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Scaling for Shock Response of Equipment in Different Submarines

This article presents scaling rules developed to predict the response of submarine equipment subjected to underwater chemical explosions. The computer was used as a surrogate for shock tests. A simplified model of a hull section was used to contain frame-mounted single degree of freedom equipment. A general scaling rule has been developed to handle the spread in the shock response attributable to the charge weight, equipment weight, and equipment frequency, where the shock response is the absolute maximum acceleration of the equipment mass as a function of the shock factor for a given charge weight. The article also examines those cases where a new hull is derived from an original hull by the linear scaling law. The solution of the shock response is well known when the internal equipment has also been linearly scaled. A new general scaling rule is developed for those cases when the equipment is not linearly scaled, that is, the equipment and charge weight used with the original hull remains unchanged when installed in the linearly scaled hull or a completely different equipment and charge weight are used with the new hull. It is emphasized that the test sections were short and devoid of typical equipment present in a real compartment. The results, nevertheless, provide trends and ratios in shock design values, not necessarily absolute design numbers. The approach taken in developing these scaling rules could be useful for enhancing field data that may exist for a given class of boat to allow greater usage of these data for different equipment subject to a variety of charge weights, attack geometries, and other boats.

INTRODUCTION

The purpose of this article is to examine scaling relationships for submarine-like structures containing internal equipment subject to underwater chemical explosions. The internal equipment are modeled as single degree of freedom frame-mounted systems such that the weight ranges from 15 to 35 kips over a frequency range of 15 to 35 Hz. The charge weight ranges from 600 lb. to 3,625 lb. of TNT. First, the conventional linear scaling between different hulls is examined, where the hull geometry, equipment weight and frequency, and charge size are all scaled by a linear factor. Next, scaling rules are introduced for the case where the hull geometry remains fixed but the charge weight and equipment may vary. Finally, scaling rules are developed for the case where the hull geometry is scaled linearly but the equipment either remains unchanged or completely different equipment is installed in the new hull. The details of a 33-ft. hull, called model B, are shown in Fig. 1. The University of Maryland HULL code, which is similar to codes used in the past by the U.S. Navy in modeling submarine-like structures, is the principal means used in the creation of the mathematical models. This code calculates the time response of the equip-

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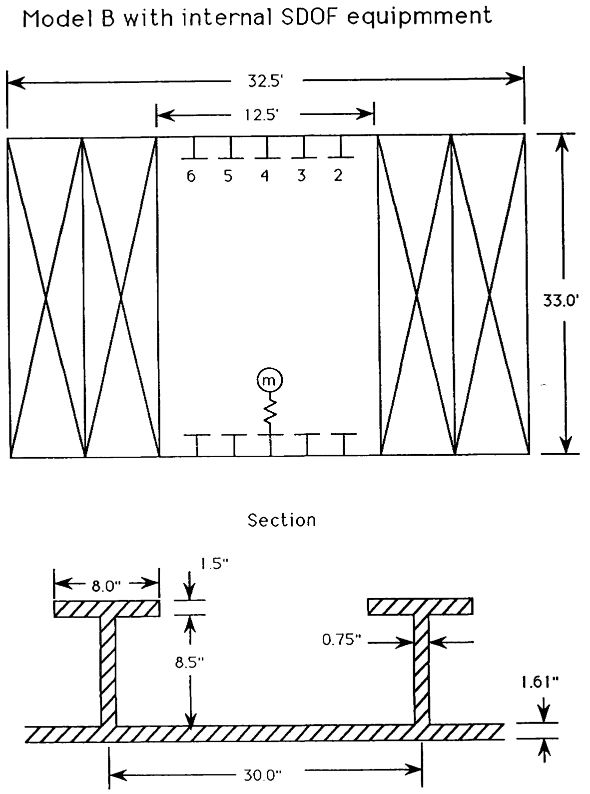

FIGURE 1 Details of model B.

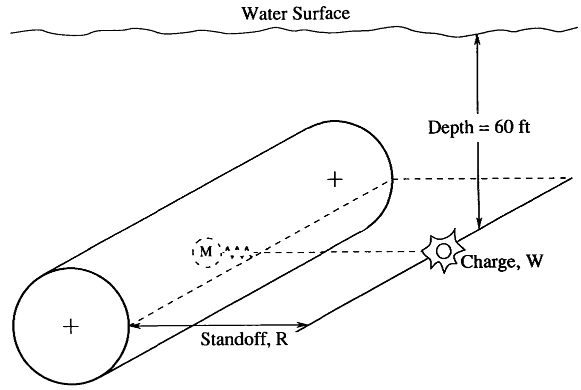

FIGURE 2 Schematic of the shot geometry.

ment and their base supports internal to a submarine-like, ring-stiffened pressure hull when the hull is subjected to an underwater chemical explosion. The pressure hull, the underwater explosion, and the fluid–structure interaction are all modeled with sufficient detail to provide a realistic environment for the study of shock excited internal equipment. It is noted that model B has more than 1,100 degrees of freedom.

