of the optimized control utilized.

The paper's sections are structured as follows: Section 2 is a description of the system. Section 3 shows the design procedures of the fuzzy controller. Section 4 shows the proposed optimized control strategy. Section 5 presents the steps of the details of the experimental real-time platform. The result discussion is also presented in this section. Finally, Section 6 presents a general conclusion that will summarize the content of this work and put forward the results obtained.

## 2. Standalone PV System Description

Figure 1 shows a block diagram of a standalone PV system. The components of the system are the PV panel connected to a variable resistive load through a matching stage. The latter is composed of a boost converter. The switch of the converter is controlled by the signals generated by the proposed method.

**2.1. PV Panel Model's Mathematical Equation.** There are a few different sorts of models, such as single-diode and two-diode models [28, 29]. The two-diode model accounts for a second diode that is wired in parallel with the first diode in the circuit that functions as a single diode's equivalent. Compared to a two-diode model, the one-diode model has fewer parameters and is simpler to model. The electrical properties  $P(V)$  and  $I(V)$  of the solar panel as simulated and experimental data clearly demonstrate that the results are identical, according to the paper [29]. The one-diode model of the solar cell is used in this paper [29].

The PV panel model's mathematical equation is formulated according to the cell's number in series only, and the cell's number in parallel is equal to 1 according to our BP SX150S model in the following equation:

$$I_{pv} = I_{ph} - I_s \left( \exp \left( \frac{V_{pv} + I_{pv} R_s}{N_s V_T} \right) - 1 \right) - \left( \frac{V_{pv} + I_{pv} R_s}{R_{sh}} \right), \quad (1)$$

where  $V_T$  is equal to  $(a.k.T/q)$ .

The PV panel electrical parameters shown in equation (1) are reported in Table 1.

Table 2 lists the BP SX150S solar panel from BP solar, which is reported at the Standard Test Condition (STC, i.e.,  $25^\circ\text{C}$  and  $1000 \text{ W/m}^2$ ).

**2.2. Boost Power Converter.** The boost power converter is inserted between the PV panel and the load as an impedance-matching stage. The PV output voltage ( $V_{pv}$ ) is regulated to keep it at the nominal voltage ( $V_{MPP}$ ) by means of an MPPT controller. Voltage regulation is equivalent to controlling the opening and closing of the IGBT power switch through a pulse width modulation (PWM) technology. The IGBT switching frequency is 8 kHz.

To get the most power out of the PV panel that is now available, a new duty ratio  $D$  of the PWM signal must be generated in real-time. Table 3 lists the parameter values for the boost converter that was designed. The mathematical equation of the boost converter's output voltage and current has been given as follows [30]:

$$\begin{cases} V_o = \frac{1}{1-D} V_i, \\ I_o = (1-D) I_i, \end{cases} \quad (2)$$

where  $V_o$  and  $I_o$  are the output's voltages and currents of boost converters, respectively, and  $V_i$  and  $I_i$  are the input's voltages and currents of boost converters, respectively.

## 3. Fuzzy MPPT Controller

The fuzzy MPPT controller is an intelligent method of tracking a PV system's maximum power point. In lieu of a precise mathematical model, it uses the fuzzy set theory. The internal functioning of a fuzzy controller of the Mamdani type is based on the structure presented in Figure 2, which includes four blocks [31].

The fuzzification consists in calculating, for each real input value, the degrees of membership to the associated fuzzy sets predefined in the database of the fuzzy system. This block carries out the transformation of the real inputs into symbolic information that can be used by the inference mechanism.

The inference mechanism consists, on the one hand, in calculating the degree of truth of the different rules of the system and, on the other hand, in associating an output value to each of these rules. This output value depends on the conclusion part of the rules, which can take several forms. It can be a fuzzy proposition, and we will speak of a rule of type Mamdani "IF-THEN" in this case:

$$\text{IF } (\dots) \text{ THEN } Y \text{ is } X, \quad X \text{ is set flou.} \quad (3)$$

The defuzzification consists in replacing the set of output values of the various rules resulting from the inference by a single real value representative of this set.

The inputs and outputs of the FLC controller are represented by the triangular and trapezoidal MFs. In Table 4, the letters P and N stand for positive and negative linguistic

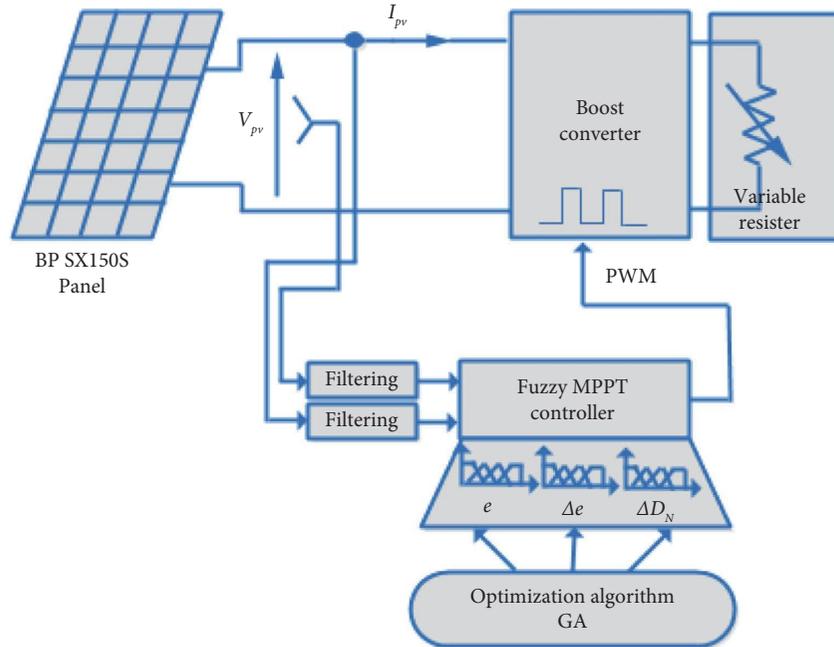


FIGURE 1: The illustrative diagram of the implemented PV system.

TABLE 1: PV panel electrical parameters.

Parameters	Designations
$V_T$	Diode thermal voltage
$a$	Diode ideality factor
$K$	Boltzmann constant
$T$	Cell's temperature
$Q$	Electron charge
$I_{ph}$	Light generated current
$I_s$	Diode saturation current
$R_s$	Series equivalent resistances
$R_{sh}$	Parallel equivalent resistances

variables, respectively. The letters B, S, and ZE additionally stand for Big, Small, and Zero. Five separate linguistic variables are allocated to each input variable,  $e(k)$  and  $\Delta e(k)$ . As a result, there are 25 different fuzzy rules in the suggested set of fuzzy rules. The entire set of fuzzy rules is presented in Table 4 [31].

The two input variables that are characterized by the following expressions at a sampling instant  $k$  are the error equation (4) and error variation equation (5):

$$e(k) = \frac{P_{pv}(k) - P_{pv}(k-1)}{V_{pv}(k) - V_{pv}(k-1)}, \quad (4)$$

$$\Delta e(k) = e(k) - e(k-1). \quad (5)$$

Based on these two inputs, the FLC calculates the subsequent operating point using MFs and a rule table. The operating point will be on the right or left side of the MPP, depending on whether E is positive or negative. When E equals zero, the MPP is reached. The operational point moves in the MPP direction according to the e-input.

The duty ratio  $D$  is computed as follows:

TABLE 2: The BP SX150S solar panel.

Parameters	
Nominal power ( $P_{MPP}$ )	150 W
Output power tolerance	$\pm 5\%$
Nominal current ( $I_{MPP}$ )	4.35 A
Nominal voltage ( $V_{MPP}$ )	34.5 V
Open circuit voltage ( $V_{oc}$ )	43.5 V
Short circuit current ( $I_{sc}$ )	4.75 A
Cells number in series ( $N_s$ )	72

TABLE 3: DC-DC boost converter specifications.

Electrical specifications	
Inductor ( $L$ )	0.6 mH
Input capacitor ( $C_i$ )	500 $\mu$ F
Output capacitor ( $C_o$ )	2200 $\mu$ F
Switching frequency ( $f_s$ )	8 kHz
IGBT	SKM50GB12T4

$$D(k) = G_D \times \Delta D_N(k) + D(k-1), \quad (6)$$

where  $\Delta D_N$  is the duty ratio at the controller output and  $G_D$  represents the factor's scaling output.

#### 4. Proposed Optimized Fuzzy-Based MPPT

Generally speaking, the following steps can be used to define the genetic algorithm.

##### 4.1. Procedure Genetic Algorithm

*Step 1* (initialization). Generate an initial population Pop ( $t$ ) of size N of chromosomes in a random manner.

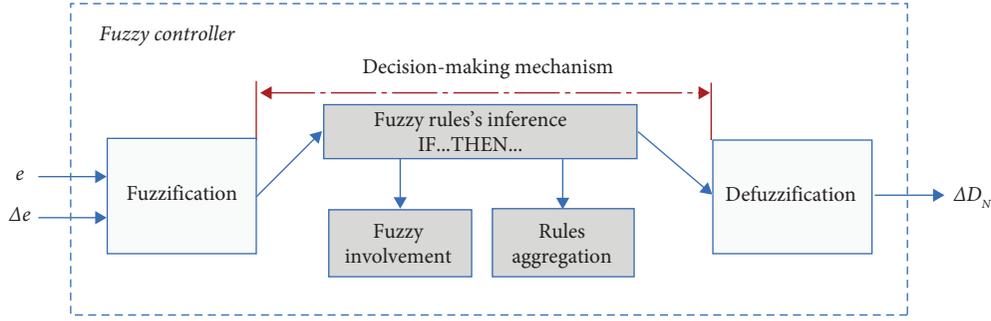


FIGURE 2: The synoptic diagram of a fuzzy controller.

TABLE 4: The rule base table for the fuzzy MPPT controller.

$e$	$\Delta e$				
	NB	NS	ZE	PS	PB
NB	ZE	ZE	PB	PB	PB
NS	ZE	ZE	PS	PS	PS
ZE	PS	ZE	ZE	ZE	NS
PS	NS	NS	NS	ZE	ZE
PB	NB	NB	NB	ZE	ZE

*Step 2 (evaluation).* Each chromosome is decoded and evaluated Pop ( $t$ ).

*Step 3 (Selection).* Production of a new population of N chromosomes with the use of a suitable selection technique, select Pop ( $t$ ) from Pop ( $t-1$ ).

*Step 4 (recombination).* According to their probability, crossover, and mutation of some chromosomes within the new population.

*Step 5 (termination).* To phase 2, as long as the problem stop condition is not satisfied.

**4.2. Structure of Chromosomes.** In this paper, the membership functions of the fuzzy MPPT controller are optimized. The inputs and output membership functions are each defined by five parameters. For a fuzzy controller with two input variables and one output variable, the membership function numbers are in order  $3 \times 5$ . Therefore, the GA chromosome's structure is given as a vector of fifteen parameter values, as shown in Figure 3.

**4.3. Initial Population.** The chromosomes of the original population are set to random variables. A sequence of random numbers  $r_{i,j}$  between 0 and 1 is created for each element  $X_{i,j}$  of particle  $i$ . Then, by projecting  $[0, 1]$  into  $[X_j^L, X_j^U]$ ,  $X_{i,j}$  is determined as follows:

$$X_{i,j} = X_j^L + r_{i,j} \times (X_j^U - X_j^L), \quad (7)$$

where  $X_j^L$  represents the inferior limit of  $X_{i,j}$  and  $X_j^U$  represents the upper limit of  $X_{i,j}$ .

**4.4. Optimization Criterion.** The goal of designing a fuzzy MPPT controller is to discover the optimal settings that minimize energy loss in the PV system, which is mostly caused by meteorological conditions. The optimization criterion's objective function given in equation (8) describes an integral squared error (ISE) function. At each learning step, an ISE value is calculated, and at the end, the best value is returned.

$$ISE = \int_0^{t_f} (e(t))^2 dt, \quad (8)$$

where  $e(t) = P_{\max}(t) - P_{pv}(t)$ ,  $P_{\max}$  is the PV panel's rated power,  $P_{pv}$  is the PV panel's instant power, and  $t_f$  is the simulation time.

**4.5. Selection Process.** The selection process is applied to the chromosomes of the algorithm. This process is the first step in the selection of the best chromosomes suitable for replication. The tournament selection is used in this work. This selection technique uses proportional selection on chromosome's pairs and then chooses from these pairs the chromosome with the best adaptation (fitness) score.

**4.6. Crossover and Mutation Operators.** In this study, for a real-coded genetic algorithm, the Laplace crossover operator is suggested [32, 33]. For the real string, a real value mutation has been created. Each parameter  $X_{i,j}$  receives an addition of a random with the probability rate pm. The direct application of this mutation can create new parameters outside the interval  $[X_j^L, X_j^U]$ . Therefore, we propose the following mutation equation (9) for keeping parameters in their range of variation:

$$X_{i,j} = \begin{cases} X_{i,j} + \text{rand}_1 \times (X_j^U - X_{i,j}) & \text{if } \text{rand}_3 < 0.5, \\ X_{i,j} + \text{rand}_2 \times (X_j^L - X_{i,j}) & \text{if } \text{rand}_3 \geq 0.5, \end{cases} \quad (9)$$

where  $\text{rand}_1$ ,  $\text{rand}_2$ , and  $\text{rand}_3$  are random numbers between  $[0, 1]$ .

## 5. Experimental Results and Discussion

The experimental test bench for the simulated PV system is depicted in Figure 4. The GREAH laboratory in France is where the hardware implementation was developed. The implementation in real time of the proposed MPPT

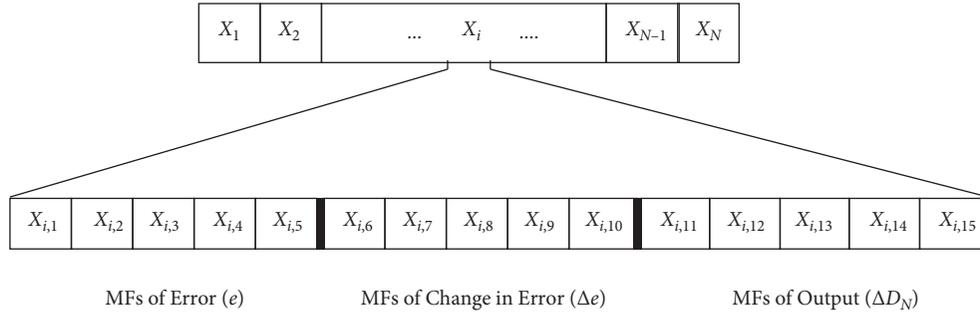


FIGURE 3: The  $N$  chromosome's real coding structure.

controller has been given by a dSPACE DS1104 board. The Simulink model of the PV panel as well as the solar radiation and temperature profiles were implemented on a Dspace 1104 board. The reference current and voltage signals computed by the panel control an adjustable DC power supply. Figure 5(a) shows the acquisition chain of the PV source emulator (EPVS). The emulated system involves the use of a boost converter to connect an EPVS to a variable DC load, and the boost's converter specs are illustrated in Table 3. The block for generating PWM signals from the duty ratio values is given in Figure 5(b). The two sensors for measuring voltage and current are Cleqee A622 and TA057, respectively. The sensors have been connected to the dSPACE acquisition board through the ADC ports. The measurements of the sensors have been filtered with digital filters implemented on the dSPACE board, as can be seen through Figure 5(a). The experimental results in Figure 6 have been given at a varying load profile. The load variation was controlled from a dSPACE signal. Figure 5(a) shows the blocks providing the load variation. The observed voltage and current signals are rescaled using gain scales of 10 and 20, respectively. The sample step time in this study is set to  $50 \mu s$ .

