

Superelement verification in complex structural models

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Received 2007

Revised 2007

Abstract. The objective of this article is to propose decision indicators to guide the analyst in the optimal definition of an ensemble of superelements in a complex structural assembly. These indicators are constructed based on comparisons between the unreduced physical model and the approximate solution provided by a nominally reduced superelement model. First, the low contribution substructure slave modes are filtered. Then, the minimum dynamical residual expansion is used to localize the superelements which are the most responsible for the response prediction errors. Moreover, it is shown that static residual vectors, which are a natural result of these calculations, can be included to represent the contribution of important truncated slave modes and consequently correct the deficient superelements. The proposed methodology is illustrated on a subassembly of an aeroengine model.

Keywords: Structural dynamics, model verification, component mode synthesis, Craig-Bampton method

1. Introduction

Complex structural models in the aeronautics and aerospace industries generally include a large number of subassemblies. A common cost saving measure is to create reduced Craig-Bampton [1] models for these substructures and to assemble them onto a residual physical structure. The resulting globally reduced model is then used to predict model responses. However, the Craig-Bampton reduction is not exact and it is well known that its precision depends on both the frequency domain of interest and the superelement slave modes retained in the individual superelement transformations. In practise, a number of difficulties are inherent in the automation of this reduction. In particular, the convergence of the predicted responses to their exact values is irregular as a function of the size of the slave basis. Moreover, a certain number of slave modes are often present corresponding to local subassembly behaviour which do not contribute significantly to the global responses but nevertheless increase memory requirements and calculation costs.

In this article, we propose a set of decision indicators to guide the designer in the optimal definition of an ensemble of superelements in a complex structural assembly. In a first step, we filter insignificant local substructure modes based on an effective mass criteria. We then perform an error localization in order to identify the superelements which are most responsible for the response prediction errors. Finally, we selectively complete the superelement slave bases with static residual vectors which are a natural result of the localization calculations. The proposed methodology is illustrated on a subassembly of an aeroengine model.

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2. Background

The objective of component mode synthesis is to approximate the dynamic behaviour of a substructure by projecting the component behaviour on an adequate subspace of response vectors. Various substructuring methods can be found in the literature [2] but only the well-known Craig-Bampton method is employed in this article [1,3]. The basis of this method is briefly described below.

Let N be the number of physical degrees of freedom (dofs) of a given substructure. These dofs are partitioned into two subsets: the subset of interface dofs denoted by the subscript j and the subset of interior dofs denoted by the subscript i . Assuming that no external forces are applied on the interior dofs, the equation of harmonic motion for an undamped substructure is given by:

$$(\mathbf{K} - \omega^2 \mathbf{M}) \mathbf{y} = \left(\begin{bmatrix} \mathbf{K}_{jj} & \mathbf{K}_{ji} \\ \mathbf{K}_{ij} & \mathbf{K}_{ii} \end{bmatrix} - \omega^2 \begin{bmatrix} \mathbf{M}_{jj} & \mathbf{M}_{ji} \\ \mathbf{M}_{ij} & \mathbf{M}_{ii} \end{bmatrix} \right) \begin{bmatrix} \mathbf{y}_j \\ \mathbf{y}_i \end{bmatrix} = \begin{bmatrix} \mathbf{f}_j \\ 0 \end{bmatrix} \quad (1)$$

where \mathbf{K} and \mathbf{M} are respectively the stiffness and mass matrices of the substructure, \mathbf{y} is the displacement vector, ω is the frequency and \mathbf{f}_j are the interface reactions. The matrices are partitioned with respect to the interface and internal dofs. In the Craig-Bampton method, a set of static constraint vectors is combined with a truncated set of k slave eigenmodes of the substructure with a fixed interface to form the reduction basis. The substructure reduction transformation is defined as:

$$\mathbf{y} = \begin{bmatrix} \mathbf{I}_{jj} & 0 \\ \Psi_{ij} & \Phi_{ik} \end{bmatrix} \begin{bmatrix} \mathbf{y}_j \\ \mathbf{q}_k \end{bmatrix} = \mathbf{T}_{CB} \begin{bmatrix} \mathbf{y}_j \\ \mathbf{q}_k \end{bmatrix} \quad (2)$$

where \mathbf{I}_{jj} is the identity matrix, $\Psi_{ij} = -\mathbf{K}_{ii}^{-1} \mathbf{K}_{ij}$ is the static transformation matrix, Φ_{ik} is the matrix of fixed interface mode shape, \mathbf{T}_{CB} the Craig-Bampton transformation matrix and \mathbf{q}_k the vector of the k generalized dofs.

The orthogonality conditions are:

$$\begin{aligned} \Phi_{ii}^T \mathbf{K}_{ii} \Phi_{ii} &= \mathbf{I}_{ii} \\ \Phi_{ii}^T \mathbf{M}_{ii} \Phi_{ii} &= \Lambda_{ii} \end{aligned} \quad (3)$$

with Λ_{ii} the diagonal matrix of eigenvalues.

Then by substituting Eqs (2) in equation (1) one can obtain the following reduced system:

$$\left(\begin{bmatrix} \bar{\mathbf{K}}_{jj} & 0 \\ 0 & \Lambda_{kk} \end{bmatrix} - \omega^2 \begin{bmatrix} \bar{\mathbf{M}}_{jj} & \mathbf{L}_{jk} \\ \mathbf{L}_{kj} & \mathbf{I}_{kk} \end{bmatrix} \right) \begin{bmatrix} \mathbf{y}_j \\ \mathbf{q}_k \end{bmatrix} = \begin{bmatrix} \mathbf{f}_j \\ 0 \end{bmatrix} \quad (4)$$

with:

$$\begin{aligned} \bar{\mathbf{K}}_{jj} &= \mathbf{K}_{jj} + \mathbf{K}_{ji} \Psi_{ij} \\ \bar{\mathbf{M}}_{jj} &= \mathbf{M}_{jj} + \mathbf{M}_{ji} \Psi_{ij} + \Psi_{ji} \mathbf{M}_{ij} + \Psi_{ji} \mathbf{M}_{ii} \Psi_{ij} \\ \mathbf{L}_{jk} &= (\mathbf{M}_{ji} + \Psi_{ji} \mathbf{M}_{ii}) \Phi_{ik} \end{aligned} \quad (5)$$

$\bar{\mathbf{K}}_{jj}$ and $\bar{\mathbf{M}}_{jj}$ are the residual matrices of substructure statically condensed to the interface dofs and \mathbf{L}_{jk} denotes the mass modal participation factor matrix.

