Shock and Vibration 16 (2009) 61–73
DOI 10.3233/SAV-2009-0454
IOS Press

Optimal occupant kinematics and crash pulse for automobile frontal impact[1]

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Received 21 September 2007
Revised 28 December 2007

Abstract. Based on a lumped-parameter model of the occupant-vehicle system, optimal kinematics of the occupant in frontal impact are investigated. It is found that for the minimization of the peak occupant deceleration, the optimal kinematics move the occupant at a constant deceleration. Based on this the optimal vehicle crash pulse is investigated. The optimal crash pulse for passive restraint systems is found to be: a positive impulse at the onset, an immediate plunge followed by a gradual rebound, and finally a positive level period. The relation of the peak occupant deceleration to the impact speed, crash deformation, and vehicle interior rattlespace is established. The optimal crash pulse for active and pre-acting restraint systems is discussed.

Keywords: Automobile front impact, optimal occupant kinematics, optimal crash pulse, restraint systems

1. Introduction

Prevention and reduction of occupant injuries in automobile crashes is an essential goal in vehicle design. The crashworthiness of automobile structures is an important issue in the design process. Engineering design of automobile structures for crashworthiness is generally executed through a combination of tests and analytical modeling and analysis. Optimal design or optimization, as an effective tool, is usually used in the vehicle design. The optimization problem of structural crashworthiness can be dealt with analytically using direct or indirect approaches [1]. The direct approach uses physical parameters of a vehicle structure as the design variables in the optimization process. As the objective function in optimization is to minimize certain injury criteria that in turn involve occupant responses, it is necessary to establish a direct relationship between these physical parameters and the occupant responses. This requires an efficient and accurate modeling and simulation of the problem, which, given the complexity of automobile structures and the interaction between the vehicle and the occupant, represents a great challenge. The indirect approach handles the problem using a two-step strategy: the optimal response for a vehicle structure is found first; then, an inverse design problem is solved for the vehicle structure according to its optimal response, or this optimal response is used as a guideline for the vehicle structure design.

The study in this paper adopts the indirect approach and deals only with the problem in the first step for frontal impact, i.e., the optimal vehicle response. The question to be addressed is: in frontal impact, for a prescribed impact speed and crash deformation of the vehicle and for given restraint characteristics, what is the optimal vehicle crash pulse that minimizes the occupant injuries?

[1]This paper is dedicated to Dr. Walter D. Pilkey, a teacher, a mentor, and a role model of us.
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ISSN 1070-9622/09/$17.00

A number of studies have been performed on the issue of the optimal vehicle crash pulse. Based on a one-mass model and a rigid multi-body model, Takahashi et al. [2] divided the vehicle crash deceleration curve into finite segments and determined the optimal deceleration curve for minimizing the occupant's injury through iterative calculations. Using a two-mass, one-dimensional model, Motozawa and Kamei [3] assumed a pure rectilinear form as the ideal pulse for the occupant deceleration. Three functions were formed to approximate the rising edge of the rectilinear form. This resulted in the optimal crash pulses having high deceleration initially followed by low or negative deceleration and constant deceleration thereafter. Based on a single-degree-of-freedom (SDOF) spring-mass model, Shi et al. [1] considered the problem of the optimal crash pulse in the occupant-vehicle relative motion domain and applied an energy relationship between the vehicle crash pulse and the occupant deceleration. It was shown that for a given crash deformation and impact speed, the optimal crash pulse is one that consists of an impulse, a subsequent zero-acceleration period, and finally a constant level period. Using a SDOF model to describe the problem, Wu et al. [4] discretized the crash pulse with respect to the crash deformation and then implemented numerical optimization to find the optimal crash pulse.

In this paper a lumped-parameter model is used to describe the occupant-vehicle system in automobile frontal impact. The optimal kinematics of the occupant in frontal impact are studied first. The optimal vehicle crash pulse is then investigated based on the optimal occupant kinematics.

2. System modeling and problem statement

Optimal design or optimization can now be performed in a virtual environment based on computational modeling, simulation, and analysis of a system. The models used for computational studies can be classified as lumped-parameter models, rigid multi-body (RMB) models, finite element (FE) models, and integrated models [5]. These models provide different levels of abstraction of a problem and thus have different applications.

Whereas automobile frontal impact can be described with various degrees of detail and accuracy by a RMB model, an FE model, or an integrated model of RMB and FE, the description of the problem with a simple, lumped-parameter model is still advantageous and desirable, as it could provide an analytical solution and lend insight into the problem. In frontal impact the occupant can be subject to injuries to various regions of the body such as the head, thorax, upper extremities, and lower extremities. While these injuries are the results of excessive stresses in respective regions induced by impact, the injury criteria used to measure these injuries are usually expressed by impact responses (such as accelerations) in respective regions. What criteria or metrics most comprehensively and accurately reflect the level of occupant protection is in itself a topic that still requires considerable investigation [1]. Nevertheless, the peak occupant thorax acceleration has been used in the automobile industry and also been one of the injury assessment values in the FMVSS 208 [6] for frontal crash occupant protection. When the prevention and reduction of injuries to the entire body are considered, the occupant can reasonably be treated as a point mass, and the peak acceleration (deceleration) of this point mass can be used as the injury criterion and serves as an indicator of the force acting on the occupant. For the investigations of crash pulse and restraint characteristics in this paper, the vehicle is treated as a single body, with only the gross motion considered.

