Dynamic analysis of a rigid body mounting system with flexible foundation subject to fluid loading

J.S. Tao[a,*], G.R. Liu[b] and K.Y. Lam[b]
[a]*Institute of High Performance Computing, 89-C Science Park Drive, #02-11/12, The Rutherford, Singapore 118261*
[b]*Department of Mechanical and Production Engineering, National University of Singapore, 10 Kent Ridge Crescent, Singapore 119260*

This paper presents an investigation of the force transmission from a rigid body mounting system to a flexible foundation with light and heavy fluid loading under the force and moment excitation. The analytical expression has been derived in which the flexible foundation effects are incorporated into a revised system stiffness matrix that is derived from the receptance matrix at mounting points. A typical case with a thin infinite plate as the foundation has been studied with the point and transfer receptances theoretically and numerically analysed in the case of light and heavy fluid loading. The results show that, compared with the rigid foundation, the force transmission is reduced and system natural frequencies are shifted. The detailed analysis demonstrates that the force reduction and frequency shifting are more obvious at low frequencies where the receptance value is significant. The study is also carried out to compare the transfer receptances from different waves in plate as it couples with water with the objective to simplify the calculation of receptance. It is found that, in the low frequency and after a short distance from driving point, the transfer receptance calculation for the heavy fluid loading can be simplified by only accounting the contribution from free wave which may easily be evaluated from the point receptance in air. It implies the plate response under heavy fluid loading could be directly derived from that with light fluid loading.

Keywords: Rigid body mounting system, thin infinite plate, receptance, force transmissibility

*Corresponding author. Tel.: 65 4708059; Fax: 65 4728960; E-mail: taojinsheng@yahoo.com.

1. Introduction

Vibration isolation of a rigid body is a basic engineering problem encountered in many areas. The conventional assumption in engineering design of isolation system is based on rigid foundation. This assumption can cause a significant error for most of cases where the foundations are flexible. A discussion of this problem for a simple isolation system can be found in reference [1]. It has been noted that the effects of elasticity of foundation should be considered for an isolation system design. However, these effects are complicated due to the coupling of six degrees of freedom of vibration for a rigid body mounting system. Ashrafiuon and Nataraj [2] derived the equations for the calculation of the elastic foundation effects without fluid loading and presented some results for an engine mounting system under an excitation of rotation unbalance. He concluded that the flexibility of the foundation seems to have significant effects on the response of engine at low speed near system and foundation's natural frequencies. No further investigation was carried out. The elastic foundation effects in his study were expressed as a revised stiffness matrix which was derived from the finite element stiffness matrix of the foundation. This method brings the difficulty in most of applications in engineering where it is difficult or impossible to get finite element stiffness matrix of the foundations especially with fluid loading. On the other hand, it is desirable to analyse the force and moment excitation separately rather than together as done by Ashrafiuon and Nataraj [2] since their system responses may be totally different.

No results have been reported in the literature for the force transmission of a rigid body mounting system with an elastic foundation subject to heavy fluid loading which is often encountered in the ship vibration control. This may be due to the complexity of the coupling among mounting system, foundation and

Shock and Vibration 8 (2001) 33–47
ISSN 1070-9622 / $8.00 © 2001, IOS Press. All rights reserved

fluid. Therefore, the question remains of whether the isolation performance of a rigid body mounting system with heavy fluid loading, such as water, would be the same as that with light fluid loading, for instance air, or significant different.

In this paper, the elastic foundation effects are derived from the receptance matrix at mounting points rather than the FEM stiffness matrix of the foundation. This receptance matrix can be directly obtained either from analytical solution for simple structure, or from finite element analysis and experiment for complex structure even with heavy fluid loading. This implies that the force transmission of a rigid body mounting system coupled with a flexible foundation and fluids can be investigated in detail as long as the mounting receptance or mobility is provided.

The analytical solution to the receptance of a rigid body mounted on a finite plate subject to heavy fluid loading can't be achieved due to the theoretical difficulty in processing the coupling between fluid and plate. In this paper, the analysis will be carried out for a thin infinite plate subject to light and heavy fluid loading. It can help to understand how the elastic foundation affects the force transmission and system natural frequencies.

The point and transfer receptances of an infinite plate with light fluid loading can be derived from the point impedance and displacement response of plate under force excitation [3]. The light fluid loading here means that the loading effects are minor and can be ignored. This is normally the case in the air. The response of an infinite plate with heavy fluid loading is more complicated and has been theoretically analysed by Crighton [4–6] and followed by Rothwell et al. [7]. However, it is well-known that the introduction of compressive fluid-loading brings the difficulties to obtain a general analytical solution in non-integral form for all the cases [7]. To a thin infinite plate with point force excitation, the analytical solutions are known only for drive point receptance, transfer receptance in far field ($r \to \infty$) at fixed frequency and very high frequency ($\omega \to \infty$) at fixed distance, where r is the distance from drive point and ω is the angular frequency. For the rigid body mounting system with mechanic excitation, the requirement is the transfer receptance in the near field and at low frequency. An analytical expression for this case is difficult. Gauss quadrature formula is applied for the integration equation in this paper to evaluate the contribution from the transfer receptance.

A case study is presented to show the significance of the flexible foundation on the rigid body response with light and heavy fluid loading. The detailed investigation is also carried out for the analysis of how receptance values from different waves in plate affect the force transmission and frequencies shifting.

2. Basic equations for a rigid body mounting system with rigid foundation

Consider a rigid body supported by vibration isolators fixed to a rigid foundation as illustrated in Fig. 1. The right-hand global coordinate system has its origin at the centre of mass of the body. The three orthogonal coordinate axes are set with Y, Z-axis parallel to the foundation and X normal to the foundation. Under the assumption of "small" motion, the rigid body mount system equation can be written as:

$$[M]\{\ddot{x}\} + [K]\{x\} = \{F\}e^{j\omega t}, \tag{1}$$

where $[M]$ is the 6×6 rigid body's mass matrix; $\{x^T\} = [x_g\ y_g\ z_g\ \theta_x\ \theta_y\theta_z]$ is the displacement vector at the centre of gravity (c.g) of rigid body; $[K]$ is the 6×6 complex stiffness matrix with rigid foundation; $\{F\}$ is the 6×1 vector of excitation forces and moments; ω is the angular frequency of excitation. The stiffness of an isolator in the three directions of its local co-ordinate system is defined by equation:

$$[k'] = [k_0](1 + i\eta), \tag{2}$$

where η is the loss factor of hysteresis damping and $i = \sqrt{-1}$. The stiffness of the isolator in the global co-ordinate system with the origin at c.g of rigid body can be expressed as:

$$\{k_i\} = [A]\{k'\}[A^T], \tag{3}$$

where $[A]$ is the transpose matrix of the Euler angle matrix [8]. Appling Eqs (2) and (3), the calculation of $[K]$ can be found in the reference [1].

The displacement vector $\{x\}$ in Eq. (1) is thus expressed as

$$\{x\} = \{F\}/\left([K] - \omega^2[M]\right). \tag{4}$$

3. Equations with flexible foundation

For a flexible base, the forces at all mounts can be written as [2]:

$$\{f_m\} = [K_e]\{x\} - [K_m]\{d_m\} \tag{5}$$

where $\{d_m\}$ is a $3n$x1 vector of foundation displacement at n mounting locations, and $[K_e]$, $[K_m]$ are stiff-

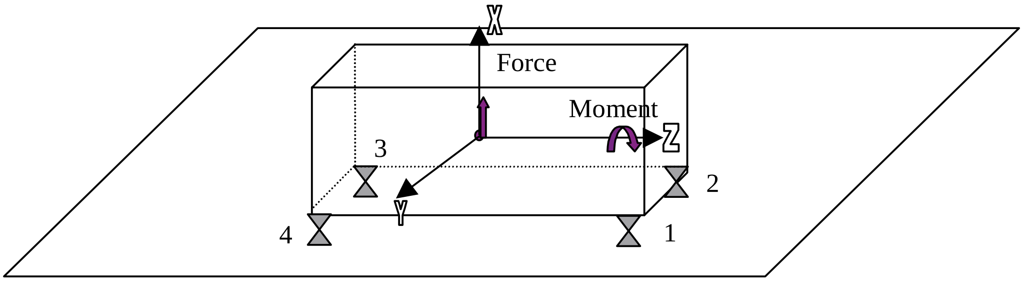


Fig. 1. Rigid body installed with compliant mounts on the flexible foundation. The origin of global co-ordinate system is at the centre of gravity of rigid body. Y and Z axes are parallel to the base and X-axis is normal to the base.

