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A Computer Program for a Canonical Problem in Underwater Shock

Finite-element/boundary-element codes are widely used to analyze the response of marine structures to underwater explosions. An important step in verifying the correctness and accuracy of such codes is the comparison of code-generated results for canonical problems with corresponding analytical or semianalytical results. At the present time, such comparisons rely on hardcopy results presented in technical journals and reports. This article describes a computer program available from SAVIAC that produces user-selected numerical results for a step-wave-excited spherical shell submerged in and (optionally) filled with an acoustic fluid. The method of solution employed in the program is based on classical expansion of the field quantities in generalized Fourier series in the meridional coordinate. Convergence of the series is enhanced by judicious application of modified Cesàro summation and partial closed-form solution.

INTRODUCTION

A canonical problem in underwater shock is the excitation of a spherical shell submerged in an acoustic fluid by a plane step-wave. The first solution, for an empty shell, was provided by Huang (1969); his results, however, contained small errors, which were later corrected by Huang et al. (1977), and by Geers (1978). In 1979, Huang solved the problem of submerged concentric spherical shells with fluid present in the region between the shells and absent inside the inner shell. Recently, Zhang and Geers (1993) solved the problem of a submerged spherical shell filled with fluid; the results were used to evaluate the accuracy of doubly asymptotic approximations (DAAs) for treating the fluid–structure interaction (Geers and Zhang, 1991).

The computer program SPHSHK/MODSUM is based on the solution method of Zhang and Geers, specialized to the case when the internal and external fluid media are the same. That method exploits the spherical geometry and load axisymmetry characterizing the problem to express the meridional variation of the shell-displacement and fluid-velocity-potential fields as Fourier–Legendre series. Enforcement of radial-motion compatibility at the shell's inner and outer surfaces is facilitated by the introduction of residual potentials (Geers, 1969, 1971, 1972; Akkas and Engin, 1980; Akkas, 1985; Akkas and Erdogan, 1993). In this article, residual potentials are defined for both exact and second-order doubly asymptotic (DAA_2) treatments of the fluid–structure interaction in order to unify the solution technique.

SPHSHK/MODSUM offers four options for treating the fluid–structure interaction:

external-acoustic/internal-acoustic
external-acoustic/internal-DAA_2

external-DAA_2/internal-acoustic
external-DAA_2/internal-DAA_2

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In addition, the internal fluid may be removed to render the shell empty. To enhance convergence of the Fourier–Legendre series, modified Cesàro summation and partial closed-form solutions were employed (Zhang and Geers, 1993). Sample numerical results are presented below in the form of surface-pressure, radial-velocity, and meridional-strain histories; additional results are provided in Geers and Zhang (1991), Zhang and Geers, (1993), and the users manual for SPHSHK/MODSUM (Ju and Geers, 1992).

PROBLEM FORMULATION

Description of the Problem

Figure 1 shows the geometry for this canonical problem. The internal/external fluid has mass density ρ and sound speed c. The shell material is elastic and isotropic, with density ρ_0 and plate velocity $c_0 = [E/\rho_0(1 - \nu^2)]^{1/2}$, where E is Young's modulus and ν is Poisson's ratio. The shell's thickness-to-radius ratio h/a is sufficiently small that thin-shell theory suffices. The point on the shell where the incident plane wave first contacts the shell is $\theta = \pi$. All formulations are nondimensional, with length normalized to a, time to a/c, and pressure to ρc^2.

Governing Modal Equations

Because the problem of Figure 1 is axisymmetric, the velocity potential and displacement fields may be expressed as (Morse and Ingard, 1968; Junger and Feit, 1986).

