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Closed-loop input shaping control of vibration in flexible structures via adaptive sliding mode control

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Abstract. Input shaping technique is widely used in reducing or eliminating residual vibration of flexible structures. The exact elimination of the residual vibration via input shaping technique depends on the amplitudes and instants of impulse application. However, systems always have parameter uncertainties which can lead to performance degradation. In this paper, a closed-loop input shaping control scheme is developed for uncertain flexible structures. The algorithm is based on input shaping control and adaptive sliding mode control. The proposed scheme does not need a priori knowledge of upper bounds on the norm of the uncertainties, but estimates them by using the adaptation technique. This scheme guarantees closed-loop system stability, and yields good performance and robustness in the presence of parameter uncertainties and external disturbances as well. Furthermore, it is shown that increasing the robustness to parameter uncertainties does not lengthen the duration of the impulse sequence. Simulation results demonstrate the efficacy of the proposed closed-loop input shaping control scheme.

Keywords: Input shaping, residual vibration, adaptive sliding mode control

1. Introduction

Numerous techniques have been proposed to reduce the residual vibration in flexible structures since the residual vibration can often have adverse affects on the operational performance and accuracy. One approach, known as input shaping [1–5], has been implemented to control the residual vibration on a variety of systems. The efficiency and effectiveness of the input shaping technique have been already confirmed in many practical systems such as a chip mounter [6], cranes [7,8], telescopic handler [9], industrial robot [10], and coordinate measuring machines [11,12].

Input shaping is implemented by a sequence of impulses, called the input shaper, together with a desired system command, and yields the system input that yields the desired motion without vibration. However, one drawback of this method is that the exact cancellation of the residual vibration depends on the amplitudes and instants of impulse application which are related to system parameters. If amplitudes or instances at impulse application are inaccurate, then the residual vibration can lead to system performance degradation. Furthermore, input shaping method does not deal with vibration excited by external disturbances.

Many design schemes have been proposed to improve the robustness of input shaping to uncertainties in the damping factors and natural frequencies of flexible structures [13–16]. However, the price for increasing robustness is to lengthen the duration of the impulse sequence which results in a slower system response. Also, the vibration caused by external disturbance has not been considered. Recently, in order to improve the robustness, a feedback controller combined with input shaping has received more attention. A method of concurrently designing the input

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shaping and feedback control for insensitivity to parameter variations is developed by [17]. Kapila et al. [18] have developed a closed-loop input shaping control method for the robustness to inaccuracies in the instants of applied impulses. However, they did not consider parameter uncertainties and exogenous disturbances. An adaptive input shaper providing robustness to parameter uncertainties by tuning the shaper to the flexible mode frequencies is explored by [19]. However, they did not consider effects caused by inaccurate amplitudes or instants of impulses, and external disturbances. A scheme of increasing the robustness of shaper without adding more impulses to the shaper is presented by [20]. The adaptive closed-loop controller is designed such that the step response of the designed model to the shaped input is tracked quite accurately. However, in this method, the input shaper is designed for the nominal system, instead of the design model which includes the nominal system and the controller. Hence, in the preshaped impulse response of the designed model, the residual vibration after the application of the last impulse is not eliminated. Furthermore, they did not consider the vibration caused by external disturbances.

On the other hand, the sliding mode control (SMC) theory [21–28] can produce the perfect trajectory tracking in the presence of parametric uncertainties and external disturbances. Therefore, in this paper, input shaping method combined with an adaptive SMC method [29–34] has been explored to reduce or eliminate vibration of flexible structures with parameter uncertainties and external disturbances. The organization of this paper is as follows. Section 2 briefly reviews the description of input shaping method. A closed-loop input shaping control based on adaptive SMC technique is developed in Section 3. The selection of sliding surface and the satisfaction of reaching conditions have been addressed. Section 4 provides results from numerical simulations, and comparison of these results to those from methods in the existing literature. Finally, a conclusion is provided in Section 5.

2. Input shaping method

Input shaping method is implemented to reduce the residual vibration of flexible systems by convolving the command input with a sequence of M impulses, also known as the input shaper. The exact elimination of the residual vibration via input shaping technique depends on the amplitudes and instants of impulse application. To illustrate and simplify the analysis, consider the case where only one flexible mode exists, i.e. $n = 1$, where n is the number of mode. Suppose a sequence of M impulses is applied to the system. These impulses have amplitudes $I_{1,i}$, $i = 1, \ldots, M$, and are applied at times $t_{1,i}$, $i = 1, \ldots, M$. If amplitudes and instances at impulse application are accurate, then system performance will result in zero vibration. From [2,3], the corresponding amplitudes and instances of M impulses can be expressed as

$$I_{1,i} = \frac{\binom{M-1}{i-1} K_1^{i-1}}{\sum\limits_{j=0}^{M-1} \binom{M-1}{j} K_1^{j}}, \quad t_{1,i} = (i-1)\frac{\pi}{\omega_1\sqrt{1-\zeta_1^2}}, \quad K_1 = e^{\frac{-\zeta_1\pi}{\sqrt{1-\zeta_1^2}}}, \quad i = 1, \ldots, M \tag{1}$$

where $I_{1,i}$ is the amplitude of the i^{th} impulse and $t_{1,i}$ is the time of the i^{th} impulse. Then, the shaped impulse sequence can be expressed

$$L_1 = I_{1,1}\delta(t) + I_{1,2}\delta(t - t_{1,2}) + \ldots I_{1,M}\delta(t - t_{1,M}) \tag{2}$$

where $t_{1,1} = 0$ and $\delta(\cdot)$ is an impulse function. It is noted that the amplitudes and the instants in Eq. (1) depend on the system parameters, i.e. the system natural frequency ω_1 and the damping ratio ζ_1. To increase the robustness to uncertainties in the natural frequency and the damping ratio, one approach is to lengthen the duration of the impulse sequence which results in a slower system response. Hence, it is desired to increase the robustness of the system without adding any impulses to the shaper.

