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Research Article

Damage Identification in Structures Based on Energy Curvature Difference of Wavelet Packet Transform

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Damage identification is of tremendous significance in engineering structures. One key issue in damage identification is to determine an index that is sensitive to the structural damage. Current damage identification indices are generally focused on dynamic characteristics such as the natural frequencies, modal shapes, frequency responses, or their mathematical combinations. In this study, based on the wavelet packet transform, we propose a novel index, the energy curvature difference (ECD) index, to identify the damage in structures. The ECD index is the summation of component energy curvature differences after a signal is decomposed using WPT. Moreover, two numerical examples are used to demonstrate the feasibility and validity of the proposed ECD index for damage identification. Stiffness reduction is employed to simulate the structural damage. The damage can be identified by the ECD index curve plot. The results of the examples indicate that the proposed ECD index is sensitive to low damage levels because even 5% stiffness reduction can be apparently identified. The proposed ECD index can be employed to effectively identify structural damage.

1. Introduction

Damage is a serious threat to structures during their service life, and a damage identification technology is of tremendous significance in engineering applications. Once a structure is damaged, the dynamic characteristics of the structure, such as the modal shapes, natural frequencies, and frequency responses, will change [1]. Various methods have been proposed for the use of dynamic responses for damage identification [2–6]. These methods are regarded as vibration-based damage identification methods. Among these methods, the approach based on wavelet transform is playing an increasingly important role because it can focus on any detail of a signal in the time or frequency domain [7, 8].

The wavelet transform is an extension of the traditional Fourier transform and is capable of performing local and self-adaptive time-frequency analyses. Therefore, the wavelet transform can reveal some hidden phenomena of a signal that other signal processing techniques fail to observe [9, 10]. This property has been introduced in the damage identification application. Numerous studies have been published regarding the use of wavelet transform for damage identification [11–15]. However, an obvious disadvantage of the wavelet transform is that the frequency resolution in the higher frequency domain is not high. It may be difficult to differentiate a signal containing high-frequency components that are very close to each other. Thus, scholars developed the wavelet packet transform (WPT) technique.

The WPT can be viewed as an extension of the traditional wavelet transform. It is capable of executing a complete level-by-level decomposition on a signal [16]. By performing the WPT on a signal, the signal can be decomposed into a series of wavelet packet components with a certain decomposition level, and the component energies can be obtained. The energy of the original signal is the summation of the component energies corresponding to different frequency bands. The WPT has been used for the damage identification of structures [17–20]. Wavelet packet energy can be obtained in different frequency bands after a signal is decomposed using WPT. The wavelet packet energy has been employed for the damage identification in ancient wood structures, beams, cable-stayed bridges, and so on [21–25]. Curvature difference can reflect the small changes in a function or a signal and has been introduced in damage identification together with wavelet packet energy. Some researchers have used curvature difference based on wavelet packet energy to locate the damage in ancient wood structures, concrete frames, and bridges [26–29].

One key issue in damage identification is to determine an index that is sensitive to the structural damage. Current damage identification indices are generally focused on dynamic characteristics such as the natural frequencies, modal shapes, frequency responses, or their mathematical combinations. In this study, based on the WPT, we propose a novel index, the energy curvature difference (ECD) index, to identify the damage in structures. The ECD index is the summation of component energy curvature differences after a signal is decomposed using WPT. It takes into account the spatial distribution of the collected signals and is sensitive to low damage levels because even a 5% stiffness reduction can be apparently identified. Moreover, two numerical examples are used to demonstrate the feasibility and validity of the proposed ECD index for damage identification. Mutation on the ECD index curve can identify the damage accurately.

The remainder of this paper is organised as follows. In Section 2, some basics of the WPT are briefly introduced. In Section 3, an ECD index based on the WPT is proposed for damage identification. In Section 4, two numerical examples are provided to illustrate the applicability of the proposed ECD index. Section 5 presents the conclusions of this study.

2. Basics of Wavelet Packet Transform

The WPT is capable of accomplishing a complete level-by-level decomposition of the signals. Wavelet packet $\psi^i_{j,k}(t)$ can be expressed as follows:

$$\psi^i_{j,k}(t) = 2^{-j/2}\psi^i\left(2^{-j}t - k\right), \quad i, j \in \mathbf{Z}^+, \ k \in \mathbf{Z}, \tag{1}$$

where i indicates the modulation parameter, j indicates the scale parameter, and k indicates the translation parameter. The wavelet packet functions possess the orthogonality property, i.e.,

$$\psi^m_{j,k}(t) \cdot \psi^n_{j,k}(t) = 0, \quad (m \neq n). \tag{2}$$

The first wavelet function is the mother wavelet function, i.e.,

$$\psi^1(t) = \psi(t). \tag{3}$$

Wavelet functions $\psi^i(t)$ can be calculated using the recursive equation

$$\begin{aligned} \psi^{2i}(t) &= \sqrt{2}\sum_{k=-\infty}^{+\infty} h(k)\,\psi^i(2t-k) \\ \psi^{2i+1}(t) &= \sqrt{2}\sum_{k=-\infty}^{+\infty} g(k)\,\psi^i(2t-k), \end{aligned} \tag{4}$$

where $h(k)$ and $g(k)$ are the quadrature mirror filters. $h(k)$ is related to the scaling function and $g(k)$ is related to the mother wavelet function.