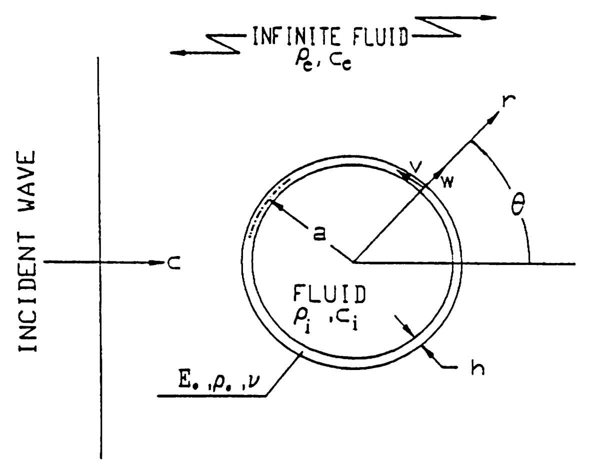

FIGURE 1 Geometry of the spherical shell problem.

$$\phi(r, \theta, t) = \sum_{n=0}^{\infty} \phi_n(r, t) P_n(\cos\theta)$$

$$\nu(\theta, t) = -\sum_{n=1}^{\infty} \nu_n(t) \frac{d}{d\theta} P_n(\cos\theta) \qquad (1)$$

$$w(\theta, t) = \sum_{n=0}^{\infty} w_n(t) P_n(\cos\theta)$$

where $P_n(\cos\theta)$ is the Legendre polynomial of order n (Abramowitz and Stegun, 1964). The governing equations for each Legendre polynomial mode are: the modal wave equation for each fluid field, the modal equations of motion for the shell, and the modal fluid–shell compatibility equations.

The modal wave equation is obtained by substituting the first of Eq. (1) into the acoustic wave equation and utilizing the orthogonality property of Legendre polynomials. The governing equation for each of the $\phi_n(r, t)$ is then

$$r^2 \frac{d^2\phi_n}{dr^2} + 2r \frac{d\phi_n}{dr} - n(n + 1)\phi_n = r^2\ddot{\phi}_n \qquad (2)$$

where an overdot denotes a time derivative. This equation pertains to both the internal and external modal velocity potentials $\phi_n^i(r, t)$ and $\phi_n^e(r, t)$, respectively.

The modal equations of motion for the shell may be written (Junger and Feit, 1986)

$$\begin{aligned} n(n + 1)\ddot{\nu}_n + \lambda_n^{VV}\nu_n + \lambda_n^{VW}w_n &= 0 \\ \ddot{w}_n + \lambda_n^{VW}\nu_n + \lambda_n^{WW}w_n &= \mu(\dot{\phi}_n^i - \dot{\phi}_n^e). \end{aligned} \qquad (3)$$

Here, $\mu = (\rho/\rho_0)(a/h)$, $\dot{\phi}_n^i - \dot{\phi}_n^e$ is the nth modal component of net pressure acting radially outward on the shell, and

$$\begin{aligned} \lambda_n^{VV} &= n(n + 1)(1 + \varepsilon)\xi_n\gamma_0 \\ \lambda_n^{VW} &= n(n + 1)(1 + \nu + \varepsilon\xi_n)\gamma_0 \\ \lambda_n^{WW} &= [2(1 + \nu) + n(n + 1)\varepsilon\xi_n]\gamma_0 \end{aligned} \qquad (4)$$

where $\varepsilon = (h/a)^2/12$, $\gamma_0 = (c_0/c)^2$, and $\xi_n = n(n + 1) - 1 + \nu$.

The modal fluid-shell compatibility equations on the inner and outer surfaces of the shell are

$$\begin{aligned} -\phi_{n,r}^i(1, t) &= \dot{w}_n(t) \\ -\phi_{n,r}^e(1, t) &= \dot{w}_n(t). \end{aligned} \qquad (5)$$

Only radial compatibility is enforced because the fluid is inviscid.

