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Research Article

Fatigue Strain and Damage Analysis of Concrete in Reinforced Concrete Beams under Constant Amplitude Fatigue Loading

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Concrete fatigue strain evolution plays a very important role in the evaluation of the material properties of concrete. To study fatigue strain and fatigue damage of concrete in reinforced concrete beams under constant amplitude bending fatigue loading, constant amplitude bending fatigue experiments with reinforced concrete beams with rectangular sections were first carried out in the laboratory. Then, by analyzing the shortcomings and limitations of existing fatigue strain evolution equations, the level-S nonlinear evolution model of fatigue strain was constructed, and the physical meaning of the parameters was discussed. Finally, the evolution of fatigue strain and fatigue damage of concrete in the compression zone of the experimental beam was analyzed based on the level-S nonlinear evolution model. The results show that, initially, fatigue strain grows rapidly. In the middle stages, fatigue strain is nearly a linear change. Because the experimental data for the third stage are relatively scarce, the evolution of the strain therefore degenerated into two phases. The model has strong adaptability and high accuracy and can reflect the evolution of fatigue strain. The fatigue damage evolution expression based on fatigue strain shows that fatigue strain and fatigue damage have similar variations, and, with the same load cycles, the greater the load level, the larger the damage, in line with the general rules of damage.

1. Introduction

Concrete fatigue strain can be a true reflection of the variation of material deformation under fatigue loading. If the relationship between the curve and the cycles is known, we can give a qualitative and quantitative description of the material fatigue strain evolution, thus providing the basis for the evaluation of material behavior. Much of the amplitude fatigue test showed that, whether ordinary concrete, lightweight aggregate concrete, high strength concrete, or fiber reinforced concrete, but also whether compression fatigue, tension fatigue, bending fatigue, uniaxial fatigue, or multiaxial fatigue, concrete longitudinal total deformation and residual deformation exhibit a very stable three-stage development law [1–6], for the rapid growth stage, the stable growth stage, and again the rapid growth stage, and this law is universal. Chen et al. [7] used a cubic polynomial fitting curve to attain correlation coefficients above 0.937, but different stress levels have different coefficients, a difference of nearly an order of magnitude. Cachim et al. [8] determined that when the concrete was under compression fatigue loading, the second phase of the concrete maximum strain rate and the logarithm of the fatigue load cycles form a linear relationship. Xie et al. [9] also fitted the second phase of fatigue strain and gained an experienced index formula. Wang et al. [10] fitted concrete experimental strain data for amplitude compressive fatigue to a two-stage nonlinear formula. Through the evolution of fatigue strain and current analytical methods of analysis, we found that the analysis methods used for fatigue strain now have low accuracy and poor adaptability, and the meaning of the parameters in the model is not clear.

Based on the analysis above, the level-S nonlinear evolution model of fatigue strain was constructed and the physical meaning of the parameters was discussed. A constant amplitude bending fatigue experiment with reinforced

Table 1: Concrete parameter values.

Cube compressive strength (MPa)	Axial compressive strength (MPa)	Splitting tensile strength (MPa)	Elastic modulus (MPa)
35.2	29.6	1.87	2.8×10^4

Table 2: Longitudinal steel parameter values.

Diameter specification (mm)	Yield strength (MPa)	Ultimate strength (MPa)	Elongation (%)	Elastic modulus (MPa)
20	439	554	18.9	2.13×10^5

concrete beams with a rectangular section was carried out in the laboratory. Then, the evolution of fatigue strain and fatigue damage of concrete in the compression zone of the experimental beam was analyzed based on the level-S nonlinear evolution model (Figure 2).

Figure 1: Testing machine control system.

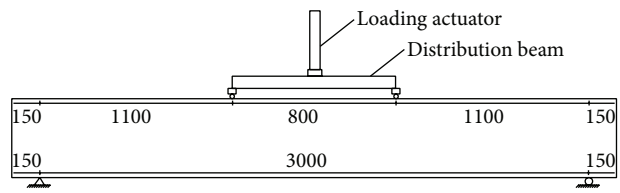

Figure 2: Experimental beam loading diagram.

