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Thermo-mechanical vibration of short carbon nanotubes embedded in pasternak foundation based on nonlocal elasticity theory

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Abstract. This study is concerned with the thermal vibration analysis of a short single-walled carbon nanotube embedded in an elastic medium based on nonlocal Timoshenko beam model. A Winkler- and Pasternak-type elastic foundation is employed to model the interaction of short carbon nanotubes and the surrounding elastic medium. Influence of all parameters such as nonlocal small-scale effects, high temperature change, Winkler modulus parameter, Pasternak shear parameter, vibration mode and aspect ratio of short carbon nanotubes on the vibration frequency are analyzed and discussed. The present study shows that for high temperature changes, the effect of Winkler constant in different nonlocal parameters on nonlocal frequency is negligible. Furthermore, for all temperatures, the nonlocal frequencies are always smaller than the local frequencies in short carbon nanotubes. In addition, for high Pasternak modulus, by increasing the aspect ratio, the nonlocal frequency decreases. It is concluded that short carbon nanotubes have the higher frequencies as compared with long carbon nanotubes.

Keywords: Short carbon nanotubes, vibration, pasternak effect, high temperature change, nonlocal timoshenko beam

1. Introduction

After seminal work of Iijima [1], many researchers have great interests on dynamic analysis of carbon nanotubes (CNTs), because of their exceptional mechanical, electrical and thermal properties [2–4]. These properties of CNTs lead to its application in the fields of nano-electronics, nano-devices, nano-composites, etc. It has been shown that the CNTs with extremely high elastic modulus and low mass density can serve as terahertz nano-resonators [5–8] in nano-electro-mechanical systems (NEMS). For example, they are thermally stable up to 2800°C in vacuum, with a thermal conductivity which is twice as large as diamond, and having an electric- current- carrying about 1000 times greater than copper wire [9].

On the other hand, most applications tend to shrink the dimensions of nano-scale devices. Researchers show that short nanotubes with open ends are required to overpass the diffusion limitation [10]. Wang et al. [11] studied the electro-chemical behavior of ultra-short carbon nanotubes. They concluded that compared with conventional long CNTs, short CNTs show much better electro-chemical performances. Lopez et al. [12] discussed structural and thermal stability of short carbon nanotubes. Seidel et al. [13] showed that short CNTs, with the lengths less than

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20 nanometers, are useful in molecular electronics and CNT field-effect transistors (CNTFETs). Yoon et al. [14] studied the vibration behavior of multi-walled carbon nanotubes embedded in an elastic medium using multiple-elastic beam model. Zhang et al. [15] developed a double-elastic beam model for transverse vibration of double-walled carbon nanotubes (DWCNTs) under axial compressive load using Euler-Bernoulli beam theory. A nonlocal elastic model for static and dynamic analysis of carbon nanotubes embedded in two-parameter Pasternak foundation applied by Ref. [16–19]. In fact, the Winkler-type elastic foundation is approximated as a series of closely spaced, mutually independent, vertical linear elastic springs. The interaction between the springs is ignored in Winkler-type foundations. Therefore, a more realistic and generalized representation of the elastic medium can be accomplished by a two-parameter foundation model [20]. However, only a limited portion of literature is concerned with the vibration analysis of carbon nanotubes considering the thermal effect and Pasternak foundation.

Moreover, the size-dependent nonlocal continuum theory is used because at small-scale length, the material microstructures, such as lattice spacing between individual atoms, become increasingly significant and thus its effect can no longer be ignored [21]. In addition, most of the previous studies treated a nanotube as a simple Euler beam and neglected the effects of transverse shear deformation and rotary inertia. In fact, the Euler-Bernoulli beam models holds only when the length of the beam is much larger than its thickness (i.e., more than 10 times). However, in many applications the length of the nanotube is insufficiently long to be simplified as an Euler beam and the influence of shear deformation and rotary inertia should be taken into account [22].

In this paper, we present a nonlocal Timoshenko beam model and derive all governing equations and boundary conditions for vibration analysis of short carbon nanotubes, using Hamilton's principle. The surrounding elastic medium is described as the Winkler- and Pasternak type foundation. Therefore, an analytically solution is used to obtain the natural frequency of short carbon nanotubes with immoveable supports. Finally, the influences of nonlocal parameter, Winkler and Pasternak shear modulus parameter, high temperature change, aspect ratio and vibration mode on vibration of short carbon nanotubes are discussed.

2. Nonlocal nanobeam model for linear analysis of short CNTs

The nonlocal elasticity theory is developed by Eringen [23,24] and Eringen and Edelen [25]. According to theory of nonlocal elasticity, the stress at a point x in a body depends not only on the strain at point x (hyper elastic case) but also on those at all other points of the body. Thus the nonlocal stress tensor σ at point x is expressed as

$$\sigma_{ij} = \int_V \lambda\left(|x' - x|, \tau\right) \varepsilon_{kl}(x') C_{ijkl} dV(x'), \tag{1}$$

$$\left(1 - \tau^2 l^2 \nabla^2\right) \sigma = t, \tau = \frac{e_0 a}{l}. \tag{2}$$