Therefore, automobile frontal impact will be described by a lumped-parameter model shown in Fig. 1, where a point mass m represents the occupant, $x(t)$ describes the gross motion of the vehicle, and $y(t)$ is the motion of the occupant relative to the vehicle. The interaction between the occupant and restrained systems, such as seat belts and airbags, is simply represented by a spring and a damper in the figure, but in general, it can be described by a control force $u(y, \dot{y}, t)$. The time history of the vehicle deceleration is usually referred to as the vehicle crash pulse $a(t)$, that is,

$$a(t) = -\ddot{x}(t). \tag{1}$$

The maximum vehicle displacement is defined as the vehicle crash deformation D_v, which is given by

$$D_v = \max_t \{x(t)\}. \tag{2}$$

The free space between the occupant and a vehicle interior surface is called the rattlespace S_0, which is the maximum allowable forward excursion for the occupant inside the vehicle, i.e.,

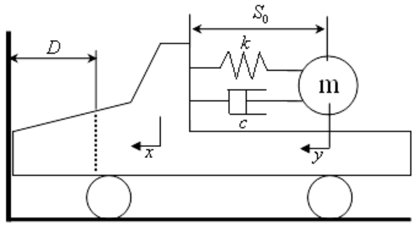

Fig. 1. A lumped-parameter model of the system.

$$\max_t\{y(t)\} \leqslant S_0. \quad (3)$$

Since this space is a major factor in the restraint system design, it is essential that S_0 be considered in addition to the vehicle crash deformation when the occupant motion during crash is investigated.

The equation of motion of the system is

$$m(\ddot{x}+\ddot{y})+u(y,\dot{y},t)=0, \quad (4)$$

with the initial conditions

$$x(0)=0,\ \dot{x}(0)=v_0, \quad (5)$$

and

$$y(0)=0,\ \dot{y}(0)=0, \quad (6)$$

where v_0 is the impact speed of the vehicle.

The question can be stated as the following problem:

Problem-1: For given restraint characteristics and prescribed impact speed and crash deformation, find the optimal vehicle crash pulse such that the peak occupant deceleration is minimized. The problem can be formulated as: find the optimal crash pulse $a_0(t)$, such that

$$J_1(a_0)=\min_a\{J_1(a)|J_2(a)\leqslant D_v,\ J_3(a)\leqslant S_0\}, \quad (7)$$

where

$$J_1=\max_t\{-[\ddot{x}(t)+\ddot{y}(t)]\} \quad (8)$$

is the peak occupant deceleration,

$$J_2=\max_t\{x(t)\} \quad (9)$$

is the maximum vehicle crash displacement, and

$$J_3=\max_t\{y(t)\} \quad (10)$$

is the maximum forward excursion of the occupant in the vehicle.

In view of the kinematics of the occupant, which is denoted as $w(t)$, *Problem-1* can be reduced to a more *generic* problem:

Problem-2: For prescribed impact speed, crash deformation, and rattlespace, find the optimal kinematics of the occupant such that the peak occupant deceleration is minimized while the maximum occupant forward displacement is bounded. The problem can be formulated as: Find the optimal kinematics of the occupant $w_0(t)$, such that

$$J_1(w_0)=\min_w\{J_1(w)|J_4(w)\leqslant D_o\}, \quad (11)$$

where

$$J_4=\max_t\{x(t)+y(t)\} \quad (12)$$

is the maximum forward displacement of the occupant, and

$$D_o = D_v + S_0 \tag{13}$$

is the allowable maximum forward excursion for the occupant with respect to the ground.

Note that unlike $a(t)$ in *Problem-1*, $w(t)$ is not a control function of the system. Instead, it is controlled or produced by $a(t)$ or $u(y, \dot{y}, t)$.

3. Optimal occupant kinematics

To find the optimal kinematics of the occupant for *Problem-2*, according to the duality or reciprocity theorem of optimization [7], the dual or reciprocal problem of *Problem-2* can be formulated as follows.

Problem-3: For prescribed impact speed, crash deformation, and rattlespace, find the optimal kinematics of the occupant such that the maximum occupant displacement is minimized while the peak occupant deceleration is bounded. The problem can be formulated as: Find the optimal kinematics of the occupant $w_0(t)$, such that

$$J_4(w_0) = \min_w \{J_4(w) | J_1(w) \leqslant A_m\}, \tag{14}$$

where A_m is the upper bound for the peak occupant deceleration.

