

Research Article Study on the Nonlinear Characteristics of a Rotating Flexible Blade with Dovetail Interface Feature

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A dynamic model is proposed in this paper for analyzing the nonlinear characteristics of a flexible blade. The dynamical equation of motion for a rotational flexible blade in a centrifugal force field is established based on the finite element method. A macro-stick-slip mechanical model of dry friction is established to simulate the constraint condition of the flexible blade. The combined motion of the external excitation and friction produces a piecewise linear vibration which is actually nonlinear. The numerical integration method is employed to calculate the vibration reduction characteristics of the nonlinear constrained rotating blade. The results show that the nonlinear dry friction force produced by the dovetail interface plays an important role in vibration reduction. And the effect of dry friction vibration reduction is significant when the rotating speed is slow or the friction coefficient is small. Besides, the magnitude of external excitation also has a great impact on the state of the friction. Therefore, some relevant experimental researches should be done in the future.

1. Introduction

In the aeroengine components, the blade has the largest number and is the most prone to accident. The blade is inevitably affected by the aerodynamic force and the centrifugal force during the process of starting, running, and stopping of the engine. This is one of the main reasons for the blade damage. The dovetail attachment structure is often used in the installation of modern aeroengine blades. This structure has the advantage of simple manufacture. And it can also use the dry friction to reduce the vibration level of blade. Thus, the high cycle fatigue damage of the blade is reduced. And the service life of the blade is longer. Therefore, the design of the blade connection structure and the construction of the dry friction mechanics model are the important contents of the research and design of aeroengine blade. In recent decades, many scholars at home and abroad have carried out in-depth theoretical analysis and experimental research on the dynamic characteristics of the blade. They have achieved fruitful results and laid a solid foundation for the further study of the later generations.

The natural characteristics of the blade have been studied in the early studies of blade. Turhan and Bulut [1] investigated the nonlinear bending vibrations of a rotating beam. The perturbation analysis was used to obtain the natural frequencies and the frequency responses. Chung and Yoo [2] used the finite element method and the discretized equations to investigate the behaviors of the natural frequencies with the variation of the rotating speed. Tsai [3] used the FEM to explore the dynamic characteristics of a single blade, 6-blade groups, and 12-blade groups and found that the vibration frequency and mode shape of single blade are in good agreement with that of the whole blade. Yan et al. [4] used the experimental mode analysis and mode correction methods to investigate the coupling vibration of the mistuned bladedisk in aeroengine and found the right working frequency range of aeroengine. Park et al. [5] investigated the vibration characteristics of the rotating blades of the wind-turbine and obtained the accurate natural frequencies of blades through the numerical method.

In order to find out the reason for blade damage and prevent it, many experts have done a lot of researches on the dynamic characteristics of the blade. Choi and Lee [6] used the modal analysis for one blade and the assembly of the blades to check the dynamic characteristics of the blades. The results showed that it is close to the resonance condition of the assembled blades when the blade is broken. Al-Bedoor and Al-Qaisia [7] used a reduced order nonlinear dynamic model to investigate the forced vibrations of a flexible rotating blade under the excitation of shaft torsional vibration. Yao et al. [8] investigated the nonlinear dynamic characteristics of the rotating blade and used the bifurcation diagram, phase portrait, and power spectrum to demonstrate that periodic motions and chaotic motions occur in nonlinear vibrations of the rotating blade under certain conditions. Lee et al. [9] developed a computational model for the dynamic characteristics of a rotor-blade system. The dynamic characteristics of the system for various system parameters were obtained. Li et al. [10, 11] established a dynamic model of rotor-blade coupling system with elastic restraints to investigate the influence of shaft bending on the coupling vibration of rotorblades system and analyze the nonlinear dynamic behavior of a continuum model. Allara [12] established different contact models to investigate the dynamic response characteristics of turbine blades and obtained the hysteresis curves of the oscillating tangential contact force versus relation tangential displacements and the dissipated energy at the contact for different contact geometries.

It is known that the damage caused by the resonance vibration of the blade is through analyzing the vibration characteristics of the blade. Proper method must be found to restrain the vibration of blade and increase the service life of blades. There are many works that have been done on the vibration reduction, but the primary method is to reduce the vibration by friction. Hartog [13] developed an ideal dry friction model which can qualitatively analyze the effect of the dry friction vibration reduction. Iwan [14] proposed the famous piecewise nonlinear hysteresis model and studied the dynamic responses of the system of a single degree of freedom and two degrees of freedom. Yang and Meng [15] used the Coulomb friction law and the macroslip model to investigate the coupling contact kinematics and developed the mathematical expression of the dry friction force. Ciğeroğlu and Özgüven [16] indicated that the microslip model can provide more accurate results by applying a quasilinearization technique. They proposed a new model about all blades around the disk, which developed the microslip method and validated feasibility of the model. Xu et al. [17, 18] presented a macroslip model to determine the dry friction force on the contact interface between the blade dovetail attachment and the disk dovetail groove. And a lumped-mass-spring model was used to explore the effect of some control parameters of a damped structure on its forced response.