The terms σ_{ij}, ε_{kl} and C_{ijkl} are the stress, strain and fourth order elasticity tensor, respectively. $\lambda(|x' - x|, \tau)$ is the Kernel function or nonlocal modulus or attenuation function incorporating into constitutive equations, $|x' - x|$ represents the distance in Euclidean form, and τ is the material constant that depends on the internal (e.g. lattice parameter, granular size, distance between C-C bonds) and external characteristic length (e.g. wave length). The values of a and l are the internal and external lengths, respectively, and classical stress tensor is defined as $t = C : \varepsilon$, where ':' represents the double dot product. The parameter e_0 is estimated such that the relations of the nonlocal elasticity model could provide satisfactory approximation of atomic dispersion curves of plane waves with those of atomic lattice dynamics [26]. In this regard, Zhang et al. [27] performed analysis of elastic interactions between Stone-Wales and divacancy defects on carbon graphene sheets. They concluded that the displacement field around defects obtained from the nonlocal continuum models and MD can match very well if e_0 is chosen to be 8.79. Duan et al. [28] reported the value of e_0 ranging from 0–19 for carbon nanotubes with nonlocal Timoshenko beam theory and using MD results. Wang studied the wave propagation in carbon nanotubes with two nonlocal continuum mechanics models: elastic Euler-Bernoulli and Timoshenko beam models and proposed $e_0 = 1\sim 14$ [29]. Shen and Zhang [30] considered e_0 in the range of 3 to 5.1 and estimated small scale effect parameter by matching the buckling torque of CNTs observed from the MD simulation results with the numerical results obtained from the nonlocal shear deformable shell model. Chan and Zhao [31] reported $e_0 = 0.23$ by considering nonlocal elasticity as an important factor in the spinning CNTs. On the other hand, Khademolhosseini et al. [32] proposed $e_0 = 0.18$ through comparison of the MD simulation results with classical and nonlocal dispersion relations. Wang et al. [33] proposed that $e_0 = 0.288$ be used in determination of the dispersion curves via elastic beam theories and the Molecular Dynamics method. In addition, Eringen [23] proposed e_0 as 0.39. On the other hand, Eringen also proposed $e_0 = 0.31$ based on the comparison of Rayleigh surface wave via nonlocal continuum mechanics and lattice dynamics. Zhang et al. [34] approximated that $e_0 = 0.82$ by matching the theoretical buckling strain obtained by the nonlocal elastic cylindrical shell model. It is clear that a large range of values for scale parameter, e_0a, is possible. The above mentioned studies indicate that reasonable choice of the value of the parameter e_0a is crucial to ensure the validity of the nonlocal models. Although, e_0 is a key parameter in the nonlocal elasticity theory, there is hitherto no rigorous study being made on estimating the scaling parameter for various physical problems. Therefore, more works, especially experimental tests, are required to determine e_0 more accurately for CNTs. In this study, the small scale coefficients were taken as $e_0a = 0.0, 0.5, 1.0, 1.5, 2.0$ (nm) for carbon nanotubes as described by Ref. [35].

3. Formulations

The displacement field equation based on Timoshenko beam theory is given as

$$u_1(x, y, z, t) = u(x, t) + z\varphi(x, t), \tag{3a}$$

$$u_2(x, y, z, t) = 0, \tag{3b}$$

$$u_3(x, y, z, t) = w(x, t), \tag{3c}$$

where u_1 and u_3 are the axial and transverse displacement of the point $(x, 0)$ on the mid-plane (i.e., $z = 0$) of the beam and $\varphi(x, t)$ denotes the rotation of the cross-section beam. The nonzero strains according to Timoshenko beam theory are expressed as

$$\varepsilon_{xx} = \frac{\partial u(x,t)}{\partial x} + z\frac{\partial \varphi(x,t)}{\partial x}, \varepsilon_{xz} = \frac{\partial w(x,t)}{\partial x} + \varphi(x,t), \tag{4}$$

where ε_{xx} and ε_{xz} are the axial and shear strain, respectively. The equations of motion of the nonlocal SWCNTs embedded in an elastic medium can be derived from the Hamilton's principle

$$\delta \int_0^{t_1} [K - (U + V)]\, dt = 0. \tag{5}$$

The strain energy of beam, U is given by

$$U = \frac{1}{2}\int_0^L \left\{ N\frac{\partial u(x,t)}{\partial x} + M\frac{\partial \varphi(x,t)}{\partial x} + Q\left(\frac{\partial w(x,t)}{\partial x} + \varphi(x,t)\right)\right\} dx, \tag{6}$$

where the normal resultant force N, bending moment M and transverse shear force Q are calculated from

$$N = \int_A \sigma_{xx} dA, \quad M = \int_A \sigma_{xx} z dA, \quad Q = \int_A \sigma_{xz} dA. \tag{7}$$

The general form of kinetic energy comes in the form below

$$K = \int_0^L \left\{ \frac{\rho A}{2}\left(\frac{\partial u(x,t)}{\partial t}\right)^2 + \frac{\rho I}{2}\left(\frac{\partial \varphi(x,t)}{\partial t}\right)^2 + \frac{\rho A}{2}\left(\frac{\partial w(x,t)}{\partial t}\right)^2 \right\} dx. \tag{8}$$

where ρ is the mass density of beam material, A is the beam's cross sectional area (circular-cross section) and I is the second moment of area about y-axis. The potential energy is equal to work done by external forces and is given by

$$V = W_E = -\frac{1}{2}\int_0^L \left\{ f(x,t)u(x,t) + q(x,t)w(x,t) + \bar{N}\left(\frac{\partial w(x,t)}{\partial x}\right)^2 + f_e w(x,t) \right\} dx \tag{9}$$