The absolute acceleration of the equipment mass as a function of the shock factor for a given charge weight is used as the measure of response, and its variation is examined to establish trends that may affect equipment design. Figure 2 is a schematic of the shot geometry where the depth of the center line of the hull and the charge are always held at 60 ft. so that the cavitation pressure remains the same in all cases. Neutral buoyancy is always maintained.

The measure of the shock intensity used herein is the square root of the acoustic approximation of the energy flux density, or shock factor SF, where

$$\mathrm{SF} = \frac{\sqrt{Q}}{R} \tag{1}$$

as shown by Cole (1948); Q is the charge weight in pounds of TNT, and R is the distance in feet between the hull and the charge.

LINEAR SCALING

Linear scaling is the process whereby a new hull is sized from an existing hull by the linear ratio of the hull diameters. Thus, the hull thickness and all other scantlings are scaled by the same scaling factor. If d is the diameter of the prototype hull, and D is the diameter of the scaled hull, then

$$D = Ld \tag{2}$$

where L is the linear scaling factor. This scaling factor is also used to scale the kinematic descriptors, the charge weight, standoff distance, and shock factor as summarized in Table 1. Note that the scaled quantities are expressed in uppercase notation.

The general linear scaling law was applied to model B that represented the prototype hull containing 20-kip, 20-Hz equipment as shown in Fig. 1. The velocity of the equipment relative to its moving base point was obtained for a 1740-lb. charge weight and a shock factor of 0.15. Two linearly scaled models were constructed from model B; model SBS in which $L = 26/33$, and model LBS in which $L = 40/33$. The equipment weight, equipment frequency, charge weight, and shock factor for the SBS model were calculated as 9.782 kips, 25.385 Hz, 851 lb., and 0.133, respectively. The corresponding values for model LBS, were 35.618 kips, 16.50 Hz, 3098.8 lb., and 0.165. An overlay of the relative velocity response of each equipment in models B, SBS,

Table I Scaling Relationships

Time:	$T = Lt$	Frequency:	$F = f/L$
Velocity:	$V = v$	Acceleration:	$A = a/L$
Charge weight:	$Q = L^3q$	Shock factor:	$\mathrm{SF} = \sqrt{L}\mathrm{SF}$

and LBS, is shown in Fig. 3. Note that time was scaled for the response of the SBS and SBL models. The closeness of fit of these responses supports the validity of the general linear scaling law as well as the adequacy of the performance of the HULL code.

CHARGE WEIGHT SCALING

Consider an equipment attached to a given hull as shown in Fig. 1 subject to a fixed charge weight and a varying shock factor. Figure 4 is a plot of two typical linear least squares fits through the equipment response. As the charge weight changes, the slope of the line through the data also changes. The acoustic pressure appears to be the key variable that affects the equipment response. A scaling rule was constructed by dividing the slope of the line representing the least squares fit of the data for the charge weight Q_a by the acoustic pressure, where the acoustic pressure is defined as $Q_a^{3/8}/R$. Thus,

$$s_a \frac{R}{Q_a^{(3/8)}} = s_a \frac{Q_a^{(1/8)}}{SF} \tag{3}$$

where s_a is the slope of the least square line for charge weight Q_a. For equal shock factor, the slope s_b for charge weight Q_b, is related to the slope for charge weight Q_a as

$$s_b = s_a \left(\frac{Q_a}{Q_b}\right)^{(1/8)}. \tag{4}$$

By way of example, consider Fig. 5(a) that shows the response data plotted for model B with 20-kip, 20-Hz equipment mounted to the hull frame. Straight lines join the data points for charge weights ranging from 600 to 3,625 lb. Figure 5(b) shows the linear least squares fit for each charge weight data in Fig. 5(a). Figure 5(c) shows the results if all of the slopes are scaled to the 1,160-lb. charge by the scaling rule of Eq. (4).