PROBLEM SOLUTION

Modal Response Equations

Zhang and Geers (1993) solved the governing modal equations by separating the external field as $\phi_n^e(r, t) = \phi_n^0(r, t) + \phi_n^s(r, t)$, where $\phi_n^0(r, t)$ pertains to the (known) incident wave and $\phi_n^s(r, t)$ pertains to the (unknown) scattered wave. Then they solved Eq. (2) for the scattered and internal fields to obtain the residual potential equations

$$\begin{aligned} \underline{\phi}_{n,r}^s(t) + \dot{\underline{\phi}}_n^s(t) + \underline{\phi}_n^s(t) + \underline{\psi}_n^s(t) &= 0 \\ \underline{\phi}_{n,r}^i(t) - \dot{\underline{\phi}}_n^i(t) + \underline{\phi}_n^i(t) + \underline{\psi}_n^i(t) &= 0 \end{aligned} \tag{6}$$

in which an underscore denotes location on the surface $r = 1$ and the residual potentials $\underline{\psi}_n^s(t)$ and $\underline{\psi}_n^i(t)$ are related to the velocity potentials $\underline{\phi}_n^s(t)$ and $\underline{\phi}_n^i(t)$ by

$$\begin{aligned} \sum_{m=0}^{n} \Gamma_{mn}\underline{\psi}_{n,n-m}^s &= \sum_{m=1}^{n} m\Gamma_{nm}\underline{\phi}_{n,n-m}^s \\ \sum_{m=0}^{n} (-1)^m\Gamma_{nm}\underline{\psi}_{n,n-m}^i &= \sum_{m=1}^{n} (-1)^m m\Gamma_{nm}\underline{\phi}_{n,n-m}^i \\ &\quad + (-1)^n \sum_{m=0}^{n} \Gamma_{nm}[\underline{\psi}_{n,n-m}^i - 2\underline{\phi}_{n,n-m+1}^i \\ &\quad - m\dot{\underline{\phi}}_{n,n-m}^i]_{t-2} \end{aligned} \tag{7}$$

where the subscript $n - m$ denotes $(n - m)$-fold differentiation in time and $\Gamma_{nm} = (n + m)!/[2^m m!(n - m)!]$. The first of Eq. (7) is an ordinary differential equation for $\underline{\psi}_n^s(t)$ and the second is a delayed-differential equation for $\underline{\psi}_n^i(t)$.

Finally, in order to improve the conditioning of the assembled equations, Zhang and Geers introduced the integrated variables $V_n(t)$, $W_n(t)$, $\underline{\Phi}_n^s(t)$, $\underline{\Psi}_n^s(t)$, $\underline{\Phi}_n^i(t)$, and $\underline{\Psi}_n^i(t)$, defined by

$$\begin{aligned} v_n = V_{n,n} \quad w_n = W_{n,n} \quad \underline{\phi}_n^s = \underline{\Phi}_{n,n}^s \\ \underline{\psi}_n^s = \underline{\Psi}_{n,n} \quad \underline{\phi}_n^i = \underline{\Phi}_{n,n}^s \quad \underline{\psi}_n^i = \underline{\Psi}_{n,n}^i. \end{aligned} \tag{8}$$

The algebraic integration of Eqs. (3), (5), (6), (7), and (8) then produced the modal response equations

$$\begin{aligned} n(n + 1)V_{n,n+2} + \lambda_n^{VV}V_{n,n} + \lambda_n^{VW}W_{n,n} &= 0 \\ W_{n,n+2} + \lambda_n^{VW}V_{n,n} + \lambda_n^{WW}W_{n,n} & \\ + \mu[\underline{\Phi}_{n,n+1}^s - \underline{\Phi}_{n,n+1}^i] &= -\mu\underline{p}_n^0 \\ W_{n,n+1} - \underline{\Phi}_{n,n+1}^s - \underline{\Phi}_{n,n}^s - \underline{\Psi}_{n,n}^s &= \dot{\underline{u}}_n^0 \\ W_{n,n+1} + \underline{\Phi}_{n,n+1}^i - \underline{\Phi}_{n,n}^i - \underline{\Psi}_{n,n}^i &= 0 \\ \sum_{m=0}^{n} \Gamma_{nm}(m\underline{\Phi}_{n,n-m}^s - \underline{\Psi}_{n,n-m}^s) &= 0 \\ \sum_{m=0}^{n} (-1)^m\Gamma_{nm}(m\underline{\Phi}_{n,n-m}^i - \underline{\Psi}_{n,n-m}^i) & \\ = (-1)^n \sum_{m=0}^{n} \Gamma_{nm}[\underline{\Psi}_{n,n-m}^i + (m + 2)\underline{\Phi}_{n,n-m}^i & \\ - 2W_{n,n-m+1}]_{t-2} & \end{aligned} \tag{9}$$