Assume that $f(t)$ is a time signal. If $f(t)$ is decomposed to the jth level, it can be expressed as

$$f(t) = \sum_{i=0}^{2^j-1} f^i_j(t), \tag{5}$$

where f^i_j represents the ith order wavelet packet component signal at level j. f^i_j can be expressed by a linear combination of wavelet packet functions $\psi^i_{j,k}(t)$ in the following manner:

$$f^i_j(t) = \sum_{k=-\infty}^{+\infty} c^i_{j,k}(t) \cdot \psi^i_{j,k}(t), \tag{6}$$

where $c^i_{j,k}(t)$ represents the wavelet packet coefficient, expressed as

$$c^i_{j,k}(t) = \int_{-\infty}^{+\infty} f(t) \cdot \psi^i_{j,k}(t)\,\mathrm{d}t. \tag{7}$$

WPT is a generalization of wavelet decomposition that offers a richer range of possibilities for signal analysis. In wavelet transform, a signal is split into an approximation and a detail. The approximation is then itself split into a second-level approximation and detail, and the process is repeated. In WPT, the details as well as the approximations can be split. This yields more different ways to encode the signal. The decomposition tree of WPT is plotted in Figure 1, where A represents the approximations and D represents the details, and j denotes the decomposition level.

The energy of signal $f(t)$ at the jth decomposition level is defined as

$$E_j = \int_{-\infty}^{+\infty} f^2(t)\,\mathrm{d}t = \sum_{m=0}^{2^j-1}\sum_{n=0}^{2^j-1}\int_{-\infty}^{+\infty} f^m_j(t) \cdot f^n_j(t)\,\mathrm{d}t. \tag{8}$$

Substituting (2) and (6) into (8) yields

$$E_j = \sum_{i=0}^{2^j-1} E^i_j = \sum_{i=0}^{2^j-1}\int_{-\infty}^{+\infty}\left[f^i_j(t)\right]^2 \mathrm{d}t, \tag{9}$$

where E^i_j represents the signal energy stored in the ith order component signal at the jth level. E^i_j is regarded as the component energy of the WPT, and it can be expressed as follows:

$$E^i_j = \int_{-\infty}^{+\infty}\left[f^i_j(t)\right]^2 \mathrm{d}t. \tag{10}$$

The physical implication of (10) is that the total energy stored in the original signal can be decomposed into a summation of the component energies, where each component corresponds to an individual frequency band.

3. Damage Identification Based on Energy Curvature Difference

One key issue of damage identification is to determine an index that is sensitive to structural damage. In this study,

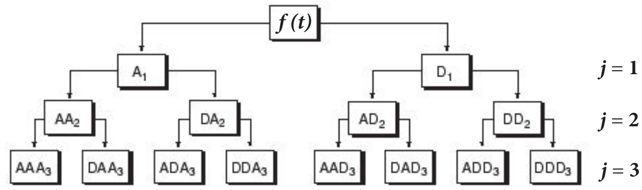


Figure 1: Decomposition tree of WPT (j is the decomposition level).

we propose an energy curvature difference (ECD) index to identify the damage in structures. The wavelet packet energy of a signal can be expressed as

$$E = E_j^0 + E_j^1 + E_j^2 + \cdots + E_j^i + \cdots + E_j^{2^j-1}, \tag{11}$$

where i represents the order of the component signal and j represents the decomposition level. We use the node acceleration responses of the structures as the signals to be analysed. These acceleration signals can be conveniently obtained by using acceleration sensors.

In practice, the curvature of a function $f(x)$ is usually obtained by using the second-order difference equation:

$$f''(x_i) = \frac{(f(x_{i+1}) - f(x_i))/(x_{i+1} - x_i) - (f(x_i) - f(x_{i-1}))/(x_i - x_{i-1})}{((x_{i+1} - x_i) + (x_i - x_{i-1}))/2}, \tag{12}$$

where x_{i-1}, x_i, and x_{i+1} represent three different points of variable x.

If the distance between x_{i-1} and $x_i$equals the distance between x_i and x_{i+1}, i.e.,

$$x_i - x_{i-1} = x_{i+1} - x_i = h, \tag{13}$$

then in this case, (12) can be simplified as

$$f''(x_i) = \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2}. \tag{14}$$

The curvature is essentially a second-order difference of a function. Compared with the original function, the curvature is more sensitive to minor changes in the variable. This is an advantage of the curvature concept. Now, we introduce the concept of curvature into the wavelet packet energy for achieving damage identification. The component energy curvature is defined as

$$\left(E_{j,k}^i\right)'' = \frac{\left(E_{j,k+1}^i - E_{j,k}^i\right)/d_{k,k+1} - \left(E_{j,k}^i - E_{j,k-1}^i\right)/d_{k-1,k}}{\left(d_{k,k+1} + d_{k-1,k}\right)/2}, \tag{15}$$

where $E_{j,k}^i$ denotes the energy related to the collected signal component $f_j^i(t)$ of the kth node and $(E_{j,k}^i)''$ represents the curvature of $E_{j,k}^i$. Symbol $d_{k,k+1}$ represents the distance between the node k and node k+1, and $d_{k-1,k}$ represents the distance between the node k-1 and node k. In particular, if $d_{k,k+1}$ is equal to $d_{k-1,k}$, i.e.,

$$d_{k,k+1} = d_{k-1,k} = d, \tag{16}$$

then in this situation, (15) can be simplified as

$$\left(E_{j,k}^i\right)'' = \frac{E_{j,k+1}^i - 2E_{j,k}^i + E_{j,k-1}^i}{d^2}. \tag{17}$$

From (17), it can be observed that, by using the curvature technique, the component energy change of a single node is transformed to the distribution change of the curvature values of three adjacent nodes. The spatial distribution of the component energies has been taken into account in (17). This technique can amplify the changes in a signal.

