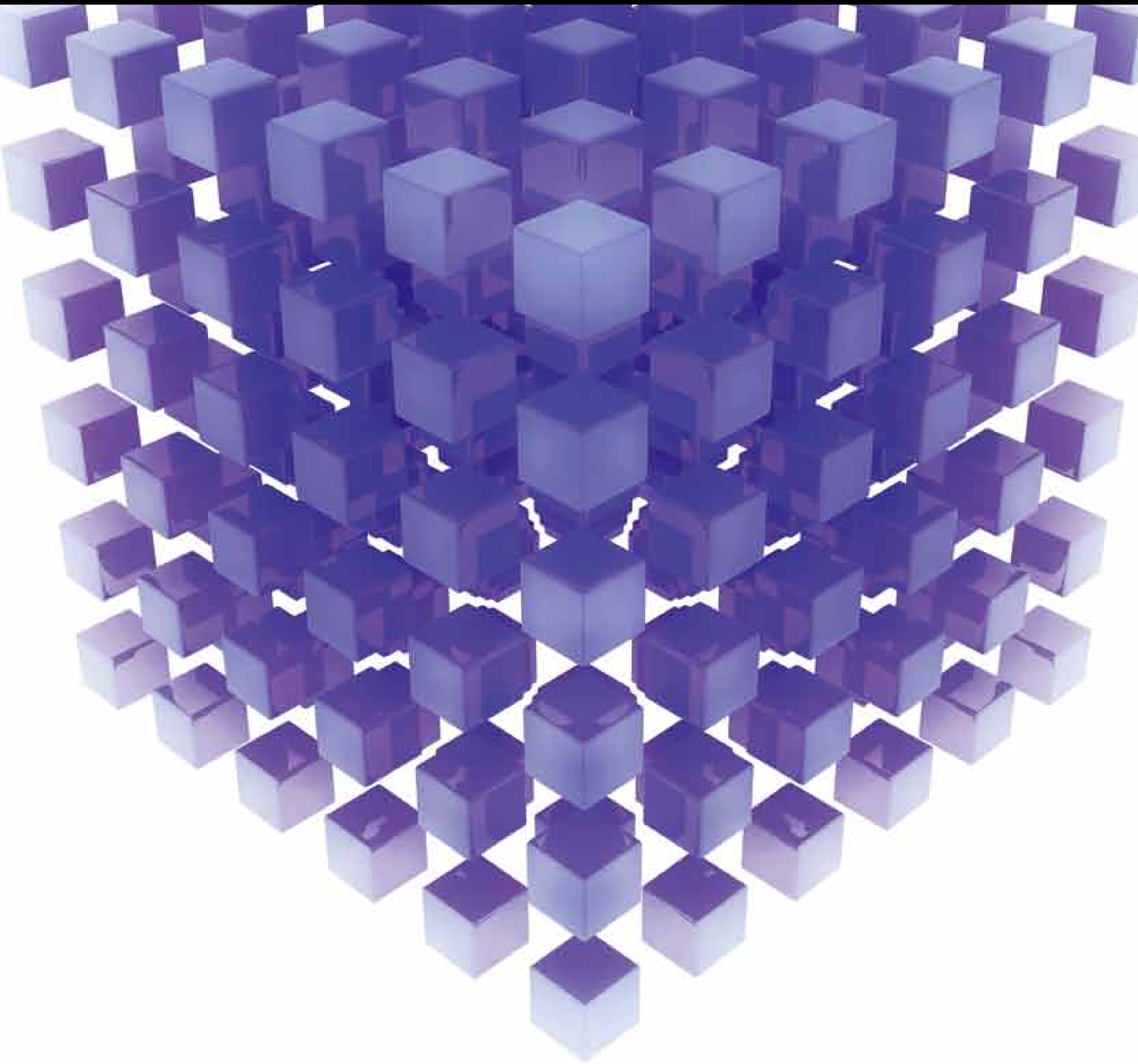
Mathematical Problems in Engineering

Service Optimization and Control

Guest Editors: Tsan-Ming Choi, Pui-Sze Chow, and Kannan Govindan



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Editorial Board

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Editorial

Service Optimization and Control

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Service operations are critical in modern business. Service providers, such as those in the telecommunication industry and other retailing sectors, are providing services to the market with a goal of optimizing their own profit. However, a service operation is different from the operations which manage the "physical products." For example, many traditionally critical decisions such as the optimal "physical product" production quantity and optimal "product" quality control are no longer relevant [1, 2]. This calls for a totally new engineering mindset and novel analytical models in order to optimize the performance of the respective service systems.

This special issue is devoted to publishing the latest and significant results on scientific research in service optimization and control. This special issue puts high emphasis on the advance of optimization methods and real world applications from a systems engineering perspective. Thus, there are both analytically focused studies as well as empirical analyses. It is also interesting to note that a couple of papers are adopting both the analytical and the quantitative empirical approaches. Thus, we can see the multidisciplinary as well as multimethodological nature of the service optimization related research.

After conducting rigorous peer review, this special issue has accepted 14 technical papers related to service optimization and control. We describe each one of them concisely as follows.

In "*A capacitated location-allocation model for flood disaster service operations with border crossing passages and probabilistic demand locations,*" motivated by the consequences of flood disasters, S. A. Mirzapour et al. explore the proper locations of relief rooms to provide the needed service. They formulate the problem as a p-center location problem. They model the problem as a mixed integer nonlinear optimization problem with the capacitated facility location-allocation consideration. They simultaneously consider the probabilistic distribution of demand locations and the line barrier in the given region. They try to minimize the maximum expected weighted distance from the relief rooms to all the demand regions. This optimization objective helps to decrease the evacuation time of people from the affected areas before flood occurrence. To illustrate the applicability of their model, they present a real case study.

In "*Service quality of online shopping platforms: a case based empirical and analytical study,*" T. M. Choi et al. study the service business model of online shopping platforms (OSPs). They focus on customer service aspect of OSPs and examine the specific case on http://www.taobao.com/index_global.php. They identify the customer perceptions of the service quality associated with Taobao's functions. They also explore these functions' impacts on customer loyalty. From an empirical study, they find that the "fulfillment and responsiveness" function is most significantly related to the customer loyalty. Based on this empirical finding, they conduct an analytical study and derive the optimal service level on this most critical function. By employing the safety first optimization objective, they prove that the optimal service level uniquely exists. Their further analysis reveals that a bigger optimal service level results if the customer loyalty is positively correlated to the service level. In addition, they find that the optimal service level is independent of the respective profit target under the safety first objective, the source of uncertainty, and the risk preference of the OSP company.

In "*Performance analysis and optimization of an adaptive admission control scheme in cognitive radio networks,*" S. Jin et al. study the cognitive radio networks with the secondary user (SU) packets. To be specific, they propose an adaptive admission control method with a system access probability for the SU packets. They introduce an adaptive factor which helps to adjust the system access probability. By building a discrete-time preemptive queueing model with adjustable joining rate, they conduct analytical exploration. To derive the steady-state distribution of the queueing model, they construct a two-dimensional Markov chain model. They conduct a computational study and present numerical findings on the impacts brought by the adaptive factor. They further derive an optimal pricing mechanism for the system.

In "*Emergency department staffing: a separated continuous linear programming approach,*" X. Wang explores overcrowding problem in the emergency department of hospital. She argues that the shortage of staff members in the emergency department is the root problem. Based on this argument, she constructs a new analytical model to address the emergency department staffing problem. To be specific, she models the problem as a separated continuous linear programming problem. She develops an efficient algorithm to determine the optimal emergency department staffing level under the total cost minimization objective.

In "*A model for assessing the service quality of university library websites,*" C. M. Wu et al. explore the criteria for assessing the service quality of library websites. They examine the problem from university students' viewpoints. Based on the fuzzy Delphi method, they employ the ANP approach to generate the priority weights of criteria. In total, a list of 12 web-based service criteria is identified according to the empirical inputs from over 3000 university students. These criteria include choices for searching for information, protection of personal information, website availability, and so forth. Different from prior research which ignores the important interdependence among criteria, C. M. Wu et al. propose the ANP approach which can capture this interdependence and they argue that their proposed results are more accurate.

In "*Crowdsourcing new product design on the web: an analysis of online designer platform service,*" X. Dai et al. study the business model called the Designer Platform Service (DPS). In fact, DPS is a combined mechanism of crowdsourcing and group buying on the web. It helps boost the growth of entrant fashion designers and links designing tasks with the real world market. They conduct an analytical optimization study on how the optimal pricing and minimum production quantity decisions are made in the DPS. They consider factors such as the entrant designer's objective, the specific decision sequences, and the structures of demand. They investigate the problem by developing the model as a Stackelberg game and derive the respective equilibrium solutions.

In "*Review on the research for separated continuous linear programming: with applications on service operations,*" X. Wang conducts a technical review on the research for Separated Continuous Linear Programming (SCLP). She examines several important formulations of SCLP. She discusses the SCLP related duality theory and solution method. She argues that most results on duality theory can provide the analytical conditions for SCLP to exhibit strong duality. She further reveals that most solution approaches for SCLP belong to either the simplex-like or the discretization-based category. She finds that the simplex-like approach helps to get the exact optimal solution but is computationally very time-consuming whereas the discretization-based methods are fast but can only lead to the approximate solutions.

In "*Cooperative advertising in a supply chain with horizontal competition,*" Y. He et al. examine the advertising strategies for a supply chain with two suppliers and one buyer. They study the supply chain system in three different cases: (i) each supply chain agent makes individual decisions independently, (ii) the retail buyer is integrated with one manufacturer, and (iii) both manufacturers are horizontally integrated. They analytically find that the manufacturer's optimal advertising efforts are independent of the participation rates offered to the retail buyer in all three cases. They argue that when the retail buyer is integrated with one manufacturer, the other manufacturer's optimal advertising efforts will not be affected. For the case when both manufacturers are horizontally integrated, they reveal that these two manufacturers would reduce the advertising efforts to avoid conflict.

Incentive alignment contracting is an important measure in supply chain management. In "*Contract strategies in competing supply chains with risk-averse suppliers,*" B. Li et al. study analytically the equilibrium contracting strategies in the presence of two competing supply chains. In their model, each supply chain includes one risk-averse supplier and one risk-neutral retailer. The supplier in each supply chain acts as the Stackelberg leader and can choose either the wholesale pricing contract or the revenue sharing contract to offer to the retailer. They study the impacts brought by factors such as competition density and degree of risk aversion on the suppliers on the equilibrium contract choices. They interestingly find that it is always an optimal decision for the first-moving supplier to choose the revenue sharing contract if the second-moving supplier chooses the wholesale pricing contract. They further reveal that the optimal retail price under the revenue sharing contract is lower than the one under the wholesale pricing contract. They also find that there is a threshold policy on the degree of risk aversion for the suppliers which influences their optimal choice on the individual contract types. These ultimately would affect the contract strategies under the competing supply chains scenario.

In "*Congestion service facilities location problem with promise of response time,*" D. Hu et al. study the congestion service facilities location problem in which there is a response time promise. They explore the problem via a queueing model which assumes the customer demands are generated at each node and requests for service arrive as a random variable following the Poisson process. They consider the response time to include sojourn time and travel time. They propose a mixed integer nonlinear programming model for locating service facility with the promise of response time. They take the locations of the service facilities and the number of servers at each facility as the control variables. In their optimization model, the objective function is to maximize the demand being served within the promised response time. They propose a hybrid algorithm which combines greedy and genetic algorithms to solve the problem. They conduct a number of computational experiments to test the performance of the algorithm and propose that the response time promise has a critical effect on the optimal location decision.

In "*On advertising games and spillover in service systems,*" L. Xu et al. analytically model the advertising competition game between a dominant service provider and some smaller service providers. They consider the scenario when the dominant service provider enjoys a larger market share, and the other smaller service providers only share the remaining "smaller" market share equally. They explore three advertising game models, which include the cooperative game, the Boxed Pig game, and the Prisoner's Dilemma game. For each game, they derive the conditions for having an equilibrium. They reveal that the advertising spillover and the number of the smaller service providers affect substantially the equilibria. They also discuss the research implications.

In "*Order allocation research of logistics service supply chain with mass customization logistics service,*" W. Liu et al. observe that customers have a high demand for specialized and customized logistics services. Motivated by the industrial practice in logistics service supply chain (LSSC) management, they study the order allocation between a logistics service integrator (LSI) and several functional logistics service providers (FLSPs). They focus on the logistics service under mass customization. They formulate a multiobjective order allocation model of LSSC. The problem is constrained with respect to factors such as customer demand, customer order decoupling point, and order difference tolerance coefficient. Their analysis reveals that the LSI prefers FLSPs in the presence of a higher scale effect coefficient. They also show that setting a high relationship cost coefficient does not necessarily yield enhanced results. They further demonstrate that for the FLSPs, a continuous improvement of large-scale operational capacity is desirable. Moreover, they argue that if the LSSC's comprehensive order allocation performance is high, then the LSI will offer cost compensation in order to improve the LSSC's level of satisfaction.

Motivated by industrial practices price comparison service (PCS) websites, in "*The impact of price comparison service on pricing strategy in a dual-channel supply chain,*" Q. Xu et al. examine the pricing strategies of retailers and supplier in a dual-channel supply chain influenced by the presence of PCS. They categorize the problem with respect to the signal availability of PCS and formulate three specific cases. For each case, they analytically derive the corresponding optimal pricing strategy. They further conduct a numerical analysis. Their study shows several important insights, which include the following: when the retailers are all affected by the PCS, the supplier is more willing to reduce the availability of price information.

In "*Sales forecasting for fashion retailing service industry: a review,*" N. Liu et al. conduct a comprehensive literature review on the topic of fashion retail sales forecasting. They focus on exploring the advantages and the disadvantages of different kinds of fashion retail sales forecasting models. They also study the real world applications of the fashion retail sales forecasting models. From the reviewed literature, they comment that, over the past 15 years, the pure statistical methods are not popularly examined in the literature. In fact, most studies are focusing on the deployment of artificial intelligence methods and some hybrid models. They argue that the use of hybrid methods for fashion retail sales forecasting is trendy and timely, and it is still a promising area for further explorations.

We believe that this special issue presents many interesting and innovative research on service optimization and control related problems. We hope the published papers will provide a good foundation for future scientific research related to service optimization and control.

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Kannan Govindan

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Research Article

Crowdsourcing New Product Design on the Web: An Analysis of Online Designer Platform Service

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A designer is a core resource in the fashion industry. Successful designers need to be creative and quick to understand the business and wider environment in which they are operating. The *Designer Platform Service* (DPS), which combines the mechanism of crowdsourcing and group buying on the web, provides a platform for entrant designers to try their abilities in the real market practice. Freelance designers post design samples or sketches of products on the website of DPS, and consumers may preorder the products (each at a fixed price) online based on the design information. Once the number of ordering reaches or passes a certain threshold, that is, the minimum production quantity (MPQ), DPS will arrange for production and delivery according to the orders received. This novel service boosts the growth of entrant designers and links designing works with real markets directly. We are interested in how the price and MPQ decisions are made in DPS, with consideration of the entrant designer's objective, decision sequences, and customer demand structures. We develop Stackelberg games to model and derive the equilibrium solutions under individual scenarios. Our findings suggest feasibility of the DPS business model.

1. Introduction

Fashion is among the most important creative industries that enjoy a great deal of attention these days. Fashion designers are the core of value creation in the fashion industry, as they are normally the source of creativity. In the UK alone, the designer sector produced £700 million income in 2003, of which £75 million (11 percent) was from license fees to designers [1]. The fashion industry is also a highly competitive industry, where product life cycles are short, economies gained by product differentiation are built on brand image, and product styling can be quickly imitated [2]. Designers have to play a dual role as creative individuals and as entrepreneurs in any provided complicated situation.

Entrant designers are an important source of innovation in the fashion industry. Yet, the professional lives of fashion entrant designers are particularly hard. It takes time and resources for advertising and promotion so that a new designer label can sustain a share in the market, whereas most new start-up designers are lacking those resources [3]. However, in the market practice, resources are often allocated on those established designers, as the market has proved the attractiveness of their products, so that the resources allocated could be expected with higher and less risky return. Without advertising and promotion, most entrant designers would start their career with a relatively low and uncertain demand for their design, until they build up their name and cultivate a market. The low and uncertain demand faced by entrant designers hinders their design from being carried to production, as this demand may be far lower than the minimum production quantity which is commonly applied in textile manufacturing. Even if the production can be made, the small production quantity and uncertain demand would lead to high production and inventory cost.

In the past decade, the internet has bred a new approach called ”crowdsourcing” that help individuals make their innovative ideas feasible. Named by Howe [4], crowdsourcing refers to the method that solicits contributions from a large group of people from an online community in order to make a certain service, idea or product available or feasible. Howe [4] also defined the sponsoring crowd as "the new pool of cheap labor: everyday people using their spare cycles to create content, solve problems, and even do corporate R&D." Many works have been done recently, on crowdsourcing contests [5] and noncompetitive crowdsourcing ideation initiatives [6–8]. For example, the crowdsourcing t-shirt platform, teespring.com, provides a way for people to create and sell t-shirts online to raise money. Here is how it works: first, people design a t-shirt and post it on the website, choose a goal, and set a price to launch a campaign page; second, people send the campaign page to supporters and let them preorder the t-shirts towards the campaign goal (preordering is free: buyers will only be charged if the goal is reached); third, people can continue to sell shirts past the goal until the campaign ends. Once it does, the platform handle production and shipment; people will get a check for the profit. The business model of teespring.com, which combines the mechanism of crowdsourcing and group buying on the web, has provided a platform for entrant designers to try their abilities in the real market practice. In this business model, there are two key decisions to make, the price of the product and the target volume for production (equal to the campaign goal in the case of teespring.com), which are similar to the widely discussed problem of "joint pricing-production decisions" in operation management [9, 10].

To better illustrate how this business model works, we consider a simplified case, Designer Platform Service (DPS), which works as a retailer connecting designers with customers and designers with manufacturers. Similarly, DPS crowdsources the product design tasks to the entrant designer community and allows the customers to make purchases in the way of group buying. First, entrant designers post design samples or sketches of products on the website of DPS, and then consumers preorder the product (each at a fixed price) online based on the product information. The entrant designer decides the selling price of the product, while the DPS decides the minimum production quantity (MPQ). Only when the preorder quantity reaches or passes the MPQ threshold will DPS arrange for production and delivery. Preorder customers then make their payment and get the products delivered. Otherwise, no production will be arranged, and no one will be charged. This novel service provides variety to customers, boosts the growth of entrant designers, and links designing works with real markets. In the whole process, we are most interested in how the price and MPQ decisions are made within DPS, with consideration of entrant designer's objective, decision sequences, and customer demand structures.

Through the study in the form of Stackelberg games and by considering various scenarios, we investigate the decisions in the mechanism with B2C transactions: an entrant designer utilizes DPS to promote her design to consumer, and we reveal the price-discovery mechanism in this e-market. We consider that in our model, the two key players, the entrant designer and DSP, have very different objectives. Due to the special stage of an entrant designer in her professional career, her priority mission is to leverage the creativity with market acceptance, so that she can later find out a way to boost her design to profitability. Our findings provide insights on how the mechanism protects DPS in pricing decision in a highly versatile and risky market, and how the mechanism provides a feasible path for entrant designers to test and adjust to the market with limited loss. Moreover, our findings suggest feasibility of the DPS business model.

The paper contributes in providing insights of equilibrium decisions in the combined business model of crowdsourcing and group buying. The paper is new in investigating a creative business model which could significantly utilize creativity and reduce cost. The paper is also innovative in considering business players having nonmonetary objectives. By investigating the two key decisions in the DSP, we show how an online platform could raise a business by crowdsourcing designs, arranging manufacturing and selling to group buying customers. Specifically, we derive optimal decisions for the two parties under the different objectives of the entrant designer, decision sequences, and market demands, through which we show how an entrant designer could benefit from DSP. The findings provide guidelines to practitioners in platform service and designers and help utilize creation with profit.

The remainder of the paper is organized as follows. In Section 2, we review the related literature. We construct the Stackelberg game models in Section 3 and find the best response decisions in Section 4. In Section 5, we derive the equilibrium decisions of the games with specific demand functions. We discuss and conclude in Section 6 with limitations and future research directions.

2. Literature Review

This paper relates to the literature (e.g., [11–13]) in service platform and group buying business on the web. Web-based group buying mechanisms are being widely used in both business-to-business (B2B) and business-to-consumer (B2C) transactions. Li and Lee [12] show that with the uncertain peer-produced services quality, a monopolistic platform provider has no incentive in offering multiple quality classes of service, while two competing platform providers may offer identical service contracts but still receive nonnegative profit. Anand and Aron [14] studied a monopolist offering Web-based group buying under different kinds of demand uncertainty. They derive the monopolist's optimal group-buying schedule under varying conditions of heterogeneity in the demand regimes and compare its profits with those obtained under the more conventional posted-price mechanism.

Pricing decision and determination of the minimum production quantity with price-dependent demand are the main focus of the models explored in this paper. In marketing and operations management literature, there is a huge amount of related studies under various settings and we review some of them as follows. For instance, Pasternack [15] studies the pricing decision and return policy for perishable products. Cai et al. [16] explore from the game-theoretic perspective the pricing scheme of a supply chain with dual channels that compete with one another. The authors show that a simple price discount contract can achieve channel coordination. In particular, the pricing issue has aroused the attention of many researchers when the demand is price-dependent. Petruzzi and Dada [17] investigate the optimal selling price and stocking quantity under the newsvendor domain with price-dependent demand. Yang et al. [18] consider an inventory system for noninstantaneous deteriorating items with price-dependent demand and develop an algorithm to solve the corresponding optimal price and order quantity when partial backlog is allowed. Chen et al. [19] suggest some coordination schemes for a supply chain with lead time consideration and price-dependent demand. Chiu et al. [20] suggest a mechanism for coordinating the pricing and stocking decisions for a supply chain with price-dependent demand.

To make use of the economy of scale, it is common for suppliers to impose some minimum quantity requirement on orders. Such practice is prevalent in the apparel industry, and Fisher and Raman [21] provide a well-studied example of minimum order quantity imposition in the industry. The classical inventory literature studies the minimum quantity in the form of lot sizing, economic order quantity, and batch ordering problems (see, e.g., [22]). Recently, Chow et al. [23] investigate how imposition of minimum order quantity affects a fashion supply chain adopting quick response strategy. Their findings suggest that the order constraint has different impacts on different supply chain agents and thus should be set carefully for the sake of the whole supply chain.

3. Problem Description

We consider the situation that an online retailer (R) provides a designer platform service (DPS) to an entrant designer (D) to display one of her (throughout the paper, we employ the pronoun "she" for the designer and "he" for the retailer for ease of presentation) own-designed apparel to customers. The retail price of the product, $p > 0$, is endogenous and decided by D. Customers who are willing to buy the product will preorder at p online before a specified deadline of ordering. The demand of the product, which is reflected by the total quantity of the preorders placed, is price-dependent and is given by

$$y(p;x) = z(p)\,x, \tag{1}$$

where $0 \leq z(p) < +\infty$ and $dz(p)/dp := z'(p) < 0$ for all $p \geq 0$, and $x \geq 0$ is a random variable with probability density function $f(x)$ and cumulative distribution function $F(x)$. Without further notification, we assume that $dF(x)/dx = f(x) > 0$ for all $x \geq 0$.

The total production cost with production quantity q is given by

$$C(q) = g + hq, \tag{2}$$

where $g > 0$ is the fixed cost and $h > 0$ is the variable cost of production. We note that the unit product cost is given by

$$c(q) = \frac{C(q)}{q} = \frac{g}{q+h}; \tag{3}$$

and hence

$$\frac{dc(q)}{dq} = -gq^{-2} < 0, \quad \forall q > 0. \tag{4}$$

Specifically, $c(q)$ is strictly decreasing in q for all $q > 0$. To justify the fixed cost of production, R requires a minimum production quantity (MPQ) $Q > 0$, such that if $y(p;x) \geq Q$, then R will proceed to arrange a manufacturer for production and all the orders placed will be fulfilled, and the customers pay for the products. Otherwise, the product will not be produced and customers will not be charged. Mathematically, the production quantity q is given by

$$q(p,Q;x) = \begin{cases} y(p;x), & \text{if } y(p;x) \geq Q, \\ 0, & \text{o.w.} \end{cases} \tag{5}$$

We consider that the profit of selling the product is shared by R and D with the former sharing α, whilst the latter sharing $(1-\alpha)$ of the total profit, where $0 \leq \alpha \leq 1$. As a remark, different ways of profit allocation under DPS project could result in different optimal solutions of Q and p. However, the effects of the allocation schemes are *out of the scope of this paper*.

For any given $Q > 0$ and $p > 0$, the profit of R is given by

$$\begin{aligned} \pi(Q,p;x) &= \begin{cases} \alpha\left[py(p;x) - C(y(p;x))\right], & \text{if } y(p;x) \geq Q, \\ 0, & \text{o.w,} \end{cases} \\ &= \begin{cases} \alpha\left[(p-h)z(p)x - g\right], & \text{if } xz(p) \geq Q, \\ 0, & \text{o.w.} \end{cases} \end{aligned} \tag{6}$$

Taking the expectation of $\pi(Q,p;x)$ on x, the expected profit of R is given by

$$E\left[\pi(Q,p)\right] = \alpha\int_{Q/z(p)}^{\infty}\left\{(p-h)z(p)x - g\right\}f(x)\,dx. \tag{7}$$

The feasibility of DPS depends on whether the total order quantity received is greater than the MPQ. We define the *feasibility probability of DPS* as the probability of the total order quantity being greater than the MPQ; that is,

$$\Psi(p,Q) = \Pr\left\{x \geq \frac{Q}{z(p)}\right\} = 1 - F\left[\frac{Q}{z(p)}\right]. \tag{8}$$

The sales volume is given by

$$q(p,Q;x) = \begin{cases} y(p;x) = xz(p), & \text{if } xz(p) \geq Q, \\ 0, & \text{o.w.} \end{cases} \tag{9}$$

Correspondingly, the expected sales volume is given by

$$E\left[q(p,Q)\right] = z(p)\int_{Q/z(p)}^{\infty} xf(x)\,dx. \tag{10}$$

As a remark, R must be profitable in order for him to participate in DPS. In other words, we need to have $\pi(Q,p;x) > 0$ for all $x \geq Q/z(p)$, which is equivalent to $\alpha[(p-h)z(p)x - g] > 0$ for all $x \geq Q/z(p)$. Simplifying

Table 1: The list of scenarios being considered in this paper.

Scenario	Decision sequence		Follower's problem	Leader's problem
	Leader	Follower		
Project-oriented R-led	Retailer	Project-oriented designer	For any given Q, to find $p^* = \arg\max_{p\geq 0}\{\Psi(p,Q)\}$ s.t. $\alpha\pi(Q,p)\geq 0$, for all $x\geq 0$.	Anticipating p^*, to find $Q^* = \arg\max_{Q\geq 0}\{E[\pi(Q,p^*)]\}$
Market-oriented R-led	Retailer	Market-oriented designer	For any given Q, to find $p^* = \arg\max_{p\geq 0}\{E[q(p,Q)]\}$ s.t. $\alpha\pi(Q,p)\geq 0$, for all $x\geq 0$.	Anticipating p^*, to find $Q^* = \arg\max_{Q\geq 0}\{E[\pi(Q,p^*)]\}$
Project-oriented D-led	Project-oriented designer	Retailer	For any given p, to find $Q^* = \arg\max_{Q\geq 0}\{E[\pi(Q,p)]\}$	Anticipating Q^*, to find $p^* = \arg\max_{p\geq 0}\{\Psi(p,Q^*)\}$ s.t. $\alpha\pi(Q^*,p^*)\geq 0$, for all $x\geq 0$
Market-oriented D-led	Market-oriented designer	Retailer	For any given p, to find $Q^* = \arg\max_{Q\geq 0}\{E[\pi(Q,p)]\}$	Anticipating Q^*, to find $p^* = \arg\max_{p\geq 0}\{E[q(p,Q^*)]\}$ s.t. $\alpha\pi(Q^*,p^*)\geq 0$, for all $x\geq 0$

the above yields $(p-h)z(p)x \geq g$ for all $x \geq Q/z(p)$. By putting $Q = xz(p)$, we need to have $(p-h)Q \geq g$ or $p \geq h + g/Q$. Moreover, for any given Q and p, if $x < Q/z(p)$ for all $x \geq 0$, then both D and R have zero profit. Therefore, there should exist some x such that $x \geq Q/z(p)$.

For any given pair of (Q,p), the pair of (Q,p) is said to be feasible if (i) $p \geq h + g/Q$ and if (ii) there exists some x such that $x \geq Q/z(p)$. The following summarize condition (i) of (Q,p) being feasible. Let

$$p_0(Q) = h + \frac{g}{Q} \tag{11}$$

(C1): $p \geq p_0(Q) > h$ for any given Q.

For condition (ii), the specific condition depends on the exact form of $f(x)$.

4. The Best Response Decisions of DPS

There are two decision variables under DPS, namely Q and p. The optimal values of Q and p may be different with different decision sequences. Therefore, in this paper we consider two scenarios, one with R making the decision first, whilst the other with D being the first mover. A game with R as the first mover is referred to as an R-led (game), whereas the one with D as the first mover is referred to as a D-led (game). The objectives of the two parties are also different. $R$is to maximize his expected profit, which must be nonnegative. For D, whether her product can be sold or not, or how many customers will buy her product are more important than the profit she is to earn from selling her product. Therefore, we consider the following two types of D whose objectives are (1) maximizing the feasibility probability of DPS project; and (2) maximizing the expected market size, which is equivalent to maximizing the expected sales volume. We refer to D, having the first objective, as "Project-oriented Designer," whilst the one with the second objective is "Market-oriented Designer." Table 1 summarizes the scenarios that are considered in this paper.

As D and R make decisions sequentially, we apply the Stackelberg game in analyzing the model in which the first mover acts as the game leader and the late mover acts as the game follower. Moreover, we apply backward induction to obtain the optimal solutions of the players in the game. Propositions 1 to 4 provide the best response solutions to various scenarios.

Proposition 1. *Under the "Project-oriented R-led" scenario:*

(a) *for any fixed $Q > 0$, the DPS feasibility probability, $\Psi(Q,p)$, is decreasing in p for $p > 0$, and the best response price that maximizes $\Psi(Q,p)$ is uniquely given by $p_1^*(Q) = p_0(Q)$,*

(b) *if the best response MPQ that maximizes the expected profit of R (denoted by $Q_1{}^*$) exists and satisfies the first-order optimality condition, it is given by*

$$Q_1{}^* = \arg\left\{Q \geq 0 : Qz\left(h + \frac{g}{Q}\right) + gz'\left(h + \frac{g}{Q}\right) = 0\right\}. \tag{12}$$

Proof. All proofs are relegated in the Appendix. □

Proposition 2. *Under the "Market-oriented R-led" scenario:*

(a) *for any fixed $Q > 0$, the expected sales volume, $E[q(Q,p)]$, is decreasing in p for $p > 0$, and the best response price that maximizes the expected market size is uniquely given by*

$$p_2^*(Q) = p_0(Q), \tag{13}$$

(b) *if the best response MPQ that maximizes the expected profit of R (denoted by $Q_2{}^*$) exists and satisfies the first-order optimality condition, it is given by*

$$Q_2{}^* = \arg\left\{Q > 0 : Qz\left(h + \frac{g}{Q}\right) + gz'\left(h + \frac{g}{Q}\right) = 0\right\}. \tag{14}$$

Propositions 1 and 2 assert that when R is the first mover in DPS, the best response decisions are the same whether D is project-oriented or market-oriented. In particular, under the two R-led scenarios, it is always optimal for D to set the retail price as low as possible [i.e., $p_0(Q)$]. Such result is natural as the objective of D is not related to the expected profit she is to earn from DPS.

Proposition 3. *Under the "Project-oriented D-led" scenario:*

(a) *for any fixed $p > p_0$*

(a-i) $E[\pi(Q, p)]$ *is strictly concave in Q if and only if*

$$(p-h)\,z(p)\,f\left[\frac{Q}{z(p)}\right] + [(p-h)Q - g]\,f'\left[\frac{Q}{z(p)}\right] > 0, \quad \forall Q > 0; \tag{15}$$

(a-ii) *a sufficient condition for strict concavity of* $E[\pi(Q, p)]$ *is* $f'[Q/z(p)] \geq 0$ *for all* $Q > 0$; *and*

(a-iii) *if* $f'[Q/z(p)] \geq 0$, *the best response MPQ that maximizes the expected profit of R is uniquely given by* $Q_3^*(p) = g/(p-h)$;

(b) *if the best response price that maximizes the feasibility probability of DPS exists and satisfies the first-order optimality condition, it is given by*

$$p_3^* = \arg\left\{p > p_0 : z(p) + (p-h)\,z'(p) = 0\right\}. \tag{16}$$

Proposition 4. *Under the "Market-oriented D-led" scenario:*

(a) *for any fixed $p > p_0$,*

(a-i) $E[\pi(Q, p)]$ *is strictly concave in Q if and only if*

$$(p-h)\,z(p)\,f\left[\frac{Q}{z(p)}\right] + [(p-h)Q - g]\,f'\left[\frac{Q}{z(p)}\right] > 0, \quad \forall Q > 0; \tag{17}$$

(a-ii) *a sufficient condition for strict concavity of* $E[\pi(Q, p)]$*is* $f'[Q/z(p)] \geq 0$ *for all* $Q > 0$; *and*

(a-iii) *if* $f'[Q/z(p)] \geq 0$, *the best response MPQ that maximizes the expected profit of R is uniquely given by*

$$Q_4^*(p) = \frac{g}{p-h} \tag{18}$$

(b) *if the best response price, that maximizes the expected sales volume, exists and satisfies the first-order optimality condition, it is given by*

$$p_4^* = \arg\{p > p_0 : \zeta(p) = 0\}, \tag{19}$$

where

$$\zeta(p) = (p-h)\,z'(p)\left((p-h)^2 z^2(p)\int_{g/[(p-h)z(p)]}^{\infty} x f(x)\,dx + g^2 f\left[\frac{g}{(p-h)\,z(p)}\right]\right) + g^2 z(p)\,f\left(\frac{g}{(p-h)\,z(p)}\right). \tag{20}$$

On the other hand, Propositions 3 and 4 indicate that when D is the first mover in DPS (i.e., the two D-led scenarios), if the best response MPQ exists and is positive, then we have $Q_3^*(p) = Q_4^*(p) = g/(p-h)$.

As a remark, it is the lowest MPQ that promises nonnegative profit. In addition, the smaller the MPQ, the higher the unit price that D should meet, which means more profit from excess order to MPQ. Therefore, regardless of the objective of D, the best response function of R is given by

$$Q_0(p) = \frac{g}{p-h}. \tag{21}$$

Notice that g is the fixed cost for production, whilst $(p-h)$ can be viewed as the unit "profit margin" for each piece of apparel under DPS. In other words, the retailer should set the MPQ in the way that such a corresponding profit can justify the fixed production cost.

Moreover, we note that the existence of the best response retail price and the best response MPQ depend on the functions $z(p)$ and $f(x)$; there may be situations where no optimal solutions exist. There is no general condition to ensure the existence of them. Propositions 1 to 4 show the analytical forms of the optimal retail price and the optimal MPQ in case they exist.

5. Analysis with Specific Demand Functions

To obtain more insight, we consider specific forms of $z(p)$ and $F(x)$ in this section. Specifically, we consider that x follows a uniform distribution $U[l, u]$, where $0 < l < u$. For $z(p)$, we consider two specific types that are commonly adopted in the literature (e.g., [17]), namely

(1) additive form:

$$z_A(p) = \begin{cases} a - bp, & \text{for } 0 < p < \dfrac{a}{b}, \\ 0, & \text{for } p \geq \dfrac{a}{b}, \end{cases} \tag{22}$$

for $a > bh > 0$ (as $p > h$, if $a > bh$ does not hold, then $p > a/b$ and $z_A(p) = 0$); and

(2) multiplicative form: $z_M(p) = ap^{-b}$ for $a > 0$ and $b > 0$.

Table 2: Equilibrium decisions with additive form demand function $z_A(p) = a - bp$.

Scenario	Condition(s) for existence of the equilibrium	Equilibrium Q	Equilibrium p
Project-oriented R-led Market-oriented R-led Project-oriented D-led	$g < (a-bh)^2u/4b$	$2bg/(a-bh)$	$(a+bh)/2b$
Market-oriented D-led	$g < (a-bh)^2u/4b$ and $p_{L,4A} < \widehat{p}_{4A} < p_{H,4A}$	$g/(\widehat{p}_{4A} - h)$	Given by $\widehat{p}_{4A}$

Next, we study the equilibriums with two demand forms separately.

5.1. Additive Form Demand Function. With $z(p) = z_A(p) = a-bp$, Propositions 5 to 7 detail the equilibrium decisions and the conditions for the existence of the equilibrium decisions for DPS under different scenarios.

Proposition 5. *For the two "R-led" scenarios:*

(a) *there exists feasible* (Q, p) *only if* $g < (a-bh)^2u/4b$ *and* $Q > gb/(a-bh)$,

(b) *for* $g < (a-bh)^2u/4b$, *the expected profit of R is unimodal in Q for all* $Q > 0$, *and the equilibrium MPQ is uniquely given by* ${Q_{1A}}^* = 2gb/(a-bh) > gb/(a-bh)$. *Correspondingly, the equilibrium price is uniquely given by* ${p_{1A}}^* = (a+bh)/2b > h$.

Let $\Delta = u[u(a-bh)^2 - 4bg]$, $p_{L,3A} = (a+bh)/2b - \sqrt{\Delta}/2ub$, and $p_{H,3A} = (a+bh)/2b + \sqrt{\Delta}/2ub$.

Proposition 6. *For the "Project-oriented D-led" scenario:*

(a) *the best response MPQ of R is given by* $Q_0(p) = g/(p-h)$ *and the pair* $(Q_0(p), p)$ *is feasible only if* $g < u(a-bh)^2/4b$,

(b) *for* $g < u(a-bh)^2/4b$, *the DPS feasibility probability is strictly concave in p, the equilibrium retail price is* ${p_{3A}}^* = (a+bh)/2b > h$, *and the equilibrium MPS is* ${Q_{3A}}^* = 2bg/(a-bh)$.

Proposition 7. *For the "Market-oriented D-led" scenario, for* $g < u(a-bh)^2/4b$:

(a) *the expected sales volume function* $E[q(p, Q_0(p))]$ *given by* (10) *is strictly concave in p,*

(b) *let* $\widehat{p}_{4A} = \arg_p\{g^2(2a+bh-3bp) - bu^2(p-h)^3(a-bp)^2 = 0\}$. *If* $p_{L,3A} < \widehat{p}_{4A} < p_{H,3A}$, *then the equilibrium p is* $\widehat{p}_{4A}$ *and the equilibrium MPS is* ${Q_{4A}}^* = g/(\widehat{p}_{4A} - h)$; *otherwise,* $(Q_0(p), p)$ *is not feasible for all* $p > 0$.

Table 2 summarizes the equilibrium decisions under various scenarios and the respective conditions required.

With $z(p) = z_A(p) = a - bp$, we found that $g - (a-bh)^2u/4b < 0$ is the common condition for the existence of the equilibrium solutions for all scenarios. As $g - (a-bh)^2u/4b < 0$ is strictly increasing with g, h, and b and is strictly decreasing with a and u, the condition $g - (a-bh)^2u/4b < 0$ implies that the equilibrium decision may not exist when g, h, and/or b are big, and/or a and/or u are small. We argue such results as follows. When g and/or h are big, it is not profitable for R under DPS, and hence, R will not participate with DPS. When b is big, the product demand is very sensitive to the price such that it is not possible to set a price to make q no less than MPQ and R with profitable. When a and/or u are small, the demand base is small. Therefore, again, it is not possible to set a price to make q no less than MPQ and R profitable.

For the equilibrium decisions, they are the same for the scenarios Project-oriented R-led, Market-oriented R-led, and Project-oriented D-led. However, the equilibrium decisions of the Market-oriented D-led scenario are different from the other three scenarios. Moreover, for the Market-oriented D-led scenario, there is an extra condition, $p_{L,4A} < \widehat{p}_{4A} < p_{H,4A}$ for the existence of the equilibrium solutions. This condition gives an upper bound and a lower bound of the equilibrium p. We argue such results as follows. Being an entrant designer, D cares less about maximizing her own profit. Rather, she aims at either maximizing the expected sales volume, or the feasibility probability of the DPS. Both objectives can be fulfilled by maximizing the demand, which is also favorable to the profit-maximizing R. In other words, the objective of D is consistent with that of R; therefore the effect of decision sequence becomes negligible. Besides, whether D is project- or market-oriented, it is also optimal for her to set p as small as possible to maximize the market size whilst keeping R profitable. From the perspective of R, as discussed in the previous section, the optimal MPQ is the one that barely justifies the fixed production cost. This reflects that it is always optimal for him to make DPS feasible so that he can have the opportunity to gain profit. Hence, the equilibrium decisions are almost the same under different decision sequences and different objectives of D when there is no limitation on setting p. However, because of the extra condition for the equilibrium p, the equilibrium decisions for the Market-oriented D-led scenario are different from the other three scenarios.

5.2. Multiplicative Form Demand Function. With $z(p) = z_M(p) = ap^{-b}$, Propositions 8 to 10 detail the equilibrium decisions and the conditions for the existence of the equilibrium decisions for DPS under different scenarios.

Proposition 8. *For the two "R-led" scenarios*

(a) *the best response retail price of D is given by* $p_0(Q) = h + g/Q$;

(b) *for* $0 < b \leq 1$, *the expected profit of R,* $E[\pi_{1M}(Q)]$, *is strictly decreasing in* Q *and* $E[\pi_{1M}(0)] = +\infty$; *and*

Table 3: Equilibrium decisions with multiplicative form demand function $z_M(p) = ap^{-b}$.

Scenario	Condition(s) for existence of equilibrium	Equilibrium Q	Equilibrium p
Project-oriented R-led	$b > 1$ and $g(b-1)[hb/(b-1)]^b/ah < u$	$(b-1)g/h$	$bh/(b-1)$
Market-oriented R-led			
Project-oriented D-led	$b > 1$ and $l < g(b-1)[hb/(b-1)]^b/ah < u$		
Market-oriented D-led			

(c) *for $b > 1$, the expected profit of the retailer is unimodal in Q for all $Q > 0$;*

(c-i) *if $g(b-1)[hb/(b-1)]^b/ah \geq u$, $(Q, p_0(Q))$ is infeasible for all $Q > 0$; and*

(c-ii) *otherwise, the equilibrium MPQ and the equilibrium p are given by $Q_{1M}{}^* = (b-1)g/h$ and $p_{1M}{}^* = bh/(b-1)$, respectively.*

Let $\Delta_{3A} = u[u(a-bh)^2 - 4bg]$, $p_{L,3A} = (a+bh)/2b - \sqrt{\Delta_{3A}}/2ub$, and $p_{H,3A} = (a+bh)/2b + \sqrt{\Delta_{3A}}/2ub$. Moreover, let

$$K_{3M}(p) = \frac{gp^b}{a(p-h)} < u, \tag{23}$$

and denote $p_{3Ml}{}'$, $p_{3Mu}{}'$, $p_{3Mu}{}''$, $p_{3Mu}{}'''$, $p_{3Ml}{}''$, $p_{3Ml}{}'''$, $p_{3Ml}{}'$, and $p_{3Mu}{}'$ that satisfy (i) $p_{3Ml}{}' > p_{3Mu}{}' > h$, (ii) $K_{3M}(p_{3Ml}{}') = l$ and (iii) $K_{3M}(p_{3Mu}{}') = u$; and $p_{3Mu}{}''$, $p_{3Mu}{}'''$, $p_{3Ml}{}''$, and $p_{3Ml}{}'''$ satisfy (i) $h < p_{3Mu}{}'' < p_{3Ml}{}'' \leq p_{3Ml}{}''' < p_{3Mu}{}'''$, (ii) $K_{3M}(p_{3Mu}{}'') = K_{3M}(p_{3Mu}{}''') = u$, (iii) $K_{3M}(p_{3Ml}{}'') = K_{3M}(p_{3Ml}{}''') = l$, (iv) $l < K_{3M}(p) < u$ for all $p_{3Mu}{}'' < p < p_{3Ml}{}''$ or $p_{3Ml}{}''' < p < p_{3Mu}{}'''$, and (v) $K_{3M}(p) \leq l$ for all $p_{3Ml}{}'' < p < p_{3Ml}{}'''$.

Proposition 9. *For the "Project-oriented D-led" scenario:*

(a) *the best response MPQ of R is $Q_0(p) = g/(p-h)$;*

(b) *for $b \leq 1$, the feasibility probability $\Psi(p)$ is strictly increasing in p for all $p_{3Mu}{}' < p < p_{3Ml}{}'$ and $\Psi(l)$= 1 for all $p \geq p_{3Ml}{}'$,*

(c) *for $b > 1$,*

(c-i) *if $(g(b-1)/ah)(hb/(b-1))^b \geq u$, then the pair $(Q^*_{3M}(p), p)$ is infeasible;*

(c-ii) *if $l < (g(b-1)/ah)(hb/(b-1))^b < u$, then $\Psi(p)$ is unimodal in p and is maximized at $p = hb/(b-1)$;*

(c-iii) *if $(g(b-1)/ah)(hb/(b-1))^b < l$, then $\Psi(p)$ is strictly increasing in p for all $p_{3Mu}{}'' < p < p_{3Ml}{}''$, is strictly decreasing in p for all $p_{3Ml}{}''' < p < p_{3Mu}{}'''$, and $\Psi_{3M}(p) = 1$ for all $p_{3Ml}{}'' < p < p_{3Ml}{}'''$.*

Proposition 9 asserts that for the Project-oriented D-led scenario, the equilibrium decision exists only if $b > 1$ and $l < (g(b-1)/ah)(hb/(b-1))^b < u$, and the equilibrium Q and equilibrium p are given by $(b-1)g/h$ and $bh/(b-1)$, respectively.

Proposition 10. *Under the "Market-oriented D-led" scenario:*

(a) *for $b \leq 1$, the expected sales volume $E[q(p, Q_0(p))]$ is strictly increasing in p for all $p_{3Mu}{}' < p < p_{3Ml}{}'$, and $E[q(p, Q_0(p))] = (u-l)/2$ for all $p \geq p_{3Ml}{}'$.*

(b) *for $b > 1$,*

(b-i) *if $l < (g(b-1)/ah)(hb/(b-1))^b < u$, then $E[q(p, Q_0(p))]$ is unimodal in p and is maximized at $p = hb/(b-1)$; and*

(b-ii) *if $(g(b-1)/ah)(hb/(b-1))^b < l$, then $E[q(p, Q_0(p))]$ is strictly increasing in p for all $p_{3Mu}{}'' < p < p_{3Ml}{}''$, is strictly decreasing in p for all $p_{3Ml}{}''' < p < p_{3Mu}{}'''$, and $\Psi_{3M}(p) = 1$ for all $p_{3Ml}{}'' < p < p_{3Ml}{}'''$.*

Proposition 10 asserts that for the Market-oriented D-led scenario, the equilibrium decision exists only if $b > 1$ and $l < (g(b-1)/ah)(hb/(b-1))^b < u$, and the equilibrium Q and equilibrium p are given by $(b-1)g/h$ and $bh/(b-1)$, respectively.

Table 3 summarizes the best response solutions under various scenarios and the respective conditions required.

With $z(p) = z_M(p) = ap^{-b}$, we found that the optimal solutions are almost the same whether D is project- or market-oriented. They are also independent of the decision sequence. Similar to the case with $z(p) = z_A(p) = a - bp$, we argue such results as follows. Being an entrant designer, D cares less about maximizing her own profit. Rather, she aims at either maximizing the expected sales volume or the feasibility probability of the DPS. Both objectives can be fulfilled by maximizing the demand, which is also favorable to the profit-maximizing R. In other words, the objective of D is consistent with that of R; therefore, the effect of decision sequence becomes negligible. Besides, whether D is project- or market-oriented, it is also optimal for her to set p as small as possible to maximize the market size whilst keeping R profitable. Hence, the optimal solutions are almost the same under different decision sequences and different objectives of D. From the perspective of R, as discussed in the previous section, the optimal MPQ is the one that barely justifies the fixed production cost. This reflects that it is always optimal for him to make DPS feasible so that he can have the opportunity to gain profit.

The optimality conditions provide some other insights to the equilibrium decisions. To be specific, $b > 1$ is required in order to have equilibrium decisions for all four scenarios: $g(b-1)[hb/(b-1)]^b/ah < u$ is required for the two R-led scenarios and $l < g(b-1)[hb/(b-1)]^b/ah < u$ is required for the two D-led scenarios. As l does not include the equilibrium Q and the equilibrium p, the value of l is irrelevant in deriving the equilibrium decisions for the two R-led scenarios, but l becomes relevant in deriving the equilibrium decisions for the two D-led scenarios. Moreover, it is more probable an equilibrium decision could be derived for two R-led scenarios than for the two D-led scenarios.

6. Discussion

This paper concerns a service platform provided by an online retailer to an entrant designer. By formulating the problem as various Stackelberg games, we first explore the best response retail price of the entrant designer and the best response MPQ of the retailer, under different designer's objectives and decision sequences in a general demand function setting, and then we explore the equilibrium decisions of the games, under different designer's objectives and decision sequences by considering two specific demand function setting structures. Essentially, the retailer should set the MPQ that justifies the fixed production cost, whilst the entrant designer should set the retail price as low as possible to enlarge the market demand whilst the retail is still profitable.

The current paper bares several limitations including but not limited to the below. First, this paper considers the simple case that the product is sold at a fixed price at all quantities, whereas there can be various ways in organizing the group selling; a possible extension is to explore how different pricing schemes may bring benefits to all parties in the system. For instance, a pricing scheme with quantity discount, that is, different levels of discounts at different total order quantities is a good candidate for future research. Second, this paper considers the system with single designer and single platform. The literature in crowdsourcing suggests that competition among source providers, which is commonly observed in reality, is good for resource allocation. Accordingly, this study can be extended to explore the effect of competition amongst multiple designers and/or platforms. For other extensions, it is interesting to explore the optimal solutions of the DPS when a well-established designer who cares about profit maximization is involved.

Appendix

Mathematical Proofs

Note: for brevity, we omit one of the variables p or q in $E[q(p,Q)]$, $E[\pi(Q,p)]$, and $\Psi(p,Q)$ in the proofs.

Proof of Proposition 1. Under the "Project-oriented R-led" scenario:

(a) for any given $Q > 0$, the DPS feasibility probability is given by $\Psi(p) = 1 - F[Q/z(p)]$, and we have

$$\frac{d\Psi(p)}{dp} = f\left[\frac{Q}{z(p)}\right]\left[\frac{Qz'(p)}{z^2(p)}\right] \leq 0, \quad \text{as } z'(p) < 0,$$
$$f(x) \geq 0, \quad \forall x \geq 0. \tag{A.1}$$

Thus, the optimal retail price that maximizes the expected sales volume is the smallest possible value of the retail price, which is equal to $p_0(Q)$ by (C1).

(b) anticipating that D would set the retail price as $p_2^*(Q) = p_0(Q)$, the retailer's expected profit when having Q as the MPQ and the corresponding first derivative in Q are given by the below, respectively:

$$\begin{aligned} E[\pi(Q)] &= \alpha\int_{Q/z[p_2^*(Q)]}^{\infty}\left[\left(\frac{g}{Q}\right)z(p_2^*(Q))x - g\right]f(x)\,dx \\ &= \alpha g\left[\frac{z(h+g/Q)}{Q}\int_{Q/z(h+g/Q)}^{\infty} xf(x)\,dx \right. \\ &\quad \left. + F\left(\frac{Q}{z(h+g/Q)}\right) - 1\right], \end{aligned} \tag{A.2}$$

$$\begin{aligned} dE\frac{[\pi(Q)]}{dQ} &= -\left(\frac{\alpha g}{Q^3}\right)\left[Qz\left(h+\frac{g}{Q}\right) + gz'\left(h+\frac{g}{Q}\right)\right] \\ &\quad \times \int_{Q/z(h+g/Q)}^{\infty} xf(x)\,dx. \end{aligned} \tag{A.3}$$

By the participation constraint of R, that is, $\pi(Q,p;x) > 0$, we have $E[\pi(Q)] > 0$. Therefore, $\int_{Q/z(h+g/Q)}^{\infty} xf(x)dx > (Q/z(h+g/Q))(1 - F(Q/z(h+g/Q))) > 0$. From (A.3), we then have $dE[\pi(Q)]/dQ|_{Q=Q_2^*} = 0 \Leftrightarrow Q_2^* z(h + g/Q_2^*) + gz'(h + g/Q_2^*) = 0$. □

Proof of Proposition 2. Under the "Market-oriented R-led" scenario:

(a) for any given $Q > 0$, the expected sales volume is given by $E[q(p)] = z(p)\int_{Q/z(p)}^{\infty} xf(x)dx$ and we have $dE[q(p)]/dp = z'(p)[\int_{Q/z(p)}^{\infty} xf(x)dx + Q^2 f[Q/z(p)]/z^2(p)] \leq 0$, as $z'(p) < 0$, and $f(x) \geq 0$ for all $x \geq 0$. Thus, the optimal retail price that maximizes the expected sales volume is the smallest possible value of the retail price, which is equal to $p_0(Q)$ by (C1).

(b) same as Proposition 1(b),

□

Proof of Proposition 3. (a) For any fixed $p > p_0$, the expected profit of R is given by

$$\begin{aligned} E[\pi(Q)] &= \alpha\int_{Q/z(p)}^{\infty}[(p-h)z(p)x - g]f(x)\,dx \\ &= \alpha\left((p-h)z(p)\int_{Q/z(p)}^{\infty} xf(x)\,dx \right. \\ &\quad \left. + gF\left[\frac{Q}{z(p)}\right] + g\right). \end{aligned} \tag{A.4}$$

The first and second derivatives of $E[\pi(Q)]$ w.r.t. Q can be easily derived as follows, respectively:

$$\frac{dE[\pi(Q)]}{dQ} = -\frac{\alpha f[Q/z(p)][(p-h)Q-g]}{z(p)}, \quad (A.5)$$

$$\frac{d^2E[\pi(Q)]}{dQ^2} = -\alpha\left((p-h)z(p)f\left[\frac{Q}{z(p)}\right] + [(p-h)Q-g]f'\left[\frac{Q}{z(p)}\right]\right)\times\left(z^2(p)\right)^{-1}. \quad (A.6)$$

(a-i) A direct observation from (A.6). (a-ii) If $f'[Q/z(p)] \geq 0$ for all $Q > 0$, then

$$(p-h)z(p)f\left[\frac{Q}{z(p)}\right] + [(p-h)Q-g]f'\left[\frac{Q}{z(p)}\right] \geq (p-h)z(p)f\left[\frac{Q}{z(p)}\right] > 0. \quad (A.7)$$

Noticed that $p \geq p_0(Q) > h$ and $f(x) > 0$. So by (a-i), $E[\pi(Q)]$ is strictly concave in Q. (a-iii) A direct result by solving the first-order condition $dE[\pi(Q)]/dQ = 0$ from (A.5).

(b) Anticipating that the retailer sets the minimum processing quantity as $Q_3^*(p)$, the probability of DPS feasibility is given by

$$\Psi(p) = 1 - F\left[\frac{Q_3^*(p)}{z(p)}\right] = 1 - F\left[\frac{g}{(p-h)z(p)}\right]. \quad (A.8)$$

The first derivative of $\Psi(p)$ is

$$\frac{d\Psi(p)}{dp} = \left[\frac{g}{(p-h)^2z^2(p)}\right]\left[z(p)+(p-h)z'(p)\right] \times f\left[\frac{g}{(p-h)z(p)}\right]. \quad (A.9)$$

The first-order condition $d\Psi(p_3^*)/dp = 0$ is equivalent to $z(p_3^*) + (p_3^* - h)z'(p_3^*) = 0$ by direct observation from (A.9). □

Proof of Proposition 4. (a) Same as the proofs for Proposition 3(a).

(b) Anticipating that the retailer sets the minimum processing quantity as $Q_4^*(p)$, the expected sales volume is given by $E[q(p)] = z(p)\int_{g/[(p-h)z(p)]}^{\infty} xf(x)dx$.

It can be easily shown that the first derivative of $E[q(p)]$ is in the form:

$$\frac{dE[q(p)]}{dp} = \frac{\zeta(p)}{\left[(p-h)^3z^2(p)\right]}. \quad (A.10)$$

The optimal price that maximizes the expected sales volume (denoted by p_4^*), if exists, should satisfy the first-order condition: $dE[q(p_4^*)]/dp = 0$, which is equivalent to $\zeta(p) = 0$ by direct observation from (A.10). □

Proof of Proposition 5. (a) From (C1), we need to have $p \geq h + g/Q$. By the property of $z_A(p)$, we also require $0 < p < a/b$. Considering both together we have $Q > gb/(a-bh)$. Next, there exists some x such that $x \geq K_{1A}(Q)$, where $K_{1A}(Q) = Q/z_A(p_{1A}{}^2)$. Since $l < x < u$, we need to have $K_{1A}(Q) < u$, which is equivalent to $Q^2 - (a-bh)uQ + bgu < 0$. We prove by contradiction that $g \leq (a-bh)^2u/4b$. Suppose $g > (a-bh)^2u/4b$. Then

$$\begin{aligned} Q^2 - (a-bh)uQ + bgu &> Q^2 - (a-bh)uQ + \frac{(a-bh)^2u^2}{4} \\ &= \left[Q - \frac{(a-bh)u}{2}\right]^2 \geq 0. \end{aligned} \quad (A.11)$$

A contradiction! So we need to have $g \leq (a-bh)^2u/4b$.

(b) The profit of the retailer when setting MPQ as Q is given by

$$\pi_{1A}(Q) = \begin{cases} [p_{1A}{}^*(Q) - h]\,xz_A[p_{1A}{}^*(Q)] - g, & \text{for } x \geq K_{1A}(Q), \\ 0, & \text{o.w.,} \end{cases} \quad (A.12)$$

where $p_{1A}{}^*(Q) = p_0(Q) = h + g/Q$ by Propositions 1 and 2. Taking expectation, we have

$$\begin{aligned} E[\pi_{1A}(Q)] &= \int_{K_{1A}(Q)}^{u}\left([p_{1A}{}^*(Q)-h]\,xz_A[p_{1A}{}^*(Q)] - g\right)f(x)\,dx \\ &= \frac{g[u-K_{1A}(Q)]}{(u-l)}\left(\left[\frac{a-bh}{Q} - \frac{bg}{Q^2}\right]\left[\frac{u+K_{1A}(Q)}{2}\right] - 1\right). \end{aligned} \quad (A.13)$$

The first and second derivative of $E[\pi_{1A}(Q)]$ can be easily derived as follows, respectively:

$$\frac{dE[\pi_{1A}(Q)]}{dQ} = \frac{g\left[u^2 - K_{1A}{}^2(Q)\right][-(a-bh)Q + 2bg]}{2(u-l)Q^3}, \quad (A.14)$$

$$\frac{d^2E[\pi_{1A}(Q)]}{dQ^2} = \frac{g\left[u^2 - K_{1A}{}^2(Q)\right][(a-bh)Q - 3bg]}{(u-l)Q^4}. \quad (A.15)$$

From (A.15), for $Q > 0$, $d^2E[\pi_{1A}(Q)]/dQ^2 < 0 \Leftrightarrow Q < 3bg/(a-bh)$ (as $u^2 > K_{1A}{}^2(Q)$). Thus, $E[\pi_{1A}(Q)]$ is strictly concave for Q in $0 < Q < 3bg/(a-bh)$ and is convex for

$Q \geq 3bg/(a-bh)$. From (A.14), $dE[\pi_{1A}(Q)]/dQ = 0 \Leftrightarrow 3bg/(a-bh) > Q = 2bg/(a-bh) > 0$ (i.e., a unique $Q > 0$ satisfying $dE[\pi_{1A}(Q)]/dQ = 0$). Therefore, $E[\pi_{1A}(Q)]$ is unimodal in Q for $Q > 0$. Thus, $Q_{1A}{}^* = 2bg/(a-bh)$ is unique. Then, by direct manipulation, $p_{1A}{}^* = p_{1A}{}^*(Q_{1A}{}^*) = (a+bh)/2b$. □

Proof of Proposition 6. Under the "Project-oriented D-led" scenario with $z(p) = z_A(p) = a - bp$:

(a) by Proposition 3, the optimal MPQ is $Q_0(p) = g/(p-h)$, which satisfies (C1). Since $l \leq x \leq u$ and to ensure $(Q_0(p), p)$ being feasible, we need to have $Q_0(p)/z_A(p) < u$, which is equivalent to

$$\eta(p) = ubp^2 - u(a+bh)p + uha + g < 0. \tag{A.16}$$

(b) Suppose $g < u(a-bh)^2/4b$. For any given $p_{L,3A} < p < p_{H,3A}$, the DPS feasibility probability is given by

$$\begin{aligned}\Psi_3(p) &= 1 - F\left[\frac{Q_{3A}{}^*(p)}{z_A(p)}\right] \\ &= -\frac{g}{(u-l)(p-h)(a-bp)} + \left[1 + \frac{u}{(u-l)}\right].\end{aligned} \tag{A.17}$$

The first and second derivatives of $\Psi_3(p)$ w.r.t. p can be derived easily as follows:

$$\frac{d\Psi(p)}{dp} = \frac{g}{(u-l)(p-h)^2(a-bp)^2}(a+hb-2bp), \tag{A.18}$$

$$\begin{aligned}\frac{d^2\Psi(p)}{dp^2} &= -\frac{2g}{(u-l)(p-h)^3(a-bp)^3} \\ &\quad \times \left[b(p-h)(a-bp) + (a+hb-2bp)^2\right] < 0,\end{aligned} \tag{A.19}$$

for all $p_{L,3A} < p < p_{H,3A}$. So $\Psi_3(p)$ is strictly concave in p for $p_{L,3A} < p < p_{H,3A}$. Solving the first-order condition, from (A.18), we have the optimal retail price: $p_{3A}{}^* = (a+hb)/2b$, which satisfies the constraint as $p_{L,3A} < p_{3A}{}^* < p_{H,3A}$. Correspondingly, the optimal MPS in this case is: $Q_{3A}{}^* = Q_0(p_{3A}{}^*) = 2bg/(a-bh)$. □

Proof of Proposition 7. Under the "Market-oriented D-led" scenario with $z(p) = z_A(p) = a - bp$:

(a) For $g < u(a-bh)^2/4b$, for any given $p_{L,3A} < p < p_{H,3A}$, the expected sales volume is given by

$$\begin{aligned}E[q(p)] &= z_A(p)\int_{Q_0(p)/z_A(p)}^{u} xf(x)\,dx \\ &= \Gamma(p) \\ &:= \frac{(a-bp)u^2}{2(u-l)} - \frac{g^2}{2(u-l)(p-h)^2(a-bp)}.\end{aligned} \tag{A.20}$$

The first and second derivatives of $E[q(p)]$ w.r.t. p are given by the below, respectively:

$$\frac{d\Gamma(p)}{dp} = \frac{\left[g^2(2a+bh-3bp) - bu^2(p-h)^3(a-bp)^2\right]}{2(u-l)(p-h)^3(a-bp)^2}, \tag{A.21}$$

$$\begin{aligned}\frac{d^2\Gamma(p)}{dp^2} &= \frac{-g^2}{3(u-l)(p-h)^4(a-bp)^3} \\ &\quad \times \left[18b^2\left(p - \frac{(2a+bh)}{3b}\right)^2 + (a-bh)^2\right] \\ &< 0 \quad \forall p > 0.\end{aligned} \tag{A.22}$$

Therefore, $\Gamma(p)$ is strictly concave in p for all $p > 0$. Hence, there is a unique maximum of $\Gamma(p)$. Solving the first-order condition by (A.21), the unique optimal p that maximizes $\Gamma(p)$ is given by

$$\widehat{p}_{4A} = \arg_p\left\{g^2(2a+bh-3bp) - bu^2(p-h)^3(a-bp)^2 = 0\right\}. \tag{A.23}$$

If $p_{L,3A} < \widehat{p}_{4A} < p_{H,3A}$, then $p = \widehat{p}_{4A}$ is the optimal solution of maximizing $E[q(p)]$, and the corresponding optimal MPQ is $Q_{4A}{}^* = Q_0(\widehat{p}_4) = g/(\widehat{p}_4 - h)$.

Next, from the proof of Proposition 7, $\lim_{p\to p_{H,3A}} Q_0(p)/z_A(p) = \lim_{p\to p_{L,3A}} Q_0(p)/z_A(p) = u$, or equivalently, $\lim_{p\to p_{H,3A}}\eta(p) = \lim_{p\to p_{L,3A}}\eta(p) = 0$ ($\eta(p)$ is given by (A.16)). Therefore, $\lim_{p\to p_{H,3A}} E[q(p)] = \lim_{p\to p_{L,3A}} E[q(p)] = 0$. If $\widehat{p}_{4A} \leq p_{L,3A}$ or $\widehat{p}_{4A} \geq p_{H,3A}$, then by the strict concavity of $\Gamma(p)$, $\Gamma(p) < 0$ for all $p_{L,3A} < p < p_{H,3A}$. By definition, $E[q(p)] \geq 0$. Therefore, $E[q(p)] \neq \Gamma(p)$ and $(Q_0(p), p)$ are not feasible for all $p > 0$ in this case. □

Proof of Proposition 8. Let $K_{1M}(Q) = Q[h + g/Q]^b/a$.

(a) From Propositions 1 and 2, for any $Q > 0$, the designer will set the retail price as $p_{1M}{}^*(Q) = p_0(Q) = h + g/Q$. Next, for $(Q, p_0(Q))$ being feasible, as $l < x < u$, we need to have $K_{1M}(Q) < u$. Moreover, we have $dK_{1M}(Q)/dQ = [h + g/Q]^{b-1}\{h + g(1-b)/Q\}/a$.

(b) The expected profit of the retailer is given by

$$\begin{aligned}&E[\pi_{1M}(Q)] \\ &= \int_{K_{1M}(Q)}^{u}\left(\left[p_{1M}{}^*(Q) - h\right]xz_M\left[p_{1M}{}^*(Q)\right] - g\right)f(x)\,dx \\ &= \frac{g[u - K_{1M}(Q)]}{(u-l)}\left[\frac{a}{Q}\left(\frac{g}{Q} + h\right)^{-b}\left(\frac{u + K_{1M}(Q)}{2}\right) - 1\right].\end{aligned} \tag{A.24}$$

The first derivative of $E[\pi_{1M}(Q)]$ is

$$\frac{dE[\pi_{1M}(Q)]}{dQ} = -agQ^{-3}\left[h+\frac{g}{Q}\right]^{-b-1}\left[u^2-K_{1m}{}^2(Q)\right]\frac{hQ-(b-1)g}{2(u-l)}. \quad (A.25)$$

For $0 < b \le 1$, we have $hQ-(b-1)g \ge hQ > 0$; so $dE[\pi_{1M}(Q)]/dQ < 0$ for all Q. Therefore, $E[\pi_{1M}(Q)]$ is strictly decreasing in Q and the optimal MPS should be the smallest possible value of Q. For $Q = 0$, $K_{1M}(0) = 0$ (as $K_{1M}(Q) = \{Q^{1/b}h + gQ^{1/b-1}]\}^b/a$ and $0 < b \le 1$), and hence $E[\pi_{1M}(0)] = +\infty$.

(c) For $b > 1$, $dE[\pi_{1M}(Q)]/dQ > 0$ for $Q < (b-1)g/h$, $dE[\pi_{1M}(Q)]/dQ = 0$ for $Q = (b-1)g/h$, and $dE[\pi_{1M}(Q)]/dQ < 0$ for $Q > (b-1)g/h$. Therefore, $E[\pi_{1M}(Q)]$ is unimodal for $Q > 0$.

(c-i) $dK_{1M}(Q)/dQ < 0$ for all $Q < g(b-1)/h$; $dK_{1M}(Q)/dQ = 0$ for all $Q = g(b-1)/h$; and $dK_{1M}(Q)/dQ > 0$ for all $Q > g(b-1)/h$. Therefore, $K_{1M}(Q)$ is minimized at $Q = g(b-1)/h$. As $K_{1M}(g(b-1)/h) = g(b-1)[hb/(b-1)]^b/ah$, if $g(b-1)[hb/(b-1)]^b/ah \ge u$, then the Retailer's participation constraint is not satisfied for any $Q > 0$; that is, $K_{1M}(Q) > u$ for all $Q > 0$.

(c-ii) If $g(b-1)[hb/(b-1)]^b/ah < u$, the optimal MPQ is uniquely given by $Q_{1M}{}^* = (b-1)g/h$, and by direct manipulation, the optimal retail price is uniquely given by $p_{1M}{}^* = p_{1M}{}^*(Q_{1M}{}^*) = bh/(b-1)$. □

Proof of Proposition 9. (a) Under the "Project-oriented D-led" scenario with $z(p) = z_M(p) = ap^{-b}$, by Proposition 3, the optimal MPQ is $Q_0(p) = g/(p-h)$, which satisfies (C1). Since $l \le x \le u$, to ensure $(Q_{3M}{}^*(p), p)$ being feasible, we need to have $Q_0(p)/z_M(p) < u$, which is equivalent to

$$K_{3M}(p) := \frac{gp^b}{a(p-h)} < u. \quad (A.26)$$

(b) and (c) We have $dK_{3M}(p)/dp = (gp^b/a(p-h)^2)[(b-1)p-bh]$. Next, anticipating $Q_0(p)$, the DPS feasibility probability is given by

$$\Psi_{3M}(p) = 1 - F\left[\frac{Q_0(p)}{z_M(p)}\right]. \quad (A.27)$$

If $l < K_{3M}(p) < u$,

$$\Psi_{3M}(p) = -\frac{g}{a(u-l)}\left(\frac{p^b}{p-h}\right)+\left(1+\frac{l}{u-l}\right); \quad (A.28)$$

and the first derivative of $\Psi_{3M}(p)$ w.r.t. p is

$$\frac{d\Psi_{3M}(p)}{dp} = -\frac{gp^{b-1}}{a(u-l)(p-h)^2}\left[(b-1)p-hb\right]. \quad (A.29)$$

If $l \ge K_{3M}(p)$, $\Psi_{3M}(p)$= 1; and if $u \le K_{3M}(p)$, $\Psi_{3M}(p) = 0$ (note that $(Q_{3M}{}^*(p), p)$ is not feasible in this case, so it can be ignored in the rest of the proof).

For $b \le 1$, as $dK_{3M}(p)/dp < 0$ for all $p > 0$, there exist unique $p_{3Ml}{}'$ and unique $p_{3Mu}{}'$, such that (i) $p_{3Ml}{}' > p_{3Mu}{}' > h$, (ii) $K_{3M}(p_{3Ml}{}') = l$, and (iii) $K_{3M}(p_{3Mu}{}') = u$. Observed from (A.29) that $d\Psi_{3M}(p)/dp > 0$ for all $p > 0$ satisfying $l < K_{3M}(p) < u$. Therefore, $\Psi_{3M}(p)$ is strictly increasing in p for all $p_{3Mu}{}' < p < p_{3Ml}{}'$, and $\Psi_{3M}(l) = 1$ for all $p \ge p_{3Ml}{}'$ ($(Q_0(p), p)$ is not feasible for any $p < p_{3Mu}{}'$).

For $b > 1$, $dK_{3M}(p)/dp < 0$ for all $h < p < hb/(b-1)$; $dK_{3M}(p)/dp = 0$ for $p = hb/(b-1)$; and $dK_{3M}(p)/dp > 0$ for all $p > hb/(b-1)$. Thus, $-K_{3M}(p)$ is unimodal in p for $p > h$, and is maximized at $p = \hat{p}_{3M} := hb/(b-1)$. As $K_{3M}(\hat{p}_{3M}) = (g(b-1)/ah)(hb/(b-1))^b$, if $(g(b-1)/ah)(hb/(b-1))^b \ge u$, then $K_{3M}(p) \ge u$ for all $p > h$. Therefore, the pair $(Q_0(p), p)$ is feasible if and only if $(g(b-1)/ah)(hb/(b-1))^b < u$.

For $b > 1$ and $(g(b-1)/ah)(hb/(b-1))^b < u$, as $K_{3M}(h) = +\infty$, there exist $p_{3Mu}{}''$ and $p_{3Mu}{}'''$ such that (i) $h < p_{3Mu}{}'' < p_{3Mu}{}'''$, (ii) $K_{3M}(p_{3Mu}{}'') = K_{3M}(p_{3Mu}{}''') = u$, and (iii) $K_{3M}(p) < u$ for all $p_{3Mu}{}'' < p < p_{3Mu}{}'''$. If $(g(b-1)/ah)(hb/(b-1))^b > l$, then $l < K_{3M}(p) < u$ for all $p_{3Mu}{}'' < p < p_{3Mu}{}'''$. Otherwise, there exist $p_{3Ml}{}''$ and $p_{3Ml}{}'''$, such that (i) $p_{3Mu}{}'' < p_{3Ml}{}'' \le p_{3Ml}{}''' < p_{3Mu}{}'''$, (ii) $K_{3M}(p_{3Ml}{}'') = K_{3M}(p_{3Ml}{}''') = l$, (iii) $l < K_{3M}(p) < u$ for all $p_{3Mu}{}'' < p < p_{3Ml}{}''$ or $p_{3Ml}{}''' < p < p_{3Mu}{}'''$, and (iv)$K_{3M}(p) \le l$ for all $p_{3Ml}{}'' < p < p_{3Ml}{}'''$.

Next, for $b > 1$ and $l < K_{3M}(p) < u$, $d\Psi_{3M}(p)/dp > 0$ for all $h < p < hb/(b-1)$; $d\Psi_{3M}(p)/dp = 0$ for $p = hb/(b-1)$; and $d\Psi_{3M}(p)/dp < 0$ for all $p > hb/(b-1)$.

Combining the above findings, for $b > 1$ and $(g(b-1)/ah)(hb/(b-1))^b < u$, if $(g(b-1)/ah)(hb/(b-1))^b > l$, then $\Psi_{3M}(p)$ is unimodal in p and is maximized at $p = hb/(b-1)$. Otherwise, $\Psi_{3M}(p)$ is strictly increasing in p for all $p_{3Mu}{}'' < p < p_{3Ml}{}''$, is strictly decreasing in p for all $p_{3Ml}{}''' < p < p_{3Mu}{}'''$, and $\Psi_{3M}(p) = 1$ for all $p_{3Ml}{}'' < p < p_{3Ml}{}'''$. □

Proof of Proposition 10. By Proposition 3(a), anticipating $Q_0(p) = g/(p-h)$, the expected sales volume is given by

$$E[q_{4M}(p)] = \Gamma_1(p) := \int_{K_{3M}(p)}^{u} xf(x)\,dx = \frac{u^2}{2(u-l)} - \frac{g^2p^{2b}}{2a^2(u-l)(p-h)^2}, \quad (A.30)$$

for $l < K_{3M}(p) < u$, where $K_{3M}(p)$ is given by (23),

$$E[q_{4M}(p)] = \frac{u-l}{2}, \quad \text{for } K_{3M}(p) \le l,$$
$$E[q_{4M}(p)] = 0, \quad \text{for } K_{3M}(p) \ge u. \quad (A.31)$$

The first and second derivatives of $\Gamma_1(p)$ wr.t. p are derived below, respectively:

$$\frac{d\Gamma_1(p)}{dp} = -\frac{g^2p^{2b-1}}{a^2(u-l)(p-h)^3}\left[(b-1)p-bh\right]. \quad (A.32)$$

For $b \le 1$, observing from (A.32), $d\Gamma_1(p)/dp > 0$ for all $p > 0$. Therefore, $E[q_{4M}(p)]$ is strictly increasing in p for all $l < K_{3M}(p) < u$. From the proof of Proposition 8,

$dK_{3M}(p)/dp < 0$ for all $p > 0$, for $b \leq 1$. Thus, $l < K_{3M}(p) < u$ implies $p_{3Mu}{}' < p < p_{3Ml}{}'$, and $K_{3M}(p) \leq l$ implies $p \geq p_{3Ml}{}'$, where $K_{3M}(p_{3Ml}{}') = l$ and $K_{3M}(p_{3Mu}{}') = u$ (see the proof of Proposition 8 for details). Moreover, $E[q_{4M}(p)] < E[q_{4M}(p_{3Ml}{}')] = (u-l)/2$ for all $p_{3Mu}{}' < p < p_{3Ml}{}'$, and $E[q_{4M}(p)] = (u-l)/2$ for all $p \geq p_{3Ml}{}'$.

For $b > 1$ and $l < K_{3M}(p) < u$, $d\Gamma_1(p)/dp > 0$ for all $h < p < hb/(b-1)$; $d\Gamma_1(p)/dp = 0$ for $p = hb/(b-1)$; and $d\Gamma_1(p)/dp < 0$ for all $p > hb/(b-1)$. Next, from the proof of Proposition 8:

(i) for $b > 1$, the pair $(Q_0^*(p), p)$ is feasible if and only if $(g(b-1)/ah)(hb/(b-1))^b < u$;

(ii) for $b > 1$ and $(g(b-1)/ah)(hb/(b-1))^b < u$, $K_{3M}(p) < u$ for all $p_{3Mu}{}'' < p < p_{3Mu}{}'''$;

(iii) for $b > 1$ and $l < (g(b-1)/ah)(hb/(b-1))^b < u$, $l < K_{3M}(p) < u$ for all $p_{3Mu}{}'' < p < p_{3Mu}{}'''$ and

(iv) for $b > 1$ and $(g(b-1)/ah)(hb/(b-1))^b \leq l$, $l < K_{3M}(p) < u$ for all $p_{3Mu}{}'' < p < p_{3Ml}{}''$ or $p_{3Ml}{}''' < p < p_{3Mu}{}'''$, and $K_{3M}(p) \leq l$ for all $p_{3Ml}{}'' < p < p_{3Ml}{}'''$.

Combining the above findings, for $b > 1$ and $(g(b-1)/ah)(hb/(b-1))^b < u$, if $(g(b-1)/ah)(hb/(b-1))^b > l$, then $E[q_{4M}(p)]$ is unimodal in p and is maximized at $p = hb/(b-1)$. Otherwise, $E[q_{4M}(p)]$ is strictly increasing in p for all $p_{3Mu}{}'' < p < p_{3Ml}{}''$, is strictly decreasing in p for all $p_{3Ml}{}''' < p < p_{3Mu}{}'''$, and $E[q_{4M}(p)] = (u-l)/2$ for all $p_{3Ml}{}'' < p < p_{3Ml}{}'''$. □

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Research Article

A Capacitated Location-Allocation Model for Flood Disaster Service Operations with Border Crossing Passages and Probabilistic Demand Locations

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Potential consequences of flood disasters, including severe loss of life and property, induce emergency managers to find the appropriate locations of relief rooms to evacuate people from the origin points to a safe place in order to lessen the possible impact of flood disasters. In this research, a p-center location problem is considered in order to determine the locations of some relief rooms in a city and their corresponding allocation clusters. This study presents a mixed integer nonlinear programming model of a capacitated facility location-allocation problem which simultaneously considers the probabilistic distribution of demand locations and a fixed line barrier in a region. The proposed model aims at minimizing the maximum expected weighted distance from the relief rooms to all the demand regions in order to decrease the evacuation time of people from the affected areas before flood occurrence. A real-world case study has been carried out to examine the effectiveness and applicability of the proposed model.

1. Introduction

Due to various occurrences of disasters such as floods, hurricanes, and earthquakes which lead to enormous property damages and human injuries, disaster management has recently become a crucial issue. Within disaster management tasks, finding the appropriate locations of relief rooms can help emergency managers to ensure public safety and well-being in tragic situations. Thus, transportation specialists play a prominent role in the emergency management process which focuses on people evacuation from disaster events in order to mitigate injury and loss of life. In this regard, there has been an increasing interest among practitioners and scholars in the field of emergency facility location problems. Compared to all disasters, flood is more possible to be predicted and prevented. Therefore, it is vital for flood disaster managers to find the appropriate locations of relief rooms or emergency medical services and allocate them to populations in order to provide efficient services.

A restricted region is a limitation in an area in which the geographic features obstruct the construction of relief rooms. Generally, a restricted planar area is divided into three categories: forbidden regions, congested regions, and regions where neither placement nor travelling is permitted through them. Lakes, mountains, highways, and military zones are some practical examples of these restricted regions. Hamacher and Nickel [1] have performed an extensive review of facility location problems with restrictions. These limitations were incorporated by Klamroth [2] into a model in which a fixed line barrier in a region divides it into two half-planes and two passages located along the line barrier provide communication between these subregions. The model proposed by Klamroth [2] is a specific formulation, and it may not be appropriate for cases that have regional probability distributions of customers or different objective functions. In the real world, when we are dealing with establishing permanent emergency service facilities for future disasters over a long term planning horizon, it is essential to consider the regional probability distribution of customers due to the uncertainties of events and partial information and data. Therefore, a more applicable model which provides better solutions would be beneficial for overcoming this weakness.

To the best of our knowledge, few research activities which consider barriers in emergency facility location problems have been accomplished, due to the computational complexities associated with these problems. Moreover, far too little attention has been paid to developing a practical mathematical model of emergency facility location that considers probabilistic customer regions and a line barrier with border crossings simultaneously. In this research, in order to address the abovementioned problems, we develop a mixed integer nonlinear programming (MINLP) model of a capacitated facility location-allocation problem (CFLAP) in an area in which a fixed line barrier and probabilistic customer regions are taken into account.

The rest of this paper is organized as follows: Section 2 provides a comprehensive literature review on related topics. Section 3 gives an explanation of the problem, and Section 4 presents the proposed mathematical model. In order to show the applicability of the proposed model, a suitable case study is illustrated in Section 5, and the model is validated in Section 6. Finally, Section 7 presents some concluding remarks and outlines a number of future research directions.

2. Literature Review

2.1. Emergency Facility Location Problems. The emergency facility location models are categorized into three different types: P-median, P-center, and maximal covering location problems. Since the objective of this research is to design an effective response strategy for disaster management organizations in order to reduce casualties, a planar center location problem (PCLP) would best suit our purpose. Actually, whenever the average cost of servicing the customers is less important than ensuring that no customer receives poor quality of services, the minimax location problem is typically proposed [3]. The procedure of the PCLP model, which is in fact a minimax problem, looks for the center in order to minimize the maximum service time, cost, or loss. Research in this area started with Sylvester [4] which was the first paper that considered the Weber problem with the PCLP model. Elzinga and Hearn [5] then introduced the Euclidean center problem with equal weights and offered an efficient solution technique for their proposed model. Francis et al. [6] made it possible to stress the importance of different customers by assigning unequal weights to them. Mehrez et al. [7] applied a single objective optimization to determine the location of a new hospital. The solutions of this problem were generated by a qualitative approach based on the evaluation of experts' judgments. After that, Daskin et al. [8] proposed an error-bound driven demand point aggregation for the minimax problem with rectilinear distance. Hurtado et al. [9] came out with some constraints on the minimax problem and used a time algorithm to solve it.

