Review

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Vibration Isolation: A Review, I. Sinusoidal and Random Excitations

This article reviews the engineering theory and technology associated with the use of discrete isolators to protect vibration-sensitive equipment or machinery against sinusoidal or random excitation. Special attention is given to protection against low-frequency (<5 Hz) and high-frequency (>100 Hz) excitation. Both passive and active isolators systems are discussed. © 1994 John Wiley & Sons, Inc.

VIBRATION CONTROL

Vibration control is the reduction of unwanted vibration in a machine or in a structural system. Like noise control, vibration control is usually categorized into methods that affect the source, the path, or the receiver, see Fig. 1. An example of source control is machinery balancing (Harris, 1987, Chap. 39). An example of receiver control is detuning, that is, assuring that the receiver natural frequencies do not coincide with any forcing frequencies. An example of path control is the use of vibration isolators, which are represented in Fig. 1 by discrete connections between the receiver and the path and between the source and the path. As such, vibration isolation is a subset of vibration control, being only one of an array of methods available to the vibration control engineer.

Although most vibration engineers spend most of their time on vibration control, it is a topic that receives little coverage in textbooks or monographs. The only textbook that gave extensive coverage to vibration control has long been out of print (MacDuff and Curreri, 1958). Most current coverage is found in handbooks, for example Beranek and Ver (1992, Chap. 28), although a new monograph is under preparation by Mead (1993).

ISOLATION AGAINST SINUSOIDAL VIBRATION

Vibration isolation is the technique of controlling vibration by interposing compact, resilient connections between the vibration source and its surrounding structure, source isolation, or between the surrounding structure and the vibration receiver, receiver isolation. In this context, compact means small compared to the vibration wavelength. These compact resilient connections will be called isolators, although other terms, such as antivibration mounts, are common. Isolators gain their resilience from their shape or their material, or both. Typical isolators employ metallic springs, polymer blocks, or trapped volumes of air (Harris, 1987, Chap. 29).

The simplest analytical model for an isolator is the single degree-of-freedom (SDOF) translational spring-mass system (Fig. 2).

The force transmissibility for the source isolation problem is defined as

$$T_{\mathrm{F}} = \frac{\hat{F}_t}{\hat{F}} \tag{1}$$

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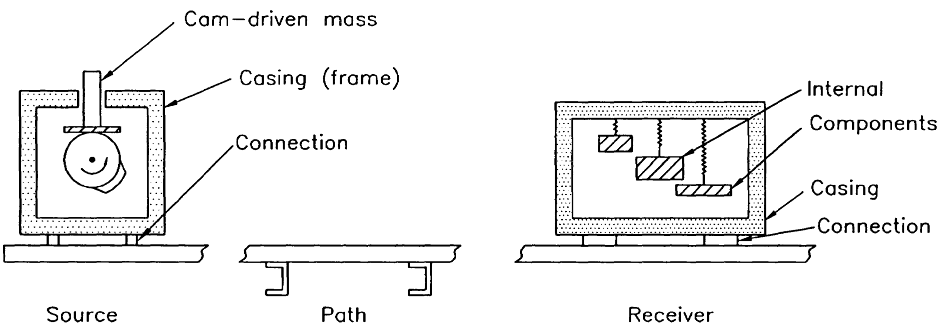

FIGURE 1 Typical arrangement of a vibration source, a vibration transmission path, and a receiver of vibration; used by permission from C. M. Harris (1991).

and the motion transmissibility for the receiver isolation problem is defined as

$$T_M = \frac{\hat{x}}{\hat{y}}. \tag{2}$$

All textbooks point out that $T_F = T_M$ but few show that this result is not fortuitous but is a necessary consequence of the reciprocity inherent in all linear elastic systems (Ungar, 1991). Because of this identity, the symbol T_M will be used for force transmissibility as well as motion transmissibility.

