Research Article

Data-Driven and Precursor-Group Uncertainty Propagation of Lattice Kinetic Parameters in UAM Benchmark

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A new data-driven sampling-based framework was developed for uncertainty quantification (UQ) of the homogenized kinetic parameters calculated by lattice physics codes such as TRITON and Polaris. In this study, extension of the database for the delayed neutron data (DND) is performed by exploring more delayed neutron experiments and adding additional isotopes/actinides to the data libraries. Afterwards, the framework is utilized to obtain a deeper knowledge of the kinetic parameters' sensitivity and uncertainty. The kinetic parameters include precursor-group-wise delayed neutron fraction (DNF) and decay constant. Input uncertainties include nuclear data (i.e., cross-sections) and DND (i.e., precursor group parameters and fractional delayed neutron yield). It is found that kinetic parameters, especially DNFs, have large uncertainties. The DNF uncertainty is driven by the cross-section uncertainties for LWR designs, while decay constant uncertainty is dominated by the DND uncertainties. The usage of correlated U-235 thermal DND in the UQ process significantly reduces the DND uncertainty contribution on the kinetic parameters. Large void fraction and presence of neutron absorber (e.g., control rod) increase the DNF uncertainty due to the hardening of neutron spectrum. High correlation between the DNF groups ($\beta_1, \dots, \beta_6$) is observed, while the decay constant groups ($\lambda_1, \dots, \lambda_6$) show weak correlation to each other and also to DNF groups. The DNF uncertainties of the dominant precursor group 4 for PWR, BWR, and VVER are about 7.5%, 9.4%, and 7.6%, respectively. The DNF uncertainty grows to larger values after fuel burnup. Kinetic parameters' values and uncertainties provided here can be efficiently used in subsequent core calculations, point reactor kinetics, and other applications.

1. Introduction

Uncertainty quantification (UQ) and sensitivity analysis (SA) are vital for assessing model's input and output. Uncertainty propagation is a typical UQ task that involves propagating the uncertainty in the model parameters into the model output. SA is the process of quantifying the effect of changing each input factor on the model's output. Both SA and UQ are used in tandem in various applications. SA can be classified in different categories such as local methods, regression-based methods, and variance-based methods [1, 2]. UQ can be implemented in different ways such as deterministic/sensitivity-based, stochastic/Monte Carlo-based, and reduced-order methods [3–5].

Nuclear reactor kinetic parameters describe the behavior of a specific type of neutrons inside the reactor called delayed neutrons. Delayed neutrons from their name are emitted later after the fission process by some fission product isotopes called delayed neutron precursors. Although delayed neutrons form small fraction (i.e., ~1%) compared to the prompt neutrons, they are very important for reactor control. Kinetic parameters have different classifications/definitions in different studies. However, most of the studies consider the delayed neutron fraction and the decay constant as the two main kinetic parameters to describe delayed neutrons. Delayed neutron fraction (DNF) expresses the fraction of delayed neutrons from the total number of neutrons emitted, while decay constant describes the timely decaying behavior of the isotope [6, 7]. These two quantities gained great

importance in the previous decades in which many experiments have been conducted to analyze the delayed neutron behavior emitted from the fission of different actinides.

Delayed neutron experiments started back to 1940s [8, 9]. The idea behind these experiments is to irradiate a sample of an actinide (e.g., U-235, U-238, and Pu-239) with thermal or fast neutrons causing fission in that sample. The experiments are usually performed on tiny samples (few grams) using high intensity neutron sources to prevent neutron multiplication within the sample. In addition, the irradiation time should be instantaneous to capture the short-lived precursors. Indeed, the last group of precursors is usually not reported in the experiments, due to the difficulty in measuring this group as well as the large uncertainty associated with its measurement. The delayed neutron activity is then monitored and analyzed to fit the precursor groups. If an exponential decay is assumed to represent the delayed neutron activity after a burst fission, then the response y (e.g., count rate) can be written with the independent time t as

$$y = \sum_{i=1}^N a_i e^{-\lambda_i t} \quad (1)$$

where N is the optimum number of periods/groups that minimizes the difference between the left and right hand sides. The parameters a_i and λ_i are the fitting parameters which can be determined by least-squares. The study by [6] is among the first studies in this area, which concluded that $N = 6$ is the optimum number of precursor groups. Following Keepin's work, several studies have been conducted to report the values of the fitting parameters for different actinides using the suggested 6-group Keepin's model. Examples of these experiments are provided in the next section. The 6-group model is not the only known model. Other models with 5, 7, and 8 groups have been investigated as well. The 8-group model suggested by Spriggs and Campbell [10, 11] gained more interest than the other two models (i.e., 5 and 7 groups). This model improves the time representation of the 6-group model by adding two additional groups and should reduce the uncertainty in the calculations. However, the 6-group model is still widely used due to the large number of experiments that have been performed to validate it compared to the recent 8-group model. Therefore, our focus will be on the 6-group Keepin's model.

A framework to propagate the uncertainty of delayed neutron data (DND) and neutron cross-sections into the homogenized kinetic parameters was developed previously by Radaideh et al. [12] in the SCALE code system [13]. The framework methodology is described in detail in that paper, where a set of DND for 7 actinides was collected and used in the UQ process. The framework was applied on a simplified PWR pin-cell geometry for demonstration and a comparison of different DND sets for U-235 was conducted. The framework was then utilized in different applications such as reduced order modeling with Gaussian processes [14] and variance decomposition using Sobol indices [15]. In this study, a deep and comprehensive analysis of the uncertainty of the homogenized precursor-group-wise kinetic parameters is presented. First, a set of DND uncertainties for 20

actinides is developed based on a rigorous review of the delayed neutron experiments. The new set contains data from a wide range of experiments and review studies. The uncertainty of the kinetic parameters is analyzed based on the combination of DND and neutron cross-section uncertainties in different aspects such as the effects of data type (DND vs. neutron cross-sections) and the effects of operating conditions such as void fraction, control rod, and gadolinium absorber. The impact of DND correlation on the uncertainty is investigated, which was neglected in the previous studies. Fuel depletion and burnup effects on the kinetic parameters' uncertainty are also explored. Finally, a set of kinetic parameters' values and uncertainties are suggested for applications (e.g., core calculations and accident analysis) based upon the lattice models in UAM benchmark [16]. Since this framework is developed mainly to analyze kinetic parameters in precursor-group form, the parameters that are explored are the DNF and the decay constant for the six precursor groups.

The remaining sections of this study are organized as follows: Section 2 presents a survey and data collection of the DND for different actinides, followed by the presentation of the mathematical formulation of the kinetic parameters and their relation to the cross-sections and DND. Afterwards, the tools and implementation of the UQ approach are described, along with a brief description of the selected lattice models used for testing purposes. Section 3 presents the results of this study which include the following: the impact of DND correlation matrix, sensitivity to operating conditions, and the burnup effect on the kinetic parameters' values and uncertainties, accompanied by a summary and discussion of the main findings. Finally, the conclusions of this study are presented in Section 4.

2. Materials and Methods

2.1. Delayed Neutron Data. In this subsection, DND parameters and their uncertainty are discussed. As mentioned in the introduction, the study performed by Keepin on the delayed neutrons of major actinides is considered one of the earliest and known studies in the area [6]. Most of the subsequent delayed neutron studies validated the results reported by Keepin. The reported delayed neutron parameters include group decay constant, group relative yield, and absolute delayed neutron yield from the fast fission of Th-232, U-233, U-235, U-238, Pu-239, and Pu-240 and thermal fission of U-233, U-235, and Pu-239. The experiment was conducted at Los Alamos National Laboratory using a bare U-235 metal assembly as the neutron source. Following Keepin's experiment, a plethora of experiments have been conducted to determine the delayed neutron yield and group parameters for different actinides, with a focus on U-235 DND parameters. For example, Cox et al. [17] conducted a study on Cf-252, Cox [18] analyzed the delayed neutron emission from Pu-241, Gudkov et al. [19] performed a delayed neutron study based on irradiating various actinide samples in a fast reactor, Loaiza et al. [20] measured the group parameters of Np-237 and U-235 from fast fission, and Saleh et al. [21] used the Texas A & M research reactor to study the delayed neutron

emission from thermal fission of U-235, Np-237, Am-241, and Am-243.

Tuttle [7] in his work reviewed the previous delayed neutron experiments that occurred before his study to report DND for the actinides with relevancy to reactor calculations. The study evaluated and revised a large number of previous experimental data and suggested a set of recommended values of DND. This makes Tuttle's work [7] a valuable source of DND to this study. It is worth mentioning that Tuttle's work relied extensively on Keepin's data, and both of these studies are still widely used in reactor physics applications. Other studies are selected for isotopes which are not reported by Tuttle/Keepin or if the data by Tuttle is not accurate due to the limited experiments available at that time for a particular isotope. Waldo et al. [25] reported group parameters for some isotopes that have been rarely studied such as U-232, Pu-238, and Cm-245. This study is also used in our data library. For other actinides at which there is no experimental data for their group parameters, data from Wilson and England [27] is used. The data reported by [27] is computational based that comes from simulating the activity of the precursors, following a burst fission of each actinide, and then using least-squares fit to calculate the group parameters. It is worth mentioning that such computational data is used here for isotopes that are less relevant to LWR applications such as Th-227, U-234, and Pa-231. First, we need to define the DND parameters of interest in this study as follows:

- (1) Group fractions ($a_{j,i}$): this parameter represents the relative abundance or fractional delayed neutron yield for the i^{th} precursor group that results from fission in isotope/actinide j . This parameter is expressed in normalized form (i.e., the sum of all precursor group fractions should be unity).
- (2) Group decay constant ($\lambda_{j,i}$): this parameter represents the effective decay constant of the precursor isotopes in the i^{th} precursor group that results from fission in isotope/actinide j . Group $i = 1$ is the longest-lived group, while group $i = 6$ contains the short-lived precursors.
- (3) Absolute delayed neutron yield ($\bar{\nu}_d$): this parameter expresses the average number of delayed neutrons emitted per fission (thermal or fast) in each actinide.
- (4) Fractional delayed neutron yield (Y): this parameter is the ratio of $\bar{\nu}_d$ to the average number of neutrons (prompt and delayed) emitted per fission in each actinide (i.e., $\bar{\nu}$).

The first three parameters, especially $\bar{\nu}_d$, are measured in delayed neutron experiments. The fourth parameter is not usually reported, since it requires $\bar{\nu}$, which is rarely reported in delayed neutron experiments. Consequently, a data source for $\bar{\nu}$ is needed. In brief, the criteria followed for dataset construction are as follows:

- (i) In general, the group parameters ($a_{j,i}$, $\lambda_{j,i}$) and the absolute delayed neutron yield for most of the fast fission data are taken from Tuttle [7] due to their high accuracy.

- (ii) The group parameters and the absolute delayed neutron yield for thermal fission data of U-233, U-235, and Pu-239 are taken from [6]. Indeed, Tuttle [7] suggested using Keepin's data for thermal fission on the basis of its quality.
- (iii) It is preferred to use the absolute delayed neutron yield from the same study as the group parameters, since the group fractions ($a_{j,i}$) are calculated (i.e., normalized) using the measured delayed neutron yield.
- (iv) If the isotope data is not available in either [7] or [6], a different source is used for the group parameters and the absolute delayed neutron yield.
- (v) If there is no experimental data available for the group parameters of a specific isotope, a computational-based data is selected from [27]. The computational data has no uncertainty and no effect on the UQ results.
- (vi) If there is no experimental data available for $\bar{\nu}_d$ for a specific isotope in the delayed neutron experiments, the value and its uncertainty are taken from SCALE data and covariance libraries which are based on ENDF-B/VII.1.
- (vii) All $\bar{\nu}$ and its uncertainty for all isotopes are taken from SCALE data and covariance libraries based on ENDF-B/VII.1.
- (viii) Exceptions to the previous points are minimal, and they are mentioned in the appropriate place in the text.

