Hindawi
Shock and Vibration
Volume 2018, Article ID 6508695, 8 pages
https://doi.org/10.1155/2018/6508695

Research Article

Robust Estimation of the Unbalance of Rotor Systems Based on Sparsity Control of the Residual Model

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Received 30 November 2017; Revised 6 February 2018; Accepted 17 July 2018; Published 14 August 2018

Academic Editor: Enrico Zappino

The vibration signals of rotating machinery are frequently disturbed by background noise and external disturbances because of the equipment's particular working environment. Thus, robustness has become one of the most important problems in identifying the unbalance of rotor systems. Based on the observation that external disturbance of the unbalance response often displays sparsity compared with measured vibration data, we present a new robust method for identifying the unbalance of rotor systems based on model residual sparsity control. The residual model is composed of two parts: one part takes regular measurements of noise, while the other part evaluates the impact of external disturbances. With the help of the sparsity of external disturbances, the unbalance identification is converted into a convex optimization problem and solved by a sparse signal reconstruction algorithm. Experiment results have shown that the proposed method is robust and effective in identifying the unbalance of rotor systems in a complex environment, improving the precision of unbalance estimation and simplifying the balancing process.

1. Introduction

Rotor unbalance, one of the most frequently seen faults in rotating machinery, needs to be modified at regular intervals to ensure safe and stable operation of the equipment. Applications that can be commonly seen are steam turbines, compressors, wind turbines, and aeroengines. Balancing theory has been thoroughly studied, and several useful balancing methods have been developed in the last decades [1–6].

The most popular balancing techniques, including influence coefficient method and modal-based balancing method, have obtained balancing masses by least squares or improved least-squares methods. The main drawback of least squares or improved least squares is that they are sensitive to noise present in the vibration response. Unfortunately, the vibration response is frequently disturbed by background noise and external disturbances because of the equipment's particular working environment. Consequently, robustness has become one of the most important problems in estimating the unbalance of rotor systems. Furthermore, the equipment can rarely run steadily when unbalance faults exist. Therefore, components that have no relationship with unbalance faults could be included in the modified vibration response, a phenomenon that also requires enhancing the robustness of unbalance estimation.

Researchers have developed several approaches to improve the performance of unbalance estimation in noise environment. Sinha et al. [7] has proposed a robust method for unbalance estimation using measured pedestal vibration along withan a priori model of both the rotor and the fluid bearings. Pennacchi et al. [8–10] has introduced an automatic procedure for unbalance estimation based on robust regression methods, which mainly focuses on high breakdown point and bounded-influence estimators. The primary principle of the methods in [8–10] is to reduce the influence of the external disturbance on unbalance estimation using a selected penalty function, aiming to address the problem that traditional balancing methods require substantial practical experience in field balancing. However, the distribution of the external disturbance should be considered in the selection of the penalty function, and limitations could exist in the application of more complicated nonlinear models. The most recent study by Nauclér and Söderström [11] present a novel method that reformulates unbalance estimation problem as a linear estimation procedure, taking disturbances into account.

Sparsity, an important characteristic of vibration signals, has been successfully applied in signal processing, signal compression, and pattern recognition [12, 13]. Recent research and experimental results have proved the effectiveness of sparsity in reducing the influence of outliers [14–16]. Either being sparse naturally or being represented sparsely on a selected basis, superior signal modeling or reconstruction performance could be obtained by controlling the sparsity of model residuals and with the application of a sparse signal reconstruction algorithm.

The practical operation of field balancing has proved the explicit sparsity of the external disturbance and background noise that affect the unbalance estimation of a rotor system compared with the collected vibration response. A new robust method for estimating the unbalance of rotor systems based on model residual sparsity control is presented in this paper. The residual model is composed of two parts. One part takes regular measurements of noise, while the other part evaluates the impact of external disturbances. With the help of the sparsity of external disturbances, rotor unbalance identification is converted into a convex optimization problem and solved by a sparse signal reconstruction algorithm. Experimental results have shown that the proposed method is robust and can effectively identify the unbalance of rotor systems in a complex environment.

In this paper, the main principle of rotor balancing is briefly introduced, and the improved regression model and sparsity signal reconstruction algorithm are presented, followed by experimental demonstration of the effectiveness and robustness of the proposed unbalance estimation approach, which leads to a positive and promising conclusion.

