

Research Article

Multistability in Horizontal Platform System with and without Time Delays

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Chaotic behavior and bifurcation analysis of horizontal platform systems (HPS) have been investigated widely by many researchers. However, the multistable features of such systems have not been investigated, and hence we identified the multistable parameter and investigated the coexisting attractors of the HPS. To understand the effects of time delays on the nonautonomous and autonomous HPS, we introduced a constant time delay in the state feedback variable. Investigation of the bifurcation of the time delayed HPS with time delay and parameters reveals that the system behavior differs between the autonomous and nonautonomous HPS. To investigate the multistability existence in time delayed HPS, we plot the bifurcation of the autonomous HPS and show the multistability and coexisting attractors.

1. Introduction

Mathematical models of most of the physical systems show complex nonlinear behaviors. The superposition principle is very useful in linear analysis, which is not valid in the case of nonlinear analysis. The uncertainty with system models often demands the nonlinear dynamical models to be analyzed without linearization. Since mass, damper, and spring are the basic components of a vibratory system, nonlinearity may be introduced into the governing differential equation through any of these components. Chaos modeling has applications in many areas in science and engineering [1]. Some of the common applications of chaotic systems in science and engineering are chemical reactors, Brusselators, dynamos, tokamak systems, biology models, neurology, ecology models, and memristive devices. Chaotic phenomenon was observed in several mechanical systems [2]. Chaos may be described as bounded random like motion exhibited by deterministic nonlinear systems characterized by their sensitivity to initial conditions. Not all nonlinear systems exhibit chaos nor does chaos occur for all combinations of system parameters and initial conditions for a given system [3].

The horizontal platform system (HPS) holds a freely rotating platform about its horizontal axis especially in offshore applications. An accelerometer is placed over the platform in order to sense the displacement; accordingly, the platform is balanced by producing inverse torque to maintain the stability of HPS. Horizontal platform system (HPS) exhibits chaotic behavior on some specified conditions [4]. Chaotic motion and bifurcation analysis of a HPS under specified conditions with its dynamical response as a function of the torque is discussed in [5]. An adaptive sliding mode controller (ASMC) is designed and implemented to control the chaotic behavior in HPS with parameter uncertainties. HPS with model and parameter uncertainties with external disturbances is investigated, the chaotic behavior of the system is controlled using HPS synchronization with time varying control [6], and the dynamic analysis of a fractional order horizontal platform system (FOHPS) and its fractional order finite time chaos control design are investigated in [7].

Motivated by the above mentioned discussions, we are interested in analyzing the HPS with autonomous and nonautonomous modeling. Most of the earlier literature has discussed and analyzed the regular bifurcation pattern of the HPS, but none has presented the multistable behavior of the

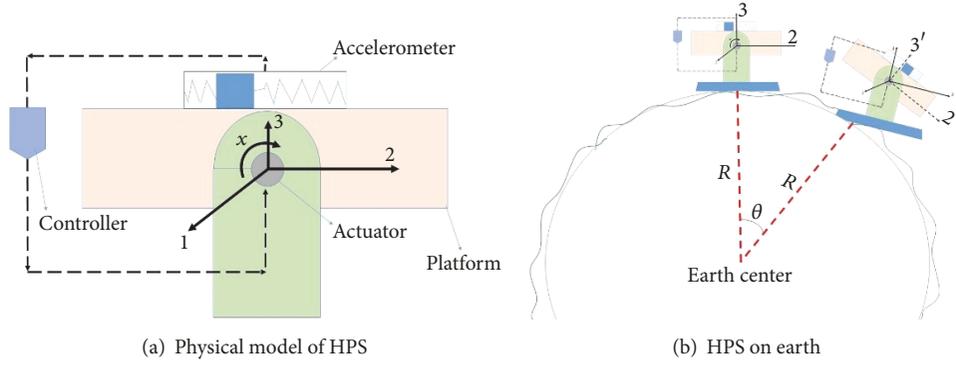


FIGURE 1

HPS which we have presented here. We also discuss the effect of constant time delays on the nonlinear behavior of the HPS and show the effect of time delays on autonomous and nonautonomous HPS.

2. Mathematical Modeling of HPS

We have used the HPS model described in [4–7] as given by Figures 1(a) and 1(b). While the platform varies from horizontal position, the accelerometer measures and sends it to the controller and the controller will produce inverse torque in order to stabilize the platform; the actuator will execute the inverse torque.

