

## Research Article

# The Study of Three-Dimensional Granular Stream Flowing through the Test Hopper-Shaped Target

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The experiments are carried out in a three-dimensional channel with a screw conveyor, which plays the role of granular drives for the granular flow system and determines the injection of granular in the test target section. The jam-to-dense transition of granular flow is studied with the different inclination angle. The results show that, with a fixed diameter of hopper orifice and initial filling position, there is a change from jam to dense when the inclination angle larger than  $22^\circ$ . Variation of the flow rate with elevated frequency of the screw conveyor is further studied. The flow pattern is changed from dilute to dense with increasing rotation frequency of the screw rod. When the rotation frequency is larger than 5 Hz, the flow is dense. The dynamic balance of the interface between dilute to dense granular is observed in the main target section. We further research the dynamic interface by measuring the highest and lowest location with time and also simulate the gravity flow rate and screw conveyor flow rate with EDEM. From the results, we find that the interface between dilute flow and dense flow is influenced by the combined action of crew conveyor flow and dense gravity flow.

## 1. Introduction

An accelerator-driven subcritical system (ADS) is a subcritical blanket driven by neutrons produced when a high-intensity proton beam bombards a high-power spallation target, and the concept has been developed over 30 years [1–4]. The spallation target, usually compared to the heart of the accelerator-driven subcritical system (ADS), produces primary neutron sources for the subcritical core. In the early days of spallation target development, several solid targets at about 100-KW beam power levels were operated during the 1980s. In these solid targets, heat removal was limited by the heat conduction of the target material and by convection cooling. Therefore, the ability to increase the target power was limited. If the heat deposited can be dealt offline, the upper limit of the target power will be increased significantly. Based on this concept, there are various kinds of targets that have been designed, constructed, and operated,

such as heavy liquid metal targets in about 1-MW megawatt pilot experiment (MEGAPIE) and spallation neutrons sources (SNS) projects [5, 6].

In China, the Chinese initial accelerator-driven subcritical system (C-ADS) project, run by the Chinese Academy of Science, started in 2011 and the ultimate objective for this program is to run a 1000 MW subcritical core driven by a target under 10 mA/1.5 GeV proton beam in the Continuous Wave (CW) operation mode [7, 8]. Recently, a 4.3 mA/5.3 MeV CW proton beam has been successfully obtained [9]. So far, following the various spallation targets mentioned above, the researcher of the Institute of Modern Physics (IMP) proposes a new concept for a high-power spallation target: the gravity-driven dense granular target, which has a compact structure and the potential for high-power operation [7]. The target material is a large collection of discrete solid particles, namely, granular materials which are chosen as the coolant at the same time, bearing higher

beam power. This target is expected to be attractive in the aspects of very high heat removal capacity and long operation life, as well as low chemical toxicity and radioactive toxicity.

In the gravity-driven dense granular target, the flow pattern is the key issue when discussing the heat removal and the wear of the granular target. When the granular flow is dilute, the wear of the grains in the elevator will be considerable large due to different kinds of collisions; when the flow pattern is dense, the heat removal of the system is stable and continuous. In addition, there are very few studies on the effect of successive turning and incline tube in the granular flow system. As we know from the theory research about granular flow, the flow rate of the granular target through the bottlenecks fluctuates with time because of the forming and breaking of dynamic arches [10–14]. The experimental study on the three-dimensional granular through the turning and inclined pipe is missing. Our study can be a typical paradigm for the dilute-to-dense transition of granular flow under gravity form three-dimensional hopper.

This paper focus on the phase transition features in the granular hopper flow system with successive turning and incline tube. The experiments are carried out in a three-dimensional pipe with a screw conveyor which is used as granular drives. The screw conveyor actively supplies the energy to sustain the circulation. The jam-to-dense transition of granular flow is studied with the different inclination angle.

The incline angle “theta” is the angle between the inclined pipe and the horizontal plane. The flow pattern was changed from dilute to dense with increasing elevate frequency. The rotation of screw rod of screw conveys is driven by a motor. Therefore, we consider the frequency of the rotation of screw rod as a variable to investigate the transformation between dilute flow and dense flow. The dynamic balance of the interface between dilute to dense granular is observed in the main target section.

## 2. Experimental Setup

*2.1. Description of the Experiment Setup.* The experiments are carried out in a three-dimensional tube with a screw conveyor which is used as a granular drive. The screw conveyor is mounted on a platform with an angle of  $60^\circ$  to the horizon (see Figure 1). The upper part of the screw conveyor is connected to a reservoir that stores steel beads. The experiment target is established with specially shaped glass spacers between a metal base and a metal cover plate. To continuously measure the flow rate online, a microwave flow meter is applied here. When particles flow to the bottom of the loop, the screw conveyor lifts them back to the top and the spheres grains reenter into the test target section under gravity.

The schematic experimental loop is shown in Figure 1(b). At the beginning, spheres grains are poured into the screw conveyor feed inlet hopper. Then, the screw conveyor is worked; spheres are elevated by using a screw conveyor in the ascent stage, then through the reservoir and flow into the experiment hopper region under gravity from

the upper annular duct where beam pipe is located, and finally, through the inclination tube back to the feed inlet hopper of the screw conveyor.