$$\{d_m\} = \begin{bmatrix} \begin{bmatrix} R_{m11xx} & R_{m11xy} & R_{m11xz} \\ R_{m11yx} & R_{m11yy} & R_{m11yz} \\ R_{m11zx} & R_{m11zy} & R_{m11zz} \end{bmatrix} & \begin{bmatrix} R_{m12xx} & R_{m12xy} & R_{m12xz} \\ R_{m12yx} & R_{m12yy} & R_{m12yz} \\ R_{m12zx} & R_{m12zy} & R_{m12zz} \end{bmatrix} & \cdots R_{m1n} \\ \vdots & \vdots & \cdots R_{m2n} \\ \vdots & \vdots & \ddots \quad \vdots \\ R_{mn1} & \cdots & \cdots R_{mnn} \end{bmatrix} \{f_m\} \tag{12}$$

ness matrices given by

$$[K_e] = \begin{bmatrix} K_1 & -k_1 r_1 \\ \vdots & \vdots \\ k_n & -k_n r_n \end{bmatrix}_{(3n\times 6)}, \tag{6}$$

$$[K_m] = \begin{bmatrix} K_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & k_n \end{bmatrix}_{(3n\times 3n)}, \tag{7}$$

where $[r_n]$ is a 3×3 skew-symmetric matrix which can be expressed as

$$[r_n] = \begin{bmatrix} 0 & -z_n & y_n \\ z_n & 0 & -x_n \\ -y_n & x_n & 0 \end{bmatrix}. \tag{8}$$

$x_n,\ y_n,\ z_n$ here are the co-ordinates of the nth mount. In Eq. (5), $\{d_m\}$ can be expressed as $[R_m]\{f_m\}$, where $[R_m]$ is the receptance matrix at mounting points. Equation (5) can be re-written as:

$$\{f_m\} = ([E] + [K_m][R_m])^{-1}[K_e]\{x\} \tag{9}$$

where $[E]$ is an identity matrix.

The basic equation for a rigid body mounting system with a flexible base can be expressed:

$$[M]\{\ddot{x}\} + [K]\{x\} - [K_e^T]\{d_m\} = \{F\}e^{j\omega t} \tag{10a}$$

Replacing $\{d_m\}$ with $[R_m]\{f_m\}$ and applying Eq. (9), the Eq. (10a) can be rewritten as:

$$[M]\{\ddot{x}\} + [K']\{x\} = \{F\}e^{j\omega t}. \tag{10b}$$

$[K']$ here is the revised system stiffness matrix which can be expressed as:

$$[K'] = [K] - [K_e^T][R_m] ([E] + [K_m][R_m])^{-1}[K_e] \tag{11}$$

where $[K]$ is the stiffness matrix with the rigid foundation in Eq. (1).

From Eq. (11), the revised system stiffness matrix can be obtained from the receptance matrix $[R_m]$ at mounting points rather than the stiffness matrix of the foundation. Other matrices in Eq. (11) can be obtained from the rigid body parameters, co-ordinates of mounting locations and the stiffness of isolators.

Matrix $[R_m]$ is a $3n \times 3n$ matrix and defined by: Eq. (12).

Where $\{f_m\} = [f_{mx1},\ f_{my1},\ f_{mz1},\ \cdots,\ f_{mxn},\ f_{myn},\ f_{mzn}]^T$ is a vector of forces at each mount in three directions. $\{d_m\} = [d_{mx1},\ d_{my1},\ d_{mz1},\ \cdots,\ d_{mxn},\ d_{myn}, d_{mzn}]^T$ is a vector of displacements at each mount in three directions. The diagonal elements of $[R_m]$ are referred as drive-point receptance representing the displacement at the driving points. Other elements are referred as transfer receptance representing the displacement at other points except driving point. Matrix $[R_m]$ can also be derived from the mobility matrix by dividing it by $-j\omega$.

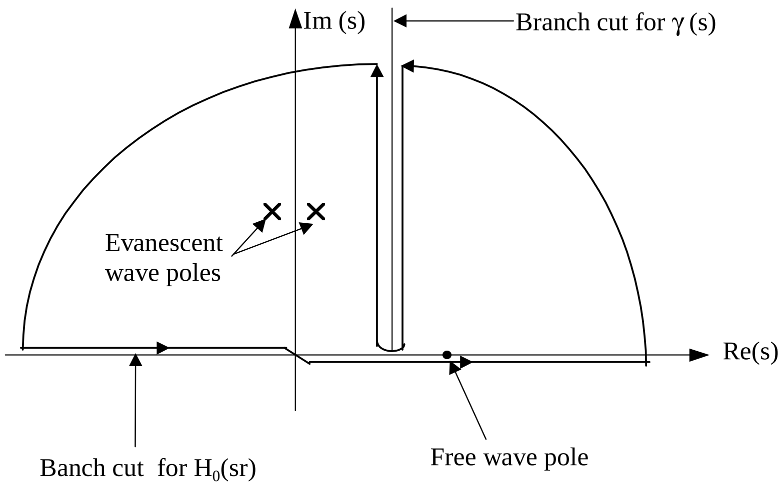


Fig. 2. The integration contour used to evalute the transfer receptance. Since the analysis is below the critical frequency, the leaky wave poles can be ignored.

$$\begin{bmatrix} d_{m1x} \\ d_{m1y} \\ d_{m1z} \\ \vdots \\ \vdots \\ d_{mnz} \end{bmatrix}_{(3nx1)} = \tag{13}$$

$$\begin{bmatrix} \begin{bmatrix} R_{m11xx} & 0 & 0 \\ 0 & \varepsilon \cdot R_{m11xx} & \varepsilon' \cdot R_{m11xx} \\ 0 & \varepsilon' \cdot R_{m11xx} & \varepsilon \cdot R_{m11xx} \end{bmatrix} & \begin{bmatrix} R_{m12xx} & 0 & 0 \\ 0 & \varepsilon \cdot R_{m12xx} & \varepsilon' \cdot R_{m12xx} \\ 0 & \varepsilon' \cdot R_{m12xx} & \varepsilon \cdot R_{m12xx} \end{bmatrix} & \cdots R_{m1n} \\ \vdots & \vdots & \cdots R_{m2n} \\ \vdots & \vdots & \ddots \quad \vdots \\ R_{mn1} & \cdots & \cdots R_{mnn} \end{bmatrix} \begin{bmatrix} f_{m1x} \\ f_{m1y} \\ f_{m1z} \\ \vdots \\ \vdots \\ f_{mnz} \end{bmatrix}_{(3nx1)}$$

4. Receptance of an infinite plate subject to light and heavy fluid loading

The theoretical solution to the receptance for a finite plate coupled with heavy fluid can't be achieved. The discussion here will be concentrated on a rigid body mounted on a thin infinite plate subject to the light and heavy fluid loading. It can provide some understanding of how the coupling among the mounting system, plate, and fluids happen.

4.1. Simplification of receptance matrix

For a thin infinite plate and with small deflection, matrix $[R_m]$ can be simplified based on the following assumptions:

– Horizontal forces (Y, Z directions) will not cause the displacement in the vertical direction (X direction).
– Vertical force (X direction) can only generate the displacement in the same direction.
– Flexural stiffness of plate (X direction) is much smaller than the compression stiffness (Y, Z directions).

With these assumptions, the point and transfer receptances in horizontal directions (Y, Z directions) can be expressed as small items ε, ε' multiplied by that in vertical direction (X direction). Equation (12) is thus rewritten as Eq. (13).

Where ε, ε' can be expressed as the ratio of flexural stiffness to compression stiffness of plate and written as

$$\begin{bmatrix} d_{m1x} \\ d_{m1y} \\ d_{m1z} \\ \vdots \\ \vdots \\ d_{mnz} \end{bmatrix}_{(3nx1)} = \begin{bmatrix} \begin{bmatrix} A_{l0} & 0 & 0 \\ 0 & \varepsilon \cdot A_{l0} & \varepsilon' \cdot A_{l0} \\ 0 & \varepsilon' \cdot A_{l0} & \varepsilon \cdot A_{l0} \end{bmatrix} & \cdots\cdots & \begin{bmatrix} A_{ln} & 0 & 0 \\ 0 & \varepsilon \cdot A_{ln} & \varepsilon' \cdot A_{ln} \\ 0 & \varepsilon' \cdot A_{ln} & \varepsilon \cdot A_{ln} \end{bmatrix} \\ \vdots & \vdots \cdots & \vdots \\ \vdots & \vdots \ddots & \vdots \\ \begin{bmatrix} A_{ln} & 0 & 0 \\ 0 & \varepsilon \cdot A_{ln} & \varepsilon' \cdot A_{ln} \\ 0 & \varepsilon' \cdot A_{ln} & \varepsilon \cdot A_{ln} \end{bmatrix} & \vdots \;\; \vdots & \begin{bmatrix} A_{l0} & 0 & 0 \\ 0 & \varepsilon \cdot A_{l0} & \varepsilon' \cdot A_{l0} \\ 0 & \varepsilon' \cdot A_{l0} & \varepsilon \cdot A_{l0} \end{bmatrix} \end{bmatrix} \begin{bmatrix} f_{m1x} \\ f_{m1y} \\ f_{m1z} \\ \vdots \\ \vdots \\ f_{mnz} \end{bmatrix}_{(3nx1)} \quad (20)$$

$$\varepsilon \sim \frac{h^2}{12(1-\nu^2)}, \quad (14)$$

$$\varepsilon' \sim \frac{\nu h^2}{12(1-\nu^2)}. \quad (15)$$

where h is the thickness of the plate and ν is the Poisson ratio.