Figure 3 shows how T_M varies with frequency, more precisely, with frequency ratio, Ω/ω, and with damping (both viscous damping, ζ, and hysteretic damping, η). Versions of Fig. 3 have appeared in undergraduate vibration textbooks from Den Hartog (1934) to Thomson (1993). It is unfortunate that even contemporary vibration textbooks tend to stop with Fig. 3, more or less where Den Hartog stopped 60 years ago. In fact, it has been argued elsewhere (Nelson, 1991) that the SDOF model is not a good design model and that a two degree-of-freedom (2DOF) model is preferable (Fig. 4). Nevertheless, the SDOF model has an extensive literature (e.g. Snowdon, 1979, contains over 200 references) and many useful trends can be learned from its study. For example

- in the vicinity of resonance ($\Omega/\omega \approx 1$), the *larger* the damping, the *smaller* the T_M;

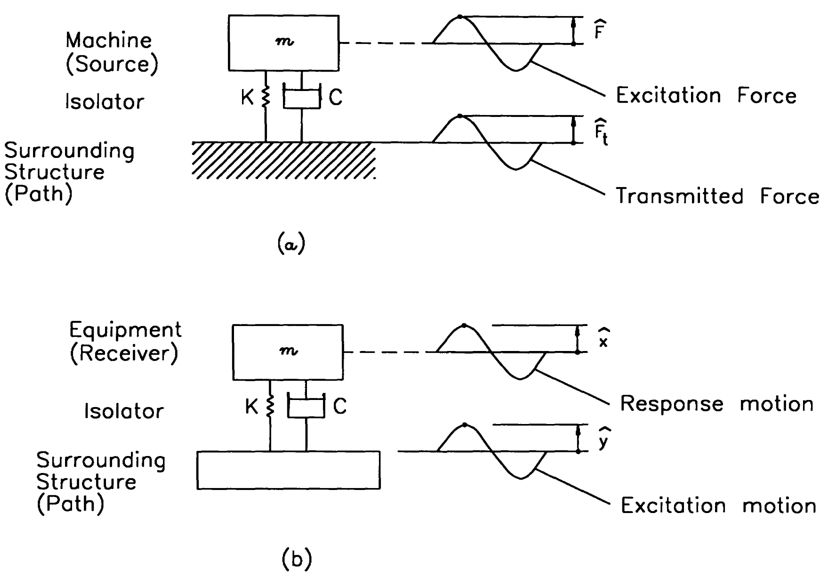

FIGURE 2 Single degree-of-freedom spring-mass-damper representation for (a) source isolation and (b) receiver isolation.

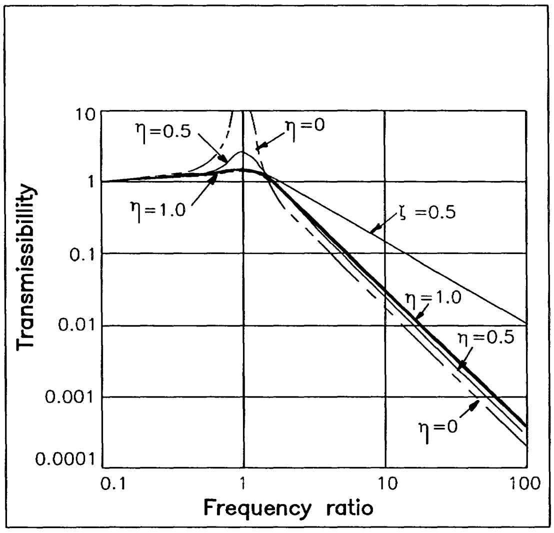

FIGURE 3 Transmissibility of single degree-of-freedom system with viscous damping (ζ) or hysteretic damping (η) vs. frequency ratio (forcing frequency, Ω/natural frequency, ω).

- isolation ($T_M < 1$) begins at $\Omega/\omega = \sqrt{2}$ for *all* values of damping;
- in the region of isolation ($\Omega/\omega > \sqrt{2}$) the *larger* the damping, the *larger* the T_M (however, for hysteretic damping this effect is so small as to be a negligible consideration in design).

In addition, for the frequency region $\Omega/\omega \gg 1$:

- if the isolator is *undamped,* T_M falls off at 40 dB/decade (12 dB/octave);
- if the isolator has *hysteretic damping,* T_M falls off at 40 dB/decade;
- if the isolator has appreciable *viscous damping,* T_M falls off at 20 dB/decade (6 dB/octave).

