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A Modified Eigenstructure Assignment Technique for Finite Element Model Updating

This article deals with an extended application of the constrained eigenstructure assignment method (CEAM) to finite element model updating. The existing formulation is modified to accommodate larger systems by developing a quadratic linear optimization procedure that is unconditionally stable. Further refinements include the updating of the mass matrix, a hysteretic damping model, and the introduction of elemental correction factors. Six numerical test cases, dealing with effects of damping and measurement noise, mode shape incompleteness, and discretization differences, were conducted in the case of a 3-D frame model with 114 coordinates. The performance of the CEAM was evaluated systematically for both the purpose of error location and the global correction of the initial model. The same cases were also studied using another model updating approach, namely the response function method (RFM). It was found that the CEAM had a number of distinct advantages, such as yielding a noniterative direct solution, requiring much less computing power, and providing acceptable results for cases, that could not be handled using the RFM.

INTRODUCTION

In spite of extensive research over the past 15 years, the state of the art in finite element (FE) model updating is far from maturing and no reliable and generally applicable procedures have been formulated so far. Several review articles (Natke, 1988; Ibrahim, 1988; Imregun and Visser, 1991; Mottershead and Friswell, 1993) reveal a wealth of updating algorithms, but the success seems to be case dependent and the applicability bounded by the skill of the analyst in choosing a *correct* set of parameters. In any case, two somewhat related approaches are now emerging as accepted state of the art tools: the inverse eigensensitivity method (IESM) (Zhang and Lallement, 1987) and the response function method (RFM) (Lin and Ewins, 1990; Visser and Imregun, 1991).

A review of the case studies reported in the literature unveils a fundamental problem: a particular solution is usually nonunique and a generated solution does not necessarily represent a true physical meaning (Imregun, 1995). Furthermore, the rate of success seems to depend on the particulars of the cases studied as well as on the skill of the analyst. Part of the problem stems from the iterative nature of both methods and the nu-

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merical difficulties that are encountered in such situations.

A recently proposed technique, the so-called constrained eigenstructure assignment method (CEAM), has the advantage of being direct and hence of not requiring any iterations for the convergence of the updating parameters, although the solution stage may contain iterative scchemes such as nonlinear optimization (Schulz and Inman, 1994). Eigenstructure assignment was first introduced in the field of control theory. Moore (1976) formulated the necessary and sufficient conditions for simultaneous eigenvalue and eigenvector assignment using state feedback for the case of distinct eigenvalues. Srinathkumar (1978) addressed the problem of pole assignment in linear time-invariant multivariable systems using output feedback. Andry et al. (1983) applied the eigenstructure assignment technique for a linear mechanical system for parameter identification. Minas and Inman (1988) and Inman and Minas (1990) applied the constrained assignment technique for the correction of damping and stiffness matrices of an FE model. They also used the pole placement method for systems with unknown mode shapes. Zimmerman and Widengren (1990) used a modified algorithm that allowed a symmetric eigenstructure assignment when correcting the damping and stiffness matrices. Their method required the solution of a general algebraic Riccati matrix equation, the size of which depended on the number of assigned modes, thus requiring very considerable CPU power for large-order systems. Finally, Schulz and Inman (1994) used the eigenstructure assignment technique with a number of constraints that were related to the physical properties of the system to be updated. They considered small-order systems that were symmetric, banded, and bounded. The constraints were built into a nonlinear optimization procedure that preserved the desired properties of the updated model.

The work presented here is an extension of Schulz and Inman's article (1994) and the following features are common to both studies. The correction of the FE model is based on a subset of modes and frequencies, as it would be in the case of measured data because of various practical limitations. The symmetry of the system matrices, the connectivity information, and other conditions, such as positive-definite mass matrix and maximum allowable change in design variables, are introduced in the form of constraints. Although the solution is bounded by the limited variability of the design parameters, no formal functional constraints exist because the limits can only be expressed in terms of inequalities.

The primary purpose of the current work is to make the methodology applicable to large systems, not only by developing a quadratic linear optimization procedure instead of the nonlinear one, but also by reducing the number of unknowns via an error representation that involves one design parameter per individual FE matrix. Although there are further variants of this latter feature, such as the allocation of design parameters to physical quantities, they will not be explored here. The main disadvantages of a nonlinear optimization procedure are the extensive CPU requirements and slow convergence.

A further objective of the study is to compare the performance of the CEAM against one of the more established model updating techniques, such as the RFM.

REVIEW OF CEAM FORMULATION

Although the following formulation is very similar to that given by Schulz and Inman (1994), two differences can be noticed: the mass matrix is included in the updating process, and a hysteretic damping model is used instead of a viscous one.

Consider an N degree of freedom spatial model with structural damping,

$$M\ddot{X} + (K + iH)\mathbf{X} = 0, \qquad (1)$$

where M, K, and H are the mass, stiffness, and hysteretic damping matrices, and $\mathbf{X}$ is the displacement vector.

Let us assume that it is possible to find an updated system that satisfies the equation of motion,

$$[M + \overline{M}]\ddot{\mathbf{X}} + [(K + \overline{K}) + i(H + \overline{H})]\mathbf{X} = 0, \qquad (2)$$

where $\overline{M}, \overline{K}$, and $\overline{H}$ denote the updated mass, stiffness, and hysteretic damping matrices. Two conditions will be imposed on the updated model.