Elgar 5500, a programmable DC power supply, has been used to emulate the electrical characteristics of the PV panel. With the help of this panel emulation, it is possible to make up for the absence of the PV panel and simulate variations in the different profiles. A dSPACE DS1104 controller with a  $50 \mu s$  sample period is utilized to create the PV panel Simulink model. The boost converter is controlled by the last via PWM signals with an 8 kHz switching frequency. A DAC output is used to create the analog signal (0-10 V range) needed to command the Elgar 5500 source. The practical properties of the EPVS, as determined by adjusting the power DC supply's output current, are shown in Figure 7

**5.1. Optimized Fuzzy MPPT Controller Using dSPACE Implementation.** As discussed in the introduction, the reason for implementing the fuzzy MPPT approach is to verify the proposed algorithm experimentally using a dSPACE control board. The fuzzy MPPT controller is computed and optimized under Matlab/Simulink. The control system is set up in accordance with the configuration described in Section 3.

Five membership functions are used to calculate the output variable ( $\Delta D_N$ ) for the FLC as well as the input variables ( $e$  and  $\Delta e$ ). The variation's ranges for  $\Delta e$ ,  $e$ , and the output are  $[-50, 50]$ ,  $[-35, 5]$ , and  $[-1.5, 1]$ , respectively.

The dSPACE board is the appropriate hardware prototyping improvement solution for doing real-time simulations in many domains and prototyping high-speed digital controllers. These controllers use the MATLAB real-time interfacing toolbox to connect the SIMULINK model to the actual hardware models. Control desk software is used as an acquisition management tool to facilitate real-time analysis of system performance and visualization of PV output waveforms. It is simple to adjust the controller's settings, manage the output load, or change the PV panel's model's mimicked weather patterns using the control desk software. The obtained currents and voltages may also be easily stored and displayed. The main points in the fuzzy MPPT-GA designing in this article are as follows:

- (1) GA learning algorithm
- (2) Experimental testing results

**5.1.1. GA's Learning Profile.** It is important to note that the optimal parameters of the fuzzy MPPT controller obtained by the GA are strongly related to the adopted methodology. The richer the learning profile and the more real variations in atmospheric conditions are taken into account, the higher the performance of the GA should be. At each step of the algorithm, we have to compromise between exploring the search space to avoid stagnating in a local optimum and exploiting the best individuals obtained to reach better values in the surroundings. If the individuals of a population are too similar, the following populations may become more and more homogeneous. In this case, the evolution of a population may be reduced to the evolution of a single dominant individual, thus reducing the exploration of the search space (premature convergence). In order to be able to efficiently search, it is, therefore, required to maintain a balance between the exploitation of the good solutions encountered and the exploration of unknown areas. An excess of exploitation can lead to a premature convergence (boggling down in a local optimum), just as an excess of exploration could lead to a quasi-random search (no convergence).

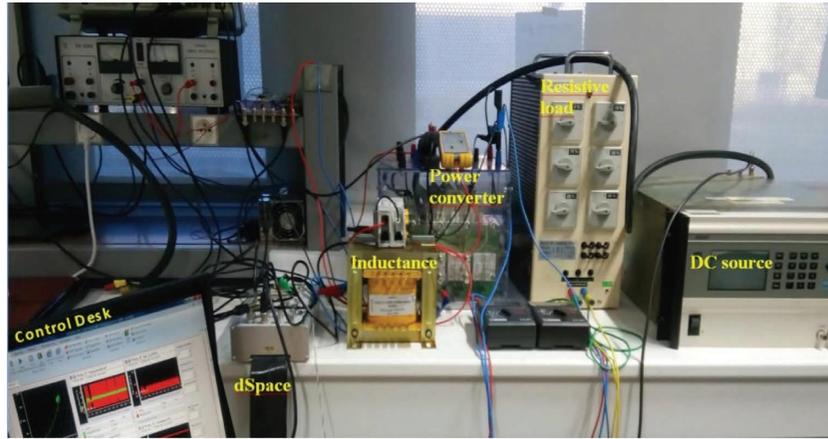


FIGURE 4: The experimental real-time platform.

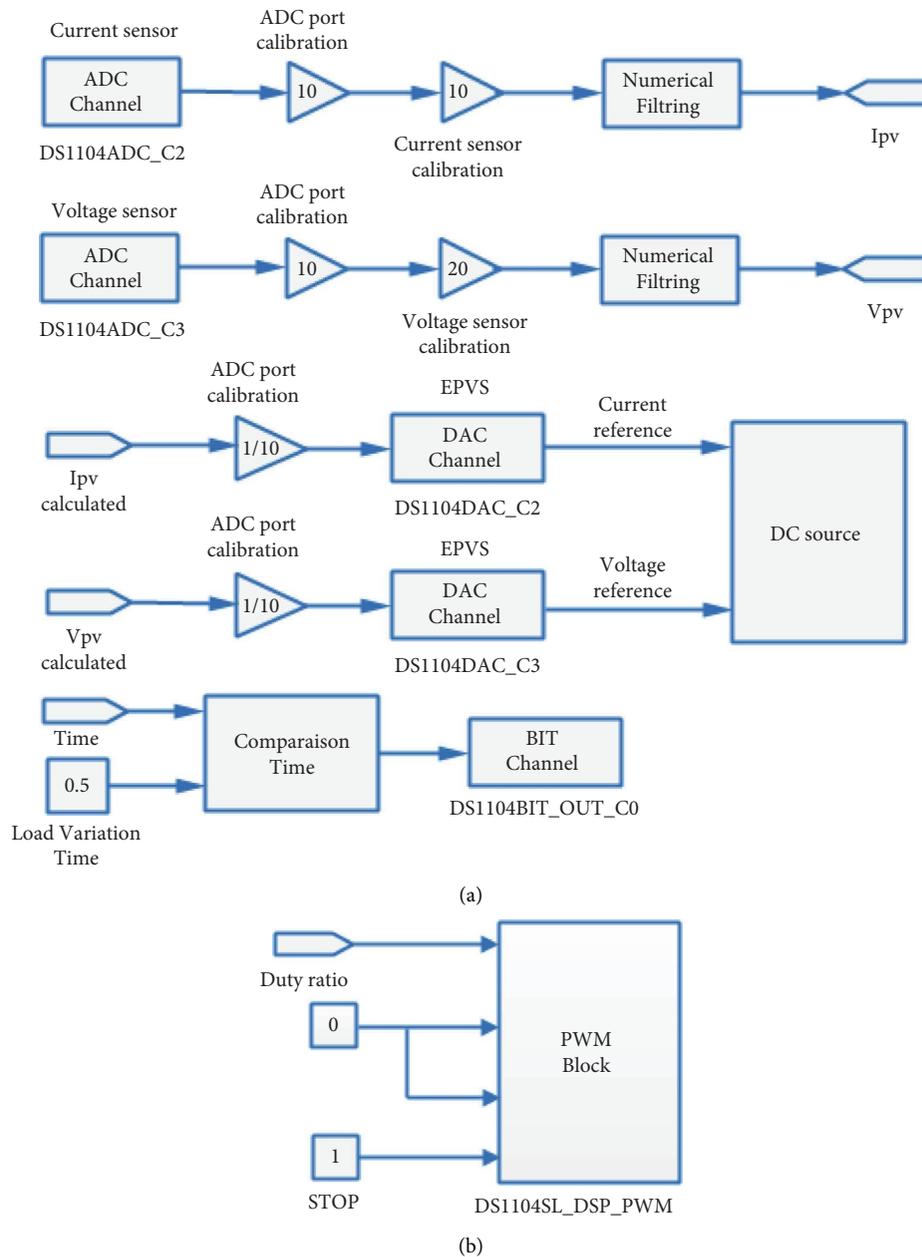


FIGURE 5: (a) ADC and DAC conversion blocks and (b) the PWM block.

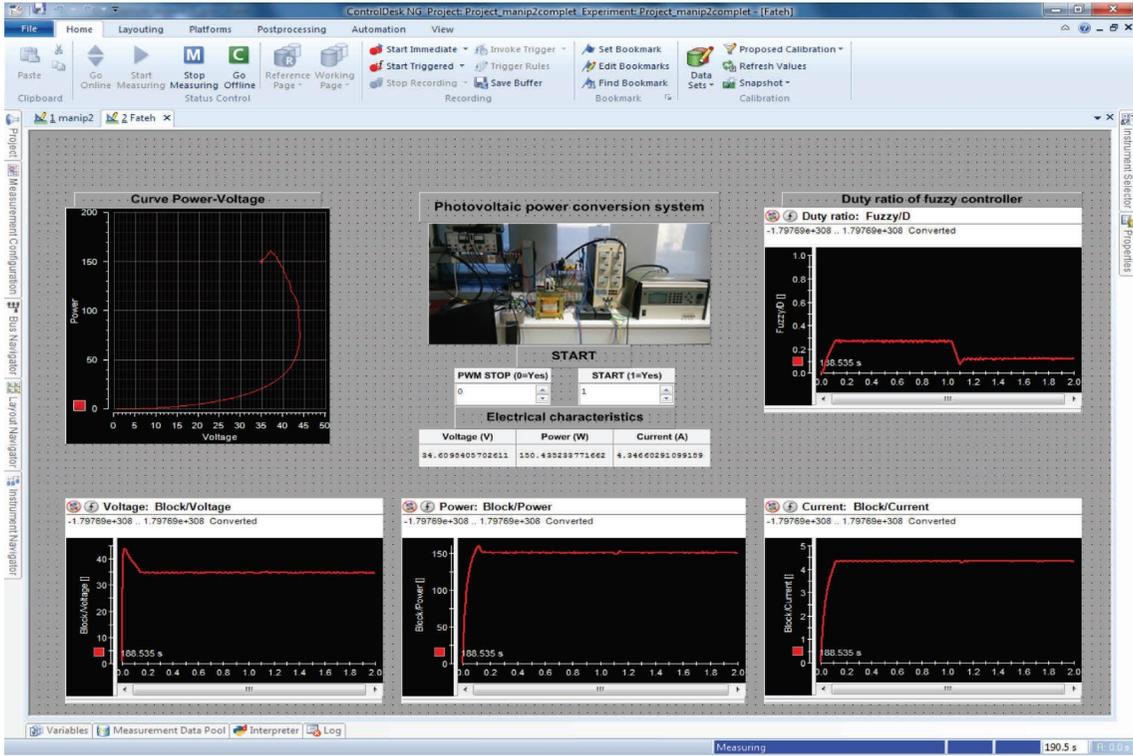


FIGURE 6: The control desk interface of the EPVS system.

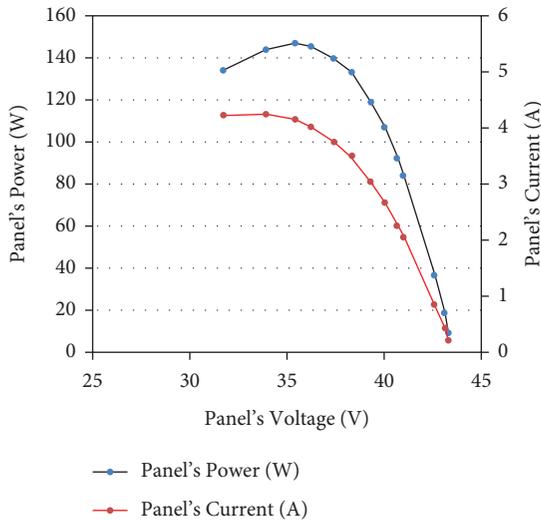


FIGURE 7: EPVS's experimental electrical characteristics.

The learning profile of the GA adopted is important for an optimal parameter result and must include any scenario that the system may encounter in the irradiance variations and from the values returned by the objective function ISE. A selection of the best chromosome (individuals) has been made by the algorithm. For this purpose, the parameters of the best chromosome were introduced to the fuzzy MPPT controller.

The value in the learning profile begins at  $600 \text{ W/m}^2$  and changes at  $1100 \text{ W/m}^2$  after 2 s; this value is maintained until 4 s after another change occurs to reach the value of  $800 \text{ W/m}^2$ .

The simulation results are given at a population of 20 chromosomes, and the training of the algorithm has been given at 200 iterations. The values of the fitness function ISE evolve in a decreasing way from iteration 01 to iteration 50. Beyond this iteration, the evolution of the curve is constant until it reaches the value ISE 0.372. The fact of training the algorithm beyond 50 iterations tells us that the algorithm has found a global optimum. The evolution of the ISE function curve per iteration is shown in Figure 8.

**5.1.2. Experiments Testing Results.** After the learning process is accomplished, we can see the tracking performance corresponding to the optimal solution obtained by the GA. The experimental testing results are obtained using the following two scenarios. The studied EPVS system has been evaluated in a 2 s total duration with a  $50 \mu\text{s}$  fixed step size during the testing phases.

- (i) In the first test, the solar irradiance and temperature are kept constant at STC conditions, but an instantaneous load change is performed at 1s. The resistance goes from a value of  $20 \Omega$  to  $14 \Omega$ .
- (ii) In the second test, the load is kept constant at  $R = 20 \Omega$ . However, the goal of this test is to study the impact of the fast-changing of solar irradiance on the performance of the proposed MPPT algorithm.

(1) *Performance Test at Standard Conditions and Fast Varying Load Conditions.* An interface controls the

parameters of the experimental test bench of the EPVS system and thus displays the curves and results of the electrical characteristics. The control of parameters is given by two buttons: START and PWM STOP. In order to activate the control part, the START value is set to 1, and the PWM STOP value is set to 0. The values of the electrical characteristics represented are voltage (V), power (W), and current (A). The curves shown are the power-voltage curve, the duty cycle of the proposed controller, and thus the voltage, power, and current.