The 2DOF system shown in Fig. 4 can also be used as a model of a two-stage isolation system (Fig. 5).

Two-stage isolation systems are widely used to isolate equipment against high frequency vibration because T_M falls off at 80 dB/decade at high frequencies (Fig. 6).

Snowdon (1968, Chap. 3) contains an analysis of a two-stage isolation system that employs elastomeric isolators. Isolator staging can be extended beyond two; in at least one case, a nine-stage isolation system has been designed and used.

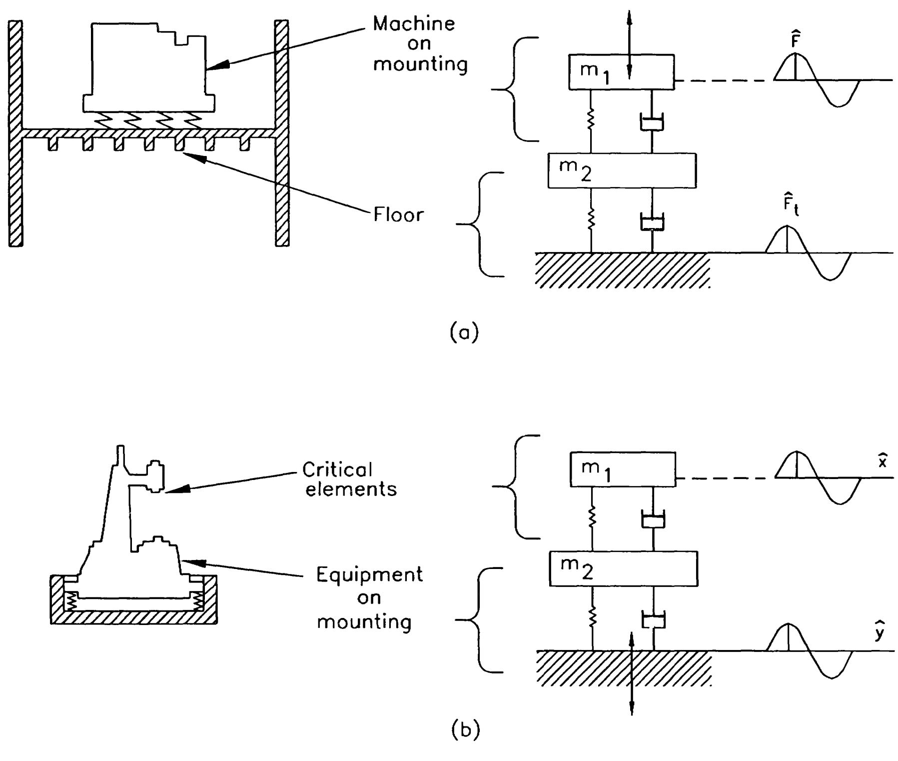

FIGURE 4 Two degree-of-freedom spring-mass-damper representation for (a) source isolation and (b) receiver isolation. Used by permission from Macinante (1984).

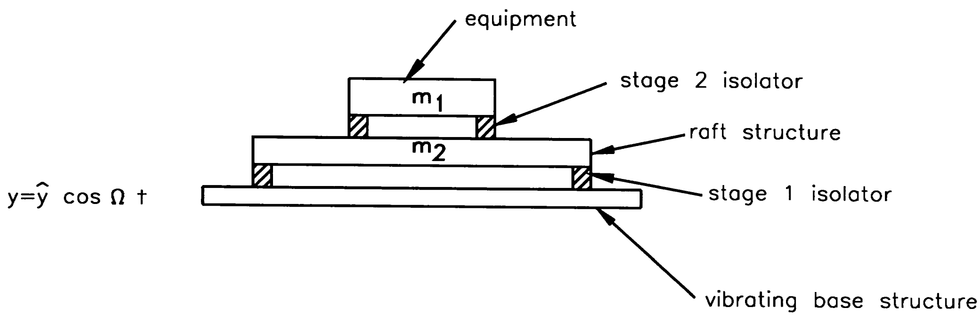

FIGURE 5 A two-stage isolation system (equipment on a raft).