These constitute six equations for six unknowns, given the modal components of incident-wave pressure and normal fluid-particle velocity at the external surface of the shell.

Improved Modal Convergence

Straightforward superposition of even a large number of modal responses does not exhibit satisfactory convergence for certain quantities, such as pressure. Hence, Zhang and Geers (1993) utilized two convergence-enhancement techniques to obtain accurate point response histories. One of these is a modified Cesàro summation (Apostol, 1957), that multiplies each modal response by a weighting factor to reduce Gibbs type oscillations.

The other technique is partial closed-form solution, in which each modal response [e.g., $w_n(t)$] is expressed as the sum of an initial modal response [e.g., $w_n^*(t)$] and a complementary modal response [e.g., $w_n^+(t)$]. The initial modal responses are solutions to Eq. (9) when only the terms in those equations that dominate early time response are retained; it turns out that the initial modal responses can be summed in closed form to produce, for example, $w^*(\theta, t)$. The complementary modal responses are solutions to a set of equations complementary to Eq. (9), which yield, for example, $w_n^+(t) = w_n(t) - w_n^*(t)$. This set is

$$
\begin{aligned}
&n(n+1)V_{n,n+2} + \lambda_n^{VV} V_{n,n} + \lambda_n^{VW} W^{+}_{n,n} = -\lambda_n^{VW} w_n^{*} \\
&W^{+}_{n,n+2} + \lambda_n^{VW} V_{n,n} + \lambda_n^{WW} W^{+}_{n,n} + \mu[\underline{\Phi}^{s+}_{n,n+1} - \underline{\Phi}^{i+}_{n,n+1}] = -\lambda_n^{WW} w_n^{*} \\
&W^{+}_{n,n+1} - \underline{\Phi}^{s+}_{n,n+1} - \underline{\Phi}^{s+}_{n,n} - \underline{\Psi}^{s}_{n,n} = \underline{\Phi}^{s*}_{n,n} \\
&W^{+}_{n,n+1} + \underline{\Phi}^{i+}_{n,n+1} - \underline{\Phi}^{i+}_{n,n} - \underline{\Psi}^{i}_{n,n} = \underline{\Phi}^{i*}_{n,n} \qquad (10) \\
&\sum_{m=0}^{n} \Gamma_{nm}(m\underline{\Phi}^{s+}_{n,n-m} - \underline{\Psi}^{s}_{n,n-m}) = -\sum_{m=1}^{n} m\Gamma_{nm}\underline{\Phi}^{s*}_{n,n-m} \\
&\sum_{m=0}^{n} (-1)^m \Gamma_{nm}(m\underline{\Phi}^{i+}_{n,n-m} - \underline{\Psi}^{i}_{n,n-m}) = -\sum_{m=1}^{n} (-1)^m m\Gamma_{nm}\underline{\Phi}^{i*}_{n,n-m}
\end{aligned}
$$

in which the initial modal responses on the right are determined by modal decomposition of the corresponding closed-form solutions:

$$
\begin{aligned}
\dot{w}^{*}(\theta, t) &= -\lambda^{-1}P_I(1 - \cos\theta)(1 - e^{-\lambda\mu\tau})H(\tau) \\
\underline{\dot{\phi}}^{s*}(\theta, t) &= -\lambda^{-1}P_I(1 - e^{-\lambda\mu\tau})H(\tau) - P_I \cos\theta[1 - \lambda^{-1}(1 - e^{-\lambda\mu\tau})H(\tau)] \\
\underline{\dot{\phi}}^{i*}(\theta, t) &= [(\lambda - 1)/\lambda]P_I(1 - \cos\theta) \cdot (1 - e^{-\lambda\mu\tau})H(\tau) \qquad (11)
\end{aligned}
$$

where λ is one (two) if the shell is empty (filled), $\tau = t - \cos\theta$ and $H(\tau)$ is the Heaviside step-function.