The results represented in this interface have been given for a temperature of 25°C and sunshine 1000 W/m<sup>2</sup> as well as a fast-varying resistance load. Figure 6 shows the control desk interface of the EPVS system.

Figures 6 and 9 show the results of the generated EPVS power of the proposed fuzzy MPPT-GA. In the presented results, during the transient load step of  $R = 20 \Omega$  between 0 s and 0.5 s, it can be seen that the proposed fuzzy MPPT-GA converges rapidly to the MPP at a time of 0.08 s with a tracking error of 3% and a steady state-error of 99.87% and then the power is maintained around 150 W with an extremely low steady-state error of about  $\pm 0.13\%$ . The total error between the tracking error and the steady-state error is about 96.87%.

In the second transient load step of  $R = 14 \Omega$  between 0.5 s and 1 s, it is clearly observed that the proposed fuzzy MPPT-GA shows better power generation, which much perfectly rated around 150 W with an extremely low steady-state error of about  $\pm 0.13\%$  (99.87%) as depicted in Figures 6 and 9. It can be concluded that the proposed fuzzy MPPT-GA completely follows the maximum power of 150 W to the STC test profile and also to the fast-varying load conditions.

Figure 10 shows the result of the duty ratio of the proposed fuzzy MPPT-GA at the STC. It can be observed a variation in the duty ratio value as soon as 0.5 s. In the presented results, during the transient load step of  $R = 20 \Omega$  between 0 s and 0.5 s, it can be seen that the proposed fuzzy MPPT-GA generates a duty ratio of the value  $D = 0.25$ . It is evident that the proposed fuzzy MPPT-GA generates the corresponding new duty ratio  $D$  ( $D = 0.15$ ) for each transaction period in the second transient load level of  $R = 14 \Omega$  between 1 s and 2 s, which establishes the new position of the desired MPP inversely. In this case, the duty ratio of the proposed fuzzy MPPT-GA is optimally adjusted. It is clearly shown that a perfect duty ratio is generated in order to keep the power produced at its desired value.

(2) *Performance Test under Fast-Changing Solar Irradiation.* The test at standard conditions, as well as at the fast variation of the load, has been successfully performed. Now, a test at fast variation solar irradiation has been performed. The load was held fixed at  $R = 20 \Omega$ . The choice of the value between  $R = 20 \Omega$  or  $R = 14 \Omega$  is not important since the result given in Figures 6 and 9 clearly shows that the power is maintained at 150 W with an error of 0.13% at the static regime for both the resistance values.

Figure 11 shows the irradiance profile, which varies between 1000 W/m<sup>2</sup> and 1100 W/m<sup>2</sup>, and both the

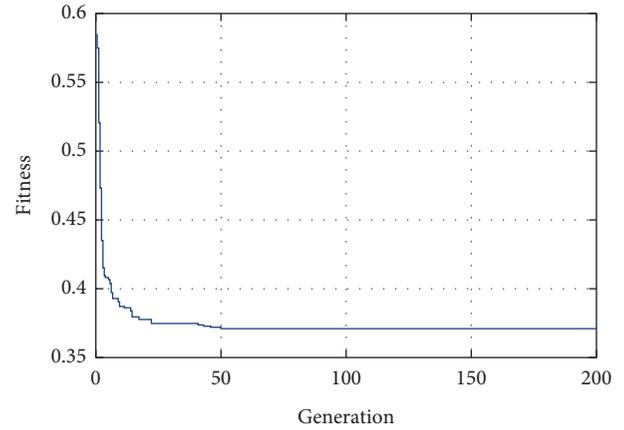


FIGURE 8: Evolution of the fitness function curve.

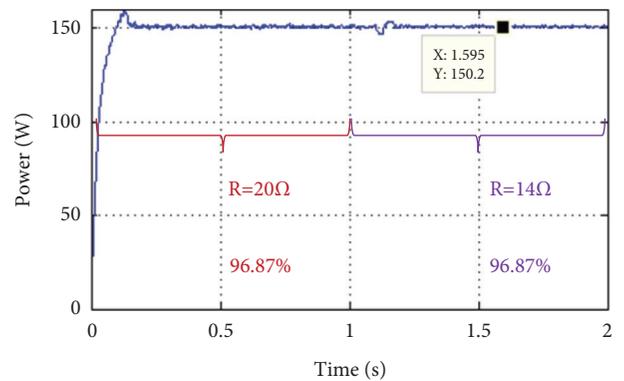


FIGURE 9: The experimental result of PV power under STC and fast-varying load.

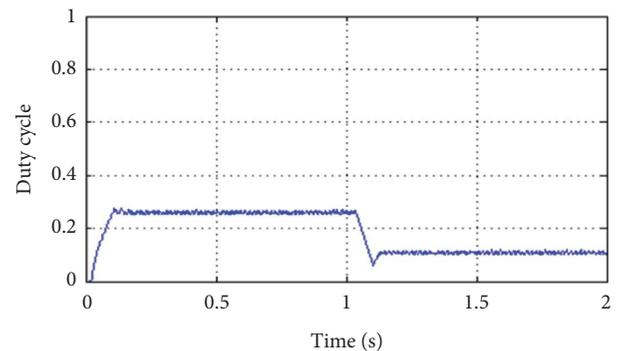


FIGURE 10: The experimental result of the duty ratio  $\Delta D_N$  under STC and fast-varying load.

temperature and the load charge are kept constant during the experiments at 25°C and  $R = 20 \Omega$ , respectively.

The experimental performance test has been carried out at the fixed charge  $20 \Omega$  and the fast-changing solar irradiance, as shown in Figure 11. In this test case, the objective has been to evaluate and observe the response of the proposed fuzzy MPPT-GA only to the fast-changing solar irradiance and at the constant ambient temperature 25°C.

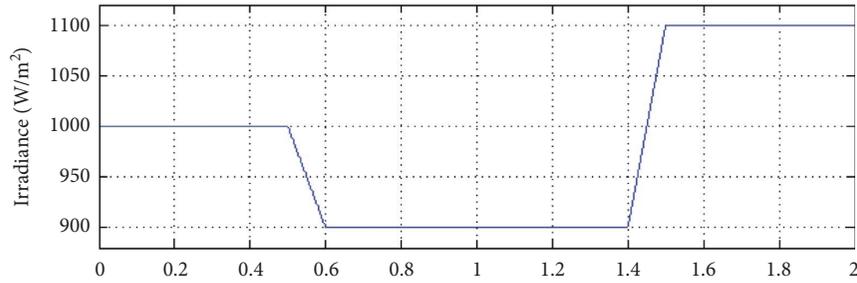


FIGURE 11: The irradiance profile.

Figure 12 shows the results of the generated EPVS power of the proposed fuzzy MPPT-GA. In the presented results, during the transient irradiance value between 0 s and 0.5 s, it can be seen that the proposed fuzzy MPPT-GA converges very rapidly to the MPP at a time of 0.06 s with a tracking error of 2.6% and a steady-state error of 99.85% and then the power is maintained around 150 W for a 1000 W/m<sup>2</sup> with an extremely low steady-state error of about  $\pm 0.10\%$ . The total error between the tracking error and the steady-state error is about 97.25%. In the second step, the transient irradiance value is between 0.5 s and 1.4 s. It can be seen that the proposed fuzzy MPPT-GA also converges at the MPP with an extremely low steady-state error of about  $\pm 0.12\%$  (99.82%). In the last step, during the transient period of 1.4 s to 2 s, it can be seen that the proposed fuzzy MPPT-GA also converges at the MPP with an extremely low steady-state error of about  $\pm 0.14\%$  (99.79%). The efficiency of the EPVS system in the three transient irradiance values has been given at 98.95%. It can be clearly observed that the result, when the MPP abruptly changes, is that the proposed fuzzy MPPT-GA may immediately push the EPVS system to the new MPP.

Figure 13 shows the result of the duty ratio of the proposed fuzzy MPPT-GA under fast-changing solar irradiation. It can be clearly observed that a new value of the duty cycle has been generated at each variation of the irradiance. In this case, the duty ratio of the proposed fuzzy MPPT-GA has been optimally adjusted in order to keep the operating point at each instant at the MPP.

Now, the proposed fuzzy MPPT-GA has been compared with a few strategies which have been published in recent works to optimize FLC-based MPPT meths. Authors in [18, 24, and 25] have performed the performance tests of standalone PV systems only at STC conditions with a fixed resistance load. The results of tests for a profile of variable irradiance that present overshoots when changing the values of irradiance more or less different from one paper to another have been presented in [18, 19, and 21].

For a more logical comparison, the optimization algorithms should use the same objective function (equation (7)), the same initial population, and also the same number of iterations. The advantages of our design strategy, which provides the best objective function (ISE) value at the conclusion of the optimization process, have been shown in Figure 8.

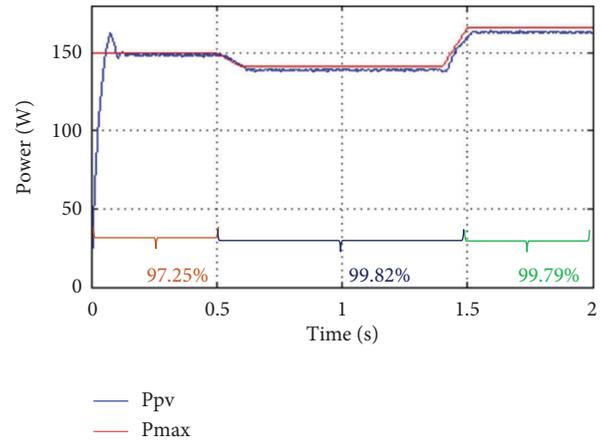
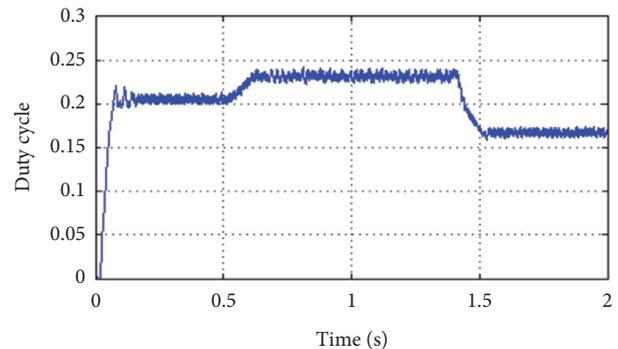


FIGURE 12: The experimental result of PV power under fast-changing solar irradiation.

FIGURE 13: The experimental result of the duty ratio  $\Delta D_N$  under fast-changing solar irradiation.

## 6. Conclusion

This paper presents a modeling and experimental validation of a standalone PV system. The system under investigation consists of a DC power source that simulates a solar panel, a DC/DC boost converter, a resistive load, and a real-time maximum power point tracking controller integrated into a dSPACE DS1104 board. The GA has been successfully applied in this study to optimize the productivity of a fuzzy MPPT algorithm by improving the controller's membership function parameters. From the objective function values obtained in the simulation, it is seen that the GA provides

a good approach for designing efficient fuzzy MPPT. The following are the main highlights of the current work:

- (i) The proposed GA algorithm's learning profile was extremely important. The results obtained during the tests using the various profiles clearly show that the parameters of the fuzzy controller's input/output membership functions determined during the optimization process accurately follow the GMPP with a steady-state error of around  $\pm 0.13\%$ .
- (ii) Similar to that, a reference signal has been used to compare the proposed approach. The outcomes demonstrated the suggested algorithm's capability to track the GMPP with quicker convergence and fewer power fluctuations than before. The suggested fuzzy controller optimized-based MPPT's viability and efficacy have been tested experimentally, and the findings unmistakably show that it is capable of tracking the GMPP with an average efficiency of 98.66% and an average tracking time of 0.08 s and 0.06 s under the STC and the fast-changing solar irradiance, respectively.

The outcomes demonstrated that the suggested approach was workable and that it was capable of tracking the maximal power point with high efficiency that exceeded 97.8% in all tested scenarios. The method proposed has been described in some detail and can be used in other similar systems.

## Data Availability

The data supporting the findings of the current study are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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## Research Article

# Intelligent System for Optimal Geometric Design Using Fuzzy Soft PDE

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In this research, a methodology is discussed to develop an intelligent system for optimal geometric design for the culinary product. An optimum geometric design meets certain specified criteria in the most efficient or cost-effective way possible. The criterion for an optimum design will depend on the specific goals and constraints of the specific problem. In this methodology, partial differential equation and weighted Bonferroni mean are consolidated to develop the intelligent system for optimal geometric designs for the required product with the blend of fuzzy soft sets. Fuzzy soft sets allow capturing the incorporation of subjective or personal opinions into decision-making processes, as well as the consideration of multiple conflicting criteria. A parameter known as the smoothness parameter is used to control the shape of the optimal geometric model. The smoothness parameter, used as a fuzzy number, is important in this developed system as it fulfills the requirements for the desired intelligent system for product design according to the industries' demands. To verify the credibility of this system, an illustrated example is presented to design a culinary product, which is profitable for the hotel industry.

## 1. Introduction

Computer-aided design (CAD) is a technique discussed by Sarcar et al. in [1]. This technique is used in the creation, modification, analysis, and optimization of the geometric modeling of products with the help of computers. As mentioned in [2], the fewer number of design variables and the intuitive control of the design for the user are the key aspects of efficiency in systems used for the mathematical modeling of three-dimensional surfaces. Surface modeling, solid modeling, and particle system modeling are the different types of three-dimensional modeling discussed in [3].

Authors discussed in [4], when one surface patch of a set of patches is varied, it creates difficult situations such as maintaining the smooth continuity of the shape. According to Frey and Borouchaki in [5], a surface can be defined geometrically by moving a line in space in a direction other than the line itself. The surface under consideration can be defined as the area swept by the movement. If either the line or the movement is curved, the resulting surface is curved.

The direction of the line is known as the  $g$  direction in mathematics, and the direction of movement of the line is known as the  $h$  direction. The  $h$  parameter notation is crucial in CAD concepts and plays an important role in generating surfaces as discussed in [6]. Patches are classified based on the shape of their originating curves. As mentioned in [7], Bezier splines produce a Bezier patch, and two B-spline curves produce a B-spline patch, as discussed in [8]. A patch will inherit features such as control points from its original spline curves. As a result, the number and position of a patch's control points are determined by the control points associated with each of the spline curves. These control points can be used to modify the patch in the same way that control points can be used to manipulate a curve.