2. Influence Coefficient Balancing Method

The influence coefficient method, the most popular and widely used approach for rotor balancing because of its experimental characteristic, was developed based on linear vibration theory, which assumes a linear correlation between vibration and unbalance mass [10, 17, 18]. In practice, the balancing rotating speed, the number of measuring planes, and the number of balancing planes are selected according to the structure of the machine. Supposing there are n_r rotating speeds, n_m measuring planes, and n_b balancing planes, the balancing masses can be obtained from the following overdetermined system:

$$y + X\theta = e, \tag{1}$$

where y is the measured vibration response and is $2n_\mathrm{m}n_\mathrm{r} \times 1$ complex vector that shall be expressed as follows:

$$y = \left\{\{\xi(\Omega_1)\}^T \cdots \left\{\xi\left(\Omega_j\right)\right\}^T \cdots \left\{\xi\left(\Omega_{n_\mathrm{r}}\right)\right\}^T\right\}^T, \tag{2}$$

where $\xi(\Omega_j)$ is a $2n_\mathrm{m} \times 1$ complex vector representing the vibration response at the jth rotating speed, and

$$\begin{aligned}\xi\left(\Omega_j\right) = &\left\{\left\{\xi_\mathrm{v}^{(1)}\left(\Omega_j\right)\ \xi_\mathrm{h}^{(1)}\left(\Omega_j\right)\right\} \cdots \left\{\xi_\mathrm{v}^{(k)}\left(\Omega_j\right)\ \xi_\mathrm{h}^{(k)}\left(\Omega_j\right)\right\}\right.\\ &\left.\cdots \left\{\xi_\mathrm{v}^{(n_\mathrm{m})}\left(\Omega_j\right)\ \xi_\mathrm{h}^{(n_\mathrm{m})}\left(\Omega_j\right)\right\}\right\}^T,\end{aligned} \tag{3}$$

where $\xi_\mathrm{v}^{(k)}(\Omega_j)$ and $\xi_\mathrm{h}^{(k)}(\Omega_j)$ are the vibration responses along the vertical direction and the horizontal direction, respectively, in the kth measuring plane, at the jth rotating speed.

(i) X is the $2n_\mathrm{m}n_\mathrm{r} \times 1$ influence matrix for all n_r rotating speeds:

$$X = \left[[C(\Omega_1)]^T \cdots \left[C\left(\Omega_j\right)\right]^T \cdots \left[C\left(\Omega_{n_\mathrm{r}}\right)\right]^T\right]^T, \tag{4}$$

and $C(\Omega_j)$ is the influence matrix at the jth rotating speed:

$$C\left(\Omega_j\right) = \begin{bmatrix} \alpha_{11} & \cdots & \alpha_{1w} & \cdots & \alpha_{1n_\mathrm{b}} \\ \vdots & \ddots & & & \vdots \\ \alpha_{k1} & & \alpha_{kw} & & \alpha_{kn_\mathrm{b}} \\ \vdots & & & \ddots & \vdots \\ \alpha_{n_\mathrm{m}1} & \cdots & \alpha_{n_\mathrm{m}w} & \cdots & \alpha_{n_\mathrm{m}n_\mathrm{b}} \end{bmatrix}, \tag{5}$$

where α_{kw} is the 2×1 complex influence vector of the wth balancing plane on the kth measuring plane.

(ii) $\theta = [\theta_1 \cdots \theta_k \cdots \theta_{n_\mathrm{b}}]^T$ is an $n_\mathrm{b} \times 1$ balancing masses vector.

(iii) e is a $2n_\mathrm{m}n_\mathrm{r} \times 1$ complex vector that represent the model errors.

3. The Improved Regression Model

Generally, (1) can be seen as a multiple linear regression problem:

$$\begin{aligned} y_i &= x_{i,1}\theta_1 + \cdots + x_{i,k}\theta_k + e_i = X_i^T\theta + e_i,\\ i &= 1,\ 2,\ \ldots,\ 2n_\mathrm{m}n_\mathrm{r}, \end{aligned} \tag{6}$$

where $i = 1,\ 2,\ \ldots,\ 2n_\mathrm{m}n_\mathrm{r}$ is the number of vibration responses collected from the rotor system. In regression analysis, y_i is usually named as the response variable, $x_{i,j}\,(j = 1,\ 2,\ \ldots,\ k)$ is the explanatory variable, θ_i is the regression coefficient, and e_i is the model residual. Generally, the regression coefficients are obtained by least squares, which minimizes $\sum_i e_i^2$. Unfortunately, the least squares method is sensitive to outliers and often gives unsatisfactory results.