The basic assumption is that the horizontal platform rests on spherical joint which enables movement in coupled manner when it is displaced in one rotational axis. $x(t)$ represents the rotation of the platform in relation to the earth. Consider A_1 , A_2 , and A_3 are the inertia moments of the platform for axes 1, 2, and 3, respectively. D denotes the damping coefficient, R represents radius of the earth, r is the proportional constant of the accelerometer, g denotes gravity, and $F \cos \omega t$ is the harmonic torque.

The governing law is Newton's 2nd law of motion for rotational motion:

$$\sum M = I\ddot{x}. \quad (1)$$

The net moment acting on a rigid body is equivalent to the product of the moment of inertia and angular acceleration about an axis.

$$\sum M = A_1\ddot{x}. \quad (*A)$$

When the horizontal platform is displaced with a relative angular displacement " x ," the moments produced on the consequence are the resisting moment due to damping (fluidic friction, Coulomb friction), the moment due to accelerometer, and the coupling effect.

The damping torque is

$$T_d = D\dot{x}. \quad (*B)$$

Force acting due to accelerometer is

$$\cos(90 - x) = \frac{f}{mg} \quad (2)$$

$$f = mg \cos(90 - x).$$

The offset distance between center of mass of accelerometer and the center of mass of platform is " a ." The accelerometer torque moment is

$$M = f \cdot a \quad (3)$$

$$M = mga \cos(90 - x).$$

Consider $ma = r$,

$$M = rg \sin x. \quad (*C)$$

Moment equation with angular velocities in terms of Euler angles and Euler rates is

$$L = I_x \dot{p} + I_{xz} \dot{r} + qr(I_z - I_y) - I_{xz} pq, \quad (4)$$

where $q = \dot{\theta} \cos \phi + \dot{\psi} \cos \theta \sin \theta$,

$$r = \dot{\psi} \cos \theta \cos \phi - \dot{\theta} \sin \phi$$

$$\theta = \text{Pitch angle}$$

$$\phi = \text{Roll angle}$$

$$\psi = \text{Yaw angle}.$$

Since $I_{xz} = 0$ and $I_x \dot{p} = A_1 \ddot{x}$,

$$L = A_1 \ddot{x} + qr(I_z - I_y)$$

$$L = A_1 \ddot{x} + (\dot{\theta} \cos \phi + \dot{\psi} \cos \theta \sin \theta) \quad (6)$$

$$\cdot (\dot{\psi} \cos \theta \cos \phi - \dot{\theta} \sin \phi) \cdot (A_2 - A_3).$$

We know that $\dot{\psi} = 0$

$$L = A_1 \ddot{x} - \dot{\theta}^2 \cos \phi \sin \phi \cdot (A_2 - A_3)$$

$$= A_1 \ddot{x} - \frac{\dot{\theta}^2 R}{R} \cos \phi \sin \phi \cdot (A_2 - A_3). \quad (7)$$

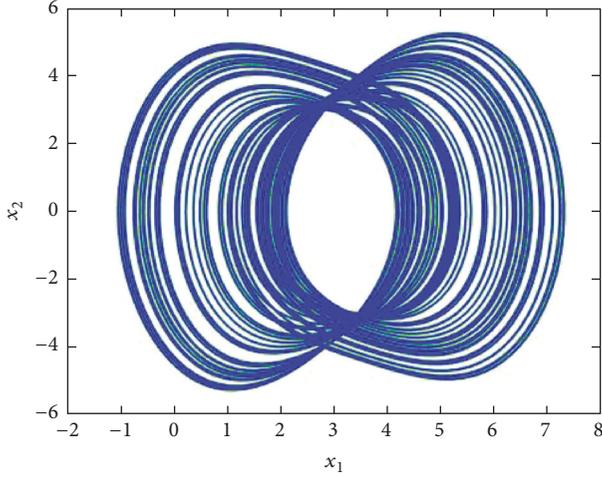


FIGURE 2: Phase portrait of the HPS.

Since $\theta^2 R = 3g$, $\phi = x$,

$$L = A_1 \ddot{x} - \frac{3g}{R} \cos x \sin x \cdot (A_2 - A_3). \quad (*D)$$

Equation of motion can be formulated using (*A), (*B), (*C), and (*D):

$$\begin{aligned} A_1 \ddot{x}(t) + D\dot{x}(t) + rg \sin x(t) \\ - \frac{3g}{R} (A_2 - A_3) \cos x(t) \cdot \sin x(t) = F \cos \omega t. \end{aligned} \quad (*)$$

Let us define the new states as $x_1(t) = x(t)$, $x_2(t) = \dot{x}(t)$ and new parameters as $a = D/A_1$, $b = rg/A_1$, $l = (3g/A_1 R)(A_2 - A_3)$, $h = F/A_1$, and (*) can be rewritten as