The global reduced model is obtained by assembling the individual superelements in accordance with the definition of their interface dofs. Note that in the Craig-Bampton method, the subset of interface dofs is commonly completed with some interior master dofs in order to improve the superelement. We will not consider this case in this paper.

3. Superelement verification methodology

The proposed methodology provides quantitative decision-making indicators for optimally defining superelements and takes place in three phases: (1) filtering of low contribution substructure slave modes, (2) localisation of deficient superelements, and (3) correction of the deficient superelements.

3.1. Filtering

The selection of the k slave modes is often based upon a frequency cutoff criterion. The common rule of thumb consists in including the modes contained in a band two times larger than the highest frequency of interest of the complete physical model. Even if this method often gives good results, it leads to retain modes which do not contribute significantly to the global dynamical behaviour or to neglect modes above the cutoff frequency which have an important contribution.

Starting from k slave modes retained with a frequency cutoff criterion, the objective of this first phase is to filter the slave modes of the substructures which have the weakest participation in the global modes in order to reduce calculation costs without a significant loss of predictivity in the analysed frequency range. Towards this end, two indicators prove to be useful: a global mode indicator and an effective mass indicator.

3.1.1. Global mode indicator

The eigenvectors of the reduced assembled system in generalized coordinates contain the coefficients of the linear combination of the modes of the substructures which compose each mode of the assembled structure. Thus, an examination of these coefficients should allow to evaluate the participation of the individual slave eigenvectors and then to identify the low value vectors.

Consider a substructure ss . In the p^{th} global mode of the assembled reduced system, one can isolate the k generalized coordinates associated with the substructure ss . These k generalized coordinates are used to form the vector ${}_p\mathbf{q}_k^{(ss)}$. Each coordinate of this vector corresponds to a slave mode of the substructure. For the considered substructure, it is thus possible to compare the relative participation of a slave mode ν to the p^{th} global mode with the following global mode indicator:

$${}_pI_\nu^{(ss)} = \frac{|{}_p q_\nu^{(ss)}|}{\sum_{r=1}^k |{}_p q_r^{(ss)}|} \quad (6)$$

where ${}_p q_\nu^{(ss)}$ is the ν^{th} component of ${}_p\mathbf{q}_k^{(ss)}$.

For a modal basis with P eigenvectors, the participation of a slave mode ν to the P eigenvectors will be estimated with:

$$I_\nu^{\text{part}} = \max_{p \in P} ({}_p I_\nu^{(ss)}) \quad (7)$$

3.1.2. Effective modal mass indicator

In structural dynamics, the purpose of the effective modal mass is to select modes which have large reaction forces with respect to a rigid interface. When used during a base-driven analysis [4], the rigid interface is the base of a structure where the shaker acts and when used in the Craig-Bampton method [5], the interface is defined by the junction dofs connecting the studied substructure with the rest of the structure.

Solving Eq. (4), we obtain the relation between the displacement \mathbf{y}_j at the interface and the interface reactions \mathbf{f}_j :

$$\mathbf{f}_j = \left(\bar{\mathbf{K}}_{jj} - \omega^2 \left[\bar{\mathbf{M}}_{jj} + \sum_{\nu=1}^k \frac{1}{\left(\frac{\omega_\nu}{\omega}\right)^2 - 1} \mathbf{L}_\nu \mathbf{L}_\nu^T \right] \right) \mathbf{y}_j \quad (8)$$

\mathbf{L}_ν is the ν^{th} column of the mass modal participation factor matrix \mathbf{L}_{jk} specified Eq. (6). The effective mass matrix of the mode ν is:

$$\left(\bar{\mathbf{M}}_{jj}^{\text{eff}} \right)^\nu = \mathbf{L}_\nu \mathbf{L}_\nu^T \quad (9)$$

Moreover, it can be shown that:

$$\sum_{\nu=1}^k \left(\bar{\mathbf{M}}_{jj}^{\text{eff}} \right)^\nu = \bar{\mathbf{M}}_{jj} - \bar{\mathbf{M}}_{jj}^{\text{eq}} - \bar{\mathbf{M}}_{jj}^{\text{res}} \quad (10)$$

where $\bar{\mathbf{M}}_{jj}^{\text{eq}} = \mathbf{M}_{jj} - \mathbf{M}_{ji} \mathbf{M}_{ii}^{-1} \mathbf{M}_{ij}$ is a negligible mass matrix and $\bar{\mathbf{M}}_{jj}^{\text{res}}$ is a residual mass matrix resulting from the modal truncation.

A slave mode will be considered as global if the associated effective mass matrix contributes significantly to the rigid body mass matrix relative to the interface dofs $\bar{\mathbf{M}}_{jj}$. In practise, the effective mass matrices are calculated for a six dofs node connected to the interface nodes with rigid elements and are extracted during the normal mode analysis. A measure of importance could be the comparison of the six diagonal terms of $\left(\bar{\mathbf{M}}_{jj}^{\text{eff}}\right)^\nu$ with the corresponding components of $\bar{\mathbf{M}}_{jj}$. A more useful scalar quantity is calculated with [5]:

$$I_\nu^{\text{eff}} = \frac{\text{tr} \left(\left(\bar{\mathbf{M}}_{jj}^{\text{eff}} \right)^\nu \right)}{\text{tr} \left(\bar{\mathbf{M}}_{jj} \right)} \quad (11)$$

However, the concept of a local mode is very subjective. It depends on the size of the studied structure, of the choice of substructures and the zones of the structure with interesting dynamic behaviour. It is not possible to define infallible global modes indicators and only observing the effective mass can lead to eliminate modes which are not local. This tool must be used with precaution. It seems that the coupling of a substructure with the remainder of the system can take place by transmission of inertia but also by a high elastic strain within the junctions. It would be judicious to study a complementary tool taking into account the stiffness of the system.

3.2. Localizing reduced modelling errors

We now propose an error indicator for localizing the reduced substructures whose modal slave basis has been truncated too severely. The method of error localization was initially developed as a decision-making tool in model updating [6–8]. The localization method consists in detecting the modelling errors in numerical simulations which are responsible for differences between analysis and experimental measurements. In the present case, we will generalise the notion of error localization to the comparison of two analytical results, one exact and the other approximate. The localization methodology should then indicate which reduced substructures are most responsible for the global modelling errors due to their truncated slave bases.

The error localization method applied here is based on the minimum dynamic residual expansion (MDRE) [9]. In the original method, the experimental eigenvectors, measured on subset dofs, are expanded to the complete set of physical dofs. The mass and stiffness matrices of each substructure are then used to evaluate the corresponding residual energies and to localize modelling errors. Details of this procedure can be found in the cited article.

