Research Article

Coincidence Summing Factor Calculation for Volumetric $\gamma$-ray Sources Using Geant4 Simulation

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1. Introduction

To measure and analyze radioactive materials we need a high-resolution detector which is HPG, but when it is used to detect photons, the possibility of detecting more than one photon at the same time is possible especially with a multi-energy source, thus the CS effect is involved. The effect of CS usually loses peak count, so, corrected counts are needed to correct due to the CS effect. This problem exists when the distance between the radioactive sample (a source or environmental sample or nuclear materials that emit multiline radiation) and the detector is small. Also, if the size of the detector is large, the absorption of photons will be higher and therefore the CS effect will be present. ICRM was the first to report this effect in the 1980s [1].

The correction factor (CF) of the CS depends on the calculation of peak-total efficiency or direct total efficiency and that is discussed by many authors by different methods and techniques such as total efficiency calculation using the Monte Carlo code [2], matrix form equations to calculate the total efficiency [3, 4], the GESPECOR code [5, 6], the code KORSUM and modified KORSUM code [7, 8], and the EFFTRAN code using the efficiency transfer technique or ETNA program [9, 10]. Total efficiency calculations sometimes become more difficult, especially for volumetric radioactive sources, where the changing of source or detector shape needed different complicated equations lead to less accuracy and long time for calculations, so the current technique determine the CS correction factor for different geometric sources using two options of GS without total efficiency calculation.

Kajimoto et al. calculated the CS correction for $^{24}$Na point source in case of close detector-to-source geometry using EGS5 code Motor Carlo simulation [11]. Yucel et al. used a semiempirical formula to calculate the true CS corrections based on the total efficiency calculation and this method can be applied without any difficulty to Ge detectors for coincident nuclide [12]. Taibi et al. used the MCNP5 code...
Monte Carlo simulation to evaluate the true CS corrections for volumetric Eu-152 sources in gamma-ray spectroscopy, the results were confirmed with the TrueCoinc software, and a good agreement was obtained [13]. The problem of true CS correction for each of the point and volumetric sources was investigated through many previous works especially with the use of environmental sources or samples with low activity [14–25].

This work aims to utilize the Geant4 simulation (GS) to evaluate the correction factor for different volumetric ¹⁵² Eu sources (cylindrical and Marinelli beaker) using two options: monoenergetic and radionuclide tracks. In monoenergetic track (MT), the primary events have a monoenergetic energy while in a radionuclide track (RT), the complete decay scheme must be taken in the simulation and the primary events have multlines based on the decay scheme of radioactive sources.

### 2. Experimental Work

In this work, a coaxial HPGe spectrometry (Figure 1) has been used; its volume approximately was 110 cm³ with a wide range of energy from 40 kV to 10 MeV for γ-ray detection. The detector dimensions are tabulated in Table 1. As mentioned in the radioactive sources used as shown in Figure 2, different volumetric sources with different shapes filled with radioactive liquid, the activity of them referred to 15 Sep 1989 are tabulated in Table 2. The detector efficiency and energy was calibrated by using radioactive two point sources (Cs-137, Co-60, and Eu-152) before the measurement.

The measured time was too long to obtain high and sufficient counts under each peak (see Figure 3). The analysis of the spectrum is performed by the region of interest (ROI) selecting. This is defined by ROI or the area of interest. The determination of ROI by choosing the start and end channels of the peak as required. The analysis using Genie-2000 software provides integrated count calculation as well

#### Table 1: Main technical characteristics of HPGe detector provided by the company.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crystal diameter</td>
<td>48 mm</td>
</tr>
<tr>
<td>Crystal length</td>
<td>54.5 mm</td>
</tr>
<tr>
<td>End cap distance</td>
<td>5 mm</td>
</tr>
<tr>
<td>Entrance window</td>
<td>1.5 mm</td>
</tr>
<tr>
<td>Dead layer</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>The detector relative efficiency</td>
<td>24%</td>
</tr>
<tr>
<td>The resolution (FWHM) at 1.33 MeV</td>
<td>1.95 keV</td>
</tr>
</tbody>
</table>

Figure 1: Illustration of the experimental setup.
as the background count calculations depend on ROI. In addition, the Genie-2000 software has a big facility to carry out automatic search as well as peak area calculations using the changing in peak fitting using an interactive fit-peak.