It can be found from Eq. (13) that $[R_m]$ is only the function of point and transfer receptances in the vertical direction (X direction). These receptances can be derived from the frequency response of the plate subject to a harmonic load of unit magnitude in the direction normal to the plate.

4.2. *Receptance matrix of mounting system with light fluid loading*

For an infinite plate, the general equation of motion governing the transverse displacement $w(r,t)$ of plate under fluid loading and unit driving force on one side is [3]:

$$D\left(\Delta^4 - k_f^4\right) w(r) = -p(r,0) + \frac{\delta(r)}{2\pi r} \quad (16)$$

where $D = fracEh^3 12(1-\nu^2)$ is the flexural rigidity of plate. E is the Young's modulus. $k_f = (\rho_s h\omega^2/D)^{1/4}$ is the flexural wave number, ρ_s is the material density and ω is the forcing angular frequency.

Under the light fluid loading, e.g. in the air, the fluid loading effect can be ignored and the displacement w can be obtained as [3]:

$$w(r,t) = \frac{i}{8\omega\sqrt{\rho_s hD}} \left[-H_0^{(1)}(k_f r) + \frac{2i}{\pi} K_0^{(1)}(k_f r)\right] e^{j\omega t} \quad (17)$$

where $H_0^{(1)}$ is the Hankel function of the first kind of order zero; $K_0^{(1)}$ is the modified Hankel function of order zero.

The drive-point receptance under light fluid loading A_{l0} is the value of w with $r = 0$ which can be derived from the point impedance of force excitation [3]:

$$A_{l0} = w(0) = \frac{i\sqrt{3}}{4\rho_s c_p \omega h^2} \quad (18)$$

where $c_p = \left[\frac{E}{\rho_s(1-\nu^2)}\right]^{1/2}$ is the phase velocity of compression wave. The transfer receptance A_{lj} between first and jth driving point is written as:

$$A_{lj} = w(r_j) = \frac{i}{8\omega\sqrt{\rho_s hD}} \left[-H_0^{(1)}(k_f r_j) + \frac{2i}{\pi} K_0^{(1)}(k_f r_j)\right] \quad (19)$$

For an infinite plate, the drive-point receptance at all points should be the same and equal to A_{l0}. The transfer receptance between two points are the same which causes the symmetry of $[R_m]$. Equation (13) can thus be simplified as Eq. (20).

4.3. *Receptance matrix of mounting system with heavy fluid loading*

In the case of heavy loading, e.g. water, the solution of the plate response under the point force driving can be solved by Hankel transform and the solution in terms of transform parameter s can be obtained as [3]:

$$w(r,t) = \frac{i}{4\pi\omega} \int_c \frac{H_0^{(1)}(sr)\gamma(s)ds}{\overline{Z}_a(s) + \overline{Z}_p(s)} e^{j\omega t} \quad (21)$$

where

$$\overline{Z}_a(s) = \frac{\rho\omega}{\sqrt{k^2 - s^2}} \quad (22)$$

and

$$\overline{Z}_p(s) = -i\omega\rho_s h\left(1 - \frac{s^4}{k_f^4}\right). \quad (23)$$

The exact result for Eq. (21) is difficult and analytical solutions are available only for simple cases. The drive-point receptance with heavy fluid can be derived from the point mobility as obtained by Crighton [5]. At

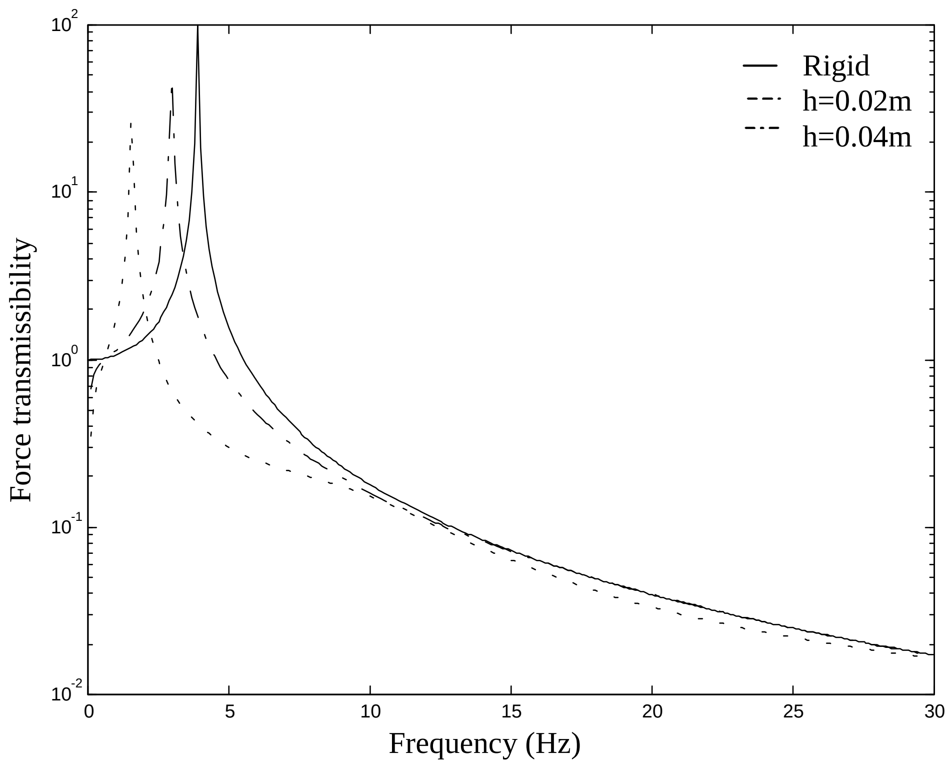


Fig. 3. Vertical force transmission at mount 1-infinite plate with light fluid loading under the unit vertical force excitation (F_z) at c.g. of rigid body.

Table 1
Mounting locations of isolators

	X (m)	Y (m)	Z (m)
c.g.	0	0	0
Mount 1	−1	1	2
Mount 2	−1	−1	2
Mount 3	−1	−1	−2
Mount 4	−1	1	−2

low frequency approximation with $\omega/\omega_c \ll 1$, this receptance can be defined as:

$$A_{h0} = w(0) = A_{l0}\frac{4}{5}\left(\frac{\rho_s h}{\rho}k_f\right)^{2/5}\left(1 - i\tan\frac{\pi}{10}\right) \tag{24}$$

where $\omega_c = \frac{\sqrt{12}c^2}{hc_p}$ is the coincidence frequency at which the phase speed of flexural waves coincides with the sound speed of the ambient fluid. For reference, this frequency for a half-inch steel plate in water is about 20 kHz.

Compared to point-drive receptance, the calculation of transfer receptance is more complicated. Rothwell and Purshouse [7] evaluated the transfer mobility of a point-excited fluid-loaded plate. The plate vibration is expressed in terms of a sum of residues of poles and a branch-line integral as shown in Fig. 2 for Eq. (21) and expressed as:

$$\begin{aligned} A_h(r,\omega) &= A_{he}(r,\omega) + A_{hl}(r,\omega) \\ &\quad + A_{hf}(r,\omega) + A_{ha}(r,\omega) \end{aligned} \tag{25}$$

where $A_{he}(r,\omega)$, $A_{hl}(r,\omega)$, $A_{hf}(r,\omega)$ denote the residue components deriving from the evanescant, leaky and free wave poles respectively. The total receptances contributed from these residues are:

$$\begin{aligned} &A_{hl,e,f}(r,\omega) \\ &= (i/2D)\sum_{n=1\sim 5}\alpha_n s_n H_0^{(1)}(s_n r) \\ &\quad \left\{\left(s_n^4 - k_f^4\right)\left(s_n^2 - k^2\right) + (\rho/\rho_s h)\gamma(s_n)k_f^4\right\} \end{aligned} \tag{26}$$

where $\gamma(s_n) = +(s^2 - k^2)^{1/2}$ for $|s| > |k|$, and for $\gamma(s_n) = -i(k^2 - s^2)^{1/2}$ for $|s| < |k|$. $\alpha_n^{-1} = dP(s_n)/ds$ and

$$\begin{aligned} &P(s_n) \\ &= \left(s_n^4 - k_f^4\right)^2\left(s_n^2 - k^2\right) - (\rho/\rho_s h)^2 k_f^8. \end{aligned} \tag{27}$$