1. The updated matrices must remain real, symmetric, and preserve the initial connectivity information.
2. The initial and updated models must have the same m modes, characterized by natural frequencies ω_r and mode shapes ϕ_r where $r = 1, \ldots, m$. At this stage it will be

assumed these are error free and that they can be obtained from a modal analysis of the measured data.

Using Eqs. (1), (2), and the two constraints above, we get

$$(M\omega_r^2 + K + iH)\phi_r + (\overline{M}\omega_r^2 + \overline{K} + i\overline{H})\phi_r = 0. \tag{3}$$

Equation (3) can be written as

$$[M\omega_r^2 + K + iH \quad I] \begin{bmatrix} \phi_r \\ (\overline{M}\omega_r^2 + \overline{K} + i\overline{H})\phi_r \end{bmatrix} = \Gamma_r \mathbf{\Psi}_r = 0. \tag{4}$$

Using a QR decomposition, the vector $\mathbf{\Psi}_r$ becomes

$$\mathbf{\Psi}_r = \begin{bmatrix} V_r \\ \overline{V}_r \end{bmatrix} \mathbf{e}_r, \tag{5}$$

where the vectors forming the columns of the $2N \times N$ matrix $\begin{bmatrix} V_r \\ \overline{V}_r \end{bmatrix}$ are the orthonormal basis for the null space of the matrix Γ_r and $\mathbf{e}_r$ is an $N \times 1$ vector of complex coefficients. Combining Eqs. (4) and (5) we get

$$\phi_r = V_r \mathbf{e}_r. \tag{6}$$

If all N ($>m$) modes are measured,

$$\mathbf{e}_r = V_r^{-1}\phi_r. \tag{7}$$

However, if only $m < N$ modes are available,

$$\mathbf{e}_r = V_r^{\oplus}\phi_r, \tag{8}$$

where $\oplus$ denotes the pseudoinverse of V.

Again, combining Eqs. (4) and (5), one obtains

$$(\overline{M}\omega_r^2 + \overline{K} + i\overline{H})\phi_r = \overline{V}_r \mathbf{e}_r. \tag{9}$$

Substituting Eq. (6) into (9) gives

$$(\overline{M}\omega_r^2 + \overline{K} + i\overline{H})V_r \mathbf{e}_r = \overline{V}_r \mathbf{e}_r. \tag{10}$$

Extending Eq. (10) into full matrix form yields

$$\overline{M} VE\Omega^2 + (\overline{K} + i\overline{H})VE = \overline{V}E. \tag{11}$$

Rearranging Eq. (11) we get

$$[\overline{M} \quad \overline{H} \quad \overline{K}] \begin{bmatrix} VE\Omega^2 \\ iVE \\ VE \end{bmatrix} = \overline{V}E, \tag{12}$$

where $[\overline{M} \quad \overline{H} \quad \overline{K}]$ is $N \times 3N$, $[VE\Omega^2 \quad VE \quad iVE]^T$ is $m \times 3N$, and $\overline{V}E$ is $N \times m$.

Separating (12) into its real and imaginary parts,

$$[\overline{M} \quad \overline{H} \quad \overline{K}] \begin{bmatrix} (VE)_R(\Omega_R^2 - \Omega_I^2) - 2(VE)_I\Omega_R\Omega_I & (VE)_I(\Omega_R^2 - \Omega_I^2) + 2(VE)_R\Omega_R\Omega_I \\ -(VE)_I & (VE)_R \\ (VE)_R & (VE)_I \end{bmatrix} = [(\overline{V}E)_R \quad \overline{V}E)_I],$$

or

$$[\overline{M} \quad \overline{H} \quad \overline{K}]\, G = Q, \tag{13}$$

where the real-valued matrices $[\overline{M} \quad \overline{H} \quad \overline{K}]$, G, and Q are $N \times 3N$, $3N \times 2m$, and $N \times 2m$, respectively.

The uniqueness of the solution and the importance of imposing constraints is discussed in some detail by Schulz and Inman (1994). It is reported that, in the general case, an optimization method will have to be used to solve Eq. (13), especially for FE model updating when constraints are needed to preserve the form and connectivities of the mass, stiffness, and damping matrices. The next section will deal with the derivation of such a solution procedure.

FORMULATION OF A QUADRATIC SOLUTION PROCEDURE

A solution to Eq. (13) is sought by finding the minimum of an objective function J defined as

$$J = \| [\overline{M} \quad \overline{H} \quad \overline{K}]\, G - Q \|_f, \tag{14}$$

subject to various constraints on matrices $\overline{M}, \overline{H}$, and $\overline{K}$, $\| \quad \|_f$ denoting the Frobenius norm of a matrix.

There are at least two ways of minimizing J: the first one is to use a nonlinear technique that results in an iterative scheme; the second one, which will be developed here, is to use a quadratic formulation that has the added advantage of being unconditionally stable.

Let us assume that

$$J = 1/2\Theta^T A\Theta + \mathbf{B}^T\Theta, \tag{15}$$

where A is a $3N^2 \times 3N^2$ matrix and $\mathbf{B}$ is a $3N^2 \times 1$ vector. The partial derivatives are given by

$$\frac{\partial J}{\partial\Theta_i\partial\Theta_j} = A_{ij} \quad \text{and} \quad \left.\frac{\partial J}{\partial\Theta_i}\right|_{\Theta_j=0} = \mathbf{B}_i. \tag{16}$$

Let $C = [\overline{M} \quad \overline{H} \quad \overline{K}]$ and $S = CG - Q$. With the new notation Eq. (15) becomes

$$J = \text{trace}(S^T S) = S_{iq}S_{iq} \quad i = 1, \ldots, N \quad \text{and} \quad q = 1, \ldots, 2m, \tag{17}$$

where the repeated index means summation over the index.