Several suitable friction models are obtained through the studies of the above. And there are some suitable methods which can be used to study the possibility of friction vibration reduction. Wang and Chen [19] used the HBM to explore the vibration characteristics of blade and computed the accurate steady-state response of blade with damper. Ding et al. [20, 21] presented an analytical method for determining the steadystate response of a system with dry friction damper. Sinha [22] discussed the transient response of the rotor with the blades deforming. And the Numerical results were presented for the highly nonlinear impact dynamics problem of hard rub with Coulomb friction. Cao et al. [23] analyzed the 2D friction contact problem of a flexible blade and found that the gap between the tips and the rotating speed of the blade significantly influence the dynamics of the system. Wang and Long [24] established the mechanical model of bladed disks with random mistuning of hysteretic dry friction damping, and used an incremental harmonic balance method to analyze the effects of parameters of bladed disks on its forced response. Zhang et al. [25] developed the constitutive relation of dry friction force for blade-root damper based on a microslip friction model and used the harmonic balance method to analyze the effect of dry friction on vibration performances of mistuned bladed disk system. Pust et al. [26] used a harmonic excitation near the natural frequency of blade to act on a blade system with damping element and investigated the vibration response characteristics of blade with dry friction dampers. Zhang et al. [27] described an efficient method to predict the nonlinear steady-state response of a complex structure with multiscattered friction contacts and analyzed the nonlinear response of the blade with underplatform dampers. Ozaydin and Ciğeroğlu [28] used onedimensional macroslip friction model with constant normal load to model the dry friction damper and investigated the effect of dry friction damping on vibration attenuation of helicopter tail shaft.

As can be seen from the previous references, the damage of blade mostly occurs in the resonance region. And most present studies simplify blade as the lumped-mass model. The deviation of this model from the actual shape of the blade is large. In order to simulate the actual shape of flexible blade more accurately and reduce the resonance peak, the discrete model of the blade based on the finite element method is established. And the ideal dry friction mechanical model is used to simulate the boundary conditions of the blade. This conforms to the dynamic environment of the rotating flexible blade. The effects of the parameters of blade system like the rotating speed, the friction coefficient, and the amplitude of external excitation on the vibration reduction characteristics of the blade are studied and the transient response of the blade is analyzed in this paper.

2. The Establishment of Dynamic Model

The structural diagram of the rotating blade with dovetail interface is shown in the Figure 1(a). In the model, R, Ω , and L stand for the radius of the disk, the rotating speed of blade, and the length of blade, respectively. $\omega(x, t)$ is the transverse displacement of the blade. The force analysis of the infinitesimal body dx is shown in Figure 1(b). M and V stand for the bending moment and the shearing force, respectively.

According to the knowledge of mechanics of elasticity, the influence of shearing deformation is considered. When the blade is deformed by the external excitation in the rotating state, the total energy equation Π can be written as follows:

$$\Pi = \frac{1}{2} \int_{R}^{R+L} EI\left(\frac{\partial^{2}\omega_{b}(x,t)}{\partial x^{2}}\right)^{2} dx$$
$$+ \frac{1}{2} \int_{R}^{R+L} \frac{GA}{k} \left(\frac{\partial \omega_{s}(x,t)}{\partial x}\right)^{2} dx$$



(a) The structural representation of the rotating blade (b) The force analysis of the infinitesimal body

FIGURE 1: The continuous model of the blade with dovetail interface.

(1)

$$+ \frac{1}{2} \int_{R}^{R+L} N_{\Omega} \left(\frac{\partial \omega(x,t)}{\partial x} \right)^{2} dx + \frac{1}{2} \int_{R}^{R+L} \rho A \left(\frac{\partial \omega(x,t)}{\partial t} \right)^{2} dx,$$

where the first item on the right of equal sign is the bending strain energy. The second is the shearing strain energy. The third is the centrifugal strain energy. And the fourth is the kinetic energy of blade. $\omega(x, t)$ is the transverse displacement resulting from the deformation of blade; $\omega_b(x, t)$ is the transverse displacement caused by the bending deformation; $\omega_s(x, t)$ is the transverse displacement due to the shearing displacement; what is more $\omega(x, t) = \omega_b(x, t) + \omega_s(x, t)$; *E*, *G*, and *I* denote Young's Modulus, the modulus of shearing, and the moment of inertia, respectively; N_{Ω} is the centrifugal force of the blade, *A* is the cross-sectional area, and ρ is density and *k* is the correction factor for considering the fact that the actual shearing strain and shearing stress are not uniformly distributed.

