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T-S fuzzy control of uncertain chaotic vibration

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Abstract. We present in this paper a novel and unified control approach that combines intelligent fuzzy logic methodology with predictive method for controlling chaotic vibration of a class of uncertain chaotic systems. We first introduce prediction into each subsystem of Takagi Sugeno (T-S) fuzzy IF-THEN rules and then present a unified T-S predictive fuzzy model for chaos control. The proposed controller can successfully stabilize the chaos and track the desired targets. The simulation results illustrate its effectiveness.

Keywords: Chaos, Takagi Sugeno fuzzy model, nonlinear predictive control

1. Introduction

Chaos and its control are an active research area in the field of nonlinear control. The aim of researching chaos control is to control the chaotic system to the periodic orbit or equilibrium point [1,5]. Up to now, some control methods such as the Ott-Grebogi-Yorke (OGY) control method [6], time delay feedback control method [7,8], predictive control method [9–11], backstepping control method [12], adaptive control method [13], incursive control method [14], high order control method [15,16] and sliding model control [18] are successfully used to suppress or stabilize chaos. However, the existing control methods also have some disadvantages. Most of these methods require a variable system parameter which is usually unavailable in the control of real systems such as the control of the permanent magnet synchronous motor drive system (PMSM) [19]. Moreover, it is very difficult to apply control methods especially for those systems with uncertain dynamics.

In recent years, fuzzy logic control (FLC) has been found one of the most popular and conventional tools in functional approximations [20,21]. The essential part of a FLC is a set of the linguistic control rules related by the dual concepts of fuzzy implication and the compositional rule of inference [22]. FLC provides an effective way to design control system that is one of the important applications in the area of control engineering and it is particularly suitable for those systems with uncertain dynamics.

Some fuzzy modeling and control techniques of uncertain chaotic systems have been proven successful [24–35]. In the most of these methods and techniques, fuzzy control is based on the Lyapunov stabilization criterion and impulsive control approach. The main disadvantage is that they have an important computational cost.

In this paper, our suggestion is yet another alternative by means of improving the fuzzy logic control method via the prediction-based feedback control concept. The fuzzy controller design algorithm is developed for the control of chaotic system with dynamics uncertainties. More precisely, we present a systematic design procedure of fuzzy model-based controller with guaranteed stability and improved tracking performance for the uncertain chaotic

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system. The proposed method utilizes the fuzzy model whose antecedent parts are conventional fuzzy IF part and the consequent parts are local linear state space models of the given system. The global fuzzy logic controller is constructed by combining such local fuzzy-model-based controller with a global predictive controller. The design procedure is mainly composed of two parts: One is to design the local compensators for each local linear state model; the other is to design a supervisory controller whose input is determined by the predictive control theory. By doing so, the overall robustness and tracking ability of the entire closed-loop system can be significantly improved. Moreover, one can design a global stable fuzzy logic controller without finding a common Lyapunov function. Furthermore, it guarantees the stability for the original underlying nonlinear system rather than just the local T-S fuzzy models used for approximation. In the modeling phase, the T-S fuzzy model is used in conjunction with the Levenberg–Marquardt algorithm for the model parameters optimization. In the controller design phase, based on the fuzzy model established above, a prediction-based controller is designed for the stabilization of the uncertain chaotic vibration on the equilibrium and on the unstable periodic orbit with guaranteed of stability criteria. As an advantage of the proposed method, the algorithm could run even if the underlying system is presented by few input-output data pairs and the use of the Levenberg-Marquardt algorithm increases the efficiency of the method to determine all the fuzzy model parameters which allows the stabilization of the trajectory on the nearest desired target.

This paper is organized as follows: The proposed T-S fuzzy model is first detailed in Section 2. Based on this model, a fuzzy predictive controller is developed in Section 3. In Section 4, applications to specific examples such as the logistic equation and the Hénon system are given to show the advantages and effectiveness of the proposed approach. Finally, conclusions are drawn in Section 5.

2. The proposed T-S fuzzy model

Fuzzy modeling techniques have been successfully used to model nonlinear systems where mathematical model of the plant is not available. Typically, the modeling process consists of constructing from a set of N data pairs of observations a model that can accurately represents the uncertain system in the form of rules.

Among the various fuzzy modeling approaches proposed in the literature, the Takagi-Sugeno fuzzy system is the most used in control and identification of chaotic systems [24]. It has gained popularity because of its excellent capability to describe uncertain systems and to approximate a large class of nonlinear systems. The basic idea of this methodology is to decompose a non linear system into a number of linear or nearly linear subsystems.

