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Research Article

Delay-Range-Dependent H_∞ Control for Automatic Mooring Positioning System with Time-Varying Input Delay

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Aiming at the economy and security of the positioning system in semi-submersible platform, the paper presents a new scheme based on the mooring line switching strategy. Considering the input delay in switching process, H_∞ control with time-varying input delay is designed to calculate the control forces to resist disturbing forces. In order to reduce the conservativeness, the information of the lower bound of delay is taken into account, and a Lyapunov function which contains the range of delay is constructed. Besides, the input constraint is considered to avoid breakage of mooring lines. The sufficient conditions for delay-range-dependent stabilization are derived in terms of LMI, and the controller is also obtained. The effectiveness of the proposed approach is illustrated by a realistic design example.

1. Introduction

For oil and gas exploration in waters deeper than 300 meters, floating platforms such as drillship and semisubmersible platforms are used. These platforms must be kept at a desired location accurately to accomplish the exploration tasks, so it is important to design appropriate positioning system. Dynamic positioning (DP) and mooring positioning system have been used for many years [1]. Sørensen et al. researched the thruster-assisted position mooring (PM) based on DP [2, 3]. The mooring system provides effectively passive control in moderate weather conditions, while the thruster can perform active control to assist the mooring system in severe environmental conditions. So, PM is often considered to be the most cost-effective and feasible positioning method. However, PM also has some shortcomings.

Compared with mooring system, although the positioning precision is improved, the cost of initial construction, usage, and maintenance is increased. Compared with DP system, the maneuverability is decreased. So, the first motivation of this paper is to propose a new positioning scheme, which not only improves the positioning precision, but also needs no extra hardware devices in order to ensure economy.

Compared with the conventional mooring system, the new scheme adds active mooring line release control to improve the positioning accuracy and uses four three-sprocket windlasses instead of twelve single-sprocket windlasses to control twelve mooring lines release based on mooring line switching. Because windlasses are the intrinsic devices in mooring system, compared with PM, the new scheme saves the thruster installation and maintenance cost; besides, fewer windlasses are adopted, so the total cost of the positioning system is reduced and the economy is guaranteed. Meanwhile, by setting switching threshold of mooring lines, the new scheme can optimize the tension distribution of the mooring lines to avoid their breakage caused by uneven tension distribution, so the security of positioning system is improved.

There exists time delay inevitably in the switching process of mooring lines. The time-delay characteristics can decrease the stability of positioning system and even induce the instability. Therefore, the second motivation of this paper is to design a reasonable control method to solve the problem caused by time delay. In order to ensure the platform has good stable operation performance, delay-range-dependent H_∞ control with time-varying input delay is adopted to calculate

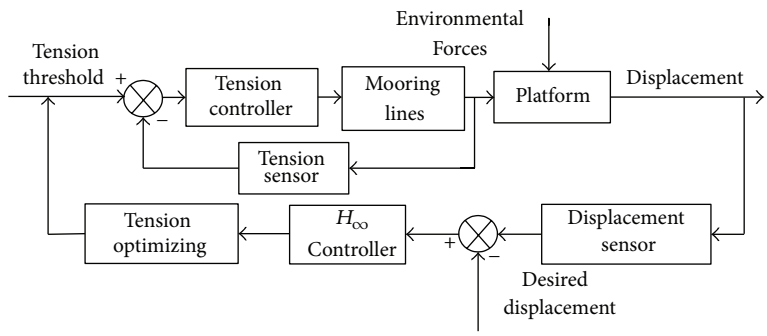

Figure 1: Principle of automatic mooring positioning system.

the control forces to resist environmental disturbances. A delay-dependent H_∞ controller ensures asymptotic stability and a prescribed H_∞ performance level of the closed-loop system for any delays smaller than a given bound. Generally speaking, delay-dependent ones can usually provide less conservative results than delay-independent ones. The H_∞ performance index and the upper bound of the delay are usually two performance indexes to be used to judge the conservatism of the derived conditions. Recent research efforts are more focused on delay-dependent stabilization and H_∞ control; see, for example, Li et al. [4–6], Li et al. [6], Lee et al. [7], and Xu et al. [8]. A free-weighting matrix method is proposed in Gao and Wang [9] and Li et al. [10] study the delay-dependent stability for systems with time-varying delay, in which system transformation and the bounding techniques on some cross terms are not included, thus avoiding the conservatism caused by model transformations; for this reason, the conservatism is reduced, but there are still problems to study. Recently, some much less conservative delay-dependent robust H_∞ control conditions are proposed for uncertain linear systems with state-delay based on new Lyapunov function. However, the range of time-varying delay regarded in previous papers is from 0 to an upper bound, and the Lyapunov functions chosen usually ignore the terms of lower bound of delay, which may lead to considerable conservativeness. In practice, the range of delay may vary in range for which the lower bound is more than zero.

In this case, Li et al. [11] employed the free-weighting matrix method to investigate the robust stability and H_∞ control for systems with interval time-varying delay. But the results were obtained by ignoring some useful terms in the derivative of Lyapunov functional. In addition, the criteria are only available to systems with fast time-varying delay. The stability problem for systems with time-varying delay in a range is studied by Jiang and Han [12]. They choose an appropriate Lyapunov function and a new method to estimate the upper bound of the derivative of Lyapunov function. Some improved delay-range-dependent stability criteria are derived based on the new Lyapunov function and the consideration of range for time delays that are applicable to both fast and slow time-varying delay. However, only stability for the nominal system is analyzed, while the H_∞ performance and controller synthesis have not been investigated. In order to avoid the mooring line breakage, the input constraint should be considered, and this is the third motivation of the paper.

The main contributions of the paper are summarized as follows: (1) a new positioning scheme based on mooring line switch method is proposed, this scheme can improve the positioning precision without the need for thrusters, and the new scheme employs four three-sprocket windlasses instead of twelve single-sprocket windlasses to control twelve mooring lines to reduce the total cost of the positioning system; (2) the delay-range-dependent H_∞ control under the input constraint is designed to ensure closed-loop system is asymptotically stable with a prescribed level of disturbance attenuation and to guarantee the input constraint, and to the best of authors' knowledge, it is the first attempt to apply the delay-range-dependent H_∞ control to the positioning system of the offshore platform; (3) an appropriate Lyapunov function is constructed, which includes the information of the lower bound and upper bound of delay. Because the method we study does not ignore some useful terms, the conservativeness of the time-delay system is reduced and the permitted delay time is extended. The desired controllers can be obtained by solving a set of LMIS. And a design example is given to demonstrate the effectiveness and the advantages of the developed method.