Based on the previous criteria, a total of 20 actinides and their DND are used in the framework. Table 1 lists the measured delayed neutron group parameters for the isotopes whose delayed neutrons are assigned to both thermal and fast sets. The data for the other isotopes whose either fast or thermal fission set is listed in Table 2 is in Appendix A. The reference used to obtain the data is reported for each isotope. Separating the data into thermal and fast fission sets is based on data availability and differences in $\bar{\nu}_d$ between thermal and fast fissions. According to Tuttle [7], it is recommended to use two sets if there is a considerable difference in $\bar{\nu}_d$ (see, e.g., U-235 in Table 3) emitted from a thermal and fast fission of an isotope. This approach is adopted only if there is experimental data that investigates the delayed neutron emission from fast and thermal fission of the same isotope. These two conditions are applicable only for the five isotopes in Table 1. The measured values of $\bar{\nu}_d$ are given in Table 3. As mentioned previously, all $\bar{\nu}$ values are taken from SCALE libraries for both fast and thermal ranges, except for Cf-252 spontaneous fission, which is obtained from [31]. In overall data tables shown in this study, "F" refers to fast fission, "T" to thermal fission, and "S" to spontaneous fission.

2.2. Mathematical Definitions. Modern lattice physics codes generate the needed kinetic parameters for a given material composition and for a given energy group. The calculation is usually performed by summation of the DND for each precursor isotope and then performs weighting by real/adjoint

TABLE 1: Measured delayed neutron group parameters for the isotopes whose delayed neutrons are assigned to both thermal and fast sets.

Isotope	No.	$a_{j,i}$ (F ^a)	$\lambda_{j,i}$ (F)	$a_{j,i}$ (T ^a)	$\lambda_{j,i}$ (T)	Source
U-233	1	0.086 ± 0.004	0.0126 ± 0.0006	0.086 ± 0.003	0.0126 ± 0.0001	F: [7]
	2	0.274 ± 0.007	0.0334 ± 0.0021	0.299 ± 0.004	0.0337 ± 0.0006	T: [6]
	3	0.227 ± 0.052	0.1310 ± 0.0070	0.252 ± 0.040	0.1386 ± 0.0058	
	4	0.317 ± 0.016	0.3020 ± 0.0360	0.278 ± 0.020	0.3254 ± 0.0306	
	5	0.073 ± 0.021	1.2700 ± 0.3900	0.051 ± 0.024	1.1271 ± 0.4435	
	6	0.023 ± 0.010	3.1300 ± 1.0000	0.034 ± 0.014	2.5023 ± 0.4246	
U-235	1	0.038 ± 0.004	0.0127 ± 0.0003	0.033 ± 0.003	0.0124 ± 0.0003	F: [7]
	2	0.213 ± 0.007	0.0317 ± 0.0012	0.219 ± 0.009	0.0305 ± 0.0010	T: [6]
	3	0.188 ± 0.024	0.1150 ± 0.0040	0.196 ± 0.022	0.1114 ± 0.0041	
	4	0.407 ± 0.010	0.3110 ± 0.0120	0.395 ± 0.011	0.3014 ± 0.0118	
	5	0.128 ± 0.012	1.4000 ± 0.1200	0.115 ± 0.009	1.1363 ± 0.1546	
	6	0.026 ± 0.004	3.8700 ± 0.5500	0.042 ± 0.008	3.0137 ± 0.3276	
Np-237	1	0.032 ± 0.003	0.0123 ± 0.0009	0.040 ± 0.002	0.0129 ± 0.0006	F: [20]
	2	0.238 ± 0.006	0.0284 ± 0.0005	0.233 ± 0.017	0.0324 ± 0.0010	T: [21]
	3	0.175 ± 0.008	0.0971 ± 0.0070	0.190 ± 0.010	0.1048 ± 0.0019	
	4	0.360 ± 0.017	0.2960 ± 0.0140	0.322 ± 0.027	0.3410 ± 0.0130	
	5	0.150 ± 0.014	0.9140 ± 0.0580	0.193 ± 0.007	0.8500 ± 0.0600	
	6	0.045 ± 0.006	3.2000 ± 0.1300	0.021 ± 0.004	3.2000 ± 0.1300 ^b	
Pu-239	1	0.038 ± 0.004	0.0129 ± 0.0003	0.035 ± 0.009	0.0128 ± 0.0006	F: [7]
	2	0.280 ± 0.006	0.0311 ± 0.0007	0.298 ± 0.035	0.0301 ± 0.0022	T: [6]
	3	0.216 ± 0.027	0.1340 ± 0.0040	0.211 ± 0.048	0.1238 ± 0.0088	
	4	0.328 ± 0.015	0.3310 ± 0.0180	0.326 ± 0.033	0.3254 ± 0.0367	
	5	0.103 ± 0.013	1.2600 ± 0.1700	0.086 ± 0.029	1.1216 ± 0.3866	
	6	0.035 ± 0.007	3.2100 ± 0.3800	0.044 ± 0.016	2.6971 ± 0.4722	
Am-243	1	0.017 ± 0.005	0.0124 ± 0.0036	0.024 ± 0.013	0.0131 ± 0.0002	F: [22]
	2	0.314 ± 0.013	0.0311 ± 0.0031	0.291 ± 0.011	0.0311 ± 0.0009	[23]
	3	0.205 ± 0.011	0.1510 ± 0.0020	0.221 ± 0.008	0.1070 ± 0.0070	[24]
	4	0.335 ± 0.011	0.3921 ± 0.0067	0.384 ± 0.016	0.3240 ± 0.0210	T: [21]
	5	0.088 ± 0.009	0.8955 ± 0.0405	0.051 ± 0.004	0.9140 ± 0.0310	
	6	0.041 ± 0.008	2.4493 ± 0.0433	0.029 ± 0.007	2.4493 ± 0.0433 ^b	

^a In this table: F refers to fast fission and T refers to thermal fission.

^b Due to lack of data for group 6, the measured data of the fast group is used in the thermal group.

neutron flux [12]. TRITON, which is a lattice physics code in the SCALE code system, calculates the adjoint/fission-weighted kinetic parameters for a given material composition using a 2-energy-group structure. The calculated kinetic parameters can then be used for core calculations. The fission-weighted precursor-group-wise DNF (β_i^{FW}) can be calculated using the nuclide-dependent DND of the precursor group i and then weighting by fission rate as follows:

$$\beta_i^{FW} = \frac{\sum_j \beta_{j,i} \int dr \int dE \bar{\nu} \Sigma_f^j \varphi(r, E)}{\sum_j \int dr \int dE \bar{\nu} \Sigma_f^j \varphi(r, E)}, \quad (2)$$

and in discretized form

$$\beta_i^{FW} = \frac{\sum_j \beta_{j,i} \sum_m \sum_g \bar{\nu} \Sigma_f^{j,m,g} \varphi_{m,g} V_m}{\sum_j \sum_m \sum_g \bar{\nu} \Sigma_f^{j,m,g} \varphi_{m,g} V_m}, \quad (3)$$

where the indices i, j, m , and g refer to the precursor group, nuclide in each precursor group, cell (node), and energy

group indices, respectively. The delayed neutron fraction of nuclide j related to the precursor group i is expressed by $\beta_{j,i}$, $\bar{\nu}$ is the average number of neutrons produced per fission, Σ_f is the fission cross-section, $\varphi(r, E)$ is the space and energy dependent neutron flux, $\varphi_{m,g}$ is the multigroup neutron flux, and V_m is the m^{th} cell volume. We can write $\beta_{j,i}$ as follows:

$$\beta_{j,i} = a_{j,i} Y_j, \quad (4)$$

where $a_{j,i}$ is the fractional group yield of the precursor group i in isotope j and Y_j is the fractional delayed neutron yield emitted due to fission in isotope j , which is defined as

$$Y_j = \frac{\bar{\nu}_d}{\bar{\nu}}. \quad (5)$$

As discussed in the previous section, the information available in the literature and data libraries provides uncertainties for $a_{j,i}$, $\bar{\nu}_d$, and $\bar{\nu}$ for various actinides. To account for the differences between the delayed and prompt neutron

TABLE 2: Delayed neutron group data for isotopes delayed neutrons are assigned to either thermal or fast fission set.

Isotope	No.	a_{ji}	λ_{ji}	Isotope	No.	a_{ji}	λ_{ji}
Th-227 (T)*	1	0.163 ± 0.000	0.0125 ± 0.0000	Th-229 (T)*	1	0.109 ± 0.000	0.0125 ± 0.0000
	2	0.264 ± 0.000	0.0308 ± 0.0000		2	0.223 ± 0.000	0.0315 ± 0.0000
	3	0.109 ± 0.000	0.0661 ± 0.0000		3	0.116 ± 0.000	0.0603 ± 0.0000
	4	0.283 ± 0.000	0.1995 ± 0.0000		4	0.295 ± 0.000	0.1997 ± 0.0000
	5	0.156 ± 0.000	0.4691 ± 0.0000		5	0.206 ± 0.000	0.5058 ± 0.0000
	6	0.024 ± 0.000	2.0235 ± 0.0000		6	0.052 ± 0.000	1.7652 ± 0.0000
Th-232 (F) [7]	1	0.034 ± 0.003	0.0124 ± 0.0003	Pa-231 (F)*	1	0.078 ± 0.000	0.0125 ± 0.0000
	2	0.150 ± 0.007	0.0334 ± 0.0016		2	0.213 ± 0.000	0.0307 ± 0.0000
	3	0.155 ± 0.031	0.1210 ± 0.007		3	0.112 ± 0.000	0.0591 ± 0.0000
	4	0.446 ± 0.022	0.3210 ± 0.016		4	0.304 ± 0.000	0.1877 ± 0.0000
	5	0.172 ± 0.019	1.2100 ± 0.1300		5	0.243 ± 0.000	0.4383 ± 0.0000
	6	0.043 ± 0.009	3.2900 ± 0.4400		6	0.052 ± 0.000	2.1462 ± 0.0000
U-232 (T) [25]	1	0.120 ± 0.013	0.0128 ± 0.0001	U-234 (F)*	1	0.054 ± 0.000	0.0125 ± 0.0000
	2	0.300 ± 0.032	0.0350 ± 0.0003		2	0.216 ± 0.000	0.0304 ± 0.0000
	3	0.307 ± 0.040	0.1439 ± 0.0059		3	0.102 ± 0.000	0.0690 ± 0.0000
	4	0.259 ± 0.034	0.3960 ± 0.0450		4	0.342 ± 0.000	0.2051 ± 0.0000
	5	0.016 ± 0.001	1.3500 ± 0.0000		5	0.219 ± 0.000	0.5090 ± 0.0000
	6	0.000 ± 0.000	0.0000 ± 0.0000		6	0.066 ± 0.000	2.2513 ± 0.0000
U-236 (F) [19]	1	0.027 ± 0.005	0.0125 ± 0.0000	U-238 (F) [7]	1	0.013 ± 0.001	0.0132 ± 0.0004
	2	0.193 ± 0.034	0.0338 ± 0.0000		2	0.137 ± 0.003	0.0321 ± 0.0009
	3	0.179 ± 0.040	0.1390 ± 0.0000		3	0.162 ± 0.030	0.1390 ± 0.0070
	4	0.431 ± 0.089	0.3310 ± 0.0000		4	0.388 ± 0.018	0.3580 ± 0.0210
	5	0.132 ± 0.028	1.0600 ± 0.0000		5	0.225 ± 0.019	1.4100 ± 0.1000
	6	0.037 ± 0.015	2.4900 ± 0.0000		6	0.075 ± 0.007	4.0200 ± 0.3200
Pu-238 (F) [25]	1	0.043 ± 0.009	0.0126 ± 0.0001	Pu-240 (F) [7]	1	0.028 ± 0.004	0.0129 ± 0.0006
	2	0.308 ± 0.068	0.0303 ± 0.0004		2	0.273 ± 0.006	0.0313 ± 0.0007
	3	0.115 ± 0.070	0.0851 ± 0.0120		3	0.192 ± 0.078	0.1350 ± 0.0160
	4	0.178 ± 0.040	0.1970 ± 0.0230		4	0.350 ± 0.030	0.3330 ± 0.0460
	5	0.328 ± 0.073	0.3560 ± 0.0510		5	0.128 ± 0.027	1.3600 ± 0.3000
	6	0.033 ± 0.005	$2.5582 \pm 0.0000*$		6	0.029 ± 0.009	4.0400 ± 1.1600
Pu-241 (T) [7]	1	0.010 ± 0.003	0.0128 ± 0.0002	Pu-242 (F) [7]	1	0.004 ± 0.001	0.0128 ± 0.0003
	2	0.229 ± 0.006	0.0299 ± 0.0006		2	0.195 ± 0.032	0.0314 ± 0.0013
	3	0.173 ± 0.025	0.1240 ± 0.0130		3	0.161 ± 0.048	0.1280 ± 0.0090
	4	0.390 ± 0.050	0.3520 ± 0.0180		4	0.412 ± 0.153	0.3250 ± 0.0200
	5	0.182 ± 0.019	1.6100 ± 0.1500		5	0.218 ± 0.087	1.3500 ± 0.9900
	6	0.016 ± 0.005	3.4700 ± 1.7000		6	0.010 ± 0.003	3.7000 ± 0.4400
Am-241 (T) [21]	1	0.036 ± 0.002	0.0122 ± 0.0005	Cm-245 (T) [25]	1	0.024 ± 0.002	0.1334 ± 0.0001
	2	0.309 ± 0.015	0.0318 ± 0.0016		2	0.302 ± 0.028	0.0303 ± 0.0001
	3	0.195 ± 0.008	0.1110 ± 0.0070		3	0.091 ± 0.029	0.1040 ± 0.0140
	4	0.331 ± 0.039	0.3000 ± 0.0170		4	0.294 ± 0.056	0.2110 ± 0.0110
	5	0.110 ± 0.005	0.8900 ± 0.0230		5	0.230 ± 0.031	0.5370 ± 0.0730
	6	0.018 ± 0.004	$2.4568 \pm 0.0000*$		6	0.059 ± 0.004	$2.8893 \pm 0.0000*$
Cf-252 (S) [26]	1		0.005 ± 0.001			$0.0125 \pm 0.0000*$	
	2		0.233 ± 0.029			0.0259 ± 0.0011	
	3		0.213 ± 0.038			0.1136 ± 0.0261	
	4		0.385 ± 0.049			0.3466 ± 0.0520	
	5		0.133 ± 0.027			1.3863 ± 0.2772	
	6		0.031 ± 0.008			$3.2396 \pm 0.0000*$	