Indeed, the Eq. (9) is extended form of potential energy in Ref. [36]. In this equation, the effects of thermal field and two-parameter elastic medium are considered. In above equation, negative sign indicates that work is done on the body, $f(x,t)$ and $q(x,t)$ are the axial and transverse distributed forces (measured per unit length), $\bar{N}$ is applied compressive force and f_e is the density of reaction force of elastic foundation and expressed as

$$f_e = K_W w(x,t) - K_G \frac{\partial^2 w(x,t)}{\partial x^2} \tag{10}$$

The terms K_W and K_G represent the Winkler and shear modulus (shear layer foundation stiffness) of the elastic medium, respectively. By using calculus of variation and substituting Eqs (6), (8) and (9) into Eq. (5), the Hamilton's principle can be represented as

$$\begin{aligned} 0 = \int_0^t \int_0^L &\left\{ m_0 \left(\frac{\partial u}{\partial t}\frac{\partial \delta u}{\partial t} + \frac{\partial w}{\partial t}\frac{\partial \delta w}{\partial t} \right) + m_2 \frac{\partial \varphi}{\partial t}\frac{\partial \delta \varphi}{\partial t} - N\frac{\partial \delta \varphi}{\partial x} - M\frac{\partial \delta w}{\partial x} \right. \\ &\left. - Q\left(\frac{\partial \delta w}{\partial x} + \partial \delta \varphi \right) + f(x,t)\delta u + q(x,t)\delta w + \bar{N}\left(\frac{\partial w}{\partial x}\frac{\partial \delta w}{\partial x} \right) + f_e \delta w \right\} dxdt. \end{aligned} \tag{11}$$

The mass inertia m_0 and m_2 are defined by

$$m_0 = \int_A \rho dA = \rho A, \quad m_2 = \int_A \rho z^2 dA = \rho I. \tag{12}$$

With integrating by parts of Eq. (11) setting the coefficient of δu, δw and $\delta\varphi$ to zero leads to equations of motion

$$\frac{\partial N}{\partial x} + f(x,t) = m_0 \frac{\partial^2 u}{\partial t^2}, \tag{13a}$$

$$\frac{\partial Q}{\partial x} + q(x,t) - K_W w + K_G \frac{\partial^2 w}{\partial x^2} - \bar{N}\frac{\partial^2 w}{\partial x^2} = m_0 \frac{\partial^2 w}{\partial t^2}, \tag{13b}$$

$$\frac{\partial M}{\partial x} - Q = m_2 \frac{\partial^2 \varphi}{\partial t^2}. \tag{13c}$$

It is assumed that the axial and transverse distributed forces are equal to zero

$$f(x,t) = q(x,t) = 0. \tag{14}$$

Differentiating Eq. (13c) once related to x and substituting into Eq. (13b), we obtain the nonlocal bending moment, M, and shear force, Q, in the Timoshenko beam theory

$$M = EI\frac{\partial \varphi}{\partial x} + \mu\left[\bar{N}\frac{\partial^2 w}{\partial x^2} + K_W w - K_G \frac{\partial^2 w}{\partial x^2} + m_0 \frac{\partial^2 w}{\partial t^2} + m_2 \frac{\partial^3 \varphi}{\partial x \partial t^2} \right], \tag{15}$$

$$Q = K_S GA\left(\frac{\partial w}{\partial x} + \varphi \right) + \mu \frac{\partial}{\partial x}\left[\bar{N}\frac{\partial^2 w}{\partial x^2} + K_W w - K_G \frac{\partial^2 w}{\partial x^2} + m_0 \frac{\partial^2 w}{\partial t^2} \right]. \tag{16}$$

where E is the Young's modulus, G is the shear modulus or modulus of rigidity and K_s is the Timoshenko's shear correction that accounts for non-uniform shear stress distribution through the thickness of the beam. By substituting Eqs (15) and (16) into Eq. (11), we obtain the complete form of equations of motion

$$EA\frac{\partial^2 u}{\partial x^2} + \mu m_0 \left(\frac{\partial^4 u}{\partial x^2 \partial t^2}\right) = m_0 \frac{\partial^2 u}{\partial t^2}. \tag{17a}$$

$$EI\frac{\partial^2 \varphi}{\partial x^2} - K_S GA\left(\frac{\partial w}{\partial x} + \varphi\right) + \mu m_2 \frac{\partial^4 \varphi}{\partial x^2 \partial t^2} = m_2 \frac{\partial^2 \varphi}{\partial t^2}. \tag{17b}$$

$$\begin{aligned}&\frac{\partial}{\partial x}\left[K_S GA\left(\frac{\partial w}{\partial x} + \varphi\right)\right] + K_W w - K_G \frac{\partial^2 w}{\partial x^2} - \bar{N}\frac{\partial^2 w}{\partial x^2} \\ &\quad + \mu \frac{\partial^2}{\partial x^2}\left[\bar{N}\frac{\partial^2 w}{\partial x^2} + K_W w - K_G \frac{\partial^2 w}{\partial x^2} + m_0 \frac{\partial^2 w}{\partial t^2}\right] = m_0 \frac{\partial^2 w}{\partial t^2}.\end{aligned} \tag{17c}$$

The boundary conditions of Eqs (17a–c) are written as follows:

$$\left(EA\frac{\partial u}{\partial x} + \mu m_0 \ \frac{\partial^3 u}{\partial x \partial t^2}\right)\delta u\Big|_0^L = 0, \tag{18}$$

$$\left\{K_S GA\left(\frac{\partial w}{\partial x} + \varphi\right) - \bar{N}\frac{\partial w}{\partial x} + \mu\frac{\partial}{\partial x}\left[\bar{N}\frac{\partial^2 w}{\partial x^2} + K_W w - K_G \frac{\partial^2 w}{\partial x^2} + m_0 \ \frac{\partial^2 w}{\partial t^2}\right]\right\}\delta\varphi\Big|_0^L = 0, \tag{19}$$

$$\left\{EI\frac{\partial \varphi}{\partial x} + \mu\left[\bar{N}\frac{\partial^2 w}{\partial x^2} + K_W w - K_G \frac{\partial^2 w}{\partial x^2} + m_0 \frac{\partial^2 w}{\partial t^2} \ + m_2 \frac{\partial^3 \varphi}{\partial x \partial t^2}\right]\right\}\delta w\Big|_0^L = 0. \tag{20}$$