From Figure 2, we can see that the geometry of the test target is similar to a cylindrical hopper with a conical outlet and a coaxial beam pipe inserted. The experiment hopper is connected to an entrance section with  $D_1 = 50$  mm. The main section target has a width  $D_2 = 40$  mm and length  $L_1 = 800$  mm. The main section connects with an exit section with a width  $D_3 = 15$  mm and length  $L_2 = 100$  mm. The out flow of the target section is controlled by a gate. The gate widths equal to  $D_3 = 15$  mm. At the final exit of the vertical section connects the tube with a width  $D_4 = 40$  mm and length  $L_3 = 100$  mm via a turning. A MF3000 microwave flow meter which is installed on this section is used to record the mass flow of steels falling out of the vertical main target. The flow meter has a precision more than 3% and repeatability more than 2%. The bottom of the hopper is connecting to an incline pipe with an angle of  $30^\circ$  to the horizon. The incline tube is connected to the feed inlet which shores initial steel beads. The diameter and mass of a single stainless steel bead is about  $1.5 \pm 0.001$  mm and 0.014 g, respectively.

The flow rate of the grains discharging from hoppers has been investigated through experiments and simulations for a long time [15–17]. When granular materials discharge from the hopper constantly by gravity, the flow rate is independent of the grains filling height, which is different with the fluids [16–18]. Also, the flow rate is independent of  $D$  if  $D$  is no  $< 2.5$  times of  $D_0$  and also greater than  $D_0 + 30d$ , where  $d$  is the diameter of the particle and  $D_0$  and  $D$  are the diameters of the orifice and hopper, respectively [18, 19]. Thus, the tube is designed as follows: (1)  $D_3 > 4d$ ,  $D_3$  is the diameter of the orifice at the bottom of the hopper, and  $d$  is the particle diameter. (2)  $L_1 > 2.5D_3$ ,  $L_1 > D_3 + 30d$ , and  $L_1$  is the length of the hopper. If the geometry is well designed, the flow rate of the hopper can be constant.

From the inset in Figure 2, we also can see that the spheres flow down along the annular section between hopper and beam pipe, while the beam goes through the inner one. The spheres are heated by beams while spallation reaction happens there and the deposited heat is removed. Particles flow steadily in such a structure, which behave similarly like flow of sand in an hourglass. The movement of the particles makes it possible to remove internal deposit heat continuously.

*2.2. Numerical Model and Parameters.* The EDEM has been widely used as an effective method to solve problems of granular materials in static packaging or flows [20–23]. We carry out Discrete Element Method (DEM) simulation on this work problem. Since the system involves dense flow state, the interactions between the particles are given by the Hertz–Mindlin contact model [24–26] and the integration of the motion equation is carried out by using the Velocity-Verlet scheme [27]. According to this model, the normal forces  $F_{ij_n}$  and tangential forces  $F_{ij_t}$  between particle  $i$ -particle  $j$  are formulated as

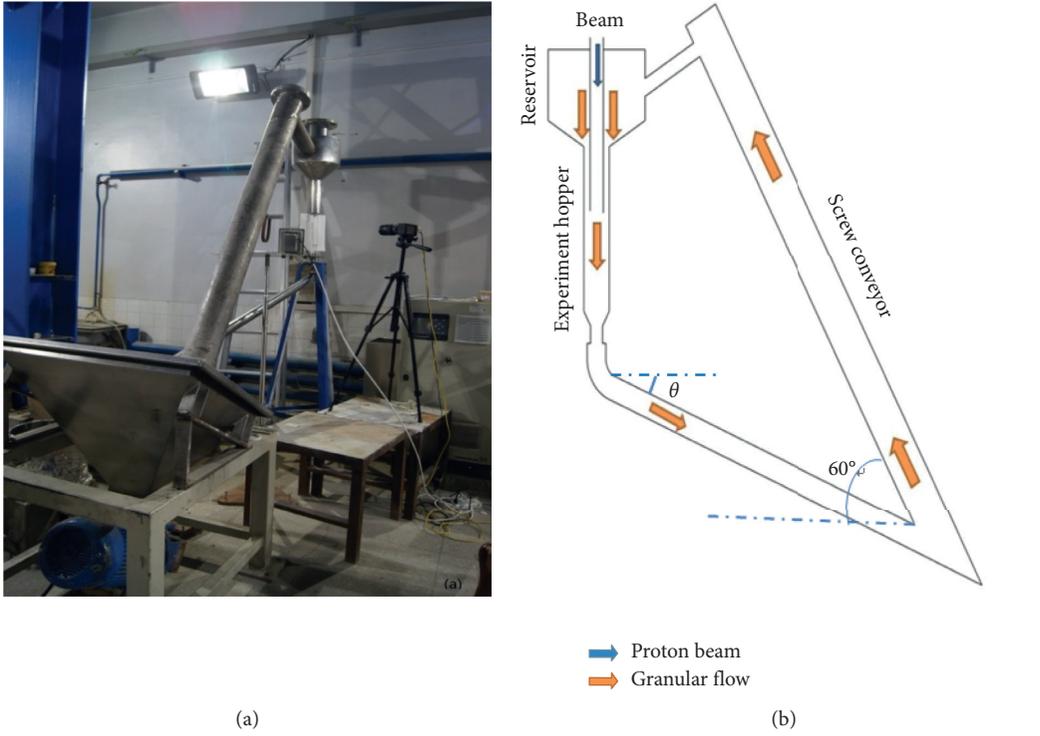


FIGURE 1: (a) The experimental loop of granular flow; (b) a schematic of the granular loop.