Although the aforementioned input shaper was presented for a system with one mode ($n = 1$), it can be extended to include additional modes, i.e. $n > 1$. Singer and Seering [2] pointed out that multi-mode systems ($n > 1$) can be dealt with by convolving the impulse sequence for each individual mode with one another. The additional modes can be handled by simply designing impulse sequences for each mode individually, then convolving the sequences for each individual mode with one another to arrive at a composite shaped impulse sequence. After applying the

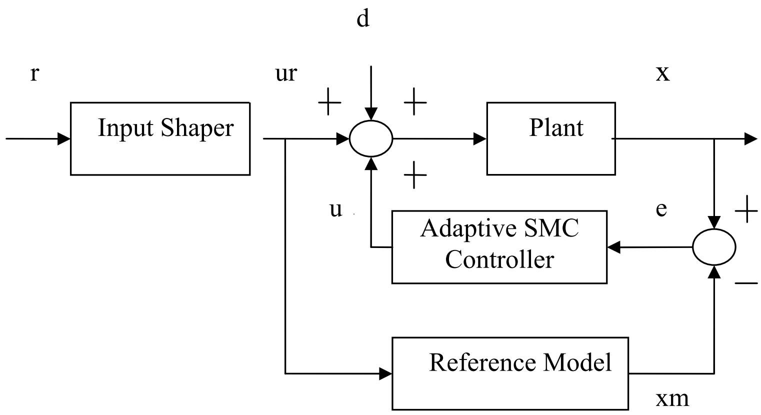


Fig. 1. Proposed control scheme.

resulting composite sequence to the system, the system performance will result in zero vibration. The composite shaped input impulse sequence L_{mult} for a flexible structure with n modes can be expressed as

$$L_{mult} = L_1 * \ldots * L_n \tag{3}$$

where L_p is the impulse sequence for the p^{th} mode of the system and can be expressed as $L_p = I_{p,1}\delta(t) + I_{p,2}\delta(t - t_{j,2}) + \ldots I_{p,M}\delta(t - t_{p,M})p = 1, \ldots, n$, and $*$ is the convolution operator.

Remark 1: For a flexible structure with n modes, the amplitudes $I_{p,i}$, and instances $t_{p,i}$, $i = 1, \ldots, M$, of impulse sequences for the p^{th} mode can be expressed as Eq. (4) just like Eq. (1).

$$I_{p,i} = \frac{\binom{M-1}{i-1} K_p^{i-1}}{\sum\limits_{j=0}^{M-1} \binom{M-1}{j} K_p^j}, t_{p,i} = (i-1)\frac{\pi}{\omega_p\sqrt{1-\zeta_p^2}}, K_p = e^{\frac{-\zeta_p \pi}{\sqrt{1-\zeta_p^2}}}, p = 1, \ldots, n; i = 1, \ldots, M \tag{4}$$

where ω_p and ς_p are the natural frequency and the damping ratio for the p^{th} mode, respectively.

3. Proposed approach

In this section, an adaptive sliding mode control method in combination with input shaping control is proposed for a flexible structure with n modes. After applying the proposed scheme to the flexible system, the closed-loop system will behave like the reference model with input shaper and exactly eliminate the residual vibration even in the presence of parameter uncertainties and external disturbances.

3.1. System description

The proposed scheme is shown in Fig. 1. An input shaper is implemented outside of the feedback loop, which is designed for the reference model with n modes and achieves the exact elimination of residual vibration. The amplitudes and instants of impulses used for each mode can be obtained from Eq. (4) with the natural frequency ω_{pm} and the damping ratio ζ_{pm} of the reference model. The feedback controller based on adaptive SMC is designed to make the closed-loop system behave like the reference model with input shaper and eliminate the residual vibration. This is an effective method of implementing input shaping technique for satisfactory performance and robustness when parameter uncertainties and external disturbances occur simultaneously in the process.

In Fig. 1, the output of input shaper u_r which convolves the reference input $r(t)$ with an input shaper (or a composite shaped input impulse sequence) L_{mult}, in Eq. (3) can be expressed as

$$u_r(t) = r(t) * L_{mult} \tag{5}$$

where $r(t) \in R$ represents the reference command and $*$ is the convolution operator.