2. Materials and Methods

2.1. Experimental Design

2.1.1. Experimental Overview. In this paper, four reinforced concrete beam experiments were carried out to test the changes in the fatigue total strain and fatigue residual strain of the concrete along with the increase in cycles. Experimental beams were divided into two groups. The first group was used to do a static load test (one beam), and the second group was used to do an amplitude fatigue test (three beams). All experiments were carried out on 250 kN tonnage fatigue test machine in the structural laboratory of Chongqing Jiaotong University.

2.1.2. Experimental Materials and Specimen Design. The concrete strength is 30 MPa. Six blocks at a size of 150 mm × 150 mm × 150 mm were poured. Three of these blocks were used to test the splitting tensile strength, and the other three were used to test the cube compressive strength. The other six blocks at a size of 150 mm × 150 mm × 300 mm were poured. Three of these blocks were used to test axial compressive strength, and the other three were used to test the elastic modulus. The average of the experimental results is shown in Table 1. Longitudinal steel used in beams is HRB335 grade threaded bars with the diameter of 20 mm. Stirrups and erect ribs are HPB300 round bars with a diameter of 8 mm. Mechanical properties of longitudinal steel are shown in Table 2. The reinforced concrete beam used in the experiment has a size of 200 mm (width) × 400 mm (height) × 3300 mm (length). Measurement of concrete fatigue strain is performed mainly through resistance strain gauges prepasted on the concrete compression surface area of the beam.

2.1.3. Test Conditions. The experimental beam number is compiled for JL-1 to JL-4. JL-1 was used to test static load, and JL-2 to JL-4 beams were used for fatigue testing under different load levels to determine fatigue strain information throughout the life cycle. Specific test conditions are shown in Table 3.

2.1.4. Experimental Device and Loading Method. The static load test and fatigue test were both carried out using the online mode of a computer and electrohydraulic servo loading equipment. The test machine control system is shown in Figure 1. The static load test has two purposes, determining the cracking load P_{cr} and the ultimate load P_u and judging whether the reinforcement of the beam is suitable. Before the formal static load test, the beams should first be preloaded three times. The formal static load test begins after completion of the preload test. The static load test is conducted by grading the load, with a load increment of 20 kN. At each loading, concrete strain values were observed by the controlling software of YE2539 and the high-speed static strain meter. To check that the data acquisition system is working properly and the testing load control system is safe and reliable, the beams should first also be preloaded three times before the formal fatigue test. Because the test was controlled by load amplitude, first, load to load by $(P_{max} + P_{min})/2$ in the form of a graded load was tested and then the fatigue tests began after setting the amplitude to $(P_{max} - P_{min})/2$ and entering the load frequency and the number of the next cycle. The concrete strain was collected when the fatigue cycle ranged to 1 time, 1000 times, 10000 times, 20000 times, 50000 times, 100000 times, 200000 times, and every 200000 times thereafter. Loading frequency is 5 Hz.

The cracking load P_{cr} is 40 kN, and the ultimate load P_u is 220 kN. Through static load test, the reinforcement of the beam is suitable.

Table 3: Test conditions.

Beam number	Loading system			Loading instructions
JL-1	Static load test			Get cracking load P_{cr} and the ultimate load P_u
	Fatigue lower limit (P_{min})	Fatigue upper limit (P_{max})	Amplitude (ΔP)	Lower limit, upper limit, and amplitude of fatigue test
JL-2	$0.1P_u$	$0.6P_u$	$0.5P_u$	Load to destruction with fatigue load
JL-3	$0.1P_u$	$0.7P_u$	$0.6P_u$	
JL-4	$0.1P_u$	$0.8P_u$	$0.7P_u$	

