

Research Article

An Identification Method for the Unbalance Parameters of a Rotor-Bearing System

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This paper is to be submitting an identification method for the unbalance parameters of a rotor-bearing system. In the method, the unbalance parameters identification problem is formulated as the unbalance force reconstruction which belongs to solving deconvolution problem, in which the unbalance force is expressed in the time domain. The unbalance response is expressed by the convolution integral of Green's function and the unbalance force. In order to avoid the unstable solution arising from the noisy responses and the deconvolution, a regularization method is adopted to stabilize the solution. Meanwhile, a searching of the sensitive measured point has also been carried out to confirm the robustness of the method. Numerical example and a test rig have been used to illustrate the proposed method.

1. Introduction

In rotating machinery systems, undesirable vibration is caused by unbalance parameters which are inevitable due to asymmetric geometry, material inhomogeneity, manufacturing tolerances, and elastic deformations of the flexible rotor shaft during operations. However, reliable estimates of unbalance parameters are difficult to obtain with theoretical models due to many impact factors. On the contrary, identification methods based on experimental data from actual test conditions provide reliable unbalance characterization which avoids the complexity of exact system modeling and analysis. It is for this reason that designers of rotating machinery mostly rely on the experimentally identified unbalance parameters in their instability analysis.

Over the years, identification of the unbalance parameters is fundamental for the balance of a rotor-bearing system. The method of mechanical balancing is adopted to isolate the vibration arising from the inherent rotor unbalance, in which small amounts of unbalance mass are added and/or removed

at specific locations to gain a satisfactory state of balance for a machine. Thus, identification technology of the unbalance parameters is improved with the development of the balancing technology [1–6]. Nevertheless, using trial weights to find the unbalance parameters in balancing field is often a time-consuming and expensive task because of requiring many machinery start-ups. These rotating machines have a high capital cost and hence the trend in the unbalance parameters identified is to reduce the number of test runs required.

Some researchers introduced optimization algorithm and inversed technology to identify the unbalance parameters. Han et al. [7] identified the unbalance parameters of a rotor-bearing system based on Kriging surrogate model and evolutionary algorithm. Tiwari and Chakravarthy [8] presented a method to identify the unbalance parameters based on the measured force response. Edwards et al. [3, 9, 10] considered the identification of the unbalance parameters based on the unbalance force in many works. De Queiroz [11] presented an unbalance force identification method to identify the unbalance parameters based on the measured

harmonic response. In these methods, unbalance parameters are all ascribed to the unbalance force. The unbalance force is identified through measured response. However, the inverse problem of the unbalance force identification is commonly ill posed [12] and noisy responses will induce serious errors in the process of identifying the unbalance parameters. So developing some inversed techniques for avoiding the ill posed problem has become the mainstream methods [13, 14].

In the paper, the unbalance parameters identification is formulated as the unbalance force identification problem. An identification method for the unbalance parameters of a rotor-bearing system based on the unbalance force reconstruction is proposed to avoid the ill condition from the measured response and the deconvolution. In the method, the unbalance force acting on rotor-bearing systems is to be determined based on the measured time domain responses coming from transient analysis. The ill condition problem of the unbalance force reconstruction will be treated by the Tikhonov regularization which can provide efficient and numerically stable solutions. In order to search for the sensitive measured point, the Finite Element Method (FEM) is adopted for the forward analysis to obtain transient responses with assumed unbalance parameters. Finally, numerical example and a test rig have been used to illustrate the identification method for the unbalance parameters.

2. Theories of the Unbalance Force Reconstruction

2.1. Dynamical Model of Rotor-Bearing Systems with the Unbalance Force. The linearized motion equation of rotor-bearing systems is expressed as

$$\mathbf{M}\ddot{\mathbf{q}} + (\omega\mathbf{T} + \mathbf{D})\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{F}, \quad (1)$$

where ω is the shaft rotational speed, \mathbf{F} is the unbalance force vector, \mathbf{q} is the transient response vector, and \mathbf{M} , \mathbf{T} , \mathbf{D} , and \mathbf{K} are the mass, gyroscopic, damping, and stiffness matrices.

In the paper, the rotor model of rotor-bearing systems is built by Euler beam elements and the consistent matrices' approach described by Genta [15] and the bearing dynamic parameters of the sliding bearing can be obtained based on the full Navier-Stokes equation [16, 17] or Reynolds equation [18, 19].