$$(1) \quad \begin{cases} F_{ij_n} = k_n \delta_{ij_n} - r_n v_{ij_n}, \\ k_n = \frac{4}{3} E \sqrt{2r \delta_{ij_n}}, \\ r_n = -2 \sqrt{\frac{5}{6}} \beta \sqrt{2E \sqrt{\frac{r}{2}} \delta_{ij_n}} m, \\ F_{ij_t} = k_t \delta_{ij_t} - r_t v_{ij_t}, \\ k_t = 4G \sqrt{2r \delta_{ij_t}}, \\ r_t = -2 \sqrt{\frac{5}{6}} \beta \sqrt{2E \sqrt{\frac{r}{2}} \delta_{ij_t}} m, \end{cases}$$

where  $E$  is Young's modulus,  $r$  is the radius of the particles,  $G$  is the shear modulus,  $\beta = \ln(e) / (\sqrt{\ln^2 + \pi^2})$  (where  $e$  is the coefficient of restitution of spheres),  $r$  is radius of the spheres, and  $m$  is the mass of one particle.  $\delta_{ij_n}$  and  $\delta_{ij_t}$  are normal and tangential displacement vectors, with  $\delta_{ij_n}$  and  $\delta_{ij_t}$  being their modulus, respectively.  $v_{ij_n}$  and  $v_{ij_t}$  are normal and tangential relative velocities between the granular  $i$  and  $j$ . If there is friction, the Coulomb yield criterion  $|F_{ij_t}| \leq |\mu_s F_{ij_n}|$  is satisfied by truncating the magnitude of  $F_{ij_t}$ . As a result, if  $|F_{ij_t}| \geq |\mu_s F_{ij_n}|$ ,  $|F_{ij_t}| = |\mu_s F_{ij_n}|$  [28].  $F_{ij_t}$  is the friction between two contacting particles  $i$  and  $j$ , and  $\mu_s$  is the particle-particle friction coefficient.

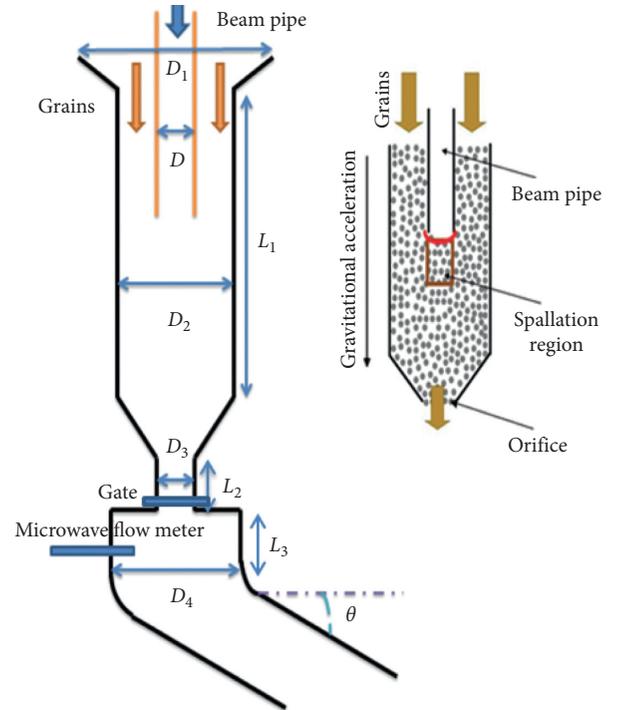


FIGURE 2: Experimental target model design.

Then, in gravity field, the equations of motion of the spheres  $i$  can be determined by Newton's second law of motion:

$$\begin{cases} m_i a_i = \sum_j (F_{ij_n} + F_{ij_t}) + m_i g, \\ I_i \dot{\omega}_i = \sum_j \left[ \frac{-r_i}{r_{ij}} \gamma_{ij} \times (F_{ij_n} + F_{ij_t}) \right], \end{cases} \quad (2)$$

where  $I_i$  is the moment of inertia of the sphere  $i$ . The parameters for the discrete element method simulation and the material properties are shown in Table 1.

Particle-wall friction coefficient and particle-particle friction coefficient are used in the simulations. These parameters cannot be determined accurately yet.

### 3. Experiment Observations and Simulation

#### 3.1. Influence of the Inclination Angle on Particle Mass Flow.

We first researched the experiment result of the flow rate change with the incline angle, and then, variation of flow rate with elevated frequency of the screw conveyor is further studied. From the results, we found that the dynamic balance of the interface between dilute to dense granular is observed in the main target section. The incline angle “theta” is the only adjustable parameter. As shown in Figure 2, the “theta” is the angle between the inclined pipe and the horizontal plane. In our experiment, the jam flow can be defined as the status where the flow rate is zero or approximate to zero, while the dense flow is the status where the flow rate will not change with the increasing  $\theta$ . The unstable region is the status where jam-to-dense transition of the flow rate increases with  $\theta$ .