In the last two decades, the field of disaster management has gained increasing attention. Many authors have conducted several studies on predisaster preparedness and postdisaster responsiveness, including Altay and Green III [10], Simpson and Hancock [11], Ichoua [12], and Shishebori and Jabalameli [13]. Caunhye et al. [14] and Galindo and Batta [15] performed an extensive review of optimization models in emergency logistics for disaster operations management. Sherali et al. [16] proposed a capacitated facility location-allocation model in order to minimize the congested evacuation time by finding the optimal locations of evacuation shelters under hurricane or flood conditions. Chang et al. [17] studied the problem of locating relief rooms and allocating relief resources for flood emergency preparation. They proposed two stochastic models under possible flood scenarios in which the demand locations are uncertain and depend on the level of flood in different scenarios. In addition, Sheu [18] presented an earthquake relief model that forecasts the demand of affected regions and coordinates the provision of relief supplies. Mete and Zabinsky [19] dealt with an optimization approach for locating and distributing medical supplies under demand and transportation uncertainties for disaster situations. Some other recent relevant studies on this issue include Jiang et al. [20], Rawls and Turnquist [21], Yi et al. [22], Halper and Raghavan [23], Wu et al. [24], Xu et al. [25], and Duran et al. [26].

2.2. Facility Location Problems under Uncertainty. When addressing the strategic decision of establishing permanent facilities to prepare for future disasters over a long term planning horizon, information is uncertain due to the time lag; so considering the location of demands as a known point with certainty leads to poor model performance [27]. The first research in the probabilistic Weber problem was done by Cooper [28] who focused on a single facility location problem. After that, Katz and Cooper [29] approximated the expected distance of transportation via Euclidean distance in which the customer locations were assumed to have a normal distribution. They proposed an algorithm to minimize the total expected distance between the new facility and existing facilities. Later on, for the case of rectilinear distance, Wesolowsky [30] considered three different probability distributions of bivariate normal, bivariate symmetric exponential, and bivariate uniform for customer locations in solving the probabilistic Weber problem.

Although some studies have recently been done for the case of multifacility minimax location problems, much less attention has been given to the probabilistic formulation of minimax problems. Stochastic demand locations were first considered by Carbone and Mehrez [31] for the minimax location problem where the locations of demand points were normally distributed with predetermined means and variances. An emergency location problem with a weighted minimax objective function has been presented by Berman et al. [32] in which the weights associated with each demand point were not given and each of them was assumed to have a uniform distribution. They proved that the objective function is convex for parameters of the uniform distribution, and, thus, the model can be solved by using standard optimization techniques. The minimax regret location problem has been investigated by Averbakh and Bereg [33] in the case of uncertain weights and coordinates of demand points. They proposed a model with two objective functions of the median and center problems by using rectilinear distance. Foul [34] applied a mathematical model with the minimax objective function to find the unique location where the demand points follow a bivariate uniform distribution. Then, Pelegrín et al. [35] proposed a comprehensive framework for the 1-center problem where the weights of demands have an arbitrary probability distribution. They have studied a variety of problems using a number of objective functions.

After that, Durmaz et al. [36] presented a model on the probabilistic capacitated multifacility Weber problem in a two-dimensional region by assuming that the locations of customers have some multivariate probability distributions. Later on, Canbolat and Von Massow [37] shared their work on locating emergency facilities with random demands. They proposed an approach that explicitly addresses the uncertainty with respect to where an emergency will occur by minimizing the maximum risk through the minimization of the maximum expected distance. Then, Hosseinijou and Bashiri [38] formulated an appropriate model to represent the minimax single facility location problem in which demand areas are weighted and their coordinates have a bivariate uniform distribution. They proposed a model which finds the optimal location of the transfer point such that the maximum expected weighted distance from the facility point to all demand points through the transfer point is minimized.

2.3. Facility Location Problems with Barriers. In many cases of facility location problems, difficulties in solution methodologies occur when barriers limit the establishment of new facilities in a specified region. The Weber problem with a circular barrier was first presented by Katz and Cooper [39]. Since then, researchers have studied a variety of location problems in the presence of barriers using different types of objective function. They showed that the objective functions of these problems are nonconvex and offered some heuristic methods for solving them. Klamroth and Wiecek [40] focused on solving a multiobjective median problem (MOMP) with the minisum objective function by assuming a line barrier in an area and showed that their proposed model is nonconvex. Many researchers have applied different heuristic algorithms for the Weber problem that considers the convex or nonconvex polyhedral barriers in a region (e.g., [41–43]). The location problem has also been extended by Klamroth [44] who suggested that the single circular barrier can be subdivided into some feasible convex regions. By using this method, the nonconvex objective function of the problem has been transformed into some convex objective functions in each region. When the number of subdivided convex regions increases, this method is not suitable, since dividing the convex regions becomes awkward. To overcome this complexity, a genetic algorithm has been proposed by Bischoff and Klamroth [45] based on the Weiszfeld technique to solve the model with a large number of demand points. Bischoff et al. [46] presented two alternative location-allocation heuristics for the multifacility Weber problem with barriers.

The locations of optimum points for a given number of passages in two cases of capacitated and incapacitated facility location problems have been discussed by Huang et al. [48]; their model tried to minimize the transportation cost. Increasingly, more authors have addressed the planar center location problem in the presence of barriers in which the objective function is to find a facility that minimizes the maximum distance (e.g., [49–51]). A different perspective of using a wave front approach was proposed by Frieß et al. [52] for the minimax location problem in the presence of barriers. Canbolat and Wesolowsky [53] developed a model by considering a probabilistic line barrier that occurs randomly on the horizontal route. Their objective was to locate a single new facility to minimize the sum of the expected distances from all customer locations. After that, Canbolat and Wesolowsky [54] presented a new experimental approach for the Weber problem by using the Varignon frame. Their proposed method tried to minimize the sum of the weighted distances from the facility to customer points by considering barriers in a region. Then, Canbolat and Wesolowsky [47] presented a planar single facility location problem in the presence of border crossing. They developed the models for minisum and minimax problems in the presence of a fixed line barrier which divides a region into two subregions.

In short, although previous studies have proposed different approaches to address the facility location-allocation problem in the presence of a barrier, they have somehow neglected the probabilistic nature of customer locations. Moreover, very limited research activities have been carried out on multifacility location problems; albeit in many real-world applications, more than a single facility is required to be established. Therefore, an appropriate mathematical model is proposed in this paper in order to address these issues.

3. Problem Definition and Formulation

3.1. Facility Location Problem in the Presence of a Line Barrier. It is important to introduce some preliminary definitions before formally defining the problem. Based on Klamroth [2], the facility location problem in the presence of a line barrier is described as follows.

Let L be a line and $\{P_k = (x^k, y^k) \epsilon L \mid k \epsilon K := \{1, \ldots, K\}\}$ be a set of points on L. Then, $B_L := L \setminus \{P_1, \ldots, P_k\}$ is called a fixed line barrier with passages. The feasible region θ is defined as the union of two closed half-planes θ_1 and θ_2 on both sides of B_L. $P_k = (x^k, y^k)$ is the coordinate of a border crossing passage. Since the connections are located on a fixed horizontal line barrier, we set $y^k = \varphi$, $k = 1, \ldots, K$. A set of existing demand regions with locations of X_j, $j = 1, \ldots, J$ and positive weights of w_j is given in θ. Then, $d_p^B(X_i, X_j)$ is the p-norm barrier distance between X_i and X_j in the presence of a fixed line barrier where communication between two subplanes is allowed only through the K passages. The basic minimax model with barriers can be stated as follows:

$$\text{Min } F(X_i) = \text{Max}\left\{w_j \cdot d_p^B(X_i, X_j)\right\}, \quad i = 1,\ldots,I,\; j = 1,\ldots,J. \tag{1}$$

Any demand region that is in the same half-plane will not be affected by the presence of the barrier. The other demand regions that are not in the same half-plane with the new facility will connect to the new facility through just one of the passages along the line barrier. For this problem, the rectilinear distance, $p = 1$, is considered to compute the distance between the new emergency facilities and all demand regions.

3.2. Stochastic Weighted Emergency Facility Location Problem in the Presence of a Line Barrier. Assume that each demand region j has a coordinate of $X_j = (U_j, V_j)$ such that U_j, V_j are independent random variables and each of them has a bivariate uniform distribution. This assumption is realistic in real-world cases wherein the geographical distribution of population in different regions is uniform in square or rectangular areas [38]. Furthermore, a uniform distribution provides a "building block" to deal with a situation which is not precisely approximated [34].

The problem is to find the optimum location of the new facility (x_i^*, y_i^*) such that the maximum expected weighted distance from the facility to all demand regions, $EF(X_i^*)$, is minimized:

$$\begin{aligned}\text{Min } EF(X_i^*) &= \text{Max}\left\{E\left[w_j \cdot d_1^B\left((x_i^*, y_i^*), (U_j, V_j)\right)\right]\right\}\\ &= \text{Max}\left\{w_j \cdot E\left[d_1^B\left((x_i^*, y_i^*), (U_j, V_j)\right)\right]\right\}\\ &\qquad i = 1,\ldots,I,\; j = 1,\ldots,J,\end{aligned} \tag{2}$$

where $E[\cdot]$ represents the expected value. The above mentioned problem is illustrated in Figure 1. The weights of emergency needs in each demand region are defined as a function of two factors. The first factor is related to the qualitative criterion of demand region j which can be the quality of road construction from a demand region to the new facility point whereas the next one is concerned with the quantitative criterion of demand region j which can be a function of population, potential of emergency situation occurrence, and other issues. The weight Q_j of the qualitative factor has an inverse relation with w_j which means that w_j will reduce whenever the quality of road construction increases and this causes the new facility to be located farther from demand region j. The weight P_j of the quantitative factor has a direct relation with w_j; for instance, whenever the population of demand region j increases, w_j will increase too and the new emergency facility will tend to be located nearer to this customer area. In order to use both quantitative and qualitative factors in the total weight of demand region j, they must be defined in the same range to be comparable. One method of doing so is normalizing both types of factors to be in the domain of zero to one. Based on the experience

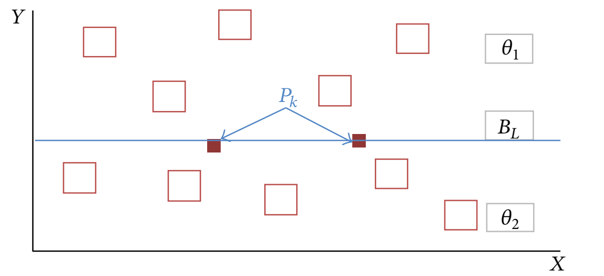

Figure 1: Demand locations with a fixed line barrier and two passages on the plane.

of emergency managers, both quantitative and qualitative factors have the same significance.

In this paper, the case of uniformly distributed demand regions in several rectangles is considered. It uses the rectilinear distance concept for measuring distances. Imagine a number of demand regions j in which U_j, V_j are independent random variables in $[a_j, b_j]$ and $[c_j, d_j]$ with probability density functions $f_u(U_j) = 1/(b_j - a_j)$ and $f_v(V_j) = 1/(d_j - c_j)$, respectively. Each area is a demand region, and it is desirable to find the optimum locations of emergency services among all these regions. The objective is to find the optimal locations of emergency facilities to provide efficient services for people. The mathematical expression for the expected distance between new facility i and demand region j is presented as

$$\begin{aligned}E\left[d\left(X_i, X_j\right)\right] &= E\left[\left|U_j - x_i\right| + \left|V_j - y_i\right|\right]\\ &= E\left[\left|U_j - x_i\right|\right] + E\left[\left|V_j - y_i\right|\right].\end{aligned} \tag{3}$$

Let $E[|U_j - x_i|]$ and $E[|V_j - y_i|]$ be denoted by $H_i(x)$ and $G_i(y)$, respectively. Then, we have

$$\begin{aligned}H_i(x) &= E\left[\left|U_j - x_i\right|\right] = \int_{-\infty}^{\infty}\left|U_j - x_i\right| \cdot f_u\left(U_j\right) d_u\\ &= \frac{1}{b_j - a_j}\int_{a_j}^{b_j}\left|U_j - x_i\right| d_u,\end{aligned} \tag{4}$$

$$\begin{aligned}G_i(y) &= E\left[\left|V_j - y_i\right|\right] = \int_{-\infty}^{\infty}\left|V_j - y_i\right| \cdot f_v\left(V_j\right) d_v\\ &= \frac{1}{d_j - c_j}\int_{c_j}^{d_j}\left|V_j - y_i\right| d_v.\end{aligned} \tag{5}$$

The closed-form expression for the expected distance $E[d(X_i, X_j)]$ was developed by Foul [34], which can be written as

$$H_i(x) = \begin{cases} -x_i + \dfrac{a_j + b_j}{2} & \text{if } x_i \le a_j \\[2ex] \dfrac{\left(x_i - a_j\right)^2 + \left(x_i - b_j\right)^2}{2 \cdot \left(b_j - a_j\right)} & \text{if } a_j \le x_i \le b_j \\[2ex] x_i - \dfrac{a_j + b_j}{2} & \text{if } x_i \ge b_j. \end{cases} \tag{6}$$

$G_i(y)$ can be defined like $H_i(x)$ by substituting x_i, a_j, b_j with y_i, c_j, d_j, respectively. With this expression, the problem is formulated as follows:

$$\text{Min } EF(X_i) = \text{Max } \{w_j \cdot [H_i(x) + G_i(y)]\}, \quad i = 1, \ldots, I, \; j = 1, \ldots, J. \tag{7}$$

Equation (7) can be transformed into an equivalent constrained nonlinear program which is presented here:

$$\begin{array}{ll} \text{Min} & Z \\ \text{Subject to} & Z \geq w_j \cdot [H_i(x) + G_i(y)], \\ & i = 1, \ldots, I, \; j = 1, \ldots, J. \end{array} \tag{8}$$

4. Proposed Mathematical Model

In this section, a MINLP model is proposed for CFLAP. The proposed MINLP model aims at finding the suitable locations of some emergency facilities in the presence of a fixed line barrier such that the expected weighted rectilinear distance between the emergency facilities and the farthest demand regions is minimized. The following assumptions and notations are used in formulating the MINLP model for CFLAP.

Assumptions. (i) The demand regions' coordinates have a bivariate uniform distribution because customer populations (residential areas) are normally dispersed or structured in square or rectangular areas.

(ii) There is a line barrier in the area which is fixed.

(iii) Two emergency service facilities are required to be established. This will ensure a more balanced allocation of the demand regions to the emergency service facilities since the entire region is divided into two half-planes by a line barrier.

Notations

I: Number of new facilities

J: Number of demand regions or customer regions

K: Number of passages

w_j: Positive weight of each existing demand region j

t_{ij}: Binary variable: 1, if demand region j is assigned to new facility i, 0, otherwise

h_{ijk}: Binary variable: 1, if demand region j is served by new facility i through passage k, 0, otherwise

l_{ij}: Binary variable: 1, if demand region j and new facility i are located in different half-planes, 0, otherwise

g_i: Binary variable: 1, if new facility i is located in the upper half-plane, 0, otherwise

q_j: Binary parameter: 1, if demand region j is located in the upper half-plane, 0, otherwise

Ca_i: Maximum capacity of new facility i

$S1_{ij}$: Binary variable: 1, if $x_i \leq a_j$, 0, otherwise

$S2_{ij}$: Binary variable: 1, if $a_j < x_i < b_j$, 0, otherwise

$S3_{ij}$: Binary variable: 1, if $x_i \geq b_j$, 0, otherwise

$F1_{ij}$: Binary variable: 1, if $y_i \leq c_j$, 0, otherwise

$F2_{ij}$: Binary variable: 1, if $c_j < y_i < d_j$, 0, otherwise

$F3_{ij}$: Binary variable: 1, if $y_i \geq d_j$, 0, otherwise

$S\acute{1}_{jk}$: Binary variable: 1, if $x^k \leq a_j$, 0, otherwise

$S\acute{2}_{jk}$: Binary variable: 1, if $a_j < x^k < b_j$, 0, otherwise

$S\acute{3}_{jk}$: Binary variable: 1, if $x^k \geq b_j$, 0, otherwise

$F\acute{1}_j$: Binary variable: 1, if $\varphi \leq c_j$, 0, otherwise

$F\acute{2}_j$: Binary variable: 1, if $c_j < \varphi < d_j$, 0, otherwise

$F\acute{3}_j$: Binary variable: 1, if $\varphi \geq d_j$, 0, otherwise.

Based on the above assumptions and notations, the objective function and constraints will be described.

4.1. Objective Function. The objective function tries to minimize Z while satisfying constraint (9). Since Z needs to be minimized, the shortest possible path through one of the border crossings along the barrier will be selected. The objective function can be formed as follows:

$$\begin{aligned} &\text{Min} \quad Z \\ &\text{where} \quad Z \geq w_j \cdot t_{ij} \cdot \Bigg[\Bigg[\Bigg(\sum_{k=1}^{K} \Big[T_k(x^k) + R_k(y^k) + |x^k - x_i| \\ &\qquad + |y^k - y_i|\Big] \cdot h_{ijk}\Bigg) \cdot l_{ij}\Bigg] \\ &\qquad + \big[[H_i(x) + G_i(y)] \cdot (1 - l_{ij})\big]\Bigg]. \end{aligned} \tag{9}$$

Note that the expected distance from the passage coordinates along the line barrier to the demand regions is denoted by $T_k(x^k)$ and $R_k(y^k)$ which are equal to $E[|U_j - x^k|]$ and $E[|V_j - y^k|]$ and they can be defined like $H_i(x)$ and $G_i(y)$ by substituting x_i with x^k and y_i with y^k, respectively.

4.2. Constraints. The constraints of this problem are formulated as follows.

4.2.1. Expected Distance. Based on the obtained closed-form expression in (6), the expected distance is computed in different ways depending on the location of the new facility and the interval of demand regions. Since this model will be solved by the GAMS software (full academic version 22.1), it is required to convert the formulas of $H_i(x)$ into a few constraints by introducing new binary variables of $S1_{ij}$, $S2_{ij}$, and $S3_{ij}$. This makes it easier for the software to perform the computation process. Likewise, new binary variables of $F1_{ij}$, $F2_{ij}$, and $F3_{ij}$ for $G_i(y)$ are introduced. $T_k(x^k)$ and $R_k(y^k)$ can be manipulated like $H_i(x)$ and $G_i(y)$ by replacing $S1_{ij}$, $S2_{ij}$,

$S3_{ij}$ with $S\acute{1}_{jk}$, $S\acute{2}_{jk}$, $S\acute{3}_{jk}$ and $F1_{ij}$, $F2_{ij}$, $F3_{ij}$ with $F\acute{1}_j$, $F\acute{2}_j$, $F\acute{3}_j$, respectively. These constraints can be written as follows:

$$2x_i \cdot S1_{ij} - 2a_j \cdot S1_{ij} \le x_i - a_j \quad \forall i, j, \tag{10}$$

$$2x_i \cdot S3_{ij} - 2b_j \cdot S3_{ij} \ge x_i - b_j \quad \forall i, j, \tag{11}$$

$$S1_{ij} + S2_{ij} + S3_{ij} = 1 \quad \forall i, j, \tag{12}$$

$$2y_i \cdot F1_{ij} - 2c_j \cdot F1_{ij} \le y_i - c_j \quad \forall i, j, \tag{13}$$

$$2y_i \cdot F3_{ij} - 2d_j \cdot F3_{ij} \ge y_i - d_j \quad \forall i, j, \tag{14}$$

$$F1_{ij} + F2_{ij} + F3_{ij} = 1 \quad \forall i, j, \tag{15}$$

$$2x^k \cdot S\acute{1}_{jk} - 2a_j \cdot S\acute{1}_{jk} \le x^k - a_j \quad \forall j, k, \tag{16}$$

$$2x^k \cdot S\acute{3}_{jk} - 2b_j \cdot S\acute{3}_{jk} \ge x^k - b_j \quad \forall j, k, \tag{17}$$

$$S\acute{1}_{jk} + S\acute{2}_{jk} + S\acute{3}_{jk} = 1 \quad \forall j, k, \tag{18}$$

$$2\varphi \cdot F\acute{1}_j - 2c_j \cdot F\acute{1}_j \le \varphi - c_j \quad \forall j, \tag{19}$$

$$2\varphi \cdot F\acute{3}_j - 2d_j \cdot F\acute{3}_j \ge \varphi - d_j \quad \forall j, \tag{20}$$

$$F\acute{1}_j + F\acute{2}_j + F\acute{3}_j = 1 \quad \forall j. \tag{21}$$

The first three constraints (constraints (10), (11), and (12)) are related to $H_i(x)$. Whenever the variable x_i is less than a_j ($x_i \le a_j$), the binary variable $S1_{ij}$ in constraint (10) will be equal to one; otherwise, it makes the binary variable to be equal to zero. Constraint (11) ensures that the expected distances between facilities and demand regions are calculated by the third formula of (6) whenever the variable x_i is more than b_j ($x_i \ge b_j$); otherwise $S3_{ij}$ will be equal to zero. If neither $S1_{ij}$ nor $S3_{ij}$ is equal to one, then the binary variable $S2_{ij}$ must be equal to one due to constraint (12). Constraint (12) also guarantees that just one of these cases can occur for the same new facility and customer region; assignment of two or three binary variables to one will lead to infeasibility and is not allowed. The rest of the constraints (constraints (13) to (21)) follow the same interpretation for $G_i(y)$, $T_k(x^k)$, and $R_k(y^k)$, respectively.

4.2.2. The Position of New Facilities and Customer Regions. Constraint (22) indicates whether the new facility is located in the upper half-plane or not:

$$2g_i \cdot y_i - y_i \ge 2g_i \cdot \varphi - \varphi, \quad \forall i. \tag{22}$$

Constraint (23) shows whether the customer region and the new facility are both located in the same half-plane or not:

$$l_{ij} = \left| q_j - g_i \right|, \quad \forall i, j. \tag{23}$$

Constraints (22) and (23) help the objective function to compute the expected distance between the new facilities and the customer regions in different cases.

4.2.3. Demand Allocation. According to constraint (24), each demand region must be served just by one of the new facilities:

$$\sum_{i=1}^{I} t_{ij} = 1, \quad \forall j. \tag{24}$$

The next constraint assures that the shortest path through just one of the passages will be selected to serve the allocated customer region in a different half-plane:

$$\sum_{k=1}^{K} h_{ijk} = l_{ij}, \quad \forall i, j. \tag{25}$$

4.2.4. Capacity. Constraint (26) guarantees that each new facility cannot serve more than its corresponding capacity:

$$\sum_{j=1}^{J} w_j \cdot t_{ij} \le Ca_i, \quad \forall i. \tag{26}$$

The modified objective function with respect to constraints (10) to (21) looks as follows:

$$\begin{aligned}
&\text{Min} \quad Z \\
&\text{where} \quad Z \ge w_j \cdot t_{ij} \\
&\quad \cdot \Bigg[\Bigg(\Bigg[\sum_{k=1}^{K} \Bigg[\left(-x^k + \frac{a_j + b_j}{2} \right) \cdot S\acute{1}_{jk} \\
&\qquad + \left(\frac{\left(x^k - a_j\right)^2 + \left(x^k - b_j\right)^2}{2 \cdot \left(b_j - a_j\right)} \right) \cdot S\acute{2}_{jk} \\
&\qquad + \left(x^k - \frac{a_j + b_j}{2} \right) \cdot S\acute{3}_{jk} \\
&\qquad + \left(-y^k + \frac{c_j + d_j}{2} \right) \cdot F\acute{1}_j \\
&\qquad + \left(\frac{\left(y^k - c_j\right)^2 + \left(y^k - d_j\right)^2}{2 \cdot \left(d_j - c_j\right)} \right) \cdot F\acute{2}_j \\
&\qquad + \left(y^k - \frac{c_j + d_j}{2} \right) \cdot F\acute{3}_j \\
&\qquad + \left| x^k - x_i \right| + \left| y^k - y_i \right| \Bigg] \cdot h_{ijk} \Bigg] \cdot l_{ij} \Bigg) \\
&\quad + \Bigg(\Bigg[\Bigg[\left(-x_i + \frac{a_j + b_j}{2} \right) \cdot S1_{ij} \\
&\qquad + \left(\frac{\left(x_i - a_j\right)^2 + \left(x_i - b_j\right)^2}{2 \cdot \left(b_j - a_j\right)} \right) \cdot S2_{ij} \\
&\qquad + \left(x_i - \frac{a_j + b_j}{2} \right) \cdot S3_{ij} \\
&\qquad + \left(-y_i + \frac{c_j + d_j}{2} \right) \cdot F1_{ij} \\
&\qquad + \left(\frac{\left(y_i - c_j\right)^2 + \left(y_i - d_j\right)^2}{2 \cdot \left(d_j - c_j\right)} \right) \cdot F2_{ij} \\
&\qquad + \left(y_i - \frac{c_j + d_j}{2} \right) \cdot F3_{ij} \Bigg] \cdot \left(1 - l_{ij}\right) \Bigg) \Bigg].
\end{aligned} \tag{27}$$

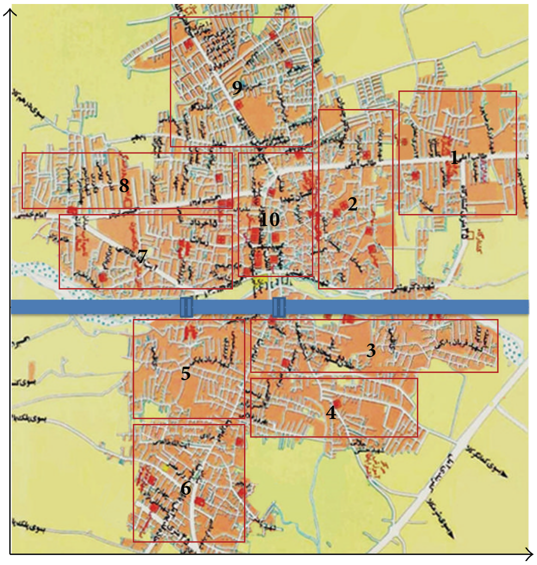

Figure 2: The illustration of the demand regions and passages.

5. A Case Illustration

To examine the practicality and effectiveness of the proposed model for CFLAP, a case is illustrated for the emergency facility location problem. One of the flood disaster management activities in the Amol city, Mazandaran province, northern part of Iran, was selected as a case study. In recent years, severe floods have occurred in the northern part of Iran. In 1999, floods in this area left 34 dead and 15 missing after more than a week and presumed dead. More than 100 people were injured and various areas were damaged with estimated losses of 15 million dollars [55]. Recently, the frequency of flood occurrence in the Mazandaran province, Iran, has increased and an appropriate model is required to find the suitable locations of relief rooms in order to decrease the evacuation time of people from the affected areas to a safe place before flood occurrence to reduce the possible impact of flood disasters.

The Amol city is located at 36°23′N latitude and 52°20′E longitude in northern Iran. This city is situated on the Haraz river bank, and it is less than 20 kilometers south of the Caspian sea and 10 kilometers north of the Alborz mountain. Based on the 2010 census, its population was around 224,000 with 61,085 families. As shown in Figure 2, the Haraz river divides the city into two subplanes, and two bridges are located along the river to connect the two subregions with each other. With respect to the geographic information system (GIS) map of the city, it has been divided into ten demand areas. As we mentioned before, the weights of emergency needs in each demand region are a function of both qualitative and quantitative criteria. The population of each area, as a quantitative criterion, was obtained from the statistics center of Iran and is shown in Table 1; the data for quality of road construction were gained from the GIS map of Amol which was prepared by the Department of Housing and Urban Development of Mazandaran Province. Table 2 presents the data for the qualitative factor.

Table 1: Population data in different regions.

j	Population	P_j
1	13464	0.057132
2	26139	0.110916
3	13984	0.059339
4	25432	0.107916
5	23916	0.101483
6	8888	0.037715
7	12972	0.055044
8	35072	0.148822
9	38392	0.162910
10	37405	0.158722

For the sake of normalizing the quantitative weight P_j of each demand region, the population number of each area is divided by the total population. For the qualitative factor, the quality of road construction is categorized into 5 groups which are high quality (HQ), good quality (GQ), acceptable quality (AQ), minor repair required (MNR), and major repair required (MJR). Since this factor is inversely related with w_j, the best quality construction receives the significance of 1 and the worst quality construction gets the maximum significance score which is equal to 5. The weight Q_j of the qualitative factor is computed similarly as the quantitative criterion. Since the significance of both criteria is the same, the final weight of each area is obtained by adding together half of the weights of the qualitative and quantitative factors.

The coordinates of ten demand regions with their corresponding final weights are tabulated in Table 3, while Table 4 gives the coordinates of the passages. It is noted that the establishment of two relief rooms with a maximum capacity percentage of 0.545 and 0.46 is required to cover all the demand regions. The proposed MINLP model contains 292 integer variables and 4 noninteger variables in terms of its complexity. Hence, it has been implemented in the GAMS 22.1 software using the BARON solver in order to identify the optimal values of x_i^* and y_i^*. BARON was created at the University of Illinois which implements global optimization algorithms of a branch and bound type and solves nonconvex problems to obtain global solutions [56]. Computations were performed using a system with a Core 2 duo 2.1 GHz CPU and 2 GB RAM and the computation time for solving the model was about 11 minutes. The time complexity grows exponentially with the increasing number of passages or new facilities.

The optimal locations of the emergency facilities and the value of the objective function are reported in Table 5. In addition, Table 6 shows the optimal corresponding allocation clusters of the two new emergency facilities. For a better understanding, Figure 3 illustrates the optimum locations of the new facilities and their corresponding allocation. Based on the results, we recommend that demand regions 1, 2, 8, 9, and 10 are served by the first new facility and the rest of the demand areas are served by the second new facility. The results will be useful for emergency managers to decrease the evacuation time in affected areas before flood occurrence.

Table 2: Data for quality of road construction.

j	HQ (%)	GQ (%)	AQ (%)	MNR (%)	MJR (%)	Score (%) of each area	Q_j
1	0.38	40.91	37.5	20.19	0	275.46	0.089512
2	1.65	39.76	35.76	11.53	9.41	281.62	0.091514
3	0	11.76	28.43	25.49	34.31	382.32	0.124236
4	1.37	6.65	23.29	39.33	29.35	388.61	0.126280
5	2.96	20	28.89	24.44	23.7	345.89	0.112398
6	0.72	11.15	40.66	24.75	22.7	357.5	0.116171
7	2.78	59.45	30.87	6.45	0	240.09	0.078018
8	5.97	68.1	20.71	3.36	1.37	224.59	0.072981
9	1.54	70.51	26.15	1.79	0	228.17	0.074145
10	15.79	14.04	14.04	10.53	45	353.11	0.114744

Table 3: Parameter values for the demand regions.

j	$[a_j, b_j]$	$[c_j, d_j]$	w_j
1	[14.6, 19.1]	[11.4, 17.1]	0.073322
2	[11.4, 14.5]	[8.9, 15]	0.101215
3	[9.2, 18.2]	[5.9, 8]	0.0917875
4	[9.2, 15.4]	[3.8, 5.8]	0.117098
5	[4.7, 9.1]	[4.7, 8]	0.1069405
6	[4.7, 9.1]	[0.3, 4.6]	0.076943
7	[1.9, 8.4]	[8.9, 11.4]	0.066531
8	[0.5, 8.4]	[11.5, 13.6]	0.1109015
9	[6.2, 11.3]	[13.7, 18.2]	0.1185275
10	[8.6, 11.3]	[9.4, 13.6]	0.136733

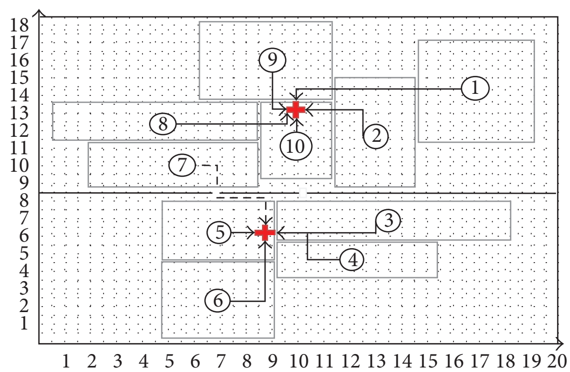

Figure 3: Illustration of the computation results.

Table 4: The coordinates of passages.

k	x^k	y^k
1	6.9	8.5
2	10.1	8.5

Table 5: Optimal solutions for the locations of relief rooms.

x_1^*	y_1^*	x_2^*	y_2^*	$F(X_i^*)$
9.8	13.312	8.6	6.139	0.682

Table 6: Optimal allocation of demand areas to the two new facilities.

(i, j)	t_{ij}	(i, j)	t_{ij}
1, 1	1	2, 1	0
1, 2	1	2, 2	0
1, 3	0	2, 3	1
1, 4	0	2, 4	1
1, 5	0	2, 5	1
1, 6	0	2, 6	1
1, 7	0	2, 7	1
1, 8	1	2, 8	0
1, 9	1	2, 9	0
1, 10	1	2, 10	0

Table 7: Coordinates of randomly selected areas.

j	$[a_j, b_j]$	$[c_j, d_j]$	w_j
1	[14.6, 19.1]	[11.4, 17.1]	0.159
3	[9.2, 18.2]	[5.9, 8]	0.198
5	[4.7, 9.1]	[4.7, 8]	0.234
7	[1.9, 8.4]	[8.9, 11.4]	0.145
9	[6.2, 11.3]	[13.7, 18.2]	0.264

6. Validation of the Model

Model validation is essential for evaluating the proposed model to make sure that the chosen new facility locations truly minimize the maximum value of distance from the stochastic demand regions to the chosen new facility points within the defined area. Based on the validation procedure of Hosseinijou and Bashiri [38], ten demand points were randomly generated within each demand region. Then, the objective functions of the recommended emergency facility locations were compared with the objective function of the proposed model.

Some efforts of validating the model are presented here. Ten scenarios were created from five randomly selected areas in Table 3. A random data toolbox in the Minitab software was used for generating random point data in each area. The weight of each point in the scenarios is equal to the weight of

Table 8: Validation results of the proposed model.

Scenario	$X_1 = (U_1, V_1)$ [14.6, 19.1] × [11.4, 17.1]	$X_3 = (U_3, V_3)$ [9.2, 18.2] × [5.9, 8]	$X_5 = (U_5, V_5)$ [4.7, 9.1] × [4.7, 8]	$X_7 = (U_7, V_7)$ [1.9, 8.4] × [8.9, 11.4]	$X_9 = (U_9, V_9)$ [6.2, 11.3] × [13.7, 18.2]	$F(X_i^*)$	|Error|
1	(18.43, 16.78)	(12.05, 7.74)	(7.96, 7.07)	(5.26, 11.26)	(7.95, 15.57)	1.160	0.05
2	(19.10, 13.44)	(13.65, 6.13)	(8.78, 7.62)	(3.75, 9.69)	(10.96, 16.86)	1.147	0.037
3	(16.57, 16.69)	(10.67, 7.55)	(4.86, 5.87)	(7.05, 9.27)	(7.69, 14.57)	1.092	0.018
4	(17.42, 12.74)	(13.48, 7.8)	(5.46, 6.5)	(3.45, 10.45)	(9.93, 16.23)	1.189	0.079
5	(16.67, 15.29)	(10.71, 7.57)	(5.03, 7.44)	(2.35, 9.89)	(7.46, 13.93)	1.049	0.061
6	(14.97, 12.32)	(15.95, 6.95)	(4.97, 4.84)	(2.66, 10.68)	(10.47, 17.96)	1.425	0.315
7	(15.66, 14.60)	(12.09, 7)	(5.16, 7)	(6.85, 10.15)	(7.5, 17.14)	1.062	0.048
8	(16.55, 13.11)	(13.51, 7.06)	(8.99, 7)	(2.17, 9.81)	(10.69, 16.83)	1.179	0.069
9	(16.30, 14.64)	(16.65, 6.87)	(6.55, 7.44)	(8.18, 9.53)	(10.57, 17.98)	1.144	0.034
10	(18.28, 13.50)	(14.47, 6.54)	(6.03, 6.33)	(4.25, 9.31)	(10.27, 15.64)	1.087	0.023

Table 9: Analysis of validation results for the proposed model.

$\overline{F(X_i^*)}$	1.153
$\lvert\overline{F(X_i^*)} - EF(X_i^*)\rvert$	0.043
$\dfrac{\lvert EF(X_i^*) - \overline{F(X_i^*)}\rvert}{EF(X_i^*)}$	3.87%

its allocated area. Table 7 presents the randomly selected areas for validation of the proposed model. In each of the scenarios, the actual value of the objective function is computed since the customer location is a point. The maximum capacity percentage of the two new facilities is set as 0.425 and 0.58, and the optimum expected value of the objective function for the five selected areas is equal to 1.11. The validation results of the proposed model are tabulated in Table 8. The actual value of the objective function for each scenario is listed in column 7; column 8 shows the error of our model, which is the difference between the actual value in column 7 and the expected value of the objective function for the five selected regions.

The final analysis of the model validation is reported in Table 9. The first row is related to the average of the objective function values for the ten scenarios which are extracted from column 7 of Table 8. The second row indicates the average error between the expected objective function value of the proposed model, and the obtained average amount in the first row. The average error percentage of the model is presented in the last row. As can be seen in Table 9, the average error percentage is equal to 3.87% which shows that the proposed CFLAP model is a good estimator of the actual objective function value and its solution is near to optimal. Hence, the proposed stochastic model is valid.

In order to highlight the advantages of the proposed model as compared to the most relevant existing literature, a comparison has been made with the model developed by Canbolat and Wesolowsky [47]. The results are summarized in Table 10.

As shown, the work of Canbolat and Wesolowsky [47] was dedicated to a single facility location problem where deterministic demand coordinates were utilized. In addition, their model was tested using a numerical example. In contrast, this study has developed a stochastic model that considers probabilistic demand locations for a multifacility location-allocation problem. The applicability of the proposed model has been evaluated using an actual case study, thus justifying its usefulness in the real-world context.

7. Conclusions

In this study, we have considered the planar center location-allocation problem for locating some emergency facilities in the presence of a fixed line barrier in a region. Although some studies have been done on facility location problems with barriers, little attention has been devoted to the probabilistic formulation of these problems where demand coordinates are distributed according to a bivariate probability distribution. In this paper, we have proposed a MINLP model for CFLAP by considering probabilistic demand regions and a fixed line barrier simultaneously, thus making the model more realistic. This study aims at finding the optimum locations of two relief rooms and their corresponding allocation which leads to minimizing the maximum expected weighted distance from the relief rooms to all the demand regions. A real-world case study has been carried out in order to show the effectiveness of the proposed model and the problem has been solved by the GAMS 22.1 software in order to find the optimum solution.

As an approximation, uniform distributions have been used to represent demand region locations. Thus, one possible area for future research is to formulate the problem with the exact probability distributions based on goodness-of-fit tests. In addition, considering the minimax regret objective function for hazardous location problems is another possible extension that can be studied.

Table 10: Comparison results.

Canbolat and Wesolowsky [47]	This study
Deterministic coordinates of demand locations	Stochastic coordinates of demand locations
Single facility location	Multifacility location
Numerical example	Real-world application

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Research Article

On Advertising Games and Spillover in Service Systems

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Motivated by the industry cases, we model the advertising competition between the dominant service provider and small service providers in one service market, where the dominant service provider has a major market share and the other small service providers share the remainder of market equally. Based on this setting, we discuss three advertising game models, that is, cooperative game, Boxed Pig game, and Prisoner's game, derive the conditions for different advertising games, and characterize their equilibria. To be specific, it is found that the advertising spillover and the number of the small service providers directly determines the advertising game equilibria, while other market parameters, to some extent, can affect the results of the advertising game equilibria. According to our theoretical findings, some management insights and suggestions are given from both the academic and practical perspectives.

1. Introduction

When you watch TV, listen to radio, read newspapers and magazines, or just surf on the internet, various kinds of exquisite advertisements immediately appear into your eyes. They compete to deliver a claim to customers that "buy me, you deserve!" Undoubtedly, advertising plays an important role in the product sales [1], as well as in enhancing customers' stickiness to the objective brand [2]. In the homogenization of service market, in order to be differentiated from other competitors, many service providers spend heavily on advertising; hoping to obtain special favor of consumers through constructing brand of company, advertising has been becoming a major strategy of nonprice competitions [3]. In general, service providers use two kinds of advertising to promote sales and to strengthen their brands, namely, generic advertising and brand advertising. The effect of generic advertising is to increase category sales while not mentioning the sponsoring brand. In contrast, brand advertising aims to gain market share by providing information about brand's value to consumers [4, 5].

Service providers adopt different advertising strategies in different market structures: a monopolistic service provider chooses the optimal advertising from its own perspective; in the oligopolistic market, however, service providers cannot achieve the expected objectives if they only consider their own advertising decisions. In order to ensure market shares and strengthen market positions, service providers, in this oligopolistic market, must take into account other service providers' advertising strategies. For example, China Life Insurance Co., Ltd. is a famous insurance company, which provides a variety of insurance products to customers and once occupied most of the shares of insurance market. However, with the powerful competitors such as Ping An Insurance company of China, Ltd. and Tai Kang Life Insurance Co., Ltd. entering into the insurance market, China Life Insurance Co., Ltd. has to adjust the marketing strategy and pay more attention on strengthening brand image in advertising, which aims to consolidate the market share in the fierce competition.

Furthermore, service providers' advertising strategies are also influenced by their upstream suppliers. In order to obtain the spillover effects of advertisings, the upstream suppliers always induce downstream service providers to invest more on advertising. For instance, the suppliers can offer subsidies to incentivize the downstream service providers to advertise more, in order to improve the overall channel profits and competitive advantages.

In this paper, we try to answer the following research questions about the advertising investment issues under a competitive market environment. How should service providers decide the optimal advertising investment in a competitive environment? If one service provider acts a dominate player in the market, should it invest in advertising first or wait the small service providers to advertise? As for a small service provider, what strategy should it adopt, advertising or waiting? And what is the most efficient strategy for the whole market?

The remainder of this paper is organized as follows. In Section 2, we review the related literature. Section 3 describes the problem and presents the advertising game models. The analysis of the game models is given in Section 4. Finally, we conclude the paper in Section 5.

2. Literature Review

There are mostly three areas of research relevant to our work: first, duopolistic advertising, second competitive game models on advertising, and third cooperative game models of advertising.

A stream of the literature has used game theoretical models to study the advertising investment issue in a duopoly setting. Espinosa & Mariel [6] find that, for the informative advertising competition, the outcomes from advertising are closer to the collusive outcomes in a feedback equilibrium. But in the case of predatory advertising, they find that expenditures are inefficiently high in a feedback equilibrium and the open-loop solution is more efficient. Prasad & Sethi [7] analyze the optimal advertising spending in a duopolistic market, develop a differential advertising game, in which the retailers' dynamic behavior is based on the Sethi stochastic advertising model and Lanchester combat model, and derive the closed-loop Nash equilibrium for symmetric and asymmetric competitors. Consistent with previous studies, Bass et al. [8] suggest that in a dynamic duopoly, generic advertising is more important than brand advertising for the long-run market shares, but in the short-term period, brand advertising is effective and market potential saturation leads to a decline in generic advertising over time.

However, little attention has been paid to advertising competition in an oligopolistic market setting, which is more common in practice. In addition, most of existing studies are based on the differential game theory, which only covers the dynamics features in advertising competition [9]. More research methods on advertising competition should be adopted to investigate other features of advertising competition, in particular, other game theories in the oligopoly market. Some recent literature has put effort to fill this gap. Brady [10] uses the Cournot game model to discuss the retailers' optimal advertising decisions, and the simulation analysis shows that small changes in model parameters can significantly influence the retailers' advertising behavior. Viscolani [11] suggests that two retailers, producing substituting products in the homogenization market, advertise to influence the product demand and interfere negatively with each other. He derives the pure-strategy Nash equilibrium and analyzes its characteristics in an advertising game. Nevertheless, these studies still concentrate on the duopoly market setting. Considering an oligopolistic market with multiple retailers who sell and advertise for a homogenous product, Norman et al. [12] find that the advertising intensity is related to the market concentration but make no discussion on the issue how retailers of different size make their optimal advertising decisions in the oligopolistic market.

Another stream of the related literature is on the cooperative advertising in the vertical supply chains. For instance, He et al. [13] use a differential Stackelberg game model to investigate the cooperative advertising between the manufacturers and retailers in the supply chain. He compares the advertising and pricing decisions in between the vertical integrated channel and coordinated channel. Seyedesfahani et al. [14] discuss three vertical noncooperative advertising games, including Nash, Stackelberg, and a cooperative game. They find that in the cooperative game, manufacturers and retailers can obtain the largest profits. However, none of the existing literature considers how the cooperative advertising in the supply chain level affects the advertising competition in the downstream market. Our paper contributes to this literature by considering the cooperation between the service providers. Moreover, few studies have investigated repeated competition game in the advertising competition.

In this paper, we construct the advertising models between the dominant service providers and small service providers in a market, discuss three advertising game models, that is, cooperative game, Boxed Pig game and Prisoner's game (in Boxed Pig game, a big, dominant pig and a little, weaker pig are locked into a Skinner box. At one end is a food dispenser. Ten units of food/energy are dispensed when a panel at the other end of the box is pressed. The pigs must each decide whether to wait by the food dispenser or to expend 2 units of energy to press the panel and run to the food. The big pig can run faster and eat faster than the little pig, so the little pig knows it will do better if it gets to the dispenser before the big pig. The pigs' actions are simultaneous, so the pigs must predict each other's strategy and then decide on their own best strategy. For more details, please refer to Rasmussen (1989) and in a conventional version of the game, two prisoners are interrogated separately about a bank robbery. If one confesses to the robbery and the other does not, the one who confesses is released, and the other receives a stiff sentence. If both confess, they receive moderate sentences. If neither confesses, they receive mild sentences for a lesser crime. Each player follows a dominant strategy, which is a strategy that is the player's best response, the one with the highest payoff, not matter what strategy the other player chooses. Assuming at atmosphere of distrust and competition, the dominant strategy equilibrium of the game is for both prisoners to confess, even though the Pareto superior outcome, the one in which neither party can be made better off, is, for both to keep quiet. For more details, please refer to Mudambi (1996)) and derive the conditions for different advertising games and their equilibrium, respectively. The main contributions of this paper are summarized as follows. First of all, the paper considers the advertising competitions among service providers and divides them into two types according to the service providers' size. Besides, this paper also considers the advertising subsidies from upstream service providers to downstream service providers affecting the competitions in the oligopolistic market.

3. Model Developments

3.1. Problem Descriptions and Notations. We consider a competitive market consisting of $M + 1$ risk-neutral service providers, among which a dominant service provider (denoted by D for short) has a majority market share w_D, and the other M small service providers share the remaining market share $1 - w_D$ equally. Each service provider's market share is denoted by w_i $(i = 1, 2, \ldots, M)$, and $w_D \gg w_i$, $w_D + \sum_{i=1}^{M} w_i = 1$. The dominant service provider D and small service providers play advertising investment games. We take the large service provider D and one classical small service provider S as examples to analyze the service providers' advertising strategies.

We model the problem as a Nash game between service providers D and S. Service provider D (or S) can decide to advertise with probability p (or q) or to wait with probability $1 - p$ (or $1 - q$). And it is assumed that the other $M - 1$ small service providers act in the same way where they do not invest in advertising and wait service providers D or S to provide advertising. Here, the wait strategy means that the service provider does not invest in advertising and wait other service providers to provide advertising. The other $M - 1$ small service providers can share the advertising effects of service provider D and S, which is consistent with real cases, and this is so-called "spillover effect." (Spillover effect refers to the extent to which information provided in messages changes beliefs about attributes that are not mentioned in the messages [15].) We assume that θ is advertising spillover rate, and $\theta = \alpha M$ $(0 < \theta < 1)$.

Based on the service providers D and S' different choice of advertising strategies, cooperative advertising and waiting, the service providers' decision pairs can be divided into four cases.

Case 1 (cooperative advertising). The total investment on advertising is I (including labor, material, and technique), where aI is for the service provider D and bI for the service provider S, where $a + b = 1$. We assume service provider D has more advertising resources and will invest more on advertisement. To capture this feature, we denote l $(l > 1)$ to be the resource coefficient such that $aI = lbI$. Letting k denote the effect of cooperative advertising, where $k > 1$, the revenue stemmings from the cooperative advertising are $R_{D1} = kblI$ and $R_{S1} = kbI$.

Case 2 (service provider D advertises, but service provider S waits). Service provider D's advertising investment is aI, the advertising effects coefficient is k_D, the total revenue from service provider D's advertising is $R_{D2} = blk_DI$.

Table 1: Advertising Game Matrix between service providers D and S.

		Provider S	
Provider D	Strategy (probability)	Advertising (q)	Waiting $(1 - q)$
	Advertising (p)	π_{D1}, π_{S1}	π_{D2}, π_{S2}
	Waiting $(1 - p)$	π_{D3}, π_{S3}	π_{D4}, π_{S4}

Case 3 (service provider S advertises, but service provider D waits). Retail S's advertising investment is bI, advertising effects coefficient is k_S, the total revenue is $R_{S3} = bk_SI$, and $k > k_D > k_S$.

Case 4 (noncooperative advertising (Prisoner's dilemma)). Both service provider D and service provider S choose to wait, and the payoff is $R_{i4} = 0$, $i = D, S$.

Because of the advertising spillover problem, the service provider who offers advertising cannot obtain the whole advertising effect. Therefore, in order to cultivate the service providers to advertise, the supplier offers a subsidy rate λ to the service provider who invests on advertising for his products, so that the total subsidy amount is λR. (In this paper, we do not consider the supplier's decisions and his revenue, we mainly study the impact of the subsidy on both service providers' decisions. For the subsidy on the revenue, in fact, it is related to the advertising investment I indirectly, because the revenue is a function of advertising investment. It is noted that this setting can solve the subsidy difference between the service providers D and S using one subsidy rate.) Suppose the subsidy rate λ is the same for different advertising effects. Throughout the paper, we use he and she to indicate service providers D and S, respectively.

3.2. Advertising Game Models. In the previously mentioned settings, we can construct the payoff matrix of service providers D and S as shown in Table 1. Obviously, the other small service providers act in the same way as service provider S.

Based on the previously mentioned analysis, we obtain the service providers D and S' payoff functions as follows.

Case 1. Both service providers D and S choose to invest in advertising.

Both service providers D and S' payoff functions are given as follows:

$$\pi_{D1} = (1 - \theta) kblI - blI + \lambda kblI, \quad (1)$$

$$\pi_{S1} = (1 - \theta) kbI - bI + \lambda kbI, \quad (2)$$

where the first term in (1) is service provider D's revenue from the advertising strategy, the second term is the advertising expenses, and the third term is the subsidy from the supplier. Similarly, (2) indicates service provider S's payoff function.

With the same logic, in Case 2, service providers D and S' payoff functions are given as follows:

$$\pi_{D2} = (1-\theta) k_D blI - blI + \lambda k_D blI, \quad (3)$$

$$\pi_{S2} = \frac{\theta}{M} k_D blI, \quad (4)$$

where the first term in (3) is service provider D's revenue from the advertising strategy, the second term is the advertising expenses, and the third term is the subsidy from the supplier. In (4), since service provider S chooses to wait, instead of offering advertisement, there is no cost incurred on advertising.

In Case 3, service providers D and S' payoff functions are given as follows:

$$\begin{aligned} \pi_{D3} &= \frac{\theta}{M} \beta k_S blI, \\ \pi_{S3} &= (1-\theta) k_S bI - bI + \lambda k_S bI, \end{aligned} \quad (5)$$

where the term $(\theta/M)\beta$ indicates the advertising spillover rate that service provider D obtains from service provider S's advertising and β $(\beta > 1)$ is the service provider D's spillover factor. In this case, service provider D chooses to wait and the small service provider S chooses to offer advertisement.

In Case 4, no service provider chooses to offer advertisement; thus the service providers' payoff functions are

$$\pi_{D4} = \pi_{S4} = 0. \quad (6)$$

Since both service providers choose to wait, there is no revenue generated. This leads to a well-known game result-Prisoner's dilemma.

4. Game Model Analysis

In this section, we will analyze the equilibrium of cooperative game and Boxed Pig game and then further study the deterioration of the Boxed Pig game.

4.1. Cooperative Equilibrium. In this advertising game, service provider D has complete information on his own and incomplete information on service provider S. Service provider D's payoff difference between advertising and waiting strategy is $\Delta\pi_D$, and whether he chooses to advertise or not lies on his expected payoff differences.

According to the analysis in Section 4.1, his payoff difference is

$$\Delta\pi_D = \sum_{i=1}^{4} p_i q_i \pi_{Di} (p=1) - \sum_{i=1}^{4} p_i q_i \pi_{Di} (p=0) \geq 0, \quad (7)$$

where $\pi_{Di}(p = 1)$ is service provider D's payoff when he choose to invest in advertising, and $\pi_{Di}(p = 0)$ represents service provider D's payoff when he choose to wait.

If and only if $\Delta\pi_D \geq 0$ holds, service provider D chooses the cooperative advertising strategy.

Where $i = 1, 2, 3,$ and 4, $p_1 = p_2 = p$, $p_3 = p_4 = 1-p$, $q_1 = q_3 = q$, and $q_2 = q_4 = 1-q$.

Substituting the previously mentioned conditions into $\Delta\pi_D$, we obtain

$$\begin{aligned} \Delta\pi_D = \Big\{ & q\left[(1-\theta)k + \lambda k\right] + (1-q) \\ & \times \left[(1-\theta)k_D - 1 + \lambda k_D\right] - q\frac{\theta}{M}\frac{k_S\beta}{l} \Big\} blI. \end{aligned} \quad (8)$$

Since $\theta = \alpha M$ and $\Delta\pi_D \geq 0$, the sufficient and necessary conditions for service provider D to choose to advertise are

$$\begin{aligned} \theta_D &\leq 1 - \frac{q\alpha(k_S\beta/l) - \lambda kq - \lambda k_D(1-q) + 1}{kq(1+\delta)^{n-1} + k_D(1-q)} = \overline{\theta}_D, \\ M_D &\leq \frac{1}{\alpha} - \frac{q\alpha(k_2\beta/l) - \lambda kq - \lambda k_D(1-q) + 1}{\alpha kq + \alpha k_D(1-q)} = \overline{M}_D, \end{aligned} \quad (9)$$

respectively.

Service provider S's payoff difference between advertising and waiting strategy is $\Delta\pi_S$. For service provider S, the sufficient and necessary condition for her to choose the cooperative advertising strategy is

$$\Delta\pi_S = \sum_{i=1}^{4} p_i q_i \pi_{Si} (q=1) - \sum_{i=1}^{4} p_i q_i \pi_{Si} (q=0) \geq 0. \quad (10)$$

Substituting the previously mentioned state probability condition into $\Delta\pi_S$, we obtain

$$\begin{aligned} \Delta\pi_S = \Big\{ & p\left[(1-\theta)k + \lambda k\right] + (1-p) \\ & \times \left[(1-\theta)k_S - 1 + \lambda k_S\right] - q\frac{\theta}{M}k_D l \Big\} bI \geq 0. \end{aligned} \quad (11)$$

In a similar way, since $\theta = \alpha M$ and $\Delta\pi_S \geq 0$, we can derive that the sufficient and necessary conditions for service provider S to choose the cooperative advertising strategy are

$$\begin{aligned} \theta_S &\leq 1 - \frac{p\alpha k_D l - \lambda kq - \lambda k_S(1-p) + 1}{kp + k_S(1-p)} = \overline{\theta}_S, \\ M_S &\leq \frac{1}{\alpha} - \frac{p\alpha k_D l - \lambda kq - \lambda k_S(1-p) + 1}{\alpha kp + \alpha k_S(1-p)} = \overline{M}_S. \end{aligned} \quad (12)$$

With the previous analytical results, we can derive the following theorem.

Theorem 1. *The sufficient and necessary conditions for service providers D and S to choose the cooperative advertising strategy are that there exists a spillover rate upper bound, that is,* $\overline{\theta}_i$ *and* $i = D, S$*, meanwhile, an upper bound for the number of small service provider, that is,* $\overline{M}_i$ *and* $i = D, S$*, under which both players choose to advertise simultaneously. Otherwise, both service providers D and S choose to wait.*

Considering that both service providers D and S have the same cooperation preferences; that is, the probabilities of cooperative advertising strategy are the same for service providers D and $S(p = q)$ and given the advertising spillover rate θ, the subsidy λ, and the advertising effect of service provider $SblI$, we can derive the following conditions for service provider D and S to cooperate on advertising:

$$\overline{\theta_S} \le \overline{\theta_D}, \qquad \overline{M_S} \le \overline{M_D}. \tag{13}$$

Under the previously mentioned conditions, the binding constraint conditions for service provider D and S are

$$\begin{aligned} \theta_e &\le \overline{\theta_S} = 1 - \frac{p\alpha k_D l - \lambda kq - \lambda k_S (1-p) + 1}{kp + k_S (1-p)}, \\ M_e &\le \overline{M_S} = -\frac{p\alpha k_D l - \lambda kq - \lambda k_S (1-p) + 1}{\alpha kp + \alpha k_S (1-p)}. \end{aligned} \tag{14}$$

The equilibrium of both service providers choosing the cooperative advertising strategies is the most efficient state. As long as the condition $(1-\theta)k \ge 1$ holds, $\Delta\pi = \pi_{D1} + \pi_{S1} - (\pi_{D2} + \pi_{S2}) \ge 0$. This result indicates that the efficiency of the cooperative advertising strategy equilibrium is no less than that in Boxed Pig game.

The previously mentioned analysis also indicates that when the advertising spillover rate θ is not very large, the subsidy parameter λ and the discount rate δ are larger; the cooperative advertising strategy will be the equilibrium outcome.

4.2. Equilibrium of Boxed Pig Game and Deterioration. Boxed Pig game cannot attain the Pareto optimality, but the equilibrium is still more efficient compared to that of Prisoner's dilemma. In the following, we aim to analyze the equilibrium conditions of Boxed Pig game and how it deteriorates into Prisoner's dilemma game.

Suppose that both service providers D and S have the same willingness of cooperative advertising strategy, that is, $p = q$; thus in certain condition of the spillover rate θ, the subsidy parameter λ and the advertising effect blI of service provider S, we can derive the following conditions in which only one service provider chooses to offer advertisement, but the other service providers will choose to wait and reap the payoff:

$$\overline{\theta_S} \le \theta_e \le \overline{\theta_D} \quad \text{or} \quad \overline{M_S} \le M_e \le \overline{M_D}. \tag{15}$$

More specifically, the previously mentioned conditions can be rewritten as

$$\begin{aligned} 1 &- \frac{p\alpha k_D l - \lambda kp - \lambda k_S (1-p) + 1}{kp + k_S (1-p)} \\ &\le \theta_e \\ &\le 1 - \frac{q\alpha (k_S \beta / l) - \lambda kq - \lambda k_D (1-q) + 1}{kq + k_D (1-q)}, \end{aligned} \tag{16}$$

or

$$\begin{aligned} \frac{1}{\alpha} &- \frac{p\alpha k_D l - \lambda kp - \lambda k_S (1-p) + 1}{\alpha kp + \alpha k_S (1-p)} \\ &\le M_e \\ &\le \frac{1}{\alpha} - \frac{q\alpha (k_S \beta / l) - \lambda kq - \lambda k_D (1-q) + 1}{\alpha kq + \alpha k_D (1-q)}. \end{aligned} \tag{17}$$

By the previously mentioned analytical results, we can easily obtain the conditions in which Boxed Pig game deteriorates into Prisoner's dilemma equilibrium, that is, (waiting, waiting):

$$\theta_e > \overline{\theta_D} \quad \text{or} \quad M_e > \overline{M_D}. \tag{18}$$

In the previous conditions, either service provider chooses to wait the other service provider to offer advertisement; however, neither service provider chooses to offer advertisement eventually.

4.3. Analyses on Equilibrium of Cooperative Game and Boxed Pig Game. The advertising strategy equilibrium depends on the values of θ_e, $\overline{\theta_D}$, $\overline{\theta_S}$, M_e, $\overline{M_D}$, and $\overline{M_S}$. Next, we will employ the comparative static analysis to discuss factors relevant to θ and M, such as the subsidy λ, the advertising effects parameters k, k_D, and k_S, the resource power l, and the discount factor δ.

With respect to (9) and (12), taking first-order derivatives of $\overline{\theta_D}$, $\overline{\theta_S}$, $\overline{M_D}$, and $\overline{M_S}$ on λ, respectively, we have,

$$\begin{aligned} \frac{\partial \overline{\theta_D}}{\partial \lambda} &> 0, \qquad \frac{\partial \overline{\theta_S}}{\partial \lambda} > 0, \\ \frac{\partial \overline{M_D}}{\partial \lambda} &> 0, \qquad \frac{\partial \overline{M_S}}{\partial \lambda} > 0. \end{aligned} \tag{19}$$

The previous results show that first-order derivatives on the subsidy λ are bigger than zero, which implies that the spillover rate both service providers can tolerate and the number of small service providers are increasing with respect to the subsidy λ from the supplier. In other words, with the subsidy increases, both service providers D and S have larger spillover rates and prefer to offer advertisement. Then, we obtain Proposition 2 as follows.

Proposition 2. *The more the subsidy (λ) from the supplier is, the more both service providers prefer to choose to offer advertisement.*

This proposition has an implication that the subsidy from the supplier makes both service providers able to tolerate a larger spillover rate θ and the number of the small service providers, because both service providers can compensate this loss stolen by the small service providers with the subsidy.

With respect to (9) and (12), taking first-order derivatives of $\overline{\theta_D}$, $\overline{\theta_S}$, $\overline{M_D}$, and $\overline{M_S}$ on k_D and k_S, respectively, we have

$$\begin{gathered} \frac{\partial\overline{\theta_D}}{\partial k_D} > 0, \qquad \frac{\partial\overline{\theta_S}}{\partial k_S} > 0, \\ \frac{\partial\overline{M_D}}{\partial k_D} > 0, \qquad \frac{\partial\overline{M_S}}{\partial k_S} > 0. \end{gathered} \tag{20}$$

The previous results show that with the advertising effects increase, service providers D and S prefer to cooperate on advertising more. No fairness preferences are considered, both service providers maximize their payoff. Then, we can obtain Proposition 3 as follows.

Proposition 3. *The more the advertising effect* (k_D, k_S) *is, the more both service providers prefer to choose to offer advertisement.*

This proposition implies that the advertising effect makes both service providers tolerate a market with a large spillover rate θ and where a number of the small service providers free-ride, because both service providers can obtain more payoff from the advertising strategy.

With respect to (9) and (12), taking first-order derivatives of $\overline{\theta_D}$, $\overline{M_D}$ on k_S, and $\overline{\theta_S}$, $\overline{M_S}$ on k_D, respectively, we obtain the following results:

$$\begin{gathered} \frac{\partial\overline{\theta_D}}{\partial k_S} < 0, \qquad \frac{\partial\overline{M_D}}{\partial k_S} < 0, \\ \frac{\partial\overline{\theta_S}}{\partial k_D} < 0, \qquad \frac{\partial\overline{M_S}}{\partial k_D} < 0. \end{gathered} \tag{21}$$

The previous results show that the first-order partial derivatives are less than 0. This finding means that when service provider D knows that service provider S has a higher advertising effect, service provider D will choose to wait, because service provider S will offer advertisement to maximize his payoff, and vice versa. On the other hand, this easily tends towards a Boxed Pig game, where the service provider with less advertising effect will wait for service providers with more advertising effect to offer advertisement, so that he could share the advertising spillover effects. We obtain the following proposition.

Proposition 4. *The more the service provider D's advertisement advertising effect is, the less service provider S is willing to offer advertisement, and vice versa.*

The results are in accord with real cases: if one service provider has more advertising resource, the other service provider would prefer to believe that the service provider who has more advertising resource will offer advertisement; as a result, this service provider will choose to wait for the advertising spillover. Theoretically, when his rival has a more advertising effect, the service provider can tolerate a smaller advertising spillover rate θ and the number of the small service provider M, and vice versa.

With respect to (9) and (12), taking first-order derivatives of $\overline{\theta_D}$, $\overline{\theta_S}$, $\overline{M_D}$, and $\overline{M_S}$ on l, we obtain that

$$\begin{gathered} \frac{\partial\overline{\theta_D}}{\partial l} > 0, \qquad \frac{\partial\overline{M_D}}{\partial l} > 0, \\ \frac{\partial\overline{\theta_S}}{\partial l} < 0, \qquad \frac{\partial\overline{M_S}}{\partial l} < 0. \end{gathered} \tag{22}$$

The previous conditions indicate that given service provider S's advertising investment, the more the service provider D invests, the less the number of the service providers and the spillover rate that service provider S can tolerate.

Proposition 5. *The more the service provider D's advertising resource is, the larger the advertising spillover rate he can tolerate. Meanwhile, the less the advertising spillover rate and the number of the small service provider that service provider S can tolerate.*

This proposition shows that when the service provider D's advertising resource affects service provider D and S' tolerance of the advertising spillover rate and the number of the small service provider in the market simultaneously, and with the advertising resource increases, service provider D can tolerate a market with a larger advertising spillover; however, service provider S can obtain fewer payoff; then she will choose to wait service provider D to offer advertisement.

With respect to (9) and (12), taking the first-order derivative of $\overline{\theta_D}$ and $\overline{M_D}$ on β, we obtain

$$\frac{\partial\overline{\theta_D}}{\partial \beta} < 0, \qquad \frac{\partial\overline{M_D}}{\partial \beta} < 0. \tag{23}$$

This result shows that given his advertising resource, in order to obtain more revenue from the advertising, service provider D has to reduce the influencing factor of advertising spillover and lessen small service providers he can tolerate in the market.

Proposition 6. *At equilibrium (waiting, advertising), the larger the spillover rate of service provider D can obtain from the advertising is, the smaller the advertising spillover rate and the number of the small service providers service provider D can tolerate.*

Let $k_1 = \gamma k_2$ and $\gamma > 1$, where γ denotes the comparison of advertising resource between service providers D and S. A larger comparison ratio means a larger advertising resource difference between service providers D and S.

Combing $k_1 = \gamma k_2$ into (9) and (12) and taking the first-order derivatives of $\overline{\theta_D}$ and $\overline{\theta_S}$ on γ, we can obtain that

$$\frac{\partial\overline{\theta_D}}{\partial \gamma} > 0, \qquad \frac{\partial\overline{\theta_S}}{\partial \gamma} < 0. \tag{24}$$

From the previous conditions, we can derive the following proposition.

Proposition 7. *The larger the advertising resource comparison ratio γ is, the larger the advertising spillover rate θ service provider D can tolerate and service provider D chooses to cooperate on advertising, and vice versa.*

This proposition indicates that if the advertising can bring more revenues to service providers in the market, more service providers choose to cooperate on advertising, not only the dominate service provider D, but also the follower service providers.

Moreover, when the dominant service provider D can obtain more revenue from the advertising that small service providers, that is, a larger γ, service provider D will choose to offer advertisement, no matter whether the small service provider responses.

5. Conclusions and Future Research

Motivated by the industry case given in the introduction section, we constructed the advertising models between the dominant service provider and small service providers in a market. In our model, the dominant service provider has a major market share, while M small service providers have the remaining market share. Based on this setting, we discussed three advertising game models (i.e., cooperative game, Boxed Pig game, and Prisoner's game) and characterized the conditions for different advertising games and their equilibria, respectively. To be specific, it is found that the advertising spillover and the number of the small service providers directly determines the advertising game equilibria, and other market parameters, to some extent, can also affect the results of the advertising game equilibria as well.

According to our theoretical findings, we make the following suggestions on choices of the advertising strategies for the supplier, the dominate service provider and small service providers. For service providers, first of all, they can realize resource complementarities and achieve the advertising synergy effects through a cooperative advertising. On the contrary, non-cooperative advertising is not a wise alternative, because the spillover effect can improve service provider's short-term profits at the expense of the long-term profits. Therefore, service providers should cooperate with each other in order to achieve long-term and whole supply chain revenue instead of free ride.

For suppliers, first, the larger the number of small service provider M is, the higher the probability for small service providers choosing to wait is. Therefore, it is necessary to reduce the number of service providers in the market in order to promote downstream service providers to cooperate on advertising. Second, suppliers should consider the type of service providers and offer different subsidies to the dominant service provider and small service providers. Finally, suppliers should foster relatively large service providers because large service providers tend to offer advertisement actively, and large service provider can signal advertising strategy to small service providers to cooperate with them.

This analysis can be extended in several directions. In practice, the service quality is the basic condition for the service providers to invest more in advertising; then a research on service quality is an interesting question in future [16]. The supplier and service providers cooperate and compete on advertising in many different forms. A subsidy between the supplier and service providers is only one of them. Thus, a more comprehensive investigation of interaction between the supplier and service providers would be a promising research direction. Furthermore, we only consider an advertising decision on whether to offer an advertisement or not in our current model. However, why and how to offer advertisement are not covered. It would be interesting to further analyze the specific advertising strategies of different service providers.

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Review Article

Sales Forecasting for Fashion Retailing Service Industry: A Review

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Sales forecasting is crucial for many retail operations. It is especially critical for the fashion retailing service industry in which product demand is very volatile and product's life cycle is short. This paper conducts a comprehensive literature review and selects a set of papers in the literature on fashion retail sales forecasting. The advantages and the drawbacks of different kinds of analytical methods for fashion retail sales forecasting are examined. The evolution of the respective forecasting methods over the past 15 years is revealed. Issues related to real-world applications of the fashion retail sales forecasting models and important future research directions are discussed.

1. Introduction

Inventory planning is a fundamental part of fashion retail operations. Proper retail inventory management, which helps to balance supply and demand, relies heavily on accurate forecast of future demand. In fact, sales forecasting refers to predicting future demand (or sales), assuming that the factors which affected demand in the past and are affecting the present will still have an influence in the future. It is an important task but is very difficult to accomplish.

In the fashion retailing industry, which is defined as the retailing business of fashion products including apparel, shoes, and fashion beauty products, forecasting itself can be treated as a "service" which represents the set of analytical tools that facilitate the companies to make the best decisions for predicting the future. Undoubtedly, a good forecasting service system can help to avoid understocking or overstocking in retail inventory planning, which further relates to other critical operations of the whole supply chain such as due date management, production planning, pricing [1, 2], and achieving high customer service level [3]. In order to achieve economics sustainability under a highly competitive environment, a company should adopt a consumer-demand driven "pull" operational strategy which means forecasting becomes a critically important task.

Compared to other retailing service industries, it is well argued that sales forecasting is a very difficult task in fashion retailing because fashion product's demand is highly volatile with ever-changing taste of the consumers and the fashion product's life cycle is very short [4, 5]. In addition, the sales of fashion products are strongly affected "stochastically" by seasonal factors, fashion trend factors [6], and many tricky variables (e.g., weather, marketing strategy, political climate, item features, and macroeconomic trend). These, together with the fact that fashion retailers are carrying a large number of stock-keeping-units (SKUs) with limited historical sales data, all make sales forecasting challenging and call for more sophisticated and versatile analytical tools. On the other hand, it is known that the fashion apparel supply chain is a relatively long one which includes upstream cotton plants, fiber manufacturers, apparel factories, distributors, wholesalers, and retailers. As a consequence, the notorious bullwhip effect [7] will have a particularly strong influence on the fashion supply chain. Since forecasting is a critical factor relating to the presence and significance of the bullwhip effect, improving forecasting can help reduce the bullwhip effect which directly enhances the efficiency of the fashion supply chain.

From the above discussions, it is crystal clear that fashion retail sales forecasting is a truly important topic in practice. Over the past decade, a number of research studies have been reported in the literature. However, each forecasting method has its limits and drawbacks. For example, the traditional statistical methods depend highly on the time series data's features and this will affect the forecasting accuracy a lot. Artificial intelligence (AI) methods can perform better in terms of accuracy than the traditional statistical forecasting models but they usually require a much longer time and a larger requirement on computational power. Thus, many researchers propose to combine multiple methods together to form a new "hybrid method" to achieve an efficient and effective forecasting task.

In this paper, we select and review a set of papers from the literature. In order to have a comprehensive collection of papers, we employ the popular and powerful research portals of http://scholar.google.com/ and http://www.sciencedirect.com/ and search objectively by keywords of "fashion forecasting," "apparel forecasting," "textile forecasting," "clothing forecasting," and other combinations with the keywords of "predict/prediction/forecast" to replace "forecasting." We then filter the searching outcomes and keep the peer-refereed papers which are written in English, together with some papers suggested by reviewers and our own peers, to compile this review paper. Notice that this paper is different from the recently published literature [8] in which they have different focal points. To be specific, [8] mainly discusses different kinds of general methods in relation to the market feature in the fashion industry whereas this paper focuses specifically on exploring and comparing the technical contents of the reviewed analytical models. As a remark, this paper can be viewed as an extension to the previous review paper in [9] with a much more comprehensive review and more in-depth discussions of the topic.

The organization of this paper is given as follows. We review the pure statistical fashion retail sales forecasting methods in Section 2. We discuss the pure AI based fashion retail sales forecasting methods in Section 3. We explore various different forms of hybrid fashion retail sales forecasting methods in Section 4. We investigate the applications of forecasting methods in the fashion retail industry in Section 5. We conclude the paper with a discussion of the evolution of methods as well as the future research directions in Section 6.

2. Statistical Fashion Sales Forecasting Methods

Traditionally, fashion sales forecasting is accomplished by the statistical methods. In fact, a lot of statistical methods have been used for sales forecasting, which include linear regression, moving average, weighted average, exponential smoothing (used when a trend is present but not linear), exponential smoothing with trend, double exponential smoothing, Bayesian analysis, and so forth.

Statistical time series analysis tools such as ARIMA and SARIMA are widely employed in sales forecasting [10]. Since these methods have a closed form expression for forecasting, it is simple and easy to implement and the results can be computed very quickly. In the literature, Green and Harrison [11] apply a Bayesian approach to explore forecasting for a mail order company which sells ladies dresses. After that, Thomassey et al. [12] use item classification to examine the accuracy of sales forecasting for new items. They find that a larger number of item families and pertinent classification criteria are required in the respective forecasting procedure in order to achieve an improved forecasting precision. They conclude that product family and aggregated forecasting are more accurate than the individual item's forecasting. Recently, Mostard et al. [13] also consider the forecasting problem based on a case study of a mail order apparel company. They propose a "top-flop" classification method and argue that it performs better than other methods. Furthermore, they find that the expert judgment methods outperform the advance demand information method for a small group of products. Another recent work [14] examines the applicability of a Bayesian forecasting model for fashion demand forecasting. It is found that the proposed hierarchical Bayesian approach yields superior quantitative results compared to many other methods.

Despite being popularly used for their simplicity and fast speed, it is well known that the statistic methods suffer a few problems. First, the selection of the right statistical methods is an uneasy task. It requires an "expert" knowledge. Second, in terms of performance, they do not usually lead to very promising results. In particular, compared to the more sophisticated methods such as AI methods, statistical models' performance is usually worse. Third, fashion sales are affected by multiple factors such as the fashion trends and seasonality and exhibit a highly irregular pattern [9], which implies that the pure statistical methods may fail to achieve a desirable forecasting outcome.

3. AI Fashion Retail Sales Forecasting Methods

As discussed in Section 2, the pure statistical models have deficiency in conducting fashion retail forecasting, in order to improve forecasting accuracy. AI methods emerge with the advance of computer technology. In fact, AI models can efficiently derive "arbitrarily nonlinear" approximation functions directly from the data. Popular methods such as artificial neural network (ANN) models [15] and fuzzy logic models are commonly employed in the literature and they are the first kind of models being employed for fashion retail sales forecasting. To be specific, ANN models have been developed and they provide satisfactory results in different domains [16–18]. In the literature of fashion sales forecasting, Frank et al. [3] explore the use of ANN model for conducting fashion retail sales forecasting. Comparing it with two other statistical methods in terms of forecasting result, it is found that ANN model achieves the best performance. Afterwards, the evolutionary neural network (ENN) model, which is a promising global searching approach for feature and model

Table 1: The summary of AI methods-based fashion retail sales forecasting.

Method	Paper	Area	Findings
ANN	[3]	Sufficient data	ANN model outperforms the two statistical based models.
Fuzzy	[15]	Short term Sufficient data	Multivariate fuzzy analysis is better compared to that of univariate analysis for short term forecasting.
	[28]	Color	Fuzzy colour prediction system is better than the traditional approach, but it only applies to single-colour prediction case.
ENN	[19]	Low demand uncertainty and weak seasonal trends Short term	Performance of ENN is better than the traditional SARIMA model for products with features of low demand uncertainty and weak seasonal trends.
ELM	[31]	Color, size, and price as significant factors	ELM outperforms several sales forecasting methods which are based on backpropagation neural networks.
EELM	[32]	Fast forecasting	EELM is versatile in which it can be used for short, medium to long-term predictions with both time series and non-time series data.

selection, has been used in fashion sales forecasting. To be specific, Au et al. [19] employ ENN to search for the ideal network structure for a forecasting system, and then an ideal neural networks structure for fashion sales forecasting is developed. They report that the performance of their proposed ENN model is better than the traditional SARIMA model for products with features of low demand uncertainty and weak seasonal trends.

The theory of fuzzy sets is proposed by Zadeh [20] and it has been applied in a lot of areas (e.g., [21–27]). In fashion retail sales forecasting, Sztandera et al. [15] construct a novel multivariate fuzzy model which is based on several important product variables such as color, time, and size. In their proposed model, grouped data and sales values are calculated for each size-class combination. Compared with several statistical models such as Winters' three parameter exponential smoothing model (W3PES), the neural network model, and the univariate forecasting models, they find that their proposed multivariable fuzzy logic model is an effective sales forecasting tool. In fact, the good performance of the fuzzy logic based models comes from their ability to identify nonlinear relationships in the input data. In addition, the multivariate fuzzy logic model performs better in comparison to the univariate counterparts. Later on, Hui et al. [28] explore the demand prediction problem in terms of fashion color forecasting. They propose a fuzzy logic system which integrates preliminary knowledge of colour prediction with the learning-based fuzzy colour prediction system to conduct forecasting. They report several promising results of their proposed method.

Despite the fact that ANN and ENN models perform well in terms of yielding high forecasting accuracy (as indicated by performance measures such as the mean-squared error), these forecasting models require a very long time to complete the forecasting task. In other words, they are very time consuming. The reason behind such a drawback comes from the fact that these models are all utilizing the gradient-based learning algorithms such as the backpropagation neural network (BPNN). To overcome this problem, the extreme learning machine (ELM) based models have emerged. In fact, ELM is known to be a super fast method and it can successfully prevent problems associated with stopping criteria, learning rate, learning epochs, local minima, and the over-tuning from happening. In the literature, ELM has been employed in fashion sales forecasting and its performance is proven to be better than many backpropagation neural networks based methods [29, 30]. Actually, Sun et al. [31] pioneer the use of ELM for sales forecasting in fashion. They investigate the relationship between sales amount and the significant factors which affect demand (e.g., design factors). However, ELM has its most critical drawback of being "unstable" as it can generate different outcome in each different run. To overcome this issue, an extended ELM method (EELM) is proposed in [32] which computes the forecasting result by repeatedly running the ELM for multiple times. Of course, the number of repeating times is an important parameter in EELM and it can be estimated.

Table 1 summarizes the representative papers using pure AI methods for conducting fashion retail sales forecasting.

Even though ELM and EELM are faster than the classical ANN and ENN based forecasting models, they are far from perfect. In particular, ELM is unstable, and EELM still needs a substantial amount of time to conduct prediction. In other words, there are cases in which they might not work well (e.g., EELM with multiple repeated runs of learning machine (LM) cannot complete the forecasting task to be done within any given time constraint imposed by the users [9]). The same applies to other pure statistical and pure AI methods. As a consequence, pinpointing on different perspectives, various hybrid models are developed in the literature to enhance fashion retail sales forecasting.

4. Hybrid Methods for Fashion Sales Forecasting

Hybrid forecasting methods are usually developed based on the fact that they can utilize the strengths of different models together to form a new forecasting method. As such, many of them are considered to be more efficient than the pure statistical models and pure AI models. It is not surprising that in recent years, a number of research works examine hybrid forecasting methods, for example, [53, 55–59]. Hybrid methods employed in the fashion forecasting literature often combine different schemes such as fuzzy model, ANN, and ELM with other techniques such as statistical models, the grey model (GM), and so forth. In the following, we review the literature on hybrid methods in multiple subsections.

4.1. Fuzzy Logic Based Hybrid Methods. Vroman et al. [33] are the pioneers in studying fuzzy based hybrid fashion forecasting method. They derive a fuzzy-adaptive model which controls the weighting factors of an exponential-smoothing statistical "Holt-Winter" forecasting method. They show that the proposed fuzzy hybrid model outperforms the conventional Holt-Winter method. They also advocate that their proposed method can be applied for new item fashion sales forecasting. After that, Thomassey et al. [34] use fuzzy logic concept to perform fashion forecasting. Their new model allows automatic learning of the nonlinear explanatory variables' influence. Notice that their model requires a subjective expert judgment for the learning process which poses a challenge for its real-world application in the fashion retailing industry. Thomassey et al. [37] propose a forecasting system which is based on multiple models such as fuzzy logic, neural networks, and evolutionary procedures. They argue that the result is versatile in processing the uncertain data. Recently, Yesil et al. [39] apply a hybrid fuzzy model to fast fashion forecasting. To be specific, they combine the fuzzy logic model and the statistical model to conduct forecasting. In their hybrid method, they calculate final forecast for weekly demand based on the weighted average of forecasts that are generated by multiple methods. They argue that their proposed method achieves high accuracy.

4.2. Neural Network Based Hybrid Methods. In neural network (NN) hybrid models, Vroman et al. [40] employ a NN model with corrective coefficients of the seasonality feature for mean-term forecasting horizon. They argue that their proposed hybrid method can also conduct forecasting for short and discontinuous time series. They report good results with their proposed NN hybrid model and believe that the outstanding performance comes from the NN's ability of mapping the nonlinear relation between data inputs and output. Thomassey and Happiette [41] develop a hybrid neural clustering and classification scheme for conducting sales forecasting of new apparel items. Their model can increase the accuracy of midterm forecasting in comparison with the mean sales profile predictor. ANN can also be combined with other techniques like Grey method (GM) and autoregressive technique. For instance, a two-stage dynamic forecasting model, which contains neural network and auto regressive technique, is applied for fashion retail forecasting in Ni and Fan [43]. In their model, Ni and Fan use neural network to establish a multivariable error forecasting model. Their model develops the concept of "influence factors" and divides the "impact factors" into two distinct stages (long term and short term). The computational experiment shows that the multivariable error forecasting model can yield good prediction results for fashion retail sales forecasting problems. Aksoy et al. [38] combine the fuzzy method and neural networks to form a new system called the adaptive-network based fuzzy inference system. Their proposed new system combines the advantages of both systems, namely, the learning capability of the neural networks and the generalization capability of the fuzzy logic technique, and establishes the hybrid powerful system. Most recently, Choi et al. [44] apply an ANN and GM based hybrid model for fashion sales forecasting with respect to color. They compare ANN, GM, Markov regime switching, and GM+ANN hybrid models. They reveal that the GM(1,1) and ANN hybrid model is the best one for forecasting fashion sales by colors in the presence of very few historical data.

