Development of an equivalent force method and an application in simulating the radiated noise from an operating diesel engine

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Received 4 June 1997
Revised 4 April 1999

In this paper a new methodology is presented for applying measured accelerations and forces as excitation on a structural finite element model in order to perform a forced frequency response analysis. The computed vibration constitutes the excitation for an acoustic boundary element analysis. The new developments presented in this paper are associated with: the equivalent force method that can prescribe the acceleration at certain parts of the structure; the integration within a single process of test data that define the excitation, with the vibration analysis, and the acoustic prediction; the utilization of the new technology in simulating the noise radiated from a running engine and determining the effects of design changes. Numerical results for noise radiated from a running engine are compared to test data for a baseline design. The effect of two structural design modifications on the radiated noise is computed, and conclusions are deduced.

Keywords: Boundary element analysis, finite element analysis, engine noise

1. Introduction

The objective of this work is to simulate numerically the noise radiated from a running engine, and identify the impact of design changes to the emitted noise [7, 9–11,13]. A finite element model, including the cylinder block, the cylinder head, the flywheel housing, the gearcase, and the crankshaft is constructed. It is utilized to compute the structural vibration under an operating load. Two major excitation sources are considered: the gas forces and the inertia loads [16]. The gas forces are applied as pressure loads on the cylinder walls and the cylinder head. The inertia loads are applied as measured triaxial acceleration at the bearing caps. Measured acceleration is often employed as a measurable for defining the loads applied on the block of an internal combustion engine [8,12]. The necessity of prescribing the structural vibration at points on the structure other than the support locations led to the development of an "equivalent force" method. In a general purpose finite element software [14] the large mass method is recommended for enforcing the desired motion at locations of interest [14,15]. A concentrated mass, at least 10^3 times the structural mass must be attached to each individual degree of freedom where the motion must be enforced. A force equal to the large mass times the desired acceleration must be applied in that particular degree of freedom as excitation. The large mass methodology works very well for base excitation where the enforced motion is specified at the support location of the structure, however, it is not applicable to the problem of interest. If the large mass method is used to enforce the accelerations at the bearing caps, the structural normal modes of the engine will be altered and the natural frequencies will be shifted. An initial attempt to utilize the large mass approach resulted in normal modes for the engine block that did not correlate with the available test data. Therefore, the necessity to perform the analysis led to the development of an "equivalent force" method. A methodology is developed for computing an equivalent force for each one of the prescribed degrees of freedom. The equivalent forces result in the desired vibration at the

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ISSN 1070-9622 / $8.00

measurement locations when they are applied on the structural finite element model. There is no requirement for a large mass to be attached to the location of enforced displacement. Therefore, the modal basis of the structure is not altered. In addition, the computation of the structural vibration by the equivalent force method is integrated within an acoustic boundary element analysis process. Thus, vibration test data, structural finite element analysis, and acoustic boundary element noise computations are combined into an integrated simulation process. The dynamic characteristics of the structure are represented in terms of the normal modes and the natural frequencies and comprise part of the input to the combined analysis process. The measured accelerations and the pressure combustion loads are also components of the input. The equivalent force method computes the structural vibration based on: the normal modes and natural frequencies of the structure; the prescribed acceleration and pressure loads; and the structural damping.

The vibration results on the outer surface of the structure are utilized as boundary conditions for the acoustic analysis. Since the equivalent force method is integrated within a noise prediction process, there is no need for an external transfer of data. The noise prediction is based on an indirect variational boundary element formulation [5,6,17,18]. The selected boundary element method can model ribs and other thin appendices that are often encountered in an engine block [18]. The computed vibration, the discretization of the structural model, and the discretization of the acoustic boundary element model are utilized in computing the emitted noise. The development, utilization, application, and validation of this approach to simulating the noise radiated from the block of an operating diesel engine are presented.

2. Mathematical formulation and numerical implementation

2.1. *Equivalent force method*

In order to physically present how the large mass method operates and demonstrate the error which is introduced in the modal basis, a simply supported beam structure can be used (Fig. 1). In this example the vibration along the y-direction in the middle of the beam is considered to be prescribed. In the large mass method, a large mass is attached to the corresponding degree of freedom (d.o.f.). The addition of the large mass introduces artificial nodes in the normal modes of the system. In the equivalent force method a mechanical load is applied along the prescribed d.o.f. instead of a large mass (Fig. 2). The value of the load is such that it results in the desired motion for the prescribed d.o.f. The process is similar to an algorithm utilized to enforce constraints in a non-linear static and dynamic structural response process [3,4].

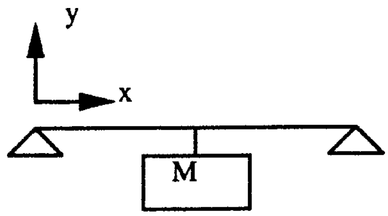


Fig. 1. Large mass method for a simply supported beam.

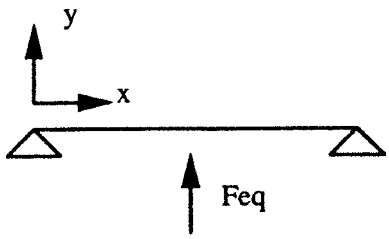


Fig. 2. Equivalent force method for a simply supported beam.