The inertia block is a special case of Fig. 5 (Fig. 7). Protecting equipment against low frequency vibrations (<5 Hz) with metal springs or polymer blocks can lead to very high aspect ratio isolators and concomitant problems with high deflection and low lateral stability. By using a massive inertia block ($m_2 \gg m_1$), the isolator aspect ratio can be reduced and, in addition, the amount of static and dynamic displacement can be decreased. The thing to remember is that inertia blocks permit the design of more practical and more stable isolator systems; they seldom lead to appreciably lower values of T_M.

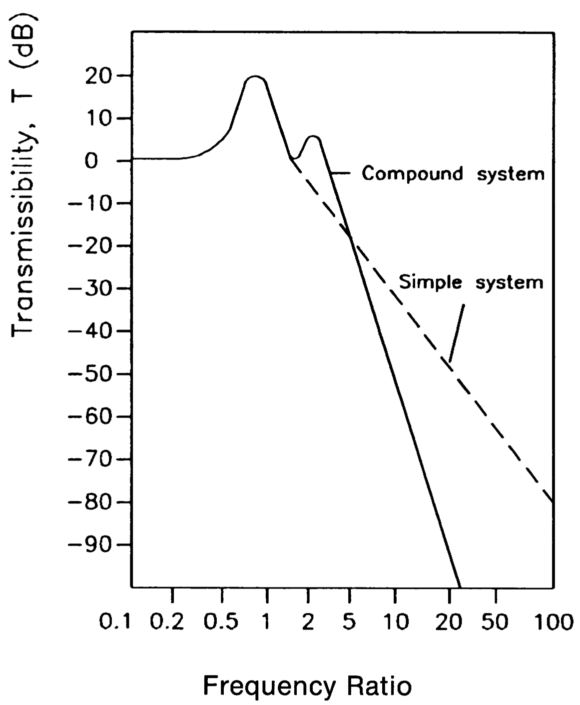

FIGURE 6 Transmissibility of a two-stage isolation system such as that shown in Fig. 5. The location of the second resonant peak depends on the mass ratio, m_1/m_2.

Low frequency isolation without inertia blocks can be obtained by using air springs (Harris, 1987, Chap. 33). Special mechanical systems, usually variations of the Stewart Platform or the Mallock Suspension, can also provide low frequency isolation without using an inertia block. Lately, a mechanical isolator that contains a negative stiffness element has shown the ability to isolate vibrations down to 1 Hz (Platus, 1993).

Only a few real systems are dominated by motion in one dimension and thus allow the use of SDOF models. Most real systems are better modeled as a piece of equipment vibrating about its center of mass in two or more of the six possible rigid mode DOF (Fig. 8).

In general, all six modes are coupled. Harris (1987, Chap. 3) gives the six coupled equations of motion. Symmetry and judicious choice of isolator stiffness can uncouple some, and in special cases, all of the DOF (Macinante, 1984, Chap. 7). Such uncoupling is desirable because then one can make the uncoupled natural frequencies equal and thus simplify the problem of achieving good isolation of these DOF.

These particular results, both for high and low frequency, contain a generic assumption; namely, that the excitation force or the excitation motion is ideal, that is, the dynamics of the

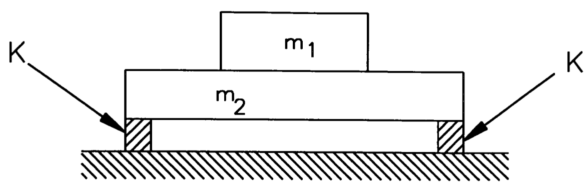

FIGURE 7 Diagram of a piece of equipment (m_1) hard mounted to an inertia block (m_2).

