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Research Article

Vibration Analysis and Optimization of a Rectangular Plate with Flanging Hyperellipse Cutout

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Vibration analysis and optimization of a rectangular plate with a flanging hyperellipse cutout is investigated in this paper, numerically. In the analysis, finite element method (FEM) is applied to perform parametric studies on various plates in different boundary conditions, addressing the influence of different cutout parameters (area, shape, flanging height, position, and rotation) on the first- and second-order natural frequencies of the rectangular plate and providing references for the optimum frequency design. Then, maximization of frequency or the difference of two consecutive frequencies of the rectangular plate is carried out using Multi-island Genetic Algorithm, aiming to achieve the best dynamic characteristics. The results show that different cutout parameters have great influence on vibration performance of the plate, the existing of the flanging increases the out-of-plane stiffness of the plate. Additionally, the nature frequency of the plate has been improved obviously for different models with the optimal design of the cutout.

1. Introduction

In the actual engineering structure, such as aerospace, ship, and marine, plate structures are considered as major engineering substructures, especially wherever the weight is a main issue. It is often necessary to fix up an appropriate number of cutouts based on experience for structural maintenance, weight reduction, or performance optimization. On one hand, stress concentration occurs around the cutouts, and the static strength of the component is reduced; on the other hand, the quality and stiffness distribution of the structure will change for the existing cutouts, which have a great influence on the mechanical properties such as natural frequency and structural load-bearing performance of the structure. Therefore, analysis of the mechanical properties of the structure with cutouts has been a hot issue in the research, which is of great significance in the engineering structure design.

Many studies [1–3] have been conducted on the stress, deformation, vibration, and buckling characteristics of structures with different type of cutouts (square, circular, elliptic, and triangular shapes). In some engineering structures, the plate structure may be subjected to in-plane compressive loading, in which case the buckling phenomenon becomes a critical design criterion. For a rectangular plate with cutout, Komur et al. [4] presented a buckling analysis of a woven-glass-polyester laminated composite plate with a circular/elliptical cutout. The results show that buckling loads are decreased by increasing both c/a and b/a ratios. The increase of cutout positioned angle causes decrease of buckling loads. Ovesy and Fazilati [5] investigated the effects of cutouts on the buckling critical stresses as well as natural frequencies of a plate with central cutout. A numerical study was conducted using the finite element method to determine the effects of square and rectangular cutouts on the buckling behavior of a 16-ply quasi-isotropic graphite/epoxy symmetrically laminated rectangular composite plate [6]. Also, the natural frequencies of every structural element are of great significance in order to avoid the so-called resonance conditions. The influences of a super-ellipse cutout on the frequency of a rectangular plate or composite laminate plate were examined by Wang et al. [7, 8]. Numerical results show that the plate frequencies are highly dependent upon the shape, size, position, and orientation of the cutout. Moreover, the effect is also highly related to the boundary conditions of the plate. Frequency optimization of the plate with cutout is also a concerned research topic. Wang and Wu [9] presented an effective numerical technique for determining the optimal location of a cutout in rectangular plates for maximum fundamental frequency of vibration. The Ritz method was employed for the vibration analysis, and generalized reduced gradient (GRG) method is used to determine the optimal values of location coordinates of the cutout. Pedersen [10] studied the optimization of a cutout of given area which is placed in the interior of a plate with an arbitrary external boundary. Optimal designs were obtained iteratively using mathematical programming, where each of the redesigns is based on finite element (FE) analysis and sensitivity analysis. Wang and Yu [11, 12] carried out maximization of eigenfrequency of a rectangular plate with a parameterization ellipse cutout, and the cutout shape optimization is performed by using Genetic Algorithm and a combined mathematical programming algorithm, respectively.

When punching cutouts on the structure in practice, there will be a certain height of flanging around the cutouts simultaneously. Few studies about the influence of flanging cutouts on structural dynamic performance have been reported, and the author of this paper has firstly studied the influence of different parameters of circular [13] and circle-rectangular [14] cutout on the vibration frequency of a rectangular plate since 2015. Considering an effective weight loss of structure and combining with shape optimization of cutout, this paper focuses on the influence analysis and optimization of structural dynamic performance of a rectangular plate with flanging hyperellipse cutout, and the boundary of the cutout is described with several parameterization parameters, including the possibility of going from an ellipse to a rectangle or even to a triangle. In this paper, the influence of different cutout parameters (area, shape, flanging height, position, and rotation) on the first and second order natural frequencies of the rectangular plate in different boundary constraints will be studied firstly, providing references for the optimum frequency design of the structure. Then, the optimal design of the rectangular plate is carried out under the constraint condition of cutout area unchanged. The first and the second natural frequency or the difference between the first two natural frequencies is maximized through optimization process, aiming to achieve the best dynamic characteristics.