2. Scheme Illustration

The semisubmersible platform which often works in the deep sea generates sway, surge, and yaw motions under the environment forces. The platform cannot get back to the original location when it generates horizontal motions, so the positioning system should be installed to offset external forces and to decrease the horizontal motions of the platform. In order to guarantee the security and performance of the platform, the automatic mooring positioning system which has function of active mooring lines release control, balancing tension distribution, and emergency releasing is used in this project. The principle of new positioning system is illustrated in Figure 1.

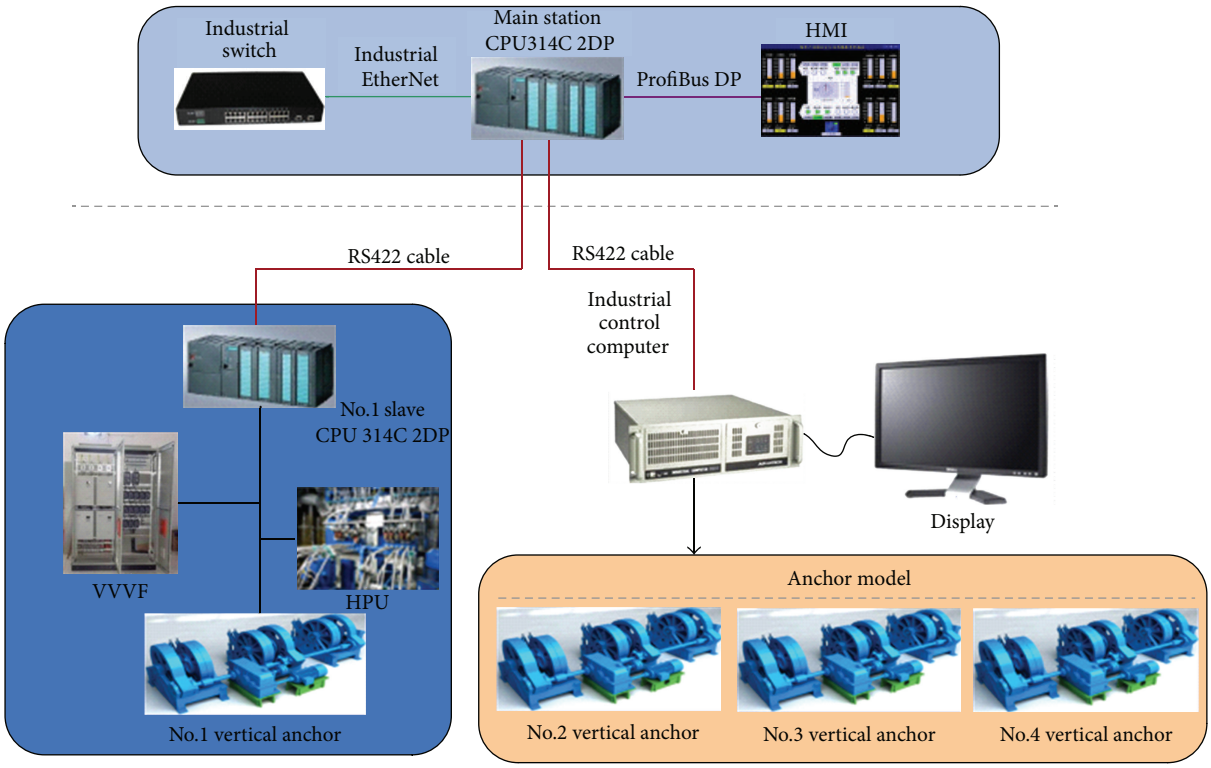

Figure 2: Test and simulation diagram of automatic mooring positioning system.

Firstly, delay-range-dependent H_∞ controller is used to calculate control forces that can guarantee the platform is in desired position under the environmental disturbances; secondly, the tension optimization algorithm with the objective function of minimum square sum of tension differences is designed to generate the twelve tensions for twelve mooring lines, and the optimization results are regarded as switching threshold of mooring lines to avoid the breakage of mooring lines because of uneven tension distribution. Finally, the servo mechanisms control the retracting or releasing of the mooring line to reach thresholds derived by optimizing algorithm. Twelve mooring lines arranged symmetrically are used in the mooring positioning system. While the original scheme that twelve anchor windlasses drive twelve mooring lines is improved into the scheme that proposing switching strategy to control twelve mooring lines driven by four anchor windlasses which has three sprockets. Figure 2 is the schematic diagram.

The central control station sends the control instructions to four anchor windlasses at the same time, that is to say, one anchor windlass can only drive one mooring line at given time; the other two mooring lines are in the off state. When the tension of the mooring line which connects with the driven device reaches the given threshold, the central station sends command to control driven device of the anchor windlass to switch to the second mooring line, and the positive or negative rotation of anchor windlass sprocket is used to control the retracting or releasing of the mooring line until the second mooring line reaches its threshold; then the driven device is connected to the third mooring line. The other three anchor windlasses have the same process.

3. Mathematical Modeling

Under the action of marine environment disturbances, the motions of the platform are separated into wave-frequency (WF) reciprocating motions and low-frequency (LF) slow-drift motions [13]. The WF motions only affect movement gestures of platform; the displacements of platform generated by WF motions are very small, while the LF motions are the dominant factors that make platform drift away from the reference position. It is not desirable to counteract the WF motions. The reason is that compensation of the WF components requires excessive control energy and intensifies the mooring lines attrition, so only the LF model is considered. The nonlinear LF model in surge, sway, and yaw of the platform is given by [14]

$$\mathbf{M}\dot{\mathbf{v}} + \mathbf{C}_{\mathbf{RB}}(\mathbf{v})\mathbf{v} + \mathbf{C}_{\mathbf{A}}(\mathbf{v}_r)\mathbf{v}_r + \mathbf{D}_L\mathbf{v} + \mathbf{D}_{NL}(\mathbf{v}_r, \mathbf{r}_r)\mathbf{v}_r = \boldsymbol{\tau},$$
$$\dot{\boldsymbol{\eta}} = R(\psi)\nu. \tag{1}$$

Note that the movement velocity of the platform for platform positioning system is really small, so damping force

can be approximated to linear force. The linear LF model can be formulated as

$$\begin{aligned} \dot{\boldsymbol{\eta}} &= \mathbf{R}(\psi)\mathbf{v}, \\ \mathbf{M}\dot{\mathbf{v}} + \mathbf{D}\mathbf{v} &= \Gamma + \mathbf{R}^{\mathrm{T}}(\psi)\mathbf{b}, \end{aligned} \tag{2}$$

where $\boldsymbol{\eta} = [x, y, \varphi]^{\mathrm{T}}$ is the position and heading vector relative to the earth-fixed frame, $\mathbf{v} = [u, v, r]^{\mathrm{T}}$ represents the vector of velocities relative to the body-fixed frame, $\mathbf{R}(\psi)$ is the coordinate transformation matrix, $\mathbf{M}$ is the system inertia matrix including added mass, $\mathbf{D}$ is damping matrix including mooring line damping, $\Gamma = [F_x, F_y, N_z]^{\mathrm{T}}$ is the vector of resultant forces and moment of mooring lines tensions and environment forces in surge, sway, and yaw direction, and $\mathbf{b} \in \Re^3$ represents unmodeled environment forces and moment. Assuming small yaw rotations about the desired heading angle, the rotation matrix $\mathbf{R}(\psi)$ can be approximated by the identity matrix, $\dot{\boldsymbol{\eta}} = \mathbf{R}(\psi)\mathbf{v} \cong \mathbf{I}\mathbf{v}$. According to the above assumption, the linear LF state-space model can be formulated as