Using Eq. (3) and applying convolution operator, Eq. (5) can be rewritten as

$$u_r(t) = \left(\prod_{p=1}^{n} I_{p,1}\right) r(t) + I_{1,2}\left(\prod_{p=2}^{n} I_{p,1}\right) r(t - t_{1,2}) + \ldots + \left(\prod_{p=1}^{n} I_{p,M}\right) r\left(t - \sum_{p=1}^{n} t_{p,M}\right) \tag{6}$$

or

$$u_r(t) = \sum_{k=0}^{2n-1} a_k r(t - t_k) \tag{7}$$

where

$$a_0 = \prod_{p=1}^{n} I_{p,1},\ \ a_1 = I_{1,2}\prod_{p=2}^{n} I_{p,1}, \ldots, a_{2n-1} = \prod_{p=1}^{n} I_{p,M},\ \ t_0 = 0,\ \ t_1 = t_{1,2}, \ldots,\ \text{and}\ t_{2n-1} = \sum_{p=1}^{n} t_{p,M}.$$

From Fig. 1 and using Eq. (7), the output of the reference model with n modes can be expressed as

$$\dot{x}_m(t) = A_m x_m(t) + B_m\left[\sum_{k=0}^{2n-1} a_k r(t - t_k)\right] \tag{8}$$

where $x_m(t) \in R^{2n}$ is reference model states,

$$A_m = diag\left(\begin{bmatrix} 0 & 1 \\ -\omega_{pm}^2 & -2\varsigma_{pm}\omega_{pm} \end{bmatrix}\right) \quad p = 1, \ldots, n,$$

and B_m are constant matrices of appropriate dimensions, ω_{pm} and ς_{pm} represent the nature frequency and the damping ratio of the reference model, respectively. It is assumed that the pair (A_m, B_m) is controllable.

From Fig. 1 and using Eq. (7), the output of the plant with n modes can be expressed as

$$\dot{x}(t) = Ax(t) + Bu(t) + B\left[\sum_{k=0}^{2n-1} a_k r(t - t_{dk})\right] + Bd(t) \tag{9}$$

where $x(t) \in R^{2n}$ are plant states, $u(t) \in R$ is the control input, $u(t) \in R^m$ is the external disturbance,

$$A = diag\left(\begin{bmatrix} 0 & 1 \\ -\omega_{po}^2 & -2\varsigma_{po}\omega_{po} \end{bmatrix}\right) \quad p = 1, \ldots, n$$

and B are constant matrices of appropriate dimensions, ω_{po} and ς_{po} represent the nature frequency and the damping ratio of the plant, respectively.

Remark 2: In order to make a comparison between the proposed method and the method by [18], the time of impulse application is assumed to be inaccurate, i.e. $t_{dk} \neq t_k$.

The parameter variations and input matrix uncertainty considered here are defined as follows:

$$\Delta A = A - A_m \tag{10}$$

$$\Delta B = B - B_m = B_m D_B \text{ and } \|D_B\| \leqslant \delta_B < 1 \tag{11}$$

It is noted that the uncertain matrix ΔA for multi-mode systems ($n > 1$) does not satisfy the so-called matching condition [21]. The designed sliding surface may depends on it and its performance can become intolerable one. Therefore, we will introduce a similar transformation to overcome this problem.

Define a similar transformation as

$$x_m(t) = T_m z_m(t) \tag{12}$$

Without loss of generality, the reference model Eq. (8) can be transformed into the controllable canonical form as

$$\dot{z}_m(t) = A_{mc} z_m(t) + B_{mc}\left[\sum_{k=0}^{2n-1} a_k r(t - t_k)\right] \tag{13}$$

where

$$A_{mc} = \begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \\ * & \dots & \dots & * \end{bmatrix} \text{ and } B_{mc} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}.$$

Then, the plant Eq. (9) can be rewritten as

$$\begin{aligned} \dot{z}(t) &= (A_{mc} + \Delta A_c)z(t) + B_{mc}(1 + D_B)u(t) \\ &\quad + B_{mc}(1 + D_B)\left[\sum_{k=0}^{2n-1} a_k r(t - t_{dk})\right] + B_{mc}(1 + D_B)d(t) \end{aligned} \tag{14}$$

where

$$\Delta A_c = \begin{bmatrix} 0 & \dots & \dots & 0 \\ \vdots & & & \vdots \\ 0 & \dots & \dots & 0 \\ \bar{*} & \dots & \dots & \bar{*} \end{bmatrix}.$$

It is clear to see that the parameter uncertainty ΔA_c satisfies the matching condition, i.e.

$$\Delta A_c = B_{mc} D_A. \tag{15}$$

The objective of this paper is, based on adaptive SMC, to propose a closed-loop input shaping scheme such that the closed-loop system will behave like the reference model with input shaper and exactly eliminate the residual vibration even in the presence of parameter uncertainties and external disturbances.

To facilitate further development, we define the errors between the plant and model outputs (or states) as

$$e(t) = z(t) - z_m(t) \tag{16}$$

Differentiating Eq. (16) and substituting Eqs (13) and (14) into the resulting equation yields

$$\dot{e}(t) = A_{mc} e(t) + B_{mc}(1 + D_B)u(t) + B_{mc} D_f(z, t) \tag{17}$$

where $D_f(z, t)$ is the lumped perturbation and can be expressed as

$$D_f(z, t) = D_A z(t) + D_B \sum_{k=0}^{2n-1} a_k r(t - t_{dk}) + \sum_{k=0}^{2n-1} a_k [r(t - t_{dk}) - r(t - t_k)] + (1 + D_B)d(t) \tag{18}$$

According to this lumped perturbation, it is assumed that there exist known positive constants c_0 and c_1 such that

$$\|D_f(z, t)\| \leqslant c_0 + c_1 \|z(t)\| \tag{19}$$

If it is possible to design a control law that makes the error dynamics Eq. (17) have a stable zero steady state solution, i.e. $e(t) \to 0 \Rightarrow z(t) \to z_m(t)$ or $x(t) \to x_m(t)$, then the closed-loop system will exactly eliminate the residual vibration like the reference model. Since SMC provides good ability to reject disturbances and remain robust to parameter perturbations while tracking a desired trajectory, it can be more useful for this purpose than other strategies in the literature.