The component energy curvature difference is defined as

$$\Delta\left(E_{j,k}^i\right)'' = \left(E_{j,k}^i\right)''_d - \left(E_{j,k}^i\right)''_u, \tag{18}$$

where subscript d denotes a damaged structure and subscript u denotes an undamaged structure.

Now we perform a summation of the component energy curvature differences, and the summation is defined as the ECD index, expressed as

$$ECD = \sum_{i=0}^{2^j-1} \Delta\left(E_{j,k}^i\right)'', \tag{19}$$

where i is the order of the wavelet packet frequency band, j is the wavelet packet decomposition level, and k is the signal collection node number.

By substituting (18) into (19), the ECD index can also be expressed as follows:

$$ECD = \sum_{i=0}^{2^j-1} \left[\left(E_{j,k}^i\right)''_d - \left(E_{j,k}^i\right)''_u\right]. \tag{20}$$

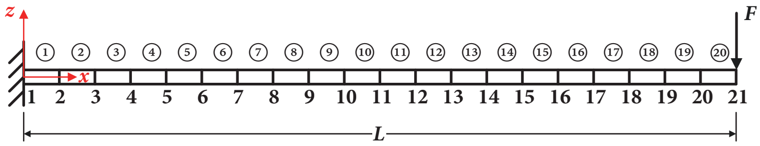


FIGURE 2: Cantilever beam under the action of an impact force.

FIGURE 3: Force-time history curve for Example 1.

FIGURE 4: Location of damage element in case L1 in Example 1.

The numerical value of the ECD index is regarded as an indicator of the damage identification in structures. In practice, we collect the node acceleration response signals of the undamaged structure and damaged structure. Then, we perform WPT on the collected signals. With the calculation of the wavelet packet component energies, we can obtain the ECD indices. Subsequently, we can plot the ECD indices of different nodes in one figure. A mutation on the plot is indicative of damage in a structure. A sudden change in the ECD plot is indicative of a damage occurrence. The location and severity of the damage can be reflected by the ECD index plot.

The proposed method requires signals of both damaged and undamaged structure, which sometimes could be difficult to obtain. In practice, with acceleration sensors installed appropriately, the signals of both damaged and undamaged structure are accessible. Besides, finite elements method is commonly used in engineering and we can simulate the damage in a fine meshed finite element model. The proposed method can give accurate results for a finite element model with damage. This will be very useful in the initial design stage of a structure. Thus, the proposed method has extent of engineering values.

4. Numerical Examples

To demonstrate the feasibility of the proposed ECD index for damage identification of structures, the following two numerical examples are considered. Both the location and severity of the damage are investigated with the proposed damage identification index.

Example 1. Consider a cantilever beam under the action of an impact force, as shown in Figure 2. The length of the beam is $L = 1$ m. The section moment of inertia of the beam is $I = 1.6{\times}10^{-7}\ \mathrm{m}^4$. Young's modulus of the material is $E = 210$GPa, the mass density is$\rho = 7850\mathrm{kg/m}^3$, and Poisson's ratio is $\nu = 0.3$. It is assumed that there is an impact force F acting at the endpoint of the beam in the vertical direction. The force-time history is plotted in Figure 3. The beam is discretized by 21 nodes and meshed with 20 elements. The node acceleration responses are calculated using the software ANSYS 15.0 and are regarded as the collected signals. The sampling frequency employed in this example is 10^5 Hz. The signals are decomposed to level 4 with Db10 wavelet, and 16 component energies are generated in total.

The element stiffness reduction can be used to simulate the structural damage. In practice, it is convenient to use Young's modulus reduction to characterize damage. In this example, we use two test cases to investigate the application of the proposed ECD index in damage location identification, as presented in Table 1. The locations of the damage elements in the finite element model are illustrated in Figures 4 and 6, and the corresponding ECD index curves are plotted in Figures 5 and 7, respectively. The damaged elements are displayed using a different color in the finite element model that it is employed for the verification of the proposed index.

We use four test cases to investigate the application of the proposed ECD index in damage severity identification, as presented in Table 2. The damage level of specific elements increases from 5% to 20%. The ECD curves corresponding to all the four cases are plotted in Figure 8.

The decomposition tree of WPT in Example 1 is plotted in Figure 9. The tree consists of several knots. Knot (j, i) represents the ith order wavelet packet component signal at decomposition level j.

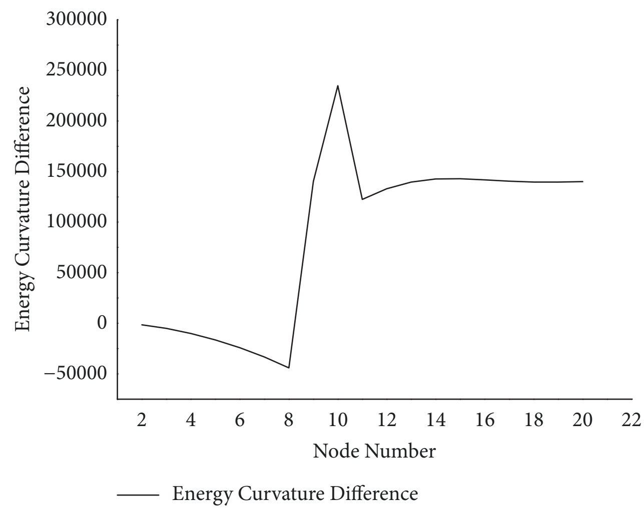


Figure 5: ECD index curve for damage location identification in case L1 in Example 1.