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{B}w(t), \tag{3}$$

where $\mathbf{x} = [\mathbf{v}^{\mathrm{T}}, \boldsymbol{\eta}^{\mathrm{T}}]^{\mathrm{T}}$ is the state-space vector, $\mathbf{u}(t)$ is the control input in horizontal direction, $w(t)$ is a 3-dimensional disturbance vector consisting of current steady flow wind, turbulent wind, and wave drift loads, and $\mathbf{y}$ is a 3-dimensional measurement vector containing the surge and sway positions and yaw angle. The system matrix is $\mathbf{A} = \begin{pmatrix} -\mathbf{M}^{-1}\mathbf{D} & \mathbf{0}_{3\times3} \\ \mathbf{I}_{3\times3} & \mathbf{0}_{3\times3} \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} \mathbf{M}^{-1} \\ \mathbf{0}_{3\times3} \end{pmatrix}$. Considering the time delay in switching process, the platform model including input delay is formulated as

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t-\tau(t)) + \mathbf{B}w(t), \quad t > 0, \\ x(t) &= \varphi(t), \quad t \in [-\tau_2, 0]; \end{aligned} \tag{4}$$

the time delay $\tau(t)$ is a time-varying continuous function that satisfies

$$0 \le \tau_1 \le \tau(t) \le \tau_2, \quad \dot{\tau}(t) \le \mu, \tag{5}$$

where τ_1, τ_2, μ are constants. Note that τ_1 may not be equal to 0. In order to make sure of mooring lines safety, we should add the input constraint:

$$|u(t)| \le u_{\max}. \tag{6}$$

4. Controller Design

4.1. Delay-Range-Dependent H_∞ Controller Design. We are interested in designing a memoryless state feedback controller:

$$u(t) = Kx(t), \tag{7}$$

where K is the constant matrix to be designed. Therefore, the closed loop system with H_∞ norm can be described by

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + BKx(t-\tau(t)) + Bw(t), \quad t > 0, \\ z(t) &= C\mathbf{x}(t) + DKx(t-\tau(t)), \\ x(t) &= \varphi(t), \quad t \in [-\tau_2, 0]. \end{aligned} \tag{8}$$

The purpose of this paper is to develop delay-dependent H_∞ conditions such that, for any $\tau(t)$ satisfying (5),

(1) the closed-loop system is asymptotically stable;

(2) under zero initial condition, the closed-loop system guarantees that $\|z\|_2 < \gamma\|w\|_2$ for all nonzero $w \in L_2[0, \infty)$ and a prescribed scalar $\gamma > 0$;

(3) the input constraint (6) is guaranteed.

Theorem 1. *Given scalars $0 \le \tau_1 < \tau_2$, $\mu, \varepsilon > 0$, the initiative mooring positioning system* (8) *satisfying* (5) *and* (6) *is asymptotically stable if there exist constants $\lambda_2, \lambda_3 \neq 0$, positive definite matrices P, Q_i $(i = 1, 2, 3)$, R_j $(j = 1, 2)$, T_i $(i = 1, 2, 3)$, and matrices with appropriate dimension M_i, N_i, S_i $(i = 1, \ldots, 4)$, such that the following LMI holds:*

$$\begin{bmatrix} -I & \sqrt{\varepsilon}K \\ * & -u_{\max}^2 P \end{bmatrix} < 0 \tag{9}$$

$$\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} & N_3^{\mathrm{T}} + M_1 & N_4^{\mathrm{T}} - S_1 & \Psi_{15} & T_1 B & \tau_2 N_1 & \tau_{12} S_1 & \tau_{12} M_1 & C^{\mathrm{T}} \\ * & \Psi_{22} & \Psi_{23} & \Psi_{24} & \Psi_{25} & T_2 B & \tau_2 N_2 & \tau_{12} S_2 & \tau_{12} M_2 & K^{\mathrm{T}} D^{\mathrm{T}} \\ * & * & -Q_1 + M_3 + M_3^{\mathrm{T}} & -S_3 + M_4^{\mathrm{T}} & M_5^{\mathrm{T}} & 0 & \tau_2 N_3 & \tau_{12} S_3 & \tau_{12} M_3 & 0 \\ * & * & * & -Q_2 - S_4 - S_4^{\mathrm{T}} & S_5^{\mathrm{T}} & 0 & \tau_2 N_4 & \tau_{12} S_4 & \tau_{12} M_4 & 0 \\ * & * & * & * & \Psi_{55} & T_3 B & \tau_2 N_5 & \tau_{12} S_5 & \tau_{12} M_5 & 0 \\ * & * & * & * & * & -\gamma^2 I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\tau_2 R_1 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\tau_{12}(R_1 + R_2) & 0 & 0 \\ * & * & * & * & * & * & * & * & -\tau_{12} R_2 & 0 \\ * & * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0, \tag{10}$$

where

$$\begin{aligned}
\Psi_{11} &= T_1A + A^{\mathrm{T}}T_1 + \sum_{i=1}^{3} Q_i + N_1 + N_1^{\mathrm{T}},\\
\Psi_{12} &= T_1BK + A^{\mathrm{T}}T_2 - N_1 + N_2^{\mathrm{T}} + S_1 - M_1,\\
\Psi_{15} &= P + N_5^{\mathrm{T}} + A^{\mathrm{T}}T_3 - T_1,\\
\Psi_{22} &= -(1-\mu)Q_3 - N_2 - N_2^{\mathrm{T}} + S_2 + S_2^{\mathrm{T}}\\
&\quad - M_2 - M_2^{\mathrm{T}} + T_2BK + K^{\mathrm{T}}B^{\mathrm{T}}T_2,\\
\Psi_{23} &= -N_3^{\mathrm{T}} + S_3^{\mathrm{T}} + M_2 - M_3^{\mathrm{T}},\\
\Psi_{24} &= -N_4^{\mathrm{T}} - S_2 + S_4^{\mathrm{T}} - M_4^{\mathrm{T}},\\
\Psi_{25} &= -N_5^{\mathrm{T}} + S_5^{\mathrm{T}} - M_5^{\mathrm{T}} - T_2 + K^{\mathrm{T}}B^{\mathrm{T}}T_3,\\
\Psi_{55} &= -\tau_2R_1 + \tau_{12}R_2 - 2T_3,\\
\tau_{12} &= \tau_2 - \tau_1.
\end{aligned} \tag{11}$$