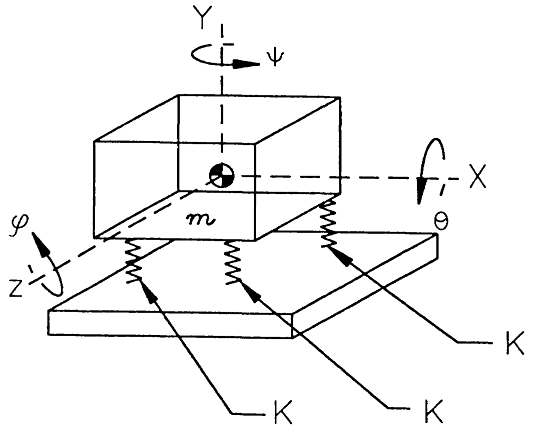

FIGURE 8 Model of a rigid piece of equipment (m) mounted on four isolators of equal stiffness (k). There are six degrees-of-freedom about the center of mass of the equipment: three in translation (x, y, z) and three in rotation (θ, ψ, ϕ).

excitation are not affected by the dynamics of the response. This assumption becomes more and more untenable as the excitation frequency increases further and further into the audio range. Simulation of high frequency (>100 Hz) isolator systems is better served by replacing the ideal source by a linear source (Ungar and Dietrich, 1966). Figure 9 shows a piece of equipment, a receiver (R), being driven by a linear source (S) through a massless isolator (I). It is also conventional (White and Walker, 1982, Chap. 26) to replace the motion transmissibility, T_M, by the insertion loss, D.

$$D = 20 \log \frac{\hat{v}_{R,HM}}{\hat{v}_{R,SM}} \tag{3}$$

where $\hat{v}_{R,HM}$ = the velocity amplitude of the receiver (R) when it is hard mounted (HM) to the source; and $\hat{v}_{R,SM}$ = the velocity amplitude of the receiver (R) when it is soft mounted (SM) to the source.

For the case of Fig. 9, it can be shown that

$$D = 20 \log \left| 1 + \frac{M_I}{M_S + M_R} \right| \tag{4}$$

where M_I = mobility of the isolator (for a massless spring of stiffness k, $M_I = i\Omega/k$); M_S = mobility of the linear source (ratio of the free velocity of the source to the blocked force of the source); and M_R = mobility of the receiver.

Even though the simulation of Fig. 9 is still simplified, Eq. (4) gives much better guidance for achieving isolation at high frequencies. In particular

1. if one lets the receiver (the vibration sensitive equipment) have a natural frequency of ω, then when $\Omega = \omega$, $M_R \to \infty$ and $D \to 0$, that is, there will be no isolation at undamped equipment resonances.
2. A high level of isolation ($D \gg 1$) will be achieved if k is chosen small enough so that

$$|M_I| \gg |M_S + M_R|. \tag{5}$$

In other words, the role of an isolator is to introduce a mobility mismatch between the source and the receiver. The larger the mismatch, the higher the isolation.

3. If M_S is not too large, one can also increase the mismatch by making M_R smaller. This can be done in three ways:
   (a) if the equipment is stiffness controlled (i.e., $\Omega/\omega \ll 1$), make the equipment stiffer;
   (b) if the equipment is mass controlled (i.e., $\Omega/\omega \gg 1$), make the equipment more massive;
   (c) if the equipment is damping controlled (i.e., $\Omega/\omega \approx 1$), augment the equipment damping.

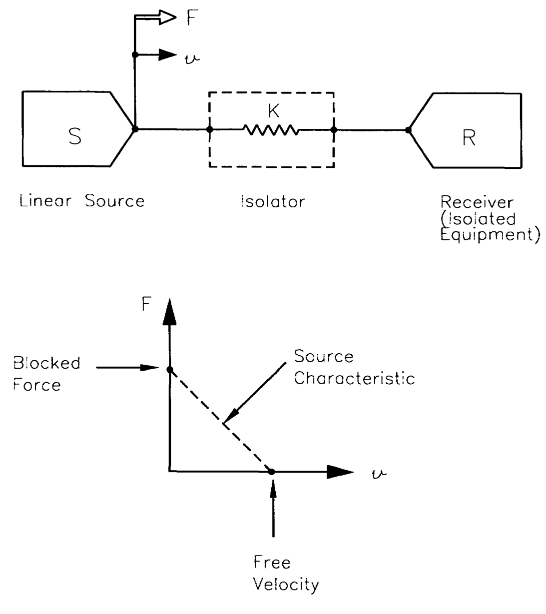

FIGURE 9 Schematic of a linear source (S) driving a piece of equipment (R) through a massless isolator (I).