3.2. Design of switching surface

In this paper, the switching function is defined as

$$s(t) = Ge(t) \tag{20}$$

where $G \in R^{1\times 2n}$.

Next, the system Eq. (17) is transformed into the regular form as

$$\dot{e}_1(t) = A_{11}e_1(t) + A_{12}e_2(t) \tag{21}$$

$$\dot{e}_2(t) = A_{21}\, e_1(t) + A_{22}\, e_2(t) + (1 + D_B)u(t) + D_f(z,t) \tag{22}$$

where

$$e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \in \begin{bmatrix} R^{2n-1} \\ R \end{bmatrix}, A_{mc} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \text{ and } B_{mc} = \begin{bmatrix} 0_{(2n-1)\times 1} \\ 1 \end{bmatrix}.$$

Without any loss of generality, consider the switching function as

$$s(t) = e_2(t) + K_s e_1(t) \tag{23}$$

where K_s is a constant matrix. Eq. (21) can be considered as a new state equation, and the new input on the switching surface $s(t) = 0$ can be described in the state feedback form as

$$e_2(t) = -K_s\, e_1(t) \tag{24}$$

Since the pair (A_m, B_m) is controllable, the gain matrix K_s can be designed by pole placement method. From Eqs (20) and (23), the matrix G can be expressed as

$$G = \begin{bmatrix} K_s & 1 \end{bmatrix} \tag{25}$$

3.3. Design of adaptive sliding mode controller

After designing the switching surface, the next phase is to design the control law such that sliding mode is reached and maintained thereafter. In this section, an appropriate control law is proposed to satisfy the reaching condition $s(t)\dot{s}(t) < -\sigma\,|s(t)|$. If the control law is selected as

$$u(t) = -(GB_{mc})^{-1}GA_{mc}e(t) - (GB_{mc})^{-1}(\eta + \sigma)sgn(s(t)) \tag{26}$$

where

$$\eta = \frac{1}{1-\delta_B}(\sigma\delta_B + \delta_B\,\|GA_{mc}e(t)\| + \|GB_{mc}\|\,(c_0 + c_1\,\|z(t)\|), \quad 1-\delta_B > 0 \text{ and } \sigma > 0, \tag{27}$$

then the reaching condition is guaranteed. This result is summarized in Theorem 1 and is presented in the Appendix.

From Theorem 1, it shows that Eq. (27) is an important condition to guarantee the satisfaction of the reaching condition. However, in practice, it is not easy to obtain the upper bound of lumped perturbation due to the complexity of structure of uncertainties. In other word, the values of constants c_0 and c_1 in Eq. (27) are difficult to be obtained. Therefore, we proposed simple adaptation laws to obtain two adaptation parameters $\hat{c}_0$ and $\hat{c}_1$ about unknown constants c_0 and c_1, respectively. Define $\hat{\eta}$ as adaptation parameter about η in Eq. (27), which can be expressed as

$$\hat{\eta} = \frac{1}{1-\delta_B}(\sigma\delta_B + \delta_B\,\|GA_{mc}e(t)\| + \|GB_{mc}\|\,(\hat{c}_0 + \hat{c}_1\,\|z(t)\|), \quad \delta_B < 1 \text{ and } \sigma > 0 \tag{28}$$

Then, the control law Eq. (26) can be modified as

$$u(t) = -(GB_{mc})^{-1}GA_{mc}e(t) - (GB_{mc})^{-1}(\hat{\eta} + \sigma)sgn(s(t)) \tag{29}$$

Now, we will estimate the adaptation parameters $\hat{c}_0$ and $\hat{c}_1$ in Eq. (28) by the adaptation laws as

$$\dot{\hat{c}}_0(t) = \alpha_0^{-1}\,\|GB_{mc}\|\;|s(t)| \tag{30}$$

$$\dot{\hat{c}}_1(t) = \alpha_1^{-1} \|GB_{mc}\| \ \|z(t)\| \ |s(t)| \tag{31}$$

where $\tilde{c}_0 = \hat{c}_0 - c_0$ and $\tilde{c}_1 = \hat{c}_1 - c_1$ are the adaptation error of each parameter , α_0 and α_1 are adaptation gains with positive constant values. Since we assume that c_0 and c_1 are constants, the adaptation parameters can be obtained by

$$\hat{c}_0 = \hat{c}_{0i} + \alpha_0^{-1} \int_{t_0}^{t} \|GB_{mc}\| \ |s| \ dt \tag{32}$$

$$\hat{c}_1 = \hat{c}_{1i} + \alpha_1^{-1} \int_{t_0}^{t} \|GB_{mc}\| \ \|z\| \ |s| \ dt \tag{33}$$

where $\hat{c}_{0i}$ and $\hat{c}_{1i}$ are the initial values of $\hat{c}_0$ and $\hat{c}_1$ respectively. By choosing appropriate $(\hat{c}_{0i}, \hat{c}_{1i})$ and (α_0, α_1), we can adjust the rate of parameter adaptation. With adaptation laws Eqs (30) and (31), the control law Eq. (29) will ensure the reaching condition is satisfied. This result is summarized in Theorem 2 and is presented in the Appendix.

Remark 3: In Theorem 2, although the adaptive upper bound of the lumped uncertainty does not converge to the real one, the switching surface maintains zero by the control law after arriving at the sliding mode as long as the adaptive upper bound is not less than real one. The adaptation parameters converge to some values depending on the values of initial conditions $(\hat{c}_{0i}, \hat{c}_{1i})$ and adaptation gains (α_0, α_1).