The following approach is employed for deriving the values of the equivalent forces from the prescribed displacements or accelerations. In the frequency domain the acceleration is equal to $-\omega^{2*}$ displacement. The structural system of equations can be written [2]

$$\left[-\omega^2[M] + \mathrm{i}\omega[C] + [K]\right]\{u\} = \{f\}, \tag{1}$$

where $[M]$ is mass matrix, $[C]$ is damping matrix, $[K]$ is stiffness matrix, $\{u\}$ is displacement of vibration, $\{f\}$ is force excitation, and ω is frequency of analysis $\times 2\pi$. This equation can be written in terms of the modal d.o.f.

$$\begin{aligned}&\left[-\omega^2[I] + \mathrm{i}\omega[\Phi]^{\mathrm{T}}[C][\Phi] + [\omega_i^2]\right]\{\eta\}\\&\quad = [\Phi]^{\mathrm{T}}\{f\} \Rightarrow [St]\{\eta\} = \{\Phi f\},\end{aligned} \tag{2}$$

where $[I]$ is identity matrix, $[\Phi]$ is modal matrix, $\{\eta\}$ is modal d.o.f., $[\omega_i^2]$ is diagonal matrix containing the eigenvalues as diagonal terms, $\{\Phi f\}$ is modal force vector, and $[St]$ is finite element modal system of equations. The modal approach for the frequency response computations was chosen for two reasons:

(i) It expedites the computational process in a multiple frequency analysis, which is often performed in typical powertrain applications.
(ii) Since the equivalent force method has been integrated within the noise computational process, it is considerably more efficient to import in the acoustic process the modal basis of the structure, rather than the structural matrices.

When the equivalent forces are applied to the finite element model, the finite element equations become:

$$[St]\{\eta_e\} = [\Phi]^{\mathrm{T}} \begin{Bmatrix} 0 \\ \cdot \\ f_{ei} \\ \cdot \\ \cdot \\ f_{ej} \\ \cdot \end{Bmatrix} = \{\Phi f_e\}, \tag{3}$$

where $\{\eta_e\}$ are modal degrees of freedom derived as response to the equivalent force excitation, f_{ei}, f_{ej} are equivalent forces applied to the prescribed degrees of freedom, and $\{\Phi f_e\}$ is vector of modal equivalent forces. The vector of the equivalent forces has non-zero entries only on the i-th and j-th degrees of freedom. In deriving the equations, for simplicity, accelerations are considered to be applied to only two d.o.f. i, j. In practice an acceleration value can be prescribed to any desired number of d.o.f. The first step in determining the equivalent forces is to apply a unit force excitation to each one of the d.o.f. where the accelerations will be prescribed. The corresponding modal response can be computed:

$$[St]\{\eta_i\} = [\Phi]^{\mathrm{T}} \begin{Bmatrix} 0 \\ \cdot \\ (1.0, 0.0) \\ \cdot \\ \cdot \\ 0 \\ \cdot \end{Bmatrix} = \{\Phi f_i\} \text{ and}$$

$$[St]\{\eta_j\} = [\Phi]^{\mathrm{T}} \begin{Bmatrix} 0 \\ \cdot \\ 0 \\ \cdot \\ \cdot \\ (1.0, 0.0) \\ \cdot \end{Bmatrix} = \{\Phi f_j\}, \tag{4}$$

where $\{\eta_i\}$ and $\{\eta_j\}$ are modal response to a unit load applied at the i-th or the j-th d.o.f., respectively, $\{\Phi f_i\}, \{\Phi f_j\}$ are modal forces corresponding to a unit load applied to the i-th or the j-th d.o.f. The corresponding physical displacements can be derived as:

$$[\Phi]\{\eta_i\} = \begin{Bmatrix} \cdot \\ \cdot \\ u_{ii} \\ \cdot \\ \cdot \\ u_{ji} \\ \cdot \end{Bmatrix} \text{ and}$$

$$[\Phi]\{\eta_j\} = \begin{Bmatrix} \cdot \\ \cdot \\ u_{ij} \\ \cdot \\ \cdot \\ u_{jj} \\ \cdot \end{Bmatrix}, \tag{5}$$

where u_{ab} is response of the a-th d.o.f. due to a unit load applied on the b-th d.o.f. Then each one of the Eq. (4) is multiplied by the corresponding yet unknown equivalent force, resulting in:

$$[St]f_{ei}\{\eta_i\} = [\Phi]^{\mathrm{T}} f_{ei} \begin{Bmatrix} \cdot \\ \cdot \\ (1.0, 0.0) \\ \cdot \\ \cdot \\ 0 \\ \cdot \end{Bmatrix} \text{ and}$$

$$[St]f_{ej}\{\eta_j\} = [\Phi]^{\mathrm{T}} f_{ej} \begin{Bmatrix} \cdot \\ \cdot \\ 0.0 \\ \cdot \\ \cdot \\ (1.0, 0.0) \\ \cdot \end{Bmatrix}, \tag{6}$$

where f_{ei}, f_{ej} are values of the yet unknown equivalent forces associated with the i-th and j-th d.o.f. The corresponding physical responses will be:

$$f_{ei}[\Phi]\{\eta_i\} = f_{ei} \begin{Bmatrix} \cdot \\ \cdot \\ u_{ii} \\ \cdot \\ \cdot \\ u_{ji} \\ \cdot \end{Bmatrix} \text{ and}$$

$$f_{ej}[\Phi]\{\eta_j\} = f_{ej}\left\{\begin{array}{c} \cdot \\ \cdot \\ u_{ij} \\ \cdot \\ \cdot \\ u_{jj} \\ \cdot \end{array}\right\}. \qquad (7)$$