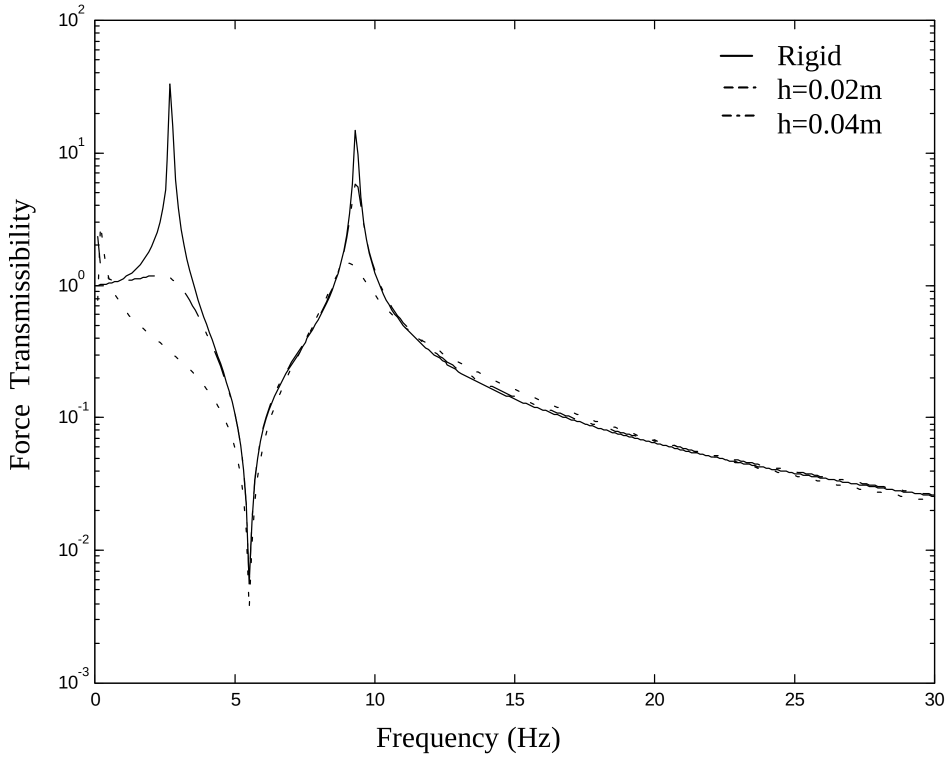


Fig. 4. Vertical force transmission at mount 1-infinite plate with light fluid loading under the unit moment excitation around z-axis at c.g. of rigid body.

Table 2
System natural frequencies with rigid foundation

Resonance mode	Original system (Hz)
f_x	3.9
f_y	2.73
f_z	3.97
$f_{y\alpha}$	9.25
$f_{z\beta}$	8.12
f_γ	9.34

$A_{ha}(r,\omega)$ denotes the branch-line integral and expressed as:

$$A_{ha} = \frac{\rho k_f^4}{2\pi\rho_s hD}\int_k^{k+i\infty}\frac{sH_0^{(1)}(sr)\gamma(s)}{P(s)}ds. \quad (28)$$

The plate's response in the far field at fixed frequency and in the high frequency at fixed position has been analysed and presented by Rothwell and Purshouse [7]. However, for rigid body mounting system, the requirement is the response at low frequency and in the near field.

Under the low frequency approximation $(\rho/\rho_s k_f h)^{1/5} \ll 1)$ [5], there have only two evanescant wave poles that are close to the imaginary axis, one free wave pole which lies on the real axis and zero leaky wave poles. The values of these poles in the s plane are shown in Fig. 2 and can be expressed as [3]:

$$s_1 = k_f(\rho/\rho_s k_f h)^{1/5} \qquad \text{(free wave)} \quad (29)$$

$$s_2 = k_f(\rho/\rho_s k_f h)^{1/5} e^{i2\pi/5} \qquad \text{(evanescant wave)} \quad (30)$$

$$s_3 = k_f(\rho/\rho_s k_f h)^{1/5} e^{i3\pi/5} \qquad \text{(evanescant wave)} \quad (31)$$

The residue contribution to receptance for free wave and evanescant wave poles in Eq. (26) can be simplified as:

$$A_{hl,e}(r,\omega) = (-i/2D)\sum_{n=1\sim 3}\alpha_n s_n H_0^{(1)}(s_n r) \left\{s_n^5 + (\rho/\rho_s h)\gamma(s_n)k_f^4\right\} \quad (32)$$

where $\alpha_n^{-1} = dP(s)/ds|_{s=s_n} = 10s_n^9$.

The analytical expression of $A_{ha}(r,\omega)$ in Eq. (28) is difficult in the near field and numerical calculation is required. To apply Gauss quadrature formula, Equation (28) should be transformed from s to s' plan with $s' = -i(s-k)$. The transformed $A_{ha}(r,\omega)$ can be

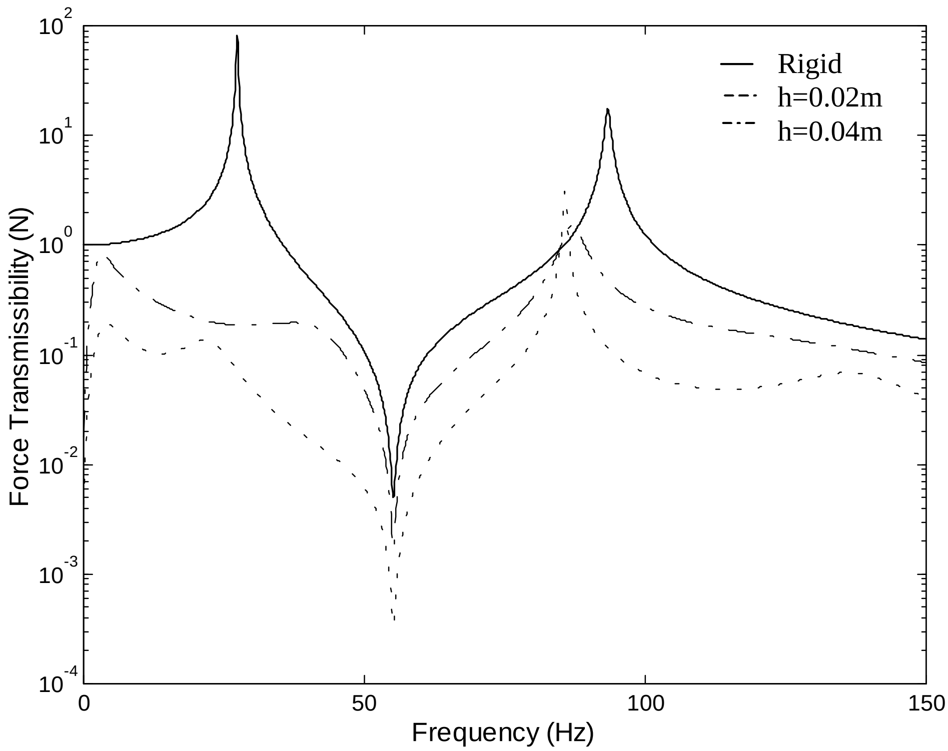


Fig. 5. Vertical force transmission at mount 1-infinite plate with light fluid loading under the unit moment excitation around z-axis at c.g. of rigid body. The isolator stiffness in all directions is enlarged 100 times as compared with that in Fig. 4.

written as:

$$A_{ha} = \frac{i\rho k_f^4}{2\pi\rho_s hD} \int_0^\infty \frac{(k+is')H_0^{(1)}(kr+is'r)\gamma(k+is')}{P(k+is')}ds' \tag{33}$$

where $\gamma(k+is') = +(2ids'-s'^2)^{1/2}$ since $|s|$ is always larger than $|k|$. The value of the poles in s' can be thus expressed as:

$$s_1' = -i(\delta - k) \qquad \text{(free wave)} \tag{34}$$

$$s_2' = \delta\sin(2\pi/5) + i(j - \delta\cos 2\pi/5) \qquad \text{(evanescant wave)} \tag{35}$$

$$s_3' = \delta\sin(3\pi/5) + i(k - \delta\cos 3\pi/5) \qquad \text{(evanescant wave)} \tag{36}$$

where $\delta = (\rho/\rho_s k_f h)^{1/5}$. The positions of poles will change with the variation of k. As $k = \delta$, $\delta\cos 2\pi/5$ and $\delta\cos 3\pi/5$, the value of poles will be real and in the integration range. The integrand will have singularities. However, in most of cases, there would no singularities in the range of integration.

5. Case study

Consider a rigid body with 4 mounting points as shown in Fig. 1. The rigid body has a mass of 10000kg and its moments of inertia are 17770, 17770 and 7110 kg.m^2. The four isolators are identical and are arranged symmetrically with respect to the centre of mass. The isolators are located with their co-ordinates as listed in Table 1 and are oriented with their principle elastic directions co-ordinated with X, Y, Z axes. The foundation is a thin infinite steel plate with thickness 0.02 m, 0.04 m, density 7800 kg/m^3 and Young's modulus 2.1×10^{11} N/m^2. The flexural rigidity D (bending stiffness) is calculated as 1.9×10^5 N/m and 1.5×10^6 N/m for 0.02 m and 0.04 m plates respectively.