To find the theoretical solution of *Problem-3,* denote

$$z(t) = x(t) + y(t), \tag{15}$$

which is the absolute motion of the occupant with respect to an inertial frame (ground) during impact. Consider the motion of the system from the initial time $t = 0$ to the instant $t = T_0$ when the occupant (m) comes to the first rest. That is,

$$v_0 + \int_0^{T_0} \ddot{z}(t)dt = 0. \tag{16}$$

The velocity of the occupant $\dot{z}(t)$ starts with v_0 at time $t = 0$ and decreases to 0 at time $t = T_0$, i.e.,

$$\dot{z}(t) \geqslant 0, \quad 0 \leqslant t \leqslant T_0. \tag{17}$$

Therefore, for $0 \leqslant t \leqslant T_0$, the displacement of the occupant $z(t)$, which is given by

$$z(t) = \int_0^t \dot{z}(\tau)d\tau, \tag{18}$$

increases monotonically with respect to time and reaches its maximum value at $t = T_0$, that is,

$$\max_{t\in[0,\,T_0]} \{z(t)\} = z(T_0). \tag{19}$$

To minimize the maximum occupant forward displacement J_4

$$J_4 = \max_{t\in[0,\,T_0]} \{x(t) + y(t)\} = \max_{t\in[0,\,T_0]} \{z(t)\} = z(T_0), \tag{20}$$

T_0 should be minimized. According to the definition of T_0 (Eq. (16)), if the deceleration of the occupant $-\ddot{z}(t)$ takes the maximum allowable value A_m, that is,

$$\ddot{z}(t) = -A_m, \tag{21}$$

T_0 is minimized, and

$$T_0 = \frac{v_0}{A_m}. \tag{22}$$

This means that in order to minimize the peak occupant displacement, the occupant deceleration should remain constant at the value of A_m. This is the optimal kinematics of the occupant for *Problem-3*.

Based on the duality or reciprocity between the *Problem-2* and *Problem-3*, the optimal kinematics of the occupant for *Problem-2* can be stated as: In order to minimize the peak occupant deceleration, the occupant deceleration should remain constant at a value of A_m that is given by

$$A_m = \frac{v_0^2}{2D_o} = \frac{v_0^2}{2(D_v + S_0)}. \tag{23}$$

This is also the optimal kinematics of the occupant for *Problem-1*. Note that the optimal kinematics of the occupant is irrespective of the shape of crash pulse.

4. Optimal crash pulse

The optimal crash pulse depends upon restraint systems that isolate, attenuate, or control the vehicle impact to the occupant. In terms of the manner in which a restraint system exerts its action on the occupant, restraint systems can be categorized into three types: passive, active, and pre-acting [5]. In this paper a safety device is considered to be passive if it generates a force only when it is being externally excited. It is active if it can generate the force not only from the external action exerted on it, but also from the internal power source it has.

An example of passive systems is an ordinary knee bolster that provides a cushion between the knee and the front interior of the vehicle. The cushion generates a dynamic reaction force on the occupant's knee only when it is compressed. If the knee is not in contact with the cushion, the cushion will apply no action on it. The reaction force of a cushion depends on its deformation and deformation rate. The action of a passive restraint system on the occupant can be expressed as

$$u = u(y, \dot{y}). \tag{24}$$

An airbag is an active restraint system. The deployment of an airbag is controlled by an inflator that is an internal power source. Therefore, the force of a deploying airbag on the occupant is time-dependent, although it also depends on the interaction with the occupant. The action of an active restraint system on the occupant can be expressed as

$$u = u(y, \dot{y}, t). \tag{25}$$

A safety device, such as a seatbelt pre-tensioner, can start to act before the onset of impact and will be referred to as a pre-acting system. A seatbelt pre-tensioner removes the slack in the belt and applies an initial load to the occupant before the occupant moves forward and stretches the belt. A pre-acting mechanism can be incorporated into a passive or an active restraint system. Its action on the occupant can be described as

$$u = u(y, \dot{y}, t, t_0), \tag{26}$$

where $t_0 < 0$ denotes the time instant at which the device starts to exert action.

4.1. Linear passive restraint characteristics

4.1.1. Optimal crash pulse

Suppose the onset of the vehicle impact is at $t = 0$. Consider the motion of the occupant right after the onset of impact ($t = 0^+$) until it stops for the first time ($t = T_0$). With the initial conditions:

$$z(0^+) = 0,\ \dot{z}(0^+) = v_0, \tag{27}$$

the optimal kinematics of the occupant according to Eq. (21) is

$$\begin{array}{l} \ddot{z}(t) = -A_m \\ \dot{z}(t) = v_0 - A_m t \\ z(t) = v_0 t - \frac{1}{2} A_m t^2 \end{array}. \tag{28}$$