Figure 3 shows the variations of the mean flow rate versus  $\theta$ . From the figure, we can see that when  $\theta \leq 20^\circ$ , the flow is jam, and the interaction of the particles is dominated by the surface friction factor. When  $\theta \geq 22^\circ$ , the flow is dense, and the interaction of the particles is dominated by multiparticle collision. Interestingly, the transition shows an unstable region at  $20^\circ \leq \theta \leq 22^\circ$ . In this region, the start flow state can be dense and the final flow can be jam. On the basis of the abovementioned discussion, the flow state of granular flow strongly depends on the inclination angle. The angle of transition between jam to dense is also affected by the particles density and the material friction coefficient.

The most wide experiments and simulations that predict the dense flow rate of grains through an orifice have been investigated for a long time. Now, the widely accepted conclusion about the flow rate is that its dependence on different parameters was proposed by Beverloo et al. [29]. The Beverloo law has form (3) for three-dimensional (3D) hoppers with round outlet and form (4) for two-dimensional (2D) hoppers.

$$W = C\rho_b \sqrt{g} (D - kd)^{2.5}, \quad (3)$$

$$W = C\rho_b \sqrt{g} (D - kd)^{1.5}, \quad (4)$$

where  $W$  is the average mass discharge rate through the orifice.  $C$  and  $k$  are empirical discharge and shape coefficients, respectively, and need to be determined

TABLE 1: DEM parameters and material properties in this work.

Parameter	Symbol	Value
Poisson’s ratio	$\nu$	0.25
Young’s modulus (GPa)	$E$	193
Shear modulus (GPa)	$G$	80
Dissipative constant	$\beta$	-0.016
Particle-wall friction coefficient	$\mu$	0.3
Particle-particle friction coefficient	$\mu_s$	0.3
Particle diameter	$d$	1.5 mm
Density	$\rho$	7850 kg/m <sup>3</sup>
Gravity	$g$	9.8 m/s <sup>2</sup>

Note: Poisson’s ratio, Young’s modulus, shear modulus, and coefficient of restitution.

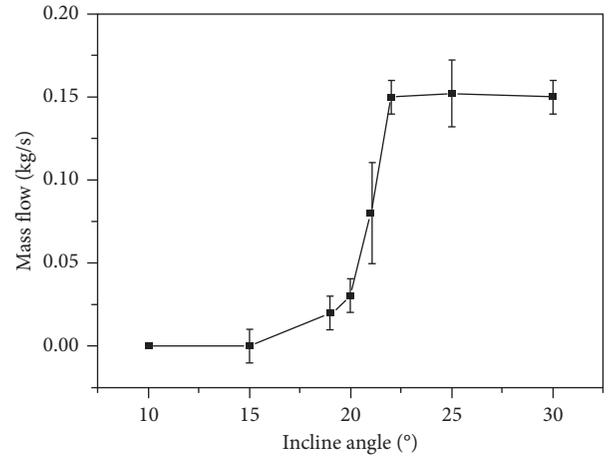


FIGURE 3: Variation of flow rate with an inclination angle.

experimentally. Usually, the so-called discharge coefficient  $C$  reflects the influence of material properties, and it was determined to be in a range between 0.55 and 0.65. The shape coefficient  $k$  is dependent on the particle shape and slope of the conical-bottom hopper. Nedderman and laokul [30] found the value of  $k$  to be approximately  $1.5 \pm 0.1$  for monosized spherical particles.  $\rho_b$  is the bulk density, and  $g$  is the acceleration of gravity.  $D$  is the diameter of the orifice at the bottom of the bopper, and  $d$  is the particle diameter. Equation (3) is known as the Beverloo law, and its validity has been tested for monosized granular samples with  $d$  larger than 0.5 mm and  $D$  big enough to avoid intermittencies in the flow due to jamming. This means that the flow rate of grains through orifices has been found to follow the Beverloo law only for  $D \gg d$ , well beyond the critical value below which the flow can be interrupted due to the formation of arches or domes [10]. The influence of the internal pipe can be neglected when the distance from the bottom of the pipe to the orifice is large than two times of the diameter of the hopper [31]. Although there have been numerous studies about hopper flow formerly, the details of flow in a three-dimensional hopper with a cycle loop is still unclear.