There are generally two methods for the numerical discretization of the total energy which is shown in (1). The first is the numerical discretization by using its modal functions. The second is the discretization of the finite element method by using the shape functions. The plane beam element is used to discretize the blade in this paper. And the whole blade is divided into 30 elements. The stiffness matrix and the mass matrix of the element are derived from the local coordinate system *oxy*. The transverse displacement of an element is $\omega^e = \omega_b^e + \omega_s^e$ in the discrete modal considering the effect of the shearing deformation.

In the local coordinate system *oxy*, the strain energy caused by bending and shearing deformation of any element can be written as follows:

$$U_{\rm b}^{e} = \frac{1}{2} \int_{(i-1)l}^{il} EI\left(\frac{\partial^{2}\omega_{\rm b}^{e}}{\partial x^{2}}\right)^{2} \mathrm{d}x,$$

$$U_{\rm s}^{e} = \frac{1}{2} \int_{(i-1)l}^{il} \frac{GA}{k} \left(\frac{\partial \omega_{\rm s}^{e}}{\partial x}\right)^{2} \mathrm{d}x,$$
(2)

where *l* is the length of an element, *i* is the ordinal number of the elements, and i = 1, 2, 3, ..., n. In this paper n = 30.

The blade is affected by the centrifugal force under the working condition. And the centrifugal strain energy which is produced by any elements can be shown as

$$U_{c}^{e} = \frac{1}{2} \int_{(i-1)l}^{il} N_{i}^{e}(x_{e}) \dot{\omega}^{e^{2}}(x_{e}, t) dx_{e}, \qquad (3)$$

where x_e is the distance from a point in an element to the front end of the element; $\omega^e(x_e, t)$ is the transverse displacement of this point; $N_i^e(x_e)$ is the centrifugal force of this point and can be written as follows:

$$N_{i}^{e}(x_{e}) = \int_{(i-1)l+x_{e}}^{L} \rho A \Omega^{2} \left[R + (i-1)l + x_{e} \right] \mathrm{d}x.$$
(4)

 $N_{\rm b}(x), N_{\rm s}(x)$ are the element shape functions. And $\delta_{\rm b}^{e}$, $\delta_{\rm s}^{e}$ are the element nodal displacement arrays. They are substituted into the element strain energy equations after the bending strain energy $U_{\rm b}^{e}$, the shearing strain energy $U_{\rm s}^{e}$, and the centrifugal strain energy $U_{\rm c}^{e}$ of the element are obtained. The bending stiffness matrix $\mathbf{k}_{\rm b}^{e}$, the shearing stiffness matrix, and the centrifugal stiffness matrix $\mathbf{k}_{\rm c}^{e}$ are obtained. Therefore, the element stiffness matrix \mathbf{k}^{e} is obtained by integrating the above three matrices.

The kinetic energy of any element consists of two parties. It can be expressed as given below:

$$T^{e} = T_{b}^{e} + T_{s}^{e}$$

$$= \frac{1}{2} \int_{(i-1)l}^{il} \rho A \left(\frac{\partial \omega_{b}}{\partial t}\right)^{2} dx$$

$$+ \frac{1}{2} \int_{(i-1)l}^{il} \rho A \left(\frac{\partial \omega_{s}}{\partial t}\right)^{2} dx,$$
(5)

where T_b^e and T_s^e are the kinetic energy caused by the bending deformation and the shearing deformation, respectively. Similarly, the mass matrices m_b^e and m_s^e can be obtained based on the kinetic energy of the element. Then, the two mass matrices are integrated to obtain the element mass matrix \mathbf{m}^e .

