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Research Article

The Influence of Slowly Varying Mass on Severity of Dynamics Nonlinearity of Bearing-Rotor Systems with Pedestal Looseness

Mian Jiang , Jigang Wu, and Shuangqi Liu

Hunan Provincial Key Laboratory of Health Maintenance for Mechanical Equipment, Hunan University of Science and Technology, Xiangtan 411201, China

Correspondence should be addressed to Mian Jiang; mjiang@cvm.ac.cn

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Nonlinearity measure is proposed to investigate the influence of slowly varying mass on severity of dynamics nonlinearity of bearing-rotor systems with pedestal looseness. A nonlinear mathematical model including the effect of slowly varying disk mass is developed for a bearing-rotor system with pedestal looseness. The varying of equivalent disk mass is described by a cosine function, and the amplitude coefficient is used as a control parameter. Then, nonlinearity measure is employed to quantify the severity of dynamics nonlinearity of bearing-rotor systems. With the increasing of looseness clearances, the curves that denote the trend of nonlinearity degree are plotted for each amplitude coefficient of mass varying. It can be concluded that larger amplitude coefficients of the disk mass varying will have more influence on the severity of dynamics nonlinearity and generation of chaotic behaviors in rotor systems with pedestal looseness.

1. Introduction

Rotor systems with slowly varying mass are fundamental components of various machines in the textile, paper, process, and cable industries. With the increasing of the velocity of the machine, its efficiency is usually decreased by vibrations and chaotic motion appeared in the rotor systems. When pedestal looseness occurs in the rotor systems, it has great impacts on the stability and work performance because of the existing complicated vibration characteristics and chaotic behaviors [1, 2]. Usually, the effect of slowly mass variation is neglected for vibration response of rotor systems, and only few results are published for this topic [3–5]. However, experimental and practical investigations show that the varying mass of the rotor has to be taken into consideration to explain some nonlinear phenomena of dynamics appearing in such systems with faults [6]. In diagnosing pedestal looseness of rotor systems with slowly varying mass, it is very important to investigate the influence of slowly varying mass on nonlinear dynamical behaviors of these rotor systems.

Extensive research has been achieved on the analysis and diagnosis of pedestal looseness in rotor systems in the past decades [7–15]. Goldman and Muszynska [16, 17] developed a bilinear model for an unbalanced rotor/bearing/stator system with looseness faults, and chaotic characteristics of responses were observed. Chu and Tang [1] analyzed the periodic, quasi-periodic, and chaos characteristics of a rotor-bearing system with pedestal looseness in which rotating speed and imbalance are considered as the control parameters. Ji and Zu [18] analyzed the vibration characteristics of an autonomous bearing-rotor system with supporting looseness using the multiscale method. These mathematical models based methods mainly focus on the impacts of pedestal looseness on the nonlinear vibration characteristics of rotor systems; the influence of varying disk mass is not considered and analyzed. As a powerful technique, finite element (FE) method is also employed to analyze the dynamics of bearing-rotor systems with pedestal looseness [19–21]. Although different mass, moment inertia, and looseness clearance are included easily to study the influences on the dynamic characteristics, the variances of severity of dynamical nonlinearity are not investigated yet. Recently, nonlinearity measure based assessment method for pedestal looseness of rotor systems is proposed by Jiang et al. [22]. The values of nonlinearity measure denote

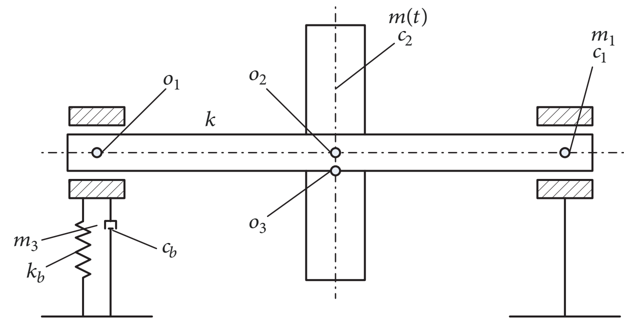


Figure 1: Bearing-rotor system with single pedestal looseness.

the severity of nonlinearity in the dynamics of rotor systems with different looseness clearances, which can also reflect the different dynamical behaviors such as periodic, quasi-periodic, and chaotic motions. It provides a numerical tool to discuss the influence of slowly varying mass on severity of dynamics nonlinearity of bearing-rotor systems.

In this paper, nonlinearity measure is employed to investigate the influence of slowly varying mass on severity of dynamics nonlinearity of bearing-rotor systems with pedestal looseness. A nonlinear mathematical model that includes the effect of slowly disk mass varying is developed for a bearing-rotor system with pedestal looseness. Then, nonlinearity measure method is employed to quantify the severity of nonlinearity degree for dynamics of bearing-rotor systems with different looseness clearances. The amplitude coefficient of disk mass varying is used as a control parameter to perform a detailed investigation of nonlinear dynamical behaviors of the bearing-rotor system. With a given amplitude coefficient, looseness clearances are used to simulate the dynamics of rotor systems and conduct the nonlinearity measure. According to the trend of nonlinearity degree, it can be found that a big amplitude coefficient of the disk mass varying will have more impacts on the severity of dynamics; chaotic behaviors will be observed more easily in rotor system with the increasing of looseness clearances.

This paper is organized as follows. The governing equations considering slowly varying mass for a bearing-rotor system with pedestal looseness are developed in Section 2. Assessment of severity of nonlinearity via nonlinearity measures is presented in Section 3. The influences of disk mass varying on dynamical behaviors are discussed in Section 4.1, while those on values of nonlinearity measure are investigated in Section 4.2. Concluding remarks are discussed in Section 5.

2. Governing Equations for Bearing-Rotor Systems with Pedestal Looseness

The considered rotor system with pedestal looseness and time-dependent disk mass varying is shown in Figure 1. There are two identical oil film bearings at both sides of the rotor system, and the shaft sections are considered to be elastic and massless. The equivalent lumped mass in the position of the bearings is m_1 and the lumped mass of the pedestal involving the looseness is m_3. The lumped mass in the position of the disk is assumed to be time-variant which is defined as a cosine function:

$$m(\tau) = m_0 (1 + \lambda \cos(\omega\tau)) \quad \tau = \varepsilon t \;\; \varepsilon \le 1, \tag{1}$$

where m_0 is the average mass of the disk, λ is the amplitude coefficient of mass varying, ω is the angular velocity of the rotor system, τ is the mass varying time, and ε is the time coefficient of mass varying.