From the result, the mass flow rate which fluctuates with the change of time in dense flow was simulated with EDEM. As noticed in Figure 4, this behavior can be explained as follows: In the dense flow state, the flow of particles is caused

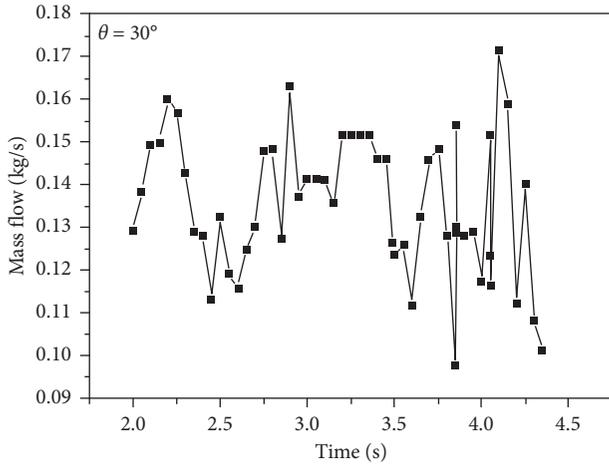


FIGURE 4: Simulation of the dense flow rate of the gravity-driven flow.

by continuous dynamic arches forming and breaking. The high fluxes in the time series correspond to the breaking of dynamic arches. Periodic variation of mass flow rate of the particles is caused by the periodicity of the arch camber. However, due to the randomness of the flow rate of the particles, the periodicity of the arch is uncertain. As a result, the wave curve is formed as shown in Figure 4. We use the Beverloo law for three-dimensional (3D) hoppers with a round outlet to calculate the average dense flow rate as follows:

$$W = C\rho_b\sqrt{g}(D - kd)^{2.5} = 0.58 * 7850 * 0.5 * \sqrt{9.8}(0.015 - 1.5 * 0.0015)^{2.5} = 0.135 \text{ kg/s}$$

From Figure 4, we can see that the average mass flow rate of the particles is about 0.14 kg/s. This is the same with which calculated by the Beverloo formula.

### 3.2. Influence of Elevated Frequency on Particle Mass Flow.

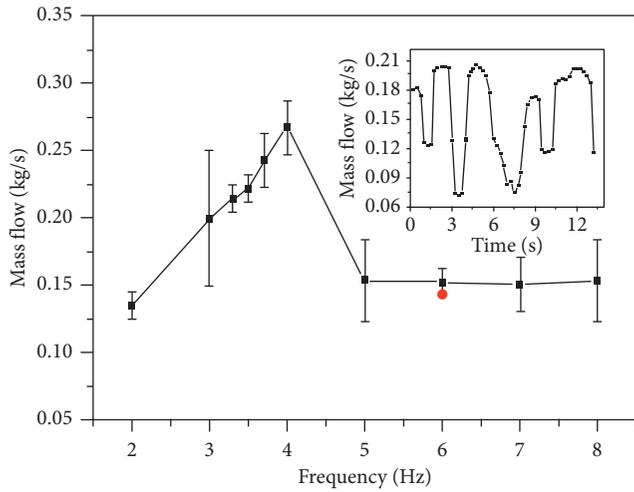
The inclination angle “theta” is fixed angle  $30^\circ$  to ensure that the flow rate of the particles through the incline pipe is dense. The variations of the mean flow rate versus elevator frequency of the screw conveyor ( $f$ ) are shown in Figure 5(a). The rotation of the screw rod of the screw conveyor is driven by a motor. Therefore, we consider the rotation frequency of the screw rod as a variable to investigate the transformation between dilute flow and dense flow. Also, we named the rotation frequency of the screw rod as elevator frequency of the screw conveyor. When the rotation frequency of the screw rod is small, the screw rod rotates slowly and the numbers of particles provided by the screw conveyor are relatively small; when the rotation frequency of screw rod is large, the screw rod rotates fast. But, the numbers of particles provided by the screw conveyor keep constant because the friction force of the particles on blades and the rotation force of the spiral blades will be balanced. The dilute-to-dense transition of granular flow can be seen in the experiment. It can be observed that mass flow increases with the elevator frequency increasing from 2 to 4 Hz and reaches a maximum value of 0.26 kg/s at  $f=4$  Hz. The flow is dilute flow, and the interaction of the particles is dominated by the two-particle

collision. The flow is dense when  $f \geq 5$  Hz. The interaction of the particles is dominated by multiparticle collision. From the inset in Figure 5(a), we can see the momentary values of the mass flow rate from the microwave flow meter in the dense flow condition. We calculated the average mass flow rate of the experiment as 0.15 kg/s. This is slightly greater than the theoretical value which was calculated by the Beverloo formula. The transition shows an unstable region at  $4 \leq f \leq 5$ , the start flow state can be dilute, and the final flow can be dense. On the basis of the abovementioned result, when  $f \geq 5$  Hz, the flow is dense, and when  $f \leq 4$  Hz, the flow is dilute. As noticed in Figure 5(b), the flow rate of the screw conveyor fluctuates with time due to the periodic rotation of the rod. Periodic variation of the mass flow rate strongly depends on the geometry structure of the rod of the screw conveyor when  $f=15$  Hz. The outlet diameter of the screw conveyor is 169 mm which is far greater than the outlet of the hopper. We also observed the dilute flow from the exit of the screw conveyor from the experiment when  $f < 4$  Hz. The average of the mass flow rate of the screw conveyor in the dense flow of the simulation is 0.16 kg/s. It is found that the experiment dense flow rate of the screw conveyor is equal to a value with which we averaged the dense gravity flow rate.