A T-S fuzzy model consists of a set of IF-THEN rules that have the form:

Plant Rule j :

$$\begin{aligned}&\textit{IF } x_1 \textit{ is } A_1^j \ldots \textit{ and } x_n \textit{ is } A_n^j,\\ &\textit{THEN } y^j(x(i)) = a_{j,0} + a_{j,1}x_1(i) + \cdots + a_{j,n}x_n(i) \textit{ For } j = 1, \ldots, m.\end{aligned} \tag{1}$$

where n, m represent the order of the system and the number of the IF-THEN rules, respectively; $a = (a_{j,0}, a_{j,1}, \ldots, a_{j,n})$ is the parameter set in the consequent part and $y^j(x(i))$ is the local output of the jth rule.

The global output is inferred as a weighted average value of the outputs estimated by all the fuzzy rules, i.e.:

$$\begin{aligned}y(i) &= F(x(i))\\ &= \frac{\sum_{j=1}^{m} y^j(x(i))\mu_j(x(i))}{\sum_{j=1}^{m} \mu_j(x(i))}\\ &= \frac{\sum_{j=1}^{m} a_{j,0}\mu_j(x(i))}{\sum_{j=1}^{m} \mu_j(x(i))} + \frac{\sum_{j=1}^{m} a_{j,1}x_1\mu_j(x(i))}{\sum_{j=1}^{m} \mu_j(x(i))} + \cdots + \frac{\sum_{j=1}^{m} a_{j,n}x_n\mu_j(x(i))}{\sum_{j=1}^{m} \mu_j(x(i))}\end{aligned} \tag{2}$$

where $\mu_j(\cdot)$ denote the fuzzy membership functions. It is assumed, that they are well-defined and verify $\sum_{j=1}^{m} \mu_j(x) \neq 0$ for all x.

Let a regressive vector

$$\xi(x) = [\xi_1(x), \xi_2(x), \ldots, \xi_m(x), x_1\xi_1(x), x_1\xi_2(x), \ldots, x_1\xi_m(x), \ldots, x_n\xi_1(x), x_n\xi_2(x), \ldots, x_n\xi_m(x)]^T \tag{3}$$

with

$$\xi_j(x) = \frac{\mu_j(x)}{\sum_{j=1}^{m} \mu_j(x)} \tag{4}$$

and a parameters vector:

$$\theta = [a_{1,0}, a_{2,0}, \ldots, a_{m,0}, a_{1,1}, a_{2,1}, \ldots, a_{m,1}, \ldots, a_{1,n}, a_{2,n}, \ldots, a_{m,n}]^T \tag{5}$$

Equation (2) can be rewritten as follows

$$y(i) = F(x(i)) = \theta^T \xi(x(i)) \tag{6}$$

The identification of the number of fuzzy rules, the parameters vector θ and the appropriate basis function $\xi(x)$ will complete the modeling process. For that, and for a given chaotic system described by a set of input-output data pairs: $((x(i), y_d(i)), i = 1, 2, \ldots .N)$, we have to minimize a quadratic function e_r which quantifies the error between the observed output $y_d(i)$ and the fuzzy system output $y(i)$.

$$e_r = \frac{1}{2}\sum_{i=1}^{N} e_i^2 = \frac{1}{2}\sum_{i=1}^{N} [y(i) - y_d(i)]^2 \tag{7}$$

The Fuzzy model identification scheme is indicated in Fig. 1. Starting with a number of rules, a vector θ and membership function parameters arbitrarily chosen, the algorithm runs until all the parameters stop moving and a desired precision e_{min} is reached. This indicates that the value of the error is minimized, so the algorithm has found a minimum. However, when the maximum number of iteration is reached without finding the optimal parameters, the rules numbers is increased and the algorithm is reinitialized.