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= -ax_2(t) - b \sin x_1(t) + l \cos x_1(t) \sin x_1(t) \\ &\quad + h \cos \omega t. \end{aligned} \quad (8)$$

The autonomous form of the HPS defined in (8) can be given by

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= -ax_2(t) - b \sin x_1(t) + l \cos x_1(t) \sin x_1(t) \\ &\quad + h \cos x_3(t) \\ \dot{x}_3(t) &= \omega. \end{aligned} \quad (9)$$

For $A_1 = 0.3$, $A_2 = 0.5$, $A_3 = 0.2$, $D = 0.4$, $r = 0.11559633$, $g = 9.8$, $R = 6378000$, $F = 3.4$, $\omega = 1.8$, the new parameters are solved as $a = 1.333$, $b = 3.776$, $l = 4.6 \times 10^{-6}$, $h = 11.333$. For initial condition $[-3.4, 2.1, 0]$, the HPS shows chaotic behavior. The phase portrait is shown in Figure 2.

2.1. Dynamic Analysis of HPS. In this section we analyze the dynamic properties of HPS (9) like equilibria points, Lyapunov exponents, and bifurcation diagrams. The system does not have a fixed equilibrium point as the system model is nonautonomous.

Lyapunov exponents of a nonlinear system define the convergence and divergence of the states. Lyapunov exponents (LEs) are necessary and more convenient for detecting chaos in dynamical systems, and the existence of a positive Lyapunov exponent confirms the chaotic behavior of the system [8–12]. Time series based LEs calculation methods like Wolf algorithm [9], Jacobian method [10], and neural network algorithm [11] are popularly known ways of calculating Lyapunov exponents for integer and fractional order systems. To calculate the LEs of the HPS we use the Wolf method [9] and the total simulation time taken to calculate LEs is 20000s. The LEs of the HPS are calculated as $L_1 = 0.2198$, $L_2 = 0$, $L_3 = -1.5531$, and one positive LE (L_1) confirms that HPS exhibits chaotic oscillations.

To analyze the impact of the parameters on the HPS, we derive and investigate the bifurcation plots. By fixing all the other parameters, we vary parameter h between $[9.8, 12.2]$, and Figures 3(a) and 3(b) show the bifurcation plots for states x_1 and x_2 , respectively. The initial condition for the first iteration is fixed as $[-3.4, 2.1, 0]$ and is changed in every iteration as the end value of the state trajectory. The bifurcation plots show regions of period-5 limit cycle ($9.9 \leq h \leq 10.05$), period-4 limit cycle for ($11.83 \leq h \leq 11.9$), period-2 limit cycle for ($11.91 \leq h \leq 12.1$), and period-1 limit cycle for ($12.1 \leq h \leq 12.2$). As can be seen from Figure 3, the HPS takes period halving exit from chaos and period doubling limit cycle route to chaos. Furthermore an in-depth observation of Figure 3(a) gives a clue of hysteresis and bistability existence by showing discontinuous bifurcation.

2.2. Multistability in HPS. Multistability in mechanical systems is important in analyzing the system performance and its nonlinear behavior. A model of a vibroimpact oscillator, known as impact pair, composed of a point mass free to move inside a periodically driven box is investigated for its multistable property [13]. The statistical mechanics of shape transitions in small elastic elliptical plates and shells driven by noise and their multistable feature is analyzed in [14]. Multistability and controllability of multistability in the presence of noise and transition to chaos in noise and quasi-periodically driven systems are of significance [15–17]. Multistable features of HPS are investigated in this section and the bifurcation diagram is obtained by plotting local maxima of the coordinate x_2 in terms of the parameter that is increased (or decreased) in tiny steps in the range of $9.8 \leq h \leq 12.2$. This strategy, known as forward and backward continuation, represents a simple way to localize the window in which the system develops multistability. The existence of multistability can be confirmed by comparing the forward (Figure 4(a) red) and backward (Figure 4(a) black) bifurcation diagrams. Figure 4(b) shows the coexisting attractors for various values of h and initial conditions $[3.4, -2.1, 0]$ (red plot), $[3.4, -2.1, -1.8]$ (blue plot).

3. Time Delayed HPS (TDHPS)

Time delayed differential equations play an important role in most of the engineering applications [18]. Time delay effects on mechanical systems have gained interest in recent years.