After obtaining the strain energy and the kinetic energy of the element, the total energy Π of the discretized model can be written as follows:

$$\Pi = \sum_{i=1}^{n} U_{b}^{i} + \sum_{i=1}^{n} U_{s}^{i} + \sum_{i=1}^{n} U_{c}^{i} + \sum_{i=1}^{n} T^{i}.$$
 (6)

The element stiffness matrix \mathbf{k}^e and the element mass matrix \mathbf{m}^e are transformed from the local coordinate system to the whole coordinate system. Then, the whole stiffness matrix \mathbf{K} and the whole mass matrix \mathbf{M} are obtained by means of the set of element matrices. The damping of the whole blade system is expressed by Rayleigh damping; its expression is

$$\mathbf{C} = \alpha \mathbf{M} + \beta \mathbf{K}.$$
 (7)

 α and β are the coefficients of Rayleigh damping. They can be expressed as follows:

$$\alpha = \frac{2\left(\xi_2/\omega_2 - \xi_1/\omega_1\right)}{\left(1/\omega_2^2 - 1/\omega_1^2\right)},\tag{7.1}$$

$$\beta = \frac{2\left(\xi_2\omega_2 - \xi_1\omega_1\right)}{\left(\omega_2^2 - \omega_1^2\right)},\tag{7.2}$$

where ω_1 and ω_2 are the first-order and the second-order natural angular frequency of the blade, respectively; ξ_1 and ξ_2 are the damping coefficients corresponding to the two natural frequencies, respectively.

3. The Establishment of Mechanical Model

The tenons are closely connected together to form the contact interfaces due to the centrifugal force of the blade. There are complicated nonlinear forces at the contact interfaces. In order to research the effect of the nonlinear force on the natural characteristics of the blade, an ideal dry friction model is established to simulate the nonlinear force of the tenon joint. The mechanical model of the dry friction is illustrated in Figure 2, where γ is tenon angle and ϕ and η are the angle between the contact interface and the vertical direction. A₁ and A₂ are the contact interfaces of the tenon; x(t) is the horizontal displacement of the blade-root; d_{u} and $d_{\rm v}$ are the displacement of the contact interfaces A₁ and A₂, respectively; N_{Ω} is the centrifugal force of blade; N_1 and N_2 are the normal pressure on the two contact interfaces; k_1 , f_1 , and μ_1 are the shearing stiffness, the friction force, and the friction coefficient of the contact interface A₁, respectively. k_2 , f_2 , and μ_2 stand for the shearing stiffness, the friction force, and the friction coefficient of the contact interface A_2 , respectively. w_1 and w_2 are the displacement of the dry friction damper.

In order to facilitate analysis and calculation, the following simplifications are made in this paper.

(1) The influence of the change of the friction force on the normal pressure acting on the contact interface is ignored.

(2) The effects of the twisting, the spanwise displacement, and the installation angle of the blade are neglected. Only the transverse displacement of the blade is considered.

(3) It is assumed that the contact interfaces are always in contact with each other. No separation occurs under the action of the centrifugal force.



FIGURE 2: The mechanical model of the dry friction.

The displacements of the two contact interfaces are obtained from the geometric relation in Figure 2:

$$d_{\rm u} = \frac{x(t)}{\cos \gamma},$$

$$d_{\rm v} = \frac{x(t)}{\cos \gamma}.$$
(8)

As shown in Figure 2, when the elasticity of the shearing spring is smaller than the slipping friction force, the contact interfaces are in sticking state. On the contrary, the contact interfaces are in slipping state. Therefore, the mathematical expression of the dry friction force can be written as follows:

$$f_{1} = \begin{cases} k_{1} (d_{u} - w_{1}) & k_{1} |d_{u} - w_{1}| < \mu_{1} N_{1} \\ \mu_{1} N_{1} \operatorname{sgn} (\dot{w}_{1}) & k_{1} |d_{u} - w_{1}| \ge \mu_{1} N_{1}, \end{cases}$$

$$f_{2} = \begin{cases} k_{2} (d_{v} - w_{2}) & k_{2} |d_{v} - w_{2}| < \mu_{2} N_{2} \\ \mu_{2} N_{2} \operatorname{sgn} (\dot{w}_{2}) & k_{2} |d_{v} - w_{2}| \ge \mu_{2} N_{2}. \end{cases}$$

$$(9)$$

According to the mechanical balance and neglecting the influence of the friction force, the positive pressure acting on the contact interface can be expressed as follows:

$$N_1 = N_2 = \frac{N_\Omega}{2\cos\gamma}.$$
 (10)

Assuming that the rotating speed is Ω , the centrifugal force of the blade is

$$N_{\Omega} = \int_{R}^{R+L} \rho A \Omega^2 x \, \mathrm{d}x. \tag{11}$$

It can be seen that the dry friction force is segmented in the period of blade vibration from its mathematical expression. Taking the contact interface A_1 as an example, the contact interfaces of the tenon-mortise will be in sticking state when the elasticity $k_1|d_u-w_1|$ is smaller than the slipping friction force $\mu_1 N_1$. Then the speed of the damper is $\dot{w}_1 = 0$. The contact interfaces of the tenon-mortise will be in slipping state when the elasticity $k_1|d_u-w_1|$ is bigger than or equal to the slipping friction force $\mu_1 N_1$. Then the speed of the damper is $\dot{w}_1 = \dot{d}_u$. Therefore, the speed of the damper can be used to judge the state of the contact interfaces.

$$\dot{w}_1 = \begin{cases} 0 & \text{sticking state} \\ \dot{d}_u & \text{slipping state.} \end{cases}$$
(12)



TABLE 1: The default parameters of the blade system.