Since this is a linear analysis, the two Eq. (6) can be added together resulting into:

$$[St](f_{ei}\{\eta_i\} + f_{ej}\{\eta_j\}) = [\Phi]^{\mathrm{T}}\left\{\begin{array}{c} \cdot \\ \cdot \\ f_{ei} \\ \cdot \\ \cdot \\ f_{ej} \\ \cdot \end{array}\right\}. \qquad (8)$$

The corresponding combined physical response will become:

$$f_{ei}[\Phi]\{\eta_i\} + f_{ej}[\Phi]\{\eta_j\} = f_{ei}\left\{\begin{array}{c} \cdot \\ \cdot \\ u_{ii} \\ \cdot \\ \cdot \\ u_{ji} \\ \cdot \end{array}\right\} + f_{ej}\left\{\begin{array}{c} \cdot \\ \cdot \\ u_{ij} \\ \cdot \\ \cdot \\ u_{jj} \\ \cdot \end{array}\right\}. \qquad (9)$$

The response at the i-th and the j-th d.o.f. must be equal to the prescribed value, therefore:

$$f_{ei}\left\{\begin{array}{c} u_{ii} \\ u_{ji} \end{array}\right\} + f_{ej}\left\{\begin{array}{c} u_{ij} \\ u_{jj} \end{array}\right\} = \left\{\begin{array}{c} \bar{u}_i \\ \bar{u}_j \end{array}\right\} \Rightarrow$$
$$\left[\begin{array}{cc} u_{ii} & u_{ij} \\ u_{ji} & u_{jj} \end{array}\right]\left\{\begin{array}{c} f_{ei} \\ f_{ej} \end{array}\right\} = \left\{\begin{array}{c} \bar{u}_i \\ \bar{u}_j \end{array}\right\}, \qquad (10)$$

where $\bar{u}_i, \bar{u}_j$ are prescribed vibrations along the i-th and j-th d.o.f. The equivalent loads can be computed from the system of Eq. (10). The results are based on the structural response due to unit loads on the defined d.o.f., and the prescribed vibration values. In the engine analysis combustion pressure loads must be applied on the cylinder walls along with the measured vibration at the bearing caps. The combustion pressure loads can also be accounted in the development of the equivalent force method. Specifically, if a predefined load (combustion loads) is part of the excitation, then the vibration due to that load only is computed first

$$[St]\{\eta_n\} = [\Phi]^{\mathrm{T}}\left\{\begin{array}{c} \cdot \\ \cdot \\ F_n \\ \cdot \end{array}\right\}, \qquad (11)$$

where F_n is predefined load (the pressure combustion load in the application of interest), u_{in}, u_{jn} are responses to the predefined load. The corresponding physical displacement will be:

$$[\Phi]\{\eta_n\} = \left\{\begin{array}{c} \cdot \\ \cdot \\ u_{in} \\ \cdot \\ \cdot \\ u_{jn} \\ \cdot \end{array}\right\}. \qquad (12)$$

Since the equivalent forces (Eq. (8)), and the predefined load (Eq. (11)) will act simultaneously on the system, the equation for the total displacement becomes:

$$\left[\begin{array}{cc} u_{ii} & u_{ij} \\ u_{ji} & u_{jj} \end{array}\right]\left\{\begin{array}{c} f_{ei} \\ f_{ej} \end{array}\right\} + \left\{\begin{array}{c} u_{in} \\ u_{jn} \end{array}\right\} = \left\{\begin{array}{c} \bar{u}_i \\ \bar{u}_j \end{array}\right\} \Rightarrow$$
$$\left[\begin{array}{cc} u_{ii} & u_{ij} \\ u_{ji} & u_{jj} \end{array}\right]\left\{\begin{array}{c} f_{ei} \\ f_{ej} \end{array}\right\} = \left\{\begin{array}{c} \bar{u}_i \\ \bar{u}_j \end{array}\right\} - \left\{\begin{array}{c} u_{in} \\ u_{jn} \end{array}\right\}. \qquad (13)$$

The equivalent forces can be computed from Eq. (13). The calculated values comprise the excitation in Eq. (2) along with the predefined loads. The resulting response will then demonstrate the measured acceleration values at the d.o.f. where the equivalent loads are applied.

2.2. Integration with an indirect variational acoustic boundary element formulation

The structural vibration computed through the equivalent force method comprises the boundary conditions for the noise analysis. The indirect variational boundary element method is used for acoustic computations [5,6,17,18]. It is based on the principle that the vibration is the source for the generation of noise. Through an integral equation it associates the radiated noise to the vibration on the surface of the structure which generates the noise

$$p(\vec{r}_{dr}) = \int_{S_a} \delta p(\vec{r}_a)\frac{\partial G(\vec{r}_a, \vec{r}_{dr})}{\partial n_a} - G(\vec{r}_a, \vec{r}_{dr})\delta dp(\vec{r}_a)\,\mathrm{d}S(\vec{r}_a), \qquad (14)$$

where $\delta p, \delta dp$ are primary acoustic variables on the surface of the vibrating structure, the former is related

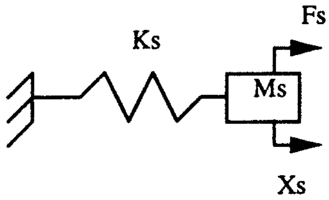


Fig. 3. Single degree of freedom system representing the structure.