From Eq. (4)

$$u(y, \dot{y}) = m A_m. \tag{29}$$

Suppose the characteristics of the restraint system can be expressed by a linear spring and a linear damper. That is,

$$u(y, \dot{y}) = ky + c\dot{y}. \tag{30}$$

Then,

$$ky + c\dot{y} = mA_m, \tag{31}$$

with the initial conditions

$$y(0^+) = 0. \tag{32}$$

The solution of Eq. (31) is

$$\begin{aligned} y(t) &= \frac{mA_m}{k}(1 - e^{-\frac{k}{c}t}) \\ \dot{y}(t) &= \frac{mA_m}{c}e^{-\frac{k}{c}t}. \\ \ddot{y}(t) &= -\frac{kmA_m}{c^2}e^{-\frac{k}{c}t} \end{aligned} \tag{33}$$

From Eqs (15), (28) and (33), for $0^+ \leqslant t \leqslant T_0$, the motion of the vehicle can be expressed as

$$\ddot{x}(t) = -A_m\left(1 - \frac{km}{c^2}e^{-\frac{k}{c}t}\right), \tag{34a}$$

$$\dot{x}(t) = v_0 - A_m\left(t + \frac{m}{c}e^{-\frac{k}{c}t}\right), \tag{34b}$$

$$x(t) = v_0 t - \tfrac{1}{2}A_m t^2 - \tfrac{mA_m}{k}(1 - e^{-\frac{k}{c}t}). \tag{34c}$$

Note that

$$x(0^+) = 0,\ \dot{x}(0^+) = v_0 - A_m m/c. \tag{35}$$

Compare to Eq. (5), from $t = 0$ to $t = 0^+$, the velocity change of the vehicle is

$$\Delta v = x(0) - x(0^+) = A_m m/c. \tag{36}$$

To produce this velocity change, the acceleration of the vehicle needs to have an impulse at the initial time ($t = 0$). Theoretically, this impulse can be represented by a Dirac delta function $\delta(t)$; that is,

$$I_x(t) = \frac{A_m m}{c}\delta(t), \tag{37}$$

where $\delta(t)$ is given by

$$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases}, \tag{38}$$

and

$$\int_{-\infty}^{\infty} \delta(t)dt = 1. \tag{39}$$

As such, from the onset of impact ($t = 0$) to the first stop ($t = T_0$), the optimal vehicle acceleration pulse is

$$\ddot{x}(t) = -A_m\left[1 + \frac{m}{c}\delta(t) - \frac{km}{c^2}e^{-\frac{k}{c}t}\right],\ 0 \leqslant t \leqslant T_0. \tag{40}$$

Since the impulse $I_x(t)$ results in only a velocity change but no change in displacement of the vehicle from $t = 0$ to $t = 0^+$, Eq. (34c) still holds.

Note that the exponential terms in Eqs (33), (34) and (40) diminish with time. After a sufficiently long time they can be neglected.

4.1.2. Determination of A_m

If the restraint characteristics are such that the maximum forward excursion of the occupant relative to the vehicle reaches the prescribed rattlespace, according to Eq. (23),

$$A_m = \frac{v_0^2}{2(D_v + S_0)}. \tag{41}$$

It follows from Eq. (34c) that

$$\max_t\{x(t)\} = x(T_0) \approx D_v + S_0 - \frac{mA_m}{k} = D_v, \tag{42}$$

where the exponential term is neglected. Then,

$$S_0 = \frac{mA_m}{k} = \frac{A_m}{\omega^2}, \tag{43}$$

where

$$\omega^2 = \frac{k}{m}, \tag{44}$$

which can be considered as the natural frequency of the restraint-occupant system. From Eqs (41) and (43)

$$\omega^2 = \frac{v_0^2}{2S_0(D_v + S_0)}, \tag{45}$$

which presents a requirement for the restraint characteristics. This means that when the natural frequency of the restraint-occupant system satisfies Eq. (45), the rattlespace will be fully used for the occupant's excursion with respect to the vehicle.

In general, however, restraint systems are designed such that

$$\max_t\{y(t)\} < S_0, \tag{46}$$

i.e., the rattlespace is not fully used. In this case it follows from Eqs (23) and (34c) that

$$\max_t\{x(t)\} = x(T_0) \approx \frac{v_0^2}{2A_m} - \frac{mA_m}{k} = D_v, \tag{47}$$

where the exponential term in Eq. (34c) is neglected also. From Eq. (47),

$$A_m = \frac{kD_v}{2m}\left(\sqrt{1 + \frac{2mv_0^2}{kD_v^2}} - 1\right) = \frac{\omega^2 D_v}{2}\left(\sqrt{1 + \frac{2v_0^2}{\omega^2 D_v^2}} - 1\right), \tag{48}$$

which indicates that the peak occupant deceleration or the amplitude of constant deceleration depends upon the impact speed, vehicle crash deformation, and the natural frequency of the restraint-occupant system.