In the present work, the experimental eigenvectors are replaced by the eigenvectors evaluated on the reduced structure and the expansion is replaced by the transformation used to recover the global physical displacements.

Let \mathbf{M} and \mathbf{K} be the mass and stiffness matrices of the global physical structure including the ensemble of substructures; $\mathbf{M}^{(ss)}$ and $\mathbf{K}^{(ss)}$ are the physical mass and stiffness matrices of the substructure ss ; $\tilde{\mathbf{M}}^r$ and $\tilde{\mathbf{K}}^r$ are the mass and stiffness matrices of the assembled reduced system and $(\tilde{\lambda}_\nu, \tilde{\mathbf{y}}_\nu)$ the eigensolutions of the mode ν after transformation to recover the complete physical displacements. $\tilde{\lambda}_\nu$, $\tilde{\mathbf{y}}_\nu$ and the mass and stiffness matrices previously cited are assumed to be known.

The aim is to localize the reduced substructures of the model $(\tilde{\mathbf{M}}^r, \tilde{\mathbf{K}}^r)$ which are insufficiently accurate.

We define the residual force for the ν^{th} eigenmode by:

$$\Delta \mathbf{f}_\nu = \left(\mathbf{K} - \tilde{\lambda}_\nu \mathbf{M} \right) \tilde{\mathbf{y}}_\nu \quad (12)$$

and the corresponding residual displacements as:

$$\mathbf{r}_\nu = \mathbf{K}^{-1} \Delta \mathbf{f}_\nu \quad (13)$$

The residual energies of the mode ν for the substructure ss are given by:

$$e_\nu^{(ss)} = \left(\mathbf{r}_\nu^{(ss)} \right)^T \mathbf{K}^{(ss)} \mathbf{r}_\nu^{(ss)} \quad (14)$$

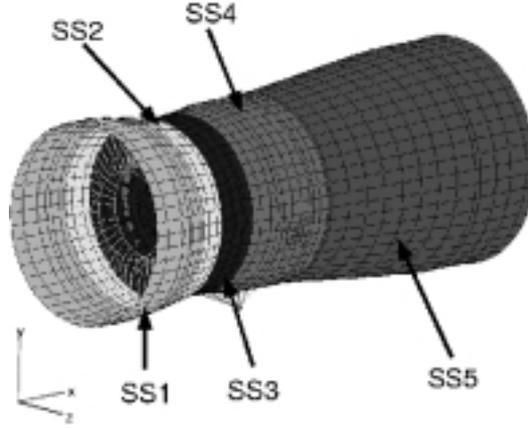


Fig. 1. Aeroengine casing and substructuring zones.

where $\mathbf{r}_\nu^{(ss)}$ is the vector of residual displacements on the dofs of the substructure ss for the mode ν .

We can now define an error localization indicator I_ν^{err} obtained by dividing the residual energy of the substructure ss by the total residual energy:

$$I_\nu^{\text{err}} = \frac{e_\nu^{(ss)}}{(\tilde{\mathbf{y}}_\nu)^T \mathbf{K} \tilde{\mathbf{y}}_\nu} \quad (15)$$

Remarks

- Large values of I_ν^{err} indicate that the corresponding reduced model bears significant reduction errors while small values indicate relatively small errors.
- Although this localization indicator allows the erroneous reduced models to be identified, it does not provide any direct indication concerning what information is missing.
- We will see in the next section that the residual displacement vectors do contain information concerning the missing slave eigenmodes and that they can be effectively used to improve the corresponding model reduction.

3.3. Superelement improvement

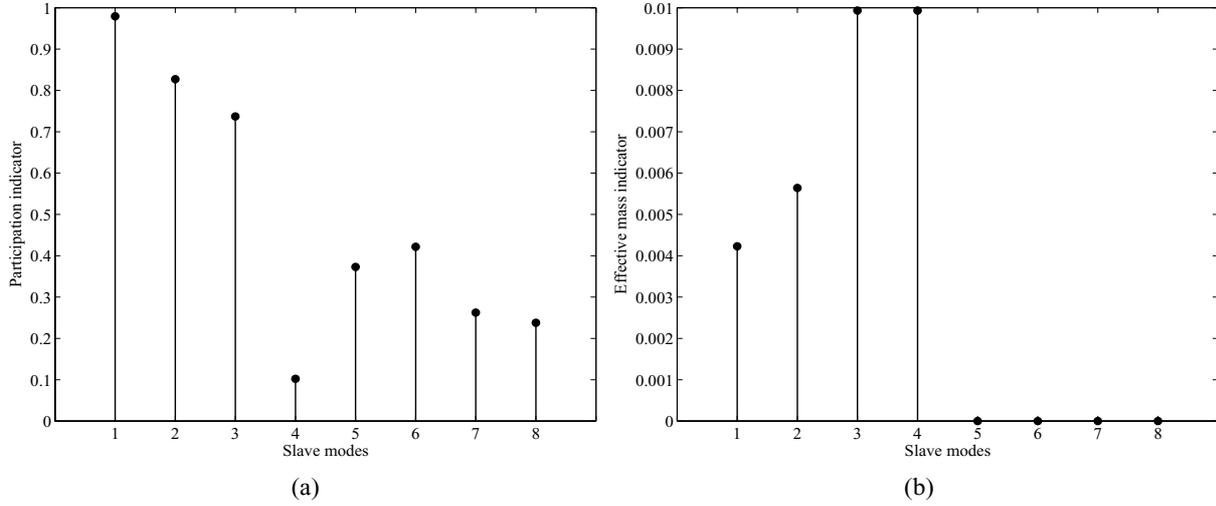
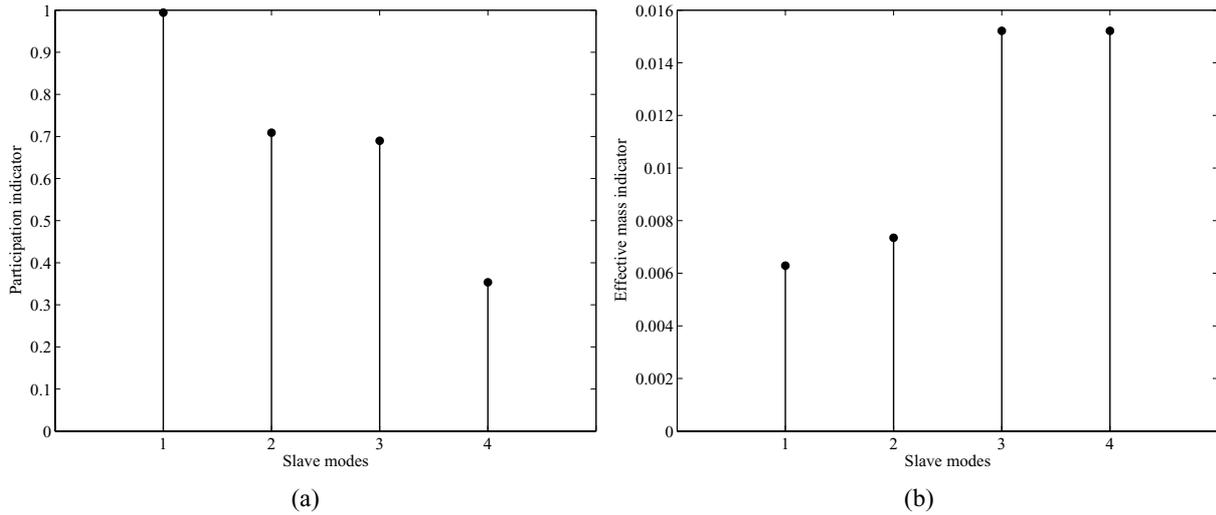
The most natural method to improve an erroneous superelement is simply to include more constrained slave eigenmodes in the Craig-Bampton transformation. Without an effective error localization method, the analyst is obliged to more or less blindly improve all of the superelements in this way. Moreover, even with a localization indicator, the erroneous superelement will traditionally still be improved in this way. Experience shows that this approach is often delicate due to the presence of both local modes and the need to limit the slave basis to only the essential eigenmodes.