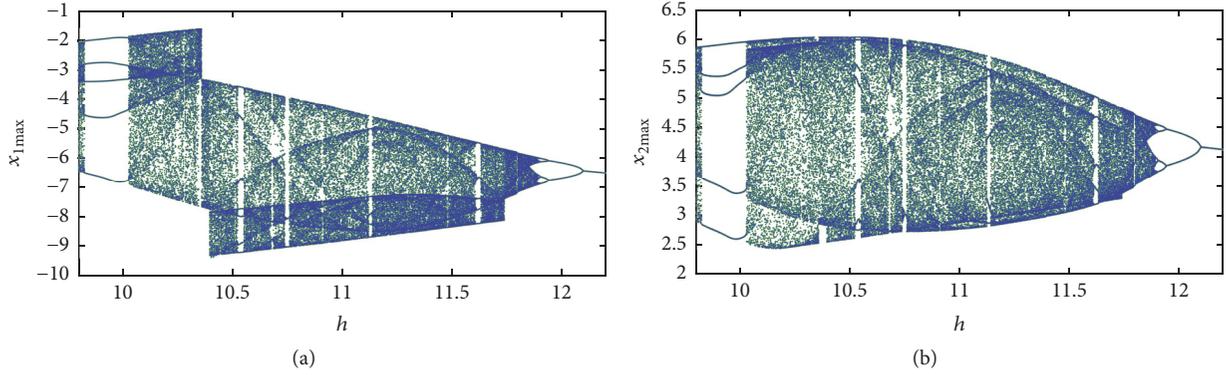


FIGURE 3: Bifurcation of the HPS (9) with the parameter h ((a) x_1 and (b) x_2).

The increased efficiency requirements in the control inputs and controller feedback have demanded the analysis of the effect of time delays on the system. The time delays in both controllers and actuators have become a serious problem and, for instance, all digital controllers, analogue antialiasing, and reconstruction filters exhibit a certain time delay during operation, and the hydraulic actuators and human machine interaction usually show even more significant time delays [19]. Considering these facts, we are interested in analyzing the time delay effects on the HPS dynamics.

3.1. Time Delayed Nonautonomous HPS. We first introduce feedback state delay for the state variable x_1 and modify the nonautonomous HPS (8) as

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= -a_\tau x_2(t) - b_\tau \sin x_1(t - \tau) \\ &\quad + l_\tau \cos x_1(t - \tau) \sin x_1(t - \tau) \\ &\quad + h_\tau \cos \omega_\tau t. \end{aligned} \quad (10)$$

and for the constant time delay $\tau = 0.1$, parameter values $a_\tau = 1.333$, $b_\tau = 3.776$, $l_\tau = 4.6 \times 10^{-6}$, $h_\tau = 11.333$, $\omega_\tau = 2$, and initial conditions $[-3.4, 2.1, 0]$, the time delayed nonautonomous HPS (TDNHPS) shows chaotic behavior as shown in Figure 5.

As discussed in the HPS, the TDNHPS also does not have a fixed point as the system is nonautonomous. To calculate the Lyapunov exponents of the TDNHPS, there are several algorithms proposed in the literature [20–22]. In this paper we adopted the technique employing the synchronization of identical systems coupled by linear negative feedback mechanism [21] for finding the exact Lyapunov exponents that are calculated as $L_1 = 0.2311$, $L_2 = -1.5245$.

To analyze the impact of the time delays on the proposed TDNHPS, we derive the bifurcation of the system with the time delay τ varied between $[0.0001, 1]$ and the system parameters fixed at their respective values for chaotic behavior, and the initial condition for first iteration is $[-3.4, 2.1, 0]$ and is changed to the end values of the state trajectory in every iteration. Figure 6 shows the bifurcation plot of TDNHPS with time delay, and as can be seen from the figure the system

shows multiple chaotic regions for $0.0001 \leq \tau \leq 0.01$, $0.15 \leq \tau \leq 0.35$, $0.4 \leq \tau \leq 0.72$, and $0.77 \leq \tau \leq 0.9$. The TDNHPS takes period doubling limit cycle route to chaos and inverse period doubling exit from chaos.

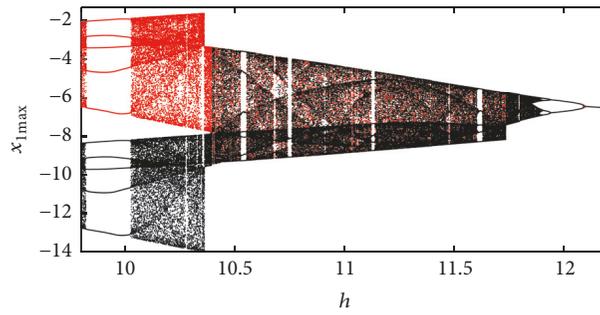
Figure 7 shows the bifurcation of the TDNHPS with the angular frequency of the excitation. As seen from the figure the TDNHPS shows discontinuous bifurcation with chaotic oscillations for $1.77 \leq \omega_\tau \leq 2.05$, $2.35 \leq \omega_\tau \leq 2.57$, and $2.6 \leq \omega_\tau \leq 2.7$ and takes period doubling route to chaos and period halving exit from chaos.