*Computational data taken from [27].

TABLE 3: Measured values of absolute delayed neutron yield ($\bar{\nu}_d$) for the selected actinides plus uncertainty collected from different sources.

Isotope	$\bar{\nu}_d$ (F)	Source	$\bar{\nu}_d$ (T)	Source
Th-227	-	-	0.00769 ± 0.00115	SCALE [13]
Th-229	-	-	0.01621 ± 0.00243	SCALE [13]
Th-232	0.05470 ± 0.00120	[7]	-	-
Pa-231	0.01110 ± 0.00110	[27]	-	-
U-232	-	-	0.00437 ± 0.00033	[25]
U-233	0.00729 ± 0.00019	[7]	0.00664 ± 0.00018	[7]
U-234	0.01060 ± 0.00120	[7]	-	-
U-235	0.01650 ± 0.00075	[28]	0.01580 ± 0.00075	[28]
U-236	0.02310 ± 0.00260	[7]	-	-
U-238	0.04510 ± 0.00061	[7]	-	-
Np-237	0.01260 ± 0.00070	[21]	0.01290 ± 0.00040	[20]
Pu-238	0.00461 ± 0.00073	[25]	-	-
Pu-239	0.00664 ± 0.00013	[7]	0.00624 ± 0.00024	[7]
Pu-240	0.00960 ± 0.00110	[7]	-	-
Pu-241	-	-	0.01560 ± 0.00160	[7]
Pu-242	0.02280 ± 0.00250	[7]	-	-
Am-241	-	-	0.00490 ± 0.00020	[21]
Am-243	0.00860 ± 0.00050	[29]	0.00840 ± 0.00040	[21]
Cm-245	-	-	0.00592 ± 0.00039	[25]
Cf-252*	0.00812 ± 0.00053	[26]	-	-

*The data for Cf-252 is the delayed neutron yield resulting from spontaneous fission.

fission spectra, β^{FW} is multiplied by the so-called importance factor to obtain adjoint-weighted DNF (which is the main focus of this study). The adjoint-weighted DNFs are more accurate than the fission-weighted DNFs, where the adjoint flux is used as additional weighting function (i.e., the importance factor) for the fission-weighted DNF. To simplify the notation for the reader, we will use β_i ($i = 1, \dots, 6$) to refer to the adjoint-weighted group-wise DNF, which can be written as

$$\beta_i = \frac{\sum_j \beta_{j,i} \sum_m \sum_g \bar{\nu} \Sigma_f^{j,m,g} \varphi_{m,g} V_m \sum_{g'} \chi_d^{g',i} \varphi_{g'}^*}{\sum_j \sum_m \sum_g \bar{\nu} \Sigma_f^{j,m,g} \varphi_{m,g} V_m \sum_{g'} \chi^{g'} \varphi_{g'}^*}, \quad (6)$$

or in compact form

$$\beta_i = \beta_i^{FW} \frac{\sum_{g'} \chi_d^{g',i} \varphi_{g'}^*}{\sum_{g'} \chi^{g'} \varphi_{g'}^*}, \quad (7)$$

where g' is energy group index, $\chi^{g'}$ is the total fission spectrum for energy group g' , and $\chi_d^{g',i}$ is the delayed neutron fission spectrum that corresponds to precursor group i and the energy group g' . The homogenized decay constant for each precursor group (λ_i) is only weighted by fission rate and it can be calculated in a similar manner to the DNF in terms of nuclear data as follows:

$$\lambda_i = \frac{\sum_j \lambda_{j,i} \beta_{j,i} \sum_m \sum_g \bar{\nu} \Sigma_f^{j,m,g} \varphi_{m,g} V_m}{\beta_i \sum_j \sum_m \sum_g \bar{\nu} \Sigma_f^{j,m,g} \varphi_{m,g} V_m}, \quad (8)$$

where $\beta_{j,i}$ is indicated by (4) and $\lambda_{j,i}$ is the effective decay constant for delayed neutron group i after fission in nuclide j . Consequently, the group-wise kinetic parameter responses of interest to this study are classified into two main categories as follows:

- (1) Adjoint-weighted group-wise DNF, β_i , where $i = 1, 2, \dots, 6$
- (2) Fission-weighted group-wise decay constant, λ_i , where $i = 1, 2, \dots, 6$

In Monte Carlo or sampling-based UQ, for a general response u , the first and second statistical moments (mean, variance) can be calculated for n samples using the following relations:

$$\bar{u} = \frac{1}{n} \sum_{i=1}^n u_i, \quad (9)$$

$$\sigma_u^2 = \frac{1}{n-1} \sum_{i=1}^n (u_i - \bar{u})^2. \quad (10)$$

The correlation coefficient between any two responses (u, v) can be defined as

$$\rho_{u,v} = \frac{\sum_i (u_i - \bar{u})(v_i - \bar{v})}{\sqrt{\sum_i (u_i - \bar{u})^2} \sqrt{\sum_i (v_i - \bar{v})^2}}, \quad (11)$$

where $\bar{u}$ and $\bar{v}$ are the statistical mean of the variables u and v , respectively.

FIGURE 1: The uncertainty in $\bar{\nu}$ (left) and fission cross-section (right) based on the 56-group covariance library in the SCALE code system.

2.3. Implementation and Test Models. Based on the discussion so far, we found that determining the uncertainty in the kinetic parameters relies on two main components: (1) the mathematical formulation of the response and (2) propagating the uncertainty in the microscopic input data. The first component can be performed by a lattice physics code, while the second one can be performed by either sensitivity-based UQ or sampling-based UQ. In this study, we implemented our approach using the SCALE code system [13, 32], where TRITON calculates the kinetic parameters in the homogenized form as discussed in Section 2.2, and Sampler propagates the uncertainty of the DND and neutron cross-sections mentioned in Section 2.1. TRITON is a multipurpose lattice physics code in SCALE. TRITON can be used for both 2D and 3D modeling. For 2D, the T-DEPL sequence in TRITON couples the NEWT deterministic solver for neutron transport calculations with the depletion solver ORIGEN. The solution obtained by NEWT includes the real and adjoint neutron fluxes which can be used in weighting the kinetic parameters. And since TRITON performs depletion calculations, then the effect of fuel burnup on the kinetic parameters' values and uncertainties can be explored. TRITON can be coupled easily with other sequences to perform sensitivity and uncertainty analyses such as TSUNAMI-2D and Sampler. Therefore, TRITON (T-DEPL) is selected as the main neutronic code in this study.

Sampler is a super-sequence that performs uncertainty analysis of other SCALE sequences (e.g., TRITON) through statistical sampling. In Sampler, all model input parameters are stochastic and follow a random distribution, causing the responses of SCALE sequences to be stochastic as well.

Sampler supports two main categories of input uncertainty: (1) nuclear data uncertainties and (2) model (i.e., input file) parameters. Nuclear data uncertainties include microscopic cross-section uncertainties in continuous/multigroup form, fission yield, and decay data for depletion calculations. The uncertainty of nuclear data is represented by precomputed random perturbation factors, which are sampled from the SCALE covariance libraries (56-group or 252-group). The nuclear data is assumed to follow a multivariate normal distribution. A set of 1000 random perturbation factor libraries was generated by the SCALE development team and saved in the data directory. This reduces the computational time needed for sampling but limits the user access to the data in which more than 1000 samples cannot be used. However, most of the applications demonstrated that responses converge within 200-300 nuclear data samples. Examples of the fission cross-section and $\bar{\nu}$ uncertainties based on the 56-group covariance library are plotted in Figure 1 for relevant actinides. The second category of input uncertainty involves sampling the model parameters such as fuel density, isotope concentration, temperature, or any other input parameter in the SCALE input/model file. Unlike the first category, this category can be accessed by the user in which the distribution type (e.g., normal, uniform, and beta) and its parameters can be defined by the user as well as the number of samples.

In this work, we added a new subcategory to the nuclear data uncertainties, which consists of the DND uncertainties that include group fractions, group decay constant, and fractional delayed neutron yield. Similar to other nuclear data forms, 1000 randomly sampled libraries are generated based on univariate normal distribution. The random libraries

FIGURE 2: Uncertainty propagation of the nuclear and DND in the SCALE code system.

contain random DND for 20 actinides based on the assembled data in Section 2.1. No correlation is considered between the DND during sampling due to the limited information in literature about the DND correlation. We present a brief analysis on the impact of the DND correlation matrix on the uncertainty of the kinetic parameters in Section 3.1, but our base data does not consider correlation, as will be discussed later. However, it is worth mentioning that this DND correlation matrix should not be confused with the correlation resulting from the normalization condition of $a_{j,i}$. All $a_{j,i}$ values in both fast and thermal fission sets are renormalized after sampling to ensure their sum to unity, which introduces correlation between the $a_{j,i}$ groups.