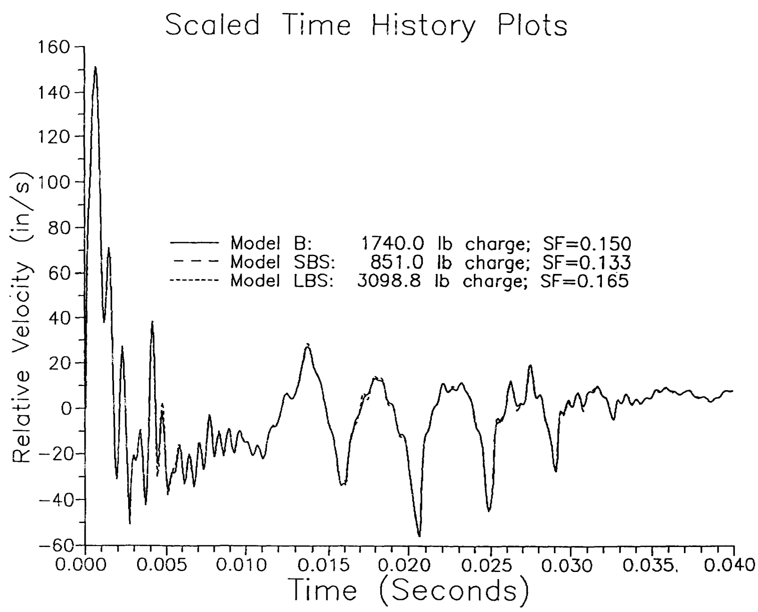

FIGURE 3 Relative velocity response of each equipment in models B, SBS, and LBS.

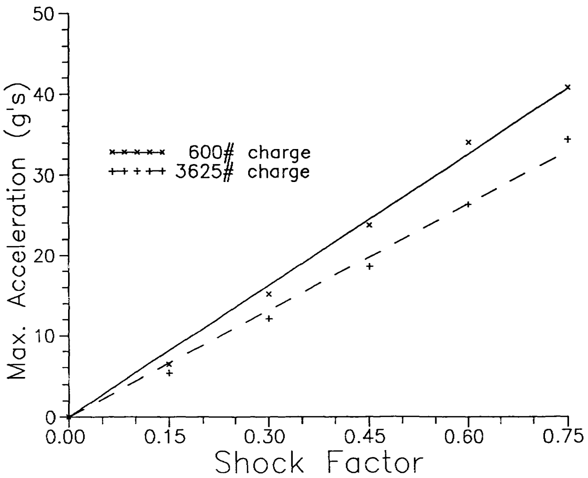

FIGURE 4 Typical linear least squares fit through response data.

EQUIPMENT WEIGHT SCALING

Consider the following general scaling rule to find the response slope s_b for equipment weight W_b knowing the slope s_a for equipment weight W_a subject to the condition that the charge weight and equipment frequency remain unchanged:

$$s_b = s_a\left(\frac{W_a}{W_b}\right)^n. \qquad (5)$$

One may reason that because two cubes of different weight scale by a factor of 1/3 and the response data are plotted as a function of the square root of the charge weight as shown in Eq. (1), $n = 1/6$ might prove to be a good estimate for the exponent n. This scaling rule was examined over a wide range of conditions for model B. The equipment weight ranged from 15 to 35 kips while the equipment frequency was held to either 20 or 30 Hz. Three charge weights and five shock factors ranging from 0.15 to 0.75 in 0.15 increments were used. The data include the actual peak acceleration response and the slope of the least squares fit through the response data for the given charge weight. The exponent n in Eq. (5) that satisfied the actual slopes was calculated for all combinations of the weight ratios. In the case of the 20-Hz equipment, an overall average of the data for the three charge weights produced $n = 0.1648$, and $n = 0.2417$ for the 30-Hz equipment.

Equation (5) was tested against the data by using $n = 1/6$ and the 25-kip equipment as the