Figure 6: Locations of damage elements in case L2 in Example 1.

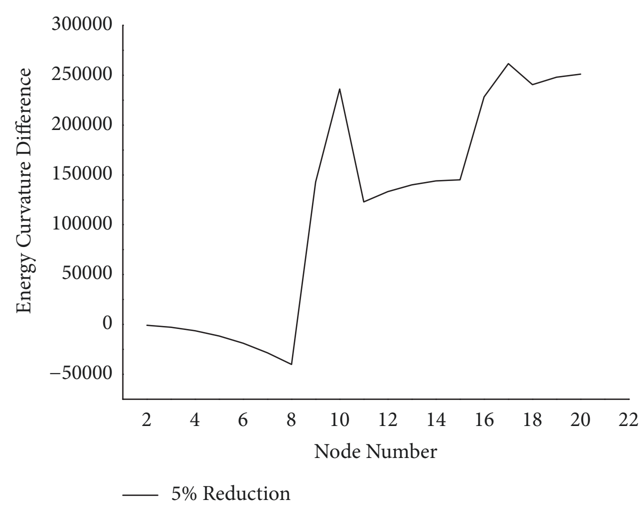


Figure 7: ECD index curve for damage location identification in case L2 in Example 1.

For graphical illustration, the wavelet packet coefficients corresponding to the 16th node in case S1 of knots (1,0), (1,1), (2,0), (2,1), (3,0), (3,1), (4,0), and (4,1) are presented in Figure 10.

From the plots in Figures 5 and 7, it can be observed that the proposed ECD index can identify the damage locations accurately. There is an obvious mutation on the ECD index curve at the location where the damage is present. Moreover, the ECD index is sensitive to low damage levels because even 5% stiffness reduction can be apparently reflected by the ECD index curve. From the plot in Figure 8, it can be observed that the damage severity can be reflected by the amplitude of the mutation point. For the same damage locations with different damage levels, the amplitudes of the mutation are higher for the cases that are more severely damaged. The results of this example indicate that the proposed ECD index can be employed to identify the damage location and damage severity effectively.

Example 2. Now, we consider a spatial frame structure under the action of an impact force, as shown in Figure 11. The dimensions are as follows: $h_1 = 1.6$ m, $h_2 = 1.5$ m, $l = 1.8$ m, and $w = 0.9$ m. The frame section is plotted in Figure 12. The section dimensions are $b = 0.1$ m and $t = 0.009$ m. Young's modulus of the material is $E = 210$ GPa, the mass density is $\rho = 7850 \text{kg/m}^3$, and Poisson's ratio is $\nu = 0.3$. It is assumed that there is an impact force acting at point D in the z direction. The force-time history is plotted in

Table 1: Test cases for damage location identification in Example 1.

Case No.	Damage element	Damage location	Damage level	Young's modulus
L1	Element 9	Between node 9 and node 10	5%	210 GPa×95%
L2	Element 9	Between node 9 and node 10	5%	210 GPa×95%
	Element 16	Between node 16 and node 17	5%	210 GPa×95%

Table 2: Test cases for damage severity identification in Example 1.

Case No.	Damage element	Damage location	Damage level	Young's modulus
S1	Element 9	Between node 9 and node 10	5%	210 GPa×95%
	Element 16	Between node 16 and node 17	5%	210 GPa×95%
S2	Element 9	Between node 9 and node 10	10%	210 GPa×90%
	Element 16	Between node 16 and node 17	10%	210 GPa×90%
S3	Element 9	Between node 9 and node 10	15%	210 GPa×85%
	Element 16	Between node 16 and node 17	15%	210 GPa×85%
S4	Element 9	Between node 9 and node 10	20%	210 GPa×80%
	Element 16	Between node 16 and node 17	20%	210 GPa×80%

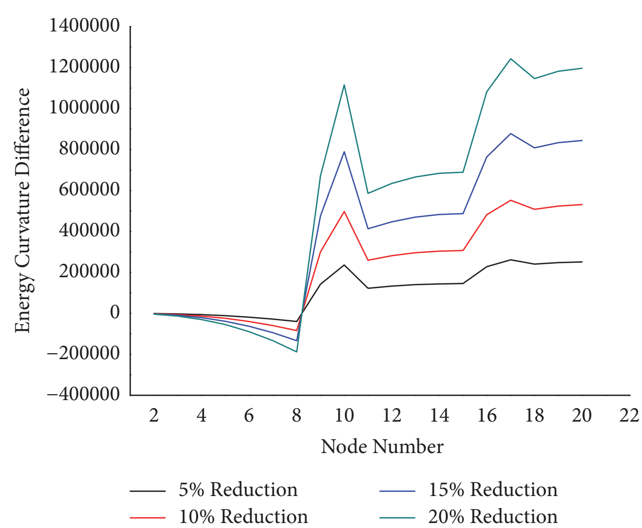


Figure 8: ECD index curve for damage severity identification in cases S1 to S4 in Example 1.