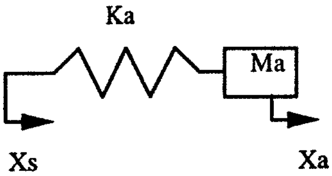


Fig. 4. Single degree of freedom system representing the acoustic medium.

to the acoustic pressure and the latter is associated with the vibration velocity, $\vec{r}_a$ and $\vec{r}_{dr}$ are position vectors associated with a point on the surface of the structure and the data recovery point where the noise level is being computed, and G is Green's function. The primary variables are computed first in the boundary element methodology. A linear system of equations is generated and all the primary variables are computed from the vibration information. Once the primary variables are computed, Eq. (14) is utilized to evaluate the acoustic response at any point in space.

A simple representation of how vibration and acoustic computations are linked can be demonstrated by two single degree of freedom systems, one representing the structure, and another representing the acoustic medium. In Figs 3 and 4, Ks is stiffness of structure, Ms is mass of structure, Xs is structural vibration, Fs is structural excitation, Ka is stiffness of acoustic system, Ma is mass of acoustic stiffness, Xa is response of acoustic system, representing the radiated noise. The finite element method is used to model the structural system. The equivalent force method provides the excitation, and the structural vibration is computed first. Then it is applied as excitation to the acoustic system, and the boundary element method is employed to compute the acoustic response (i.e., noise).

The equivalent force method has been integrated with the indirect variational boundary element method. Since a structural finite element library is not available within the acoustic code, instead of assembling or importing the structural matrices, the modal basis is imported in order to represent the structural system. Once the vibration is computed (using Eqs (2) and (13)), the acoustic velocities are generated on the surface of the boundary element model using the relationship:

$$u_{\mathrm{a}} = \hat{n}\vec{u}_{\mathrm{st}}, \tag{15}$$

where u_{a} is acoustic velocity, $\hat{n}$ is unit normal, and $\vec{u}_{\mathrm{st}}$ is structural velocity. The structural and the acoustic models do not require equivalent discretization or coinciding nodes. A mapping technique is used for generating the acoustic velocities from the structural vibration, and the indirect variational boundary element method is utilized for computing the radiated noise.

3. Application, validation

The equivalent force method is utilized to perform the structural vibration and the acoustic analysis for an engine block of an inline six cylinder diesel engine. The excitation is comprised by the measured triaxial acceleration at the bearing caps and the combustion pressure loads applied on the cylinder head and the cylinder walls. The equivalent force method is integrated with an indirect variational boundary element formulation. The normal modes and the natural frequencies of the structure are imported into the acoustic prediction software from a structural finite element modal analysis. The test data for the measured acceleration and the combustion pressure loads are also part of the input. The computed radiated acoustic power is compared successfully to test data. The effect on the radiated noise of two design changes introduced to the engine assembly is computed and the information is utilized in making decisions with respect to the design.

A structural finite element model including the cylinder block, the cylinder head, the flywheel housing, the gearcase, and the crankshaft are constructed. A modal analysis is performed initially to determine the natural frequencies and the mode shapes. Table 1 summarizes the percentage of difference in the natural frequencies between the first 13 measured and the computed normal modes. The results demonstrate good correlation between test and finite element analysis. The largest differences are observed at nodes dominated by total block motion. The skirt modes demonstrate very good correlation. The modal basis is computed up to a frequency of 2500 Hz, and 66 normal modes are extracted. The modal information is imported into the acoustical analysis process. The combination of measured triaxial data at the bearing caps, and combustion pressure loads at the cylinder

Table 1

Correlation table between test and numerical normal modes

Mode	1	2	3	4	5	6	7	8	9	10	11	12	13
% Diff	3.5	5.5	–0.8	–0.6	0.2	–0.3	4.3	5.8	4.8	–0.2	0.1	1.6	0.6