3.2. Time Delayed Autonomous HPS (TDAHPS). The time delayed autonomous system for the HPS (9) is derived by introducing the constant time delay in the feedback of state variable x_1 given as

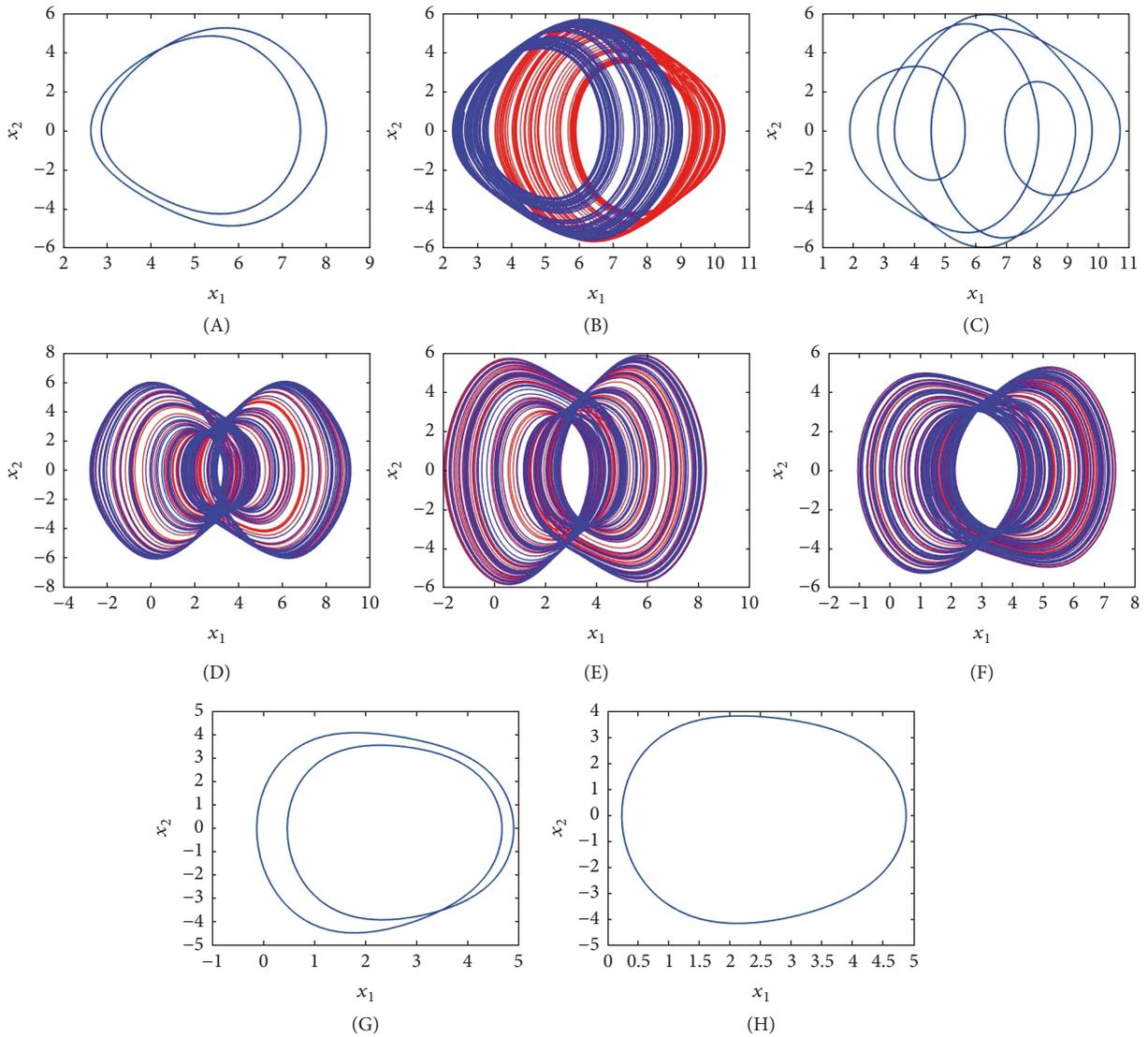
$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= -ax_2(t - \tau) - b \sin x_1(t - \tau) \\ &\quad + l \cos x_1(t - \tau) \sin x_1(t - \tau) \\ &\quad + h \cos x_3(t) \\ \dot{x}_3(t) &= \omega. \end{aligned} \quad (11)$$

Like TDNHPS, the TDAHPS also shows chaotic behavior for the same conditions and parameter values, and the chaotic attractor is shown in Figure 8.