Use of the two convergence-enhancement techniques just described yields, for $0 \leq t < 2$, point response histories for meridional displacement, radial displacement, external surface pressure, and internal surface pressure:

$$
\begin{aligned}
v(\theta, t) &= -\sum_{n=1}^{N} C_n v_n(t) \frac{d}{d\theta} P_n(\cos\theta) \\
w(\theta, t) &= w^{*}(\theta, t) + \sum_{n=0}^{N} C_n w_n^{+}(t) P_n(\cos\theta) \\
\underline{p}^{e}(\theta, t) &= \underline{p}^{0}(\theta, t) + \underline{\dot{\phi}}^{s*}(\theta, t) + \sum_{n=0}^{N} C_n \underline{\dot{\phi}}_n^{s+}(t) P_n(\cos\theta) \qquad (12) \\
\underline{p}^{i}(\theta, t) &= \underline{\dot{\phi}}^{i*}(\theta, t) + \sum_{n=0}^{N} C_n \underline{\dot{\phi}}_n^{i+}(t) P_n(\cos\theta)
\end{aligned}
$$

where $C_0 = C_1 = C_2 = 1$ and $C_n = (N + 1 - n)/(N - 1)$ for $n > 2$. For $t \geq 2$, a partial closed-form solution is no longer used because the initial responses no longer dominate the complete solution and the procedure becomes more complicated. Hence, Eq. (9) comes into play, and point response histories for $t \geq 2$ are given by Eq. (12) with $w^{*}(\theta, t) = \underline{\dot{\phi}}^{s*}(\theta, t) = \underline{\dot{\phi}}^{i*}(\theta, t) = 0$ and with $w_n^{+}(t)$, $\underline{\dot{\phi}}_n^{s+}(t)$, and $\underline{\dot{\phi}}_n^{i+}(t)$ replaced by $w_n(t)$, $\underline{\dot{\phi}}_n^{s}(t)$, and $\underline{\dot{\phi}}_n^{i}(t)$, respectively.

Shell Stress and Strain

The stress and strain fields in the shell are also of interest. From Junger and Feit (1986), these are given by

$$
\begin{aligned}
\sigma_{\theta\theta}(z, \theta, t) &= \frac{\rho_0 c_0^2}{\rho c^2} [\varepsilon_{\theta\theta}(z, \theta, t) + \nu\varepsilon_{\varphi\varphi}(z, \theta, t)] \\
\sigma_{\varphi\varphi}(z, \theta, t) &= \frac{\rho_0 c_0^2}{\rho c^2} [\varepsilon_{\varphi\varphi}(z, \theta, t) + \nu\varepsilon_{\theta\theta}(z, \theta, t)] \\
\varepsilon_{\theta\theta}(z, \theta, t) &= \frac{\partial v}{\partial\theta}(\theta, t) + w(\theta, t) + z\left[\frac{\partial v}{\partial\theta}(\theta, t) - \frac{\partial^2 w}{\partial\theta^2}(\theta, t)\right] \qquad (13) \\
\varepsilon_{\varphi\varphi}(z, \theta, t) &= v(\theta, t)\cot\theta + w(\theta, t) + z\left[v(\theta, t) - \frac{\partial w}{\partial\theta}(\theta, t)\right]\cot\theta.
\end{aligned}
$$

Convergence-enhanced solutions for shell strains are provided by SPHSHK/MODSUM, as follows.