The proposed localization procedure yields residual displacement vectors as intermediate results. These vectors can be used to enrich the Craig-Bampton transformation for an erroneous superelement. The first step is to decompose each residual vector into two parts, one part which is affine to the space of static interface modes and one part which is affine to the space of truncated slave eigenmodes. We then remove the contribution of static displacements at the interfaces which we assumed to be already correctly represented:

$$\tilde{\mathbf{R}} = \mathbf{R} - \begin{bmatrix} \mathbf{I}_{jj} \\ \Psi_{ij} \end{bmatrix} \mathbf{R}_j \quad (16)$$

where \mathbf{R} is the basis of the residual vectors, \mathbf{R}_j is the basis of the residual vectors restricted to the interface dofs.

Note that the resulting residual basis is not necessarily orthonormal. Indeed, the truncated slave eigenmodes of the reduced substructure can contribute to different eigenmodes of the global structure. Consequently, they can appear in several residuals. To this purpose, a singular value decomposition of the matrix of residuals $\tilde{\mathbf{R}}$ is performed [11]:

Fig. 2. SS1 filtering results: (a) I_{ν}^{part} , (b) I_{ν}^{eff} .Fig. 3. SS2 filtering results: (a) I_{ν}^{part} , (b) I_{ν}^{eff} .

$$\tilde{\mathbf{R}} = \mathbf{U} \Sigma \mathbf{V}^T \quad (17)$$

where \mathbf{U} and \mathbf{V} are two orthonormal real matrices and Σ is a diagonal matrix of the singular values of $\tilde{\mathbf{R}}$. We extract the vectors columns from \mathbf{U} and preserve the first m vectors corresponding to the m largest singular values. An enriched Craig-Bampton transformation is then defined by: $\mathbf{T}_{\text{CBE}} = [\mathbf{T}_{\text{CB}}, \mathbf{U}_1, \dots, \mathbf{U}_m]$ which replaces the initial transformation basis \mathbf{T}_{CB} .

4. Numerical application

The proposed methodology is now applied to the aeroengine casing model shown in Fig. 1. The physical finite element model is mainly discretized with shell and beam elements. The model contains 28,590 dofs and is under free-free boundary conditions. The global physical model is divided into five substructures indicated Fig. 1.

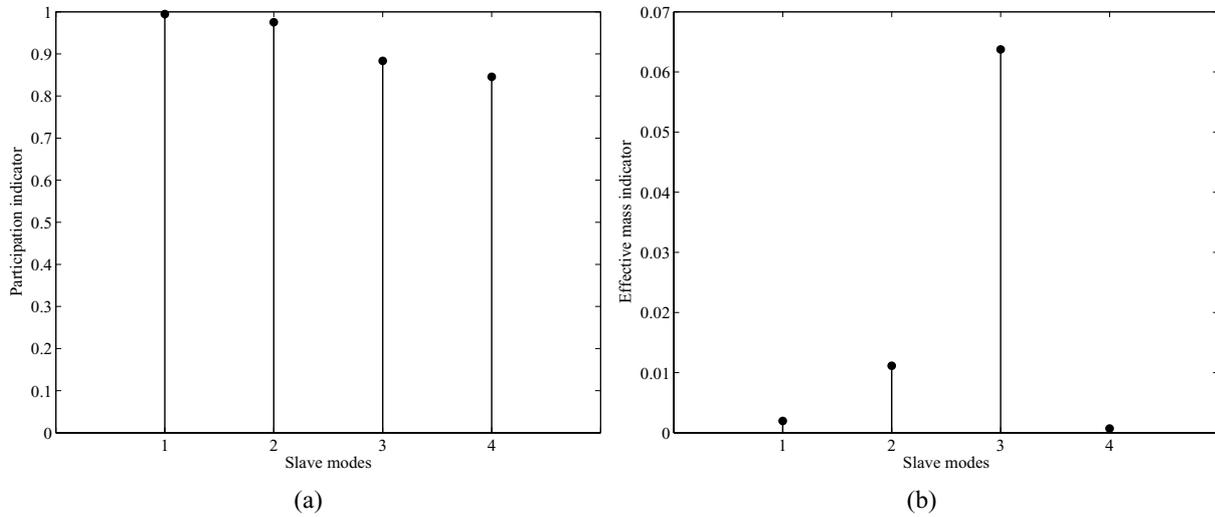


Fig. 4. SS3 filtering results: (a) I_{ν}^{part} , (b) I_{ν}^{eff} .