Using the definition of matrix S,

$$S_{iq} = C_{ij}G_{jq} - Q_{iq} \quad i = 1, \ldots, N \quad q = 1, \ldots, 2m, j = 1, \ldots, 3N. \tag{18}$$

Inserting Eq. (18) into (17),

$$J = C_{ij}G_{jq}C_{il}G_{lq} - 2C_{ij}G_{jq}Q_{iq} + Q_{iq}Q_{iq} \quad i = 1, \ldots, N, \quad q = 1, \ldots, 2m; j, l = 1, \ldots, 3N. \tag{19}$$

By differentiating Eq. (19) with respect to C_{lm} and using Eq. (16) we get

$$A_{ijkl} = 2\delta^i_k G_{jq}G_{lq} \quad \text{and} \quad \mathbf{B}_{ij} = -2G_{jq}Q_{iq} \quad q = 1, \ldots, 2m, \quad i,k = 1, \ldots, N; j,l = 1, \ldots, 3N, \tag{20}$$

where δ is the Kronecker delta function.

The size of matrix A, $3N^2 \times 3N^2$, is prohibitive for any practical application to be considered. However, as in most updating studies, there is no particular need to update the individual elements of the global mass, stiffness, and damping matrices. Common practice is to assign correction factors, the so-called p values, to the individual FE matrices and to compute those to obtain the required global changes. In other words, it is assumed that the errors are proportional to the elemental matrices

$$\overline{M} = \sum_{i=1}^{L} p_i^m M_i, \quad \overline{H} = \sum_{i=1}^{L} p_i^h H_i, \quad \overline{K} = \sum_{i=1}^{L} p_i^k K_i, \tag{21}$$

where L is the number of individual FEs in the model and p_i^m, p_i^k, and p_i^h are the ith correction factors for the mass, stiffness, and damping matrices, respectively.

Referring to Eq. (13), let us define

$$G = \begin{bmatrix} G1 \\ G2 \\ G3 \end{bmatrix},$$

where $G1$, $G2$, and $G3$ are $N \times 2m$ matrices. Inserting Eq. (21) into (13) one obtains

$$\left(\sum_{i=1}^{L} p_i^m M_i\right) G1 + \left(\sum_{i=1}^{L} p_i^h H_i\right) G2 + \left(\sum_{i=1}^{L} p_i^k K_i\right) G3 = Q. \tag{22}$$

Equation (22) can explicitly be written as:

$$\left[\begin{Bmatrix} (M_1G_1)_{1,1} \\ \cdot \\ (M_1G_1)_{N,1} \\ (M_1G_1)_{2,1} \\ \cdot \\ \cdot \\ (M_1G_1)_{N,N} \end{Bmatrix}, \begin{Bmatrix} (M_2G_1)_{1,1} \\ \cdot \\ (M_2G_1)_{N,1} \\ (M_2G_1)_{2,1} \\ \cdot \\ \cdot \\ (M_2G_1)_{n,n} \end{Bmatrix}, \right.$$

$$\ldots, \begin{Bmatrix} (M_LG_1)_{1,1} \\ \cdot \\ (M_LG_1)_{N,1} \\ (M_LG_1)_{2,1} \\ \cdot \\ \cdot \\ (M_LG_1)_{N,N} \end{Bmatrix}, \begin{Bmatrix} (H_1G_2)_{1,1} \\ \cdot \\ (H_1G_2)_{N,1} \\ (H_1G_2)_{2,1} \\ \cdot \\ \cdot \\ (H_1G_2)_{N,N} \end{Bmatrix},$$

$$
\ldots, \left\{\begin{matrix} (H_L G_2)_{1,1} \\ \cdot \\ (H_L G_2)_{N,1} \\ (H_L G_2)_{2,1} \\ \cdot \\ \cdot \\ (H_L G_2)_{N,N} \end{matrix}\right\}, \left\{\begin{matrix} (K_1 G_3)_{1,1} \\ \cdot \\ (K_1 G_3)_{N,1} \\ (K_1 G_3)_{2,1} \\ \cdot \\ \cdot \\ (K_1 G_3)_{N,N} \end{matrix}\right\}, \ldots,
$$

$$
\left.\left\{\begin{matrix} (K_L G_3)_{1,1} \\ \cdot \\ (K_L G_3)_{N,1} \\ (K_L G_3)_{2,1} \\ \cdot \\ \cdot \\ (K_L G_3)_{N,N} \end{matrix}\right\}\right] \left\{\begin{matrix} P_1^m \\ \vdots \\ P_L^m \\ P_1^h \\ \vdots \\ P_L^h \\ P_1^k \\ \vdots \\ P_L^k \end{matrix}\right\} = \left\{\begin{matrix} Q_{1,1} \\ \cdot \\ Q_{N,1} \\ Q_{2,1} \\ \cdot \\ \cdot \\ Q_{N,N} \end{matrix}\right\}.
$$

In short matrix notation, Eq. (22) becomes

$$
A \times \mathbf{P} = \mathbf{B}, \tag{23}
$$

where A is a $2Nm \times 3L$ matrix, $\mathbf{P}$ is a vector of $3L \times 1$ unknowns, and $\mathbf{B}$ is a $2Nm \times 1$ vector. Therefore the initial $3N^2 \times 3N^2$ problem has now been transformed to an overdetermined problem of size $2Nm \times 3L$ where m is the number of measured modes and L is the number of FEs in the mathematical model. The $3L \times 1$ vector of correction factors can be found by applying a singular value decomposition to matrix A.