For the proposed fuzzy model, we have chosen Gaussian membership functions with centers and relative widths respectively designed by parameters $c_{j,i}$ and $\sigma_{j,i}$. The advantage of using this type of membership functions is to ensure continuity that is required in conventional optimization techniques.

$$\mu_j(x(i)) = \prod_{k=1}^{n} \exp\left[-\frac{1}{2}\left(\frac{x_k(i) - c_{j,i}}{\sigma_{j,i}}\right)^2\right] \tag{8}$$

The model consequent parameters $a_{j,0}, a_{j,1}, \ldots, a_{j,n}$ $(j = 1, \ldots, m)$; the centers and relative widths of the Gaussian membership's function are adjusted in an iterative manner using the Levenberg-Marquardt algorithm [36]. This very efficient optimization algorithm has the capability to converge to the optimum of any quadratic nonlinear function with a good approximation. It consists to the iterations

$$\alpha^{\nu+1} = \alpha^{\nu} - (J_\alpha^{\nu T} J_\alpha^{\nu} + \lambda I)^{-1} J_\alpha^{\nu T} e_i \tag{9}$$

where J_α^{ν} represents the Jacobian at the iteration ν, defined by

$$J_\alpha^{\nu} = \left[J_{a_{j,k}}^{\nu} = \frac{\partial e_i^{\nu}}{\partial a_{j,k}}(k = 0, \ldots, n), J_{c_{j,l}}^{\nu} = \frac{\partial e_i^{\nu}}{\partial c_{j,l}}, J_{\sigma_{j,l}}^{\nu} = \frac{\partial e_i^{\nu}}{\partial \sigma_{j,l}}(l = 1, \ldots, n)\right] \tag{10}$$

α is a vector whose elements are the different parameters of the fuzzy model, λ is a small positive scalar and I is the identity matrix.

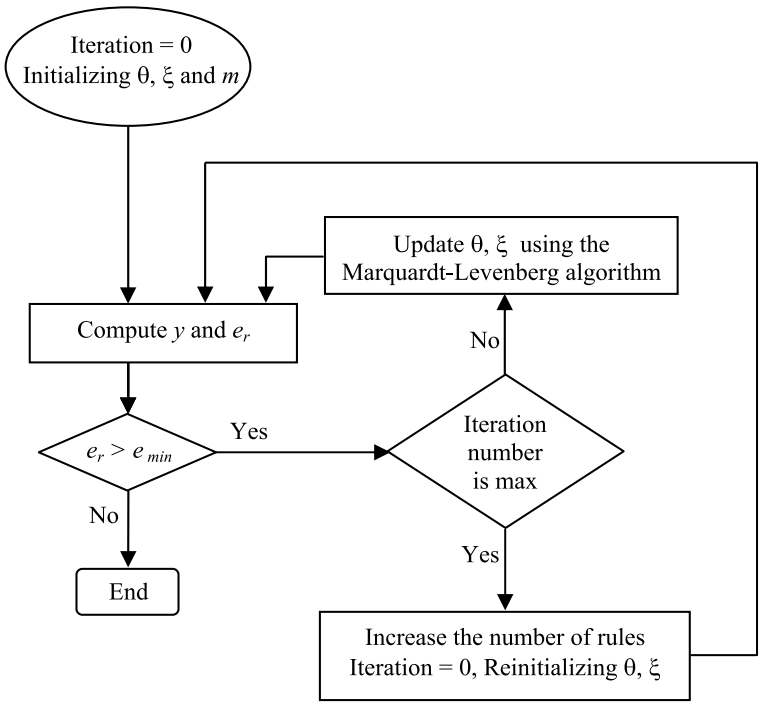


Fig. 1. The proposed T-S Fuzzy model identification scheme.

3. The proposed T-S fuzzy controller

The objective of control is to drive the chaotic system with parameter uncertainties from a chaotic regime to a regular attractor such as unstable fixed points or unstable periodic orbits. Using a one step ahead prediction and based on the fuzzy model given by Eq. (6), fuzzy state of the future output $y_p(i)$ is obtained as follows

$$y_p(i) = F(y(i)) = \theta^T \xi(y(i)) \tag{11}$$

Let $\bar{y}$ to be the desired target to be stabilized, it satisfies

$$\bar{y} = F(\bar{y}) = \theta^T \xi(\bar{y}) \tag{12}$$

The linearization of $F(y(i))$ around $\bar{y}$ is given by

$$F(y(i)) = \bar{y} + A\,[y(i) - \bar{y}] \tag{13}$$

where A represents the derivative of the uncontrolled system evaluated around the desired point $\bar{y}$. It can be approximated by

$$A = D_y F(\bar{y}) = \frac{y(i+1) - \bar{y}}{y(i) - \bar{y}} = \frac{\delta y(i+1)}{\delta y(i)} \tag{14}$$

The purpose of control is to assure the system asymptotically converges towards $\bar{y}$ with only extremely small applied force u. The fuzzy control law is determined by the difference between the future fuzzy output $y_p(i)$ and the current fuzzy output $y(i)$. It will be in the form

$$u(i) = K\,[y_p(i) - y(i)] = K\,[F(y(i)) - y(i)] \tag{15}$$

where K represents the gain.