The DND random libraries are written in human-readable format which means that the user can easily replace them by other data libraries generated using other DND sources or sampling methods. The user can increase the number of libraries/samples to larger than 1000, even though the convergence study reported in [12] showed that 200-300 random samples of DND are more than sufficient for convergence. The only restriction is that the user-generated libraries (name, directory, and format) should be consistent with the original libraries. Another advantage of this format is the ability to replace the globally sampled random libraries with point-wise perturbed libraries, which can be used for local sensitivity analysis of the kinetic parameters. The previous discussion on TRITON, Sampler, and DND is summarized in the flowchart in Figure 2.

For LWR applications, three sample cases of PWR, BWR, and VVER lattice designs are selected from the OECD/NEA Uncertainty Analysis in Modelling (UAM) Benchmark for design, operation, and safety analysis of LWRs [16]. The PWR design is based on the Three Mile Island (TMI-1) 15×15 lattice design. The specifications of PWR pin-cell and lattice geometries are shown in Figures 3(a) and 3(b). The BWR lattice is

based on the Peach Bottom 2 (PB-2) design which is a 7×7 lattice. The BWR lattice model is shown in Figure 3(c). The VVER lattice design is based on the Kozloduy-6 VVER-1000 hexagonal lattice. The VVER lattice model is shown in Figure 3(d). All of the prescribed designs are modeled in 2D using TRITON with reflective boundary conditions. The PWR pin-cell in Figure 3(a) is considered as a test model, which will be used in some sensitivity studies (unless mentioned otherwise). For burnup calculations, all lattice models are burned to 2000 days (~ 3 cycles) using power density of 33.6 kW/kgU for PWR, 25 kW/kgU for BWR, and 42.6 kW/kg for VVER. Additional details regarding the configuration, dimensions, and operating conditions for these designs can be found in the UAM benchmark report [16].

3. Results and Discussion

3.1. Impact of DND Correlation. Correlation between the group parameters is important for uncertainty propagation. Unfortunately, very limited resources are available for the correlation between the group parameters for actinides. Since correlation analysis requires repeating the experiment several times to infer the parametric correlation, a correlation matrix for the U-235 group parameters is reported by Loaiza and Haskin [30] and shown in Table 4. The correlation coefficients were calculated based on nonlinear least-squares fits to 11 pulsed irradiations of a highly enriched U-235 sample (93% enrichment) of Godiva. The correlation is significant between the group parameters, especially between the adjacent groups. The correlation can be highly positive (λ_3, λ_4) or highly negative (a_4, λ_3). Surprisingly, most of the significant correlation values ($|\rho_{i,j}| > 0.7$) involve correlation between (a_i, λ_j) rather than (a_i, a_j) or (λ_i, λ_j).

FIGURE 3: Sketch of the LWR test models used in this study: (a) PWR pin-cell, (b) PWR 15x15 lattice, (c) BWR pin-cell, (d) BWR 7x7 lattice, (e) VVER-1000 pin-cell, and (f) VVER-1000 lattice, based on the UAM benchmark [16].

TABLE 4: The correlation matrix between the group parameters of U-235 thermal fission as reported by [30].

	a_1	λ_1	a_2	λ_2	a_3	λ_3	a_4	λ_4	a_5	λ_5	a_6	λ_6
a_1	1	0.53	0.13	0.09	0.5	0.33	-0.33	0.49	-0.05	0.4	-0.19	0.03
λ_1	0.53	1	0.18	-0.16	-0.04	0.05	-0.11	-0.02	0.3	0.05	-0.4	0.13
a_2	0.13	0.18	1	0.67	0.23	0.72	-0.66	0.34	-0.36	0.17	-0.26	-0.08
λ_2	0.09	-0.16	0.67	1	0.36	0.88	-0.8	0.57	-0.35	0.05	-0.22	0.13
a_3	0.5	-0.04	0.23	0.36	1	0.63	-0.68	0.93	-0.63	0.76	-0.17	0.45
λ_3	0.33	0.05	0.72	0.88	0.63	1	-0.98	0.82	-0.45	0.36	-0.44	0.28
a_4	-0.33	-0.11	-0.66	-0.8	-0.68	-0.98	1	-0.85	0.49	-0.39	0.44	-0.31
λ_4	0.49	-0.02	0.34	0.57	0.93	0.82	-0.85	1	0.51	0.64	-0.29	0.47
a_5	-0.05	0.3	-0.36	-0.35	-0.63	-0.45	0.49	0.51	1	-0.52	0.28	0.09
λ_5	0.4	0.05	0.17	0.05	0.76	0.36	-0.39	0.64	-0.52	1	-0.39	0.49
a_6	-0.19	-0.4	-0.26	-0.22	-0.17	-0.44	0.44	-0.29	0.28	-0.39	1	-0.28
λ_6	0.03	0.13	-0.08	0.13	0.45	0.28	-0.31	0.47	0.09	0.49	-0.28	1

The results of the application of this correlation matrix on the kinetic parameters are shown in Table 5 for a PWR pin-cell considering beginning of life (BOL) without fuel burnup. Table 5 shows a comparison between the kinetic parameters' uncertainties for two cases. In the uncorrelated case in which all actinides' DND parameters (including U-235) are sampled independently, notice that the uncorrelated case still includes

the inherent correlation resulting from $a_{j,i}$ normalization as mentioned before in Section 2.3. The correlated case includes additionally the parametric correlation in Table 4, which is used for U-235 DND during sampling. It is worth mentioning that the correlation is applied to the U-235 thermal fission data only. This means that the fast fission parameters for U-235 and the DND for all other actinides

TABLE 5: Comparison of the kinetic parameters evaluation for a PWR pin-cell (BOL) when considering correlated U-235 thermal DND with the uncorrelated case.

Parameter	Uncorrelated case			Correlated case		
	Mean	Std*	Std/mean(%)	Mean	Std*	Std/mean(%)
β_1	2.120E-04	1.533E-05	7.2	2.123E-04	3.894E-06	1.8
β_2	1.419E-03	4.883E-05	3.4	1.418E-03	9.279E-06	0.7
β_3	1.299E-03	9.208E-05	7.1	1.300E-03	2.901E-05	2.2
β_4	2.708E-03	6.466E-05	2.4	2.706E-03	2.023E-05	0.8
β_5	8.949E-04	4.568E-05	5.1	8.958E-04	1.837E-05	2.1
β_6	3.005E-04	3.798E-05	12.6	3.013E-04	7.592E-06	2.5
λ_1	1.250E-02	2.368E-04	1.9	1.249E-02	5.430E-05	0.4
λ_2	3.081E-02	7.729E-04	2.5	3.082E-02	1.887E-04	0.6
λ_3	1.149E-01	3.278E-03	2.9	1.150E-01	1.051E-03	0.9
λ_4	3.100E-01	9.152E-03	3.0	3.100E-01	3.224E-03	1.0
λ_5	1.228E+00	1.029E-01	8.4	1.233E+00	3.161E-02	2.6
λ_6	3.319E+00	2.410E-01	7.3	3.305E+00	1.108E-01	3.4

*All kinetic parameters' uncertainties in this table are caused by DND uncertainty ONLY. The cross-section uncertainties are not propagated.

remain uncorrelated. In addition, the results reported in Table 5 include the uncertainty due to the DND only, as the nuclear data source (i.e., cross-sections) is turned off to isolate its effect. It is obvious from the results in Table 5 that the correlation significantly reduces the DND uncertainty effect on the kinetic parameters. All 12 kinetic parameters experience a decrease in their uncertainty after including the correlation between the U-235 data. Most of the DNF groups (β_i) experience uncertainty decrease of more than 50% after including the correlation. In general, it can be confirmed at least based on this correlation test that the DND uncertainty contribution will decrease after including the correlation between the groups. However, the authors decided to use the uncorrelated data as the reference data in all subsequent analyses for these reasons as follows:

- (i) There is no agreement in the literature about a correlation matrix to be used for uncertainty propagation [22], as all sources that are used in the data collection in Section 2.1 did not report any correlation data.
- (ii) As reported by [30], the correlation was calculated based on 11 samples, which are indeed less than the number of parameters (i.e., 12 parameters) to be analyzed. This implies that this correlation could change with larger number of samples.

It can be concluded in general that DND correlation could decrease the uncertainty in the kinetic parameters. We are still reporting kinetic parameters' uncertainty information based on DND correlation for the reader in Table 6 for the PWR lattice. All subsequent analyses focus on uncorrelated DND for all actinides (unless stated otherwise).

3.2. Sensitivity to Data and Operating Conditions. The effect of the uncertainty source on the kinetic parameters' uncertainty is studied and the results are plotted in Figure 4(a) for β_i and in Figure 5(a) for λ_i . In this study, two main uncertainty sources are considered: (1) nuclear data uncertainty (cross-sections, fission yield, and decay data) and this source

is referred to as "XS" and (2) nuclide-dependent DND (group fractions, group decay constant, and fractional delayed neutron yield). The results are presented in terms of the coefficient of variation or relative uncertainty ($\sigma/\mu\%$) for each kinetic parameter. It can be observed that XS uncertainties dominate DNF (β_i) uncertainties for the following responses: β_2 , β_4 , and β_5 , while the uncertainty of the other three groups is driven by DND. Since β_2 and β_4 form about 60% of the total DNF, and they are both driven by XS uncertainties, then we can conclude that nuclear data uncertainties control the DNF uncertainty. However, Figure 5(a) shows that decay constant parameters demonstrate an opposite behavior to DNF, as DND contribution dominates the uncertainty in all group-wise decay constants (λ_i). This analysis is considered generic in which the total uncertainty is decomposed into two main categories. A thorough analysis of the variance is also important, where the variance of each kinetic parameter is decomposed into portions attributable to each DND and XS parameter, similar to what has been done before on DND by Sobol indices [15]. The correlation and high dimensionality of the neutron cross-sections add more challenges to this variance decomposition, and this topic features one of our future studies.

Since cross-section uncertainties contribute to the major DNF groups, a numerical test case is performed to obtain more information about the corresponding isotope. Figures 4(b)-5(b) show the kinetic parameters' relative uncertainty for two test cases. The first case is a PWR pin-cell with low U-235 enrichment, while the second case has the same geometry and conditions but with very high U-235 enrichment (> 90 wt% U-235). This test is used to isolate the effect of the cross-section covariances of U-238. A significant reduction in DNF uncertainty can be observed, especially for the key groups β_2 and β_4 . Notice that the mean value between the two cases is slightly different as the high enriched case has less delayed neutron emission (i.e., U-238 has larger $\bar{\nu}_d$ than U-235). For example, for the low enriched case, $\beta_4 = 2.693 \times 10^{-3} \pm 2.010 \times 10^{-4}$, while, for the high enriched case,

FIGURE 4: Precursor-group-wise DNF (β_i) relative uncertainty (in %) for various sensitivity cases at BOL: (a) data source (PWR pin-cell), (b) initial fuel content (PWR pin-cell), (c) void fraction (BWR lattice), and (d) neutron absorber (PWR lattice).

$\beta_4 = 2.509 \times 10^{-3} \pm 9.822 \times 10^{-5}$. This reduction implies that U-238 covariances are a major reason for the large uncertainty in the DNF for the LWR case. In Figure 5(b), the decay constants, which are mostly driven by DND, do not experience a significant change when moving to the highly enriched case, except for λ_6 that increased from 8% to 11%. By looking at Figure 1, it can be observed that U-238 uncertainty in σ_f and $\bar{\nu}$ is substantially larger than the other actinides, which causes the large uncertainty in DNFs. Thus additional improvements to U-238 covariances are needed to obtain less uncertain DNFs for LWR applications.