NUMERICAL STUDY

CEAM Case Studies

It is now proposed to apply the CEAM to the case of a 3-D frame that is modeled using 3-D 12 degree of freedom beam elements. This example was already used by Ziaei-Rad and Imregun (1996) to investigate the accuracy required of experimental data for model updating and it is one of the standard cases for comparative studies. Four different models were created for the purposes of conducting parametric studies. Model FE1 had 20 elements, for 10 of which the Young's modulus was increased by 8%. Model FE2 had also had 20 elements, but this time a 30% change was introduced into the X and Y moments of inertia (Fig. 1). X1 and X2 were considered to be error-free reference models, the latter being double the size of the former. The aim of the numerical studies was to correct models FE1 and FE2 using simulated experimental data obtained from models X1 and X2. The various models used are summarized in Table 1.

The main objective of the case study is to examine the performance of the CEAM from both numerical efficiency and updating quality viewpoints. Also, comparisons will be made against another model updating technique, namely the RFM. The mass matrix was excluded from the updating process for all cases but the fifth one, because its inclusion created numerical problems when there were no associated mass errors.

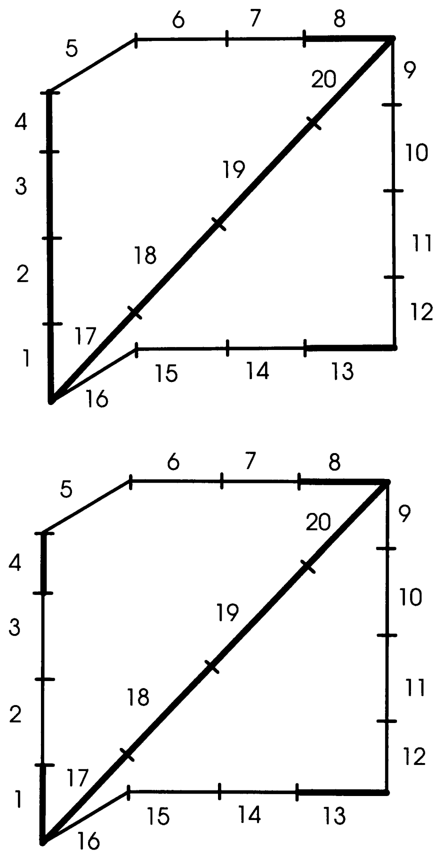

FIGURE 1 (a) The frame structure, model FE1 (8% increase in Young's modulus for the elements shown in bold). (b) The frame structure, model FE2 (30% increase in I_x and I_y for the elements shown in bold).

Table 1. Target and Initial FE Models of Frame Structure

Model	FE1 (Initial)	FE2 (Initial)	X1 (Target)	X2 (Target)
No. elements	20	20	20	40
Mesh size	19 nodes	19 nodes	19 nodes	39 nodes
No. DOFs	114	114	114	234
Errors	8% in E	30% in I_x and I_y	None	None

E, Young's modulus; I_x and I_y, moments of inertia.

Case 1: Assigning Real and Complete Modes. The first case studied is a straightforward check of the formulation whereby the first 10 modes of model X1 are assigned to model FE1 without the presence of any damping or experimental noise. In this particular case, the p values indicate the exact location of the error (Fig. 2) and the response obtained from the updated model is identical to that of the reference model X1 (Fig. 3). However, this is an expected result because

1. the problem is overdetermined with 40 unknowns (mass and stiffness p values for the 20 FEs) and 10 complete modes, each containing 115 data items;
2. the changes made are directly proportional to the correction factors and hence Eq. (21) is exact in this particular case; and
3. the initial and target models have identical meshes and hence the discretization errors cancel each other.

Case 2: Effect of Damping. The same exercise was repeated by including 1% hysteretic damping in both models, i.e., FE1 and X1. The damping in model X1 was forced to be nonproportional by considering some of the elements only. The initial damping in FE1 was assumed to be proportional by allocating a damping matrix for each element. In this case the number of unknowns is $3 \times 20 = 60$ and the elements of the target mode shape vector are complex. Using again 10 complete modes, the errors were identified exactly, including those associated with the nonproportional damping (Fig. 4). As a direct consequence, the responses obtained from the reference and updated models were found to be identical. This result is somewhat encouraging because other updating approaches are known to be prone to numerical problems in similar cases (Imregun et al., 1995a,b).

Case 3: Effect of Measurement Noise. The undamped and damped cases above were repeated for 5% random noise that was added to the simulated frequency response functions (FRFs) obtained from X1. The target eigenproperties were then obtained by applying a global rational fraction curve-fitting algorithm to the polluted FRFs. The computed p values are shown in Fig. 5 and it is immediately seen that there is little correspondence between the actual errors and the proposed corrections. The responses obtained from the target (X1), initial (FE1), and updated models are overlaid in Fig. 6. Although noise has an obvious detrimental effect on the updating quality, the updated model still shows a marked improvement over the initial one, indicating that the model has been corrected in some global sense without particular emphasis on the location or magnitude of

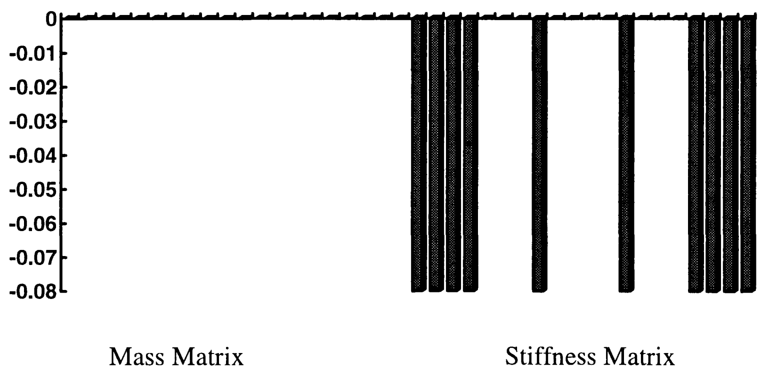

FIGURE 2 Computed p values for case 1 (CEAM).