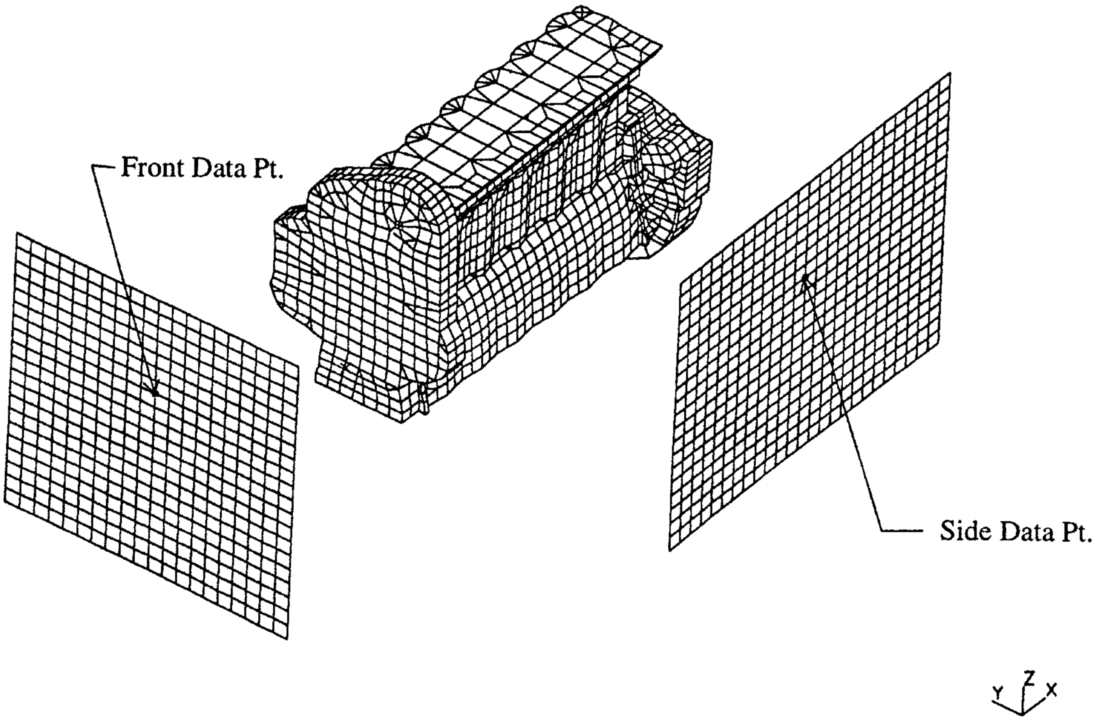


Fig. 5. Acoustic boundary element model and data recovery planes.

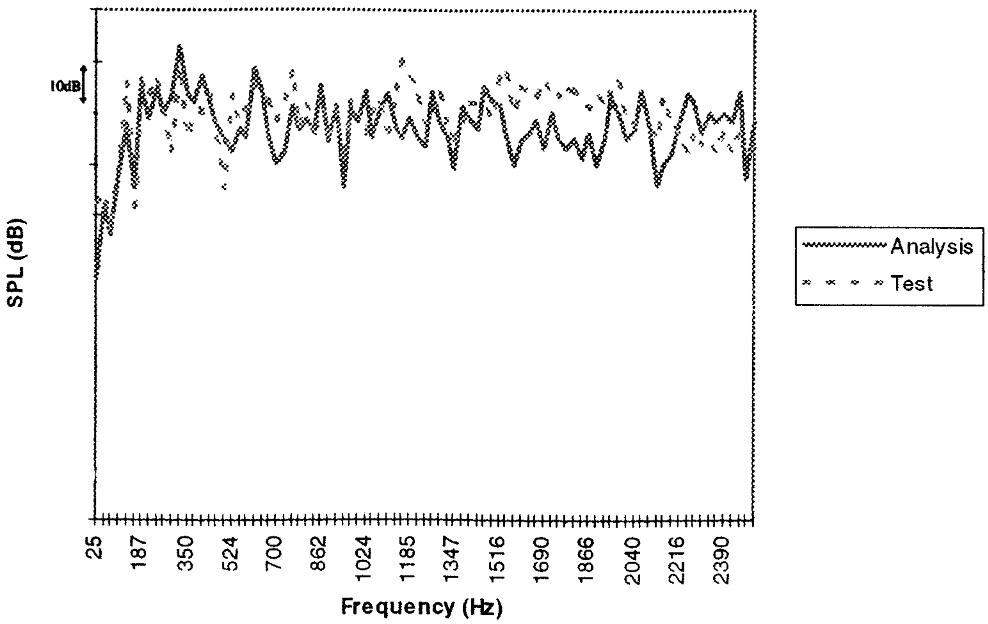


Fig. 6. Numerical and test data for the acoustic SPL at a point in front of the engine.

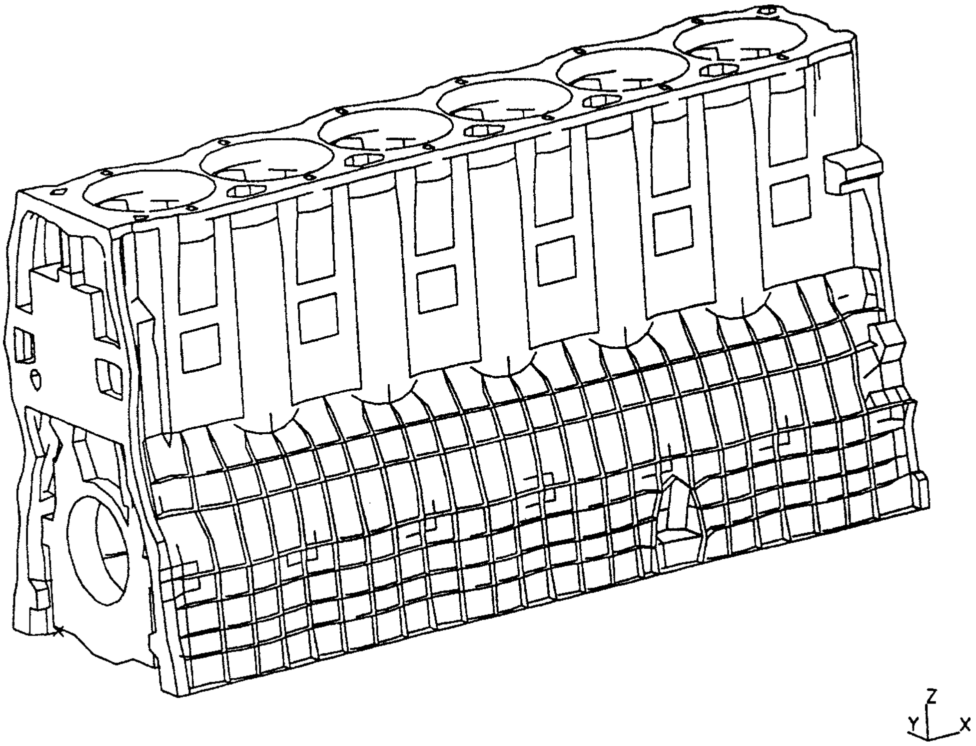


Fig. 7. Additional ribbing on the block.