2. Model of Rectangular Plate with Flanging Hyperellipse Cutout

As shown in Figure 1, a flanging ellipse cutout is opened in the center of a rectangular thin plate. In order to reduce the stress concentration, the cutout boundary must be smooth enough, which is described by hyperelliptic equation:

$$\left(\frac{x}{a}\right)^{\eta} + \left(\frac{y}{b}\right)^{\eta} = 1, \tag{1}$$

where a and b are half axle length of the ellipse along x, y axis directions, respectively, and η is an exponential of hyperelliptic equations, for example, ellipse ($\eta = 2$) or rectangular ($\eta = 5$). In order to describe the location and rotation at the same time, the cutout boundary is described in parameter form:

$$\begin{cases} x = x_0 + a\cos(\theta)\cos(t)^{2/\eta} - b\sin(\theta)\sin(t)^{2/\eta}, \\ y = y_0 + a\sin(\theta)\cos(t)^{2/\eta} + b\cos(\theta)\sin(t)^{2/\eta}, \end{cases} \tag{2}$$

where (x_0, y_0) represents the center coordinates of the ellipse and $t \in (0, 2\pi)$ is auxiliary parameter. Area of the cutout is

$$A_c = 4\int_0^a b\left(1 - \left(\frac{x}{a}\right)^{\eta}\right)^{1/\eta} dx = \frac{2ab}{\eta} B\left(\frac{1}{\eta}, \frac{1}{\eta}\right), \tag{3}$$

where $B(1/\eta, 1/\eta)$ is a beta function, when $\eta = 2$, $B(1/2, 1/2) = \pi$. Then, the cutout rate of the plate is defined as

$$\phi = \frac{2ab}{LW\eta} B\left(\frac{1}{\eta}, \frac{1}{\eta}\right). \tag{4}$$

Considering that the boundaries of the rectangular plate can be clamped (C), simple support (S), or free (F), we use 4 letters to describe the boundary support situation, beginning from the left edge ($x = -L/2$) and according to the order of counterclockwise. Suppose the size of the rectangular plate is 450 (L) × 300 (W) × 2 mm, elastic modulus of the material is 70 GPa, Poisson's ratio is 0.3, and mass density is 2700 kg/m^3.

3. Frequency Analysis

Influence of different cutout parameters (area, shape, flanging height, position, and rotation) on the first (f_1) and second (f_2) order natural frequencies of the rectangular plate in different boundary constraints are analyzed in this section. During the analysis, ANSYS, which is known as general purpose finite element software, was preferred as numerical tool, and SHELL181 element type was used to produce for mesh structure.

In this paper, the first-order vibration frequency (f_1) and the second-order vibration frequency (f_2) are the first two frequencies of the structural vibration, respectively, in the order of frequency values from small to large. Figures 2 and 3, respectively, show the oscillatory process of the first two modes of the plate with a central flanging ellipse cutout under the CFFF constraint, with 10 frames in 0.5 seconds, and the initial situation is not displayed. As can be seen from the figures, the first-order frequency is a bending vibration around the y-axis, and the second-order frequency is a torsional vibration about the x-axis. The blue to red color in the figures indicates the amplitude of different points on the board during the corresponding vibration, with blue being zero and red being the largest value. Moreover, it is found that the first two-order vibration modes have not changed with the variations of structural parameters, which is

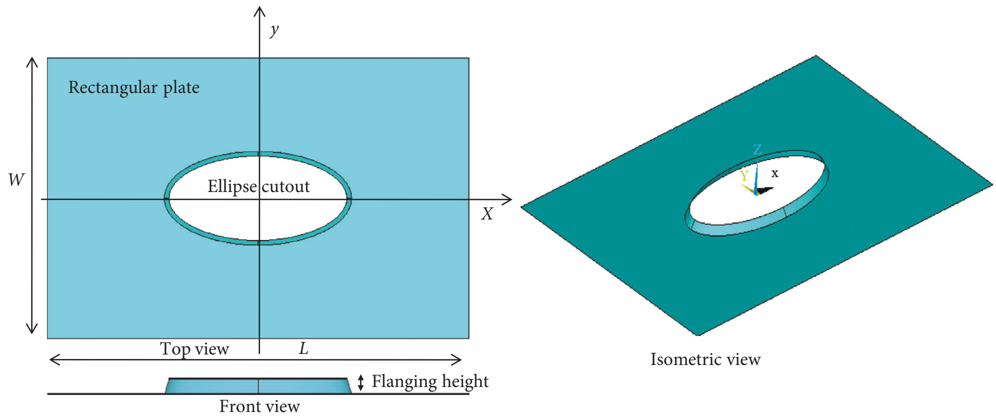


FIGURE 1: A rectangular plate with a central flanging ellipse cutout.

also applicable to the vibration of the structure under the remaining boundary constraints.