In Figure 1, c_1, c_2 are the equivalent damping coefficients in the positions of bearing and disk, respectively, while k is the stiffness coefficient of the shaft. It is assumed that the left support has the single pedestal looseness; the maximum static gap of the looseness is δ. The foundation to the pedestal is equivalent to a spring-damping system with the stiffness coefficient k_b and damping coefficient c_b, which can be expressed using the following piecewise linear structure [1]:

$$k_b = \begin{cases} k_{b1} & y_4 < 0 \\ 0 & 0 \le y_4 \le \delta \\ k_{b3} & y_4 > \delta; \end{cases}$$
$$c_b = \begin{cases} c_{b1} & y_4 < 0 \\ c_{b2} & 0 \le y_4 \le \delta \\ c_{b3} & y_4 > \delta. \end{cases} \tag{2}$$

It is assumed that the horizontal and vertical displacements in the right-bearing position are x_1, y_1, in the disk position x_2, y_2, and in the left-bearing position x_3, y_3. The horizontal movement of the left pedestal is small and considered to be negligible. Its displacement in the vertical direction is y_4. The governing equations $N(\omega)$ for the bearing-rotor system with left pedestal looseness can then be written as follows:

$$N(\omega):\begin{cases} m_1\ddot{x}_1 + c_1\dot{x}_1 + k(x_1 - x_2) = F_x(x_1, y_1, \dot{x}_1, \dot{y}_1) \\ m_1\ddot{y}_1 + c_1\dot{y}_1 + k(y_1 - y_2) = F_y(x_1, y_1, \dot{x}_1, \dot{y}_1) - m_1 g \\ \dfrac{d}{dt}(m(\tau)\dot{x}_2) + c_2\dot{x}_2 + k(x_2 - x_1) + k(x_2 - x_3) = m(\tau)e_b\omega^2\cos(\omega t) \\ \dfrac{d}{dt}(m(\tau)\dot{y}_2) + c_2\dot{y}_2 + k(y_2 - y_1) + k(y_2 - y_3 - y_4) = m(\tau)\left(e_b\omega^2\sin(\omega t) - g\right) \\ m_1\ddot{x}_3 + c_1\dot{x}_3 + k(x_3 - x_2) = F_x(x_3, y_3 - y_4, \dot{x}_3, \dot{y}_3 - \dot{y}_4) \\ m_1\ddot{y}_3 + c_1\dot{y}_3 + k(y_3 + y_4 - y_2) = F_y(x_3, y_3 - y_4, \dot{x}_3, \dot{y}_3 - \dot{y}_4) - m_1 g \\ m_3\ddot{y}_4 + c_b\dot{y}_4 + (ky_4 + k_b y_4) = -F_y(x_3, y_3 - y_4, \dot{x}_3, \dot{y}_3 - \dot{y}_4) - m_3 g, \end{cases} \quad (3)$$

where e_b is the unbalance and ω is the rotating speed.

In (3), $F_x(x_1, y_1, \dot{x}_1, \dot{y}_1)$ and $F_y(x_1, y_1, \dot{x}_1, \dot{y}_1)$ denote the horizontal and vertical force components of the oil film in the right bearing, while $F_x(x_3, y_3 - y_4, \dot{x}_3, \dot{y}_3 - \dot{y}_4)$ and $F_y(x_3, y_3 - y_4, \dot{x}_3, \dot{y}_3 - \dot{y}_4)$ are those of the left bearing. The nonlinear forces components from the journal bearing are obtained under the short bearing theory [23]:

$$F_x = sf_x, \quad F_y = sf_y, \quad (4)$$

where F_x, F_y denote the horizontal and vertical force components of the oil film. The adjustment factor $s = \mu\omega RL[R/c]^2[L/2R]^2$, ω is the rotating speed, R is the radius of bearing, L is the length of bearing, c is the bearing clearance, and μ is the oil viscosity. The force components f_x, f_y can be obtained using the following equations:

$$\begin{bmatrix} f_x \\ f_y \end{bmatrix} = \frac{\sqrt{(x - 2\dot{y})^2 + (y + 2\dot{x})^2}}{1 - x^2 - y^2} \times \begin{bmatrix} 3xV - \sin\beta E - 2\cos\beta S \\ 3yV + \cos\beta E - 2\sin\beta S \end{bmatrix}, \quad (5)$$

where V, E, S, β can be represented as

$$\begin{aligned} V(x, y, \beta) &= \frac{2 + (y\cos\beta - x\sin\beta)E}{1 - x^2 - y^2}, \\ S(x, y, \beta) &= \frac{x\cos\beta + y\sin\beta}{1 - (x\cos\beta + y\sin\beta)^2}, \\ E(x, y, \beta) &= \frac{2}{\sqrt{1 - x^2 - y^2}}\left(\frac{\pi}{2} + \arctan\left(\frac{(y\cos\beta - x\sin\beta)}{\sqrt{1 - x^2 - y^2}}\right)\right), \\ \beta(x, y) &= \arctan\frac{y + 2\dot{x}}{x - 2\dot{y}} - \frac{\pi}{2}\operatorname{sign}\left(\frac{y + 2\dot{x}}{x - 2\dot{y}}\right) - \frac{\pi}{2} \cdot \operatorname{sign}(y + 2\dot{x}). \end{aligned} \quad (6)$$

Because $d/dt = (d/d\tau)(d\tau/dt) = \varepsilon(d/d\tau)$, (3) can be rewritten as follows:

$$N(\omega):\begin{cases} m_1\ddot{x}_1 + c_1\dot{x}_1 + k(x_1 - x_2) = F_x(x_1, y_1, \dot{x}_1, \dot{y}_1) \\ m_1\ddot{y}_1 + c_1\dot{y}_1 + k(y_1 - y_2) = F_y(x_1, y_1, \dot{x}_1, \dot{y}_1) - m_1 g \\ \ddot{x}_2 + c_2(\tau)\dot{x}_2 + k(\tau)(x_2 - x_1) + k(\tau)(x_2 - x_3) = e_b\omega^2\cos(\omega t) \\ \ddot{y}_2 + c_2(\tau)\dot{y}_2 + k(\tau)(y_2 - y_1) + k(\tau)(y_2 - y_3 - y_4) = e_b\omega^2\sin(\omega t) - g \\ m_1\ddot{x}_3 + c_1\dot{x}_3 + k(x_3 - x_2) = F_x(x_3, y_3 - y_4, \dot{x}_3, \dot{y}_3 - \dot{y}_4) \\ m_1\ddot{y}_3 + c_1\dot{y}_3 + k(y_3 + y_4 - y_2) = F_y(x_3, y_3 - y_4, \dot{x}_3, \dot{y}_3 - \dot{y}_4) - m_1 g \\ m_3\ddot{y}_4 + c_b\dot{y}_4 + (ky_4 + k_b y_4) = -F_y(x_3, y_3 - y_4, \dot{x}_3, \dot{y}_3 - \dot{y}_4) - m_3 g, \end{cases} \quad (7)$$