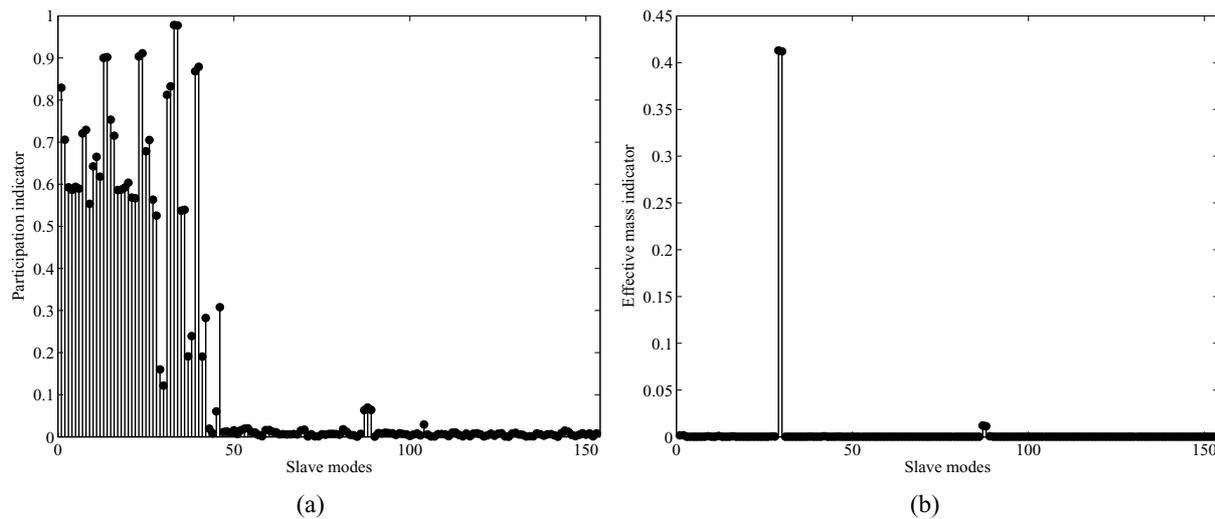


Fig. 5. SS5 filtering results: (a) I_{ν}^{part} , (b) I_{ν}^{eff} .

The substructures SS1, SS2, SS3 and SS5 will be integrated as Craig-Bampton superelements while the residual substructure SS4 will remain a physical substructure.

The frequency range of analysis of the whole model is $[0, 300 \text{ Hz}]$. To determine the number of superelement slave modes to be retained for each substructure, we use the rule which consists in including the modes contained in a band two times larger than the highest frequency of interest of the complete physical model. After the analysis of each substructure on $[0, 600 \text{ Hz}]$, 8 slave modes are retained for SS1, 4 modes for SS2, 4 modes for SS3 and 153 modes for SS5.

4.1. Slave mode filtering results

Using the substructures previously described, we carried out a modal analysis of the assembled reduced model. We then apply the filtering strategy described in Section 3.1 to each substructures in order to remove the local modes which do not take part in the modal behaviour of the assembled structure. Obtained results are presented Figs 2,

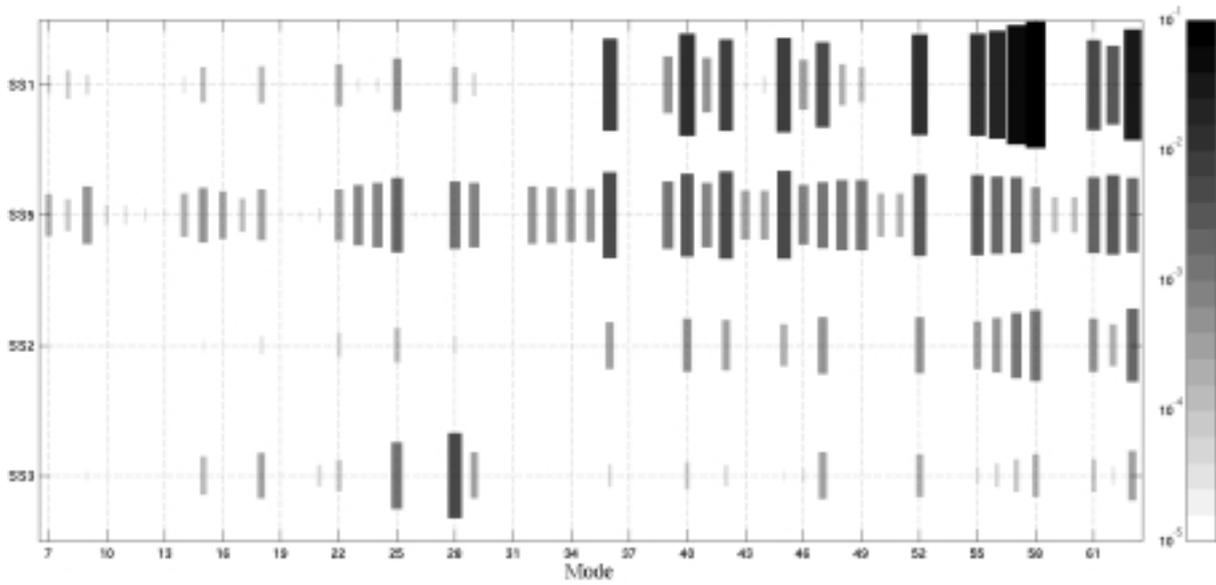


Fig. 6. Nominal superelement error localization.

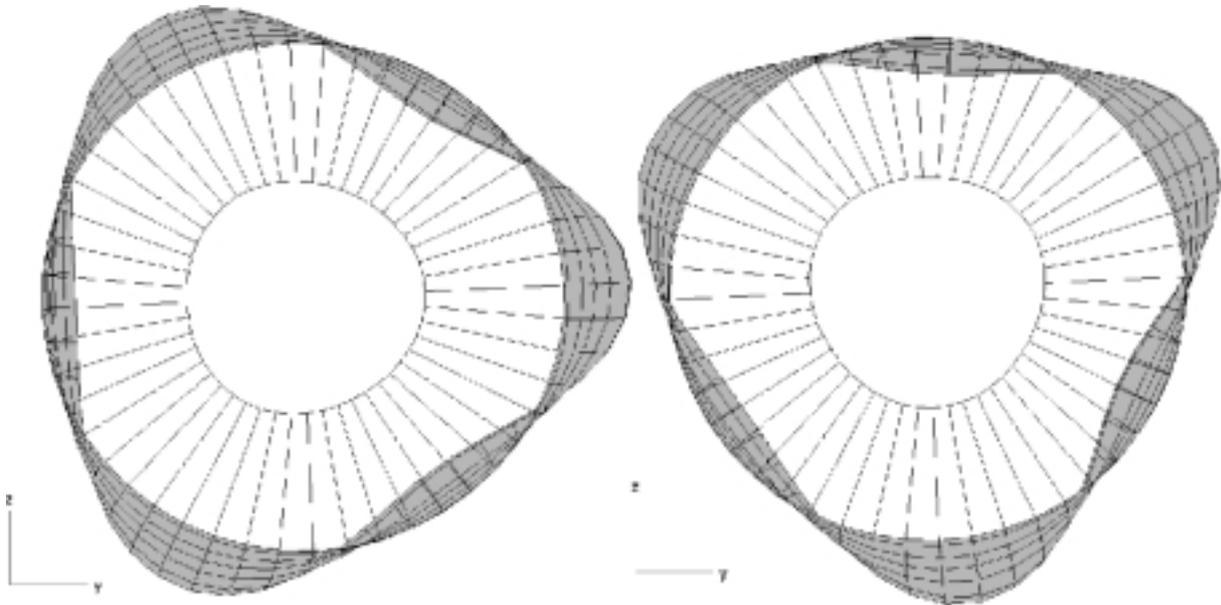


Fig. 7. Deflections of the two first residual vectors.