The isolators are soft in vertical (X) direction (1.5×10^6 N/m) and stiff in horizontal (Y, Z) directions (3×10^6 N/m). Thus, the stiffness of isolators in X direction (K_x) is the same as that of the bending stiffness of 0.04 m plate but 8 times of that of 0.02 m plate. The calculated static deflection of isolators in X direction is 0.0125 m (0.5 in). The damping loss factor η is 0.01. The Possion ration is 0.27. The system resonance modes with the rigid foundation are calculated and listed in Table 2.

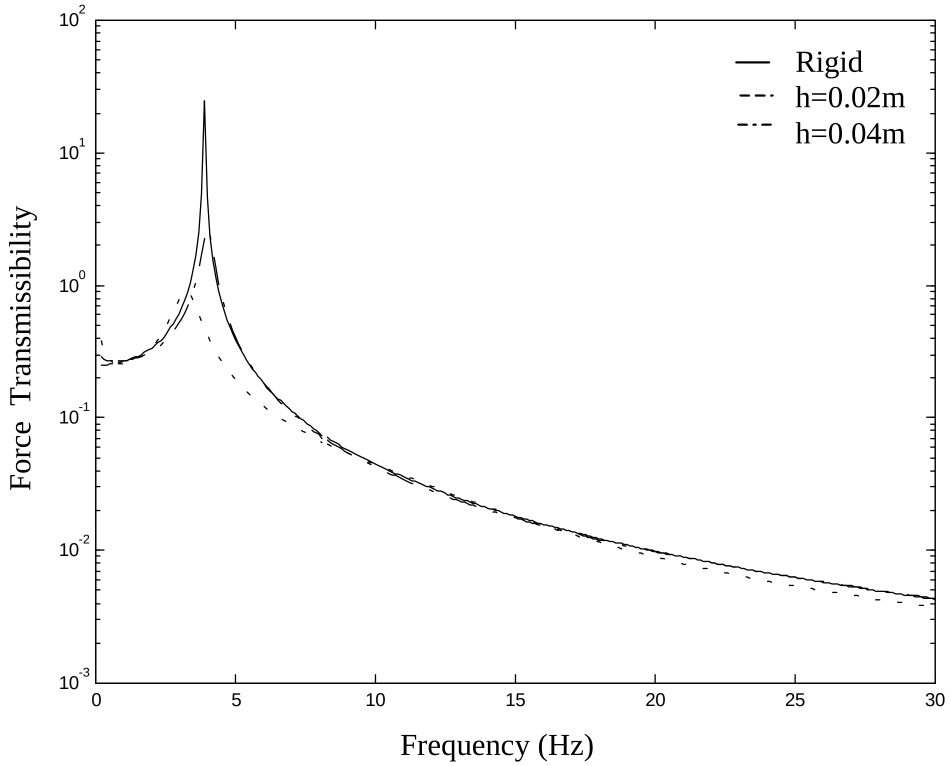


Fig. 6. Vertical force transmission at mount 1-infinite plate with heavy fluid loading under the unit vertical force excitation (F_z) at c.g. of rigid body.

5.1. *Frequency response of mounting system with fluid loading subject to force and moment excitation*

Frequency response of above mounting system with light fluid loading (air) subject to force and moment excitation has been calculated. Figure 3 illustrates the force transmissibility at mount 1 of the rigid body mounting system under different plate thickness. It is found that, the thinner the plate the less vertical force transmission from rigid body to the foundation. The rigid foundation has the highest force transmission. These results are similar to that obtained by Ashrafiuon & Nataraj [2] which based on the stiffness matrix of foundation. Figure 4 shows the vertical force transmission at mount 1 under the unit moment excitation in Z-direction (M_z). It is interesting to found that the force transmission is significantly reduced near the frequency at translational mode, 2.73 Hz (f_y) but much less at the rotation mode, 9.25 Hz ($f_{y\alpha}$) and no effect at anti-resonance frequency. These foundation effects are strong frequency dependent. As the excitation frequency is much higher than the natural frequencies in Figs 3, 4, the force transmission gets close to that with a rigid foundation.

Another phenomenon found in Figs 3, 4 is that the system natural frequencies have been shifted to lower values. This shifting is more obvious as the isolators become stiffer in all directions. Figure 5 shows the force transmission as K_x, and K_y, K_z are enlarged 100 times to 1.5×10^8 N/m and 3×10^8 N/m respectively. In this case, the isolator is much stiffer than foundation that is similar to the case where the machine is rigidly mounted on the plate. The interesting thing is that the frequency shifting is much larger for translational mode than that for rotational mode but no effect for anti-resonance mode. The reduction of force transmission is also no longer limited only near the system natural frequencies. It implies that, as the isolator stiffness is around the bending stiffness of plate, the elasticity of foundation only affects the force transmission near the system natural frequencies. However, as an isolator becomes much stiffer as compared with the foundation, this effect can be extended to a wider frequency range.

When the plate subjects to heavy fluid loading, such as the water, the frequency response of the rigid body mounting system will be significantly changed. Figures 6, 7 illustrate the vertical force transmission of the rigid body mounting system subject to water loading

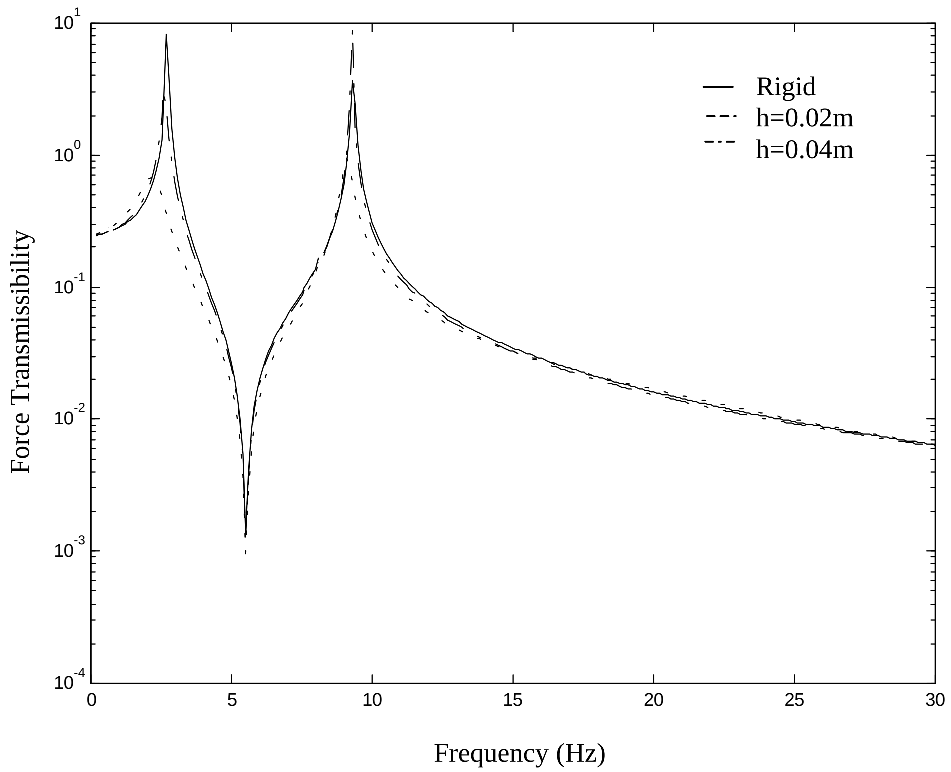


Fig. 7. Vertical force transmission at mount 1-infinite plate with heavy fluid loading under the unit moment excitation around z-axis at c.g. of rigid body.

under unit force and moment excitation. Due to the low frequency limitation applied for Eqs (29), (30) and (31), the range of frequencies calculated in this case is from 0 to 30 Hz. It is found that the elastic foundation effects are significantly reduced with the force transmission increased and frequency shifting decreased as compared with that in Figs 3, 4.

5.2. *Effects of mounting receptance with light fluid loading*

It is interesting to investigate how the mounting receptance affects the frequency response of the mounting system. To a single mass and spring system on a rigid foundation, the system natural frequency should keep the same even with the damping of isolator provided this damping is frequency independent. However, the frequency shifting will happen as the damping factor varies with the frequency. It is the case when a single mass and spring system is mounted on an infinite plate. This is because the point drive receptance in Eq. (18) is a pure imaginary value which implies that the effects of a thin infinite plate can be considered as a damping adding on the mounting system. This damping effect is inversely proportional to the frequency and square of the plate thickness. To a rigid body mounting system, there exists the transfer receptance besides the point receptance. However, the contribution of point receptance is still significant which is shown in Figs 8,9. For the transfer receptance, the imaginary part can also be considered as a damping effect while the real part is the combination of mass and spring effects. These receptances vary with frequency. Figure 8 present the real and imaginary parts of the transfer receptance along with the variation of frequency. It shows that the receptance values at low frequencies are significant which implies that the frequency shifting and force reduction should be larger. This can explain why the frequency shifting and force reduction in Figs 4, 5 are more obvious at translational modes and less at rotational modes since their frequencies are higher.