To eliminate the chattering behavior, the control law Eq. (29) is modified to be

$$u(t) = -(GB_{mc})^{-1}GA_{mc}e(t) - (GB_{mc})^{-1}(\hat{\eta} + \sigma)sat\left(\frac{s(t)}{\varepsilon}\right) \tag{34}$$

where ε is the boundary layer thickness.

Remark 4: Once the saturation function is used in control law Eq. (34), the switching surface $s(t)$ will not be equal to zero for all time and the adaptive parameter will slowly increase. In order to overcome this disadvantage, we can modify the adaptive laws Eqs (30) and (31) by following σ-modification algorithm [30].

$$\dot{\hat{c}}_0(t) = \begin{cases} 0 & \text{if } |s(t)| \leqslant \varepsilon \\ \alpha_0^{-1} \|GB_{mc}\| \ |s(t)| & \text{if } |s(t)| > \varepsilon \end{cases} \tag{35}$$

$$\dot{\hat{c}}_1(t) = \begin{cases} 0 & \text{if } |s(t)| \leqslant \varepsilon \\ \alpha_1^{-1} \|GB_{mc}\| \ \|z(t)\| \ |s(t)| & \text{if } |s(t)| > \varepsilon \end{cases} \tag{36}$$

4. Illustrative examples

Example 1. Consider a plant with parameter uncertainties [20]. The uncertain plant and the nominal system are expressed as

$$P_o(s) = \frac{1}{s^2 + 0.4s + 1} \text{ and } P_n(s) = \frac{1}{s^2 + 0.84s + 6},$$

respectively. The designed model is expressed as

$$P_m(s) = \frac{(s + 6)}{(s + 5)(s^2 + 0.84s + 6)}.$$

In their method, the input shaper is designed for the nominal system $P_n(s)$ instead of the designed model $P_m(s)$ which includes the nominal system and controller. The step response of the designed model to the shaped input is tracked quite accurately by the closed-loop system. However, the impulse response of the designed model to the shaped input, Fig. 2(a), clearly shows that the residual vibration is not eliminated, and the purpose of using the input shaper is lost.

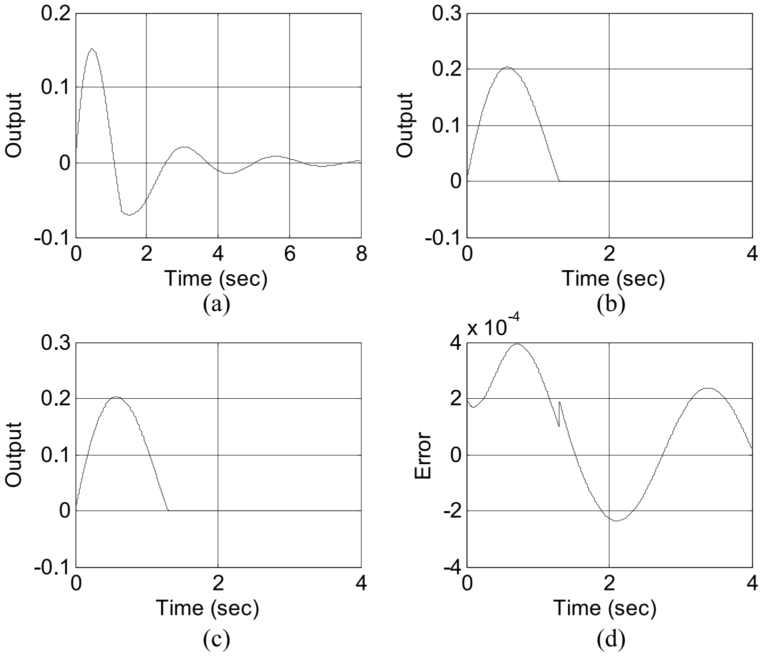


Fig. 2. (a) Impulse response of the model to the shaped input [20] (b) Response with exact cancellation (c) Response with parameter variations and external disturbance (d) Output error.

Next, the proposed approach is applied to the plant with parameter uncertainties and an external disturbance to the system $P_n(s)$. The reference model is selected as the nominal system $P_n(s)$ with model parameters $\omega_{1m} = 2.45$ rad/s and $\zeta_{1m} = 0.1715$. The reference model with the input shaper Eq. (13) can be expressed as

$$\dot{z}_m(t) = A_{mc} z_m(t) + B_{mc} a_0 r(t) + B_{mc} a_1 (t - t_1) \tag{37}$$

with

$$A_{mc} = \begin{bmatrix} 0 & 1 \\ -6 & -0.84 \end{bmatrix}, B_{mc} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, a_0 = 0.6334, a_1 = 0.3666.$$