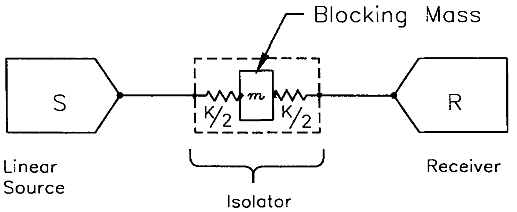

FIGURE 10 Schematic of a linear source (S) driving a piece of equipment (R) through an isolator containing a blocking mass (m).

This high-frequency model can be expanded to include an isolator with mass and damping (Snowdon, 1979, Section 6.2). A comprehensive method for designing isolators based on this approach is given in SAE (1962).

At even higher frequencies, isolators begin to behave as wave guides and it is helpful to think of the vibration as structure-borne noise and to treat the problem with the methods described in Cremer and Heckl (1988). In particular, at these frequencies a concentrated mass that is inserted into an isolator can effectively block the transmission of wave energy, especially if the waves are flexural. Figure 10 is a schematic of such a blocking mass. A practical example of an isolator with a self-contained blocking mass is described by Young and Hanners (1973).

A second generic assumption of the above technology is that the isolators or isolator systems are passive, that is, they receive no external energy or information. When they do receive such inputs, the system is active. Active vibration has received much impetus recently from progress in control algorithms (e.g., neural, fuzzy, or adaptive) and in sensors and actuators (e.g., piezoelectric polymer sheets).

A schematic of an active isolation system applied to a SDOF system is shown in Fig. 11 and a typical enhancement of isolation performance is shown in Fig. 12.

Active isolators were first used to protect instruments against low frequency vibration, for example, Tsutsumi (1964) describes an early example of the active isolation of an instrument test platform against slow tilting caused by earth motion and building distortion. With the development of actuators with high frequency response, it has become possible to extend the performance of active isolators into the low audio range. Also, in using active isolators, it must be remembered that the isolation system may have to transmit static loads. Hence active isolators with low stiffness at low frequency are usually used in parallel with passive isolators, see Watters et al., (1988) where active isolators are used in parallel with passive elastomeric isolators to reduce the T_F of a diesel engine by an additional 20 dB in the frequency band 20–100 Hz.

It is more difficult to achieve broad-band active isolation than narrow-band and, as a result, active isolation is most likely to be used in cases where the excitation is periodic. This is especially true for those cases where high performance demands justify the added initial cost and maintenance. A review of active control of machinery isolators can be found in von Flotow (1988).

As was mentioned previously, the textbook and monograph literature in vibration control is thin; this is also true in vibration isolation. Crede

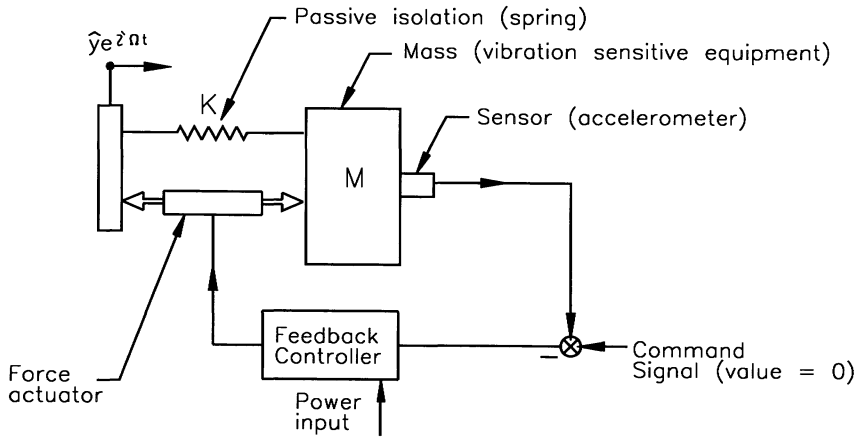

FIGURE 11 Schematic of an active vibration isolator employed in parallel with a passive isolator of stiffness k.