3.1. Influence of Cutout Area on the Frequency. Influence of a central flanging ellipse cutout area on the frequency is studied firstly. The half axle length a of the ellipse is constant, and b changes with different cutout area. The flanging height h is 6 mm. Table 1 shows the first two natural frequencies of the rectangular plate in CFFF. For comparison, the natural frequency of the plate without cutout in CFFF is also simulated. From Table 1, the results show that the natural frequency of the plate has changed with various cutout areas. Comparing with the natural frequency of the rectangular plate without cutout, the first-order frequency increases continuously with the increase of the cutout area until $\phi = 0.14$ and then decreases; however, the value is still greater than the case of without cutout. For the second-order frequency, the frequency increases with a small cutout area, then continues to decrease with the increase of the cutout area, and there will be cases that the frequency will be lower than the case of without cutout. The corresponding vibration mode remains unchanged with the increase of the cutout area.

Figure 4 shows the variation of the first two natural frequencies of the plate with the increase of cutout rate under other different boundaries. With the increase of the cutout rate, it can be seen that the first two frequencies increase continuously under different boundary constraints and are all larger than the corresponding natural frequencies without cutout. The influence of flange cutout on the frequency of the plate is also related to the boundary conditions, and the influence of CCCC constraint is the highest.

3.2. Influence of Ellipse Shape on the Frequency. Assuming that the central ellipse cutout has the same cutout rate (eg., $a = 100$ mm, $b = 50$ mm, $\phi = 0.116$), the influence of ellipse shape on the frequency is investigated. The two half axle length a and b can take different values, which are not independent and must satisfy Equation (4). And the flange height $h = 6$ mm.

Table 2 lists the variation of the first- and second-order natural frequency results of the clamped-free plate. With the increase of a and decrease of b gradually, the first-order frequency increases continuously, and the second-order frequency decreases firstly, and then significantly increases when $a = 125$ mm and $b = 40$ mm.

Figure 5 plots variations of the natural frequencies with a central ellipse cutout of different shape under different boundary constraints. With the increase of a and decrease of b gradually, it can be found that the first-order frequency decreases and the second-order frequency increases for most boundary constraints, in addition to CFCF boundary, the results of which are contrary. When $a = 125$ mm and $b = 50$ mm, the second frequency under CSCS suddenly dropped, which is needed for further consideration.

3.3. Influence of Flanging Height on the Frequency. To study the influence of different flange height on the frequency of the plate, the cutout area is kept unchanged ($a = 100$ mm, $b = 50$ mm), and the cutout location is still located in the center.

Table 3 shows the variations of the first two frequencies with the change of the flanging height. From Table 3, it can be seen that the changes of the flanging height have a large influence on the frequency of the CFFF rectangular plate. As the flanging height increases, the first two frequencies of the rectangular plate are continuously increasing. Comparing with the corresponding frequency of the plate without cutout (see the first row of Table 1), it can be found that when the flanging height is small, the frequency value of the plate is smaller than that of the plate without cutout. When the flange height increases to a certain extent, the frequency value of the plate is larger than that of the plate without cutout. As shown in Table 3, the first-order frequency value is greater than the corresponding noncutout condition when $h \geq 2$ mm. For the second-order frequency, the flange height $h \geq 10$ mm is required.

The vibration frequency results under different constraints are shown in Figure 6. As can be seen from Figure 6, with the increase of the flange height, the first two-order frequency of the rectangular plate shows an overall trend of increasing. Comparing with the non-flanged plate ($h = 0$ mm), the first two-order vibration