Here, $\bar{N}$ represents the axial force on the CNTs and is expressed as

$$\bar{N} = N_m + N_\theta \tag{21}$$

where N_m is the axial force due to the mechanical loading prior to buckling and N_θ is the axial force due to the influence of temperature change. Here, the theory of thermal elasticity mechanics is adopted because the Young's modulus of SWCNT is insensitive to temperature change in the tube at temperatures of less than approximately $1100^\circ K$, but decreases at high temperature [37]. In addition, the high thermal conductivity of CNTs leads to the uniform and constant axial force, N_θ, as below [38]

$$N_\theta = -\frac{EA}{1 - 2\upsilon}\alpha_x \theta, \tag{22}$$

where α_x is the coefficient of thermal expansion in the direction of x-axis, υ is the Poisson's ratio and θ denotes the change in temperature. Here changes for high temperature environment will be considered. By considering the boundary conditions for immoveable supports, $u\,(0, t) = u\,(L, t) = 0$, the axial force due to mechanical loading will be zero.

4. Analytical solution for vibration of short carbon nanotubes

By the application of the separation of variables, we can assume periodic solutions of the form $\varphi(x, t) = \phi(x)e^{i\omega t}$ and $w(x, t) = W(x)e^{i\omega t}$ for vibration analysis of short carbon nanotubes, where $\phi(x)$ and $W(x)$ are the mode shapes and ω is the frequency of natural vibration [36]. Therefore, with substituting $\varphi(x, t)$ and $w(x, t)$ into Eqs (17b) and (17c), we obtain

$$\frac{d}{dx}\left(EI\frac{d\phi}{dx}\right) - K_S GA\left(\frac{dW}{dx} + \phi\right) + m_2\omega^2\left(\phi - \mu\frac{d^2\phi}{dx^2}\right) = 0. \tag{23}$$

$$\begin{aligned}&\frac{d}{dx}\left[K_S GA\left(\frac{dW}{dx}+\phi\right)\right]+K_W W-K_G\frac{d^2W}{dx^2}-\bar{N}\frac{d^2W}{dx^2}+\mu\bar{N}\frac{d^4W}{dx^4}+\mu K_W\frac{d^2W}{dx^2}\\&\quad-\mu K_G\frac{d^4W}{dx^4}-\mu m_0\omega^2\frac{d^2W}{dx^2}+m_0\omega^2 W=0.\end{aligned} \tag{24}$$

Differentiating Eq. (24) once, substituting for $\frac{d\phi}{dx}$ and by some simplifications, we obtain

$$A\frac{d^6W}{dx^6}+B\frac{d^4W}{dx^4}+C\frac{d^2W}{dx^2}+DW=0, \tag{25}$$

where

$$\begin{aligned}A&=\mu\left(EI-\mu m_2\omega^2\right)\left(\frac{\bar{N}-K_G}{K_S GA}\right),\\B&=\left(EI-\mu m_2\omega^2\right)\left(1-\frac{\bar{N}+K_G-\mu K_W+\mu m_0\omega^2}{K_S GA}\right)\\&\quad+\mu\left(\frac{\bar{N}-K_G}{K_S GA}\right)\left(m_2\omega^2-K_S GA\right),\\C&=\left(m_2\omega^2-K_S GA\right)\left(1-\frac{\bar{N}+K_G-\mu K_W+\mu m_0\omega^2}{K_S GA}\right)+K_S GA\\&\quad+\left(EI-\mu m_2\omega^2\right)\left(\frac{m_0\omega^2+K_W}{K_S GA}\right),\\D&=\left(m_2\omega^2-K_S GA\right)\left(\frac{m_0\omega^2+K_W}{K_S GA}\right).\end{aligned} \tag{26}$$

It is noticeable that by ignoring the thermal and elastic medium parameters, the governing differential equation in Ref. [36] is derived. Considering the boundary conditions for simply-supported short-SWCNT with immoveable ends as

$$w(0,t)=w(L,t)=0,\quad M=0 \text{ at } x=0 \text{ and } x=L. \tag{27}$$

By substituting $\varphi(x,t)$ and $w(x,t)$ into Eqs (19) and (20), the natural boundary conditions for linear vibration are derived as below

$$\left\{K_S GA\left(\frac{dW}{dx}+\phi\right)-\bar{N}\frac{dW}{dx}+\mu\bar{N}\frac{d^3W}{dx^3}-\ \mu m_0\omega^2\frac{dW}{dx}\right\}\Bigg|_0^L=0. \tag{28a}$$

$$\left\{EI\frac{d\phi}{dx}+\mu\left[\bar{N}\frac{d^2W}{dx^2}+K_W W-K_G\frac{d^2W}{dx^2}-m_0\omega^2 W\ -m_2\omega^2\frac{d\phi}{dx}\right]\right\}\Bigg|_0^L=0. \tag{28b}$$

The general solution can be considered as follows:

$$W(x)=\sum_{m=1}^{\infty}\sin\frac{m\pi}{L}x, \tag{29}$$

where m is the mode number. The above solution can satisfy all boundary conditions. With substituting Eqs (26) and (29) into Eq. (25), we calculate the natural frequencies for different cases. The frequency equation in general

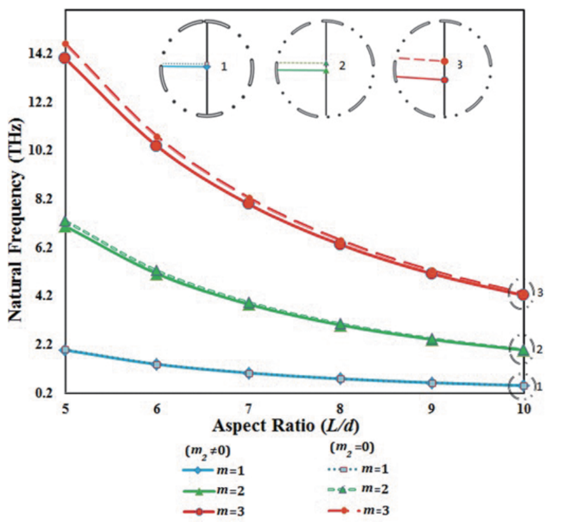

Fig. 1. Variation of natural frequency with aspect ratio L/d for different change in mode numbers, m, with/without rotary inertia, m_2, for short-SWCNT, ($K_W = K_G = \bar{N} = 0$).