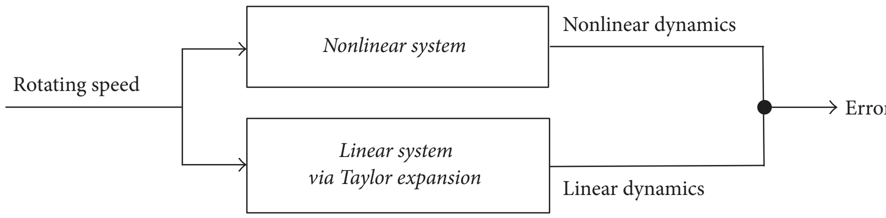


Figure 2: Setup for nonlinearity measure for dynamics of rotor systems.

where $c_2(\tau) = (c_2 + \varepsilon m'(\tau))/m(\tau)$, $k(\tau) = k/m(\tau)$, and $m'(\tau) = dm(\tau)/dt = -\lambda\omega m_0 \sin(\omega\tau)$.

3. Assessment of Severity of Nonlinearity via Nonlinearity Measures

In this paper, nonlinearity measures [24] are employed to assess the severity of dynamics nonlinearity of rotor-bearing systems with pedestal looseness. The nonlinearity measure represents an approach to systematically quantify the degree of nonlinearity for dynamical systems, which is already applied to assessment for pedestal looseness of rotor-bearing systems by Jiang et al. [22]. The fundamental idea underlying nonlinearity measure is to compare the dynamic behaviors of the nonlinear systems (3) with a linear system via Taylor expansions in an appropriate setup [24]. The setup for nonlinearity measures can be depicted in Figure 2.

The critical point is to find a linear approximated system on the vicinity of the equilibrium position (x_0, y_0) by Taylor expansion. In (3), oil film force components are the source of nonlinearity and have nonlinear characteristics. Then, Taylor expansions are employed to obtain the linear approximations of the oil film force components. These linear approximations will have a good accuracy because of the small perturbations on oil films from the vibrations amplitudes or velocity. Therefore, a linear system $L(\omega)$ developed to approximate the dynamics of (3) is given as follows:

$$L(\omega): \begin{cases} m_1\ddot{x}_1 + c_1\dot{x}_1 + k(x_1 - x_2) = \widetilde{F}_x(x_1, y_1, \dot{x}_1, \dot{y}_1) \\ m_1\ddot{y}_1 + c_1\dot{y}_1 + k(y_1 - y_2) = \widetilde{F}_y(x_1, y_1, \dot{x}_1, \dot{y}_1) - m_1 g \\ \ddot{x}_2 + c_2(\tau)\dot{x}_2 + k(\tau)(x_2 - x_1) + k(\tau)(x_2 - x_3) = e_b\omega^2\cos(\omega t) \\ \ddot{y}_2 + c_2(\tau)\dot{y}_2 + k(\tau)(y_2 - y_1) + k(\tau)(y_2 - y_3 - y_4) = e_b\omega^2\sin(\omega t) - g \\ m_1\ddot{x}_3 + c_1\dot{x}_3 + k(x_3 - x_2) = \widetilde{F}_x(x_3, y_3 - y_4, \dot{x}_3, \dot{y}_3 - \dot{y}_4) \\ m_1\ddot{y}_3 + c_1\dot{y}_3 + k(y_3 + y_4 - y_2) = \widetilde{F}_y(x_3, y_3 - y_4, \dot{x}_3, \dot{y}_3 - \dot{y}_4) - m_1 g \\ m_3\ddot{y}_4 + c_b\dot{y}_4 + (ky_4 + k_b y_4) = -\widetilde{F}_y(x_3, y_3 - y_4, \dot{x}_3, \dot{y}_3 - \dot{y}_4) - m_3 g, \end{cases} \quad (8)$$

where $\widetilde{F}_x, \widetilde{F}_y$ denote the liner approximations of force components F_x, F_y. They are obtained via Taylor expansion for force components on the vicinity of the equilibrium position. Linearizing the nonlinear terms on the static equilibrium position allows a linear system (8) to be derived. The details of this procedure can be found clearly in [22].

The nonlinearity measures quantify the differences between the dynamical responses of the nonlinear system $N(\omega)$ and linear system $L(\omega)$. This can provide information regarding how well the dynamical behaviors of the nonlinear system $N(\omega)$ resemble that of the linear model $L(\omega)$. According to the definition proposed by Schweickhardt and Allgöwer [24], the following definition is used because a linear system is obtained in advance:

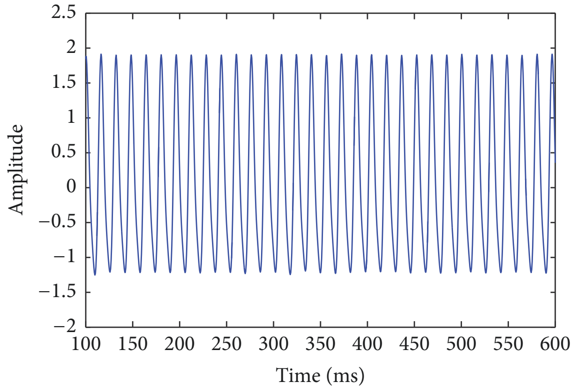
$$\phi_N^\omega = \sup_\omega \frac{\|N(\omega) - L(\omega)\|}{\|N(\omega)\|}, \quad (9)$$

where $N(\omega)$ is the nonlinear system, $L(\omega)$ is the linear model, and ϕ_N^ω denotes the value of the nonlinearity quantification using the input ω. A constant rotational speed ω is considered to be the input of (9) in this paper, and a simple definition of nonlinearity measure is given for engineering applications with lower computational requirement:

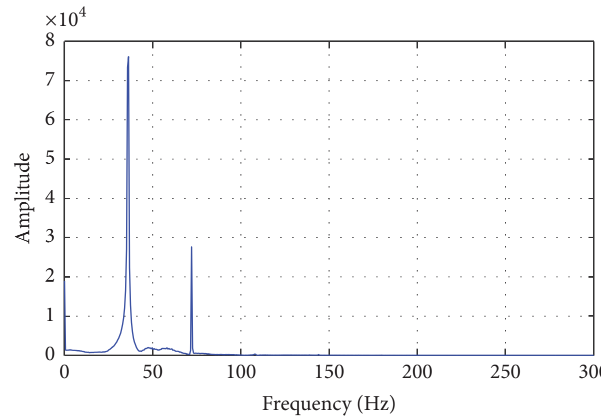
$$\phi_N^\omega = \frac{\|N(\omega) - L(\omega)\|}{\|N(\omega)\|}. \quad (10)$$

The value of ϕ_N^ω corresponds to the percentwise deviation of the dynamics of linear model $\mathbf{L}(\omega)$ from the dynamics of nonlinear system $N(\omega)$. In (9) and (10), $\|(\cdot)\|$ is the norm with the following definition:

$$\|x(\cdot)\| = \sqrt{\int_0^\infty |x(t)|^2\, dt}, \quad (11)$$



(a) Vibration amplitudes



(b) Vibration spectrums

FIGURE 3: The horizontal direction response of the disk ($\lambda = 0.1$, $\delta = 0.00002$ m).

where $x(t)$ denotes the nonlinear response signals that describe the dynamics of the systems. In principle, any norm can be used for the following considerations. In order to stress that fact (and for readability purposes), we will denote a norm using $\|(\cdot)\|$, without explicitly specifying which norm is used.

The result of the nonlinearity evaluation ϕ_N^{ω} will satisfy $\phi_N^{\omega} \geq 0$. It is quite challenging to discuss the high-dimensional system (3) and (8) in an analytical way. Therefore, the calculations of nonlinearity measure resorted to numerical methods, where the fourth order Runge-Kutta method is used to integrate the dynamic systems. The same procedure of nondimensionalization in [22] is used to transform the systems into new ones, while a smaller marching step is chosen to ensure a stable solution and to avoid the numerical divergence at the point where the damping and stiffness parameters are discontinuous.

4. The Influence of Disk Mass Varying on Values of Nonlinearity Measure

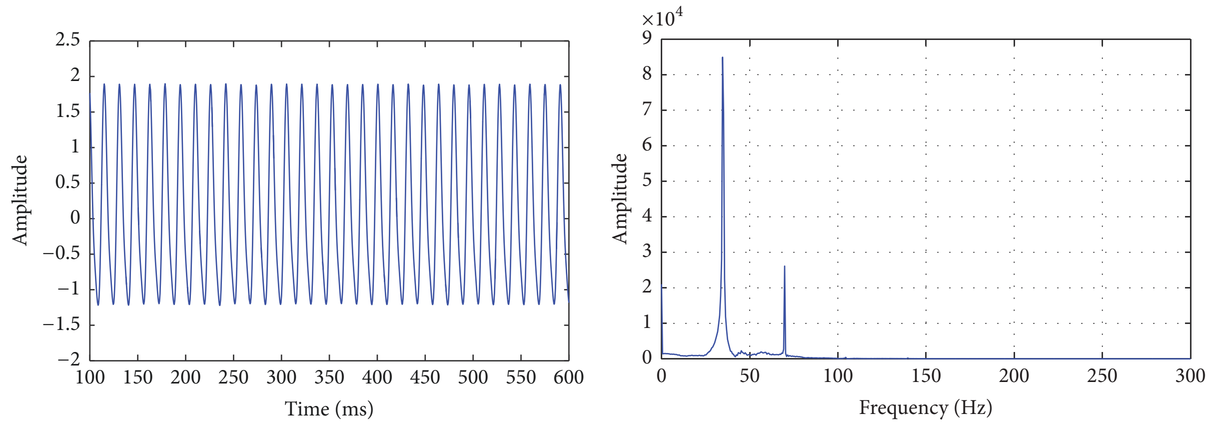
The influence of disk mass varying on values of nonlinearity measures are discussed in this section. The amplitude coefficient λ of mass varying is used as a control parameter to perform a detailed investigation of nonlinear dynamics of the bearing-rotor system. The influences on dynamical behaviors for the amplitude coefficient varying are discussed and compared at the first stage, while the impacts on the trend of nonlinearity evaluation of rotor systems with pedestal looseness will be given at the second stage. The values of parameters used in simulations and nonlinearity measures are given in Table 1.

TABLE 1: Values of simulation parameters.

Parameter	Value
m_0	4 kg
ε	0.05
m_1	32.1 kg
m_3	10 kg
c_1	1050 N·s/m
c_2	2100 N·s/m
k	2.5×10^7 N/m
ω	2100 rpm
e	0.5×10^{-4} m
k_{b1}	7.5×10^9 N/m
k_{b3}	7.5×10^7 N/m
μ	0.018 pa°s
c_{b1}	350 N·s/m
c_{b2}	100 N·s/m
c_{b3}	500 N·s/m
R	0.025 m
L	0.012 m
c	0.00011 m

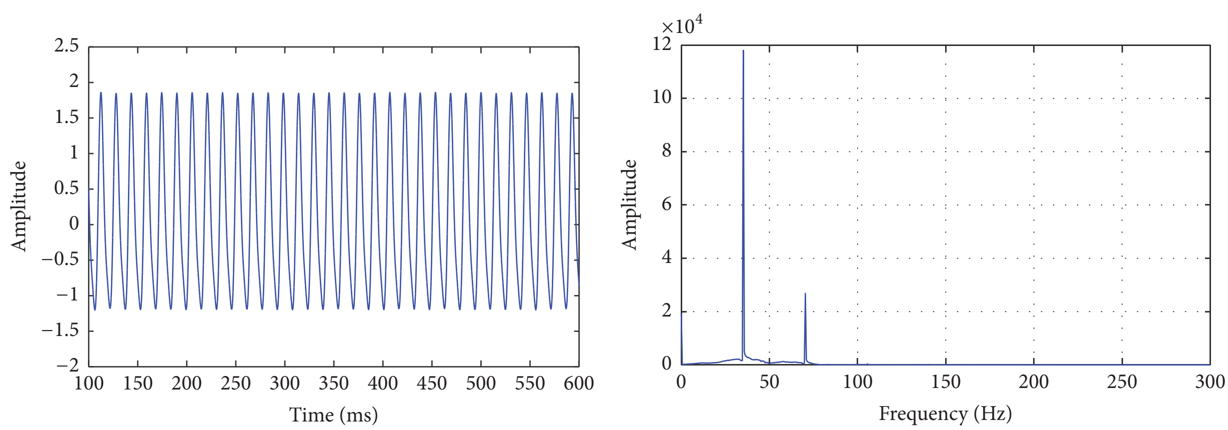
4.1. The Influence of Disk Mass Varying on Dynamical Behaviors. The amplitude coefficient λ of mass varying is an important parameter to have the impact on the dynamical behaviors of rotor systems with pedestal looseness. In the process of discussions for the influence of mass varying, the amplitude coefficients $\lambda = 0.1$, $\lambda = 0.3$, $\lambda = 0.5$ are used to simulate the nonlinear system (8). Looseness clearance δ is given different values from 0 to 0.0035 m to simulate the dynamics of the rotor system; the periodic, quasi-periodic, and chaotic behaviors are often observed in the bearing-rotor system. Because of the varying disk mass, the bearing-rotor system with looseness clearance 0.00002 m begins to generate the quasi-periodic behaviors, while that with looseness clearance 0.0001 m begins to have the chaotic behaviors. Then, the looseness clearances 0.00002 m, 0.0001 m are chosen to express the dynamical behaviors of the rotor systems. The vibration amplitudes in the positions of disk and pedestal looseness are collected and the amplitude spectrums in frequency domain are also given in Figures 3–14.