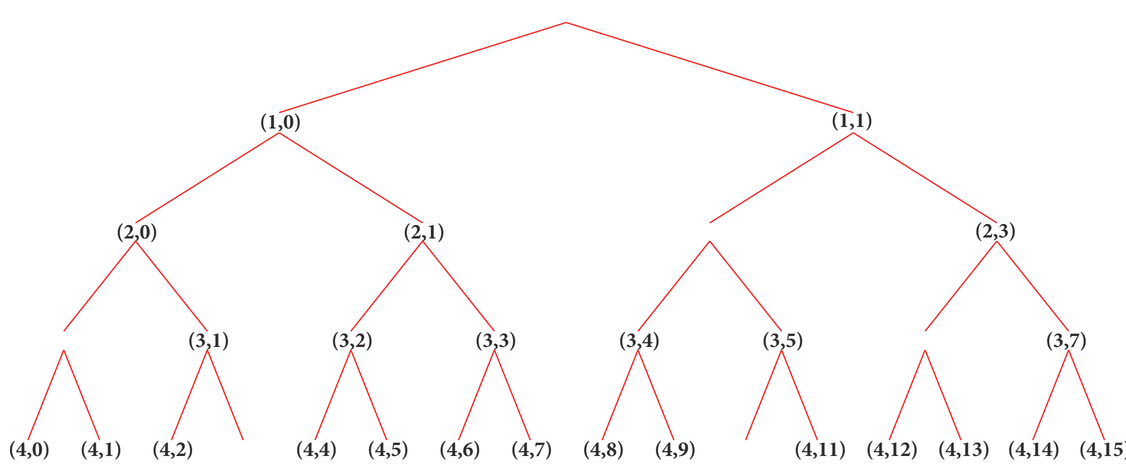
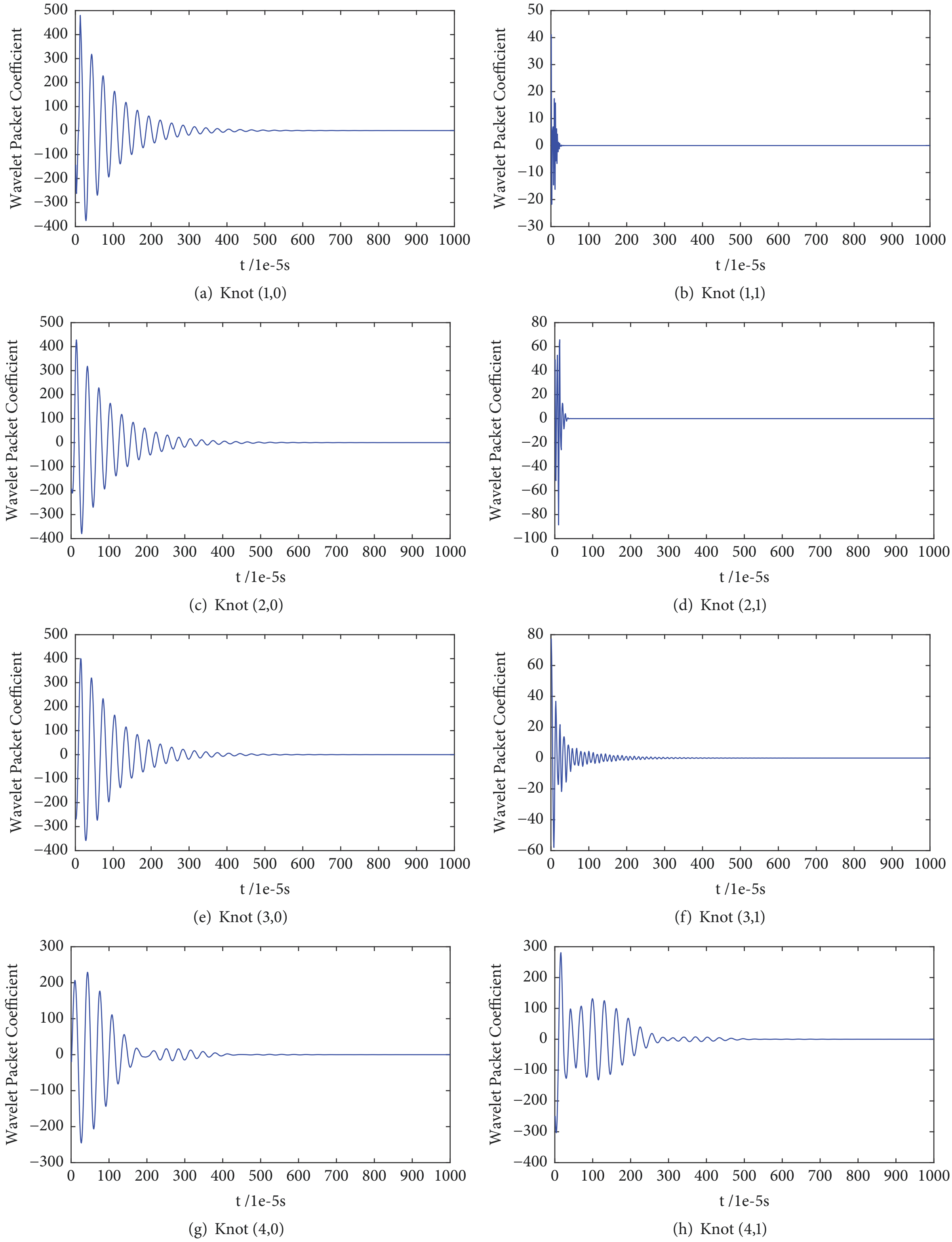


Figure 9: Decomposition tree of WPT in Example 1.



(a) Knot (1,0)
(b) Knot (1,1)
(c) Knot (2,0)
(d) Knot (2,1)
(e) Knot (3,0)
(f) Knot (3,1)
(g) Knot (4,0)
(h) Knot (4,1)

FIGURE 10: Wavelet packet coefficients corresponding to the 16th node in case S1 in Example 1.