Note that expressions of A_m by Eq. (41) and by Eq. (48) are identical if Eq. (43) is substituted into Eq. (39). According to Eqs (33) and (48), Eq. (46) means that

$$\omega^2 > \frac{v_0^2}{2S_0(D_v + S_0)}, \tag{49}$$

which is a requirement for the restraint characteristics such that the rattlespace is not fully used.

It can be inferred that if

$$\omega^2 < \frac{v_0^2}{2S_0(D_v + S_0)}, \tag{50}$$

the excursion of the occupant will exceed the space limit, which violates the rattlespace constraint.

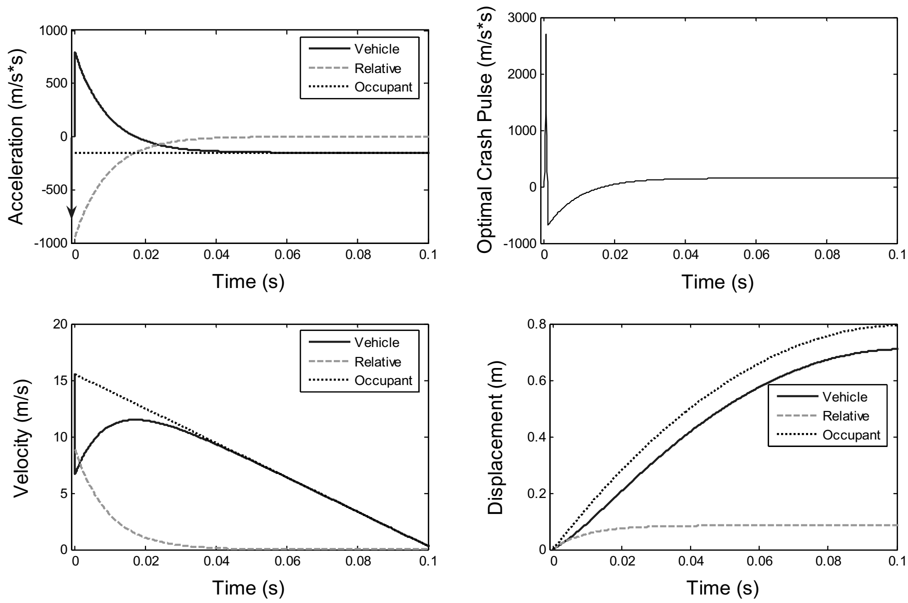

Fig. 2. Optimal crash pulse and the motions of the vehicle and the occupant ($\zeta = 0.2$).

4.1.3. Illustration of optimal crash pulse

From Eq. (40), the optimal crash pulse can be expressed as

$$a(t) = A_m \left[1 + \frac{m}{c}\delta(t) - \frac{km}{c^2}e^{-\frac{k}{c}t}\right] = A_m \left[1 + \frac{1}{2\zeta\omega}\delta(t) - \frac{1}{4\zeta^2}e^{-\frac{\omega}{2\zeta}t}\right],\ 0 \leqslant t \leqslant T_0, \tag{51}$$

where

$$\zeta = \frac{c}{2\omega m} = \frac{c}{2\sqrt{km}}, \tag{52}$$

which is referred to as the damping ratio of the restraint-occupant system. The introduction of ω and ζ results in the reduction of parameters in Eq. (51) and thus eliminates the need to provide a value for m, which varies from occupant to occupant. According to Eq. (51), $\zeta = 0.5$ is the critical damping that affects the initial shape of the optimal crash pulse.

For the illustration of the optimal crash pulse, choose the values for the parameters in Eq. (51) to be representative of automobile frontal impact [1]: $v_0 = 15.56$ m/s (35 mph), $D_v = 0.71$ m (28 in), and $\omega = 42.43$ rad/s. In order to exhibit the effect of the damping on the initial shape of the crash pulse, choose $\zeta = 0.20$, $\zeta = 0.50$, and $\zeta = 0.70$, respectively. Correspondingly, the time histories of the acceleration, velocity, and the displacement of the vehicle and the occupant and the relative motion between them are shown in Figs 2–4, respectively. Note that the impulse of delta function at $t = 0$ means that the acceleration of the vehicle at $t = 0$ is negatively infinite, but in a figure only a finite value can be displayed. Therefore, the impulse at the initial time is represented by an arrow in Figs 2–4 instead.

While the impulse at the onset is expressed by a Dirac delta function, the amplitude of the crash pulse or the amplitude of the deceleration that a vehicle can attain is limited in engineering reality. In order to provide a visible display of the optimal crash pulse, the impulse at the initial time can be approximated by a finite-amplitude pulse. As such, the optimal crash pulses for the three different damping ratios are shown in Figs 2–4 also. The common features of these optimal crash pulses can be abstracted as follows: a jump to a large positive value at the onset, an immediate plunge followed by a gradual rebound, and then a positive level period for the rest of time. The plunge part of the curve depends upon the system damping, however. As shown in Figs 2–4, when the damping ratio changes from 0.2 to 0.5 and to 0.7, the plunge part of the curve drops to a negative value, to zero, and to a positive value, respectively.