4.3. ELM Based Hybrid Methods. The extreme learning machine (ELM) is quick in conducting forecasting [45]. Despite the fact that it is not perfect because of its unstable nature, its "fast speed" makes it a very good candidate to be a component model for more advanced hybrid model for fashion forecasting. For example, Wong and Guo [42] propose a novel learning algorithm-based neural network to first generate initial sales forecast and then use a heuristic fine-tuning process to obtain more accurate final sales forecast. Their learning algorithm integrates an improved harmony search algorithm and an extreme learning machine to improve the network generalization performance. They claim that the performance of their proposed model is superior to the traditional ARIMA models and two recently developed neural network models for fashion sales forecasting. Xia et al. [46] examine a forecasting model based on extreme learning machine model with the adaptive metrics. In their model, the inputs can solve the problems of amplitude changing and trend determination, which in turn helps to reduce the effect of the over fitting of networks. Yu et al. [47] use ELM and Grey relational analysis (GRA) to develop a fashion color forecasting hybrid method [47]. Their computational result with real empirical data proves that their proposed model outperforms several other competing models in forecasting fashion color.

4.4. Other Hybrid Methods. In addition to the types of hybrid methods reviewed above, some other innovative forecasting combined methods are also reported in the literature on fashion sales forecasting. For example, Choi et al. [48] employ a hybrid SARIMA wavelet transform (SW) method for fashion sales forecasting. Using real data and artificial data, they show that with relatively weak seasonality and highly variable seasonality factor, their proposed SW method outperforms the classical statistical methods. They conclude to say that the SW method is suitable for conducting volatile demand forecasting in fashion. Thomassey and Fiordaliso [49] develop a hybrid method which is based on an existing clustering technique and a decision tree classifier. Their proposed hybrid method is useful for estimating the sales profiles of new items in fashion retail in which there is no historical sales data. Ni and Fan [43] establish a combined method which includes autoregression and decision tree method (called ART method). They propose that this hybrid method performs very well for fashion sales forecasting. Table 2 summarizes the reviewed hybrid methods.

5. Applications in Fashion Industry

Sales forecasting is a real-world problem in fashion retailing. From the perspective on applications and implementation, various issues are identified.

First, in terms of the forecasting horizon, most of the existing forecasting models are suitable for middle-term and long-term forecasting. However, short-term forecasting, including the very short term forecasting such as real-time forecasting, is not yet fully explored. This kind of short-term forecasting is very important given the nature of the fashion industry (the fashion trend is unpredictable, and the lead time is very short). From the literature review, we find that the fuzzy logic based technique has been adopted for short-term fashion sales forecasting, the methods [15, 34, 36, 38], and so forth. Thus, we argue that for real-world implementations, fuzzy logic based models, together with other speedy models (such as statistical methods), can be good candidates for real-world implementation as a short-term retail sales forecasting system.

Second, regarding the product type to be forecasted, two kinds of products are involved, namely, the existing product and a new product. Compared to the existing products forecasting, prediction on new product forecasting seems to be much more complicated and difficult, due to the absence of historical sales data. In the current literature, some papers study the new item forecasting (e.g., [60–64]), but very few papers explore the new item forecasting in fashion industry, and exceptions include the following: (i) an item classification method is used in [12], a neural networks and classification combined method is reported in [41], and a fuzzy and Holt Winter hybrid method is examined in [33], and an ANN based hybrid method is proposed in [37]. Obviously, the AI method is used frequently for new item forecasting. It is the case because the AI method can better catch the characteristics of the data and get a more accurate result. Notice that the classification method is applied to new item forecasting for such a reason, too. When predicting a new item, the information and data are very limited. In order to get more information, we have to wisely extract the useful information from the available data. Therefore, a systematic classification scheme is critical for this task.

Third, in terms of speed, in general, statistical methods can output the forecasting results very quickly. AI methods are usually more time consuming. In the past, the lead time in the fashion industry is a bit longer than now, and the lead time can be ten months or even one year. However, the fashion industry has changed and fast fashion companies like ZARA, H&M, and Mango are adopting quick response strategy with a very short lead time (e.g., 2 weeks in Zara for some products). As a result, forecasting result must be available within a very short time for any forecasting application for these companies. From the reviewed literature, we observe that owing to the high speed of ELM [45], it can be a good candidate to function under "fast fashion forecasting" domain, together with statistical methods. In addition, the fuzzy combiner method can also be used to explore the problem for fast fashion forecasting [39], in which the combiner generates forecasts by combining the forecasts of different methods through fuzzy logic.

6. Conclusion and Future Research Directions

In this paper, we have conducted a comprehensive review of the literature on fashion retail sales forecasting. We have explored the advantages and the drawbacks of different kinds of analytical methods for fashion retail sales forecasting. We have also examined the pertinent issues related to real-world applications of the fashion retail sales forecasting models. From the reviewed literature above, we prepare Table 3 which summarizes the fashion forecasting literature with respect to the pure statistical models, the pure AI models, and the hybrid models.

From Table 3, despite being popularly employed in the industry, it is interesting to observe that the pure statistical methods are not popularly studied in the literature over the past 15 years. The reasons are as follows. (i) They are already well-explored, and (ii) they are not sufficient to yield sophisticated forecasting result by themselves. In fact, new studies all move to AI and the hybrid models. The pure AI models are studied a few times over the past 15 years. However, obviously, pure AI (with a single method) models are also not sufficient to generate most accurate forecasting result with respect to the feature of fashion sales forecasting. As a result, the hybrid model based papers appear most frequently, especially over the past four years. Thus, we believe that it is still a timely topic to explore more advanced hybrid models for fashion retail sales forecasting.

Finally, we conclude this paper with the discussions of a few future research directions below.

(1) For fashion retail sales forecasting, regarding the data source, there are three kinds of data, namely, the time series data, cross-section data, and panel data. The time series data, which is collected over discrete intervals of time, is widely used in fashion forecasting and the methods applied to time-series data are also well developed. Cross-section data is collected over sample units in a particular time period and panel data follows individual microunits over time. These two kinds of data are not yet fully used for fashion sales forecasting. Recently, a forecasting method using the panel data is developed in [54] and it will be an interesting future research direction to explore the use of these different types of data for fashion sales forecasting.

(2) Color is one critical element in fashion and it is highly related to the inventory and production planning of fashion apparel products. However, from the reviewed literature, only very few prior studies have examined color forecasting (such as [44, 50–52, 65]). Thus, more studies on this topic can be conducted. In addition, on a related area, no prior study has examined how fashion pattern design and

Table 2: The summary of hybrid methods-based fashion retail sales forecasting models.

Method		Paper	Domain	Findings
Fuzzy	Holt Winter	[33]	New items	The proposed fuzzy-adaptive model controls the weight factors of an exponential-smoothing forecasting method, and it can be applied to new item sales forecasting.
	CCX	[34]	Mean term	It uses fuzzy logic abilities to map the nonlinear influences of explanatory variables to conduct forecasting, but expert judgment is required for the learning process.
		[35]	Mean term	It allows a textile company to obtain mean term forecasting to pass commands to providers.
	NN	[36]	Short term	The method performs short-term forecasting by readjusting mean-term model forecasts from load real sales.
	Distribution of aggregated forecast and classification	[37]	New items Insufficient data	The method (items forecasting model based on distribution of aggregated forecast and classification) estimates the item sales of the same family without requiring historical data.
	NN	[38]	Short term	The model promotes greatly the accuracy of forecasting results for the horizon of one month.
	NN	[39]	Fast forecasting	Using fuzzy logic, the combiner calculates final forecast for each week's demand as a weighted average of forecasts that are generated by different methods. The combined forecast achieves better accuracy than any of the individual forecasts.
ANN	CCX	[40]	Mean term	Considering noisy data and multiple explanatory variables (controlled, available or not) related to the sales behavior, the proposed model performs well.
	Classification	[41]	New items	Neural clustering and classification model globally increases the accuracy of midterm forecasting in comparison with the mean sales profile predictor.
	ELM + Harmony search	[42]	Mean term	The learning algorithm integrates an improved harmony search algorithm and an extreme learning machine to improve the network generalization performance and is better than traditional ARIMA models and two recently developed neural network models in fashion forecasting.
	ART	[43]	Two stages: long term and short term	Combining the ART model the and error forecasting model based on neural network, an adjustment improving model which can be applied to the fashion retail forecasting is developed.
	GM	[44]	Color trend Insufficient data	GM+ANN hybrid models are examined in the domain of color trend forecasting with a limited amount of historical data. The GM+ANN hybrid model is the best one for forecasting fashion sales by colors where only very few historical data is available.
ELM	Statistic	[45]	Fast forecasting	A comparison with other traditional methods has shown that the ELM fast forecasting model is quick and effective.
	Metrics	[46]	Sufficient data	The adaptive metrics of inputs can solve the problems of amplitude changing and trend determination and reduce the effect of the overfitting of the neural networks. The model outperforms autoregression (AR), ANN, and ELM models.
	GRA	[47]	Color trend	With real data analysis, the results show that the ANN family models, especially for ELM with GRA, outperform the other models for forecasting fashion color trend.
SARIMA	Wavelet	[48]	Highly volatile sales	For real data with relatively weak seasonality and highly variable seasonality factor, the SW hybrid model performs well.
Decision tree	Clustering	[49]	Mean term	The proposed model based on existing clustering technique and decision tree classifier is useful to estimate sales profiles of new items in the absence of historical sales data.
	Auto-regressive technique	[43]	Short term	A two-stage dynamic short-term forecasting model is proposed

Table 3: The evolution of topics over time.

	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Pure statistical models						[12]			[49]							[14]
Pure AI models						[3]	[15]	[28]			[19, 31, 50]		[51, 52]		[32]	
Hybrid models	[33]			[40]	[34, 35]			[36, 37]		[41]			[42, 53]	[43, 45, 48]	[38, 39, 46, 47]	[44, 54]

other design factors affect demand and the respective sales forecasting mechanism. It is another interesting topic for further studies.

(3) In fashion retail system, the sales of the apparel product are strongly influenced by the calendar factor, for example, holiday. It can be observed easily that the sales in National day's holidays in Hong Kong and Black Friday holidays in the USA will go up very quickly and highly. On one hand, the demands on these specific dates are much more volatile and difficult to predict. On the other hand, the revenue that can be generated during these periods of time can be huge. As a consequence, how to precisely forecast the demand during special dates/events becomes crucial to fashion retailors. This becomes another topic open for future research.

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Research Article

Performance Analysis and Optimization of an Adaptive Admission Control Scheme in Cognitive Radio Networks

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In cognitive radio networks, if all the secondary user (SU) packets join the system without any restrictions, the average latency of the SU packets will be greater, especially when the traffic load of the system is higher. For this, we propose an adaptive admission control scheme with a system access probability for the SU packets in this paper. We suppose the system access probability is inversely proportional to the total number of packets in the system and introduce an Adaptive Factor to adjust the system access probability. Accordingly, we build a discrete-time preemptive queueing model with adjustable joining rate. In order to obtain the steady-state distribution of the queueing model exactly, we construct a two-dimensional Markov chain. Moreover, we derive the formulas for the blocking rate, the throughput, and the average latency of the SU packets. Afterwards, we provide numerical results to investigate the influence of the Adaptive Factor on different performance measures. We also give the individually optimal strategy and the socially optimal strategy from the standpoints of the SU packets. Finally, we provide a pricing mechanism to coordinate the two optimal strategies.

1. Introduction

Nowadays, radio spectrum is one of the scarcest and most invaluable resources for wireless communications [1]. However, actual measures show that the utilization of the spectrum is very low in practical networks [2]. This underutilization of the spectrum motivated the researchers to explore how to improve the allocation of the spectrum. As a result of this tendency, cognitive radio networks have emerged as a promising technology for solving the problem of the spectrum underutilization [3].

There are two types of users in cognitive radio networks, namely, Primary Users (PUs) and Secondary Users (SUs) [4]. The PUs access the licensed spectrum with a preemptive priority. The SUs can make opportunistic use of the licensed spectrum whenever the spectrum is not occupied by the PUs.

In recent years, research interest in cognitive radio networks has grown rapidly, and a great amount of research has dealt with the system performance of cognitive radio networks [5].

In [6], Su and Zhang developed a Markov chain to obtain the aggregate throughput of the SUs with two channel sensing policies. In [7], Kim et al. analyzed a carrier sense multiple access strategy in multichannel cognitive radio networks with a three-dimensional Markov chain. They derived the throughput and the packet delay of the SUs. Moreover, priority queueing systems have been widely adopted in performance studies of cognitive radio networks, since the priority queueing systems are suitable to model the nonidentical behaviors of different types of customers that join the systems [8]. In [9], Do et al. applied an M/G/1 preemptive priority queueing scheme to analyze the average waiting time of SU packets in the system. In [10], Smith et al. considered an $M/M/N/N$ preemptive priority queue with two types of customers. They investigated the mean number and the blocking probabilities for both kinds of users in cognitive radio networks.

The above mentioned research works on cognitive radio networks have focused on the system access strategy with 1 persistent scheme for the SUs. That is to say, the SU packets are supposed to join the system no matter how many packets are available in the system. Obviously, this kind of system access strategy will lead to a greater latency of the SUs. In networks, greater latency (also called time delay) may be potential source of poor performance, even of instability [11–13]. In order to control the access of SU packets in cognitive radio networks, in [14], Li and Han proposed an access threshold for the SU packets. They assumed the SU packets would not join the system when the queue length of the SU packets was equal to or greater than the access threshold. They supposed that the queue length could be received in a broadcast message sent out from the base station. With numerical results, they gave the individually and socially optimal access thresholds for the SU packets. However, the observable queue assumption will increase the system overhead due to the necessity of a signaling scheme. In [15], Turhan et al. assumed that a newly arriving SU packet would be admitted to join the system only when the total number of packets in the system was smaller than an access threshold. One drawback of the models in [14, 15] is that the access threshold was fixed and a newly arriving SU packet would definitely join the system when the number of packets did not exceed the access threshold. For better performance, we should adjust the system access of a newly arriving SU packet adaptively according to the traffic load of the system.

On the other hand, most of the researches on the performance evaluation of cognitive radio networks was performed in continuous-time domain. However, as nowadays communication systems are digital [16], it would be more suitable to use discrete-time system models rather than their continuous-time counterparts when analyzing or designing digital transmitting systems [17, 18].

In this paper, in order to control the system access of SU packets adaptively, we propose an admission control scheme with a system access probability. By introducing an Adaptive Factor, we assume the system access probability for a newly arriving SU packet is inversely proportional to the total number of packets in the system. We call this admission control scheme an adaptive admission control scheme. Based on the working principle of the adaptive admission control scheme, by considering the digital nature of modern communication, we build a discrete-time preemptive queueing model with priority and adjustable joining rate. We exactly evaluate the system performance by examining the blocking rate, the throughput, and the average latency of the SU packets. Furthermore, we compare the individually optimal strategy and the socially optimal strategy for the SU packets and propose a pricing mechanism for the SU packets.

The remainder of this paper is organized as follows. An adaptive admission control scheme for the SU packets and the system model are proposed in Section 2. The performance analysis is carried out in Section 3. In Section 4, the formulas for the performance measures, such as the blocking rate, the throughput and the average latency of the SU packets are obtained. Moreover, numerical results are provided to show the influence of the Adaptive Factor on different performance measures. In Section 5, the individually optimal strategy and the socially optimal strategy for the SU packets are compared, and a pricing mechanism is proposed to coordinate the two optimal strategies. Finally, conclusions are drawn in Section 6.

2. System Model for an Adaptive Admission Control Scheme

2.1. An Adaptive Admission Control Scheme in Cognitive Radio Networks. In this paper, we consider one licensed spectrum with a single channel in a kind of centralized cognitive radio network, in which there is a central controller that can allocate the spectrum for the PUs and the SUs in the network. The PU packets have preemptive priority to occupy the channel, and the SU packets can only make opportunistic use of the channel. Obviously, the greater the number of SU packets in the system is, the longer the latency of an SU packet and the higher the administration cost will be. Therefore, we propose an adaptive admission control scheme for the SU packets. The working principle for the adaptive admission control scheme is illustrated in Figure 1.

In Figure 1, there are five points to be emphasized as follows.

(1) In order to decrease the latency of the SU packets, a finite buffer with capacity of K $(K > 0)$ is set for the SU packets. On the other hand, to satisfy the latency requirement of the PUs to a maximum extend, no buffer is prepared for the PUs.

(2) In the adaptive admission control scheme, the central controller counts the packets in the system periodically. When a new SU packet arrives at the system, the central controller will admit this SU packet with probability $\beta_i = 1/(\alpha i + 1)$ or refuse this SU packet with probability $(1 - \beta_i)$, where i is the number of packets in the system and α is the Adaptive Factor. We call β_i the system access probability, which is inversely proportional to the number of packets in the system.

(3) When an SU packet is admitted to join the system, if the channel is being occupied by another packet (a PU packet or an SU packet), this SU packet will queue in the buffer. If the buffer is full, this SU packet will be blocked.

(4) When a PU packet joins the system, if the channel is idle, this PU packet will definitely occupy the channel directly; if the channel is being occupied by another PU packet, the newly arriving PU packet will be blocked; if the channel is being occupied by an SU packet, the newly arriving PU packet will interrupt the transmission of this SU packet and occupy the channel immediately.

(5) When the transmission of an SU packet is interrupted, this SU packet will return back to the buffer of the SUs and is queued at the head. If the buffer of the SUs is full, the SU packet queued at the end will be

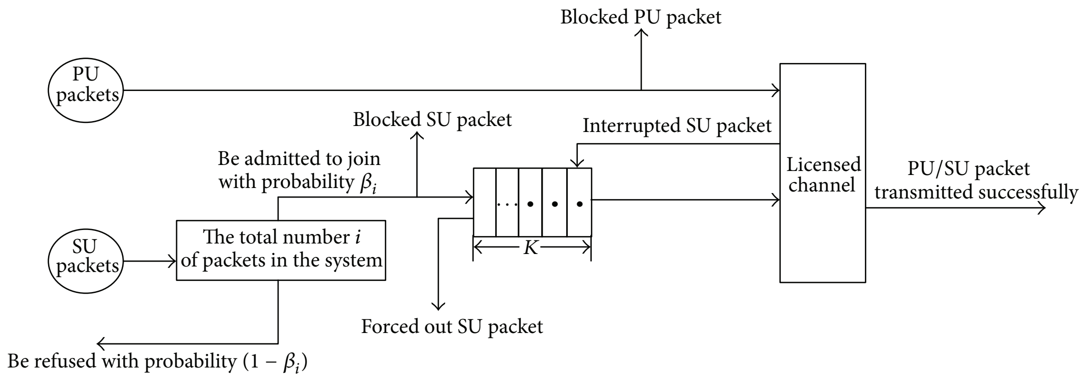

Figure 1: Adaptive admission control scheme in cognitive radio networks.

forced to leave the system. If the new admission of an SU packet and the interruption of an SU packet occur simultaneously, then due to having only one vacancy in the buffer, the interrupted SU packet will join the system, and the newly admitted SU packet will be blocked by the system. Hence, the interrupted SU packet is assumed to have a higher priority than the newly admitted SU packet.

From the above description, it is clear that the system access probability is dependent on the total number of packets in the system. The larger the number of packets in the system is, the less the possibility that a newly arriving SU packet will be admitted to join the system. On the other hand, the Adaptive Factor can adjust the impact of the total number of SU packets on the system access probability. Obviously, this adaptive admission control scheme can reduce the average latency of the SU packets.

2.2. System Model. Based on the adaptive admission control scheme proposed in this paper, a preemptive priority queueing model with adjustable joining rate can be built.

To capture the digital nature of modern networks, we consider the queueing model in discrete-time field. The time axis is divided into slots with equal length. The slot boundaries are marked as $n\,(n = 1, 2, \ldots)$. The arrivals of the packets can occur immediately after the beginning instant of a slot, and the departures of the packets can occur just prior to the end of a slot. Taking the instant $t = n$ as an example, the arrival of a packet is supposed to occur at (n, n^+), and the departure of a packet is supposed to occur at (n^-, n). That is to say, an early arrival system (EAS) is considered. In order to avoid ambiguity, we assume that a new SU packet arriving during the interval of $((n+1), (n+1)^+)$ will be admitted or refused by the system based on the number of packets in the system at the instant $t = n^+$.

The interarrival times and transmission times for both the two kinds of packets (PU packets and SU packets) are assumed to be independent, identically distributed (i.i.d) random variables following geometrical distributions [14]. The arriving intervals of the PU packets and the SU packets are supposed to follow geometrical distributions with parameters λ_1 $(0 < \lambda_1 < 1, \overline{\lambda}_1 = 1 - \lambda_1)$ and λ_2 $(0 < \lambda_2 < 1, \overline{\lambda}_2 = 1 - \lambda_2)$, respectively. According to the adaptive admission control scheme, when the total number of packets in the system is i, we denote the actual joining rate γ_i of the SU packets that are admitted to join the system as $\gamma_i = \lambda_2/(\alpha i + 1)$, where α is the Adaptive Factor. The transmission times of the PU packets and the SU packets are assumed to follow geometrical distributions with parameters μ_1 $(0 < \mu_1 < 1, \overline{\mu}_1 = 1 - \mu_1)$ and μ_2 $(0 < \mu_2 < 1, \overline{\mu}_2 = 1 - \mu_2)$, respectively. The traffic intensity of the PU packets and the SU packets are defined as ρ_1 $(\rho_1 = \lambda_1/\mu_1)$ and ρ_2 $(\rho_2 = \lambda_2/\mu_2)$, respectively.

Let $L_n = i$ $(i = 0, 1, 2, \ldots, K+1)$ be the total number of packets in the system at the instant $t = n^+$, and let $L_n^{(1)} = j$ $(j = 0, 1)$ be the number of PU packets in the system at the instant $t = n^+$. $\{L_n, L_n^{(1)}\}$ constitutes a two-dimensional Markov chain [13]. The state spaces of this Markov chain are given as follows:

$$\boldsymbol{\Omega} = (0,0) \cup \{(i,j) : 1 \le i \le K+1,\ j = 0, 1\}, \tag{1}$$

where state $(0,0)$ denotes there is no packet in the system; state $(i, 0)$ denotes that there are i SU packets and no PU packet in the system; state $(i, 1)$ denotes that there are $(i-1)$ SU packets and one PU packet in the system.

3. Performance Analysis

We define the system phase as the total number of packets in the system. Let $\mathbf{Q}$ be the state transition probability matrix for the system phases. $\mathbf{Q}$ can be given as a $(K+2) \times (K+2)$ block-structured matrix as follows:

$$\mathbf{Q} = \begin{pmatrix} \mathbf{Q}_{0,0} & \mathbf{Q}_{0,1} & \mathbf{Q}_{0,2} & & & & \\ \mathbf{Q}_{1,0} & \mathbf{Q}_{1,1} & \mathbf{Q}_{1,2} & \mathbf{Q}_{1,3} & & 0 & \\ & \mathbf{Q}_{2,1} & \mathbf{Q}_{2,2} & \mathbf{Q}_{2,3} & \mathbf{Q}_{2,4} & & \\ & \ddots & \ddots & \ddots & \ddots & & \\ 0 & & & \mathbf{Q}_{K,K-1} & \mathbf{Q}_{K,K} & \mathbf{Q}_{K,K+1} & \\ & & & & \mathbf{Q}_{K+1,K} & \mathbf{Q}_{K+1,K+1} & \end{pmatrix}, \tag{2}$$

where $\mathbf{Q}_{u,v}$ is the transition probability matrix from the system phase u to the system phase v, $u = 0, 1, \ldots, K+1$, $v = 0, 1, \ldots, K+1$.

$\mathbf{Q}$ can be discussed according to different system phases as follows.

(1) At the instant $t = n^+$, if the system phase $u = 0$, that is, there is no packet in the system, the system phase will be $v\,(v = 0, 1, 2)$ at the instant $t = (n + 1)^+$.

For the system phase $v = 0$, namely, there is no packet in the system at the instant $t = (n+1)^+$, the transition probability matrix $\mathbf{Q}_{0,0}$ is in fact a vector with only one scalar value given as follows:

$$\mathbf{Q}_{0,0} = \overline{\lambda}_1\overline{\lambda}_2. \tag{3}$$

For the system phase $v = 1$, namely, there is one SU packet or one PU packet in the system at the instant $t = (n + 1)^+$, the transition probability matrix $\mathbf{Q}_{0,1}$ is a row vector with two elements given as follows:

$$\mathbf{Q}_{0,1} = \left(\overline{\lambda}_1\lambda_2, \lambda_1\overline{\lambda}_2\right). \tag{4}$$

For the system phase $v = 2$, namely, there is one SU packet and one PU packet in the system at the instant $t = (n + 1)^+$, the transition probability matrix $\mathbf{Q}_{0,2}$ is a row vector with two elements given as follows:

$$\mathbf{Q}_{0,2} = \left(0, \lambda_1\lambda_2\right). \tag{5}$$

(2) At the instant $t = n^+$, if the system phase $u = 1$, that is, there is one packet in the system, the system phase will be $v\,(v = 0, 1, 2, 3)$ at the instant $t = (n + 1)^+$.

The system phase $v = 0$ means there is no packet in the system at the instant $t = (n + 1)^+$. For this case, the packet (one PU packet or one SU packet) in the system leaves, and there is no packet arrival. So, the transition probability matrix $\mathbf{Q}_{1,0}$ is a column vector with two elements given as follows:

$$\mathbf{Q}_{1,0} = \begin{pmatrix} \overline{\lambda}_1\left(1-\gamma_1\right)\mu_2 \\ \overline{\lambda}_1\left(1-\gamma_1\right)\mu_1 \end{pmatrix}. \tag{6}$$

The system phase $v = 1$ means there is one packet in the system at the instant $t = (n + 1)^+$. For this case, either the packet (one PU packet or one SU packet) in the system does not leave, and there is no packet arrival, or the PU packet in the system does not leave, and there is one PU packet arrival, or the packet (one PU packet or one SU packet) in the system leaves, and there is one packet (one PU packet or one SU packet) arrival. So, the transition probability matrix $\mathbf{Q}_{1,1}$ can be given as follows:

$$\mathbf{Q}_{1,1} = \begin{pmatrix} \overline{\lambda}_1\left(\left(1-\gamma_1\right)\overline{\mu}_2 + \gamma_1\mu_2\right) & \lambda_1\left(1-\gamma_1\right)\mu_2 \\ \overline{\lambda}_1\gamma_1\mu_1 & \left(1-\gamma_1\right)\left(\lambda_1\mu_1 + \overline{\mu}_1\right) \end{pmatrix}. \tag{7}$$

The system phase $v = 2$ means there are two packets in the system at the instant $t = (n + 1)^+$. For this case, either the SU packet in the system does not leave, and there is one packet (one PU packet or one SU packet) arrival, or the PU packet in the system does not leave, and there are two packet (one PU packet and one SU packet) arrivals, or the PU packet in the system does not leave, and there is one SU packet arrival, or the packet (one PU packet or one SU packet) in the system leaves, and there are two packet (one PU packet and one SU packet) arrivals. So, the transition probability matrix $\mathbf{Q}_{1,2}$ can be given as follows:

$$\mathbf{Q}_{1,2} = \begin{pmatrix} \overline{\lambda}_1\gamma_1\overline{\mu}_2 & \lambda_1\left(\left(1-\gamma_1\right)\overline{\mu}_2 + \gamma_1\mu_2\right) \\ 0 & \gamma_1\left(\lambda_1\mu_1 + \overline{\mu}_1\right) \end{pmatrix}. \tag{8}$$

The system phase $v = 3$ means there are three packets in the system at the instant $t = (n + 1)^+$. For this case, the SU packet in the system does not leave, and there are two packet (one PU packet and one SU packet) arrivals. So, the transition probability matrix $\mathbf{Q}_{1,3}$ can be given as follows:

$$\mathbf{Q}_{1,3} = \begin{pmatrix} 0 & \lambda_1\gamma_1\overline{\mu}_2 \\ 0 & 0 \end{pmatrix}. \tag{9}$$

(3) At the instant $t = n^+$, if the system phase $2 \le u \le K-1$, that is, there are u $(2 \le u \le K - 1)$ packets in the system, the system phase will be v $(v = u - 1, u, u + 1, u + 2)$ at the instant $t = (n + 1)^+$.

The system phase $v = u-1$ means there are $(u-1)$ packets in the system at the instant $t = (n + 1)^+$. For this case, one of the packets in the system leaves, and there is no packet arrival. So the transition probability matrix $\mathbf{Q}_{u,u-1}$ can be given as follows:

$$\mathbf{Q}_{u,u-1} = \begin{pmatrix} \overline{\lambda}_1\left(1-\gamma_u\right)\mu_2 & 0 \\ \overline{\lambda}_1\left(1-\gamma_u\right)\mu_1 & 0 \end{pmatrix}. \tag{10}$$

Similar to the matrix structures shown in (7)–(9), the transition probability matrix $\mathbf{Q}_{u,v}$ for $v\,(v = u, u + 1, u + 2)$ can be given as follows:

$$\mathbf{Q}_{u,u} = \begin{pmatrix} \overline{\lambda}_1\left(\left(1-\gamma_u\right)\overline{\mu}_2 + \gamma_u\mu_2\right) & \lambda_1\left(1-\gamma_u\right)\mu_2 \\ \overline{\lambda}_1\gamma_u\mu_1 & \left(1-\gamma_u\right)\left(\lambda_1\mu_1 + \overline{\mu}_1\right) \end{pmatrix}, \tag{11}$$

$$\mathbf{Q}_{u,u+1} = \begin{pmatrix} \overline{\lambda}_1\gamma_u\overline{\mu}_2 & \lambda_1\left(\left(1-\gamma_u\right)\overline{\mu}_2 + \gamma_u\mu_2\right) \\ 0 & \gamma_u\left(\lambda_1\mu_1 + \overline{\mu}_1\right) \end{pmatrix}, \tag{12}$$

$$\mathbf{Q}_{u,u+2} = \begin{pmatrix} 0 & \lambda_1\gamma_u\overline{\mu}_2 \\ 0 & 0 \end{pmatrix}. \tag{13}$$

(4) At the instant $t = n^+$, if the system phase $u = K$, that is, there is only one vacancy in the buffer, the system phase will be $v\,(v = K - 1, K, K + 1)$ at the instant $t = (n + 1)^+$.

Similar to the matrix structures shown in (10) and (11), the transition probability matrix $\mathbf{Q}_{K,K-1}$ and $\mathbf{Q}_{K,K}$ can be given as follows:

$$\mathbf{Q}_{K,K-1} = \begin{pmatrix} \overline{\lambda}_1\left(1-\gamma_K\right)\mu_2 & 0 \\ \overline{\lambda}_1\left(1-\gamma_K\right)\mu_1 & 0 \end{pmatrix}, \tag{14}$$

$$\mathbf{Q}_{K,K} = \begin{pmatrix} \overline{\lambda}_1\left(\left(1-\gamma_K\right)\overline{\mu}_2 + \gamma_K\mu_2\right) & \lambda_1\left(1-\gamma_K\right)\mu_2 \\ \overline{\lambda}_1\gamma_K\mu_1 & \left(1-\gamma_K\right)\left(\lambda_1\mu_1 + \overline{\mu}_1\right) \end{pmatrix}. \tag{15}$$

The system phase $v = K+1$ means there are $(K+1)$ packets in the system at the instant $t = (n+1)^+$; that is, the system is full. For this case, none of the packets in the system leaves, and there is at least one packet arrival; or one of the packets in the system leaves, and there are two packet (one SU packet and one PU packet) arrivals. So the transition probability matrix $\mathbf{Q}_{K,K+1}$ can be given as follows:

$$\mathbf{Q}_{K,K+1} = \begin{pmatrix} \overline{\lambda}_1 \gamma_K \overline{\mu}_2 & \lambda_1 (1-(1-\gamma_K)\mu_2) \\ 0 & \gamma_K (\lambda_1 \mu_1 + \overline{\mu}_1) \end{pmatrix}. \quad (16)$$

(5) At the instant $t = n^+$, if the system phase $u = K+1$, that is, there is no vacancy in the buffer, the system phase will be v $(v = K, K+1)$ at the instant $t = (n+1)^+$.

Similar to the matrix structure shown in (10), the transition probability matrix $\mathbf{Q}_{K+1,K}$ can be given as follows:

$$\mathbf{Q}_{K+1,K} = \begin{pmatrix} \overline{\lambda}_1 (1-\gamma_{K+1}) \mu_2 & 0 \\ \overline{\lambda}_1 (1-\gamma_{K+1}) \mu_1 & 0 \end{pmatrix}. \quad (17)$$

The system phase $v = K+1$ means that there are $(K+1)$ packets in the system at the instant $t = (n+1)^+$; that is, the system is full. For this case, none of the packets in the system leaves, or one of the packets in the system leaves, and there is at least one packet arrival including also forced leaving of SU packet due to arrival of PU packet. So the transition probability matrix $\mathbf{Q}_{K+1,K+1}$ can be given as follows:

$$\mathbf{Q}_{K+1,K+1} = \begin{pmatrix} \overline{\lambda}_1 (\gamma_{K+1} \mu_2 + \overline{\mu}_2) & \lambda_1 \\ \overline{\lambda}_1 \mu_1 \gamma_{K+1} & \lambda_1 \mu_1 + \overline{\mu}_1 \end{pmatrix}. \quad (18)$$

The structure of the transition probability matrix $\mathbf{Q}$ indicates that the two-dimensional Markov chain $\{L_n, L_n^{(1)}\}$ is nonperiodic, irreducible, and positive recurrent [17]. Let $\pi_{i,j}$ be the steady-state distribution of the two-dimensional Markov chain, which can be given as follows:

$$\pi_{i,j} = \lim_{n\to\infty} P\{L_n = i, L_n^{(1)} = j\}. \quad (19)$$

Let $\mathbf{\Pi}_i$ be the steady-state probability vector for the system being at phase i. $\mathbf{\Pi}_i$ can be given as follows:

$$\mathbf{\Pi}_i = \begin{cases} \pi_{0,0}, & i = 0, \\ (\pi_{i,0}, \pi_{i,1}), & 1 \le i \le K+1. \end{cases} \quad (20)$$

Combining the equilibrium equation and the normalization condition in the above Markov chain, we have

$$\begin{gathered} (\mathbf{\Pi}_0, \mathbf{\Pi}_1, \ldots, \mathbf{\Pi}_K, \mathbf{\Pi}_{K+1}) \mathbf{Q} = (\mathbf{\Pi}_0, \mathbf{\Pi}_1, \ldots, \mathbf{\Pi}_K, \mathbf{\Pi}_{K+1}), \\ (\mathbf{\Pi}_0, \mathbf{\Pi}_1, \ldots, \mathbf{\Pi}_K, \mathbf{\Pi}_{K+1}) \mathbf{e} = 1, \end{gathered} \quad (21)$$

where $\mathbf{e}$ is a one's column vector.

Equation (21) is a linear system of equations with $2\times(K+1)+1$ unknowns. By using a Gaussian elimination method to solve the linear equations, we can obtain the steady-state distribution $\pi_{i,j}$ defined in (19).

4. Performance Measures and Numerical Results

4.1. Performance Measures. In this subsection, we give some performance measures for cognitive radio networks with an adaptive admission control scheme as follows.

We define the average system access rate θ of the SU packets as the average number of SU packets that are admitted to join the system per slot. In the adaptive admission control scheme proposed in this paper, the probability that a newly arriving SU packet is admitted to join the system is dependent on the Adaptive Factor and the total number of packets in the system at the arrival instant. So the average system access rate θ can be given as follows:

$$\theta = \lambda_2 \pi_{0,0} + \sum_{i=1}^{K+1} \frac{\lambda_2}{\alpha i + 1} (\pi_{i,0} + \pi_{i,1}). \quad (22)$$

We define the blocking rate P_B of the SU packets as the average number of admitted SU packets that are blocked by the system per slot. An admitted SU packet will be blocked by the system in three cases: (1) Suppose that the channel is occupied by an SU packet and the buffer of the SU packets is already full in the previous slot. At current slot, the SU packet occupying the channel is not transmitted completely; or the SU packet occupying the channel is transmitted successfully, but a PU packet joins the system and occupies the channel. (2) Suppose that the channel is occupied by a PU packet, and the buffer of the SU packets is already full in the previous slot. At current slot, the PU packet occupying the channel is not transmitted completely; or the PU packet occupying the channel is transmitted successfully, but a new PU packet joins the system and occupies the channel. (3) Suppose that the channel is occupied by an SU packet, and there is only one vacancy in the buffer of the SU packets in the previous slot. At current slot, the SU packet occupying the channel is not transmitted completely and is interrupted by a newly arriving PU packet. The interrupted SU packet returns back to the buffer and occupies the only one vacancy. Therefore, the blocking rate P_B of the SU packets can be given as follows:

$$\begin{aligned} P_B = {} & \frac{\lambda_2}{\alpha(K+1)+1} \\ & \times ((\overline{\mu}_2 + \lambda_1 \mu_2) \pi_{K+1,0} + (\overline{\mu}_1 + \lambda_1 \mu_1) \pi_{K+1,1}) \\ & + \frac{\lambda_2}{\alpha K + 1} \lambda_1 \overline{\mu}_2 \pi_{K,0}. \end{aligned} \quad (23)$$

We define the throughput S of the SU packets as the average number of SU packets transmitted successfully per slot. An SU packet can be transmitted successfully if and only if this SU packet is admitted to join the system, not blocked by the system, and not forced to leave the system before the transmission is completely finished. The blocking rate of the SU packets can be obtained in (23). When a PU packet arrives at the system during the transmission time of an SU packet, if the buffer of the SUs is full, the SU packet queueing at the

Table 1: Common parameters setting in the numerical results.

Parameter	Value
Slot	1 ms
Average packet size	2750 Bytes
Data rate in physical layer	11 Mbps

end of the buffer will be forced to leave the system. Therefore, the throughput S of the SU packets is given as follows:

$$S = \theta - P_B - \lambda_1 \overline{\mu}_2 \pi_{K+1,0}. \quad (24)$$

We define the latency of an SU packet as the time period from the instant an SU packet is admitted to join the system to the instant that the SU packet is successfully transmitted. In fact, the latency of an SU packet is the sojourn time of that SU packet.

Let $L_n^{(2)}$ be the number of SU packets in the system at the instant $t = n^+$, and let $L^{(2)} = \lim_{n\to\infty} L_n^{(2)}$ be the number of SU packets in steady state. We can get the average number $E[L^{(2)}]$ of the SU packets as follows:

$$E\left[L^{(2)}\right] = \sum_{i=0}^{K+1} iP\left\{L^{(2)} = i\right\} = \sum_{i=1}^{K+1} i\pi_{i,0} + \sum_{i=0}^{K} i\pi_{i+1,1}. \quad (25)$$

By using Little's law [19], the average latency δ of the SU packets can be given as follows:

$$\delta = \frac{E\left[L^{(2)}\right]}{S}. \quad (26)$$

4.2. Numerical Results. In the numerical results, we consider the 2.4 GHZ spectrum band in Wi-Fi based wireless networks. Following the IEEE 802.11 b/g standard and referencing the parameter setting in [7], the common parameters used in the numerical results are summarized in Table 1.

Based on Table 1, the transmission rate for the packets can be calculated as $\mu_1 = \mu_2 = 0.5$.

Moreover, the data set for the Adaptive Factor is set as $\alpha = \{0.0, 0.1, \ldots, 0.9, 1.0\}$, in which the case of $\alpha = 0$ indicates the conventional system access strategy without any admission control. In this way, the influence of the Adaptive Factor α on the system performance and on the efficiency of the proposed adaptive admission control scheme can be shown.

At the same time, in order to distinctively investigate the influences of the arrivals for the PU packets and the SU packets on the performance measures, we set the arrival rates of the PU packets and the SU packets as $\lambda_1 = \{0.2, 0.3\}$ and $\lambda_2 = \{0.4, 0.6\}$. Additionally, as an example, we assume the buffer capacity of the SU is fixed at $K = 10$. We remark here that with a larger buffer capacity of the SUs, both the throughput and the average latency of the SU packets will be higher.

Figure 2 shows how the throughput S of the SU packets changes with respect to the Adaptive Factor α for the different arrival rates.

From Figure 2, we observe that, for the same arrival rate λ_1 of the PU packets and the same arrival rate λ_2 of the SU

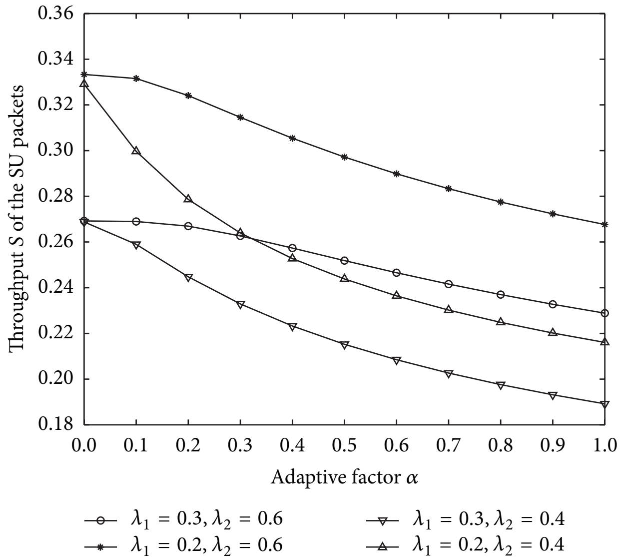

Figure 2: Throughput S of the SU packets versus Adaptive Factor α.

packets, the throughput S of the SU packets will decrease as the Adaptive Factor α increases. The reason is that the larger the Adaptive Factor is, the less likely it is that a newly arriving SU packet will be admitted to join the system, so the throughput of the SU packets will be lower.

On the other hand, from Figure 2, we see that for the same arrival rate λ_1 of the PU packets and the same Adaptive Factor α, the throughput S of the SU packets will increase as the arrival rate λ_2 of the SU packets increases. The reason is that the larger the arrival rate of the SU packets is, the more SU packets can join the system, so the throughput of the SU packets will be lower.

Moreover, we find that for the same arrival rate λ_2 of the SU packets and the same Adaptive Factor α, the larger the arrival rate λ_1 of the PU packets is, the smaller the throughput S of the SU packets will be. This is because as the arrival rate of the PU packets increases, the possibility that the channel is occupied by a PU packet is higher; then the possibility for an SU packet occupying the channel will be lower. On the other hand, the possibility for an SU packet being interrupted by PU packets will be higher. As a result, the throughput of the SU packets will decrease.

We examine the influence of the Adaptive Factor α on the average latency δ of the SU packets for the different arrival rates in Figure 3.

In Figure 3, we conclude that for the same arrival rate λ_1 of the PU packets and the same arrival rate λ_2 of the SU packets, the average latency δ of the SU packets will decrease as the Adaptive Factor α increases. The reason is that the larger the Adaptive Factor is, the lower the possibility that a newly arriving SU packet will be admitted to join the system is, the less the number of SU packets in the system will be, and this will result in a decrease in the average latency of the SU packets.

On the other hand, from Figure 3, we find that for the same arrival rate λ_1 of the PU packets and the same Adaptive

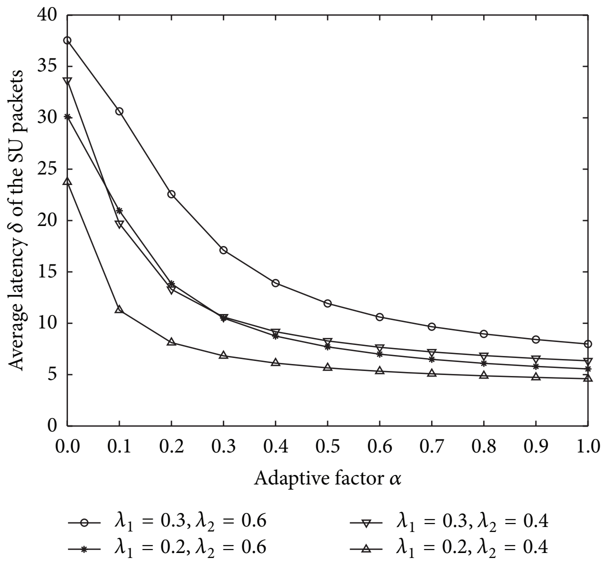

Figure 3: Average latency δ of the SU packets versus Adaptive Factor α.

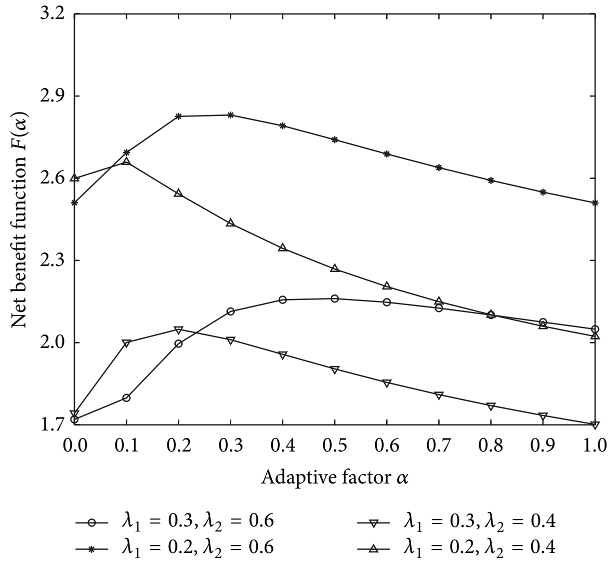

Figure 4: Net benefit function $F(\alpha)$ versus Adaptive Factor α.

Factor α, the average latency δ of the SU packets will increase as the arrival rate λ_2 of the SU packets increases. The reason is that the larger the arrival rate of the SU packets is, the more SU packets can join the system and wait in the buffer, and this will induce an increase in the average latency of the SU packets.

Moreover, we see that for the same arrival rate λ_2 of the SU packets and the same Adaptive Factor α, the larger the arrival rate λ_1 of the PU packets is, the longer the average latency δ of the SU packets will be. This is because as the arrival rate of the PU packets increases, the possibility that the channel is occupied by a PU packet will increase, so the time length for an SU packet waiting in the buffer will increase. This will make the average latency of the SU packets increase.

When the Adaptive Factor is set to be $\alpha = 0$ in Figures 2 and 3, we can obtain the system performance of the conventional system access strategy without any admission control. In the adaptive admission control scheme we can conclude from the trends of the performance measures of the SU packets, that the average latency of the SU packets is reduced significantly, while the throughput of the SU packets will be decreased slightly.

Additionally, from Figures 2 and 3, we know that as the Adaptive Factor α increases and the average latency δ of the SU packets will decrease, but also the throughput S of the SU packets will decrease. So we conclude that there is a trade-off when setting the Adaptive Factor α. In order to formulate the joint optimal problem of throughput S and average latency δ of the SU packets, we construct a net benefit function $F(\alpha)$ as follows:

$$F(\alpha) = C_1 S - C_2 \delta, \tag{27}$$

where C_1 and C_2 are supposed to be the reward of the throughput S and the cost of the average latency δ of the SU packets, respectively.

From (27), the optimal Adaptive Factor α^* can be given as follows:

$$\alpha^* = \arg\max \{F(\alpha)\}, \tag{28}$$

where "arg max" stands for the argument of the maximum [20].

By setting $C_1 = 10$ and $C_2 = 0.03$ as an example, we plot how the net benefit function $F(\alpha)$ changes with respect to the Adaptive Factor α in Figure 4.

As expected we can see in Figure 4, that there is a maximum net benefit when the Adaptive Factor is set to the optimal value for all the arrival rates of packets. For example, when $\lambda_1 = 0.3, \lambda_2 = 0.6$, the optimal Adaptive Factor is $\alpha^* = 0.5$ and the maximum net benefit is $F(\alpha^*) = 2.16$; when $\lambda_1 = 0.2, \lambda_2 = 0.6$, the optimal Adaptive Factor is $\alpha^* = 0.3$ and the maximum net benefit is $F(\alpha^*) = 2.83$; when $\lambda_1 = 0.3, \lambda_2 = 0.4$, the optimal Adaptive Factor is $\alpha^* = 0.2$ and the maximum net benefit is $F(\alpha^*) = 2.05$; when $\lambda_1 = 0.2, \lambda_2 = 0.4$, the optimal Adaptive Factor is $\alpha^* = 0.1$ and the maximum net benefit is $F(\alpha^*) = 2.66$.

5. Performance Optimization

In the adaptive admission control scheme proposed in this paper, when an SU packet arrives at the system, this SU packet may be not admitted by the system. Even this SU packet is admitted to join the system; it may be blocked by the system. In other word, the transmission for an SU packet is not guaranteed. From the view point of the SU packets, it is necessary to make optimization for their actions to obtain the maximum benefit. So, in this section, we firstly give some assumptions, and then we compare the individually optimal strategy and the socially optimal strategy for the SU packets. At last, in order to coordinate the two optimal strategies, we propose a pricing mechanism.

5.1. Assumptions for Performance Optimization. In this subsection, we give some assumptions that will be used in the following optimizations.

(1) We assume that a newly arriving SU packet is not aware the number of packets already in the system and does not know whether to be admitted by the system before making any decisions. This assumption is different from the observable queue case in [14]. Additionally, an SU packet will either irrevocably join the system or not join the system at all.

(2) Let R be the reward for a successful transmission of an SU packet. Since the admission to the system for the SU packets is not guaranteed, we refer the arrival of a new SU packet as a trial. There is a cost T ($T < R$) associated with each trial. That is to say, when a new SU packet arrives at the system, it will pay a cost T for trying to join the system, no matter whether or not it will be transmitted successfully.

(3) We denote the potential arrival rate of the SU packets as Λ.

5.2. Comparison between Individually and Socially Optimal Strategies. We firstly discuss the individually optimal strategy [20] for the SUs. From the view point of a single SU packet considered in this paper, there is a mixed trial strategy described with a fraction q, $0 \le q \le 1$, which is the probability for an SU packet trying to join the system. We denote the individually optimal trial rate as λ_e and the individually optimal trial probability as q_e. Then we have $\lambda_e = q_e\Lambda$, where Λ is the potential arrival rate of the SU packets.

Both of the optimal trial probability q_e and the actual joining rate γ_i introduced in Section 2 will impact the accessibility of the SU packets. With the optimal trial probability q_e, an SU packet will reach Nash equilibrium [20]. With the actual joining rate γ_i introduced in the admission control scheme, the access of the SU packets can be controlled adaptively based on the number of packets already in the system.

We can obtain the expression for the probability $\varepsilon(\lambda_2)$ that an SU packet can be successfully transmitted as follows:

$$\varepsilon(\lambda_2) = \frac{S}{\lambda_2}, \tag{29}$$

where S is the throughput of the SU packets given in (24), and λ_2 ($0 < \lambda_2 < 1$) is the arrival rate of the SU packets.

The individual net benefit $B_I(\lambda_2)$ for an SU packet who tries to join the system is given by

$$B_I(\lambda_2) = \varepsilon(\lambda_2)(R - T) - (1 - \varepsilon(\lambda_2))T = \varepsilon(\lambda_2)R - T. \tag{30}$$

Considering the complexity of the individual net benefit function, we explore the monotonic property of $B_I(\lambda_2)$ in (30) with numerical results. Taking the parameters used in Figures 2 and 3 and setting $R = 60$, $T = 35$, $\lambda_1 = 0.3$, $\lambda_2 \in (0.01, 0.50]$ as an example, we show how the individual net benefit $B_I(\lambda_2)$ changes with respect to the arrival rate λ_2 of the SU packets for the different Adaptive Factors α in Figure 5.

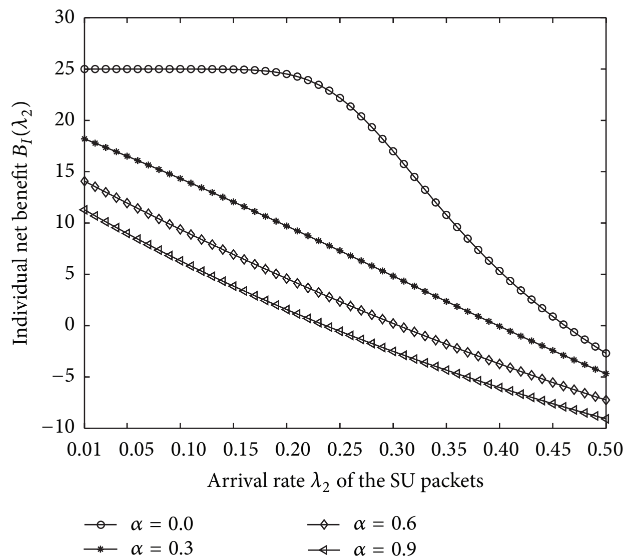

Figure 5: Dependency of the individual net benefit $B_I(\lambda_2)$ on λ_2.

As shown in Figure 5, the individual net benefit for an SU packet is monotonely decreasing as the arrival rate of the SU packets increases. We consider three cases for the optimal trial strategy of a single SU packet as follows:

(1) $B_I(0^+) \le 0$. In this case, even if no other SU packets join the system, the SU packet who tries to join the system will not get a positive benefit. So, the trial strategy with probability $q_e = 0$ is an optimal strategy and no other optimal strategy is possible.

(2) $B_I(\Lambda) \ge 0$. In this case, even if all the potential arrival SU packets try to join the system, all of them will get nonnegative benefits. So, the trial strategy with probability $q_e = 1$ is an optimal strategy and no other optimal strategy is possible.

(3) $B_I(0^+) > 0$ and $B_I(\Lambda) < 0$. In this case, if all the SU packets join the system with probability $q = 1$, the SU packet who tries to join the system will get a negative net benefit. So $q = 1$ is not an optimal strategy. On the other hand, if all the SU packets join the system with probability $q = 0$, the SU packet who tries to join the system will get a positive net benefit. So $q = 0$ is not an optimal strategy too. Therefore, there is an optimal trial probability $q_e = \lambda_e/\Lambda$, where λ_e can be obtained by solving the equation $B_I(\lambda_e) = 0$.

Thereupon, we discuss the socially optimal strategy [20]. The social net benefit $B_S(\lambda_2)$ is defined as follows:

$$B_S(\lambda_2) = \lambda_2(\varepsilon(\lambda_2)R - T). \tag{31}$$

We also explore the monotonic property of $B_S(\lambda_2)$ in (31) with numerical results. By applying the same parameters as used in Figure 5, we show how the social net benefit $B_S(\lambda_2)$ changes with respect to the arrival rate λ_2 of the SU packets for the different Adaptive Factors α in Figure 6.

From Figure 6, we conclude that as the arrival rate of the SU packets increases, the function of the social net benefit

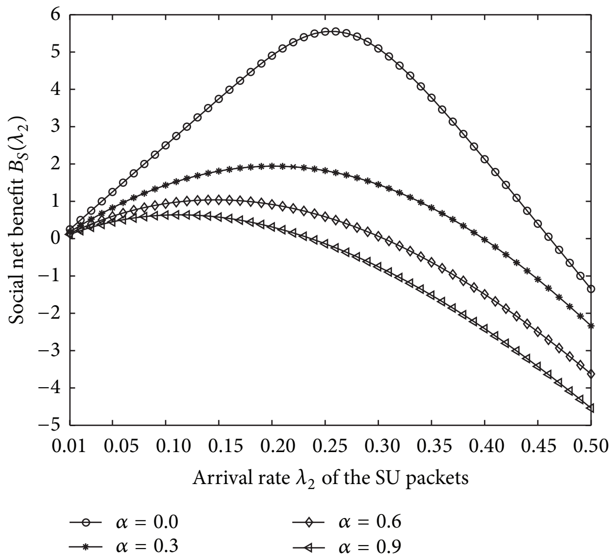

Figure 6: Function $B_S(\lambda_2)$ of the social net benefit.

Table 2: Numerical results for the individually and socially optimal strategies.

α	λ_e		q_e		λ^*	q^*
	min	max	min	max		
0.00	0.46	0.47	0.92	0.94	0.26	0.52
0.30	0.39	0.40	0.78	0.80	0.20	0.40
0.60	0.30	0.31	0.60	0.62	0.15	0.30
0.90	0.23	0.24	0.46	0.48	0.11	0.22

shows an upper convex behavior. Hence, the socially optimal trial rate λ^* can be given as follows:

$$\lambda^* = \arg\max_{0\le\lambda_2\le\Lambda} \{B_S(\lambda_2)\}. \tag{32}$$

Then we can get the socially optimal trial probability q^* as follows:

$$\begin{aligned} q^* &= 1, \quad \lambda^* \ge \Lambda, \\ q^* &= \frac{\lambda^*}{\Lambda}, \quad 0 < \lambda^* < \Lambda. \end{aligned} \tag{33}$$

By setting the potential arrival rate for the SU packets as $\Lambda = 0.5$ in Figures 5 and 6, for the different Adaptive Factors α, we obtain the value ranges of the individually optimal trial rate λ_e and the values of the socially optimal trail rate λ^*. We can also calculate the value ranges of the individually optimal trial probability q_e with $q_e = \lambda_e/\Lambda$ and the values of the socially optimal trail probability q^* with $q^* = \lambda^*/\Lambda$. We summarize these numerical results in Table 2.

In Table 2, the estimates for different numerical results are accurate to two decimal places.

From Table 2, we find that, for all the Adaptive Factors α, the socially optimal trail rate λ^* is smaller than the individually optimal trial rate λ_e, and the socially optimal trial probability q^* is smaller than the individually optimal trial probability q_e. These conclusions are consistent with the results given in [14, 20].

On the other hand, we also observe that as the Adaptive Factor α increases, both of the optimal trail rates and the optimal trial probabilities show a decreasing trend. The reason is that the larger the Adaptive Factor is, a newly arriving SU packet is less likely to be admitted by the system. In order to reduce the trial cost of the refused SU packets, the optimal trail rate and the optimal trial probability will be lower.

5.3. Pricing Mechanism. In order to oblige the single SU packet to adopt socially optimal strategy, we give a pricing mechanism by subtracting an extra fee f from the reward for the SU packet with successful transmission. When the extra fee f is imposed, the net benefit $B_T(\lambda_2)$ for an SU packet who tries to join the system can be given as follows:

$$B_T(\lambda_2) = \varepsilon(\lambda_2)(R - f) - T. \tag{34}$$

By setting $\lambda_2 = \lambda^*$ in (34), we can obtain the net benefit $B_T(\lambda^*)$ as follows:

$$B_T(\lambda^*) = \varepsilon(\lambda^*)(R - f) - T. \tag{35}$$

By solving $B_T(\lambda^*) = 0$, the extra fee f can be given as follows:

$$f = R - \frac{T}{\varepsilon(\lambda^*)}. \tag{36}$$

Specially, in the case of $\lambda^* = \Lambda$, the extra fee f will be equal to or less than $(R - T/\varepsilon(\Lambda))$.

For example, by using the numerical results given in Table 2, we can calculate the extra fee f with (36) for different Adaptive Factors α as follows: when the adaptive Factor α is 0, the extra fee f is 26.45; when the adaptive Factor α is 0.30, the extra fee f is 13.03; when the adaptive Factor α is 0.60, the extra fee f is 9.92; when the adaptive Factor α is 0.90, the extra fee f is 8.53.

Conclusively, as the Adaptive Factor α increases, the extra fee f shows a decreasing trend. The reason is that as the Adaptive Factor increases, the gap between the individually and the socially optimal trail rate shown in Table 2 will decrease, so the extra fee f will be reduced accordingly.

6. Conclusions

In order to reduce the SU packets' greater latency due to larger number of SU packets access to the system without any restrictions, in this paper, we proposed an adaptive admission control scheme for the SUs in cognitive radio networks. We introduced an Adaptive Factor into the admission control scheme so as to control the system access behavior of the SU packets adaptively. The system access probability of a newly arriving SU packet is determined by the Adaptive Factor and the total number of packets already in the system at the arriving instant. Based on the working principle of the adaptive admission control scheme and the priority of the PUs in cognitive radio networks, we built a discrete-time preemptive queueing model with priority and adjustable joining rate. We constructed a two-dimensional Markov chain and gave the transition probability matrix of the Markov chain to exactly analyze the queueing model. Accordingly, we derived the formulas for the different performance measures. Moreover, we gave the individually optimal strategy and the socially optimal strategy for the SU packets, and then a pricing mechanism was presented to coordinate the two optimal strategies.

In this paper, we assumed the interarrival and transmission times for the packets to follow geometric distributions [14]. As a future work, we will extend the system model by considering some nongeometric distributions for the interarrivals and the transmission times of the packets.

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Research Article

Congestion Service Facilities Location Problem with Promise of Response Time

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In many services, promise of specific response time is advertised as a commitment by the service providers for the customer satisfaction. Congestion on service facilities could delay the delivery of the services and hurts the overall satisfaction. In this paper, congestion service facilities location problem with promise of response time is studied, and a mixed integer nonlinear programming model is presented with budget constrained. The facilities are modeled as M/M/c queues. The decision variables of the model are the locations of the service facilities and the number of servers at each facility. The objective function is to maximize the demands served within specific response time promised by the service provider. To solve this problem, we propose an algorithm that combines greedy and genetic algorithms. In order to verify the proposed algorithm, a lot of computational experiments are tested. And the results demonstrate that response time has a significant impact on location decision.

1. Introduction

Facility location is a critical decision for a wide range of public and private firms. For example, in public sectors, location decisions for fire stations, ambulances, and other emergency service centers relate directly to the safety of citizens' lives and properties. In private sectors, industries need to locate warehouses, distribution centers, retail outlets, and so forth. These locating decisions concern both costs and competitiveness. In short, the success or failure of both public and private facilities partly depends on the locations chosen for these facilities. Facility location theory has been studied in various forms for hundreds of years. The study formally started by Weber to position a single warehouse [1]. Then, many traditional location models are proposed, including p-median problem [2], covering problem [3, 4], and p-center problem [2]. Following these early investigations, the studies of location theory become popular in recent years. Yang and Zhang [5] studied chain stores location problem with bounded linear consumption expansion function on paths. Yu et al. [6] expanded capacitated facility location problem with consideration of serve radius and economic benefit. Ma et al. [7] studied facility location problem, combined with the feature of demand. Liu et al. [8] presented a location model that assigns online demands to the capacitated regional warehouses. For a survey on models and methods, please see the book edited by Daskin [9] and the review done by ReVelle et al. [10].

The studies that mentioned above mainly concentrate on travel time, physical distance, or some other related travel cost. They assume that facilities are sufficiently large to meet any demand immediately. However, facilities could be congested frequently. To address this issue, congestion facility location problem begins to be considered since Maximum Expected Covering Location Problem (MEXCLP) by Daskin [11] who introduced this model in connection with the location of ambulances and assumed that the probability of each server being busy is predetermined. Then Marianov and Serra [12] introduced queueing theory to address the congestion facility location problems. They applied a constraint to ensure that there is at least a server available on demand with probability α. Huang et al. [13] studied the connections of network that are modeled as M/G/1 queues. Decision variables include selecting connections, assigning flows to the connections and sizing their capacities. To solve this model, they developed an algorithm based on Lagrangian relaxation.

Hu et al. [14] examined bi-objective model based on flow interception problem. The model is formulated from the view of M/M/c queuing system. Service quantity and quality are simultaneously considered as objectives. T. Drezner and Z. Drezner [15] studied the server distribution problem with the objective to minimize the combined travel time and waiting time at the facility for all customers. In their method, the distribution of demand among the facilities is governed by the gravity rule. Other references on the subject include [16–18].

In the literature described above, the models are developed with the objective of optimizing congestion indicators, such as waiting time [14, 15, 18], response time [17], work load [16], and relative congestion costs [13]. However, in many service sectors, congestion indicators are often specified by the service providers. For example, Marianov and Serra [12] considered that the waiting time-limit and queueing length-limit are predetermined.

In real life, response time is often a key competitive priority and represents the firm's commitment for the customer satisfaction. Therefore, specific response time guarantees are often advertised. For examples, Yonghe King, famous for soya bean milk and youtiao, reduces its promised response time from 30 to 20 minutes to enhance its competitiveness. Some take away restaurants often give a discount of food if it is not served within promised time. Jingdong on-line mall promises that customers are able to receive goods within 24 hours after the orders. Within a few years' competition, Jingdong has already taken a leading position in China. The similar situation is common in reality but has not been investigated thoroughly. To keep the promise, proper planning of the facilities is one of the most important actions for service providers. Ideally, the more the service resources they have, the shorter the response time the customer can get. However, due to budget limitation in real life, reasonable facilities location and servers allocation are needed. Therefore, developing a model that addresses this issue could be a main contribution for the current study.

In view of the above described, we consider that the response time-limit is predetermined, which is an important characteristic of our model that distinguishes from others. In this case the objective of model we propose is to maximize requests served within a constant response time in consideration of budget limitation. We concentrate on two parts which affect response time. (1) Travel time is determined by the distance between the service facility and demand node. (2) Sojourn time in facilities is affected by many issues, such as shortage in supply, service interruption (power cut or water cut), strike, and other accidents, but one of the most common is service congestion.

Many existing works [11–14, 16, 17] appoint the value of servers' number at each location in advance. Ideally, the more servers deployed at one facility location, the more effective it could be, but in reality the increasing number of servers also leads to more costs, which is often an important concern for decision makers. Another contribution of our work is that servers' number of each potential facility location could be determined endogenously by the proposed model.

Most closely related to the work in this paper is the study of Berman and Drezner [19]. They introduced an m servers allocation problem. In contrast to the research by Berman and Drezner, there are two differences between their model and ours. First, there are different objectives. In their model, the objective function is to minimize average response time, simple and explicitly convex in the number of servers. In view of the reality of response time-limit, however, our goal is to maximize requests served within a given response time. It is much more complicated and needs further analysis. Second, more generalized constraints. Berman and Drezner assumed that facility location incurs no cost and the total number of servers is known in advance. This is not the case in our problem. We propose constrained budget on the facility location and servers. When location cost is zero, our constraints are reduced to theirs, that is; our constraints are more generalized.

With the consideration of the limited budget, the difficulty of our problem is to solve two contradicting objectives. On the one hand, it is desirable to locate facilities as many as possible so that total travel time for customers could be reduced. On the other hand, with locating costs increasing and budget constraint, the available budget for servers is decreasing, which leads to lower service efficiency and longer waiting time. Therefore, when the demand is high and the travel distance is relatively short, the solution is expected to include fewer facility locations and more servers at each location. When travel time is relatively long, the solution is expected to include more locations and fewer servers at each location. Thus the key to maximize demands satisfied within promised response time lies in the balance of servers' costs and facilities location costs.

This paper is organized as follows. We formulate the problem and provide some analysis in Section 2. In Section 3, we present solution algorithms. Computational results and sensitivity analysis are included in Section 4. In the last section, we provide conclusions and suggestions for future research.

2. Formulation of the Model

In this section, to solve facility location problem with response time-limit, a mixed integer model is formulated. A service facility is modeled as an M/M/c queuing system. Customers arrive at the facility according to a Poisson process. Service time is exponentially distributed. Each facility has multiple servers and the number of servers is a decision variable in our model. It is assumed that services at different facilities are homogenous, which means that the efficiency of every server is the same. In real life, customers generally have no idea of congestion at each facility before they arrive at the service facility so they choose the closest facility. Location and servers both incur costs, and servers' costs are assumed to be linear with the number of servers.

Now, we denote index sets by the following:

(i) I: set of demand nodes, denoted by $i \in I$,

(ii) J: set of candidate locations, denoted by $j \in J$.

Next, we denote parameters by the following:

(i) f_i: demand at node i, yielding to random Poisson distribution with parameter λ_i,

(ii) w_j: sojourn time at facility j, including waiting time and service time,

(iii) t_{ij}: response time of facility j to demand node i,

(iv) T: response time that service facilities have promised,

(v) d_{ij}: travel time between facility j and demand node i,

(vi) B: budget limitation,

(vii) a_j: cost of candidate facility location j,

(viii) b: cost of unit server,

(ix) μ: average service rate per unit server.

The decision variables of the problem areThe decision variables of the problem are

(i) $y_j = \begin{cases} 1 & \text{if a facility is located at node } j\,(j \in J) \\ 0 & \text{otherwise} \end{cases}$

(ii) $x_{ij} = \begin{cases} 1 & \text{if facility } j \text{ serves demand node } i \\ 0 & \text{otherwise.} \end{cases}$

(iii) c_j: the number of servers at facility j.

Given the previous definitions, the location model can be formulated as follows:

$$\text{maximize } Z = E\left(\sum_{i \in I}\sum_{j \in J} f_i P\left(t_{ij} \leqslant T\right) x_{ij}\right), \tag{1}$$

subject to

$$x_{ij} \leqslant y_j, \quad \forall i \in I,\ j \in J \tag{2}$$

$$\sum_{j \in J} x_{ij} = 1, \quad \forall i \in I \tag{3}$$

$$\sum_{k \in J \mid d_{ik} \leqslant d_{ij}} x_{ik} \geqslant y_j, \quad \forall i \in I,\ j \in J \tag{4}$$

$$\sum_{j \in J} \left(a_j y_j + b \cdot c_j\right) \leqslant B \tag{5}$$

$$c_j \leqslant M y_j, \quad \forall j \in J \tag{6}$$

$$\sum_{i \in I} \lambda_i x_{ij} < c_j \mu, \quad \forall j \in J \tag{7}$$

$$x_{ij} \in \{0, 1\}, \quad \forall i \in I,\ j \in J \tag{8}$$

$$y_j \in \{0, 1\}, \quad \forall j \in J \tag{9}$$

$$c_j \in \mathbb{N}, \quad \forall j \in J \tag{10}$$

The model has the objective of maximizing expected demand rates that are served within promised time T. Constraints (2) ensure that facility j can serve demand node i only if facility j is open. Constraints (3) guarantee that demand node i is served by one and only one facility. Constraints (4) assure that each demand node is served by the closest open facility. Constraint (5) is a budget limitation, spending on location and servers. Parameter M, which is a very large number in constraints (6), can be chosen as $M = \lfloor B/b \rfloor$ ($\lfloor a \rfloor$ denotes the largest positive integral that is no more than a). Constraints (6) ensure that servers can be deployed at facility j only if facility j is open. Constraints (7) prevent infinite waiting time. Constraints (8) and (9) are binary constraints, and the last constraints preserve the positive integer restrictions on decision variables of the number of servers at each open facility.

Since service response time includes travel time and sojourn time $t_{ij} = d_{ij} + w_j$, the objective function Z can be transformed as follows:

$$\begin{aligned} Z &= E\left(\sum_{i \in I}\sum_{j \in J} f_i P\left(t_{ij} \leqslant T\right) x_{ij}\right) \\ &= E\left(\sum_{i \in I}\sum_{j \in J} f_i P\left(w_j \leqslant T - d_{ij}\right) x_{ij}\right), \end{aligned} \tag{11}$$

Let $T_{ij} = T - d_{ij}$, $Z = \sum_{i \in I} \sum_{j \in J} \lambda_i F(T_{ij}) x_{ij}$.

$F(T_{ij})$ is a probability distribution function, denoting the probability that demand node i's sojourn time at facility j is not larger than T_{ij}. So the response time restriction in the objective function is transformed to the sojourn time restriction.

For further analysis, we introduce some queuing theory relevant to our research.

In an M/M/c queuing system, the following parameters are always known:

μ: average service rate,

λ: average arrival rate,

c: the number of servers.

Let $\rho = \lambda/\mu$ and $\rho_c = \rho/c$. p_0, the probability that no demand sojourns in system, is denoted by

$$\begin{aligned} p_0 &= \left(\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!\left(1 - \rho_c\right)}\right)^{-1}, \\ p_c &= \frac{\rho^c}{c!} p_0, \end{aligned} \tag{12}$$

expressing the probability that the number of demands sojourning in system is c.

Then, $F(t)$, the probability that sojourn time in system is not larger than t, is shown as follows [20]:

when $c = 1$, $F(t) = 1 - e^{-(\mu - \lambda)t}$;

when $c > 1$,

$$F(t)=\begin{cases}1-\left(1+\dfrac{p_c\mu t}{1-\rho_c}\right)e^{-\mu t}, & \rho=c-1,\\ 1-\left(1+\dfrac{p_c}{(c-1-\rho)(1-\rho_c)}\right)e^{-\mu t}+\dfrac{p_c}{(c-1-\rho)(1-\rho_c)}e^{-c\mu(1-\rho_c)t}, & \rho\neq c-1.\end{cases} \tag{13}$$

As we know, distribution function of waiting time t is

$$W_q(t)=1-\frac{p_c}{1-\rho_c}e^{-\mu(c-\rho)t}, \tag{14}$$

and delay probability, the probability that at least one demand is in system for service, is

$$\mathrm{DT}(c)=1-W_q(0)=\frac{p_c}{1-\rho_c}. \tag{15}$$

Let

$$g(c)=\mathrm{DT}(c)\,e^{-\mu(c-\rho)t}. \tag{16}$$

Lemma 1. *$g(c)$ is a convex function.*

Proof. Second derivative of $g(c)$ is

$$g''(c)=\mathrm{DT}''(c)\,e^{-\mu(c-\rho)t}+\mu^2\mathrm{DT}(c)\,e^{-\mu(c-\rho)t}-2\mathrm{DT}'(c)\,e^{-\mu(c-\rho)t}. \tag{17}$$

As $\mathrm{DT}(c)$ is a nonincreasing function in c [21], $\mathrm{DT}'(c)\leqslant 0$ and $\mathrm{DT}''(c)\geqslant 0$. Then, $g''(c)\geqslant 0$ is resulted, and Lemma 1 can be proved. □

Theorem 2. *Sojourn time's distribution function $F(t)$ is a concave function in c.*

Proof. From Lemma 1, $g(c)$ is a convex function. Then, waiting time distribution function $W_q(t)=1-g(c)$ is a concave function in c; hence,

$$\frac{\partial^2 W_q(t)}{\partial c^2}\leqslant 0. \tag{18}$$

Sojourn time w is made up of two parts, waiting time w_q and service time s, $w=w_q+s$. Then $F(t)$ can be expressed as follows:

$$\begin{aligned}F(t)&=P\{w\leqslant t\}\\&=P\{w_q+s\leqslant t\}\\&=\int_0^t P\{w_q\leqslant t-t_s\}\,dP\{s\leqslant t_s\}.\end{aligned} \tag{19}$$

Since it is an M/M/c system, service time s yields to a negative exponential distribution, and $F(t)$ can be changed to

$$\begin{aligned}F(t)&=\int_0^t W_q(t-t_s)\,d\left(1-e^{-\mu t_s}\right)\\&=\int_0^t W_q(t-t_s)\,\mu e^{-\mu t_s}dt_s,\end{aligned} \tag{20}$$

$$\frac{\partial^2 F(t)}{\partial c^2}=\int_0^t\frac{\partial^2 W_q(t-t_s)}{\partial c^2}\mu e^{-\mu t_s}dt_s. \tag{21}$$

From

$$\frac{\partial^2 W_q(t-t_s)}{\partial c^2}\leqslant 0, \tag{22}$$

$$\frac{\partial^2 F(t)}{\partial c^2}\leqslant 0, \tag{23}$$

and Theorem 2 is resulted. □

According to the proof of Theorem 2, $F(t)$ owns the following two properties:

(1) $F(t)$ is a non-decreasing function in c,

(2) $F(t)$ is a concave function in c, that is to say, $F_{c+1}(t)-F_c(t)$ is a nonincreasing function in c.

The properties are similar to [19], and therefore we can get the optimal solution of servers allocation by greedy algorithm in the condition that location decisions have been made. Now, the problem is transformed to the model only with the location decision.

The model we have built is a combinatorial and nonlinear optimization problem. Without taking account of congestion situation, the waiting time is zero, and then the problem can be reduced to the maximum covering location problem. Thus the maximum covering location problem is a special case of this problem. It is known that the maximum covering location problem is NP-hard [9]. Clearly, this problem is also NP-hard. In order to solve large size problems, heuristics are considered our choice.

3. Algorithms

To solve the traditional location-allocation problem, various algorithms have been provided. Exact solution methods for location problem have been proposed and investigated in previous reports. Holmberg [22] embedded the dual ascent method in a branch-and-bound framework. Sasaki et al.

[23] proposed enumeration-based approach. de Camargo et al. [24] used benders decomposition method for the uncapacitated multiple allocation hub location problem.

Traditional location problems are NP-hard. Increase in the polynomial order of the problem leads to an exponential explosion in the computation time, and exact methods can only solve small instances of the presented problem. Thus, most of the literature applies efficient heuristics to solve the problem, which includes local search, greedy heuristic, simulate annealing, tabu search and Lagrangian relaxation, and so forth. Hybrid methods have also been studied recently [18, 19, 25]. For a more complete review of these algorithms, the reader can be referred to [26].

Algorithms for solving the congestion servers location-allocation problem have been discussed by Berman and Drezner [19] and Aboolian et al. [18]. Berman and Drezner developed three heuristic approaches, namely tabu search, simulated annealing and genetic algorithms. Compared with the precise results of total enumeration, the three heuristics are showed to perform very well, especially the genetic algorithm, which obtains the results with the smallest gap and in the shortest computation time. Aboolian et al. presented descent algorithm and genetic algorithm. Large-scale numerical examples illustrated that genetic algorithm performs more efficiently than descent algorithm. These works have proved that genetic algorithm is more efficient to solve congestion servers location-allocation problem than other heuristics presented in [18, 19].

This problem can be decomposed in two smaller problems: at a higher level, named as master problem (MP), the location decisions are made; while at an inferior level, known as subproblem (SP), the determination of the servers' number at each location is done. The MP is a 0-1 integer programming problem, while the SP is an allocation problem. We consider a hybrid algorithm of combining genetic algorithm with greedy algorithm to solve the model. Genetic algorithm is applied to solve MP, and precise solution of SP can be obtained by greedy algorithm. The results of our algorithm for small size problems are compared with the results obtained from numerical algorithm, in order to verify the performance of the algorithm. Although there are alternative algorithms in the literature [18, 19], our choice is driven by the following reasons. Firstly, genetic algorithm is proved to be more efficient to solve congestion facility location problem than other heuristics [18, 19]. Secondly, due to the properties we proved in Section 2, results of servers allocation are optimal by greedy algorithm.

3.1. Precise Algorithm of Servers Allocation. Greedy algorithms have been comprehensively used in location problems [27–30]. In these works, greedy algorithms are developed to determine location decision and the solutions are approximations. However, different from other literature, greedy algorithm in this paper is used to solve servers allocation problem. Due to sojourn time distribution function's special properties, we can get precise results of servers' number at each facility location provided that location decision has been made. Assuming that location decision is S, now compute the number of servers at each location with the budget constraint. Procedure is as follows.

(1) For each node j $(j \in S)$, compute average arrival rate $\overline{\lambda}_j$. $\overline{\lambda}_j = \sum_{i \in L_j(S)} \lambda_i$, where $L_j(S)$ is the set of demand nodes from which facility $j \in S$ is the closest in S.

(2) For each node j $(j \in S)$, find the minimum number of servers necessary to serve the customer flows. This minimum number is

$$k_j = \left\lfloor \frac{\overline{\lambda}_j}{\mu} \right\rfloor, \tag{24}$$

in which k_j is the largest integral that is no more than $\overline{\lambda}_j/\mu$.

(3) If

$$\sum_{j \in S} \left(b \cdot k_j + a_j \right) > B, \tag{25}$$

in which B is the given budget, b is unit server's costs and a_j is the location costs of facility j, there is no feasible solution, and the computing should stop.

If

$$B - b < \sum_{j \in S} \left(b \cdot k_j + a_j \right) \leqslant B, \tag{26}$$

the number of servers is k_j, and stop.

If

$$\sum_{j \in S} \left(b \cdot k_j + a_j \right) \leq B - b, \tag{27}$$

go to step 4 to allocate the rest servers.

(4) Compute the objective increase through increasing the number of servers by one at facility j; that is,

$$\Delta_j = \sum_{i \in L_j(S)} \lambda_i \left(F_{k_j+1}(t_i) - F_{k_j}(t_i) \right), \tag{28}$$

in which $L_j(S)$ expresses the set of demand nodes served by facility j. Compute Δ_j for each $j \in S$, choose the maximum Δ_{j^*} $(j^* \in S)$, and then let $k_{j^*} = k_{j^*} + 1$. Repeat Procedure (4) until the rest of budget is not enough to pay for another server.

3.2. Genetic Algorithm. Genetic algorithm was first applied to location-allocation problems by Hosage and Goodchild in 1986 [31]. It has been widely applied to solve location problems. Correa et al. [32] proposed a genetic algorithm for solving a capacitated p-median problem. Jaramillo et al. [33] introduced genetic algorithm for solving uncapacitated and capacitated fixed charge problems, the maximum covering problem, and competitive location models. Balakrishnan et al. [34] presented a hybrid approach of combining genetic

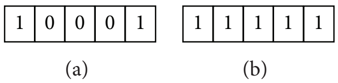

Figure 1: Binary representation of an individual's chromosome.

algorithm with dynamic programming for the dynamic facility location problem. Genetic algorithm is also proved to be efficient to solve congestion facility location problem [18, 19, 35].

In this section, we propose a genetic algorithm to find facility locations. In the proposed method, good chromosomes are identified through crossover, mutation and selection operations. We apply greedy algorithm to decide the number of servers at each location. Crossover and mutation we use are similar as the methods described in the literature mentioned above, which is not the main focus of our work. Only the major operations that we have made are described in this section.

3.2.1. Code. A candidate solution is represented as follows: the representation scheme developed is a $|J|$-bit binary string as the chromosome structure, where $|J|$ is the number of potential facility locations. A value of 1 for the jth bit implies that a facility is located at the jth location. For instance, considering a problem with 5 potential facility locations, the binary representation of an individual's chromosome is illustrated in Figure 1. Figure 1(a) shows the situation when only two facilities are located in potential locations one and five. Figure 1(b) illustrates the situation when facilities are located at all of the potential facility locations.