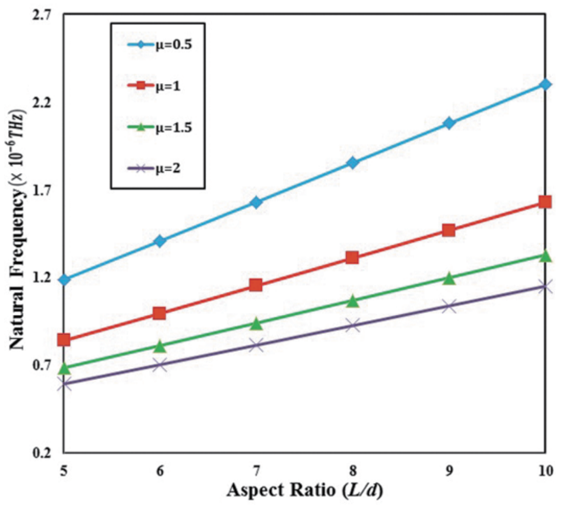

Fig. 2. Change of natural frequency with aspect ratio L/d for different change in nonlocal parameter.

form is written as follows:

$$\begin{aligned}
&-\left[\mu\left(EI-\mu m_2\omega^2\right)\left(\frac{\bar{N}-K_G}{K_SGA}\right)\right]\left(\frac{m\pi}{L}\right)^6 \\
&+\left[\left(EI-\mu m_2\omega^2\right)\left(1-\frac{\bar{N}+K_G-\mu K_W+\mu m_0\omega^2}{K_SGA}\right)\right. \\
&\left.+\mu\left(\frac{\bar{N}-K_G}{K_SGA}\right)\left(m_2\omega^2-K_SGA\right)\right]\left(\frac{m\pi}{L}\right)^4-\left[\left(EI-\mu m_2\omega^2\right)\left(\frac{m_0\omega^2+K_W}{K_SGA}\right)\right. \\
&\left.+K_SGA+\left(m_2\omega^2-K_SGA\right)\times\left(1-\frac{\bar{N}+K_G-\mu K_W+\mu m_0\omega^2}{K_SGA}\right)\right]\left(\frac{m\pi}{L}\right)^2 \\
&+\left(m_2\omega^2-K_SGA\right)\left(\frac{m_0\omega^2+K_W}{K_SGA}\right)=0.
\end{aligned} \tag{30}$$

5. Numerical results

Here, we present numerical solutions for the vibration of short-SWCNTs, considering the effects of thermal field and Pasternak elastic medium. The following values of effective properties are used [36]

$$\rho = 2300\ \text{Kg/m}^3, E = 1000\ \text{Gpa}, d = 1\times 10^{-9}\ \text{m}, K_S = 0.877, \upsilon = 0.19, G = 420\ \text{Gpa} \tag{31}$$

Here, we consider the aspect ratios (length-to-diameter, L/d) of short carbon nanotubes in the range of 5 to 10. In the following, five different cases are studied in order to consider the different parameters:

Case (1). $\boldsymbol{\mu = 0, K_G = 0, K_W = 0, \bar{N} = 0}$

In this case, the natural frequency for local problem is calculated. The effect of aspect ratio and rotary inertia in local frequency is shown in Fig. 1. It depicts that with increasing in aspect ratio (L/d), the natural frequencies decrease but with increasing in mode number (m), the natural frequencies increase. Moreover, the natural frequencies

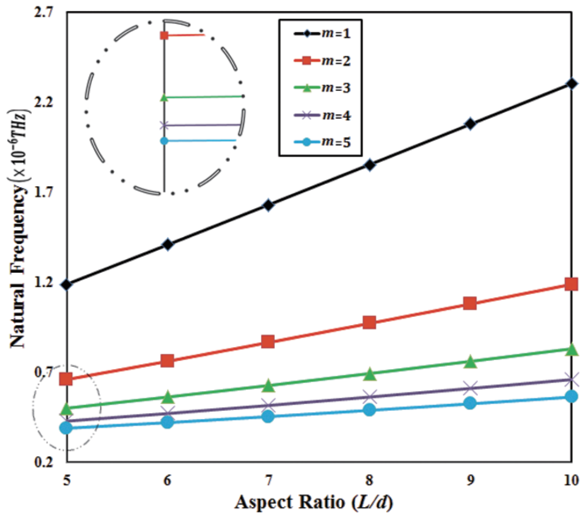

Fig. 3. Mode number effect on natural frequency of a short-SWCNT for different aspect ratio L/d and $\mu = 0.5$.