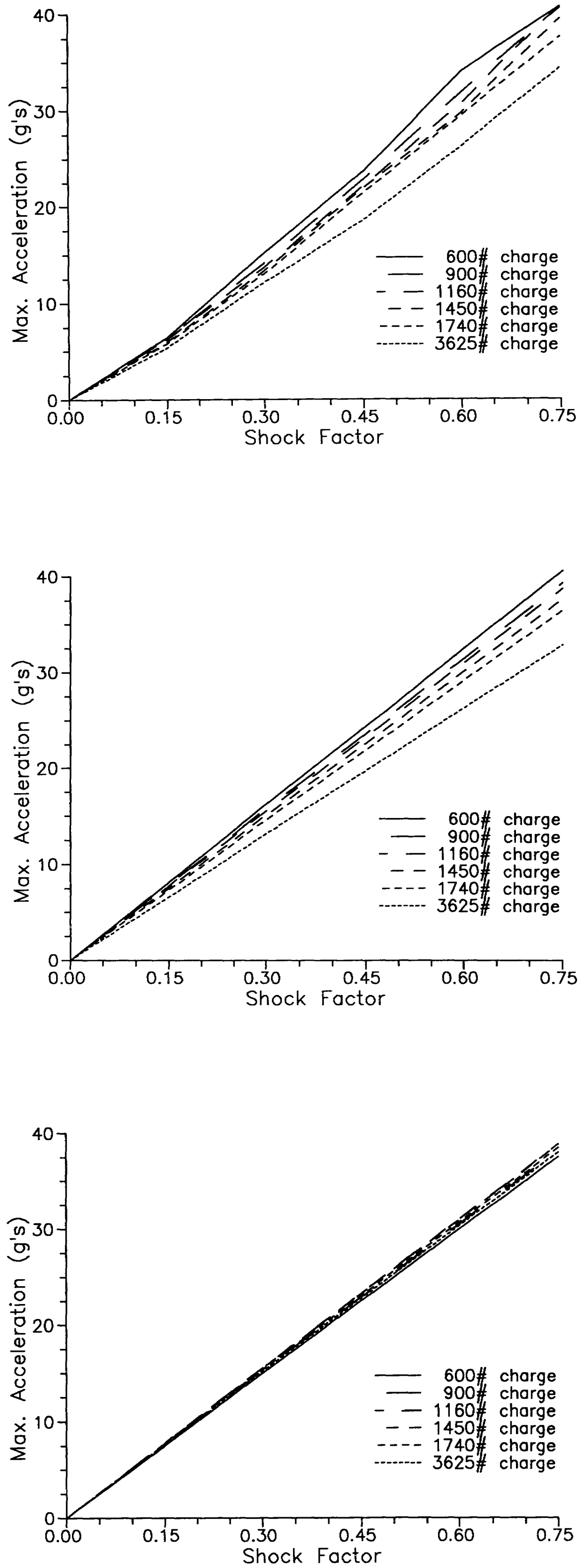

FIGURE 5 (a) Response data for fixed equipment weight and frequency, varying charge weights; (b) linear least squares fit for each charge weight; (c) slopes scaled to the 1160-lb. charge weight by Eq. (4).

Table II Scaled Slopes for Model B Scaled on 25 Kip Using $n = 1/6$

	1160#	Percent Error	1740#	Percent Error	3625#	Percent Error
			20 Hz			
15k	49.6304	0.19	46.5146	0.88	41.9986	1.77
20k	49.7928	0.13	46.8593	0.14	42.2374	1.21
25k	49.7273	0.00	46.9253	0.00	42.7534	0.00
30k	49.2088	1.04	46.8408	0.18	43.0428	0.68
35k	48.6975	2.07	46.5949	0.70	43.1867	1.01
			30 Hz			
15k	94.6099	1.87	90.3958	1.52	83.8887	1.22
20k	94.1621	1.39	90.1273	1.22	83.8633	1.19
25k	92.8713	0.00	89.0394	0.00	82.8755	0.00
30k	90.9845	2.03	87.5623	1.66	81.4362	1.74
35k	88.9039	4.27	85.5378	3.93	79.7023	3.83