BWR systems encounter considerable variation in void fraction axially, which causes the coolant density to change drastically from bottom to top of the channel. Investigation of the effect of the coolant density on the kinetic parameters and their uncertainty is performed based on the BWR lattice

in Figure 3(c) with four different coolant densities (ρ_{cool}). The results are plotted in Figure 4(c) for β_i and in Figure 5(c) for λ_i . The effect of void fraction is more influential on the uncertainty of β_2 , β_4 , and β_5 . The uncertainty of DNF tends to decrease with increasing the coolant density. The uncertainty in β_4 is about 8% at the highest coolant density (0.8 g/cm^3) and about 11% at the lowest (0.2 g/cm^3). With low moderation, fast fission and capture in U-238 are enhanced compared to U-235 thermal fission. Therefore, the effect of U-238 fission cross-section covariances increases, causing the DNF uncertainty to increase for low density values. It is worth mentioning that the fast fission of U-238 slightly increases the raw/mean value of β_i , as U-238 emits more delayed neutrons than U-235 as shown in Table 3. The uncertainty in the decay constant in Figure 5(c) shows less sensitivity to the void fraction changes than the DNF groups.

FIGURE 5: Precursor-group-wise decay constant (λ_i) relative uncertainty (in %) for various sensitivity cases at BOL: (a) data source (PWR pin-cell), (b) initial fuel content (PWR pin-cell), (c) void fraction (BWR lattice), and (d) neutron absorber (PWR lattice).

A final sensitivity study for the effect of the neutron absorber on the kinetic parameters' uncertainty is shown in Figures 4(d)-5(d). The results are performed on the PWR lattice. The base case is the original case described in Figure 3(b). The rodged case involves a full control rod insertion in all 16 guide tubes. The gadolinium case involves distributing a total of 24 gadolinium rods symmetrically across the lattice with 5% Gd_2O_3 in each rod. The results show again that the decay constant uncertainty is less sensitive to the presence of neutron absorber than the DNF. Similar to the void fraction effect, the presence of control rod and gadolinium absorber hardens the neutron spectrum, which increases the Pu-239 breeding and U-238 fast fission, causing the DNF uncertainty to increase. The uncertainty of the gadolinium case is bracketed between the base and rodged cases, as the control rod absorption is stronger than the gadolinium rod.

In general, we can connect this discussion to the previous data effect as follows: since DNFs show more sensitivity to the nuclear data, operating conditions (e.g., void fraction and control rod) have more effect on DNF uncertainties. And since decay constants are more sensitive to DND, changing operating conditions has less effect on their uncertainty. To summarize, lattice designs with large heterogeneity (e.g., 3D, MOX fuel, water rods, and control rods) should be modeled explicitly for accurate analysis of kinetic parameters. Also, all results discussed in this subsection are performed at BOL without fuel depletion, which is highlighted in the following subsection.

3.3. Burnup Analysis. It is important to study how kinetic parameters and their uncertainties change as a function of burnup, since the fuel composition changes after depletion

FIGURE 6: Variation of the precursor-group-wise DNF (β_i) as a function of burnup plus 1σ uncertainty for the PWR lattice.

FIGURE 7: Variation of the precursor-group-wise decay constant (λ_i) as a function of burnup plus 1σ uncertainty for the PWR lattice.

and new actinide isotopes appear in the fuel, which contributes to delayed neutron emission. Figures 6 and 7 show the group-wise DNF (β_i) and group-wise decay constant (λ_i) as a function of burnup, respectively. The results presented in this subsection are based on the PWR lattice depleted to about 65 GWD/MTU with an average power of 33.6 kW/kgU. It is worth mentioning that the uncertainty band in Figures 6 and 7 corresponds to one standard deviation around the mean value.

We can observe that β_i raw/mean value decreases with burnup due to the fuel composition changes. As fuel depletes,

the concentration of plutonium isotopes increases and the U-235 concentration decreases. Pu-239 emits less delayed neutrons than U-235, as its delayed neutron yield is around one-third of U-235 as indicated in Table 3. Therefore, as burnup increases, delayed neutron emission is dominated by plutonium isotopes (e.g., Pu-239 and Pu-241). The uncertainties in β_i groups also increase with burnup for multiple reasons: (1) the change in fuel composition introduces new cross-section covariances from other actinides, which contribute to the uncertainty, (2) the increase in plutonium content has larger uncertainties in its fission cross-sections

FIGURE 8: Correlation matrix of the kinetic parameters for the PWR lattice at BOL and EOL.

and $\bar{\nu}$ than U-235 (see Figure 1), and (3) the depletion of U-235 increases the likelihood of fast fission in U-238, which also has larger cross-section covariances (see Figure 1). Based on some preliminary tests (not shown here for brevity), the uncertainty contribution from DND does not change significantly with burnup, and the large increase in DNF uncertainty after depletion in Figure 6 results mainly from nuclear data. To explain further, by depleting the highly enriched case mentioned in Section 3.2 and Figure 6(b), an insignificant change in β_i uncertainty with burnup is found, as U-235 is the main delayed neutron emitter across the cycle. Once again, U-238 covariances are expected to cause the substantial rise in DNF uncertainty after burnup. For decay constant parameters, the raw/mean value shows almost a constant behavior with burnup (e.g., λ_1, λ_2), which is different from the decreasing trend of the DNFs. The burnup-dependent relative uncertainty also remains nearly constant for most of the λ_i groups. The uncertainties of λ_5 and λ_6 show a slight decrease and increase toward EOL, respectively. In general, since we observed that decay constant uncertainty is driven by DND, and the uncertainty of the group decay constant parameters for Pu-239 and U-235 are close (see Table 1), the effect of burnup on decay constant is generally smaller than DNF.

3.4. Summary Data. For uncertainty propagation of kinetic parameters in subsequent applications, it is important to have a correlation matrix between the kinetic parameters, so that a covariance matrix can be constructed. The correlation matrices for all kinetic parameters are plotted for BOL and EOL (~ 65 GWD/MTU) in Figure 8 for the PWR lattice. The tabulated and numerical values for the correlation coefficients are presented in Appendix C. The matrix shows Pearson correlation coefficient between the group-wise parameters β_i and λ_i . This correlation matrix of the kinetic parameters (i.e., output) should not be confused with the correlation matrix listed in Table 4, which corresponds to the fundamental DND (i.e., input). Notice that DNFs (β_i) are highly correlated at BOL. This correlation between DNF groups is magnified

when moving to EOL. The correlation between β_i is expected to be due to the correlation between the cross-sections, as the uncertainty in β_i is driven by nuclear data covariances. It is worth mentioning that the highly enriched case in Section 3.2 shows a much weaker correlation at BOL between DNF groups, implying that U-238 covariances play a major role in the significant correlation behaviour between DNFs. Also, the correlation matrix at BOL shows weak correlation between β_i and λ_i groups, and this weak correlation continues toward EOL. In addition, λ_i groups show weak correlation to each other, which could be due to using uncorrelated DND, and this weak correlation does not change at larger burnup values.

According to the previous analysis, we observe that kinetic parameters, especially DNFs, have high uncertainties when considering both fundamental neutron cross-sections and DND uncertainties. The cross-section uncertainties largely contributed to DNF uncertainties, while DND contributed more to the uncertainty in decay constants. The previous conclusions are subjective to the DND values and uncertainties collected in this study (which are the best available in the literature), the 56-group covariance library in the SCALE-6.2.3 code system, and LWR fuel composition, geometry, and operating conditions. Using any different combinations of the previous factors could significantly change the kinetic parameters' values and uncertainties. The summary of the kinetic parameters and their uncertainties as calculated by this study is reported in Appendix B for the PWR lattice only. This is done only to ensure conciseness of this paper (since the tables of kinetic parameters with burnup have large size). The detailed results for BWR and VVER systems will appear in the first-author's PhD thesis [33]. For PWR in Appendix B, there are two data sets: the first set (recommended) is based on using uncorrelated DND in the UQ process, while the second set includes correlated DND for U-235 thermal data using the correlation matrix in Table 4 [30]. The numerical values are reported at six burnup steps. In this section, final results of the kinetic parameters for the three systems are presented in Table 7 at BOL. For PWR,

TABLE 7: Summary of kinetic parameters' values and uncertainties at BOL for various LWR lattice models in UAM benchmark.

Parameter	PWR			BWR			VVER		
	Mean	Std	Std/mean(%)	Mean	Std	Std/mean(%)	Mean	Std	Std/mean(%)
β_1	2.105E-04	1.766E-05	8.4	2.053E-04	1.828E-05	8.9	2.093E-04	1.762E-05	8.4
β_2	1.409E-03	8.974E-05	6.4	1.400E-03	1.050E-04	7.5	1.411E-03	8.964E-05	6.4
β_3	1.290E-03	1.250E-04	9.7	1.293E-03	1.425E-04	11.0	1.292E-03	1.253E-04	9.7
β_4	2.685E-03	2.016E-04	7.5	2.704E-03	2.550E-04	9.4	2.684E-03	2.037E-04	7.6
β_5	8.851E-04	1.100E-04	12.4	9.212E-04	1.424E-04	15.5	8.841E-04	1.118E-04	12.6
β_6	2.980E-04	5.065E-05	17.0	3.146E-04	5.921E-05	18.8	3.024E-04	5.089E-05	16.8
$\lambda_1(s^{-1})$	1.249E-02	2.402E-04	1.9	1.250E-02	2.474E-04	2.0	1.248E-02	2.374E-04	1.9
$\lambda_2(s^{-1})$	3.079E-02	7.865E-04	2.6	3.079E-02	7.942E-04	2.6	3.075E-02	7.714E-04	2.5
$\lambda_3(s^{-1})$	1.147E-01	3.567E-03	3.1	1.155E-01	3.799E-03	3.3	1.146E-01	3.552E-03	3.1
$\lambda_4(s^{-1})$	3.096E-01	9.954E-03	3.2	3.115E-01	1.051E-02	3.4	3.092E-01	9.855E-03	3.2
$\lambda_5(s^{-1})$	1.224E+00	1.072E-01	8.8	1.229E+00	1.069E-01	8.7	1.215E+00	1.063E-01	8.8
$\lambda_6(s^{-1})$	3.303E+00	2.580E-01	7.8	3.336E+00	2.623E-01	7.9	3.282E+00	2.531E-01	7.7

the dominant DNF group β_4 uncertainty is about 7.5%. For BWR, the β_4 uncertainty is 9.5%, while for VVER the relative uncertainty is 7.6%. In general, PWR and VVER have lower uncertainties in their kinetic parameters than the BWR (see Table 7).

The fundamental difference between the UQ approach adopted in this study and previous studies is the explicit and individual treatment of each precursor group for the kinetic parameters. This allows capturing more realistic uncertainty behaviour in the kinetic parameters. Majority of the previous studies relied on the eigenvalue or the k-ratio approach which allows β_{eff} estimation through calculating the prompt (k_p) and effective (k_{eff}) multiplication factors. Due to the dependency of β_{eff} on k_{eff} , it is expected that the k-ratio approach will be driven by cross-section uncertainties as observed before by [34, 35]. Our precursor-group-based approach also explored the decay constant uncertainty, which is rarely considered in previous studies. Although any differences in results between this study and others can be attributed to differences in data libraries, geometry, and material composition, the difference in the methodology is the main reason. Based on the results presented in this section, kinetic parameters' values and uncertainties are sensitive to the fuel composition more than any other factor (which in turn affects the DND and cross-sections). Consequently, the authors recommend repeating the calculations in this study for any new reactor design (especially advanced reactor designs) or covariance data library available to see their effect on the final uncertainty. The data listed in this study can be used for testing purposes and for LWR designs that are close/similar to the designs in UAM benchmark.