From the first of Eq. (11), the closed-form initial-response solutions for the step-wave-excited shell's radial displacement and its first and second meridional derivatives are

$$
\begin{aligned}
w^{*}(\theta, t) &= -\lambda^{-1}P_I(1 - \cos\theta) \cdot \left[\tau - \frac{1}{\lambda\mu}(1 - e^{-\lambda\mu\tau})\right] H(\tau) \\
\frac{\partial w^{*}}{\partial\theta}(\theta, t) &= -\lambda^{-1}P_I(1 - \cos\theta) \cdot \sin\theta\,(1 - e^{-\lambda\mu\tau})H(\tau) - \lambda^{-1}P_I \sin\theta\left[\tau - \frac{1}{\lambda\mu}(1 - e^{-\lambda\mu\tau})\right] H(\tau) \qquad (14) \\
\frac{\partial^2 w^{*}}{\partial\theta^2}(\theta, t) &= -\mu P_I(1 - \cos\theta)\sin^2\theta\, e^{-\lambda\mu\tau} H(\tau) - \lambda^{-1}P_I[(1 - \cos\theta)\cos\theta + 2\sin^2\theta](1 - e^{-\lambda\mu\tau}) H(\tau) \\
&\quad - \lambda^{-1}P_I \cos\theta\left[\tau - \frac{1}{\lambda\mu}(1 - e^{-\lambda\mu\tau})\right] H(\tau).
\end{aligned}
$$

With these solutions and the solutions to Eq. (10), the last two of Eq. (13) and the first two of Eq. (12) yield for meridional and azimuthal strain

$$
\begin{aligned}
\varepsilon_{\theta\theta}(z, \theta, t) &= w^*(\theta, t) - z \frac{\partial^2 w^*}{\partial \theta^2}(\theta, t) \\
&\quad + \sum_{n=0}^{N} C_n \Big\{ w_n^+(t) P_n(\cos\theta) \\
&\quad - [(1+z)v_n(t) + z w_n^+(t)] \frac{d^2}{d\theta^2} P_n(\cos\theta) \Big\} \\
\varepsilon_{\varphi\varphi}(z, \theta, t) &= w^*(\theta, t) - z \cot\theta \frac{\partial w^*}{\partial \theta}(\theta, t) \\
&\quad + \sum_{n=0}^{N} C_n \Big\{ w_n^+(t) P_n(\cos\theta) \\
&\quad - \cot\theta [(1+z)v_n(t) + z w_n^+(t)] \frac{d}{d\theta} P_n(\cos\theta) \Big\}.
\end{aligned}
\tag{15}
$$

DAA_2 Residual Potentials

DAA_2-based computation in SPHSHK has been configured to fit within the framework of the acoustic computation by redefining the residual potentials such that Eq. (6) become equivalent to the DAA_2 equations (Geers and Zhang, 1991). The latter are [cf., Eq. (6)]

$$
\begin{aligned}
&\dot{\phi}_{n,r}^s(t) + n\phi_{n,r}^s(t) + \ddot{\phi}_n^s(t) + (n+1)\dot{\phi}_n^s(t) \\
&\qquad + n(n+1)\phi_n^s(t) = 0 \\
&\ddot{\phi}_{o,r}^i(t) + 3\dot{\phi}_{o,r}^i(t) + 6\phi_{o,r}^i(t) \\
&\qquad + \dddot{\phi}_o^i(t) + 2\ddot{\phi}_o^i(t) = 0 \\
&\dot{\phi}_{n,r}^i(t) + (n+1)\phi_{n,r}^i(t) + \ddot{\phi}_n^i(t) \\
&\qquad + n\dot{\phi}_n^i(t) + n(n+1)\phi_n^i(t) = 0, \quad n > 0.
\end{aligned}
\tag{16}
$$

The resulting DAA_2 relations that link the integrated residual potentials and the integrated velocity potentials [see Eq. (8)] may be found to be [cf., the last two of Eqs. (9) and (10)]