2.2. Analytical Method

2.2.1. Fatigue Strain Evolution Model. A simple formed level-S nonlinear strain evolution model was proposed, which can not only describe a three-stage variation well but also cover a different stress level. The model can consider the initial strain, instability speed, and the proportion of the total fatigue life of third phase (fatigue strain acceleration phase) [11, 12]. The model is as follows:

$$\varepsilon^n = \varepsilon^0 + \alpha \cdot \left(\frac{\beta}{\beta - n/N_f} - 1 \right)^{1/p}. \quad (1)$$

In formula (1), ε^0 and ε^n are initial strain and the fatigue strain when fatigue load cycle has moved n cycles. The parameters n, N_f are the fatigue load cycles and fatigue life, respectively. α, β, and p are equation parameters.

If you take ε^n as the maximum fatigue strain ε^n_{max} or fatigue residual strain ε^n_{res} after n cycles and take ε_0 as the initial maximum strain ε^0_{max} or initial residual strain ε^0_{res}, then formula (1) becomes (2) and (3) as follows:

$$\varepsilon^n_{max} = \varepsilon^0_{max} + \alpha \cdot \left(\frac{\beta}{\beta - n/N_f} - 1 \right)^{1/p}, \quad (2)$$

$$\varepsilon^n_{res} = \varepsilon^0_{res} + \alpha \left(\frac{\beta}{\beta - n/N_f} - 1 \right)^{1/p}. \quad (3)$$

Formula (2) is the maximum fatigue strain evolution model, and formula (3) is the fatigue residual strain evolution model.

2.3. Physical Meaning and the Range of Model Parameters. The initial maximum strain ε^0_{max} and initial residual strain ε^0_{res} are caused mainly by initial defects in the material and the preloading of the member (preloading a general load to the fatigue upper limit). Generally, distinguishing between the two is difficult, so, in this paper, the two are, respectively, represented by fatigue maximum strain ε^1_{max} and residual strain ε^1_{res} when stress reaches the upper limit stress the first time, and the values are generally obtained by experiment. According to the actual experimental data, Wang et al. [10] determined that the relationship between the two is $\varepsilon^1_{res} = 0.25(\varepsilon^1_{max}/\varepsilon_{unstab})^2$, where ε_{unstab} is the total strain of the concrete unstable state.

To study the range of parameters α, β, and p and their impact on the evolution of fatigue strain curves, formulas (2) and (3) can be divided by ultimate fatigue strain on both sides at the same time, so the two formulas become as follows:

$$\frac{\varepsilon^n_{max}}{\varepsilon^f_{max}} = \frac{\varepsilon^0_{max}}{\varepsilon^f_{max}} + \frac{\alpha}{\varepsilon^f_{max}} \cdot \left(\frac{\beta}{\beta - n/N_f} - 1 \right)^{1/p}, \quad (4)$$

$$\frac{\varepsilon^n_{res}}{\varepsilon^f_{res}} = \frac{\varepsilon^0_{res}}{\varepsilon^f_{res}} + \frac{\alpha}{\varepsilon^f_{res}} \left(\frac{\beta}{\beta - n/N_f} - 1 \right)^{1/p}. \quad (5)$$

In formula (4), ε^f_{max} is the extreme of the maximum fatigue strain. In formula (5), ε^f_{res} is the extreme of fatigue residual strain. Formulas (4) and (5) are the normalized evolution model of maximum fatigue strain and fatigue residual strain, respectively.