4. Conclusions

In this study, an extension of a previously developed framework for UQ of the six-group kinetic parameters is presented. The framework is data driven where each precursor group is treated separately to calculate its delayed neutron fraction. A comprehensive set of DND is collected from various delayed neutron experiments, which report group fractions, group decay constant, and delayed neutron yield for different

actinides. Two major sources of uncertainty are considered: neutron cross-sections and nuclide-dependent DND. The calculations are performed using TRITON lattice physics code and Sampler in the SCALE code system. Kinetic parameters' sensitivity and uncertainty from different perspectives are investigated and the following conclusions can be drawn from this study:

- (i) Weighted kinetic parameters', especially precursor-group DNFs (β_i), have large uncertainties. Precursor groups 5 and 6 are usually characterized by high uncertainty.
- (ii) Sources for correlation data between the DND group parameters are limited. The usage of correlated DND for U-235 in the UQ process reduces the uncertainty in the kinetic parameters significantly (due to the DND source). However, kinetic parameters' uncertainties due to uncorrelated DND are considered as the base data due to the unavailability of correlation info for most of the actinides.
- (iii) The cross-section uncertainties dominate the uncertainty in the DNF groups (for LWR designs), mainly due to U-238 covariances (as revealed by a numerical test with high U-235 enrichment). On the other hand, the DND shows larger contribution to the decay constant uncertainties.
- (iv) Large void fraction and presence of neutron absorber (e.g., control rod) increase the DNF uncertainty due to the neutron spectrum hardening.
- (v) High correlation between the DNF responses (β_i) is observed, while decay constant groups show weak correlation to each other and also to DNF groups. This conclusion is observed when considering uncorrelated DND during the UQ process.
- (vi) The uncertainty in the dominant DNF group β_4 for PWR, BWR, and VVER is about 7.5%, 9.5%, and 7.6%, respectively. The uncertainty in DNF groups grows to larger values as burnup increases, due to the depletion of U-235 and introduction of new actinides into the fuel composition.

TABLE 8: Correlation matrix for kinetic parameters in PWR lattice at different burnups "based on uncorrelated DND": (1) 0 GWD/MTU, (2) 10 GWD/MTU, (3) 20 GWD/MTU, (4) 30 GWD/MTU, (5) 40 GWD/MTU, and (6) 50 GWD/MTU.

(1)	β_1	β_2	β_3	β_4	β_5	β_6	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	(2)	β_1	β_2	β_3	β_4	β_5	β_6	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
β_1	1	0.43	0.16	0.43	0.36	0.21	-0.01	0.06	0.09	0.06	-0.06	0.12	β_1	1	0.63	0.37	0.57	0.45	0.36	0.01	0.06	0.17	0.11	-0.03	0.11
β_2	0.43	1	0.33	0.81	0.68	0.45	0.02	0.09	0.23	0.22	0.05	0.21	β_2	0.63	1	0.58	0.84	0.71	0.58	0.04	0.07	0.29	0.25	0.07	0.19
β_3	0.16	0.33	1	0.47	0.49	0.34	0.07	0.1	0.2	0.21	0.09	0.15	β_3	0.37	0.58	1	0.65	0.62	0.51	0.09	0.09	0.3	0.25	0.13	0.2
β_4	0.43	0.81	0.47	1	0.83	0.57	0.06	0.11	0.3	0.29	0.1	0.26	β_4	0.57	0.84	0.65	1	0.88	0.71	0.08	0.12	0.39	0.34	0.14	0.29
β_5	0.36	0.68	0.49	0.83	1	0.57	0.06	0.11	0.31	0.3	0.08	0.27	β_5	0.45	0.71	0.62	0.88	1	0.7	0.09	0.14	0.39	0.35	0.14	0.34
β_6	0.21	0.45	0.34	0.57	0.57	1	0.09	0.05	0.22	0.22	0.08	0.08	β_6	0.36	0.58	0.51	0.71	0.7	1	0.11	0.08	0.33	0.3	0.12	0.16
λ_1	-0.01	0.02	0.07	0.06	0.06	0.09	1	0.02	0.06	0.04	-0.03	-0.01	λ_1	0.01	0.04	0.09	0.08	0.09	0.11	1	0.04	0.08	0.04	-0.01	0.01
λ_2	0.06	0.09	0.1	0.11	0.11	0.05	0.02	1	0.05	0.01	0.01	0.04	λ_2	0.06	0.07	0.09	0.12	0.14	0.08	0.04	1	0.05	0.03	0.03	0.06
λ_3	0.09	0.23	0.2	0.3	0.31	0.22	0.06	0.05	1	0.13	0.03	0.12	λ_3	0.17	0.29	0.3	0.39	0.39	0.33	0.08	0.05	1	0.17	0.06	0.18
λ_4	0.06	0.22	0.21	0.29	0.3	0.22	0.04	0.01	0.13	1	0.06	0.12	λ_4	0.11	0.25	0.25	0.34	0.35	0.3	0.04	0.03	0.17	1	0.1	0.18
λ_5	-0.06	0.05	0.09	0.1	0.08	0.08	-0.03	0.01	0.03	0.06	1	0.04	λ_5	-0.03	0.07	0.13	0.14	0.14	0.12	-0.01	0.03	0.06	0.1	1	0.08
λ_6	0.12	0.21	0.15	0.26	0.27	0.08	-0.01	0.04	0.12	0.12	0.04	1	λ_6	0.11	0.19	0.2	0.29	0.34	0.16	0.01	0.06	0.18	0.18	0.08	1
(3)	β_1	β_2	β_3	β_4	β_5	β_6	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	(4)	β_1	β_2	β_3	β_4	β_5	β_6	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
β_1	1	0.76	0.55	0.66	0.5	0.48	0.04	0.04	0.22	0.15	-0.01	0.06	β_1	1	0.82	0.65	0.71	0.53	0.54	0.07	0.01	0.22	0.15	-0.02	0.01
β_2	0.76	1	0.72	0.85	0.69	0.65	0.06	0.03	0.32	0.25	0.06	0.13	β_2	0.82	1	0.79	0.86	0.69	0.69	0.08	0.01	0.31	0.23	0.04	0.08
β_3	0.55	0.72	1	0.75	0.68	0.63	0.11	0.07	0.38	0.27	0.13	0.19	β_3	0.65	0.79	1	0.81	0.72	0.7	0.13	0.05	0.41	0.26	0.11	0.17
β_4	0.66	0.85	0.75	1	0.89	0.8	0.11	0.11	0.44	0.36	0.14	0.29	β_4	0.71	0.86	0.81	1	0.89	0.84	0.13	0.1	0.45	0.36	0.13	0.28
β_5	0.5	0.69	0.68	0.89	1	0.78	0.11	0.15	0.44	0.38	0.16	0.37	β_5	0.53	0.69	0.72	0.89	1	0.82	0.14	0.15	0.46	0.38	0.16	0.38
β_6	0.48	0.65	0.63	0.8	0.78	1	0.13	0.1	0.4	0.34	0.13	0.2	β_6	0.54	0.69	0.7	0.84	0.82	1	0.14	0.11	0.43	0.35	0.12	0.22
λ_1	0.04	0.06	0.11	0.11	0.11	0.13	1	0.04	0.1	0.04	0	0.04	λ_1	0.07	0.08	0.13	0.13	0.14	0.14	1	0.05	0.1	0.05	0	0.07
λ_2	0.04	0.03	0.07	0.11	0.15	0.1	0.04	1	0.05	0.04	0.05	0.08	λ_2	0.01	0.01	0.05	0.1	0.15	0.11	0.05	1	0.05	0.06	0.06	0.1
λ_3	0.22	0.32	0.38	0.44	0.44	0.4	0.1	0.05	1	0.2	0.07	0.22	λ_3	0.22	0.31	0.41	0.45	0.46	0.43	0.1	0.05	1	0.22	0.07	0.25
λ_4	0.15	0.25	0.27	0.36	0.38	0.34	0.04	0.04	0.2	1	0.13	0.21	λ_4	0.15	0.23	0.26	0.36	0.38	0.35	0.05	0.06	0.22	1	0.13	0.23
λ_5	-0.01	0.06	0.13	0.14	0.16	0.13	0	0.05	0.07	0.13	1	0.12	λ_5	-0.02	0.04	0.11	0.13	0.16	0.12	0	0.06	0.07	0.13	1	0.15
λ_6	0.06	0.13	0.19	0.29	0.37	0.2	0.04	0.08	0.22	0.21	0.12	1	λ_6	0.01	0.08	0.17	0.28	0.38	0.22	0.07	0.1	0.25	0.23	0.15	1
(5)	β_1	β_2	β_3	β_4	β_5	β_6	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	(6)	β_1	β_2	β_3	β_4	β_5	β_6	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
β_1	1	0.84	0.69	0.72	0.54	0.57	0.08	0	0.21	0.14	-0.03	-0.01	β_1	1	0.85	0.72	0.74	0.54	0.6	0.08	-0.01	0.17	0.11	-0.06	-0.04
β_2	0.84	1	0.81	0.86	0.68	0.7	0.08	0	0.29	0.2	0.01	0.05	β_2	0.85	1	0.83	0.86	0.68	0.72	0.08	-0.01	0.25	0.16	-0.03	0.01
β_3	0.69	0.81	1	0.83	0.73	0.73	0.13	0.04	0.4	0.25	0.08	0.16	β_3	0.72	0.83	1	0.85	0.74	0.76	0.13	0.03	0.37	0.2	0.03	0.13
β_4	0.72	0.86	0.83	1	0.89	0.86	0.14	0.09	0.44	0.34	0.1	0.27	β_4	0.74	0.86	0.85	1	0.89	0.88	0.15	0.09	0.4	0.29	0.05	0.24
β_5	0.54	0.68	0.73	0.89	1	0.84	0.15	0.15	0.46	0.36	0.14	0.39	β_5	0.54	0.68	0.74	0.89	1	0.86	0.16	0.15	0.44	0.33	0.1	0.38
β_6	0.57	0.7	0.73	0.86	0.84	1	0.15	0.11	0.43	0.34	0.1	0.23	β_6	0.6	0.72	0.76	0.88	0.86	1	0.15	0.12	0.41	0.31	0.06	0.22
λ_1	0.08	0.08	0.13	0.14	0.15	0.15	1	0.04	0.11	0.05	0	0.09	λ_1	0.08	0.08	0.13	0.15	0.16	0.15	1	0.04	0.1	0.05	0	0.11
λ_2	0	0	0.04	0.09	0.15	0.11	0.04	1	0.06	0.07	0.07	0.11	λ_2	-0.01	-0.01	0.03	0.09	0.15	0.12	0.04	1	0.07	0.08	0.08	0.12
λ_3	0.21	0.29	0.4	0.44	0.46	0.43	0.11	0.06	1	0.21	0.06	0.27	λ_3	0.17	0.25	0.37	0.4	0.44	0.41	0.1	0.07	1	0.2	0.04	0.27
λ_4	0.14	0.2	0.25	0.34	0.36	0.34	0.05	0.07	0.21	1	0.12	0.24	λ_4	0.11	0.16	0.2	0.29	0.33	0.31	0.05	0.08	0.2	1	0.09	0.23
λ_5	-0.03	0.01	0.08	0.1	0.14	0.1	0	0.07	0.06	0.12	1	0.16	λ_5	-0.06	-0.03	0.03	0.05	0.1	0.06	0	0.08	0.04	0.09	1	0.16
λ_6	-0.01	0.05	0.16	0.27	0.39	0.23	0.09	0.11	0.27	0.24	0.16	1	λ_6	-0.04	0.01	0.13	0.24	0.38	0.22	0.11	0.12	0.27	0.23	0.16	1

TABLE 9: Correlation matrix for kinetic parameters in PWR lattice at different burnups "based on correlated $U-235$ DND": (1) 0 GWD/MTU, (2) 10 GWD/MTU, (3) 20 GWD/MTU, (4) 30 GWD/MTU, (5) 40 GWD/MTU, and (6) 50 GWD/MTU.