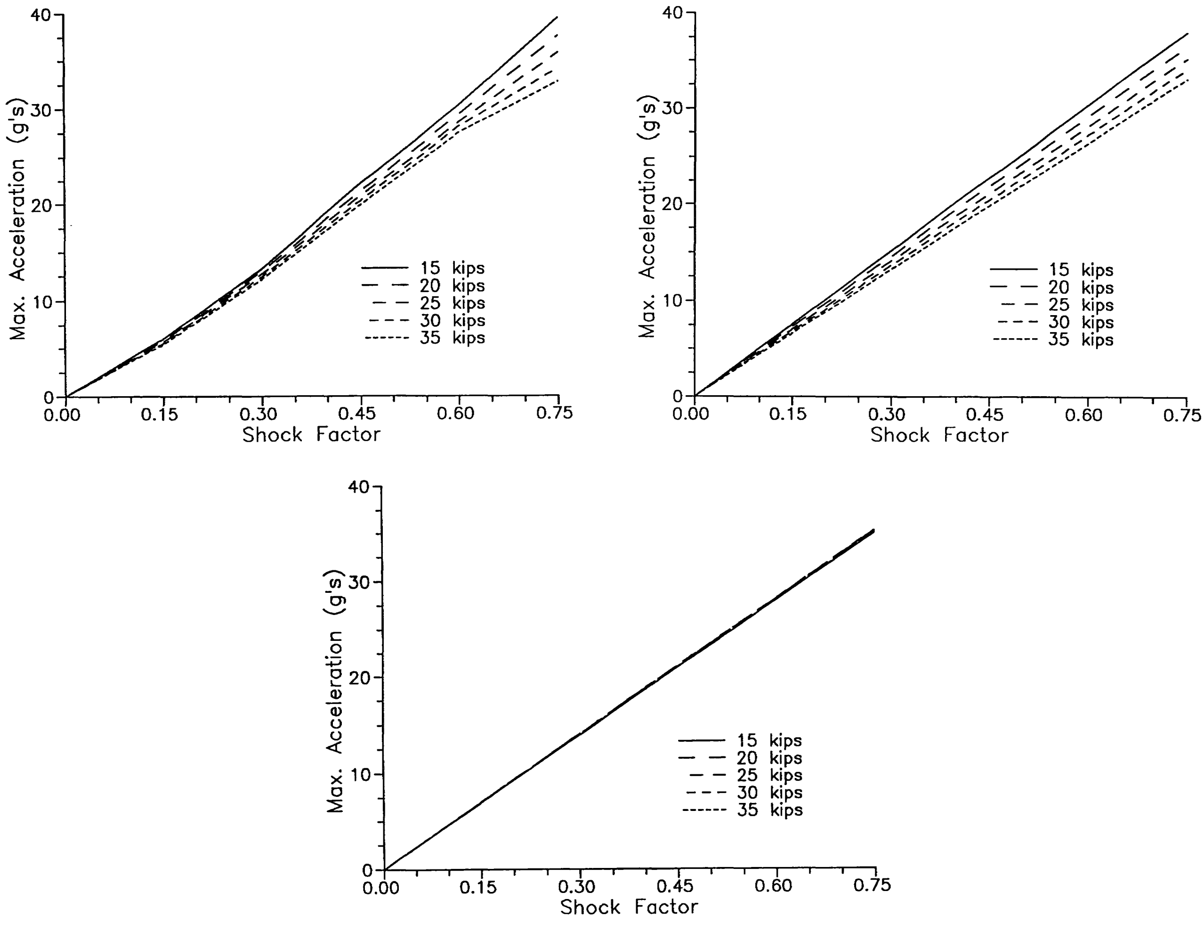

FIGURE 6 (a) Response data for fixed equipment frequency and charge weight, varying equipment weight; (b) linear least squares fit for each equipment weight; (c) slopes scaled to the 25-kip equipment weight.

reference weight. The results are summarized in Table 2. The largest error for the range of parameters is 4.27%, indicating that the scaling rule for the equipment weight is relatively insensitive to changes in the exponent n in Eq. (5). For example, consider the response of 20-Hz equipment subject to a charge weight of 1740 lb. of TNT for five different equipment weights as shown in Fig. 6(a). The linearized response curves are shown in Fig. 6(b) and the scaled responses using Eq. (5) with $n = 1/6$ are shown in Fig. 6(c).

EQUIPMENT FREQUENCY SCALING

A scaling rule for the equipment fixed base natural frequency was developed in a similar way. Consider the general form of the rule as follows:

$$s_b = s_a\left(\frac{f_b}{f_a}\right)^n \qquad (6)$$

in which s_a = reference slope for a given charge weight, equipment weight, and equipment frequency f_a; s_b = slope for the same charge weight, equipment weight, but the equipment frequency is now f_b.

Response data were obtained for three charge weights, where the equipment frequency ranged from 15 to 35 Hz in increments of 5 Hz, and the equipment weight ranged from 15 to 25 kips in 5-kip increments. The required values of n in Eq. (6) that satisfy these data were calculated for all combinations of the frequency ratios. The overall averages were calculated for the 15-, 20-, and the 25-kip equipment weights were 1.6610, 1.6161, and 1.5649, respectively. Table 3 shows the scaled slopes for $n = 1.6$ using the 25-Hz equipment as the reference frequency. The errors are generally less than 5%, except for three instances, the largest being 6.74%. Figure 7 shows the response data for 25-kip equipment and 1740-lb. TNT charge weight, and Fig. 7(a) shows the linearized response curves. The scaled responses are shown in Fig. 7(c) for $n = 1.6$ in Eq. (6).

GENERAL SCALING RULES

Same Hull

The scaling rules expressed by Eqs. (4)–(6) are combined to form the general scaling rule for the same hull:

$$s_b = s_a\left(\frac{Q_a}{Q_b}\right)^{(1/8)}\left(\frac{W_a}{W_b}\right)^{(1/6)}\left(\frac{f_b}{f_a}\right)^{(1.6)} \qquad (7)$$

where $n = 1/6$ is used for the equipment weight scaling exponent, and $n = 1.6$ for the frequency

Table III Scaled Slopes for Model B Scaled on 25 Hz Using $n = 1.6$

	1160#	Percent Error	1740#	Percent Error	3625#	Percent Error
			15 kip			
15 Hz	73.9978	4.93	69.3920	5.72	62.4264	6.74
20 Hz	77.2287	0.78	72.3802	1.66	65.3530	2.37
25 Hz	77.8358	0.00	73.6055	0.00	66.9400	0.00
30 Hz	76.9523	1.14	73.5247	0.11	68.2320	1.93
35 Hz	76.7924	1.34	73.6554	0.07	68.4134	2.20
			20 kip			
15 Hz	72.0228	1.34	67.7016	2.95	60.7785	5.85
20 Hz	73.8541	1.16	69.6486	0.16	62.6476	2.95
25 Hz	73.0046	0.00	69.7909	0.00	64.5518	0.00
30 Hz	73.0107	0.01	70.0227	0.38	65.0180	0.72
35 Hz	71.6435	1.86	68.8981	1.24	64.0306	0.81
			25 kip			
15 Hz	70.0969	0.13	65.9290	1.99	59.3361	4.86
20 Hz	71.0642	1.25	67.0599	0.31	61.0980	2.04
25 Hz	70.1853	0.00	67.2661	0.00	62.3678	0.00
30 Hz	69.3732	1.16	66.5108	1.12	61.9065	0.74
35 Hz	66.9675	4.58	64.4307	4.22	60.0135	3.77