FIGURE 3: The amplitude-frequency response curves of blade at different rotating speeds.

The integral of the above equation is obtained:

$$w_1 = \begin{cases} c_1 & \text{sticking state} \\ d_u + c_2 & \text{slipping state,} \end{cases}$$
(13)

where c_1, c_2 are constant number. But the values of c_1, c_2 are unknown. Therefore, the influence of c_1 and c_2 can be eliminated by using the displacement difference between the former and later moments of the damper in the process of iterative calculation. Then the expression from formula (13) can be obtained:

$$\Delta w_{1s} = w_{1s} - w_{1(s-1)} = \begin{cases} 0 & \text{sticking state} \\ \Delta d_{us} & \text{slipping state,} \end{cases}$$
(14)

where *s* represents the order of iterations. From (14), it can be seen that the displacement difference $\Delta w_{1s} = 0$ when the contact interfaces of the tenon-mortise are in sticking state and $\Delta w_{1s} = \Delta d_{us}$ when the contact interfaces of the tenon-mortise are in slipping state.

The friction forces of the two contact interfaces are all along the contact interface, but in the opposite direction. The components of the friction force in vertical and horizontal direction are

$$f_{\rm v} = f_1 \sin \gamma - f_2 \sin \gamma,$$

$$f_{\rm h} = f_1 \cos \gamma + f_2 \cos \gamma.$$
(15)

The geometric relationship of the tenon shows that $f_v = 0$. The stiffness matrix **K**, the mass matrix **M**, and the damping matrix **C** of the system are known. The dynamic equation of the system can be written as follows:

$$M\ddot{X} + C\dot{X} + KX = Q(t) - F_{h}(x, t).$$
 (16)

where $\mathbf{Q}(t)$ is the external excitation and $\mathbf{F}_{h}(x,t)$ is the friction vector.

4. The Natural Characteristics

In this paper, the default parameters of rotating blades are shown in Table 1.

The rotating speed of blade is $\Omega = 200 \text{ rad/s}$. The friction coefficients of the contact interfaces are equal: $\mu_1 = \mu_2 = 0.2$. The harmonic excitation $\mathbf{Q}(t) = P_a \sin(\omega t)$ is used to simulate the aerodynamic excitation, where the excitation amplitude $p_a = 500 \text{ N}$ and the angular frequency of excitation $\omega = 1600 \text{ rad/s}$. This section will analyze the effect of the rotating speed of blade, the friction coefficient, and the excitation amplitude on the nonlinear characteristics of blade, respectively.

4.1. The Effect of Rotating Speeds. Figure 3 shows the amplitude-frequency response curve of the blade-tip at different rotating speeds. And each curve corresponds to different rotating speeds. In Figure 3(a), the amplitude-frequency



FIGURE 4: The responses of blade-root at different rotating speeds ((a) is the time-domain response, (b) is the frequency spectrum, and (c) is the phase diagram).

response curve of the blade shows a certain hard nonlinear phenomenon when the rotating speed of blade is less than 350 rad/s. The resonance peak of the blade has a tendency of shift to the right. And the value is obviously lower than that of the linear phenomenon. At the same time, the resonance peak of blade increases with the rotating speed. When the rotating speed is higher than 350 rad/s, the amplitude-frequency response curve is no longer nonlinear. However, the resonant frequency of the blade increases gradually. And the resonance peak decreases gradually with the rotating speed. It can also be seen from the diagram that when the rotating speed is lower, the blade also has a large response amplitude in the low-frequency area. Figure 3(b) shows the amplitudefrequency response curve of blade at slow speed. In addition to the higher response amplitude at low frequencies, the amplitude-frequency response curve exhibits a soft nonlinear phenomenon when the excitation angular frequency ranges from 600 rad/s to 900 rad/s. There is the same trend as the result which is obtained in [15]. In the current excitation frequency range, which is far less than the first natural frequency, the blade should behave as a rigid body. But the

soft nonlinear phenomenon disappeared immediately with the increase of the rotating speed of blade.