(1)	β_1	β_2	β_3	β_4	β_5	β_6	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	(2)	β_1	β_2	β_3	β_4	β_5	β_6	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
β_1	1	0.89	0.78	0.84	0.73	0.74	0.21	0.22	0.42	0.43	0.31	0.39	β_1	1	0.89	0.77	0.79	0.65	0.68	0.22	0.1	0.41	0.36	0.23	0.28
β_2	0.89	1	0.9	0.97	0.9	0.9	0.26	0.29	0.58	0.57	0.42	0.5	β_2	0.89	1	0.87	0.91	0.8	0.82	0.24	0.13	0.54	0.46	0.32	0.37
β_3	0.78	0.9	1	0.9	0.85	0.86	0.24	0.26	0.7	0.56	0.42	0.5	β_3	0.77	0.87	1	0.89	0.82	0.83	0.23	0.15	0.69	0.5	0.36	0.44
β_4	0.84	0.97	0.9	1	0.95	0.95	0.3	0.32	0.66	0.64	0.48	0.56	β_4	0.79	0.91	0.89	1	0.94	0.94	0.29	0.21	0.66	0.59	0.44	0.54
β_5	0.73	0.9	0.85	0.95	1	0.95	0.31	0.32	0.7	0.66	0.54	0.58	β_5	0.65	0.8	0.82	0.94	1	0.93	0.29	0.25	0.67	0.6	0.49	0.59
β_6	0.74	0.9	0.86	0.95	0.95	1	0.31	0.32	0.7	0.65	0.5	0.6	β_6	0.68	0.82	0.83	0.94	0.93	1	0.29	0.23	0.67	0.58	0.45	0.57
λ_1	0.21	0.26	0.24	0.3	0.31	0.31	1	0.08	0.28	0.23	0.19	0.21	λ_1	0.22	0.24	0.23	0.29	0.29	0.29	1	0.04	0.23	0.21	0.16	0.21
λ_2	0.22	0.29	0.26	0.3	0.32	0.32	0.08	1	0.24	0.25	0.19	0.21	λ_2	0.1	0.13	0.15	0.21	0.25	0.23	0.04	1	0.15	0.19	0.14	0.18
λ_3	0.42	0.58	0.7	0.66	0.7	0.7	0.28	0.24	1	0.51	0.41	0.48	λ_3	0.41	0.54	0.69	0.66	0.67	0.67	0.23	0.15	1	0.46	0.34	0.46
λ_4	0.43	0.57	0.56	0.64	0.66	0.65	0.23	0.25	0.51	1	0.39	0.44	λ_4	0.36	0.46	0.5	0.59	0.6	0.58	0.21	0.19	0.46	1	0.33	0.42
λ_5	0.31	0.42	0.42	0.48	0.54	0.5	0.19	0.19	0.41	0.39	1	0.43	λ_5	0.23	0.32	0.36	0.44	0.49	0.45	0.16	0.14	0.34	0.33	1	0.41
λ_6	0.39	0.5	0.5	0.56	0.58	0.6	0.21	0.21	0.48	0.44	0.43	1	λ_6	0.28	0.37	0.44	0.54	0.59	0.57	0.21	0.18	0.46	0.42	0.41	1
(3)	β_1	β_2	β_3	β_4	β_5	β_6	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	(4)	β_1	β_2	β_3	β_4	β_5	β_6	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
β_1	1	0.88	0.77	0.76	0.59	0.65	0.21	0.01	0.35	0.29	0.13	0.14	β_1	1	0.87	0.77	0.76	0.57	0.63	0.19	-0.01	0.3	0.23	0.06	0.07
β_2	0.88	1	0.86	0.88	0.73	0.77	0.21	0.03	0.46	0.36	0.2	0.22	β_2	0.87	1	0.86	0.87	0.7	0.75	0.18	0	0.4	0.29	0.11	0.13
β_3	0.77	0.86	1	0.88	0.78	0.81	0.21	0.07	0.63	0.42	0.25	0.34	β_3	0.77	0.86	1	0.87	0.77	0.8	0.19	0.04	0.56	0.35	0.16	0.26
β_4	0.76	0.88	0.88	1	0.92	0.93	0.26	0.14	0.61	0.51	0.33	0.46	β_4	0.76	0.87	0.87	1	0.91	0.92	0.24	0.11	0.55	0.44	0.24	0.39
β_5	0.59	0.73	0.78	0.92	1	0.91	0.27	0.19	0.63	0.53	0.4	0.55	β_5	0.57	0.7	0.77	0.91	1	0.9	0.25	0.16	0.58	0.47	0.3	0.51
β_6	0.65	0.77	0.81	0.93	0.91	1	0.25	0.16	0.62	0.51	0.35	0.48	β_6	0.63	0.75	0.8	0.92	0.9	1	0.23	0.14	0.56	0.44	0.25	0.41
λ_1	0.21	0.21	0.21	0.26	0.27	0.25	1	0.02	0.19	0.17	0.1	0.19	λ_1	0.19	0.18	0.19	0.24	0.25	0.23	1	0.01	0.15	0.14	0.07	0.18
λ_2	0.01	0.03	0.07	0.14	0.19	0.16	0.02	1	0.09	0.15	0.11	0.15	λ_2	-0.01	0	0.04	0.11	0.16	0.14	0.01	1	0.08	0.13	0.1	0.14
λ_3	0.35	0.46	0.63	0.61	0.63	0.62	0.19	0.09	1	0.39	0.25	0.41	λ_3	0.3	0.4	0.56	0.55	0.58	0.56	0.15	0.08	1	0.33	0.18	0.37
λ_4	0.29	0.36	0.42	0.51	0.53	0.51	0.17	0.15	0.39	1	0.24	0.38	λ_4	0.23	0.29	0.35	0.44	0.47	0.44	0.14	0.13	0.33	1	0.17	0.33
λ_5	0.13	0.2	0.25	0.33	0.4	0.35	0.1	0.11	0.25	0.24	1	0.36	λ_5	0.06	0.11	0.16	0.24	0.3	0.25	0.07	0.1	0.18	0.17	1	0.31
λ_6	0.14	0.22	0.34	0.46	0.55	0.48	0.19	0.15	0.41	0.38	0.36	1	λ_6	0.07	0.13	0.26	0.39	0.51	0.41	0.18	0.14	0.37	0.33	0.31	1
(5)	β_1	β_2	β_3	β_4	β_5	β_6	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	(6)	β_1	β_2	β_3	β_4	β_5	β_6	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
β_1	1	0.87	0.76	0.75	0.56	0.63	0.17	-0.02	0.25	0.19	0.01	0.03	β_1	1	0.87	0.76	0.75	0.55	0.62	0.13	-0.03	0.19	0.14	-0.04	-0.02
β_2	0.87	1	0.86	0.87	0.69	0.75	0.16	-0.01	0.34	0.24	0.05	0.08	β_2	0.87	1	0.85	0.87	0.68	0.74	0.13	-0.02	0.28	0.17	-0.02	0.04
β_3	0.76	0.86	1	0.87	0.76	0.79	0.18	0.03	0.5	0.3	0.1	0.22	β_3	0.76	0.85	1	0.87	0.75	0.79	0.15	0.03	0.42	0.23	0.03	0.17
β_4	0.75	0.87	0.87	1	0.9	0.91	0.22	0.09	0.51	0.39	0.17	0.35	β_4	0.75	0.87	0.87	1	0.9	0.91	0.2	0.09	0.44	0.32	0.08	0.3
β_5	0.56	0.69	0.76	0.9	1	0.89	0.23	0.15	0.54	0.42	0.23	0.48	β_5	0.55	0.68	0.75	0.9	1	0.88	0.21	0.15	0.48	0.35	0.15	0.44
β_6	0.63	0.75	0.79	0.91	0.89	1	0.21	0.13	0.51	0.39	0.19	0.36	β_6	0.62	0.74	0.79	0.91	0.88	1	0.19	0.12	0.45	0.33	0.1	0.31
λ_1	0.17	0.16	0.18	0.22	0.23	0.21	1	0.01	0.13	0.12	0.05	0.17	λ_1	0.13	0.13	0.15	0.2	0.21	0.19	1	0.01	0.11	0.1	0.02	0.17
λ_2	-0.02	-0.01	0.03	0.09	0.15	0.13	0.01	1	0.08	0.12	0.09	0.14	λ_2	-0.03	-0.02	0.03	0.09	0.15	0.12	0.01	1	0.08	0.12	0.09	0.14
λ_3	0.25	0.34	0.5	0.51	0.54	0.51	0.13	0.08	1	0.29	0.13	0.34	λ_3	0.19	0.28	0.42	0.44	0.48	0.45	0.11	0.08	1	0.24	0.08	0.31
λ_4	0.19	0.24	0.3	0.39	0.42	0.39	0.12	0.12	0.29	1	0.12	0.3	λ_4	0.14	0.17	0.23	0.32	0.35	0.33	0.1	0.12	0.24	1	0.07	0.26
λ_5	0.01	0.05	0.1	0.17	0.23	0.19	0.05	0.09	0.13	0.12	1	0.27	λ_5	-0.04	-0.02	0.03	0.08	0.15	0.1	0.02	0.09	0.08	0.07	1	0.22
λ_6	0.03	0.08	0.22	0.35	0.48	0.36	0.17	0.14	0.34	0.3	0.27	1	λ_6	-0.02	0.04	0.17	0.3	0.44	0.31	0.17	0.14	0.31	0.26	0.22	1

TABLE 10: Kinetic parameters' mean value (left column) and uncertainty (right column) at different burnup steps for the PWR lattice when considering uncorrelated DND.