In fact, the unbalance force caused by the unbalance parameters can be structured by

$$\begin{aligned} \mathbf{F} &= \begin{pmatrix} f_x \\ f_y \end{pmatrix} = \begin{bmatrix} \cos \omega t & -\sin \omega t \\ \sin \omega t & \cos \omega t \end{bmatrix} \begin{pmatrix} A \cos \varphi \\ A \sin \varphi \end{pmatrix}, \\ A &= m e \omega^2, \end{aligned} \quad (2)$$

where t is time, the unbalance magnitude of m and the unbalance phase φ defined relative to the shaft marker are the unbalance parameters, and e is a radius of the chosen reference plane.

In many practical situations, the radius e is predetermined to some arbitrary chosen reference plane; hence e is known. The unbalance parameters (m, φ) can be determined uniquely

by the unbalance force (f_x, f_y). Thus, the unbalance parameters identification can be formulated into the unbalance force reconstruction.

2.2. The Unbalance Force Reconstruction. Considering the system that acted on the unbalance force is a liner and time-invariant system, the transient response of (1) at a direction (x) by a single unbalance force can also be expressed by a convolution integral function that is multiplied by the unbalance force function and the corresponding Green's kernel in time domain by Liu and Han [20] and Liu et al. [21], which is expressed by

$$u(x, t) = \int_0^t g(x, t - \tau) f(\tau) d\tau, \quad (3)$$

where $u(x, t)$ is the transient response in x direction, $g(x, t)$ is the response of a unit pulse of rotor-bearing systems, named the corresponding Green's function, and $f(t)$ is the time history of the unbalance force acting on the rotor-bearing system which needs to be identified.

When this convolution integral in the time domain is discretized to Q evenly spaced sample points, (3) can be expressed by a matrix form of

$$\begin{aligned} &\begin{bmatrix} u(x, t_1) \\ u(x, t_2) \\ \vdots \\ u(x, t_Q) \end{bmatrix} \\ &= \begin{bmatrix} g(t_1) & 0 & \cdots & 0 \\ g(t_2) & g(t_1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g(t_Q) & g(t_{Q-1}) & \cdots & g(t_1) \end{bmatrix} \begin{bmatrix} f(t_0) \\ f(t_1) \\ \vdots \\ f(t_{Q-1}) \end{bmatrix} \Delta t, \end{aligned} \quad (4)$$

where $u(x, t_j)$, $g(t_j)$, and $f(t_j)$ are the displacement, Green's function, and unbalance force at the time $t_j = j\Delta t$, ($j = 0, 1, \dots, Q$), respectively. Δt is the time interval.

Equation (4) can be equivalent to

$$\mathbf{U} = \mathbf{G} \cdot \mathbf{F}. \quad (5)$$

Because the rotor-bearing system is linear, the total displacement can be gained by the linear superposition method. When a rotor-bearing system is acted on multi-source unbalance forces in two radial directions (x and y),

the identification problem of multisource unbalance forces is expressed by

$$\begin{aligned} & \begin{bmatrix} \mathbf{U}^{(x1)} \\ \mathbf{U}^{(y1)} \\ \vdots \\ \mathbf{U}^{(xM)} \\ \mathbf{U}^{(yM)} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{G}^{(x1 x1)} & \mathbf{G}^{(x1 y1)} & \dots & \mathbf{G}^{(x1 xN)} & \mathbf{G}^{(x1 yN)} \\ \mathbf{G}^{(y1 x1)} & \mathbf{G}^{(y1 y1)} & \dots & \mathbf{G}^{(y1 xN)} & \mathbf{G}^{(y1 yN)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{G}^{(xM x1)} & \mathbf{G}^{(xM y1)} & \dots & \mathbf{G}^{(xM xN)} & \mathbf{G}^{(xM yN)} \\ \mathbf{G}^{(yM x1)} & \mathbf{G}^{(yM y1)} & \dots & \mathbf{G}^{(yM xN)} & \mathbf{G}^{(yM yN)} \end{bmatrix} \begin{bmatrix} \mathbf{F}^{(x1)} \\ \mathbf{F}^{(y1)} \\ \vdots \\ \mathbf{F}^{(xN)} \\ \mathbf{F}^{(yN)} \end{bmatrix}, \quad (6) \end{aligned}$$

where M is the number of points used to measure displacement and N is the number of unbalance forces sources. $\mathbf{U}^{(ym)}$, ($m = 1, 2, \dots, M$), and $\mathbf{F}^{(yn)}$, ($n = 1, 2, \dots, N$) are the m th measured displacement and the n th unbalance force, respectively, and $\mathbf{G}^{(ym yn)}$ is Green's function matrix coming from the n th unbalance force to the m th measured displacement.

For presentation purposes, (4) and (6) are shown using the matrix form as (5) uniformly.