Parameter β, called the destabilizing factor, is not an independent parameter and is relevant to the parameters p and α. When the circulation ratio n/N_f is 1, the curve of formulas (4) and (5) passes the point (1.1). If the coordinate is put into formulas (4) and (5), we will obtain parameter β calculation formulas for the fatigue strain evolutionary models (4) and (5) as follows:

$$\beta_1 = \left(\frac{\left(1 - \varepsilon^0_{max}/\varepsilon^f_{max}\right)}{\left(\alpha/\varepsilon^f_{max}\right)} \right)^{-p} + 1, \quad (6)$$

$$\beta_2 = \left(\frac{\left(1 - \varepsilon^0_{res}/\varepsilon^f_{res}\right)}{\left(\alpha/\varepsilon^f_{res}\right)} \right)^{-p} + 1. \quad (7)$$

The paper takes the maximum fatigue strain evolution model (4) as the example to study the influence of parameters on the fatigue strain evolution curve. First, we take $\varepsilon^0_{max}/\varepsilon^f_{max}$ and $\alpha/\varepsilon^f_{max}$ as fixed values to study the impact of different p on the curve and then to study the different effects of α on the curve by taking the set $\varepsilon^0_{max}/\varepsilon^f_{max}$ and p. The result shows that parameter p affects the curve convergence speed of the level-S nonlinear strain model. Specifically, the greater the p value is, the faster the convergence of the third phase of fatigue strain curve is. Therefore, p is called the instability speed factor. From the paper, p ranges in the recommendations for [2, 8]. The result shows that α affects the proportion of the third stage (convergence stage) of fatigue strain in the total

Table 4: Coefficients of evolution equation.

Beam number	Load level	$\varepsilon_{\max}^{n}$ α	β	p	Fitting accuracy (R^2)
JL-2	0.6	63.23	1.33	25.30	0.9872
JL-3	0.7	84.25	1.61	35.94	0.9931
JL-4	0.8	34.34	2.02	39.25	0.9825
Beam number	Load level	$\varepsilon_{\text{res}}^{n}$ α	β	p	Fitting accuracy (R^2)
JL-2	0.6	48.43	2.94	5.55	0.9834
JL-3	0.7	57.57	1.73	8.95	0.9933
JL-4	0.8	67.26	1.32	8.94	0.9950

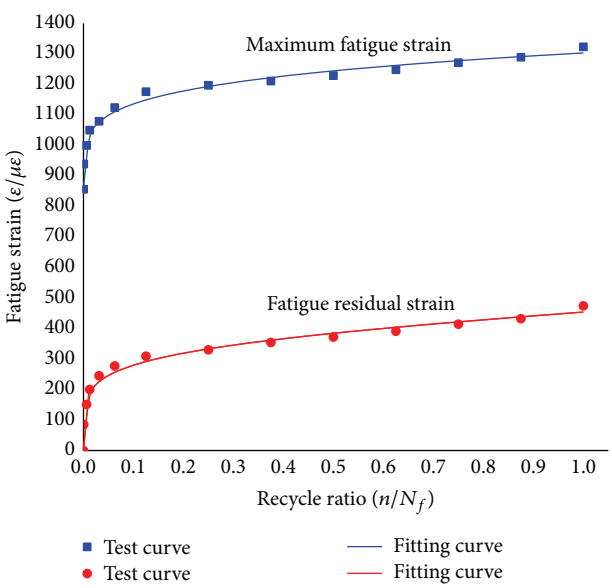

Figure 3: Testing and fitting results of fatigue strain of JL-2.

fatigue life. Specifically, the larger the α value, the smaller the proportion of the acceleration phase.

3. Results and Discussion

3.1. Evolutionary Analysis of Concrete Fatigue Strain. In this paper, concrete fatigue strain obtained in the experiment was analyzed by a level-S nonlinear strain evolution model. The fatigue maximum strain and fatigue residual strain of different load levels were fitted according to formulas (2) and (3). Fitting results and testing results are shown in Figures 3–5. Table 4 shows the coefficient of fatigue strain evolution equation at each stress level.