We consider the situation where the fuzzy system's output trajectory is in a chaotic state, and we want to design a controller u, to be fed into the fuzzy equation, so as to stabilize the following controlled system

$$\begin{aligned} y(i+1) &= F(y(i)) + u(i) \\ &= F(y(i)) + K\left[F(y(i)) - y(i)\right] \\ &= (K+1)F(y(i)) - Ky(i) \end{aligned} \tag{16}$$

By applying the classical criteria of stability given by $\left|\frac{\delta y(i+1)}{\delta y(i)}\right| < 1$, we obtain

$$\left|(K+1)\left.\frac{\partial F(y(i))}{\partial y(i)}\right|_{y=\bar{y}} - K\right| < 1 \tag{17}$$

i.e.

$$|(K+1)A - K| < 1 \tag{18}$$

Due to the fact that fixed points or periodic orbits are unknown because there is no mathematical model of the chaotic system, in this sense, and since stabilization is guaranteed in a neighborhood of the stabilized orbit, we assume that in the vicinity of the desired point, value of A can be approximated by

$$A = \frac{F(y(i)) - F(y(i-1))}{y(i) - y(i-1)} = \frac{y(i+1) - y(i)}{y(i) - y(i-1)} \tag{19}$$

only if the condition

$$|y(i) - y(i-1)| < \varepsilon \tag{20}$$

is verified with ε is a small positive number.

4. Numerical examples

In this section, the validity and effectiveness of the proposed T-S fuzzy modelization and control method are examined through the simulation of chaotic systems with uncertain dynamics. The control objective is for a system to follow a given desired trajectory and to produce a control input vector $u(i)$ such that the trajectory error approaches 0 as $i \to \infty$.

4.1. Example 1

In this example, we apply the both fuzzy modeling and controlling phases to the data pairs of an uncertain chaotic system given by the logistic equation of the form

$$y(i+1) = 4y(i)(1 - y(i)) + c \tag{21}$$

First, we discuss the general case when $c = 0$.

This system is chaotic without control. The trajectory of the true system given by input-output data pairs is shown in Fig. 2 for the iteration i from $i_0 = 0$ to $i_f = 1000$.

Based on the data generated by the true system, the fuzzy model for this uncertain chaotic system is obtained by linearizing the nonlinear system over a number of operating points in the phase plane. As shown in Fig. 2, the state variable is between 0 and 1 for all i $(i_0 \leqslant i \leqslant i_f)$. Therefore, to focus our theoretical studies, we consider that the uncertain chaotic system is of order $n = 1$, represented by $N = 1000$ input-output data pairs and modelized on $y = \{0; 0.5; 1\}$ which results in three Gaussian membership functions $(m = 3)$. The centers of these membership functions are initially distributed uniformly over the range of data $[0, 1]$, and the relative widths are chosen so as to get a reasonable coverage. The Levenberg-Marquardt algorithm is used for search of best parameters $a_{j,i}, c_{j,i}$ and $\sigma_{j,i}$.

The plot of input output map of the resulting T-S fuzzy system compared to the plot of the true system, initial and final membership functions and the modeling error are shown in Figs 3(a-d).

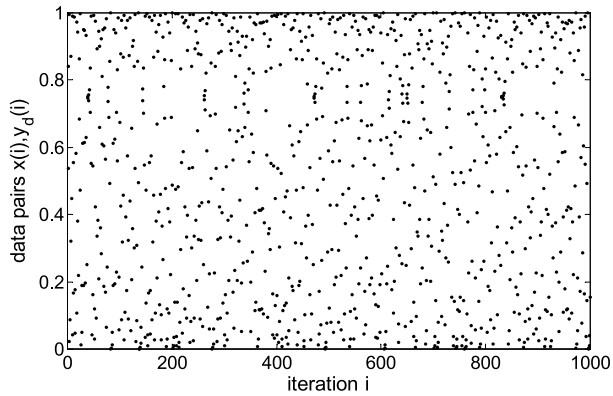


Fig. 2. The input-output data pairs of the true system.