In order to reconstruct \mathbf{F} from \mathbf{U} in real rotor-bearing systems in the time domain, the transient response \mathbf{U} and Green's function \mathbf{G} need to be known. Usually, in actual project, the transient response \mathbf{U} can be measured by sensors. Green's function \mathbf{G} can be obtained by some numerical computation method. However, the measured response often contains noise. It is one major reason that the problem of identifying the unbalance force is usually ill posed. The ill posed problem can be dealt with through A Tikhonov regularization method [22, 23].

2.3. Regularization. In general, the identified unbalance force is inaccurate because of the noise data of measured displacement which can produce amplified errors of identified result in inverse problems. And Green's matrix is sensitive to these errors.

Considering the noise data of measured displacement, (5) can be expressed as follows:

$$\mathbf{U}_\delta = \mathbf{G} \cdot \mathbf{F} + \boldsymbol{\delta}, \quad (7)$$

where \mathbf{U}_δ is measured displacement containing noise data. \mathbf{F} is theoretical value of identified unbalance force. $\boldsymbol{\delta}$ is the noise data of measured displacement.

When there is no additional information in the process of unbalanced force identification, a singular value decomposition (SVD) in the method of regularization is used to deal with the noise data of measured displacement [20]. Then the decomposition of \mathbf{G} is as follows:

$$\mathbf{G} = \mathbf{W} \Omega \mathbf{V}^T = \sum_{i=1}^k \mathbf{w}_i \sigma_i \mathbf{v}_i^T, \quad (8)$$

where $\mathbf{W} = [\mathbf{w}_1 \ \mathbf{w}_2 \ \dots \ \mathbf{w}_k]$, $\mathbf{V} = [\mathbf{v}_1 \ \mathbf{v}_2 \ \dots \ \mathbf{v}_k]$ are matrices with orthonormal columns, $\mathbf{W}^T \mathbf{W} = \mathbf{V}^T \mathbf{V} = \mathbf{I}_k$ and in which $\Omega = \text{Diag}(\sigma_i)$, ($i = 1, 2, \dots, k$), has nonnegative diagonal singular values appearing in nonincreasing order. In addition, this decomposition is particularly helpful because it can give a formulation of the identified unbalance force:

$$\begin{aligned} \tilde{\mathbf{F}} &= \mathbf{G}^{-1} \mathbf{U}_\delta = \mathbf{V} \text{Diag}(\sigma_i^{-1}) \mathbf{W}^T \mathbf{U}_\delta \\ &= \mathbf{F} + \sum_{i=1}^k \sigma_i^{-1} (\mathbf{w}_i^T \boldsymbol{\delta}) \mathbf{v}_i. \end{aligned} \quad (9)$$

Through (9), it showed that the identified unbalance force will be affected by the noise data of measured displacement $\boldsymbol{\delta}$ which can be dealt with by the small singular value σ_i . In the method of regularization, the value σ_i^{-1} is multiplied by a filter function $f(\alpha, \sigma_i)$ to overcome this problem. Then the stable and approximate solution $\tilde{\mathbf{F}}_\alpha$ for the unknown unbalance force can be obtained by

$$\begin{aligned} \tilde{\mathbf{F}} &= \mathbf{G}^\alpha \cdot \mathbf{U}_\delta = \mathbf{V} \text{Diag}(f(\alpha, \sigma_i) \sigma_i^{-1}) \mathbf{W}^T \mathbf{U}_\delta \\ &= \sum_{i=1}^k f(\alpha, \sigma_i) \sigma_i^{-1} (\mathbf{w}_i^T \mathbf{U}_\delta) \mathbf{v}_i. \end{aligned} \quad (10)$$

If the filter function is selected by (11), (10) will be expressed by (12):

$$f(\alpha, \sigma) = 1 - \frac{4}{\pi} \arctan(e^{-\sigma/\alpha}), \quad (11)$$

$$\begin{aligned} \tilde{\mathbf{F}}_\alpha &= \sum_{i=1}^k \left(1 - \frac{4}{\pi} \arctan(e^{-\sigma_i/\alpha}) \right) \sigma_i^{-1} (\mathbf{w}_i^T \mathbf{U}_\delta) \mathbf{v}_i \\ &= (\mathbf{G}^T \mathbf{G} + \alpha \mathbf{I})^{-1} \mathbf{G}^T \mathbf{U}_\delta. \end{aligned} \quad (12)$$

3. Solving Strategy

In actual practice, the ill posed inverse problem can be dealt with by regularization described above. It is often effective to select effective measurements that are sensitive to the parameter variation to avoid the ill posedness. The proposed solving strategy for identifying the unbalance parameters of a rotor-bearing system based on the unbalance force reconstruction is outlined in Figure 1.