Given the above limitations of least squares, robust estimators have been developed to enhance the robustness. The M-Huber estimator, GM estimator, and least trimmed squares (LTS) estimator, the three most common robust estimators, are used widely. In general, robust estimators reduce the impact of contaminated data by using different penalty functions to obtain robustness, which is useful in unbalance estimation because these penalty functions can automatically determine weight coefficients, while weight coefficients should be manually given by experts, as in traditional methods like weighted least squares. A more detailed and comprehensive description of using these robust estimators to identify unbalance can be found in [10]. However, the proper penalty function is still necessary in the investigation of the vibration data.

Rotor balancing requires that the equipment should run steadily in the whole run-up process. Therefore, the vibration response is usually collected from different rotating speeds and on several balancing planes. In the normal case, only a small part of the vibration response is contaminated, and hence, the external disturbance or outliers explicitly display sparsity. Instead of de-emphasizing the contaminated data in vibration response like robust estimators, an alternative approach is to measure the contaminated data in the regression model (6), which is useful as the distribution of the contaminated data can be used to instruct the selection of balancing planes and balancing speeds, both of which have significant influence on the efficiency and precision of dynamic balancing.

To take advantage of the observation that the contaminated data are sparse compared with the collected vibration response, an alternative model is proposed as follows:

$$\begin{aligned} y_i &= x_{i,1}\theta_1 + \cdots + x_{i,k}\theta_k + \varepsilon_i + o_i = X_i^T\theta + \varepsilon_i + o_i, \\ i &= 1,\ 2, \ldots,\ 2n_{\mathrm{m}}n_{\mathrm{r}}, \end{aligned} \tag{7}$$

where $\{\varepsilon_i\}_{i=1}^{2n_{\mathrm{m}}n_{\mathrm{r}}}$ are zero-mean i.i.d. random variables typically representing model residuals and $\{o_i\}_{i=1}^{2n_{\mathrm{m}}n_{\mathrm{r}}}$ are variables representing external disturbance and outliers. If the vibration response is contaminated, $o_i \neq 0$, and otherwise $o_i = 0$. Therefore, $\{o_i\}_{i=1}^{2n_{\mathrm{m}}n_{\mathrm{r}}}$ is a sparse vector. The matrix form of (7) is

$$y = X\theta + o + \varepsilon = [X,\ I]\begin{bmatrix}\theta \\ o\end{bmatrix} + \varepsilon. \tag{8}$$

4. Robust Estimation of the Unbalance of Rotor System Based on Sparse Signal Recovery Algorithms

With both the regression coefficients θ and vector o unknown, (8) is an underdetermined problem. Taking advantage of the sparsity of vector o, (8) can be converted to the optimization problem:

$$\min_{\theta,o}\left[\|y - X\theta - o\|_2^2 + \lambda\|o\|_0\right], \tag{9}$$

where $\|o\|_0$ denotes the ℓ_0-norm and λ is a regularization parameter that controls the sparsity of vector o. Recent research on sparse representation and compressive sensing has used the fact that the ℓ_1-norm is a convex approximation of ℓ_0-norm. Thus, optimization problem (9) can be approximated by the convex optimization problem:

$$\min_{\theta,o}\left[\|y - X\theta - o\|_2^2 + \lambda\|o\|_1\right], \tag{10}$$

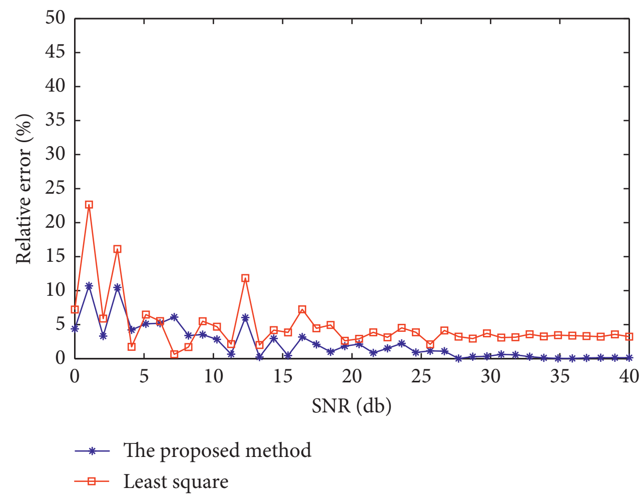

Figure 1: The relative error under different noise levels.