The smoothness and extendibility of the surface's topology are the two most important issues in the computer modeling of surfaces. As mentioned in [9], patches, which are mathematically truly curved, are the solution for smooth surfaces. They must, however, be constructed with grid-like rows and columns. According to the author's explanation in

[10], surfaces are subdivided to make them smooth. To achieve the smooth curvature of the surface, the simple polygonal model is subdivided into smaller polygons in this technique. The extent to which the original polygons are subdivided is determined by the curvature of the end surface. When a curve or surface is infinitely subdivided, the result is truly curved and is known as a limit curve or limit surface as discussed in [11]. The most important aspect of the subdivision technique is that it allows to manipulate the shape of the model with both the original polygonal model and the subdivision model. When the original polygonal model is used, the subdivision result is also modified. Moreover, if the subdivision model is manipulated, the changes will have no effect on the original polygon model.

The surface generation method based on partial differential equations, mentioned in [12], is another efficient mathematical model for representing real-world objects that addresses some of the drawbacks discussed in the preceding methods. It is particularly appealing due to the fact that it allows for intuitive manipulation of the shape of the object with minimal user interaction. With a slight change in its design parameters, it generates a wide range of shapes from a single equation. This technique, also known as the PDE method discussed in [13, 14], entails solving a suitable partial differential equation over a set of boundary conditions. A design parameter is also included in the equation to give the user more flexibility in changing the shape of the object with less effort. This method's rendering process is also simple, as it entails solving a mathematical equation to generate points and plot them to create the desired surfaces and shapes. Patches of the surfaces generated by this method can be used to create objects with complex shapes.

According to Ugail and Wilson in [14, 15], geometric modeling, using the spline approach for design optimization, has the disadvantage of making it difficult to maintain a smooth transition between adjacent patches during the design process. It happens because they typically represent an object shape as a mesh of rectangular curvilinear parts. The parameterizations of a product's shape are considered relatively simple if it is made up of standard geometric constructs such as circles, squares, cubes, and cylinders. However, in general, most products cannot be completely constructed using these standard shapes alone but must also include free form surfaces in their design to achieve the desired shape. Regardless of how minor these surfaces are in the overall shape of the object, they may be critical if they are part of the object's functionality. It should be noted that the PDE method is similar in some ways to previously established surface design methods such as Bezier patches and rational B-splines discussed in [15–18]. However, PDE's global smoothing approach, in conjunction with its elliptic boundary-value formulation, clearly distinguishes it from those traditional spline-based techniques. When compared to the hundreds of control points required by existing spline-based techniques, PDE requires only a small set of design variables. As mentioned in [19], Ugail et al. research in the area of interactive design demonstrated that PDE surfaces could be defined and

manipulated efficiently in real time. Authors also demonstrated that surface manipulation could be performed in an interactive environment with an interactively defined set of parameters, discussed in [19–22]. These parameters, by definition, make their effect on the geometry of the surface obvious.

Regarding interactive design using the PDE method, Bloor and Wilson showed in [23] that, this method can produce simple surfaces in an interactive environment. This work was then extended by Ugail et al., mentioned in [19, 20]. They used four boundary conditions in the PDE method to generate surface. Later, their work was extended by Ugail et al., discussed in [24], in which they used six boundary conditions to generate the geometric surface. It was easier to work with six boundary conditions than with four boundary conditions, as a more curvier geometric model can be generated and a complex geometric model can be generated more easily. In this article, their work is further extended by using eight boundary conditions to develop the complex geometric model with the help of a set of interactively defined parameters. The boundary conditions are responsible to control the overall form of the surface of the designed model, while a set of parameters control all the coordinates of the boundary conditions. One of the parameters involved in the designing of the desired model is the modified smoothness parameter, responsible for the smoothness and quality of the geometric model, which is developed with the help of the fuzzy soft matrix (FSM) along with weighted Bonferroni mean (WBM).

The fuzzy soft matrix (FSM) is a very useful tool to gather requirements of the industry for designing their products, while weighted Bonferroni mean (WBM) blend the requirements of industry into a single entity. According to Zadeh discussed in [25], fuzzy sets theory is useful for modeling uncertain circumstances better than standard theories. In 1999, Molodtsov introduced the concept of soft set, discussed in [26], and successfully solved some uncertainties. Later, Maji et al. [27] modified the concept of soft set theory and successfully applied it to decision-making problems. In 2001, Maji et al. in [28] presented the concept of the fuzzy soft set by combining the concept of the fuzzy set with the soft set. One of the main advantages of fuzzy soft sets is that they allow for the incorporation of subjective or personal opinions into decision-making processes, as well as the consideration of multiple conflicting criteria. This can be particularly useful in situations where there is a lack of objective data or where multiple stakeholders have different viewpoints or priorities.

Fuzzy soft sets have been applied in a wide range of fields, including decision analysis, artificial intelligence, information retrieval, and control systems. They are often used in conjunction with other techniques, such as fuzzy logic or artificial neural networks, to improve the accuracy and robustness of decision-making systems. This theory opened the path to many new concepts. Neog and Sut mentioned in [29] used fuzzy soft complement and fuzzy soft matrix operation to solve decision-making problems. Sut in [30] utilizes fuzzy soft relations in a decision-making problem.

Cagman et al. in [31] proposed a decision-making method that makes use of a fuzzy soft aggregation operator and a cardinal set. Celik and Yamak in [32] used fuzzy soft set theory with a fuzzy aggregation operator to diagnose patients' diseases. In 2018, Beg et al. [33] developed a time-dependent model to analyse human attributes by fuzzy soft matrix (FSM) and Bonferroni mean (BM). BM was developed by Bonferroni in [34], which was helpful in the multiple comparison test. Yager in [35, 36] and Beliakov et al. in [37] discussed useful properties of BM to capture the interrelationships among arguments. Using the concept of BM with fuzzy soft matrix, Beg et al. explained in [33] and defined Bonferroni fuzzy soft matrix (BFSM) and weighted Bonferroni fuzzy soft matrix (WBFSM) for data representation. They also used WBFSM for design making.

An optimum geometric design is a design that meets certain specified criteria in the most efficient or cost-effective way possible. The criteria for an optimum design will depend on the specific goals and constraints of the problem at hand and may include factors such as strength, stiffness, weight, cost, and manufacturability.

In this article, the concept of WBFSM is used to calculate the modified smoothness parameter used in the partial differential equation. Since the requirements for designing an intelligent system to generate a geometric model for an industry can only be provided in the form of fuzzy soft set and single value is required for smoothness parameter, so this intelligent system was needed to convert fuzzy soft set in a single number. For this purpose, WBFSM is used in this article as it is an ideal tool to blend multiple values in a single value.

To verify this intelligent system, an example is taken from the hotel industry to design a drinking glass for fizzy drinks. According to a case study discussed in [38], drinking behavior of a person highly depends on the design of a glass. If the glass has an effective design and is easy to drink from, then the person will drink more than usual. The design of a glass also depends on the type of drinks. It is also discussed in the same case study, how design of glass affects the drinking behavior of a person. Considering the drinking behavior and the type of drink, it can be very helpful in designing a glass. In the making of a drinking glass, many parameters are considered (such as quality, material used in glass, and price) according to the requirement of the customer. But the problem is that a customer is not able to see it until the manufacturing of the glass is complete. Sometimes, the final result is not according to the customer's requirement. For this purpose, a three-dimensional design of a drinking glass according to the required parameters is generated. The intelligent system developed in this article is used to generate multiple designs of drinking glass according to customer's requirements. After modeling these designs, multicriteria decision-making (MCDM) is used to select the glass which efficiently fulfills the customer requirements.

The structure of this article is as follows: Section 2 is devoted, the basic definitions used in this research, which will help the reader in understanding this research. Section 3 provides the detail on the PDE used in this research along with some geometric models generated using the involved

PDE method. In Section 4, forming of the smoothness parameter with the help of WBFSM is discussed in detail. In this section, we also developed an intelligent system and with the help of illustrated examples, we verified the credibility of the developed system. Finally, the last section of this article is concluded with the discussion on the benefits and utilization of the developed system in different designing industries. A path for further research is also mentioned in this section.

## 2. Preliminaries

Partial differential equation (PDE) is a tool which is used in every field of mathematics. In this article, a hybrid methodology is developed by using fuzzy sets and soft sets theory with blend of eighth order PDE with some boundary conditions and a set of parameters. In this chapter, some basic definitions are discussed which will be helpful in further discussion.

*Definition 1* [25]. Let  $X$  be a nonempty set. A fuzzy set  $A$  in  $X$  is characterized by its membership function  $\mu_A: X \rightarrow [0, 1]$ , and  $\mu_A(x)$  is interpreted as the degree of membership of element  $x$  in the fuzzy set  $A$  for each  $x \in X$ .

*Definition 2* [26]. Let  $Y$  be a universal set,  $E$  be a set of parameters, and  $A$  be a subset of  $E$ . Power set of  $Y$  is denoted by  $P(Y)$ , then the pair  $(F; A)$  is said to be a soft set over the universal set  $Y$ , where  $F$  is mapping given by  $F: A \rightarrow P(Y)$ . In fact, a soft set is a parameterized family of subsets over the universal set  $Y$ . Every set  $(F(e); e)$  represents the set of elements of the soft set  $(F; A)$ .

*Definition 3* [39]. If  $Y$  be the universal set,  $E$  be the set of parameters, and  $B \subseteq E$  and  $P^Y$  be the set of all fuzzy subsets of  $Y$ , then  $(G, B)$  is said to be the fuzzy soft set over  $Y$ , where  $G$  is a mapping such that  $G: B \rightarrow P^Y$ . Tabular representation of a fuzzy soft set is represented in Table 1.

*Definition 4* [37]. Let there be two natural numbers  $s, t$  and  $r_i \geq 0$  for  $(i = 1, 2, \dots, n)$ . Then, Bonferroni mean  $F^{s,t}$  is as follows:

$$F^{s,t} = \left( \frac{1}{n(n-1)} \sum_{i,j=1, i \neq j}^n r_i^s r_j^t \right)^{(1/(s+t))}. \quad (1)$$

The Bonferroni mean (BM) is a useful aggregative operator that is an extension of the arithmetic mean. The Bonferroni mean (BM) has the property of capturing the interrelationships between factors that are significant for multicriteria decision-making, according to Yager [35]. The following scenario for BM was studied by Dyckhoff and Pedrycz [40] for various  $s$  and  $t$  values.

(i) If  $s = 1$  and  $t = 0$ , then

$$F^{1,0} = \left( \frac{1}{n} \sum_{i=1}^n r_i \right). \quad (2)$$

(ii) If  $s = 2$  and  $t = 0$ , then

TABLE 1: Tabular representation of a fuzzy soft set  $(G, B)$ .

	$y_1$	$y_2$	$y_3$	$\dots$	$y_n$
$e_1$	$a_{11}$	$a_{12}$	$a_{13}$	$\dots$	$a_{1n}$
$e_2$	$a_{21}$	$a_{22}$	$a_{23}$	$\dots$	$a_{2n}$
$e_3$	$a_{31}$	$a_{32}$	$a_{33}$	$\dots$	$a_{3n}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$e_m$	$a_{m1}$	$a_{m2}$	$a_{m3}$	$\dots$	$a_{mn}$

$$F^{2,0} = \left( \frac{1}{n} \sum_{i=1}^n r_i \right)^{(1/2)}. \quad (3)$$

(iii) If  $s \rightarrow +\infty$  and  $t = 0$ , then

$$\lim_{s \rightarrow +\infty} F^{s,0} = \max_i \{r_i\}. \quad (4)$$

(iv) If  $s \rightarrow 0$  and  $t = 0$ , then

$$\lim_{s \rightarrow 0} F^{s,0} = \left( \prod_{i=1}^n r_i \right)^{(1/n)}. \quad (5)$$

(v) If  $s = 1$  and  $t = 1$ , then

$$F^{1,1} = \left( \frac{1}{n(n-1)} \sum_{i,j=1, i \neq j}^n r_i r_j \right)^{(1/2)}. \quad (6)$$

In equation (6), the operator's interpretation is a combination of the relational operators "and" and "average." Product activities, in particular, can be leveraged to implement two satisfaction requirements. As a result,  $r_i$  and  $r_j$  denote the degree to which the two requirements are satisfied. As a result,  $F^{1,1}(r_1, r_2, \dots, r_n)$  estimates the reference pair's average satisfaction.

*Definition 5* [35]. If  $s$  and  $t$  are two natural numbers and  $r_i \geq 0$  for all  $(i = 1, 2, 3, \dots, n)$  and for any weighted vector  $W = (w^i \geq 0)^T$  of  $r_i$ , which has condition  $\sum_{i=1}^n w_i = 1$ , then weighted Bonferroni mean is

$$WF^{s,t} = \left( \frac{1}{n(n-1)} \sum_{i,j=1, i \neq j}^n (w_i r_i)^s (w_j r_j)^t \right)^{(1/(s+t))}. \quad (7)$$

*Definition 6* [33]. Let there be a fuzzy soft matrix  $F_{mn}^h = [f_{ij}^h]$  where  $i = 1, 2, \dots, m, j = 1, 2, \dots, n$  and  $h = 1, 2, \dots, q$ . Then, Bonferroni fuzzy soft matrix (BFSM) is defined as  $F_{mn} = [f_{mn}]$ , where  $f_{mn} = F^{s,t}(f_{mn}^1, f_{mn}^2, \dots, f_{mn}^q)$ . Here,  $s$  and  $t$  are natural numbers.

*Definition 7* [33]. Let a fuzzy soft matrix be  $F_{m \times n} = [f_{i,j}]$ , where  $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$ . Then, weighted Bonferroni fuzzy soft matrix (WBFMSM) is defined as  $F_{m \times 1} = [d_{i,1}]$  for weight vector  $W = w_j$ , which is the weight of  $f_{i,j}$ .

The weight vector has the condition  $\sum_j^n w_j = 1$ . Here,  $d_{i,1} = WF^{s,t}(f_{i,1}, f_{i,2}, \dots, f_{i,n})$ .

*Definition 8* [33]. Optimum fuzzy soft constant (OFSC) is Optimum =  $\max_i(c_{i,1})$ , where  $c_{i,1}$  is from the last definition.

*2.1. Partial Differential Equation Surfaces.* Wood ward introduced in [41] that the PDE method mainly aims to provide a skillful function accepting some specific boundary and continuity conditions, and this function works as a connecting surface for primary neighboring surfaces. Follow that a second order elliptic PDE method was used by Bloor and Wilson [42] to generate blend surfaces in the field of CAGD. They then extended their work and generated PDE surfaces using fourth order PDE [19, 20], which used four boundary conditions to generate surfaces. In this article using their work as a reference, eight order PDE surfaces are generated using eight boundary conditions.

$$\left( \frac{\partial^2}{\partial \alpha^2} + \eta^2 \frac{\partial^2}{\partial \beta^2} \right)^4 \Omega(\alpha, \beta) = 0, \quad (8)$$

where  $\Omega(\alpha, \beta) = (x(\alpha, \beta), y(\alpha, \beta), z(\alpha, \beta))$  is a parametric function.