There are no defined fixed points for the system and the Lyapunov exponents of the system are calculated as $L_1 = 0.2392$, $L_2 = 0$, $L_3 = -1.5137$. The bifurcation of the TDAHPS with time delay is given in Figure 9. Unlike TDNHPS (Figure 6), the TDAHPS shows nearly continuous bifurcation for $0.05 \leq \tau \leq 0.8$ and takes routing entry to and exit from chaos with period doubling and period halving limit cycle, respectively. Also there is significant difference between the bifurcation patterns of TDNHPS and TDAHPS confirming that the approximation of the time with a third state is not an effective method to convert nonautonomous to autonomous systems when time delay effects are considered. Similarly the effect of the parameter (ω_τ) also varies between autonomous and nonautonomous HPS as can be seen from Figures 10 and 7, respectively.



(a) Multistability in HPS with the red plots showing the forward continuation (increase parameter h from 9.8 to 12.2) and black plots showing backward continuation (decrease parameter h from 12.2 to 9.8) with reinitializing of the initial conditions



(b) Coexisting attractors for various values of h and initial conditions $[3.4, -2.1, 0]$ (red plot), $[3.4, -2.1, -1.8]$ (blue plot); (A) $h = 9$; (B) $h = 9.5$; (C) $h = 10$; (D) $h = 10.5$; (E) $h = 11$; (F) $h = 11.5$; (G) $h = 12$; (H) $h = 12.2$

FIGURE 4

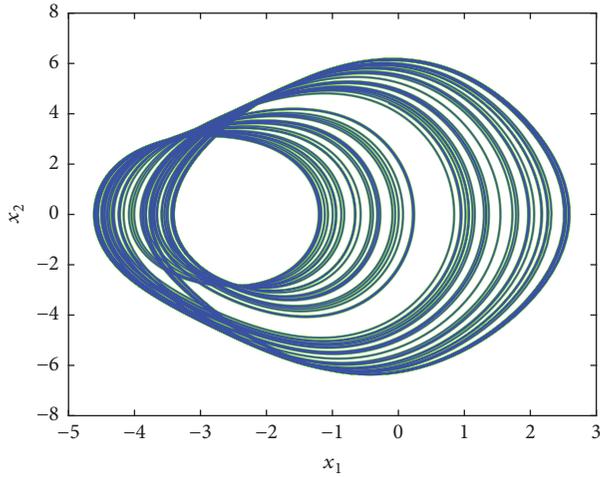


FIGURE 5: Chaotic attractor of the time delayed nonautonomous HPS.

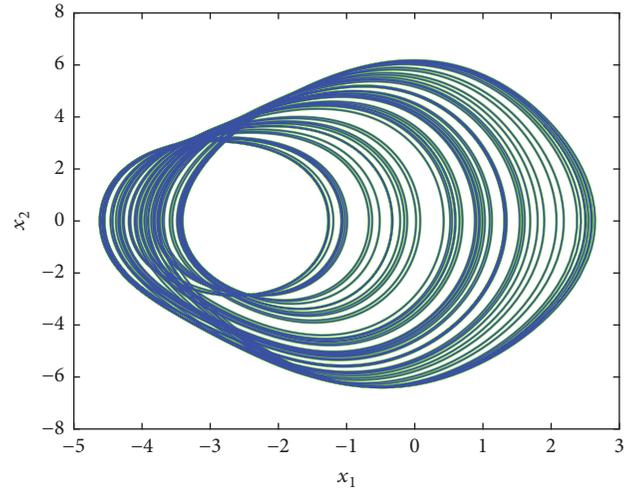


FIGURE 8: Phase portrait of the time delayed autonomous HPS.

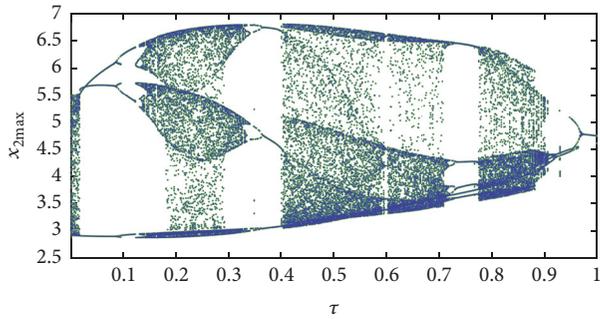


FIGURE 6: Bifurcation of the TDNHPS with time delay τ .