where $\|o\|_1$ is the ℓ_1-norm of vector o. Since o is a sparse vector, (9) is a sparse signal recovery problem. Accordingly, both the regression coefficients and vector o can be obtained by sparse signal recovery algorithms like interior point algorithms or infeasible path-following algorithms. The regression coefficients represent the unbalance mass of the rotor system, while vector o determines how each vibration response is contaminated. Hence, vector o has useful information that directly demonstrates the distribution law of external disturbances and guides the application of dynamic balancing.

The regularization parameter λ has a critical influence on the performance of the unbalance estimation, and both underregularizing and overregularization can result in loss of precision of unbalance estimation. Generally, the larger λ, the sparser the vector o. Therefore, a large λ will cause the vector o to become a zero vector, while a small λ is not sufficient to extract the external disturbance and outliers. The selection of the regularization parameter is more a matter of engineering art and can be determined by the following empirical formula [19, 20]:

$$\lambda = \frac{\hat{\sigma}\sqrt{2\log\left(2n_{\mathrm{m}}n_{\mathrm{r}}\right)}}{3}, \tag{11}$$

where $\hat{\sigma}$ is a robust estimation of the scale of residuals.

A simulation is presented to evaluate the robustness of the proposed method under noise and external disturbances. As shown in (8), the external disturbance has constant amplitude and random position, and the noise range from 0 dB to 40 dB. The relative errors of both the proposed method and least square are analyzed. It can be seen from Figure 1 that the relative error of both the proposed method and least square declines with the decrease of the noise level. Generally, the proposed method has a superior performance than least square at each noise level. When the noise level range from 0 dB to 10 dB, which can be treated as a strong background noise environment, the relative error of the proposed method stabilizes around 5%~10%. The simulation

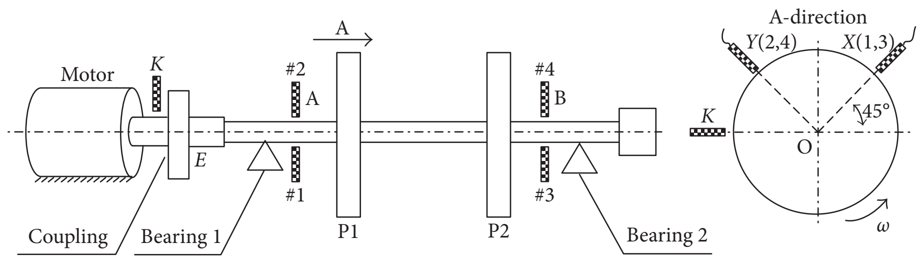

Figure 2: The structure of the test rig.