To preserve the shape of the surface, the selection of boundary conditions and parametric domain are essential for equation (8). Here,  $\Omega$  is normally taken as a rectangle, that is,  $\Omega: \alpha_0 \leq \alpha \leq \alpha_1, \beta_0 \leq \beta \leq \beta_1$ . The parameter  $\eta$  is known as a smoothness parameter which plays an important role for the shape reconstruction of a model [41].

To solve the eighth order PDE, equation (8) required eight boundary conditions so that a unique solution in  $x, y$ , and  $z$  coordinates can be obtained. Due to this, in this research, the required PDE surface is generated by the solution of equation (8) with eight positional curves taken by the given geometric shape with the help of CAGD and used as boundary conditions. An illustrated solution of equation (8) with 8 boundary conditions is drawn in Figure 1 as representing the 3D model of an apple.

Let the domain of  $\Omega$  be finite defined as  $\Omega: 0 \leq \alpha \leq 1, 0 \leq \beta \leq 2\pi$  such that

$$\begin{aligned} \Omega(0, \beta) &= \hat{h}_0(\beta), \\ \Omega(k, \beta) &= \hat{h}_k(\beta), \\ \Omega(l, \beta) &= \hat{h}_l(\beta), \\ \Omega(m, \beta) &= \hat{h}_m(\beta), \\ \Omega(n, \beta) &= \hat{h}_n(\beta), \\ \Omega(o, \beta) &= \hat{h}_o(\beta), \\ \Omega(p, \beta) &= \hat{h}_p(\beta), \\ \Omega(1, \beta) &= \hat{h}_1(\beta), \end{aligned} \quad (9)$$

$\hat{h}_0(\beta), \hat{h}_k(\beta), \hat{h}_l(\beta), \hat{h}_m(\beta), \hat{h}_n(\beta), \hat{h}_o(\beta), \hat{h}_p(\beta), \hat{h}_1(\beta)$  are the boundary conditions. The relative position of the boundary curve depends on the parameters  $0 < k < l < m < n < o < p < 1$ .



FIGURE 1: 3D model of an apple.

In this research, equation (8) is solved by using the method of separation of variables as follows: where

$$\Omega(\alpha, \beta) = \bar{q}_0(\alpha) + \sum_{i=1}^F [\bar{\sigma}_i(\alpha) \cos(i\beta) + \bar{\tau}_i(\alpha) \sin(i\beta)], \quad (10)$$

$$\bar{q}_0 = \bar{\mu}_{00} + \bar{\mu}_{01}\alpha + \bar{\mu}_{02}\alpha^2 + \bar{\mu}_{03}\alpha^3 + \bar{\mu}_{04}\alpha^4 + \bar{\mu}_{05}\alpha^5 + \bar{\mu}_{06}\alpha^6 + \bar{\mu}_{07}\alpha^7, \quad (11)$$

$$\bar{\sigma}_i = \bar{\nu}_{i1}e^{\eta i\alpha} + \bar{\nu}_{i2}e^{-\eta i\alpha} + \bar{\nu}_{i3}\alpha e^{\eta i\alpha} + \bar{\nu}_{i4}\alpha e^{-\eta i\alpha} + \bar{\nu}_{i5}\alpha^2 e^{\eta i\alpha} + \bar{\nu}_{i6}\alpha^2 e^{-\eta i\alpha} + \bar{\nu}_{i7}\alpha^3 e^{\eta i\alpha} + \bar{\nu}_{i8}\alpha^3 e^{-\eta i\alpha}, \quad (12)$$

$$\bar{\tau}_i = \bar{\xi}_{i1}e^{\eta i\alpha} + \bar{\xi}_{i2}e^{-\eta i\alpha} + \bar{\xi}_{i3}\alpha e^{\eta i\alpha} + \bar{\xi}_{i4}\alpha e^{-\eta i\alpha} + \bar{\xi}_{i5}\alpha^2 e^{\eta i\alpha} + \bar{\xi}_{i6}\alpha^2 e^{-\eta i\alpha} + \bar{\xi}_{i7}\alpha^3 e^{\eta i\alpha} + \bar{\xi}_{i8}\alpha^3 e^{-\eta i\alpha}. \quad (13)$$

Here, all the values of  $\mu$ ,  $\nu$ , and  $\xi$  are unknown.

The effect in the final shape of a model due to these boundary conditions is shown in Figure 1. A slight change in the position of the curves can change the shape of the entire model. The unknown parameters,  $\mu$ ,  $\nu$ , and  $\xi$ , also have a huge role in obtaining the final shape.

**2.2. Error Analysis.** The unknown constants are Fourier coefficients. They can be calculated using suitable numerical techniques on the given boundary conditions. The shape of the final model involves some error. The following equation shows that error:

$$\Omega(\alpha, \beta) = \bar{q}_0(\alpha) + \sum_{i=1}^F [\bar{\sigma}_i(\alpha) \cos(i\beta) + \bar{\tau}_i(\alpha) \sin(i\beta)] + \bar{R}(\alpha, \beta), \quad (14)$$

where the error is shown by  $\bar{R}$  and  $F$  is a finite natural number.

In the boundary conditions, the coefficients  $\rho(\alpha)$  and  $\tau(\alpha)$  can be calculated using the amplitude of the  $i^{\text{th}}$  mode. The role of high frequency modes to the final model is shown

by the term  $\bar{R}(\alpha, \beta)$ . Because of the finite value of  $F$ , this term has an effect on the overall shape of the model. The following equation is used to find this term:

$$\bar{R}(\alpha, \beta) = \bar{w}_1(\beta)e^{l\alpha} + \bar{w}_2(\beta)e^{-l\alpha} + \bar{w}_3(\beta)\alpha e^{l\alpha} + \bar{w}_4(\beta)\alpha e^{-l\alpha} + \bar{w}_5(\beta)\alpha^2 e^{l\alpha} + \bar{w}_6(\beta)\alpha^2 e^{-l\alpha} + \bar{w}_7(\beta)\alpha^3 e^{l\alpha} + \bar{w}_8(\beta)\alpha^3 e^{-l\alpha}. \quad (15)$$

Here,  $\bar{w}_1, \bar{w}_2, \bar{w}_3, \bar{w}_4, \bar{w}_5, \bar{w}_6, \bar{w}_7, \bar{w}_8$ , and  $l$  are obtained, by taking the difference of the original boundary conditions with the conditions fulfilled by the function in equation (10).

**2.3. Different Geometric Models Using PDE Surfaces.** The various geometrical figures can be generated from a variety of curve options. The examples mentioned demonstrate the

efficacy and comprehensibility of the PDE approach for the generation of complex geometry shapes, which supports the main goal of this study. The figures from 2 to 9 are generated using eight boundary conditions with  $\Omega: 0 \leq \alpha \leq 1, 0 \leq \beta \leq 2\pi$ , where  $x = \alpha \cos(\beta)$ ,  $y = \alpha \sin(\beta)$  and  $0 \leq z \leq 1$ . In Figure 2, only a single point is taken for the first curve and the surface is generated by joining that point to each point of the next curve. Figure 3 is also generated using a point for top and bottom curves. A circular curve is used for the first curve of Figure 4 and a point for its bottom curve which gave it the shape of a pot. To generate the shape of a peach in Figure 5, the position of the first curve is taken between the second and third curves. (Figures 6–9) are also drawn in the same manner according to the related boundaries conditions.

### 3. Intelligent Systems for Designing of the Culinary Product Based on the Fuzzy Soft Partial Differential Equation

In optimum designing of the culinary product, the Bonferroni mean has the ability to capture desirable properties among arguments' interrelationship. While the fuzzy soft set deals with the situation where uncertainty occurs, differential equation is a useful tool to analyse the rate of change of dependent variables with respect to the independent variable. Therefore, the characteristic of BM with blend of fuzzy soft sets and differential equation becomes a very useful instrument to design an optimum culinary product according to the requirement of the industry. In this section, fuzzy soft matrix is used with weighted Bonferroni mean to calculate the value of the smoothness parameter,  $\eta$ , in the PDE defined in equation (8). An example is also developed to demonstrate the reliability of the developed technique.

#### 3.1. An Intelligent System for Optimum Geometric Design.

In the previous section, equation (8) was used to generate a PDE surface using eight curves. This equation has a parameter  $\eta$ , known as the smoothness parameter, which plays an important role in the shape and smoothness of a surface. Usually,  $\eta$  is considered 1 by default. But when this PDE method is applied to design a product for some customers, a specific value of  $\eta$  is needed as it plays an important role in fulfilling the requirements of the customer. The parameter,  $\eta$ , consists of all the specifications for designing a product, for example, price, quality, design, volume, and shape. Due to the uncertainty involved during product designing, it is not possible to take  $\eta = 1$ . To develop an intelligent system for designing the required product using  $\eta$  in the PDE given in equation (8) for different requirements of the industry, the following steps should be considered.

Step 1: there are  $r$  decision makers, who provide  $r$  number of decision matrices, in the form of fuzzy soft matrices  $A_{mn}^r = [a_{mn}]$ , where  $m = 1, 2, \dots, i$  and  $n = 1, 2, \dots, j$ , according to industrial requirements to design a specific product. A weight vector  $W = \{w_1, w_2, w_3, \dots, w_n\}$  is also given according to the industry requirements.  $W$  must satisfy the conditions  $w_p > 0$  for

$p = 1, 2, 3, \dots, j$  and  $\sum_{p=1}^j \omega_p = 1$ , then using Definition 7, we calculate WBFSM for each  $A_{mn} = [a_{mn}]$ . For each decision matrix  $A_{mn}$ , a unique resultant matrix of the form  $WAs,t = [a_{mn}]$ , for  $m = 1, 2, 3, \dots, i$  and  $s, t$  are natural numbers, are developed. Each entry of  $WAs,t$  (say  $\eta_{mn}$ ) provides a unique product design by using Equation (8) as specified by the industry.

Step 2: a matrix  $E = [\eta_{mn}]$ , where  $m = 1, 2, \dots, i$  and  $n = 1, 2, \dots, j$ , will be generated by the calculated values of  $\eta_{mn}$  in step 1.

Step 3: to select the best product design between the generated designs by each  $\eta_{mn}$ , we use

$$\begin{aligned} &\text{Optimum Smoothing Parameter } (\eta^-) \\ &= \max(\max(\eta_{mn})), \end{aligned} \quad (16)$$

where  $m = 1, 2, \dots, i$  and  $n = 1, 2, \dots, j$ . This intelligent system helps obtain an optimal geometric design with the help of  $\bar{\eta}$ . This is the best design of the desired product which completely fulfills the requirements of the industry.

3.2. *Illustrated Example.* Mr. Khan is an owner of a juice bar, who wants to buy cold drink glass for serving fizzy drinks. He wants the design of the glass to be attractive but simple. Also, the glass should be of fine quality with price as low as possible, and the volume of the glass should be appropriate for one serving. He went to a glass design company. He selected five different basic designs for glass according to his given parameters which are  $P = \{p(\text{price}), v(\text{volume}), \text{ and } q(\text{quality})\}$ . Mr. Khan also gave some weight  $W = (0.4 \ 0.3 \ 0.3)$  to each of these parameters. Then, these designs were given to five experts designers  $= \{d_1, d_2, d_3, d_4, d_5\}$ . All the designers gave their suggestions for each glass according to Mr. Khan's requirements. Following is the fuzzy soft matrix for the basic design of the first glass according to the suggestions of the experts.

$$G_1 = \begin{bmatrix} 0.30 & 0.70 & 0.20 \\ 0.90 & 0.60 & 0.70 \\ 0.60 & 0.75 & 0.40 \\ 0.80 & 0.60 & 0.50 \\ 0.40 & 0.50 & 0.35 \end{bmatrix}. \quad (17)$$

Then, using the definition (2.0.9) of weighted Bonferroni fuzzy soft matrix (WBFSM), the WBFSM for the first glass design  $G_1$  is according to the given weights for the parameters, which is as follows:

$$B_1 = \begin{bmatrix} 0.1225 \\ 0.2437 \\ 0.1913 \\ 0.2102 \\ 0.1373 \end{bmatrix}. \quad (18)$$

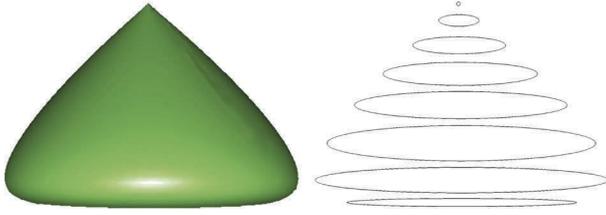


FIGURE 2: Tomb of a mosque with the following boundary condition in which  $0 \leq \beta \leq 2\pi$ :  $C_1 = (0.0\cos(\beta), 0.0\sin(\beta), 1.0)$ ,  $C_2 = (0.1\cos(\beta), 0.1\sin(\beta), 0.8)$ ,  $C_3 = (0.3\cos(\beta), 0.3\sin(\beta), 0.7)$ ,  $C_4 = (0.5\cos(\beta), 0.5\sin(\beta), 0.6)$ ,  $C_5 = (0.6\cos(\beta), 0.6\sin(\beta), 0.5)$ ,  $C_6 = (0.8\cos(\beta), 0.8\sin(\beta), 0.3)$ ,  $C_7 = (0.9\cos(\beta), 0.9\sin(\beta), 0.2)$ , and  $C_8 = (0.7\cos(\beta), 0.7\sin(\beta), 0.0)$ .

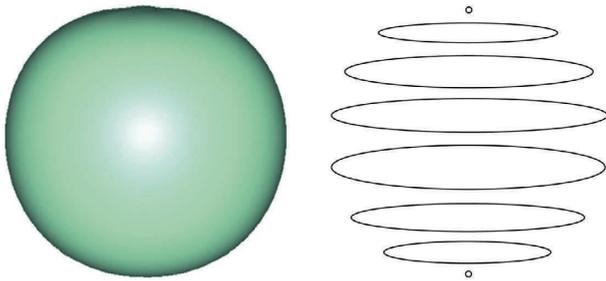


FIGURE 3: Sphere with the following boundary condition in which  $0 \leq \beta \leq 2\pi$ :  $C_1 = (0.0\cos(\beta), 0.0\sin(\beta), 1.0)$ ,  $C_2 = (0.2\cos(\beta), 0.2\sin(\beta), 0.8)$ ,  $C_3 = (0.6\cos(\beta), 0.6\sin(\beta), 0.7)$ ,  $C_4 = (1.0\cos(\beta), 1.0\sin(\beta), 0.5)$ ,  $C_5 = (0.6\cos(\beta), 0.6\sin(\beta), 0.3)$ ,  $C_6 = (0.2\cos(\beta), 0.2\sin(\beta), 0.2)$ ,  $C_7 = (0.1\cos(\beta), 0.1\sin(\beta), 0.1)$ , and  $C_8 = (0.0\cos(\beta), 0.0\sin(\beta), 0.0)$ .