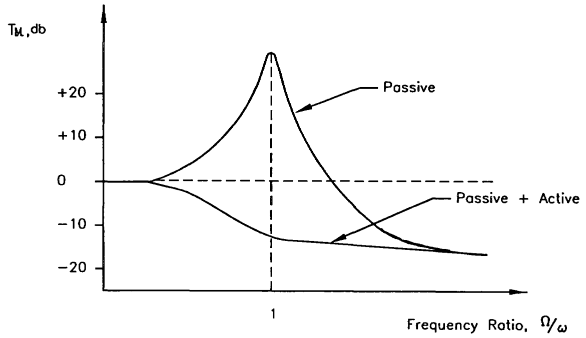

FIGURE 12 Typical reduction of the transmissibility of a passive system by the addition of an active system.

(1965) is an acknowledged classic but has been out of print for sometime. Fortunately, Macinante (1984) is in print and is a worthy successor. More recently Frolov and Furman (1990) has become available, giving insight into Russian techniques and technology in this field. For vibration isolation of mechanical equipment in buildings, Jones (1984) is extremely helpful because it provides pictures and schematics of a wide selection of practical isolation systems. In addition, it is unique in providing a long litany (backed up with photographs) of misinstalled, misaligned, and misselected isolators. Much can be learned from studying the mistakes and oversights of others.

Among these few books and monographs on vibration isolation, there are even fewer that discuss the design and use of nonlinear isolators. Frolov and Furman (1990) present a highly classical discussion of this topic while Mustin (1968) addresses one of its more practical aspects, the use of nonlinear cushioning material in the packaging of vibration-sensitive equipment.

ISOLATION AGAINST RANDOM VIBRATION

The previous discussion has been limited to vibration isolation against sinusoidal excitation, or, at most, against highly tonal excitation. A design acceptance test would usually be a sine wave whose frequency was swept slowly (2 octaves/min) over the frequency range of interest. Prior to the 1950s when power plants, particularly aircraft power plants, were primarily of the reciprocating type, the above knowledge and associated technology were adequate to solve the problems at hand.

With the advent of jet engines and rocket motors, vibration problems arose, especially in the aerospace industry where the sources of tonal excitation were overwhelmed by sources of random excitation. Examples of random sources are boundary layer turbulence and the exhaust from jet and rocket engines. These sources are characterized by features not found in the sinusoidal or highly tonal world: smooth frequency spectra over wide frequency bands and large variations in amplitude over short periods of time. The underlying theory of random vibration of mechanical systems has been well codified (Argyris and Mlejnek, 1991, Chap. 8).

Vibration testing using random inputs was applied to missile electronics in the early 1960s and extended to aircraft electronics in the 1980s. At present, random vibration testing is recognized as a good way of evaluating electronics reliability, even if the operational environment is nonrandom or even nonvibrating. For a discussion of such vibration screening see Tustin and Mercado (1984, Section 30). Note that vibration screening is usually done with the equipment or system hard mounted, that is, with isolators removed.

A fundamental result of random vibration theory is

$$S_R(\Omega) = |H(\Omega)|^2 S_E(\Omega) \tag{6}$$

where S_R is the power spectral density (PSD) of the response, S_E is the PSD of the excitation, and H is the (complex) frequency response characteristic between response and excitation. If S_R is the velocity PSD of the response and if S_E is the force PSD of the excitation, then H is the random equivalent to the mobility of the system. Similarly, if S_R is the acceleration PSD of the response and S_E is the acceleration PSD of the excitation, $|H|$ is the random equivalent of the sinusoidal motion transmissibility denoted by the symbol, τ_M.