In this method, firstly, transient response of numerical calculation based on the FEM can be obtained with assumed unbalance parameters and displacement responses numerically generated are treated as actual measurements after adding the measurement noise. Numerical simulation model of rotor-bearing systems is used to search for a strong correlation point between the displacement response and the unbalanced force by comparison of identified unbalance parameters with the assumed ones. Then, the response of a unit pulse generated numerically and the experimental response of the test rig at reference measured points are used to gain the unbalance force reconstruction, and the regularization method is adopted to deal with ill posedness.

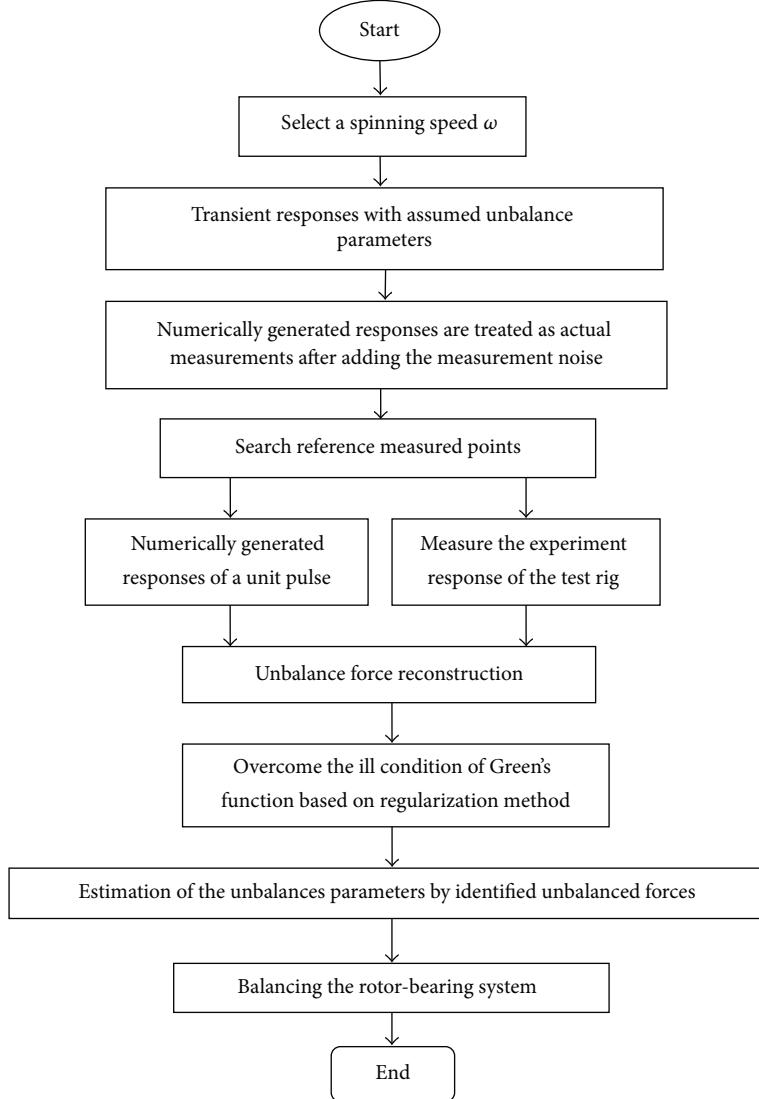


FIGURE 1: The identification method for the unbalance parameters of a rotor-bearing system.

Finally, the unbalance parameters of the rotor-bearing system are identified from the unbalance force reconstruction based on the experimental data.

4. Numerical Simulation

Before applying the proposed method to the actual test data, using the direct numerical simulation to search for strong correlation point between the displacement response and the unbalanced force, meanwhile the numerical simulation is used to examine the reliability and accuracy of the proposed method. A numerical simulation coming from the literature [1] is performed, and Figure 2 shows the parameters and the FEM model. The bearing dynamic parameters of the sliding bearing and the structural parameters of the numerical model are listed in Table 1. The numerical model is subdivided into 20 elements consisting of 21 nodes with two disks at nodes 6

and 16. The assumed unbalance parameters are 4.50 g at 30° in disc 1 and 2.20 g at 60° in disc 2.

The displacement responses in the time domain at nodes 2 and 20 are collected at 4000 rpm by using assumed unbalance parameters. 5% Gaussian noise is added in the displacement responses to simulate the actual measurement noise and to check the robustness of the proposed method for identifying the unbalance parameters. And the identification method was proposed to determine the multisource unbalance forces. The identified unbalance parameters will be compared with the corresponding actual unbalance parameters and the unbalance parameters identified by the impulse response in the literature [1] to check the effectiveness of the proposed method.