Proof. First, we show the asymptotic stability of (8) with $w(t) = 0$. Then, choose a Lyapunov functional candidate as

$$\begin{aligned}
V(t) &= x^T(t)Px(t) + \sum_{i=1}^{2}\int_{t-\tau_i}^{t} x^T(s)Q_ix(s)\,ds\\
&\quad + \int_{t-\tau(t)}^{t} x^T(s)Q_3x(s)\,ds\\
&\quad + \int_{-\tau_2}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R_1\dot{x}(s)\,ds\,d\theta\\
&\quad + \int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} \dot{x}^T(s)R_2\dot{x}(s)\,ds\,d\theta,
\end{aligned} \tag{12}$$

where $P = P^{\mathrm{T}} > 0$, $Q_i = Q_i^{\mathrm{T}} > 0$, $i = 1,2,3$, and $R_j = R_j^{\mathrm{T}} > 0$, $j = 1,2$, are to be determined.

The derivative of $V(t)$ satisfies

$$\begin{aligned}
\dot{V}(t) &= 2x^T(t)P\dot{x}(t)\\
&\quad + \sum_{i=1}^{2}\left[x^T(t)Q_ix(t) - x^T(t-\tau_i)\right.\\
&\qquad \left.\times Q_ix(t-\tau_i)\right] + x^T(t)Q_3x(t)\\
&\quad - (1-\dot{\tau}(t))x^T(t-\tau(t))\\
&\quad \times Q_3x(t-\tau(t)) + \tau_2\dot{x}^T(t)R_1\dot{x}(t)\\
&\quad - \int_{t-\tau_2}^{t}\dot{x}^T(s)R_1\dot{x}(s)\,ds\\
&\quad + (\tau_2-\tau_1)\dot{x}^T(t)R_2\dot{x}(t)\\
&\quad - \int_{t-\tau_2}^{t-\tau_1}\dot{x}^T(s)R_2\dot{x}(s)\,ds.
\end{aligned} \tag{13}$$

According to the Leibniz-Newton formula, for any appropriate dimensioned matrices N_i, S_i, and M_i, $i = 1,2,3,4$, the following equations are true:

$$\begin{aligned}
&2\left[x^T(t)N_1 + x^T(t-\tau(t))N_2 + x^T(t-\tau_1)\right.\\
&\quad \left.\times N_3 + x^T(t-\tau_2)N_4\right]\\
&\quad \times\left[x(t) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t}\dot{x}(s)\,ds\right] = 0,
\end{aligned} \tag{14}$$

$$\begin{aligned}
&2\left[x^T(t)S_1 + x^T(t-\tau(t))S_2\right.\\
&\quad \left.+ x^T(t-\tau_1)S_3 + x^T(t-\tau_2)S_4\right]\\
&\quad \times\left[x(t-\tau(t)) - x(t-\tau_2) - \int_{t-\tau_2}^{t-\tau(t)}\dot{x}(s)\,ds\right] = 0,
\end{aligned} \tag{15}$$

$$\begin{aligned}
&2\left[x^T(t)M_1 + x^T(t-\tau(t))M_2\right.\\
&\quad \left.+ x^T(t-\tau_1)M_3 + x^T(t-\tau_2)M_4\right]\\
&\quad \times\left[x(t-\tau_1) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t-\tau_1}\dot{x}(s)\,ds\right] = 0.
\end{aligned} \tag{16}$$

In addition, from the integral property and (8), we have

$$\begin{aligned}
&-\int_{t-\tau_2}^{t}\dot{x}^T(s)R_1\dot{x}(s)\,ds\\
&\quad = -\int_{t-\tau(t)}^{t}\dot{x}^T(s)R_1\dot{x}(s)\,ds\\
&\qquad - \int_{t-\tau_2}^{t-\tau(t)}\dot{x}^T(s)R_1\dot{x}(s)\,ds\\
&-\int_{t-\tau_2}^{t}\dot{x}^T(s)R_1\dot{x}(s)\,ds\\
&\quad = -\int_{t-\tau(t)}^{t}\dot{x}^T(s)R_1\dot{x}(s)\,ds\\
&\qquad - \int_{t-\tau_2}^{t-\tau(t)}\dot{x}^T(s)R_1\dot{x}(s)\,ds,
\end{aligned} \tag{17}$$

$$\begin{aligned}
&2\left[x^T(t)T_1 + x^T(t-\tau(t))T_2 + \dot{x}^T(t)T_3\right]\\
&\quad \times\left[Ax(t) + BKx^T(t-\tau(t)) - \dot{x}(t)\right] = 0.
\end{aligned} \tag{18}$$

Adding the left side of (14)–(16) and (18) into (13) and using (17), we can get

$$\begin{aligned}
\dot{V}(t) &\le 2x^T(t)P\dot{x}(t) + \sum_{i=1}^{3}x^T(t)Q_ix(t)\\
&\quad - \sum_{i=1}^{2}x^T(t-\tau_i)Q_ix(t-\tau_i) - (1-\mu)
\end{aligned}$$