From Figures 3–5 and Table 4, fatigue strain evolution formulas (2) and (3) can be good fit to the experimental data. Correlation coefficients are above 0.98. Because experimental strain data of the third stage (near destruction) are relatively small, evolution of maximum fatigue strain and fatigue residual strain both occurs by the three-stage reduced to a two-stage variation. The first phase of both is rapid growth; the second phase is nearly linear

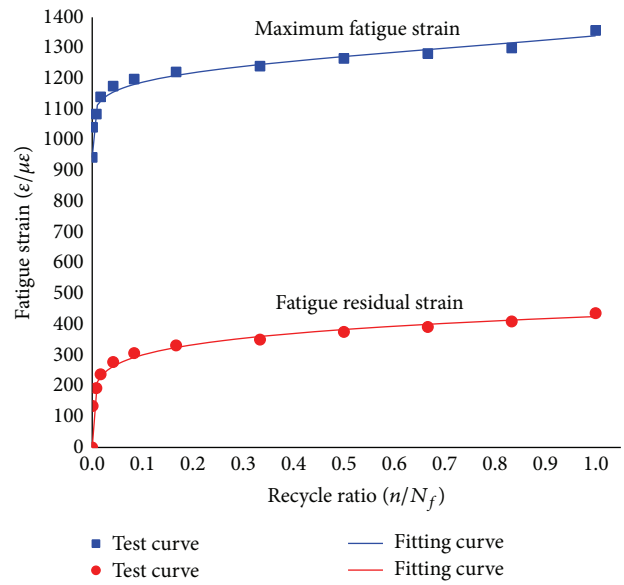

Figure 4: Testing and fitting results of fatigue strain of JL-3.

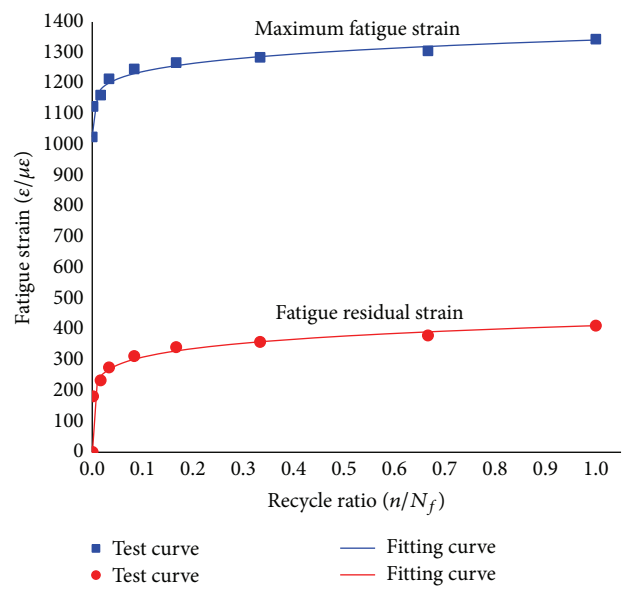

Figure 5: Testing and fitting results of fatigue strain of JL-4.

change. The model can be adapted to specific forms of change.

3.2. Evolutionary Analysis of Fatigue Damage Based on Fatigue Strain. Generally, the preloading test should be done before performing the experiment, and the preload is often the upper limit of the fatigue load. This preload will cause damage to the beam. The initial damage contemplated herein consists of two parts, the initial damage caused by material defects and the damage caused by the process of the load first loaded to the upper fatigue limit. After understanding this point,

the damage based on maximum fatigue strain is defined as follows:

$$D = \begin{cases} D_0 = \dfrac{\varepsilon_{\max}^0}{\varepsilon_{\max}^f} \\ D_0 + \dfrac{\varepsilon_{\max}^n - \varepsilon_{\max}^0}{\varepsilon_{\max}^f}. \end{cases} \tag{8}$$

In (8), D_0 is the initial damage. If we take the maximum fatigue strain evolution formula (2) into formula (8), the damage evolution equation based on the maximum fatigue strain is obtained. The damage evolution formula is as follows:

$$D = \begin{cases} D_0 = \dfrac{\varepsilon_{\max}^0}{\varepsilon_{\max}^f} \\ D_0 + \dfrac{\alpha'}{\varepsilon_{\max}^f} \cdot \left(\dfrac{\beta'}{\beta' - n/N_f} - 1 \right)^{1/p'}. \end{cases} \tag{9}$$