This paper takes the response of the blade-root as an example. The transient response characteristics of the blade are observed because of the friction force acting on the tenon of blade. In order to make the response more obvious, the excitation angle frequency is 1600 rad/s which is close to the first natural frequency of the blade. When the rotating speeds are 100 rad/s, 200 rad/s, and 600 rad/s, the time-domain response diagram, the frequency spectrum, and the phase diagram of blade-root are shown as (a_1) , (b_1) , (c_1) ; (a_2) , (b_2) , (c_2) ; and (a_3) , (b_3) , (c_3) of Figure 4, respectively. When the rotating speeds are slow, such as 100 rad/s and 200 rad/s, the time-domain waveform shows a harmonic phenomenon and the response amplitude decreases because of the nonlinear dry friction force. There are not only dominant frequency, but also obvious 3x and 5x components in the frequency spectrum. And the phase diagram of the blade-root is no longer smooth, indicating that the motion of the blade is complicated. When the rotating speed is higher, such as 600 rad/s, the time-domain waveform of blade-root shows that a simple



FIGURE 5: The variation curves of dry friction force at different rotating speeds (subscript 1 represents the variation curve of dry friction force and subscript 2 represents the hysteresis loop).

harmonic variation and the response amplitude are small. There is only dominant frequency in the frequency spectrum. It proves that there is not nonlinear phenomenon at this moment. The phase diagram changes smoothly and regularly.

In Figure 5, (a_1) and (a_2) , (b_1) and (b_2) , and (c_1) and (c_2) show the variation curves of dry friction force and the hysteresis loops when the rotating speeds are 100 rad/s, 200 rad/s, and 600 rad/s, respectively. They can reveal the variation of dry friction force at different rotating speeds. When the rotating speeds are 100 rad/s and 200 rad/s, the dry friction force shows a wave clipping phenomenon with time and will keep constant for a period after it reaches the maximum value. The maximum value of dry friction force will increase with the rotating speed. But the time of the dry friction force at its peak will gradually decrease. The dry friction force varies piecewise linearly with the displacement of blade-root under these speeds. The hysteresis loop of dry friction force is a closed parallelogram. And the area of it represents the energy consumed by dry friction. The area of the hysteresis loop (2 \times the peak of dry friction force \times slipping distance = 0.76) in Figure $5(b_2)$ is larger than that $(2 \times \text{the peak of dry friction force} \times \text{slipping distance} =$ (0.29) in Figure 5(a₂). The result illustrates that the energy consumption of dry friction is larger when the rotating speed is 200 rad/s. That makes the response amplitude of blade-root smaller. When the rotating speed of the blade is 600 rad/s, the dry friction force presents a simple harmonic variation with time and the hysteresis loop is a reciprocating straight line. There is no energy consumption at this moment.

4.2. The Effect of Friction Coefficient. The dry friction force and the vibration reduction characteristics of the blade are

directly affected by the change of friction coefficient. In order to research the effect of friction coefficient on the vibration reduction characteristics, the amplitude-frequency curves of blade-tip at different friction coefficients are obtained when the rotating speeds are 200 rad/s and 400 rad/s, respectively. It can be seen form Figure 6(a) that when the friction coefficient of the contact interfaces is small, the amplitudefrequency curves of the blade-tip exhibit a certain hard nonlinear phenomenon. And the response amplitude of the blade increases and the nonlinear phenomenon decreases gradually with the friction coefficient in resonance region. There is no longer nonlinearity in the amplitude-frequency curve of blade-tip when the friction coefficient is bigger than before. It can be obtained by comparing Figure 6(a)with Figure 6(b) that the response amplitude of the blade in resonance region is larger and the nonlinear phenomenon is relatively weak when the friction coefficients are the same and the rotating speed is higher. Therefore, the effect of dry friction vibration reduction is more obvious when the friction coefficient is small and the rotating speed is slow.

In order to further research the effect of the friction coefficient on the vibration reduction characteristics of blade. The transient response of the blade-root at different friction coefficients is observed. When the friction coefficients are 0.2, 0.4, and 0.8, the time-domain response diagram, the frequency spectrum, and the phase diagram of blade-root are shown as (a_1) , (b_1) , (c_1) ; (a_2) , (b_2) , (c_2) ; and (a_3) , (b_3) , (c_3) of Figure 7, respectively. When the friction coefficients are small, such as 0.2 and 0.4, the time-domain waveform shows a harmonic phenomenon and the response amplitude decreases because of the nonlinear dry friction force. There is not only dominant frequency, but also obvious 3x and



FIGURE 6: The amplitude-frequency response curves of blade at different friction coefficients.