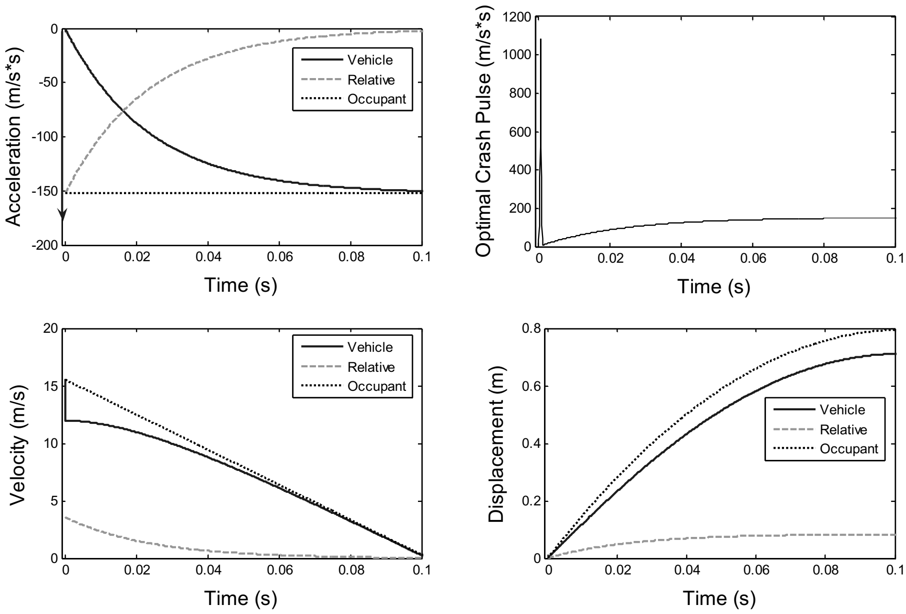

Fig. 3. Optimal crash pulse and the motions of the vehicle and the occupant ($\zeta = 0.5$).

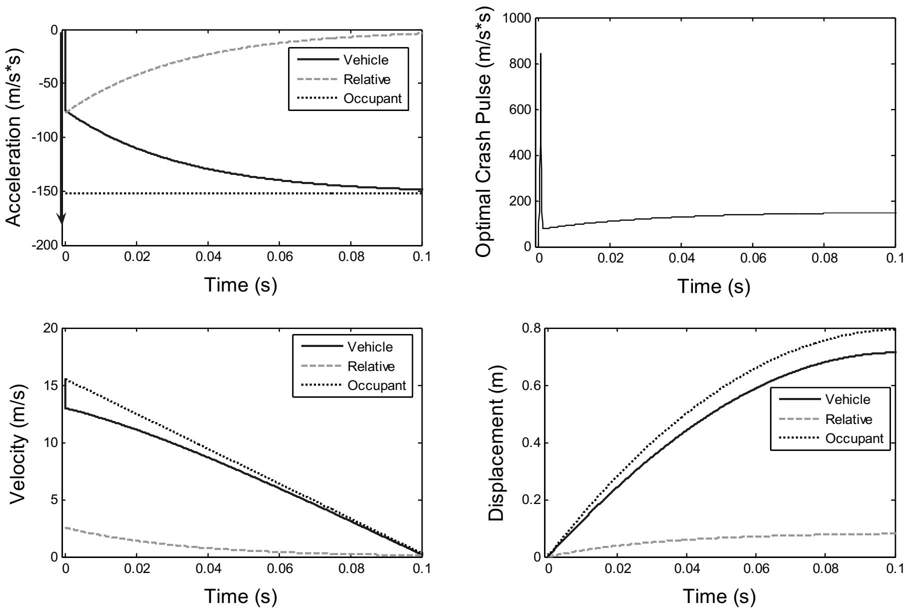

Fig. 4. Optimal crash pulse and the motions of the vehicle and the occupant ($\zeta = 0.7$).

4.2. Nonlinear passive restraint characteristics

Consider the motion of the occupant relative to the vehicle right after the onset of impact. According to Eq. (32), $y(0^+) = 0$. In order to meet Eq. (29) ($u(y, \dot{y}) = mA_m$), $\dot{y}(0^+) \neq 0$. Due to the damping in a restraint system, eventually $\dot{y}(t) \to 0$; that is, after sufficient long time, $\dot{y}(t) \approx 0$. Thus, even though the nonlinear characteristics of a passive restraint system are not explicitly described, it can be reasonably assumed that $y(t)$ can be approximated by

$$y(t) = \sum_i A_i(1 - e^{-\alpha_i t}), \tag{53}$$

where A_i depends on the stiffness and α_i depends on the stiffness and damping of the system. According to Eq. (29)

$$\begin{aligned} &u(\textstyle\sum_i A_i,\, 0) = mA_m \\ &u(0, \textstyle\sum_i \alpha_i A_i) = mA_m \end{aligned}, \tag{54}$$