The open-loop pre-shaped impulse response with accurate amplitudes and instants ($t_1 = 1.3018$ sec Eq. (1)) is shown in Fig. 2(b). The natural frequency and damping ratio of uncertain system $P_o(s)$ are given as $\omega_{1o} = 1$ rad/s and $\zeta_{1o} = 0.2$. The input shaped dynamic system Eq. (14) can be given by

$$\begin{aligned} \dot{z}(t) &= (A_{mc} + \Delta A_c) z(t) + B_{mc}(1 + D_B) u(t) \\ &\quad + B_{mc}(1 + D_B)[a_0 r(t) + a_1 r(t - t_1)] + B_{mc}(1 + D_B) d(t) \end{aligned} \tag{38}$$

with

$$\Delta A_c = \begin{bmatrix} 0 & 0 \\ 5 & 0.44 \end{bmatrix}, D_B = 0.1, d(t) = \sin(\sqrt{6}t)$$

which corresponds to resonance condition and a_0, a_1, t_1 are same as aforementioned values. It should be mentioned that the natural frequency and damping ratio have an error of more than 100%. Using Eq. (25), the gain matrix K_s and the switching surface gain matrix $G = \begin{bmatrix} 5 & 1 \end{bmatrix}$ can be obtained to place the eigenvalue of the reduced order system at -5. Then, the control law Eq. (34) is designed as

$$u(t) = -\begin{bmatrix} -6 & 4.16 \end{bmatrix} e(t) - (\hat{\eta} + 2) sat\left(\frac{s(t)}{0.01}\right) \tag{39}$$

where $\hat{\eta} = 1.11\ [0.2 + 0.1\ \|GA_{mc}e(t)\| + (\hat{c}_0 + \hat{c}_1 \|z(t)\|)\ ]$. Using Eqs (35) and (36), the adaptation parameters $\hat{c}_0(t)$ and $\hat{c}_1(t)$ can be obtained with $(\hat{c}_{0i}, \hat{c}_{1i}) = (5, 5)$ and $(\alpha_0, \alpha_1) = (20, 20)$, respectively. As shown in Fig. 2(c),

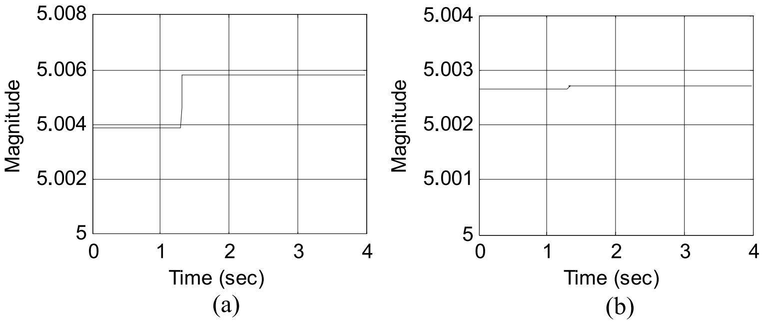


Fig. 3. (a) The adaptation parameter $\hat{c}_0(t)$ (b) The adaptation parameter $\hat{c}_1(t)$.

the response of closed-loop system behaves like that in Fig. 2(b), with exact cancellation even in the presence of both parameter variations and the resonant external disturbance. It is clearly shown that the response of the reference model to the shaped input is tracked quite accurately by the closed-loop system. Figure 2(d) shows the error between the desired and actual output. The adaptation parameters are shown in Fig. 3(a) and 3(b), respectively.

Remark 5: The purpose of using this example is to compare the results obtained from [20] and show one of advantage of the proposed method. In general, the realistic modeling errors would be in the 5–30% area.

Example 2. Consider a flexible structure with two vibratory modes [18]. The parameters of the structure are given as $\omega_{1s} = 4$ rad/sec, $\zeta_{1s} = 0.1$, $\omega_{2s} = 6$ rad/sec, and $\zeta_{2s} = 0.2$. The input shaped dynamic system Eq. (9) without parameter uncertainties and external disturbances is given by

$$\dot{x}(t) = A_s x(t) + B_s u(t) + B_s \left[\sum_{k=0}^{3} a_k r(t - t_{dk}) \right] \tag{40}$$

with

$$A_s = diag\left(\begin{bmatrix} 0 & 1 \\ -\omega_{ps}^2 & -2\zeta_{ps}\omega_{ps} \end{bmatrix} \right) \quad (p = 1, 2), B_s = \begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix}^T,$$

$a_0 = 0.3778, a_1 = 0.1955, a_2 = 0.2762, a_3 = 0.1455$. The open-loop pre-shaped impulse response with accurate amplitudes and instants ($t_1 = 0.5344$ sec, $t_2 = 0.7894$ sec and $t_3 = 1.3238$ sec which are obtained by using Eqs (4) and (7)) is shown in Fig. 4(a). The state feedback controller designed by [18] is used to improve the system performance in the presence of inaccurate impulse application instants ($t_{d1} = 0.55\ t_1$, $t_{d2} = 0.55\ t_2$ and $t_{d3} = 0.35\ t_3$). Figure 4(b) shows the response from the method by [18]. Note that the residual vibration is not eliminated.

Next, the proposed approach is applied to the plant with parameter uncertainties and external disturbance to system Eq. (40). The model parameters of the reference model are selected as $\omega_{1m} = 4$ rad/s, $\zeta_{1m} = 0.1$, $\omega_{2m} = 6$ rad/s, and $\zeta_{2m} = 0.2$. The reference model with the input shaper Eq. (13) can be expressed as

$$\dot{z}_m(t) = A_{mc} z_m(t) + B_{mc} \left[\sum_{k=0}^{3} a_k r(t - t_k) \right] \tag{41}$$

with

$$A_{mc} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -576 & -67.2 & -53.92 & -3.2 \end{bmatrix}, B_{mc} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix},$$

$a_0, a_1, a_2, a_3, t_1, t_2,$ and t_3 are same as aforementioned values. In order to show that the proposed method provides excellent robustness to parameter variation, the natural frequency and damping ratio of plant are given as $\omega_{1o} = 8$

