Research Article

Inelastic Form Factor Calculations for $^{46,48,50}\text{Ti}$ Isotopes Using Tassie Model

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Inelastic form factors of electrical transition have been calculated for $^{46,48,50}\text{Ti}$ isotopes using the Tassie model. The form factors have been calculated for different exciting energies. The harmonic oscillator (HO) wave function has been used as a single-particle wave function. The model space has been considered as $1f7/2$, $2p3/2$, $2p1/2$, and $2f5/2$. $Gx1$ has been used as effective interaction in all calculations. In all calculations, the effective charge has been considered as $1.5e$ for proton and $0.5e$ for neutron. All obtained results have been compared with data from an experiment. The calculations show the Tassie model gives a good description of longitudinal form factors of $^{46,48,50}\text{Ti}$ isotopes in $E(2^+)$ transitions as compared with experimental data, especially in the region below 2 fm^{-1} of momentum transfer, but in the $E(4^+)$, the theoretical results deviated slightly from experimental data especially in the region greater than 1.5 fm^{-1} of momentum transfer.

1. Introduction

The gamma transitions and excitation of electrons in the nuclei have been considered in the Tassie model (TM) [1]. The form factors of the inelastic longitudinal scattering are calculated. This model is reduced to the normal liquid drop model for uniform load distribution. TM is a more elastic model that attempts a nonuniform loading and mass density distribution. The transition density depends on the density of the nucleus in the ground state, in these models [1]. The density of ground state for all occupied shells including the center is formulated according to single-particle charge density in TM. The nucleons restricted within $1f7/2$, $2p3/2$, $2p1/2$, and $2f5/2$ only. Comparisons between obtained theoretical results and experimental longitudinal electron scattering form factors have been used as a stringent test of the model for transition density. Microscopic and macroscopic theories can be used to study excitation in nuclei [2]. The shell model within a restricted model space (MS) is one of the models which usefully describe nuclei's static properties,

where an effective charge is used. Calculations of form factors using only model space wave function are not always successful in reproducing electron scattering data [3]. The MS has been considered as $1f7/2$, $2p3/2$, $2p1/2$, and $2f5/2$. $Gx1$ has been used as effective interaction in all calculations. The effective charge has been considered as $1.5e$ for proton and $0.5e$ for neutron. Different excitation energies are obtained in the calculation of the inelastic electrical transition of $^{46,48,50}\text{Ti}$ isotopes. The theoretical calculations are compared with experimental data taken from reference [4] and reference [5]. In all figures, the theoretical calculations are shown as continuous curves, and the experimental data are shown as a dotted curve. The single-particle wave function of the harmonic oscillator (HO) potential with size parameter b is used to reproduce the measured ground-state root mean square charges of the radii of nuclei under study. The one body density (OBD) matrix elements $\chi^\Lambda \Gamma_f \Gamma_i (\alpha_f, \alpha_i)$ have been calculated by using the OXBASH [6] as shell model code. This research was performed at the Laboratory of the department of Physics, University of Surrey, UK.

2. Theory

The core-polarization (CP) transition density is given by the Tassie form according to the collective modes of nuclei. The Coulomb form factors for the Tassie model become [1]

$$F_J^L(q) = \sqrt{\frac{4\pi}{2Ji+1}} \frac{1}{z} \left\{ \int_0^\infty r^2 j_J(qr) \rho_{Jts}^{ms}(r) dr + N \int_0^\infty dr r^2 j_J(qr) r^{J-1} \frac{d\rho_0(r)}{dr} \right\} F_{f.s}(q) F_{C.m}(q). \quad (1)$$

The radial integral $\int_0^\infty dr r^2 j_J(qr) r^{J+1} (d\rho_0(i, f, r)/dr)$ can be written as $\int_0^\infty (d/dr) \{ r^{J+1} j_J(qr) \rho_0(r) \} dr - \int_0^\infty dr (J+1) r^J j_J(qr) \rho_0(r) - \int_0^\infty dr r^{J+1} (d/dr) j_J(qr) \rho_0(r)$.

If the first term gives zero contribution, it is possible to add the second and third term together as $-q \int_0^\infty dr r^{J+1} \rho_0(r) [(d/d(qr)) + ((J+1)/qr)] j_J(qr)$. From the recursion of spherical Bessel function [7],

$$\left[\frac{d}{d(qr)} + \frac{J+1}{qr} \right] j_J(qr) = j_{J-1}(qr),$$

$$\int_0^\infty dr r^{J+1} j_J(qr) \frac{d\rho_0(r)}{dr} = -q \int_0^\infty dr r^{J+1} j_{J-1}(qr) \rho_0(r). \quad (2)$$

The form factor, therefore, takes shape [7]

$$F_J^L(q) = \left(\frac{4\pi}{2Ji+1} \right)^{1/2} \frac{1}{z} \left\{ \int_0^\infty r^2 j_J(qr) \rho_{Jts}^{ms}(r) dr - Nq \int_0^\infty dr r^{J+1} \rho_0 j_{J-1}(qr) \right\} F_{f.s}(q) F_{C.m}(q). \quad (3)$$

The constant proportionality can be measured using the form factor $q=k$.

$$N = \frac{\int_0^\infty dr r^2 j_J(kr) \rho_{Jts}^{ms}(r) - F_J^L z \sqrt{(2Ji+1/4\pi)}}{k \int_0^\infty dr r^{J+1} \rho_0(r) j_{J-1}(kr)}. \quad (4)$$

The form factors at $q=k$ is related to the strengths of the transitions $B(CJ)$. It can be shown as [7]

$$N = \frac{\int_0^\infty dr r^2 j_J(kr) \rho_{Jts}^{ms}(i, f, r) - \sqrt{(2Ji+1)B(CJ)/(2Ji+1)!!} k^J}{k \int_0^\infty dr r^{J+1} \rho_0(i, f, r) j_{J-1}(kr)}, \quad (5)$$

$$N = \frac{\int_0^\infty dr r^{J+2} j_J \rho_{Jts}^{ms}(i, f, r) - \sqrt{(2Ji+1)B(CJ)}}{(2Ji+1) \int_0^\infty dr r^{2J} \rho_0(i, f, r)}.$$

with $(2J+1)!! = (2J+1)(2J-1)!!$.