To facilitate data fitting, if $\alpha'/\varepsilon_{\max}^f = m$, then the above equation becomes

$$D = \begin{cases} D_0 = \dfrac{\varepsilon_{\max}^0}{\varepsilon_{\max}^f} \\ D_0 + m \left(\dfrac{\beta'}{\beta' - n/N_f} - 1 \right)^{1/p'}. \end{cases} \tag{10}$$

Similarly, the definition of damage based on fatigue residual strain can be expressed as follows:

$$D = \begin{cases} D_0 = \dfrac{\varepsilon_{\text{res}}^0}{\varepsilon_{\text{res}}^f} \\ D_0 + \dfrac{\varepsilon_{\text{res}}^n - \varepsilon_{\text{res}}^0}{\varepsilon_{\text{res}}^f}. \end{cases} \tag{11}$$

If we take the fatigue residual strain evolution formula (3) into formula (11), the damage evolution equation based on the fatigue residual strain is obtained. The expression is as follows:

$$D = \begin{cases} D_0 = \dfrac{\varepsilon_{\text{res}}^0}{\varepsilon_{\text{res}}^f} \\ D_0 + \dfrac{\alpha'}{\varepsilon_{\text{res}}^f} \cdot \left(\dfrac{\beta'}{\beta' - n/N_f} - 1 \right)^{1/p'}. \end{cases} \tag{12}$$

To facilitate data fitting, if $\alpha'/\varepsilon_{\text{res}}^f = n$, then the above equation becomes

$$D = \begin{cases} D_0 = \dfrac{\varepsilon_{\text{res}}^0}{\varepsilon_{\text{res}}^f} \\ D_0 + n \left(\dfrac{\beta'}{\beta' - n/N_f} - 1 \right)^{1/p'}. \end{cases} \tag{13}$$

In this paper, concrete damage evolution in three beams was analyzed using fatigue damage evolution formulas (10) and (13). The experimental curve and the fitting curve of the three beams based on maximum fatigue strain are shown in Figures 6–8. The experimental curve and the fitting curve of the three beams based on the fatigue residual strain are shown in Figures 9–11. Fatigue damage evolution equation coefficients are shown in Table 5.

Table 5: Fatigue damage evolution equation coefficients.

Beam number	Load level	D			Fitting accuracy (R^2)
		m	β'	p'	
JL-2	0.6	0.41	6.96	5.34	0.9914
JL-3	0.7	0.38	6.21	5.84	0.9849
JL-4	0.8	0.28	3.54	6.13	0.9881
Beam number	Load level	D			Fitting accuracy (R^2)
		n	β'	p'	
JL-2	0.6	0.86	2.42	4.33	0.9859
JL-3	0.7	0.83	3.09	4.07	0.9933
JL-4	0.8	0.80	4.42	3.86	0.9950

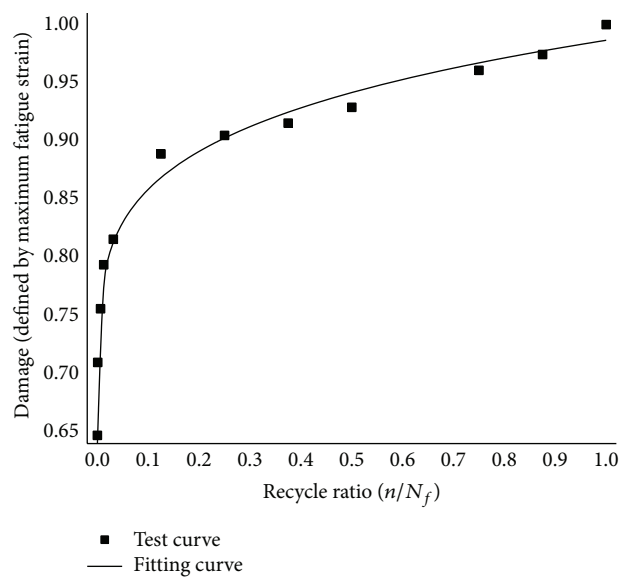

Figure 6: Damage result of JL-2 based on maximum fatigue strain.