Accordingly,

$$\begin{aligned} &\dot{y}(t) = \textstyle\sum_i \alpha_i A_i e^{-\alpha_i t} \\ &\ddot{y}(t) = \textstyle\sum_i -\alpha_i^2 A_i e^{-\alpha_i t} \end{aligned}, \tag{55}$$

From Eqs (15), (28), (53), and (55),

$$\left.\begin{aligned} &x(t) = v_0 t - \tfrac{1}{2}A_m t^2 - \textstyle\sum_i A_i(1 - e^{-\alpha_i t}) \\ &\dot{x}(t) = v_0 - A_m t - \textstyle\sum_i \alpha_i A_i e^{-\alpha_i t} \\ &\ddot{x}(t) = -A_m + \textstyle\sum_i \alpha_i^2 A_i e^{-\alpha_i t} \end{aligned}\right. ,\ 0^+ \leqslant t \leqslant T_0, \tag{56}$$

Compare to Eq. (5), from $t = 0$ to $t = 0^+$, the velocity change of the vehicle is

$$\Delta v = x(0) - x(0^+) = \sum_i \alpha_i A_i. \tag{57}$$

To produce this velocity change, the acceleration of the vehicle needs to have an impulse at the initial time ($t = 0$), which theoretically can be represented by a Dirac delta function $\delta(t)$:

$$I_x(t) = \left(\sum_i \alpha_i A_i\right) \delta(t). \tag{58}$$

As such, from the onset of impact ($t = 0$) to the first stop ($t = T_0$), the optimal vehicle acceleration pulse is

$$\ddot{x}(t) = -A_m + \sum_i \alpha_i^2 A_i e^{-\alpha_i t} - \left(\sum_i \alpha_i A_i\right) \delta(t),\ \ 0 \leqslant t \leqslant T_0. \tag{59}$$

Since the impulse $I_x(t)$ results in only a velocity change but no change in displacement of the vehicle from $t = 0$ to $t = 0^+$, the expression of vehicle displacement in Eq. (56) still holds for $0 \leqslant t \leqslant T_0$.

Comparing Eq. (59) with Eq. (40), we can see that for passive restraint characteristics, whether linear or nonlinear, the basic characteristics of the optimal crash pulses are the same. That is, the acceleration of the vehicle at $t = 0$ is negatively infinite; after $t = 0$, it jumps back and then decays; after a period of time, it basically remains constant at a negative value.

4.3. *Active restraint systems*

Suppose for an active restraint system the passive part and the active mechanism are separable, i.e.,

$$u(y, \dot{y}, t) = g(y, \dot{y}) + h(t), \tag{60}$$

where $g(y, \dot{y})$ corresponds to the passive characteristics and $h(t)$ represents the active mechanism. From Eq. (4),

$$g(y, \dot{y}) = mA_m - h(t). \tag{61}$$

The motion of the occupant relative to the vehicle can be determined by solving this equation. The optimal crash pulse can then be found, depending both on the passive characteristics and on the active mechanism.

Theoretically, by optimally designing the active mechanism $h(t)$ alone, an active restraint system may be able to provide required protection to the occupant so that the optimal kinematics of the occupant is attained and the peak occupant deceleration is minimized, even though the crash pulse is not optimized.

4.4. Pre-acting restraint systems

A pre-acting mechanism exerts an action on the occupant before the onset of impact. It can be designed to move the occupant in the opposite direction of impact. If a free space is available for the occupant to move backward after pre-action, the space for the occupant to move forward during impact will become larger. This means that additional rattlespace is created. This additional rattlespace will reduce the peak occupant deceleration for either passive or active restraint systems.

With the additional rattlespace being considered, the problem of the optimal crash pulse for pre-acting restraint systems can be treated in the same way as for passive and active systems.

It has been shown that a pre-acting system is superior to an active system, which in turn is superior to a passive system, in terms of their limiting or optimal performance in the impact isolation or attenuation [8].

4.5. Numerical solution of optimal crash pulse

When restraint characteristics are nonlinear, or when restraint characteristics are linear but the motion of the vehicle is subject to other constraints such as a limit on the amplitude of the deceleration, the optimal crash pulse cannot be readily obtained analytically. Instead, numerical optimization can be utilized to find solutions. In order to use numerical optimization to obtain the optimal crash pulse, discretize the time interval [0, T_0] into identical subintervals. In addition, represent the crash pulse $a(t)$ on each subinterval by a constant value, that is,

$$a\ (t) = a_i \quad for \quad (1-i)\ \Delta t \leqslant t < i\,\Delta t,\ i = 1,\ldots,N, \tag{62}$$

where N is the number of subintervals, Δt is the length of each subinterval, and $\Delta t = T_0/N$. Denote

$$\mathbf{A} = [a_1 a_2 \ldots\, a_N]^T\,, \tag{63}$$

as the vector of design variables. The motion of the occupant relative to the vehicle is discretized accordingly