The transfer receptance also varies with the distance from the driving point. Figure 9 illustrates the receptances along with the changes of the distance from the driving point. It is found that transfer receptance fluctuates and gradually disappear. The proper location of mounting point can minimise the real part of receptance (mass $+$ spring), leaving the plate controlled by damp-

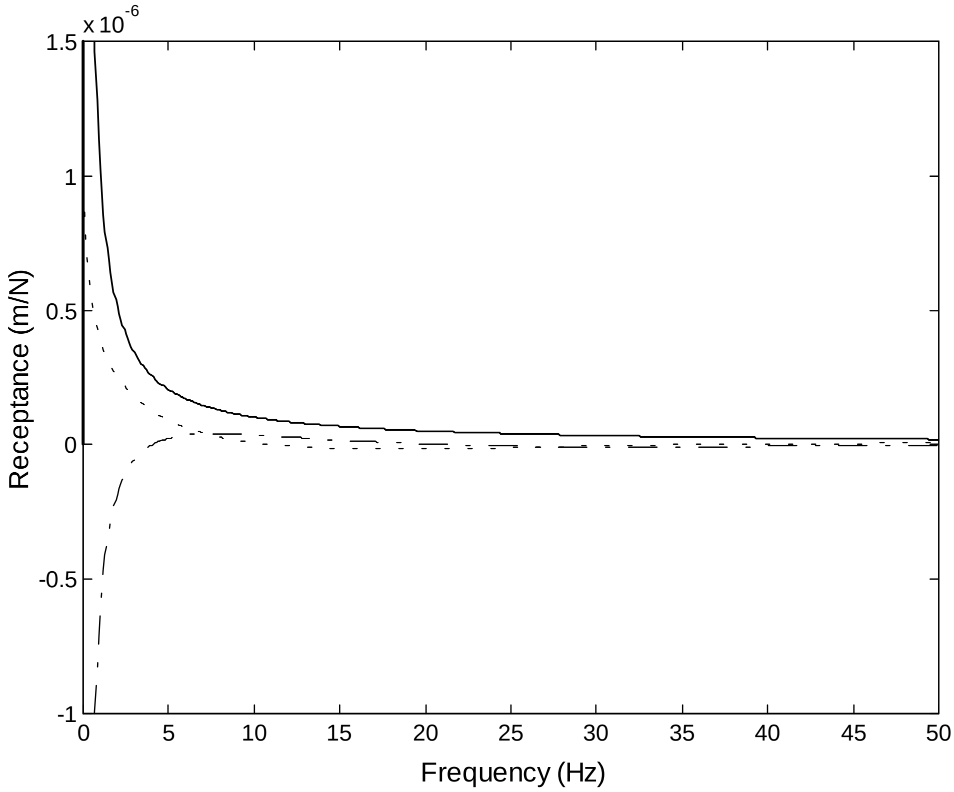


Fig. 8. The comparison of real and imaginary parts of receptance with the variation of frequency under the light fluid loading. The distance from driving point is 4 m with plate thickness 0.04 m. —, point receptance (pure imaginary); — — -, real part of transfer receptance; — - —, the imaginary part of transfer receptance.

ing. This can reduce the resonance effect of system and be beneficial to the vibration control.

5.3. *Effects of mounting receptances from different waves with heavy fluid loading*

As a plate coupled with heavy fluid, the transfer receptance calculation becomes complicated since there is more than one wave in the plate. It is interesting to investigate the transfer receptances of different waves in the near field and at low frequency to see which wave is dominant for the contribution of receptance. This investigation may cause a significant simplification for the calculation of receptance. Rothwell & Purshouse [7] investigated this but only concentrated on the far field and relatively high frequency. Figure 10 (a, b, c, d, e, f) shows the values of receptance from the contribution of different waves with plate thickness 0.02 m, 0.04 m at different frequencies (4 Hz, 50 Hz, 100 Hz) coupled with water. For comparison, the point receptance is also given which is a straight line in the figure. It is found that, in the near field, the transfer receptance from evanescent waves and that from acoustic contribution as defined in Eq. (28) decay quickly to an insignificant level for all plate thickness and frequencies, leaving only the point drive receptance and transfer receptance from free wave. This distance is only about 3 m from source in this case. It demonstrates that in the far field (only a few meters) the response of the fluid-loaded plate is dominated by the free wave. Thus, as the mounting points of rigid body are separated with a space more than 3 meters in this case, only the transfer receptance from free wave needs to be considered. Figure 11 illustrates the force transmission with unit moment excitation after ignoring the receptance from evanescent wave and that from acoustic contribution. Compared with Fig. 7, the force transmission in all frequencies seems no much difference.

Figure 10 shows the receptance values of different waves when the plate subjects to heavy fluid loading (water). It is interesting to normalise these receptance values with that of point receptance in vacuo. Figure 12 shows these normalised values for different waves. The plate thickness is 0.04 m and frequency is 10 Hz. It is

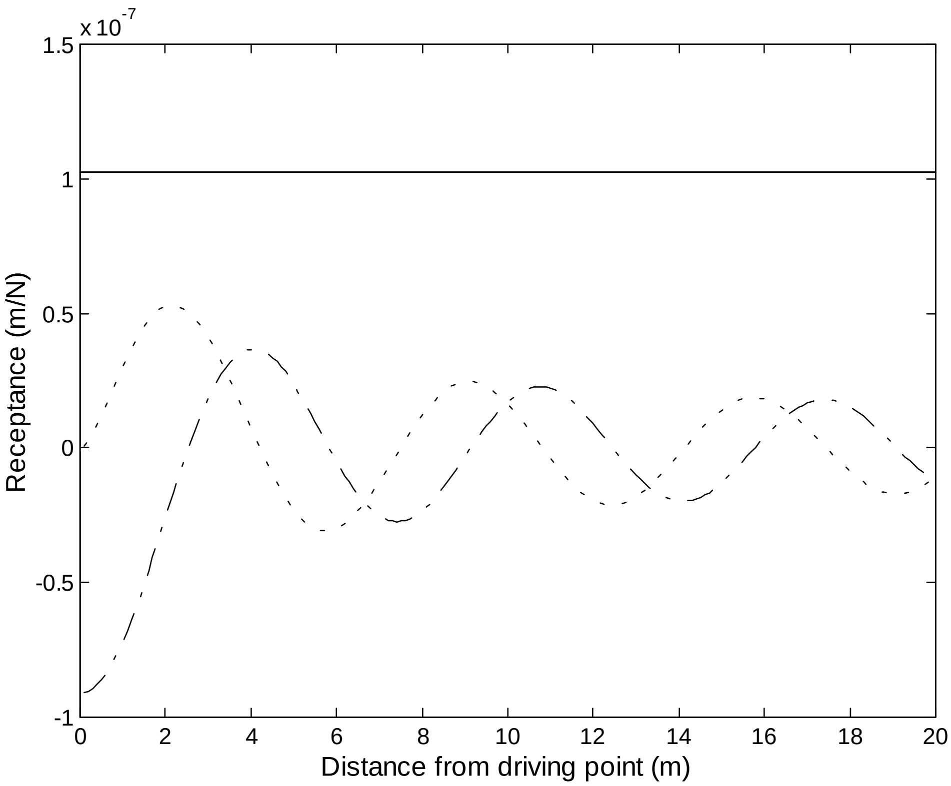


Fig. 9. The comparison of real and imaginary parts of receptance with the variation of distance from driving point under the light fluid loading. Frequency is 50 Hz with plate thickness 0.04 m. —, point receptance (pure imaginary); — — -, real part of transfer receptance; — - —, the imaginary part of transfer receptance.

found that the normalised value for free wave becomes a constant which is about 0.36 after a short distance while the values for evanescent wave and acoustic contribution decays quickly in a short distance. The further investigation shows that this constant varies with the frequencies: 0.4 for frequency 25 Hz and 0.42 for frequency 50 Hz. It implies that, for a certain frequency and a short distance away from the driving point, the displacement or velocity response of plate with heavy fluid loading may be able to be derived from that with light fluid loading by just multiplying a constant.

6. Conclusion

This paper presents an investigation of the force transmission from a rigid body mounting system to a flexible foundation subject to light and heavy fluid loading and under the force and moment excitation. The analytical expression has been derived in which the flexible foundation effects are incorporated into a revised system stiffness matrix which is derived from the receptance matrix at mounting points. This receptance (or mobility) matrix can be directly measured or obtained from FEM simulation by applying a unit force excitation at mounting points. It provides a practical method in engineering to evaluate the isolation performance of a mounting system with a flexible foundation and coupled with fluids. Due to the theoretical difficulty for the analysis of a finite plate, the further study is carried out for a thin infinite plate as the foundation with light and heavy fluid loading. The point and transfer receptance matrix is simplified and derived from the theoretical and numerical analysis of the plate response under a unit force excitation.