3, 4 and 5. It is clear that the two indicators are not in total agreement, a compromise is thus made to select the slave modes. Based on these criteria, we decided to remove 4 modes for SS1 (modes 4 to 8), 1 mode for SS2 (mode 4), 0 modes for SS3 and 107 modes for SS5 (modes 43 to 45, 47 to 86 and 90 to 153). Even if useful modes are unfortunately deleted, the missing information will be restituted thanks to the correction step described in Section 3.3.

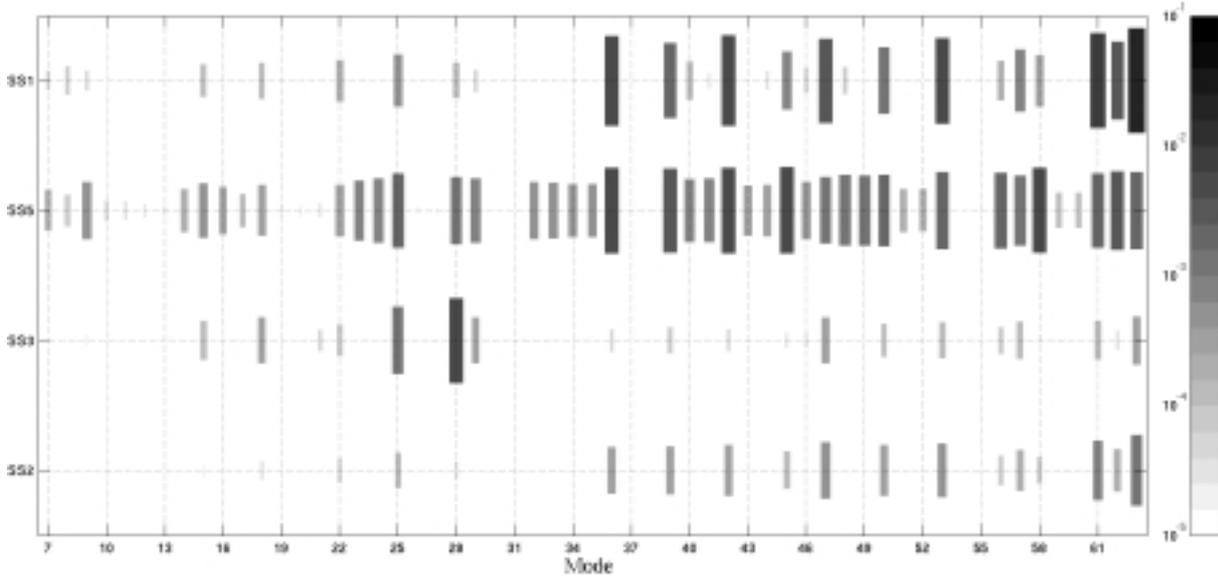


Fig. 8. Improved superelement error localization.

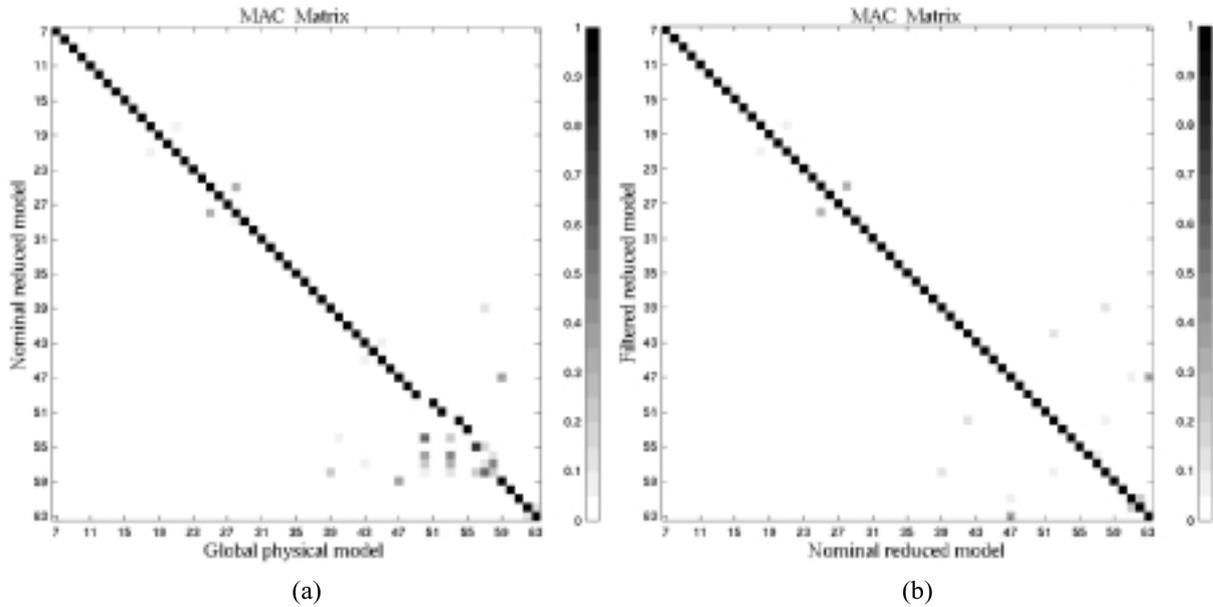


Fig. 9. MAC matrices matching before (a) and after filtering (b).

4.2. Superelement localization results

The proposed error localization indicator allows us to identify the substructures which contribute most to the global model prediction errors as seen Fig. 6 with the patch matrix. The more the ratio given by Eq. (15) is important the more the colour of the patch goes black and the size of the patch increases. In this case, we identify SS1 as being the origin of the principal errors of the global model. I_{ν}^{eff} is about 10^{-2} for several modes of this substructure and the maximum value is 0.086. This result is significant since it corresponds to an error about 8.56%. The errors in the other structures are lower than 0.5%.