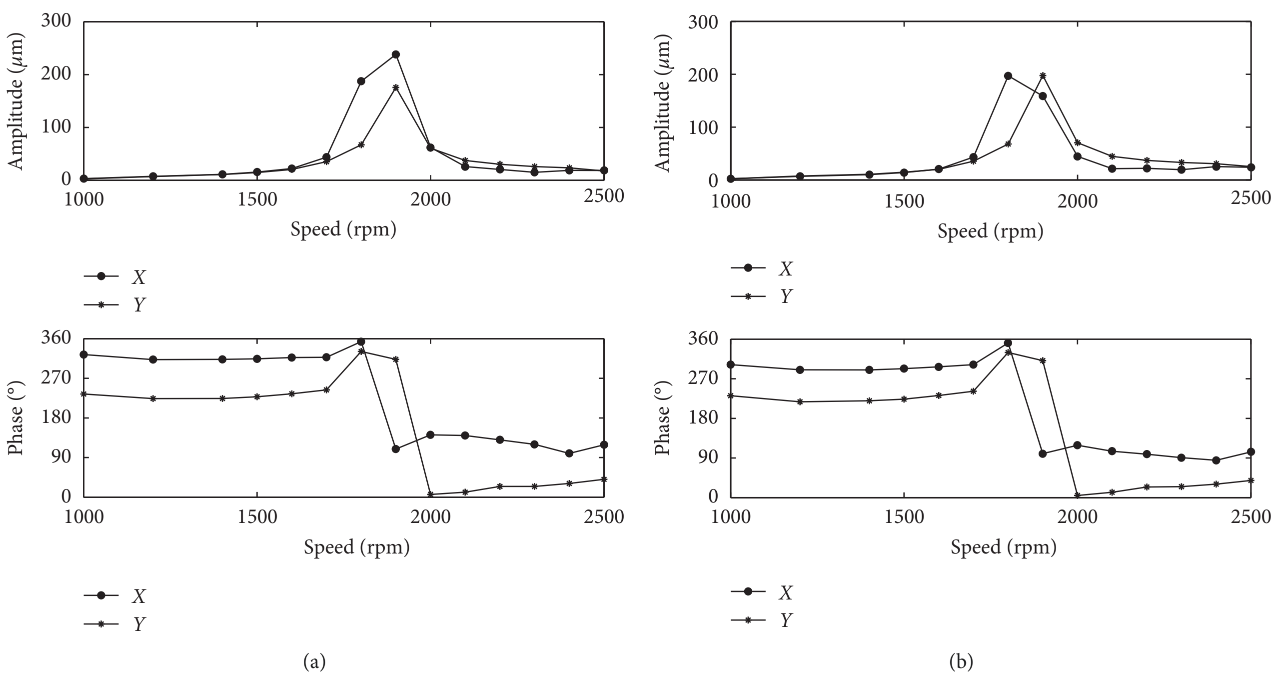

Figure 3: The vibration response of measuring planes A and B. X and Y correspond to the horizontal and vertical direction, respectively. (a) Vibration response of measuring plane A. (b) Vibration response of measuring plane B.

results indicated that the proposed method has a satisfactory robustness under noise environment.

5. Experiments and Discussion

In this section, the proposed robust unbalance estimation method is verified in the identification of the balancing mass of a test rig. The structure sketch of the test rig and the configuration of transducer are illustrated in Figure 2, where transducers #1–#4 measure the vibration of the cross sections A and B, while transducer #5 is a key phase sensor that collects key phase pulses to compute the rotating speed of the rotor, and P1 and P2 are two balancing planes. The first-order critical speed of the rotor system is about 1880 rpm. In order to solve problem (10), the widely used optimization MATLAB toolbox CVX is adopted [21, 22].

5.1. Case of One Unbalance. In order to eliminate the influence of other faults and concentrate on investigating the performance of the method to estimate unbalance mass, the vibration response due to unbalance mass is obtained by two run-up processes. The original vibration response of several different rotating speeds is collected by one run-up process, and one known unbalance mass is applied in balancing plane P1 to simulate the unbalance fault. Then, the rotor system is started up again, and the vibration signals of the rotor are measured with applied unbalance mass. With the vibration response of the above two run-up processes, the experimental vibration response due to applied unbalance mass is obtained in the speed range of 1000 rpm~2500 rpm and shown in Figure 3.

The influence matrix is usually obtained by applied trial weights or analytical models. Here, the influence matrix is estimated by experimental tests, as analytical models are complicated and often not sufficiently accurate enough. Since the vibration response is complex data, the performance of the proposed method is evaluated by the global relative error, which can be derived by the Euclidean norm and stated as

$$\eta_G = \sqrt{\frac{[\hat{\theta} - \theta]^{*T} [\hat{\theta} - \theta]}{\theta^{*T} \theta}}, \quad (12)$$

where θ is the true value of unbalance mass and $\hat{\theta}$ is the estimated unbalance mass.

The unbalance mass estimation results generated by means of least squares (LS), M-Huber estimator, GM estimator, and the proposed method are shown in Table 1. Similar results are obtained by robust estimators, and these robust estimators can improve the precision of unbalance estimation in some way compared with the LS method. However, the relative errors are still unsatisfactory, especially the estimated phase. The proposed robust estimation method based on sparse signal recovery algorithms shows better performance with a global relative error of 11.6% ($\lambda = 2.478$). The different estimation results of LS, robust estimators, and the proposed method suggest that the collected vibration response of the rotor system is contaminated by outliers or external disturbances, and the proposed method based on sparse signal recovery algorithms is more robust than the robust estimators.