The comparison results of the numerical simulation in Figures 3 and 4 show that the unbalance forces identified by the proposed method are in accord with the assumed and the literature's ones. The identified unbalance parameter is

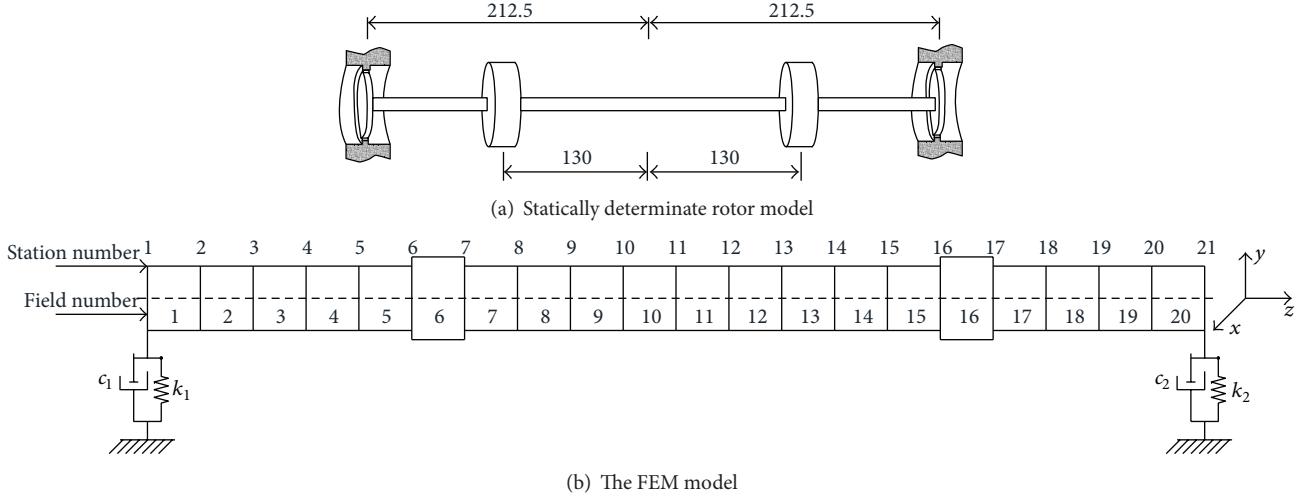


FIGURE 2: The model for the numerical simulation.

TABLE 1: Parameters of the numerical model.

Parts	Parameters	Specifications	
Shaft	Length (mm)	425.0	
	Diameter (mm)	10.0	
	Mass (kg)	4	
	Diametral inertia ($\text{kg}\cdot\text{m}^2$)	0.0046	
	Polar inertia ($\text{kg}\cdot\text{m}^2$)	0.00786	
Disk	Diameter (mm)	74.0	
	Thickness (mm)	25.0	
The sliding bearing	Stiffness coefficients ($\text{MN}\cdot\text{m}^{-1}$)	Damping coefficients ($\text{kN}\cdot\text{s}\cdot\text{m}^{-1}$)	
The first bearing	k_{xx}	24.10	c_{xx}
	k_{xy}	-3.19	c_{xy}
	k_{yx}	72.20	c_{yx}
	k_{yy}	45.40	c_{yy}
The second bearing	k_{xx}	25.80	c_{xx}
	k_{xy}	-3.78	c_{xy}
	k_{yx}	81.20	c_{yx}
	k_{yy}	52.60	c_{yy}

gained through identified unbalance forces, the comparison of magnitude and phase is given in Table 2, and the identified unbalance parameters show excellent agreement in magnitude as well as phase with the assumed and the literature's ones. From these discussions, the proposed method can identify the unbalance parameters reliably and effectively; at the same time, this proposed method is also robust to the noise data of measured displacement.

5. Experimental Example

It is aimed at gaining the unbalance parameters of a rotor-bearing system from the measured displacement response. The test rig considered in this paper shown in Figure 5 is an INV-1612 rotor system by Coinv (China Orient Institute of

Noise & Vibration). The test rig is driven through a motor which can produce some different speeds and directions. The test rig is composed of a disk and two sliding bearings. There are 16 equally spaced M4-threaded holes at a radius of 30 mm of the disk. The bearing dynamic parameters of the sliding bearing and the structural parameter of the test rig are listed in Table 3. The transient response in both the horizontal and vertical directions (x and y) can be measured by the proximity (displacement) probes mounted in Figure 5. The key-phase signal provides the once-per-revolution signal which serves to locate the angular position of the shaft. And the measured signals are processed by the INV303/306 software. Displacement responses are obtained from the test rig at 2000 rpm, shown in Figure 6.