The results with a rigid body mounted on a thin infinite plate are presented. It indicates that the coupled system natural frequencies have been shifted to lower values and the force transmission has been reduced for both light and heavy fluid loading as the foundation is flexible. The further analysis shows that the shifting of system natural frequency and force reduction is more obvious at low frequency where the receptance value is significant.

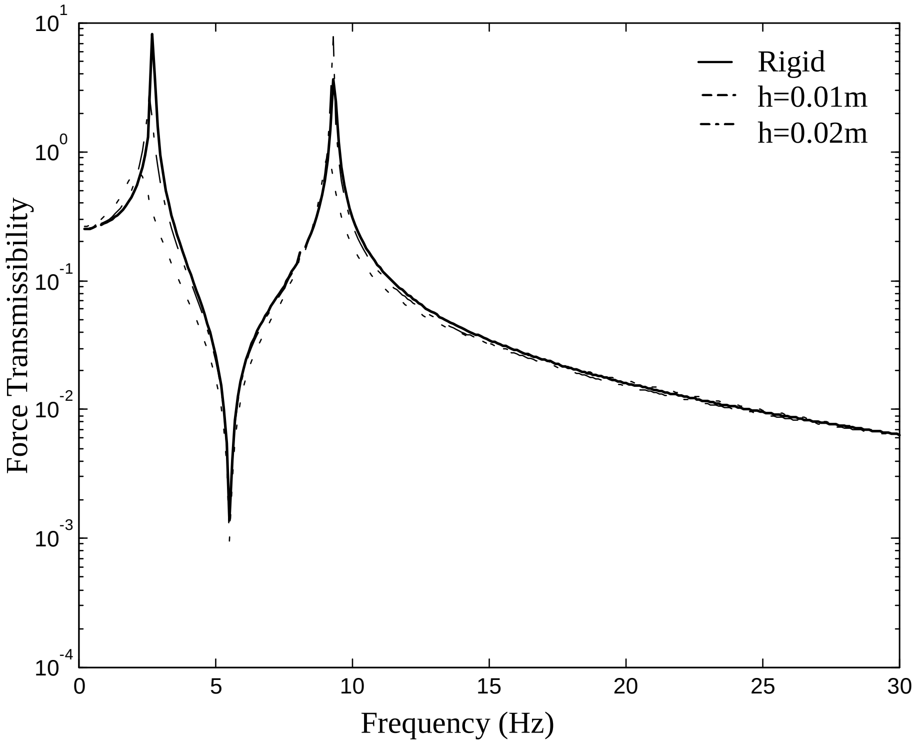


Fig. 11. Vertical force transmission at mount 1-infinite plate with heavy fluid loading under the unit moment excitation around z-axis at c.g. of rigid body. The receptance from evanescent wave and acoustic contribution is ignored.

The detailed investigation is also carried out for the different waves existed in the plate with the objective to simplify the calculation of the receptance with heavy fluid loading. The results show that the effects from free wave play a dominant role while the effects from evanescent wave and acoustic contribution limits only in a few meters from driving point. This free wave receptance may be able to be derived from the point receptance in vacuo since the ratio between them becomes a constant for a certain frequency and after a short distance from driving point.

Nomenclature

A_{l0} = point receptance at mounting point with light fluid loading

A_{lj} = transfer receptance between mounting points with light fluid loading

A_{h0} = point receptance at mounting point with heavy fluid loading

A_{h} = transfer receptance between mounting points with heavy fluid loading

A_{he} = transfer receptance of evanescant wave between mounting points with heavy fluid loading

A_{hf} = transfer receptance of free wave between mounting points with heavy fluid loading

A_{hl} = transfer receptance of leak wave between mounting points with heavy fluid loading

A_{he} = transfer receptance of acoustic contribution between mounting points with heavy fluid loading

c = sound velocity in the fluid

c_p = phase velocity of compression wave in plate

D = flexural rigidity of plate

E = Young's modulus

$f_x,\ f_y,\ f_z$ = translation modes of rigid body mounting system

$f_{x\alpha},\ f_{y\beta},\ f_{z\gamma}$ = rotation modes of rigid body mounting system

h = thickness of plate

k_f = flexural wave number

k = wave number of excitation force

η = hysteresis damping factor

ω = angular frequency of excitation force

ω_c = coincidence frequency

ν = Poisson ratio

r = distance away from driving point

ρ_s = material density of plate

ρ = density of fluid

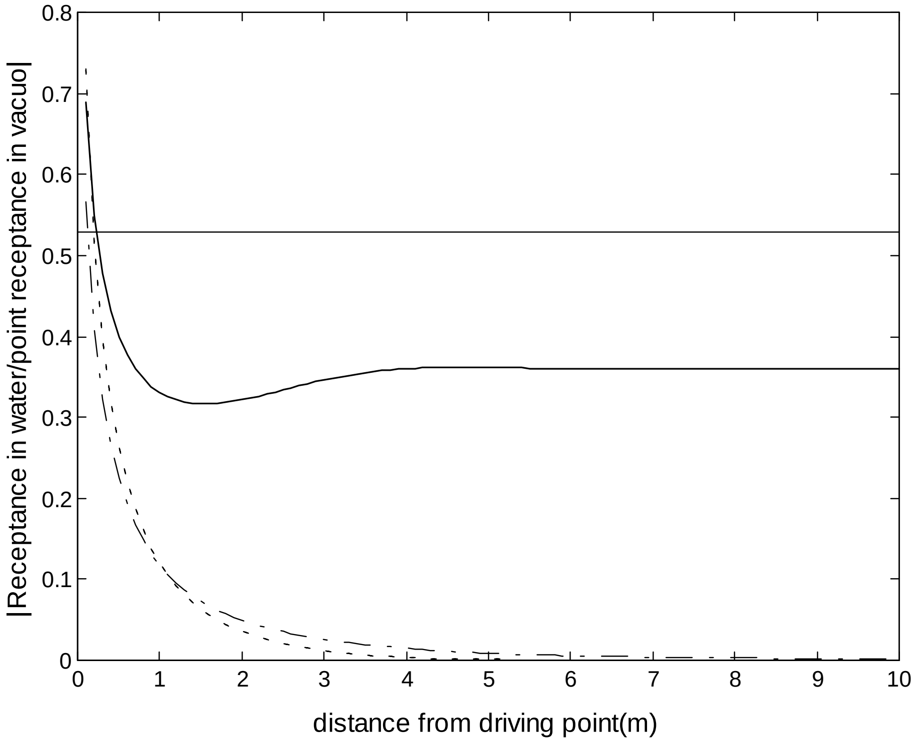


Fig. 12. The comparison of receptance in water and in vacuo. __ the free wave contribution; — — -, the evanescent wave contribution; – - –, the acoustic contribution; —, the point drive receptance. The frequency is 10 Hz and plate thickness is 0.04 m.

p = sound pressure in the fluid

Matrices

$[A]$ = transpose matrix of Euler angle matrix
$[K]$ = stiffness matrix of mounting system with rigid foundation
$[K']$ = revised stiffness matrix of mounting system with flexible foundation
$[k_0]$ = stiffness matrix of isolator in three directions of its local co-ordinate system
$[k']$ = stiffness matrix of isolator in three directions of its local co-ordinate system with damping
$[k_i]$ = stiffness matrix of isolator in the global co-ordinate system
$[M]$ = rigid bady mass matrix
$[r_n]$ = location matrix of mounting points
$[R_m]$ = receptance matrix at mounting points

Vectors

$\{d_m\}$ = displacement at all mounts
$\{f_m\}$ = force at all mounts
$\{F\}$ = excitation force and moment
$\{x\}$ = displacement vector at centre gravity of rigid body

References

[1] C.M. Harris and C.E. Crede, *Shock and Vibration Handbook*, (2nd ed.), McDRAW-HILL Inc., 1976.
[2] Ashrafiuon and C. Nataraj, Dynamic analysis of engine-mount system, *Journal of Vibration and Acoustics* **114** (1992), 79–83.
[3] M.C. Junger and D. Feit, *Sound, Structure and Their Interaction*, (2nd ed.), M.I.T. Press, Cambridge, Massachusetts, 1986.
[4] D.G. Crighton, Force and moment admittance of plates under arbitrary fluid loading, *Journal of Sound and Vibration* **20**(2) (1972), 209–218.
[5] D.G. Crighton, Point admittance of an infinite thin elastic plate under fluid loading, *Journal of Sound and Vibration* **54**(3) (1977), 389–391.
[6] D.G. Crighton, Approximations to the admittances and free wavenumbers of fluid-loaded panels, *Journal of Sound and Vibration* **68**(1) (1980), 15–33.
[7] D.J. Rothwell and M. Purshouse, The transfer admittance of a point-excited fluid-loaded plate, *Journal of Sound and Vibration* **120**(3) (1987), 431–443.
[8] D.T. Greenwoon, *Principles of Dynamics*, (2nd ed.), Prentics-Hall, Inc., Englewood Cliffs, New Jersey, 1979.