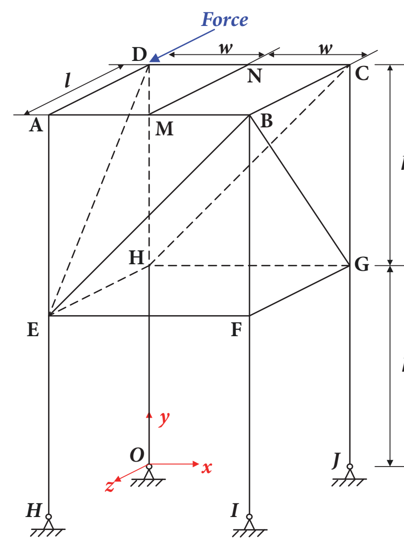


FIGURE 11: Spatial frame structure under the action of an impact force.

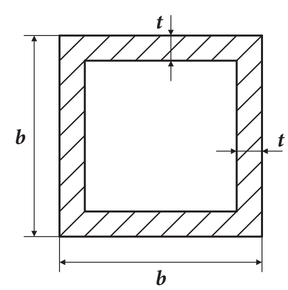


FIGURE 12: Illustration of the frame section.

FIGURE 13: Force-time history curve for Example 2.

Figure 13. The structure is meshed with 760 beam elements. The node acceleration responses are calculated using the software ANSYS 15.0 and are regarded as the collected signals. The sampling frequency employed in this example is 2000 Hz. The signals are decomposed to level 4 with Db10 wavelet, and 16 component energies are generated in total.

It is assumed that the damage occurs in the EF part. In the finite element model, the EF part is discretized by 36 elements. The element close to point E is numbered 1, and the element close to point F is numbered 36. The length of each element is 0.05 m. There are totally 37 nodes located in the EF part. The node at point E is numbered as the 1st node, and the node at point F is numbered as the 37th node. The element numbers and node numbers in the EF part are illustrated in Figure 14.

The element stiffness reduction can be used to simulate the structural damage. In practice, it is convenient to use Young's modulus reduction to characterize damage. In this example, we use two test cases to investigate the application of the proposed ECD index in damage location identification, as presented in Table 3. The locations of the damage elements in the finite element model are illustrated in Figures 15 and 17, and the corresponding ECD index curves are plotted in Figures 16 and 18, respectively. The damaged elements are displayed using a different color in the finite element model that it is employed for the verification of the proposed index.

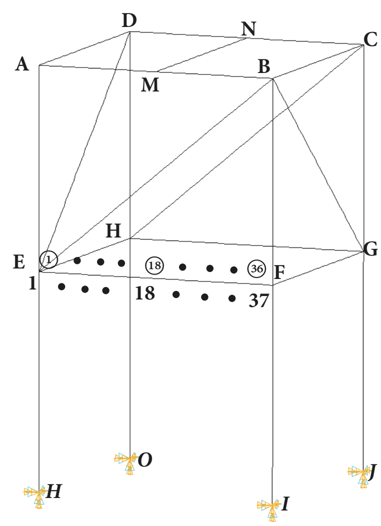


Figure 14: Element numbers and node numbers of the EF part.

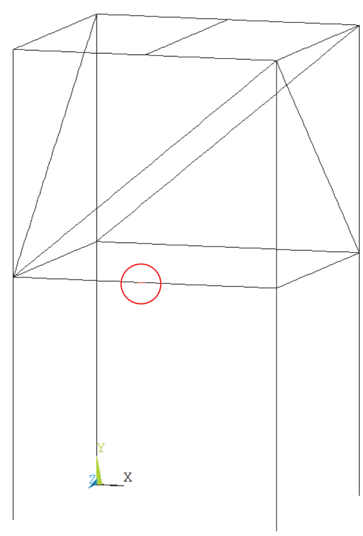


Figure 15: Location of damage element in case L1 in Example 2.

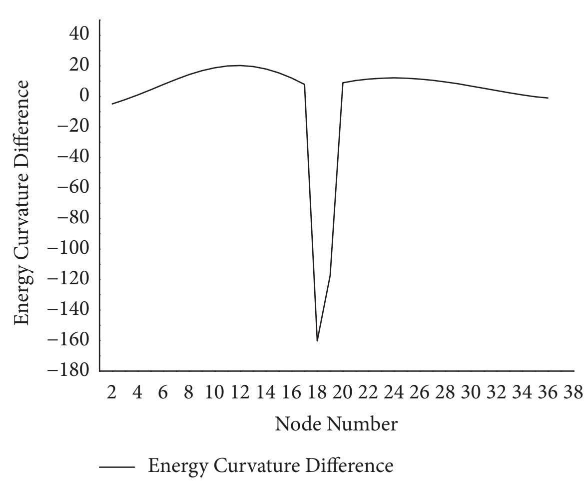


Figure 16: ECD index curve for damage location identification in case L1 in Example 2.

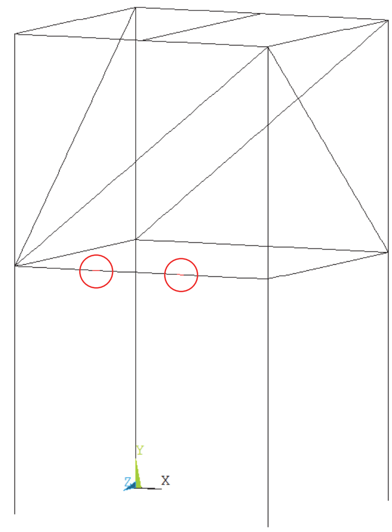


Figure 17: Locations of damage elements in case L2 in Example 2.