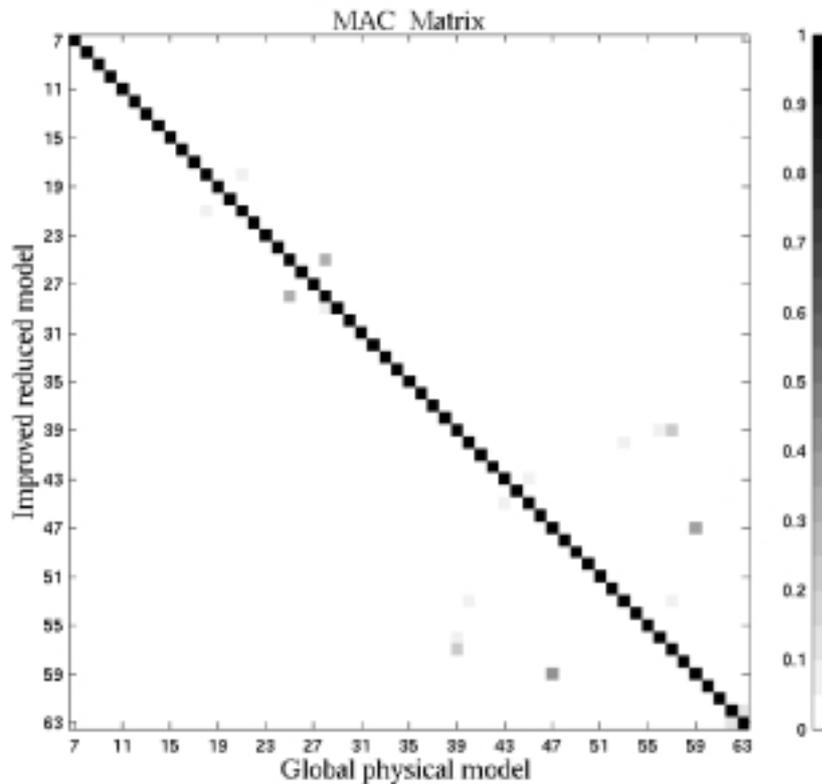


Fig. 10. MAC matrix after improvement.

4.3. *Superelement correction results*

We decide to improve the superelement SS1 by enriching its transformation as depicted in Section 3.3. The singular value decomposition allows us to choose the number of vectors to be reinjected and we decide to retain the first two vectors. Figure 7 shows the deflection shapes of the retained residual vectors. In the present case, these vectors correspond directly to a set of truncated slave modes (modes 11 and 12). The associated eigenfrequencies of these modes are respectively 664.153 Hz and 664.157 Hz. As the cutoff frequency was 600 Hz they were not initially taken into account.

4.4. *Final comparison results*

After the correction of the substructure SS1, the modal analysis of the improved reduced-order model is carried out over the frequency range $[0, 300 \text{ Hz}]$. We obtain 63 modes. The error localization method is then used to requalify this model as seen in Fig. 8. The error indicators are significantly smaller than those obtained in the first localization. The most important is 0.0173 that is to say 1.73%.

In order to evaluate the improvement made to the initially reduced model and the quality of the final model, both are compared with the physical finite element model using the MAC (Modal Assurance Criteria) (Figs 9 and 10). Here, a mode of a reference model is considered to be not paired with another mode of the compared model if no MAC values are above 70%. One can observe on the MAC matrix Fig. 9(a) that the initial reduced model had significant errors for some eigenmodes. In particular, four modes are not paired. Concerning the frequencies of the paired modes, the maximum absolute relative error was 0.97% and the mean absolute relative error was

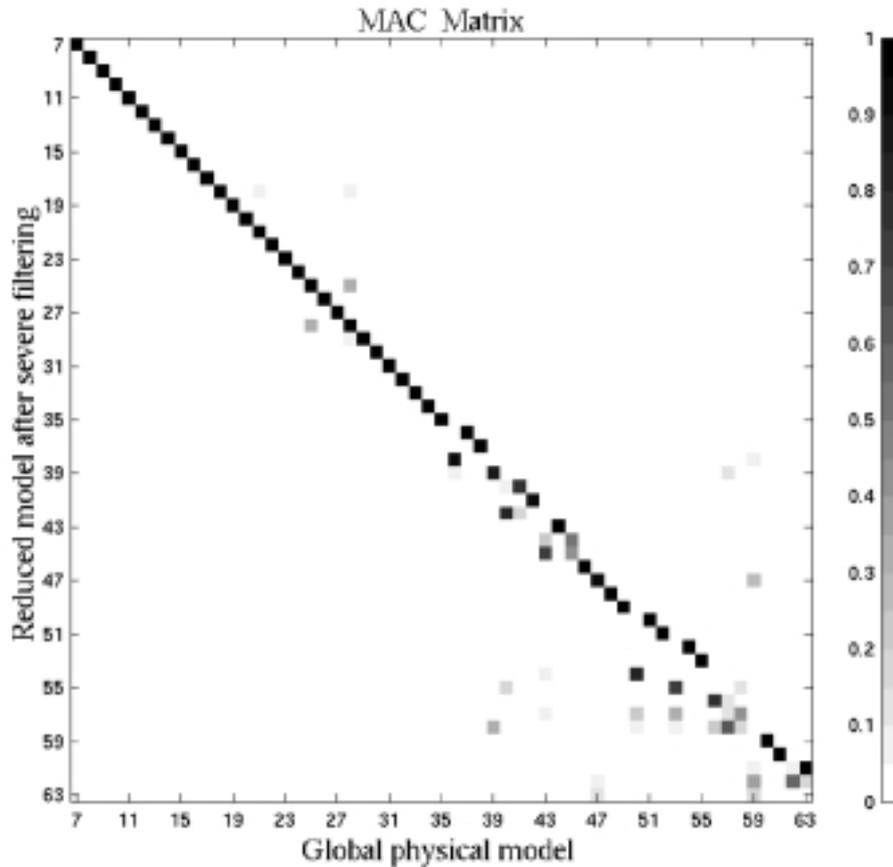


Fig. 11. MAC matrix after sever filtering.

0.13%. Figure 9(b) shows the comparison between the initial reduced model and the one obtained after filtering. No significant deterioration's can be noticed. The MAC matrix 10 indicates that the improved model correlates well with the reference model with all modes paired. The maximum absolute relative error in frequency is 0.90% and the mean value is 0.14%.