External DAA_2, $0 \le t < 2$:

$$\underline{\Psi}_{n,n+1}^s + n\underline{\Psi}_{n,n}^s - n^2\underline{\Phi}_{n,n}^{s+} = n^2\underline{\Phi}_{n,n}^{s*}.$$

Internal DAA_2, $0 \le t < 2$, $n = 0$:

$$\underline{\Psi}_{0,2}^i + 3\underline{\Psi}_{0,1}^i + 6\underline{\Psi}_{0,0}^i - 3\underline{\Phi}_{0,1}^{i+} + 6\underline{\Phi}_{0,0}^{i+} = 3\underline{\Phi}_{0,1}^{i*} - 6\underline{\Phi}_{0,0}^{i*}.$$

Internal DAA_2, $0 \le t < 2$, $n > 0$:

$$\underline{\Psi}_{n,n+1}^i + (n+1)\underline{\Psi}_{n,n}^i + (n+1)^2\underline{\Phi}_{n,n}^{i+} = -(n+1)^2\underline{\Phi}_{n,n}^{i*}.$$

External DAA_2, $t \ge 2$:

$$\underline{\Psi}_{n,n+1}^s + n\underline{\Psi}_{n,n}^s - n^2\underline{\Phi}_{n,n}^s = 0.$$

Internal DAA_2, $t \ge 2$, $n = 0$:

$$\underline{\Psi}_{0,2}^i + 3\underline{\Psi}_{0,1}^i + 6\underline{\Psi}_{0,0}^i - 3\underline{\Phi}_{0,1}^i + 6\underline{\Phi}_{0,0}^i = 0.$$

Internal DAA_2, $t \ge 2$, $n > 0$:

$$\underline{\Psi}_{n,n+1}^i + (n+1)\underline{\Psi}_{n,n}^i + (n+1)^2\underline{\Phi}_{n,n}^i = 0. \tag{17}$$

Thus, the computational procedure for DAA_2 treatments of the external and/or internal fluid–structure interaction is identical to that for the acoustic treatments.

REPRESENTATIVE CALCULATIONS

Use of SPHSHK/MODSUM

Computation with SPHSHK/MODSUM is very simple (Ju and Geers, 1992). SPHSHK performs the modal response calculations for any selected group of contiguous modes from $n = 0$ through $n = 8$. Acoustic or DAA_2 treatments of the external and internal fluid–structure interactions may be selected in any combination. Computations may be performed for any duration, subject to data storage capacity. The parameter data are the ratio of shell thickness to radius, Poisson's ratio for the shell material, the ratio of shell density to fluid density, and the ratio of shell plate velocity to fluid sound speed.

MODSUM computes physical responses at desired θ-locations using partial closed-form solution and modified Cesàro summation of the modal responses generated by SPHSHK. At each θ-location specified, MODSUM generates response histories for quantities selected from the following list: external and internal surface pressure; radial and meridional velocity; meridional and azimuthal membrane strain; meridional and azimuthal bending strain; meridional and azimuthal strain at the external surface; and meridional and azimuthal strain at the internal surface.

Numerical Results

Sample response histories are presented here for a steel shell submerged in and containing water. The thickness-to-radius ratio for the shell is 0.01, Poisson's ratio is 0.3, the ratio of shell density to fluid density is 7.7, and the ratio of shell plate velocity to fluid sound speed is 3.715.

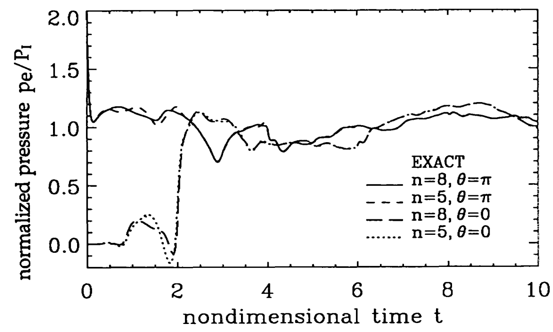

FIGURE 2 External-surface pressure histories for acoustic treatments of the fluid–structure interaction.