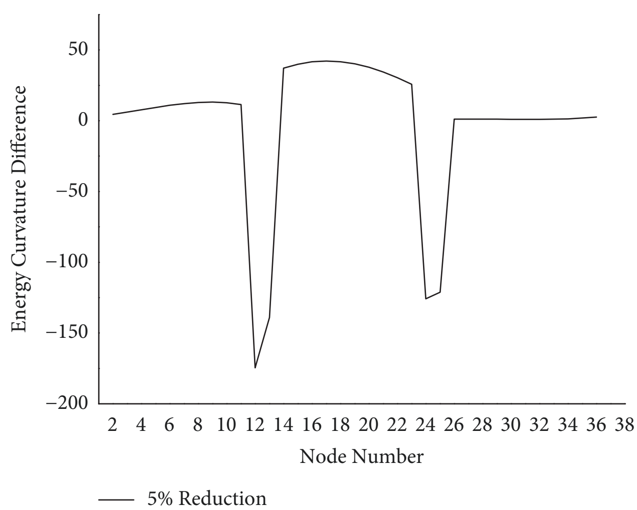


Figure 18: ECD index curve for damage location identification in case L2 in Example 2.

We use four test cases to investigate the application of the proposed ECD index in damage severity identification, as presented in Table 4. The damage level of specific elements increases from 5% to 20%. The ECD curves corresponding to all the four cases are plotted in Figure 19.

The decomposition tree of WPT in Example 1 is plotted in Figure 20. The tree consists of several knots. Knot (j, i) represents the ith order wavelet packet component signal at decomposition level j.

For graphical illustration, the wavelet packet coefficients corresponding to the 24th node in case S1 of knots (1,0), (1,1), (2,0), (2,1), (3,0), (3,1), (4,0), and (4,1) are presented in Figure 21.

From the plots in Figures 16 and 18, it can be observed that the proposed ECD index can identify the damage locations accurately. There is an obvious mutation on the ECD index curve at the location where the damage is present. Moreover, the ECD index is sensitive to low damage levels because even 5% stiffness reduction can be apparently reflected by the ECD index curve. From the plot in Figure 19, it can be observed that the damage severity can be reflected by the amplitude of the mutation point. For the same damage locations with different damage levels, the amplitudes of the mutation are higher for the cases that are more severely damaged. The results of this example indicate that the proposed ECD index can be employed to identify the damage location and damage severity effectively.

5. Conclusions

In this study, we propose an ECD index based on the WPT to identify a structural damage. The node acceleration responses of the undamaged and damaged structures are collected. We perform WPT on the signals and compute the component

Table 3: Test cases for damage location identification in Example 2.

Case No.	Damage element	Damage location	Damage level	Young's modulus
L1	Element 18	Between node 18 and node 19	5%	210 GPa×95%
L2	Element 12	Between node 12 and node 13	5%	210 GPa×95%
	Element 24	Between node 24 and node 25	5%	210 GPa×95%

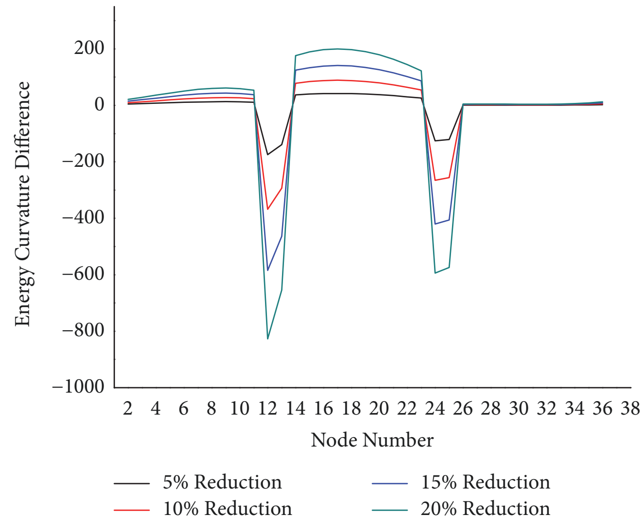


Figure 19: ECD index curve for damage severity identification in cases S1 to S4 in Example 2.