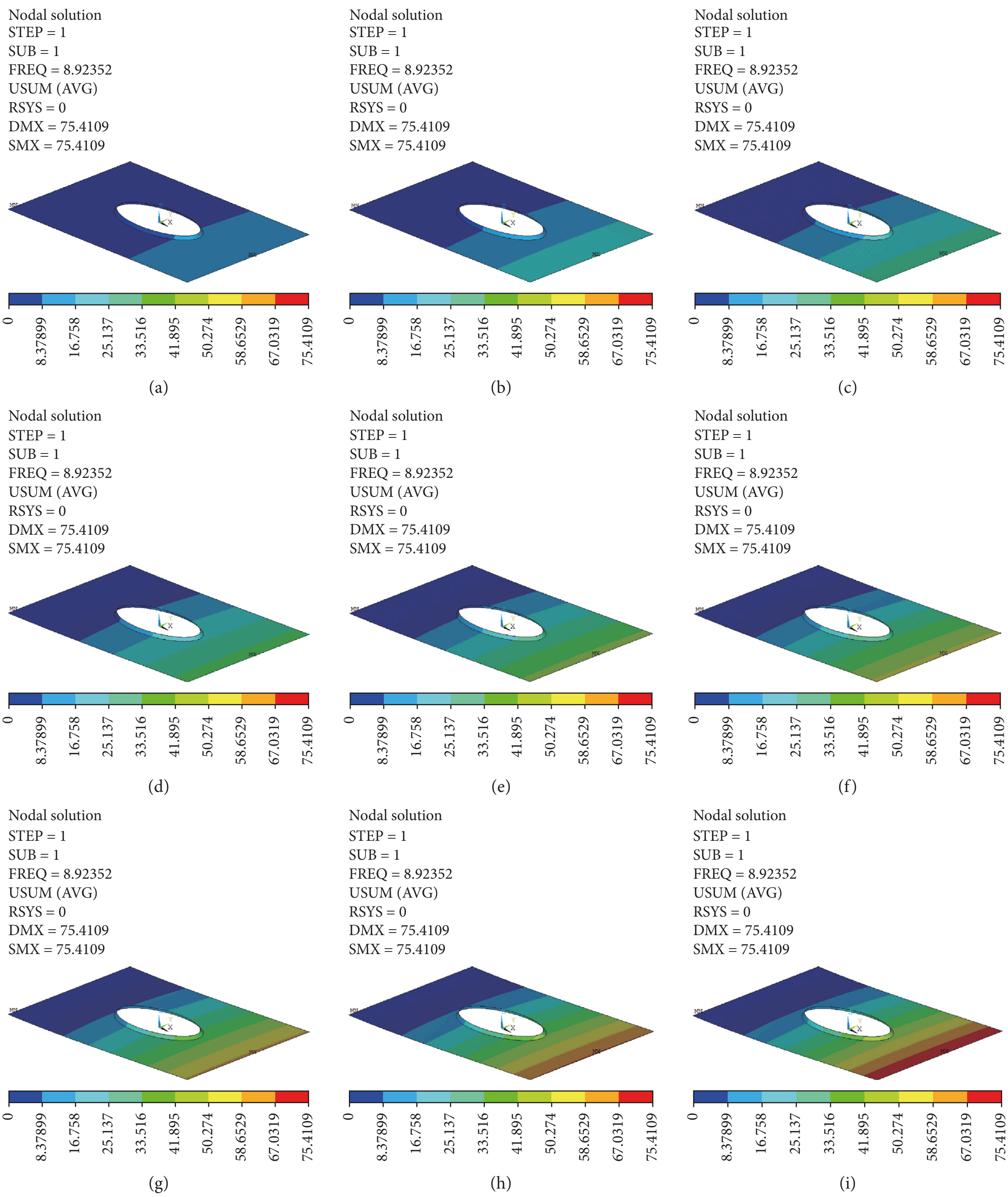


Figure 2: Oscillatory process of the plate for the first frequency in CFFF. (a) Time = 0.05. (b) Time = 0.1. (c) Time = 0.15. (d) Time = 0.2. (e) Time = 0.25. (f) Time = 0.3. (g) Time = 0.35. (h) Time = 0.4. (i) Time = 0.45.

frequency of the flanged plate is larger than that of the nonflanged plate. Similar to the ribbed stiffeners, the out-of-plane stiffness of the rectangular plate is increased by the presence of flanging, whereas the first two-order vibrations of the rectangular plate mainly show out-of-plane vibrations. At the same time, it is noted that when the height of the flange reaches a certain value, the frequency of the rectangular plate is not obviously changed, indicating that the influence of the flange height on the frequency is decreasing.

3.4. Influence of Cutout Location on the Frequency. The influence of cutout location on the frequency of the rectangular plate is studied, assuming that the position of the cutout center moves in the positive direction of x-axis. The