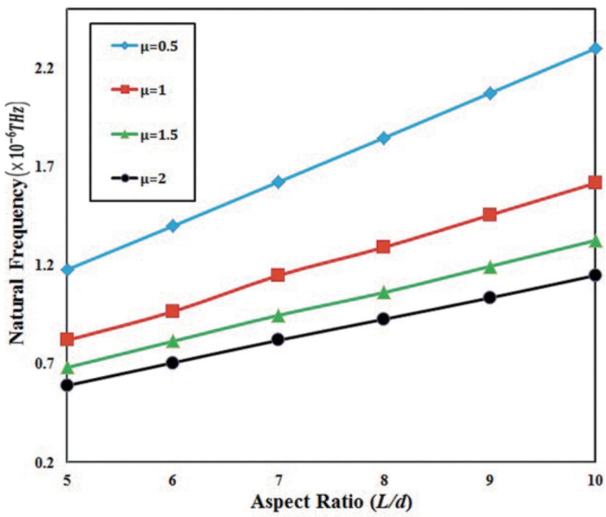

Fig. 4. Variation of natural frequency with aspect ratio L/d for different nonlocal parameter, ($K_W = 10^3$ N/m^2, $m = 1$).

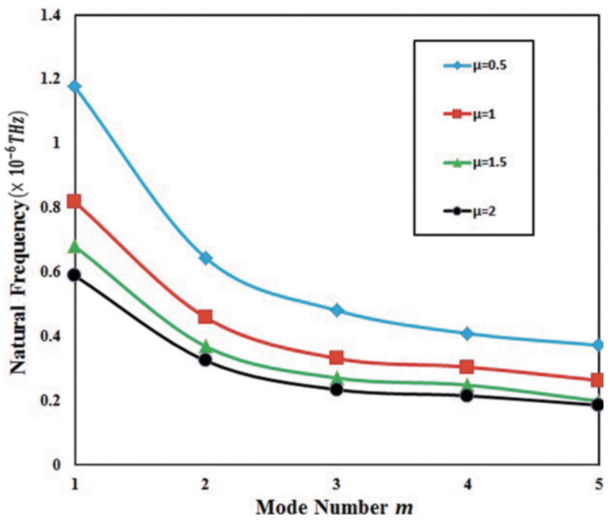

Fig. 5. The effect of mode number on natural frequency for different nonlocal parameter μ, ($K_W = 10^3$ N/m^2, $L/d = 5$).

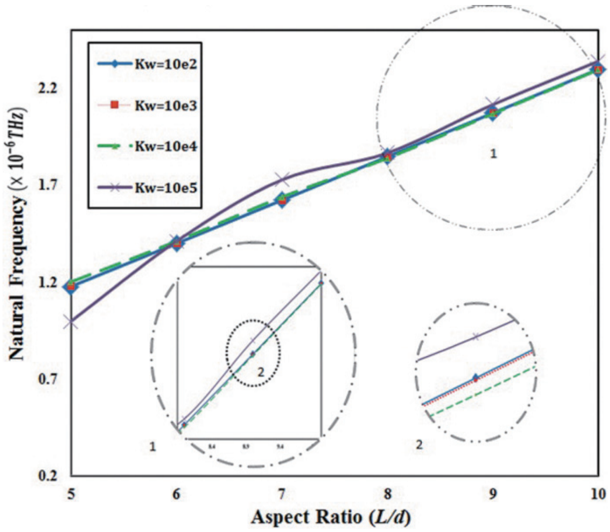

Fig. 6. The effect of Winkler modulus K_W on natural frequency with different aspect ratio for ($m = 1, \mu = 0.5$).

in the presence of rotary inertia are less than its counterpart without the effect of rotary inertia. Therefore, rotary inertia leads to diminish the amount of natural frequencies. This is the most basic case that has been considered.

Case (2). $\boldsymbol{K_G = 0, K_W = 0, \bar{N} = 0}$

In nonlocal cases in which the nonlocal parameter is considered, with increasing in aspect ratio, the amount of natural frequencies increase but it is lower than the local values (case1). The Winkler and shear modulus parameters are equal to zero. Also the effect of thermal field is not considered. Figure 2 shows the variation of natural frequencies for different nonlocal parameter. It is clear that increasing in nonlocal parameter (μ) in different aspect ratio leads to reducing natural frequency. In Fig. 3, the change of mode number with different aspect ratio has been considered. The nonlocal parameter is constant.

It can be shown that by increasing in mode number, unlike the previous case, the natural frequencies significantly reduce.

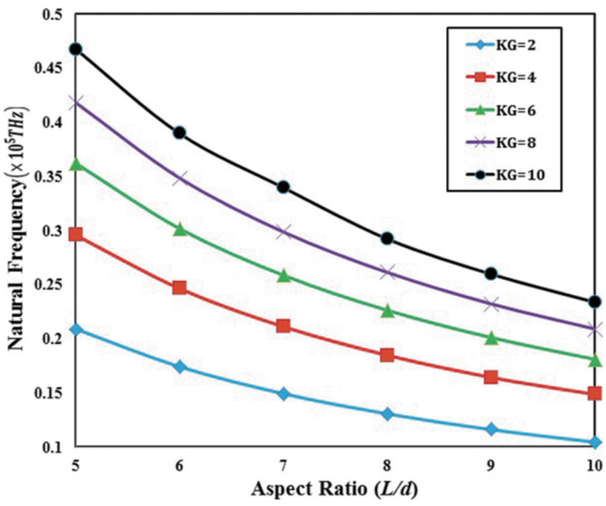

Fig. 7. The effect of shear modulus parameter K_G on natural frequency of a short-SWCNT for different aspect ratio L/d, ($K_W = 10^2$ N/m^2, $m = 1$, $\mu = 0.5$).