Seen from Figures 6–11, from the whole process of damage development, the concrete damage increases as the circulation ratio increases; however, initially, the concrete damage grows rapidly, and when in the middle stage, it is a nearly linear change. Similar to strain evolution, it has a two-stage variation. From Table 5, we know that the model has high accuracy. With the same load cycles, the greater the load level, the larger the damage, in line with the general rules of damage.

4. Conclusions

The conclusions are as follows:

(1) By analyzing the shortcomings and limitations of the existing fatigue strain evolution equations, the

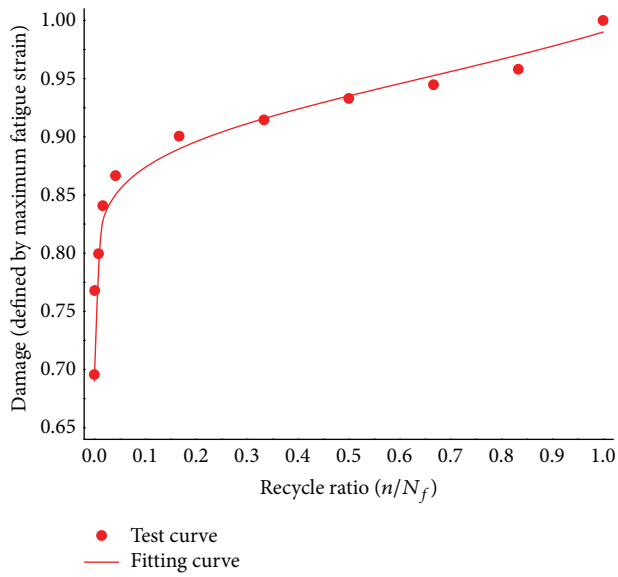

Figure 7: Damage result of JL-3 based on maximum fatigue strain.

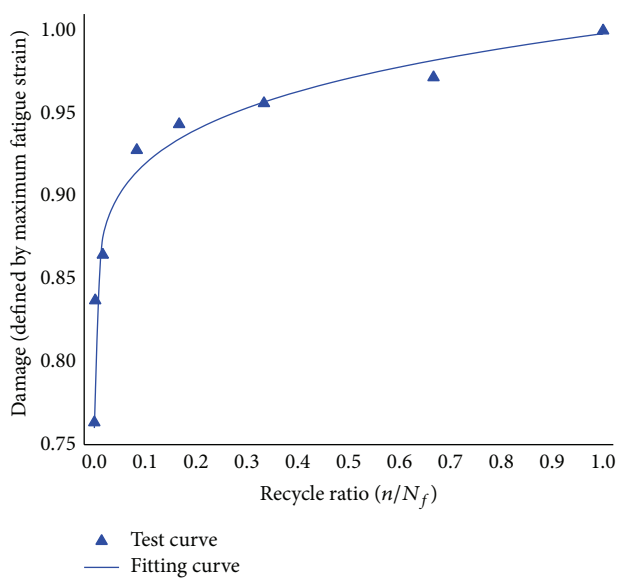

Figure 8: Damage result of JL-4 based on maximum fatigue strain.