When the looseness clearances $\delta = 0.00002$ m, Figures 3–5 display the vibration amplitudes and its spectrums at the position of the disk with varying amplitude coefficients, respectively. Similarly, they are given in Figures 6–8 for



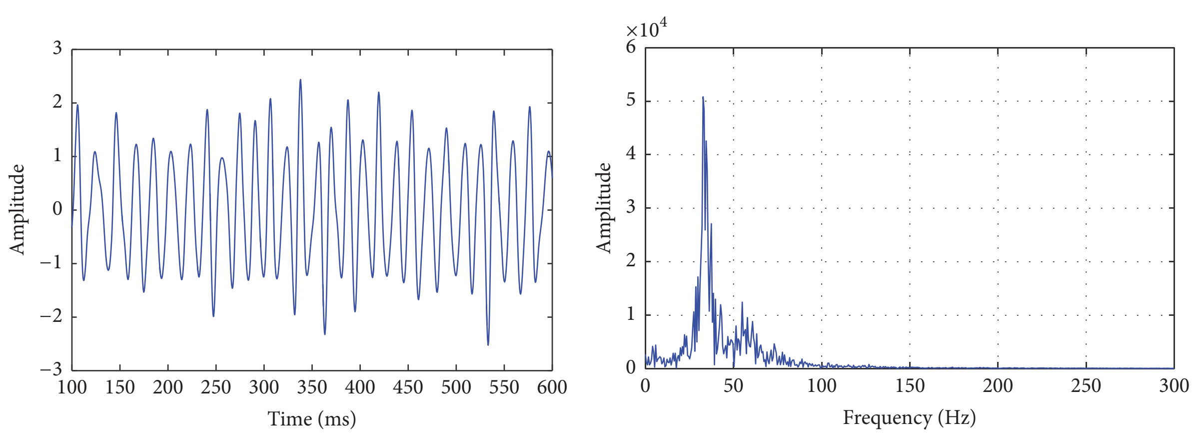
(a) Vibration amplitudes (b) Vibration spectrums

FIGURE 4: The horizontal direction response of the disk ($\lambda = 0.3$, $\delta = 0.00002$ m).



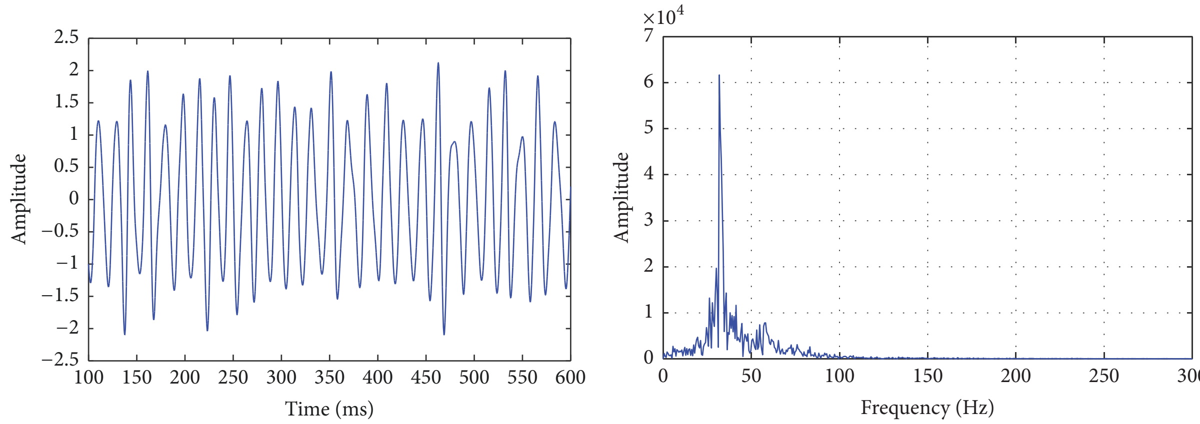
(a) Vibration amplitudes (b) Vibration spectrums

FIGURE 5: The horizontal direction response of the disk ($\lambda = 0.5$, $\delta = 0.00002$ m).



(a) Vibration amplitudes (b) Vibration spectrums

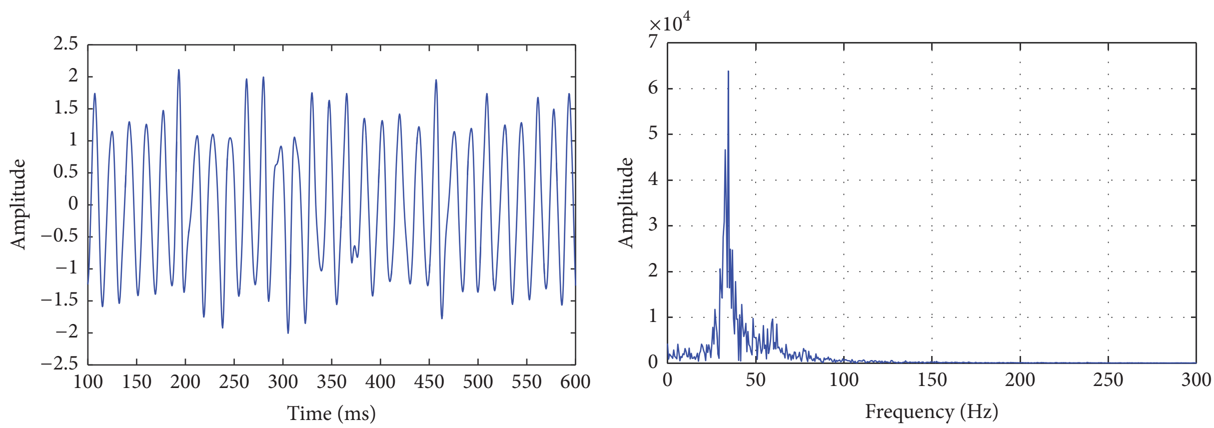
FIGURE 6: The horizontal direction response of the disk ($\lambda = 0.1$, $\delta = 0.0001$ m).