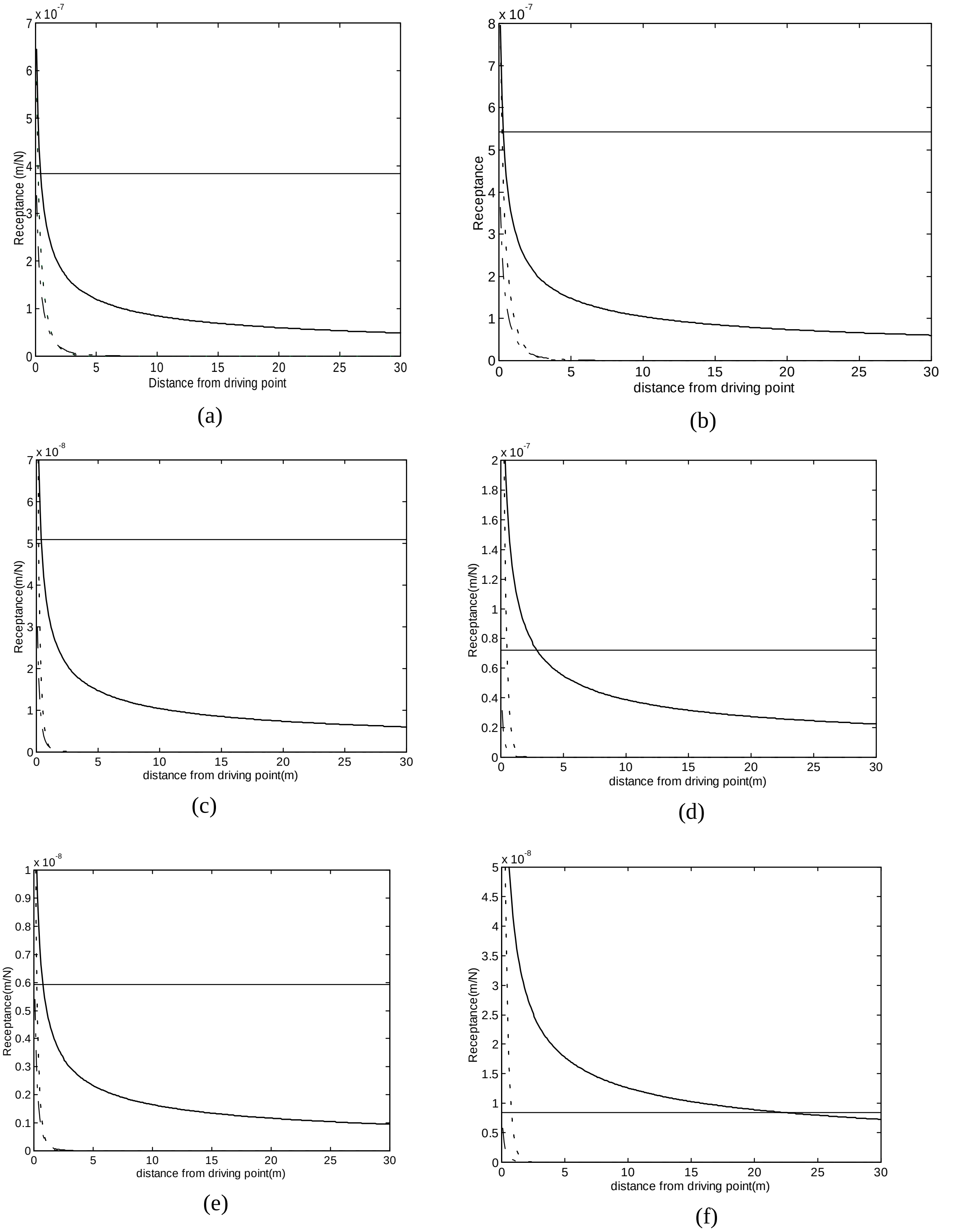
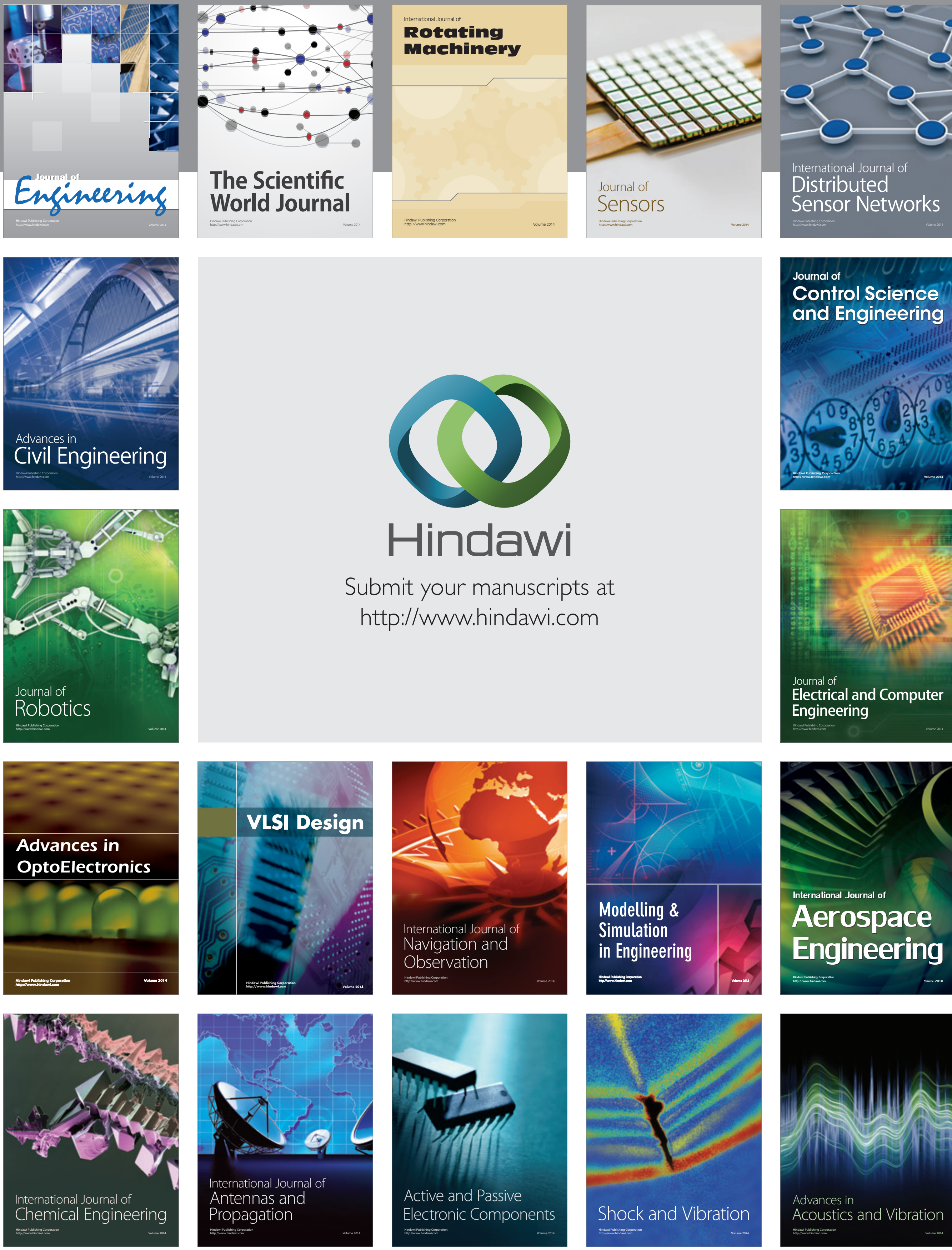


Fig. 10. The contribution to the response (receptance) of steel pates in water from different waves at frequencies 4 Hz (a, b), 50 Hz (c, d) and 100 Hz (e, f,). The left side figure for each frequency is with plate thickness 0.02 m while right side is 0.04 m.__ , free wave contribution; − − −, evanescent wave contribution; − - −, the acoustic contribution. —, point drive receptance.



Journal of Engineering
The Scientific World Journal
International Journal of Rotating Machinery
Journal of Sensors
International Journal of Distributed Sensor Networks
Advances in Civil Engineering
Journal of Control Science and Engineering
Hindawi
Submit your manuscripts at
http://www.hindawi.com
Journal of Robotics
Journal of Electrical and Computer Engineering
Advances in OptoElectronics
VLSI Design
International Journal of Navigation and Observation
Modelling & Simulation in Engineering
International Journal of Aerospace Engineering
International Journal of Chemical Engineering
International Journal of Antennas and Propagation
Active and Passive Electronic Components
Shock and Vibration
Advances in Acoustics and Vibration