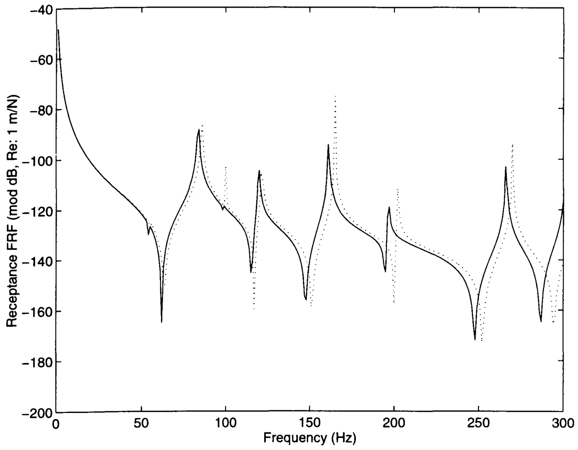

FIGURE 3 Initial, target, and updated responses for case 1 (CEAM). Target and updated responses are identical; (···) initial, (---) target, (—) updated.

the initial discrepancy. This finding is in line with those of many other studies that use a formulation similar to Eq. (21).

Case 4: Effect of Mode Shape Incompleteness. One of the well-known problems in model updating is the size incompatibility between the experimental and theoretical models. The difficulties arise because of poor accessibility and due to the lack of reliable methods for measuring the rotational degrees of freedom. This latter situation is simulated here by removing the rotational coordinates from the target mode shapes. Three different sets of results, corresponding to 40-, 60-, and 80-mode assignments, are shown in Fig. 7. The adverse effect of coordinate reduction is obvious and it can, to a certain extent, be compensated for by using more and more modes in the assignment, although no updated model is able to match the target one exactly.

Case 5: Effect of Localized Changes. Model FE2 was used to investigate the effect of localized changes. In this case, the discrepancies between X1 and FE2 are not directly proportional to the p values and hence Eq. (21) is no longer exact in this particular case. In other words, the moment of inertia errors are not global in nature because not all elements of the individual stiffness matrices are affected the same way. As the inertia errors are associated with the mass matrix, this was also included in the updating process. Initial calculations produced p values that were not rep-

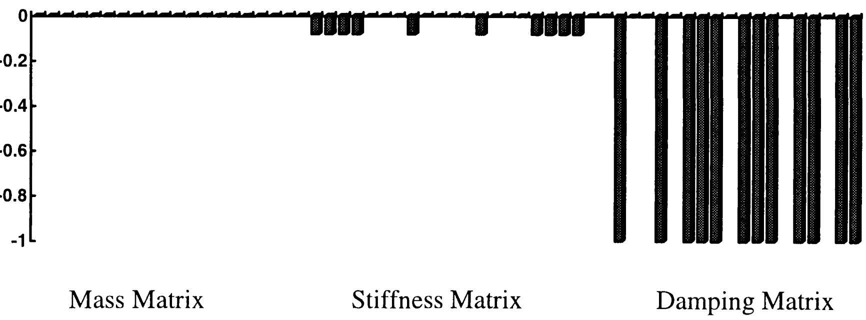

FIGURE 4 Computed p values for case 2 (CEAM).

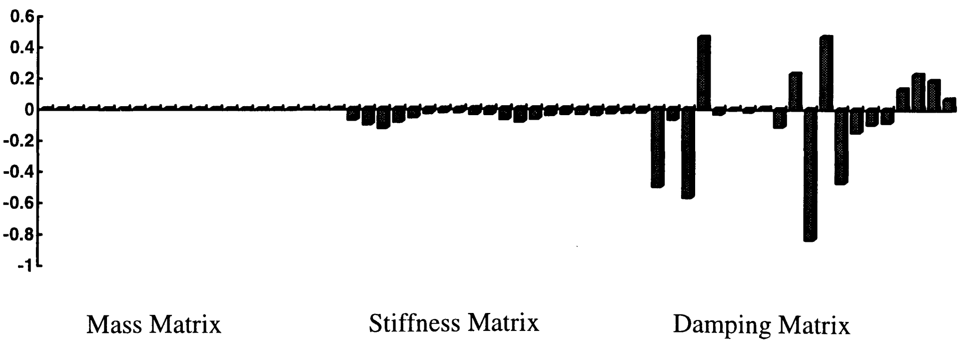

FIGURE 5 Computed p values for case 3 (CEAM).