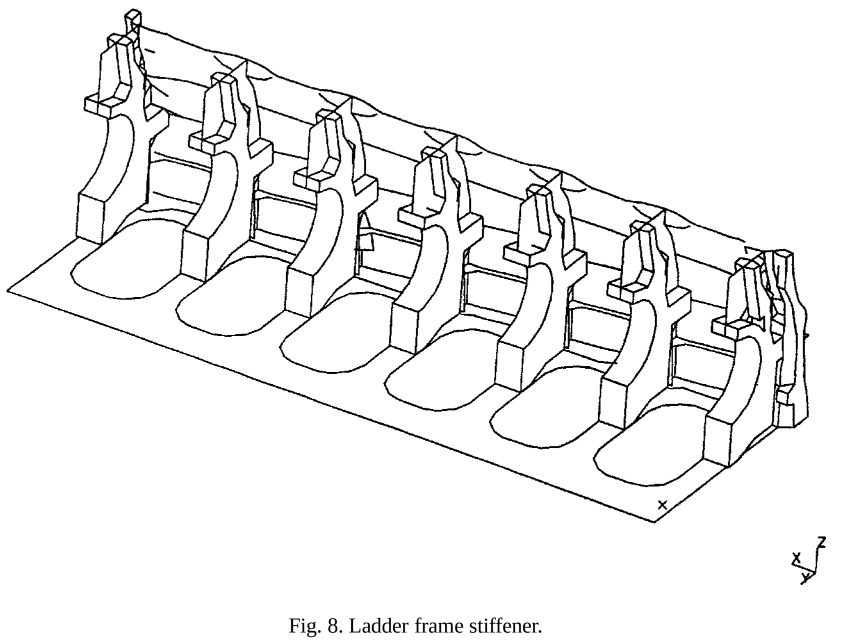


Fig. 8. Ladder frame stiffener.

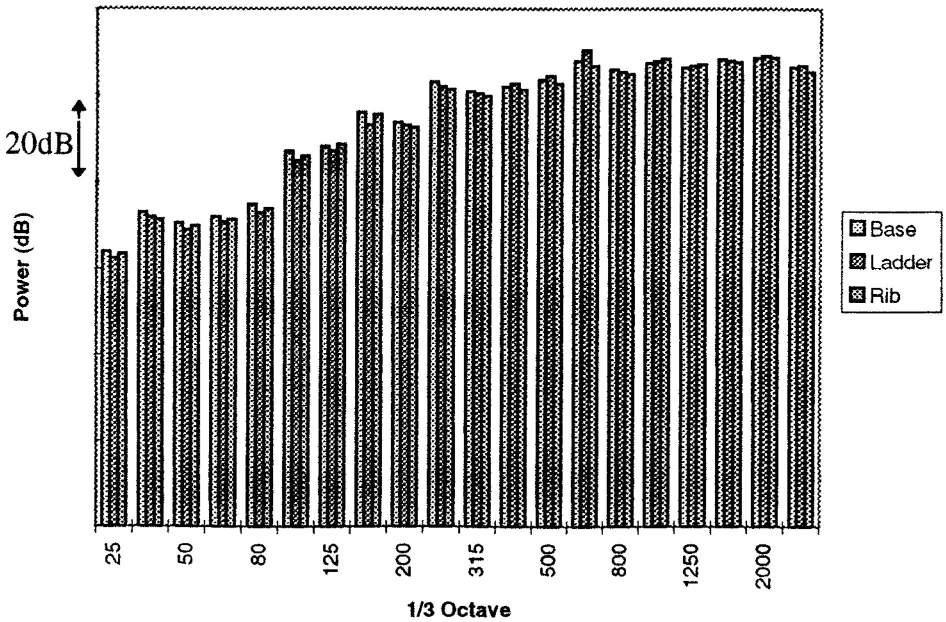
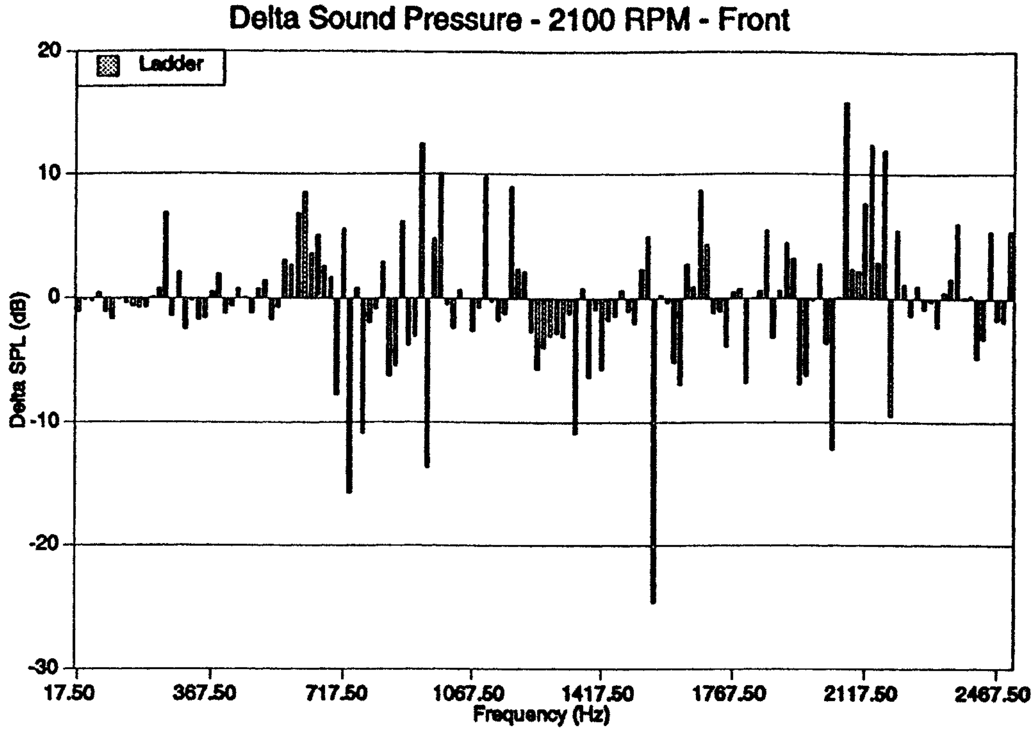