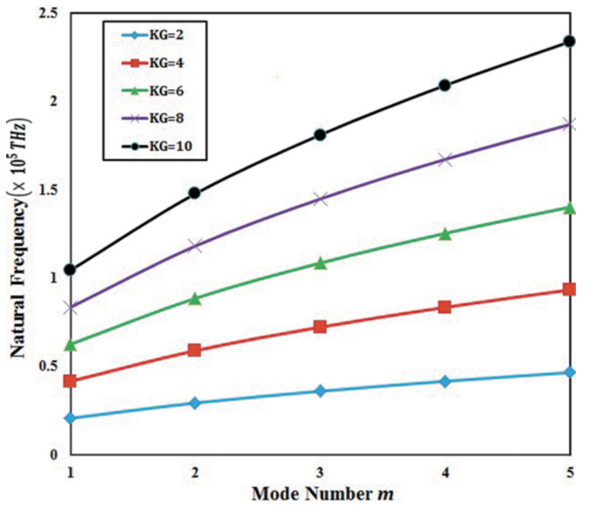

Fig. 8. The natural frequency of a short-SWCNT with different mode number m and shear modulus parameter K_G, ($K_W = 10^2$ N/m^2, $\mu = 0.5$, $L/d = 5$).

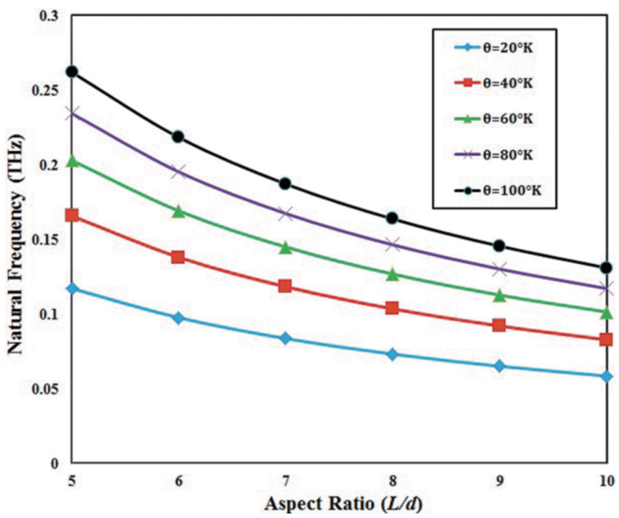

Fig. 9. The effect of temperature on frequency of a short-SWCNT with different aspect ratio L/d, ($K_G = K_W = 0$, $m = 1$, $\mu = 0.5$).

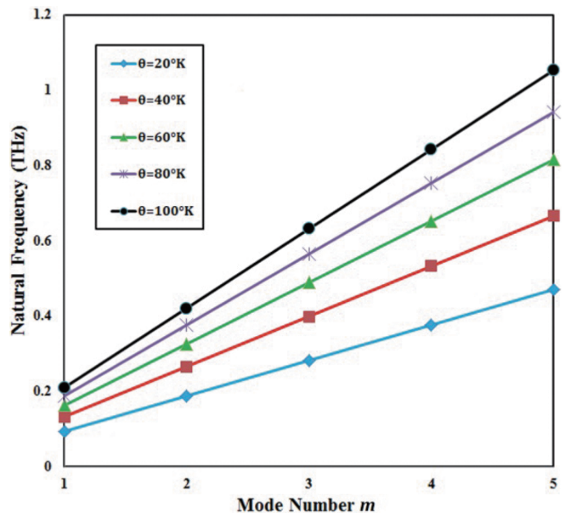

Fig. 10. Change of mode number m on natural frequency for different temperature, ($\mu = 0.5$, $L/d = 5$, $K_W = K_G = 0$).

Case (3). $\boldsymbol{K_G = 0, \bar{N} = 0}$

In the absence of Pasternak foundation and thermal effect, we consider the effect of Winkler modulus in constitutive equation. The scale coefficients are taken as $e_0a \leqslant 2$ [35]. Figure 4 illustrates the effect of K_W on natural frequency with different nonlocal parameter for different aspect ratio. It is clear that by increasing in nonlocal parameter (μ), the natural frequencies decrease but with increasing in aspect ratio (L/d), the natural frequencies increase. In this case, by using Winkler foundation, the natural frequencies are slightly reduced with respect to case (2).

Figure 5 shows the effect of mode number m on natural frequency in a fixed aspect ratio using Winkler modulus parameter. It is clear that by increasing in mode number, the natural frequencies decrease.

The effect of Winkler foundation K_W on natural frequency with different aspect ratio is shown in Fig. 6. It is noted that natural frequency for short carbon nanotubes with higher Winkle modulus (e.g. $K_W = 10^5$ N/m^2) are significantly affected by aspect ratio in comparison with lower Winkler modulus parameter (e.g. $K_W = 10^2$ N/m^2).

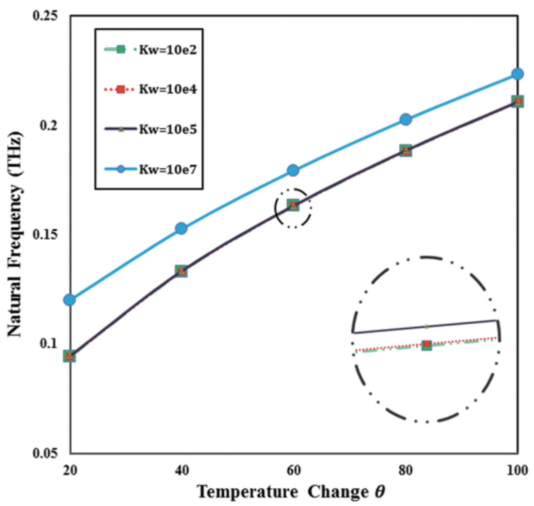

Fig. 11. Variation of natural frequency with temperature for different Winkler modulus K_W. ($\mu = 0.5, K_G = 0, m = 1$).