(a) Vibration amplitudes

(b) Vibration spectrums

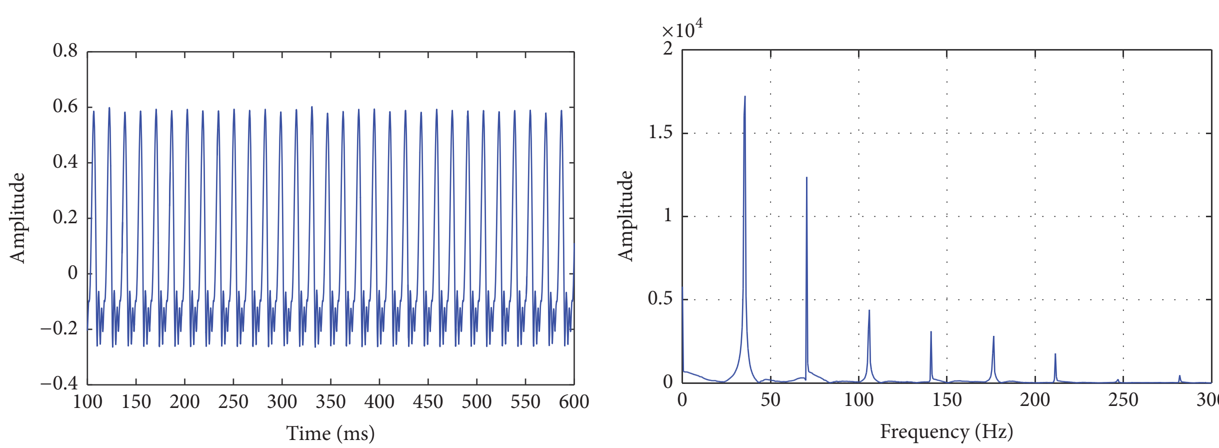
FIGURE 7: The horizontal direction response of the disk ($\lambda = 0.3$, $\delta = 0.0001$ m).



(a) Vibration amplitudes

(b) Vibration spectrums

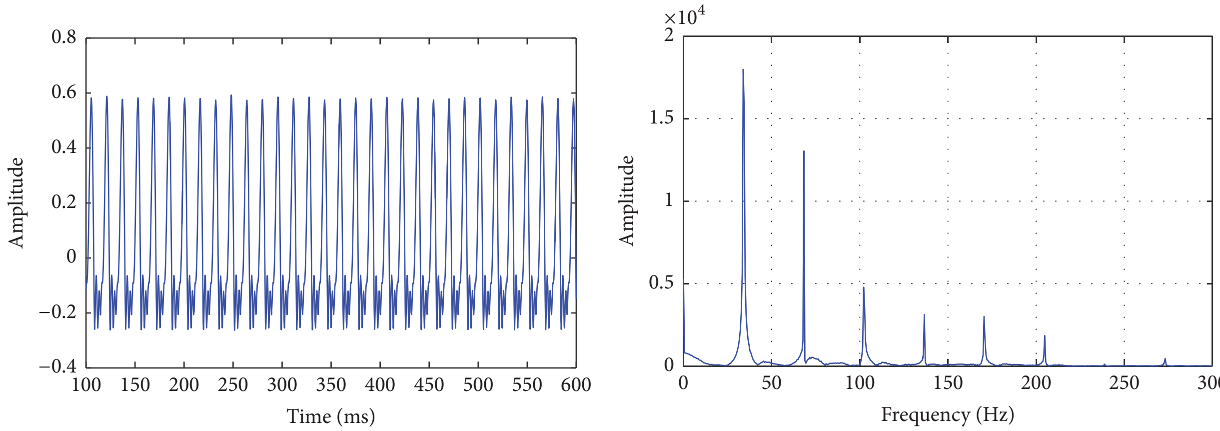
FIGURE 8: The horizontal direction response of the disk ($\lambda = 0.5$, $\delta = 0.0001$ m).



(a) Vibration amplitudes

(b) Vibration spectrums

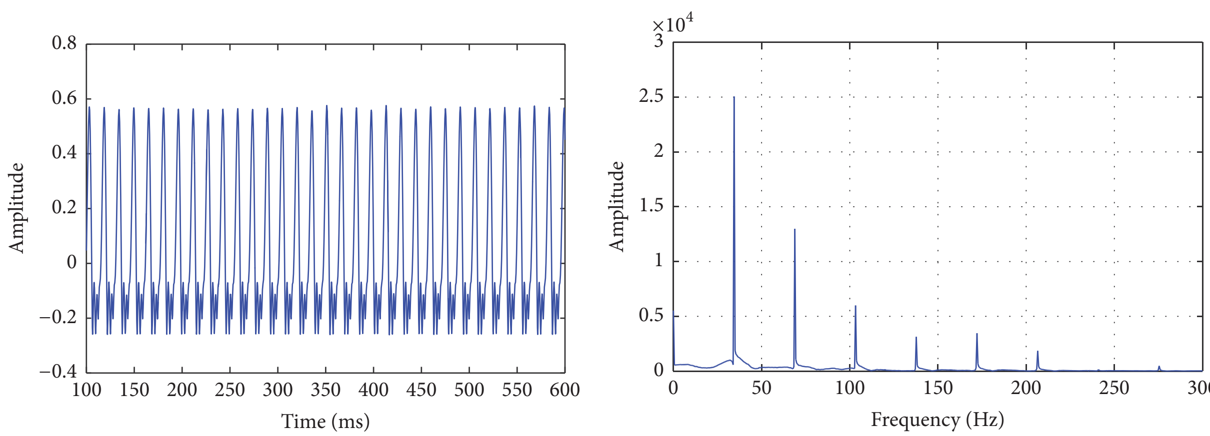
FIGURE 9: The horizontal direction response of the position of pedestal looseness ($\lambda = 0.1$, $\delta = 0.00002$ m).