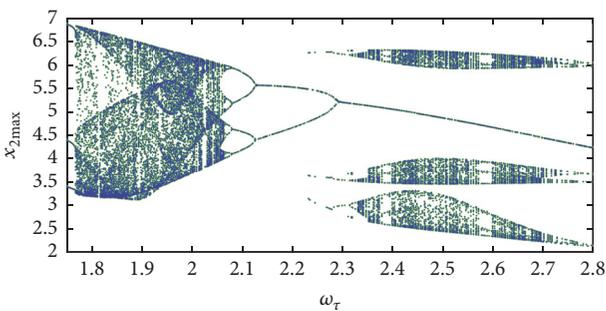


FIGURE 7: Bifurcation of the TDNHPS with parameter ω_τ .

3.3. Multistability in Time Delayed Autonomous HPS. To analyze the multistability existence in time delayed autonomous HPS, we derive and analyze the bifurcation of TDAHPS with parameter h_τ . The initial condition for the first iteration is taken as $[-3.4, 2.1, 0]$ and is reinitialized to the last value of the state variables at the end of every iteration, plotting local maxima of the coordinate x_2 in terms of the parameter h_τ that is increased from 9.8 to 13 (forward continuation) and decreased from 13 to 9.8 (backward continuation) in tiny steps. The constant time delay of the TDAHPS is fixed at 0.1. Figure 11 shows the multistability in TDAHPS, and as seen from the figure there are coexisting attractors for

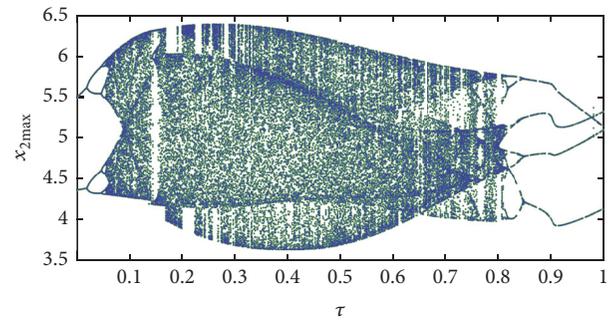


FIGURE 9: Bifurcation of the TDAHPS with time delay τ .

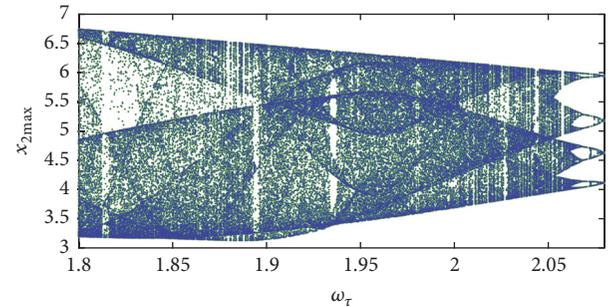


FIGURE 10: Bifurcation of the TDAHPS with parameter ω_τ .

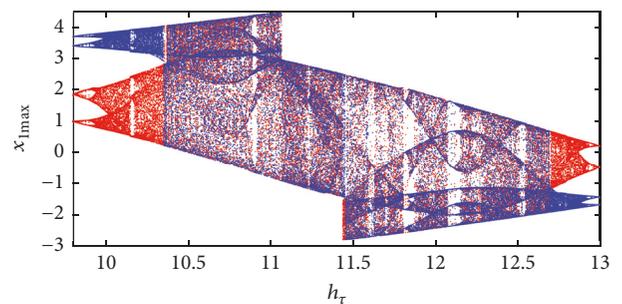


FIGURE 11: Multistability in TDAHPS with the red plots showing the forward continuation (increase in parameter h_τ from 9.8 to 13) and blue plots showing backward continuation (decrease in parameter h_τ from 13 to 9.8) with reinitializing of the initial conditions.