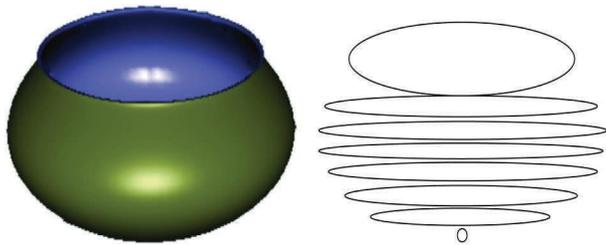


FIGURE 4: A cooking pot with the following boundary condition in which  $0 \leq \beta \leq 2\pi$ :  $C_1 = (0.7\cos(\beta), 0.7\sin(\beta), 1.0)$ ,  $C_2 = (0.5\cos(\beta), 0.5\sin(\beta), 0.95)$ ,  $C_3 = (0.6\cos(\beta), 0.6\sin(\beta), 0.7)$ ,  $C_4 = (0.7\cos(\beta), 0.7\sin(\beta), 0.6)$ ,  $C_5 = (0.8\cos(\beta), 0.8\sin(\beta), 0.5)$ ,  $C_6 = (1.0\cos(\beta), 1.0\sin(\beta), 0.4)$ ,  $C_7 = (0.4\cos(\beta), 0.4\sin(\beta), 0.2)$ , and  $C_8 = (0.1\cos(\beta), 0.1\sin(\beta), 0.0)$ .

Now, for matrix  $G_1$ , there are five different values. Each value defines a smoothness parameter  $\eta_{mn}$  for a separate glass. Each value will be used in equation (8) as  $\eta_{mn}$  to get a slightly different glass design. Here, the values are  $\eta_{11} = 0.1225$ ,  $\eta_{21} = 0.2437$ ,  $\eta_{31} = 0.1913$ ,  $\eta_{41} = 0.2102$ , and  $\eta_{51} = 0.1373$ . Putting these values in equation (8), the following designs are generated:

As it can be seen from the data  $\eta_{21} > \eta_{41} > \eta_{31} > \eta_{51} > \eta_{11}$ , so the glass designed in Figure 10(b) with the value  $\eta_{21}$  is the best between these five glass designs described in Figures 10(a)–10(e). The difference between these glasses is

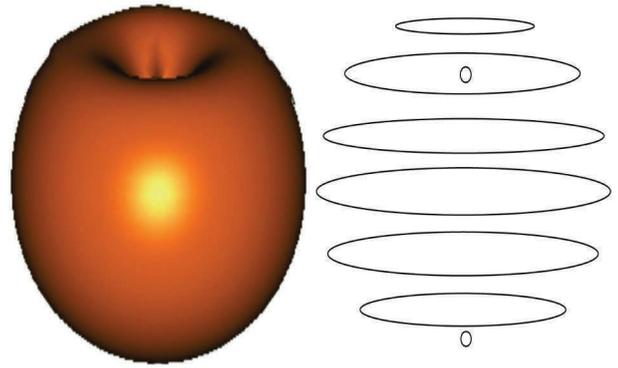


FIGURE 5: A peach model with the following boundary condition in which  $0 \leq \beta \leq 2\pi$ :  $C_1 = (0.0\cos(\beta), 0.0\sin(\beta), 0.8)$ ,  $C_2 = (0.3\cos(\beta), 0.3\sin(\beta), 1.0)$ ,  $C_3 = (0.5\cos(\beta), 0.5\sin(\beta), 0.7)$ ,  $C_4 = (0.8\cos(\beta), 0.8\sin(\beta), 0.6)$ ,  $C_5 = (1.0\cos(\beta), 1.0\sin(\beta), 0.5)$ ,  $C_6 = (0.8\cos(\beta), 0.8\sin(\beta), 0.3)$ ,  $C_7 = (0.3\cos(\beta), 0.3\sin(\beta), 0.2)$ , and  $C_8 = (0.0\cos(\beta), 0.0\sin(\beta), 0.0)$ .

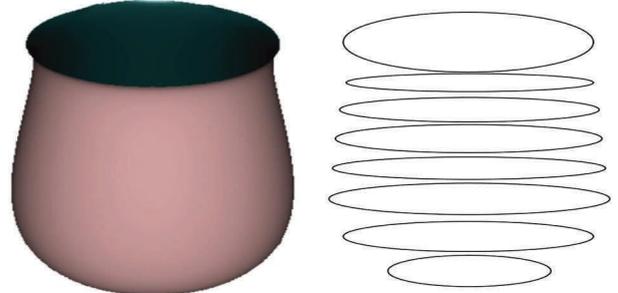


FIGURE 6: A glass bowl with the following boundary condition in which  $0 \leq \beta \leq 2\pi$ :  $C_1 = (0.8\cos(\beta), 0.8\sin(\beta), 1.0)$ ,  $C_2 = (0.7\cos(\beta), 0.7\sin(\beta), 0.9)$ ,  $C_3 = (0.8\cos(\beta), 0.8\sin(\beta), 0.7)$ ,  $C_4 = (0.85\cos(\beta), 0.85\sin(\beta), 0.6)$ ,  $C_5 = (0.9\cos(\beta), 0.9\sin(\beta), 0.5)$ ,  $C_6 = (1.0\cos(\beta), 1.0\sin(\beta), 0.3)$ ,  $C_7 = (0.9\cos(\beta), 0.9\sin(\beta), 0.15)$ , and  $C_8 = (0.5\cos(\beta), 0.5\sin(\beta), 0.0)$ .

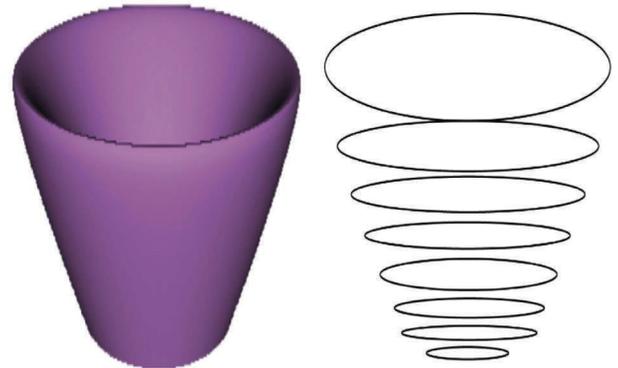


FIGURE 7: Drinking glass with the following boundary condition in which  $0 \leq \beta \leq 2\pi$ :  $C_1 = (1.0\cos(\beta), 1.0\sin(\beta), 1.0)$ ,  $C_2 = (0.9\cos(\beta), 0.9\sin(\beta), 0.8)$ ,  $C_3 = (0.7\cos(\beta), 0.7\sin(\beta), 0.7)$ ,  $C_4 = (0.6\cos(\beta), 0.6\sin(\beta), 0.6)$ ,  $C_5 = (0.5\cos(\beta), 0.5\sin(\beta), 0.5)$ ,  $C_6 = (0.4\cos(\beta), 0.4\sin(\beta), 0.3)$ ,  $C_7 = (0.3\cos(\beta), 0.3\sin(\beta), 0.2)$ , and  $C_8 = (0.2\cos(\beta), 0.2\sin(\beta), 0.0)$ .

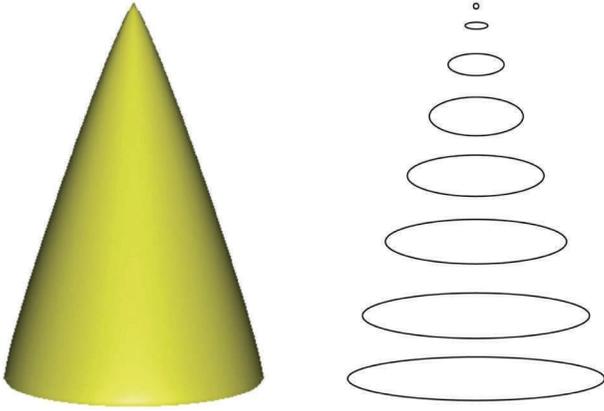


FIGURE 8: A cone model with the following boundary condition in which  $0 \leq \beta \leq 2\pi$ :  $C_1 = (0.0\cos(\beta), 0.0\sin(\beta), 1.0)$ ,  $C_2 = (0.1\cos(\beta), 0.1\sin(\beta), 0.9)$ ,  $C_3 = (0.2\cos(\beta), 0.2\sin(\beta), 0.8)$ ,  $C_4 = (0.4\cos(\beta), 0.4\sin(\beta), 0.6)$ ,  $C_5 = (0.6\cos(\beta), 0.6\sin(\beta), 0.5)$ ,  $C_6 = (0.8\cos(\beta), 0.8\sin(\beta), 0.3)$ ,  $C_7 = (0.9\cos(\beta), 0.9\sin(\beta), 0.2)$ , and  $C_8 = (1.0\cos(\beta), 1.0\sin(\beta), 0.0)$ .

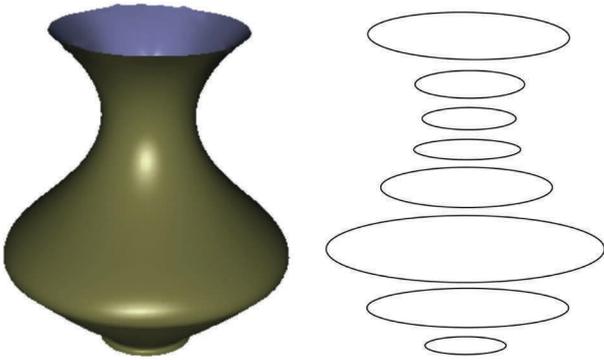


FIGURE 9: Vase with narrow mouth with the following boundary condition in which  $0 \leq \beta \leq 2\pi$ :  $C_1 = (0.5\cos(\beta), 0.5\sin(\beta), 1.0)$ ,  $C_2 = (0.3\cos(\beta), 0.3\sin(\beta), 0.8)$ ,  $C_3 = (0.25\cos(\beta), 0.25\sin(\beta), 0.7)$ ,  $C_4 = (0.3\cos(\beta), 0.3\sin(\beta), 0.6)$ ,  $C_5 = (0.4\cos(\beta), 0.4\sin(\beta), 0.5)$ ,  $C_6 = (0.75\cos(\beta), 0.75\sin(\beta), 0.3)$ ,  $C_7 = (0.5\cos(\beta), 0.5\sin(\beta), 0.2)$ , and  $C_8 = (0.15\cos(\beta), 0.15\sin(\beta), 0.0)$ .

so small that one cannot detect it just by looking at their pictures, but when the data are observed, it can be seen that the glass with the best quality and moderate volume is selected. Here, volume of 0.5 is approximately equal to 250 ml that is also considered as one serving of a drink. If the given data of Figure 10(e) are observed, it has the exact volume of one serving, but the quality of the material of glass is very low, so it is not a good choice for the customer.

The fuzzy soft matrix for the second design of the glass according to the suggestions of the experts is as follows:

$$G_2 = \begin{bmatrix} 0.10 & 0.65 & 0.30 \\ 0.60 & 0.53 & 0.70 \\ 0.40 & 0.60 & 0.50 \\ 0.50 & 0.57 & 0.60 \\ 0.80 & 0.48 & 1.00 \end{bmatrix}. \quad (19)$$

Similarly, WBFSM for the matrix  $G_2$  is as follows:

$$B_2 = \begin{bmatrix} 0.0982 \\ 0.2016 \\ 0.1631 \\ 0.1835 \\ 0.2485 \end{bmatrix}. \quad (20)$$

Here,  $\eta_{12} = 0.0982$ ,  $\eta_{22} = 0.2016$ ,  $\eta_{32} = 0.1631$ ,  $\eta_{42} = 0.1835$ , and  $\eta_{52} = 0.2485$ . Putting these values of  $\eta_{mn}$  in equation (8), the generated designs are as follows:

Here,  $\eta_{52} > \eta_{22} > \eta_{42} > \eta_{32} > \eta_{12}$ . So,  $\eta_{52}$  gives the best glass from the abovementioned five glasses shown in Figures 11(a)–11(e). It can be noticed from the data that  $\eta_{52}$  has the highest quality and moderate volume for one serving.

The fuzzy soft matrix for the third glass design is as follows:

$$G_3 = \begin{bmatrix} 0.20 & 0.25 & 0.35 \\ 0.70 & 0.58 & 0.50 \\ 1.00 & 0.48 & 0.65 \\ 0.40 & 0.25 & 0.50 \\ 0.10 & 0.10 & 0.30 \end{bmatrix}. \quad (21)$$

For matrix  $G_3$ , the WBFSM is as follows:

$$B_3 = \begin{bmatrix} 0.0862 \\ 0.1973 \\ 0.2336 \\ 0.1255 \\ 0.0500 \end{bmatrix}. \quad (22)$$

Here,  $\eta_{13} = 0.0862$ ,  $\eta_{23} = 0.1973$ ,  $\eta_{33} = 0.2336$ ,  $\eta_{43} = 0.1255$ , and  $\eta_{53} = 0.0500$ . Figures 12(a)–12(e) designs are generated by the values of  $\eta_{mn}$  after putting them in equation (8):

As  $\eta_{33} > \eta_{23} > \eta_{43} > \eta_{13} > \eta_{53}$ , so the selected design for  $G_3$  seen in Figure 12(c) is the design made by  $\eta_{33}$ . This design has the highest quality, and the volume is close to 250 ml.