3.2.2. Crossover, Selection, and Mutation Operations. Randomly select two members from the parental population and merge them to produce offspring. If the offspring is better than the worst parental population member and different from its parent, replace the worst member by the offspring. Repeat above crossover $|J|$ times. Select $|J|$ members from parent and child populations with best objectives. A random mutation pattern is generated for each chromosome. For each gene of the chromosome, a uniform random number between 0 and 1 is generated. If this number is less than a given number p_m, the gene mutates from 0 to 1 or from 1 to 0. If the individual after mutation is not satisfied with budget constraint, the mutation is considered as invalid.

3.2.3. Feasible Operation. We define a candidate solution (individual) for our problem as feasible if the total cost including location and minimum servers costs are no more than the given budget. As initial or crossover individual may produce infeasible solution, for crowds diversity and avoiding local optimum solutions, randomly select one gene, mutating from 1 to 0 until satisfying budget constraint.

The algorithm process is shown in Figure 2.

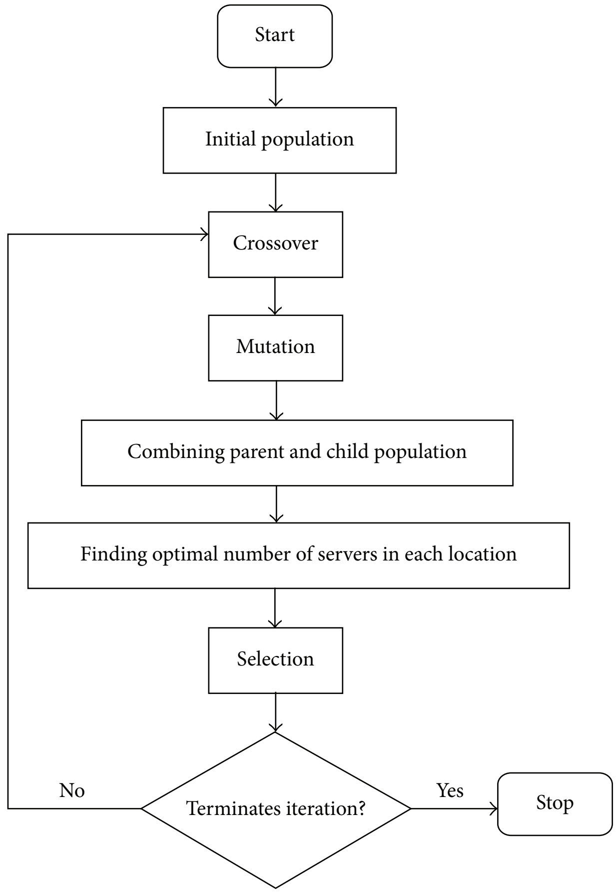

Figure 2: Process flowchart.

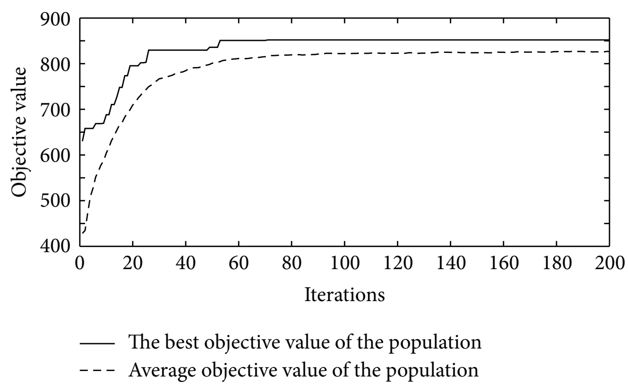

Figure 3: The convergence of genetic algorithm.

4. Example and Computational Experience

The algorithm is coded in Matlab 7.0, and computational experiments are conducted on a PC with 2.2 GHz Intel Pentium Dual E2200 and 2.00 GB RAM. Demand nodes are randomly generated on a [0, 50][0, 50] plane. Candidate facility locations are defined an dconsidered as demand nodes. Each location cost and average demand rate are both random values which vary between 20 and 50. Unit server cost $b = 8$, and average service rate of unit server $\mu = 8$. We set the parameters of genetic algorithm as follows: population

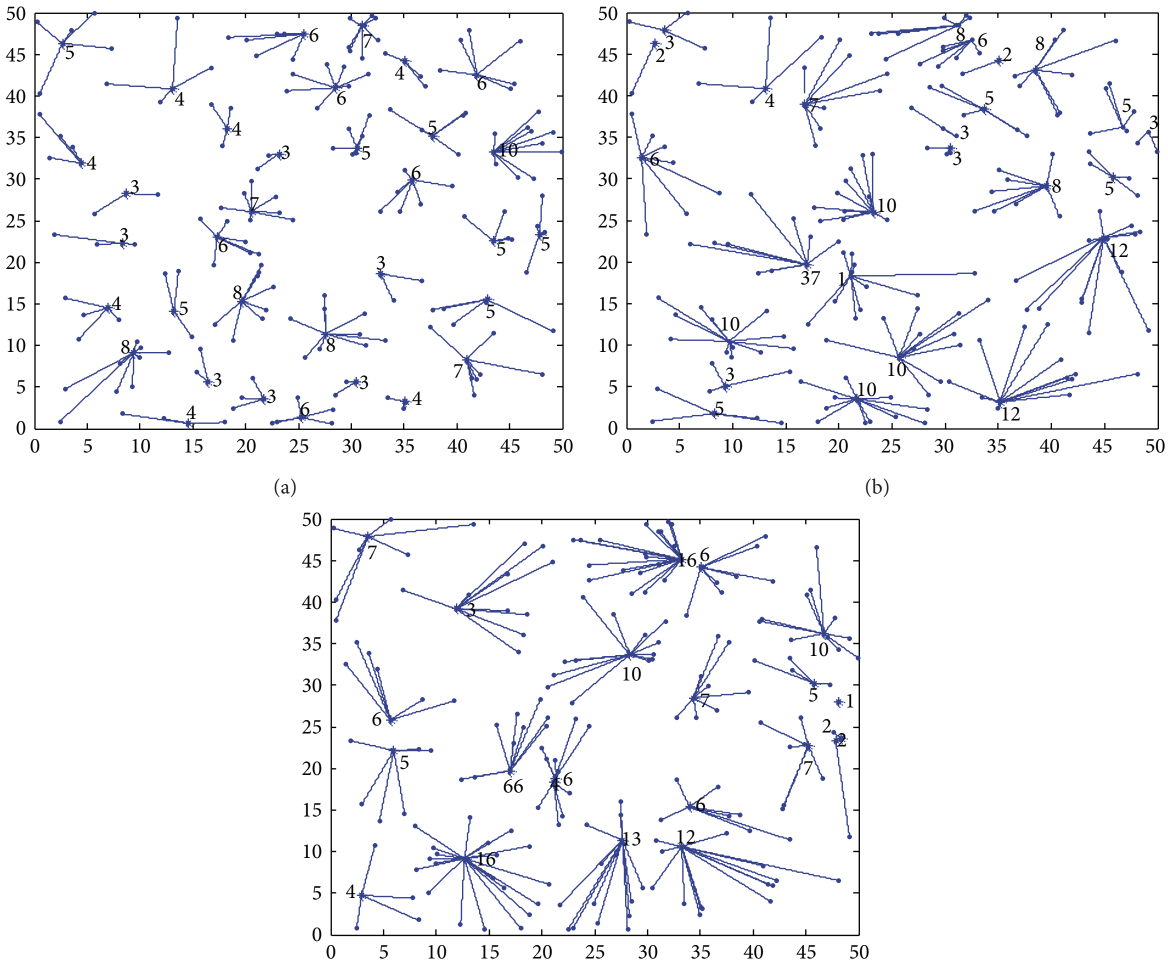

FIGURE 4: Optimization results with different T. (a) $T = 5$, (b) $T = 15$, and (c) $T = 20$.

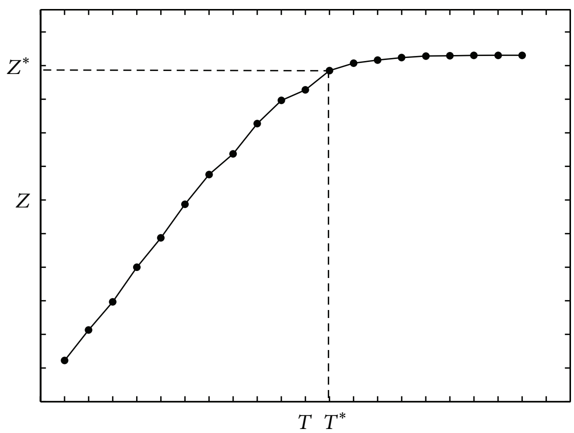

FIGURE 5: Sensitivity analysis with respect to promised response time T.

size is equal to node, maximum number of generation is equal to 200, and the mutation probability $p_m = 0.1$. Figure 3 shows the convergence of this algorithm (the number of demand nodes $|I| = 50$, $B = 900$, $T = 5$).

Table 1 compares the results of genetic and exact algorithm. Optimal solutions are obtained by numerical algorithm but the computation time increases exponentially with the number of facilities. Considering computation time, the numbers of the facility locations in the test problems are set to be a value no more than 6; that is,

$$\frac{\left(B - b\sum_{i\in I}\lambda_i/u\right)}{20} < 7. \tag{29}$$

seven test problems of varying size are constructed. For each test problem, 50 experiments are randomly generated, leading to a total of 350 experiments. The gap is measured by $(Z^* - Z)/Z^*$, where Z^* is the optimum value and Z is the objective value by our algorithm. From results, the range of gaps is from 0% to 9.65%, and the average gap shifts from 0 to 2.09%. In these experiments, the performance is quiet good and genetic algorithm finds the optimal solution at least 50%.

Then, we set budget $B = 2500$, and the number of demand points $|I| = 200$. Figures 4(a), 4(b) and 4(c) respectively show the facility locations, demand allocation and servers distribution under different promised response time $T = 5, 15$ and 20. The number beside location point means the number

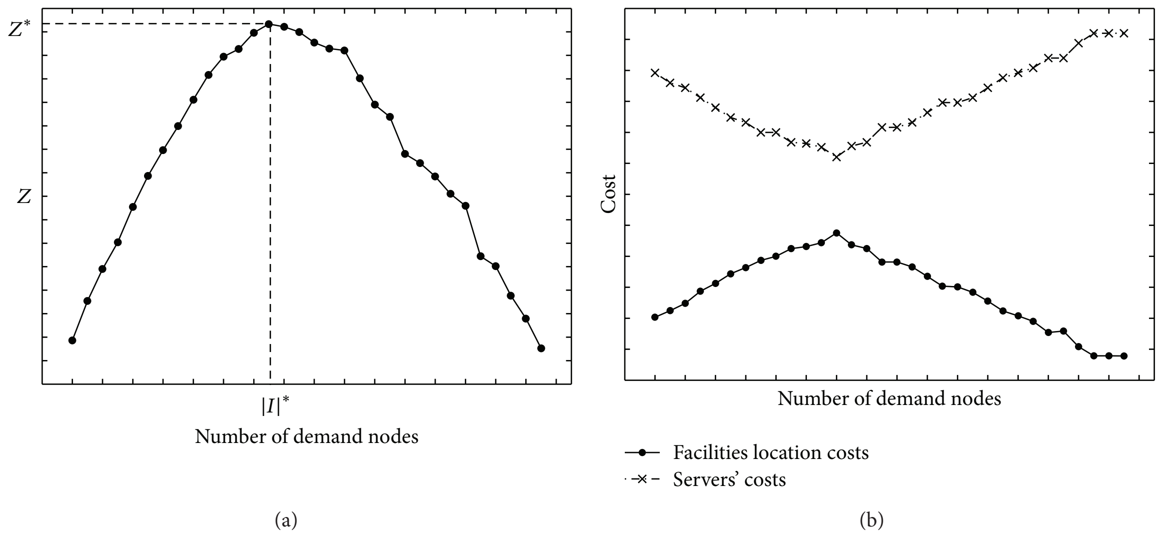

Figure 6: Sensitivity analysis with respect to number of demand nodes.

Table 1: Results of genetic and exact algorithms.

Node	Budget	Genetic algorithm					Numerical algorithm time (S)
		Maximum gap	Minimum gap	Average gap	Optimum solution	Time (S)	
30	350	0	0	0	100%	14.82	45.3
40	400	3.87%	0	1.61%	50%	24.72	78.4
50	450	9.65%	0	2.09%	50%	30.64	523.5
55	480	8.17%	0	1.41%	60%	31.12	335.5
60	500	5.21%	0	0.72%	80%	35.03	487.2
70	550	3.35%	0	1.12%	50%	40.8	1537.6
80	650	6.94%	0	1.23%	70%	55.78	7643.3

of servers at the facility. When $T = 5$, it is desirable to locate more facilities and distribute less servers at each facility. In Figure 4(b), when $T = 15$, we find that the number of facilities is equal to 34, servers at each location are no more than 8, and the decentralized service makes travel time reduced. With the increase of T, the number of facilities is smaller and servers are pooling to reduce service time. Figure 4 illustrates that the promised response time has a strong impact on the location decisions.

In the second set of experiments, we are interested in testing the effect of different parameters. The parameters studied are originally set as follows: promised response time $T = 5$, number of demand nodes $|I| = 30$ and budget $B = 600$. We use three instances for above three parameters sensitivity analysis. For each instance, we vary one parameter under study while keeping two other parameters as original values. The results are presented in Figures 5–7.

Figure 5 presents objective value analysis with respect to promised response time. We find that as promised response time T increases, the objective value, the number of customers served within T, increases as well. However, the objective value Z increases little after the promised response time threshold. Figure 5 shows that when $T > T^*$, the number of customers served within T barely changes. Therefore, through Figure 5 we characterize the reasonable range of promised response time for the service provider, which is no more than T^*. And there is no need to make a longer promise response time decision. The reasons are as follows: (i) longer promised response time ($T > T^*$) has little impact on quantity of customers served within T. (ii) longer promised response time makes service facilities less attractive to customers and weakens their competitiveness.

In Figure 6, we show the sensitivity analysis with respect to the number of demand nodes varying. In Figure 6(a), customers served within promised response time increase initially and then decrease with demand nodes increase. When demand is small, there are enough facilities and servers for service requirements. Although increasing demand leads to reduction of percentage of demand served within T, the impact of percentage reduction is smaller than the increase of demand. Therefore, we can see the result in Figure 6(a) that the objective value is increasing with demand increase before point $|I| = |I|^*$. Z is maximized at point $|I| = |I|^*$. After that point, the percentage of demand served within T drops sharply as $|I|$ increases. Therefore, increasing demand leads to lower demand served within promised time instead. Figure 6(a) illustrates that a service provider may choose service area to provide high-quality service to a limited number

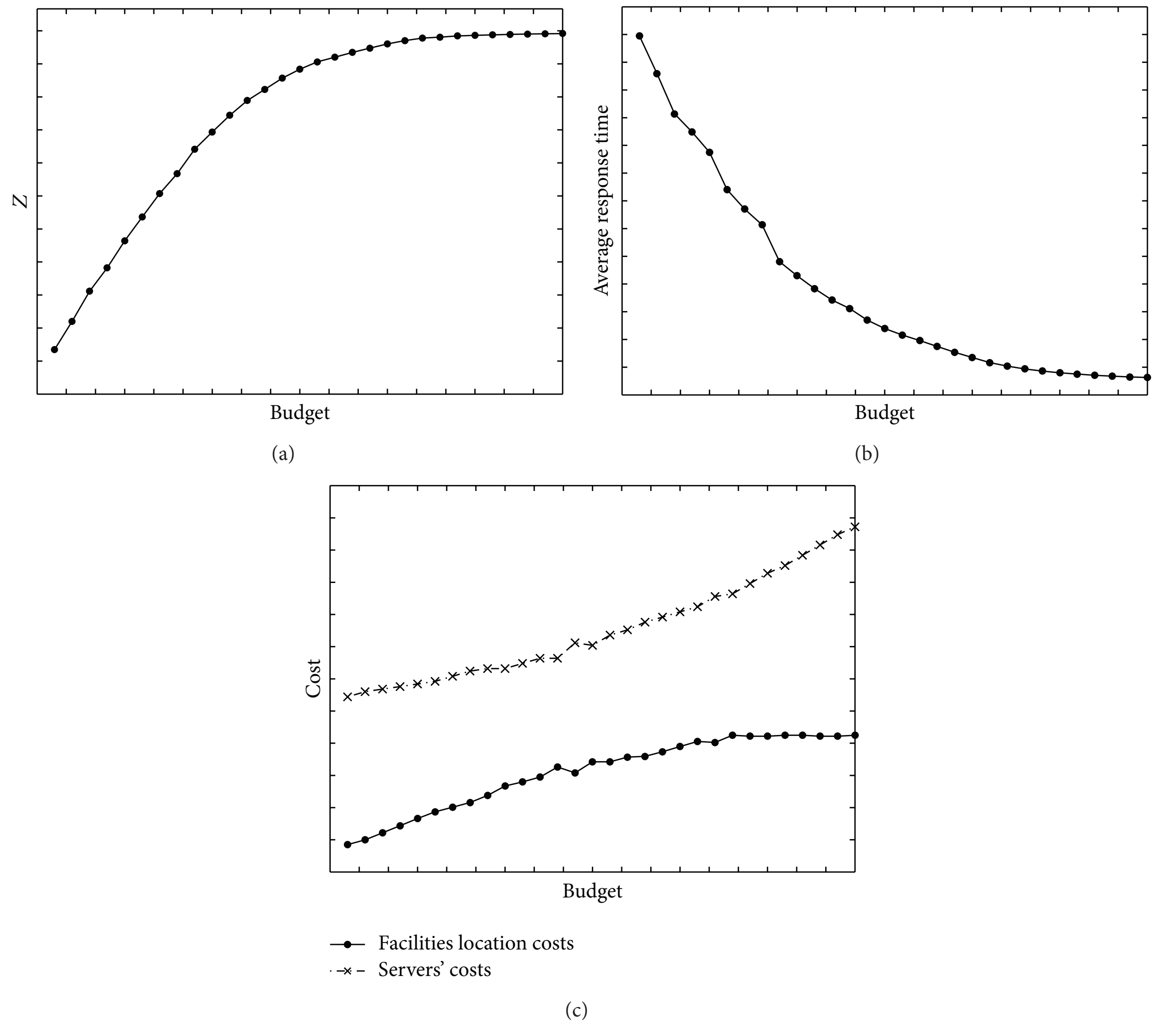

Figure 7: Sensitivity analysis with respect to budget.

of customers, or it may provide lower-quality service to a large number of customers. Therefore, for the service provider, more demand may not be better, which is consistent with the reality. In real life, take away service, such as KFC Delivery, and McDelivery, provides service in designated areas, not all the neighborhoods or streets. The above situation is also our further research on servers location and service areas selection.

Figure 6(b) demonstrates the trade-off between facilities location costs and servers' costs with the increasing demand when the budget is given. From Figure 6(b), as demand expanding, a service provider should increase investment in the construction of new facilities and decrease investment in service efficiency. As demand continues to increase, capital investment in facilities and servers is just converse, from which we can estimate that the demand is too large for the servers to serve and the average waiting time must be longer than the average travel time. Therefore, the action of increasing the number of servers should be taken. The reasons that an increase in the demand results in a decrease in the facility location costs, we think, are as follows: (i) As the budget constraint, increasing servers costs must lead to a decrease in location costs. (ii) Decrease locations will result in the number of servers increase at per facility, and pooling of servers can also decrease waiting time. Consequently, if demand is too small or too large, then we can predict that facilities investment should be small and funds should be focused on hiring more service personnel and purchasing or leasing more equipment for service.

Figure 7 illustrates the variation of satisfied customers, average response time and costs of location and servers with different budget. From Figures 7(a) and 7(b), with the budget increasing, we observe that the increment of Z and decrement of average response time are reduced gradually. We estimate that the objective function and the average response time may have concave-convex quality of the budget. Figure 7(c) demonstrates the investment in facility location and servers with different budgets. Although the investment in locations and servers both increase, their increments are different. The investment in facility location is approximately concave in the budget, whereas the investment in servers is convex. That is, the increment of facility location costs is decreasing, and the increment of serves costs is increasing with the budget increase. There exists a budget threshold point B^*, on which the increments of facility location and servers are equal. We can estimate that when the budget is small, the increment of location costs is much more than the increase of servers' costs. At this moment, if additional budget is allocated, the management would devote most of attention and budget to increasing distribution of firms. Gradually, with the budget increase, the gap between the increment of location costs and servers' costs becomes smaller and smaller. When budget $> B^*$, increments would be inverse. That means that the investment in service efficiency begins to pay more attention after necessary facilities and service resources construction, if the budget is still abundant.

5. Conclusions and Future Research

In this paper, a model of servers location-allocation problem with promise of response time has been presented. The objective of the model is to maximize the demands satisfied in promised response time, through finding proper locations and according servers. Because of budget constraint, balancing of location costs and servers' costs could decrease response time. System waiting time distribution function is proved to be concave in the number of servers. According to this property, precise algorithm for allocation of servers can be obtained by greedy algorithm. We propose a hybrid algorithm that combines greedy and genetic algorithms, which is proved to be fine compared with exact results by lots of computational experiments.

Finally, the future research could be extended as follows.

(1) More efficient algorithms need to be studied deeply. For example, hybrid methods (also called matheuristics) may be explored as efficient methods for congestion facility location problem.

(2) General service time distribution could be studied instead of exponential distribution in this paper.

(3) The model can be reformulated with incorporation of service response time restrictions, which can be stated as the probability of waiting for no more than x people is greater than α.

(4) All facilities are assumed to be independent in this paper. In future research, cooperative service could be considered.

(5) Customers are assumed to visit the closest open facility in this paper. In future research, the possibility of dynamically assigning a customer to a facility depending on current facility loadings could be studied with an appropriate dispatching rule.

Acknowledgments

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Research Article

Order Allocation Research of Logistics Service Supply Chain with Mass Customization Logistics Service

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This paper studies the order allocation between a logistics service integrator (LSI) and multiple functional logistics service providers (FLSPs) with MCLS. To maximize the satisfaction of FLSPs, minimize the total cost of LSI, and maximize the customized degree, this paper establishes a multiobjective order allocation model of LSSC that is constrained by meeting customer demand, customer order decoupling point, and order difference tolerance coefficient. Numerical analysis is performed with Lingo 12 software. This paper also discusses the influences of scale effect coefficient, order difference tolerance coefficient, and relationship cost coefficient on the comprehensive order allocation performance of the LSSC. Results show that LSI prefers FLSPs with better scale effect coefficients and does not need to set an extremely high order difference tolerance coefficient. Similarly, setting a high relationship cost coefficient does not necessarily correspond to better results. For FLSPs, the continuous improvement of large-scale operational capacity is required. When the comprehensive order allocation performance of the LSSC is high, the LSI should offer cost compensation to improve the satisfaction of the LSSC.

1. Introduction

Outsourcing service, such as information technology services, financial services, and logistics services, has become increasingly popular in recent years due to the fact it appears to be more profitable. Many service integrators and functional service providers develop a long-term service purchasing/supplier relationships to satisfy customers demand and obtain more profit with integrated services and then form a service supply chains (SSC). Specially, in a logistics service supply chain (LSSC), a logistics service integrator (LSI) provides customized logistics services by integrating the service capacities of multiple functional logistics service providers (FLSPs) [1]. For instance, Baogong Logistics Company, one of the largest LSI in China, integrates more than 500 storage companies, more than 1,200 highway transportation companies, and over 500 manual loading and unloading companies as its FLSPs and then accomplishes the integrated logistics services for its customers such as Procter & Gamble, and Philips.

In recent years, customer requirements for specialized and customized logistics services have increased. Many logistics enterprises have also begun considering a change in logistics service mode. These logistics enterprises have attempted to provide mass customization logistics service (MCLS) instead of only mass logistics service [2]. MCLS is one of the most important trends in logistics service. The capability of offering MCLS to customers has become the key to improving market competitiveness. In the MCLS environment, many logistics enterprises spontaneously form a logistics service supply chain (LSSC) through unions and integrations to meet customized demand and offer large-scale services [1, 3]. As the core enterprise of LSSC, the logistics service integrator (LSI) integrates the advantages of other functional logistics service providers (FLSPs) in different processes and different functions and delivers the integrated service to customer. For instance, on 11 November, 2011, more than 2.67 million packages were delivered all over the country by the Yuantong Express Company in China; this number is more than four times the number of packages delivered on the same day in 2010. These 2.67 million packages were delivered not only within 3 days to 5 days to customers in 31 provinces in the country but also under customized logistics service. This process brought huge pressure to the Yuantong Express Company. To complete the courier service, Yuantong Express Company cooperated with other FLSPs (e.g., SF Express and ZTO Express) by outsourcing some customer orders.

Under the MCLS environment, the LSI integrates different customer orders and allocates such orders to multiple FLSPs. In contrast to existing order allocation methods, the LSI needs to consider the influence of customer order decoupling points (CODPs). Each customer order has a unique CODP; therefore, when the LSI integrates multiple customer orders, the cost scale effect caused by large-scale services and the different CODPs of these orders should be considered. We also need to consider the minimum total cost of LSI and FLSP satisfaction in order allocation processes. Hence, a multiobjective decision-making problem exists. This problem involves addressing the diversity of customer orders, minimizing the total cost of the LSI, and considering the optimal satisfaction of FLSPs. This paper studies the order allocation problem (OAP) between LSI and multi-FLSPs, considers the characteristics of the MCLS by modeling and analysis, and explores the influence of related factors on the comprehensive performance of order allocation. Moreover, several management insights are provided.

This paper is organized as follows. Section 2 presents the literature review, where existing research on mass customization and order allocation are systematically summarized. Section 3 demonstrates the model building. An order allocation model of LSSC under the MCLS environment is established. Section 4 characterizes the model solution. A method for solving the multiobjective programming model is provided. Section 5 presents the numerical analysis. The influence of related factors on the comprehensive performance of order allocation is discussed in this section. Section 6 provides the main conclusions, management insights, and implications for LSI and FLSPs. Section 7 presents the limitations of the study and discusses suggestions for further research in this field.

2. Literature Review

This paper involves two main research areas: mass customization and order allocation. The literature review focuses on these two areas and elaborates their research progress and inadequacies.

2.1. Mass Customization. Studies on mass customization mainly include mass customization production and mass customization service. Mass customization production is primarily for manufacturing industry and mass customization service is mainly for service industry.

2.1.1. Mass Customization Production. Most mass customization production studies have focused on the supply chain of the manufacturing industry [4, 5]. The main content of these studies is centered on the design of a delay system. Reference [6] proposed five kinds of postponement strategy: tags delay, packaging delay, assembly delay, manufacturing delay, and time delay. Thereafter, scholars studied the problem of choosing the appropriate postponement strategy in terms of the applicable conditions of time delay and cost delay in manufacturing [7]. Moreover, several scholars examined the influence of implementing postponement strategies on the cost of delay system [8, 9].

Studies on the decision-making problem of CODP have increased in recent years. In enterprise manufacturing activities, CODP is the transition point from make-to-stock to make-to-order [10]. In our paper, CODP indicates the procedure to start customization service. The positioning model of CODP has progressed from single CODP to multi-CODP [11, 12] and from static CODP to dynamic CODP [13, 14]. Given the increasing number of studies on supply chains, several scholars have investigated that the CODP problem undersupply chain environments and achieved fruitful research results. For example, reference [15] studied the influence of CODP selection on inventory costs when the lead time and universal module average holding cost are the same regardless of CODP position, which considers inventory and other costs in the model. Reference [16] examined the best CODP selection model around the supply chain environment.

2.1.2. Mass Customization Service. The increasing adoption of the service supply chain in recent years has attracted scholars in studying the CODP problem of the service industry under mass customization environments. These previous studies mainly examined how a single service enterprise positions the CODP in service processes while considering mass customization costs and ignoring inventory costs. Moreover, there is a special issue on service optimization and control in mathematical problems in engineering which includes a hot topic of mass customization service. There are also some scholars dedicating research perspective to specific industries, such as logistics services industry. References [17, 18] explored the scheduling model of LSSC in the context of MCLS.

2.2. Order Allocation on Supply Chain. To date, most OAPs are solved by using multiobjective linear programming or nonlinear programming methods, which spread around the produce supply chain. References [19, 20] studied the application of genetic algorithm for optimizing order allocation. Current studies on customer order allocation mainly consider the service level and minimizing procurement costs [13, 19, 21]. And most studies have focused on the manufacturing industry supply chain. Several scholars have investigated the integrated multiobjective decision-making method, which considers both supplier selection and OAPs [22]. In addition to the solving method, several scholars have focused on the order allocation model, such as research on the order allocation model of supply chains with FLSP load cases [17, 18, 23].

The growth of LSSC has directed considerable research attention on its order allocation. Several studies have examined the OAP in two-echelon LSSC [12, 19]. Reference [1] investigated an emergency order allocation model based on multiproviders in two-echelon LSSC. Considering the multi-echelon nature of LSSC, references [17, 18] conducted a study on the order allocation of three-echelon LSSC.

The literature review indicates that the MCLS environment has been left unexamined even though scholars have focused on the OAP of LSSC. The MCLS needs to consider the cost advantages of mass service and customized special requirements; therefore, the order allocation in this case is clearly distinct from general cases. Furthermore, cost control is not the most important decision-making objective when we consider the customization requirements. Thus, this paper focuses on the OAP around the MCLS environment by modeling its characteristics and exploring the influence of related factors on the comprehensive performance of order allocation. Furthermore, several management insights are also provided.

3. Model Building

Section 3 introduces the model assumptions and variables. To maximize the satisfaction of FLSPs, minimize the total cost of the LSI, and maximize the customized degree of customers, we established a multiobjective order allocation model of LSSC that is constrained by meeting the demand, CODP, and order difference tolerance coefficient (Section 3.2).

3.1. Model Description. We considered a two-echelon LSSC, which consists of one LSI and many FLSPs. After receiving orders from multiple customers, the LSI analyzes the customized and universal service requirements of these customer orders and allocates them to more than one FLSP. Every logistic service for customer orders comprises multiple service processes. Each process needs one kind of service that can be provided by many cooperative FLSPs. The service capability of each FLSP is possibly different. We assumed that $[a_{ijk}, b_{ijk}]$ denotes the service ability interval provided by FLSP ith for customer jth in the procedure kth; x_{ijk} stands for the jth customer's order assigned to the ith FLSP in the kth procedure.

Table 1 shows the notations of the model. The other assumptions of the model are as follows.

(1) Each FLSP i $(i = 1, 2, \ldots, n)$ can provide mass service and customization service for the LSI. The total service procedures of customer j are L_j $(j = 1, 2, \ldots, M)$, and the order demand is D_j.

(2) K_j represents the procedure to start the customization service of the jth customer; this service is not necessarily the same for all customers. The LSI needs to consider the K_j of all customers and set a common CODP, which is signified by K' for all customers. K' is mass service, and K' is customization service. K' should be set before K_0 $(K_0 = \min\{K_1, K_2, \ldots, K_M\})$ because the CODP must be set before all K_j. Given the characteristics of the MCLS, Procedure 1 must be mass service; hence, $2 \le K' \le K_0$).

(3) Given the importance of different customizations for each customer, the weight of the jth customer is β_j $(\sum_{j=1}^{M} \beta_j = 1)$.

(4) Customer orders are assigned to FLSPs through the LSI. FLSPs will accomplish the order assigned by the LSI. No game exists between LSI and FLSPs.

3.2. Model Building. Previous order allocation models are mostly based on the costs and orders service level [13, 16, 24]. While conducting the order allocation of the LSSC under the MCLS environment, the LSI is required to consider multiobjectives, such as the total cost of the LSI, the total satisfaction of FLSPs, and the satisfaction of customized demand. To maximize the satisfaction of FLSPs, minimize the total cost of the LSI, and maximize the customization degree for customer demand, this paper establishes an order allocation model of LSSC that is constrained by meeting the demand, CODP, and order difference tolerance coefficient. The greatest difference between the previous model and ours is that we considered the impact of changing CODP on order allocation result, so that it can reflects the order allocation characteristics on MCLS better. The specific modeling procedure is described in the following sections. Sections 3.2.1 to 3.2.4 discuss the building processes of several aspects, namely, the objective function of FLSP satisfaction, LSI total cost, the objective function of customized degree for customer demand, and the three constraints, respectively.

3.2.1. Maximizing the Satisfaction of FLSPs. Maximizing the satisfaction of FLSPs is considered the first process by the LSI in order allocation. Based on the satisfaction function of reference [1], the satisfaction of the ith FLSP for the jth customer in the kth procedure is as follows:

$$\rho_{ijk} = \begin{cases} \dfrac{b_{ijk}}{x_{ijk}}, & x_{ijk} > b_{ijk}, \\ \rho_{ijk}^{0} + \left(1 - \rho_{ijk}^{0}\right) * \dfrac{x_{ijk} - a_{ijk}}{b_{ijk} - a_{ijk}}, & a_{ijk} \le x_{ijk} \le b_{ijk}, \\ \dfrac{x_{ijk}}{a_{ijk}} * \rho_{ijk}^{0}, & 0 \le x \le a_{ijk}, \end{cases} \quad (1)$$

where ρ_{ijk}^{0} is the initial satisfaction of the ith FLSP in the kth procedure when the order value is equal to the minimum service capacity (1). Each FLSP has different service capacity intervals while conducting mass service and customization service, which can be expressed as $k \le K'$, $[a_{ijk}, b_{ijk}] = [a_{ijk1}, b_{ijk1}]$; $k > K'$, $[a_{ijk}, b_{ijk}] = [a_{ijk2}, b_{ijk2}]$. In the same manner, each FLSP has different initial satisfaction on mass service and customization service. This initial satisfaction can be expressed as $k \le K'$, $[a_{ijk}, b_{ijk}] = [a_{ijk1}, b_{ijk1}]$; $k > K'$, $[a_{ijk}, b_{ijk}] = [a_{ijk2}, b_{ijk2}]$. When $0 \le x \le a_{ijk}$, the satisfaction of the FLSP increases with increasing order value. Similarly, when $a_{ijk} \le x_{ijk} \le b_{ijk}$, the satisfaction of the FLSP increases

Table 1: Model notations.

Notations	Description
$[a_{ijk}, b_{ijk}]$	The service ability interval of the ith FLSP for jth customer in the kth procedure
$[a_{ijk1}, b_{ijk1}]$	The value of $[a_{ijk}, b_{ijk}]$ when the kth procedure is mass service
$[a_{ijk2}, b_{ijk2}]$	The value of $[a_{ijk}, b_{ijk}]$ when the kth procedure is customization service
x_{ijk}	The jth customer's order assigned to the ith FLSP in the kth procedure
τ	Scale effect coefficient
D_j	The demand of the jth customer
L_j	The total service procedures of the jth customer
β_j	The weight of the jth customer
ρ_{ijk}	The satisfaction of the ith FLSP for the order of the jth customer in the kth procedure
ρ_{ijk}^0	The initial satisfaction of the ith FLSP in the kth procedure when the order is equal to the minimum service capacity of ith FLSP
ρ_{ijk1}^0	The value of ρ_{ijk}^0 when the kth procedure is mass service
ρ_{ijk2}^0	The value of ρ_{ijk}^0 when the kth procedure is customization service
c_{ijk1}	The unit operating cost of the ith FLSP for the jth customer when the kth procedure is mass service
c_{ijk2}	The unit operating cost of the ith FLSP for the jth customer when the kth procedure is customization service
ε_{i1}	The weight of the single satisfaction of the ith FLSP
ε_{i2}	The weight of the overall satisfaction of the ith FLSP
λ_{ij}	The preference of the ith FLSP for the jth customer
k	The kth service procedure
K_j	The procedure to start the customization service of the jth customer
K_0	The minimum of all K_j
K'	The CODP for all customers
θ	Order difference
ω	Order difference tolerance coefficient of LSI
δ	Relationship cost coefficient of LSI
f_{jk}	The order quantity of the jth customer in the kth procedure
Z_1	The total satisfaction of FLSPs in LSSC
Z_2	The total cost of LSI in LSSC
Z_3	The total customized degree of LSSC
Z_2^*	The optimal solution of objective function Z_2 when not considering the objective function Z_1 and Z_3
F_1^*	The optimal solution of objective function Z_1 when not considering the objective function Z_2 and Z_3
F_3'	The value of Z_3, when the optimal solution is F_1^*
F_3^*	The optimal solution of objective function Z_3 when not considering the objective function Z_1 and Z_2
F_1'	The value of Z_1, when the optimal solution is F_3^*
α_1	Weight coefficients of the objective function Z_1 in weighted objective function
α_2	Weight coefficients of the objective function Z_3 in weighted objective function
Z	The objective function synthesized by Z_1 and Z_3 is also called the comprehensive performance of LSSC
Z^*	The optimal solution of Z

with increasing order value (the satisfaction maximum is one). However, when $x_{ijk} \geq b_{ijk}$, the satisfaction of the FLSP decreases with increasing order value because the order exceeds the upper limit of the service capacity.

Given that each FLSP has different preferences for each customer, satisfaction comprises single satisfaction and overall satisfaction. The weight of the single satisfaction of the ith FLSP is ε_{i1}, and the weight of the overall satisfaction of the ith FLSP is ε_{i2} $(\varepsilon_{i1} + \varepsilon_{i2} = 1)$.

(1) Single satisfaction: the satisfaction of the FLSP for the order allocation result of one customer. Assuming that the preference of the ith FLSP for the jth customer is λ_{ij}, $\sum_{j=1}^{M} \lambda_{ij}$ is equal to one, thus, the single satisfaction of the ith FLSP is $\varepsilon_{i1}(\sum_{j=1}^{M} \lambda_{ij}(1/L_j) \sum_{k=1}^{L_j} \rho_{ijk})$.

(2) Overall satisfaction: the satisfaction of the FLSP for the order allocation result of all customers. If the FLSP can service two customers, the FLSP expects that the satisfaction for both customers is high. Usually, we use the average satisfaction of multiple customers to represent the overall satisfaction. Thus, the overall satisfaction of the ith FLSP is $\varepsilon_{i2}((1/M) \sum_{j=1}^{M}(1/L_j) \sum_{k=1}^{L_j} \rho_{ijk})$.

The total satisfaction of ith FLSP is $\varepsilon_{i1}(\sum_{j=1}^{M}\lambda_{ij}(1/L_j)\sum_{k=1}^{L_j}\rho_{ijk}) + \varepsilon_{i2}((1/M)\sum_{j=1}^{M}(1/L_j)\sum_{k=1}^{L_j}\rho_{ijk})$.

Therefore, the objective function of maximizing the satisfaction of FLSPs is expressed as follows:

$$\max Z_1 = \frac{1}{n}\sum_{i=1}^{n}\left(\varepsilon_{i1}\left(\sum_{j=1}^{M}\lambda_{ij}\frac{1}{L_j}\sum_{k=1}^{L_j}\rho_{ijk}\right) + \varepsilon_{i2}\left(\frac{1}{M}\sum_{j=1}^{M}\frac{1}{L_j}\sum_{k=1}^{L_j}\rho_{ijk}\right)\right). \tag{2}$$

3.2.2. Minimizing the Total Cost of the LSI. In the order allocation process, the LSI also considers minimizing the cost. The total cost of the LSI is related not only to the number of orders assigned but also to the position of the CODP. When CODP approaches the customer, the quantity of procedures and order of mass services increase; the order of mass service is also obvious. Therefore, the scale effect is more noticeable.

The total cost of the LSI consists of two parts, namely, mass service cost and customization service cost. The unit operating cost of the ith FLSP for the jth customer is c_{ijk1} when the kth procedure is mass service (3). The unit operating cost of the ith FLSP for the jth customer is c_{ijk2} when the kth procedure is customization service (4). Consider

$$c_{ijk1} = A_{ik} - B_{ik}x_{ijk}, \tag{3}$$

$$c_{ijk2} = C_i, \tag{4}$$

where c_{ijk1} displays a scale effect, which decreases with the increasing of x_{ijk} (3). A larger x_{ijk} corresponds to a smaller c_{ijk1}. However, because of the characteristics of customization, c_{ijk2} does not change with the variation of x_{ijk}.

When the procedure of mass service is K' because of the existence of the scale effect, an increase in K' corresponds to a smaller c_{ijk1}. Hence, we introduce the scale effect coefficient τ, which is related to the number of procedures. The cost of the ith FLSP for the jth customer in all procedures is expressed as follows:

$$\sum_{k=1}^{K'}c_{ijk1} * x_{ijk} - \tau K'\sum_{k=1}^{K'}c_{ijk1} * x_{ijk} = \left(1-\tau K'\right)\sum_{k=1}^{K'}c_{ijk1} * x_{ijk}. \tag{5}$$

The cost of all FLSPs is summarized. Thereafter, the objective function of minimizing the total cost of the LSI is obtained:

$$\min Z_2 = \sum_{i=1}^{n}\sum_{j=1}^{M}\left(\left(1-\tau K'\right)\sum_{k=1}^{K'}c_{ijk1} * x_{ijk} + \sum_{k=K'+1}^{L_j}c_{ijk2} * x_{ijk}\right). \tag{6}$$

3.2.3. Maximizing the Customized Degree for Customer Demand. Under the MCLS environment, each customer will intend to maximize their own customized degree. Therefore, the customized degree for customer demand is one of the most important objectives. However, given that the customized degree for customer demand will affect the total cost of the LSI, the former cannot be too high. Mass customization involves reducing total cost as much as possible and maximizing the customized demand (i.e., increase the cost of customization services). Thus, we multiply the ratio of the customization service cost and total cost by the weight to represent the customized degree of each customer. We then summarize the customized degree of each customer to obtain the customized degree of all customers:

$$\max Z_3 = \sum_{j=1}^{M}\beta_j\left(\left(\sum_{i=1}^{n}\sum_{k=K'+1}^{L_j}c_{ijk2} * x_{ijk}\right) \times \left(\sum_{i=1}^{n}\left(\left(1-\tau K'\right)\sum_{k=1}^{K'}c_{ijk1} * x_{ijk} + \sum_{k=K'+1}^{L_j}c_{ijk2} * x_{ijk}\right)\right)^{-1}\right). \tag{7}$$

3.2.4. Order Allocation Constraints. In the order allocation process, the LSI also needs to meet several constraints. First, the total order quantity of all FLSPs should be equal to the customer demand in the kth procedure. Second, the K' (CODP) should be between the second procedure and the K_0 procedure ($K_0 = \min\{K_1, K_2, \ldots, K_M\}$). Third, given that the service procedures of each customer and the procedures to start the customization service are different, the LSI needs to consider the order difference θ of customers while serving multiple customers. To conduct mass customization service, the order difference cannot be extremely high. That is, θ cannot exceed the upper limit ω. Finally, x_{ijk} must be nonnegative. The four constraints can be expressed as (8), (9), (10), and (11):

$$\sum_{i=1}^{n}\sum_{j=1}^{M}x_{ijk} = \sum_{j=1}^{M}f_{jk}, \tag{8}$$

$$2 \le K' \le K_0, \tag{9}$$

$$\theta = \frac{1}{M}\sum_{j=1}^{M}\frac{K_j - K'}{K_j} \le \omega, \tag{10}$$

$$x_{ijk} \ge 0, \quad i = 1,2,\ldots,n;\; j = 1,2,\ldots,M;\; k = 1,2,\ldots,L_j. \tag{11}$$

As shown in (10), θ can be expressed as (12):

$$\theta = \frac{1}{M}\sum_{j=1}^{M}\frac{K_j - K'}{K_j}. \tag{12}$$

Equation (12) shows that θ is related to K_j and K'. If K_j is equal to K', θ is zero. A larger gap between K_j and K' corresponds to a higher θ value.

Combining the three objective functions given in Sections 3.2.1 to 3.2.3, we establish a multiobjective order allocation model under the MCLS environment ((13) to (19)). Consider

$$\max Z_1 = \frac{1}{n}\sum_{i=1}^{n}\left(\varepsilon_{i1}\left(\sum_{j=1}^{M}\lambda_{ij}\frac{1}{L_j}\sum_{k=1}^{L_j}\rho_{ijk}\right) + \varepsilon_{i2}\left(\frac{1}{M}\sum_{j=1}^{M}\frac{1}{L_j}\sum_{k=1}^{L_j}\rho_{ijk}\right)\right), \tag{13}$$

$$\min Z_2 = \sum_{i=1}^{n}\sum_{j=1}^{M}\left(\left(1-\tau K'\right)\sum_{k=1}^{K'}c_{ijk1} * x_{ijk} + \sum_{k=K'+1}^{L_j}c_{ijk2} * x_{ijk}\right), \tag{14}$$

$$\max Z_3 = \sum_{j=1}^{M}\beta_j\left(\left(\sum_{i=1}^{n}\sum_{k=K'+1}^{L_j}c_{ijk2} * x_{ijk}\right) \times \left(\sum_{i=1}^{n}\left(\left(1-\tau K'\right)\sum_{k=1}^{K'}c_{ijk1} * x_{ijk} + \sum_{k=K'+1}^{L_j}c_{ijk2} * x_{ijk}\right)\right)^{-1}\right), \tag{15}$$

$$\text{s.t} \quad \sum_{i=1}^{n}\sum_{j=1}^{M}x_{ijk} = \sum_{j=1}^{M}f_{jk}, \tag{16}$$

$$2 \le K' \le K_0, \tag{17}$$

$$\theta = \frac{1}{M}\sum_{j=1}^{M}\frac{K_j - K'}{K_j} \le \omega, \tag{18}$$

$$x_{ijk} \ge 0, \quad i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, M;\ k = 1, 2, \ldots, L_j. \tag{19}$$

In the objective functions, (13) maximizes the satisfaction of FLSPs; (14) minimizes the total cost of the LSI; (15) maximizes the customized degree for customer demand.

For the constraints, (16) implies that the service capacity supply is equal to the order demand in each procedure; (17) means that K' (CODP) should be between the second procedure and K_0 procedure. Equation (18) indicates that the order difference should be less than the order difference tolerance coefficient. Equation (19) denotes that the order assigned to each FLSP must be nonnegative.

4. Model Solution

The model is a multiobjective programming problem with three objectives and three constraints. Multiobjective programming problems have numerous mature solutions, such as the evaluation function method including linear weighting method, reference target method, maximin method [9], goal programming method [25], delaminating sequence method [26], and subordinate function method [27]. For the specific issues, it is necessary to choose an appropriate solution method to solve the practical problems. Given that conflicts and incommensurability between goals exists, finding an absolute optimal solution is difficult. Therefore, a compromise between the various objectives is commonly performed to obtain the Pareto-optimal solution for decision makers. Specifically, two approaches can be used to solve multiobjective programming problem. One is to balance between objectives, assign weight to each of the objective functions and switch the multiobjective functions to a single objective function. The other method is to reduce the number of objective functions and transform the reduced objective functions into constraints. However, on MCLS, the LSI should not only consider the total cost target but also the satisfaction target and the customized degree target. Generally, supply chain management emphasizes the strategic partnership. In order to maintain long-term cooperation with FLSPs and customers, in order allocation, the LSI should keep the total cost within a specific range instead of merely seeking for the minimum. Maximizing the satisfaction and the customized degree should be the target. As a result, we transform the total cost objective into a constraint to reduce the number of objective functions and transform the target of satisfaction customized degree into a single objective, after which the model is solved.

Based on the previous solving methods, in order allocation, considering these actual situations, we introduce a parameter called the relationship cost coefficient δ into our model. This parameter is used to represent the increasing cost limit of the LSI to enhance the cooperation and satisfaction of FLSPs and customers. Hence, Z_2 can become a constraint. Thereafter, we choose a linear weighting method, which is the most typical method, to transform the multiobjective Z_1, Z_3 into a single objective Z and solve the model. The specific steps in solving the model are as follows.

Step 1. The consideration of the objective functions Z_1, Z_3 is set aside to consider the case of objective function Z_2. Thereafter, the relationship cost coefficient δ is introduced. The objective can then be translated into a constraint:

$$Z_2 \le (1+\delta) Z_2^*. \tag{20}$$

This approach denotes that the LSI can minimize and maintain the total cost within a certain range. $\delta = 0.2$ means that the LSI can exist with a total cost less than 1.2 times of Z_2^* to increase the satisfaction and customized degree. The LSI considers this input to be conducive in the completion of orders.

Step 2. The model has Z_1 and Z_3. The linear weighing method is used to transform the objective Z_1 and Z_3 into a new objective function max $Z = \alpha_1 Z_1 + \alpha_2 Z_3$ $(\alpha_1 + \alpha_2 = 1)$. Determining the weight (α_1, α_2) is the main problem of this approach. α_1, α_2 can be determined by the following equation:

$$\alpha_1 = \frac{F_3^* - F_3'}{F_1^* - F_1' + F_3^* - F_3'},$$
$$\alpha_2 = \frac{F_1^* - F_1'}{F_1^* - F_1' + F_3^* - F_3'}. \quad (21)$$

Step 3. After determining α_1 and α_2, the model is transformed into a single-objective planning model:

$$\max Z = \alpha_1 Z_1 + \alpha_2 Z_3,$$
$$\text{s.t} \quad Z_2 \le (1+\delta) Z_2^*,$$
$$\sum_{i=1}^{n} \sum_{j=1}^{M} x_{ijk} = \sum_{j=1}^{M} f_{jk},$$
$$2 \le K' \le K_0,$$
$$\theta = \frac{1}{M} \sum_{j=1}^{M} \frac{K_j - K'}{K_j} \le \omega,$$
$$x_{ijk} \ge 0, \quad i = 1, 2, \ldots, n; \; j = 1, 2, \ldots, M; \; k = 1, 2, \ldots, L_j. \quad (22)$$

In solving the objective function Z, the x_{ijk} obtained is the optimal solution Z^*. Z reflects the comprehensive performance of the LSSC. A bigger Z corresponds to better comprehensive performance.

5. Numerical Analysis

This section illustrates the validity of the model by numerical analysis, explores the influence of related factors on time scheduling results, and provides several management insights. First, we use the Lingo 12 software to implement the numerical analysis. The results are shown in Section 5.1. From the perspective of LSI order management decision, we choose and study the influence of following factors (τ, ω, δ) on order allocation decision (Section 5.2 to Section 5.5). τ is an important factor to reflect the scale effect on MCLS. ω is also an important factor to reflect the influence of order difference on LSI's decision making. δ indicates the cost volatility with which the LSI can live. We will study the influence of the three (τ, ω, δ) on order allocation separately.

5.1. Model Solution. Figures 1 and 2 show that each customer (A, B, and C) has an order and needs the LSI Q to provide transportation services. Q assigns the three orders to the FLSPs (a, b, c, d, and e). The demand of each customer is 60, 100, and 80. The number of service procedures of each customer is 8, 7, and 7. $K_1 = 6$, $K_2 = 5$, $K_3 = 5$, $K_0 = \min(K_1, K_2, K_3) = 5$, $\tau = 0.05$, $\omega = 0.4$, and $\delta = 20\%$. The values of other parameters are shown in Tables 2 and 3.

Table 2: Parameter data (1).

$[a_{ijk}, b_{ijk}]$	$[a_{ijk1}, b_{ijk1}]$	$[a_{ijk2}, b_{ijk2}]$
$i = 1$	$[40, 80]$	$[20, 40]$
$i = 2$	$[30, 70]$	$[20, 35]$
$i = 3$	$[35, 90]$	$[25, 40]$
$i = 4$	$[30, 60]$	$[25, 50]$
$i = 5$	$[30, 80]$	$[10, 40]$
c_{ijk}	c_{ijk1}	c_{ijk2}
$i = 1$	$10 - 0.02x_{ijk}$	15
$i = 2$	$11 - 0.02x_{ijk}$	18
$i = 3$	$9 - 0.02x_{ijk}$	16
$i = 4$	$12 - 0.02x_{ijk}$	20
$i = 5$	$14 - 0.02x_{ijk}$	22
ρ_{ijk}	ρ_{ijk1}	ρ_{ijk2}
$i = 1$	0.2	0.3
$i = 2$	0.35	0.35
$i = 3$	0.15	0.3
$i = 4$	0.3	0.2
$i = 5$	0.25	0.3
ε_i	ε_{i1}	ε_{i2}
$i = 1$	0.6	0.4
$i = 2$	0.5	0.5
$i = 3$	0.7	0.3
$i = 4$	0.4	0.6
$i = 5$	0.35	0.65

Table 3: Parameter data (3).

λ_{ij}	$j = 1$	$j = 2$	$j = 3$
$i = 1$	0.3	0.3	0.4
$i = 2$	0.2	0.4	0.4
$i = 3$	0.35	0.35	0.3
$i = 4$	0.25	0.35	0.4
$i = 5$	0.3	0.4	0.3
β_j	0.3	0.4	0.3
D_j	60	100	80
K_j	6	5	5
L_j	8	7	7

By using the solution method in Section 4, Tables 4 and 5 show the result and performance of the order allocation, respectively.

5.2. Influence of Scale Effect Coefficient τ on Order Allocation. To analyze the influence of τ on the related factors of order allocation $(Z_2, \text{CODP}, Z_3, Z)$, we make τ equal to 0, 0.025, 0.05, 0.075, 0.1, 0.125, and 0.15. The data is shown in Table 6. Figures 3 to 6 are plotted with the data from Table 6.

Figure 3 shows that τ has significant influence on the total cost of LSI Z_2. A greater τ corresponds to a smaller Z_2. The scale effect of the cost is more obvious with increasing τ; thus, the total cost decreases.

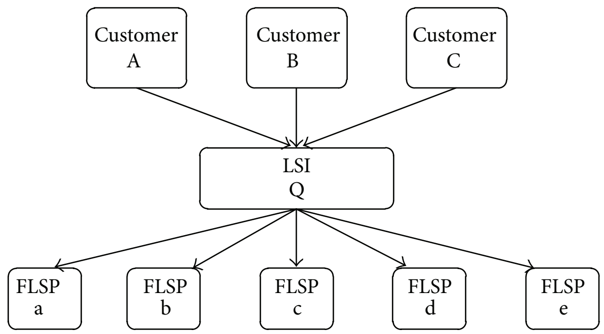
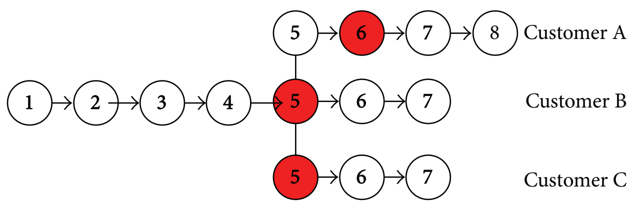


Figure 1: Structure of the LSSC.



Note: The arrows indicate the flow of the order allocation.

Figure 2: Logistics service procedure of customer A, B, and C. Note: The red circle denotes the procedure that starts the customization service of each customer.

Table 4: Order allocation results.

Procedure	1–5			6–7			8
Customer Order value FLSP	A	B	C	A	B	C	A
a	0	4.31	0	31.59	18.20	37.79	31.59
b	0	6.33	0	6.14	37.86	0.53	6.14
c	60	4.99	80	4.63	40	37.82	4.63
d	0	4.93	0	6.41	0.35	0	6.41
e	0	79.44	0	11.23	3.60	3.86	11.23

Table 5: Order allocation performance.

Total cost of LSSC	17472.020
CODP	5
Customized degree	0.522
Z	0.391

Table 6: Influence of τ on order allocation.

τ	Z_2	Z_3	CODP	Z
0	20280	0.596	4	0.403
0.025	18960.000	0.524	5	0.388
0.05	17472.020	0.522	5	0.391
0.075	16320.000	0.580	5	0.417
0.1	15000.000	0.638	5	0.469
0.125	13680.000	0.646	5	0.472
0.15	12360.000	0.783	5	0.582

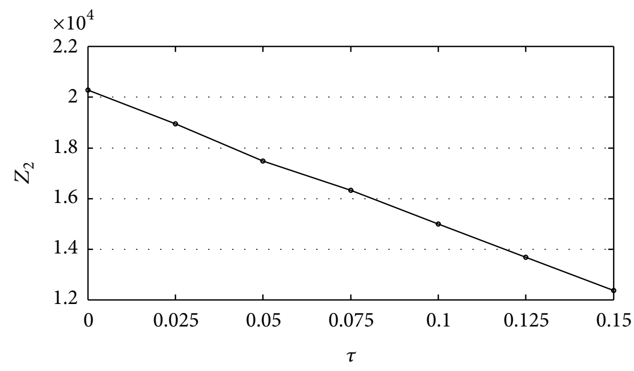


Figure 3: Influence of τ on Z_2 (total cost of LSI).

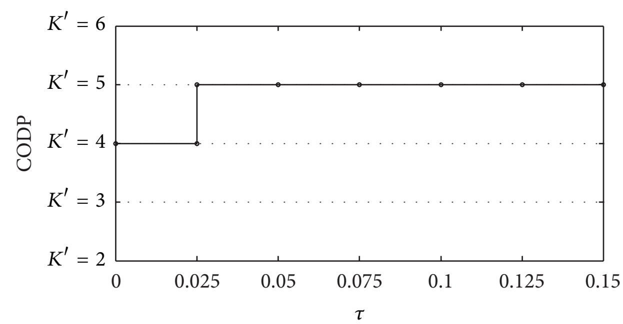


Figure 4: Influence of τ on CODP.

Figure 4 shows that τ has a certain influence on CODP. If $\tau = 0$, the scale effect is nonexistent and the CODP is in the fourth procedure. With $\tau \geq 0.025$, the CODP changes to the fifth procedure because the scale effect of cost is more obvious with increasing τ. Therefore, the LSI will delay the CODP to acquire a considerable scale effect.

If $\tau < 0.05$, the customized degree Z_3 decreases with increasing τ, which is contrary to $\tau \geq 0.05$ (Figure 5). Z_3 initially decreases and then increases. This observation differs from the observation of previous studies that an increasing τ corresponds to a more obvious scale effect and smaller cost of mass service. Furthermore, previous studies show that the customized degree is equal to the ratio of the customization service cost and total cost (mass service cost plus customization service cost). Therefore, the customized degree should always increase.

Our results differ from those of previous studies that did not include the CODP. The LSI will delay the CODP to acquire the scale effect, which leads to a significant increase in mass service cost and a decrease in customized degree (Figure 4). Furthermore, the CODP becomes stable when τ increases to a certain value. A greater τ corresponds to a more obvious scale effect. The increasing extent of the mass service cost becomes smaller and the customized degree becomes larger. The mass service cost decreases and the CODP moves backward with increasing τ. The customized degree initially decreases and then increases.

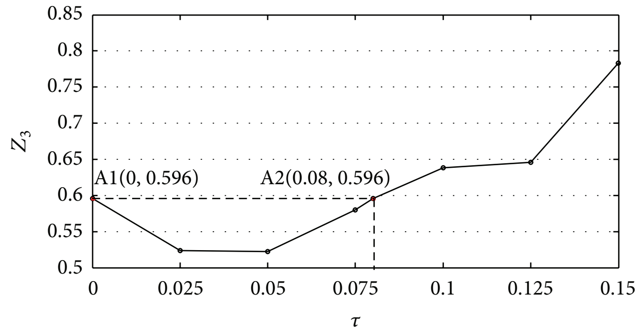


FIGURE 5: Influence of τ on Z_3 (customized degree).

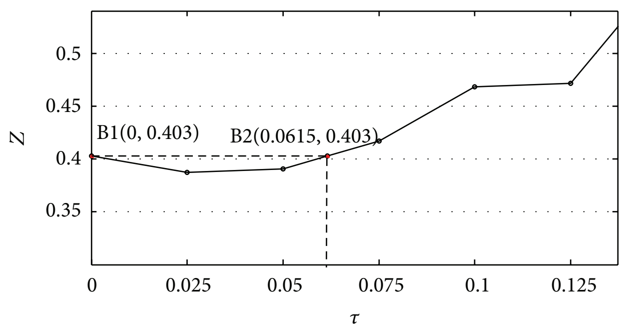


FIGURE 6: Influence of τ on Z (comprehensive performance of LSSC).

TABLE 7: Influence of ω on order allocation.

ω	Z_2	Z_3	CODP	Z
0.4	17472.020	0.522	5	0.391
0.45	17472.020	0.522	5	0.391
0.5	17499.530	0.613	5	0.471
0.55	17499.530	0.613	5	0.471
0.6	17499.530	0.613	5	0.471
0.65	17640.000	0.681	4	0.561
0.7	17640.000	0.681	4	0.561
0.8	17640.000	0.681	4	0.561
0.9	17640.000	0.681	4	0.561
1	17640.000	0.681	4	0.561

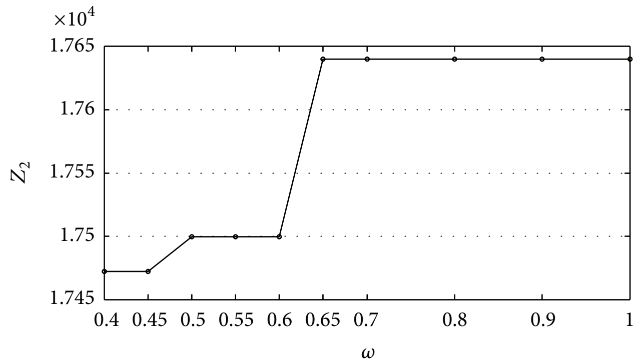


FIGURE 7: Influence of ω on Z_2 (total cost of LSI).

If $\tau < 0.025$, Z decreases with increasing τ, which is contrary to $\tau \geq 0.025$ (Figure 6). The comprehensive performance of LSSC initially decreases and then increases.

Further analysis shows that when Z is greater than 0.403 ($\tau = 0.0615$), Z increases with increasing τ; thus, the LSI prefers the FLSP with a bigger τ. When Z is less than 0.403 ($\tau = 0.0615$), two τ's equal one performance. The LSI still prefers the FLSP with a bigger τ because a bigger τ leads to smaller total costs.

5.3. Influence of Order Difference Tolerance Coefficient ω on Order Allocation. To analyze the influence of ω on the related factors of order allocation (Z_2, CODP, Z_3, Z), we equate ω to 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.8, 0.9, and 1. The data is shown in Table 7. Figures 7 to 10 are plotted by using the data from Table 7.

The order difference tolerance coefficient ω has a certain influence on the total cost of the LSI (Figure 7). The total cost of the LSI increases with increasing ω. The order difference also increases with increasing ω. The CODP should be set before $K_0 = \min\{K_1, K_2, \ldots, K_M\}$. The CODP may move forward, thus increasing the procedure of customization service and total cost. Therefore, the LSI should choose the customer with low order difference and maintain the CODP in the same point to reduce total cost.

When ω is greater than 0.65, the total cost is stable at 17,640 (Figure 7). Increasing ω with no limitations is unnecessary because of the upper limit ($\omega = 0.65$ in numerical analysis). Consequently, the LSI should set a reasonable upper limit for ω in the MCLS.

When ω is less than 0.65, the CODP is in the fifth procedure; when ω is larger than 0.65, the CODP is in the fourth procedure (Figure 8). These findings show that ω has a certain influence on CODP. A larger ω corresponds to a larger CODP range. The CODP moves forward with increasing ω to a certain value.

Figure 9 demonstrates that ω affects Z_3. Z_3 increases with increasing ω. The CODP moves to the second procedure, thus increasing the procedure of customization service. Consequently, the customized degree will also increase. When ω is greater than 0.65, increasing ω with no limitation is unnecessary because of the upper limit. Thus, the LSI should set a reasonable upper limit for ω in the MCLS.

Figure 10 indicates that Z increases steadily with increasing ω and becomes stable at 0.561. The comprehensive performance of LSSC shows a step growth instead of a linear growth with increasing ω. This observation is attributed to the forward movement of the CODP to the second procedure. Variables ω and Z have ranges; however, the CODP cannot move forward indefinitely because it must move after the second procedure, which determines the maximum value of ω; ω is not a constraint if it is greater than the maximum value.

5.4. Influence of Relationship Cost Coefficient δ on Order Allocation. To analyze the influence of δ on the related factors of order allocation (Z_2, CODP, Z_3, Z), we equated δ to 0.15,

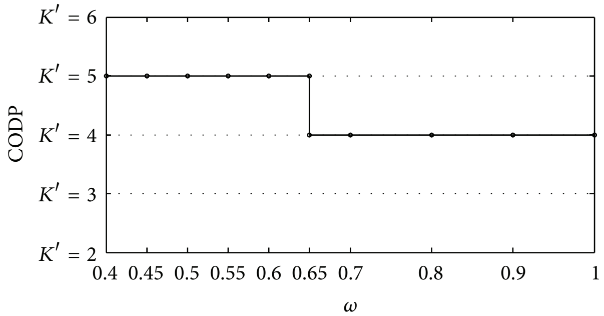


Figure 8: Influence of ω on CODP.

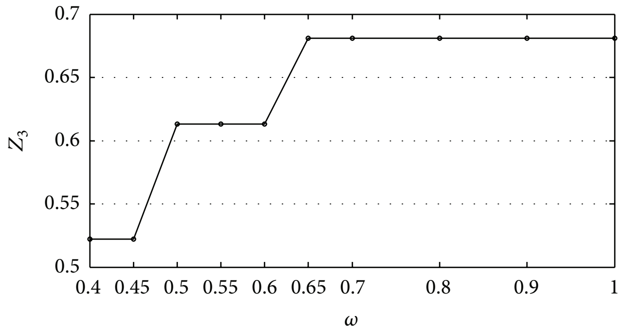


Figure 9: Influence of ω on Z_3 (customized degree).

Table 8: Influence of δ on order allocation.

δ	Z_2	Z_3	CODP	Z
0.15	15400.000	0.575	5	0.421
0.2	17472.020	0.522	5	0.391
0.25	17990.84	0.570	5	0.413
0.3	18887.720	0.652	4	0.497
0.35	19845.000	0.668	4	0.465
0.4	20580.000	0.663	4	0.477
0.45	21002.100	0.685	4	0.494

0.2, 0.25, 0.3, 0.35, 0.4, and 0.45. The data are shown in Table 8. Figures 11 to 14 are plotted by using the data from Table 8.

Figure 11 shows that δ has a significant influence on the total cost of the LSI. A larger δ corresponds to a larger Z_2. This conclusion is consistent with the conclusion of Liu et al. [1]. A larger δ corresponds to a greater Z_2 range, thus increasing the total cost.

Figure 12 indicates that δ has a certain influence on the CODP. The CODP moves forward to the second procedure with increasing δ. A larger δ corresponds to a greater Z_2 range, which increases the procedure of the customization service and decreases the procedure of the mass service. Therefore, the CODP will move forward.

When $\delta < 0.2$, Z_3 decreases along with increasing δ, which is contrary to $\delta \geq 0.2$ (Figure 13). The relationship cost coefficient δ influences the customized degree positively when δ is greater than a certain value.

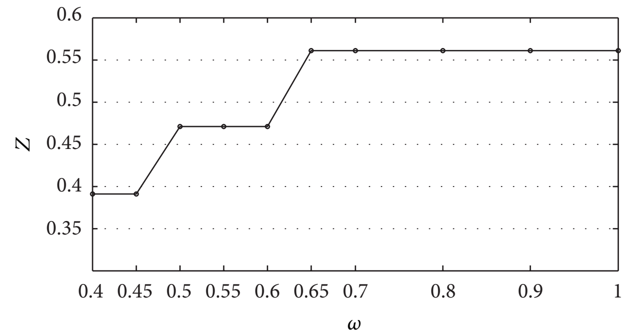


Figure 10: Influence of ω on Z (comprehensive performance of LSSC).

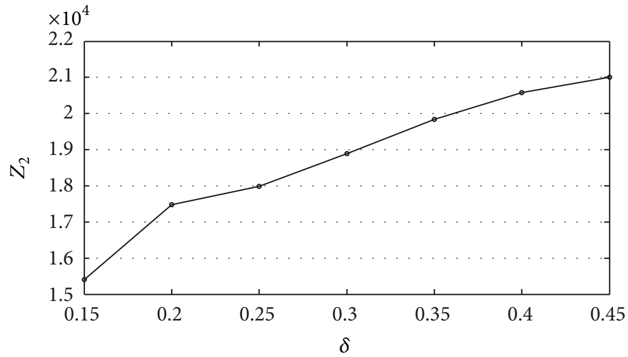


Figure 11: Influence of δ on Z_2 (total cost of LSI).

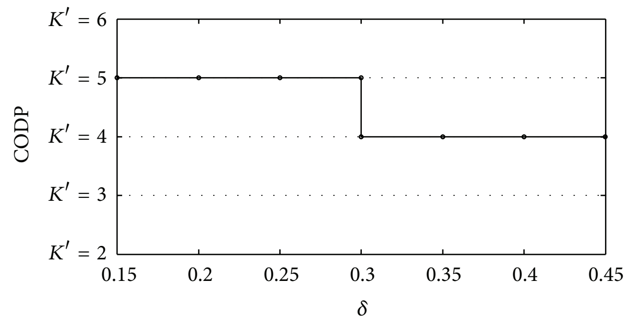


Figure 12: Influence of δ on CODP.

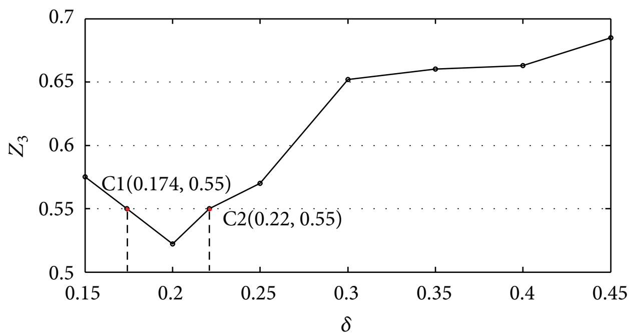


Figure 13: Influence of δ on Z_3 (customized degree).

Further analysis shows that if the customized degree is greater than 0.575, Z_3 increases with increasing δ (Figure 13). Thus, the LSI prefers to increase the value of δ. However, when the customized degree is less than 0.575, two δs will be present. For this case, the LSI will prefer the smaller δ. For example, when the customized degree is 0.55, two δ will be present: C1 (0.174,0.55) and C2 (0.22,0.55) (Figure 13). In this case, the LSI will choose C1.

Figure 14 shows that Z decreases before increasing with increasing δ. The breaking point is when $\delta = 0.2$. This observation is different from previous studies that indicate that a greater δ corresponds to a looser constraint. Hence, Z will increase.

This observation is attributed to the two parts of Z: customized degree and satisfaction. Figure 13 shows that if δ is less than 0.2, the customized degree decreases with increasing δ. Consequently, Z decreases. When δ is more than 0.2, Z increases with increasing customized degree (Figure 13).

Figure 14 shows that Z only exhibits a positive correlation with δ when Z is greater than 0.421. When Z is less than 0.421, Z initially decreases and then increases. When Z is less than 0.421, two δ's are present. For this case, the LSI will prefer the smaller δ. For example, when Z equals 0.4, two δ's exist: D1 (0.18, 0.4) and D2 (0.22, 0.4). In this case, the LSI will choose D1.

5.5. Relationship of Order Allocation Results and Related Factors. In this section, the relationship of order allocation results (total cost Z_2, customized degree Z_3, CODP, and comprehensive performance Z) and related factors (scale effect coefficient τ, order difference tolerance coefficient ω, and relationship cost coefficient δ) is summarized.

6. Main Conclusions and Management Insights

In this section, we elaborate the main conclusions and implications for researchers. We also explore the management insights for LSI and FLSP and provide some suggestions.

6.1. Main Conclusions. On the basis of the analysis on Section 5, we obtained the following conclusions concerning order allocation under an MCLS environment.

(1) The total cost of LSI Z_2 decreases with increasing scale effect coefficient τ and increases with increasing order difference tolerance coefficient ω and relationship cost coefficient δ. However, the influence of ω on Z_2 is finite. Z_2 becomes stable when ω increases to a certain value.

(2) The CODP moves back to the customers with increasing scale effect coefficient τ and moves forward to the second procedure with increasing order difference tolerance coefficient ω and relationship cost coefficient δ. LSI can move the CODP forward and increase the customized degree by increasing the value of δ. When τ is greater, the LSI prefers moving the CODP backward and acquiring the scale effect.

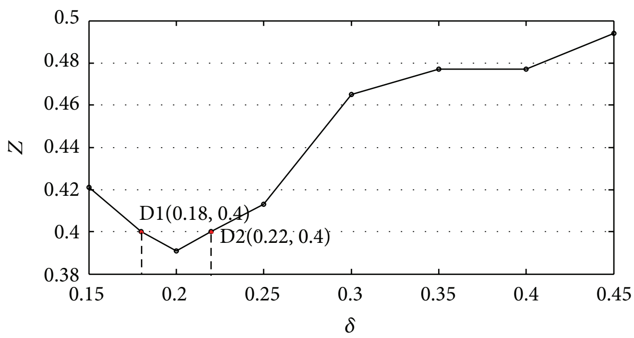


Figure 14: Influence of δ on Z (comprehensive performance of LSSC).

(3) The customized degree Z_3 initially decreases before increasing with increasing scale effect coefficient τ and relationship cost coefficient δ. When Z_3 is greater than a certain value, τ and δ have a positive correlation with Z_3. ω always has a positive influence on Z_3. When ω increases to a certain value, Z_3 tends to be stable.

(4) The comprehensive performance of order allocation Z initially decreases before increasing with increasing scale effect coefficient τ and relationship cost coefficient δ. When Z is greater than a certain value, τ and δ have a positive correlation with Z; ω always has a positive effect on Z. When ω increases to a certain value, Z tends to be a stable.

6.2. Implications for Researchers. In line with academic research, this paper elaborates on the management of order allocation under the MCLS environment and the influences caused by allocation results. This paper provides a theoretical basis for the study of control methods about the performance of order allocation in MCLS. For instance, the type of order allocation method needs to be determined in choosing the FLSP. How will the comprehensive performance of the LSSC improve with minimum cost? How will the contradiction between cost and customization be solved? What factors have more influence on the comprehensive performance of the LSSC? This study provides the necessary theoretical basis for further research in the application and scheduling of the theoretical and empirical studies of LSSC under the MCLS environment.

6.3. Implications for Managers. LSI and FLSP managers should understand the factors that need focus and establish long-term strategic partnership for better cooperation.

The LSI should be careful in order allocation and choose the FLSP with a greater scale effect. The order difference tolerance factor should not be set extremely high. Excessive order difference tolerance results in a higher comprehensive performance of order allocation; however, excessive order difference tolerance can also lead to excessive total LSI cost. The order difference tolerance has a reasonable upper limit. Beyond this upper limit, the comprehensive performance of order allocation and the degree of customized experience initially decrease before increasing with increasing relationship cost coefficient. Therefore, a bigger relationship cost coefficient of the LSI does not indicate a better setup; instead, we should consider both the customized degree of the target and comprehensive performance.

For FLSP, the scale effect coefficient is important to enterprises for order allocation. Thus, enterprises need to improve large-scale operational capabilities continuously. When the comprehensive performance is high, FLSPs should request for cost compensation from the LSI by increasing the relationship cost coefficient, which improves FLSP satisfaction.

7. Research Limitations and Future Work

Under increasing customer demand for specialized and customized logistics services, the competitiveness of the LSSC depends on its ability to meet the need for customized requirements with the operating expense of mass service. The satisfaction of FLSPs will be maximized by minimizing the total cost of the LSI and maximizing the customized degree for customer demand. This paper establishes an order allocation model for the LSSC that is constrained by meeting the demand, CODP, and order difference tolerance coefficient. Numerical analysis is conducted with Lingo 12 software. The influences of the scale effect coefficient, order difference tolerance coefficient, and relationship cost coefficient on the order allocation and comprehensive performance of the LSSC are discussed. Management insights are also proposed.

However, the order allocation model also has some disadvantages. For instance, we assumed that each service process in order allocation requires only one service capability, such as transport service capability. In many cases, logistics service is the combination of different services including transportation, storage, loading, and unloading; each process may also be supported by a variety of capacities. The equivalent problem of each capacity needs to be considered in detail in the future. We also assumed that customer demand and service capacity are stable; however, customer demand may be random in actuality. Order allocation with random customer demand is an important direction for future studies. Furthermore, we assumed that the members of the LSSC have long-term cooperation, transparency, and mutual trust. In reality, we need to consider the game behavior among members because of its influence on order allocation. These problems need to be explored in detail in future research.

Acknowledgment

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Research Article

Contract Strategies in Competing Supply Chains with Risk-Averse Suppliers

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This paper investigates the equilibrium contract strategies of two competing supply chains. Each chain is composed of one risk-averse supplier and one risk-neutral retailer. The two suppliers, as Stackelberg leaders, can choose either wholesale price or revenue sharing contract. We compare the outcomes obtained from different scenarios and study the impact of competition density and risk attitudes on the suppliers' contract choices. We find that it is always optimal for the first-moving supplier to choose revenue sharing contract if the follower chooses wholesale price contract. We also find that the retail price under revenue sharing contract is lower than that under wholesale price contract. Compared to Pan et al. (2010), we find that a threshold policy with respect to the degree of product substitutability holds in a sequential competition environment. There also exists a threshold policy with respect to the degree of risk aversion for the suppliers to choose their individual contract types and the corresponding chain-to-chain contract strategies.

1. Introduction

Nowadays, with the development of information technology, many "pure" service companies have arisen, such as Expedia, Alibaba.com, Crip.com, and China Mobile. In these companies, no physical products are sold—what they can provide are intangible products, that is, services. Even some companies, who traditionally produce and sell physical products, are changing to be service providers. One typical example is IBM, who uses its LotusLive Connections to apply a new business model named "Software-as-a-Service (SaaS)", under which customers need only "pay a subscription fee to use a software over the internet" [1]. To find more customers, these service companies rely on agents/retailers and hence form service supply chains. For example, China Mobile contracts with agents to sell its telecommunication services. As the key supply chain member, China Mobile is very powerful. They determine the service charges, the service contracts, and so on. Retailers just follow China Mobile's decisions to earn some agent fee.

Similar phenomena can be observed in nonservice supply chains. For example, in the petrol retailing industry, Caffarra and Mattei [2] find that UK oil companies hold the right of selecting the organizational structures (and hence the contract parameters such as the wholesale price) in their retail networks. It is suggested that the petrol suppliers would be significantly "rewarded or penalized" according to their contract strategies. For the other industries, such as the electronic industry [3, 4], the television industry [5], and the agriculture industry [6], the determination of contract type and/or parameters are continuously discussed and investigated. This indicates that the competing suppliers, especially those who sell their products through retailers, must be very careful at the contract design stage so as to gain some competitive advantages.

In practice, different suppliers may choose different kinds of contracts with retailers. A variety of contractual relationships have been discussed in the literature, for example, wholesale-price contract [7], revenue sharing contract [8], quantity-discount contract [9], and consignment contract

[10]. See Cachon [11] for the literature review on supply chain contracts. Among these studies, we note that Pan et al. [8] study the contract choice problems (revenue sharing versus wholesale price) by assuming two competing manufacturers make their decisions simultaneously, which is reasonable but ignores the more common sequential-moving supply chains. Therefore, in this paper, we relax their assumption and examine scenarios where suppliers have imbalanced power, so that suppliers can act as Stackelberg leaders. Such supply chains can be found in many industries, for example, the domestic appliance industry, tourism industry, telecommunication industry, and beverage industry. Moreover, we note that more and more suppliers are concerning the loss minimization of their companies; thus, it is important to investigate the role of risk aversion in the competing supply chains. Tsay [12] points out that the "risk sensitivity" has an important impact on the decisions of the supply chain members. However, few studies have addressed risk aversion in chain-to-chain competition environments. Thus, in this paper, we relax the risk-neutral assumption of Pan et al. [8] to examine scenarios where suppliers are risk-averse, and analyze the effects of the degree of risk aversion on the suppliers' contract decisions. Readers hence can view this paper as an extension of that of Pan et al. [8] with the consideration of sequential-moving supply chains and risk-averse suppliers.

Our analysis leads to the following insights. (I) Revenue sharing contract is the best option for the first-moving supplier if the follower chooses wholesale price contract, regardless of the degree of competition density and risk aversion. However, if the follower chooses revenue sharing contract, it is possible for the leader to choose wholesale price contract. (II) There exist threshold policies with respect to the degree of competition density and risk aversion for the suppliers to choose contract strategies. The retail price under revenue sharing contract is lower than that under wholesale price contract. Thus, if the follower could not afford the loss due to low retail price, he/she has incentives to switch to wholesale price contract.

The remainder of this paper is organized as follows. Section 2 briefly reviews related literature. Section 3 presents our basic model. Section 4 derives close form equilibrium solutions for different contract structures, including the WW, WR, RW, and RR cases. Section 5 compares the equilibrium profits under different cases and identifies the conditions under which the suppliers should choose wholesale price contract or revenue sharing contract. Finally, we summarize the paper and suggest research extensions in Section 6.

2. Literature Review

This paper is related to the work on chain-to-chain competition. McGuire and Staelin [13] investigate the effect of product substitutability on the suppliers' decision of integrating with an outside retailer or not. Trivedi [14] looks into two manufacturers and two common retailers and analyzes the effect of product and channel substitution. Boyaci and Gallego [15] consider a market comprising two supply chains and explore the value and limitation of channel coordination. Wu et al. [16, 17] extend the seminal work of McGuire and Staelin [13] by considering demand uncertainty. They build competitive newsvendor models to investigate the channel design problems. Allon and Federgruen [18] build multiple stage models to study competition in the service industries. Shou et al. [19] study the chain-to-chain competition problems with the consideration of supply uncertainty. Chiu et al. [20] consider price-dependent demand and study methods to avoid price war among multiple retailers. Xie et al. [21] consider the selection of supply chain structures and quality improvement strategies of two supply chains. Li et al. [22] investigate the channel choice problem in two competing supply chains with asymmetric cost information. Different from the aforementioned literature, we study the contract strategies with the consideration of sequential-moving supply chains and risk-averse suppliers.

The literature on supply chain contract is also related. Cachon [11] gives an excellent review on supply chain contract and coordination. This work has extensively reviewed the one-to-one or one-to-N supply chain structures (see, e.g, [23–25]). One conclusion is that the wholesale-price contract is Pareto inferior for the supply chain, which leads to the double marginalization effect. To eliminate the double marginalization effect, manufacturers need to design sophisticated contracts, such as quantity flexibility contract, quantity discount contract, and revenue sharing contract. However, are sophisticated contracts always better for all the supply chains?

Recently, some papers find that the degree of competition has a significant impact on the contract choice decision. Ha and Tong [26] investigate contracting and information sharing problems in two competing supply chains. They highlight that contract type is a key driver of the value of information sharing. Differing from Ha and Tong [26], Ha et al. [27] assume imperfect demand signals, production diseconomy, and both Cournot and Bertrand competition with differentiated products. Cachon and Kök [9] study the problem of contract choice in the supply chain structure comprising two competing manufacturers and a single retailer. They show that sophisticated contracts (e.g., quantity discount, two-part tariff contract) may make manufacturers worse off and the retailer substantially better off. Pan et al. [8] discuss different contract strategies (wholesale price contract versus revenue sharing contract) under both manufacturer dominated and retailer dominated scenarios in a supply chain with either two-one structure or one-two structure. Zhao and Shi [10] consider two supply chains, each with multiple upstream suppliers producing complementary products and selling to a single buyer. They focus on the channel structure (integration versus decentralization) issues and the contract strategies (wholesale price contract versus revenue sharing contract) issues. Ai et al. [28] demonstrate the impact of demand uncertainty and retailers' forecasting precision on contract decisions of the manufacturers in two competing supply chains. Feng and Lu [7] analyze contract strategies in a supply chain comprising two competing manufacturers and two competing retailers. They derive the outcomes of Stackelberg games via generalized Nash bargaining scheme. Li et al. [29] explore the effect of supply chain structure and competition at both manufacturer and retailer levels on contract choices of two competing supply chains.