### 3.3. The Interface Morphology between Dilute Flow and Dense Flow.

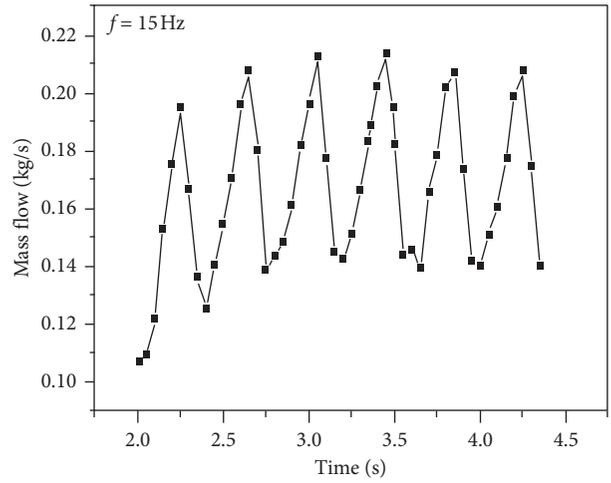
To illustrate the alternation between dense gravity flow and screw conveyor flow more clearly, we focus on the time interval from 2 s to 2.45 s. Figure 6 shows several typical snapshots during this time period. It is observed that the upper hopper is dilute and the lower part is dense. The upper part of the hopper is determined by screw conveyor flow, and the lower is determined by gravity flow. The screw conveyor flow is influenced by periodic rotation of the screw rod. The dense gravity flow rate is influenced by the periodic dynamic arch camber. At  $t=2$  s (see Figure 6(a)), the system is in the low mass flow rate of both dense gravity flow and screw conveyor flow. The interface between dilute flow and dense flow located at the lowest location that the ruler scale is 15 cm, and with the increase in time, the interface starts rising. Until  $t=2.25$  s (see Figure 6), the system is in high mass flow, and the interface lies on the highest location that the ruler scale is 24 cm. The different value ( $H$ ) between the highest and lowest location is 9 cm. We calculate the  $H$ ,  $H = Q_{\max} - Q_{\min} / \rho\pi(D_2/2)^2 * \Delta t$ , of the screw conveyor flow (see Figure 5(b)) and the dense gravity flow (see Figure 4), respectively. We calculate the different values ( $H$ ) between the highest and lowest location from the highest and lowest mass flow rate of the screw conveyor flow  $H_1=5.8$  cm and the dense gravity flow  $H_2=2.7$  cm. The value of  $H_1 + H_2$  is nearly equal to the experiment value  $H$ .

Figure 7 shows the simulation results between dense gravity flow and screw conveyor flow from 2.0 s to 2.5 s. It can be observed that the interface located on the highest location when the gravity flow and screw conveyor flow has the high mass flow. This is corresponding with Figure 6(d). On the contrary, when the interface lies in the lowest location, the gravity flow and screw conveyor flow have the low mass flow rate. This is corresponding with Figures 6(a) and 6(f). The



-- Experiment  
 -•- Theory

(a)



(b)

FIGURE 5: (a) Variation of the flow rate with elevated frequency. (b) Mass flow of the screw conveyor with time at  $f=15$  Hz simulated with EDEM.



(a)



(b)



(c)

FIGURE 6: Continued.

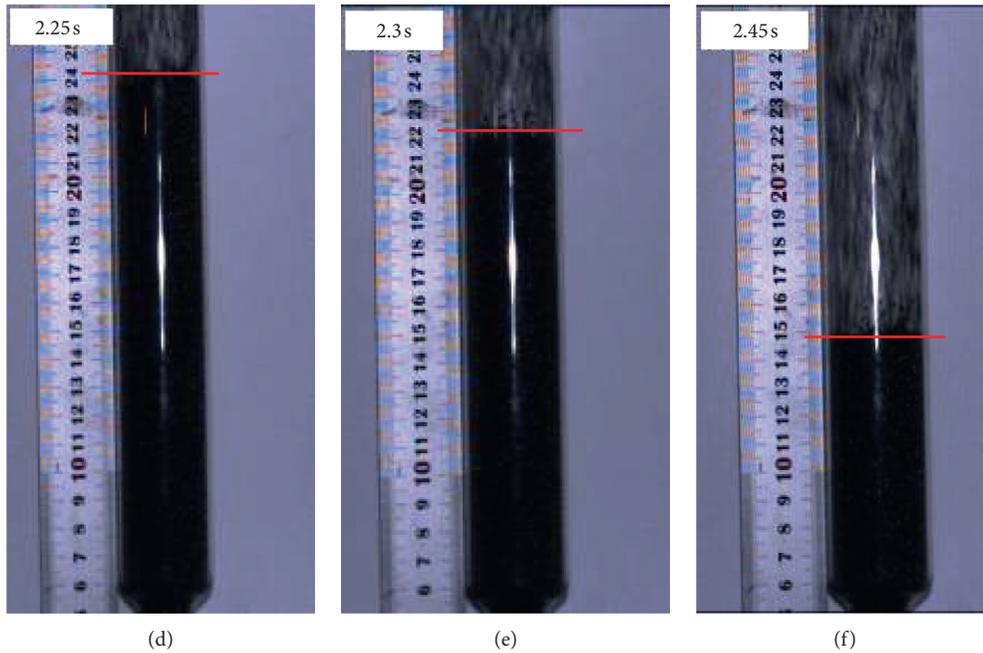


FIGURE 6: Experiment flow snapshots were taken with a high-speed camera: (a–d) interface between dilute flow and dense flow rose with time. (d–f) interface declined with time. (a) and (f) the lowest location of the interface. (d) The highest location of the interface. (b), (c), and (e) the middle location of interface at different time.