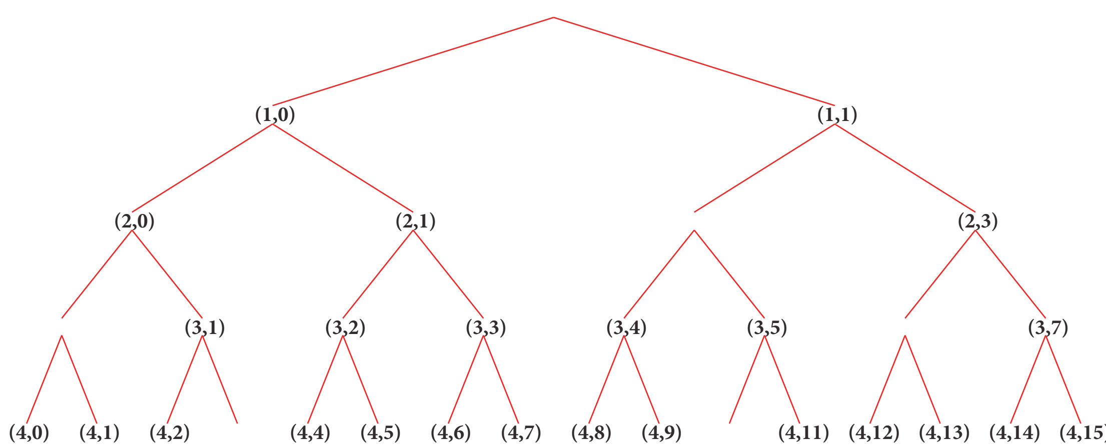
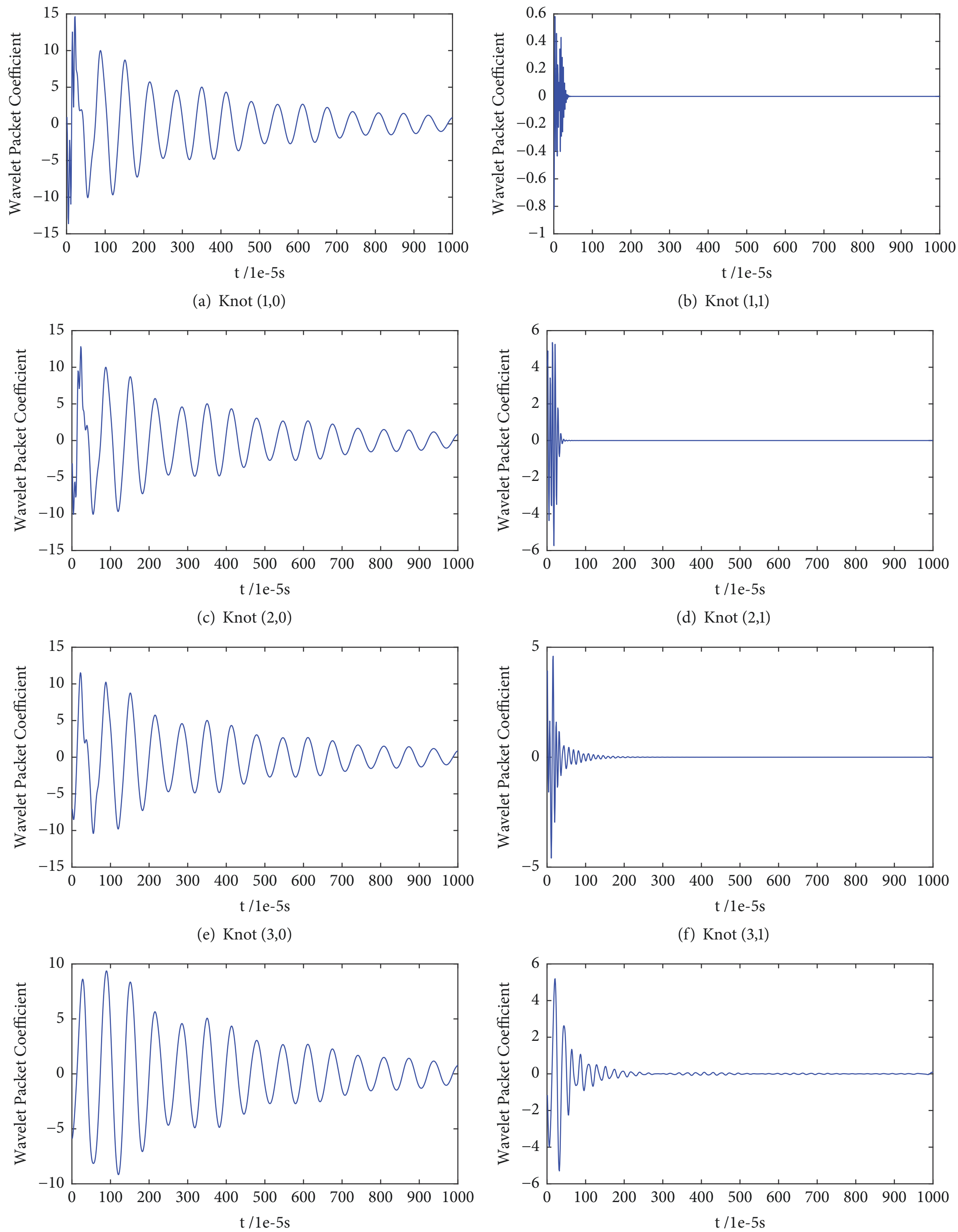


Figure 20: Decomposition tree of WPT in Example 2.

Table 4: Test cases for damage severity identification in Example 2.

Case No.	Damage element	Damage location	Damage level	Young's modulus
S1	Element 12	Between node 12 and node 13	5%	210 GPa×95%
	Element 24	Between node 24 and node 25	5%	210 GPa×95%
S2	Element 12	Between node 12 and node 13	10%	210 GPa×90%
	Element 24	Between node 24 and node 25	10%	210 GPa×90%
S3	Element 12	Between node 12 and node 13	15%	210 GPa×85%
	Element 24	Between node 24 and node 25	15%	210 GPa×85%
S4	Element 12	Between node 12 and node 13	20%	210 GPa×80%
	Element 24	Between node 24 and node 25	20%	210 GPa×80%



(a) Knot (1,0)
(b) Knot (1,1)
(c) Knot (2,0)
(d) Knot (2,1)
(e) Knot (3,0)
(f) Knot (3,1)
(g) Knot (4,0)
(h) Knot (4,1)

Figure 21: Wavelet packet coefficients corresponding to the 24th node in case S1 in Example 2.

energy of each decomposed component signal. Then the ECD index can be obtained. With the illustration of the ECD index curve, the damage location and damage severity can be identified effectively. Two numerical examples are used to demonstrate the applicability of the proposed ECD index for damage identification. The ECD index is sensitive to low damage levels because even 5% stiffness reduction can be apparently identified by it. The mutation on the ECD index curve can identify the damage accurately, which is demonstrated by the illustration of damage elements in the finite element model. The results of this study indicate that the proposed ECD index can be employed to effectively identify the damage to structures for engineering applications.

Conflicts of Interest

There are no conflicts of interest related to this paper.

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