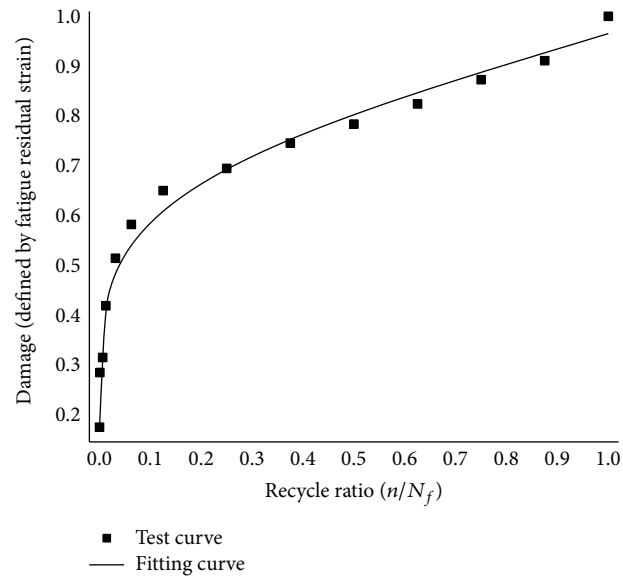

Figure 9: Damage result of JL-2 based on fatigue residual strain.

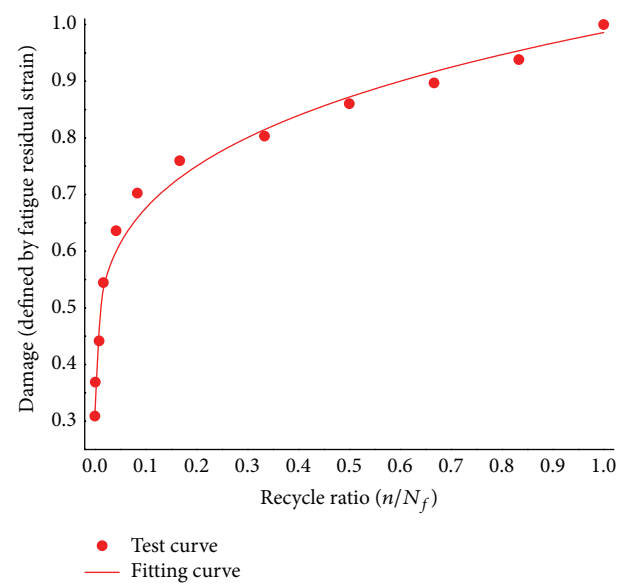

Figure 10: Damage result of JL-3 based on fatigue residual strain.

level-S nonlinear evolution model of fatigue strain was constructed, and the physical meaning of the parameters was discussed.

(2) Test data analysis by the level-S nonlinear strain evolution model shows that, initially, fatigue strain grows rapidly and that, when in middle stage, it is a nearly linear change. The model has strong adaptability and high accuracy and can reflect the evolution of fatigue strain.

(3) The fatigue damage evolution expression based on fatigue strain shows that fatigue strain and fatigue damage have similar variations and that, with the same load cycles, the greater the load level is, the larger the damage is, in line with the general rules of damage.

Competing Interests

The authors declare that they have no competing interests.

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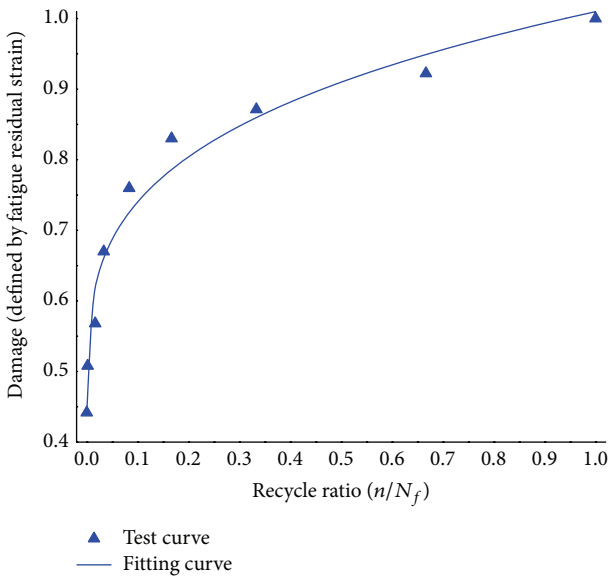

Figure 11: Damage result of JL-4 based on fatigue residual strain.

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