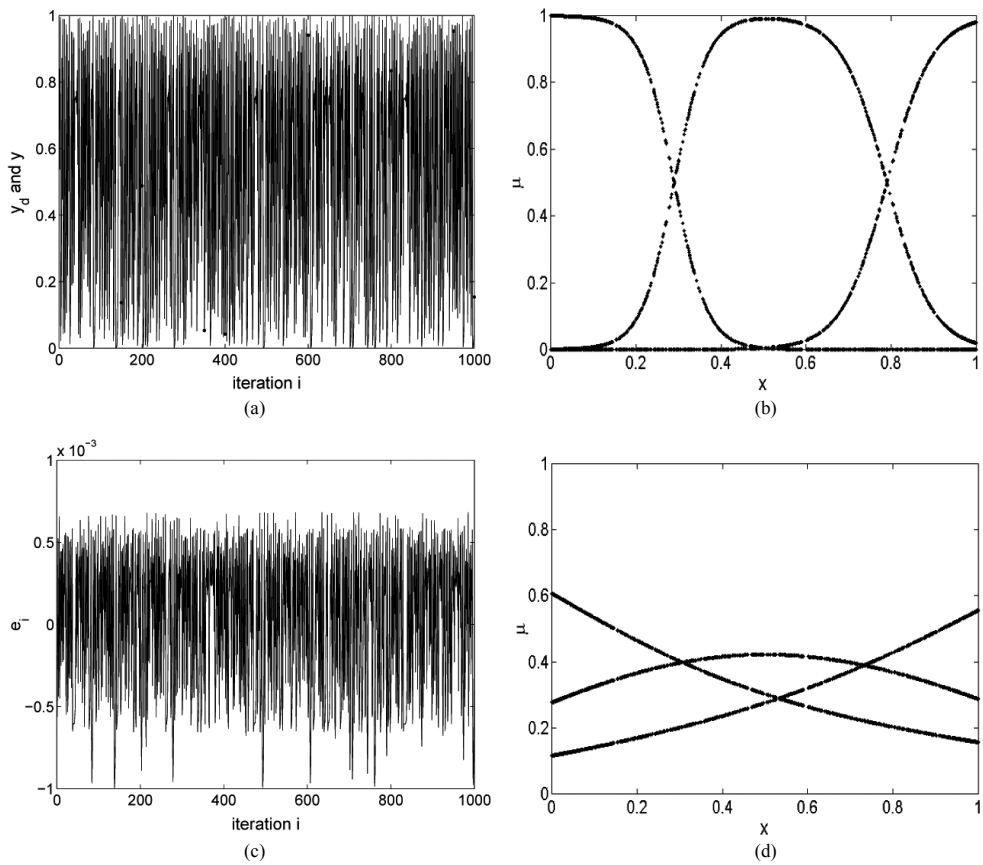
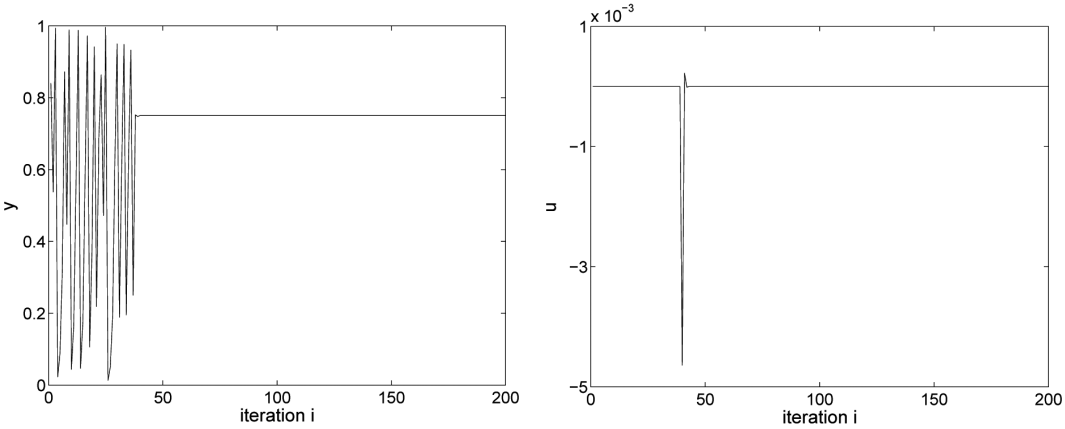


Fig. 3. T-S Fuzzy modeling of input-output data pairs. (a) Input-output data and fuzzy model. (b) Initial membership functions. (c) Modeling error. (d) Final membership functions.