In the literature above, most studies assume that manufacturers are risk-neutral. When facing random demand, however, the manufacturers will be more concerned with the risk associated with demand uncertainties. The inclusion of risk into decision making has gained increasing interest in supply chain studies (e.g., [30–34]). Different methods have been used to study risk aversion; for example, Hsieh and Lu [35] characterize each retailer's risk-embedded objects and compare them with conditional value-at-risk. They study the manufacturer's return policy and the retailers' decisions. Caliskan-Demirag et al. [36] model risk aversion by adopting the conditional-value-at-risk (CVaR) decision criterion, and analyze the manufacturer's rebate amount decisions and the retailer's joint inventory and pricing decisions.

Some other papers develop the risk aversion model in a mean-variance framework. Chen and Seshadri [37] consider a single period model in which multiple risk-averse retailers purchase a single product from a common distributor with a mean-variance utility approach. Choi et al. [38] carry out a mean-variance analysis of the newsvendor problems. Choi and Chow [39] study quick response program using a mean-variance approach. Xiao and Choi [40] focus on channel structure strategies in a two-echelon system consisting of two manufacturers and two retailers, where all players are risk-averse. Wei and Choi [32] explore the use of wholesale price contract and profit sharing contract to coordinate supply chains under the mean-variance decision framework. Chiu et al. [41] carry out a mean-variance analysis of supply chains under target sales rebate contracts. Chiu et al. [42] explore the performance of sales rebate contract in fashion supply chains under a mean-variance framework. Ma et al. [43] examine the impact of bargaining powers and the retailer's risk attitudes using a CVaR framework. Li et al. [44] conduct a mean variance analysis of a fast fashion supply chain with return policy. Ma et al. [45] study the inventory control problem for a loss-averse retailer with financial constraint in a finite periodic review system. Shen et al. [46] explore how markdown money policy performs in a two-stage TC/fashion supply chain with an upstream risk-averse manufacturer and a downstream risk-neutral retailer under the mean-variance framework. See Chiu and Choi [47] for an excellent review of this line of literature.

Our paper is most closely related to Cachon and Kök [9], Pan et al. [8], Feng and Lu [7], and Li et al. [29]. We extend their models in two dimensions. First, we examine scenarios where manufacturers have imbalanced power, so that manufacturers can move first. Second, we examine scenarios where manufacturers are risk-averse and investigate the impact of the degree of risk aversion on the contract choice decisions. Referring to Xiao and Choi [40], we model the risk aversion with mean-variance utility functions in this paper.

3. Model Settings

Consider a market in which two supply chains (denoted by SC_1 and SC_2) compete with each other. Each supply chain is composed of a single supplier (denoted by M_i, $i = 1, 2$) and a single exclusive retailer (denoted by R_i, $i = 1, 2$). Without loss of generalization, we assume SC_1 is the market leader who produces and launches a product first. This is a common phenomenon in practice, for example, China Mobile launches its M-Zone service earlier than China Unicom, whose competing product is Up-Power (admittedly, it is possible to investigate the timing decisions of the two supply chains, named as endogenous leadership game; see Wang et al. [3, 4, 48] for further information. However, here our focus is the contract type choice and the impact of risk attitudes we leave the timing decisions for future research). We assume the two chains compete strictly in retail price, p_i, which directly determines their respective customer demand, $\widetilde{D_i}$. Referring to McGuire and Staelin [13], we assume the following linear demand function:

$$\widetilde{D_i} = \widetilde{A_i} - p_i + \theta p_{3-i}, \tag{1}$$

where $i = 1, 2$. $\widetilde{A_i}$ denotes the stochastic market potential for product i, with the mean $\overline{A_i} > 0$ and variance σ_i^2. The parameter $\theta \in (0, 1)$ represents the degree of product substitutability, which can be viewed as a measurement of demand competition density [26]. When $\theta \to 0$, there is no cross market influence, implying that the products are independent. When $\theta \to 1$, there is a perfect product substitution.

Towards the contract types adopted in each supply chain, we consider wholesale price contract (denoted by W) and revenue sharing contract (denoted by R). The reasons for choosing these two contract types are as follows. First, both wholesale price contract and revenue sharing contract are widely studied and used in theory and practice; we can thus compare our results directly with previous literature such as Pan et al. [8] and Zhao and Shi [10]. Second, these contract types represent the "simple" contract and the "sophisticated" contract, respectively.

Under a wholesale price contract, w_i is the wholesale price of product i, p_i is the retail price, and u_i is the unit profit margin of retailer i, where $u_i = p_i - w_i$. Under a revenue sharing contract, the retailer keeps a portion of channel revenue ($\phi_i p_i$), and $(1-\phi_i)p_i$ is the fraction the supplier earns. Moreover, as Pan et al. [8] have pointed out, the supplier who chooses a revenue-sharing contract should make sure that the retailer's profit for each unit of product sold is not less than the marginal profit under a wholesale price contract, that is, $\phi_i p_i - w_i \geq u_i$. As the leader of a supply chain, service providers are authorized to decide whether wholesale price contract or revenue sharing contract should be offered to the exclusive retailer. Similar assumption has been made by Pan et al. [8]. Combing the suppliers' choices, in the two supply chains, there will be four possible scenarios: WW, WR, RW, and RR, where WW denotes that two supply chains are restricted to choose the wholesale price contract; WR means that supplier 1 (M_1) chooses the wholesale-price contract, while supplier 2 (M_2) chooses the revenue sharing contract. RW and RR are similar. Our model includes a two-level game. On the one hand, there is a Stackelberg game between SC_1 and SC_2. The market leader SC_1 determines his/her contract strategies to maximize its own profit, anticipating the follower SC_2's possible actions. On the other hand, the individual members of each supply chain maximize their own profits, and there is a Stackelberg relationship between the supplier and the retailer.

In particular, similar to Xiao and Choi [40] and Choi et al. [49], we model the risk attitudes of the suppliers with mean-variance utility function as follows:

$$U^l_{M_i}\left(\pi^l_{M_i}\right) = E\left(\pi^l_{M_i}\right) - \lambda_i \operatorname{Var}\left(\pi^l_{M_i}\right), \tag{2}$$

where $U^l_{M_i}(\pi^l_{M_i})$ denotes the utility function of supplier i and $E(\pi^l_{M_i})$ and $\operatorname{Var}(\pi^l_{M_i})$ represent the mean and the variance of supplier i's expected profit, respectively. l = WW, WR, RW, and RR and the parameter $\lambda_i \geq 0$ denotes the constant absolute risk aversion of supplier i. The larger λ_i is, the more conservative supplier i's behavior will be.

4. Equilibrium Solutions

In this section, we derive the close form solutions for different contract structures, including WW, WR, RW, and RR. We use backward induction to find the equilibriums for all of the participants.

4.1. Symmetric Subgame 1: WW. In the WW case, both M_1 and M_2 are restricted to choose wholesale price contracts. We consider SC_2's decisions first. Given wholesale price w_2^{WW}, R_2 determines u_2^{WW} (note that $u_i^{WW} = p_i^{WW} - w_i^{WW}$, so R_i's decision of u_i^{WW} is equivalent to the decision of retail price p_i^{WW}) by maximizing his/her expected profit:

$$\max_{u_2^{WW}\geq 0} \pi_{R2}^{WW} = u_2^{WW}\left[\overline{A_2} - \left(u_2^{WW} + w_2^{WW}\right) + \theta\left(u_1^{WW} + w_1^{WW}\right)\right]. \tag{3}$$

It can be shown that π_{R2}^{WW} is concave with respect to u_2^{WW}; thus, the best response u_2^{WW} can be written as

$$u_2^{*WW}\left(w_2^{WW}, u_1^{WW}, w_1^{WW}\right) = \frac{\overline{A_2} - w_2^{WW} + \theta\left(u_1^{WW} + w_1^{WW}\right)}{2}. \tag{4}$$

Anticipating the R_2's actions, M_2 determines wholesale price to maximize his/her utility:

$$\max_{w_2^{WW}\geq 0} U_{M_2}^{WW}\left(\pi_{M_2}^{WW}\right) = \left(w_2^{WW} - c_2\right) \times D_2\left(u_2^{*WW}\right) - \frac{1}{2}\lambda_2\sigma_2^2\left(w_2^{WW} - c_2\right)^2, \tag{5}$$

where c_i denotes the manufacturing cost of product i, $i = 1, 2$. The first-order condition of (5) leads to the best response wholesale price:

$$w_2^{*WW}\left(u_1^{WW}, w_1^{WW}\right) = c_2 + \frac{\overline{A_2} - c_2 + \theta\left(u_1^{WW} + w_1^{WW}\right)}{2 + 2\lambda_2\sigma_2^2}. \tag{6}$$

We now turn to SC_1's decisions. Anticipating SC_2's actions, M_1 and R_1 of SC_1 determine their best responses sequentially. Using similar event sequence, we derive the equilibriums with respect to u_1^{WW} and w_1^{WW}. For the convenience, let $\Delta_i = 1 + \lambda_i\sigma_i^2$, $i = 1, 2$. We have

$$\begin{aligned} u_1^{*WW}\left(w_1^{WW}\right) &= \frac{4\Delta_2\overline{A_1} + 2\theta\Delta_2\overline{A_2} + \theta\left(2\Delta_2 c_2 - c_2 + \overline{A_2}\right)}{4\Delta_2\theta^2 + 2\theta^2 - 8\Delta_2} + \frac{\left(\theta^2 + 2\Delta_2\theta^2 - 4\Delta_2\right)w_1^{WW}}{4\Delta_2\theta^2 + 2\theta^2 - 8\Delta_2}, \\ w_1^{*WW} &= \frac{2\Delta_2\left(2c_1 + 2\overline{A_1} + 4c_1\lambda_1\sigma_1^2 - c_1\theta^2 + \theta\overline{A_2} + \theta c_2\right)}{8\Delta_1\Delta_2 - 2\theta^2\left(2\Delta_2 + 1\right)} + \frac{\theta\left(\overline{A_2} - c_2 - c_1\theta\right)}{8\Delta_1\Delta_2 - 2\theta^2\left(2\Delta_2 + 1\right)}. \end{aligned} \tag{7}$$

We use superscript WW to represent the optimal solutions in the WW case. Substituting w_1^{*WW} back to $u_1^{*WW}(w_1^{WW})$, $w_2^{*WW}(u_1^{WW}, w_1^{WW})$, $u_2^{*WW}(w_2^{WW}, u_1^{WW}, w_1^{WW})$, we get all the equilibriums u_1^{*WW}, w_2^{*WW}, u_2^{*WW} and the corresponding production quantities and profits.

4.2. Hybrid Subgame 1: WR. The WR case represents that the M_1 chooses wholesale price contract while the M_2 chooses revenue sharing contract. Similar to the WW case, we first analyze SC_2's decisions. As pointed out by Pan et al. [8], under a revenue sharing contract, the supplier M_2 should make sure that $\phi_2 p_2^{WR} - w_2^{WR} \geq \nu_2$, where $\nu_2 = u_2^{*WW}$; otherwise, the retailer will prefer a wholesale price contract to a revenue sharing contract. As for M_2, he/she expects that the retailer shares the revenue as small as possible such that $\phi_2^{WR} p_2^{WR} - w_2^{WR} = \nu_2$. Therefore, given ϕ_2^{WR}, the decision of p_2^{WR} and w_2^{WR} is equivalent. The profit maximization problem of M_2 is

$$\begin{aligned} \max_{p_2^{WR}\geq 0} U_{M_2}^{WR}\left(\pi_{M_2}^{WR}\right) &= \left(p_2^{WR} - c_2 - \nu_2\right)\left[\overline{A_2} - p_2^{WR} + \theta\left(w_1^{WR} + u_1^{WR}\right)\right] \\ &\quad - \frac{1}{2}\lambda_2\sigma_2^2\left(p_2^{WR} - c_2 - \nu_2\right)^2. \end{aligned} \tag{8}$$

The first-order condition of (8) leads to the best response p_2^*:

$$p_2^{*WR}\left(u_1^{WR}, w_1^{WR}\right) = c_2 + \nu_2 + \frac{\overline{A_2} - c_2 - \nu_2 + \theta\left(u_1^{WR} + w_1^{WR}\right)}{2 + \lambda_2\sigma_2^2}. \tag{9}$$

Then, M_1 and R_1 of SC_1 determine their best response conditional on the response of SC_2, $p_2^{*WR}(u_1^{WR}, w_1^{WR})$. It can be shown that

$$u_1^{*WR}\left(w_1^{WR}\right) = \frac{(1+\Delta_2)\left(\overline{A_1} - w_1^{WR}\right)}{2\left[(1+\Delta_2) - \theta^2\right]} + \frac{\theta\left(\overline{A}_2 + c_2\Delta_2 + \nu_2\Delta_2 + \theta w_1^{WR}\right)}{2\left[(1+\Delta_2) - \theta^2\right]},$$

$$w_1^{*WR} = \frac{\theta\left[\overline{A_2} - c_1\theta + \Delta_2(c_2+\nu_2)\right]}{2\left[\Delta_1(1+\Delta_2) - \theta^2\right]} + \frac{(1+\Delta_2)\left(\overline{A_1} - c_1 + 2c_1\Delta_1\right)}{2\left[\Delta_1(1+\Delta_2) - \theta^2\right]}. \quad (10)$$

We use superscript WR to represent the optimal solutions in the WR case. Substituting w_1^{*WR} into $u_1^{*WR}(w_1^{WR})$ and $p_2^{*WR}(u_1^{WR}, w_1^{WR})$ yields all the equilibriums in the WR case.

4.3. Hybrid Subgame 2: RW. The RW case represents that M_1 chooses revenue sharing contract while M_2 chooses wholesale price contract. The detail of solution process is similar to that in WR case. The best responses u_2^{RW}, w_2^{RW}, and p_1^{RW} are given by

$$u_2^{*RW}\left(w_2^{RW}, p_1^{RW}\right) = \frac{\overline{A_2} - w_2^{RW} + \theta p_1^{RW}}{2}$$

$$w_2^{*RW}\left(p_1^{RW}\right) = c_2 + \frac{\overline{A_2} - c_2 + \theta p_1^{RW}}{2 + 2\lambda_2\sigma_2^2},$$

$$p_1^{*RW} = \frac{2\Delta_2\left(2\overline{A_1} + \theta\overline{A_2}\right) + \theta\left(2c_2\Delta_2 - c_2 + \overline{A_2}\right)}{2\left[2\Delta_1\Delta_2 - \theta^2(1+2\Delta_2) + 2\Delta_2\right]}, - \frac{(c_1+\nu_1)\left(2\theta^2\Delta_2 + \theta^2 - 4\Delta_2 + 4\Delta_2\lambda_1\sigma_1^2\right)}{2\left[2\Delta_1\Delta_2 - \theta^2(1+2\Delta_2) + 2\Delta_2\right]}, \quad (11)$$

where $\nu_1 = u_1^{*WW}$. Substituting p_1^{*RW} into $w_2^{*RW}(p_1^{RW})$ and $u_2^{*RW}(w_2^{RW}, p_1^{RW})$ yields all the equilibriums in the RW case.

4.4. Symmetric Subgame 2: RR. The RR case represents that both M_1 and M_2 choose revenue sharing contract. Note that M_1 and M_2 have to guarantee R_1 and R_2 to obtain at least the same profits as those they can obtain under a wholesale price contract. We derive the best response retail price of SC_2 first as

$$p_2^{*RR}\left(p_1^{RR}\right) = c_2 + \nu_2 + \frac{\overline{A_2} - c_2 - \nu_2 + \theta p_1^{RR}}{2 + 2\lambda_2\sigma_2^2}. \quad (12)$$

Anticipating the SC_2's actions, SC_1 determines his best response p_1^{RR} as

$$p_1^{*RR} = \frac{2\Delta_2\overline{A_1} - (c_1+\nu_1)\left(\theta^2 - 2\Delta_2 + 4\Delta_2\lambda_1\sigma_1^2\right)}{2\left(2\Delta_2 - \theta^2 + \Delta_2\lambda_1\sigma_1^2\right)} + \frac{\theta\left[(2\Delta_2 - 1)(c_2+\nu_2) + \overline{A_2}\right]}{2\left(2\Delta_2 - \theta^2 + \Delta_2\lambda_1\sigma_1^2\right)}. \quad (13)$$

Substituting p_1^{*RR} back yields all the equilibriums in the RR case.

5. Analysis of Contract Strategies

In this section, we compare the suppliers' equilibrium profits under different cases and identify the conditions under which they choose wholesale price contract or revenue sharing contract. We also would like to find the equilibrium contract structure. According to previous literatures such as Choi et al. [38], Xiao and Choi [40], and Pan et al. [8], the degree of product substitutability and risk attitudes are the critical factors; therefore, we assume a symmetric scenario in the sense that $\overline{A_1} = \overline{A_2} = \overline{A}$, $\sigma_1 = \sigma_2 = \sigma$, $c_1 = c_2 = c$. Then their impacts are ruled out, leaving M_1 and M_2 differing from each other via their risk attitudes: λ_1 and λ_2. We find that under this assumption, all of the optimal $U_{M_i}^{l}$, $l = $ WW, WR, RW, RR, $i = 1, 2$, can be expected as a function of a common factor $(\overline{A} - c + c\theta)^2$. Then, the comparison of the utilities depends only on three parameters: the degree of product substitutability (θ), the degree of risk aversion (λ_i), and the variance of stochastic market base (σ). Moreover, the parameters λ_i and σ always arise together with the form of $\lambda_i\sigma^2$, therefore, to focus on the impact of risk aversion, we first set σ as a given positive constant. Next we investigate how the degree of product substitutability and the risk attitudes of the suppliers influence the supply chain contract decisions and compare our results with the previous literature. We analyze symmetric and asymmetric risk attitudes of the two suppliers.

5.1. Impact of Product Substitutability (Risk-Neutral Suppliers). To obtain analytical comparison results with respect to the degree of product substitutability, we assume $\lambda_1 = \lambda_2 = \lambda = 0$, in other words, both suppliers are risk-neutral. We analyze the incentive of a supplier to switch from wholesale price contract (revenue sharing contract) to revenue sharing contract (wholesale price contract). Lemmas 1 and 2 describe M_1's (M_2's) preference given M_2's (M_1's) choice.

Lemma 1. *From M_1's perspective, (i) if M_2 chooses wholesale price contract, then it is optimal for M_1 to choose revenue sharing contract. θ has no impact. (ii) If M_2 chooses revenue sharing contract, then M_1 is better to choose revenue sharing contract when $0 < \theta \leq 0.69$, otherwise, wholesale price contract.*

Table 1: Stability analysis of risk-neutral suppliers' perspectives.

Original structure	M_1 New structure	M_1 Change in profits	M_2 New structure	M_2 Change in profits
WW	RW	$0 < \theta < 1, \uparrow^*$	WR	$0 < \theta \leq 0.55, \uparrow$ $0.55 < \theta < 1, \downarrow$
RW	WW	$0 < \theta < 1, \downarrow$	RR	$0 < \theta \leq 0.51, \uparrow$ $0.51 < \theta < 1, \downarrow$
WR	RR	$0 < \theta \leq 0.69, \uparrow$ $0.69 < \theta < 1, \downarrow$	WW	$0 < \theta \leq 0.55, \downarrow$ $0.55 < \theta < 1, \uparrow$
RR	WR	$0 < \theta \leq 0.69, \downarrow$ $0.69 < \theta < 1, \uparrow$	RW	$0 < \theta \leq 0.51, \downarrow$ $0.51 < \theta < 1, \uparrow$

*When the utility of new structure is more than the original structure, we denote it by "↑", otherwise, "↓".

Lemma 2. *From M_2's perspective, (i) if M_1 chooses wholesale price contract, then M_2 is better off with revenue sharing contract under $0 < \theta \leq 0.55$, otherwise, wholesale price contract. (ii) If M_1 chooses revenue sharing contract, then M_2 is better off with revenue sharing contract under $0 < \theta \leq 0.51$, otherwise, wholesale price contract.*

Based on Lemmas 1 and 2, Table 1 summarizes the suppliers' incentives to switch from one structure to another one.

This leads to the equilibrium contract strategies summarized in Theorem 3.

Theorem 3. *Assuming risk-neutral suppliers, the equilibrium contract strategies for the two competing supply chains is RR if θ is smaller than $\hat{\hat{\theta}}$, and RW otherwise.*

Thus, there exists a unique threshold value of $\hat{\hat{\theta}}$ which guides M_2's choice of contract type. When competition is tense, that is, θ is large, it is better for M_2 to adopt a different contract type. As the degree of product substitutability decreases, competition becomes not so tense, then it is optimal for the market follower to copy the leader's strategy: adopting a revenue sharing contract. Note that, with simultaneous competition, Pan et al. [8] show that it is optimal for the suppliers to choose revenue-sharing contract when there is no tense competition. Here, we verify this finding in a sequential competition environment. The underlying reason lies in their different preferences towards the "low-price-high-volume" strategy. To further explain our findings, we compare the retail prices in different scenarios and have the following lemma.

Lemma 4. *Assuming risk-neutral suppliers, the following orders always hold:*

$$p_1^{*RR} - p_1^{*WW} < 0; \quad p_2^{*RR} - p_2^{*WW} < 0; \quad p_2^{*RR} < p_1^{*RR}. \tag{14}$$

As we show in Lemma 4, the retail price under revenue sharing contract is lower than that under wholesale price contract. Market leader has to decide a low retail price to snatch a large market share, while market follower has the right of cutting down the leader's retail price and hence

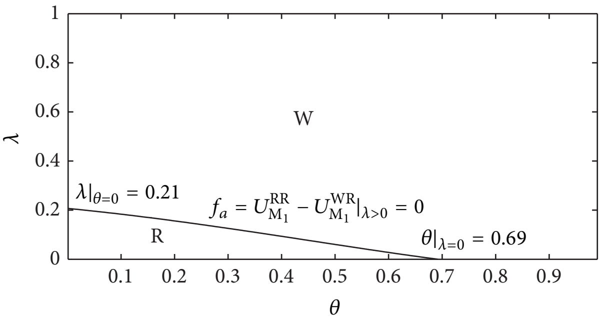


Figure 1: M_1's preference given M_2 choosing revenue sharing contract ($\lambda > 0$).

enjoys a second-mover advantage in the price competition ($p_2^{*RR} < p_1^{*RR}$). This behavior will not occur when there is no information interaction under simultaneous competition, as Pan et al. [8] have studied. On the other hand, facing tense competition, the follower's profits with "low-price-high-volume" strategy may not compensate its loss due to low retail price. Thus, the follower has to switch to the wholesale price contract so as to enjoy a higher profit margin. This is consistent with Pan et al. [8].

5.2. Impact of Product Substitutability and Risk Attitudes (Identical Risk-Averse Suppliers). We now turn to the case $\lambda_1 = \lambda_2 = \lambda > 0$, that is, both suppliers are risk-averse, but they are identical. With this assumption, suppliers' choices are influenced by the degree of product substitutability and risk aversion. We find that given M_2's choice of wholesale price contract, M_1 always prefers revenue sharing contract. This is consistent with our previous finding shown in Lemma 1. However, it is not easy to analytically find M_1's preference when M_2 chooses revenue sharing contract. We conduct numerical studies and observe similar curves; see Figure 1 for illustration. The parameters are $\overline{A} = 1$, $\sigma = 1$, $c = 0.5$, $\lambda \in (0,1)$, and $\theta \in (0,1)$.

The rectangular area $\{(\theta, \lambda) \mid 0 < \theta < 1, 0 < \lambda < 1\}$ is divided into two distinct regions by the curve $f_a = U_{M_1}^{RR} - U_{M_1}^{WR}|_{\lambda>0} = 0$, where "W" denotes that M_1 prefers wholesale price contract and "R" denotes that M_1 prefers

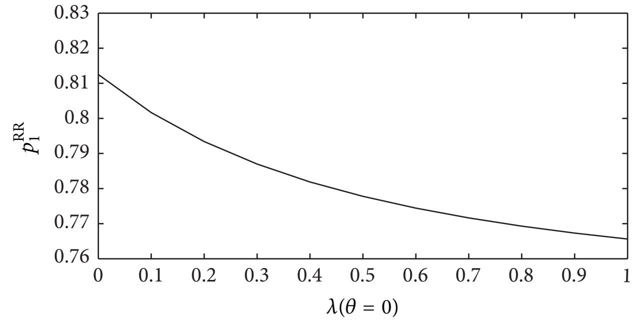


Figure 2: Impact of λ on p_1^{*RR}.

revenue sharing contract. From Figure 1 we can obtain the following observations. (i) If $\lambda = 0$, that is, both suppliers are risk-neutral, given M_2 choosing revenue sharing contract, M_1 prefers revenue sharing contract when $0 < \theta \leq 0.69$, otherwise, wholesale price contract. This is consistent with Lemma 1. (ii) As the degree of risk aversion increases (e.g., $\lambda > 0.21$), given M_2 choosing revenue sharing contract, the wholesale price contract is attractive for M_1 no matter what the value of θ is. This observation can be explained as follows. When M_1 becomes more and more conservative (λ becomes larger and larger), it can be observed that p_1^{*RR} is decreasing (see Figure 2). Thus, to avoid the retail price dropping too much, M_1 tends to replace the revenue sharing contract by the wholesale price contract, owing to the fact that the retail price under the revenue sharing contract is always lower than that under the wholesale price contract (see Lemma 4 and Figure 3).

We now also illustrate the follower M_2's preference given M_1's choice, by assuming the same numerical parameters.

Similar observations can be found in Figures 4(a) and 4(b). "W" denotes that M_2 prefers wholesale price contract, and "R" denotes that M_2 prefers revenue sharing contract. We can obtain the following observations. (i) In Figure 4(a), if $\lambda = 0$, given M_1 choosing wholesale price contract, M_2 prefers revenue sharing contract, when $0 < \theta \leq 0.55$, otherwise, wholesale price contract. In Figure 4(b), if $\lambda = 0$, given M_1 choosing revenue sharing contract, M_2 prefers revenue sharing contract, when $0 < \theta \leq 0.51$, otherwise, wholesale price contract. This is consistent with Lemma 2. (ii) As the degree of risk aversion increases (e.g., $\lambda > 0.19$), regardless of M_1 choosing wholesale price contract or revenue sharing contract, it is optimal for M_2 to choose the wholesale price contract. A possible reason is that as M_2 becomes more and more conservative, he/she tends to avoid the retail price dropping too much (p_2^{*RR} is decreasing in λ). We obtain equilibrium contract strategies illustrated by Figure 5.

In Figure 5, "RW" denotes that M_1 chooses revenue sharing contract and M_2 chooses wholesale price contract. "RR" denotes that both M_1 and M_2 choose revenue sharing contract. We first verify the findings summarized in Theorem 3, that is, "RR" is the equilibrium contract strategy when $0.51 < \theta < 1$, and "RW" is the equilibrium contract strategy when $0.51 < \theta < 1$. Then, we find that if the degree of risk aversion and product substitutability are both low, "RR" sustains as the equilibrium contract strategy. Recall that when the competition intensity is low, suppliers tend to keep a relatively low retail price to stimulate demand by adopting revenue sharing contract. However, when suppliers are very conservative, as we have observed, the retail price will become too low. Consequently, M_2 may switch to the wholesale price contract, which guarantees a larger profit margin. This differentiates our work from that of Pan et al. [8], in which the impact of risk attitude is not studied.

5.3. Verification with Asymmetric Risk Attitudes (Heterogeneous Risk-Averse Suppliers). In this subsection, we assume that the risk aversion between two suppliers is asymmetric, that is, $\lambda_1 \neq \lambda_2$. Interestingly, given M_2's choice of wholesale price contract, it can be shown that M_1 always prefers revenue sharing contract. Combining our previous findings with symmetric risk attitudes, this phenomenon is rather robust.

Consider the following:

$$\begin{aligned} & U_{M_1}^{RW} - U_{M_1}^{WW}\Big|_{\lambda_1 \neq \lambda_2} \\ &\quad = \left(\overline{A} - c + c\theta\right)^2 \left(4 + 4\lambda_2\sigma^2 - 2\lambda_2\theta^2\sigma^2 - 3\theta^2\right)^2 \\ &\qquad \times \left(2\theta\lambda_2\sigma^2 + 3\theta + 4 + 4\lambda_2\sigma^2\right)^2 \\ &\qquad \times \left(256\Gamma\left(1 + \lambda_2\sigma^2\right)\left(\Gamma + 2\lambda_1\lambda_2\sigma^4 + 2\lambda_1\sigma^2\right)^2\right)^{-1} > 0, \end{aligned} \tag{15}$$

where $\Gamma = 4 - 2\lambda_2\theta^2\sigma^2 + 2\lambda_1\lambda_2\sigma^4 - 3\theta^2 + 4\lambda_2\sigma^2 + 2\lambda_1\sigma^2$.

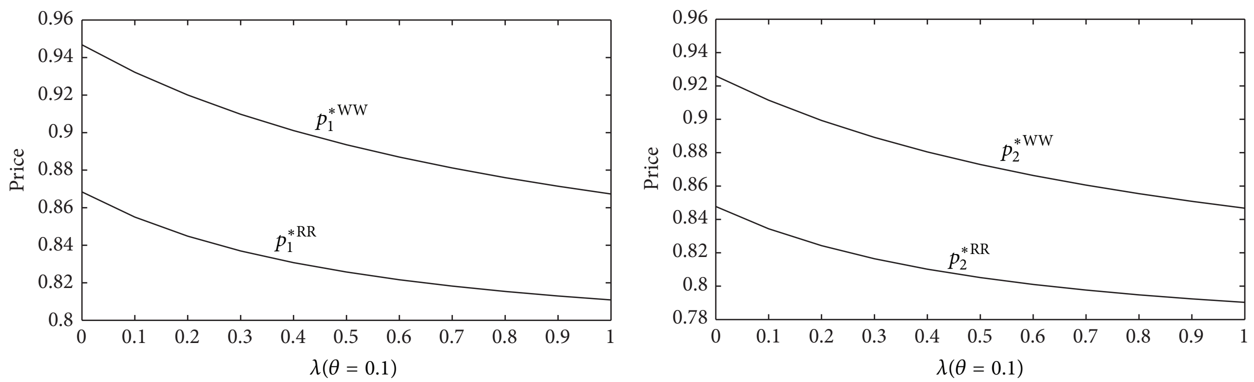
Clearly, $U_{M_1}^{RW} - U_{M_1}^{WW}|_{\lambda_1 \neq \lambda_2} > 0$ always holds regardless of the value of λ_1, λ_2, and θ.

Now, we analyze M_1's preference given M_2's choice of revenue sharing contract. The parameters are $\overline{A} = 1$, $\sigma = 1$, $c = 0.5$, $\theta \in (0, 1)$, $\lambda_2 \in (0, 20)$, and $\lambda_1 = 0, 0.1$.

As observed from Figures 6(a) and 6(b), when λ_1 increases, it becomes more likely for M_1 to choose wholesale price contract. A possible reason is that M_1 tend to avoid sharply lowered retail price. Another observation is when λ_2 is large, it is better for M_1 to choose revenue sharing contract. The reason lies in M_2's possible choice of wholesale price contract when he/she is very conservative. It has been shown that M_1's optimal strategy is revenue sharing, given M_2's choice wholesale price contract.

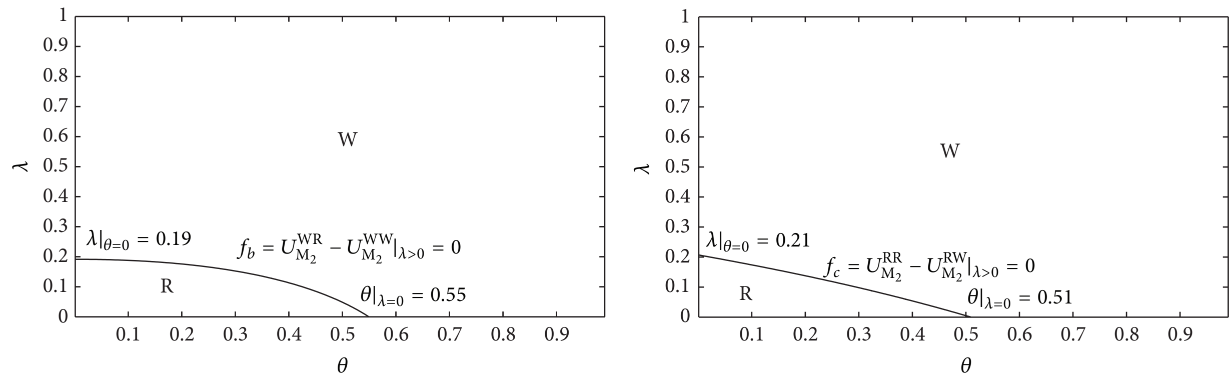
We now also illustrate M_2's preference given M_1's choice under asymmetric risk attitudes. First of all, assuming M_1 chooses wholesale price contract, we use the following parameters: $\overline{A} = 1$, $\sigma = 1$, $c = 0.5$, $\theta \in (0, 1)$, $\lambda_1 \in (0, 20)$, and $\lambda_2 = 0, 0.1$. As seen from Figures 7(a) and 7(b), the outcomes can be classified into two distinct regions by the critical curves $f_g = 0$, $f_h = 0$, $f_i = 0$, respectively. Again, with larger λ_2, M_2 tends to choose wholesale price contract. Assuming M_1 chooses revenue sharing contract, the observation is similar.

5.4. Verification with Stochastic Market Variance. In this sub-section, we investigate the role that stochastic market variance (σ) plays in the suppliers' contract choices. First,



(a) (b)

Figure 3: Comparison of the retail prices with $\lambda_1 = \lambda_2 = \lambda > 0$.



(a) (b)

Figure 4: (a) M_2's preference given M_1's choice of wholesale price contract ($\lambda > 0$). (b) M_2's preference given M_1's choice of revenue sharing contract ($\lambda > 0$).

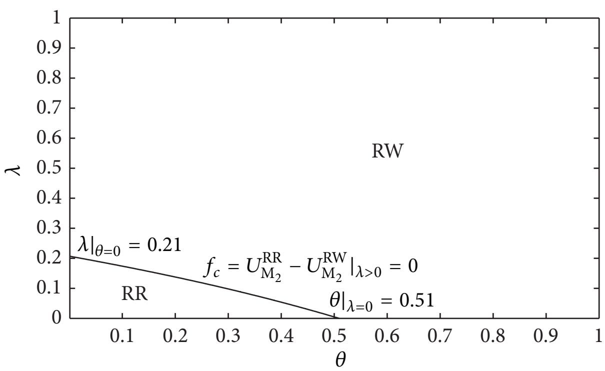
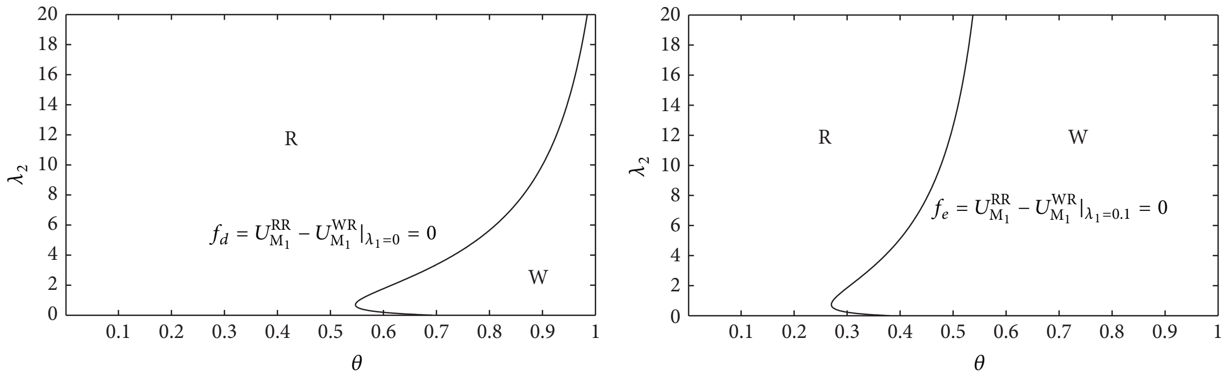


Figure 5: Equilibrium contract strategies of suppliers ($\lambda > 0$).

we examine M_1's preference. Given M_2's choice of wholesale price contract, it can be shown that M_1 always prefers revenue sharing contract. Second, we illustrate M_1's preference given M_2's choice of revenue sharing contract and M_2's preference given M_1's choice of wholesale price contract or revenue sharing contract. The parameters are $\overline{A} = 1$, $c = 0.5$, $\theta \in (0, 1)$, $\sigma \in (0, 2)$, and $\lambda = 0.1, 0.5$. As seen from Figure 8(a), the rectangular area $\{(\theta, \sigma) \mid 0 \leq \theta < 1, 0 \leq \sigma < 2\}$ is divided into two parts by the curves $f_j(\theta, \sigma) = 0$, $f_k(\theta, \sigma) = 0$, $f_l(\theta, \sigma) = 0$. In the upper right part (i.e., region I) of the area, all functions $f_j(\theta, \sigma)$, $f_k(\theta, \sigma)$, and $f_l(\theta, \sigma)$ are less than zero. In contrast, in the lower left part (i.e., region II) of the area, all functions $f_j(\theta, \sigma)$, $f_k(\theta, \sigma)$, and $f_l(\theta, \sigma)$ are greater than zero. These observations imply that if the competition density is small and the market variance is also small, revenue sharing contract will be the best option for the two suppliers. This indicates a threshold policy with respect to σ for the suppliers to select the contract type. Figure 8(b) shows similar curves. Possible explanation is as follows. Recall that the comparison of the suppliers' utilities depends only on θ, λ_i, and σ, and the parameters λ_i and σ always arise together with the form of $\lambda_i \sigma^2$. Thus, keeping λ_i as a constant, the impact of σ is similar to that of λ_i shown previously (threshold type).

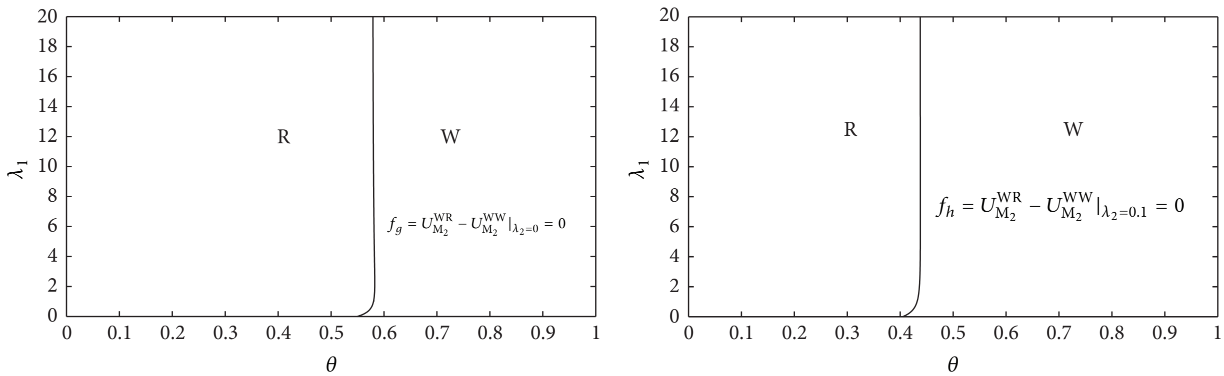
6. Conclusion Remarks and Future Research

In this paper, we consider two sequential-moving supply chains, where each supply chain is composed of one risk-averse supplier and one risk-neutral retailer. We investigate the impact of competition intensity and risk aversion on



(a) (b)

FIGURE 6: (a) M_1's preference given M_2's choice of revenue sharing contract ($\lambda_1 = 0$). (b) M_1's preference given M_2's choice of revenue sharing contract ($\lambda_1 = 0.1$).



(a) (b)

FIGURE 7: (a) M_2's preference given M_1's choice of wholesale price contract ($\lambda_2 = 0$). (b) M_2's preference given M_1's choice of wholesale price contract under ($\lambda_2 = 0.1$).

the contract choice problems. We study by deriving and comparing the equilibriums under the symmetric subgames (including WW and RR case) and hybrid subgames (including WR and RW case), respectively.

Our paper differs from the previous studies of contract choice in two important ways. First, we consider contract choice in the context of two sequential-moving supply chains. Second, we assume that two competing suppliers are risk-averse and analyze how risk aversion of one supplier affects his/her and rival's decisions. We show that the retail prices under revenue sharing contract are lower than that under wholesale price contract. Thus, when the degrees of product substitutability and risk aversion are beyond a threshold value; the second-moving supplier may suffer from too low retail price and hence switch from revenue sharing contract to wholesale price contract. These results complement the current literature that assumes players are risk-neutral and help to explain why sophisticated contracts (e.g., revenue sharing contract) have not completely eliminated the Pareto inferior wholesale-price contract in reality.

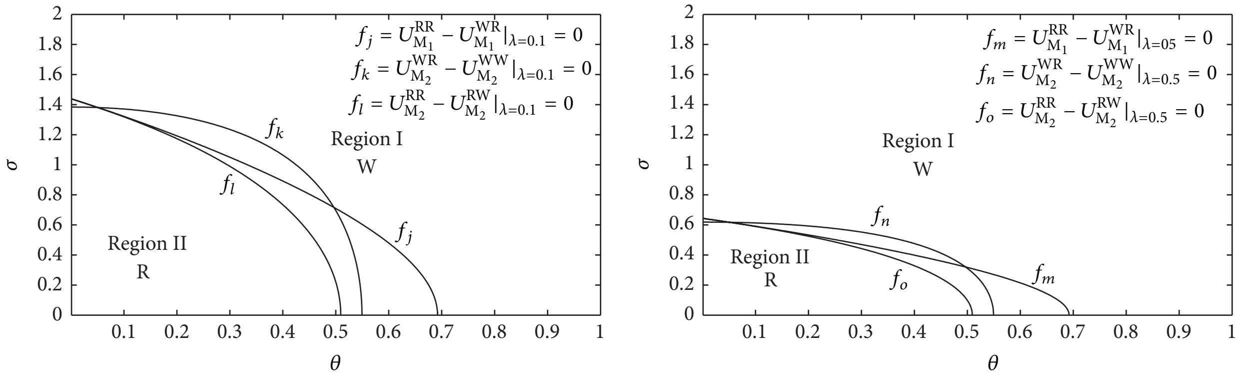
We would like to mention two potential directions for future research. First, it is worthy studying the other risk aversion models (e.g., [31, 50]) to see whether our insights hold or not. Second, we have assumed that information is common knowledge. Relaxing this assumption would lead to studies with asymmetric information and would generate insights on the value of information. As Ha and Tong [26] have pointed out, the value of information may significantly influence the choice of contract type.

Appendices

A. Proof of Lemma 1

Proof. (i) Given M_2's choice of wholesale price contract, we compare the utilities of M_1 under revenue sharing contract and wholesale price contract as follows:

$$U_{M_1}^{RW} - U_{M_1}^{WW}\Big|_{\lambda=0} = \frac{(3\theta+4)^2\left(\overline{A} - c + c\theta\right)^2}{256\left(4-3\theta^2\right)} > 0. \quad \text{(A.1)}$$



(a) (b)

FIGURE 8: (a) M_1's and M_2's preference given ($\lambda = 0.1$). (b) M_1's and M_2's preference given ($\lambda = 0.5$).

(ii) Given M_2's choice of revenue sharing contract, we compare the utilities of M_1 under revenue sharing contract and wholesale price contract, finding that

$$U_{M_1}^{RR} - U_{M_1}^{WR}\Big|_{\lambda=0} = \frac{\left(\overline{A} - c + c\theta\right)^2 \Phi_1(\theta)}{4096\,(2-\theta^2)\,(4-3\theta^2)^2}, \tag{A.2}$$

where

$$\begin{aligned}\Phi_1(\theta) &= 2048 + 1024\theta - 4736\theta^2 - 3712\theta^3 \\ &\quad + 1616\theta^4 + 2040\theta^5 + 441\theta^6 \\ &= 441(\theta - 1.22)(\theta - 0.69)(\theta + 1.18) \\ &\quad \times (\theta + 1.74)(\theta + 1.05)(\theta + 2.57).\end{aligned} \tag{A.3}$$

As $0 < \theta < 1$, we have $U_{M_1}^{RR} - U_{M_1}^{WR}|_{\lambda=0} \geq 0$ if $0 < \theta \leq 0.69$, otherwise, $U_{M_1}^{RR} - U_{M_1}^{WR}|_{\lambda=0} < 0$. □

B. Proof of Lemma 2

Proof. (i) Given M_1's choice of wholesale price contract, we compare the utilities of M_2 under revenue sharing contract and wholesale price contract and find that

$$U_{M_2}^{WR} - U_{M_2}^{WW}\Big|_{\lambda=0} = -\frac{\left(\overline{A} - c + c\theta\right)^2 \Phi_2(\theta)}{16384(8 - 10\theta^2 + 3\theta^4)^2}, \tag{B.1}$$

where

$$\begin{aligned}\Phi_2(\theta) &= 423\theta^8 + 1800\theta^7 - 23232\theta^6 - 56832\theta^5 + 122224\theta^4 \\ &\quad + 102912\theta^3 + 50176\theta^2 - 24576\theta - 16384 \\ &= 423(\theta - 0.55)(\theta - 1.28)(\theta - 6.7)(\theta + 1.07) \\ &\quad \times (\theta + 1.33)(\theta + 8.77)\left(\theta^2 + 1.62\theta + 0.66\right).\end{aligned} \tag{B.2}$$

As $0 < \theta < 1$, we have $U_{M_2}^{WR} - U_{M_2}^{WW}|_{\lambda=0} \geq 0$ if $0 < \theta \leq 0.55$.

Otherwise, $U_{M_2}^{WR} - U_{M_2}^{WW}|_{\lambda=0} \leq 0$.

(ii) Given M_1's choice of revenue sharing contract, we compare the utilities of M_2 with revenue sharing contract and wholesale price contract as follows:

$$U_{M_2}^{RR} - U_{M_2}^{RW}\Big|_{\lambda=0} = \frac{\left(\overline{A} - c + c\theta\right)^2 \Phi_3(\theta)}{4096(8 - 10\theta^2 + 3\theta^4)^2}, \tag{B.3}$$

where

$$\begin{aligned}\Phi_3(\theta) &= 81\theta^8 - 1224\theta^7 + 488\theta^6 + 10432\theta^5 + 6512\theta^4 \\ &\quad - 15232\theta^3 - 15104\theta^2 + 2048\theta + 4096 \\ &= 81(\theta - 0.51)(\theta - 14)(\theta - 3.69)(\theta - 1.29) \\ &\quad \times (\theta + 1.1)(\theta + 1.36)\left(\theta^2 + 1.93\theta + 1\right).\end{aligned} \tag{B.4}$$

As $0 < \theta < 1$, we have $U_{M_2}^{RR} - U_{M_2}^{RW}|_{\lambda=0} \geq 0$ if $0 < \theta \leq 0.51$, otherwise, $U_{M_2}^{RR} - U_{M_2}^{RW}|_{\lambda=0} \leq 0$. □

C. Proof of Lemma 4

Proof. Consider the following:

$$\begin{aligned} p_1^{*RR} - p_1^{*WW}\Big|_{\lambda=0} &= -\frac{\left(\overline{A} - c + c\theta\right)\left(32 + 40\theta + 4\theta^2 - 9\theta^3\right)}{32\left(8 - 10\theta^2 + 3\theta^4\right)} < 0, \\ p_2^{*RR} - p_2^{*WW}\Big|_{\lambda=0} &= -\frac{\left(\overline{A} - c + c\theta\right)\left(64 + 80\theta - 4\theta^2 - 20\theta^3 - 3\theta^4\right)}{64\left(8 - 10\theta^2 + 3\theta^4\right)} < 0,\end{aligned}$$

$$
\begin{aligned}
& p_2^{*RR} - p_1^{*RR}\Big|_{\lambda=0} \\
& \quad = -\frac{\theta^2\left(\overline{A}-c+c\theta\right)\left(60-2\theta-39\theta^2\right)}{64\left(8-10\theta^2+3\theta^4\right)} < 0, \\
& D_2^{*RR} - D_1^{*RR}\Big|_{\lambda=0} \\
& \quad = \frac{\theta^2\left(1+\theta\right)\left(\overline{A}-c+c\theta\right)\left(60-2\theta-39\theta^2\right)}{64\left(2-\theta^2\right)\left(4-3\theta^2\right)} > 0.
\end{aligned}
\tag{C.1}
$$

□

Acknowledgments

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Research Article

Service Quality of Online Shopping Platforms: A Case-Based Empirical and Analytical Study

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Customer service is crucially important for online shopping platforms (OSPs) such as eBay and Taobao. Based on the well-established service quality instruments and the scenario of the specific case on Taobao, this paper focuses on exploring the service quality of an OSP with an aim of revealing customer perceptions of the service quality associated with the provided functions and investigating their impacts on customer loyalty. By an empirical study, this paper finds that the "fulfillment and responsiveness" function is significantly related to the customer loyalty. Further analytical study is conducted to reveal that the optimal service level on the "fulfillment and responsiveness" function for the risk averse OSP uniquely exists. Moreover, the analytical results prove that (i) if the customer loyalty is more positively correlated to the service level, it will lead to a larger optimal service level, and (ii) the optimal service level is independent of the profit target, the source of uncertainty, and the risk preference of the OSP.

1. Introduction

Observing from the popular and growing trend of electronic commerce (e-commerce), there is no doubt that business such as those selling fashion-related products can now use the Internet to interact with customers and gain the competitive edge. The critical determinants of success in e-commerce cover not only the low price strategy but also its service quality (i.e., e-service quality) [1, 2]. In the literature, Zeithaml [2] defines e-service quality as an overall customer assessment and judgment of e-service delivery in the virtual marketplace. To measure the customer perceptions of service quality, the SERVQUAL model, which was first developed by Parasuraman et al. [3], has been widely adopted. The SERVQUAL model includes five dimensions, namely, tangibles, reliability, responsiveness, assurance, and empathy. Previous studies on the measurement of e-service quality focus on the "rewording" of the original scale items of the application of the SERVQUAL model. Yet, service researchers ought to pay extra attention to e-services in the field of service quality, because assessing service quality in e-commerce might be different from that in physical marketplace service [4]. As such, it is necessary to reformulate the SERVQUAL scale items in e-commerce context [5].

With the prevailing trend of online shopping in Hong Kong and China, http://taobao.com/ becomes one of the hottest online shopping platforms (OSPs) despite its keen competition with internationally renowned companies such as yahoo auction and eBay as well as domestic companies such as Jingdong and Dangdang. Yahoo auction and eBay enable consumers to search for products globally and hence have their niche. However, it is interesting to observe that http://taobao.com/ has successfully penetrated into the Hong Kong and Macau e-tailing markets by its price leadership, special features, and a wide product range. More importantly, http://taobao.com/ also enjoys the geographic advantage of pooling local sellers together with a low local delivery cost, and convenient and unobstructed communication with seller. The exclusive functions featured in http://taobao.com/ include (i) Searching Functions, (ii) Guarantee of Trade Safety, (iii) Aliwangwang, (iv) Alipay, and (v) Taodot. All these features of http://taobao.com/ are then able to

Table 1: Featured functions of http://taobao.com/.

Featured functions	Descriptions
Search function	Search by keywords, show items in icons or lists or by default arrangement, sort searching product according to price, credibility, transaction volume, and store location, and lastly sort the production categories into brand new, hot items, product from Taobao mall, second handed, or auction
Guarantee of trade safety	Each seller of http://taobao.com/ is required to sign the Guarantee of Trade Safety Agreement which ensures that s/he commits to provide guaranteed transaction service to customers. Customers are able to request for refund if they receive any incorrect goods. The terms of guarantee include 7 days of unconditional return, no counterfeits, 3 times refund for counterfeits, maintenance within 30 days of purchase, and express delivery
Aliwangwang	It is the pervasive communication between buyer and seller prior to the purchase through its embedded proprietary instant chat program. Customers can enquire about products, engage in bargaining, display of product image in the messenger window, and make queries of the delivery status and logistic message
Alipay	It is an escrow-based online payment platform. It aims to enable fast money transaction with higher security and privacy
Taodot	It is an online platform that provides the value adding service for Alipay users. This convenient tool enables Taobao users who do not have an RMB account to pay for their online transactions

differentiate it from other competing OSPs and obtain its niche. Table 1 summarizes the major featured functions of http://taobao.com/.

For the research goal and contribution: first, by modifying the SERVQUAL model, this paper attempts to derive the instrument dimensions of e-service quality from the functions featured on http://taobao.com/ and to develop a research model to examine how its e-service quality dimensions affect the customer loyalty. To achieve this research objective, empirical data is collected from 195 online consumers (who have purchased fashion products (as a remark, in this paper, in order to be more concrete, we focus on examining the consumers' experience in purchasing fashion products) on http://taobao.com/). Second, based on the empirical finding, this paper further analytically develops and explores the optimal service decision on the function which is critical to the OSP. We believe that the research findings derived from this paper can contribute to the literature on service management, and they also provide important managerial insights on OSP's operations.

2. Literature Review

2.1. Review of Related Empirical Studies and Models. SERVQUAL is a multiple-item scale for measuring customer perception of service quality [3]. According to those five dimensions of the SERVQUAL model, we note that the tangibles dimension means the physical facilities and the appearance of personnel; the reliability dimension refers to the company's ability to perform the promised service dependably and accurately; the responsiveness dimension relates to the willingness to help customers and provide prompt service; the assurance dimension means the employee knowledge base which induces customer trust and confidence; and finally, the empathy dimension is about caring and individualized attention provided to customers by the service provider.

Recently, SERVQUAL scale has been employed to measure the corresponding system service quality in e-commerce [6, 7]. Most existing research on e-service quality measurement focuses on rewording the SERVQUAL scale items (e.g., [8–10]). The challenges in measuring web-based service quality mainly come from the differences between web-based and traditional customer service. Parasuraman and Grewal [4] suggest that revisions of the classical SERVQUAL dimensions are necessary because for web-based service quality, customers interact with technology rather than the traditional service personnel. Moreover, several studies have proposed that the SERVQUAL scale items should be reformulated in the online shopping context [5, 11].

As a remark, there are criticisms over the applicability of the SERVQUAL model. Among them, one important critique is on the fact that SERVQUAL's five dimensions are not universally applicable, and that the model fails to draw on established economic, statistical, and psychological theories [12]. In fact, the most significant criticism is the stability of dimensions and items across different industries [13]. Other criticisms include (i) its focus on the process of service delivery rather than the outcomes of the service encounter, and (ii) the probable respondent error from the reversed polarity of items in the scale in the consumer survey [12, 14]. Parasuraman et al. [15] also comment that the SERVQUAL is far from perfect as it is not general enough, but they argue that it does provide the basic skeleton for others to study service quality even though some degree of customization is probably needed (e.g., adding context-specific items). Although SERVQUAL's construct validity is being challenged, it is still the most widely used model employed to measure customer expectations and perceptions of service quality.

E-service quality is defined as how customers judge and evaluate the e-service being delivered to them [11]. On the evaluation of websites service quality, Parasuraman et al. [16] introduce the E-S-QUAL and E-RecS-QUAL scales which extend the SERVQUAL model in evaluating the service quality in the "online shopping context." E-S-QUAL is a multiple-item scale consisting of efficiency (8 items), system availability (4 items), fulfillment (7 items), and privacy (3 items) dimensions. E-S-QUAL provides the first formal definition of Web site service quality and it is a framework for consumer evaluation of electronic services. Notice that Zeithaml [2]

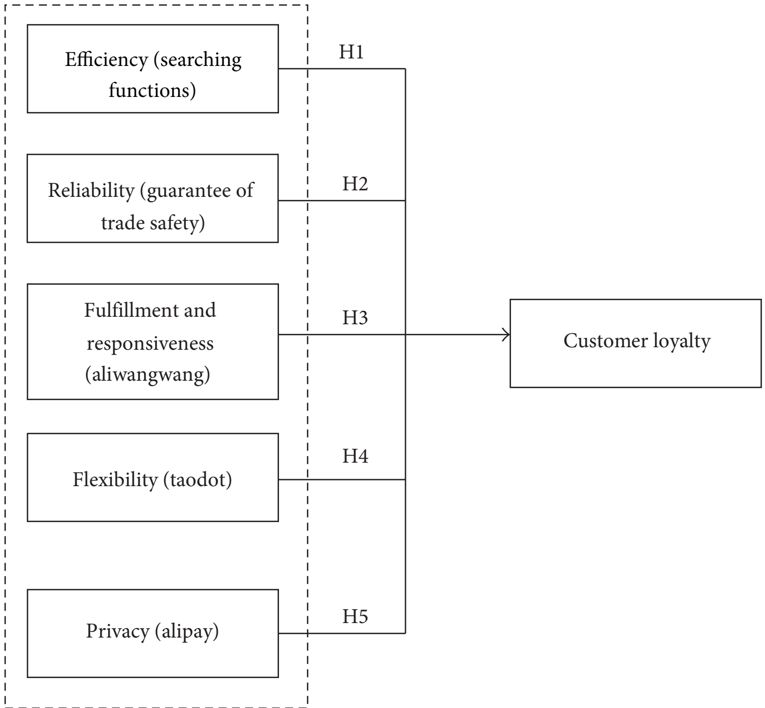

Figure 1: The proposed ESQ-OSP e-service quality model (Customized for http://taobao.com/'s case).

suggests that e-service quality is a multidimensional construct, and the electronic service recovery construct involves dimensions different from the core e-service quality. From e-service perspective, Parasuraman et al. [16] develop E-RecS-QUAL specifically to measure e-service quality, and this scale brings out a new perspective by utilizing valuable consumer information on service recovery issues. Actually, E-RevS-QUAL is a subscale that contains items focusing on handling service problems and inquiries, and being effective to those who used to have nonroutine behaviors on the online store. The basic E-S-QUAL Scale (relevant for a Web site's entire customer base) is a four-dimensional, 22-item scale, whereas E-RecS-QUAL (relevant for the portion of the customer base with recovery service experience) is a three-dimensional scale with 7 items. E-RevS-QUAL is used to stimulate and facilitate additional scholarly research on e-service quality and also assist practitioners in systematically assessing and improving e-service quality together with E-S-QUAL [16].

Notice that Parasuraman et al. [16] cannot evaluate the validity and psychometric properties of the scale in their study due to the insufficient sample size [17]. The main reason of the limited sample size is that the number of consumers who encounter service problems is relatively smaller than those who do not. Hence, their study is neither fully generic nor absolutely reliable. Moreover, E-S-QUAL is still a recent model (2005) which requires more time to verify. As a remark, observe that there are different versions of the modified E-S-QUAL for identifying the best e-service quality dimensions with the goal of examining different online businesses [17]. Given the limitation of the existing models, this paper proposes and explores an empirical model which incorporates the service quality dimensions extracted from SERVQUAL, E-S-QUAL, and E-RevS-QUAL which we believe can appropriately represent and study the service quality in an OSP such as http://taobao.com/. (For some more recent literature on service quality related to online shopping, see Rossitier [18], Ding et al. [19], Ha and Stoel [20], and Hu and Qiang [21].)

Regarding the research motivation, notice that prior studies in the literature have suggested that perceived service quality positively influences customer satisfaction and purchase intentions [22, 23]. However, to the best of our knowledge, in the context of online shopping, the impacts of how service quality dimensions affect overall e-service quality and the customer loyalty are still under-explored. This paper, hence, contributes to the literature by partially filling this research gap. Moreover, by building the empirical data-based analytical model, we explore the optimal service decision for the OSP and derive various important insights.

2.2. Empirical Hypotheses Development. After combining and synthesizing the existing construct of both service quality and e-service quality, based on the reviewed literature, we propose a perceived *e-service quality construct for an OSP* (ESQ-OSP) with http://taobao.com/ taken as the specific case. The proposed e-service quality model comprises 5 dimensions extracted from SERVQUAL, E-S-QUAL, and E-RevS-QUAL models, namely, efficiency, reliability, fulfillment and responsiveness, flexibility, and privacy as shown in Figure 1.

We now describe each dimension under the proposed ESQ-OSP model. First, in Parasuraman et al. [16], efficiency is reflected by whether the web site is simple to use, structured properly, and requires a minimum amount of information to be inputted by the customer. As a result, the online shipping platform is efficient if (i) it is easy for customers to find what they need, (ii) it enables customers to complete a transaction quickly, and (iii) the information provided by the web site is well organized. Thus, service quality of the searching functions of http://taobao.com/ can reflect its efficiency. Therefore, we have the following hypothesis.

H1: efficiency positively influences the customer loyalty of http://taobao.com/.

Reliability here refers to the consistency of performance and dependability of companies [3]. Reliability can make customers recognize the consistency and credibility of the company. In e-commerce, it is vital to make customers trust the promises that the OSP made. Obviously, "The Guarantee of Trade Safety" function of http://taobao.com/ is related to ensuring reliability, and it is hypothesized to influence customer loyalty:

H2: reliability positively influences the customer loyalty of http://taobao.com/.

Fulfillment refers to the extent in which the website's promises on order delivery and items availability are fulfilled. To be specific, fulfillment is reflected by whether the OSP can ensure items being available to delivery within an acceptable time frame and guarantee quick delivery as promised [16]. For http://taobao.com/, the functions of Aliwangwang are able to represent fulfillment. Responsiveness means the effective handling of problems and returns through the site. It provides customers with convenient options for returning items. The website promises to handle product returns and offers a meaningful guarantee so that customers would feel more comfortable [16]. As one of the service quality dimensions, contacting the seller through Aliwangwang reflects the responsiveness of http://taobao.com/. Therefore, it is hypothesized that:

H3: "fulfillment and responsiveness" positively influence the customer loyalty of http://taobao.com/.

Flexibility refers to the choice of payment methods. For example, when purchasing products in http://taobao.com/, Taodot is an innovative way for the users to pay online safely. Consumers can buy Taodot to deposit/add values in their Alipay account. It is safer and shows its flexibility if consumers are in Hong Kong or Macau. Then, the following hypothesis is proposed.

H4: flexibility positively influences the customer loyalty of http://taobao.com/.

Privacy refers to the degree to which the website is safe and customer information is well-protected. This dimension holds an important position in e-commerce and is critical in e-service. Customers perceive significant risks in the virtual environment of e-commerce stemming from the possibility of improper use of their personal financial data. The privacy function in http://taobao.com/ is mainly associated with Alipay. Accordingly, it is hypothesized that:

H5: privacy positively influences the customer loyalty of http://taobao.com/.

2.3. Review of Related Analytical Models. In the literature on analytical research, there are many recent studies related to the optimal service decision. For example, Xiao et al. [24] explore the optimal service strategy with optimal pricing decisions in a retail supply chain with risk averse players. They consider the situation in which the retailer has to choose whether to provide a service guarantee (SG) or to provide no service guarantee (NSG). They relate the optimal choice to the supplier's degree of risk aversion and the consumer's sensitivity to service reliability. They find that endogenization of wholesale pricing decision will raise the retailer's motivation to adopt SG strategy if the consumer is sufficiently risk averse. Choi [25] examines the optimal service charge for consumer product returns under mass customization operations. He explores two cases which cover the scenario when the mass customization company is risk-neutral and risk-averse. He derives the closed-form expression of the optimal return service charge for each case. He analytically shows how the mass customization company's degree of risk aversion affects the optimal return service charge policy. In addition, he derives the analytical conditions under which it is optimal for the mass customization company to offer a free return with a full refund policy. Hu and Qiang [21] investigate an online supply chain analytical model which is consisted of manufacturers, express service providers, and electronic retailers. They consider the situation in which consumers who purchase products from the electronic retailers can choose to adopt the express delivery service or simply pick up the products themselves. At the same time, the express service companies can invest on service quality improvement. They develop the variational inequality formulation and generate insights via exploring the equilibrium. As reviewed above, it is evidenced that analytical modeling research is commonly adopted to explore optimal service decisions and strategies in the literature. However, to the best of our knowledge, none of them explore the problem associated with the empirical service quality framework. Thus, we believe that this paper is the first one which investigates the service quality problem for OSPs by combining both empirical and analytical approaches.

3. Data Collection and Empirical Data Analysis

A consumer questionnaire survey is conducted. 250 questionnaires are distributed via social network websites under a convenience sampling approach. Receivers are invited to complete the questionnaire if they know http://taobao.com/ and have visited it. 190 valid questionnaires are received and employed for statistical analysis finally.

The questionnaire is divided into 8 parts. The first part is used to collect the demographic data of the respondents. The second part is employed to examine the purchase behavior of the respondents on http://taobao.com/. The rest of the questionnaire is divided into 6 subparts to investigate the perceptions of http://taobao.com/ users on the 5 functions provided by http://taobao.com/ and their loyalty towards the

OSP. The 7-point Likert scale is used, in which 1 represents "strongly disagree" and 7 represents "strongly agree".

3.1. Descriptive Statistics. Among all the completed questionnaires received, 64% of the respondents have purchased fashion products on http://taobao.com/ before while 36% have not. Thus, the remaining analysis will be based on the responses of these 64% of the collected sample in order to get valid data. To be specific, these consumers are considered as http://taobao.com/'s customers (for fashion products). The percentage of the http://taobao.com/ customers who browse products on http://taobao.com/ daily occupies 8%. 13%, 11%, and 26% of http://taobao.com/'s customers browse products on http://taobao.com/ 2-3 times a week, once a week, and 2-3 times per month, respectively. The percentages of the http://taobao.com/'s customers who buy products via http://taobao.com/ daily, 2-3 times a week, once a week, and 2-3 times a month are 3%, 2%, 2%, and 11%, respectively.

Regarding spending, 75% of the http://taobao.com/ users spend less than \$500 on http://taobao.com/ monthly. 19% of them spend \$500-1000 on Tobao.com monthly. 2%, 1%, and 3% of the users spend \$1000–1500, \$1500–2000, and more than \$2000 on http://taobao.com/ monthly, respectively. Regarding the users shopping experiences on http://taobao.com/, ranking their enjoyment (in a 1–7 scale with 7 being the best), more than half of the respondents score their enjoyment level strictly above 4 (64%). 36% of them score as score 5 which represents the greatest group, 23% of them score as score 6, and 5% of them rank it 7. The percentage of score 4, which means neutral excitement, occupies 15%. The percentages of low scores 1, 2, and 3 occupy 5%, 5%, and 11%, respectively.

3.2. Hypotheses Testing. Before testing hypotheses, we first conduct the reliability test to assess the internal consistency of the variables. Reliability test is measured by the Cronbach's alpha in which the alpha value of 0.7 or above is widely recognized as an acceptable level of reliability [27]. The Cronbach alpha values for Searching functions, Guarantee of Trade Safety, Aliwangwang, Alipay, and Taodot are found to be 0.933, 0.919, 0.956, 0.958, and 0.921, respectively. All of them are higher than the acceptable level. Besides, the seven questions of customer loyalty are tested in this part. The Cronbach alpha value of the customer loyalty is 0.905 which also exceeds the acceptable level. Thus, we conclude that all these dimensions are sufficiently reliable. Next, we proceed to test the relationships of the five featured functions and customer loyalty by performing multiple linear regression analysis. It is shown that there is a statistically significant relationship between customer loyalty and the Aliwangwang function. The related statistical results of hypotheses are summarized in Table 2. As such, hypothesis H3 is supported (more details on the statistical analysis can be found in thesis research of Kwok [26]).

4. Empirical Findings and Insights

From the above empirical analysis, first of all, we know that the factor of "fulfillment and responsiveness (Aliwangwang function)" is the most critical one as it most strongly affects customer loyalty of http://taobao.com/'s customers (who purchased fashion products before). Observe that this result is consistent with the findings in the traditional service context which suggests that customers expect quick feedback on their requests and suggestions for improvements [2]. For http://taobao.com/'s case, it can be explained by the function of Aliwangwang which provides instant messaging for http://taobao.com/'s users to send messages to the sellers. This enhances the fulfillment of http://taobao.com/ users on queries and needs related to product delivery and item availability. This function also can reduce the customers' doubt on the product that they want to buy and further confirm their purchase decision. In addition, the customers can contact the sellers whenever the sellers are online with a simple click. This function extends the accessibility of the sellers/online stores without the control of the store operating hours. With Aliwangwang, http://taobao.com/ users are able to obtain adequate product information even if it is not given on the product web page. It increases the convenience of http://taobao.com/. Since it is a unique feature provided by http://taobao.com/ but not its close competitors, it becomes a major niche for http://taobao.com/.

Second, observe that only one hypothesis among the proposed five is proven to be true statistically. A possible reason may be given as follows. Unlike the traditional physical store or other kinds of online electronic retailers such as http://www.amazon.com/, etc., http://taobao.com/ is an online marketplace in which there are thousands of independent retailers. As a result, customers' perception on the service quality and their loyalty towards the OSP may be affected by their buying experience with the individual retailers. Owing to the fundamental differences, many hypotheses which apply for the more traditional bricks-and-mortar retailing/electronic retailing do not hold for the OSPs such as http://taobao.com/. In addition, notice that in http://taobao.com/, communication with individual sellers is especially important. In fact, customers would like to gather more information so that they can make better choice from a large variety of independent retailers selling similar products on the platform. This accounts for the support of Hypothesis 3 (which is about Aliwangwang and related to communication aspect).

Third, we know that a higher level of "fulfillment and responsiveness (Aliwangwang function)" will lead to a higher level of customer loyalty. According to Edvardsson et al. [28], customer loyalty will positively affect the revenue of the OSP, it will be thus interesting to analytically examine whether there exists an optimal level of "fulfillment and responsiveness" for the OSP. We, hence, further conduct an analytical study in the next section.

5. Analytical Service Optimization Model and Findings

Based on the empirical findings from Section 4, we proceed to build an analytical model for further analysis. First, from the linear regression analysis conducted in Section 4 (see Table 2), we notice that the level of customer loyalty l is related

Table 2: Summary of regression analysis.

Corresponding hypothesis	Independent variable	Standardized coefficient	t-value	Sig.
Hypothesis 1	Efficiency	−0.057	−0.575	0.573
Hypothesis 2	Reliability	0.087	0.896	0.383
Hypothesis 3	Fulfillment & responsiveness	0.582	2.255	0.038**
Hypothesis 4	Flexibility	0.062	0.406	0.690
Hypothesis 5	Privacy	0.307	1.391	0.182

Remarks:
dependent variable: customer loyalty.
**hypothesis is supported.

to the level of "fulfillment and responsiveness" λ in a linear function as follows,

$$l = a + b\lambda, \tag{1}$$

where $a, b > 0$.

Second, since it is intuitive that a higher level of customer loyalty l will "stochastically" (it means expectedly, the revenue should be higher but the exact magnitude is unknown) lead to a higher revenue $R(l)$ for the OSP such as http://taobao.com/, we have the following:

$$R(l) = G(l) + \varepsilon, \tag{2}$$

where (i) $G(l)$ is an increasing function of l. For analytical tractability, we further assume $G(l)$ as a concave function. (ii) ε is a random variable which captures the noise (i.e., uncertainty) associated with $R(l)$. Following the literature [29], we model it to follow a symmetric distribution such as the normal distribution with a zero mean and constant variance σ^2.

Third, providing a higher level of "fulfillment and responsiveness" λ requires resource. We, hence, define the respective cost as $C(\lambda)$ which is an increasing function of λ. Similar to $G(l)$, we assume $C(\lambda)$ as a convex function for analytical tractability. Notice that the above functional definitions are all intuitive and rather general.

With the above model, we can derive the profit $\pi(\lambda)$, the expected profit $E[\pi(\lambda)]$, and the variance of profit $V[\pi(\lambda)]$ as functions of λ as follows:

$$\begin{aligned} \pi(\lambda) &= R(l) - C(\lambda) \\ &= G(a + b\lambda) - C(\lambda) + \varepsilon, \\ E[\pi(\lambda)] &= G(a + b\lambda) - C(\lambda), \\ V[\pi(\lambda)] &= \sigma^2. \end{aligned} \tag{3}$$

Because e-business is known to be a risky operation, we argue that the OSP has concern on risk and possesses a risk averse attitude [24]. As such, we employ the following mean-variance safety first measure (SFM(λ)) as the objective function (for the modeling of risk averse optimization objectives, see Xiao and Choi [30], Chiu and Choi [31], Chiu et al. [32], Choi and Chiu [33], Choi et al. [34], Choi [35], and Shen et al. [36]) for the OSP as follows:

$$\mathrm{SFM}(\lambda) = \frac{(E[\pi(\lambda)] - \beta)}{\sqrt{V[\pi(\lambda)]}}, \tag{4}$$

where β is the expected profit target that the OSP wants to achieve.

For the meaning behind SFM(λ), please notice that by Bienayme-Tchebycheff inequality, we have the following inequality:

$$P(|\pi(\lambda) - E[\pi(\lambda)]| \geq E[\pi(\lambda)] - \beta) \leq \frac{V[\pi(\lambda)]}{(E[\pi(\lambda)] - \beta)^2}, \tag{5}$$

$$\Longrightarrow P(\pi(\lambda) \leq \beta) \leq \frac{V[\pi(\lambda)]}{(E[\pi(\lambda)] - \beta)^2} \equiv \frac{1}{\mathrm{SFM}(\lambda)^2}. \tag{6}$$

Thus, the RHS of (6) gives the analytical upper bound for $P(\pi(\lambda) \leq \beta)$. Obviously, maximizing SFM(λ) yields a minimized upper bound for $P(\pi(\lambda) \leq \beta)$ and hence SFM(λ) is called the safety first measure (see [37]).

With the above model, we define (7), and then have Proposition 1. Notice that $S_G(\lambda)$ and $S_C(\lambda)$ are the first order derivatives of $G(a + b\lambda)$ and $C(\lambda)$, respectively, and they also represent the corresponding slopes as follows:

$$\begin{aligned} \lambda^*_{\mathrm{SFM}} &= \arg\max_{\lambda} \mathrm{SFM}(\lambda), \\ \lambda^*_{\mathrm{EP}} &= \arg\max_{\lambda} E[\pi(\lambda)], \\ S_G(\lambda) &= \frac{dG(a + b\lambda)}{d\lambda}, \\ S_C(\lambda) &= \frac{dC(\lambda)}{d\lambda}. \end{aligned} \tag{7}$$

Proposition 1. *λ^*_{SFM} uniquely exists and it is equal to* $\arg_{\lambda}\{S_G(\lambda)/S_C(\lambda) = 1\}$.

Proof of Proposition 1. All proofs are placed in the Appendix. □

Proposition 1 first presents the uniqueness feature of the optimal level of fulfillment and responsiveness (i.e. optimal service level) which maximizes the safety first objective. It then reveals that this optimal service level is the one which equalizes $S_G(\lambda)$ and $S_C(\lambda)$. In other words, when the rates of change of $G(a + b\lambda)$ and $C(\lambda)$ are the same, the corresponding service level λ is optimal. From Proposition 1, we have Proposition 2.

Proposition 2. *(a) A larger b implies a larger λ^*_{SFM}. (b) λ^*_{SFM} is independent of β and σ.*

Observe that a larger b means that the customer loyalty is more positively correlated to the service level λ. Proposition 2(a) hence indicates that it will lead to a bigger optimal service level because the optimal solution refers to the one which balances the increase of revenue $S_G(\lambda)$ and rise of cost $S_C(\lambda)$. It is also interesting to note from Proposition 2(b) that λ^*_{SFM} is independent of β and σ. This finding can be explained by the fact that the variance of profit $V[\pi(\lambda)]$ is a constant which is independent of the service level λ. Thus, the corresponding optimal service level is not related to σ as well as the expected profit target β, and will in fact behave like the risk neutral case as shown in Proposition 3 below.

Proposition 3. *λ^*_{EP} uniquely exists and $\lambda^*_{SFM} = \lambda^*_{EP}$.*

Proposition 3 shows a rather surprising result that the risk averse and risk neutral OSPs will have the same optimal service levels for this particular case. In other words, irrespective of the risk preference, the optimal service level is the same. This finding is important because it shows the fundamental difference between different kinds of operations management problems. For example, for inventory planning under uncertainty, the optimal inventory decision for the risk averse and risk neutral cases will be different. However, in service optimization, the situation can be different as shown here.

6. Concluding Remarks

We conclude by discussing some research implications derived from the discussions above. First, the empirical findings suggest that in order to enhance customer loyalty, online stores should develop marketing strategies to enhance "fulfillment and responsiveness" since they strongly affect customer loyalty. http://taobao.com/ has devoted valuable corporate resources to it and we argue that http://taobao.com/ can further improve the functions of Aliwangwang to achieve even better communication between the sellers and http://taobao.com/ users. Some probable enhancements including inserting flash games and converting to smartphone apps can all help to attract the http://taobao.com/ users to use Aliwangwang. Furthermore, http://taobao.com/ can make use of Aliwangwang to obtain personalized data related to the customers' preferences. As a result, a higher degree of web site customization (e.g., based on past purchases and other customers' information collected) can be provided to individual customers.

Second, for the empirical result driven analytical studies, we find that the optimal service level on "fulfillment and responsiveness" function for the risk averse OSP uniquely exists and can be expressed as the service level which balances the increase of revenue $S_G(\lambda)$ and the rise of cost $S_C(\lambda)$. We further show that if the customer's loyalty is more positively correlated to the service level λ, it will lead to a larger optimal service level. Moreover, the optimal service level under the safety first objective is independent of profit target β and uncertainty threshold σ. Finally, we analytically prove that the optimal service level does not depend on the risk preference of the OSP.

For future research, there are two directions. Empirically, it is useful to examine more functions of OSP such as http://taobao.com/. This paper only focuses on the several core functions specifically adopted by http://taobao.com/ and there are some more which deserve further explorations. Besides, unlike the traditional bricks-and-mortar and electronic retailers, the operation of http://taobao.com/ involves thousands of independent sellers. As a result, the customers' perceived service quality of the platform would be considerably affected by their buying experience with individual sellers. In the future, it is worth investigating the modification of the existing service quality-scale measurement to cater for this specific type of new business model. The second direction for further studies relates to the theoretical analysis based on more comprehensive economics analytical models. We believe that the findings revealed by this paper can lay the foundation for many further empirical and analytical studies.

Appendix

All Proofs of Propositions

Proof of Proposition 1. From (4), we have

$$\mathrm{SFM}(\lambda) = \frac{(E[\pi(\lambda)] - \beta)}{\sqrt{V[\pi(\lambda)]}}. \tag{A.1}$$

Since $V[\pi(\lambda)] = \sigma^2$, (A.2) becomes

$$\mathrm{SFM}(\lambda) = \frac{(E[\pi(\lambda)] - \beta)}{\sigma}. \tag{A.2}$$

$$\begin{aligned}\frac{d\mathrm{SFM}(\lambda)}{d\lambda} &= \frac{dE[\pi(\lambda)]/d\lambda}{\sigma}\\ &= \frac{d[G(a+b\lambda) - C(\lambda)]/d\lambda}{\sigma}.\end{aligned} \tag{A.3}$$

$$\frac{d^2\mathrm{SFM}(\lambda)}{d\lambda^2} = \frac{1}{\sigma}\left(\frac{d^2G(a+b\lambda)}{d\lambda^2} - \frac{d^2C(\lambda)}{d\lambda^2}\right). \tag{A.4}$$

Since $G(a + b\lambda)$ is concave and $C(\lambda)$ is convex, $d^2\mathrm{SFM}(\lambda)/d\lambda^2 < 0$ which means $\mathrm{SFM}(\lambda)$ is concave, and λ^*_{SFM} uniquely exists. In addition, notice that $\lambda^*_{\mathrm{SFM}} = \arg_\lambda[d\mathrm{SFM}(\lambda)/d\lambda = 0]$. Thus, we have $\lambda^*_{\mathrm{SFM}} = \arg_\lambda\{S_G(\lambda)/S_C(\lambda) = 1\}$. □

Proof of Proposition 2. Part (a): notice that $\lambda^*_{\mathrm{SFM}} = \arg_\lambda\{S_G(\lambda)/S_C(\lambda) = 1\}$ and

$$\begin{aligned}S_G(\lambda) &= \frac{d[G(a+b\lambda)]}{d\lambda}\\ &= \frac{d(a+b\lambda)}{d\lambda}\frac{dG(a+b\lambda)}{d(a+b\lambda)} = b\frac{dG(a+b\lambda)}{d(a+b\lambda)}.\end{aligned} \tag{A.5}$$

Thus, $\lambda^*_{\mathrm{SFM}} = \arg_\lambda\{S_G(\lambda)/S_C(\lambda) = 1\} = \arg\{S_C(\lambda) = b(dG(a+b\lambda)/d(a+b\lambda))\}$.

Define $f(\lambda) = S_C(\lambda)/(dG(l)/dl)|_{l=a+b\lambda}$. Consider

$$f'(\lambda) = \frac{1}{\left((dG(l)/dl)|_{l=a+b\lambda}\right)^2} \times \left(\left(\left.\frac{dG(l)}{dl}\right|_{l=a+b\lambda}\right)\frac{dS_C(\lambda)}{d\lambda} - S_C(\lambda)\frac{d}{d\lambda}\left(\left.\frac{dG(l)}{dl}\right|_{l=a+b\lambda}\right)\right). \quad (A.6)$$

We have

(a) $dS_C(\lambda)/d\lambda = d^2C(\lambda)/d\lambda^2 > 0$ [$\because C(\lambda)$ is convex in λ],

(b) $(dG(l)/dl)|_{l=a+b\lambda} > 0$ [$\because G(l)$ is increasing in l],

(c) $S_C(\lambda) = dC(\lambda)/d\lambda > 0$ [$\because C(\lambda)$ is increasing in λ],

(d) $(d/d\lambda)((dG(l)/dl)|_{l=a+b\lambda}) = [d^2G(l)/dl^2 \cdot d(l = a + b\lambda)/d\lambda]_{l=a+b\lambda} = b(d^2G(l)/dl^2)_{l=a+b\lambda} < 0$ [$\because G(l)$ is concave in l].

Therefore, we have $f'(\lambda) > 0$ for all λ. In other words, $f(\lambda)$ is increasing in λ. Therefore, a larger b implies a largerλ^*_{SFM}.