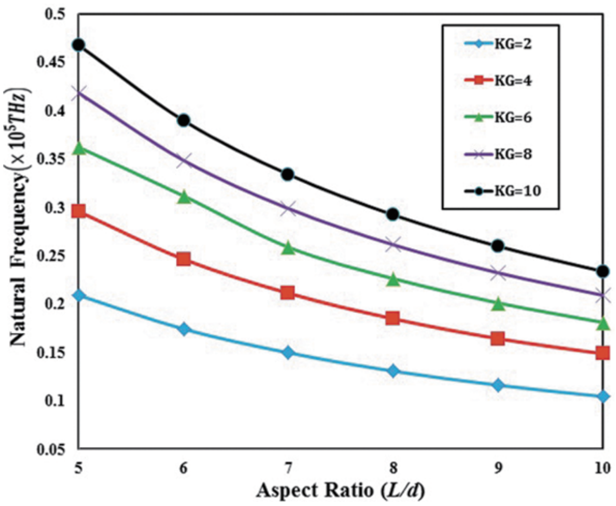

Fig. 12. Change of aspect ratio L/d and effect of Pasternak foundation K_G on frequency of a short-SWCNT, ($K_W = 10^2$ N/m^2, $\theta = 100^\circ K$, $m = 1, \mu = 0.5$).

In general, natural frequencies increase due to increasing the Winkler modulus parameter. For $K_W \leqslant 10^5$ N/m^2, the plots are almost coincided on each other.

Case (4). $\bar{\boldsymbol{N}} = \mathbf{0}$

Effects of elastic foundation on the natural frequency of short single-walled carbon nanotubes with surrounding elastic medium modeled as Pasternak foundation are shown in Fig. 7. In this case, a value of $K_W = 100$ is taken and the shear modulus parameter K_G is varied from 0 to 10. These values are taken from Ref. [18] for the analysis of double-layered graphene sheets embedded in polymer matrix. It is assumed that the shearing layer stiffness of the foundation is one-tenth of the value of Winkler modulus [39]. Moreover, by increasing in aspect ratio, the natural frequencies increase significantly as compared with the case (3). As illustrated in Fig. 7, by increasing the shear modulus parameter K_G in a constant aspect ratio, the natural frequencies of a short single-walled carbon nanotubes increase significantly and the frequencies are higher than the values in Fig. 4. The variation of natural frequencies with mode number for different shear modulus parameter is illustrated in Fig. 8. Here, nonlocal parameters and aspect ratio are assumed to be constant. Figure 8 depicts that with increasing in Pasternak constant, the natural frequencies increase.

Also, it is clear that the effect of mode number m on natural frequencies is similar to shear modulus parameter. Therefore, it is noticeable that by increasing in shear modulus parameter K_G, the natural frequencies increase significantly.

Case (5)

This general cases is included all previous cases. In addition, the temperature effect has been considered here. It is reported that all the coefficients of thermal expansion for single walled carbon nanotube are negative at low temperature and are positive at high temperature [40]. Thus thermal expansion coefficient for short carbon nanotubes is taken as $+1.1 \times 10^{-6\circ} K$ [41]. In this case, the temperature changes are assumed to be uniform. Figures 9–12 are presented in order to review the effect of different parameter in frequency of a short-SWCNT. In Fig. 9, the effect of temperature change in natural frequency is considered. The Winkler and shear modulus parameter are equal to zero. It is clear that with increasing in temperature, the natural frequencies increase. In addition, the length-to-diameter parameter leads to decrease the natural frequencies.

The effect of mode number on natural frequency for different temperature change is shown in Fig. 10. Here, the nonlocal parameter and aspect ratio are constant. Therefore, with increasing in mode number m and temperature change, the frequencies increase significantly.

The simultaneously effects of Winkler modulus K_W and temperature change on natural frequency are shown in Fig. 11. It is clear that by increasing in temperature, θ, and Winkler modulus, K_W, the natural frequencies increase and for low Winkler modulus, the diagrams are almost coincided on each other and the frequencies response are identical. As compared with Fig. 9, using Winkler foundation leads to decrease natural frequencies. As mentioned before, the natural frequencies with higher Winkler modulus (e.g. $K_W = 10^7$ N/m^2) are significantly affected by aspect ratio in comparison with lower Winkler modulus parameter (e.g. $K_W = 10^2$ N/m^2).

Effects of aspect ratio along with Pasternak foundation and a fix nonlocal parameter are shown in Fig. 12. It is realized that with increasing in aspect ratio, natural frequencies decrease but in this case, with difference in temperature, natural frequency doesn't change significantly.

6. Conclusions

In this paper, the linear vibration characteristics of a short SWCNT embedded in Pasternak foundation in thermal environments were investigated. Analytical solution was used to solve the constitutive equations. The main results of this paper are obtained as follows:

An increase in nonlocal parameter leads to decrease the natural frequency. For a short SWCNT, with increasing the Winkler modulus in a constant aspect ratio, the natural frequencies increase. By comparing with the Winkler medium, for the Pasternak foundation, the frequency significantly increased. In addition, when a short SWCNT is subjected to various parameters simultaneously, the effect of Pasternak foundation on natural frequency is more significant. In this case, the effects of temperature change and Winkler foundation are negligible. For a short SWCNT, by increasing the temperature, in a constant nonlocal parameter, the natural frequencies increase. However, the frequency values are lower than the results obtained in case (2). Therefore, we realize that two-parameter elastic medium plays a very important role in frequencies of short SWCNT. Generally, it is concluded that the effect of Pasternak foundation on natural frequency is more significant as compared to the effects of thermal loading, Winkler modulus and nonlocal parameter and should be considered in vibration analysis of short carbon nanotubes.

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