Now, for the fourth glass design, the fuzzy soft matrix is as follows:

$$G_4 = \begin{bmatrix} 0.20 & 0.10 & 0.30 \\ 0.80 & 0.60 & 0.50 \\ 0.40 & 0.37 & 0.60 \\ 0.10 & 0.20 & 0.10 \\ 1.00 & 0.40 & 0.80 \end{bmatrix}. \quad (23)$$

The weighted Bonferroni fuzzy soft matrix (WBFSM) for fuzzy soft matrix  $G_4$  is as follows:

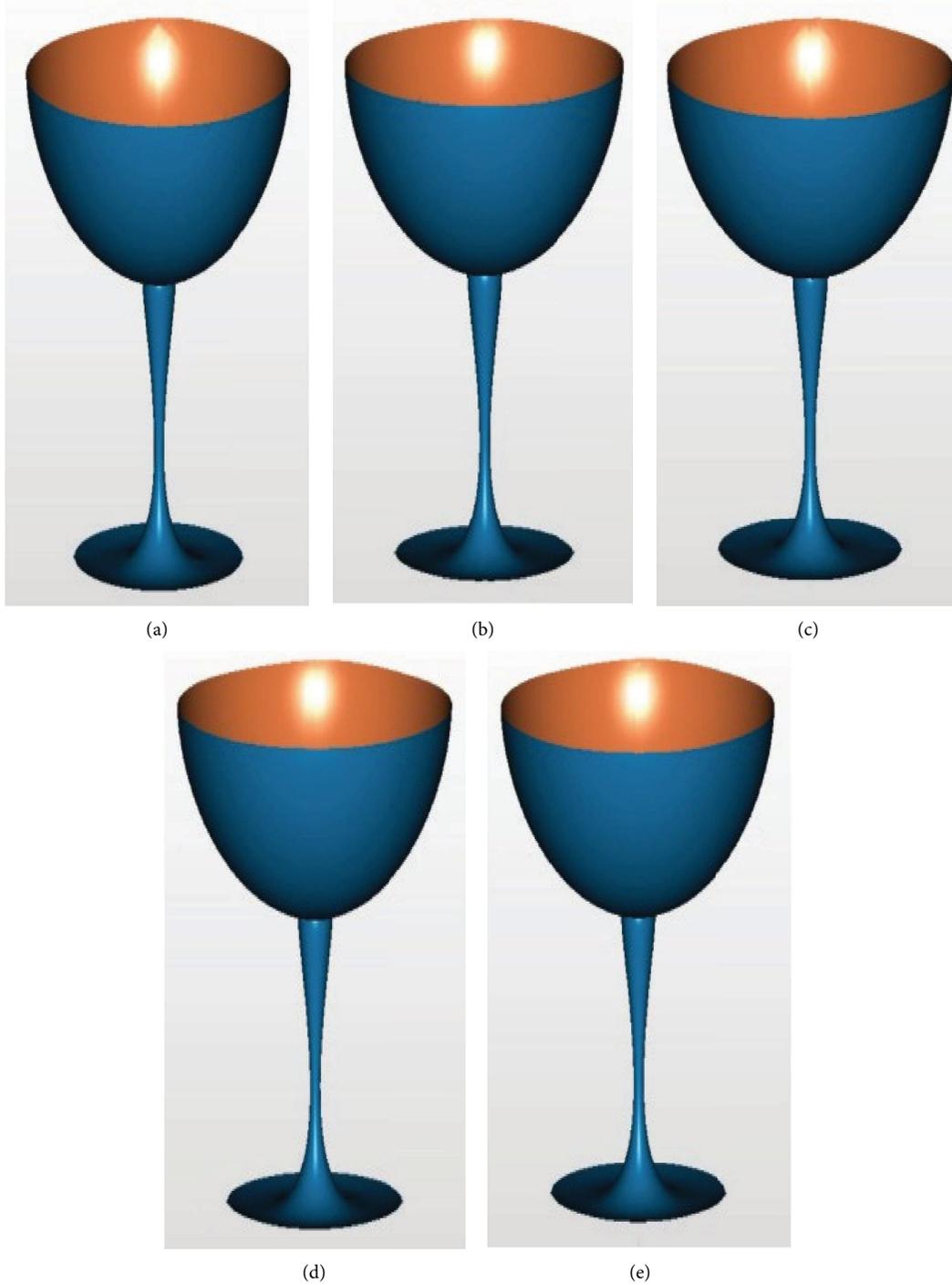


FIGURE 10: (a) First glass model when the smoothness parameter is  $\eta_{11} = 0.1225$ . (b) First glass model when the smoothness parameter is  $\eta_{21} = 0.2437$ . (c) First glass model when the smoothness parameter is  $\eta_{31} = 0.1913$ . (d) First glass model when the smoothness parameter is  $\eta_{41} = 0.2102$ . (e) First glass model when the smoothness parameter is  $\eta_{51} = 0.1373$ .

$$B_4 = \begin{bmatrix} 0.0640 \\ 0.2102 \\ 0.1489 \\ 0.0424 \\ 0.2400 \end{bmatrix} .$$

(24)

$\eta_{14} = 0.0640$ ,  $\eta_{24} = 0.2102$ ,  $\eta_{34} = 0.1489$ ,  $\eta_{44} = 0.0424$ , and  $\eta_{54} = 0.2400$ . Putting these values in equation (8) for  $\eta_{mn}$  gives slightly different designs.

$\eta_{54} > \eta_{24} > \eta_{34} > \eta_{14} > \eta_{44}$ . Similarly, by observing the data given for this design, it is noticed that  $\eta_{54}$  has the finest quality with volume closer to 250 ml. So, the glass generated

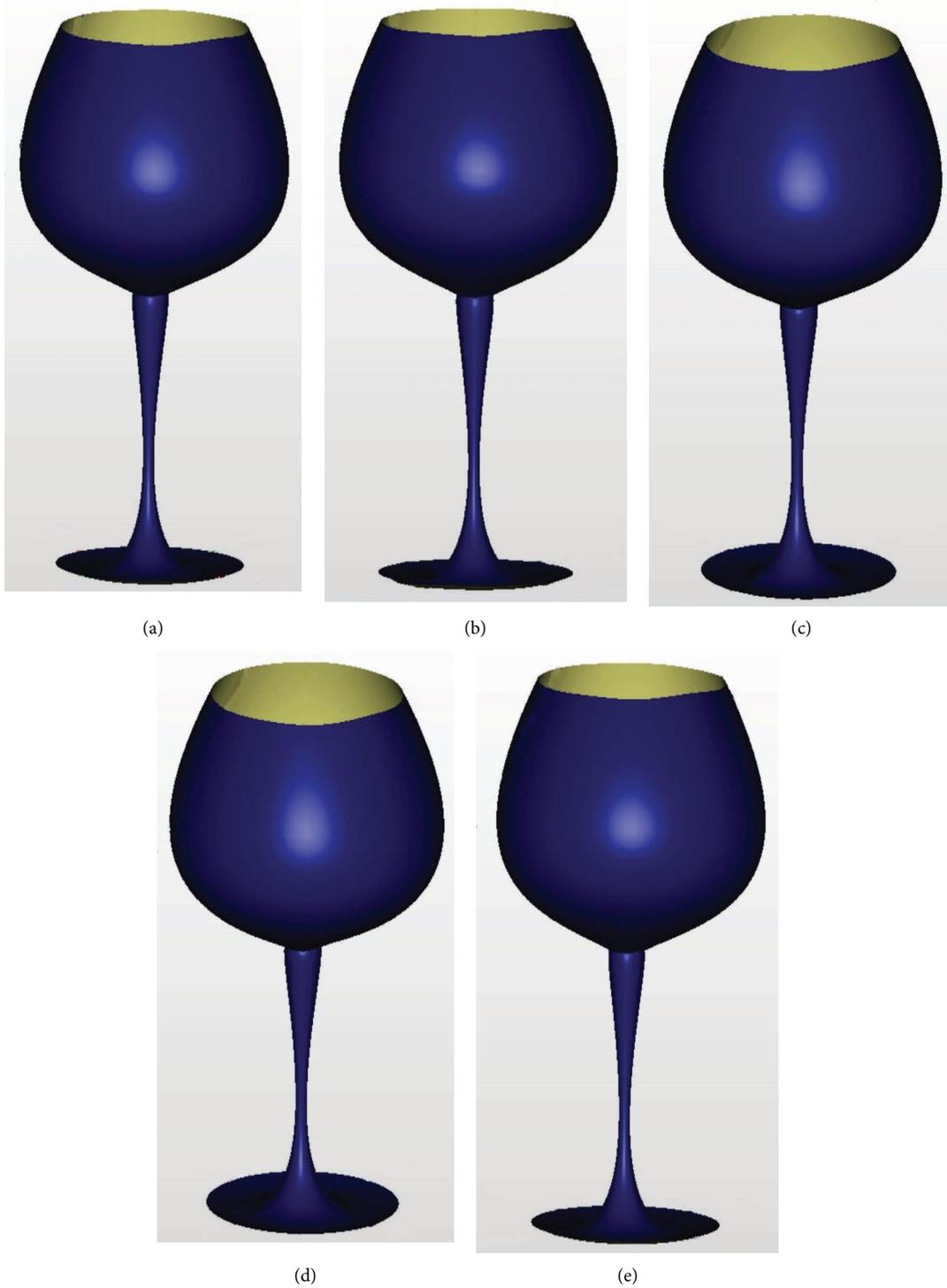


FIGURE 11: (a) Second glass model when the smoothness parameter is  $\eta_{12} = 0.0982$ . (b) Second glass model when the smoothness parameter is  $\eta_{22} = 0.2016$ . (c) Second glass model when the smoothness parameter is  $\eta_{32} = 0.1631$ . (d) Second glass model when the smoothness parameter is  $\eta_{42} = 0.1835$ . (e) Second glass model when the smoothness parameter is  $\eta_{52} = 0.2485$ .

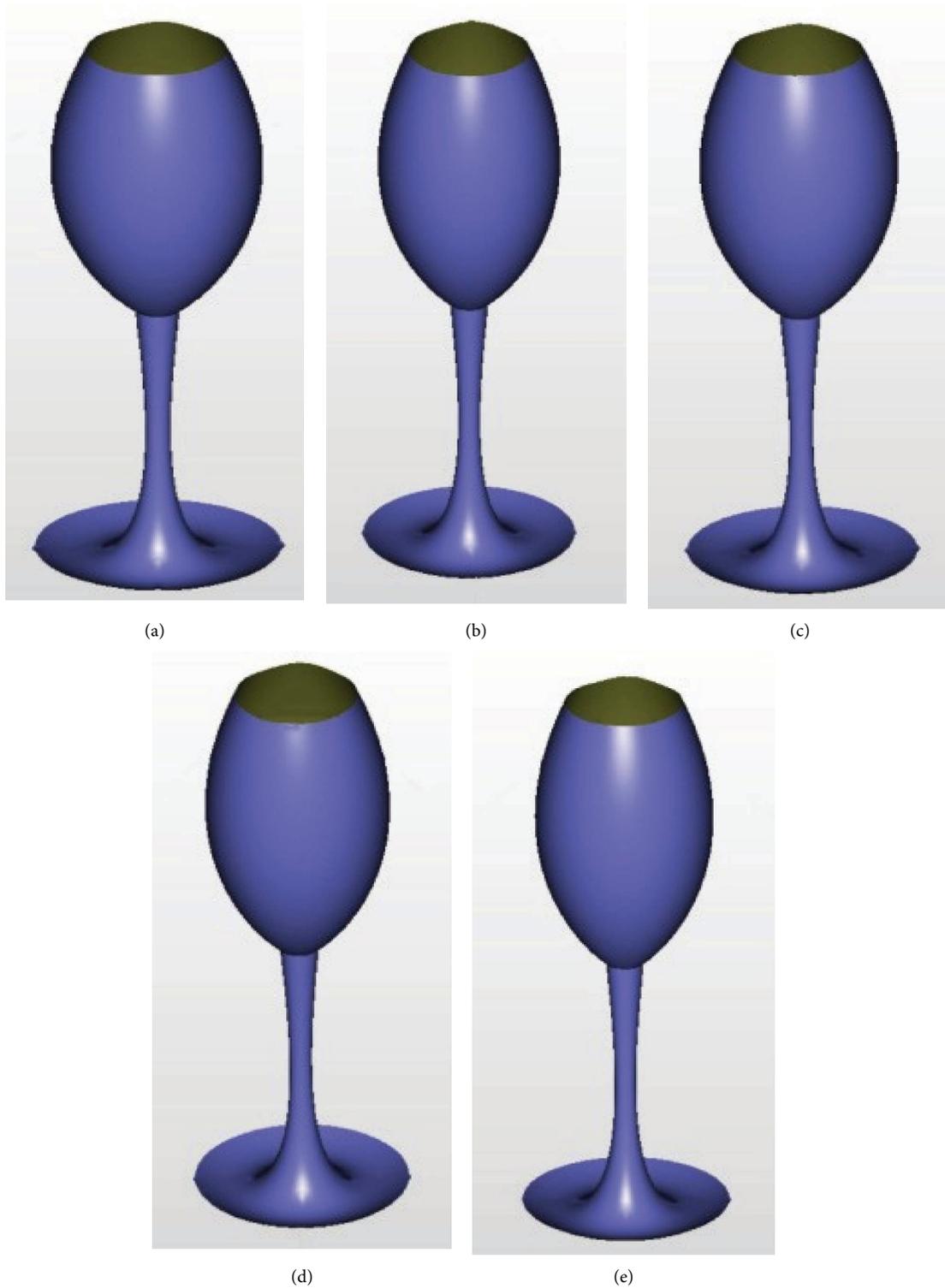


FIGURE 12: (a) Third glass model when the smoothness parameter is  $\eta_{13} = 0.0862$ . (b) Third glass model when the smoothness parameter is  $\eta_{23} = 0.1973$ . (c) Third glass model when the smoothness parameter is  $\eta_{33} = 0.2336$ . (d) Third glass model when the smoothness parameter is  $\eta_{43} = 0.1255$ . (e) Third glass model when the smoothness parameter is  $\eta_{53} = 0.0500$ .