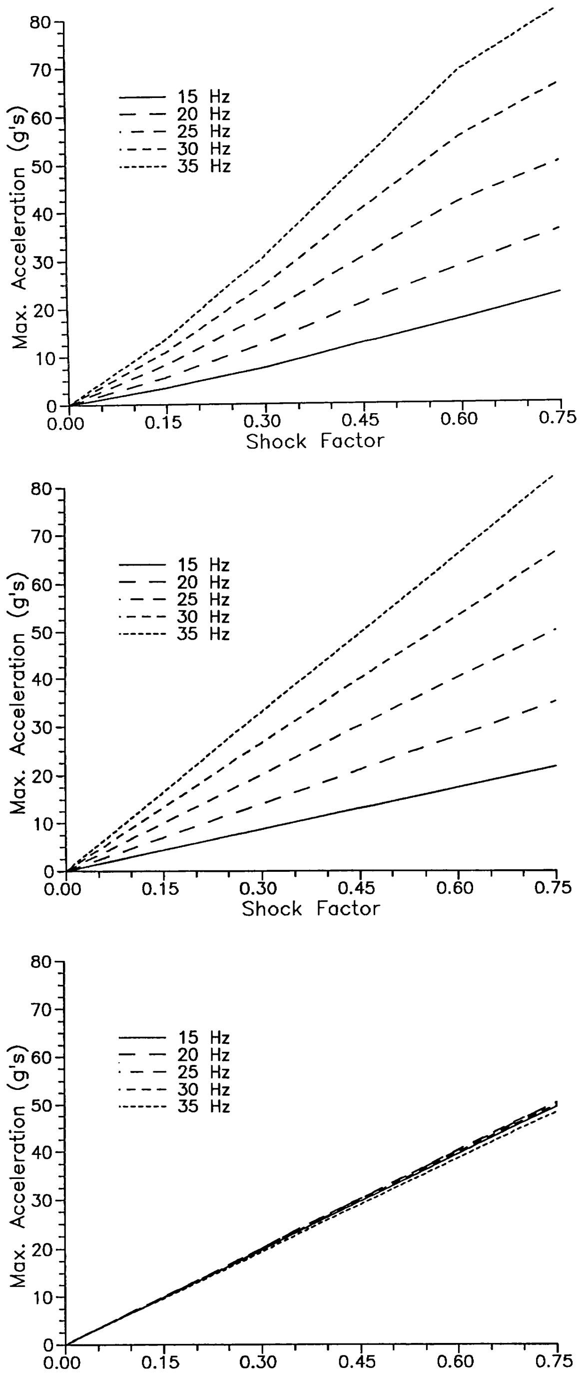

FIGURE 7 (a) Response data for fixed equipment weight and charge weight, varying equipment frequency; (b) linear least squares fit for each equipment frequency; (c) slopes scaled to the 25-Hz frequency.

scaling. The ranges recommended for applying this rule are: 600–3,625 lb. for the charge weight; 15–35 kips for the equipment weight; and 15–35 Hz for the equipment fixed base frequency.

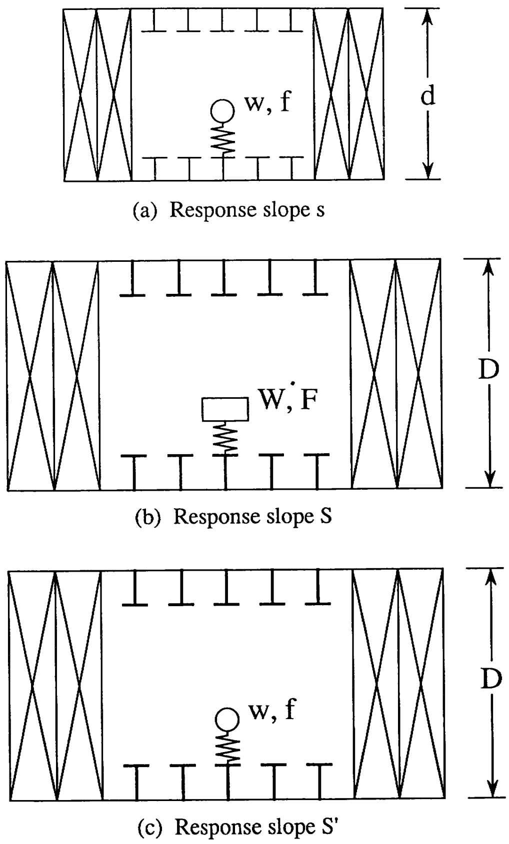

FIGURE 8 (a) Original hull model and equipment; (b) linearly scaled hull and equipment; (c) linearly scaled hull, original equipment.