The estimated residual vibration response of both measuring planes A and B is demonstrated in Figure 4. As shown, compared with the original vibration, the amplitude of vibration signals before and after the first-order critical speed has been reduced significantly after balancing. Because of the resonance vibration phenomenon, the vibration amplitude around the first-order critical speed still exists, but has been reduced to an acceptable level.

In order to study the distribution of the external disturbance and outliers in the unbalance response, motivated by the idea of signal-to-noise ratio, the useful information ratio is introduced to evaluate the quality of collected vibration response. Considering the bivariate characteristic of complex data, the useful information ratio is given by

$$\chi_u = \sqrt{\frac{v_r^{*T} v_r}{v_o^{*T} v_o}}, \quad (13)$$

where v_r is the residual vibration response and v_o is the original vibration response. The useful information ratios in the speed range of 1000 rpm–2500 rpm are shown in Figure 5. The useful information ratio varies with rotating speed and has reduced to less than 0.5 around the first-order critical speed, which is consistent with the phenomenon that accurate amplitude and phase information is difficult to obtain because of the violent vibration near the critical speed. Furthermore, the variation of useful information ratio suggests that the external disturbance is mainly distributed near the first-order critical speed. Therefore, the balancing speeds must be selected away from critical speeds.

5.2. Case of Two Unbalances. In this case, two known unbalance masses are applied in balancing planes P1 and P2 separately to simulate multiplane unbalance fault. The goal is to identify precisely the applied unbalance in terms of masses and phases. The experimental vibration response by applied unbalance masses is generated in the same way as in the one-unbalance case, which is shown in Figure 6. Due to the difference between the horizontal stiffness and vertical stiffness of the rotor system, the resonance peaks appear at different rotating speeds, about 1800 rpm in horizontal direction and 1900 rpm in the vertical direction.

The influence matrix is generated by experimental tests in the same way as in the one-unbalance case. The estimated results in terms of balancing masses and phases of LS,

Table 1: The results of estimated unbalance mass.

	Mass (g)	Phase (°)	Relative error (%)
Actual	0.4	0	
LS	0.42	33.53	59.57
M-Huber	0.39	28.0	48.18
GM	0.39	334.8	43.44
The proposed method	0.38	5.98	11.6

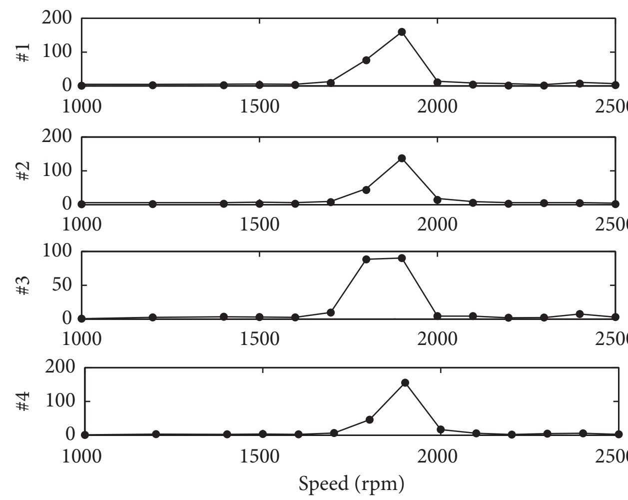

Figure 4: Estimated residual vibration response of measuring planes A and B. #1–#4 correspond to the four measuring directions.

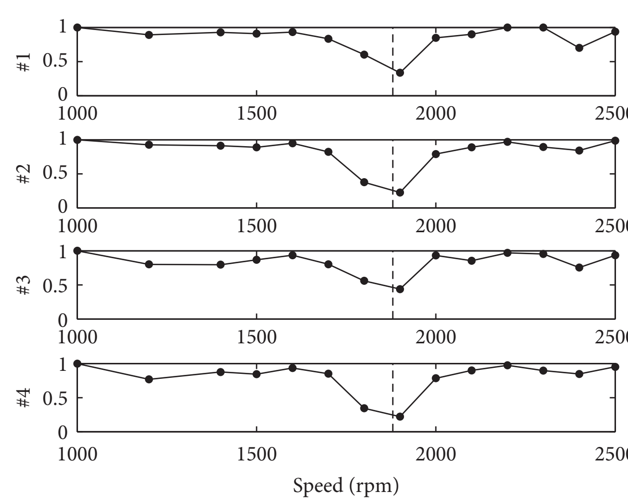

Figure 5: The useful information ratio curve of measuring planes A and B. #1–#4 correspond to the four measuring directions.