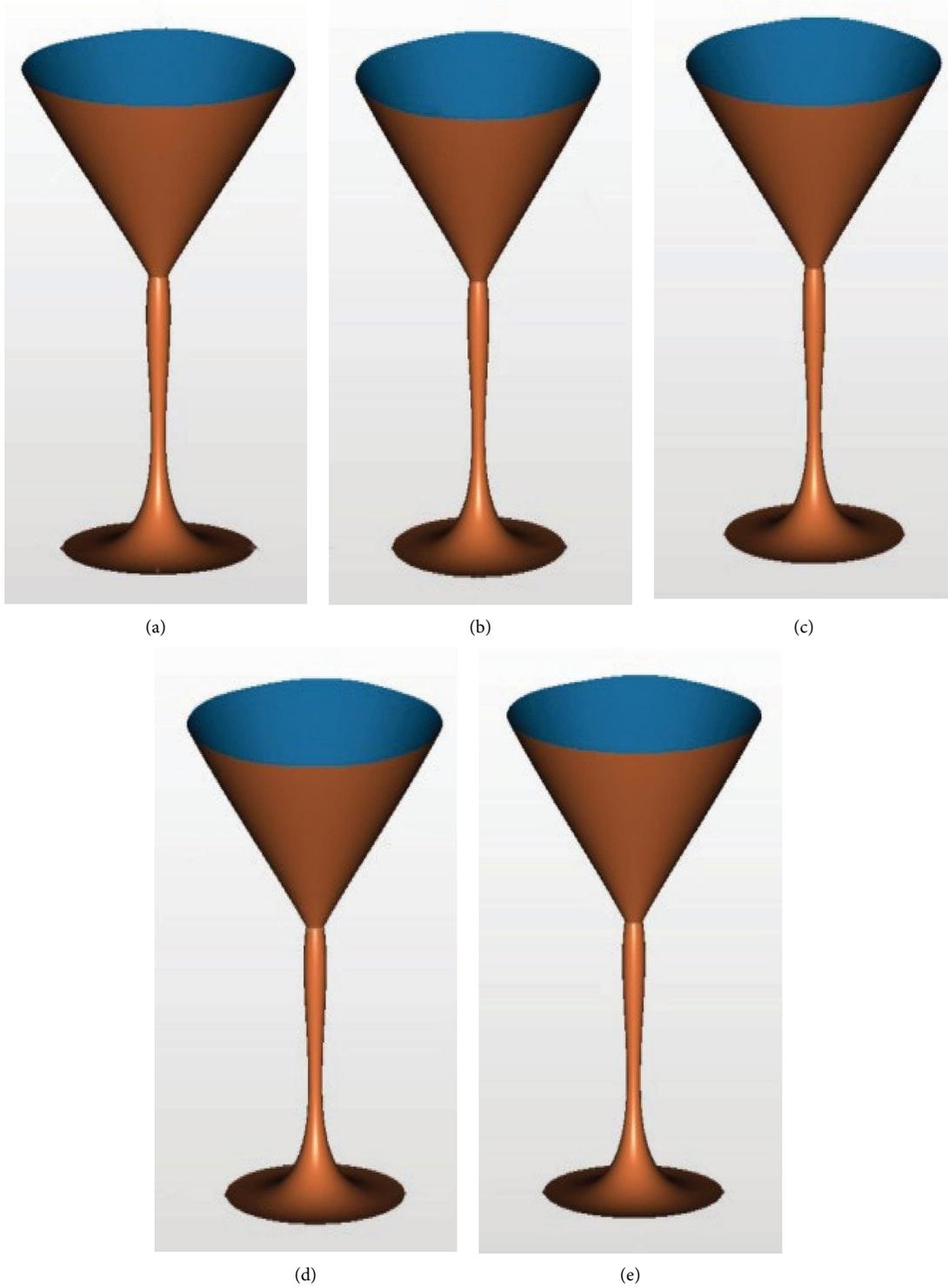


FIGURE 13: (a) Forth glass model when the smoothness parameter is  $\eta_{14} = 0.0640$ . (b) Forth glass model when the smoothness parameter is  $\eta_{24} = 0.2102$ . (c) Forth glass model when the smoothness parameter is  $\eta_{34} = 0.1489$ . (d) Forth glass model when the smoothness parameter is  $\eta_{44} = 0.0424$ . (e) Forth glass model when the smoothness parameter is  $\eta_{54} = 0.2400$ .

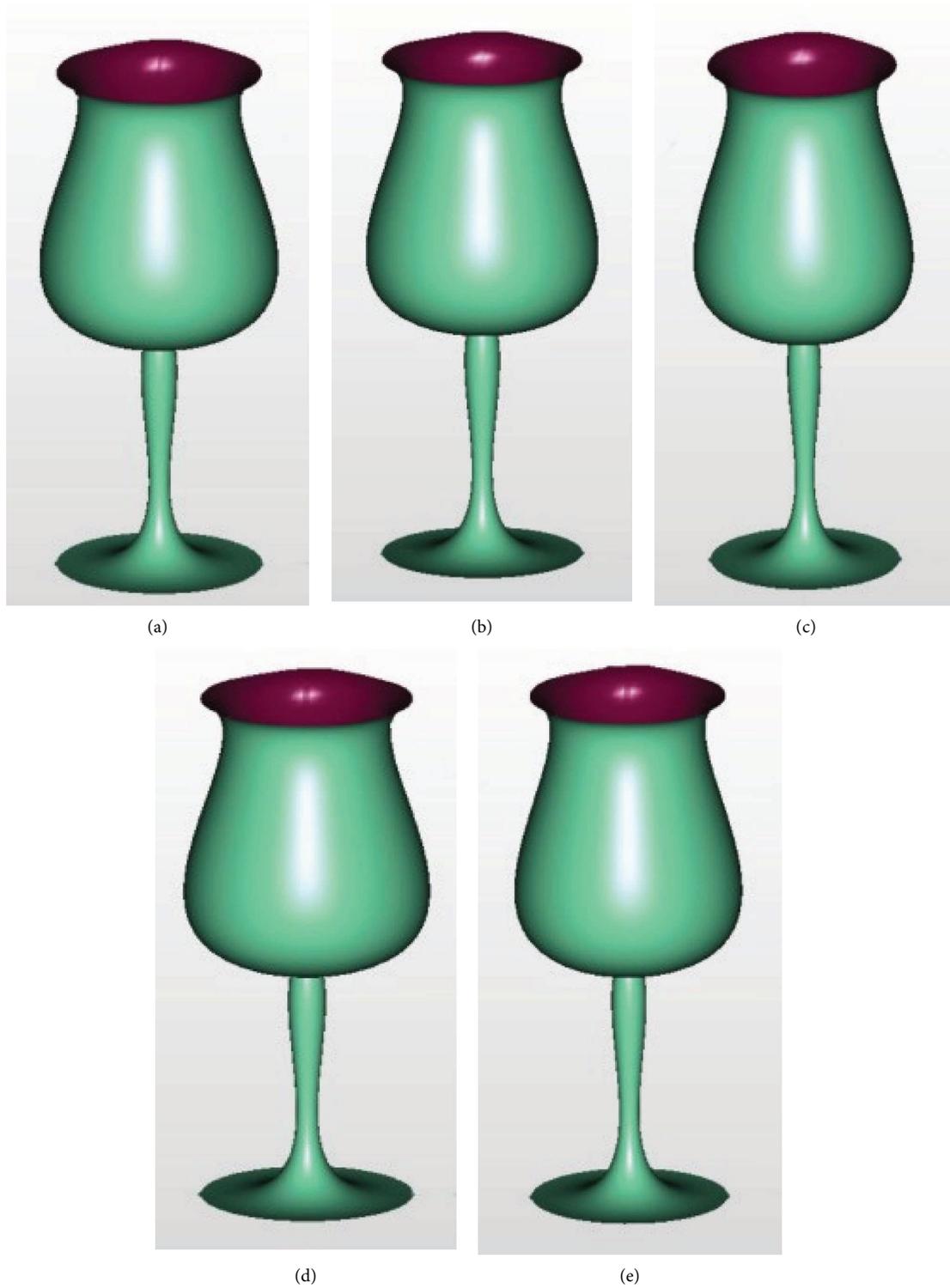


FIGURE 14: (a) Fifth glass model when the smoothness parameter is  $\eta_{15} = 0.2163$ . (b) Fifth glass model when the smoothness parameter is  $\eta_{25} = 0.2365$ . (c) Fifth glass model when the smoothness parameter is  $\eta_{35} = 0.1741$ . (d) Fifth glass model when the smoothness parameter is  $\eta_{45} = 0.1494$ . (e) Fifth glass model when the smoothness parameter is  $\eta_{55} = 0.1192$ .

in Figure 13(d), with  $\eta_{54}$ , is the selected glass from the glass design for  $G_4$  shown in Figures 13(a)–13(e).

Fuzzy soft matrix for the last glass design according to the suggestions of the designers is as follows:

$$G_5 = \begin{bmatrix} 0.90 & 0.30 & 0.80 \\ 1.00 & 0.35 & 0.83 \\ 0.60 & 0.70 & 0.30 \\ 0.50 & 0.63 & 0.25 \\ 0.40 & 0.20 & 0.50 \end{bmatrix}. \quad (25)$$

Now, for this last fuzzy soft matrix  $G_5$ , WBFSM is as follows:

$$B_5 = \begin{bmatrix} 0.2163 \\ 0.2365 \\ 0.1741 \\ 0.1494 \\ 0.1192 \end{bmatrix}. \quad (26)$$

Here,  $\eta_{15} = 0.2163$ ,  $\eta_{25} = 0.2365$ ,  $\eta_{35} = 0.1741$ ,  $\eta_{45} = 0.1494$ , and  $\eta_{55} = 0.1192$ . Putting these values for  $\eta_{mn}$  in equation (8) gives slightly different designs shown in Figures 14(a)–14(e):

$\eta_{25} > \eta_{15} > \eta_{35} > \eta_{45} > \eta_{55}$ . Here,  $\eta_{25}$  has the greatest value, so it is our selected glass for  $G_5$ . The selected glass shown in Figure 14(b) has volume closer to 250 ml and finest quality.

Now, all the values of  $\eta_{mn}$  will generate a matrix such that

$$\begin{bmatrix} 0.1225 & 0.0982 & 0.0862 & 0.0640 & 0.2163 \\ 0.2437 & 0.2016 & 0.1973 & 0.2102 & 0.2365 \\ 0.1913 & 0.1631 & 0.2336 & 0.1489 & 0.1741 \\ 0.2102 & 0.1835 & 0.1255 & 0.0424 & 0.1494 \\ 0.1373 & 0.2485 & 0.0500 & 0.2400 & 0.1192 \end{bmatrix}. \quad (27)$$

A maximum value has been selected from each column of the abovementioned matrix, and by the definition of the optimum smoothness parameter,  $\bar{\eta}_{mn}$  defined in Step 3 will be used, which is as follows:

$$\bar{\eta} = \max(0.2437, 0.2485, 0.2336, 0.2400, 0.2365), \quad (28)$$

$$\bar{\eta} = 0.2485.$$

As  $\bar{\eta} = 0.2485$ , so the glass design generated from this value is the best design according to the customer's requirement. If the data of the glass designs mentioned previously are observed, it can be seen that the final selected design has the lowest price and finest quality among all the five designs. Also, the volume of the selected glass design is slightly less than 250 ml, which is in the favor of our customer. The shape of the designed glass is also favorable for fizzy drinks. According to the case study about glass design on drinking behavior discussed in [38], it can be concluded that the selected design has the best effect on the drinking behavior of a person since the selected glass design has the

curve that makes it easy for a person to take a sip of his drink. Also, its shape does not allow the fizz in the drink to die down immediately, so a person can enjoy his drink for a long time. Hence, the selected glass design has the best result in the favor of the customer, and it also fully satisfies his requirements.

From the influence of each glass design on the drinking behavior of a person, it can be proved that the selected glass is the best choice for Mr. Khan. Looking at the design of  $G_1$  glass, it can be seen that it is shaped like a parabola. It also has a wide mouth, which makes it easier for a person to drink from it. Although it is a good shape of glass to serve juices, but it is not an ideal shape for fizzy drinks since it does not hold the fizz in the drink for a long time. Due to its wide mouth and parabolic shape, the fizz in the drink dies down fast. The design of glass  $G_3$  is like a cylinder. Although it has a curve in its shape, which makes it a good choice for fizzy drinks, but its mouth is so narrow which makes it difficult for a person to drink from it. Its narrow mouth and low volume does not make it a good choice for fizzy drinks, but it is a good choice for alcoholic drinks because these types of glasses reduce the consumption of alcoholic drinks which is also mentioned in a case study on drinking behaviors [38]. The next glass design  $G_4$  is a V-shaped glass. As it can be seen in the figure mentioned previously, this glass has a wide mouth with no curve in its shape, which makes it a bad choice for fizzy drinks. The reason for this glass not being a good choice for fizzy drinks is the same as the design of glass  $G_1$ , but unlike glass design  $G_1$ , it has a small volume, which makes it a good choice for nonfizzy drinks taken in small quantity. They are also a good choice for small serving of juice for kids. The glass design  $G_5$  has a design like a vase or a tulip flower. Although it has an attractive shape, its deep curve and narrow mouth does not make it a good choice for fizzy drinks. This type of glasses is best for serving milk drinks such as milk shakes or coconut milk. Now, on observing the selected design of the glass which is  $G_2$  glass design, it is clear that its shape is the ideal shape for fizzy drinks. It has a right amount of curve to hold the fizz in the drink for a long time and its mouth is not narrow, which allow a person to drink from it easily and enjoy his drink. On observing all the designs of the glasses, it is proven that the selected glass not only has the best shape for fizzy drinks but also fulfills the requirements of the customer, Mr. Khan.

## 4. Conclusion

In this article, an intelligent system is developed for geometric designing of products according to industrial requirements. This intelligent system is developed using the fuzzy soft partial differential equation with weighted Bonferroni mean to design the geometric models of the required product. For this purpose, a parameter known as the optimum smoothness parameter is defined, using WBFSM, in the PDE. This parameter fulfills industrial requirements and generates the optimal design of the desired product. The overall shape of the surface is determined by the boundary conditions (which are effectively defined by curves in 3-space) at the edge of the surface patch. As a result, it is

relatively simple for a designer to use this intelligent system to create objects of practical significance using standard computer devices. As this intelligent system uses close curves, rather than using standard shapes, such as ellipse, square, and cylinder, it is easier to shape it according to the need. Finally, to show the effectiveness of the developed technique, an illustrated example is developed for the hotel industry to design a drinking glass according to their requirements. From these geometric designs of the glasses, the design developed using the optimum smoothness parameter was selected, as it fulfilled the customer's requirements in the best way possible. All of this process was performed prior to the development of the product, which reduced the time and money spent on the development of the glass.

The developed intelligent system of PDE with weighted Bonferroni fuzzy soft matrix can be useful in many industries. In the future, this developed system can further be extended in the development of many products. It can be used in the development of capsule shells in pharmaceutical companies because temperature has a major effect on these capsule shells. Extreme temperature can melt them, so this PDE method can be used by taking temperature as the main parameter. Similarly, it can also be used in the manufacturing of robots, aircrafts, and home decor products.

### Data Availability

The datasets generated during and/or analyzed during the current study are available from the corresponding author upon reasonable request.

### Additional Points

*Human and Animal Rights.* We would like to mention that this article does not contain any studies with animals and does not involve any studies on human beings.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

### Authors' Contributions

The authors equally conceived the study, participated in its design and coordination, drafted the manuscript, participated in the sequence alignment, and read and approved the final manuscript.

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