(a) Vibration amplitudes

(b) Vibration spectrums

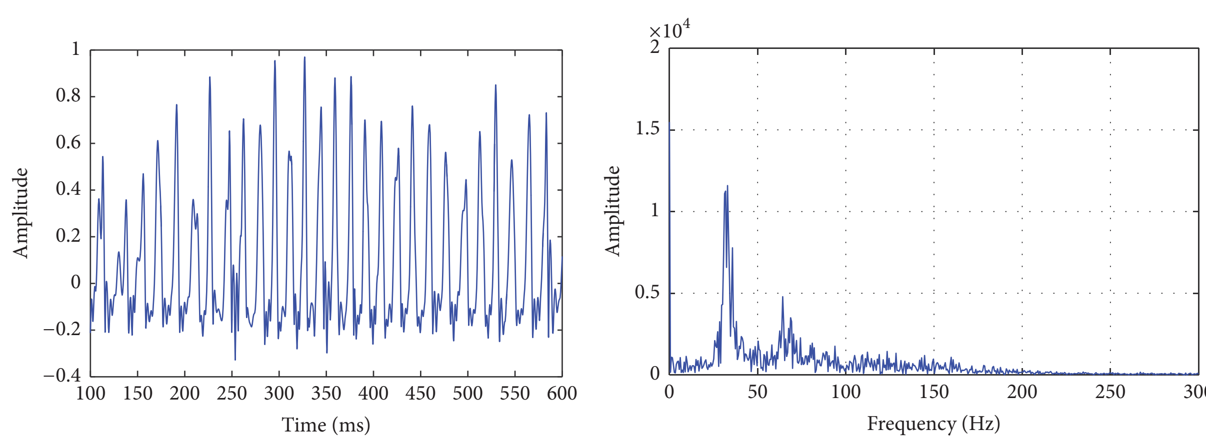
FIGURE 10: The horizontal direction response of the position of pedestal looseness ($\lambda = 0.3$, $\delta = 0.00002$ m).



(a) Vibration amplitudes

(b) Vibration spectrums

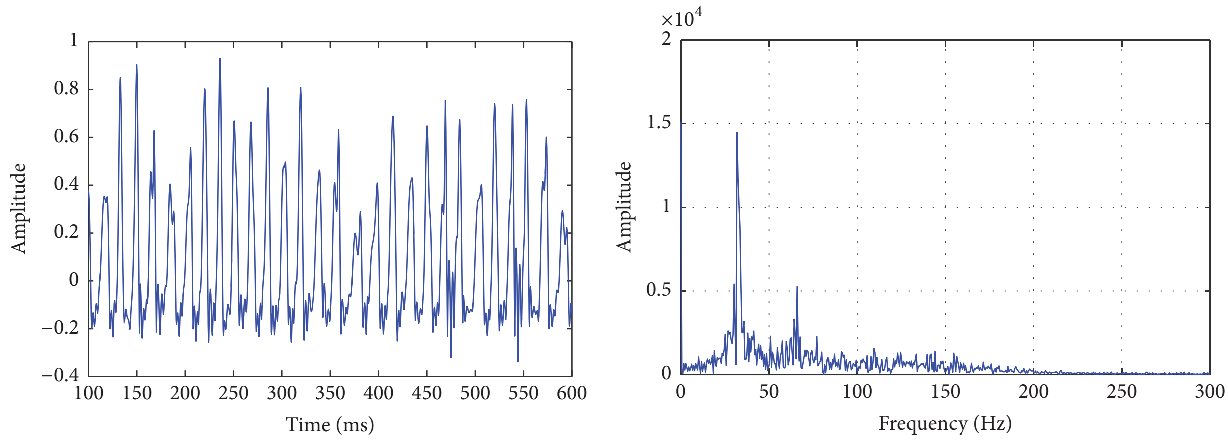
FIGURE 11: The horizontal direction response of the position of pedestal looseness ($\lambda = 0.5$, $\delta = 0.00002$ m).



(a) Vibration amplitudes

(b) Vibration spectrums

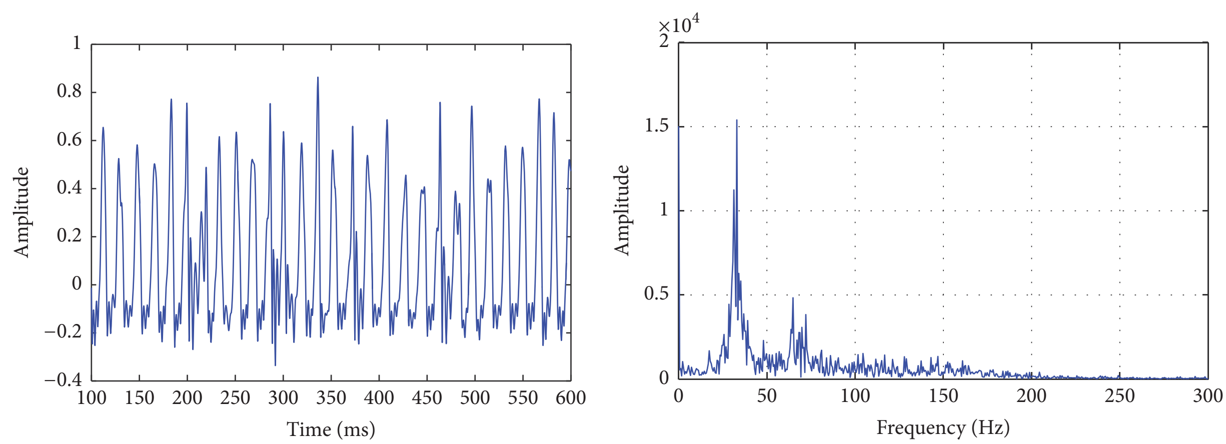
FIGURE 12: The horizontal direction response of the position of pedestal looseness ($\lambda = 0.1$, $\delta = 0.0001$ m).



(a) Vibration amplitudes

(b) Vibration spectrums

FIGURE 13: The horizontal direction response of the position of pedestal looseness ($\lambda = 0.3$, $\delta = 0.0001$ m).



(a) Vibration amplitudes

(b) Vibration spectrums

FIGURE 14: The horizontal direction response of the position of pedestal looseness ($\lambda = 0.5$, $\delta = 0.0001$ m).

the rotor system with $\delta = 0.0001$ m. For the position of the pedestal looseness, the vibration amplitudes and its spectrums with different amplitude coefficients are given in Figures 9–14 with looseness clearances $\delta = 0.00002$ m and $\delta = 0.0001$ m, respectively. The results of vibration analysis of the system with slowly disk mass varying show that their behaviors represent a perturbation of the system with constant disk mass. It can be seen that amplitude coefficients changing has significant impacts on the vibration properties of rotor systems.