$$
\begin{aligned}
&\times x^T(t-\tau(t))Q_3x(t-\tau(t))+\dot{x}^T(t)\\
&\times\left[\tau_2R_1+(\tau_2-\tau_1)R_2\right]\dot{x}(t)\\
&-\int_{t-\tau(t)}^{t}\dot{x}^T(s)R_1\dot{x}(s)\,ds\\
&-\int_{t-\tau_2}^{t-\tau(t)}\dot{x}^T(s)(R_1+R_2)\dot{x}(s)\,ds\\
&-\int_{t-\tau(t)}^{t-\tau_1}\dot{x}^T(s)R_2\dot{x}(s)\,ds\\
&+2\left[x^T(t)N_1+x^T(t-\tau(t))N_2+x^T(t-\tau_1)N_3\right.\\
&\quad\left.+x^T(t-\tau_2)N_4\right]\\
&\times\left[x(t)-x(t-\tau(t))-\int_{t-\tau(t)}^{t}\dot{x}(s)\,ds\right]\\
&+2\left[x^T(t)S_1+x^T(t-\tau(t))S_2\right.\\
&\quad\left.+x^T(t-\tau_1)S_3+x^T(t-\tau_2)S_4\right]\\
&\times\left[x(t-\tau(t))-x(t-\tau_2)-\int_{t-\tau_2}^{t-\tau(t)}\dot{x}(s)\,ds\right]\\
&+2\left[x^T(t)M_1+x^T(t-\tau(t))M_2\right.\\
&\quad\left.+x^T(t-\tau_1)M_3+x^T(t-\tau_2)M_4\right]\\
&\times\left[x(t-\tau_1)-x(t-\tau(t))-\int_{t-\tau(t)}^{t-\tau_1}\dot{x}(s)\,ds\right]\\
&+2\left[x^T(t)T_1+x^T(t-\tau(t))T_2+\dot{x}^T(t)T_3\right]\\
&\times\left[Ax(t)+BKx^T(t-\tau(t))-\dot{x}(t)\right]\\
\le{}&\zeta^T(t)\left[\Psi+\tau_2NR_1^{-1}N^T+\tau_{12}S(R_1+R_2)^{-1}S^T\right.\\
&\quad\left.+\tau_{12}MR_2^{-1}M^T\right]\zeta(t)\\
&-\int_{t-\tau(t)}^{t}\left[\zeta^T(t)N+\dot{x}^T(s)R_1\right]\\
&\quad\times R_1^{-1}\left[N^T\zeta(t)+R_1\dot{x}(s)\right]ds\\
&-\int_{t-\tau_2}^{t-\tau(t)}\left[\zeta^T(t)S+\dot{x}^T(s)(R_1+R_2)\right](R_1+R_2)^{-1}\\
&\quad\times\left[S^T\zeta(t)+(R_1+R_2)\dot{x}(s)_1\right]ds\\
&-\int_{t-\tau(t)}^{t-\tau_1}\left[\zeta^T(t)M+\dot{x}^T(s)R_2\right]\\
&\quad\times R_2^{-1}\left[M^T\zeta(t)+R_2\dot{x}(s)\right]ds,
\end{aligned}
\tag{19}
$$

where

$$
\zeta(t)=\begin{bmatrix}x(t)\\x(t-\tau(t))\\x(t-\tau_1)\\x(t-\tau_2)\\\dot{x}(t)\end{bmatrix},\qquad N=\begin{bmatrix}N_1\\N_2\\N_3\\N_4\\N_5\end{bmatrix},
$$

$$
S=\begin{bmatrix}S_1\\S_2\\S_3\\S_4\\S_5\end{bmatrix},\qquad M=\begin{bmatrix}M_1\\M_2\\M_3\\M_4\\M_5\end{bmatrix},
$$

$$
\psi=\begin{bmatrix}\psi_{11}&\psi_{12}&N_3^T+M_1&N_4^T-S_1&\psi_{15}\\*&\psi_{22}&\psi_{23}&\psi_{24}&\psi_{25}\\*&*&-Q_1+M_3+M_3^T&-S_3+M_4^T&M_5^T\\*&*&*&-Q_2-S_4-S_4^T&-S_5^T\\*&*&*&*&\psi_{55}\end{bmatrix}.
\tag{20}
$$

Since $R_i>0$, then the last three parts in (19) are less than 0, so

$$
\begin{aligned}
\dot{V}(t)\le{}&\zeta^T(t)\left[\Psi+\tau_2NR_1^{-1}N^T+\tau_{12}S(R_1+R_2)^{-1}S^T\right.\\
&\left.+\tau_{12}MR_2^{-1}M^T\right]\zeta(t).
\end{aligned}
\tag{21}
$$

By Schur complements, inequality (9) guarantees

$$
\begin{aligned}
&\Psi+\tau_2NR_1^{-1}N^T+\tau_{12}S(R_1+R_2)^{-1}S^T\\
&\quad+\tau_{12}MR_2^{-1}M^T<0;
\end{aligned}
\tag{22}
$$

then $\dot{V}(t)<0$; the asymptotic stability is established.

Next, we shall establish the H_∞ performance of the system in (8) under zero initial conditions. Firstly, define the Lyapunov function as in (12). Then, according to the above proof, the time derivative of $V(t)$ is given by

$$
\begin{aligned}
\dot{V}(t)\le{}&\widetilde{\zeta}^T(t)\left[\widetilde{\Psi}+\tau_2\widetilde{N}R_1^{-1}\widetilde{N}^T+\tau_{12}\widetilde{S}(R_1+R_2)^{-1}\widetilde{S}^T\right.\\
&\left.+\tau_{12}\widetilde{M}R_2^{-1}\widetilde{M}^T\right]\widetilde{\zeta}(t)\\
&-\int_{t-\tau(t)}^{t}\left[\widetilde{\zeta}^T(t)\widetilde{N}+\dot{x}^T(s)R_1\right]\\
&\quad\times R_1^{-1}\left[\widetilde{N}^T\widetilde{\zeta}(t)+R_1\dot{x}(s)\right]ds\\
&-\int_{t-\tau_2}^{t-\tau(t)}\left[\widetilde{\zeta}^T(t)\widetilde{S}+\dot{x}^T(s)(R_1+R_2)\right]\\
&\quad\times(R_1+R_2)^{-1}\\
&\quad\times\left[\widetilde{S}^T\widetilde{\zeta}(t)+(R_1+R_2)\dot{x}(s)_1\right]ds\\
&-\int_{t-\tau(t)}^{t-\tau_1}\left[\widetilde{\zeta}^T(t)\widetilde{M}+\dot{x}^T(s)R_2\right]\\
&\quad\times R_2^{-1}\left[\widetilde{M}^T\widetilde{\zeta}(t)+R_2\dot{x}(s)\right]ds,
\end{aligned}
\tag{23}
$$

where

$$\widetilde{M}^T = [M_1 \;\; M_2 \;\; M_3 \;\; M_4 \;\; M_5 \;\; 0]^T,$$

$$\widetilde{\Psi} = \begin{bmatrix} \psi_{11} & \psi_{12} & N_3^T + M_1 & N_4^T - S_1 & \psi_{15} & T_1 B \\ * & \psi_{22} & \psi_{23} & \psi_{24} & \psi_{25} & T_2 B \\ * & * & -Q_1 + M_3 + M_3^T & -S_3 + M_4^T & M_5^T & 0 \\ * & * & * & -Q_2 - S_4 - S_4^T & -S_5^T & 0 \\ * & * & * & * & \psi_{55} & T_3 B \\ * & * & * & * & * & 0 \end{bmatrix},$$

$$\widetilde{\zeta}(t) = \begin{bmatrix} x(t) \\ x(t-\tau(t)) \\ x(t-\tau_1) \\ x(t-\tau_2) \\ \dot{x}(t) \\ w(t) \end{bmatrix}, \qquad \widetilde{N} = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ 0 \end{bmatrix}, \qquad \widetilde{S} = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ 0 \end{bmatrix}. \tag{24}$$