Part (b): since both $S_G(\lambda)$ and $S_C(\lambda)$ are independent of β and σ, it is obvious that $\lambda^*_{\mathrm{SFM}} = \arg_\lambda\{S_G(\lambda)/S_C(\lambda) = 1\}$ is also independent of β and σ. □

Proof of Proposition 3. First notice that $E[\pi(\lambda)]$ is a concave function. Thus, λ^*_{EP} uniquely exists. Second, from (A.3), we have

$$\frac{d\mathrm{SFM}(\lambda)}{d\lambda} = \frac{dE[\pi(\lambda)]/d\lambda}{\sigma} = \frac{1}{\sigma}\cdot\frac{dE[\pi(\lambda)]}{d\lambda}. \quad (A.7)$$

Obviously, from (A.7), we have $\lambda^*_{\mathrm{SFM}} = \arg_\lambda[d\mathrm{SFM}(\lambda)/d\lambda = 0] = \arg_\lambda[dE[\pi(\lambda)]/d\lambda = 0]$, which is the same as λ^*_{EP}. Thus, $\lambda^*_{\mathrm{SFM}} = \lambda^*_{\mathrm{EP}}$. □

Authors' Contribution

All authors have good contribution to the paper and the listing follows their surnames.

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Research Article

The Impact of Price Comparison Service on Pricing Strategy in a Dual-Channel Supply Chain

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Recently, price comparison service (PCS) websites are more and more popular due to its features in facilitating transparent price and promoting rational purchase decision. Motivated by the industrial practices, in this study, we examine the pricing strategies of retailers and supplier in a dual-channel supply chain influenced by the signals of PCS. We categorize and discuss three situations according to the signal availability of PCS, under which the optimal pricing strategies are derived. Finally, we conduct a numerical study and find that in fact the retailers and supplier are all more willing to avoid the existence of PCS with the objective of profit maximization. When both of retailers are affected by the PCS, the supplier is more willing to reduce the availability of price information. Important managerial insights are discussed.

1. Introduction

Price is one of the key competitive dimensions of purchasing a product. As consumers are eager to have access to the better price information in the market, price comparison service (PCS) is naturally born. The PCS provides specific product information and price differences for consumer's reference [1]. By checking the PCS, consumers are able to compare prices with other retailers and make better decisions [2, 3]. For example, Skyscanner, an online air ticket price comparison website, helps online consumers to compare flight prices of any given route over a month period among different airline and agents. More examples of PCS are summarized in Table 1. According to Table 1, products shown in the PCS include air ticket, hotel, fashion, home product, computer, and electronics. One observation from Table 1 is noteworthy: all products shown in PCS are under fierce price competition.

Price differences and price fluctuations exert an impact on the decision making among consumers and supply chain parties. Huang and Swaminathan [4] compare various products from Amazon and BELK and find that there exists price difference under an online duopoly environment. This observation motivates us to explore the pricing strategy with consideration of PCS. Serenko and Hayes [3] state that the PCS offers tremendous benefits for consumers who may potentially receive lower prices and for online vendors (i.e., retailer), who may not only capture more price information of their rivals but also get more exposure for their brands.

Pricing issues have been extensively studied by the scholars of supply chain; however, the impact of PCS has rarely been investigated. In this study, we first examine how the signals of PCS affect the decision making of supply chain members in a dual-channel supply chain. More specifically, we mainly develop a model with consideration of PCS and assume that supplier's pricing decision is affected by the PCS, while the retailers' pricing decision can be divided into three situations: (i) neither of retailers is affected by the PCS; (ii) either of retailers is affected by the PCS; and (iii) both of retailers are affected by the PCS. According to the three situations mentioned above, we derive the optimal pricing decision of supply chain parties in such a dual-channel supply chain, and our numerical results show that the best strategies for the retailers and supplier are to avoid the existence of PCS. In addition, we find if both of the retailers are affected by the PCS, the supplier should tend to reduce the availability of price information, and if either of retailers is affected by the PCS, the retailer should tend to exclusively cooperate with PCS.

The paper is organized as follows. We show the related literature in Section 2. The model is presented in Section 3,

Table 1: Examples of PCS [3].

Name	URL	Products
Become	http://www.become.com/	Fashion, electronics, home, and so forth
BizRate	http://www.bizrate.com/	Fashion, electronics, home, and so forth
Boxz	http://www.boxz.com/	Mobile phones
Eprice	http://www.eprice.com.hk/	Electronics
MySimon	http://www.mysimon.com/	Fashion and computer
PriceGrabber	http://www.pricegrabber.com/	Computer and electronics
Shopping	http://www.shopping.com/	Fashion, electronics, home, and so forth
Shopzilla	http://www.shopzilla.com/	Fashion, electronics, home, and so forth
Smarter	http://www.smarter.com/	Fashion, electronics, home, and so forth
Skyscanner	http://www.skyscanner.com/	Air ticket and hotel

and we obtain the optimal retail prices on both channels under vertically competition in Section 4. In Section 5, we study the supplier's pricing strategies when facing two competing retailers. We further conduct numerical studies to investigate the impact of PCS on pricing and expected profit among supply chain parties in Section 6. Finally, conclusion is presented in Section 7.

2. Literature Review

In the last decade, researchers have studied many issues related to the dual-channel supply chain with traditional and internet channel (please refer to [5, 6] for more discussions and review). More recently, Yao and Liu [7] study the pricing competition between retail and e-tail distribution channels under the Bertrand and the Stackelberg price competition models. Interestingly, they find that an optimal wholesale price exists under a different market structure in which the retailer is encouraged to accommodate the additional e-tail channel. Cai [8] examines the impact of channel structure on various supply chain parties with and without coordination under a dual-channel supply chain. Chen et al. [9] investigate the contracting strategies in a dual-channel supply chain. According to their results, the wholesale price contract could coordinate the dual-channel supply chain.

Pricing is popularly investigated in supply chain management, particularly in the dual-channel supply chain [10]. An early research, conducted by Ingene and Parry [11], discusses the case of a single manufacturer selling an identical product to two competing retailers with a linear quantity discount schedule and a two-part tariff. Ingene and Parry claim that coordination is not always in the manufacturer's interest when retailers are competing. Chiang et al. [12] also examine a price competition game in a dual-channel supply chain. They find that a direct channel strategy could lead the manufacturer to be more profitable by posing a viable threat to draw customers away from the retailer. Huang and Swaminathan [4] investigate the pricing strategies in a dual-channel supply chain by assuming a stylized deterministic demand model. Under such demand function, the retailers tend to set a higher retail price in order to make more profit. Recently, Tang and Xing [13] compare the pricing behavior between online branches of traditional retailers and pure internet retailers. They conclude that the price charged by pure e-tailers for DVD titles is 14% lower than those charged by e-tailers with traditional channels. From the marketing perspective, price comparison can affect buyer's perceptions of acquisition value, transaction value, and behavioral intentions [2]. The impact of PCS on pricing decision is thus inevitable.

The service is significantly important for supply chain management. Dumrongsiri et al. [14] consider the impact of retailer's service quality on a dual-channel supply chain in which a manufacturer sells to a retailer as well as to consumers directly. They reach an interesting conclusion that a higher retailer's service quality may lead to a higher manufacturer's profit in a dual-channel supply chain. Dan et al. [15] examine the retail services in a centralized and a decentralized dual-channel supply chain using the two-stage optimization technique and Stackelberg game. Their results imply that retail services could strongly influence the manufacturer and the retailer's pricing strategies. In this paper, we examine the impact of PCS on a dual-channel supply chain. PCS is new to the literature of supply chain management, although it has been largely explored in the area of electronic business [1, 3].

The signals of PCS are closely related to information availability, namely, information asymmetry or symmetry. However, information asymmetry has been largely discussed in the literature of supply chain management. Desiraju and Moorthy [16] study information asymmetry regarding a price- and service-sensitive demand curve. They show that the coordination can be achieved by requiring service performance. Cakanyildirim and Sethi [17] find that information asymmetry regarding a manufacturer's production cost does not necessarily cause inefficiency in supply chain. Mukhopadhyay et al. [18] examine the mixed channels under information asymmetry and propose retailer to add differentiated value to the product so as to eliminate the possibility of channel conflict. In this study, we consider both information asymmetry and symmetry cases.

3. The Model

In this paper, we consider a dual-channel supply chain system in which one supplier provides the products to two retailers at the same wholesale price, and then two retailers (e.g., Wal-Mart and Amazon) sell products to consumers. Here, we consider one retailer is a physical retailer and the other is an online retailer (As a remark, the physical retailer and the online retailer may or may not be homogenous due to the signal availability of PCS. The heterogeneous structure has particularly happened in the scenario that either of retailers is affected by the PCS). To simplify, the two retailers are referred as retailer1 and retailer2. We consider the retail

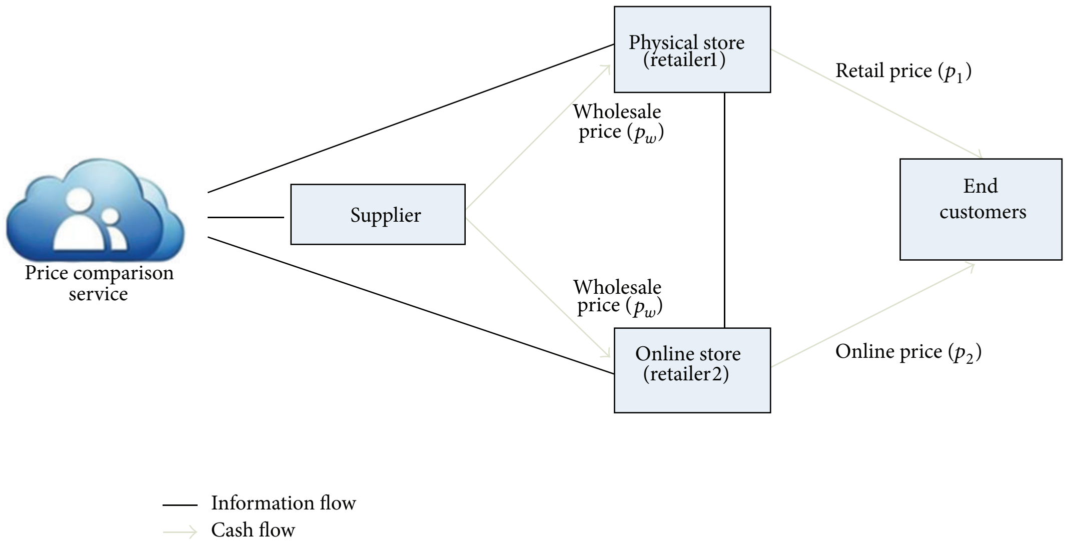

Figure 1: Dual-channel supply chain system with PCS.

prices are public information for every party in this supply chain system. All parties face the risk of price competition brought by price comparison service (PCS) (The PCS is a third party who is independent with all parties in such a supply chain), which positively affect their pricing decisions. The sequence of events is as follows. First, supplier decides wholesale price. Second, based on the wholesale price offered by supplier, both retailer1 and retailer2 determine the retail price simultaneously. As a remark, all decisions made by supply chain parties might be influenced by the PCS. This game is analyzed by using backward induction technique. This dual-channel supply chain with the PCS is depicted in Figure 1.

3.1. Notations

D_1, D_2: Market demand for retailer1 and retailer2

D_s: Market demand

p_w: Wholesale price offered by supplier

p_1, p_2: Retail prices decided by retailer1 and retailer2

d: Market size base

α: Demand sensitivity for retailer1's market price

β: Demand sensitivity for retailer2's market price

μ: Impact of PCS on demand, which follows a normal distribution $\mu \sim N(0, \sigma)$

x_0, x_1, x_2: Impact of PCS on pricing decision determined by supplier, retailer1, and retailer2, respectively

θ_m, δ_m: Adjusted coefficients of retail pricing, $m \in 1, 2, 3, 4$

λ_0, λ: Adjusted coefficients of wholesale pricing.

3.2. Basic Model. Without any loss of generality, we consider that the demand curve in the dual-channel supply chain is a linear function of the selling price, where $\mu \sim N(0, \sigma^2)$ is a random parameter to represent random uncertain information [19]. The function of market demand is expressed as follows:

$$D_i\left(p_i, p_j, \mu\right) = d_i - \alpha p_i + \beta p_j + \mu, \tag{1}$$

where $i, j \in (1, 2)$, $i \neq j$, and d, α, β are constants.

Supply chain parties might enable to receive the signal of PCS. We denote x_0, x_1, and x_2 as the degree of PCS signal the supplier and two retailers received, respectively. According to the degree of signal, the parties would make price adjustment, and the adjusted price will in turn affect the market demand. Assuming $x_i = \mu + \varepsilon_i$, both μ and ε_i are independent random variables, where ε_i follows normal distribution with mean of η, $(\eta > 0)$ and variance of ε_i, $i \in 0, 1, 2$. Covariance matrix of ε_i can be denoted as $\sum = \mathrm{diag}(\sigma, s_0, s_1, s_2)$. For example, if the price comparison exerts greater impact on retailer1, then the retailer1 will adjust its price, which will cause decrease in demand; while if the impact is much less, then the retailer1 will tend to keep the price stable, which will not affect the market demand. Prices offered by the supplier and retailers are functions of x_0, x_1, x_2, namely, $p_w = f_0(x_0)$ and $p_i = f_i(x_i)$.

Supplier adjusts its wholesale price based on the signal of PCS. However, due to asymmetry of transmission, the retailers may or may not be affected by the PCS. Here, we divide it into three situations accordingly.

Situation 1. The supplier does not share price comparison information x_0 with both of the retailers, namely, neither of retailers are not affected by the PCS information x_0 directly. Instead, they are able to adjust retail price based on respective price comparison x_i as well as wholesale price p_w. The adjusted retail price hence is $p_i = \theta_1 + \theta_2 x_i + \theta_3 p_w$, $i \in 1, 2$.

Situation 2. PCS information x_0 exerts impact on either of retailers, then the one who is affected by x_0 will adjust its price and the corresponding retail price is $p_i = \theta_1 + \theta_2 x_i + \theta_3 p_w + \theta_4 x_0$, $i = 1$ or $i = 2$.

Situation 3. PCS information x_0 exerts impact on both of retailers, namely, retailers make pricing decisions based on x_0 and x_i. The adjusted retail price is $p_i = \theta_1 + \theta_2 x_i + \theta_3 p_w + \theta_4 x_0$, $i \in 1, 2$.

4. Retail Pricing Strategy

4.1. Neither of Retailers Is Directly Affected by x_0. We consider the supplier is affected by price comparison x_0, and its updated wholesale pricing strategy is

$$p_w = \lambda_0 + \lambda x_0, \tag{2}$$

where λ_0, λ are price coefficients (see [20]). In this subsection, we consider that the supplier does not share information with the two retailers who are only affected by x_1 and x_2, respectively. The updated pricing strategies of the two retailers are hence as follows:

$$p_1 = f_1(x_1, p_w) = \theta_1 + \theta_2 x_1 + \theta_3 p_w, \tag{3}$$

$$p_2 = f_2(x_2, p_w) = \delta_1 + \delta_2 x_2 + \delta_3 p_w. \tag{4}$$

Assume that the competitive relationship between the two retailers who are well-matched in strength and the impact of PCS on each retailer are equal. Then, the expected demand of retailer1 can be expressed as:

$$E(\mu \mid x_1, x_0) = \frac{\sigma s_0 x_1 + \sigma s_1 x_0}{\sigma s_1 + \sigma s_0 + s_1 s_0}. \tag{5}$$

According to the above assumption, the formula can be further described as:

$$\begin{aligned} E(\mu \mid x_1, p_w) &= E(x_2 \mid x_1, p_w) \\ &= \frac{\sigma s_0 x_1 \lambda + \sigma s_1 (p_w - \lambda_0)}{(\sigma s_1 + \sigma s_0 + s_1 s_0)\lambda}, \end{aligned} \tag{6}$$

and the expected demand of retailer2 can be expressed as:

$$\begin{aligned} E(\mu \mid x_2, p_w) &= E(x_1 \mid x_2, p_w) \\ &= \frac{\sigma s_0 x_2 \lambda + \sigma s_2 (p_w - \lambda_0)}{(\sigma s_2 + \sigma s_0 + s_2 s_0)\lambda}. \end{aligned} \tag{7}$$

In addition, the signal of PCS is symmetric for both retailers; the expected prices of retailer1 and retailer2 are

$$E(p_2 \mid x_1, p_w) = \delta_1 + \delta_2 E(x_2 \mid x_1, p_w) + \delta_3 p_w, \tag{8}$$

$$E(p_1 \mid x_2, p_w) = \theta_1 + \theta_2 E(x_1 \mid x_2, p_w) + \theta_3 p_w. \tag{9}$$

Then, the expected profits of retailer1 and retailer2 are expressed as:

$$\begin{aligned} E(\Pi_1 \mid x_1, p_w) = {} & [d_1 - \alpha p_1 + \beta E(p_2 \mid x_1, p_w) \\ & + E(\mu \mid x_1, p_w)](p_1 - p_w), \end{aligned} \tag{10}$$

$$\begin{aligned} E(\Pi_2 \mid x_2, p_w) = {} & [d_2 - \alpha p_2 + \beta E(p_1 \mid x_2, p_w) \\ & + E(\mu \mid x_2, p_w)](p_2 - p_w). \end{aligned} \tag{11}$$

Proposition 1. *When neither of retailers is influenced by the supplier price comparison impact x_0, the profit functions $E(\Pi_1 \mid x_1, p_w)$ and $E(\Pi_2 \mid x_2, p_w)$ are strictly concave in p_1 and p_2, respectively, namely, the optimal retail prices p_1^* and p_2^* uniquely exist.*

To enhance the presentation, all proofs are relegated to the Appendices.

By solving the first-order derivative of p_1 in $E(\Pi_1 \mid x_1, p_w)$ and p_2 in $E(\Pi_1 \mid x_1, p_w)$, respectively, one can have optimal retail pricing strategies of retailer1 and retailer2 in this situation as follows:

$$\begin{aligned} p_1^* &= \frac{1}{2\alpha}\left[d_1 + \alpha p_w + \beta E(p_2 \mid x_1, p_w) + E(\mu \mid x_1, p_w)\right], \\ p_2^* &= \frac{1}{2\alpha}\left[d_2 + \alpha p_w + \beta E(p_1 \mid x_2, p_w) + E(\mu \mid x_2, p_w)\right]. \end{aligned} \tag{12}$$

By solving (2)–(12) simultaneously, one finds θ_i, $i = 1, 2, 3, 4$ and δ_i, $i = 1, 2, 3$. Consider

$$\begin{aligned} \theta_1 = {} & \frac{1-t}{2-t} \\ & - \left(\left(2\sigma\left(1-t^2\right)\left[4\left(\left(\frac{1}{\alpha+\beta} - d\right)s_1 k_2 + 2d s_2 k_1\right)\right.\right.\right. \\ & \left.\left. + d\sigma s_0\left(2s_1 + t s_2\right)\right]\right) \\ & \left. \times\left(V\left(4-t^2\right)\right)^{-1}\right), \end{aligned}$$

$$\theta_2 = \frac{\sigma s_0\left(1-t^2\right)\left[\left(2/(\alpha+\beta) - 2d\right)k_2 + d\sigma s_0\right]}{V},$$

$$\begin{aligned} \theta_3 = {} & \frac{1}{2-t} \\ & - \left(\left(2\sigma\left(1-t^2\right)\left[4\left(\left(\frac{1}{\alpha+\beta} - d\right)s_1 k_2 + 2d s_2 k_1\right)\right.\right.\right. \\ & \left.\left. + d\sigma s_0\left(2s_1 + t s_2\right)\right]\right) \\ & \left. \times\left(V\left(4-t^2\right)\right)^{-1}\right), \end{aligned}$$

$$\begin{aligned} \delta_1 = {} & \frac{1-t}{2-t} \\ & - \left(\left(2\sigma\left(1-t^2\right)\left[4\left(\left(\frac{1}{\alpha+\beta} - d\right)s_2 k_1 + 2d s_1 k_2\right)\right.\right.\right. \\ & \left.\left. + d\sigma s_0\left(2s_2 + t s_1\right)\right]\right) \\ & \left. \times\left(V\left(4-t^2\right)\right)^{-1}\right), \end{aligned}$$

$$\delta_2 = \frac{\sigma s_0 \left(1-t^2\right)\left[\left(2/\left(\alpha+\beta\right)-2d\right)k_1 + d\sigma s_0\right]}{V},$$

$$\delta_3 = \frac{1}{2-t} - \left(\left(2\sigma\left(1-t^2\right)\left[4\left(\left(\frac{1}{\alpha+\beta}-d\right)s_1k_1 + 2ds_2k_2\right) + d\sigma s_0\left(2s_2+ts_1\right)\right]\right) \times \left(V\left(4-t^2\right)\right)^{-1}\right), \tag{13}$$

where $t = \beta/\alpha$, $d_1 = d_2 = d/2$, $k_1 = (\sigma + s_0)s_1 + \sigma s_0$, $k_2 = (\sigma + s_0)s_2 + \sigma s_0$, and $V = 4k_1k_2 - t^2\sigma^2 s_0^2$.

Thus, the optimal pricing strategies of the two retailers can be obtained by substituting (13) into $p_1^* = \theta_1 + \theta_2 x_1 + \theta_3 p_w$ and $p_2^* = \delta_1 + \delta_2 x_2 + \delta_3 p_w$.

4.2. Either of Retailers Is Affected by x_0. In this situation, either of retailers is affected by price comparison information x_0. We assume that retailer1 is affected by x_0 while retailer2 is not affected by x_0 directly, therefore, we can have

$$p_1 = \theta_1 + \theta_2 x_1 + \theta_3 p_w + \theta_4 x_0, \tag{14}$$

$$p_2 = \delta_1 + \delta_2 x_2 + \delta_3 p_w. \tag{15}$$

Retailer1 enables to speculate pricing strategy of retailer2 based on x_0 and x_1. Consider

$$E\left(p_2 \mid x_1, x_0\right) = \delta_1 + \delta_2 \frac{\sigma s_0 x_1 + \sigma s_1 x_0}{\sigma s_1 + \sigma s_0 + s_1 s_0} + \delta_3 p_w. \tag{16}$$

But retailer2 enables to speculate pricing strategy of retailer1 based on x_2 and p_w. Consider

$$E\left(p_1 \mid x_2, p_w\right) = \theta_1 + \theta_2 \frac{\sigma s_0 x_2 + \sigma s_2 p_w - \sigma s_2}{\sigma s_2 + \sigma s_0 + s_2 s_0} + \theta_3 p_w + \theta_4\left(p_w - 1\right), \tag{17}$$

$$E\left(\mu \mid x_1, x_0\right) = \frac{\sigma s_0 x_1 + \sigma s_1 x_0}{\sigma s_1 + \sigma s_0 + s_1 s_0}, \tag{18}$$

$$E\left(\mu \mid x_2, p_w\right) = \frac{\sigma s_0 x_1 \lambda + \sigma s_1\left(p_w - \lambda\right)}{\left(\sigma s_1 + \sigma s_0 + s_1 s_0\right)\lambda}. \tag{19}$$

In order to reduce potential risks brought by price changes with the signal of PCS, the two retailers determine the optimal retail price by maximizing the expected profit. To this end, the expected profits for retailer1 and retailer2 are

$$E\left(\Pi_1 \mid x_1, x_0\right) = E\left[\left(\frac{d}{2} - \alpha p_1 + \beta p_2 + \mu\right)\left(p_1 - p_w\right) \mid x_1, x_0\right], \tag{20}$$

$$E\left(\Pi_2 \mid x_2, p_w\right) = E\left[\left(\frac{d}{2} - \alpha p_2 + \beta p_1 + \mu\right)\left(p_2 - p_w\right) \mid x_2, p_w\right]. \tag{21}$$

Proposition 2. *When either of retailers is influenced by the supplier price comparison impact x_0, the profit functions $E(\Pi_1 \mid x_1, x_0)$ and $E(\Pi_2 \mid x_2, p_w)$ are strictly concave in p_1 and p_2, respectively, namely, the optimal retail price p_1^* and p_2^* uniquely exist.*

By solving the first derivative of p_1 and p_2, one can have the optimal pricing strategies of the two retailers as follows:

$$p_1^* = \frac{1}{2\alpha}\left[\frac{d}{2} + \beta E\left(p_2 \mid x_1, x_0\right) + E\left(\mu \mid x_1, x_0\right) + \alpha p_w\right],$$

$$p_2^* = \frac{1}{2\alpha}\left[\frac{d}{2} + \beta E\left(p_1 \mid x_2, p_w\right) + E\left(\mu \mid x_2, p_w\right) + \alpha p_w\right]. \tag{22}$$

Then, solving (14)–(22) simultaneously, one finds θ_i, $i = 1, 2, 3, 4$ and δ_i, $i = 1, 2, 3$. Consider

$$\theta_1 = \frac{1-t}{2-t} - \frac{\sigma d\left(1-t^2\right)\left[2\left(\sigma + s_0\right)s_1 s_2 + \sigma s_0\left(2s_2 + ts_1\right)\right]}{V\left(2-t\right)}, \tag{23}$$

$$\theta_2 = \frac{\sigma s_0\left(1-t^2\right)\left[2\left(1/\left(\alpha+\beta\right)-d\right)k_2 + d\sigma s_0\right]}{V}, \tag{24}$$

$$\theta_3 = \frac{1}{2-t} - \frac{\sigma d\left(1-t^2\right)\left[2\left(\sigma + s_0\right)s_1 s_2 + \sigma s_0\left(2s_2 + ts_1\right)\right]}{\alpha V\left(2-t\right)}, \tag{25}$$

$$\theta_4 = \frac{1}{2-t} - \frac{\alpha\sigma s_1\left(1-t^2\right)\left(2k_2 + t\sigma s_1\right)}{V}, \tag{26}$$

$$\delta_1 = \frac{1-t}{2-t} - \left(\left(4\sigma^2\left(1-t^2\right)\left(\frac{1}{\alpha+\beta}-d\right) \times\left[\left(\frac{2}{\alpha+\beta}-2d\right)(\sigma+s_0)s_1s_2+\sigma s_0\right]\right) \times\left(V\left(4-t^2\right)\right)^{-1}\right), \tag{27}$$

$$\delta_2 = \frac{\sigma s_0\left(1-t^2\right)\left[\left(2/(\alpha+\beta)-2d\right)k_1+d\sigma s_0\right]}{V}, \tag{28}$$

$$\delta_3 = \frac{1}{2-t} - \left(\left(2\sigma\left(1-t^2\right)\left[2s_2\left(\frac{2}{\alpha+\beta}-2d\right)k_1+d\sigma s_0\right] +ds_1\left[\left(\frac{2}{\alpha+\beta}-2d\right)k_2+d\sigma s_0\right]\right) \times\left(\alpha V\left(4-t^2\right)\right)^{-1}\right), \tag{29}$$

where $t = \beta/\alpha$, $k_1 = (\sigma+s_0)s_1+\sigma s_0$, $k_2 = (\sigma+s_0)s_2+\sigma s_0$, and $V = 4k_1k_2 - t^2\sigma^2 s_0^2$.

The optimal pricing strategies of the two retailers can be obtained by substituting (22) to (29) into $p_1^* = \theta_1+\theta_2x_1+\theta_3p_w+\theta_4x_0$ and $p_2^* = \delta_1+\delta_2x_2+\delta_3p_w$.

4.3. Both of Retailers Are Affected by x_0. In this situation, the supplier shares price comparison information with retailers, which enables both of retailers to be affected by x_0 directly. Hence, price functions of retailer1 and retailer2 are as follows:

$$p_1 = \theta_1+\theta_2x_1+\theta_3p_w+\theta_4x_0, \tag{30}$$

$$p_2 = \delta_1+\delta_2x_2+\delta_3p_w+\delta_4x_0. \tag{31}$$

In this situation, due to price competition, the retailer enables to speculate pricing strategy of its rival according to x_1 or x_2. The new pricing strategies of retailer1 and retailer2 are

$$E\left(p_2 \mid x_1,x_0\right) = \delta_1+\delta_2\frac{\sigma s_0x_1+\sigma s_1x_0}{\sigma s_1+\sigma s_0+s_1s_0} +\delta_3p_w+\delta_4x_0, \tag{32}$$

$$E\left(p_1 \mid x_2,x_0\right) = \theta_1+\theta_2\frac{\sigma s_0x_1+\sigma s_1x_0}{\sigma s_1+\sigma s_0+s_1s_0}+\theta_3p_w+\theta_4x_0, \tag{33}$$

$$E\left(\mu \mid x_1,x_0\right) = \frac{\sigma s_0x_1+\sigma s_1x_0}{\sigma s_1+\sigma s_0+s_1s_0}, \tag{34}$$

$$E\left(\mu \mid x_2,x_0\right) = \frac{\sigma s_0x_2+\sigma s_1x_0}{\sigma s_1+\sigma s_0+s_1s_0}. \tag{35}$$

The expected profits of retailer1 and retailer2 are expressed as:

$$E\left(\Pi_1 \mid x_1,x_0\right) = E\left[\left(\frac{d}{2}-\alpha p_1+\beta p_2+\mu\right)\left(p_1-p_w\right) \mid x_1,x_0\right], \tag{36}$$

$$E\left(\Pi_2 \mid x_2,x_0\right) = E\left[\left(\frac{d}{2}-\alpha p_2+\beta p_1+\mu\right)\left(p_2-p_w\right) \mid x_2,x_0\right]. \tag{37}$$

Proposition 3. *When both of retailers are influenced by the supplier price comparison impact x_0, the profit functions $E(\Pi_1 \mid x_1,x_0)$ and $E(\Pi_2 \mid x_2,x_0)$ are strictly concave in p_1 and p_2, respectively, namely, the optimal retail prices p_1^* and p_2^* uniquely exist.*

By solving the first derivative of p_1 and p_2, one can have the optimal pricing strategies of the two retailers as follows:

$$p_1^* = \frac{1}{2\alpha}\left[\frac{d}{2}+\beta E\left(p_2 \mid x_1,x_0\right)+E\left(\mu \mid x_1,x_0\right)+\alpha p_w\right],$$
$$p_2^* = \frac{1}{2\alpha}\left[\frac{d}{2}+\beta E\left(p_1 \mid x_2,x_0\right)+E\left(\mu \mid x_2,x_0\right)+\alpha p_w\right]. \tag{38}$$

Solving (30)–(38) simultaneously, one finds θ_m, $m = 1,2,3,4$ and δ_m, $m = 1,2,3,4$. Consider

$$\theta_1 = \frac{1-t}{2-t} - \frac{\sigma d\left(1-t^2\right)\left[\left(4\sigma+ds_0\right)k_1+\sigma s_0\left(2s_1+ts_2\right)\right]}{U\left(2-t^2\right)},$$

$$\theta_2 = \frac{\sigma s_0\left(1-t^2\right)\left[2\left(1/(\alpha+\beta)-d\right)k_2+d\sigma s_0\right]}{U},$$

$$\theta_3 = \frac{1}{2-t} - \frac{\sigma d\left(1-t^2\right)\left[\left(4\sigma+ds_0\right)k_2+\sigma s_0\left(2s_2+ts_1\right)\right]}{\alpha V\left(4-t^2\right)},$$

$$\theta_4 = \frac{\alpha\sigma s\left(1-t^2\right)_0\left(2k_2+t\sigma s_0\right)}{U},$$

$$\delta_1 = \frac{1-t}{2-t} - \left(\left(4\sigma^2\left(1-t^2\right)\left(\frac{1}{\alpha+\beta}-d\right) \times\left[\left(\frac{2}{\alpha+\beta}-2d\right)(\sigma+s_0)s_1s_2 +\sigma s_0(2+t)(s_1+s_2)\right]\right) \times\left(U\left(2-t^2\right)\right)^{-1}\right),$$

$$\delta_2 = \frac{\sigma s_0 \left(1-t^2\right)\left[\left(2/\left(\alpha+\beta\right)-2d\right)k_1 + d\sigma s_0\right]}{U},$$

$$\delta_3 = \frac{1}{2-t} - \frac{\sigma d\left(1-t^2\right)\left[\left(4\sigma+ds_0\right)k_1+\sigma s_0\left(2s_1+ts_2\right)\right]}{V\left(4-t^2\right)},$$

$$\delta_4 = \frac{\alpha\sigma s_0\left(1-t^2\right)\left(2k_1+t\sigma s_0\right)}{U}, \tag{39}$$

where $t = \beta/\alpha$, $k_1 = (\sigma + s_0)s_1 + \sigma s_0$, $k_2 = (\sigma + s_0)s_2 + \sigma s_0$, $V = 4k_1k_2 - t^2\sigma^2 {s_0}^2$, and $U = 2k_1k_2 - t^2\sigma^2 s_0^2$.

The optimal pricing strategies of the two retailers can be obtained by substituting (39) into $p_1^* = \theta_1 + \theta_2 x_1 + \theta_3 p_w + \theta_4 x_0$ and $p_2^* = \delta_1 + \delta_2 x_2 + \delta_3 p_w + \delta_4 x_0$.

5. Wholesale Pricing Strategy

In this section, we investigate the supplier's pricing decision. The supplier is affected by the signal of PCS. We denote the impact as x_0. Then, it can be divided into three situations according to previous section, namely, according to whether or not the retailers are affected by x_0. We consider the wholesale pricing is $p_w = \lambda_0 + \lambda x_0$, and the supplier aims to maximize its expected profit

$$E\left(\Pi_3 \mid x_0\right) = E\left[\left(D_1 + D_2\right)p_w \mid x_0\right], \tag{40}$$

where $D_1(p_1, p_2, \mu) = d_1 - \alpha p_1 + \beta p_2 + \mu$, $D_2(p_1, p_2, \mu) = d_2 - \alpha p_2 + \beta p_1 + \mu d_1 = d_2 = d/2$; that is,

$$\begin{aligned} &E\left(\Pi_3 \mid x_0\right) \\ &\quad = E\left\{\left[d - (\alpha-\beta)\left(p_1+p_2\right) + 2\mu\right]p_w \mid x_0\right\} \\ &\quad = \left\{d - (\alpha-\beta)E\left[\left(p_1+p_2\right)\mid x_0\right] + 2E\left(\mu \mid x_0\right)\right\}p_w, \\ &E\left(x_1 \mid x_0\right) = E\left(x_2 \mid x_0\right) = E\left(\mu \mid x_0\right) = \frac{\sigma}{x_0\left(\sigma+s_0\right)}. \end{aligned} \tag{41}$$

Denoting $\sigma/(x_0(\sigma + s_0))$ as Δ, optimal wholesale pricing is discussed in the following parts.

5.1. Neither of Retailers Is Affected by x_0. In this subsection, we consider that x_0 is not correlated with the two retailers, but retailers are affected indirectly by x_0 of p_w. According to (3), (4), and (41), we can get the expected profit of supplier as follows:

$$\begin{aligned} &E\left(\Pi_3 \mid x_0\right) \\ &\quad = \{d - (\alpha-\beta)\left[\left(\theta_1+\delta_1\right) - \left(\theta_2+\delta_2\right)\Delta \right. \\ &\qquad\qquad \left. + \left(\theta_3+\delta_3\right)E\left(p_w \mid x_0\right)\right] + 2\Delta\}\, p_w. \end{aligned} \tag{42}$$

Proposition 4. *When x_0 exerts impact to neither of retailers, the profit function $E(\Pi_3 \mid x_0)$ is strictly concave in p_{w1}, and the optimal wholesale price p_{w1}^* uniquely exists.*

Solving the first-order derivative of $E(\Pi_3 \mid x_0)$ with respect to p_w, one can get

$$\begin{aligned} p_{w1}^* = {} & 8\alpha d\sigma^2 s_0\left(s_2+s_1\right)\left(\left(2-t\right)\alpha - t\beta + 1\right) \\ & \times \left(2-t\right)^2\left(1-t^2\right)\left(k_1+k_2\right) \\ & \times \left(\left(\frac{1}{\alpha+\beta} - d\right)s_1 + 2ds_2\left(1+k_1\right)\right) \\ & + \frac{\sigma^2 s_0}{\sigma+s_0}\left(2-t\right)\left(\alpha-\beta\right)\left(1-t^2\right)\left(k_1+k_2\right) \\ & \times \left(\left(\frac{2}{\alpha+\beta} - 2d\right) + d\sigma s_0\right) \\ & - \frac{\left(2-t\right)\left(d\left(\sigma+s_0\right)+2\sigma\right)}{\left(1-t\right)\left(\sigma+s_0\right)}, \end{aligned} \tag{43}$$

where $t = \alpha/\beta$, $\theta_1, \delta_1, \theta_2, \delta_2, \theta_3, \delta_3$ are the same as the results mentioned in Section 4.1.

5.2. Either of Retailers Is Affected by x_0. In this subsection, we consider either of retailers is affected by x_0. According to (14), (15), and (41), we can get the expected profit of supplier as follows:

$$\begin{aligned} &E\left(\Pi_3 \mid x_0\right) \\ &\quad = \{d - (\alpha-\beta)\left[\left(\theta_1+\delta_1\right) + \left(\theta_2+\delta_2\right)\Delta + \left(\theta_3+\delta_3\right)\right. \\ &\qquad\qquad \left. \times E\left(p_w \mid x_0\right) + \delta_4 E\left(x_0 \mid x_0\right)\right] + 2\Delta\}\, p_w. \end{aligned} \tag{44}$$

Proposition 5. *When x_0 exerts impact on either of the retailers, the supplier's profit function $E(\Pi_3 \mid x_0)$ is strictly concave in p_{w2}, and the optimal wholesale price p_{w2}^* uniquely exists.*

Solving the first-order derivative of $E(\Pi_3 \mid x_0)$ with respect to p_w, and let $dEp_w/dp_w = 0$, one can get p_{w2}^*

$$\begin{aligned} p_{w2}^* = \Bigg[& \left(4k_1k_2 - t^2\sigma^2 s_0^2\right) \\ & - \Bigg(\left(d\left(4k_1k_2 - t^2\sigma^2 s_0^2\right)\left(4-t^2\right)\right) \\ & \quad \times \left(\left(1-t\right)\left(4k_1k_2 - t^2\sigma^2 s_0^2\right) - 4\sigma\left(2-t\right)\right. \\ & \quad \left. \times \left(2\left(\frac{1}{\alpha+\beta} - d\right)s_1k_2 + ds_2k_1\right)\right)^{-1}\Bigg)\Bigg] \cdot x_0 \\ & + 4\sigma\alpha\left(\left(2-t\right)\alpha - t\beta\right)\left(1-t^2\right)\left(k_1+k_2\right) \\ & \times \left(\left(2s_1\left(\frac{1}{\alpha+\beta} - d\right) + ds_2\right) + d\sigma s_0\left(2+t\right)\left(s_1+s_2\right)\right) \\ & + \frac{2s_0\sigma^2}{\sigma+s_0}\left(2-t\right)\left(\alpha-\beta\right)\left(1-t^2\right)\left(k_1+k_2\right) \end{aligned}$$

$$\times\left(\left(\frac{1}{\alpha+\beta}-d\right)+d\sigma s_0\right)$$
$$-\left((\sigma s_1(2k_2+t\sigma s_1)(1+t))\right.$$
$$\times\left(\left(4k_1k_2-t^2\sigma^2s_0^2\right)\left(4-t^2\right)-4\sigma(2-t)(1+t)\right.$$
$$\left.\left.\times\left(2\left(\frac{1}{\alpha+\beta}-d\right)s_1k_2+ds_2k_1\right)\right)^{-1}\right), \quad (45)$$

where $t=\alpha/\beta, \theta_1, \delta_1, \theta_2, \delta_2, \theta_3, \delta_3$ are the same as the results in Section 4.2.

5.3. Both of Retailers Are Affected by x_0. In this subsection, x_0 exerts impact on both of retailers. According to (30), (31), and (41), we can get the expected profit of supplier as follows:

$$\begin{aligned}E(\Pi_3\mid x_0) &= \{d-(\alpha-\beta)[(\theta_1+\delta_1)+(\theta_2+\delta_2)\Delta+(\theta_3+\delta_3)\\ &\quad\times E(p_w\mid x_0)\\ &\quad+(\theta_4+\delta_4)E(x_0\mid x_0)]+2\Delta\}p_w.\end{aligned} \quad (46)$$

Proposition 6. *When x_0 exerts impact on both of retailers, the profit function $E(\Pi_3 \mid x_0)$ is strictly concave in p_{w3}, and the optimal wholesale price p_{w3}^* uniquely exists.*

Solving the first-order derivative of $E(\Pi_3 \mid x_0)$ with respect to p_w, one obtains the optimal wholesale price p_{w3}^* as follows:

$$p_{w3}^*=\frac{\alpha\sigma s_0\left(1-t^2\right)\left(2\left(k_1+k_2\right)+t\sigma s_0\right)}{2k_1k_2-t^2\sigma^2s_0^2}\cdot x_0$$
$$+\left(\alpha-t\beta-\frac{1}{d}\right)$$
$$\times\left[\left(\left(4\alpha\sigma^2\left(1-t^2\right)\left(\frac{1}{\alpha+\beta}-d\right)\right.\right.\right.$$
$$\times\left(\left(\frac{2}{\alpha+\beta}-2d\right)+(2+t)\sigma s_0\left(s_1+s_2\right)\right)$$
$$+\alpha\sigma^2d\left(1-t^2\right)$$
$$\left.\times\left((4\sigma+ds_0)k_1+\sigma s_0(2s_1+s_2)\right)\right)$$
$$\times\left(\alpha\left(2k_1k_2-t^2\sigma^2s_0^2\right)\left(2-t^2\right)\right.$$
$$\left.\left.\times\left(4k_1k_2-t^2\sigma^2s_0^2\right)\left(4-t^2\right)\right)^{-1}\right)-\frac{\sigma}{\sigma+s_0}$$
$$\left.\cdot\frac{\left(1-t^2\right)\sigma s_0\left((2/(\alpha+\beta)-2d)(k_1+k_2)+ds_0\right)}{2k_1k_2-t^2\sigma^2s_0^2}\right], \quad (47)$$

where $t=\alpha/\beta, \theta_1, \delta_1, \theta_2, \delta_2, \theta_3, \delta_3$ are the same as the results in Section 4.3.

6. Simulation Analysis

Assuming that $d = 200$, $\alpha = 0.5$, $\beta = 0.3$, $\mathrm{diag}(\sigma, s_0, s_1, s_2) = \mathrm{diag}(10, 2, 4, 3)$, and $\lambda = 10$, $\lambda_0 = 40$ based on the three situations discussed above, we make an analysis of how x_0, x_1, x_2 affects optimal pricing p_1^*, p_2^*, p_w^*. Based on the analysis of the three situations shown above, we are able to obtain optimal wholesale prices in different situations. The numerical results are shown in Tables 2, 3, 4, and 5.

6.1. Simulation Analysis of Retail Pricing Model. Recall that x_0, x_1, x_2 represent the degree of PCS signal on price, we take the values of x_0, x_1, x_2 from 0 to 1, where $x_0, x_1, x_2 = 0$ implies that the corresponding parties are not affected by the signal of PCS, whereas $x_0, x_1, x_2 = 1$ implies that the corresponding party are fully affected by the signal of PCS. As for the retailers, from Figure 2, we can see when x_0, x_1, x_2 take values from 0 to 1, the line of Situation 1 p_1^* is higher than those in Situations 2 and 3; the line of Situation 2 p_2^* is higher than the line of Situation 2 p_1^*; the line of Situation 3 p_1^* is the lowest one among all situations. Besides, optimal prices in every situation are increasing with x_0, x_1, x_2. From Figure 3, we can find that the line of Situation 1 $E(p_1^*)$ is the highest among all situations. When x_0, x_1, x_2 are from 0.4 to 0.7, the line of Situation 2 $E(p_1^*)$, $E(p_2^*)$ and the line of Situation 3 $E(p_1^*)$ are getting closer. Besides, optimal profits of every situation are increasing with x_0, x_1, x_2.

According to Figures 2 and 3, three observations for the retailers are noteworthy.

Observation 1. The optimal retail price of Situation 1 and its corresponding optimal profit are higher than those of Situations 2 and 3. In other words, in order to obtain a higher profit, the retailers are more willing to avoid the existence of PCS.

Observation 2. Although the retailers tend to avoid the existence of PCS, they are also willing to seek more availability of price information because the optimal retail price and their corresponding profit would be higher if the availability of pricing information is more sufficient.

Observation 3. In Situation 2, obtaining the information from the PCS could lead retailer1 to set a higher retail price and gain more profits than those of retailer2. Hence, the retailer should

Table 2: Optimal retail pricing strategies in different situations.

		$x_0 = x_1 = x_2$										
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Situation 1	p_1^*, p_2^*	174	176	178	180	183	187	189	193	198	201	205
Situation 2	p_1^*	96	103	111	120	125	130	134	140	147	152	158
	p_2^*	150	156	160	163	166	169	172	174	176	178	181
Situation 3	p_1^*, p_2^*	71	76	80	89	101	111	120	126	138	147	152

Table 3: Optimal profits in different situations.

		$x_0 = x_1 = x_2$										
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Situation 1	$E(p_1^*), E(p_2^*)$	22358	22248	22411	22468	22712	23089	23194	23564	24060	24289	24645
Situation 2	$E(p_1^*)$	7918	8083	8289	8540	8649	9758	9829	9985	10180	10301	10453
	$E(p_2^*)$	6432	6908	7382	7845	8071	8384	8624	8909	9516	9918	10099
Situation 3	$E(p_1^*), E(p_2^*)$	5659	5968	6192	6781	7548	8134	8444	8644	8850	8972	9132

Table 4: Optimal wholesale price offered by supplier under different x_0.

	x_0										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
p_{w1}^*	150	150	150	150	150	150	150	150	150	150	150
p_{w2}^*	167	182	197	212	227	242	257	272	287	302	317
p_{w3}^*	233	273	313	353	393	433	473	513	553	593	633

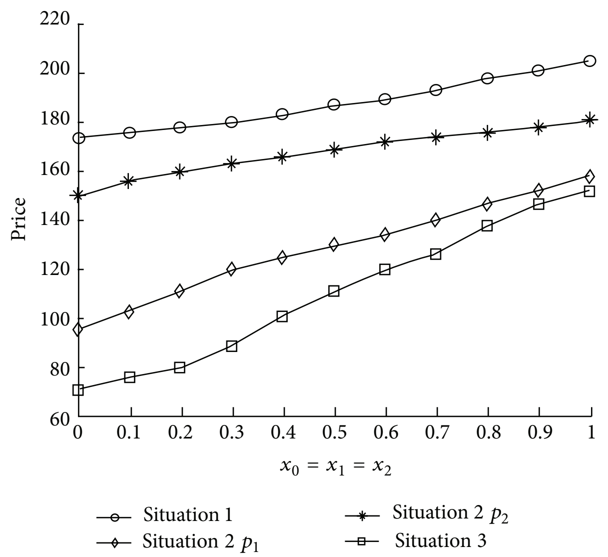

Figure 2: Optimal retail prices for 4.1, 4.2, and 4.3.

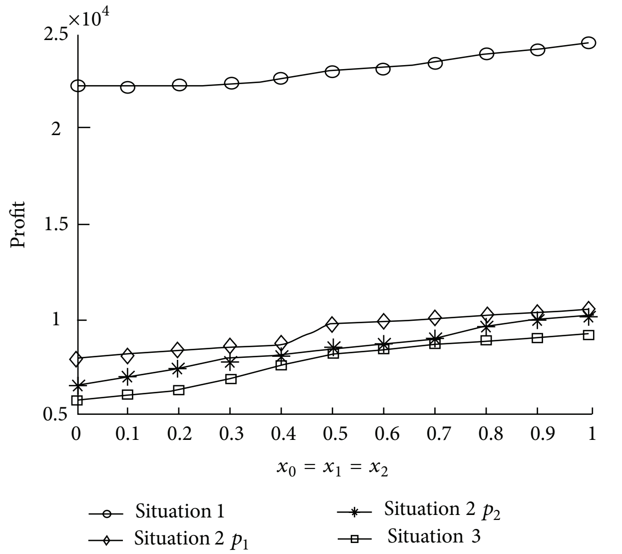

Figure 3: Optimal profits of retailers for 4.1, 4.2, and 4.3.

tend to exclusively cooperate with the PCS. This observation enhances the value of the existence of PCS.

Based on Observations 1, 2, and 3, the best strategy for the retailers is to avoid the PCS and meanwhile to increase the availability of price information. In reality, the PCS might be a third party service who is out of the control of the retailers. However, our results provide an important implication about how the retailers react if the PCS existed.

6.2. Simulation Analysis of Wholesale Pricing Model. As for the supplier, from Figure 4, we can see that the wholesale prices in every situation are linearly correlated with x_0. For certain x_0, the optimal wholesale prices in Situation 3 is higher than those in Situations 1 and 2. And the optimal wholesale prices in Situations 2 and 3 are increasing with x_0. From Figure 5, we can find that profits in Situations 2 and 3 are increasing with x_0; however, the one in Situation 1 is linearly negatively correlated with x_0.

According to Figures 4 and 5, three observations are noteworthy.

Table 5: Optimal profits of supplier under different x_0.

	x_0										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$E(p_{w1}^*)$	23400	22050	21300	20100	18450	17250	16350	15150	14100	13200	12150
$E(p_{w2}^*)$	27555	27664	28171	29468	29964	30976	31611	32096	32431	32616	33285
$E(p_{w3}^*)$	34018	38493	41316	43713	45195	46764	47773	49248	51429	52777	53805

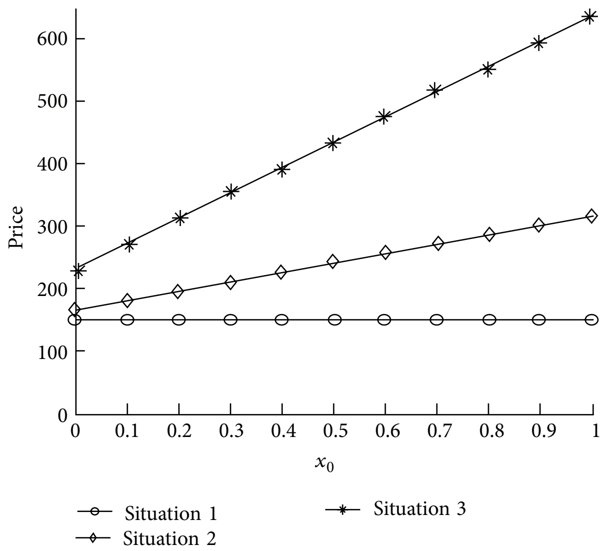

Figure 4: Optimal wholesale prices for 5.1, 5.2, and 5.3.

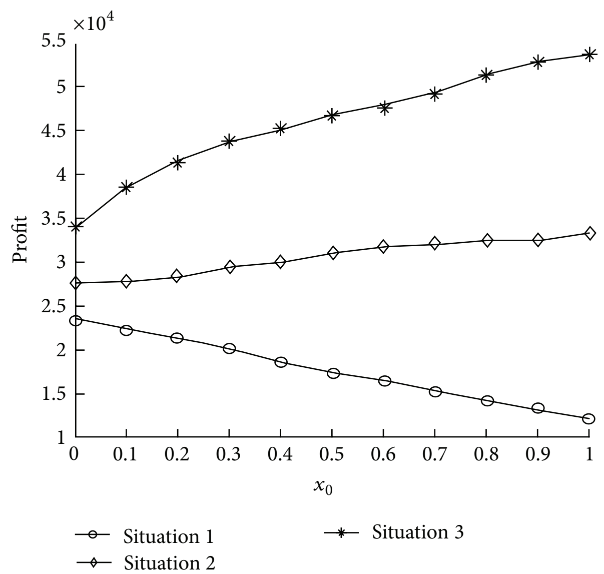

Figure 5: Optimal profits of supplier for 5.1, 5.2, and 5.3.

Observation 4. The optimal wholesale price of Situation 1 is lower than those of Situations 1 and 2, while the corresponding optimal profit is higher than those of Situations 1 and 2. This observation implies that the supplier tends to set a higher wholesale price if the PCS affects the decisions of retail price made by both of retailers. Similar to the results of retailers, the supplier is also more willing to avoid the existence of PCS.

Observation 5. We notice that, in Situation 3, both of the retailers are affected by the PCS; more availability of price information would lead to a decrease in the supplier's profit. Hence, the supplier in Situation 3 should keep the availability of price information as much as possible.

Observations 4 and 5 indicate that the supplier is more willing to avoid the impact of PCS and also increase the availability of price information; however, if both of the retailers are affected by the PCS, he should tend to reduce the availability of price information.

7. Conclusions

With the rapid development of e-commerce, the PCS enables to provide transparent price and promote rational consumption. However, the PCS would also affect the pricing decision made by supply chain parties. In this study, we derived the optimal pricing strategies with respective to three situations of signal availability of PCS. Our numerical study implied that when both of retailers are affected by the PCS, the supplier is more willing to reduce the availability of price information, and when either of retailers is affected by the PCS, the retailer should tend to exclusively cooperate with the PCS. In addition, the retailers and supplier are all more willing to avoid the existence of PCS and increase the availability of price information. This result implied that (1) if the entire supply chain belongs to an identical corporate, the corporate is better to merge the PCS; (2) more transparency in price information actually hurts the supply chain parties' and supply chain's profit. This is consistent with the industrial practice in which price war could lead to a big drop in both market and price [21].

For the research limitations, we admit that the findings are mainly based on the analytical models and numerical studies we have developed under various assumptions, such as a linear demand function with respect to the PCS. In addition, the analysis is conducted on a relatively simple dual-channel supply chain of one supplier and two competing retailers, which is an abstract version of the more complex real world supply chain. In this paper, we consider that the PCS is independent from the parties in supply chain; however, according to the practices in China, we observe that Baidu has developed new business model as a reaction to price comparison impact. It would be interesting to investigate the mechanism of how the supply chain parties should cooperate with the PCS. It is also promising to extend the model and the analysis to more complex supply chain systems (such as the ones that are risk averse agents [22–24]). It is also meaningful to investigate whether it is beneficial for the supplier to always share information from contracting perspective among supply chain parties [25, 26]. This would address another interesting topic from the perspective of social influence on the PCS in supply chain management [27].

Appendices

Proof of Proposition 1. According to (2), (4), (6), (8), and (10), we have

$$\begin{aligned}
E(\Pi_1 \mid x_1, p_w) &= [d_1 - \alpha p_1 + \beta E(p_2 \mid x_1, p_w) + E(\mu \mid x_1, p_w)] \\
&\quad \times (p_1 - p_w) \\
&= [d_1 - \alpha p_1 + \beta(\delta_1 + \delta_2 E(x_2 \mid x_1, p_w) + \delta_3 p_w)] \\
&\quad \times (p_1 - p_w) \\
&= \Bigg[d_1 - \alpha p_1 \\
&\quad + \beta\left(\delta_1 + \delta_2 \cdot \frac{\sigma s_0 x_2 \lambda + \sigma s_2 (p_w - \lambda_0)}{(\sigma s_2 + \sigma s_0 + s_2 s_0)\lambda} + \delta_3 p_w\right)\Bigg] \\
&\quad \times (p_1 - p_w) \\
&= \Bigg[d_1 - \alpha p_1 + \beta\Bigg(\delta_1 + \delta_2 \cdot \frac{\sigma s_0 x_2 + \sigma s_2 x_0}{\sigma s_2 + \sigma s_0 + s_2 s_0} \\
&\quad + \delta_3(\lambda_0 + \lambda x_0)\Bigg)\Bigg][p_1 - (\lambda_0 + \lambda x_0)] \\
&= -\alpha p_1^2 + W_1 p_1 + Y_1,
\end{aligned} \tag{A.1}$$

where $\alpha > 0$, $W_1 = d_1 + \beta(\delta_1 + \delta_2 \cdot (\sigma s_0 x_2 + \sigma s_2 x_0)/(\sigma s_2 + \sigma s_0 + s_2 s_0) + \delta_3(\lambda_0 + \lambda x_0)) + \alpha(\lambda_0 + \lambda x_0)$ and $Y_1 = -(\lambda_0 + \lambda x_0)(d_1 + \beta(\delta_1 + \delta_2 \cdot (\sigma s_0 x_2 + \sigma s_2 x_0)/(\sigma s_2 + \sigma s_0 + s_2 s_0) + \delta_3(\lambda_0 + \lambda x_0)))$.

According to (2), (3), (7), (9), and (11), we have

$$\begin{aligned}
E(\Pi_2 \mid x_2, p_w) &= [d_2 - \alpha p_2 + \beta E(p_1 \mid x_2, p_w) \\
&\quad + E(\mu \mid x_2, p_w)](p_2 - p_w) \\
&= \Bigg[d_2 - \alpha p_2 + \beta \frac{\lambda \sigma s_0 x_2 + \sigma s_2 (p_w - \lambda_0)}{(\sigma s_2 + \sigma s_0 + s_2 s_0)\lambda} \\
&\quad + \frac{\lambda \sigma s_0 x_2 + \sigma s_2 (p_w - \lambda_0)}{(\sigma s_2 + \sigma s_0 + s_2 s_0)\lambda}\Bigg](p_2 - p_w) \\
&= \Bigg[d_2 - \alpha p_2 + \beta \frac{\sigma s_0 x_2 + \sigma s_2}{\sigma s_2 + \sigma s_0 + s_2 s_0} \\
&\quad + \frac{\sigma s_0 x_2 + \sigma s_2}{\sigma s_2 + \sigma s_0 + s_2 s_0}\Bigg][p_2 - (\lambda_0 + \lambda x_0)] \\
&= -\alpha p_2^2 + W_2 p_2 + Y_2,
\end{aligned} \tag{A.2}$$

where $\alpha > 0$, $W_2 = \alpha(\lambda_0 + \lambda x_0)(d_2 + \beta(\sigma s_0 x_2 + \sigma s_2)/(\sigma s_2 + \sigma s_0 + s_2 s_0) + (\sigma s_0 x_2 + \sigma s_2)/(\sigma s_2 + \sigma s_0 + s_2 s_0))$, $Y_2 = -(\lambda_0 + \lambda x_0)(\beta((\sigma s_0 x_2 + \sigma s_2)/(\sigma s_2 + \sigma s_0 + s_2 s_0)) + (\sigma s_0 x_2 + \sigma s_2)/(\sigma s_2 + \sigma s_0 + s_2 s_0))$.

According to the nature of the quadratic function and $Y_2 < 0$, the profit functions $E(\Pi_1 \mid x_1, p_w)$ and $E(\Pi_2 \mid x_2, p_w)$ are strictly concave in p_1 and p_2, respectively. □

Proof of Proposition 2. According to (2), (15), (18), and (20), we have

$$\begin{aligned}
E(\Pi_1 \mid x_1, x_0) &= E\left[\left(\frac{d}{2} - \alpha p_1 + \beta p_2 + \mu\right)(p_1 - p_w) \mid x_1, x_0\right] \\
&= E\Bigg[\Bigg(\frac{d}{2} - \alpha p_1 + \beta(\delta_1 + \delta_2 x_2 + \delta_3(\lambda_0 + \lambda x_0) + \delta_4 x_0) \\
&\quad + \mu\Bigg)(p_1 - \lambda_0 - \lambda x_0) \mid x_1, x_0\Bigg] \\
&= -\alpha p_1^2 + W_3 p_1 + Y_3,
\end{aligned} \tag{A.3}$$

where $\alpha > 0$, W_3 represents a coefficient, and Y_3 represents constant in this quadratic function.

According to (2), (14), (17), and (19), we have

$$\begin{aligned}
E(\Pi_2 \mid x_2, p_w) &= E\left[\left(\frac{d}{2} - \alpha p_2 + \beta p_1 + \mu\right)(p_2 - p_w) \mid x_2, p_w\right] \\
&= E\Bigg[\left(\frac{d}{2} - \alpha p_2 + \beta(\theta_1 + \theta_2 x_1 + \theta_3 p_w + \theta_4 x_0) + \mu\right) \\
&\quad \times (p_2 - \lambda_0 - \lambda x_0) \mid x_2, p_w\Bigg] \\
&= -\alpha p_2^2 + W_4 p_2 + Y_4,
\end{aligned} \tag{A.4}$$

where $\alpha > 0$, W_4 represents a coefficient, and Y_4 represents constant in this quadratic function.

Similar to Proposition 1, according to the nature of the quadratic function, both $E(\Pi_1 \mid x_1, x_0)$ and $E(\Pi_2 \mid x_2, p_w)$ are strictly concave in p_1 and p_2, respectively. □

Proof of Proposition 3. According to (2), (31), (32), (34), and (36), we have

$$\begin{aligned}
E(\Pi_1 \mid x_1, x_0) &= E\left[\left(\frac{d}{2} - \alpha p_1 + \beta p_2 + \mu\right)(p_1 - p_w) \mid x_1, x_0\right] \\
&= E\left[\left(\frac{d}{2} - \alpha p_1 + \beta(\delta_1 + \delta_2 x_2 + \delta_3 p_w + \delta_4 x_0) + \mu\right)\right. \\
&\qquad \left. \times (p_1 - \lambda_0 - \lambda x_0) \mid x_1, x_0\right] \\
&= -\alpha p_1^2 + W_5 p_1 + Y_5,
\end{aligned} \tag{A.5}$$

where $\alpha > 0$, W_5 represents a coefficient, and Y_5 represents constant in this quadratic function.

According to (2), (30), (33), and (35),we have

$$\begin{aligned}
E(\Pi_2 \mid x_2, x_0) &= E\left[\left(\frac{d}{2} - \alpha p_2 + \beta p_1 + \mu\right)(p_2 - p_w) \mid x_2, x_0\right] \\
&= E\left[\left(\frac{d}{2} - \alpha p_2 + \beta(\theta_1 + \theta_2 x_1 + \theta_3 p_w + \theta_4 x_0) + \mu\right)\right. \\
&\qquad \left. \times (p_2 - \lambda_0 - \lambda x_0) \mid x_2, x_0\right] \\
&= -\alpha p_2^2 + W_6 p_2 + Y_6,
\end{aligned} \tag{A.6}$$

where $\alpha > 0$, W_6 represents a coefficient, and Y_6 represents constant in this quadratic function.

Similar to Proposition 1, the profit functions $E(\Pi_1 \mid x_1, x_0)$ and $E(\Pi_2 \mid x_2, x_0)$ are strictly concave in p_1 and p_2, respectively. □

Proof of Proposition 4. According to (3), (4), and (41), we have

$$\begin{aligned}
E(\Pi_3 \mid x_0) &= E\{[d - (\alpha - \beta)(p_1 + p_2) + 2\mu]\, p_w \mid x_0\} \\
&= \{d - (\alpha - \beta) E[(p_1 + p_2) \mid x_0] + 2E(\mu \mid x_0)\}\, p_w \\
&= \{d - (\alpha - \beta)[(\theta_1 + \delta_1) - (\theta_2 + \delta_2)\Delta \\
&\qquad + (\theta_3 + \delta_3) E(p_w \mid x_0)] + 2\Delta\}\, p_w \\
&= (\beta - \alpha)(\theta_3 + \delta_3)\, p_w^2 + W_7 p_w + Y_7,
\end{aligned} \tag{A.7}$$

where $\alpha > \beta$, W_7 represents a coefficient, and Y_7 represents constant in this quadratic function.

Similarly, according to the nature of the quadratic function, the profit function $E(\Pi_3 \mid x_0)$ is strictly concave in p_{w1}. □

Proof of Proposition 5. According to (14), (15), and (41), we have

$$\begin{aligned}
E(\Pi_3 \mid x_0) &= E\{[d - (\alpha - \beta)(p_1 + p_2) + 2\mu]\, p_w \mid x_0\} \\
&= \{d - (\alpha - \beta) E[(p_1 + p_2) \mid x_0] + 2E(\mu \mid x_0)\}\, p_w \\
&= \{d - (\alpha - \beta)[(\theta_1 + \delta_1) + (\theta_2 + \delta_2)\Delta + (\theta_3 + \delta_3) \\
&\qquad \times E(p_w \mid x_0) + \delta_4 E(x_0 \mid x_0)] + 2\Delta\}\, p_w \\
&= (\beta - \alpha)(\theta_3 + \delta_3)\, p_w^2 + W_8 p_w + Y_8,
\end{aligned} \tag{A.8}$$

where $\alpha > \beta$, W_8 represents a coefficient, and Y_8 represents constant in this quadratic function.

Similarly, according to the nature of the quadratic function, the profit function $E(\Pi_3 \mid x_0)$ is strictly concave in p_{w2}. □

Proof of Proposition 6. According to (30), (31), and (41), we have

$$\begin{aligned}
E(\Pi_3 \mid x_0) &= E\{[d - (\alpha - \beta)(p_1 + p_2) + 2\mu]\, p_w \mid x_0\} \\
&= \{d - (\alpha - \beta) E[(p_1 + p_2) \mid x_0] + 2E(\mu \mid x_0)\}\, p_w \\
&= \{d - (\alpha - \beta)[(\theta_1 + \delta_1) + (\theta_2 + \delta_2)\Delta \\
&\qquad + (\theta_3 + \delta_3) E(p_w \mid x_0) \\
&\qquad + (\theta_4 + \delta_4) E(x_0 \mid x_0)] + 2\Delta\}\, p_w \\
&= (\beta - \alpha)(\theta_3 + \delta_3)\, p_w^2 + W_9 p_w + Y_9,
\end{aligned} \tag{A.9}$$

where $\alpha > \beta$, W_9 represents a coefficient, and Y_9 represents constant in this quadratic function.

Similarly, according to the nature of the quadratic function, the profit function $E(\Pi_3 \mid x_0)$ is strictly concave in p_{w3}. □

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Research Article

Emergency Department Staffing: A Separated Continuous Linear Programming Approach

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The emergency department (ED) overcrowding is becoming serious in recent years. The shortage of ED staff is considered to be a common factor to contribute to the ED overcrowding. In this paper, we present a new model to address the ED staffing from a macroscopic perspective. The model is a kind of optimization model called separated continuous linear programming (SCLP). By using the efficient algorithm for SCLP, we can get the ED staffing level which minimizes the total cost of the patients and the ED.

1. Introduction

Emergency medical service is an important part of medical and health care systems. Its quality of service and availability directly affect the quality of health care. In recent years, the emergency department (ED) in many hospitals has experienced overcrowding [1, 2], and patients, health care workers, and hospitals in which the ED exists are adversely affected.

ED overcrowding leads to the delay in the assessment and treatment of patients [3], including the delay in hospitalization. Long residence time in ED adds additional pain, burden on patients and sometimes even leads to the unexpected death.

ED overcrowding makes health care workers overworked [4] and greatly influences their enthusiasm for work and efficiency in work, the chances of medical errors and medical disputes increasing. At the same time, ED overcrowding also affects the normal operation of the entire hospital in which the ED exists, leading to a longer hospitalization time for the emergency patients and lowering the capability of the hospital to meet the need for hospitalization of other nonurgent patients.

Many problems lead to ED overcrowding; one of them is the shortage of physicians to meet the fast-growing demand for ED. There are several reasons for the shortage of physicians; one of them is the high operational cost of ED. Physician salaries represent a large part of the operational cost in ED and there is not enough motivation for the hospital to improving their service by increasing the amount of physicians in ED in the past. The increasing degree of ED overcrowding in recent years introduces so many troubles to the hospitals and the management authority in hospitals found that they have no other choice but trying to find effective methods to alleviate ED overcrowding. Obviously, improving the quality of ED staffing decision is one approach. The management authority wants to know the least amount of physicians ED needs to provide improved quality of service, so that the degree of ED overcrowding can be alleviated to an acceptable level.

In recent years, research efforts are made to improve the ED staffing. This paper provides a new approach to balance the waiting cost of the patients and the operational cost of the EDs. The objective is to find an appropriate amount of physicians and their scheduling which can meet the ED demand with the least total cost of the patients and the EDs. The advantage of our approach is that the time needed to find the ED staffing level is shorter compared with the existing methodologies. Also, we do not make strong assumptions on the distribution of the arrival rate and the distribution of the service time of the demand to ED.

The paper is organized as follows. In Section 2, we provide a literature review on ED staffing; we also give a brief introduction to separated continuous linear programming (SCLP) on which our approach is based. In Section 3, we describe the emergency service process that we focus on. In Section 4 we construct an SCLP model to minimize the total cost of patients and the ED given a specific ED staffing level. We then solve the SCLP model with varying staffing level in ED and finally find the appropriate staffing level which results in the least total cost of patients and the ED among the possible staffing levels. In Section 5, we summarize what we get and point out some future research directions.

2. The Literature Review

2.1. Literature on ED Staffing. There are two main research methodologies to tackle the staffing in ED. One is simulation and the other is the stochastic models based on the queueing theory.

The most commonly used research method for ED staffing is simulation [5]. The reason is that the ED of a modern hospital is a highly complex system and it is difficult to use the analytic model to describe the dynamic ED reality accurately. Simulation offers a natural framework to address this issue within. Simulation models often possess high validity because they track true system behavior fairly accurately, but they yield fewer analytic inference than that of analytic models because they take both input parameters and decisions as fixed. A lot of experiments need to be performed before an approximate optimal solution can be found, and the gap between the approximation optimal solution and the true optimal solution is unknown. Also, the approximate optimal solution obtained is just a numerical solution and we have no knowledge on the interactions among ED staffing and other managerial decisions, thus makeing it difficult to pinpoint the relationship between the staffing level and the quality of service in ED.

On the other hand, although analytic models can never capture all characteristics of an actual ED process, stochastic models, based on queuing theory, are more appropriate for capturing the volatile and inherently nondeterministic ED reality, and they can be invaluable in providing managerial insight that greatly improves performance of the ED process [6]. Although the operation in ED is very complex, many researchers try to focus on the most important parts of the ED process and build simpler stochastic queueing models to study the ED staffing. These models strive to seek a balance between the tractability and validity and get some valuable results [5, 7–9].

There are three important characteristics of the demand of EDs. Firstly, the arrival rate of the patients has significant variation over the time-of-the-day, day-of-the-week, and month-of-the year, that is, time-varying arrival rate. Secondly, the patients are prioritized into emergent, urgent, and nonurgent categories by use of triage upon arrival, that is, multiclass customers. Thirdly, the patients usually visit the physician several times during their sojourn time in the ED before they are discharged or hospitalized, that is, customer feedback.

Zeltyn et al. [5] use simulation-based models to address the ED staffing with time-varying demand. They incorporate the offered-load technique and classical square-root safety-staffing principle which are based on the M/M/s queueing model. The staffing recommendations they provided are implemented by a large Israeli hospital and get the satisfactory results.

Green et al. [8] use the M/M/s queueing model as part of a Lag stationary independent period by period (SIPP) approach to determine how to vary staffing level to meet changing demand in an urban hospital ED in the United States. Their staffing recommendations have been implemented in that ED and as a result the proportion of patients who left without being seen (LWBS) decreases significantly.

Recently, Yom-Tov and Mandelbaum [9] analyze a queueing model that they call Erlang-R, where the "R" stands for ReEntrant customers, to accommodate the phenomenon that the patients return to physician several times during their sojourn within the ED. They propose time-varying square-root staffing policy which is based on the modified-offered-load and Erlang-R model. They use simulations to verify that their staffing recommendations work well in the ED settings.

One of the problems for the queueing model approach is that some special distributions for patient arrival and service process, such as the Poisson arrival and exponential service time, and some working discipline, such as First-Come-First-Serve (FCFS), need to be assumed in order to get the explicit closed-form expression for some quality measures (e.g., the fraction of patients who LWBS in an ED) on the ED service. But these assumptions do not always agree with reality.

The purpose of the present paper is to introduce models for dynamic ED staffing without explicitly calculating the quantities of interest like patients' waiting time, the fraction of patients who LWBS, and so forth, and rather developing models directed towards using efficient algorithms to handle these quantities. We do this by looking at the ED system from a macroscopic perspective and use fluid network to approximate the overloaded ED system. From our knowledge, there is no other work tackling the ED staffing by using this approach.

2.2. A Brief Introduction to SCLP. Linear programming (LP) is probably the most successful mathematical model in terms of its extremely wide range of industrial applications and its superb speed and capacity in solving very large size problems. For this reason, LP has been pushed and extended to an ever-broadening frontier. One of such frontier extensions is the so-called *separated continuous linear programming* (SCLP) which was first introduced by Anderson [10] who used it to model the job-shop scheduling problem. The following is the SCLP due to Weiss [11]:

$$\begin{aligned}
\text{(SCLP)}\quad \max\quad & \int_0^T \left[\left(\gamma + (T-t)\,c\right)' u(t) + d' x(t)\right] dt \\
\text{s.t.}\quad & \int_0^t G u(s)\, ds + F x(t) \le \alpha + t a, \\
& H u(t) \le b, \\
& u(t) \ge 0, \quad x(t) \ge 0, \quad t \in [0, T],
\end{aligned} \tag{1}$$

where $u(t)$, $x(t)$ are decision variables and are assumed to be bounded measurable functions with the measure of the break-point set being 0. $\gamma, c, d, \alpha, a, b$ are vectors, G, F, H are matrices. $'$ denotes the transpose operation. The word "separated" refers to the fact that there are two kinds of constraints in SCLP: the constraints involving integration and the instantaneous constraints [10].

SCLP has in recent years attracted considerable research attention in the field of stochastic networks, a part of queueing theory. The multiclass stochastic network is a system consists of different classes (types) of jobs which need to be processed and a set of servers which process the jobs. Jobs arrive to the system randomly. Each server can process one or more classes of jobs and the processing time for every job is different for different class of jobs. The jobs in the same class have the same characteristics such as arrival rate and service requirements. After one job is processed in one server, it may leave the network instantaneously or may become another class of job and go to another server for processing. The multiclass stochastic network is a very useful model for many real systems.

For each multiclass stochastic network, there is a corresponding deterministic fluid network, which takes only the first-order data (means and rates) from the stochastic model and assumes that the jobs circulating in the network are continuous flows instead of discrete units. With appropriate scaling, the fluid network is a limit of the stochastic network, in the sense of strong law of large numbers (refer to, e.g., [12]). Furthermore, the fluid model has played a central role in studying the stability of stochastic networks [13]. Because of these developments, the real-time control (dynamic scheduling) of a stochastic network, which is itself a quite intractable problem, can be turned into the control of a corresponding fluid network, and the latter problem takes exactly the form of SCLP. We will show in Section 4 that the problem of finding the dynamic control of the patient flow, subproblem of optimizing the ED staffing, can be formulated as an SCLP (see (20) in Section 4).

The dual of SCLP is the following problem:

$$\begin{aligned}
(\mathrm{SCLP}^*) \qquad \min \quad & \int_0^T \left[(\alpha + (T-t)\,a)' p(t) + b' q(t)\right] dt \\
\text{s.t.} \quad & \int_0^t G' p(s)\, ds + H' q(t) - (\gamma + tc) \geq 0, \\
& F' p(t) - d \geq 0, \\
& p(t) \geq 0, \qquad q(t) \geq 0, \qquad t \in [0, T],
\end{aligned} \tag{2}$$

where the dual variables, $p(t)$ and $q(t)$, are bounded measurable functions.

There exists the rich literature on duality theory and algorithms for solving SCLP. Interested reader may refer to [14] for the detailed review on the research results on SCLP. Specifically, Wang [15] suggested an algorithm which is a polynomial approximation scheme for solving SCLP. Because that algorithm is very efficient in terms of the computing time, we choose that algorithm to solve the model we propose in this paper. In the next Section 2.3, we will explain in detail how that algorithm works.

2.3. An Approximation Algorithm for SCLP. Before explaining the algorithm in [15], we first introduce some notations and conventions which we will use in the remaining part of this paper.

(i) By default, all vectors are column vectors. One exception is when we denote the solutions to the SCLP as (u, x), we mean $(u', x')'$.

(ii) $\pi = \{t_0, \ldots, t_m\}$ denotes a partition of $[0, T]$ into m segments:

$$0 = t_0 < t_1 < \cdots < t_m = T. \tag{3}$$

We use π_m to denote π when all the interval lengths in π are the same.

(iii) Given a partition $\pi = \{t_0, \ldots, t_m\}$ and a vector $\hat{f} := (\hat{f}(t_0), \hat{f}(t_1), \ldots, \hat{f}(t_m))$, where $\hat{f}(\cdot)$ is a right continuous function, the following (continuous) function

$$f(t) = \left(\frac{t_i - t}{t_i - t_{i-1}}\right) \hat{f}(t_{i-1}) + \left(\frac{t - t_{i-1}}{t_i - t_{i-1}}\right) \hat{f}(t_i), \qquad \text{for } t \in [t_{i-1}, t_i], \quad i = 1, \ldots, m \tag{4}$$

is called a piecewise linear extension of $\hat{f}$, whereas the following (right-continuous) function

$$f(t) = \begin{cases} \hat{f}(t_{i-1}), & t \in [t_{i-1}, t_i), \text{ for } i = 1, \ldots, m \\ \hat{f}(t_{m-1}), & t = T, \end{cases} \tag{5}$$

is called a piecewise constant extension of $\hat{f}$.

(iv) For problem (P), we use $v(P)$ to denote the optimal objective value of (P).

We start with introducing the following discretization of (SCLP) based on a partition of $[0, T]$, $\pi = \{t_0, \ldots, t_m\}$:

$$\begin{aligned}
(\mathrm{LP}_1(\pi)) \qquad \max \quad & \sum_{i=1}^{m} \left(\left(\gamma + \left(T - \frac{t_i + t_{i-1}}{2}\right) c\right)' \hat{u}(t_{i-1}) + d' \left[\hat{x}(t_i) + \hat{x}(t_{i-1})\right] \frac{t_i - t_{i-1}}{2} \right) \\
\text{s.t.} \quad & \alpha + t_i a - \left[G\hat{u}(t_0) + \cdots + G\hat{u}(t_{i-1}) + F\hat{x}(t_i)\right] \geq 0, \\
& i = 1, 2, \ldots, m; \\
& (t_i - t_{i-1})\, b - H\hat{u}(t_{i-1}) \geq 0, \\
& i = 1, \ldots, m; \\
& \hat{u}(t_{i-1}) \geq 0, \quad \hat{x}(t_i) \geq 0, \\
& i = 1, \ldots, m.
\end{aligned} \tag{6}$$

In addition, assume that

$$\alpha - F\hat{x}(t_0) \geq 0, \qquad \hat{x}(t_0) \geq 0. \tag{7}$$

The following Lemma and the algorithm presented later in this subsection are adapted from [15].

Lemma 1. *Suppose that $(\widehat{u}, \widehat{x})$ is a feasible solution to $(LP_1(\pi))$, with*

$$\widehat{u} = (\widehat{u}(t_0), \ldots, \widehat{u}(t_{m-1})), \qquad \widehat{x} = (\widehat{x}(t_1), \ldots, \widehat{x}(t_m)). \tag{8}$$

Let $u(t)$ be the piecewise constant extension of $(\widehat{u}(t_0)/(t_1 - t_0), \ldots, \widehat{u}(t_{m-1})/(t_m - t_{m-1}))$, and let $x(t)$ be the piecewise linear extension of $(\widehat{x}(t_0), \widehat{x})$. Then, $(u(t), x(t))$ is a feasible solution to (SCLP), and the objective value of $(\widehat{u}, \widehat{x})$ in (6) is the same as that of $(u(t), x(t))$ in (1).

The same discretization applies to (SCLP*), the dual problem. Specifically, corresponding to the partition $\pi = \{t_0, \ldots, t_m\}$, we have

$$\begin{aligned}
(\mathrm{LP}_2(\pi)) \quad \min \quad & \sum_{i=1}^{m} \left(\left(\alpha + \left(T - \frac{t_i + t_{i-1}}{2} \right) a \right)' \widehat{p}(t_{i-1}) \right. \\
& \left. + b' \left[\widehat{q}(t_i) + \widehat{q}(t_{i-1}) \right] \frac{t_i - t_{i-1}}{2} \right) \\
\text{s.t.} \quad & G'\widehat{p}(t_0) + G'\widehat{p}(t_1) + \cdots + G'\widehat{p}(t_{i-1}) \\
& + H'\widehat{q}(t_i) - (\gamma + t_i c) \geq 0, \\
& i = 1, \ldots, m; \\
& F'\widehat{p}(t_{i-1}) - (t_i - t_{i-1}) d \geq 0, \\
& i = 1, \ldots, m; \\
& \widehat{p}(t_{i-1}) \geq 0, \quad \widehat{q}(t_i) \geq 0, \\
& i = 1, \ldots, m.
\end{aligned} \tag{9}$$

In addition, assume that

$$H'\widehat{q}(t_0) - \gamma \geq 0, \quad \widehat{q}(t_0) \geq 0. \tag{10}$$

We need the following assumption.

Assumption 2. The following two linear programming problems have optimal solutions:

$$\begin{aligned}
(\mathrm{LP}_1) \quad \max \quad & c'u + d'x \\
\text{s.t.} \quad & \alpha + Ta - Gu - Fx \geq 0, \\
& Tb - Hu \geq 0 \\
& u \geq 0, \quad x \geq 0, \\
(\mathrm{LP}_2) \quad \min \quad & a'p + b'q \\
\text{s.t.} \quad & G'p + H'q - (\gamma + Tc) \geq 0, \\
& F'p - Td \geq 0, \\
& p \geq 0, \quad q \geq 0.
\end{aligned} \tag{11}$$

Algorithm 3. Let δ be the predefined precision between the objective value of the solution requested and v(SCLP).

Step 1. Check if Assumption 2 is satisfied. If yes, go to Step 2; otherwise, stop.

Step 2. Solving $(T/2m)(v(\mathrm{LP}_1) - v(\mathrm{LP}_2) + b'\widehat{q}(t_0) - d'\widehat{x}(t_0)) \leq \delta$ to get m (number of intervals in partition π_m), find the optimal solution $(\widehat{u}^*, \widehat{x}^*)$ for $(\mathrm{LP}_1(\pi_m))$. Use the extension of this solution to construct a feasible solution for (SCLP). Stop.

Note that the method to construct a feasible solution for (SCLP) from the extension of a feasible solution for $(\mathrm{LP}_1(\pi_m))$ is mentioned in Lemma 1.

To get m, we need to first solve the two linear programs (LP_1) and (LP_2). In addition, we also need to determine $\widehat{x}(t_0)$ and $\widehat{q}(t_0)$. This can be accomplished by solving the following two linear programs:

$$\begin{aligned}
\max \quad & d'\widehat{x}(t_0) \\
\text{s.t.} \quad & \alpha - F\widehat{x}(t_0) \geq 0, \\
& \widehat{x}(t_0) \geq 0, \\
\min \quad & b'\widehat{q}(t_0) \\
\text{s.t.} \quad & H'\widehat{q}(t_0) - \gamma \geq 0, \\
& \widehat{q}(t_0) \geq 0.
\end{aligned} \tag{12}$$

After getting m, we solve the linear program $(\mathrm{LP}_1(\pi_m))$ to get a feasible solution for (SCLP) which satisfies the predefined precision δ.