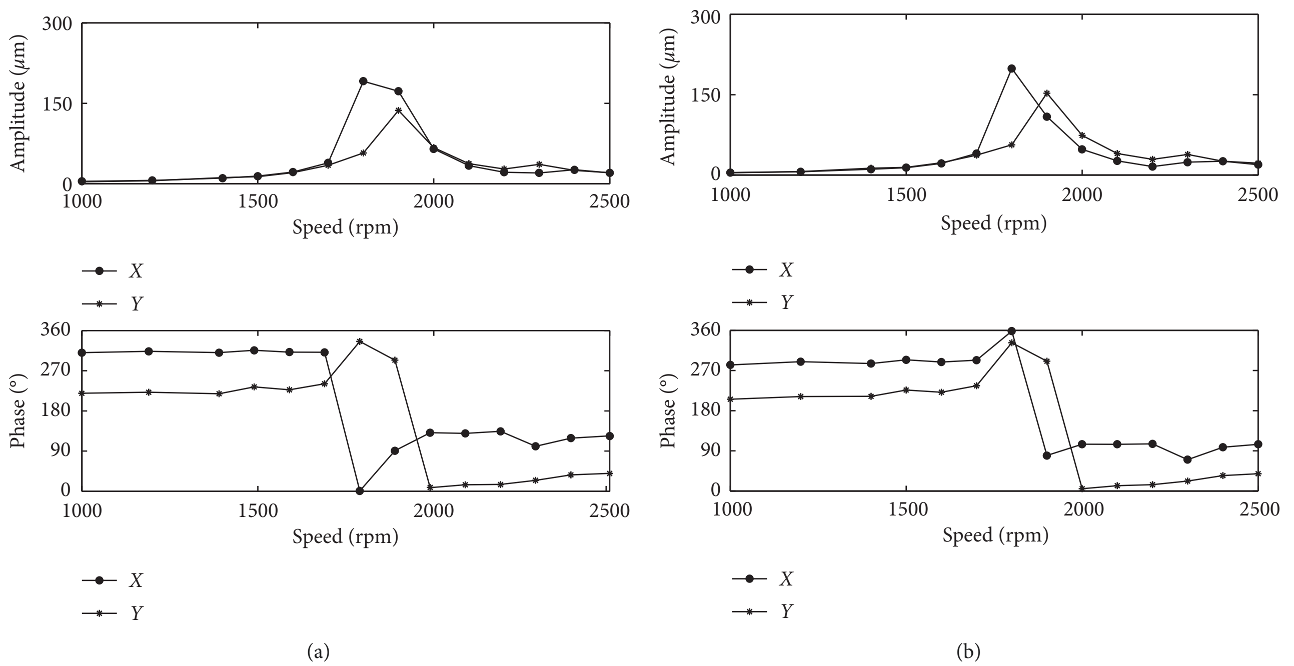

Figure 6: The vibration response of measuring planes A and B. X and Y correspond to the horizontal and vertical direction, respectively. (a) Vibration response of measuring plane A. (b) Vibration response of measuring plane B.

Table 2: The results of estimated unbalance masses.

	Balancing plane P1			Balancing plane P2		
	Mass (g)	Phase (°)	Relative error (%)	Mass (g)	Phase (°)	Relative error (%)
Actual	0.2	0		0.2	0	
LS	0.13	40.73	66.40	0.26	11.76	36.06
M-Huber	0.17	29.16	48.78	0.23	342.89	34.63
GM	0.17	29.34	49.00	0.23	341.50	37.51
The proposed method	0.17	25.24	42.8	0.22	344.26	31.55

M-Huber estimator, GM estimator, and the proposed approach are demonstrated in Table 2. The comparison of the estimated results in Table 2 with the known applied unbalance masses and phases proves that robust estimators gain a better performance than the results generated by the LS method.

The relative error is reduced from 66.40% to about 50% in balancing plane P1, and it changes smoothly in balancing plane P2. Similar results are obtained by the proposed approach compared with the robust estimators, suggesting that the proposed method can reduce the impact of external disturbance and improve the precision of unbalance estimation. The estimated results of robust estimators and the proposed approach indicate that a robust method should be adopted when the vibration response is corrupted.