The uncorrected measured photopeak efficiency of a certain energy \( E \) was evaluated by the following equation:

\[
\varepsilon_e(E) = \frac{N_e(E)/t}{P_y(E)Ae^{-\lambda T}},
\]

where, \( N_e(E) \) is the net measured counts of full-energy peak after subtracted by the background at energy \( E \), \( t \) is the actual time for experimental measurement, \( P_y(E) \) is the probability of photon emission at a given energy \( E \), \( A \) is the standard radionuclide activity in Bq, \( e^{-\lambda T} \) represent the decay factor, where \( T \) is the production time and \( \lambda \) represent the decay constant. The uncertainty in the full-energy peak (FEP) efficiency is denoted by \( \sigma_e \), and calculated by the following expression:

\[
\sigma_e = \varepsilon e \left( \left( \frac{\partial \varepsilon_e}{\partial A} \right)^2 \cdot \sigma_A^2 + \left( \frac{\partial \varepsilon_e}{\partial P} \right)^2 \cdot \sigma_P^2 + \left( \frac{\partial \varepsilon_e}{\partial N} \right)^2 \cdot \sigma_N^2 \right)^{1/2},
\]

where, \( \sigma_N, \sigma_P \), and \( \sigma_A \) are the associated uncertainties with the net count rate \( N \), emission probability \( P \), and activity \( A \), respectively. The corrected measured FEP efficiency is calculated by the following expression:

\[
\varepsilon_e = \frac{N_e(E)/t}{P_y(E)Ae^{-\lambda T}}\]
\[ \varepsilon_1 = \varepsilon_p \cdot \text{CF}, \tag{3} \]

where CF is the correction factor of the CS effect.

### 3. Geant4 Simulation (GS)

GS is a process of simulated radiation (photons, neutrons, protons, or electrons) and interact during materials. These simulations include multiple functions, including physical processes, geometry, analysis, and tracking. Physical processes contain multiple functions, including electromagnetic operations, interactions of radiation, and analyzes [17]. The simulation of the detector and the source is made in a large volume of the "world volume" as shown in Figure 4. Figure 4 represents the simulation of HPGe detector and cylindrical source and collimator. In this work, the simulated radiation was \( \gamma \)-ray photons with a radionuclide, including decay scheme (multiple energies) or using monoenergetic option (monoenergetic photons).

#### 3.1. Radionuclide Track (RT).

The decay scheme of radionuclides represents the primary events, where the decay scheme of Eu-152 source was simulated in details, including all energies emitted from this source. These energies were interacted with the HPGe crystal and the results of all interaction types (photoelectric, compton, and pair production) registered in a histogram file. After simulating the program, the run occurred with a number of events at least \( 10^7 \) events and the generated spectrum as shown in Figure 5 was obtained using root software. Gaussian distribution was used to fit each peak in the spectrum to calculate the area under this peak. The simulated FEP efficiency using the radionuclide track at certain energy, denoted by \( \varepsilon_1 \) and its equation is as follows:

\[ \varepsilon_1 = \frac{\text{Area under the peak}}{\text{Primary events} \cdot I_p}. \tag{4} \]

#### 3.2. Track 2: Monoenergetic Track (MT).

Monoenergetic gamma rays represent the primary events, where a single energy from Eu-152 source was simulated in details. These events with the sample energy were interacted with the HPGe crystal and the results of all interaction types (photoelectric, compton, and pair production) registered in a histogram file. After simulating the program, the run occurred with a number of events at least \( 10^7 \) events and the generated spectrum as shown in Figure 6 was obtained using root software. Gaussian distribution was used to fit each peak in the spectrum to calculate the area under this peak. To limit the error within 1\%, a number of particles (primary events) \( 10^7 \) are used in each simulation, otherwise the error has well gotten higher as the number goes less [15, 16]. The simulated FEP efficiency using the monoenergetic track at certain energy, denoted by \( \varepsilon_2 \) and its equation is as follows:

\[ \varepsilon_2 = \frac{\text{Area under the peak}}{\text{Primary events}}. \tag{5} \]

### 3.3. CF Calculation.

The correction factor of the CS can be estimated from the two options, where the radionuclide track contains multiple lines and is detected at the same time, so the possibility of coincidence summing exists, and in the second track (MT), just a monoenergetic line was detected or simulated so there is no coincidence summing.
The correction factor calculated by the following equation is as follows:

\[ CF = \frac{\varepsilon_2}{\varepsilon_1} \]  

**4. Results and Discussion**

First, the detector geometry and the radioactive source in [8] are modeled using the Geant4 code, where the detector crystal dimensions were 58.2 mm diameter and 79 mm length. The cylindrical source activity was 86.2 ± 1.5 kBq (date: 2013 May 1). The source is distributed uniformly inside a cylinder (100 mmφ, 149 mmH) filled with 0.1 M HCl and has an active volume of 1000 ml. The FEP efficiency was calculated by radionuclide and monoenergetic tracks, the net area under the peak was calculated and tabulated in Table 3, from this area and the number of primary events, the simulated FEP efficiency was calculated as shown in Figure 7. The FEP efficiency by using MT higher than the FEP efficiency by RT et al. discussed energies, where at 244.7 keV from 1000 ml cylindrical source, the FEP efficiency in by using MT was 0.01161, and the FEP efficiency by using RT was 0.009948, while at 1408.1 keV, the FEP efficiencies were 0.00301 and 0.00271, respectively.