5.1. Identification of the Unbalance Parameters for the Test Rig. The displacement responses in the time history from the experimental data are used as inputs and the experimental unbalance parameters have been identified. Figure 7 shows the identified unbalance forces by the proposed method. From Figure 7, it can be seen that the time cycle of identified unbalance forces is 0.03 s consistent with the rotor spin speed of 2000 rpm. From Figure 7 and (2), the unbalance parameters are calculated. The magnitude of the unbalance parameters is estimated to be 44.202 g·mm and the phase angle is 96° ahead of the shaft marker. Because the unbalance mass is fixed at 30 mm radii from the center of the disk, the magnitude of the unbalance parameters is 1.4734 g.

5.2. The Test Rig Balancing. A comparison of the vibration amplitudes before and after balancing proved the usefulness of the proposed method when the actual unbalance parameters are unknown. The unbalance mass of 1.4734 g is added to the disk in Figure 5. Figure 8 shows the new displacement responses measured in two radial directions with the balance weight fitted. The comparison of displacement responses measured before and after balancing is shown in Figure 8 and

TABLE 2: Comparison of the unbalance parameters from numerical simulation.

Unbalance parameters position	Assumed unbalance, magnitude@phase (g@degree)	Identified unbalance (from literature [1]), magnitude@phase (g@degree)	Identified unbalance, magnitude@phase (g@degree)
Disc 1	4.50@30	4.42@29.65	4.51@30.05
Disc 2	2.20@60	2.18@59.72	2.19@59.84

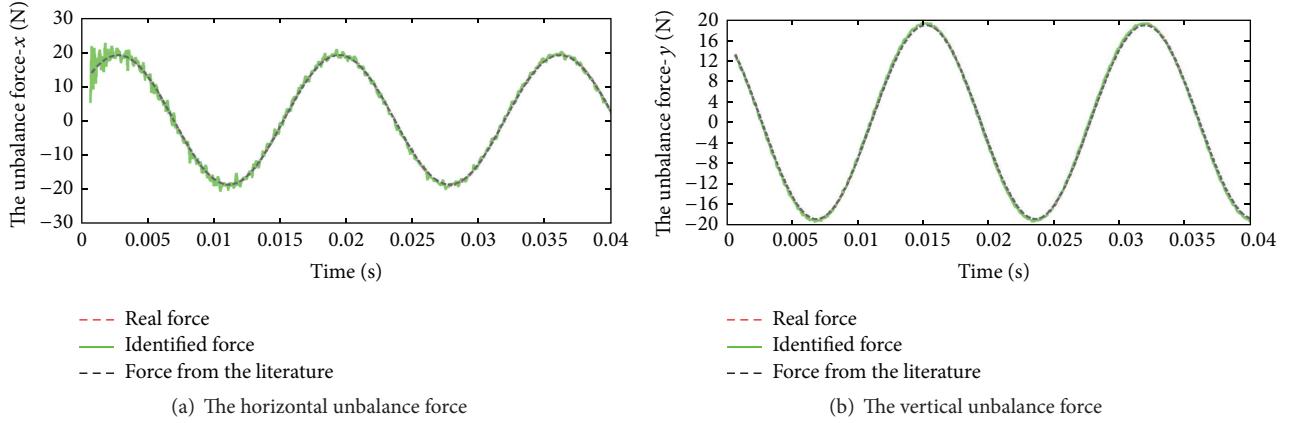


FIGURE 3: The identified unbalance forces on the first disk.

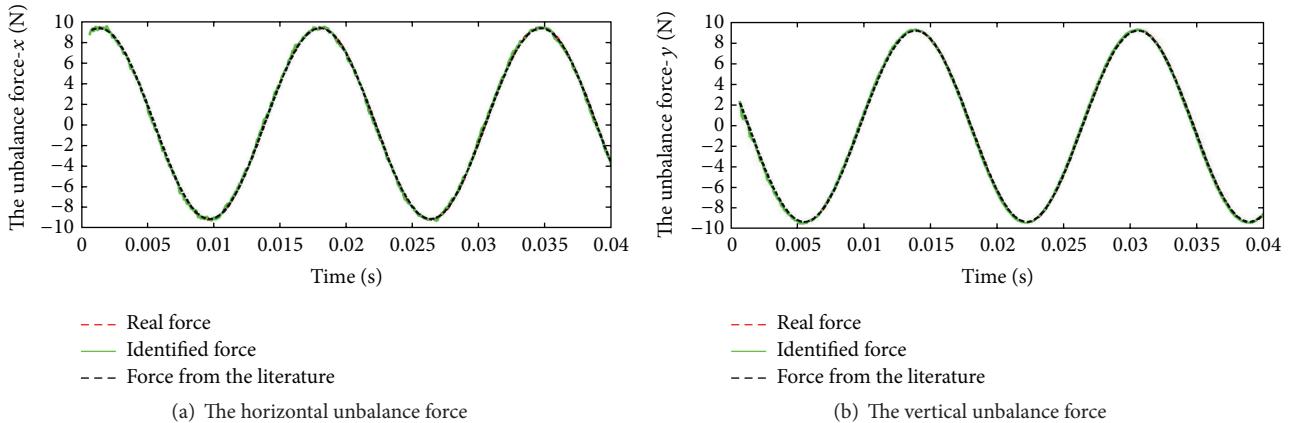


FIGURE 4: The identified unbalance forces on the second disk.