In order to apply the proposed T-S fuzzy controller, we have to determine the correction to apply in the vicinity of the unstable target to adjust the next point so it falls on the stable one. The vicinity of the desired target is determined by Eq. (20).

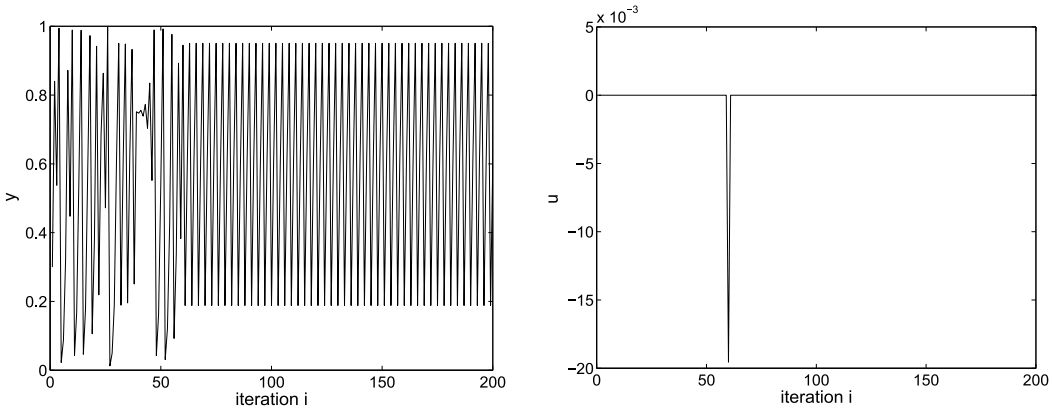
$$|y(i) - y(i-1)| < 0.01 \tag{22}$$

At iteration $i = 40$, the test (22) is verified and we get $y(i) = 0.755$.



(a) (b)

Fig. 4. T-S Fuzzy system under control. (a) Behavior of $y(i)$. (b) Control input $u(i)$.



(a) (b)

Fig. 5. T-S Fuzzy system stabilized on periodic orbit. (a) Behavior of $y(i)$. (b) Control input $u(i)$.

From Eq. (19), value of A is obtained from simulation as follows

$$A = \frac{y(i+1) - y(i)}{y(i) - y(i-1)} = \frac{0.738 - 0.755}{0.755 - 0.746} = -2.011 \tag{23}$$

The desired target is stabilized by the proposed method of control for

$$|-2.011 - 3.011 \times K| < 1 \tag{24}$$

Thus, the feedback gain K is in the range of $-1 < K < -0.335$.

In the validation phase of the identified T-S fuzzy model, once inequality Eq. (22) is satisfied, control input u becomes active and tracks the trajectory towards the fixed point for $K = -0.65$.

From Figs 4(a) and (b), one can see that the control input takes a nonzero value at iteration $i = 40$ and stabilizes the chaotic vibration on the fixed point. Numerically, fixed point was found $\bar{y} = 0.750$.

In other hand, and by choosing $K = -0.8$, the control input is activated at iteration $i = 60$ and stabilizes the chaotic trajectory on the periodic orbit as depicted in Figs 5(a) and (b).

In order to discuss the robustness of the proposed method, we add a constant parameter $(c = -0.05)$ to the original input-output data pairs and plot the output response of the controlled fuzzy system for $K = -0.65$. Figures 6(a) and (b) show that even if the original system is disturbed by an additional value, the fuzzy system converges to a new point, near the fixed one, which stabilizes the modified system. In this case, fixed point is $\bar{y} = 0.733$.