Fig. 9. Radiated sound power per 1/3 octave frequency band for three designs.

walls and the cylinder head comprise the excitation. The equivalent force method is utilized for computing the vibration of the block between 0–2500 Hz at 5 Hz increments. The vibration of the outer surface of the block is mapped on the acoustic boundary element model. This process is also automated and there is no need for external exchange of data. The acoustic boundary element model represents the geometry of the outer surface of the block. Two data recovery planes are defined at one meter distance from the block. They are positioned at its left side and in front of it. The data recovery planes represent the microphone locations in the numerical simulation. Figure 5 presents the acoustic boundary element model and the two data recovery planes. The entire analysis is performed for two load cases representing two combinations of RPM and engine load (1950 RPM at 50% load, and 2100 RPM for 100% load). Test data are available for the latter, and all the results presented here concern the latter case. Figure 6 presents numerical results and test data for the sound pressure level at a single point in the front of the engine. The results demonstrate similar trends and similar absolute values for the sound pressure level. The correlation is better for lower frequencies (below 1600 Hz). This is expected because although the modal analysis is performed up to 2500 Hz the modal information is extracted only up to the same frequency. Therefore, a modal truncation error is introduced in the high frequency range of the analysis. In addition, the test data included noise from engine accessories, which are not present in the numerical model. The correlation of the acoustic response between analysis and test data is considered very satisfactory. The effect of two design changes on the radiated noise is determined through numerical simulations. Specifically, the effect of additional ribbing on the block (Fig. 7) and the influence of a ladder frame (Fig. 8) are analyzed. The overall radiated sound power did not change significantly between the baseline and the two modified designs. The changes altered the radiated power by $+0.5$ dB and -0.2 dB, respectively. The change in the sound power per 1/3 octave frequency band is presented in Fig. 9. It can be observed that the two structural modifications considered in this work have a small effect to the total radiated power for the analyzed operating conditions. In addition, the change in the sound pressure level between the baseline and the two modified designs is presented for the two points located in the front and the left side of the engine (Figs 10 and 11, respectively). The acoustic pressure results demonstrate that the addition of ribbing on the block tends to overall reduce the SPL at the two data recovery points more than the addition of the ladder frame. It must be pointed out however, that in the comparative analysis presented in this paper the acceleration data at the bearing caps are considered the same for all three designs. Such an assumption is made because hardware is available only for the



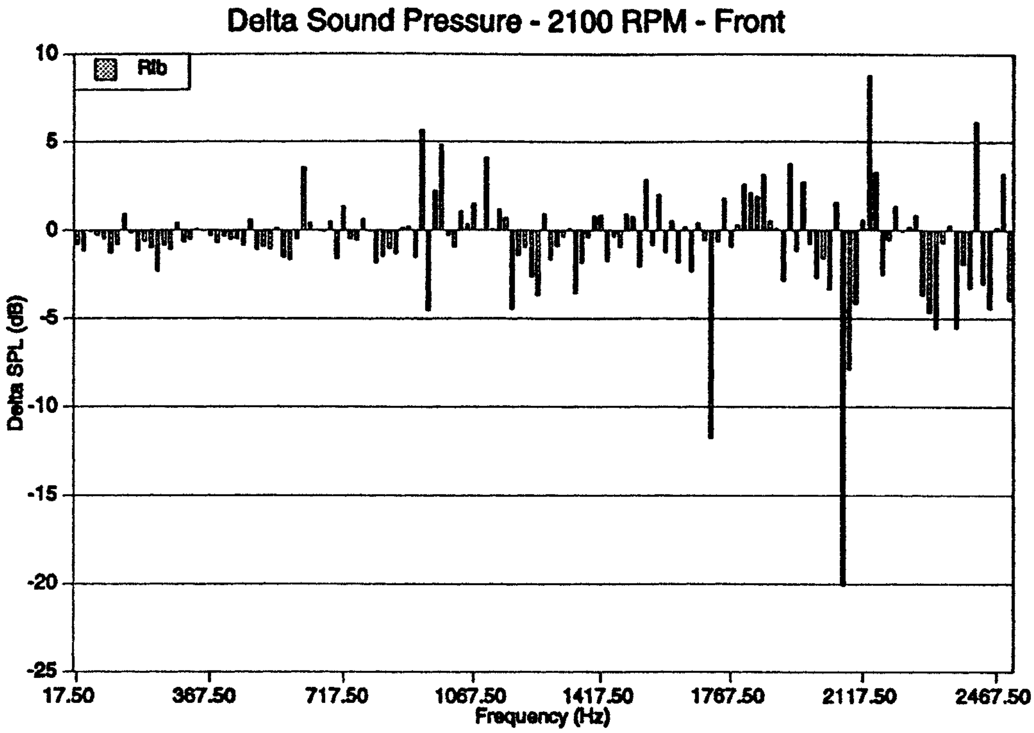


Fig. 10. Change in the acoustic response in the front of the engine.