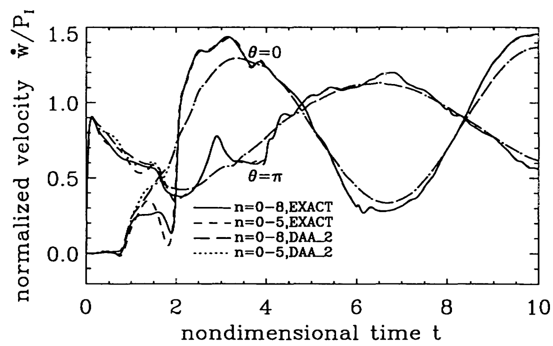

FIGURE 4 Radial-velocity histories for acoustic and DAA_2 treatments of the fluid–structure interaction.

Figures 2–5 show external-surface-pressure, radial-velocity, and meridional-strain histories at $\theta = 0$ and π for both external-acoustic/internal-acoustic and external-DAA_2/internal-DAA_2 treatments of the fluid–structure interaction. Histories for an external-DAA_2/internal-acoustic treatment would virtually coincide with their external-acoustic/internal-acoustic counterparts, and those for an external-acoustic/internal-DAA_2 treatment would virtually coincide with their external-DAA_2/internal-DAA_2 counterparts. Hence, DAA_2 treatment of the external fluid–structure interaction is more accurate than DAA_2 treatment of the internal fluid–structure interaction.

Comparison of $n = 0–5$ and $n = 0–8$ summations in Figs. 2–4 indicates that modal convergence for external pressure and radial velocity is good except during $1 < t < 2$ at $\theta = 0$. Similar comparison of strain responses indicates good convergence at all times. Figure 5 shows that bending strain is much smaller than membrane strain. Extensive discussions of numerical results for this problem are contained in Geers and Zhang (1991) and in Zhang and Geers (1993).

CONCLUSION

A computer program, SPHSHK/MODSUM, has been developed to generate response histories for the canonical underwater shock problem of a step-wave-excited spherical shell submerged in and (optionally) filled with an acoustic fluid. The program is based on a solution by generalized Fourier series, with modal convergence enhanced through Cesàro summation and partial closed-form solution. It is available on a DOS-formatted floppy disc accompanied by hardcopy documentation from SAVIAC, 2711 Jefferson Davis Highway, Suite 600, Arlington, VA 22202-4158.

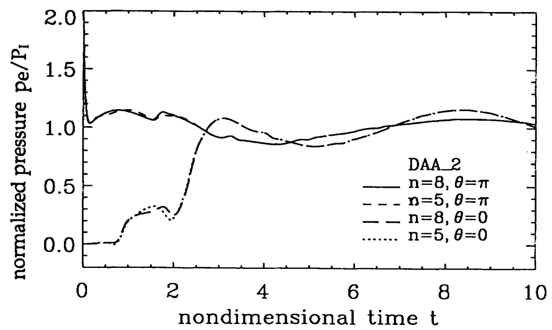

FIGURE 3 External-surface pressure histories for DAA_2 treatments of the fluid–structure interaction.

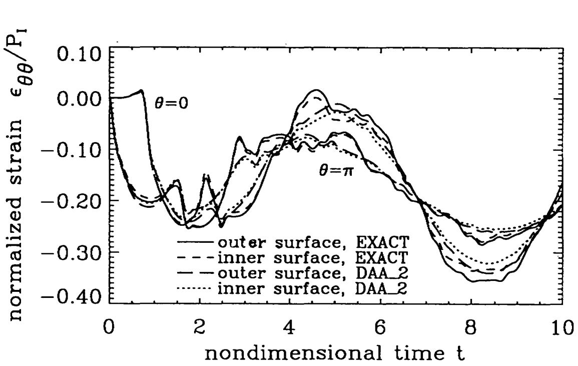

FIGURE 5 Meridional-strain histories for acoustic and DAA_2 treatments of the fluid–structure interaction.

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