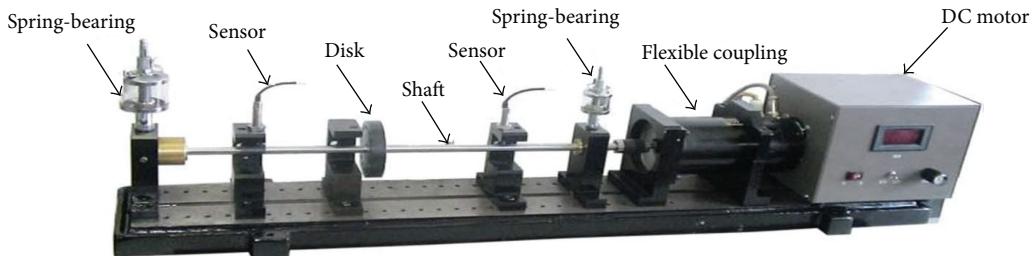


FIGURE 5: The test rig and schematic diagram.

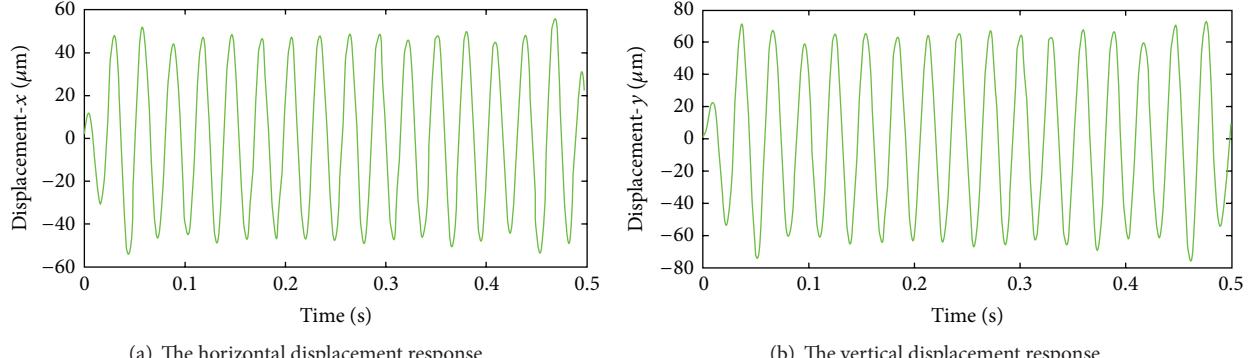


FIGURE 6: Responses at the rotor spin speed of 2000 rpm.

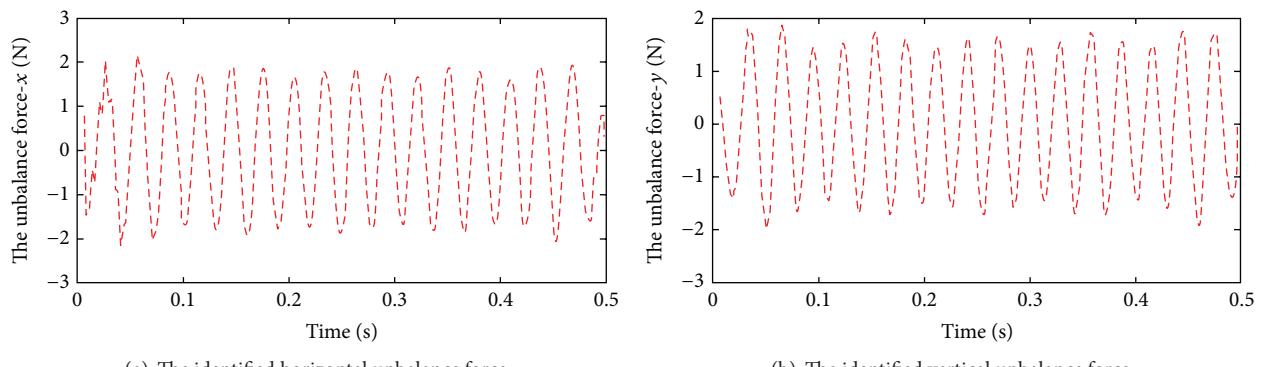


FIGURE 7: The identified unbalance forces of the test rig.