3. Results and Discussion

3.1. Form Factors of ^{46}Ti Nucleus

3.1.1. Longitudinal Form Factors of $E(2^+)$ Transition. The longitudinal form factors of electrical transition with excitation energy 0.889 MeV have been calculated. A comparison between the experimental and theoretical form factors of the $E(2^+)$ transition for ^{46}Ti is given in Figure 1(a). The theoretical form factors show three peaks. Figure 1(a) shows longitudinal $E(2^+)$ electron scattering form factors as a function of momentum transfer for ^{46}Ti . We notice that the first peak occurs at $q=0.6 \text{ fm}^{-1}$, the second peak at

$q=1.7 \text{ fm}^{-1}$, and the third peak at $q=2.6 \text{ fm}^{-1}$. It is noticed from Figure 1(a) that the TM gives very good results in the form factor of first and second peaks, and the third maximum gives good results. This indicates that the total form factors are very close to the practical data for the first peak $q < 1 \text{ fm}^{-1}$, while the second and third peaks are over-estimated. This explains that the TM is in good agreement with all momentum transfers.

3.1.2. Longitudinal Form Factors of $E(4^+)$ Transition. The longitudinal electrical transition form factors of ^{46}Ti of 2.010 MeV excitation energy have been calculated and compared with practical data. Figure 1(b) shows the form factor calculation of $E(4^+)$. This result shows two peaks. The

FIGURE 1: Inelastic longitudinal form factors of ^{46}Ti . (a) $E(2^+)$ transition, the practical data are taken from reference [4]. (b) $E(4^+)$ transition.

FIGURE 2: Inelastic longitudinal form factors of ^{48}Ti . (a) $E(2^+)$ transition, the practical data are taken from reference [4]. (b) $E(4^+)$ transition, the practical data are taken from reference [5].

calculation of this state is to ensure the theoretical form factors without the experimental data. The $E(4^+)$ gives two peaks in all momentum transfer extended from 1 to 3.

3.2. Form Factors of ^{48}Ti Nucleus

3.2.1. Longitudinal Form Factors of $E(2^+)$ Transition. The first maximum of longitudinal electrical transition form factors with excitation energy 0.984 MeV occurs at 0.65 fm^{-1} momentum transfer, the second maximum occurred at 1.5 fm^{-1} , and the third maximum occurred at 2.5 fm^{-1} of momentum transfer as shown in Figure 2(a). The first and second peaks of the theoretical calculations as shown as continuous curve are good agreement with experimental data which shows as dotted curve, but the third peak of theoretical calculations locates under the experimental data.

3.2.2. Longitudinal Form Factors of $E(4^+)$ Transition. Figure 2(b) shows good agreement with experimental data in the momentum transfer below 0.7 fm^{-1} of longitudinal electrical transition form factors with excitation energy 2.296 MeV. As shown in Figure 2(b), the theoretical result is slightly underestimated over 0.7 fm^{-1} momentum transfer.

3.3. Form Factors of ^{50}Ti Nucleus

3.3.1. Longitudinal Form Factors of $E(2^+)$ Transition. The theoretical data of this transition with excitation energy 1.554 MeV give three peaks. The first maximum occurred at 0.7 fm^{-1} , the second maximum occurred at 1.7 fm^{-1} momentum transfer, and the third maximum occurred at 2.5 fm^{-1} momentum transfer, as shown in Figure 3(a).

FIGURE 3: Inelastic longitudinal form factors of ^{50}Ti . (a) $E(2^+)$ transition, the practical data are taken from reference [4]. (b) $E(4^+)$ transition, the practical data are taken from reference [5].

Figure 3(a) shows the first peak of theoretical calculations locates over the experimental data up to 1.3 fm^{-1} , the second maximum give good agreement with experimental transfer extended from 1.5 fm^{-1} up to 1.8 fm^{-1} , and the third maximum fails to describe the form factors in the region 2.4 fm^{-1} up to 2.7 fm^{-1} of momentum transfer.

3.3.2. Longitudinal Form Factors of $E(4^+)$ Transition. The longitudinal electrical transition form factors of ^{50}Ti with excitation energy 2.657 MeV are given in Figure 3(b). All the theoretical calculations as shown in continuous curve in figure are over the experimental data which can be seen in Figure 3(b). This indicates that the theoretical form factors of $E(4^+)$ transition do not describe the experimental data because the Tassie model fails to describe $E(4^+)$ for ^{50}Ti isotope.

4. Conclusions

The Tassie model gives a good description of longitudinal form factors of $^{46,48,50}\text{Ti}$ isotope in $E(2^+)$ transitions as compared with experimental data especially in the region below 2 fm^{-1} of momentum transfer, but in the $E(4^+)$, the theoretical results deviate slightly from experimental data especially in the region greater than 1.5 fm^{-1} of momentum transfer.

Data Availability

The data are available from the Scopus website (<https://www.scopus.com/authid/detail.uri?authorId=57193342274>).

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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