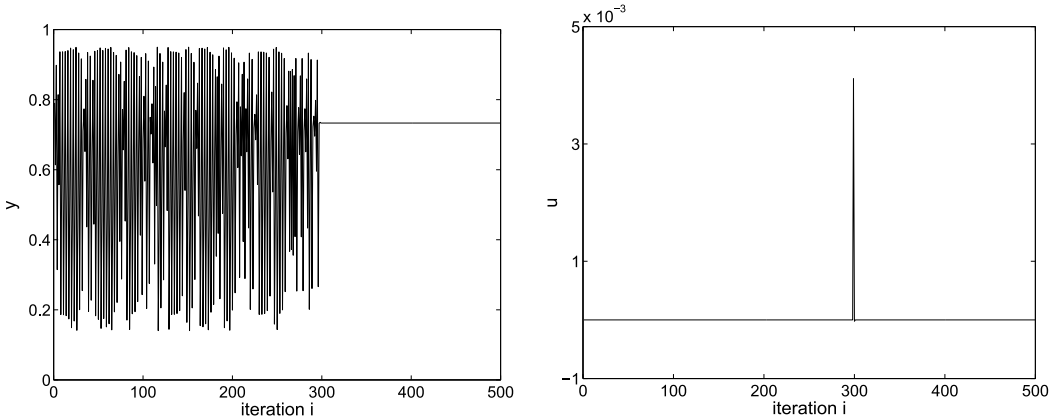


Fig. 6. Robustness of the proposed control method. (a) Behavior of $y(i)$. (b) Control input $u(i)$.

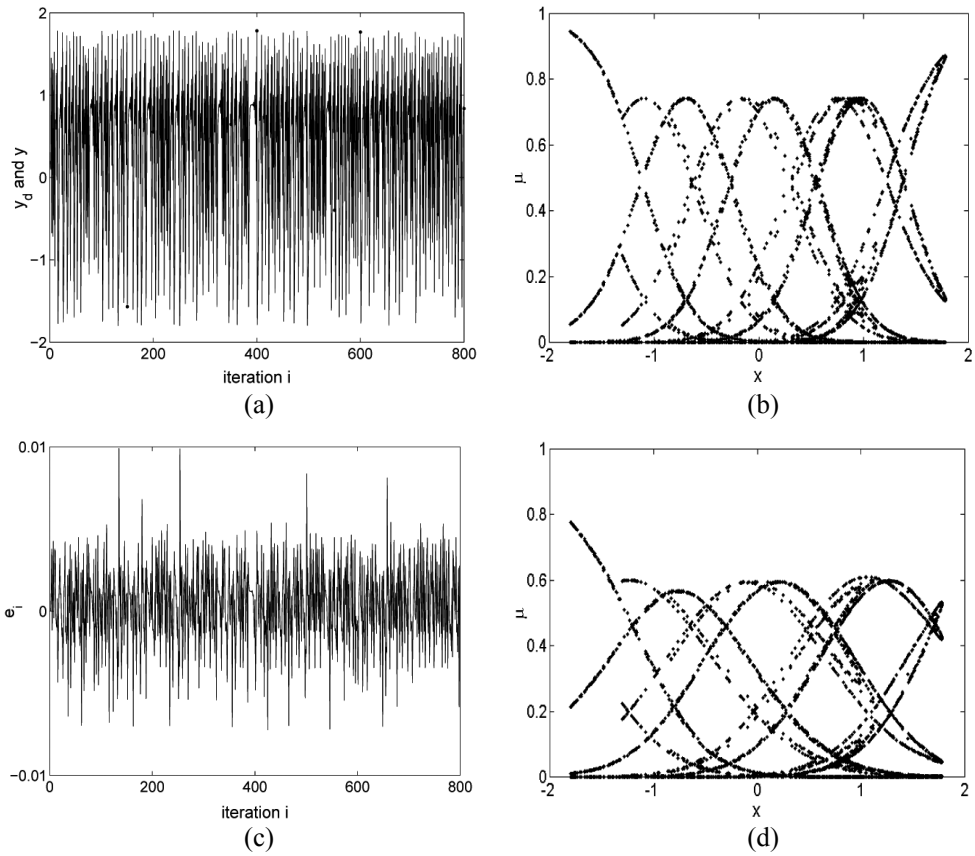


Fig. 7. T-S Fuzzy modeling of the Henon system. (a) Input-output data and fuzzy model. (b) Initial MF. (c) Modeling error. (d) Final MF.

4.2. Example 2

A simulation on higher-dimensional system such as Hénon system has been carried out. This two dimensional system is given by