The CF was calculated using equation 6 at different energies of the cylindrical $^{152}$Eu radioactive source. The calculation of CF in the present simulated technique was
compared with the modified KORSUM code [8], and the relative deviation ($\Delta\%$) was calculated by

$$\Delta\% = \left( \frac{CF_{\text{Geant4}} - CF_{\text{KORSUM}}}{CF_{\text{Geant4}}} \right) \times 100.$$  \hspace{1cm} (7)

The results were tabulated in Table 4 and showed an excellent compatibility between the two methods. This indicated that this simulation technique using Geant4 is correct to calculate the CS corrections by simple and faster computations.

Secondly, the FEP efficiency was determined experimentally using cylindrical and Marinelli beaker radioactive sources with different volume according to section 2. This efficiency was compared with the simulated FEP efficiency by using monoenergetic and radionuclide tracks as shown in Figure 8 for cylindrical sources and Figure 9 for Marinelli beaker sources. The results indicated that the experimental and radionuclide simulated FEP efficiency have almost the values energy ranges, but the monoenergetic simulated FEP efficiency is higher, that is due to the existence of the CS in experimental and radionuclide simulated methods.

Finally, from the ratio between the monoenergetic and radionuclide simulated FEP efficiency, the correction factor of the coincidence summing effect for the HPGe detector using all present radioactive sources was calculated in Table 5. It is clear that the lower volume has the higher correction factor, for example, the correction factor for cylindrical source C1 = 1.143 (400 ml) and for C2 (500 ml) is 1.112, that

### Table 4: Comparison between the present work and the modified KORSUM code and the deviation error between them.

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>Energy (keV)</th>
<th>Correction factor (CF)</th>
<th>Modified KORSUM [8]</th>
<th>Present work Geant4</th>
<th>$\Delta%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EC($\beta^+$)</td>
<td>121.78</td>
<td>1.1235</td>
<td>1.1229</td>
<td>$-0.05$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>244.7</td>
<td>1.1674</td>
<td>1.1671</td>
<td>$-0.03$</td>
<td></td>
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<tr>
<td></td>
<td>443.97</td>
<td>1.1375</td>
<td>1.138</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>867.38</td>
<td>1.17</td>
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<tr>
<td></td>
<td>964.082</td>
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<td>1.1161</td>
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<tr>
<td></td>
<td>1085.84</td>
<td>1.036</td>
<td>1.0361</td>
<td>$0.01$</td>
<td></td>
</tr>
<tr>
<td></td>
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<td>1.0978</td>
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<tr>
<td>$\beta^-$</td>
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<td>$-0.01$</td>
<td></td>
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<tr>
<td></td>
<td>1299.15</td>
<td>1.0756</td>
<td>1.0757</td>
<td>$0.01$</td>
<td></td>
</tr>
</tbody>
</table>

![Figure 8: The experimental, simulated RT, and simulated MT full-energy peak efficiency for different cylindrical volumetric 152Eu sources as a function of different $\gamma$-ray energies as a function of different $\gamma$-ray energies.](image)

![Figure 9: The experimental, simulated RT, and simulated MT full-energy peak efficiency for different Marinelli beaker volumetric 152Eu sources as a function of different $\gamma$-ray energies.](image)
is due to the absorption of source container. If the volume increases, the self-absorption will increase, and then the correction factor will decrease.

5. Conclusion

The FEP efficiency of HPGe detector was calculated by radionuclide and monoenergetic tracks in Geant4 simulation using different shapes and volumes radioactive sources. The correction factor of CS was evaluated based on the two simulated efficiencies by fast and accurate way compared with evaluating the correction factor by total efficiency calculation. The FEP efficiency by using MT is higher than the FEP efficiency by RT et al. discussed energies, where at 244.7 keV from 1000 ml cylindrical source, the FEP efficiency in by using MT was 0.01161 and the FEP efficiency by using RT was 0.009948, while at 1408.01 keV, the FEP efficiencies were 0.00301 and 0.00271, respectively. The CF was calculated for cylindrical and Marinelli beaker sources with different volumes. The lower volume has the higher correction factor in the same shape of radioactive sources, that is due to the self-absorption of the volumetric source.

Data Availability

All data are available in the manuscript.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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References

[16] E. Garcia-Torano, M. Pozuelo, and F. Salvat, “Monte Carlo calculations of coincidence-summing corrections for volume sources in gamma-ray spectrometry with Ge detectors,”

Table 5: The correction factor of the coincidence summing effect using different radioactive volumetric sources.

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>C1</th>
<th>C2</th>
<th>M1</th>
<th>M2</th>
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<td>1.136</td>
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<td>1.188</td>
<td>1.156</td>
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<td>1.119</td>
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<td>1.091</td>
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<td>1.106</td>
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