As can be seen from (1), m_0 denotes the average mass of the disk, and λ denotes the change coefficient of disk mass near the average value m_0 in the varying time. A large λ means a relative large range of the change of disk mass, and vice versa. With a constant unbalance of the disk, larger change of disk mass will give more significant impacts on the dynamics of rotor systems with pedestal looseness, which are shown in Figures 3–14. It can also be found that when a larger looseness clearance is chosen in numerical experiments, vibration signals have more nonlinear characteristics in its amplitudes and spectrums (more frequency components are generated). In detail, Figures 3–8 show that the main frequencies of the vibration signals in the position of disk have larger amplitudes with the increasing of λ and the fixed looseness clearance. On the other hand, the vibration signals in the position of pedestal have more apparent shock effect (Figures 9–14). Similarly, it can be concluded that more complex frequency components are generated in the vibration signals in the position of pedestal with an increasing λ and the fixed looseness clearance. In conclusion, large coefficient of slowly disk mass varying will lead to more influences on the severity of dynamics nonlinearity of rotor systems with pedestal looseness.

4.2. The Effect of Disk Mass Varying on Nonlinearity Measure

(1) $\lambda = 0$, $m_0 = 4$ kg, 10 kg, 30 kg, 50 kg. With a given amplitude coefficient 0, the looseness clearances from 0 to 0.0035 m are used to simulate the dynamical behaviors of rotor systems with a given disk mass. A series of values of nonlinearity

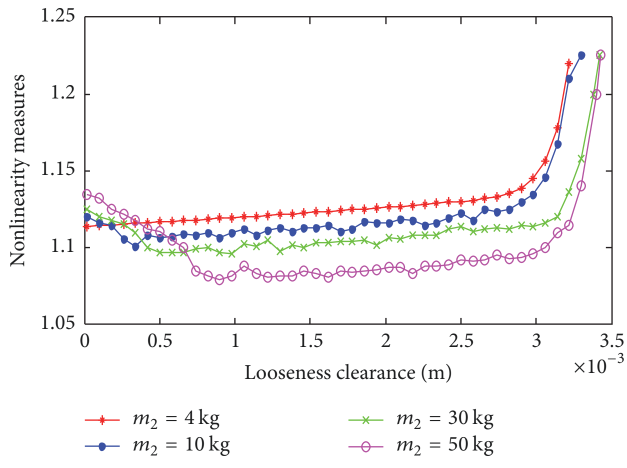


Figure 15: Curves of nonlinearity measure and looseness clearances with different disk mass.

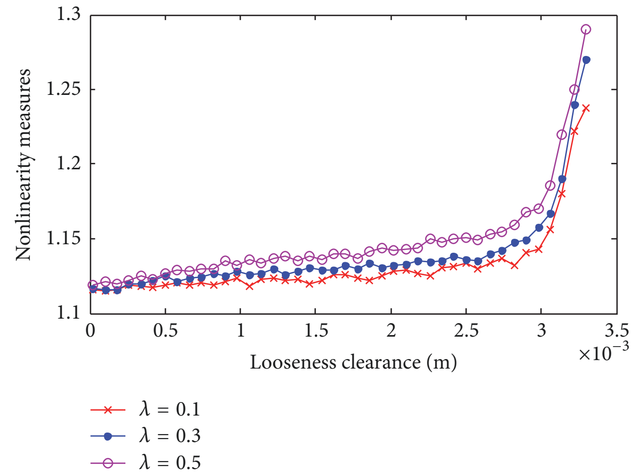


Figure 16: Curves of nonlinearity measure and looseness clearances with different amplitude coefficient.

measure are obtained and linked to be a curve for every given disk mass. Here, four different disk mass values are chosen to obtain four curves of nonlinearity measure, which are shown in Figure 15.

As can be seen from Figure 15, four curves are all described by exponential function with different fitting coefficients. It can be observed that the values of nonlinearity measure for the rotor system with pedestal looseness will have a decreasing trend, when the looseness clearances are smaller than 0.001 m. In the interval [0, 0.003 m], the values of nonlinearity measure are growing steadily. If the loose clearance is larger than 0.003 m, the nonlinearity degree will sharply grow. Particularly, the rotor system with larger disk mass will have more apparent decreasing than that with small disk mass, when the looseness clearance is increasing in the interval [0, 0.001 m]. It can be explained that, with the increasing of the disk mass, the unbalance of the larger disk may have serious influences on the bearing clearance and force components of the oil film in the bearing.

(2) $m_0 = 4\,kg$, $\lambda = 0.1, 0.3, 0.5$. With a given average mass of the disk, looseness clearances from 0 to 0.0035 m are used to simulate the dynamical behaviors of rotor systems, and a series of values of nonlinearity measure are obtained and linked to be a curve for a chosen amplitude coefficient. Three values are used to be amplitude coefficients and obtain three curves of nonlinearity measure, which is shown in Figure 16.

As can be seen from Figure 16, these three curves are all described by exponential function with different amplitude coefficients of disk mass varying. Quasi-periodic behaviors are often observed in the rotor systems with small looseness clearances. Because m_0 is small, the varying of disk mass has less impacts on dynamical behaviors of the rotor system with small looseness clearances. It can be found that the three curves are close in the interval [0, 0.0005 m] of looseness clearances. When the looseness clearances are increasing, the severity of nonlinearity for dynamics is influenced by the nonlinear elastic force and the varying of disk mass. The chaotic behaviors will emerge primarily in the dynamics of rotor system for large values of amplitude coefficient of disk mass varying, which will evidently modify the range of chaotic behaviors. With a large amplitude coefficient of disk mass varying, lateral vibrations of the disk also have irregularity behaviors; this also lead to a sharply increasing the assessment values of severity of nonlinearity of dynamics.

5. Conclusions

This paper proposed the investigation of the influence of disk mass varying on severity of dynamics nonlinearity via nonlinearity measure for bearing-rotor systems with pedestal looseness. A nonlinear mathematical model including the effect of slowly disk mass varying was developed for a bearing-rotor system with pedestal looseness. The varying of equivalent disk mass is described by a cosine function, and the amplitude coefficient is used as a control parameter. Then, nonlinearity measure was employed to quantify the severity of dynamics nonlinearity of bearing-rotor systems. With the increasing of looseness clearances, the curves that denote the trend of nonlinearity degrees were plotted for each amplitude coefficient of disk mass varying. Larger amplitude coefficients of disk mass varying will have more impacts on the severity of dynamics nonlinearity and generation of chaotic behaviors in rotor systems with pedestal looseness.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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