FIGURE 7: The responses of blade-root at different friction coefficients ((a) is the time-domain response, (b) is the frequency spectrum, and (c) is the phase diagram).



FIGURE 8: The variation curves of dry friction force at different friction coefficients (subscript 1 represents the variation curve of dry friction force and subscript 2 represents the hysteresis loop).

5x components in the frequency spectrum. And the phase diagram of the blade-root is also shown as a complicated curve. When the friction coefficient is bigger, such as 0.8, the time-domain waveform of blade-root shows a simple harmonic variation and the response amplitude is bigger than that of before. There is only dominant frequency in the frequency spectrum. And the phase diagram changes smoothly and regularly.

In Figure 8, (a_1) and (a_2) , (b_1) and (b_2) , and (c_1) and (c_2) show the variation curves of dry friction force and the hysteresis loops when the friction coefficients are 0.2, 0.4, and 0.8, respectively. They can reveal the variation of dry friction force at different friction coefficients of contact interfaces. When the friction coefficients are 0.2 and 0.4, the dry friction force shows a wave clipping phenomenon with time and will keep constant for a period after it reaches the maximum value. The maximum value of dry friction force will increase with the friction coefficient. But the time of the dry friction force at its peak will gradually decrease. The dry friction force varies piecewise linearly with the displacement of bladeroot. Meanwhile, the hysteresis loop of dry friction force is a closed parallelogram. And the area of the parallelogram $(2 \times$ the peak of dry friction force \times slipping distance) decreases gradually with the increase of the friction coefficient. The vibration dissipation by dry friction is low when the area of the parallelogram is small. The dry friction force presents a simple harmonic variation with the time and the hysteresis loop is a reciprocating straight line when the friction coefficient is 0.8. And there is no energy consumption.

4.3. The Effect of Excitation Amplitude. Considering the soft and hard nonlinear phenomena of the amplitude-frequency

curves at low and high rotating speed, the effect of different excitation amplitudes on the vibration reduction characteristics of the blade at a high or low speed is studied. Figure 9 shows the amplitude-frequency response curves of blade-tip at different excitation amplitudes when the rotating speeds are 200 rad/s and 400 rad/s, respectively. Each curve corresponds to different excitation amplitudes. It can be seen from Figure 9(a) that the nonlinear phenomenon of the amplitude-frequency curves is more obvious with the increase of excitation amplitude. But the resonance peak value of blade increases and the resonance zone also expands. There is also a large response amplitude at the low frequency when the excitation amplitude is too large. That is extremely dangerous. In Figure 9(b), the amplitude-frequency curve of the blade-tip does not show a nonlinear phenomenon when the excitation amplitude is small. With the increase of excitation amplitude, the nonlinear phenomenon is more and more obvious. And the value of resonance peak also increases gradually. It can be known by comparing Figure 9(a) with Figure 9(b) that the effect of dry friction vibration reduction is obvious in the resonant region when the rotating speed is slow. And the effect of vibration reduction is related to the value of excitation amplitude.

In order to further research the effect of excitation amplitude on the vibration reduction characteristics of blade, the transient response characteristics of the blade-root at different excitation amplitudes are observed. When the excitation amplitudes are 100 N, 500 N, and 1000 N, the timedomain response diagram, the frequency spectrum, and the phase diagram of blade-root are shown as (a_1) , (b_1) , (c_1) ; (a_2) , (b_2) , (c_2) ; and (a_3) , (b_3) , (c_3) of Figure 10, respectively. When the excitation amplitude is small, such as 100 N,



FIGURE 9: The amplitude-frequency response curves at different excitation amplitudes.



FIGURE 10: The responses of blade-root at different excitation amplitudes ((a) is the time-domain response, (b) is the frequency spectrum, and (c) is the phase diagram).

the time-domain waveform of blade-root shows a simple harmonic variation. There is only dominant frequency in the frequency spectrum. And the phase diagram is smooth and varies regularly. When the excitation amplitudes are bigger than former, such as 500 N and 1000 N, the time-domain waveform shows a harmonic phenomenon because of the nonlinear dry friction force. There is not only dominant frequency but also obvious 3x and 5x components in the frequency spectrum. And the phase diagram of the bladeroot is also shown as a complicated curve. Compared with



FIGURE 11: The variation curves of dry friction force at different excitation amplitudes (subscript 1 represents the variation curve of dry friction force and subscript 2 represents the hysteresis loop).