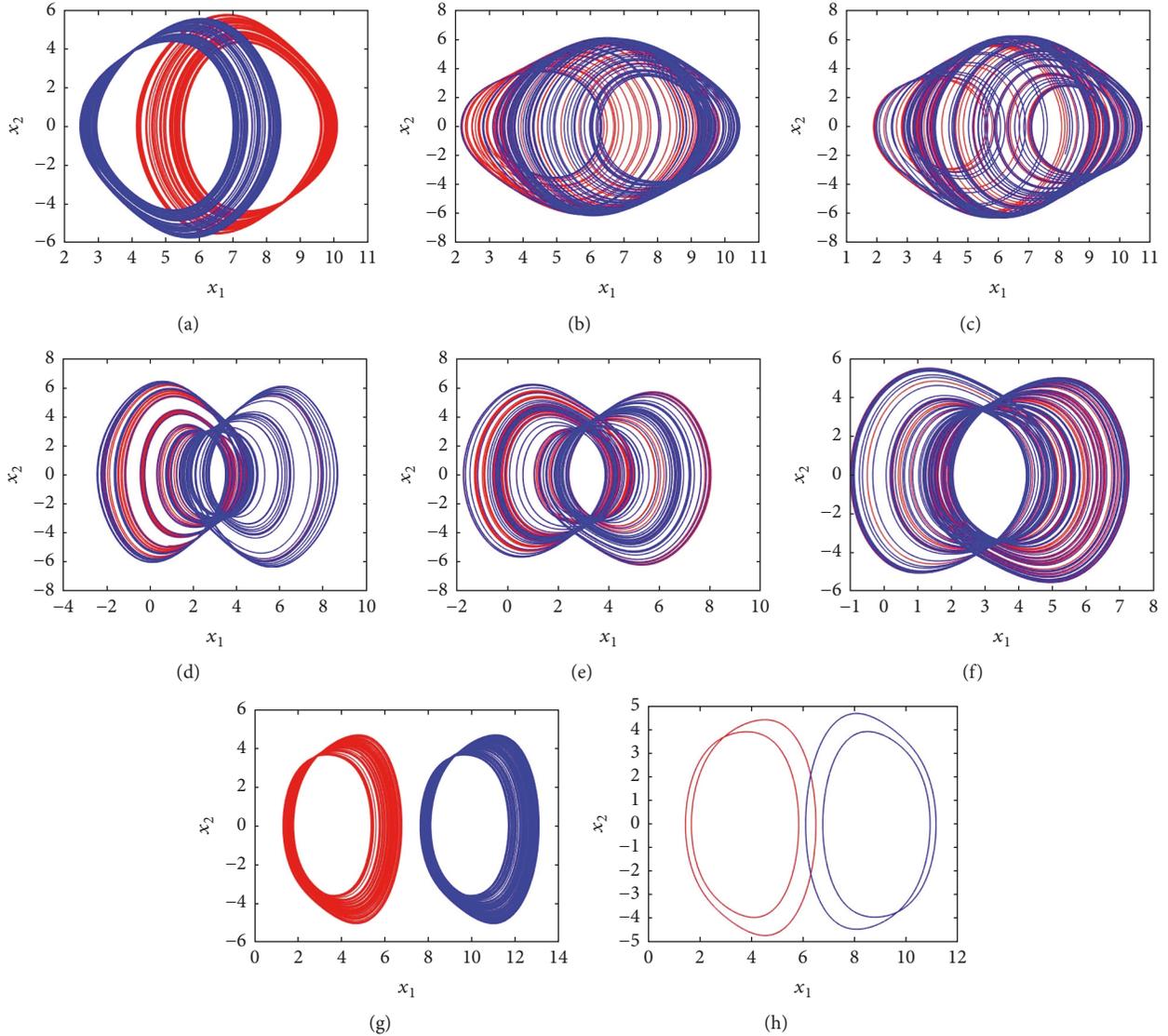


FIGURE 12: Coexisting attractors of the TDAHPS for various values of parameter h_τ . (a) $h_\tau = 10$; (b) $h_\tau = 10.5$; (c) $h_\tau = 11$; (d) $h_\tau = 11.5$; (e) $h_\tau = 12$; (f) $h_\tau = 12.5$; (g) $h_\tau = 12.8$; (h) $h_\tau = 13$.

$9.918 \leq h_\tau \leq 10.36$, $11.45 \leq h_\tau \leq 11.46$, and $12.69 \leq h_\tau \leq 12.93$. These coexisting attractors are evidence of multistability and are seen by comparing the forward (red) and backward (blue) continuation plots. Figure 12 shows the 2D phase portraits of the TDAHPS.

4. Conclusion

Bifurcation analysis and chaotic behaviors exhibited by a horizontal platform system are studied for nonautonomous nondelayed system. The bifurcation of the HPS with parameter h shows that the system shows multistability with coexisting attractors. Time delay effects on the autonomous and nonautonomous HPS are derived by introducing a constant delay with the state feedback variable x_1 . Bifurcation of the autonomous and nonautonomous TDHPS with time delay shows that the system behavior differs with the introduction

of the third state variable. Similarly the bifurcation of the parameters of the autonomous and nonautonomous TDHPS also shows difference. This clearly shows that a constant time delay can change the system behavior between the autonomous and nonautonomous systems. Multistability and coexisting attractors are seen in the TDHPS as well as the HPS.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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