Thus, we have

$$\begin{aligned} &\dot{V}(t) + z^T(t) z(t) - \gamma^2 w^T(t) w(t) \\ &\quad \le \widetilde{\zeta}^T(t) \\ &\qquad \times \Big[\widetilde{\Psi} + \tau_2 \widetilde{N} R_1^{-1} \widetilde{N}^T + \tau_{12} \widetilde{S} (R_1 + R_2)^{-1} \widetilde{S}^T \\ &\qquad\quad + \tau_{12} \widetilde{M} R_2^{-1} \widetilde{M}^T + \psi_7 + \psi_8\Big] \widetilde{\zeta}(t) \\ &\qquad - \int_{t-\tau(t)}^{t} \Big[\widetilde{\zeta}^T(t) \widetilde{N} + \dot{x}^T(s) R_1\Big] \\ &\qquad\quad \times R_1^{-1} \Big[\widetilde{N}^T \widetilde{\zeta}(t) + R_1 \dot{x}(s)\Big] ds \\ &\qquad - \int_{t-\tau_2}^{t-\tau(t)} \Big[\widetilde{\zeta}^T(t) \widetilde{S} + \dot{x}^T(s) (R_1 + R_2)\Big] \\ &\qquad\quad \times (R_1 + R_2)^{-1} \\ &\qquad\quad \times \Big[\widetilde{S}^T \widetilde{\zeta}(t) + (R_1 + R_2) \dot{x}(s)_1\Big] ds \\ &\qquad - \int_{t-\tau(t)}^{t-\tau_1} \Big[\widetilde{\zeta}^T(t) \widetilde{M} + \dot{x}^T(s) R_2\Big] R_2^{-1} \\ &\qquad\quad \times \Big[\widetilde{M}^T \widetilde{\zeta}(t) + R_2 \dot{x}(s)\Big] ds, \\ &\psi_7 = [C \;\; DK \;\; 0 \;\; 0 \;\; 0]^T [C \;\; DK \;\; 0 \;\; 0 \;\; 0], \\ &\psi_8 = \operatorname{diag}\{0 \;\; 0 \;\; 0 \;\; 0 \;\; -\gamma^2 I\}. \end{aligned} \tag{25}$$

If

$$\begin{aligned} &\Psi + A^T(\tau_2 R_1 + \tau_{12} R_2) A + \tau_2 N R_1^{-1} N^T \\ &\quad + \tau_{12} S (R_1 + R_2)^{-1} S^T + \tau_{12} M R_2^{-1} M^T < 0, \end{aligned} \tag{26}$$

which is equivalent to (9) according to Schur complements, then we get

$$\dot{V}(t) + z^T(t) z(t) - \gamma^2 w^T(t) w(t) < 0, \tag{27}$$

$$\begin{aligned} \max_{t>0} |u(t)|^2 &= \max_{t>d(t)} \left\| x^T(t-\tau(t)) K^T K x(t-\tau(t)) \right\|_2 \\ &= \max_{t>d(t)} \Big\| x^T(t-\tau(t)) P^{1/2} P^{-1/2} K^T \\ &\qquad \times K P^{-1/2} P^{1/2} \times x(t-\tau(t)) \Big\|_2 \\ &< \varepsilon \cdot \theta_{\max}\left(P^{-1/2} K^T K P^{-1/2}\right) \end{aligned} \tag{28}$$

for all nonzero $w \in L_2[0,\infty)$. Under zero initial conditions, we have $V(0) = 0$ and $V(\infty) \ge 0$. Integrating (27) yields $\|z\|_2 < \gamma \|w\|_2$, and the H_∞ performance is established.

Noting that the integral terms of (12) are more than zero, we obtain $x^T(t) P x(t) < \varepsilon$, with $\varepsilon = \gamma^2 w_{\max} + V(0)$, and we can also obtain

$$x^T(t-\tau(t)) P x(t-\tau(t)) < \varepsilon \quad \text{with } t > \tau(t). \tag{29}$$

In addition, we will show that the input constraint is satisfied. Equation (4.1) ensures $\dot{V}(t) - \gamma^2 w^T(t) w(t) < 0$. Integrating the above inequality from zero to $t > 0$, we can get

$$V(t) - V(0) < \gamma^2 \int_0^t w^T(t) w(t)\, dt < \gamma^2 \|w\|_2^2, \tag{30}$$

where $\theta_{\max}(\cdot)$ shows maximal eigenvalue. From the above inequality, the input constraint (6) is established if $\varepsilon \cdot P^{-1/2} K^T K P^{-1/2} < u_{\max}^2 I$. By Schur complements, the above inequality is equivalent to (9). This completes the proof. □

Theorem 2. *Given scalars $0 \le \tau_1 < \tau_2$, $\mu, \varepsilon > 0$, the initiative mooring positioning system* (8) *satisfying* (5) *and* (6) *is asymptotically stable and satisfies $\|z\|_2 < \gamma \|w\|_2$ for any nonzero $w \in L_2[0,\infty)$ under zero initial condition, if there exist constants $\lambda_2, \lambda_3 \ne 0$, positive definite matrices X, $\overline{P}$, $\overline{Q}_i$, $\overline{R}_j$, and matrices with appropriate dimension $\overline{M}_i$, $\overline{N}_i$ $\overline{S}_i$ ($i = 1, \ldots, 4$), Y satisfying*

$$\overline{\Psi} = \begin{bmatrix} \overline{\Psi}_{11} & \overline{\Psi}_{12} & \overline{N}_3^{\mathrm{T}} + \overline{M}_1 & \overline{N}_4^{\mathrm{T}} - \overline{S}_1 & \overline{\Psi}_{15} & BY & \tau_2\overline{N}_1 & \tau_{12}\overline{S}_1 & \tau_{12}\overline{M}_1 & X^{\mathrm{T}}C^{\mathrm{T}} \\ * & \overline{\Psi}_{22} & \overline{\Psi}_{23} & \overline{\Psi}_{24} & \overline{\Psi}_{25} & \lambda_2 BY & \tau_2\overline{N}_2 & \tau_{12}\overline{S}_2 & \tau_{12}\overline{M}_2 & Y^{\mathrm{T}}D^{\mathrm{T}} \\ * & * & -Q_1 + M_3 + M_3^{\mathrm{T}} & -\overline{S}_3 + \overline{M}_4^{\mathrm{T}} & \overline{M}_5^{\mathrm{T}} & 0 & \tau_2\overline{N}_3 & \tau_{12}\overline{S}_3 & \tau_{12}\overline{M}_3 & 0 \\ * & * & * & -\overline{Q}_2 - \overline{S}_4 - \overline{S}_4^{\mathrm{T}} & \overline{S}_5^{\mathrm{T}} & 0 & \tau_2\overline{N}_4 & \tau_{12}\overline{S}_4 & \tau_{12}\overline{M}_4 & 0 \\ * & * & * & * & \overline{\Psi}_{55} & \lambda_3 BY & \tau_2\overline{N}_5 & \tau_{12}\overline{S}_5 & \tau_{12}\overline{M}_5 & 0 \\ * & * & * & * & * & -\gamma^2 I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\tau_2\overline{R}_1 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\tau_{12}\left(\overline{R}_1 + \overline{R}_2\right) & 0 & 0 \\ * & * & * & * & * & * & * & * & -\tau_{12}\overline{R}_2 & 0 \\ * & * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0 \tag{31}$$