4.5. Second analysis

We test here the ability of the method to compensate for the deletion of important slave modes. During the filtering stage, the same slave modes are removed for SS1, SS2 and SS3 but for SS5 modes 43 to 45 and modes 47 to 153 are deleted. Thus, three modes (87 to 89) which seems global are missing. One can notice on the MAC matrix given in Fig. 11 that the reduced model after this filtering has deteriorated. Figure 12 shows the superelement localization errors. As expected SS1 and SS5 are identified to be erroneous. Then, we improve the SS1 and SS5 slave bases with residual vectors. After an observation of the singular values, two vectors are retained for SS1 and two for SS5. For SS1, the deflection shapes of the retained residual vectors are the same than before. A comparison of the two SS5 residues with the modes 87, 88 and 89 is showed Fig. 13. One can see that the second residue correlates perfectly with mode 89 and the first residue belongs to the subset formed by the eigenvectors of modes 87 and 88 which are double modes. Thus, the useful information is well recovered thanks to the correction stage. Modal analysis of the new improved model gives the same results that those showed Fig. 10.

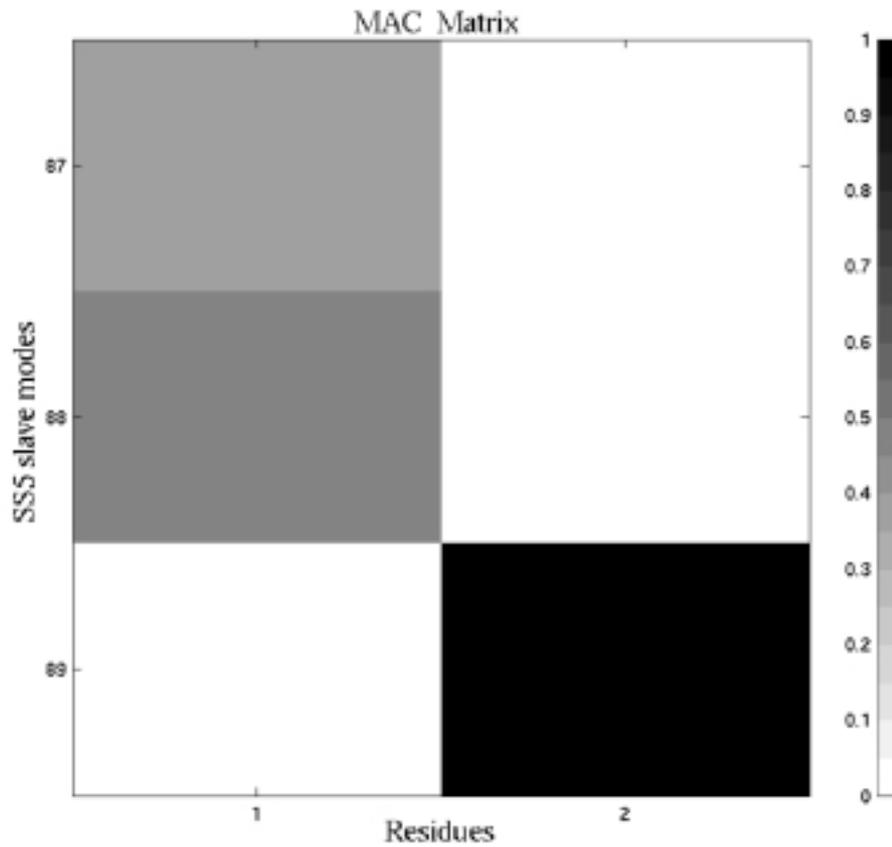


Fig. 12. Comparison between SS5 slave modes and residues.

5. Conclusion

A set of decision-making indicators has been proposed to localize and improve erroneous Craig-Bampton superelement models in a complex structure. Usually, the superelements are improved more or less blindly by including more slave modes. Firstly, two indicators allow local slave eigenvectors to be eliminated. The first one uses the values of the generalized coordinates to compare the relative participation of slave modes to the global modes. The second one uses the effective mass matrices to identify modes which favour coupling of a substructure with the others by transmission of inertia. However, the effect of a strong elastic strain at the interface is not examined. A complementary tool taking into account the stiffness of the structure would be useful. After this filtering, it is shown that an error localization using the MDRE can detect the most erroneous reduced models. A major drawback is the computing time since the method needs the inversion of the stiffness matrix of the original system. Finally, we propose an effective method to enrich the Craig-Bampton transformation using residual vectors which result from the localization strategy. The different indicators have been effectively applied to a technologically interesting structure to illustrate the proposed methodology. In this paper, only the Craig-Bampton method is studied but some of the tools, particularly the localization and correction of errors, can be applied successfully to other substructuring techniques.

Acknowledgement

The authors wish to thank the French Ministry for Research for his support in the framework of the supersonic project.

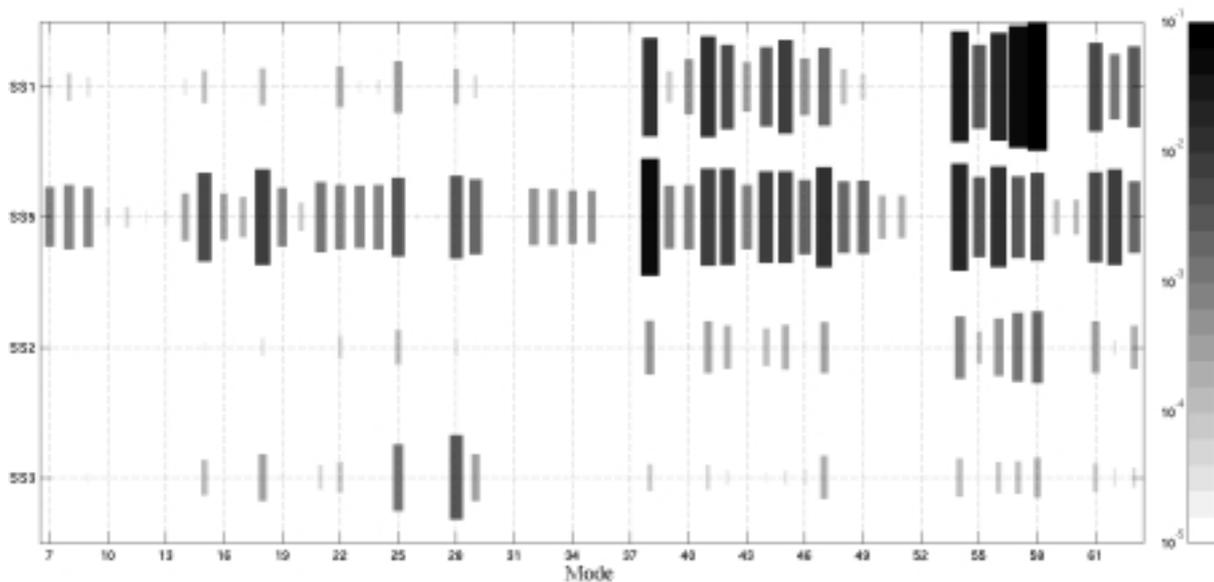


Fig. 13. Superelement error localization after severe filtering.

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