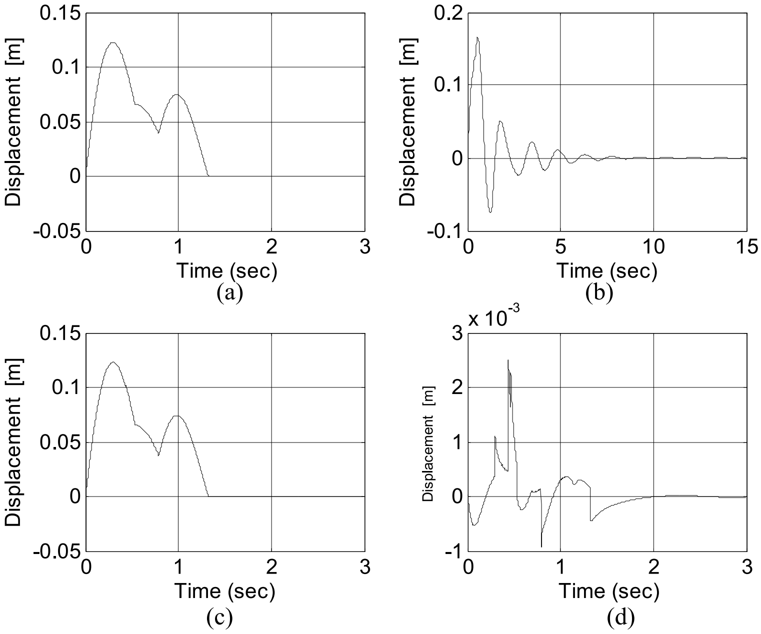


Fig. 4. (a) Response with exact cancellation (b) Response with inaccurate impulse application instance [18] (c) Response with parameter variations and external disturbance (d) Output error.

rad/s, $\zeta_{1o} = 0.2$, $\omega_{2o} = 3$ rad/s and $\zeta_{2o} = 0.1$ respectively, which the natural frequency and damping ratio have errors of 100% respectively. The input shaped dynamic system Eq. (14) is given by

$$\begin{aligned}\dot{z}(t) &= (A_{mc} + \Delta A_c)z(t) + B_{mc}(1 + D_B)u(t) \\ &\quad + B_{mc}(1 + D_B)\left[\sum_{k=0}^{3} a_k r(t - t_{dk})\right] + B_{mc}(1 + D_B)d(t)\end{aligned} \tag{42}$$

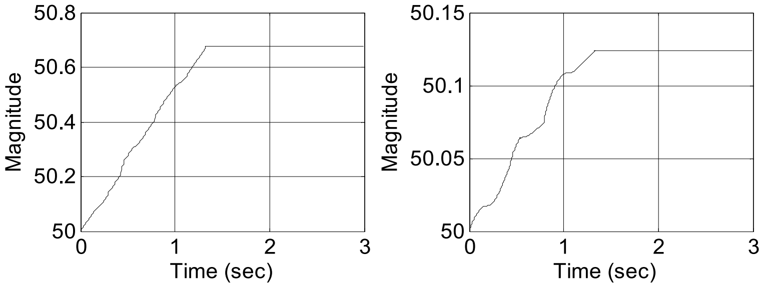
where

$$\Delta A_c = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -21 & -0.6 \end{bmatrix}, D_B = 0.1 \text{ and } d(t) = \sin(4t)$$

which corresponds to resonance condition of the first mode. According to Eq. (25), the gain matrix K_s and the switching surface gain matrix $G = \left[\ 320\ \ 152\ \ 22\ \ 1\ \right]$ can be obtained to place the eigenvalue of the reduced order system at $\left[-4\ -8\ -10\right]$. Then, the control law Eq. (34) can be designed as

$$u(t) = -\left[\ -576\ \ 252.8\ \ 98.08\ \ 18.8\ \right]\ e(t) - (\hat{\eta} + 2)sat\left(\frac{s(t)}{0.03}\right) \tag{43}$$

where $\hat{\eta} = 1.11\ [0.2 + 0.1\ \|GA_{mc}e(t)\| + (\hat{c}_0 + \hat{c}_1\ \|z(t)\|\ )]$. Using Eqs (35) and (36), two adaptation parameters $\hat{c}_0$ and $\hat{c}_1$ can be obtained when the design parameters are set as $(\hat{c}_{0i}, \hat{c}_{1i}) = (50, 50)$ and $(\alpha_0, \alpha_1) = (20, 20)$, respectively. The result from numerical simulation confirms the stability of the system Eq. (17). In Fig. 4(c), the system performs satisfactorily even in the presence of the large parameter variations and external disturbance. The reference model response to the shaped input is tracked quite accurately by the closed-loop system in case of unknown upper bound of lumped perturbation. Figure 4(d) shows the output error. The adaptation parameters are shown in Fig. 5(a) and 5(b), respectively. It clearly shows that the proposed technique has excellent vibration reduction and robustness properties in the presence of both parameter variations and the resonant external disturbance.



(a) (b)

Fig. 5. (a) The adaptation parameter $\hat{c}_0(t)$ (b) The adaptation parameter $\hat{c}_1(t)$.

5. Conclusions

In this paper, it has been shown a control scheme that incorporates input shaping method with adaptive SMC provides good performance and robustness even in the presence of both large variations in plant parameters and external disturbances. It has been shown that increasing the robustness to parameter uncertainties does not lengthen the duration of the impulse sequence. Also, the stability of closed-loop system is guaranteed in case of unknown upper bounds on norm of uncertainties. Simulation studies have confirmed the validity of the proposed closed-loop input shaping control scheme.