To evaluate the quality of measured vibration response, the useful information ratio curves are calculated and shown in Figure 7. Consistent with the one-unbalance case, it can be observed that the useful information ratio is reduced to less than 0.5 near the first-order critical speed and is stabilized above 0.8 in most rotating speeds away from the first-order critical speed. The change of the useful information ratio curves also shows that the vibration response around the first-order critical speed has been corrupted. On the other hand, comparison of the useful information ratio curves of the two-unbalance case with those of the one-unbalance case

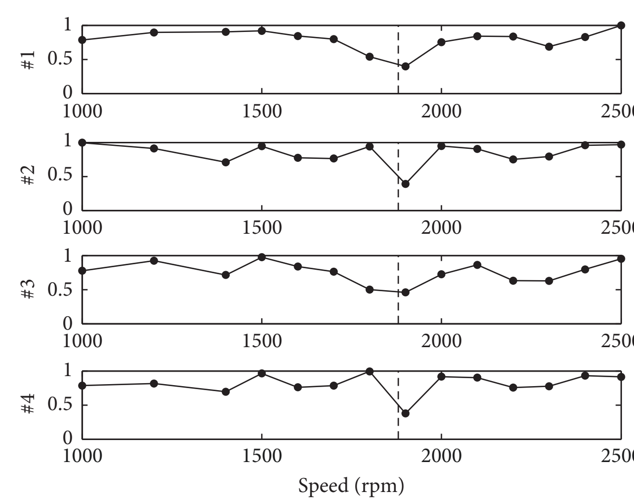

Figure 7: The useful information ratio curve of measuring planes A and B. #1–#4 correspond to the four measuring directions.

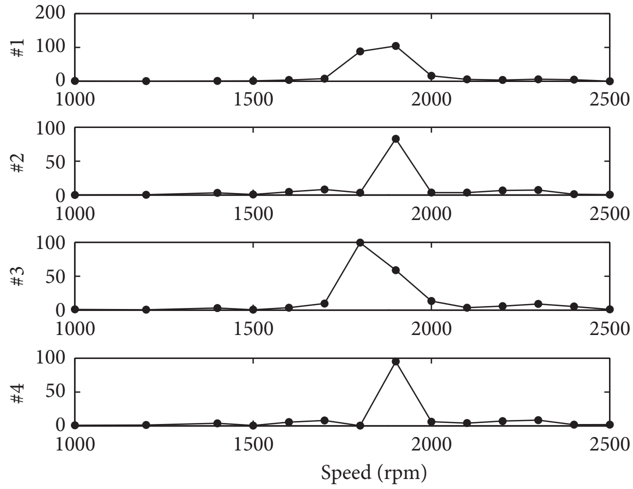

Figure 8: Estimated residual vibration response of measuring planes A and B. #1–#4 correspond to the four measuring directions.

indicates that the quality of measured vibration response of the two-unbalance case is worse than the quality of the one-unbalance case, which is the reason why the performance of the proposed method in the one-unbalance case is better than that in the two-unbalance case. Therefore, the useful information ratio curves can be adopted as an indication of the precision of unbalance estimation.

The estimated residual vibration responses of both measuring planes A and B are demonstrated in Figure 8. As for the one-unbalance case, the amplitude of vibration signals before and after the first-order critical speed has been reduced efficiently after balancing. As shown in Figures 7 and 8, at one rotating speed, the higher the quality of vibration response, the smaller the vibration residual. Thus, special attention should be paid to the collection process of vibration response.

6. Conclusions

A new robust method for estimating the unbalance of rotor systems based on sparsity control of the residual model is presented. The performance of the proposed method has been compared with several robust estimators in one-unbalance and two-unbalance cases. The experimental studies suggest that the proposed method functions more smoothly compared with LS and robust estimators like M-Huber estimator and GM estimator. The external disturbance and outliers are mainly distributed near the first-order critical speed of the rotor system. The information obtained from the useful information ratio curves could be used to guide the selection of balancing speed and the identification of the critical resonance zone of the rotor system.

Considering a real application condition, the proposed method can be used to improve the precision of unbalance estimation and give some information to simplify the balancing process by analyzing the estimation results and the useful information ratio curves.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was partly supported by the projects of National Natural Science Foundation of China (nos. 51775409, 51365051, and 51421004), the Equipment Pre-Research Sharing Technology and Domain Funds (no. 6140004030116JW08001), and National Key Research and Development Project (2017YFF0210504).

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