$$y_i = y\,(i\,\Delta t)\ \ , \ddot{y}_i = \ddot{y}\,(i\,\Delta t\,)\ \ , i = 1, \ldots\,,\ N, \tag{64}$$

and can be obtained from the numerical integration of Eq. (4). The system performance criteria of Eqs (8)–(10) can be expressed by the discretized system responses:

$$J_1(\mathbf{A}) = \max_{1\leqslant i\leqslant N}\{-(\ddot{x}_i + \ddot{y}_i)\}, \tag{65}$$

$$J_2(\mathbf{A}) = \max_{1\leqslant i\leqslant N}\{x_i\}, \tag{66}$$

and

$$J_3(\mathbf{A}) = \max_{1\leqslant i\leqslant N}\{y_i\}. \tag{67}$$

The problem of finding the optimal crash pulse can then be formulated as a numerical optimization problem:

$$\begin{aligned} &\textit{Design Variables}\text{: } \mathbf{A} \\ &\textit{Objective Function}\text{: } \min\{J_1(\mathbf{A})\} \\ &\textit{Constraints}\text{: } J_2(\mathbf{A}) \leqslant D \\ &J_3(\mathbf{A}) \leqslant S_0 \\ &\mathbf{A}_L \leqslant \mathbf{A} \leqslant \mathbf{A}_U \end{aligned} \tag{68}$$

where $\mathbf{A}_L$ and $\mathbf{A}_U$ are the lower and upper bounds on the crash pulse. A nonlinear optimization method, such as sequential quadratic programming method [9], can be used to solve this problem.

5. Discussion

The simplifications and assumptions made in the investigations may have imposed certain limitations on the analyses in this paper. These will be addressed in the following discussion.

- Automobile frontal impact induces complicated biodynamic loading to the occupant and causes injuries to various regions of the body. When the prevention and reduction of injuries to the entire body is considered, the occupant can be reasonably treated as a point mass and its peak acceleration (deceleration) can be used as the injury criterion. However, this simplification of the problem may not be able to address particular injuries. For instance, when the thoracic injuries are considered, the one-mass model used in the paper cannot describe the chest compression and the peak deceleration may not be a good predictor of AIS+3 injuries [10].
- The action of a restraint system on the occupant may lag behind the vehicle impact. For instance, the action of active restraint systems (e.g, airbags) on the occupant is controlled by the sensors that detect the vehicle impact and thus usually lags behind it. However the time delay between the action and the impact can be minimal for passive systems (e.g., seatbelts) and pre-acting systems. This time delay has not been considered in this paper. Note that the vehicle crash pulse discussed here is not necessarily the impact response of the vehicle at the front end. It should be considered as the response at the mass center of the vehicle.
- For automobile frontal impact, there are major differences between car-to-barrier impact and car-to-car impact, which include impact speed and crash deformation. While the optimal occupant kinematics (constant occupant deceleration) may be applicable to both scenarios, the optimal vehicle crash pulse obtained may not be fully suitable to the car-to-car impact because the scenario of frontal impact considered in this paper mainly pertains to the car-to-barrier impact.
- The characteristics of restraint systems under impact are so complicated that in general, they are nonlinear and time-dependent. The assumption of linear restraint characteristics leads to the finding of the theoretical optimal crash pulse. Whereas this optimal crash pulse reveals certain characteristics of the optimal crash pulse for general restraint systems, major differences between them are anticipated. Computational optimization is required for the determination of the optimal crash pulse for general restraint systems.

6. Concluding remarks

In automobile frontal impact, the optimal kinematics of the occupant is such that the occupant moves at a constant deceleration. For given restraint characteristics, it is possible for the occupant to attain optimal kinematics by optimizing the vehicle crash pulse. For passive restraint systems, the optimal vehicle crash pulse is found to be: a positive impulse at the onset, an immediate plunge followed by a gradual rebound, and finally a positive constant level period. In general, the optimal crash pulse depends upon the characteristics of particular restraint systems.

Whereas the optimal crash pulse depends upon the characteristics of a particular restraint system, its general aspects can be ascertained as follows:

- If restraint systems are passive and have certain damping, an impulse is needed at the initial time.
- If restraint systems are active and able to generate an impulsive action on the occupant initially, an initial impulse may not be necessary for the optimal crash pulse.
- The major potion of the optimal crash pulse after an initial period tends to be leveled.

The optimal crash pulse can be obtained via the optimal design of vehicle structures. It is impossible for passive structures to generate an impulse at the initial time of impact, since the response of passive structures is base upon the force-deformation relation. However, an initial impulse can be generated by an active mechanism or system if it has an internal power source and is able to ascertain the onset of impact.

It is recognized that the simplification of the problem and the assumptions made in the investigations may have imposed certain limitations on the analyses in this paper. However, the results and conclusions derived can still be used as general guidelines for the crashworthiness design of the vehicle structure.

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