Appendix

Theorem 1: Consider the error dynamics Eq. (17). If the control law is selected as

$$u(t) = -(GB_{mc})^{-1}GA_{mc}e(t) - (GB_{mc})^{-1}(\eta + \sigma)sgn(s(t)) \tag{44}$$

where

$$\eta = \frac{1}{1-\delta_B}(\sigma\delta_B + \delta_B \left\|GA_{mc}e(t)\right\| + \left\|GB_{mc}\right\|(c_0 + c_1\left\|z(t)\right\|), \quad 1-\delta_B > 0 \text{ and } \sigma > 0, \tag{45}$$

then the reaching condition is guaranteed.

Proof: Differentiating Eq. (20) and using Eq. (17)

$$\dot{s}(t) = GA_{mc}e(t) + GB_{mc}(1 + D_B)u(t) + GB_{mc}D_f(z,t) \tag{46}$$

Substituting Eq. (44) into Eq. (46)

$$\begin{aligned}\dot{s}(t) = GA_{mc}e(t) - GB_{mc}(1 + D_B)(GB_{mc})^{-1}GA_{mc}e(t) \\ -GB_{mc}(1 + D_B)(GB_{mc})^{-1}(\eta + \sigma)sgn(s(t)) + GB_{mc}D_f(z,t)\end{aligned} \tag{47}$$

or

$$\dot{s}(t) = -D_BGA_{mc}e(t) - (\eta + \sigma)sgn(s(t)) - D_B(\eta + \sigma)sgn(s(t)) + GB_{mc}D_f(z,t) \tag{48}$$

Pr-multiplying Eq. (48) by $s(t)$, we obtain

$$\begin{aligned}s(t)\dot{s} &= -\sigma|s(t)| - \eta|s(t)| - (\eta + \sigma)D_B|s(t)| - sD_BGA_{mc}e(t) + sGB_{mc}D_f(z,t) \\ &< -\sigma|s(t)| - |s(t)|\{\eta(1-\delta_B) - (\sigma\delta_B + \delta_B||GA_{mc}e(t)|| + ||GB_{mc}||||D_f(z,t)||)\}\end{aligned} \tag{49}$$

Substituting Eq. (19) into Eq. (49)

$$s(t)\dot{s}(t) < -\sigma|s(t)| - |s(t)|\{\eta(1-\delta_B) - [\sigma\delta_B + \delta_B||GA_{mc}e(t)|| + ||GB_{mc}||(c_0 + c_1||z(t)||)] \tag{50}$$

Using Eq. (45), we can obtain

$$s(t)\dot{s}(t) < -\sigma|s(t)| \tag{51}$$

It is clear that the reaching condition is achieved as Eq. (45) is satisfied. The proof is completed.

Theorem 2: Consider the error dynamics Eq. (17). If the control law Eq. (29) with adaptation laws Eqs (30) and (31) is used, then the reaching condition is satisfied.

Proof: Choose the Lyapunov function as

$$V(s,t) = \frac{1}{2}(s^T s + \alpha_0 \tilde{c}_0^2 + \alpha_1 \tilde{c}_1^2) \tag{52}$$

Then, taking the derivative of $V(s,t)$ along the trajectories of Eq. (17)

$$\begin{aligned} \dot{V}(s,t) &= s\dot{s} + \alpha_0 \tilde{c}_0 \dot{\tilde{c}}_0 + \alpha_1 \tilde{c}_1 \dot{\tilde{c}}_1 \\ &= s\,[GA_{mc}e + GB_{mc}(1+D_B)u + GB_{mc}D_f] + \alpha_0 \tilde{c}_0 \dot{\tilde{c}}_0 + \alpha_1 \tilde{c}_1 \dot{\tilde{c}}_1 \end{aligned} \tag{53}$$

Substituting Eqs (19), (28) and (29) into Eq. (53)

$$\begin{aligned} \dot{V}(s,t) &\leqslant -\sigma\,|s| - |s|\ \{\,\hat{\eta}(1-\delta_B) - [\sigma\delta_B + \delta_B\,\|GA_{mc}e\| + \|GB_{mc}\|\ (c_0 + c_1\,\|z\|)\ ]\,\} + \alpha_0 \tilde{c}_0 \dot{\tilde{c}}_0 + \alpha_1 \tilde{c}_1 \dot{\tilde{c}}_1 \\ &= -\sigma\,|s| - |s|\ \{\,\|GB_{mc}\|\ (\tilde{c}_0 + \tilde{c}_1\,\|z\|)\} + \alpha_0 \tilde{c}_0 \dot{\tilde{c}}_0 + \alpha_1 \tilde{c}_1 \dot{\tilde{c}}_1 \\ &= -\sigma\,|s| + \tilde{c}_0(\alpha_0 \dot{\tilde{c}}_0 - \|GB_{mc}\|\ |s|) + \tilde{c}_1(\alpha_1 \dot{\tilde{c}}_1 - \|GB_{mc}\|\ \|z\|\ |s|) \end{aligned} \tag{54}$$

Taking Eqs (30) and (31) as the adaptation laws, we can obtain

$$\dot{V}(s,t) \leqslant -\sigma\,|s| < 0 \text{ for } s \neq 0 \tag{55}$$

Since $\sigma > 0$, $s \to 0$ as $t \to \infty$. The proof is completed.

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