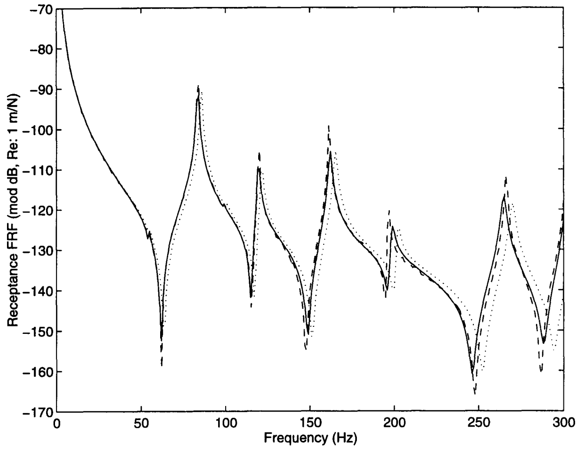

FIGURE 6 Initial, target, and updated responses for case 3 (CEAM): (· · ·) initial, (---) target, (—) updated.

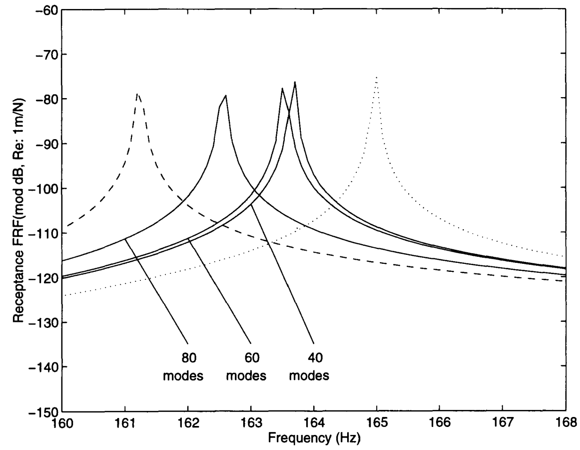

FIGURE 7 Effect of increasing the number of assigned modes, case 4 (CEAM): (· · ·) initial, (---) target, (—) updated.

resentative of the inertia discrepancies between the two models. More alarmingly, the resulting response model showed very poor agreement with the target one. It was then decided to increase the number of modes to be assigned as this approach was observed to be a cure in the previous case study. Given the practical limitations on the availbility of higher experimental modes, 30 modes of the initial model, FE2, were assigned to the updated model. The resulting p values are shown in Fig. 8 while the FRFs obtained from the updated model are compared to the initially predicted and target ones in Fig. 9. Although the error location is quite poor, the performance of the updated model is acceptable at the response level, a feature that again suggests that accurate error location is not necessarily a prerequisite for updating.

Case 6: Effect of Discretization Differences. All cases studied so far are based on a one to one correspondence between the theoretical and experimental models, a feature that cannot be achieved in practice. The fact that models FE1, FE2, and X1 have been discretized using the same mesh not only simplifies the problem of model updating significantly, but is also unrepresentative of the real engineering problem where the discrepancies between the experimental and theoretical models are not explicitly present in the theoretical model in the form of directly correctable parameters. It was therefore decided to update model FE1 using model X2, the mesh of which is double in size. Initially, it was attempted to assign the modal properties of model X2 to model FE1 directly. However, after a few attempts it was noticed that p values were all ap-

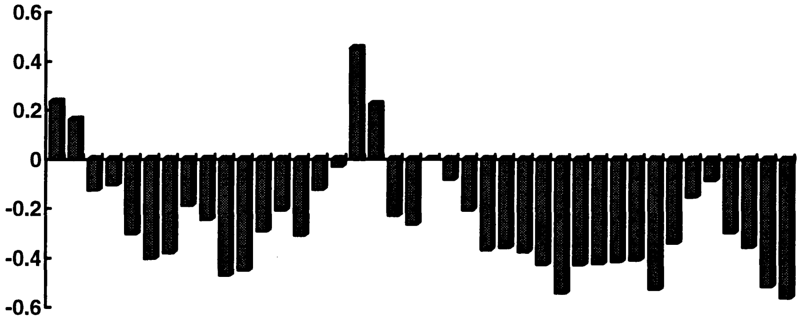

FIGURE 8 Computed p values for case 5 (CEAM).

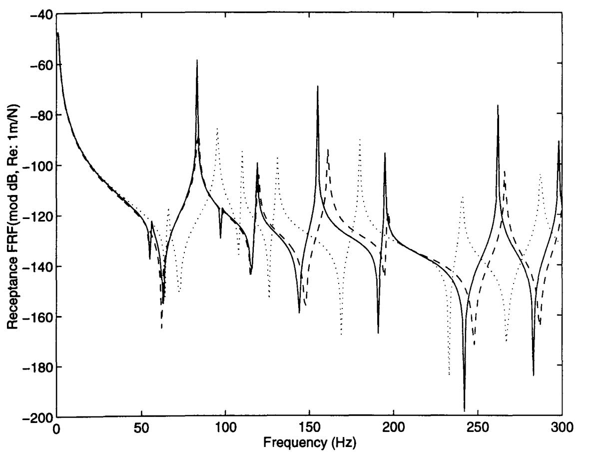

FIGURE 9 Initial, target, and updated responses for case 5 (CEAM): (···) initial, (---) measured, (—) updated.

proaching the value of -1, indicating that only the trivial solution could be found. The explanation for such behavior is relatively straightforward. As shown in Fig. 10, the assignment can only succeed if the target and initial sets belong to the same eigendomain. If the two sets are not in the same domain, it is not possible to find a set of mass and stiffness correction matrices so that the initial and updated models span the same set of eigenvalues and eigenvectors for a given frequency range. In other words, no updated model can be guaranteed until the closeness of the two sets is improved. Such an approach will be adopted here in the form of a two-stage assignment.