TABLE 3: Parameters of the test rig.

Parts	Parameters	Specifications		
Shaft	Length (mm)	490.0		
	Diameter (mm)	10.0		
	Density ($\text{kg}(\text{m}^3)^{-1}$)	7850.0		
	Young's modulus (Gpa)	205.8		
	Poisson ratio	0.3		
Disk	Diameter (mm)	80.0		
	Thickness (mm)	15.0		
The sliding bearing	Stiffness coefficients ($\text{MN}\cdot\text{m}^{-1}$)	Damping coefficients ($\text{kN}\cdot\text{s}\cdot\text{m}^{-1}$)		
The first bearing	k_{xx}	49.513	c_{xx}	200.21
	k_{xy}	-29.591	c_{xy}	21.507
	k_{yx}	6.4475	c_{yx}	702.16
	k_{yy}	35.744	c_{yy}	272.48
The second bearing	k_{xx}	200.021	c_{xx}	874.03
	k_{xy}	-69.727	c_{xy}	390.64
	k_{yx}	51.759	c_{yx}	361.34
	k_{yy}	0.1	c_{yy}	244.16

Table 4. The horizontal displacement response is attenuated by about 58.85%, and the vertical response is decreased by about 65.76%. The measured displacement responses can be reduced by applying the identified unbalance parameters based on the proposed method.

6. Conclusions

An identification method for the unbalance parameters of a rotor-bearing system is proposed. The main advantage of this method is that the states of a few measured points need to be measured only by one test which greatly reduced the number of physical experiments and cost and it is robust to the noise data of measured displacement, which is very critical to the practical situation. And the proposed method can be accurate and effective to obtain the dynamic load determined difficultly by some traditional methods.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

TABLE 4: Highest amplitude of measured points before and after balancing.

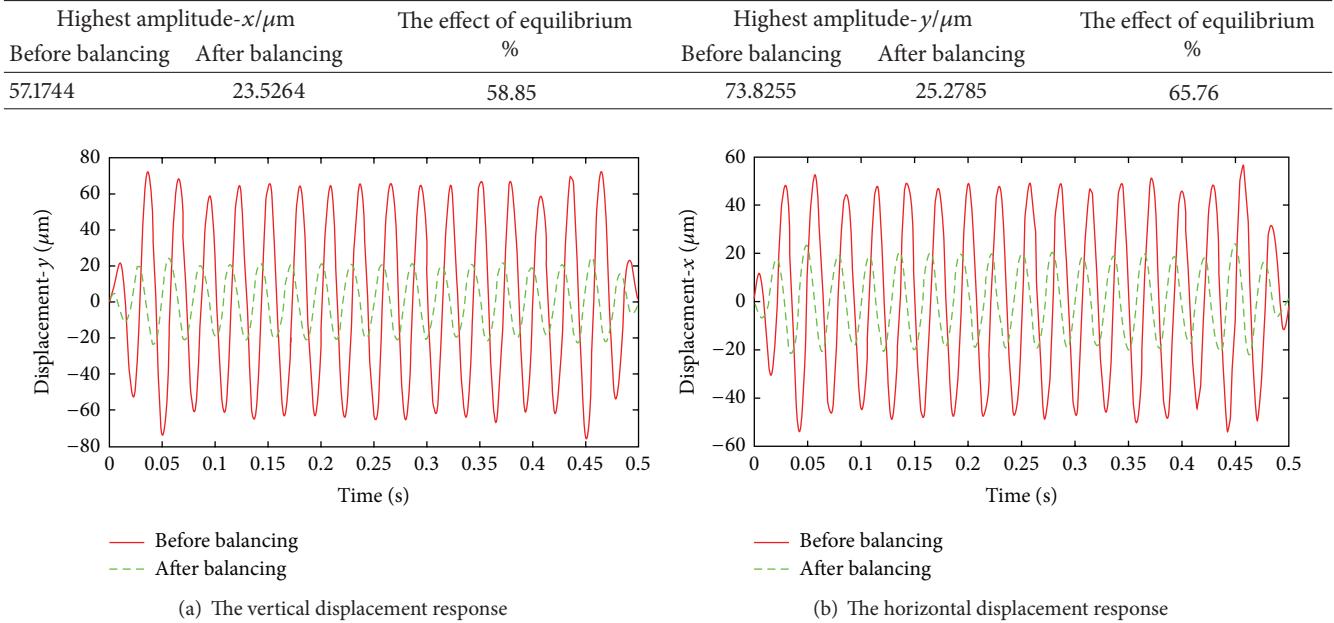


FIGURE 8: Experimental unbalance responses: before and after the balancing procedure.

Acknowledgments

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