$$\begin{bmatrix} -I & \sqrt{\varepsilon}Y \\ * & -u_{\max}^2 X \end{bmatrix} < 0, \tag{32}$$

where

$$\begin{aligned} \overline{\Psi}_{11} &= AX + XA^{\mathrm{T}} + \sum_{i=1}^{3}\overline{Q}_i + \overline{N}_1 + \overline{N}_1^{\mathrm{T}}, \\ \overline{\Psi}_{12} &= BY + \lambda_2 XA^{\mathrm{T}} - \overline{N}_1 + \overline{N}_2^{\mathrm{T}} + \overline{S}_1 - \overline{M}_1, \\ \overline{\Psi}_{15} &= \overline{P} + N_5^{\mathrm{T}} + \lambda_3 XA^{\mathrm{T}} - X^{\mathrm{T}}, \\ \overline{\Psi}_{22} &= -(1-\mu)\overline{Q}_3 - \overline{N}_2 - \overline{N}_2^{\mathrm{T}} + \overline{S}_2 + \overline{S}_2^{\mathrm{T}} - \overline{M}_2 \\ &\quad - \overline{M}_2^{\mathrm{T}} + \lambda_2 BY + \lambda_2 Y^{\mathrm{T}}B^{\mathrm{T}}, \\ \overline{\Psi}_{23} &= -\overline{N}_3^{\mathrm{T}} + \overline{S}_3^{\mathrm{T}} + \overline{M}_2 - \overline{M}_3^{\mathrm{T}}, \\ \overline{\psi}_{24} &= -\overline{N}_4^{\mathrm{T}} - \overline{S}_2 + \overline{S}_4^{\mathrm{T}} - \overline{M}_4^{\mathrm{T}}, \\ \overline{\Psi}_{25} &= -\overline{N}_5^{\mathrm{T}} + \overline{S}_5^{\mathrm{T}} - \overline{M}_5^{\mathrm{T}} - \lambda_2 X^{\mathrm{T}} + \lambda_3 Y^{\mathrm{T}}B^{\mathrm{T}}, \\ \overline{\Psi}_{55} &= -\tau_2\overline{R}_1 + \tau_{12}\overline{R}_2 - 2\lambda_3 X^{\mathrm{T}}. \end{aligned} \tag{33}$$

In addition, if the above inequalities have a feasible solution, the H_∞ state feedback controller can be given by

$$u(t) = YX^{-1}x(t). \tag{34}$$

Proof. Define the matrices $X = T_1^{-1}$, $Y = KX$, $\overline{Q}_i = XQ_iX$, $\overline{R}_j = XR_jX$, $\overline{N}_i = XN_iX$, $\overline{S}_i = XS_iX$, $\overline{M}_i = XM_iX$, $T_i = \lambda_i T_1$, $\overline{P} = XPX$ and perform congruent transformation in matrix and refer to the proof process of Theorem 1; Theorem 2 can be proved. □

Note that using the method of Theorem 2, the problems of finding the maximum allowed delay τ_2 for a given γ and τ_1 or the smallest γ for given τ_1 and τ_2 can be easily solved without the need for explicitly tuning any parameters.

We can get the required resultant forces that the offshore platform moves into the desired position by the delay-range-dependent H_∞ controller. In order to obtain the threshold tension of every mooring line, the improved dynamic hybrid framework in the next section is used.

4.2. Tension Optimizing Using Genetic Algorithm. The resultant forces in horizontal direction can be obtained by the above delay-range-dependent H_∞ controller. The tension distribution method is required to calculate tensions of twelve mooring lines. Meanwhile, the uniform distribution of tensions can avoid the mooring lines breakage and ensure the security of the platform. So the goal is optimizing tensions distribution on the basis of accurate positioning. The minimum square sum of tension differences is considered as objective function; other conditions are satisfied by the constraints:

$$\begin{aligned} \min \quad & F = \sum_{i=1}^{12}\sum_{j=1}^{12}\left(T_i - T_j\right)^2 \\ \text{Subjected to} \quad & \sum_{i=1}^{12} T_i \cos\varphi_i = -F_x \\ & \sum_{i=1}^{12} T_i \sin\varphi_i = -F_y \\ & \sum_{i=1}^{12} T_i d_i \left(-\cos\varphi_i \sin\theta_i + \sin\varphi_i \cos\theta_i\right) = -M \\ & T_{\min} \le T \le \frac{T_{\max}}{k}, \end{aligned} \tag{35}$$

where F_x is the surge force, F_y is the sway force and M is the disturbance moment in yaw direction, T_i is the tension of the ith mooring line, φ_i is the angle between the mooring

line and x-axis, d_i is the distance from mooring line fixed point and central point, and $T_{\min}$ and $T_{\max}$ are the minimal and maximum tension of mooring line. K is the safety factor of mooring line; it should always be more than 1.67 which is according to API RP-2SK for intact conditions [15]. Then we use three equality constraints to ensure the positional precision of the platform, the fourth inequality constraint shows the safety tension range of the mooring line.

We can see this optimization problem is a constraint optimization problem. So a dynamic hybrid framework which contains a search algorithm and a constraint-handling approach is used. A self-adaptive differential evolution acts as search algorithm, and Pareto dominance used in multi-objective optimization works as a constraint-handling technique. Dynamic hybrid framework transforms a constraint optimization problem into a biobjective problem by treating the degree of constraint violation as an additional objective [16]. Therefore, the original objective function and the constraint violation should be considered simultaneously when comparing the individuals in the population. And the self-adaptive differential evolution can efficiently adjust the control parameters in differential evolution and thus adapt to the search situations, the process can be divided into the next three steps [17]: (1) each target vector T_i in the population P is used to generate a trail vector u_i through the mutation and crossover operations; the control parameters are updated according to the self-adaptive scheme; (2) compute the two objective functions, for the trail vector u_i; (3) if u_i dominates T_i, the trail vector u_i will replace the target vector T_i; else no replacement occurs. The process of Pareto dominance is the same with [18].