 (b_1) , (b_2) , and (b_3) , it is found that the response amplitude of the blade-root increases with the excitation amplitude.

In Figure 11, (a_1) and (a_2) , (b_1) and (b_2) , and (c_1) and (c_2) show the variation curves of dry friction force and the hysteresis loops when the excitation amplitudes are 100 N, 500 N, and 1000 N, respectively. They can reveal the variation of dry friction force at different excitation amplitudes. The dry friction force represents a simple harmonic variation and the hysteresis loop is a reciprocating straight line when the excitation amplitude is 100 N. The dry friction does not consume energy at this moment. The dry friction force shows a wave clipping phenomenon with time and will keep constant for a period after it reaches the maximum value when the excitation amplitudes are 500 N and 1000 N. The maximum value of dry friction force will not change with the increase of the excitation amplitude. But the time of the dry friction force at its peak will gradually increase. The dry friction force varies piecewise linearly with the displacement of blade-root. Meanwhile, the hysteresis loop of dry friction force is a closed parallelogram. And the area of the parallelogram $(2 \times \text{the peak of dry friction force} \times \text{slipping})$ distance) increases gradually with the excitation amplitude. The vibration dissipation by dry friction is large when the area of the parallelogram is large.

5. Discussion

The constraining force of the blade changes with the rotating speed. The constraining force is small so that the tenonmortise is in completely loosing state when the rotating speed of the blade is slow. The blade will show the whole rigid body vibration under the action of external excitation. And the amplitude-frequency curve of the blade has a soft nonlinear phenomenon in the low-frequency range. The constraining force increases rapidly with the square of the speed ($N_{\Omega} \propto \Omega^2$). The dry friction force shows piecewise linearly change with the displacement of blade-root when the rotating speed is less than 350 rad/s. That makes the amplitude-frequency curve of blade represent a certain hard nonlinear phenomenon. When the rotating speed continues to increase, the constraining force is too large so that the tenon and mortise are tightly attached together. Therefore, the dry friction force shows linearly change with the displacement of blade-root. And there is no nonlinear phenomenon in the amplitude-frequency curve of the blade.

The dry friction can consume the energy generated by the vibration of the blade when the blade is excited by the external excitation, that reduces the response amplitude of the blade. It is known that the sliding friction force of the contact interface is proportional to the friction coefficient ($f \propto \mu$). When the friction coefficient is small, the tenon and the mortise are easy in a relatively slipping state. This will result in obvious effect of the dry friction vibration reduction. On the contrary, the tenon and the mortise will be in a sticking state for a longer time if the friction coefficient is bigger. And the effect of the dry friction vibration reduction is weakened.

Although the effect of the dry friction vibration reduction is different when the blade is at a lower speed and higher speed, respectively, the energy dissipation of dry friction is also affected by the friction coefficient. But the rotating speed of blade, the friction coefficient, and the amplitude of the external excitation are affected by each other. Therefore, the dry friction vibration reduction has different results in different situations.

6. Conclusion

In this paper, the effect of the rotating speed of the blade, the friction coefficient of contact interface, and the amplitude of the external excitation on the natural characteristics of the blade is studied, respectively, based on the ideal dry friction model and finite element method. The conclusions are as follows.

(1) The amplitude-frequency curve of the blade has a soft nonlinear phenomenon at the low-frequency range when the rotating speed is small. The soft nonlinear phenomenon disappears gradually with the increase of the rotating speed. And the amplitude-frequency curve begins to show a certain hard nonlinear phenomenon in the resonance region of the blade. There is an obvious effect of dry friction vibration reduction. But when the rotating speed is too high, there is no nonlinear phenomenon in the amplitude-frequency curve of blade and the effect of dry friction vibration reduction disappears. Therefore, it is important to pay more attention to the structure design of tenon-mortise. There are different vibration reduction effects with different structural parameters.

(2) The change of the friction coefficient has a great effect on the vibration reduction characteristics of the blade. When the friction coefficient is small, the amplitude-frequency curve of blade is easy to show nonlinear phenomenon. And the effect of the dry friction vibration reduction is obvious. But with the increase of the friction coefficient, both the nonlinear phenomenon and the effect of dry friction vibration reduction begin to decrease.

(3) The change of the excitation amplitude also has a certain effect on the vibration reduction characteristics of the blade. The nonlinear phenomenon of the amplitude-frequency curves is more obvious with the increase of the excitation amplitude. Although the effect of dry friction vibration reduction is increased, the blade response amplitude also increases. Therefore, it is very important to prevent the blade from being excited by the external excitation with too large fluctuation.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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