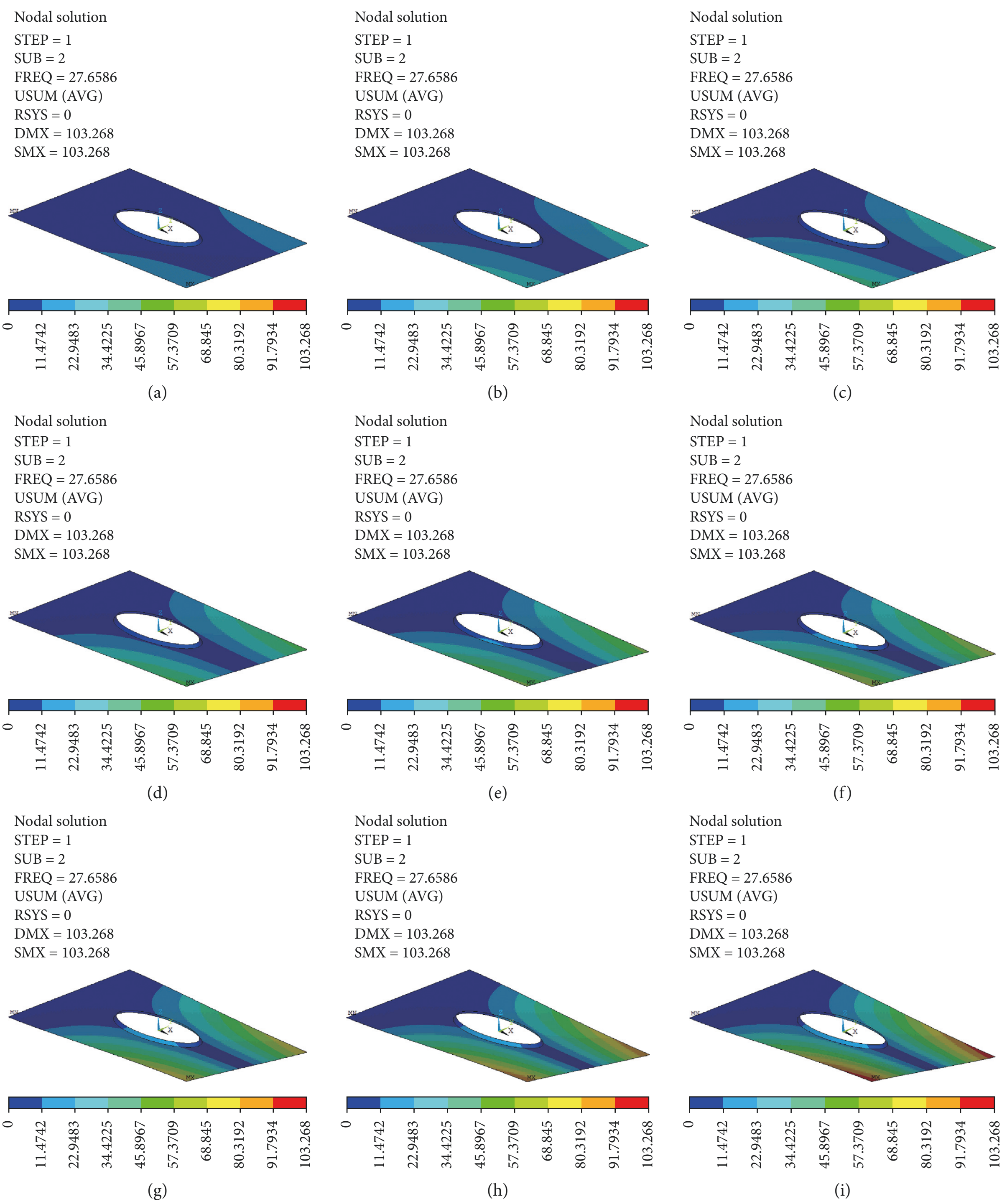


FIGURE 3: Oscillatory process of the plate for the second frequency in CFFF. (a) Time = 0.05. (b) Time = 0.1. (c) Time = 0.15. (d) Time = 0.2. (e) Time = 0.25. (f) Time = 0.3. (g) Time = 0.35. (h) Time = 0.4. (i) Time = 0.45.

cutout area is also kept unchanged ($a = 100$ mm, $b = 50$ mm), and the flange height $h = 6$ mm.

For the CFFF plate, changes of the first two-order natural frequencies with the position of the cutout center are shown in Table 4. As the central position of the cutout moves along the positive direction of the x-axis, the mass distribution of the clamped end increases, and the frequency of the rectangular plate should continue to increase for a cantilever structure, which is consistent with the data listed in Table 4.

It can be seen from Figure 7 that the first-order frequency shows a decreasing trend as the position of the opening moves in the positive direction of the x-axis, and the second-order frequency shows a trend of increasing. For the

Table 1: Influence of central cutout area on the first two frequencies of clamped-free plate.

a (mm)	b (mm)	ϕ	Natural frequency (Hz)	
			f_1	f_2
0	0	0	8.364	28.204
100	20	0.047	8.840	28.408
100	30	0.070	8.882	28.130
100	40	0.093	8.909	27.873
100	50	0.116	8.923	27.399
100	60	0.140	8.925	27.478
100	70	0.163	8.911	27.325
100	80	0.186	8.879	27.200
100	90	0.209	8.826	27.101