In summary, the previous Algorithm 3 amounts to solving five linear programming problems: (LP_1), (LP_2), (12), and $(\mathrm{LP}_1(\pi_m))$. Linear programs are known to be polynomially solvable. Hence, this algorithm is a polynomial-time algorithm.

3. The Emergency Treatment Process

The treatment process in ED constitutes a multiclass stochastic queueing network with feedback. In this network, patients arrive at the ED randomly. The critically ill patients get the medical service immediately after arrival. The other patients are assigned different priority to see the physician for the first time after triage. The service (treatment) time for each patient is random, and there are several possible outcomes after the patient sees the physician, including revisiting the physician after examination and/or testing, going to the emergency observation room (ER) within the ED (under the care of another group of physicians and nurses) or leaving ED (going home with medicine, moving to ward or dead). In the last two cases, we say the patient *leaves the system*. Figure 1 illustrates a simplified version of such an emergency treatment process in a typical ED which we will focus on in this paper.

In Figure 1, Server 1 represents physicians for treatment (hereafter, doctors); Server 2 represents the physicians for examination and testing (hereafter, testing staff). The patients arriving at the ED from outside are classified into n classes after triage based on patients' health condition. The patients in the same class are assigned the same priority which is

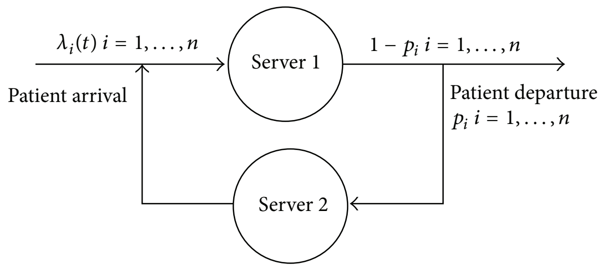

Figure 1: The emergency treatment process in a typical ED.

represented by the time deadline to see the doctor for the first time [16]; for example, class 2 patients are told after triage that they will see the doctor within 30 minutes; "30 minutes" here is the time deadline for class 2 patients and is the measure for the quality of service in a specific ED. Patients within each class are served on an FCFS basis. The critically ill patients will not queue in the network, the effect they impose on the network is occupying some critical resources, such as the physicians and the beds, during some time intervals.

The patients queueing before Server 1 and waiting for their first time to see the doctor are said in their *first stage*. After seeing the doctor, the patients either leave the system or enter their *second stage*—queueing before Server 2 and waiting for examination and/or testing. Upon getting their examination and/or testing results, the patients enter their *third stage*—queueing before Server 1 and waiting to see the doctor again. The patients leave the system (discharged or hospitalized) after they see the doctor for the second time.

The physicians and nurses who are in charge of the service within the ER are usually another group of staff. Their staffing is not the subject of this paper and we do not include the service process within the ER in Figure 1.

4. Mathematical Model for ED Staffing

We consider the ED staffing within a time interval $[0, T]$. The criterion for the appropriate ED staffing level is that the total cost for patients and the ED within the time interval $[0, T]$ is minimized. This criterion includes the quality of service requirement in ED by explicitly considering the waiting cost of patients. By balancing the cost of patients and that of the ED, we want to find an ED staffing level to cope with the time-varying demand for the ED service, which can balance the quality of service requirement and the utilization of the ED staff.

We address the ED staffing problem by two steps. In the first step, we construct the queueing model to determine the appropriate scheduling policy for patients to minimize the waiting cost of patients given a specific ED staffing level. The queueing model is an SCLP and we can solve it efficiently by using the algorithm described in Section 2.3. In the second step, by varying the ED staffing level in the SCLP, we can find the least staffing level for the ED which minimize the total cost of patients and the ED for the time interval $[0, T]$.

In building the analytic model of the ED system, one needs to decide how detailed does the description of the system have to be? The more microscopic the model is, the more accuracy is achieved, but on the other hand, usually the harder it is to handle analytically [17]. In this paper, we choose to view the ED system in a more macroscopic way. We use the deterministic fluid model to describe the performance of the overloaded ED systems. The use of the fluid model in overloaded system is justified in a number of papers, for example, in Whitt [18, 19]. The patients who left without being seen are ignored, and the ED systems try hard to accommodate the patients who wait until seeing the physicians.

In the ED process that we focus on, the arrival rate of the patients and the service time of the patients are random variables; instead of assuming that the arrival rate or the service time obeys some particular distributions (as those in the previous papers [5, 7–9]), we only take the first moments (means of the arrival rate and the service time) to describe these randomness.

Specifically, the corresponding fluid network for the ED treatment process in Figure 1 is as follows: the patients in the ED are regarded as continuous fluid instead of individual person. The arrival rate and the service time of the fluid in each server are deterministic. After visiting Server 1, the fluid proceeds to Sever 2 with deterministic percentage (probability). The fluid which do not proceed to Server 2 exits the network. After visiting Server 2, the fluid continues to visit Server 1 again. The fluid exits the network after visiting Server 1 in the second time.

Let $\lambda_i(t)$ denote the arrival rate of class i patients from outside at time t. The mean service time for class i patients in their stage 1 is η_i. After receiving service from Server 1, the class i patients will enter their second stage and queue before Server 2 with probability p_i, or leave the system with probability $1 - p_i$. The mean service time for class i patients in their second stage is γ_i. After receiving service from Server 2, the class i patients enter their third stage and queue before Server 1 again, and the mean service time for them is μ_i.

The service capacity of each doctor or testing staff is 1. Let s_j denote the number of doctors or testing staff in Server j and let $b_j(t)$ denote the service capability of Server j at time t for patients in the queues. If the critically ill patients show during $[0, T]$, they will occupy some capacity of Server j and subsequently $b_j(t)$ becomes $k_j s_j$, $k_j \in [0, 1)$, where the value of k_j depends on the condition of the critically ill patients. If no critically ill patients show during $[0, T]$, $b_j(t)$ is simply s_j.

Let ω_{ij} denote the waiting cost of class i patients in their jth stage (including the cost for the waiting time and the cost of patients' health condition changes due to the long waiting time, e.g., the additional financial cost). ω_{i1} should be big enough to reflect the cost for different class patients in order to guarantee that each patient in his first stage can get service within the time deadline the ED promises. Usually a senior physician in an ED can tell the approximate waiting cost for a specific class of patients in their specific stage and we can use this piece of information to determine the value of ω_{ij}'s. Let c_j denote the service cost for each doctor or each testing staff in Server j. The estimation of the ED's operational cost is relatively straightforward because the salaries paid to the physicians constitute most of the ED operational cost. So we can use the physicians' salaries to represent the service cost of an ED.

Let a_{ij} denote the amount of class i patients in their jth stage at time 0.

The following is the fluid model which determines the optimal scheduling policy for patients with the objective of minimizing the waiting cost of patients given a specific ED staffing level:

$$\text{Model 1:}\quad Q(s_1, s_2) = \min \sum_{i=1}^{n}\sum_{j=1}^{3}\left(\int_0^T \omega_{ij}x_{ij}(t)\right)dt + c_1 s_1 + c_2 s_2 \tag{13}$$

$$\text{s.t.}\quad x_{i1}(t) = a_{i1} + \int_0^t \lambda_i(s)\,ds - \int_0^t u_{i1}(s)\,ds,\quad i = 1,\dots,n, \tag{14}$$

$$x_{i2}(t) = a_{i2} + p_i\int_0^t u_{i1}(s)\,ds - \int_0^t u_{i2}(s)\,ds,\quad i = 1,\dots,n, \tag{15}$$

$$x_{i3}(t) = a_{i3} + p_i\int_0^t u_{i2}(s)\,ds - \int_0^t u_{i3}(s)\,ds,\quad i = 1,\dots,n, \tag{16}$$

$$\sum_{i=1}^{n}\left(\eta_i u_{i1}(t) + \mu_i u_{i3}(t)\right) \le b_1(t), \tag{17}$$

$$\sum_{i=1}^{n}\gamma_i u_{i2}(t) \le b_2(t), \tag{18}$$

$$u_{ij}(t),\quad x_{ij}(t) \ge 0,\quad i = 1,\dots,n,\quad j = 1,2,3,\quad t \in [0,T], \tag{19}$$

where $x_{ij}(t)$ is the state variable and represents the amount of class i patients in their jth stage at time t, $u_{ij}(t)$ is the control variable and represents the service rate for class i patients in their jth stage at time t, $i = 1,\dots,n$, $j = 1,2,3$. s_1, s_2 are the numbers of doctors in Server 1 and testing staff in Server 2, respectively.

The objective of (13) is minimizing the total cost for patients and the ED within the time interval $[0,T]$.

The first three sets of constraints describe the dynamic of the ED service process, that is; how the state variables $x_{ij}(t)$ are changing with the changing of the control variables $u_{ij}(t)$.

The constraint (14) states that the amount of class i patients in their first stage at time t is equal to the summation of the amount of class i patients in their first stage at time 0 and the amount of class i patients arrived from outside lessing the amount of class i patients serviced by Server 1 and exit their first stage during time interval $[0,t]$.

The constraint (15) states that the amount of class i patients in their second stage at time t is equal to the summation of the amount of class i patients in their second stage at time 0 and the amount of class i patients who enter their second stage lessing the amount of class i patients serviced by Server 2 and enter their third stage during time interval $[0,t]$.

The constraint (16) states that the amount of class i patients in their third stage at time t is equal to the summation of the amount of class i patients in their third stage at time 0 and the amount of class i patients who enter their third stage lessing the amount of class i patients serviced by Server 1 and exit their third stage during time interval $[0,t]$.

The next two sets of constraints are the service capacity constraints. The constraint (17) states that the total capacity consumed by the patients in their first and third stages cannot exceed the available capacity of Server 1, and the constraint (18) states that the total capacity consumed by the patients in their second stage cannot exceed the available capacity of Server 2.

Although there are time-varying parameters in Model 1, that is, $\lambda_i(t)$, $b_j(t)$, we can assume that within the sufficient short time interval, for example, within one hour, those parameters can be regarded as constants. This assumption is justified by the statistical analysis of the ED treatment process [9]. Because of this, we can choose a sufficient small T and replace $\lambda_i(t)$, $b_j(t)$ with λ_i, $k_j s_j$, respectively, when $t \in [0,T]$. Here, k_j can be specified based on the statistical information on the type and arrival rate of critically ill patients during $[0,T]$. With these simplifications, Model 1 reduces to the following SCLP problem:

$$\max \int_0^T \left[(-\omega_{11}\ \ -\omega_{12}\ \ -\omega_{13}\ \ \cdots\ \ -\omega_{n1}\ \ -\omega_{n2}\ \ -\omega_{n3})' x(t)\right] dt + c_1 s_1 + c_2 s_2$$

$$\text{s.t.}\quad \int_0^t \begin{pmatrix} -1 & & & & & & \\ p_1 & -1 & & & & & \\ & p_1 & -1 & & & & \\ & & & \ddots & & & \\ & & & & -1 & & \\ & & & & p_n & -1 & \\ & & & & & p_n & -1 \end{pmatrix} u(s)\,ds + \begin{pmatrix} -1 & & & & & & \\ & -1 & & & & & \\ & & -1 & & & & \\ & & & \ddots & & & \\ & & & & -1 & & \\ & & & & & -1 & \\ & & & & & & -1 \end{pmatrix} x(t)$$

$$= \begin{pmatrix} -a_{11} \\ -a_{12} \\ -a_{13} \\ \vdots \\ -a_{n1} \\ -a_{n2} \\ -a_{n3} \end{pmatrix} + t \begin{pmatrix} -\lambda_1 \\ 0 \\ 0 \\ \vdots \\ -\lambda_n \\ 0 \\ 0 \end{pmatrix},$$

$$\begin{pmatrix} \eta_1 & 0 & \mu_1 & \cdots & \eta_n & 0 & \mu_n \\ 0 & \gamma_1 & 0 & \cdots & 0 & \gamma_n & 0 \end{pmatrix} u(t) \le \begin{pmatrix} k_1 s_1 \\ k_2 s_2 \end{pmatrix},$$

$$u(t) \ge 0, \quad x(t) \ge 0, \quad t \in [0, T], \tag{20}$$

where

$$u(t) = \begin{pmatrix} u_{11} \\ u_{12} \\ u_{13} \\ \vdots \\ u_{n1} \\ u_{n2} \\ u_{n3} \end{pmatrix}, \qquad x(t) = \begin{pmatrix} x_{11} \\ x_{12} \\ x_{13} \\ \vdots \\ x_{n1} \\ x_{n2} \\ x_{n3} \end{pmatrix}, \tag{21}$$

and all the blank entries in the matrices in (20) are 0.

We can use algorithm described in Section 2.3 to solve (20) efficiently.

Then we solve the following problem to get the optimal ED staffing level:

$$\begin{aligned} \text{Model 2} \quad & \min \quad Q(s_1, s_2) \\ & \text{s.t.} \quad s_1 \ge 1, \\ & \qquad\; s_2 \ge 1, \\ & \qquad\; s_1, s_2 \text{ are integers}, \end{aligned} \tag{22}$$

where

$$Q(s_1, s_2) = \min \int_0^T \left[\begin{pmatrix} -\omega_{11} & -\omega_{12} & -\omega_{13} & \cdots & -\omega_{n1} & -\omega_{n2} & -\omega_{n3} \end{pmatrix}' x(t) \right] dt + c_1 s_1 + c_2 s_2. \tag{23}$$

Note that $Q(s_1, s_2)$ is just the objective of Model 1.

From our experience in some large hospital, s_j's are usually single-digit numbers, so we can solve Model 2 efficiently by enumerating all the possible combination of s_1 and s_2.

In summary, we can get the optimal ED staffing level by solving Model 1 and Model 2, which amounts to solving several instances of (20). As (20) is an SCLP, we can solve it in polynomial time; thus, we can get the optimized ED staffing level very quickly.

5. Conclusion and Future Work

In this paper, we propose a new approach to address the ED staffing. Our approach consists of two steps. In the first step, we construct the fluid model (Model 1 or (20)) to determine the optimal scheduling policy for patients in the ED to minimize the waiting cost of patients given a specific ED staffing level. The model is an SCLP and can be solved efficiently by utilizing the algorithm suggested by Wang [15]. Then, in the second step, we solve Model 2 to get the appropriate ED staffing level which minimizes the total cost of patients and the ED.

Our approach accommodates the three characteristics of the demand in EDs, that is, time-varying arrival rate, multi-classes customer, and customer feedback. Comparing with other existing approaches on ED staffing, our approach is more efficient to get the appropriate staffing level.

There are still some interesting problems that need explore. In this paper, we assume that the waiting costs of the patients are the linear functions of the queue lengths before Servers. How to determine the amount of ED physicians if the cost functions are other type of functions, for example, convex and increasing function of the queue lengths, is still unknown.

Also, in this paper, we only consider how to determine the amount of ED physicians who are in charge of the patients arrived from outside. In reality, there is another decision to make; that is, how to determine the amount of physicians who are in charge of the patients in ER. Although this issue is usually translated into determining the appropriate amount of beds in ER in most of the literature, directly handling this issue is still meaningful if there is some cooperation between these physicians and the physicians who are in charge of patients arrived from outside.

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Review Article

Review on the Research for Separated Continuous Linear Programming: With Applications on Service Operations

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We give a review on the research for a class of optimization model—separated continuous linear programming (SCLP). SCLP takes several similar forms and can be used to find the dynamic control of a multiclass fluid network. We review the duality theory and solution methods for it. We also present application examples of SCLP on service operations.

1. Introduction

Linear programming (LP) is probably the most successful mathematical model in terms of its extremely wide range of industrial applications and its superb speed and capacity in solving very large size problems. For this reason, LP has been pushed and extended to an ever-broadening frontier [1, 2]. One of such frontier extensions is the so-called separated continuous linear programming(SCLP).

SCLP was first introduced by Anderson [3] who used it to model the job-shop scheduling problem. The following is the formulation according to him [3]:

$$\begin{aligned} \min \quad & \int_0^T c(t)' u(t)\, dt \\ \text{s.t.} \quad & \int_0^t Gu(s)\, ds \le a(t), \\ & Hu(t) \le b(t), \\ & u(t) \ge 0, \quad t \in [0, T], \end{aligned} \tag{1}$$

where $u(t)$ is the decision variable and is assumed to be a bounded measurable function, $b(t)$, $c(t)$ are bounded measurable functions, and $a(t)$ is an absolutely continuous function. G, F, H are constant matrices. "$'$" denotes the transpose operation. The word "separated" here refers to the fact that there are two kinds of constraints in SCLP: the constraints involving integration and the instantaneous constraints [3].

There are several other similar formulations of SCLP.

Luo and Bertsimas [4] considered the following more general SCLP which they called separated continuous linear programs with side constraints (SCSCLP):

$$\begin{aligned} \min \quad & \int_0^T \left(c(t)' u(t) + g(t)' x(t) \right) dt \\ \text{s.t.} \quad & \int_0^t Gu(s)\, ds + Ex(t) \le a(t), \\ & Hu(t) \le b(t), \\ & Fx(t) \le h(t), \\ & u(t) \ge 0, \quad t \in [0, T], \end{aligned} \tag{2}$$

where $u(t)$, $x(t)$ are decision variables and are assumed to be bounded measurable functions, $b(t)$, $c(t)$, $g(t)$, and $h(t)$ are bounded measurable functions, and $a(t)$ is an absolutely continuous function. It is easy to see that when E, F are zero matrix, and $g(t) = 0$ for $t \in [0, T]$, (2) is reduced to (1).

Shapiro [5] considered the following SCLP problem:

$$\begin{aligned}
\min \quad & \int_0^T c(t)' u(t)\, dt \\
\text{s.t.} \quad & \int_0^t G(s,t)\, u(s)\, ds \le a(t), \quad t \in [0,T], \\
& H(t)\, u(t) \le b(t), \text{a.e. } t \in [0,T], \\
& u(t) \ge 0, \text{a.e. } t \in [0,T],
\end{aligned} \tag{3}$$

where $u(t)$ is the decision variable and is assumed to be a bounded measurable function, $b(t)$ and $H(t)$ are bounded measurable functions, $c(t)$ is a Lebesgue integrable function, $a(t)$ is a continuous function, and $G(s,t)$ is a continuous function such that, for every $s \in [0,T]$, it is of bounded variation as a function of $t \in [0,T]$. It is obvious that if $G(s,t)$ and $H(t)$ are constant matrices, (3) is reduced to (1).

In the work of Weiss [6], he introduced the following SCLP:

$$\begin{aligned}
\max \quad & \int_0^T \left((\gamma + (T-t)\, c)' u(t) + d' x(t)\right) dt \\
\text{s.t.} \quad & \int_0^t G u(s)\, ds + F x(t) \le \alpha + a t, \\
& H u(t) \le b, \\
& u(t), x(t) \ge 0, \quad t \in [0,T],
\end{aligned} \tag{4}$$

where $u(t)$, $x(t)$ are decision variables and are assumed to be bounded measurable functions with the measure of the breakpoint set being 0.

Wang [7] considered the following problem which she called generalized separated continuous linear programming (GSCLP):

$$\begin{aligned}
\max \quad & \int_0^T \left(c(T-t)' u(t) + d(T-t)' x(t)\right) dt \\
\text{s.t.} \quad & a(t) - \int_0^t G u(s)\, ds - F x(t) \ge 0, \\
& b(t) - H u(t) \ge 0, \\
& u(t), x(t) \ge 0, \quad t \in [0,T],
\end{aligned} \tag{5}$$

where $u(t)$, $x(t)$ are decision variables and are assumed to be bounded measurable functions. $a(t)$, $c(T-t)$ are piecewise linear, and $b(t)$, $d(T-t)$ are piecewise constant function, on $[0,T]$. It is easy to see that when $a(t)$, $c(t)$ are linear and $b(t)$, $d(t)$ are constant functions, (5) is reduced to (4).

In the recent paper by Nasrabadi et al. [8], they studied the SCLP with a fuzzy valued objective function which they called fuzzy separated continuous linear program (FSCLP):

$$\begin{aligned}
\min \quad & \int_0^T (\tilde{\gamma} + t\tilde{c})' u(t)\, dt \\
\text{s.t.} \quad & \int_0^t G u(s)\, ds \le \alpha + a t, \\
& H u(t) \le b, \\
& u(t) \ge 0, \quad t \in [0,T].
\end{aligned} \tag{6}$$

Here, $u(t)$ is the decision variable and is assumed to be a bounded measurable function. $\tilde{\gamma}$, $\tilde{c}$ are constant vectors with trapezoidal fuzzy numbers as components.

The theoretical results for SCLP are mainly on the duality theory and solution methods. The application results for SCLP are on finding the dynamic control of a multiclass fluid network which can model alot of real systems, including service systems. In this paper, we review both the theoretical results and the application results for SCLP, with the focus of application on service operations.

The rest of the paper is organized as follows. In Section 2, we review the literature on duality theory for SCLP. Section 3 is devoted to the solution methods for SCLP. Application of SCLP on service operations is discussed in Section 4. We summarized the current results on SCLP and point out some future directions in this area in Section 5.

2. Duality Theory

As with finite LP, duality theory for SCLP plays an important role in the development of solution methods for it. In the following, we will discuss the results on duality theory for SCLP according to the formulation of SCLP.

Using essentially the same method as LP's, Anderson and Nash [9] derived a dual problem of SCLP (1) as in the following:

$$\begin{aligned}
\max \quad & -\int_0^T a(t)' p(t)\, dt - \int_0^T b(t)' q(t)\, dt \\
\text{s.t.} \quad & c(t) + \int_t^T G' p(s)\, ds + H' q(t) \ge 0, \\
& p(t) \ge 0, q(t) \ge 0, \quad t \in [0,T],
\end{aligned} \tag{$D1$}$$

where $p(t)$, $q(t)$ are the decision variables and are assumed to be bounded measurable functions. But later on, some researcher found that for (1) and $(D1)$, there exist such instances that either the primal or the dual problem has no feasible or no optimal solution, while the other one has an optimal solution [10].

To avoid this difficulty, Pullan [11] introduced another dual problem of (1) as in the following:

$$\begin{aligned} \max \quad & -\int_0^T d\pi'(t)\,a(t) - \int_0^T b(t)' q(t)\,dt \\ \text{s.t.} \quad & c(t) - G'\pi(t) + H' q(t) \ge 0, \\ & \pi(t) \text{ monotonically increasing and right} \\ & \text{continuous on } [0,T] \text{ with } \pi(T) = 0, \\ & q(t) \ge 0, \quad t \in [0,T], \end{aligned} \tag{$D1^*$}$$

where $\pi(t)$, $q(t)$ are the decision variables and $q(t)$ is assumed to be a Lebesgue integrable function on $[0,T]$.

Pullan [12] developed an account of duality theory for (1) and $(D1^*)$. He introduced the definition of complementary slackness for (1) and $(D1^*)$ and proved that if $u(t)$ is feasible for (1) and $(\pi'(t), q'(t))'$ is feasible for $(D1^*)$, when $(\pi'(t), q'(t))'$ complementary slack with $u(t)$, $u(t)$ is optimal for (1) and $(\pi'(t), q'(t))'$ is optimal for $(D1^*)$. He also derived several sufficient conditions for the optimality of (1), the conditions for no duality gap between (1) and $(D1^*)$, and the conditions under which $(D1^*)$ has an optimal solution. Finally, he proved that strong duality holds between (1) and $(D1^*)$ and that there exist piecewise analytic optimal solutions for both (1) and $(D1^*)$ under the assumptions that $a(t)$, $b(t)$ and $c(t)$ are piecewise analytic and $a(t)$ is also continuous.

Luo and Bertsimas [4] formulated the dual problem of (2) as follows:

$$\begin{aligned} \max \quad & -\int_0^T a(t)' d\pi(t) - \int_0^T h(t)' dp(t) \\ & -\int_0^T b(t)' q(t)\,dt \\ \text{s.t.} \quad & c(t) - G'\pi(t) + H' q(t) \ge 0, \\ & E'\pi(t) + F' p(t) = \int_t^T g(t)\,dt, \\ & \pi(t) \text{ bounded measurable with} \\ & \text{finite variation,} \quad \pi(T) = 0, \\ & p(t) \text{ monotonically increasing and right} \\ & \text{continuous on } [0,T] \text{ with } p(T) = 0, \\ & q(t) \ge 0, \quad t \in [0,T], \end{aligned} \tag{$D2$}$$

where $\pi(t)$, $p(t)$, $q(t)$ are decision variables and $q(t)$ is assumed to be a bounded measurable function on $[0,T]$. When E, F are zero matrices and $g(t) = 0$ for $t \in [0,T]$, $(D2)$ is reduced to $(D1^*)$.

They showed that there is no duality gap between (2) and $(D2)$ under the following assumptions: $a(t)$ and $h(t)$ are continuous; $a(t)$, $c(t)$ and $h(t)$ are piecewise linear; $b(t)$ and $g(t)$ are piecewise constant; (2) is feasible and its objective value is bounded from below.

Shapiro [5] derived the Lagrangian dual of (3) as follows:

$$\begin{aligned} \max \quad & -\int_0^T a(t)' d\pi(t) - \int_0^T b(t)' q(t)\,dt \\ \text{s.t.} \quad & c(t) - G(t,t)'\pi(t) \\ & -\int_t^T (dG(t,s))'\pi(s) + H(t)' q(t) \ge 0, \\ & q(t) \ge 0, \text{a.e. } t \in [0,T], \\ & \pi(t) \text{ monotonically nondecreasing, right} \\ & \text{continuous with bounded variation} \\ & \text{on } [0,T] \text{ with } \pi(T) = 0, \end{aligned} \tag{$D3$}$$

where $\pi(t)$, $q(t)$ are decision variables and $q(t)$ is assumed to be a Lebesgue integrable function. When $G(s,t)$ and $H(t)$ are constant matrices, $(D3)$ is reduced to $(D1^*)$.

Shapiro proved that if the feasible region of (3) is nonempty, the matrix $H(t)$ is constant, and the set

$$\{u(t) : Hu(t) \le b(t), u(t) \ge 0, \text{a.e. } t \in [0,T]\} \tag{7}$$

is bounded, then the optimal objective value of (3) and $(D3)$ are the same, and there exists an optimal solution for (3).

Weiss [6] gave the following dual of (4):

$$\begin{aligned} \min \quad & \int_0^T \left((\alpha + (T-t)\,a)' p(t) + b' q(t)\right) dt \\ \text{s.t.} \quad & \int_0^t G' p(s)\,ds + H' q(t) \ge \gamma + ct, \\ & F' p(t) \ge d, \\ & p(t), q(t) \ge 0, \quad t \in [0,T], \end{aligned} \tag{$D4$}$$

where $p(t)$, $q(t)$ are decision variables and are assumed to be bounded measurable functions. Note that in this dual problem, the time is running back, that is, here t is $T-t$ in the primal problem.

Weiss introduced the following Boundary-LP, Boundary-LP*, Rates-LP(K,J), and Rates-LP$^*(K,J)$.

Boundary-LP

$$\begin{aligned} \max \quad & d' x^0 \\ \text{s.t.} \quad & F x^0 \le \alpha, \\ & x^0 \ge 0, \end{aligned} \tag{8}$$

Boundary-LP*

$$\begin{aligned} \min \quad & b' q^N \\ \text{s.t.} \quad & H' q^N \ge \gamma, \\ & q^N \ge 0, \end{aligned} \tag{9}$$

Rates-LP(K, J)

$$\begin{aligned} \max \quad & \begin{bmatrix} c' & 0 \end{bmatrix} u + \begin{bmatrix} 0 & d' \end{bmatrix} \dot{x} \\ \text{s.t.} \quad & [G \ \ 0]\, u + [I \ \ F]\, \dot{x} = a, \\ & [H \ \ I]\, u = b, \\ & \dot{x}_k \ge 0 \quad \text{for } k \notin K, \\ & u_j = 0 \quad \text{for } j \in J, \\ & u_j \ge 0 \quad \text{for } j \notin J, \end{aligned} \tag{10}$$

Rates-LP$^*(K, J)$

$$\begin{aligned} \min \quad & \begin{bmatrix} a' & 0 \end{bmatrix} p + \begin{bmatrix} 0 & b' \end{bmatrix} \dot{q} \\ \text{s.t.} \quad & \begin{bmatrix} G' & 0 \end{bmatrix} p + \begin{bmatrix} -I & H' \end{bmatrix} \dot{q} = c, \\ & \begin{bmatrix} F' & -I \end{bmatrix} p = d, \\ & \dot{q}_j \ge 0 \quad \text{for } j \notin J, \\ & p_k = 0 \quad \text{for } k \in K, \\ & p_k \ge 0 \quad \text{for } k \notin K, \end{aligned} \tag{11}$$

where K, J are the subsets of the subscripts for $x(t)(p(t))$, $u(t)(q(t))$ in (4), respectively. Weiss proved that strong duality holds between (4) and $(D4)$ under Assumption 1.

Assumption 1.

(i) The Boundary-LP/LP* have a solution x^0, q^N. Denote $K_0 = \{k : x_k^0 > 0\}$, $J_N = \{j : q_j^N > 0\}$.

(ii) The Rates-LP$(\emptyset, J_N)$ and the Rates-LP$^*(K_0, \emptyset)$ are both feasible.

Wang [7] considered the following dual of (5):

$$\begin{aligned} \min \quad & \int_0^T \left(a(T-t)' p(t) + b(T-t)' q(t) \right) dt \\ \text{s.t.} \quad & \int_0^t G' p(s)\, ds + H' q(t) - c(t) \ge 0, \\ & F' p(t) - d(t) \ge 0, \\ & p(t), q(t) \ge 0, \quad t \in [0, T], \end{aligned} \tag{D5}$$

where $p(t)$, $q(t)$ are decision variables and are assumed to be bounded measurable functions. When $a(t)$, $c(t)$ are linear $b(t)$, $d(t)$ are constant functions, $(D5)$ is reduced to $(D4)$.

Wang [7] proved that strong duality holds between (5) and $(D5)$ under Assumption 2.

Assumption 2. The following two ordinary linear programming problems have optimal solutions:

$$\begin{aligned} \min \quad & \begin{pmatrix} (T-T_1)\, a_2 \\ T_1 a_1 \end{pmatrix}' p + \begin{pmatrix} (T-T_1)\, b_2 - T_1 b_1 \\ T_1 b_1 \end{pmatrix}' q \\ \text{s.t.} \quad & \begin{pmatrix} G' & \\ G' & G' \end{pmatrix} p + \begin{pmatrix} H' & \\ & H' \end{pmatrix} q \\ & - \begin{pmatrix} \gamma + (T-T_1)\, c_1 \\ \gamma + (T-T_1)\, c_1 + T_1 c_2 \end{pmatrix} \ge 0, \\ & \begin{pmatrix} F' & \\ & F' \end{pmatrix} p - \begin{pmatrix} (T-T_1)\, d_1 \\ T_1 d_2 \end{pmatrix} \ge 0, \\ & p, q \ge 0, \\ \max \quad & \begin{pmatrix} (T-T_1)\, c_1 \\ T_1 c_2 \end{pmatrix}' u + \begin{pmatrix} (T-T_1)\, d_1 \\ T_1 d_2 - (T-T_1)\, d_1 \end{pmatrix}' x \\ \text{s.t.} \quad & \begin{pmatrix} \alpha + T_1 a_1 + (T-T_1)\, a_2 \\ \alpha + T_1 a_1 \end{pmatrix} \\ & - \begin{pmatrix} G & G \\ & G \end{pmatrix} u - \begin{pmatrix} F & \\ & F \end{pmatrix} x \ge 0, \\ & \begin{pmatrix} (T-T_1)\, b_2 \\ T_1 b_1 \end{pmatrix} - \begin{pmatrix} H & \\ & H \end{pmatrix} u \ge 0, \\ & u, x \ge 0. \end{aligned} \tag{12}$$

Nasrabadi et al. [8] consider the following dual problem of (6):

$$\begin{aligned} \max \quad & -\int_0^T (\alpha + ta)\, d\tilde{\pi}(t)' - \int_0^T b' \tilde{q}(t)\, dt \\ \text{s.t.} \quad & \tilde{\gamma} + t\tilde{c} - G' \tilde{\pi}(t) + H' \tilde{q}(t) \ge_R 0, \\ & \tilde{\pi}(t) \ \text{ is trapezoidal fuzzy number-valued,} \\ & \text{monotonically increasing, and right continuous} \end{aligned}$$

$$\text{on } [0,T] \text{ with } \tilde{\pi}(T) = 0, \quad \tilde{q}(t) \text{ is Lebesgue-integrable and trapezoidal fuzzy numbervalued,} \tag{D6}$$

where $\tilde{\pi}(t)$, $\tilde{q}(t)$ are decision variables and R denotes any arbitrary, but fixed, linear-ranking function.

They proved that strong duality holds between (6) and ($D6$) with respect to R under the assumption that (6) has an optimal solution.

3. Solution Methods

One of the motivations for studying SCLP is that under some assumptions, the solution for SCLP has some nice forms. In the following, we introduce the results in the form of the optimal solution for SCLP under various assumptions in the first subsection; we present the algorithms for solving SCLP problem in the second to fourth subsections.

3.1. The Form of the Optimal Solution for SCLP. Anderson et al. [13] pointed out that if the feasible region for (1) is nonempty and bounded, then (1) is solvable and there exists an extreme point optimal solution for (1). Furthermore, if $a(t)$, $c(t)$ are piecewise linear, with $a(t)$ also absolutely continuous, and $b(t)$ is piecewise constant, then (1) has an optimal solution in which $u(t)$ is piecewise constant on $[0,T]$.

Anderson and Philpott [14] continued to study this issue and proved that when $a(t)$ and $b(t)$ are piecewise analytic (but with $a(t)$ continuous) and $c(t)$ is piecewise constant on $[0,T]$, if the set $\{u(t) : Hu(t) \le b(t), u(t) \ge 0\}$ is bounded for each $t \in [0,T]$ and the feasible region for (1) is nonempty, then there exists an optimal solution for (1) with $u(t)$ piecewise analytic on $[0,T]$.

Pullan [15] proved that if $a(t)$, $b(t)$, $c(t)$ are piecewise analytic on $[0,T]$, with $a(t)$ also continuous, when the feasible region for (1) is nonempty and bounded, there exists an optimal extreme point solution for (1) with $u(t)$ piecewise analytic on $[0,T]$. Furthermore, if $a(t)$ is piecewise linear and $b(t)$ is piecewise constant on $[0,T]$, there exists an optimal extreme point solution for (1) with $u(t)$ piecewise constant on $[0,T]$.

Note that the results of [11, 12] do not guarantee an optimal solution of the appropriate form that is also an extreme point; the results in [15] do guarantee such a solution.

Luo and Bertsimas [4] showed that there exists an optimal solution for ($D2$) that is piecewise linear under the following assumption: $a(t)$ and $h(t)$ are continuous; $a(t)$, $c(t)$ and $h(t)$ are piecewise linear; $b(t)$ and $g(t)$ are piecewise constant; (2) is feasible and its objective value is bounded from below. Furthermore, there exists a bounded measurable optimal solution for (2) if and only if the algorithm they suggested terminates with such a solution. Also, when the feasible region for (2) is bounded and E is an identity matrix, there exists an optimal solution in which $u(t)$ is piecewise constant.

Weiss [6] proved that under Assumption 1, (4) and ($D4$) possess complementary slack optimal primal and dual solutions, with continuous piecewise linear $x(t)$, $q(t)$ and piecewise constant $u(t)$, $p(t)$.

Wang [7] proved that under Assumption 2, there exist optimal solutions for (5) in which $u(t)$ is piecewise constant and $x(t)$ is piecewise linear, or there exist a series of feasible solutions for (5) in which $u(t)$ is piecewise constant and $x(t)$ is piecewise linear and whose objective values converge to the optimal objective value of (5).

Nasrabadi et al. [8] proved that under the assumption that the feasible region for (6) is bounded and nonempty, there exist optimal solutions for (6) in which $u(t)$ is piecewise constant and $x(t)$ is piecewise linear, or there exist a series of feasible solutions for (6) in which $u(t)$ is piecewise constant and $x(t)$ is piecewise linear and whose objective values converge to the optimal objective value of (6).

3.2. Simplex-Like Methods for Solving SCLP Problem. Based on the duality theory and results on the forms of the optimal solutions for SCLP, several methods are proposed to solve SCLP exactly or approximately. Most of these algorithms fall in one of two categories: simplex-like and discretization based. In this subsection, we will discuss the simplex-like methods. The discretization methods will be discussed in the next subsection.

Anderson and Philpott [16] developed the so-called continuous-time network simplex algorithm to solve a kind of continuous network program which can be formulated as (1) with piecewise linear $a(t)$ and $c(t)$, with $a(t)$ being absolutely continuous, and piecewise constant $b(t)$. This is the first algorithm which was implemented in a computer to solve (1). Unfortunately, there is no convergence guarantee for this algorithm, and it often produces a sequence of solutions which converge to a suboptimal solution [17].

Pullan [18] proposed a simplex-like algorithm to solve (1) with piecewise linear and continuous $a(t)$, piecewise constant $b(t)$, and piecewise analytic $c(t)$. Again, there is no convergence guarantee for this algorithm.

In the recent work of Weiss [6], he proposed a simplex-like algorithm to solve (4) under Assumptions 1 and 3 as follows.

Assumption 3. The column $\begin{bmatrix} a \\ b \end{bmatrix}$ is in general position to the matrix $\begin{bmatrix} G & F & I & 0 \\ H & 0 & 0 & I \end{bmatrix}$, and the column $\begin{bmatrix} c \\ d \end{bmatrix}$ is in general position to the matrix $\begin{bmatrix} G' & H' & -I & 0 \\ F' & 0 & 0 & -I \end{bmatrix}$.

The Weiss algorithm is a parametric and recursive algorithm: start from an optimal solution valid at $T = 0$ and construct the optimal solution valid at $T \in [\theta^{r-1}T, \theta^{r}T]$, where $r = 1, \ldots, R$ and $0 = \theta^0 < \theta^1 < \cdots < \theta^R = 1$, and finally, reach the optimal solution at T. Going from optimal solution valid at $T \in [\theta^{r-1}T, \theta^{r}T]$ to that valid at $T \in [\theta^{r}T, \theta^{r+1}T]$ constitutes the pivot operation of SCLP, which involves solving several Rates-LP and Rates-LP* and/or solving several subproblems of original SCLP (4) which are themselves SCLP's with smaller size than that of (4).

Weiss managed to prove that the number of SCLP pivots needed to solve (4) is finite but increase exponentially with the problem size.

3.3. Discretization-Based Methods for Solving SCLP Problem. There are two kinds of discretization-based methods, one is solving a sequence of discretization problems for SCLP and terminated with some predefined criteria; the other is solving a specific instance of discretization problems for SCLP and terminated with the solution with some pre-defined precision.

Pullan [19] proposed an algorithm to solve (1) with piecewise linear $a(t)$, $c(t)$, with $a(t)$ also absolutely continuous, and piecewise constant $b(t)$. He formulated a new discretization for (1) which is a finite LP problem (there is another discretization problem for (1) called the standard discretization for (1) which is also a finite LP problem. The later was used before in the context of CLP in, e.g., [20, 21]). Based on the relationship between (1), $(D1^*)$, and the discretization problems he introduced, a feasible solution for (1) is obtained with $u(t)$ piecewise constant in $[0, T]$. Then, in each iteration, the existing feasible solution for (1) is used together with the solution for the discretization he introduced to produce a new feasible solution with the strictly improving objective value. The number of the breakpoints in the new feasible solution is tripled. This process continues until an optimal solution is found or the resulting feasible solution is within a predescribed limit. The algorithm either terminates in a finite number of iterations with an optimal solution, or the objective values of the resulting feasible iterative solutions converge to the optimal objective value of (1).

Following Pullan's work, Philpott and Craddock [17] proposed a similar algorithm for (1). The main difference between their algorithm and Pullan's is in the criterion for adding breakpoints in producing the new feasible solution. After each iteration, the number of breakpoints in the new feasible solution Philpott and Craddock [17] produced is at most doubled.

Luo and Bertsimas [4] provided an algorithm to solve (2) under the assumptions that $a(t)$ and $h(t)$ are continuous; $a(t)$, $c(t)$ and $h(t)$ are piecewise linear; $b(t)$ and $g(t)$ are piecewise constant; (2) is feasible and its objective value is bounded from below. The main idea of their algorithm is similar to that of Pullan [19] and Philpott and Craddock [17] for (1). The key difference is that in Pullan [19] and Philpott and Craddock [17], at each iteration, the breakpoints in $[0, T]$ are fixed so that the discretization problem for (1) is an LP problem; In this algorithm, the breakpoints of $[0, T]$ are also a decision variable (although the number of the breakpoints is fixed), so the discretization problem for (2) is a (in general) nonconvex quadratic programming problem. In each iteration, this nonconvex quadratic programming problem is solved to get a KKT solution. The series of KKT points obtained have nonincreasing objective value. These KKT solutions are used to construct a sequence of feasible solutions for (2) with piecewise constant $u(t)$, and the objective values of these feasible solutions converge to the optimal objective value of (2). If the feasible region for (2) is bounded and E is the identity matrix, the algorithm produces an optimal solution for (2) with piecewise constant $u(t)$.

Nasrabadi et al. [8] proposed an algorithm to solve (6) under the assumption that the feasible region for (6) is bounded and nonempty. Their algorithm is the counterpart of Pullan's algorithm [19] for solving (6). After each iteration, their algorithm constructs a new feasible solution for (6) with the number of breakpoints doubled.

All the discretization-based algorithms we have discussed so far solve a sequence of discretization problems for SCLP until the optimal solution is obtained or some pre-defined criteria are met. However, the following two algorithms only solve a specific instance of the discretization problem for SCLP and get a feasible solution with pre-defined quality.

The first of these algorithms is proposed by Fleischer and Sethuraman [22]. Their algorithm is used to solve a special case of multicommodity flow problem with holding costs (MHC) which can be written as (2) with $x(T) = 0$, and $a(t)$, $b(t)$, $c(t)$, $g(t)$, $h(t)$ are nonnegative constant vectors, and E is the identity matrix, with the assumption that the objective value of this (2) is bounded below by 0. It is not known if there is any optimal solution for this problem because the solution set of this problem may not be bounded. By analyzing the properties of the objective function, they introduced a discretization problem (which is an LP problem) with two given constants $\varepsilon > 0$ and $\sigma > 0$. Solving this discretization problem is essentially finding a minimum cost flow in the time-expanded network of the original network. This algorithm will produce a feasible solution with the objective value $(1 + \varepsilon)\text{opt} + \delta$, where opt is the optimal objective value of the original problem. The time complexity of this algorithm is polynomial in the input network, $1/\varepsilon$ and $\log(1/\delta) > 0$.

The second of these algorithms is proposed by Wang [7]. The pre-defined error bound between the objective value of the required solution and the optimal objective solution for (5) is denoted as δ. By analyzing the relationship between the discretizations for (5) and δ, one specific discretization problem for (5) is solved and the solution obtained is used to construct a feasible solution for (5) with piecewise constant $u(t)$ and piecewise linear $x(t)$. The difference between the objective value of this feasible solution and the optimal objective solution for (5) is bounded by δ.

3.4. Other Methods for Solving the SCLP Problem. There are still some solution methods for solving SCLP which do not fall in the previous two categories.

Chen and Yao [23] developed a myopic approach to solve the SCLP by solving a sequence of LP's over time. They gave sufficient conditions under which the myopic approach results in an optimal solution. They also showed that for the so-called Klimov's model—a multiclass network with a single server—this approach leads to the same priority policy as in Klimov's original stochastic setting, which is known as the Gittins index rule.

Avram et al. [24] solve (1) with linear $a(t)$, $c(t)$ and constant $b(t)$ by heuristic method. By using Pontryagin's maximum principle, they show that the optimal solution is the threshold type control. They proposed an algorithm that first decomposes the original problem into several very small size problems and uses maximum principle to derive the exact optimal solutions for these problems; the solutions obtained are then combined to construct an approximate solution for the original problems.

4. Application on Service Operations

SCLP has in recent years attracted considerable research attention in the field of stochastic networks. The multiclass stochastic network is a system that consists of different classes (types) of jobs which need to be processed and a set of servers which process the jobs. Jobs arrive to the system randomly or according to some probability distribution. Each server can process one or more classes of jobs, and the processing time for each server to process one job is different for different classes of jobs. The jobs in the same class have the same characteristics such as arrival rate and service requirements After one job is processed in one server, it may leave the network instantaneously or may become another class of jobs and go to another server for processing. The multiclass stochastic network is a very useful model for many real systems. For example, it can model the supply chain in which different products are delivered [25]. It can model the hub-and-spoke network in airline systems with crowdedness or congestion in the system [26]. It also can model the operations of call centers with interactive voice response [27]. It can also model health units with many patients in them [28].

For each multiclass stochastic network, there is a corresponding deterministic fluid network, which takes only the first-order data (means and rates) from the stochastic model and assumes that the jobs circulating in the network are continuous flows instead of discrete units. With appropriate scaling, the fluid network is a limit of the stochastic network, in the sense of strong law of large numbers (refer to, e.g., [29]). Furthermore, the fluid model has played a central role in studying the stability of stochastic networks [30]. Because of these developments, the real-time control (dynamic scheduling) of a stochastic network, which is itself a quite intractable problem, can be turned into the control of a corresponding fluid network, and the latter problem takes exactly the form of SCLP.

Although the most applications of SCLP mentioned in the literature are in the manufacturing system, they do have some applications in service systems which we will present in the next two subsections. Note that these applications are for the corresponding fluid networks instead of the original networks.

4.1. Optimal Dynamic Routing (Evacuation) in Communication Networks. Hajek and Ogier [31] considered a single destination communication network $\Phi = (V, A, c, d)$, where (V, A) is a finite directed graph with a set V of nodes and a set A of links. $A \subset V \times V$, d is the destination node, and $c = (c_l : l \in A)$ is a capacity assignment vector, $c_l \geq 0$ for every link l.

For each node i in V, let $x_i(t)$ denote the amount of traffic at node i at time t. A demand for the network is a pair $(x(0), r)$ where $x_i(0)$ denotes the (non-negative) initial amount of traffic at node i and r_i denotes the (non-negative) rate of traffic arriving at node i from outside the network. By convention, $x_d(t) = r_d = 0$ for all $t \geq 0$. Since the input rates r_i are not time varying, if the total delay is finite for some control, then it is possible to empty each of the nodes in finite time. Let T denote such a time point that $x(T) = 0$. Let $u_l(t)$ denote the instantaneous flow on link l at time t; the problem is finding a measurable function $u(t)$, such that the total waiting time in the network incurred by all traffic is minimized.

For every $i \in V$, let $E(i)$ denote the collection of nodes j such that $(i, j) \in A$; let $I(i)$ denote the collection of nodes k such that $(k, i) \in A$.

The problem can be formulated as follows:

$$\begin{aligned} \min \quad & \int_0^T \sum_{i \in N-d} x_i(t)\, dt \\ \text{s.t.} \quad & \dot{x}_i(t) = r_i - \sum_{j \in E(i)} u_{ij}(t) \\ & \quad + \sum_{k \in I(i)} u_{ki}(t), \quad i \in V, \\ & u_l(t) \leq c_l, \quad l \in A, \\ & u_l(t) \geq 0, \quad x_i(t) \geq 0. \end{aligned} \tag{13}$$

By integration on both sides of the first constraint from $[0, T]$, it is easy to see that this is a special case of SCLP (4).

4.2. Admission Control and Dynamic Routing for the Telephone Loss Network. Luo [32] considered a telephone loss network defined on a complete digraph $G = (V, A)$, with $V = \{1, 2, \ldots, n\}$ and $A = \{(i, j), i \neq j\}$, that is, this network consists of n different locations $i = 1, \ldots, n$ and $n \times (n-1)$ different links (i, j), for $i \neq j$. At time 0, there are some initial calls in the network. From each i, calls to j arise at a rate of λ_{ij}, and the duration of each is $1/\mu$. Calls will either be accepted or rejected. If a call from i to j is accepted, it generates reward w_{ij} and can either be routed directly to j through the link (i, j) or be routed through (i, k) to a third location k and then from k to j through (k, j). There are no other alternative routes, and once a call is accepted, it cannot be interrupted. Every link (i, j) has a capacity of c_{ij} switching circuits. Every call consumes one switching circuit on every link it uses. If a call from i to j is rejected, a penalty of v_{ij} is incurred.

For any $i \neq j$ and $k \neq j$, let $x_{ikj}(t)$ be the number of calls at time t in the network that are routed from i to j through k, and let $x_{iij}(t)$ be the number of calls at time t that are routed directly from i to j. For any $i \neq j$ and $k \neq j$, let $u_{ikj}(t)$ be the control variable that represents the rate at which calls made at time t from i to j through k, and let $u_{iij}(t)$ be the rate at which calls made at time t are routed directly from i to j. The problem is to decide whether to accept a call, and if we accept it, how are we going to route it, so that the net revenue (the sum of the weighted rewards of accepted calls less the penalty for lost calls) is maximized over a period of time $[0, T]$.

The problem can be formulated as follows:

$$\begin{aligned}
\max \quad & \int_0^T \left(\sum_{i,j} w_{ij} \sum_k x_{ikj}(t) \right. \\
& \qquad \left. - \sum_{i,j} v_{ij} \left(\lambda_{ij} - \sum_k u_{ikj}(t) \right) \right) dt \\
\text{s.t.} \quad & x_{ikj}(t) = x_{ikj}(0) \\
& + \int_0^t \left(u_{ikj}(t) - \mu x_{ikj}(t) \right) dt, \quad i \neq j, k \neq j \\
& \sum_{k \neq j} u_{ikj}(t) \leq \lambda_{ij}, \quad i \neq j \\
& \sum_{k \neq j} x_{kij}(t) + \sum_{j \neq k, i \neq k} x_{ijk}(t) \leq c_{ij}, \quad i \neq j \\
& x(t), u(t) \geq 0, \quad t \in [0, T].
\end{aligned} \tag{14}$$

With a slight abuse of notation, define $x_{ikj}(t) = x_{ikj}(t)e^{\mu t}$ and $u_{ikj}(t) = u_{ikj}(t)e^{\mu t}$. This problem can be reformulated as follows:

$$\begin{aligned}
\max \quad & \int_0^T \left(e^{-\mu t} v' u(t) + e^{-\mu t} w' x(t) \right) dt \\
\text{s.t.} \quad & x_{ikj}(t) = x_{ikj}(0) \\
& \qquad + \int_0^t u_{ikj}(t)\, dt, \quad i \neq j, k \neq j \\
& \sum_{k \neq j} u_{ikj}(t) \leq e^{\mu t} \lambda_{ij}, \quad i \neq j \\
& \sum_{k \neq j} x_{kij}(t) + \sum_{j \neq k, i \neq k} x_{ijk}(t) \leq e^{\mu t} c_{ij}, \quad i \neq j \\
& x(t), u(t) \geq 0, \quad t \in [0, T].
\end{aligned} \tag{15}$$

It is easy to see that this is an SCLP (2).

5. Conclusion

In this paper, we give a review on the research for SCLP. Several formulations of SCLP are presented, along with the related duality theory and solution method. Most results on duality theory provide the conditions under which SCLP has strong duality or has optimal solution. Most solution methods for SCLP fall in two categories: simplex-like and discretization based. The simplex-like method can get the exact optimal solution but is very time consuming; the discretization based methods are quite fast but can only get the approximate solutions in most cases.

There are still some problems needed to address in the future. For example, how to perform the sensitivity analysis for SCLP? How to define the robust SCLP? As the parameters in SCLP may not be able to get accurately in practice, the results of these research directions will definitely help to enlarge the range of the applications of SCLP.

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Research Article

Cooperative Advertising in a Supply Chain with Horizontal Competition

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Cooperative advertising programs are usually provided by manufacturers to stimulate retailers investing more in local advertising to increase the sales of their products or services. While previous literature on cooperative advertising mainly focuses on a "single-manufacturer single-retailer" framework, the decision-making framework with "multiple-manufacturer single-retailer" becomes more realistic because of the increasing power of retailers as well as the increased competition among the manufacturers. In view of this, in this paper we investigate the cooperative advertising program in a "two-manufacturer single-retailer" supply chain in three different scenarios; that is, (i) each channel member makes decisions independently; (ii) the retailer is vertically integrated with one manufacturer; (iii) two manufacturers are horizontally integrated. Utilizing differential game theory, the open-loop equilibrium-advertising strategies of each channel member are obtained and compared. Also, we investigate the effects of competitive intensity on the firm's profit in three different scenarios by using the numerical analysis.

1. Introduction

To increase the sales of their products or services, some manufacturers or service providers utilize cooperative advertising programs, through which they share a part of the retailer's advertising cost, to stimulate retailers advertising more on their products or services. Generally, advertising can be divided into national advertising and local advertising. The former focuses on building brand image about the products or services. The latter is often price oriented to stimulate consumer to purchase the products or services at once. Supported by subsidies from a manufacturer's cooperative advertising program, retailers would always increase their local advertising expenditures and thus improve their profits [1].

Surveys showed that, for many manufacturers or service providers such as General Electric, their advertising budgets to retailers via cooperative advertising programs are more three times of that they spent on national advertising [2]. Further, Dant and Berger found that 25–40% of local advertisements are cooperatively funded [3]. Total expenditures on cooperative advertising in 2000 were estimated at $15 billion, compared to $900 million in 1970, nearly a four-fold increase in real terms [4]. In 2010, about $50 billion was spent on cooperative advertising programs [5].

The tendency toward increased spending on cooperative advertising has received significant attention from researchers. The cooperative advertising models under study can be divided into two categories: static models [1, 6–12] and dynamic models [13–19]. However, these studies mainly focus on a "single-manufacturer single-retailer" framework.

Retailers in today's market are increasingly more powerful than manufacturers. Useem found that sales through Wal-Mart accounted for 17% of P&G's total sales in 2002, 39% of Tandy's, and over 10% for many other large manufacturers [20]. Tesco is the largest grocer in the United Kingdom, accounting for almost 30% of the supermarket sales [21]. Home Depot and Lowe have more than 50% of the home improvement market [22]. As retailers become more dominant, manufacturers face fierce competition among themselves. Thus, it is necessary to take the competition among manufacturers into account when studying the cooperative advertising model.

The significant contribution of this paper is that it generalizes existing cooperative advertising work on "single-manufacturer single-retailer" framework to the "two-manufacturer single-retailer" framework. This generalization has provided new analytical results about how the competition affects the advertising efforts and profit for channel member. In detail, we study the open-loop equilibrium advertising strategies of each channel member in three different scenarios, including that (i) each channel member makes decisions independently; (ii) the retailer is vertically integrated with one manufacturer; (iii) two manufacturers are horizontally integrated. Specifically, the following research questions are addressed in this paper. (i) For each scenario, what are the equilibrium advertising efforts for each channel member and what is the manufacturer's optimal participation rate for the retailer's local advertising expenditures? (ii) When the retailer integrates with one manufacturer, does the manufacturer change its decisions about national advertising expenditures and participation rates? (iii) How does the horizontal integration of two manufacturers affect the decisions of each channel member?

To answer the above questions, we focus on the cooperative advertising problem in a "two-manufacturer single-retailer" framework. The dynamic advertising models are proposed based on the Nerlove-Arrow model. Utilizing differential game theory, the open-loop equilibrium advertising strategies of each channel member are obtained and compared in three different scenarios.

The remainder of the paper is structured as follows. Previous literature related to our topic is reviewed in Section 2. Section 3 develops the proposed models, and then the equilibrium advertising efforts and participation rates in three different scenarios are discussed. Section 4 offers a numerical analysis. Conclusions and suggestions for future research are in Section 5. Proofs for all propositions in the paper are given in Appendices.

2. Literature Review

Our work is related to several research streams. First is the stream of literature that focuses on cooperative advertising, which can be divided into two main categories: static models and dynamic models. A primary static model was proposed by Berger [6], who was the first to analyze cooperative advertising. Bergen and John developed two formal models to study the effects of the participation rate offered by manufacturers [7]. By dividing advertising into national and local, Huang et al. were able to study co-op advertising models in a static supply chain framework and discuss the channel members' advertising decisions for different relationship configurations between the channel members [1, 8]. Based on the work of Huang and Li [1], Yue et al. studied the co-op advertising problem by considering a price discount in demand elasticity market circumstance [9]. Xie and Neyret proposed a more general model, including co-op advertising and pricing [10]. Further, Seyed Esfahani et al. considered vertical co-op advertising along with pricing decisions in a supply chain and proved that both the manufacturer and the retailer reach the highest profits level when they follow a cooperation strategy [12]. For the dynamic advertising models, Chintagunta and Jain extended the work of Nerlove and Arrow [23] to consider the interaction effects of manufacturer and retailer goodwill on channel sales and developed a dynamic model to study the equilibrium advertising strategies in a two-member marketing channel [13]. Jørgensen et al. provided a dynamic model for a cooperative advertising framework, which allows both channel members to make long- and short-term advertising efforts to enhance sales and consumer goodwill [14]. Further, Jørgensen et al. introduced decreasing marginal returns to goodwill and adopted a more flexible functional form for the sales dynamics [15]. Jørgensen et al. studied the cooperative advertising program in the case where the retailer's promotions can damage the brand image [16]. Extending the work of Jørgensen et al. [15], Karray and Zaccour considered a differential game model for a marketing channel formed by one manufacturer and one retailer and concluded that a cooperative advertising program can help the manufacturer mitigate the competitive impact of the private label [18]. He et al. provided a theoretical analysis of cooperative advertising plans in a dynamic stochastic supply chain [19].

The above literature is mainly focused on a "single-manufacturer single-retailer" framework. Few studies address a "multiple-manufacturer single-retailer" framework or any other framework. Kurtuluş and Toktay considered a model including two competing manufacturers and one retailer; the result revealed that the retailer can use the form of category management and the category shelf space to control the intensity of competition between manufacturers to his benefit [24]. Adida and DeMiguel studied competition in a supply chain where multiple manufacturers compete in quantities to supply a set of products to multiple risk-averse retailers who compete in quantities to satisfy the uncertain consumer demand [25]. Cachon and Kök also studied a supply chain system with competing manufacturers and a single retailer; the results showed that the properties a contractual form exhibits in a one-manufacturer supply chain may not carry over to the realistic setting in which multiple manufacturers must compete to sell their goods through the same retailer [26]. Further, Lu et al. highlighted the importance of service from manufacturers in the interactions between two competing manufacturers and their common retailer, and their result showed that as the market base of one product increases, the second manufacturer also benefits but at a lesser amount than the first manufacturer [27]. However, the abovementioned works with multiple manufacturers do not consider cooperative advertising. There are some cooperative advertising works that focus on a "multiple-retailer" framework. For example, He et al. [28, 29] extended He et al. [19] by considering the competing retailers, and their results showed that the manufacturer's support for its retailer is higher under competition than in its absence.

To our best knowledge, research relating to cooperative advertising focused on a "multiple-manufacturer single-retailer" framework in the supply chain has not been explored in literature. In this study, we investigate a cooperative advertising model using the "two-manufacturer single-retailer" framework.

3. Model Description

As shown in Figure 1, we consider a supply chain system consisting of two competing manufacturers and one retailer. The two manufacturers produce similar products with different brands which are denoted as i, $i \in \{1,2\}$ that the retailer sells simultaneously. The competition is based primarily on the use of nonprice competitive strategies, namely, the two manufacturers each advertise their products, and the retailer advertises two products simultaneously.

We introduce the additional notation in this paper (see Table 1).

As our goodwill-based model is based upon the model of Nerlove-Arrow, the changing of the stock of goodwill of product i is given by

$$\dot{G}_i(t) = U_{Mi} - \theta U_{M(3-i)} - \delta G_i, \quad i \in \{1,2\}, \tag{1}$$

where $0 < \theta < 1$ is a constant which represents the rival advertising's negative effect on goodwill, as seen in previous literature [30]. Next, $\delta > 0$ is the diminishing rate of goodwill, which captures the idea that consumers may forget the brand to some extent. National advertising mainly focuses on firm's long-term objectives such as brand awareness, image, and credibility [31]. Therefore, we only take the effect of national advertising into the stock of goodwill here. Further, the initial goodwill of the two products is denoted as

$$G_1(0) = G_{10} \geq 0, \qquad G_2(0) = G_{20} \geq 0, \tag{2}$$

and the sales $S_i(t)$ of the two brands along time t satisfy

$$S_i(t) = \max\{0, \alpha G_i + \lambda U_{Ri} - \chi U_{R(3-i)}\}, \quad i \in \{1,2\}. \tag{3}$$

In (3), α, λ, χ are all positive constants. For the sake of simplicity, we suppose that the influence coefficient is identical for these two products. In (3), the item αG_i represents the long-term effect of national advertising on sales, and the item λU_{Ri} represents the effect of the retailer's local advertising on product i. As in previous research such as Jørgensen et al. [14], we only take the promotion effect of local advertising into the functions of sales here without the stock of goodwill. The item $-\chi U_{R(3-i)}$ illustrates the rival local advertising's negative effect on sales. Next, $\lambda > \chi > 0$, which implies that the effects of rival advertising are generally smaller than the effects of one's own advertising effect, which is a fairly common assumption in the relevant literature [30].

The advertising cost functions are quadratic with respect to marketing efforts, namely,

$$C(U_{Mi}(t)) = \frac{U_{Mi}^2(t)}{2}, \quad C(U_{Ri}) = \frac{U_{Ri}^2(t)}{2}, \quad i \in \{1,2\}. \tag{4}$$

This assumption about the advertising cost function is commonly found in literature [32]. The convex cost function implies increasing marginal cost of effort.

Without considering advertising expenditures, the marginal profit of manufacturer i is assumed as $\rho_{Mi} \geq 0$, and the marginal profit of the retailer selling the product i is

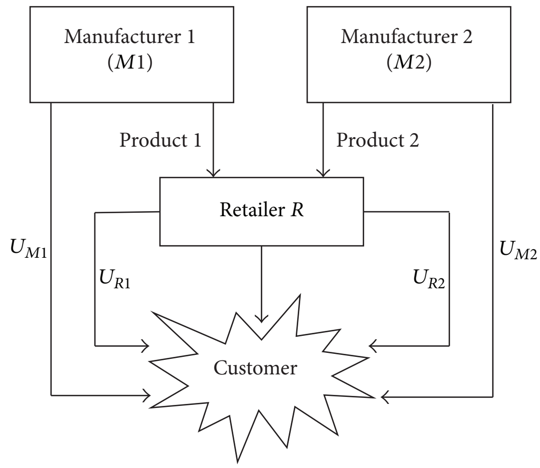

Figure 1

Table 1: Notation.

t	Time t, $t \geq 0$
$G_i(t)$	Goodwill of the product i at time t, $i \in \{1,2\}$
$U_{Mi}(t)$	Manufacturer i's national advertising efforts at time t
$U_{Ri}(t)$	Retailer's local advertising efforts for product i at time t
$S_i(t)$	Sales of product i along time t
$\theta \in [0,1]$	The national advertising's competition coefficient
$\chi \in [0,1]$	The local advertising's competition coefficient
$\phi_i \in [0,1]$	Manufacturer i's participation rate for the retailer's advertising cost
$\rho_{Mi} \geq 0$	Marginal profit of manufacturer i
$\rho_{Ri} \geq 0$	Marginal profit of the retailer sells the product i
$\delta > 0$	Diminishing rate of goodwill
$r > 0$	Discount rate of the manufacturers and the retailer
π_{Mi}, π_R	Profit functions for Mi and R, respectively
J_{Mi}, J_R	Current value of profit functions for Mi and R, respectively

$\rho_{Ri} \geq 0$. The profit functions of the two manufacturers are then

$$\pi_{Mi}(t) = \rho_{Mi} S_i(t) - \frac{1}{2} U_{Mi}^2(t) - \frac{1}{2} \phi_i U_{Ri}^2(t), \quad i \in \{1,2\}, \tag{5}$$

and the profit function of the retailer is

$$\pi_R(t) = \sum_{i=1}^{2} \left(\rho_{Ri} S_i(t) - \frac{1}{2}(1 - \phi_i) U_{Ri}^2(t) \right). \tag{6}$$

In this paper, we assume the participation rate is a constant over time for the following reasons. (i) Although much more literature assumes that the participation rate changes along time [19], a changing participation rate is so complex that there are no cooperative advertising programs in practice. In the empirical studies of Nagler [4], all the 1470 plans explicitly listed a single fixed participation rate. If a firm provides a cooperative advertising program with a changing participation rate, the manufacturer would have to know the retailer's daily advertising cost exactly, which is much more difficult than learning the whole advertising cost over a certain period of time. (ii) Even, in previous studies which model the participation rate as a function of time, the final optimal decision for participation rates were all constant over time [14, 18, 19, 28].

Please note that U_{Mi} and ϕ_i are manufacturer's decision variables, and U_{Ri} is retailer's decision variable. Then we consider a two-stage game in this paper. The manufacturers offer their participation rates for the retailer's local advertising expenditure at stage 1, and then the manufacturers and retailer determine their advertising efforts along time t simultaneously at stage 2. We firstly keep the participation rates ϕ_i $(i = 1, 2)$ as fixed, calculate the advertising efforts of the manufacturers and retailer utilizing differential game theory, and then decide the manufacturers' optimal participation rates.

3.1. Each Channel Member Makes Decisions Independently. In this scenario, each channel member makes decisions independently, and the profit functions of all channel members are given by (5) and (6). Note the profits for all the channel members changes along with time t. Each channel member, then, strives to maximize the current values of its profit. With a common discount rate $r > 0$ and for the sales $S_i \geq 0$, we have

$$\max_{U_{Mi} \geq 0, 1 \geq \phi_i \geq 0} J_{Mi} = \int_0^{+\infty} e^{-rt} \pi_{Mi}(t)\, dt, \quad i \in \{1, 2\}, \tag{7}$$

and, for the retailer, we have

$$\max_{U_{R1} \geq 0, U_{R2} \geq 0} J_R = \int_0^{+\infty} e^{-rt} \pi_R(t)\, dt. \tag{8}$$

Taking (1) into account, we get the current value Hamiltonian of two manufacturers as

$$\begin{aligned} H_{M1} &= \pi_{M1} + \mu_{11}\left(U_{M1} - \theta U_{M2} - \delta G_1\right) \\ &\quad + \mu_{12}\left(U_{M2} - \theta U_{M1} - \delta G_2\right), \\ H_{M2} &= \pi_{M2} + \mu_{21}\left(U_{M1} - \theta U_{M2} - \delta G_1\right) \\ &\quad + \mu_{22}\left(U_{M2} - \theta U_{M1} - \delta G_2\right). \end{aligned} \tag{9}$$

Similarly, we get the retailer's current value Hamiltonian as

$$\begin{aligned} H_R &= \pi_R + \mu_{31}\left(U_{M1} - \theta U_{M2} - \delta G_1\right) \\ &\quad + \mu_{32}\left(U_{M2} - \theta U_{M1} - \delta G_2\right), \end{aligned} \tag{10}$$

where μ_{i1} and μ_{i2} $(i = 1, 2, 3)$ represent the costate variables in the firm's problem corresponding to the firm's goodwill levels.

Then using the necessary conditions for equilibrium, we obtain the following results.

Proposition 1. *When each channel member makes decisions independently and the participation rates ϕ_i $(i = 1, 2)$ are fixed, the equilibrium advertising efforts for two manufacturers on their products along time t are all constants; that is,*

$$U_{Mi}^{(1)} = \frac{\alpha \rho_{Mi}}{(r + \delta)}, \quad i \in \{1, 2\}. \tag{11}$$

Proposition 1 illustrates the following facts. (i) Whatever the participation rate that the manufacturer undertakes for the retailer's advertising cost, the manufacturer's equilibrium national advertising efforts are kept the same and are just linear with its own marginal profit. The larger the marginal profit, the more the manufacturer would spend on national advertising. (ii) The manufacturer's equilibrium advertising efforts are determined by the item $\alpha\rho_{Mi}/(r + \delta)$, which is aimed at maintaining the long-term effect of advertising. Specifically, this item decreases sharply when the diminishing rate of consumer goodwill becomes very large or the decision makers are more short sighted. Therefore, when the decision makers do not feel confident in future, or the customer's goodwill diminishes quickly, the advertising efforts would drop.

Further, we obtain the retailer's equilibrium advertising efforts for two brands as follows.

Proposition 2. *When each channel member makes decisions independently and the participation rates ϕ_i $(i = 1, 2)$ are fixed, the retailer's equilibrium advertising efforts for the two brands along time t are all constants:*

$$U_{Ri}^{(1)} = \begin{cases} \dfrac{\lambda\rho_{Ri} - \chi\rho_{R(3-i)}}{1 - \phi_i} & \text{if } \lambda\rho_{Ri} - \chi\rho_{R(3-i)} \geq 0, \\ 0 & \text{else} \end{cases} \tag{12}$$

$$i \in \{1, 2\}.$$

Proposition 2 holds the following managerial implications. (i) When condition $\lambda\rho_{Ri} - \chi\rho_{R,(3-i)} \geq 0$, $i \in \{1, 2\}$ is satisfied, a higher participation rate leads the retailer to spend more on local advertising but the advertising efforts are independent of the participation rate which the other manufacturer provides to the retailer. Therefore, the manufacturer can use the participation rate to guide the retailer's advertising efforts for his product. (ii) The equilibrium local advertising efforts on product i are increased by the retailer's marginal profit of product i.

Furthermore, when condition $\lambda > \chi > 0$ is satisfied, we get the following results by (12).

(i) If condition $\lambda\rho_{R2} - \chi\rho_{R1} < 0$ holds, we have $U_{R1}^{(1)} = (\lambda\rho_{R1} - \chi\rho_{R2})/(1 - \phi_1)$ and $U_{R2}^{(1)} = 0$. Because the marginal profit which the retailer obtains from product 2 is extremely small, the benefit of $U_{R2}^{(1)}$ from product 2 does not offset the loss from product 1. Thus, the retailer would not advertise product 2. In other words, whatever participation rate manufacturer 2 offers, the retailer would never advertise product 2. Under this situation, manufacturer 2 only can change the situation of no-local-advertising efforts on his product by offering the retailer a higher margin.

(ii) If conditions $\lambda\rho_{R1}-\chi\rho_{R2} \geq 0$ and $\lambda\rho_{R2}-\chi\rho_{R1} \geq 0$ hold, we can get $U_{R1}^{(1)} = (\lambda\rho_{R1} - \chi\rho_{R2})/(1-\phi_1)$ and $U_{R2}^{(1)} = (\lambda\rho_{R2} - \chi\rho_{R1})/(1-\phi_2)$. In this situation, advertising for two brands would lead to so much gain for the retailer that the retailer would advertise both products at a certain level.

(iii) If condition $\lambda\rho_{R1} - \chi\rho_{R2} < 0$ holds, we have $U_{R1}^{(1)} = 0$ and $U_{R2}^{(1)} = (\lambda\rho_{R2} - \chi\rho_{R1})/(1-\phi_2)$. This situation is similar to the first situation; the retailer would advertise product 2, but not product 1.

When the two manufacturers' participation rates are fixed, the equilibrium advertising efforts for all channel members are given by Propositions 1 and 2. Based on these results, we can work out the stock of goodwill for the two products as well as for the current value of profits for all channel members, which is given by Proposition 3.

Proposition 3. *When each channel member makes decisions independently and their advertising efforts are kept as constants, that is,* $U_{Mi}(t) = U_{Mi}^{(1)}$ *and* $U_{Ri}(t) = U_{Ri}^{(1)}$, $i = 1,2$, *then the accumulated goodwill of two products along time t is*

$$G_i(t) = D_i e^{-\delta t} + G_{iSS}^{(1)}, \quad i \in \{1,2\}, \tag{13}$$

where $D_i = G_{i0} - (U_{Mi}^{(1)} - \theta U_{M,(3-i)}^{(1)})/\delta$ *and* $G_{iSS}^{(1)} = (U_{Mi}^{(1)} - \theta U_{M,(3-i)}^{(1)})/\delta$, $i = 1,2$. $G_{iSS}^{(1)}$ *is the steady-state goodwill for product i when* $t \to \infty$.

From (13), we obtain the following facts: (i) the steady-state goodwill for product i increases with manufacturer 1's national advertising efforts; (ii) the steady-state goodwill for product i decreases with the rival manufacturer's national advertising efforts because of the competitive effect; (iii) steady-state goodwill is only affected by the manufacturer's advertising efforts because local advertising has only an instant promotion effect that has no impact on the stock of goodwill; (iv) when the diminishing rate of goodwill becomes very large, steady-state goodwill decreases.

Substituting (11)–(13) into (7) and (8) and with the participation rates ϕ_i ($i = 1,2$) being fixed, we get the current value of manufacturer 1's profit under the equilibrium condition as follows:

$$\begin{aligned} J_{Mi}^{(1)} = {} & \frac{D_i\alpha\rho_{Mi}}{r+\delta} + \frac{\alpha\rho_{Mi}\left(U_{Mi}^{(1)} - \theta U_{M,(3-i)}^{(1)}\right)}{r\delta} \\ & + \frac{\rho_{Mi}\left(\lambda U_{Ri}^{(1)} - \chi U_{R,(3-i)}^{(1)}\right)}{r} \\ & - \frac{\left(U_{Mi}^{(1)}\right)^2 + \phi_i\left(U_{Ri}^{(1)}\right)^2}{2r}, \quad i \in \{1,2\}. \end{aligned} \tag{14}$$

The current value of the retailer's profit is

$$\begin{aligned} J_R^{(1)} = {} & \frac{D_1\alpha\rho_{R1} + D_2\alpha\rho_{R2}}{r+\delta} \\ & + \frac{\alpha\rho_{R1}\left(U_{M1}^{(1)} - \theta U_{M2}^{(1)}\right) + \alpha\rho_{R2}\left(U_{M2}^{(1)} - \theta U_{M1}^{(1)}\right)}{r\delta} \\ & - \frac{1-\phi_1}{2}\left(U_{R1}^{(1)}\right)^2 + \frac{\rho_{R1}\left(\lambda U_{R1}^{(1)} - \chi U_{R2}^{(1)}\right)}{r} \\ & + \frac{\rho_{R2}\left(\lambda U_{R2}^{(1)} - \chi U_{R1}^{(1)}\right)}{r} - \frac{1-\phi_2}{2}\left(U_{R2}^{(1)}\right)^2, \end{aligned} \tag{15}$$

where $D_i = G_{i0} - (U_{Mi}^{(1)} - \theta U_{M,(3-i)}^{(1)})/\delta$, $i \in \{1,2\}$.

Differentiating $J_{Mi}^{(1)}$ with the participation rate ϕ_i, we get optimal participation rates, from are given by Proposition 4.

Proposition 4. *When all the channel members make decisions independently, the optimal participation rates that the two manufacturers provide to the retailer under the equilibrium condition are*

$$\phi_i^{(1)} = \begin{cases} \dfrac{\lambda(2\rho_{Mi} - \rho_{Ri}) + \chi\rho_{R(3-i)}}{\lambda(2\rho_{Mi} + \rho_{Ri}) - \chi\rho_{R(3-i)}} & \text{if } \rho_{Mi} \geq \dfrac{(\lambda\rho_{Ri} - \chi\rho_{R(3-i)})}{2\lambda}, \\ 0 & \text{else} \end{cases} \quad i \in \{1,2\}. \tag{16}$$

For (16), the restraining condition $\rho_{M1} \geq (\lambda\rho_{R1} - \chi\rho_{R2})/2\lambda$ implies that manufacturer 1 is willing to provide the participation rate with the retailer only when he can obtain a large enough marginal profit. Differentiating $\phi_1^{(1)}$ from ρ_{M1}, ρ_{R1}, and ρ_{R2}, and knowing that $\rho_{M1} \geq (\lambda\rho_{R1} - \chi\rho_{R2})/2\lambda$, we find that $\partial\phi_1^{(1)}/\partial\rho_{M1} > 0$; $\partial\phi_1^{(1)}/\partial\rho_{R1} < 0$; $\partial\phi_1^{(1)}/\partial\rho_{R2} > 0$. The above expressions show that (i) when the manufacturer's marginal profit increases, he would offer a high participation rate to the retailer; (ii) when a high marginal profit would be obtained by the retailer, the manufacturer has less incentive to offer a high participation rate for the cooperative program; (iii) manufacturer 1 would offer a high participation rate if the retailer obtains a larger marginal profit from product 2.

Furthermore, substituting the optimal participation rates into (12), we find that the retailer's equilibrium local advertising efforts on the two brands are all constants, that is,

$$U_{Ri}^{(1)} = \begin{cases} \lambda\rho_{Mi} + \dfrac{1}{2}\lambda\rho_{Ri} - \dfrac{1}{2}\chi\rho_{R(3-i)}, \\ \qquad \text{if } 0 < \dfrac{(\lambda\rho_{Ri} - \chi\rho_{R(3-i)})}{2\lambda} \leq \rho_{Mi}, \\ \lambda\rho_{Ri} - \chi\rho_{R(3-i)}, \\ \qquad \text{if } \rho_{Mi} < \dfrac{(\lambda\rho_{Ri} - \chi\rho_{R(3-i)})}{2\lambda}, \\ 0 \qquad \text{else} \end{cases} \quad i \in \{1,2\}. \tag{17}$$

Equation (17) shows that the retailer's equilibrium advertising level $U_{Ri}^{(1)}$ for a product is not only linear with his own marginal profit ρ_{Ri}, but also linear with the manufacturer's marginal profit ρ_{Mi} if the conditions $0 < (\lambda\rho_{Ri} - \chi\rho_{R(3-i)})/2\lambda \le \rho_{Mi}$ hold. Supposing that $\rho_{Mi} + \rho_{Ri} = \rho_i$ is the channel marginal profit of one product, the equilibrium advertising level $U_{Ri}^{(1)}$ can be rewritten as $U_{Ri}^{(1)} = \lambda\rho_{Mi}/2 + \lambda\rho_i/2 - \chi\rho_{R,(3-i)}/2$ if and only if $0 < \lambda\rho_{Ri} - \chi\rho_{R,(3-i)} \le 2\lambda\rho_{Mi}$, $i = 1, 2$ hold. The channel marginal profit of product i is not changed in the short term; therefore, the above equations imply that the retailer's equilibrium advertising efforts are independent of the marginal profit which the retailer obtains from product i.

3.2. Retailer Integrates with a Manufacturer. In second scenario, the retailer integrates with one of the manufacturers. We assume this manufacturer is $M1$. Then, the profit function of the integration system is $\pi_{M1,R} = \pi_{M1} + \pi_R$, and the objective of integration system is

$$\max_{U_{M1}\ge 0, U_{R1}\ge 0, U_{R2}\ge 0} J_{M1,R} = \int_0^{+\infty} e^{-rt} \left(\pi_{M1} + \pi_R\right) dt. \quad (18)$$

Further, the objective of manufacturer 2 is

$$\max_{1\ge\phi_2\ge 0, U_{M2}\ge 0} J_{M2} = \int_0^{+\infty} e^{-rt} \pi_{M2} dt. \quad (19)$$

Taking state equation (1) into account, the current value Hamiltonian of the vertical integration system ($M1$ and R) is

$$\begin{aligned} H_{M1,R} = \pi_{M1} + \pi_R + \gamma_{11}\left(U_{M1} - \theta U_{M2} - \delta G_1\right) \\ + \gamma_{12}\left(U_{M2} - \theta U_{M1} - \delta G_2\right), \end{aligned} \quad (20)$$

and that of manufacturer 2 is

$$\begin{aligned} H_{M2} = \pi_{M2} + \gamma_{21}\left(U_{M1} - \theta U_{M2} - \delta G_1\right) \\ + \gamma_{22}\left(U_{M2} - \theta U_{M1} - \delta G_2\right), \end{aligned} \quad (21)$$

where γ_{i1} and γ_{i2} $(i = 1, 2)$ represent the costate variables in the channel member's problem corresponding to the firm's goodwill level.

Then using (20) and (21), we obtain the following results.

Proposition 5. *When the retailer integrates with manufacturer 1 and the participation rate ϕ_2 provided by manufacturer 2 is fixed, the equilibrium advertising efforts of manufacturer 2 are constant, that is,*

$$U_{M2}^{(2)} = \frac{\alpha\rho_{M2}}{(r+\delta)}. \quad (22)$$

Compared to Proposition 1, we find that manufacturer 2's equilibrium advertising level is the same, which implies that manufacturer 2's advertising level does not depend on the integration between manufacturer 1 and retailer.

Proposition 6. *When the retailer integrates with manufacturer 1 and the participation rate ϕ_2 provided by manufacturer 2 is fixed, manufacturer 1's equilibrium advertising efforts are constant, that is,*

$$U_{M1}^{(2)} = \begin{cases} \dfrac{\alpha\rho_1 - \theta\alpha\rho_{R2}}{r+\delta} & \text{if } \rho_1 \ge \theta\rho_{R2}, \\ 0 & \text{else}, \end{cases} \quad (23)$$

and the retailer's equilibrium advertising efforts for the two brands along time t are all constants, that is,

$$U_{R1}^{(2)} = \begin{cases} \lambda\rho_1 - \chi\rho_{R2} & \text{if } \lambda\rho_1 - \chi\rho_{R2} \ge 0, \\ 0 & \text{else}, \end{cases} \quad (24)$$

$$U_{R2}^{(2)} = \begin{cases} \dfrac{\lambda\rho_{R2} - \chi\rho_1}{1-\phi_2} & \text{if } \lambda\rho_{R2} - \chi\rho_1 \ge 0, \\ 0 & \text{else}, \end{cases} \quad (25)$$

where $\rho_1 = \rho_{M1} + \rho_{R1}$.

Proposition 6 shows the following trends. If the condition $\rho_1 > \theta\rho_{R2}$ holds, the national advertising efforts $U_{M1}^{(2)}$ are affected by the item $(\alpha\rho_1 - \theta\alpha\rho_{R2})/(r+\delta)$. From this item, we know that the larger the channel marginal profit of product 1 (ρ_1), the more the integration system would spend on national advertising for product 1. As opposed to the first scenario, in this scenario the national advertising efforts are also affected by the rival product's marginal profit ρ_{R2}. When ρ_{R2} is increased, the integration system would decrease national advertising efforts for product 1 and thus decrease product 1's adverse influence on product 2. Further, if the channel marginal profit of product 1 is too small, the integration system would not advertise product 1.

If we subtract (17) from (24), we get

$$\Delta U_{R1} = \begin{cases} \lambda\rho_{M1} & \text{if } 2\lambda\rho_{M1} < \lambda\rho_{R1} - \chi\rho_{R2}, \\ \dfrac{\lambda\rho_{R1} - \chi\rho_{R2}}{2} & \text{if } 0 < \lambda\rho_{R1} - \chi\rho_{R2} \le 2\lambda\rho_{M1}, \\ \lambda\rho_1 - \chi\rho_{R2} & \text{if } -\lambda\rho_{M1} < \lambda\rho_{R1} - \chi\rho_{R2} \le 0, \\ 0 & \text{else}, \end{cases} \quad (26)$$

where $\rho_1 = \rho_{M1} + \rho_{R1}$.