$$y(i+1) = a - y^2(i) + by(i-1) \tag{25}$$

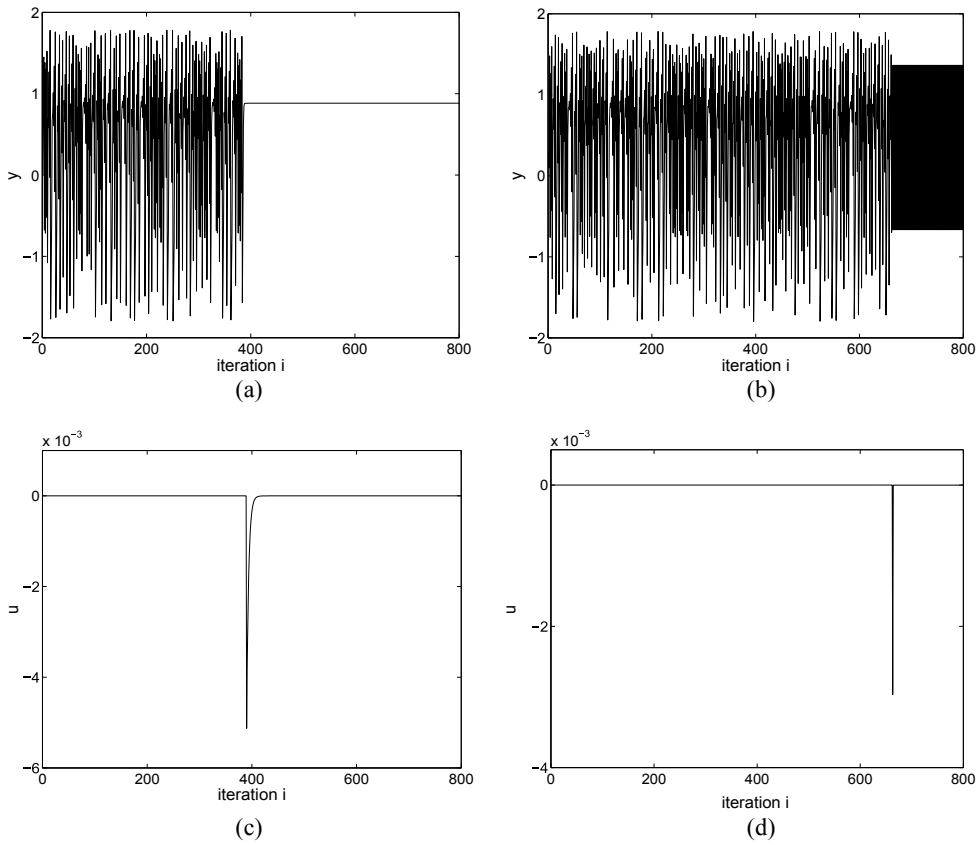


Fig. 8. The controlled T-S Fuzzy system. (a) and (c) T-S Fuzzy system stabilized on the fixed point. (b) and (d) T-S Fuzzy system stabilized on the periodic orbit.

It exhibits a chaotic regime for $a = 1.4$ and $b = 0.3$.

The T-S fuzzy system depends on two inputs: $y(i)$ and $y(i-1)$. The T-S fuzzy model is constructed using a set of $N = 800$ input-output data pairs obtained by simulation.

Since the fuzzy rules for $y(i)$ and $y(i-1)$ have the same consequent parts, then the nonlinear system (25) is linearized on $y = \{-2; 0; 2\}$ which results in seven membership functions. The results of the T-S fuzzy modeling phase are depicted in Figs 7 (a-d). They show that the optimization algorithm works very well even when the number of parameters to identify is important.

The condition of stabilization of the T-S fuzzy system on the desired targets is given by

$$-1 < K < 0.3196 \tag{26}$$

Figures 8(a-d) give the control results. They are obtained for $K = -0.9$ and for $K = -0.65$ respectively. For the first case, the T-S fuzzy system is in the vicinity of the fixed point at iteration $i = 390$. At this moment, the control input becomes active and stabilizes the system on the fixed point $\bar{y} = 0.884$. For the second case, the control is activated at iteration $i = 663$ and the T-S fuzzy system is stabilized on the periodic orbit.

5. Conclusion

In this paper, a T-S fuzzy modeling scheme and T-S fuzzy control methodology was developed and applied to chaotic systems with dynamics uncertainties, where the T-S fuzzy model is established using only the input-output data pairs of the true systems. Based on this fuzzy model, the proposed control law ensures that the chaotic state

tracks stable constant targets, which falls on fixed point or periodic orbit. The use of the Levenberg-Marquardt algorithm increases the robustness of the method.

The simulation results indicate that the proposed fuzzy controller has faster response than the conventional controller, and does not require a precise model. Moreover, it is not necessary to verify the stability condition same as for the impulsive fuzzy controller since the prediction law guarantee the stability in the neighborhood of the desired targets. However, we still have to solve the problem of modelisation and control of uncertain continuous chaotic systems. This is a topic for future research.

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