After some deliberation, it was decided to assign the first 10 eigenvalues (but not eigenvectors) of model X2 to model FE1 and to keep the eigenvectors of FE1 unchanged. This approach is

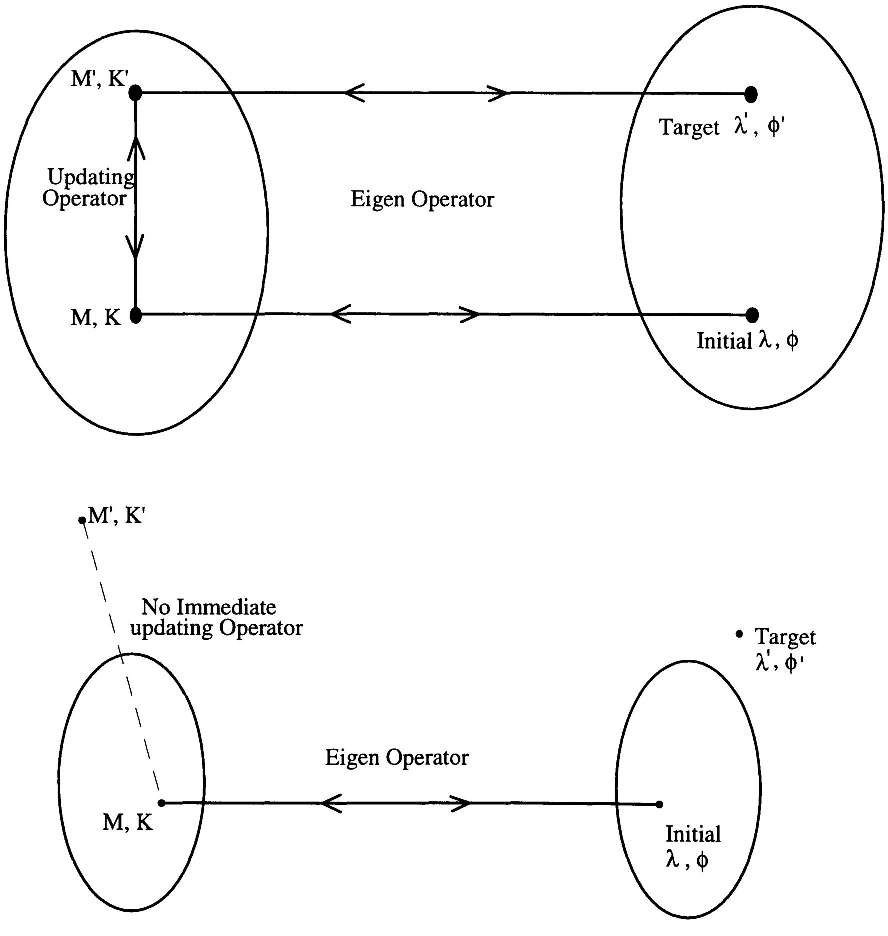

FIGURE 10 (a) Target and initial models belong to the same domain. (b) Target and initial models do not belong to the same domain.

equivalent to changing the global material properties of the system, say density and Young's modulus, because such a modification will produce shifts in the natural frequencies only. Once an updated model was obtained, a further assignment was made by using the first 10 eigenvalues and eigenvectors of X2. To force model closeness, a further 60 modes (both eigenvalues and eigenvectors) of model FE1 were self-assigned. The results of this two-stage assignment are plotted in Fig. 11 in the form of initial, target, and updated FRFs. It is interesting to note that the first mode is not particularly well corrected but the remaining part of the response shows a marked improvement. The problem lies, once again, in the closeness of the initial and target models. For the two given sets, it was not possible to find a modification that could correct the first mode.

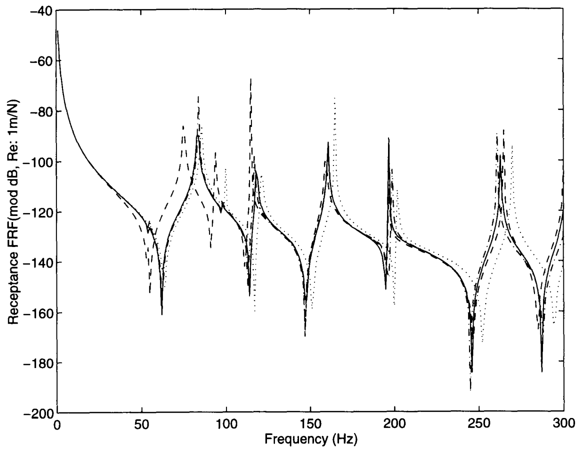

FIGURE 11 Initial, target, and the two updated responses for case 6 (two-stage CEAM): (· · ·) initial, (---) measured, (—) using eigenvalues, (-·-·-) using eigenvalues and eigenvectors.

A Comparison with RFM

It is now proposed to compare the performance of the RFM and CEAM by repeating the six cases above using the former technique. From the outset, it must be stressed that the RFM is an iterative method where the convergence of the p values cannot be guaranteed. On the other hand, the CEAM is based on the direct solution of an overdetermined set of linear equations and the optimization algorithm is unconditionally stable by virtue of being quadratic. Although both methods will yield identical results for noise-free and complete modal information cases, very significant differences can be seen in other situations. The results are listed in Table 2.