It is easy to prove that ΔU_{R1} given by (26) is nonnegative, which means that the integration between the retailer and the manufacturer would increase the retailer's equilibrium local advertising efforts for product 1.

Furthermore, combining (24) and (25), we obtain similar managerial implications as the results of Proposition 2, but we also find some differences.

(i) When $\lambda\rho_{R2} - \chi\rho_1 < 0$ holds, we have $U_{R2}^{(2)} = 0$ and $U_{R1}^{(2)} = \lambda\rho_1 - \chi\rho_{R2}$. Note that $\rho_1 = \rho_{M1} + \rho_{R1} > \rho_{R1}$, which implies that the integration between the retailer and manufacturer 1 would lead to the increase in the retailer's local advertising threshold for product 2 and would also increase the retailer's equilibrium advertising efforts for product 1.

(ii) If conditions $\lambda\rho_1 - \chi\rho_{R2} \geq 0$ and $\lambda\rho_{R2} - \chi\rho_1 \geq 0$ hold, $U_{R2}^{(2)} = \lambda\rho_1 - \chi\rho_{R2}$ and $U_{R2}^{(2)} = (\lambda\rho_{R2} - \chi\rho_1)/(1-\phi_2)$. In this situation, the retailer increases the equilibrium advertising efforts for product 1 and decreases efforts for product 2.

(iii) When $\lambda\rho_1 - \chi\rho_{R2} < 0$ holds, $U_{R1}^{(2)} = 0$ and $U_{R2}^{(2)} = (\lambda\rho_{R2} - \chi\rho_1)/(1-\phi_2)$. Note that $\rho_1 > \rho_{R1}$, which implies that the integration between the retailer and manufacturer 1 reduces the retailer's local advertising threshold for product 1 and also decreases the retailer's equilibrium advertising efforts for product 2.

We can calculate the stock of goodwill for the two products and the current value of profits for all channel members, which are given by Proposition 7.

Proposition 7. *When the retailer integrates with manufacturer M1, and their advertising efforts are kept constant, that is, $U_{Mi}(t) = U_{Mi}^{(2)}$ and $U_{Ri}(t) = U_{Ri}^{(2)}$, $i = 1, 2$, then the accumulated goodwill of the two products along time t is*

$$G_i(t) = E_i e^{-\delta t} + G_{iSS}^{(2)}, \quad i \in \{1, 2\}, \tag{27}$$

where $G_{iSS}^{(2)} = (U_{Mi}^{(2)} - \theta U_{M,(3-i)}^{(2)})/\delta$, $E_i = G_{i0} - (U_{Mi}^{(2)} - \theta U_{M,(3-i)}^{(2)})/\delta$, $i = 1, 2$. $G_{iSS}^{(2)}$ is the steady-state goodwill for product i when $t \to \infty$.

Substituting (22) through (25) and (27) into (18) and (19), and assuming that the participation rates ϕ_i $(i = 1, 2)$ are fixed, we get the current value of the integration system's profit as follows:

$$\begin{aligned} J_{M1,R}^{(2)} = {} & \frac{E_1\alpha\rho_1 + E_2\alpha\rho_{R2}}{r+\delta} \\ & + \frac{\alpha\rho_1\left(U_{M1}^{(2)} - \theta U_{M2}^{(2)}\right) + \alpha\rho_{R2}\left(U_{M2}^{(2)} - \theta U_{M1}^{(2)}\right)}{r\delta} \\ & - \frac{\left(U_{M1}^{(2)}\right)^2 + \left(U_{R1}^{(2)}\right)^2}{2r} + \frac{\rho_1\left(\lambda U_{R1}^{(2)} - \chi U_{R2}^{(2)}\right)}{r} \\ & + \frac{\rho_{R2}\left(\lambda U_{R2}^{(2)} - \chi U_{R1}^{(2)}\right)}{r} - \frac{(1-\phi_2)\left(U_{R2}^{(2)}\right)^2}{2r}, \end{aligned} \tag{28}$$

and the profit for manufacturer 2 is

$$\begin{aligned} J_{M2}^{(2)} = {} & \frac{E_2\alpha\rho_{M2}}{r+\delta} + \frac{\alpha\rho_{M2}\left(U_{M2}^{(2)} - \theta U_{M1}^{(2)}\right)}{r\delta} \\ & + \frac{\rho_{M2}\left(\lambda U_{R2}^{(2)} - \chi U_{R1}^{(2)}\right)}{r} - \frac{\left(U_{M2}^{(2)}\right)^2 - \phi_2\left(U_{R2}^{(2)}\right)^2}{2r}, \end{aligned} \tag{29}$$

where $E_i = G_{i0} - (U_{Mi}^{(2)} - \theta U_{M,(3-i)}^{(2)})/\delta$, $i = 1, 2$.

Differentiating $J_{M2}^{(2)}$ from the participation rate ϕ_2, we get the optimal participation rate, which is given by Proposition 8.

Proposition 8. *When the retailer integrates with manufacturer 1, and the advertising levels are kept as constants, that is, $U_{Mi}(t) = U_{Mi}^{(2)}, U_{Ri}(t) = U_{Ri}^{(2)}, i = 1, 2$, the optimal participation rate which manufacturer 2 provides is*

$$\phi_2^{(2)} = \begin{cases} \dfrac{\lambda(2\rho_{M2} - \rho_{R2}) + \chi\rho_1}{\lambda(2\rho_{M2} + \rho_{R2}) - \chi\rho_1} & \textit{if } \rho_{M2} \geq \dfrac{(\lambda\rho_{R2} - \chi\rho_1)}{2\lambda}, \\ 0 & \textit{else}, \end{cases} \tag{30}$$

where $\rho_1 = \rho_{M1} + \rho_{R1}$.

Subtracting (16) from (30), we have

$$\Delta\phi_2 = \begin{cases} \dfrac{4\lambda\chi\rho_{M1}\rho_{M2}}{L_1L_2} \\ \qquad \text{if } \dfrac{(\lambda\rho_{R2} - \chi\rho_{R1})}{2\lambda} \leq \rho_{M2}, \\ \dfrac{\lambda(2\rho_{M2} - \rho_{R2}) + \chi\rho_1}{\lambda(2\rho_{M2} + \rho_{R2}) - \chi\rho_1} \\ \qquad \text{if } \rho_{M2} < \dfrac{(\lambda\rho_{R2} - \chi\rho_{R1})}{2\lambda} \leq \rho_{M2} + \dfrac{\chi\rho_{M1}}{2\lambda}, \\ 0 \qquad \text{else}, \end{cases} \tag{31}$$

where $L_1 = 2\lambda\rho_{M2} + \lambda\rho_{R2} - \chi\rho_1$ and $L_2 = 2\lambda\rho_{M2} + \lambda\rho_{R2} - \chi\rho_{R1}$.

We can prove that (31) is nonnegative, which implies that manufacturer 2 would increase his participation rate to the retailer when the retailer integrates with manufacturer 1.

Further, substituting the optimal participation rate into (25), we see that the retailer's equilibrium local advertising level for product 2 is constant, that is,

$$U_{R2}^{(2)} = \begin{cases} \lambda\rho_{R2} - \chi\rho_1 & \text{if } \rho_{M2} < \dfrac{(\lambda\rho_{R2} - \chi\rho_1)}{2\lambda}, \\ \lambda\rho_{M2} + \dfrac{1}{2}\lambda\rho_{R2} - \dfrac{1}{2}\chi\rho_1 & \text{if } 0 < \dfrac{(\lambda\rho_{R2} - \chi\rho_1)}{2\lambda} \leq \rho_{M2}, \\ 0 & \text{else}, \end{cases} \tag{32}$$

where $\rho_1 = \rho_{M1} + \rho_{R1}$.

Subtracting (17) from (32), we have

$$\Delta U_{R2}=\begin{cases}-\chi\rho_{M1} & \text{if } 2\lambda\rho_{M2}+\chi\rho_{M1}<\lambda\rho_{R2}-\chi\rho_{R1}\\ \dfrac{2\lambda\rho_{M2}-(\lambda\rho_{R2}-\chi\rho_{R1})-\chi\rho_{M1}}{2} & \text{if } 2\lambda\rho_{M2}<\lambda\rho_{R2}-\chi\rho_{R1}\le 2\lambda\rho_{M2}+\chi\rho_{M1}\\ -\dfrac{\chi\rho_{M1}}{2} & \text{if } \chi\rho_{M1}<\lambda\rho_{R2}-\chi\rho_{R1}\le 2\lambda\rho_{M2}\\ -\lambda\rho_{M2}-\dfrac{1}{2}\lambda\rho_{R2}+\dfrac{1}{2}\chi\rho_{R1} & \text{if } 0<\lambda\rho_{R2}-\chi\rho_{R1}\le\chi\rho_{M1}\\ 0 & \text{else.}\end{cases} \tag{33}$$

Note that the result of (33) is less than zero; the intuition behind this can be explained as follows. When the retailer integrates with manufacturer 1, the retailer would always reduce the local advertising efforts for product 2 to decrease the competitive influence on product 1.

3.3. The Two Manufacturers Are Horizontally Integrated. When the two manufacturers integrated, it can be seen as a single firm with two different brands in the same product category. Examples in practice include Lenovo. IBM's personal computing division was acquired by Lenovo in 2004, and the PC of IBM became a subbrand of Lenovo Group named "Thinkpad." This is a historical precedent of two manufacturers behaving as a single player, yet, as far as we know, previous researches on dynamic cooperative advertising programs have never studied such scenario, a single manufacturer with two different brands. Most previous research investigated a "single-manufacturer single-retailer" supply chain with a single brand/product. When the manufacturer advertises two different brands, the result does change; therefore, the third scenario must be considered. In this scenario, the integration system's profit function is $\pi_{M1,M2} = \pi_{M1} + \pi_{M2}$, so the objective is

$$\max_{\substack{U_{M1}\ge 0, U_{M2}\ge 0\\ 1\ge\phi_i\ge 0}} J_{M1,M2} = \int_0^{+\infty} e^{-rt}\left(\pi_{M1}+\pi_{M2}\right)dt, \tag{34}$$

and the retailer's objective is

$$\max_{U_{R1}\ge 0, U_{R2}\ge 0} J_R = \int_0^{+\infty} e^{-rt}\pi_R dt. \tag{35}$$

The current value Hamiltonian of the integration system ($M1$ and $M2$) is

$$\begin{aligned}H_{M1,M2} &= \pi_{M1}+\pi_{M2}+\nu_{11}\left(U_{M1}-\theta U_{M2}-\delta G_1\right)\\ &\quad+\nu_{12}\left(U_{M2}-\theta U_{M1}-\delta G_2\right),\end{aligned} \tag{36}$$

and that of the retailer is

$$\begin{aligned}H_R &= \pi_R+\nu_{21}\left(U_{M1}-\theta U_{M2}-\delta G_1\right)\\ &\quad+\nu_{22}\left(U_{M2}-\theta U_{M1}-\delta G_2\right),\end{aligned} \tag{37}$$

where ν_{i1} and ν_{i2} ($i = 1, 2$) are the costate variables to the firm's goodwill levels.

Using the necessary conditions for equilibrium, we get the following results.

Proposition 9. *When the two manufacturers are horizontally integrated and the participation rate ϕ_i ($i = 1, 2$) is kept fixed, the equilibrium national advertising efforts for the two manufacturers are all constants, that is,*

$$U_{Mi}^{(3)}=\begin{cases}\dfrac{\alpha\left(\rho_{Mi}-\theta\rho_{M(3-i)}\right)}{r+\delta} & \textit{if } \rho_{Mi}\ge\theta\rho_{M(3-i)},\\ 0 & \textit{else}\end{cases} \qquad i\in\{1,2\}, \tag{38}$$

and the retailer's equilibrium local advertising efforts for the two products are all constants, that is,

$$U_{Ri}^{(3)}=\begin{cases}\dfrac{\lambda\rho_{Ri}-\chi\rho_{R(3-i)}}{1-\phi_i} & \textit{if } \lambda\rho_{Ri}-\chi\rho_{R(3-i)}\ge 0,\\ 0 & \textit{else}\end{cases} \qquad i\in\{1,2\}. \tag{39}$$

Note that the retailer's equilibrium local advertising efforts given by (39) are just the same as Proposition 2. This result implies that whether the two manufacturers integrate with each other or not, the retailer always keeps the same local advertising efforts for products 1 and 2 only if the participation rates are not changed.

In addition, comparing (38) with the results of Proposition 1, we have

$$\Delta U_{Mi}=\begin{cases}-\dfrac{\alpha\theta\rho_{M3-i}}{r+\delta} & \text{if } \rho_{Mi}\ge\theta\rho_{M,3-i},\\ -\dfrac{\alpha\rho_{M1}}{r+\delta} & \text{else}\end{cases} \qquad i\in\{1,2\}. \tag{40}$$

Equation (40) illustrates the following fact. When the two manufacturers integrate as a horizontal alliance, they would decrease their equilibrium advertising efforts to avoid internal conflict. Specifically, combing the conditions of (38) we have $U_{Mi} = 0$ and $U_{M,(3-i)} = (\alpha\rho_{M3-i} - \alpha\theta\rho_{Mi})/(r + \delta)$ if the condition $\rho_{Mi} < \theta\rho_{M,3-i}$ holds. This implies that when the marginal profit of one product for the manufacturer is rather low, the horizontal integration system would invest in national advertising only for the other product.

Proposition 10. *When the two manufacturers are horizontally integrated and all channel members' advertising efforts are kept as constants, that is, $U_{Mi} = U_{Mi}^{(3)}$ and $U_{Ri} = U_{Ri}^{(3)}$, then the accumulated goodwill for the two products along time t is*

$$G_i(t) = F_i e^{-\delta t} + G_{iSS}^{(3)}, \quad i\in\{1,2\}, \tag{41}$$

where $G_{iSS}^{(3)} = (U_{Mi}^{(3)} - \theta U_{M,(3-i)}^{(3)})/\delta$, $F_i = G_{i0} - (U_{Mi}^{(3)} - \theta U_{M,(3-i)}^{(3)})/\delta$, $i = 1, 2$. $G_{iSS}^{(3)}$ *is the steady-state goodwill for product i when* $t \to \infty$.

Substituting (38), (39), and (41) into (34) and (35) and with the participation rates ϕ_i $(i = 1, 2)$ fixed, we get that the current value of profit for the horizontal integration system is

$$J_{M1,M2}^{(3)} = \frac{F_1\alpha\rho_{M1} + F_2\alpha\rho_{M2}}{r+\delta} + \frac{\alpha\rho_{M1}\left(U_{M1}^{(3)} - \theta U_{M2}^{(3)}\right) + \alpha\rho_{M2}\left(U_{M2}^{(3)} - \theta U_{M1}^{(3)}\right)}{r\delta} + \frac{\rho_{M1}\left(\lambda U_{R1}^{(3)} - \chi U_{R2}^{(3)}\right)}{r} + \frac{\rho_{M2}\left(\lambda U_{R2}^{(3)} - \chi U_{R1}^{(3)}\right)}{r} - \frac{\left(U_{M1}^{(3)}\right)^2 + \left(U_{M2}^{(3)}\right)^2 + \phi_1\left(U_{R1}^{(3)}\right)^2 + \phi_2\left(U_{R2}^{(3)}\right)^2}{2r}, \tag{42}$$

and the current value of the profit for the retailer is

$$J_{R}^{(3)} = \frac{F_1\alpha\rho_{R1} + F_2\alpha\rho_{R2}}{r+\delta} + \frac{\alpha\rho_{R1}\left(U_{M1}^{(3)} - \theta U_{M2}^{(3)}\right) + \alpha\rho_{R2}\left(U_{M2}^{(3)} - \theta U_{M1}^{(3)}\right)}{r\delta} - \frac{1-\phi_1}{2}\left(U_{R1}^{(3)}\right)^2 + \frac{\rho_{R1}\left(\lambda U_{R1}^{(3)} - \chi U_{R2}^{(3)}\right)}{r} + \frac{\rho_{R2}\left(\lambda U_{R2}^{(3)} - \chi U_{R1}^{(3)}\right)}{r} - \frac{1-\phi_2}{2}\left(U_{R2}^{(3)}\right)^2, \tag{43}$$

where $F_i = G_{i0} - (U_{Mi}^{(3)} - \theta U_{M,(3-i)}^{(3)})/\delta$, $i = 1, 2$.

Differentiating $J_{M1,M2}^{(3)}$ with the participation rate ϕ_1 and ϕ_2, we get the optimal participation rates:

$$\phi_i^{(3)} = \begin{cases} \dfrac{\lambda\left(2\rho_{Mi} - \rho_{Ri}\right) + \chi\rho_{R(3-i)}}{\lambda\left(2\rho_{Mi} + \rho_{Ri}\right) - \chi\rho_{R(3-i)}} & \text{if } \rho_{Mi} \geq \dfrac{\left(\lambda\rho_{Ri} - \chi\rho_{R(3-i)}\right)}{2\lambda} \\ 0 & \text{else,} \end{cases} \quad i \in \{1, 2\}. \tag{44}$$

Note that the above expressions of participation rates are identical with the results of Proposition 4. Together with the results of Proposition 9, we find that the equilibrium local advertising efforts for the two products are identical no matter whether two manufacturers are horizontally integrated or not.

In this scenario, the equilibrium advertising efforts for the two manufacturers become lower, but the equilibrium local advertising efforts for the two products are not changed. This could lead to the phenomenon that the retailer has so much power from advertising the two products that the retailer has incentive to prevent the horizontal alliance between the two manufacturers. That is why a successful manufacturer's horizontal integration in a dominant retailer market is very rare in actual practice.

4. Numerical Analysis

In this section, we use numerical analysis to further illustrate the impact of local advertising competition on the profits for all channel members and supplement insights from these theoretical results. In our numerical analysis, we use the following values to establish ranges for model parameters: $\alpha = 12$, $r = 0.3$, $\delta = 0.2$, $\theta = 0.2$, $\lambda = 10$, $\chi = 4$, $\rho_{M1} = 7$, $\rho_{M2} = 8$, $\rho_{R1} = 5$, $\rho_{R2} = 4$, $G_{10} = 300$, and $G_{20} = 320$.

To obtain qualitative insight regarding how the current value of each channel member's profit varies as competition coefficients θ and χ vary in scenario 1, we keep other parameters fixed and draw their relationships in Figure 2.

Figure 2 suggests that, in scenario 1, the profit for each channel member decreases as competition coefficients θ and χ increase. From Figure 2, if competition coefficients θ and χ equal zero, advertising for one product would not adversely influence the sales of the other product. In this situation, all channel members would obtain the maximum profits. As the competition effects of advertising become intense, the advertising effect on sales would scale down, and the profits for all channel members would decrease.

Figure 3 illustrates the effect of the competition coefficients θ and χ on the current value of the retailer's profit in scenario 1 and scenario 3, keeping other parameters fixed.

From Figure 3, we obtain the following facts. (i) When two manufacturers are horizontally integrated, the retailer's profit would decrease compared with his profit in scenario 1. Because the retailer would obtain a larger impact on the sales of products in scenario 3, the retailer would have incentive to prevent the horizontal alliance between manufacturers. (ii) Similarly to Figure 2, as competition coefficients θ and χ increase, the retailer's profit would decrease whether the two manufacturers integrate or not.

Finally, Figure 4 illustrates the impacts of the competition coefficients θ and χ on the current value of the profit of manufacturer 2. It suggests the following tendencies: (i) similarly to Figure 2, with competition coefficients θ and χ increasing, the profit for manufacturer 2 would decrease whether manufacturer 1 integrates with the retailer or not and (ii) when manufacturer 1 integrates with the retailer, the profit for manufacturer 2 would decrease compared to his profit in scenario 1. From Figures 3 and 4, we find that regardless of which two firms (i.e., $M1$ and retailer, $M2$ and retailer, or $M1$ and $M2$) integrate their efforts, the third firm would suffer.

5. Conclusion

Previous research primarily focused on a "single-manufacturer single-retailer" framework, whereas few studies address a "multiple-manufacturer single-retailer" framework. To fill this gap, this paper investigates the advertising strategies for a "two-manufacturer single-retailer" supply chain in

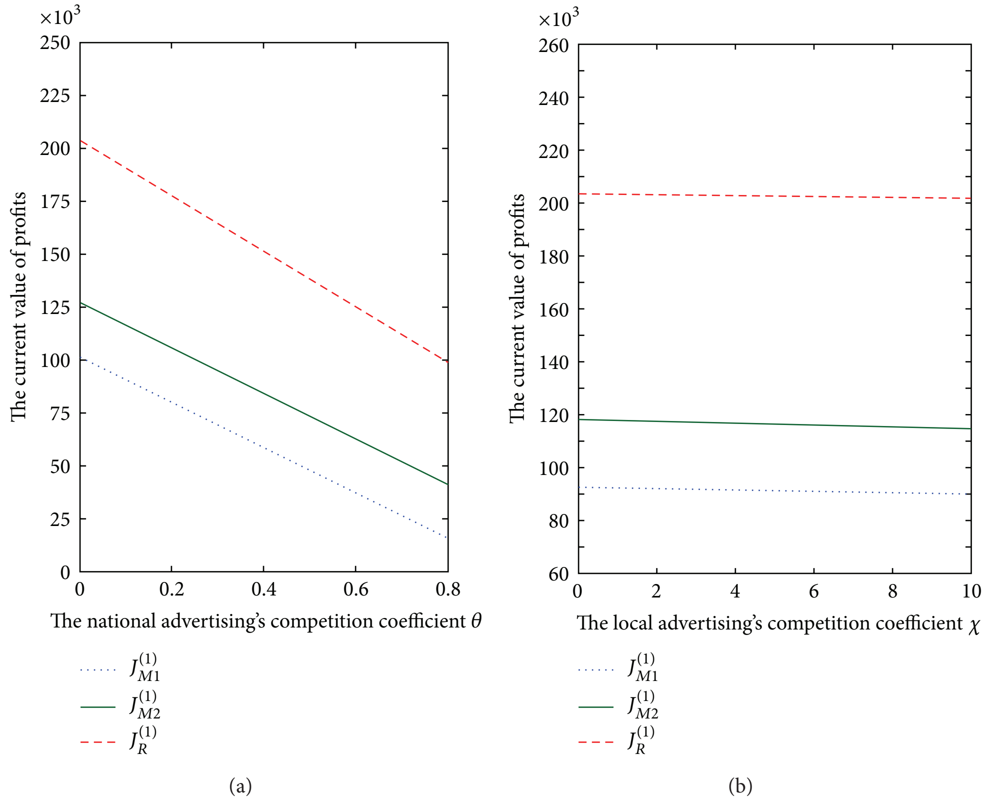

Figure 2: Relationships between profits and competition coefficients θ and χ.

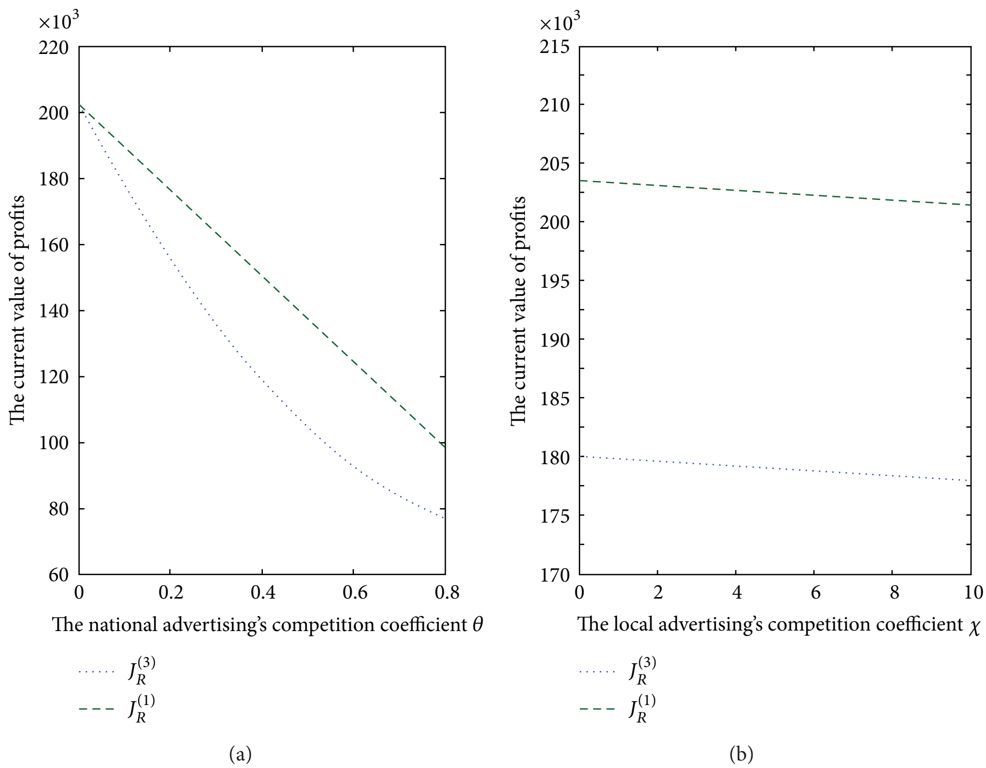

Figure 3: Relationships between the retailer's profit and the competition coefficients θ and χ.

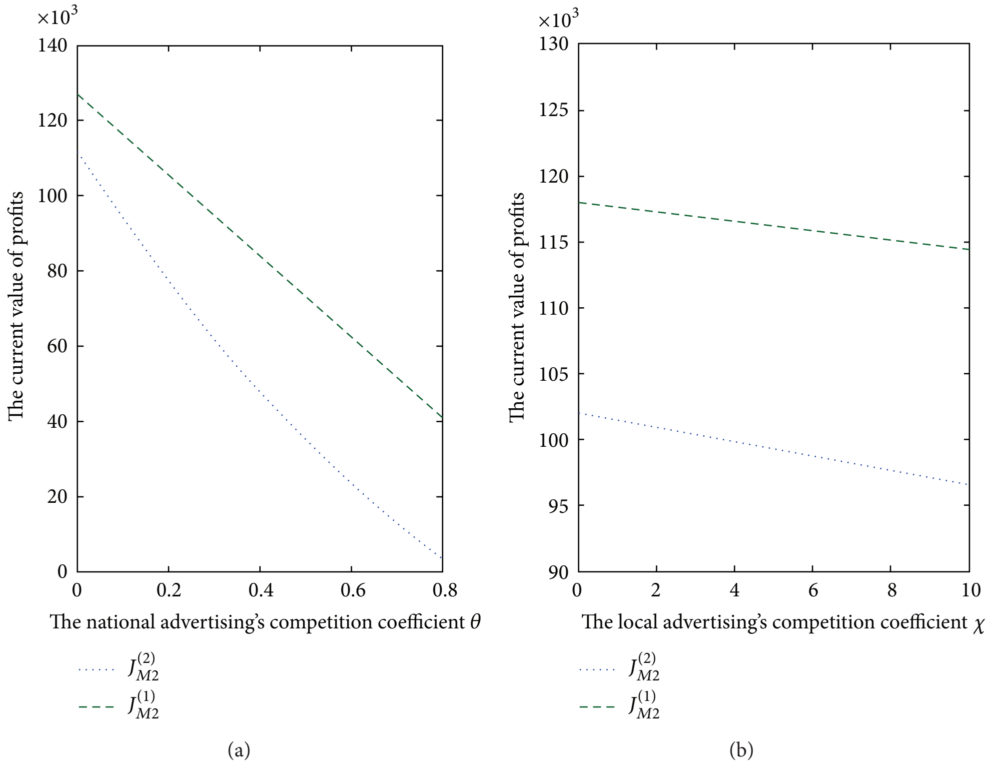

FIGURE 4: Relationships between the profit of $M2$ and the competition coefficients θ and χ.

three different scenarios: (i) each channel member makes decisions independently; (ii) the retailer integrates with one of the manufacturers; (iii) two manufacturers are horizontally integrated.

Based on the results of the three scenarios, we find the following results. (i) The manufacturer's equilibrium advertising efforts are independent of the participation rates that the two manufacturers offer to the retailer in all three scenarios. (ii) When the retailer integrates with one manufacturer, the other manufacturer's equilibrium advertising efforts would not be changed. The retailer would enhance the local advertising efforts for the integrated manufacturer and reduce the local advertising efforts for the other manufacturer. In response, the other manufacturer would offer a higher (compared to scenario 1) advertising cost participation rate to the retailer. (iii) When the two manufacturers are horizontally integrated, they would reduce the national advertising efforts to avoid internal conflict. They also offer the same advertising cost participation rate to the retailer as in scenario 1. (iv) If any two firms (i.e., $M1$ and retailer, $M2$ and retailer, or $M1$ and $M2$) are integrated, the profit of the third firm would decrease. All these insights provide important implications and guidelines for cooperative advertising program design in supply chain practice.

It should be noted that our models only consider the effects of advertising, but this situation may not always hold. In addition, it may be more interesting if we introduce the factors of pricing and quality to the cooperative advertising model. Additionally, our work on the "two-manufacturer single-retailer" framework can be extended into a "multiple-manufacturer single-retailer" framework.

Appendices

A. Each Channel Member Makes Decisions Independently

Proof of Propositions 1 and 3. When each channel member makes decisions independently, the current value Hamiltonian of manufacturer 1 is

$$\begin{aligned} H_{M1} &= \pi_{M1} + \mu_{11}\left(U_{M1} - \theta U_{M2} - \delta G_1\right) \\ &\quad + \mu_{12}\left(U_{M2} - \theta U_{M1} - \delta G_2\right). \end{aligned} \tag{A.1}$$

Then we form the Lagrangian:

$$\begin{aligned} L_{M1} &= \pi_{M1} + \mu_{11}\left(U_{M1} - \theta U_{M2} - \delta G_1\right) \\ &\quad + \mu_{12}\left(U_{M2} - \theta U_{M1} - \delta G_2\right) \\ &\quad + \eta_{11}\left(\alpha G_1 + \lambda U_{R1} - \chi U_{R2}\right) \\ &\quad + \eta_{12}\left(\alpha G_2 + \lambda U_{R2} - \chi U_{R1}\right), \end{aligned} \tag{A.2}$$

the necessary conditions for equilibrium are given by

$$\frac{\partial L_{M1}}{\partial U_{M1}} = 0, \quad (A.3)$$

$$\dot{\mu}_{11} = r\mu_{11} - \frac{\partial L_{M1}}{\partial G_1}, \quad (A.4)$$

$$\dot{\mu}_{12} = r\mu_{12} - \frac{\partial L_{M1}}{\partial G_2}, \quad (A.5)$$

$$\frac{\partial L_{M1}}{\partial \eta_{1i}} > 0, \quad \eta_{1i} \geq 0, \quad \eta_{1i}\frac{\partial L_{M1}}{\partial \eta_{1i}} = 0, \quad i = 1, 2. \quad (A.6)$$

Equation (A.3) implies

$$U_{M1} = \mu_{11} - \theta\mu_{12}. \quad (A.7)$$

Solving (A.4)–(A.6), we get

$$\begin{aligned}\dot{\mu}_{11} &= (r+\delta)\,\mu_{11} - \alpha\rho_{M1} - \alpha\eta_1,\\ \dot{\mu}_{12} &= (r+\delta)\,\mu_{12} - \alpha\eta_2.\end{aligned} \quad (A.8)$$

Equation (A.6) implies: $\eta_{1i} = 0$, then substituting $\eta_{1i} = 0$ into (A.8), we get

$$\begin{aligned}\dot{\mu}_{11} &= (r+\delta)\,\mu_{11} - \alpha\rho_{M1},\\ \dot{\mu}_{12} &= (r+\delta)\,\mu_{12}.\end{aligned} \quad (A.9)$$

Differentiating (A.7) with respect to time and substituting for μ_{11}, μ_{12} and their time derivative in (A.9), we get

$$\dot{U}_{M1} = (r+\delta)\,U_{M1} - \alpha\rho_{M1}. \quad (A.10)$$

Solving (A.10) to get the time paths of U_{M1}, we find

$$U_{M1}(t) = C_1 e^{(r+\delta)t} + \frac{\alpha\rho_{M1}}{(r+\delta)}. \quad (A.11)$$

Because there is no constraint at $t \to \infty$, U_{M1} should satisfy the free-boundary condition:

$$\lim_{t\to\infty} U_{M1}(t) < \infty. \quad (A.12)$$

Condition (A.12) implies that $C_1 = 0$. Therefore, we obtain the equilibrium advertising effort for manufacturer 1 as follows:

$$U_{M1}^{(1)} = \frac{\alpha\rho_{M1}}{r+\delta}. \quad (A.13)$$

Similarly considering manufacturer 2's profit maximizing problem, we obtain the equilibrium advertising level for manufacturer 1 as follows:

$$U_{M2}^{(1)} = \frac{\alpha\rho_{M2}}{r+\delta}. \quad (A.14)$$

For (1), we can get the general solutions of (1) as

$$G_i(t) = D_i e^{-\delta t} + G_{iSS}, \quad i \in \{1, 2\}, \quad (A.15)$$

where $G_{iSS} = (U_{Mi} - \theta U_{M,(3-i)})/\delta$, $i = 1, 2$.

D_i is an arbitrary constant. Letting $t = 0$ in (A.15) and utilizing the initial conditions of (2), we get $D_i = G_{i0} - (U_{Mi} - \theta U_{M,(3-i)})/\delta$, $i = 1, 2$.

Substituting (A.13) and (A.14) into (A.15), we find that

$$G_i(t) = D_i e^{-\delta t} + G_{iSS}^{(1)}, \quad i \in \{1, 2\}, \quad (A.16)$$

where $D_i = G_{i0} - (U_{Mi}^{(1)} - \theta U_{M,(3-i)}^{(1)})/\delta$, $i = 1, 2$ and $G_{iSS}^{(1)} = (U_{Mi}^{(1)} - \theta U_{M,(3-i)}^{(1)})/\delta$, $i = 1, 2$, when condition $0 \leq \theta \leq \min\{\rho_{M1}/\rho_{M2}, \rho_{M2}/\rho_{M1}\}$ holds, the steady-state goodwill G_{iSS} is nonnegative. □

Proof of Proposition 2. When each channel member makes decisions independently, the current value Hamiltonian of the retailer is:

$$\begin{aligned}H_R = {} & \rho_{R1}(\alpha G_1 + \lambda U_{R1} - \chi U_{R2})\\ & + \rho_{R2}(\alpha G_2 + \lambda U_{R2} - \chi U_{R1})\\ & - \frac{1}{2}(1-\phi_1)U_{R1}^2 - \frac{1}{2}(1-\phi_2)U_{R2}^2\\ & + \mu_{31}(U_{M1} - \theta U_{M2} - \delta G_1)\\ & + \mu_{32}(U_{M2} - \theta U_{M1} - \delta G_2).\end{aligned} \quad (A.17)$$

Then we form the Lagrangian

$$\begin{aligned}L_R = {} & \rho_{R1}(\alpha G_1 + \lambda U_{R1} - \chi U_{R2})\\ & + \rho_{R2}(\alpha G_2 + \lambda U_{R2} - \chi U_{R1})\\ & - \frac{1}{2}(1-\phi_1)U_{R1}^2 - \frac{1}{2}(1-\phi_2)U_{R2}^2\\ & + \mu_{31}(U_{M1} - \theta U_{M2} - \delta G_1)\\ & + \mu_{32}(U_{M2} - \theta U_{M1} - \delta G_2)\\ & + \eta_{31}(\alpha G_1 + \lambda U_{R1} - \chi U_{R2})\\ & + \eta_{32}(\alpha G_2 + \lambda U_{R2} - \chi U_{R1}).\end{aligned} \quad (A.18)$$

The necessary conditions for equilibrium are given by

$$\frac{\partial L_R}{\partial U_{R1}} = 0, \quad (A.19)$$

$$\frac{\partial L_R}{\partial U_{R2}} = 0, \quad (A.20)$$

$$\dot{\mu}_{31} = r\mu_{31} - \frac{\partial L_R}{\partial G_1}, \quad (A.21)$$

$$\dot{\mu}_{32} = r\mu_{32} - \frac{\partial L_R}{\partial G_2}, \quad (A.22)$$

$$\frac{\partial L_R}{\partial \eta_{3i}} > 0, \quad \eta_{3i} \geq 0, \quad \eta_{3i}\frac{\partial L_R}{\partial \eta_{3i}} = 0, \quad i = 1, 2. \quad (A.23)$$

Because U_{Ri} is constrained to be nonnegative, (A.19) implies that

$$U_{R1}(t) = \max\left\{0, \frac{(\lambda\rho_{R1} - \chi\rho_{R2})}{(1-\phi_1)}\right\}. \quad (A.24)$$

Through a similar proof, we find that

$$U_{R2}(t) = \max\left\{0, \frac{(\lambda\rho_{R2} - \chi\rho_{R1})}{(1-\phi_2)}\right\}. \quad (A.25)$$

Therefore we can obtain the following results:

$$U_{Ri}^{(1)} = \begin{cases} \dfrac{\lambda\rho_{Ri} - \chi\rho_{R(3-i)}}{1-\phi_i} & \text{if } \lambda\rho_{Ri} - \chi\rho_{R(3-i)} \geq 0, \\ 0 & \text{else,} \end{cases} \quad (A.26)$$

$$i \in \{1, 2\}.$$

□

Proof of Proposition 4. There are no relationships between participation rate ϕ_1 and $U_{R2}^{(1)}$ or the two manufacturer's national advertising efforts. Thus in differentiating $J_{M1}^{(1)}$ from the participation rate ϕ_1 we only consider two situations. Situation (1) when $\lambda\rho_{R1} - \chi\rho_{R2} \geq 0$ holds, $U_{R1}^{(1)} = (\lambda\rho_{R1} - \chi\rho_{R2})/(1-\phi_1)$, substituting the above expression into (20) and differentiating J_{M1} with the participation rate ϕ_1, we get manufacturer 1's optimal participation rate: $\phi_1 = [\lambda(2\rho_{M1} - \rho_{R1}) + \chi\rho_{R2}]/[\lambda(2\rho_{M1} + \rho_{R1}) - \chi\rho_{R2}]$. Since $0 \leq \phi_1 \leq 1$, the condition $2\lambda\rho_{M1} \geq (\lambda\rho_{R1} - \chi\rho_{R2})$ is required. Situation (2) if condition $(\lambda\rho_{R1} - \chi\rho_{R2}) < 0$ holds, we get $U_{R1}^{(1)} = 0$. In this situation, whatever participation rate manufacturer 1 offers, the retailer would never advertise product 1. The participation rate ϕ_1 is an arbitrary constant. We suppose $\phi_1 = [\lambda(2\rho_{M1} - \rho_{R1}) + \chi\rho_{R2}]/[\lambda(2\rho_{M1} + \rho_{R1}) - \chi\rho_{R2}]$. The participation rate ϕ_1 is useless in this situation; therefore, the participation rate could be negative.

In conclusion, we get the following results:

$$\phi_1^{(1)} = \begin{cases} \dfrac{\lambda(2\rho_{M1} - \rho_{R1}) + \chi\rho_{R2}}{\lambda(2\rho_{M1} + \rho_{R1}) - \chi\rho_{R2}} & \text{if } 2\lambda\rho_{M1} \geq \lambda\rho_{R1} - \chi\rho_{R2} \\ 0 & \text{else.} \end{cases} \quad (A.27)$$

Through a similar proof, we get manufacturer 2's optimal share rate is

$$\phi_2^{(1)} = \begin{cases} \dfrac{\lambda(2\rho_{M2} - \rho_{R2}) + \chi\rho_{R1}}{\lambda(2\rho_{M2} + \rho_{R2}) - \chi\rho_{R1}} & \text{if } 2\lambda\rho_{M2} \geq \lambda\rho_{R2} - \chi\rho_{R1} \\ 0 & \text{else.} \end{cases} \quad (A.28)$$

□

B. The Retailer Is Vertically Integrated with One Manufacturer

Proof of Proposition 5. When the retailer integrates with a manufacturer, the current value Hamiltonian of manufacturer 2 is

$$\begin{aligned} H_{M2} = {} & \rho_{M2}(\alpha G_2 + \lambda U_{R2} - \chi U_{R1}) \\ & - \frac{1}{2}U_{M2}^2 - \frac{1}{2}\phi_2 U_{R2}^2 \\ & + \gamma_{21}(U_{M1} - \theta U_{M2} - \delta G_1) \\ & + \gamma_{22}(U_{M2} - \theta U_{M1} - \delta G_2). \end{aligned} \quad (B.1)$$

Then we form the Lagrangian:

$$\begin{aligned} L_{M2} = {} & \rho_{M2}(\alpha G_2 + \lambda U_{R2} - \chi U_{R1}) - \frac{1}{2}U_{M2}^2 - \frac{1}{2}\phi_2 U_{R2}^2 \\ & + \gamma_{21}(U_{M1} - \theta U_{M2} - \delta G_1) \\ & + \gamma_{22}(U_{M2} - \theta U_{M1} - \delta G_2) \\ & + \xi_{21}(\alpha G_1 + \lambda U_{R1} - \chi U_{R2}) \\ & + \xi_{22}(\alpha G_2 + \lambda U_{R2} - \chi U_{R1}). \end{aligned} \quad (B.2)$$

At optimality, the necessary conditions are

$$\begin{gathered} \frac{\partial L_{M2}}{\partial U_{M2}} = 0, \\ \dot{\gamma}_{21} = r\gamma_{21} - \frac{\partial L_{M1}}{\partial G_1}, \\ \dot{\gamma}_{22} = r\gamma_{22} - \frac{\partial L_{M1}}{\partial G_2}, \\ \frac{\partial L_{M2}}{\partial \xi_{2i}} > 0, \quad \xi_{2i} \geq 0, \quad \xi_{2i}\frac{\partial L_{M2}}{\partial \xi_{2i}} = 0, \quad i = 1, 2. \end{gathered} \quad (B.3)$$

Proceeding as in the proof for Proposition 1, we get

$$U_{M2}^{(2)} = \frac{\alpha\rho_{M2}}{(r+\delta)}. \quad (B.4)$$

□

Proof of Propositions 6 and 7. When the retailer integrates with a manufacturer, the current value Hamiltonian for integration system is given by

$$\begin{aligned} H_{M1,R} = {} & \pi_{M1} + \pi_R + \gamma_{11}(U_{M1} - \theta U_{M2} - \delta G_1) \\ & + \gamma_{12}(U_{M2} - \theta U_{M1} - \delta G_2). \end{aligned} \quad (B.5)$$

Then we form the Lagrangian:

$$\begin{aligned} L_{M1,R} = {} & \pi_{M1} + \pi_R + \gamma_{11}(U_{M1} - \theta U_{M2} - \delta G_1) \\ & + \gamma_{12}(U_{M2} - \theta U_{M1} - \delta G_2) \\ & + \xi_{11}(\alpha G_1 + \lambda U_{R1} - \chi U_{R2}) \\ & + \xi_{12}(\alpha G_2 + \lambda U_{R2} - \chi U_{R1}). \end{aligned} \quad (B.6)$$

Proceeding as in the proof for Proposition 1, and constraining,as in most cases, the advertising efforts $U(t)$ to be nonnegative, we get the following results:

$$U_{R1}^{(2)} = \max\{0, \lambda(\rho_{M1}+\rho_{R1}) - \chi\rho_{R2}\},$$
$$U_{R2}^{(2)} = \max\left\{0, \frac{\lambda\rho_{R2}-\chi(\rho_{R1}+\rho_{M1})}{1-\phi_2}\right\}. \tag{B.7}$$

Thus, the equilibrium advertising levels for manufacturer 2 are as follows:

$$U_{M1}^{(2)} = \begin{cases} \dfrac{\alpha(\rho_{M1}+\rho_{R1})}{r+\delta} - \dfrac{\theta\alpha\rho_{R2}}{r+\delta} & \text{if } (\rho_{M1}+\rho_{R1}) \ge \theta\rho_{R2} \\ 0 & \text{else.} \end{cases} \tag{B.8}$$

Also, we obtain the equilibrium local advertising levels for the two products:

$$U_{R1}^{(2)} = \begin{cases} \lambda(\rho_{M1}+\rho_{R1}) - \chi\rho_{R2} & \text{if } \lambda\rho_1 - \chi\rho_{R2} \ge 0 \\ 0 & \text{else,} \end{cases}$$
$$U_{R2}^{(2)} = \begin{cases} \dfrac{\lambda\rho_{R2}-\chi(\rho_{R1}+\rho_{M1})}{1-\phi_2} & \text{if } \lambda\rho_{R2} - \chi\rho_1 \ge 0 \\ 0 & \text{else.} \end{cases} \tag{B.9}$$

Substituting (B.4) and (B.8) into (A.15), we find that

$$G_i(t) = E_i e^{-\delta t} + G_{iSS}^{(2)}, \quad i \in \{1,2\}, \tag{B.10}$$

where $E_i = G_{i0} - (U_{Mi}^{(2)} - \theta U_{M,(3-i)}^{(2)})/\delta$, $i = 1,2$ and $G_{iSS}^{(2)} = (U_{Mi}^{(2)} - \theta U_{M,(3-i)}^{(2)})/\delta$, $i = 1,2$.

When condition $0 \le \theta \le \min\{\rho_1/\rho_2, (\rho_1 - \sqrt{\rho_1^2 - 4\rho_{M2}\rho_{R2}})/2\rho_{R2}\}$ holds, the steady-state goodwill $G_{iSS}^{(2)}$ is nonnegative. □

Proof of Proposition 8. There are no relationships between participation rate ϕ_2 and $U_{R2}^{(2)}$ or two manufacturer's national advertising efforts. Thus in differentiating $J_{M2}^{(2)}$ from the participation rate ϕ_2 we only consider two situations. (1) When $\lambda\rho_{R2} - \chi\rho_1 \ge 0$ holds, we get $U_{R2}^{(2)} = [\lambda\rho_{R2} - \chi(\rho_{R1} + \rho_{M1})]/(1-\phi_2)$. Substituting the above expression into (38), and differentiating J_{M2} from the participation rate ϕ_2, we get manufacturer 2's optimal participation rate. $\phi_2 = [\lambda(2\rho_{M2} - \rho_{R2}) + \chi\rho_1]/[\lambda(2\rho_{M2}+\rho_{R2}) - \chi\rho_1]$. Since $0 \le \phi_2 \le 1$, the assumption $\rho_{M2} \ge (\lambda\rho_{R2} - \chi\rho_1)/2\lambda$ is required. (2) When condition $(\lambda\rho_{R2} - \chi\rho_{R1}) < 0$ holds, we get $U_{R2}^{(2)} = 0$. In this situation, whatever participation rate manufacturer 2 offers, the retailer would never advertise product 2. Thus, ϕ_2 is an arbitrary constant. Here we suppose: $\phi_2 = [\lambda(2\rho_{M2} - \rho_{R2}) + \chi\rho_1]/[\lambda(2\rho_{M2}+\rho_{R2}) - \chi\rho_1]$. The participation rate ϕ_1 is useless in this situation; therefore the participation rate could be negative.

In conclusion, we get the following results:

$$\phi_2^{(2)} = \begin{cases} \dfrac{\lambda(2\rho_{M2}-\rho_{R2}) + \chi\rho_1}{\lambda(2\rho_{M2}+\rho_{R2}) - \chi\rho_1} & \text{if } \rho_{M2} \ge \dfrac{(\lambda\rho_{R2}-\chi\rho_1)}{2\lambda} \\ 0 & \text{else.} \end{cases} \tag{B.11}$$

□

C. Two Manufacturers Are Horizontally Integrated

Proof of Proposition 9. When the two manufacturers are horizontally integrated, the current value Hamiltonian for the integration system is given by

$$\begin{aligned} H_{M1,M2} = {} & \rho_{M1}(\alpha G_1 + \lambda U_{R1} - \chi U_{R2}) \\ & + \rho_{M2}(\alpha G_2 + \lambda U_{R2} - \chi U_{R1}) \\ & - \frac{1}{2}U_{M1}^2 - \frac{1}{2}(1-\phi_1)U_{R1}^2 \\ & - \frac{1}{2}U_{M2}^2 - \frac{1}{2}(1-\phi_2)U_{R2}^2 \\ & + \nu_{11}(U_{M1} - \theta U_{M2} - \delta G_1) \\ & + \nu_{12}(U_{M2} - \theta U_{M1} - \delta G_2). \end{aligned} \tag{C.1}$$

Then we form the Lagrangian

$$\begin{aligned} L_{M1,M2} = {} & \rho_{M1}(\alpha G_1 + \lambda U_{R1} - \chi U_{R2}) \\ & + \rho_{M2}(\alpha G_2 + \lambda U_{R2} - \chi U_{R1}) \\ & - \frac{1}{2}U_{M1}^2 - \frac{1}{2}(1-\phi_1)U_{R1}^2 \\ & - \frac{1}{2}U_{M2}^2 - \frac{1}{2}(1-\phi_2)U_{R2}^2 \\ & + \nu_{11}(U_{M1} - \theta U_{M2} - \delta G_1) \\ & + \nu_{12}(U_{M2} - \theta U_{M1} - \delta G_2) \\ & + \vartheta_{11}(\alpha G_1 + \lambda U_{R1} - \chi U_{R2}) \\ & + \vartheta_{12}(\alpha G_2 + \lambda U_{R2} - \chi U_{R1}). \end{aligned} \tag{C.2}$$

At optimality, the necessary conditions are

$$\frac{\partial L_{M1,M2}}{\partial U_{M1}} = 0, \tag{C.3}$$

$$\dot{\nu}_{11} = r\nu_{11} - \frac{\partial L_{M1,M2}}{\partial G_1}, \tag{C.4}$$

$$\dot{\nu}_{12} = r\nu_{12} - \frac{\partial L_{M1,M2}}{\partial G_2}, \tag{C.5}$$

$$\frac{\partial L_{M1,M2}}{\partial \vartheta_{1i}} > 0, \quad \vartheta_{1i} \ge 0, \quad \vartheta_{1i}\frac{\partial L_{M1,M2}}{\partial \vartheta_{1i}} = 0, \quad i = 1,2. \tag{C.6}$$

In most case the advertising effort U_{M1} is constrained to be nonnegative. Thus, (C.3) implies

$$U_{M1} = \max\{0, \beta\rho_{M1} + \nu_{11} - \theta\nu_{12}\}. \tag{C.7}$$

Solving (C.4)-(C.5), we get

$$\begin{aligned} \dot{\nu}_{11} &= (r+\delta)\nu_{11} - \alpha\rho_{M1} - \alpha\vartheta_{11}, \\ \dot{\nu}_{12} &= (r+\delta)\nu_{12} - \alpha\rho_{M2} - \alpha\vartheta_{12}. \end{aligned} \tag{C.8}$$

Equation (C.6) implies, $\vartheta_{1i} = 0$; then substituting $\vartheta_{1i} = 0$ into (C.8), we get

$$\dot{\nu}_{11} = (r+\delta)\,\nu_{11} - \alpha\rho_{M1}, \tag{C.9}$$

$$\dot{\nu}_{12} = (r+\delta)\,\nu_{12} - \alpha\rho_{M2}. \tag{C.10}$$

Differentiating (C.7) with respect to time and substituting for ν_{11}, ν_{12} and their time derivative in (C.9)-(C.10), we get

$$\dot{U}_{M1} = (r+\delta)\,U_{M1} - \alpha\rho_{M1} + \theta\alpha\rho_{M2}. \tag{C.11}$$

Proceeding as in the proof of Proposition 1, we get

$$\begin{aligned} U_{M1}^{(3)} &= \max\left\{0, \frac{\alpha\rho_{M1}}{(r+\delta)} - \frac{\theta\alpha\rho_{M2}}{(r+\delta)}\right\}, \\ U_{M2}^{(3)} &= \max\left\{0, \frac{\alpha\rho_{M2}}{(r+\delta)} - \frac{\theta\alpha\rho_{M1}}{(r+\delta)}\right\}. \end{aligned} \tag{C.12}$$

Thus, the equilibrium advertising levels for two manufacturers are as follows:

$$U_{Mi}^{(3)} = \begin{cases} \dfrac{\alpha\left(\rho_{Mi} - \theta\rho_{M(3-i)}\right)}{r+\delta} & \text{if } \rho_{Mi} \ge \theta\rho_{M,3-i}, \\ 0 & \text{else} \end{cases} \qquad i \in \{1,2\}. \tag{C.13}$$

Substituting (C.13) into (A.15), we find that

$$G_i(t) = F_i e^{-\delta t} + G_{iSS}^{(3)}, \quad i \in \{1,2\}, \tag{C.14}$$

where $F_i = G_{i0} - (U_{Mi}^{(3)} - \theta U_{M,(3-i)}^{(3)})/\delta$, $i = 1,2$ and $G_{iSS}^{(3)} = (U_{Mi}^{(3)} - \theta U_{M,(3-i)}^{(3)})/\delta$, $i = 1,2$. When condition $0 \le 2\theta/(1+\theta^2) \le \min\{\rho_{M1}/\rho_{M2}, \rho_{M2}/\rho_{M1}\}$ holds, the steady-state goodwill $G_{iSS}^{(3)}$ is nonnegative.

The current value Hamiltonian for the retailer is given by

$$\begin{aligned} H_R = {} & \rho_{R1}\left(\alpha G_1 + \lambda U_{R1} - \chi U_{R2}\right) \\ & + \rho_{R2}\left(\alpha G_2 + \lambda U_{R2} - \chi U_{R1}\right) - \frac{1}{2}\left(1-\phi_1\right)U_{R1}^2 \\ & - \frac{1}{2}\left(1-\phi_2\right)U_{R2}^2 + \nu_{21}\left(U_{M1} - \theta U_{M2} - \delta G_1\right) \\ & + \nu_{22}\left(U_{M2} - \theta U_{M1} - \delta G_2\right). \end{aligned} \tag{C.15}$$

Then we form the Lagrangian:

$$\begin{aligned} L_R = {} & \rho_{R1}\left(\alpha G_1 + \lambda U_{R1} - \chi U_{R2}\right) \\ & + \rho_{R2}\left(\alpha G_2 + \lambda U_{R2} - \chi U_{R1}\right) \\ & - \frac{1}{2}\left(1-\phi_1\right)U_{R1}^2 - \frac{1}{2}\left(1-\phi_2\right)U_{R2}^2 \\ & + \nu_{21}\left(U_{M1} - \theta U_{M2} - \delta G_1\right) \\ & + \nu_{22}\left(U_{M2} - \theta U_{M1} - \delta G_2\right) \\ & + \vartheta_{21}\left(\alpha G_1 + \lambda U_{R1} - \chi U_{R2}\right) \\ & + \vartheta_{22}\left(\alpha G_2 + \lambda U_{R2} - \chi U_{R1}\right). \end{aligned} \tag{C.16}$$

Proceeding as in the proof of Proposition 2, we get

$$\begin{aligned} U_{R1}^{(3)} &= \max\left\{0, \frac{\left(\lambda\rho_{R1} - \chi\rho_{R2}\right)}{\left(1-\phi_1\right)}\right\}, \\ U_{R2}^{(3)} &= \max\left\{0, \frac{\left(\lambda\rho_{R2} - \chi\rho_{R1}\right)}{\left(1-\phi_2\right)}\right\}. \end{aligned} \tag{C.17}$$

Then we get the following results:

$$U_{Ri}^{(3)} = \begin{cases} \dfrac{\lambda\rho_{Ri} - \chi\rho_{R(3-i)}}{1-\phi_i} & \text{if } \lambda\rho_{Ri} - \chi\rho_{R(3-i)} \ge 0 \\ 0 & \text{else,} \end{cases} \qquad i \in \{1,2\}. \tag{C.18}$$

□

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Research Article

A Model for Assessing the Service Quality of University Library Websites

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Evaluating the e-service quality is essential to organizations. The future of e-libraries has a vital place in universities. Libraries need to use websites as means to provide access to information resources, news and events. The importance of assessing e-service quality of libraries is significant. Previous researchers have developed many methods for assessing e-service quality. However, most of them only focus on the independent hierarchical structure. In this paper, we would like to figure out the criteria for assessing the service quality of library websites from university students' viewpoints. According to interdependent criteria, the analytic network process (ANP) approach is employed to (i) generate the priority weights of each criterion; (ii) measure the service quality of university library websites. 12 web-based service criteria are identified according to 3144 university students' viewpoints based on fuzzy Delphi method. On the basis of past studies, we divide 12 criteria into 3 perspectives, namely, system, efficiency, and information quality to measure the service quality of university library websites. On the basis of 3 perspectives and 12 important criteria, service quality of university library websites could be measured more effectively. Moreover, the practical application to measure the service quality of the old and new versions of one university library website presented in Section 5 is generic and also suitable to be exploited for Taiwanese universities.

1. Introduction

The Internet has contributed a convenient and efficient channel for distributing information and services [1]. E-service is becoming increasingly important when determining quality of service delivery in organizations. Evaluating the e-service quality is essential to organizations [2]. Researchers start to investigate the e-service quality of different industries. Libraries are increasingly delivering collections and services electronically [3]. Yalman and Kutluca [4] point out that the Internet is one of the basic tools to provide library services. It is necessary for universities to adopt the concept of e-library and to consider electronic librarianship while restructuring their services they provided. The future of e-libraries has a vital place in universities. Besides providing information about library collection and facilities, libraries need to use websites as means to provide access to information resources, news, and events [5]. The importance of assessing e-service quality of libraries is significant. In this paper, we have identified the criteria to assess the service quality of library websites from university students' viewpoints based on fuzzy Delphi method. Additionally, the ANP approach is employed to generate the priority weights of each criterion and to measure the service quality of university library websites.

Library websites should provide students with real concern contents, information of interest as the premise of the user's demand. Libraries must move away from the view of the click rate transfer to pay attention to the needs of users [6]. As the result, 12 web-based service criteria are identified according to 3144 university students' viewpoints based on fuzzy Delphi method, including choices for searching for information according to users' preference, protection of users' personal information, availability of the website, promptness of taking care of problems, simple procedure of application, relevant content can be displayed for each item, downloading speed, promptness of search system response, accurateness of provided information, whether the website updates timely, whether the latest information is provided on the front page, and variety of electronic resources. On the basis of Hu [1], Parasuraman et al. [7], and Lee and Kozar [8], 12 criteria are taken into 3 perspectives to structure the hierarchy. Unlike previous researchers [8–11] who ignored the interdependence among criteria, this paper utilizes the ANP approach to capture the interdependence of the criteria as it appears to be one of the more feasible and accurate solutions for us to generate the priority weights of each criteria and to measure the service quality of university library websites. The result can provide a foundation for better understanding students' perceptions of e-service quality for libraries. This information is useful to universities libraries website designers and service providers and in the management of web-based services.

2. Service Quality of Website

For the purpose of this paper, the evaluating criteria are obtained by reviewing previous literatures as well as collecting students' opinions. The literatures, related to the evaluation of service quality, are described in the following. Hu [1] points out that traditional Likert scales cannot handle uncertain assessments according to human intuition for the service quality measurement, and so the fuzzy numbers is added and utilized to measure decision makers' subjective preferences. In the paper, Hu develops a genetic-algorithm-based approach to determine the importance of criteria for assessing the service quality of travel websites. Lin [10] reviews past studies to get 16 criteria for measuring the course website quality. Fuzzy analytic hierarchy process (AHP) is also applied to identify the relative weights of criteria between high and low online learning experience groups. Hu and Liao [11] employ fuzzy multiple criteria decision making approach to obtain important criteria for measuring service quality of Internet banking. Yu et al. [12] use AHP and fuzzy technique for order preference by similarity to ideal solution (TOPSIS) to rank business to consumer (B2C) e-commerce websites in e-alliance. Hsu et al. [13] proposed a process of algorithm that combined the consistent fuzzy preference relations method with ANP to evaluate e-service quality.

A set of initial criteria for measuring the service quality of university library websites are compiled from the abovementioned literatures. As the ANP approach needs more calculations and additional pairwise comparisons, the computing process would be complex if there are too many criteria [14, 15]. Moreover, some of the initial criteria are unrelated to evaluating library websites. The advantage of fuzzy Delphi method is its simplicity. All the expert opinions can be encompassed in one investigation. Hence, this method can create more effective criteria selection [16]. This paper adopts fuzzy Delphi method to identify the criteria for assessing the service quality of library websites.

3. Fuzzy Delphi Method

The Delphi method is a traditional forecasting approach that does not require large samples. It can be utilized to generate a professional consensus for complex topics [17]. The Delphi method suffers from low convergence expert opinions and more execution cost. Murry et al. [18] integrate Delphi method and fuzzy theory. Membership degree is applied to establish the membership function of each participant. Ishikawa et al. [19] also introduce fuzzy theory into Delphi method. Max-min and fuzzy integration algorithm is developed. Hus and Yang [20] apply a triangular fuzzy number to encompass expert opinions and establish a fuzzy Delphi method. The max and min values of expert opinions are taken as the 2 terminal points of triangular fuzzy numbers, and the geometric mean is taken as the membership degree of triangular fuzzy numbers to derive the statistical unbiased effect and avoid the impact of extreme values. Kuo and Chen [21] point out that the advantage of fuzzy Delphi method for collecting group decision is that every expert opinion can be considered and integrated to achieve the consensus of group decisions. Moreover, it reduces the time of investigation and the consumption of cost and time. Ma et al. [16] describe the advantage of fuzzy Delphi method is its simplicity. All the expert opinions can be encompassed in one investigation. Hence, this method can create more effective criteria selection. This paper applies fuzzy Delphi method to identify the criteria for assessing the service quality of library websites.

4. Method: Analytic Network Process

In recent years, various researchers have applied the ANP approach in many managerial areas. Liao and Chang [22] apply the ANP approach to select televised sportscasters for Olympic Games. Liao and Chang [23] combine the ANP approach and balanced scorecard (BSC) to select key capabilities of Taiwanese TV-shopping companies. Liao and Chang [24] apply the ANP approach to choose public relations personnel for Taiwanese hospitals. Lin [25] combines ANP with fuzzy preference programming (FPP) to select supplier and then allocates orders among the selected suppliers by multiobjective linear programming (MOLP). Oh et al. [26] apply ANP, and BSC to evaluate the feasibility of a new telecom service. They point out that ANP can get more realistic results. Wu et al. [27] combine ANP with conjoint analysis (CA) to simplify ANP for hospital policymakers making appropriate management policies. Wu et al. [28] apply ANP to select strategic alliance partners for the LCD industry. J. K. Chen and I. S. Chen [29] apply decision-making trial and evaluation laboratory (DEMATEL), fuzzy ANP and TOPSIS to develop a new innovation support system. Liao and Chang [30] combine ANP with BSC for measuring the managerial performance of TV companies. Liao et al. [31] select program suppliers for TV companies using ANP. Lin and Tsai [32] integrate ANP and TOPSIS to select locations for foreign direct investments in new hospitals in China. Tsai and Hsu [33] combine DEMATEL with ANP to select cost of quality models. Tseng [34] uses ANP, DEMATEL, and fuzzy set theory to obtain the relative weight of BSC factors for a university performance measurement. Yüksel and Dağdeviren [35] integrate fuzzy ANP and BSC to measure the performance of a manufacturing firm in Turkey. Liao et al. [36] use ANP and TOPSIS for assessing the performance of Taiwanese tour guides. Altuntas et al. [37] apply AHP and ANP to measure hospital service quality. Hu et al. [38] use ANP to evaluate the performance of Taiwan homestay industry. Hsu et al. [13] propose a process of algorithm that combined the consistent fuzzy preference relations method with ANP to evaluate e-service quality. They also point out that ANP is capable of addressing interdependent relationships among criteria. Kang et al. [39] apply fuzzy ANP and interpretive structural modeling (ISM) to select technologies for new product development (NPD).

From the previous literatures, we know that the ANP approach is widely applied in decision making. Compared with the AHP approach, the ANP approach is more accurate and feasible under interdependent situations. This is the reason we choose the ANP approach as our method for generating the priority weights of criteria and measuring the service quality of university library websites. The ANP approach is a comprehensive decision-making technique that captures the outcome of dependency between criteria [40]. The AHP approach serves as a starting point of the ANP approach. Priorities are established in the same way that they are in the AHP approach using pairwise comparisons. The weight assigned to each perspective and criterion may be estimated from the data or subjectively by decision makers. It would be desirable to measure the consistency of the decision makers' judgment. The AHP approach provides a measure through the consistency ratio which is an indicator of the reliability of the model. This ratio is designed in such a way that the values of the ratio exceeding 0.1 indicate inconsistent judgment [41].

5. Application

This section can be divided into 2 parts. First, the ANP approach is applied to generate the priority weights of criteria. Second, the ANP approach is employed to measure the service quality of the old and new versions of one university library website.

5.1. Generate the Priority Weights of Criteria. The ANP approach comprises the following steps [40].

Step 1 (model construction and problem structuring). The initial criteria for measuring the service quality of university library websites are compiled from the literatures mentioned in Section 2. Subsequently, the initial criteria are modified according to the opinions of university students using questionnaires developed based on Likert seven-point scale, with 1 as most unimportant and 7 as most important. Questionnaires are sent to 3144 university students to obtain their opinions about the importance of criteria. A demographic and behavioral characteristic profile of the respondents is shown in the appendix. Among the 3144 respondents, 1148 respondents (36.51%) are males and 1996 respondents are female (63.49%). About 19% of the respondents are at the graduate school level or above. Almost one-fourth of the respondents are from the College of Management, followed by the College of Social Sciences, the College of Liberal Arts, the College of Science, and the College of Life Science. 57 respondents (1.81%) access the library website more than 20 times per month. 7.63% of respondents use library website above 5 hours per week.

On the basis of fuzzy Delphi method, the geometric mean of each criterion is used to denote the consensus of the respondents' evaluation value of the criteria. According to the geometric mean values, top 12 web-based service criteria are identified, including choices for searching for information according to users' preference, protection of users' personal information, availability of the website, promptness of taking care of problems, simple procedure of application, relevant content can be displayed for each item, downloading speed, promptness of search system response, accurateness of provided information, whether the website updates timely, whether the latest information is provided on the front page, and variety of electronic resources. The contributors of the criteria are shown in Table 1. Based on Hu [1], Parasuraman et al. [7], and Lee and Kozar [8], this study proposes a hierarchy, illustrated in Figure 1, to evaluate the e-service quality of libraries in universities. Unlike previous researchers [8–11] who ignored the interdependence among criteria, in this paper, the ANP approach, which captures the interdependence, appears to be one of the more feasible and accurate solutions for us to generate the priority weights of each criteria and to measure the service quality of university library websites.

Only 44 students who access the library website more than 20 times per month and also use the library website above 5 hours per week are selected as respondents for the ANP questionnaires. We sent the ANP questionnaires to them by e-mail and personally. 34 ANP questionnaires are collected. Deducting questionnaires those consistency ratios exceeding 0.1, 25 ANP questionnaires are retained to generate the priority weights of criteria.

Step 2 (determine the perspectives and criteria weights). In this step, 25 students make a series of pairwise comparisons to establish the relative importance of perspectives. In these comparisons, a one-nine scale is applied to compare the 2 perspectives. The pairwise comparison matrix and the development of each perspective priority weight are shown in Table 2.

According to the interdependency of criteria, we apply pairwise comparisons again to establish the criteria relationships within each perspective. The eigenvector of the observable pairwise comparison matrix provide the criteria weights at this level, which will be used in the supermatrix. With respect to choices for searching for information according to users' preference, for example, a pairwise comparison within the system perspective can be shown in Table 3. According to this way, we can derive every criterion weight to obtain the supermatrix.

Table 1: Contributors of the criteria.

Criteria	Contributors
Choices for searching for information according to users' preference	[1, 13]
Protection of users' personal information	[1, 7–9, 11, 13]
Availability of the website	[1, 7]
Promptness of taking care of problems	[1, 2, 7, 11]
Simple procedure of application	[1–3, 10–13]
Relevant content can be displayed for each item	[1, 3]
Downloading speed	[3, 7, 13]
Promptness of search system response	[1–3, 10]
Accurateness of provided information	[1, 2, 8–11]
Whether the website updates timely	[10]
Whether the latest information is provided on the front page	Students proposed
Variety of electronic resources	[3, 8]

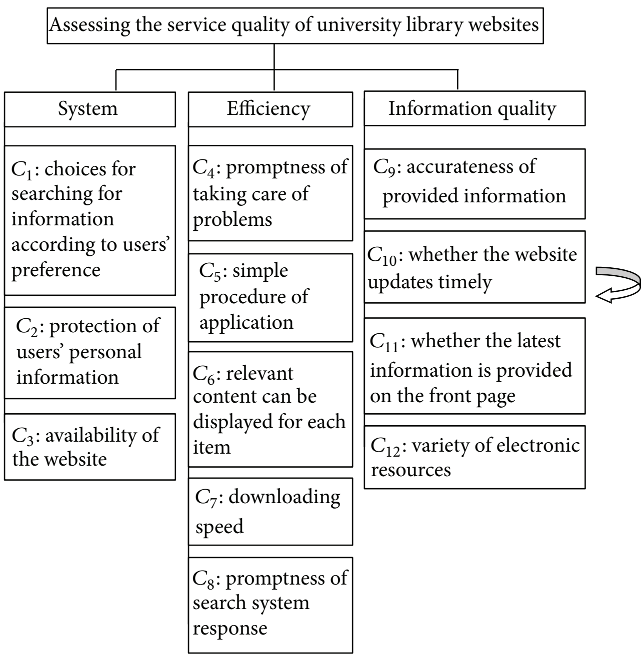

Figure 1: Hierarchy for assessing the service quality of university library websites.

Step 3 (construct and solve the supermatrix). An example of supermatrix is show in Figure 2. The components are C_k, $k = 1, \ldots, n$, and each component k has m_k elements, denoted by $e_{k1}, e_{k2}, \ldots, e_{kmk}$. The eigenvector obtained in Step 2 are grouped and located in appropriate positions in the supermatrix on the basis of the influences. The criteria weights derived from Step 2 are used to get the column of the supermatrix as shown in Table 4. Finally, the system solution is derived by multiplying the supermatrix of model variables by itself, which accounts for variable interaction, until the system's row values converge to the same value for each column of the matrix, as shown in Table 5.

$$
W = \begin{array}{cc} & \begin{array}{ccccc} C_1 & \cdots & C_k & \cdots & C_N \\ e_{11}\ \cdots\ e_{1n_1} & \cdots & e_{k1}\ \cdots\ e_{kn_k} & \cdots & e_{N1}\ \cdots\ e_{Nn_N} \end{array} \\
\begin{array}{c} C_1 \; \begin{array}{c} e_{11} \\ \vdots \\ e_{1n_1} \end{array} \\ \vdots \; \vdots \\ C_k \; \begin{array}{c} e_{k1} \\ \vdots \\ e_{kn_k} \end{array} \\ \vdots \; \vdots \\ C_N \; \begin{array}{c} e_{N1} \\ \vdots \\ e_{Nn_N} \end{array} \end{array} & \begin{bmatrix} W_{11} & \cdots & W_{1k} & \cdots & W_{1N} \\ \vdots & & \vdots & \ddots & \vdots \\ W_{k1} & \cdots & W_{kk} & \cdots & W_{kN} \\ \vdots & & \vdots & \ddots & \vdots \\ W_{N1} & \cdots & W_{Nk} & \cdots & W_{NN} \end{bmatrix} \end{array},
$$

Figure 2: An example of supermatrix.

According to Tables 2 and 5, we can aggregate the total weight of criteria as shown in Table 6. According to Table 6, protection of users' personal information is the most important factor to measure the service quality of library website, followed by choices for searching for information according to users' preference, availability of the website, accurateness of provided information, whether the website updates timely, promptness of taking care of problems, variety of electronic resources, simple procedure of application, whether the latest information is provided on the front page, promptness of search system response, relevant content can be displayed for each item, and downloading speed.

5.2. Measure the Service Quality of University Library Websites. In the second part, the ANP approach is applied to measure the service quality of the old and new versions of one university library website.

Step 1 (model construction and problem structuring). According to hierarchy, the service quality of old and new versions of one university library website is evaluated. We sent the ANP questionnaires to 50 graduate school students personally. Deducting the ANP questionnaires that the consistency ratio is exceeding 0.1, 20 ANP questionnaires are retained to assessing the service quality of library websites.

Step 2 (determine the perspectives and criteria weights). In this step, 20 graduate school students make a series of pairwise comparisons to establish the relative importance of perspectives. In these comparisons, a one-nine scale is applied to compare the 2 perspectives. The pairwise comparison matrix and the development of each perspective priority weight are shown in Table 7. According to the interdependency of criteria, we apply pairwise comparisons again to establish the criteria relationships within each perspective. The eigenvector of the observable pairwise comparison matrix provide the criteria weights at this level, which will be used in the supermatrix.

Step 3 (construct and solve the supermatrix). The criteria weights derived from Step 2 are used to get the column of the

Table 2: The pairwise comparisons of perspectives.

	System	Efficiency	Information quality	Weights
		λ_{max} = 3.0010 C.R. = 0.0007		
System	1.0000	1.0118	1.0481	0.3399
Efficiency	0.9883	1.0000	0.9456	0.3258
Information quality	0.9541	1.0576	1.0000	0.3343

Table 3: The pairwise comparisons within system perspective with respect to choices for searching for information according to users' preference.

	Protection of users' personal information	Availability of the website	Weights
	λ_{max} = 2.0000 C.R. = 0.0000		
Protection of users' personal information	1.0000	1.0524	0.5128
Availability of the website	0.9502	1.0000	0.4872

Table 4: The supermatrix before convergence.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}
C_1	0.0000	0.5046	0.4843									
C_2	0.5128	0.0000	0.5157									
C_3	0.4872	0.4954	0.0000									
C_4				0.0000	0.4018	0.3300	0.3206	0.2975				
C_5				0.3358	0.0000	0.3008	0.2648	0.2598				
C_6				0.2482	0.1830	0.0000	0.2121	0.2547				
C_7				0.1584	0.1763	0.1862	0.0000	0.1880				
C_8				0.2576	0.2389	0.1830	0.2026	0.0000				
C_9									0.0000	0.4345	0.3194	0.4406
C_{10}									0.3778	0.0000	0.3963	0.2941
C_{11}									0.2949	0.2348	0.0000	0.2653
C_{12}									0.3273	0.3307	0.2843	0.0000

Table 5: The supermatrix after convergence.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}
C_1	0.3309	0.3309	0.3309									
C_2	0.3396	0.3396	0.3396									
C_3	0.3295	0.3295	0.3295									
C_4				0.2549	0.2549	0.2549	0.2549	0.2549				
C_5				0.2282	0.2282	0.2282	0.2282	0.2282				
C_6				0.1836	0.1836	0.1836	0.1836	0.1836				
C_7				0.1494	0.1494	0.1494	0.1494	0.1494				
C_8				0.1840	0.1840	0.1840	0.1840	0.1840				
C_9									0.2870	0.2870	0.2870	0.2870
C_{10}									0.2624	0.2624	0.2624	0.2624
C_{11}									0.2101	0.2101	0.2101	0.2101
C_{12}									0.2405	0.2405	0.2405	0.2405

supermatrix as shown in Table 8. Finally, the system solution is derived by multiplying the supermatrix of model variables by itself, which accounts for variable interaction, until the system's row values converge to the same value for each column of the matrix, as shown in Table 9.

Step 4 (select the best alternative). The weight of each alternative with respect to the criteria is shown in Table 10. According to Tables 7, 9, and 10, we can aggregate the total weight of each alternative as shown in Table 11. Therefore, it is obvious that the e-service quality of new version is better than old one.

6. Conclusion

The importance of assessing e-service quality of libraries is significant. Researchers have developed many methods for

Table 6: The total weight of criteria.

	Weight of supermatrix after convergence	Perspective priority weight	Total weight
C_1	0.3309	0.3399	0.1125
C_2	0.3396	0.3399	0.1154
C_3	0.3295	0.3399	0.1120
C_4	0.2549	0.3258	0.0831
C_5	0.2282	0.3258	0.0743
C_6	0.1836	0.3258	0.0598
C_7	0.1494	0.3258	0.0487
C_8	0.1840	0.3258	0.0600
C_9	0.2870	0.3343	0.0960
C_{10}	0.2624	0.3343	0.0877
C_{11}	0.2101	0.3343	0.0702
C_{12}	0.2405	0.3343	0.0804

Table 7: The pairwise comparisons of perspectives.

	System	Efficiency	Information quality	Weights
		λ_{max} = 3.0047 C.R. = 0.0036		
System	1.0000	1.0845	1.1914	0.3623
Efficiency	0.9221	1.0000	0.8941	0.3119
Information quality	0.8394	1.1184	1.0000	0.3257

Table 8: The supermatrix before convergence.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}
C_1	0.0000	0.4063	0.4343									
C_2	0.4563	0.0000	0.5657									
C_3	0.5437	0.5937	0.0000									
C_4				0.0000	0.4276	0.3184	0.2582	0.2985				
C_5				0.2912	0.0000	0.3404	0.2756	0.2563				
C_6				0.2426	0.1826	0.0000	0.2348	0.2789				
C_7				0.1377	0.1532	0.1551	0.0000	0.1663				
C_8				0.3284	0.2366	0.1860	0.2314	0.0000				
C_9									0.0000	0.5189	0.3672	0.4794
C_{10}									0.3553	0.0000	0.4205	0.2999
C_{11}									0.2902	0.1663	0.0000	0.2207
C_{12}									0.3546	0.3148	0.2123	0.0000

Table 9: The supermatrix after convergence.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}
C_1	0.2961	0.2961	0.2961									
C_2	0.3406	0.3406	0.3406									
C_3	0.3632	0.3632	0.3632									
C_4				0.2511	0.2511	0.2511	0.2511	0.2511				
C_5				0.2257	0.2257	0.2257	0.2257	0.2257				
C_6				0.1894	0.1894	0.1894	0.1894	0.1894				
C_7				0.1321	0.1321	0.1321	0.1321	0.1321				
C_8				0.2017	0.2017	0.2017	0.2017	0.2017				
C_9									0.3168	0.3168	0.3168	0.3168
C_{10}									0.2616	0.2616	0.2616	0.2616
C_{11}									0.1872	0.1872	0.1872	0.1872
C_{12}									0.2344	0.2344	0.2344	0.2344

Table 10: The weight of each alternative with respect to criteria.

	New version of library website	Old version of library website
C_1	0.5993	0.4007
C_2	0.4453	0.5547
C_3	0.5000	0.5000
C_4	0.5547	0.4453
C_5	0.5689	0.4311
C_6	0.5000	0.5000
C_7	0.5346	0.4654
C_8	0.4453	0.5547
C_9	0.5000	0.5000
C_{10}	0.5255	0.4745
C_{11}	0.4453	0.5547
C_{12}	0.6322	0.3678

Table 11: The total weight of alternatives.

	Weight of supermatrix after convergence	Perspective priority weight	New version of library website	Old version of library website
C_1	0.2961	0.3623	0.0643	0.0430
C_2	0.3406	0.3623	0.0550	0.0685
C_3	0.3632	0.3623	0.0658	0.0658
C_4	0.2511	0.3119	0.0435	0.0349
C_5	0.2257	0.3119	0.0401	0.0304
C_6	0.1894	0.3119	0.0295	0.0295
C_7	0.1321	0.3119	0.0220	0.0192
C_8	0.2017	0.3119	0.0280	0.0349
C_9	0.3168	0.3257	0.0516	0.0516
C_{10}	0.2616	0.3257	0.0448	0.0404
C_{11}	0.1872	0.3257	0.0271	0.0338
C_{12}	0.2344	0.3257	0.0483	0.0281
	Total weight		0.5199	0.4801

assessing e-service quality. However, most of them only focus on the independent hierarchical structure. In this paper, we figure out the criteria for assessing the service quality of library websites from university students' viewpoints based on fuzzy Delphi method. The ANP approach is employed to generate the priority weights of criteria and to measure the service quality of university library websites. 12 web-based service criteria are identified according to 3144 university students' viewpoints, including choices for searching for information according to users' preference, protection of users' personal information, availability of the website, promptness of taking care of problems, simple procedure of application, relevant content can be displayed for each item, downloading speed, promptness of search system response, accurateness of provided information, whether the website updates timely, whether the latest information is provided on the front page, and variety of electronic resources. According to past studies, 12 criteria are taken into 3 perspectives to structure the hierarchy. Unlike previous researchers who ignored the interdependence among criteria, in this paper, the ANP approach capturing the interdependence appears to be one of the most feasible and accurate solutions for us to generate the priority weights of each criterion and measure the service quality of university library websites.

Finally, we find that protection of users' personal information is the most important factor to measure the service quality of library website, followed by choices for searching for information according to users' preference, availability of the website, accurateness of provided information, whether the website updates timely, promptness of taking care of problems, variety of electronic resources, simple procedure of application, whether the latest information is provided on the front page, promptness of search system response, relevant content can be displayed for each item, and downloading speed. The design of university library websites and services should be in accordance with characteristics of users. This result also helps universities libraries website designers and service providers to put extra emphasis on to maintain or increase students' perception of overall e-service quality. For example, university library websites should spend more effort on the ease of use of their websites for finding information and keeping user information safer. Library website should

Table 12: Demographic and behavioral characteristic profile of respondents ($n = 3144$).

Variable	N	%
Gender		
Male	1148	36.51
Female	1996	63.49
Age		
18–28	2626	83.52
29–39	471	14.98
40–49	47	1.50
Education		
University	2547	81.01
Graduate school	496	15.77
Ph.D. student	101	3.22
Field of Study		
College of Liberal Arts	378	12.02
College of Engineering	192	6.11
College of Medicine	141	4.48
College of Science	287	9.13
College of Electrical Engineering and Computer Science	194	6.17
College of Social Sciences	542	17.23
College of Management	753	23.95
College of Law	144	4.58
College of Life Science	247	7.86
College of Education	209	6.65
College of Bioresources and Agriculture	57	1.82
Frequency of library website visit per month		
<5	1941	61.74
5–10	564	17.93
11–15	412	13.11
16–20	170	5.41
More than 20	57	1.81
Hours of library website usage per week		
<1	1755	55.82
1–3	675	21.47
3.1–5	474	15.08
More than 5	240	7.63

get users the right information in minimal time and effort. Established online instructions can help users to search for the needed information. Provided easy access to a well-organized collection of information resources is also vital. When user assesses the library website, he or she approaches it by perceiving how easy it is to gain access to the service, in terms of the convenience of using the websites. Moreover, the practical application to measure the service quality of the old and new versions of one university library website presented in Section 5 is generic and also suitable to be exploited for Taiwanese universities.

The hierarchy proposed in this paper considers 12 critical criteria. We suggest that future research studies can incorporate more criteria in order to conduct more accurate estimates. Moreover, follow-up researchers could analyze this topic with the concept of fuzzy sets or combining ANP with other multiple criteria decision making (MCDM) approaches such as DEMATEL and TOPSIS.

Appendix

See Table 12.

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