General Linear Scaling Law

Consider the protype hull of diameter d shown in Fig. 8(a) that includes an equipment of weight w and frequency f subject to a charge weight q at a distance r from the hull. The slope of the response is

$$s = \frac{a}{(sf)} = \frac{ar}{q^{(1/2)}} \tag{8}$$

in which a is the peak acceleration experienced by the equipment. Figure 8(b) represents the linearly scaled model of Fig. 8(a), such that the hull, scantlings, equipment, and charge weight are scaled by the linear scaling factor $L = D/d$. We call this configuration model LBS or SBS as defined earlier. It follows that the response slope of these linearly scaled models is

$$S = \frac{A}{(SF)} = \frac{AR}{Q^{(1/2)}}. \tag{9}$$

Using the scaling relationships in Table 1, we obtain the general scaling rule for the response slopes as:

$$S = \frac{s}{L^{(3/2)}}. \tag{10}$$

Linearly Scaled Hull: Same Equipment, Same Charge Weight

Figure 8 shows a sequence of three hulls. The original or protype hull in Fig. 8(a), subject to a charge weight q, supports an equipment of weight w and frequency f. The response slope is s. Figure 8(b) shows the original configuration that has been linearly scaled, where the scaled equipment weight and scaled frequency are W and F, respectively, and the scaled charge weight is Q. The response slope is S. Figure 8(c) shows the hull geometry as being the same as in Fig. 8(b), but the equipment and charge weight are the same as in Fig. 8(a). We call this configuration model LB or SB. Let S' be the response slope for the model in Fig. 8(c). It follows that the scaling of the response slopes between the models in Figs. 8(b) and (c) is

$$S' = S\left(\frac{Q}{q}\right)^{(1/8)}\left(\frac{W}{w}\right)^{(1/6)}\left(\frac{f}{F}\right)^{(1.6)}. \tag{11}$$

Referring to the scaling relationships in Table 1 and Eq. (10).

$$S' = sL^{(0.975)}. \tag{12}$$

Knowing the response slope s for the equipment in the original model in Fig. 8(a), one can predict the response slope S' when this equipment is subject to the same shock when placed in the scaled hull in Fig. 8(c).

Linearly Scaled Hull: Different Equipment, Different Charge Weight

Consider the same sequence of hulls in Fig. 8. Assume that the response slope is known for model B in Fig. 8(a) and that Fig. 8(b) is again the linearly scaled model either LBS or SBS. Suppose an equipment and/or charge weight shown in Fig. 8(c) are different from those in the prototype in Fig. 8(a). Equation (11) can be used to predict the response slope for this new and different model. Thus, one could have the equipment response data for a particular system as modeled in Fig. 8(a) and predict the responses for a different charge weight, equipment weight, and equipment frequency, respectively, in a linearly scaled hull.

RESULTS

Tests were run on the validity of the general scaling rule given by Eq. (10). The charge weights ranged from 600 to 3,625 lb.; the equipment weight from 15 to 25 kips; and the frequency equal to 20 and 30 Hz. The results showed the predicted response slopes compared favorably with the actual slopes for each configuration. The errors are generally less than 1%, except for the comparison between LBS model and model B subject to a charge weight equal to 1450 lb. where the largest error was 1.42%.

Equation (12) was similarly tested. The results are shown in Fig. 9. The absolute percent errors in the response slopes are shown in Fig. 9 for models SB and LB, where the equipment weights were 15, 20, and 25 kips, and the equipment frequencies were 20 and 30 Hz, respectively. Six different charge weights were used. In

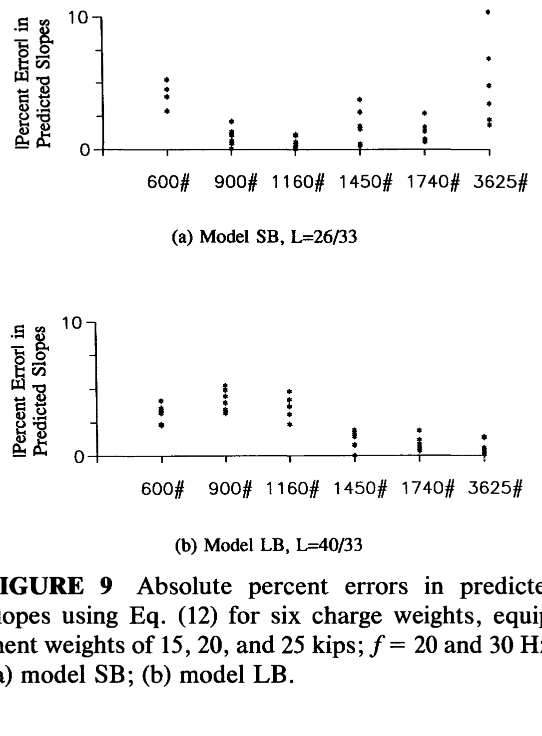

FIGURE 9 Absolute percent errors in predicted slopes using Eq. (12) for six charge weights, equipment weights of 15, 20, and 25 kips; $f = 20$ and 30 Hz; (a) model SB; (b) model LB.