Furthermore, as shown in Fig. 13, the response of a spring-mass system to a narrow-band random excitation is very closely approximated by an amplitude-modulated sine wave. This suggests that $\tau_M \approx T_M$ for narrow-band response and thus for this case, Eq. (6) can be rewritten as

$$(S_R)_{accel} \approx T_M^2 \, (S_E)_{accel}. \tag{7}$$

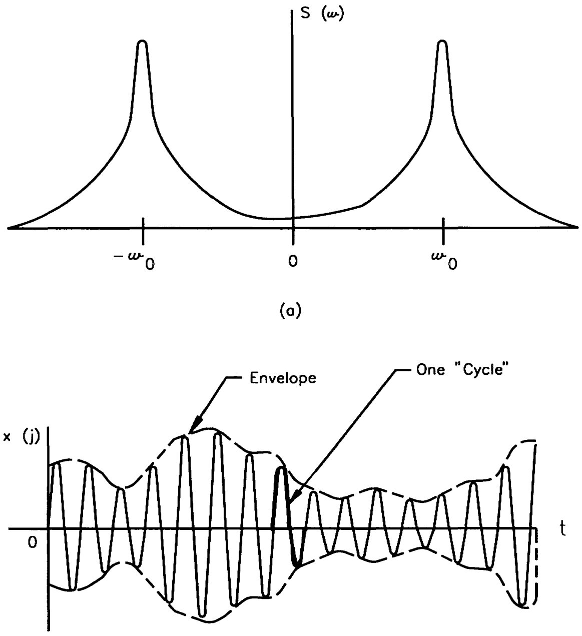

FIGURE 13 (a) Two-side PSD of a narrow-band random process; (b) time history of a narrow-band random process. Used by permission from Crandall and Mark (1963).

If the excitation is wide band and the system is linear, the wide-band response can be obtained by using Eq. (7) in a series of contiguous narrow bands (e.g., 1 Hz) that span the complete excitation bandwidth. The result of such a calculation is shown in Fig. 14.

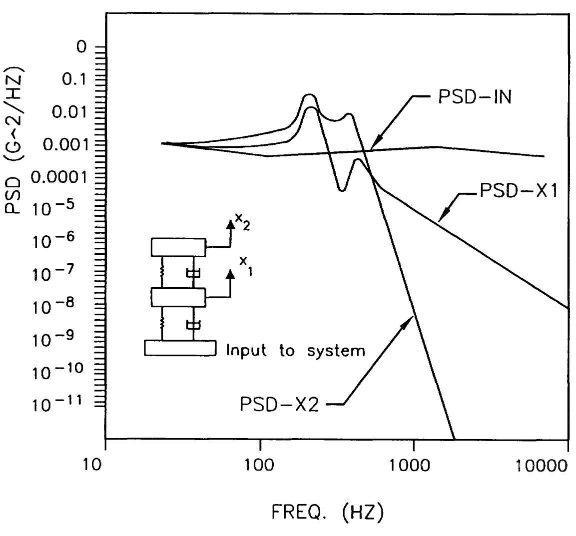

FIGURE 14 Power spectral density (PSD) of a wide-band input excitation (PSD-IN) and the responses (PSD-X1 and PSD-X2) of a two-stage isolation system.

CONCLUSIONS

It is hoped that this survey gives the practicing engineer an understanding of some of what is needed to protect vibration-sensitive equipment or machinery against sinusoidal or random excitation.

However, it is worth asking several questions before using the theory and practice described above.

1. Are vibration isolators really necessary? Not all equipment needs to be isolated to survive its vibration environment. To determine if isolators are necessary, one must know, or determine, the fragility (sensitivity) of the equipment (see Kornhauser, 1964, Chap. 5) and then compare that fragility to the excitation of the equipment.
2. What are the dominant power paths? There is little point in inserting isolators into load paths that convey only a minor portion of the vibrational power flow. Some thought, analysis, or testing can usually identify the major load paths and save expensive redesign.
3. Is the design governed by impulsive excitation rather than sinusoidal or random excitation? Vibration isolation against impulsive excitation, or shock, is an extensive subject and is not as well codified as are the subjects of this review. Part II will discuss shock isolation.

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