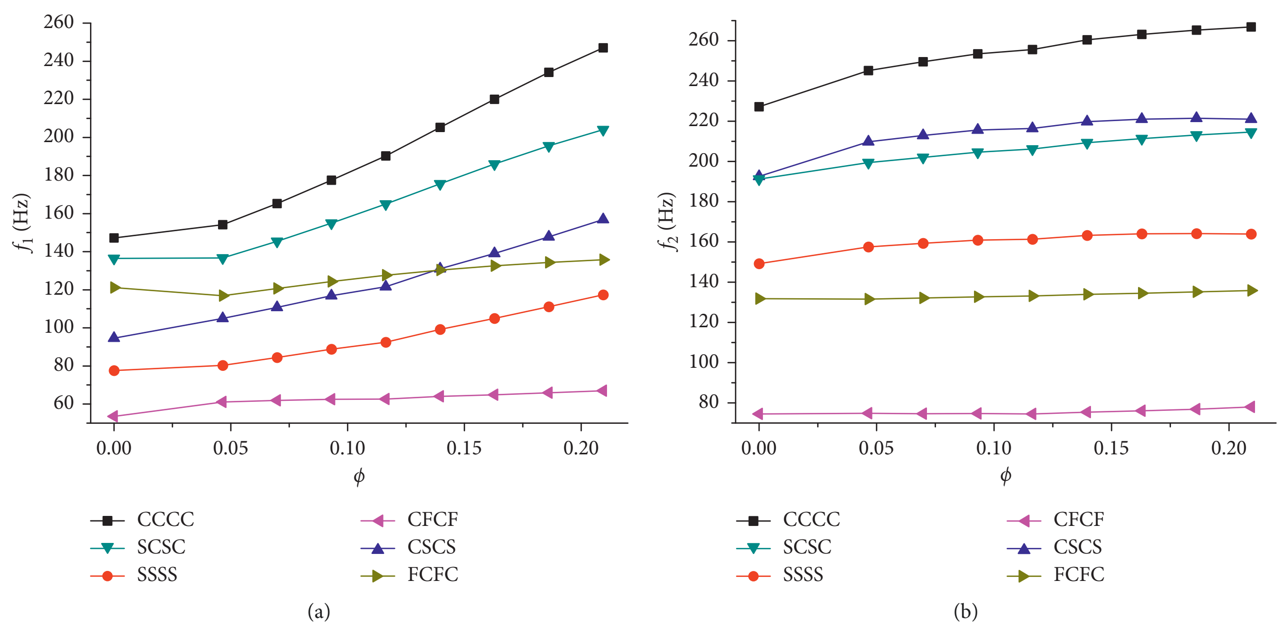


Figure 4: Variations of the natural frequencies of the plate supposed differently with various cutout areas.

Table 2: Influence of ellipse shape on the first two frequencies of clamped-free plate.

a (mm)	b (mm)	Natural frequency (Hz)	
		f_1	f_2
50	100	8.2109	27.831
55.6	90	8.3417	27.765
62.5	80	8.4665	27.709
71.4	70	8.5948	27.666
83.3	60	8.7387	27.647
100	50	8.8815	27.399
125	40	9.2156	27.718

CFCF and FCFC constraints, the change of the first two natural frequencies is insignificant.

3.5. Influence of Cutout Rotation on the Frequency. The rectangular plate with a central ellipse cutout (a = 100 mm, b = 50 mm, h = 6 mm) is used to study the influence of counterclockwise turning of the cutout on the frequency.

The influence of the rotation angle θ on the natural frequency of the rectangular plate in CFFF is shown in Table 5. The change of the rotation angle θ has a distinctly different effect on the first two-order frequencies. The first order natural frequency reaches maximum at $\theta = 0^\circ$ and then decreases gradually, while the second-order natural frequency shows the trend of increasing first, then decreasing, and then increasing again.

The first-order vibration frequency shows an increasing trend with the angle increasing for most boundary constraints; however, the result in CFCF constraints is decreasing, as shown in Figure 8. The second-order frequency decreases with the increase of the angle under the constraints of CCCC, SSSS, CSCS, and SCSC. The other two types of boundary conditions have little influence on the frequency variations.

3.6. Influence of Cutout Shape on the Frequency. It is assumed that there are different shapes of hyperellipses cutout in the center of the rectangular plate, considering the hyperelliptic equations exponential η = 1.5–5.0, respectively, and taking a = 100 mm, b = 50 mm, and h = 6 mm.