Parameter	Burnup (GWD/MTU)											
	0	10	20	30	40	50						
β_1	2.105E-04	1.766E-05	1.839E-04	1.681E-05	1.630E-04	1.843E-05	1.458E-04	2.023E-05	1.343E-04	2.154E-05	1.204E-04	2.322E-05
β_2	1.409E-03	8.974E-05	1.270E-03	1.052E-04	1.176E-03	1.301E-04	1.107E-03	1.502E-04	1.064E-03	1.630E-04	1.013E-03	1.782E-04
β_3	1.290E-03	1.250E-04	1.145E-03	1.233E-04	1.046E-03	1.325E-04	9.724E-04	1.430E-04	9.263E-04	1.508E-04	8.728E-04	1.612E-04
β_4	2.685E-03	2.016E-04	2.356E-03	2.161E-04	2.137E-03	2.389E-04	1.980E-03	2.606E-04	1.884E-03	2.763E-04	1.773E-03	2.964E-04
β_5	8.851E-04	1.100E-04	7.950E-04	1.154E-04	7.443E-04	1.233E-04	7.151E-04	1.318E-04	7.003E-04	1.384E-04	6.859E-04	1.474E-04
β_6	2.980E-04	5.065E-05	2.701E-04	4.814E-05	2.504E-04	4.888E-05	2.354E-04	5.074E-05	2.259E-04	5.253E-05	2.149E-04	5.524E-05
λ_1	1.249E-02	2.402E-04	1.253E-02	2.223E-04	1.257E-02	2.160E-04	1.260E-02	2.155E-04	1.263E-02	2.183E-04	1.267E-02	2.265E-04
λ_2	3.079E-02	7.865E-04	3.079E-02	7.287E-04	3.077E-02	7.186E-04	3.074E-02	7.279E-04	3.071E-02	7.442E-04	3.068E-02	7.779E-04
λ_3	1.147E-01	3.567E-03	1.164E-01	3.479E-03	1.180E-01	3.562E-03	1.195E-01	3.750E-03	1.207E-01	3.960E-03	1.223E-01	4.326E-03
λ_4	3.096E-01	9.954E-03	3.129E-01	9.597E-03	3.167E-01	9.667E-03	3.207E-01	9.949E-03	3.239E-01	1.028E-02	3.284E-01	1.090E-02
λ_5	1.224E+00	1.072E-01	1.238E+00	9.754E-02	1.260E+00	9.083E-02	1.286E+00	8.614E-02	1.306E+00	8.411E-02	1.333E+00	8.374E-02
λ_6	3.303E+00	2.580E-01	3.321E+00	2.456E-01	3.341E+00	2.445E-01	3.366E+00	2.507E-01	3.386E+00	2.598E-01	3.415E+00	2.774E-01

Kinetic parameters' values and uncertainties provided by this study can be efficiently used in subsequent core calculations, point reactor kinetics applications, control rod worth calculations, and many other applications. In future work, identification of the key DND and cross-sections that cause uncertainty in the kinetic parameters will be conducted using global or variance-based sensitivity analysis. Inference of the correlation between the DND should be given additional interests through simulating the decay curves of the actinides after a burst fission under nuclear data perturbations. Afterwards, uncertainty quantification of kinetic parameters will be performed for other fuel and advanced reactor design types.

Appendix

A. Delayed Neutron Group Parameters for Isotopes with a Single Fission Set

This appendix lists the group parameters for selected actinides and isotopes whose delayed neutrons are assigned to either thermal or fast set in Table 2. The list is complementary to the list given in Table 1 which contains actinides with thermal and fast fission group parameters. The data contains both measured and calculated values and the reference is indicated for each isotope. The calculated values do not have uncertainties. Measured data for U-236 is only for $a_{j,i}$, as the decay constant is reported without uncertainty. The U-232 and Pu-238 measured data have large uncertainties due to the limited experiments available. If the application contains large amount of U-232 and Pu-238 isotopes, the authors recommend replacing their data with more accurate computational/measured data if available.

B. Tables of Kinetic Parameters for PWR

In this appendix, the mean and standard deviation for kinetic parameters are reported at various burnup steps for the PWR lattice described in Figure 3(b). Two sets are given: the first includes kinetic parameters' uncertainties without considering correlation between DND, while the second set includes the correlation matrix given in Table 4 for U-235 thermal fission data.

C. Correlation Matrices of Kinetic Parameters

Correlation matrices are given at six different depletion steps in Table 8 based on uncorrelated DND and in Table 9 based on correlated U-235 thermal fission data. The correlation coefficients are based on the PWR lattice and they can be used for other LWR designs, since their correlation coefficients are close. The correlation matrices presented in Table 8 should be used in conjunction with the data in Table 10, while the correlation matrices presented in Table 9 should be used with the data in Table 6.

Data Availability

As far as the data availability, the code inputs and kinetics data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

- [1] H. C. Frey and S. R. Patil, "Identification and review of sensitivity analysis methods," *Risk Analysis*, vol. 22, no. 3, pp. 553–578, 2002.
- [2] E. Borgonovo and E. Plischke, "Sensitivity analysis: a review of recent advances," *European Journal of Operational Research*, vol. 248, no. 3, pp. 869–887, 2016.
- [3] B. M. Adams, W. Bohnhoff, K. Dalbey et al., "Dakota, a multi-level parallel object-oriented framework for design optimization, parameter estimation, uncertainty quantification, and sensitivity analysis: version 5.0 user's manual, Sandia National Laboratories," Tech. Rep. SAND2010-2183.
- [4] O. L. Matre and O. M. Knio, *Spectral Methods for Uncertainty Quantification: with Applications to Computational Fluid Dynamics*, Springer Science & Business Media, 2010.
- [5] C. J. Díez, O. Buss, A. Hoefer, D. Porsch, and O. Cabellos, "Comparison of nuclear data uncertainty propagation methodologies for PWR burn-up simulations," *Annals of Nuclear Energy*, vol. 77, pp. 101–114, 2015.
- [6] G. R. Keepin, T. F. Wimett, and R. K. Zeigler, "Delayed neutrons from fissionable isotopes of uranium, plutonium, and thorium," *Physical Review A: Atomic, Molecular and Optical Physics*, vol. 107, no. 4, pp. 1044–1049, 1957.
- [7] R. J. Tuttle, "Delayed-Neutron Data for Reactor-Physics Analysis," *Nuclear Science and Engineering*, vol. 56, no. 1, pp. 37–71, 1975.
- [8] A. H. Snell, V. A. Nedzel, H. W. Ibser, J. S. Levinger, R. G. Wilkinson, and M. B. Sampson, "Studies of the delayed neutrons. I. The decay curve and the intensity of the delayed neutrons," *Physical Review A: Atomic, Molecular and Optical Physics*, vol. 72, no. 7, pp. 541–544, 1947.
- [9] F. De Hoffmann and B. T. Feld, "Delayed neutrons from Pu239," *Physical Review A: Atomic, Molecular and Optical Physics*, vol. 72, no. 7, pp. 567–569, 1947.
- [10] G. D. Spriggs, J. M. Campbell, and V. M. Pikaikin, "An 8-group delayed neutron model based on a consistent set of half-lives," *Progress in Nuclear Energy*, vol. 41, no. 1-4, pp. 223–251, 2002.
- [11] J. M. Campbell and G. D. Spriggs, "8-group delayed neutron spectral data for Hansen-Roach energy group structure," *Progress in Nuclear Energy*, vol. 41, no. 1-4, pp. 253–283, 2002.
- [12] M. I. Radaideh, W. A. Wieselquist, and T. Kozłowski, "A new framework for sampling-based uncertainty quantification of the six-group reactor kinetic parameters," *Annals of Nuclear Energy*, vol. 127, pp. 1–11, 2019.
- [13] B. T. Rearden and M. A. Jessee, "SCALE code system," Tech. Rep., Oak Ridge National Lab.(ORNL), Oak Ridge, TN, USA, 2018.
- [14] M. I. Radaideh, W. Wieselquist, and T. Kozłowski, "Sensitivity and uncertainty analysis of the fundamental delayed neutron data in lwrs," in *Proceedings of the BEPU 2018: Best Estimate Plus Uncertainty International Conference 2018*, Lucca, Italy, 2018.
- [15] M. I. Radaideh, W. Wieselquist, and T. Kozłowski, "Variance-based sensitivity analysis of the reactor kinetic parameters," in *Proceedings of the PHYTRA4: The Fourth International Conference on Physics and Technology of Reactors and Applications*, pp. 786–797, Marrakech, Morocco, 2018.

- [16] K. Ivanov, M. Avramova, S. Kamerow et al., *Benchmark for Uncertainty Analysis in Modeling (UAM) for Design, Operation And Safety Analysis of LWRs*, vol. 1, OECD Nuclear Energy Agency, 2013.
- [17] S. Cox, P. Fields, A. Friedman, R. Sjoblom, and A. Smith, "Delayed neutrons from the spontaneous fission of Cf252," *Physical Review A: Atomic, Molecular and Optical Physics*, vol. 112, no. 3, pp. 960–963, 1958.
- [18] S. A. Cox, "Delayed-neutron studies from the thermal-neutron-induced fission of pu241," *Physical Review A: Atomic, Molecular and Optical Physics*, vol. 123, no. 5, pp. 1735–1737, 1961.
- [19] A. N. Gudkov, V. M. Zhivun, A. V. Zvonarev et al., "Measurement of the delayed-neutron yields in the fission of 233U, 236U, 237Np, 240Pu, and 241Pu by neutrons from the spectrum of a fast reactor," *Soviet Atomic Energy*, vol. 66, no. 2, pp. 115–118, 1989.
- [20] D. J. Loaiza, G. Brunson, R. Sanchez, and K. Butterfield, "Measurements of absolute delayed neutron yield and group constants in the fast fission of 235U and 237Np," *Nuclear Science and Engineering*, vol. 128, no. 3, pp. 270–277, 1998.
- [21] H. H. Saleh, T. A. Parish, S. Raman, and N. Shinohara, "Measurements of delayed neutron decay constants and fission yields from 235U, 237Np, 241Am, and 243Am," *Nuclear Science and Engineering*, vol. 125, no. 1, pp. 51–60, 1996.
- [22] G. Rudstam, P. Finck, A. Filip, A. D'Angelo, and R. McKnight, "Delayed neutron data for the major actinides," Tech. Rep., Nuclear Energy Agency, Organization for Economic Co-Operation and Development, 2002.
- [23] W. S. Charlton, T. A. Parish, and S. Raman, "Preliminary pulsing experiments to measure delayed neutron emission parameters," Tech. Rep., Oak Ridge National Lab, 1998.
- [24] M. Andoh, "The recent measurements of delayed neutron emission from minor actinide isotopes conducted at Texas A & M University," Tech. Rep., 1999.
- [25] R. W. Waldo, R. A. Karam, and R. A. Meyer, "Delayed neutron yields: Time dependent measurements and a predictive model," *Physical Review C: Nuclear Physics*, vol. 23, no. 3, pp. 1113–1127, 1981.
- [26] A. C. Wahl, "Nuclear-charge distribution and delayed-neutron yields for thermal-neutron-induced fission of 235U, 233U, and 239Pu and for spontaneous fission of 252Cf," *Atomic Data and Nuclear Data Tables*, vol. 39, no. 1, pp. 1–156, 1988.
- [27] W. B. Wilson and T. R. England, "Delayed neutron study using ENDF/B-VI basic nuclear data," *Progress in Nuclear Energy*, vol. 41, no. 1-4, pp. 71–107, 2002.
- [28] G. R. Keepin, *Physics of Nuclear Kinetics*, Addison-Wesley Pub. Co., 1965.
- [29] W. Charlton, T. Parish, S. Raman, and N. Shinohara, "Measurements of delayed neutron decay constants and fission yields from 235U, 237Np, 241Am and 243Am," in *Proceedings of the International Conference on Nuclear Data for Science and Technology, Trieste for the Italian Physical Society*, 1997.
- [30] D. J. Loaiza and F. E. Haskin, "Dominant delayed neutron precursors to model reactivity predictions for multiple fissioning nuclides," *Nuclear Science and Engineering*, vol. 134, no. 1, pp. 22–36, 2000.
- [31] P. Santi and M. Miller, "Reevaluation of prompt neutron emission multiplicity distributions for spontaneous fission," *Nuclear Science and Engineering*, vol. 160, no. 2, pp. 190–199, 2008.
- [32] S. M. Bowman, "SCALE 6: comprehensive nuclear safety analysis code system," *Nuclear Technology*, vol. 174, no. 2, pp. 126–148, 2011.
- [33] M. I. Radaideh, *A novel framework for data-driven modeling, uncertainty quantification, and deep learning of nuclear reactor simulations [Ph.D. thesis]*, University of Illinois at Urbana Champaign, 2019.
- [34] I.-A. Kodeli, *Sensitivity and uncertainty in the effective delayed neutron fraction (β_{eff})*, *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, vol. 715, 2013.
- [35] A. Aures, F. Bostelmann, I. A. Kodeli, K. Velkov, and W. Zwermann, "Uncertainty in the delayed neutron fraction in fuel assembly depletion calculations," in *Proceedings of the EPJ Web of Conferences*, vol. 146, p. 02052, EDP Sciences, 2017.