For the first two cases, both methods produced identical answers but the CEAM is seen to be about an order of magnitude faster, because the RFM needs three to four iterations for convergence. For the third case, where the measured FRF data are polluted by noise, the RFM fails to converge while the CEAM produces an updated model, the response from which is in good agreement with the target one. The fourth case, where incomplete mode shapes are used, is handled better by the RFM in the sense that both the discrepancies are identified and the agreement at the response level is good. However, it should be noted that relatively small errors still appear in the mass matrix. A comparison of the p values computed using the two methods is given in Fig. 12. The performance of the two methods is about the same for the fifth case for which the changes are in the moments of inertia. The last case, where there is no one to one correspondence between the target and the initial models, can only be dealt with via the CEAM. The agreement of the target and updated models at the response level is acceptable, except in the vicinity of the first mode.

Table 2. Computational Effort for RFM and CEAM Updating

Case	Description	CEAM (s)	RFM s/Iteration	RFM No. Iterations
1	FE1 vs. X1, 10 complete modes, no damping, no noise	100	284	3
2	FE2 vs. X1, 10 complete modes, 1% damping, no noise	106	431	4
3	FE1 vs. X1, 10 complete modes, 1% damping, 5% noise	359	No conv.	—
4	FE1 vs. X1, 60 incomplete modes, no damping, no noise	471	170	12
5	FE2 vs. X1, 50 complete modes, no damping, no noise	746	207	20
6	FE1 vs. X2, 70 complete modes, no damping, no noise	881	No. conv.	—

All CPU seconds normalized with respect to case 1, CEAM solution.

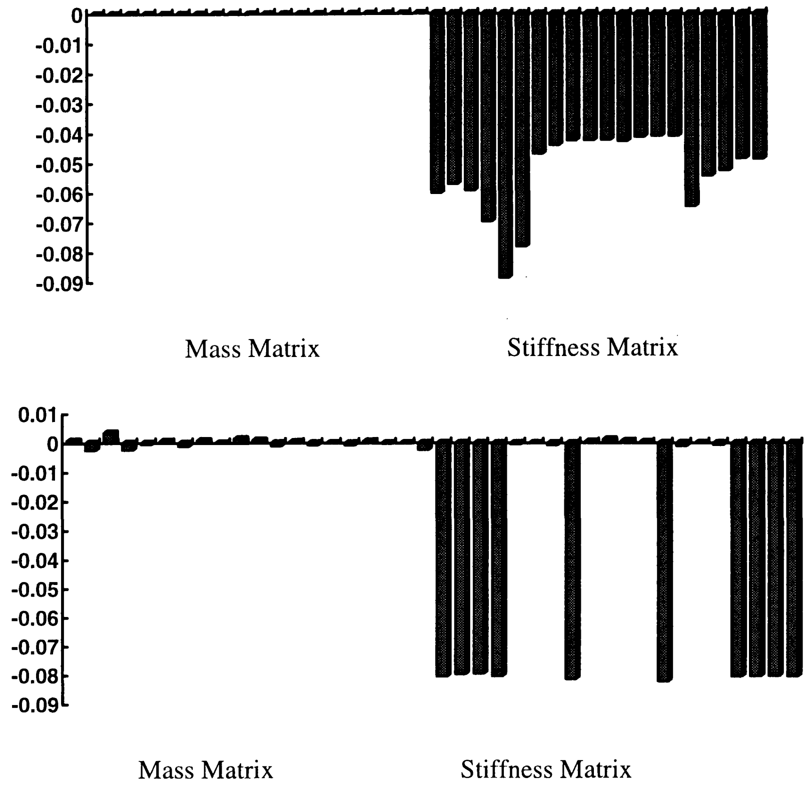

FIGURE 12 RFM (*bottom*) and CEAM (*top*) p value comparison.

CONCLUDING REMARKS

1. The existing constrained eigenstructure assignment method was modified so that it can deal with the updating of large-order systems. The resulting formulation is compatible with other updating methods in the sense that the individual mass, stiffness, and damping matrices are corrected by simple multipliers, the so-called p values.
2. The inherent difficulties associated with FE model updating are once again illustrated by the case studies that are undertaken. Using a correction factor formulation, the model can only be improved in a global sense without particular emphasis on the actual sources of discrepancy between the theoretical and experimental models. However, these globally updated models match the measured response with acceptable accuracy.
3. Numerical case studies seem to indicate that the updating of the damping matrix becomes an easier and less ill-conditioned task, by virtue of using an unconditionally stable quadratic optimization algorithm.
4. As in many other case studies, the closeness of the initial and target models is found to be a key issue for successful updating. This point is clearly illustrated by the last case study, although it was possible to employ a two-stage updating procedure to partly overcome this difficulty. However, in the general case, no updated model can be quaranteed when the initial and updated models do not span the same set of eigenvalues and eigenvectors by virtue of belonging to the same eigendomain. This observation is general and underlines one of the fundamental problems in model updating.
5. The eigenstructure assignment method yields the solution directly and hence it has a significant advantage over iterative methods such as RFM. A comparative study between the two methods reveals that the RFM requires substantially more CPU power in all cases. Also, the convergence of the RFM cannot be guaranteed in cases

where the measured FRFs are polluted by noise, or when the discretization differences are significant. On the other hand, the RFM seems to be able to cope better with incomplete measured data.

6. Present versions of RFM and CEAM can only correct the FE model in a global sense because the location of specific discrepancies cannot be achieved by formulations based on elemental correction factors. In any case, this feature is an inherent problem in model updating studies, as illustrated by the last case study: unless the discrepancies are actually present in the model to be updated in a one to one fashion, it is difficult to see how they can be remedied by changing other, albeit related, parameters.

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