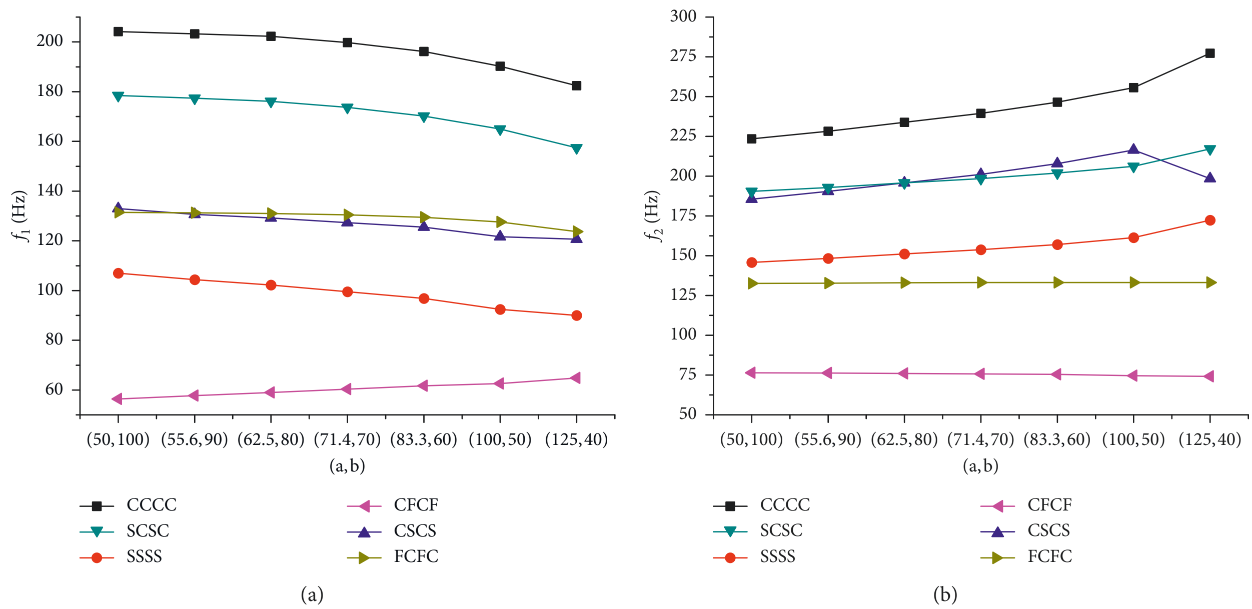


Figure 5: Variations of the natural frequencies with a central ellipse cutout of different shapes.

Table 3: Influence of flanging height on the first two frequencies of clamped-free plate.

h (mm)	Natural frequency (Hz) f_1	f_2
0	8.1411	25.851
2	8.3991	26.528
4	8.7273	27.153
6	8.8815	27.399
8	9.0155	28.044
10	9.0552	28.379

Table 6 lists the changes of the first two natural frequencies with exponential variation of the hyperelliptic equation for the CFFF-supported rectangular plate. As the shape of the cutout changes from an ellipse to a rectangle, the opening rate gradually increases. The first natural frequency gradually increases, and the second natural frequency gradually decreases.

Variations of the natural frequencies of the rectangular plate with various η are shown in Figure 9. The first-order natural frequency is gradually increased with the increase of exponential. The second-order natural frequency is also gradually increased with the increase of exponential under CCCC, SSSS, and SCSC constraints. For the CSCS boundary, the second-order frequency increases when value of η is small ($\eta \leq 2.5$) and then decreases when η becomes large. The other two types of boundary CFCF and FCFC have little influence on the frequency variations.

4. Frequency Optimization

In this paper, optimal frequency design of the rectangular plate with cutout is processed by using Multi-island Genetic Algorithm in ISIGHT software, keeping constant cutout area. From Equation (3), the size of the cutout area is related to parameters of a, b, and η. The location, rotation of the cutout, and flanging height are also taking into account in optimization process for the impact of frequency, expanding the optimization range. Parameters configuration of Multi-island Genetic Algorithm adopt the default mode in ISIGHT.

The natural frequency of the rectangular plate with cutout is maximized through the optimization process, with design variables of parameters a, b, η, x_0, y_0, θ in Equation (2) and flanging height h. Under the constant cutout area condition, the design variable b is calculated by Equation (3). If the design variables a, b, and η appear in the code string at the same time, it is not guaranteed that the constraint of the cutout area can be satisfied after the crossover and mutation operation. The optimization mathematical model is expressed as

$$\begin{aligned} &\max \quad f, \\ &\text{s.t.} \quad A = A_0, \\ &\qquad |x_0| < \frac{L}{2} - d_x - \delta, \\ &\qquad |y_0| < \frac{W}{2} - d_y - \delta. \end{aligned} \tag{5}$$

In Equation (5), f represents the first-order natural frequency f_1, the second-order frequency f_2, or the difference between the first- and second-order frequencies $(f_2 - f_1)$, respectively; A_0 is a given cutout area; d_x and d_y are the largest dimensions of the cutout boundary along the x-axis and y-axis, respectively; and δ is the minimum allowed distance between the cutout boundary and the plate edge, preventing the cutout if it is too close to the edge of the plate and affecting the accuracy of frequency calculation.