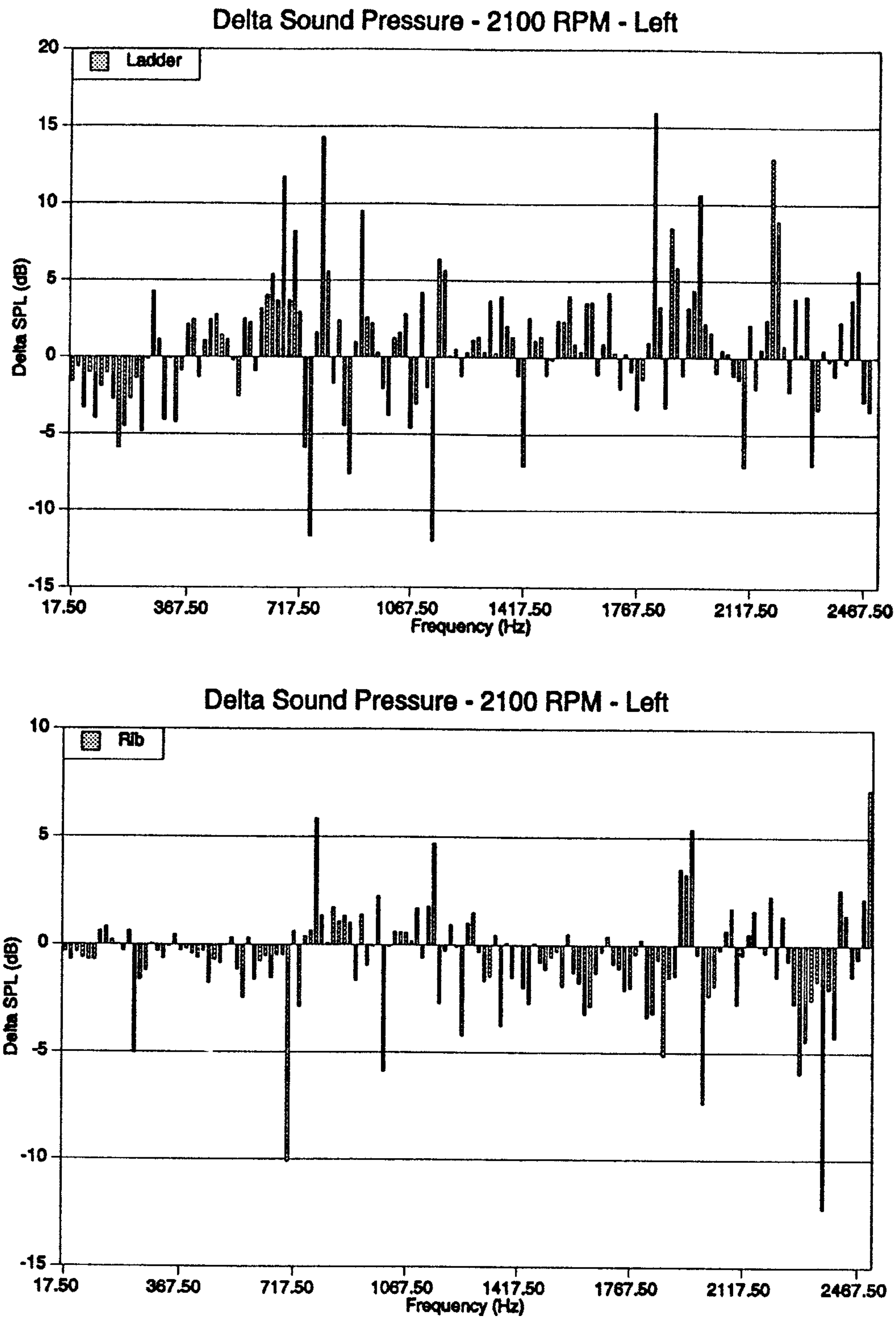


Fig. 11. Change in the acoustic response at the left side of the engine.

baseline design. In reality the structural modifications introduced in the block will affect the interaction between the block and the crankshaft, and alter the excitation transmitted to the block. Thus, the need for developing in the future a predictive capability for calculating the interaction between the crankshaft and the block through the bearings is demonstrated.

4. Conclusions

The mathematical formulation of the equivalent force method is presented. It allows to prescribe the vibration at any degree of freedom of a structural finite element model without introducing any errors in the modal basis. The integration of the equivalent force method with an indirect variational boundary element formulation is utilized in computing acoustic results for the noise radiated from a running engine. A combination of measured acceleration and combustion pressures provides the excitation. Numerical results are successfully compared to test data. The effect of design changes on the radiated noise are also computed. This application demonstrates how the equivalent force method can be utilized to integrate test data into numerical analysis. In addition, it identifies how the structural finite element method can be combined with acoustic boundary elements in performing a structural-acoustics simulation. The correlation to test data validates the numerical process. Finally, by computing numerically the effect of design changes on the radiated noise, the power of the numerical simulations is demonstrated. It allows to identify the impact of modifications on the performance of a system without the need to construct any prototype. It can therefore reduce the design cycle and lead to cost and time savings. Future developments for the overall improvement of the simulation process are also identified.

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