5. Design Example

In this section, an example is provided to demonstrate the superiority of the new positioning system and the effectiveness of the proposed delay-range-dependent H_∞ controller design method. The sea state for simulation is wind speed 26.3 m/s, significant wave height 7 m, current speed 1.03 m/s, and the disturbance direction 45°. In order to use the LMI toolbox in MATLAB availably, we use nondimensional parameters to represent the platform model. The nondimensional mass and damping matrixes are from [14]:

$$\mathbf{M} = \begin{bmatrix} 1.0653 & 0 & 0 \\ 0 & 2.0672 & -0.4093 \\ 0 & -0.4093 & 0.2108 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} 0.0853 & 0 & 0 \\ 0 & 0.0772 & 0.0153 \\ 0 & 0.0153 & 0.0048 \end{bmatrix}. \tag{36}$$

First, the surge position without active control is shown in Figure 3.

In the following, we consider the new mooring positioning scheme and use the proposed delay-range-dependent H_∞ controller to get some results. The parameters in surge motion are as follows:

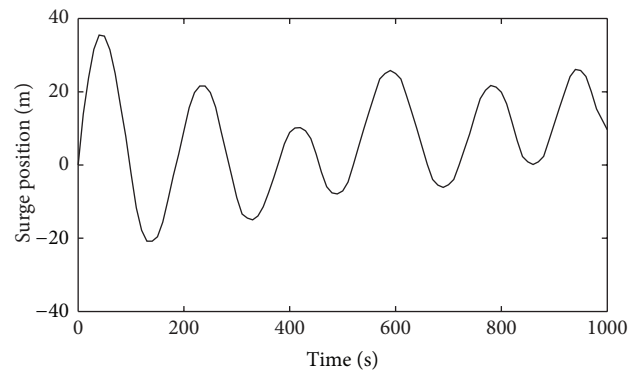

Figure 3: Surge position in traditional mooring positioning scheme.

$$A = \begin{bmatrix} -0.0801 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.0840 & -0.0193 & 0 & 0 & 0 \\ 0 & -0.2357 & -0.0603 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \tag{37}$$

$$B = \begin{bmatrix} 0.9387 & 0 & 0 \\ 0 & 0.7859 & 1.5259 \\ 0 & 1.5259 & 7.7066 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{38}$$

Suppose $\lambda_2 = 0.05$, $\lambda_3 = 50$, $\varepsilon = 1$, $\mu = 1.2$, $\tau_1 = 1$, $\tau_2 = 1.1$; by solving the optimization problem formulated in Section 4.1, an admissible control gain matrix is:

$$\mathbf{K} = Y\mathbf{X}^{-1} = \begin{bmatrix} 1.1197 & 0.5351 & 0.9392 & 0.3887 & 0.0637 & 0.0496 \\ 0.7028 & 1.3958 & 0.8900 & 0.0094 & 0.6523 & -0.1369 \\ -0.0290 & -0.1740 & 0.0243 & 0.0145 & -0.1127 & 0.0822 \end{bmatrix}. \tag{39}$$

Then, for a given γ and τ_1, by using the method we propose in this paper, the maximum allowed delay upper bound $\tau_2 = 6.8674$; for comparison, the upper bound obtained from the method without considering the information of the lower bound of delay is $\tau_2' = 2.33$. It is obvious that the conservativeness has been reduced by considering the lower bound information, the allowable delay range has been enlarged, and the delay range meets the requirement of the positioning system.

A dynamic hybrid framework is used to solve the problem of tension optimization. Tensions of twelve mooring lines obtained from the optimization procedure are 294 KN, 294 KN, 294 KN, 1473 KN, 1473 KN, 1473 KN, 2663 KN, 2663 KN, 2663 KN, 418 KN, 418 KN, and 418 KN. The maximum tension is 2663 KN, and it is less than the maximum tension 3680 KN of the traditional mooring positioning

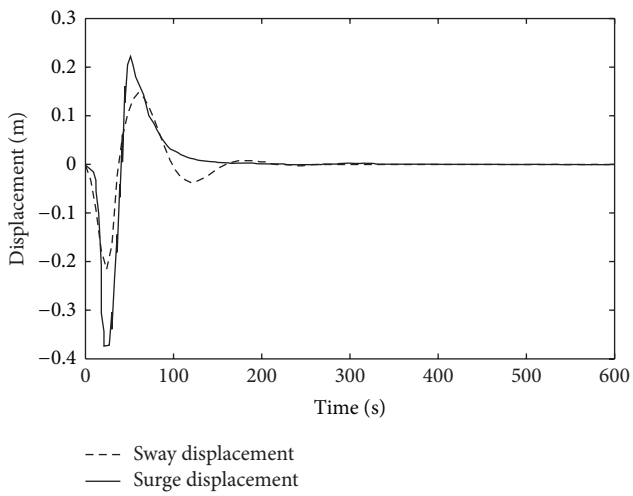

Figure 4: Position in new mooring positioning scheme.

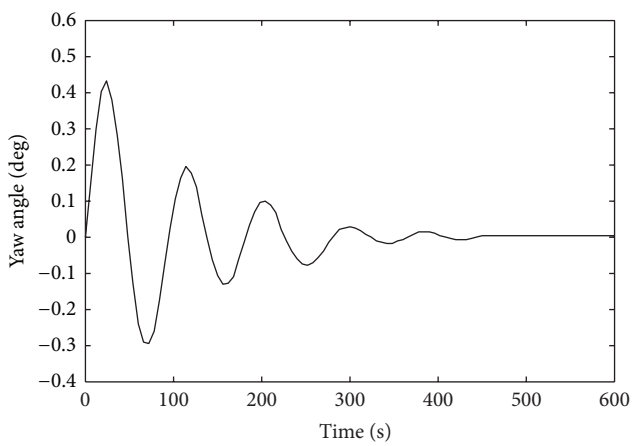

Figure 5: Yaw angle in new mooring positioning scheme.

scheme. The displacement and yaw angle obtained from the new positioning scheme are shown in Figures 4 and 5.

Compared with Figures 3 and 4, we can get that the surge motion in Figure 4 which uses the delay-range-dependent H_∞ control keeps at the desired position after some adjusting period, while the traditional positioning scheme cannot maintain in the desired location. The sway and yaw motions are in the same situation. It concludes that the new mooring positioning scheme has high positioning accuracy.

6. Conclusion

In this paper, a new mooring positioning scheme based on mooring line switching strategy has been proposed. By using delay-range-dependent H_∞ control in which the information of the lower bound of delay is taken into account, the input delay problem in the positioning scheme has been solved. A memoryless H_∞ state feedback controller to guarantee robust stability, a prescribed H_∞ performance level of the closed-loop system, and the input constraint have been obtained to generate resultant forces in horizontal direction to resist disturbing forces. A realistic design example has been illustrated, and the results show that the conservativeness has been reduced by considering the lower bound information and the allowable delay range suit for the positioning system we study. The simulation results also show that, compared with the traditional mooring positioning system, the positioning accuracy of the positioning system we present has been improved and the security of mooring lines has enhanced because of the tension optimizing method.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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