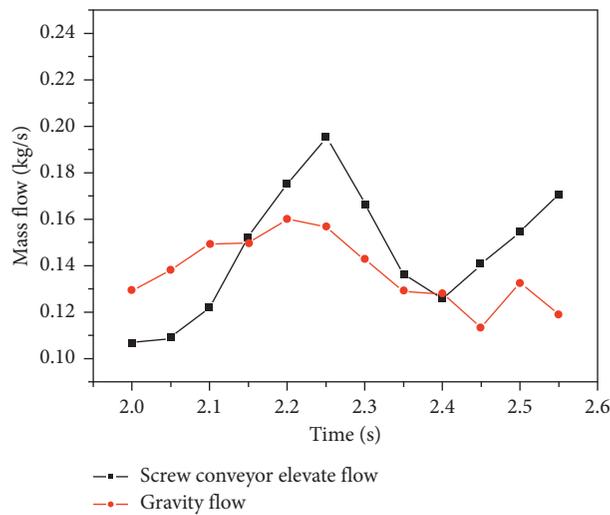


FIGURE 7: Mass flow rate between dense gravity flow and screw convey flow simulated with EDEM.

period interface between dilute flow and dense flow are influenced by the combined action of crew conveyor flow and dense gravity flow. The screw conveyor flow is influenced by periodic rotation of the screw rod. The period of the dilute flow has a relatively stable period due to the rotation of the screw shaft is stable. The period of dense flow is chaotic because the periodic dynamic arch camber is random. The periodicity of H is not a constant. Comparing Figure 7 with Figure 6, we can find that the simulation results are in qualitative agreement with experiment.

#### 4. Conclusions

In summary, the new concept for high-power spallation target-gravity-driven dense granular flow target which was proposed by the researcher of Institute of Modern Physics (IMP) is expected to use in the Chinese Accelerate Driven System (C-ADS) project. This paper studies the effect of inclination experiment target on the phase transition of granular flow. The granular flow shows a jam-to-dense transition at  $\theta = 22^\circ$ . The mass flow rate which fluctuates with

the time in dense flow is acquired from EDEM simulation. The dilute-to-dense transition of granular flow can be seen in the experiment with the increase in elevator frequency of the screw conveyor. When the frequency of the screw conveyor is larger than 5 Hz, the flow is dense. The dynamic balance of the interface between dilute to dense granular is observed in the main target section in the experiment. Periodic variation of the mass flow rate of the shaft of the screw conveyor is acquired from EDEM simulation. We find that the interface between dilute flow and dense flow is influenced by the combined action of the crew conveyor flow and gravity flow.

This work connects the elevator of the screw conveyor with the gravity-driven granular flow. The conclusion is very useful for MW spallation target for cost-effective ADS facilities. Also, results are also valuable for the optimization of transportation and processing of granular materials in industry and agriculture.

### Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

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### References

- [1] K. Furukawa, Y. Kato, T. Ohmichi et al., "The combined system of accelerator molten-salt breeder (AMSB) and molten-salt converter reactor (MSCR), Japan-us seminar on thorium fuel reactors (October 1982, Nara, Japan), thorium fuel reactors," *Journal of the Atomic Energy Society of Japan*, vol. 22, pp. 271–281, 1985.
- [2] C. D. Bowman, E. D. Arthur, P. W. Lisowski et al., "Nuclear energy generation and waste transmutation using an accelerator-driven intense thermal neutron source," *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, vol. 320, no. 1-2, pp. 336–367, 1992.
- [3] H. Nifenecker, S. David, J. M. Loiseaux, and O. Meplan, "Basics of accelerator driven subcritical reactors," *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, vol. 463, no. 3, pp. 428–467, 2001.
- [4] C. Rubbia, J. Alexandre, S. Andriamonje et al., "A European roadmap for developing accelerator driven systems (ADS) for nuclear waste incineration," ENEA Report, Energy and Sustainable Economic Development, Rome, Italy, 2001.
- [5] H. A. Abderrahim, J. Galambos, Y. Gohar et al., "Accelerator and target technology for accelerator driven transmutation and energy production," vol. 1, pp. 1–23, 2010, DOE White Paper on ADS, [http://science.energy.gov/~media/hep/pdf/files/pdfs/ADS\\_White\\_Paper\\_final.pdf](http://science.energy.gov/~media/hep/pdf/files/pdfs/ADS_White_Paper_final.pdf).
- [6] T. A. Gabriel, J. R. Haines, and T. J. McManamy, "Overview of the spallation neutron source (SNS) with emphasis on target systems," *Journal of Nuclear Materials*, vol. 318, pp. 1–13, 2003.
- [7] L. Yang and W. Zhan, "New concept for ADS spallation target: gravity-driven dense granular flow target," *Science China Technological Sciences*, vol. 58, pp. 1705–1711, 2010.
- [8] Z. Li, P. Cheng, H. Geng et al., "Physics design of an accelerator for an accelerator-driven subcritical system," *Physical Review Special Topics-Accelerators and Beams*, vol. 16, pp. 1–23, Article ID 080101, 2013.
- [9] W. Zhan, "Accelerator driven sustainable fission energy," in *Proceedings of the 7th International Particle Accelerator Conference (IPAC'16)*, JACOW, Busan, South Korea, pp. 4271–4275, May 2016.
- [10] D. Zuriguel, Y. Radzyner, and D. C. Rapaport, "Jamming during the discharge of grains from a soil described as a percolating transition," *Physical Review E*, vol. 68, pp. 30301–30304, 2003.
- [11] R. Delannay, M. Louge, P. Richard, N. Taberlet, and A. Valance, "Towards a theoretical picture of dense granular flows down inclines," *Nature Materials*, vol. 6, no. 2, pp. 99–108, 2007.
- [12] M. Hou, W. Chen, T. Zhang et al., "Global nature of dilute-to-dense transition of granular flow in a 2D channel," *Physical Review Letters*, vol. 91, pp. 24301–24304, 2003.
- [13] K. To, P.-Y. Lai, and H. K. Pak, "Jamming of granular flow in a two-dimensional hopper," *Physical Review Letters*, vol. 86, no. 1, pp. 71–74, 2001.
- [14] M. Hou, W. Chen, T. Zhang, and K. Lu, "Electric field controlled dilute-dense flow transition in granular flow through a vertical pipe," *Powder Technology*, vol. 135–136, pp. 105–111, 2003.
- [15] J. J. Drozd and C. Denniston, "Velocity fluctuations in dense granular flows," *Physical Review E*, vol. 78, Article ID 041304, 2008.
- [16] A. Tripathi and D. V. Khakhar, "Steady flow of smooth, inelastic particles on a bumpy inclined plane: hard and soft particle simulations," *Physical Review E*, vol. 81, Article ID 041307, 2010.
- [17] V. Kumaran and S. Maheshwari, "Transition due to base roughness in a dense granular flow down an inclined plane," *Physics of Fluids*, vol. 24, Article ID 053302, 2012.
- [18] A. Anand, J. S. Curtis, C. R. Wassgren, B. C. Hancock, and W. R. Ketterhagen, "Predicting discharge dynamics from a rectangular hopper using the discrete element method (DEM)," *Chemical Engineering Science*, vol. 63, no. 24, pp. 5821–5830, 2008.
- [19] H. Tao, B. Jin, W. Zhong et al., "Discrete element method modeling of non-spherical granular flow in rectangular hopper," *Chemical Engineering and Processing: Process Intensification*, vol. 49, no. 2, pp. 151–158, 2010.
- [20] N. V. Brilliantov, F. Spahn, J.-M. Hertzsch, and T. Pöschel, "Model for collisions in granular gases," *Physical Review E*, vol. 53, no. 5, pp. 5382–5392, 1996.
- [21] Y. Yu and H. Saxén, "Discrete element method simulation of properties of a 3D conical hopper with mono-sized spheres," *Advanced Powder Technology*, vol. 22, no. 3, pp. 324–331, 2011.
- [22] R. M. Nedderman, *Statics and Kinematics of Granular Materials*, Cambridge University Press, New York, NY, USA, 2005.

- [23] N. V. Brilliantov, N. Albers, F. Spahn, and T. Pöschel, "Collision dynamics of granular particles with adhesion," *Physical Review E*, vol. 76, Article ID 051302, 2007.
- [24] N. V. Brilliantov, A. V. Pimenova, and D. S. Goldobin, "A dissipative force between colliding viscoelastic bodies: rigorous approach," *EPL (Europhysics Letters)*, vol. 109, no. 1, Article ID 14005, 2015.
- [25] K. Saitoh, A. Bodrova, H. Hayakawa, and N. V. Brilliantov, "Negative normal restitution coefficient found in simulation of nanocluster collisions," *Physical Review Letters*, vol. 105, Article ID 238001, 2010.
- [26] B. C. Burman, "A discrete numerical-model for granular assemblies," *Geotechnique*, vol. 30, pp. 331–334, 1980.
- [27] L. E. Silbert, D. Ertas, G. S. Grest et al., "Granular flow down an inclined plane: bagnold scaling and rheology," *Physical Review E*, vol. 64, Article ID 051302, 2001.
- [28] P. A. Cundall and O. D. L. Strack, "A discrete numerical model for granular assemblies–reply," *Geotechnique*, vol. 30, pp. 335–336, 1980.
- [29] W. A. Beverloo, H. A. Leniger, and J. Van de Velde, "The flow of granular solids through orifices," *Chemical Engineering Science*, vol. 15, no. 3-4, pp. 260–269, 1961.
- [30] R. M. Nedderman and C. Laohakul, "The thickness of the shear zone of flowing granular materials," *Powder Technology*, vol. 25, no. 1, pp. 91–100, 1980.
- [31] J. Wan, S. Zhang, Y. Tian et al., "Influence of geometrical and material parameters on flow rate in simplified ADS dense granular-flow target: a preliminary study," *Journal of Nuclear Science and Technology*, vol. 53, no. 11, pp. 1–7, 2016.