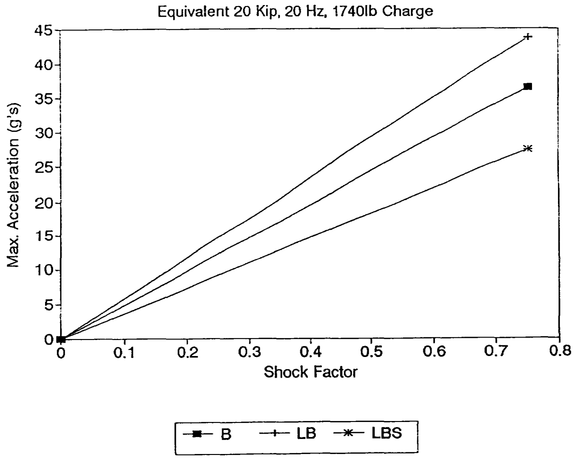

FIGURE 10 Least squares response slopes for models B, LB, and LBS.

the case of predicting the response slopes for model SB, the errors tend to be about 5% or less, except for the 15-kip, 20-Hz equipment where the error was 10.35%. The errors associated with model LB were less than 5% except in one case where it equaled 5.26%.

A word of caution is offered in predicting equipment response from fully scaled models. Recall the time history motions in Fig. 3 for the response of equipment installed in model B, LBS, and SBS. Figure 10 shows the least squares response slopes for models B, LB, and LBS. Note that higher peak accelerations are experienced by the equipment placed in the linearly scaled hull, namely model LB, and lower peak

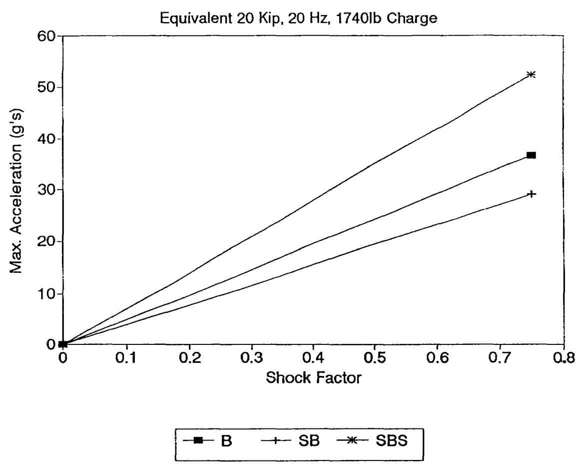

FIGURE 11 Least squares response slopes for models B, SB, and SBS.

accelerations in the case of the fully scaled model LBS. Consequently, if shock design values generated from model LBS were used without these new scaling rules to predict the actual equipment response in model B, the equipment would be underdesigned and overdesigned if model LB were used. Just the opposite occurs for models SB and SBS as shown in Fig. 11. Now the original equipment installed in model B would be underdesigned if the predicted design values generated from model SB were used without the new scaling rules and overdesigned for the case of model SBS.

CONCLUSIONS

The results of this study have demonstrated that useful information may be obtained by using a computer as an initial surrogate for shock testing purposes. These results show the relative changes in shock design values for different boats and attack geometries. It is emphasized that the test sections were small in size and devoid of typical equipment present in a real compartment. Consequently, the results provide only trends in shock design values rather than absolute design numbers.

The general linear scaling law that has been used in practice was shown to provide excellent results for the time history responses. Of course, one problem is how to predict the equipment peak accelerations for the case where the hull is fixed and the charge weight, equipment weight, and/or the equipment fixed base frequency varies. The second problem is how to predict these peak accelerations when the equipment and/or charge size are not varied, or a new equipment is installed in the scaled hull.

An attempt to answer the first problem required the collection of large amounts of computer generated data for the submarine model containing single degree of freedom frame-mounted equipment. The recommended rules for scaling on charge weight, equipment weight, and equipment fixed base frequency for a fixed hull size provides a reasonably wide numerical range. The range of errors associated with the scaling rule of Eq. (7) are within engineering accuracy. The intent of this general scaling rule is to allow greater usage of existing shock response data for different equipment subject to a variety of shock conditions.

The answer to the second and major problem was answered in two steps. The first step, represented by Eq. (10), showed how to obtain the response for the fully scaled model from the prototype model. The second part, represented by Eq. (11), produced a scaling rule that allows the charge weight, equipment weight, and/or the equipment frequency to change to either of the original values, in which case Eq. (12) is used, or to entirely new equipment that might be installed in the scaled hull, in which case Eq. (11) is used. It was shown that care must be exercised when shock design values are generated from a fully scaled model or a scaled hull containing the same equipment as the prototype model. Depending on the circumstances, the generated shock design values may be either too high or too low if these new scaling rules are ignored.

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