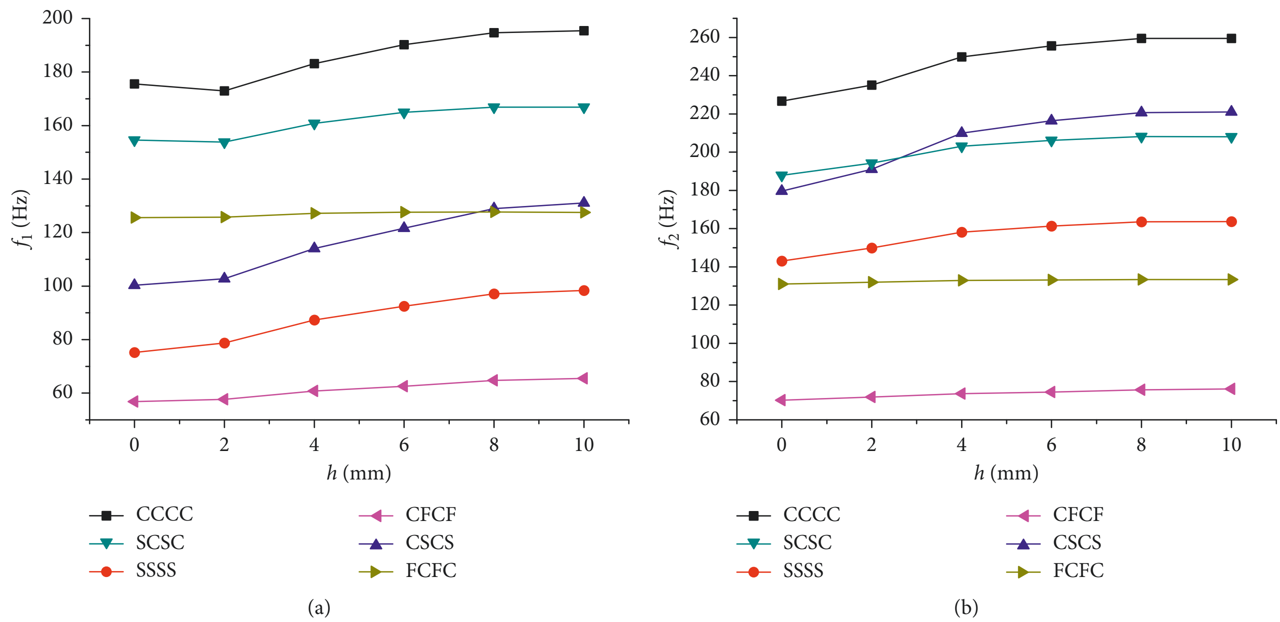


Figure 6: Variations of the natural frequencies of a rectangular plate supposed differently with various h.

Table 4: Influence of cutout location on the first two frequencies of clamped-free plate.

x (mm)	Natural frequency (Hz)	
	f_1	f_2
0	8.8815	27.399
15	8.9030	27.687
30	8.9002	27.721
45	8.9146	27.758
60	8.9456	27.799
75	8.9933	27.840

Frequency optimization of the rectangular plate in CCCC boundaries is investigated in this paper, requiring a flanging cutout design in the interior of the plate. Assuming $\delta = 25$ mm and $A_0 = 75^2\pi\,\text{mm}^2$, the first-order frequency f_1 is maximized (Model 1). When there is no cutout in the plate, the first natural frequency is 147.16 Hz. A flanging circular cutout with 75 mm radius (as indicated by the dashed line in Figure 10) and 6 mm flanging height is opened in the center of the plate firstly as the initial optimization condition; the first-order natural frequency reaches 206.49 Hz. After the optimization design (shown by the solid line in Figure 10), the first-order frequency reached 210.53 Hz, which is 4.04 Hz higher than that before optimization.

Sometimes it is desirable to control the second-order natural frequency of the rectangular plate. For this purpose, the parameters of the cutout of the rectangular plate are reoptimized, aiming to maximize the second-order natural frequency (Model 2). Keeping $\delta = 25$ mm and $A_0 = 75^2\pi\,\text{mm}^2$ unchanged, the natural frequency of the plate with a central circular cutout with a 75 mm radius (shown in dashed line in Figure 11) and 6 mm flanging height is 241.49 Hz. After optimization (as shown by the solid line in Figure 11), the second-order natural frequency of the plate can reach 313.79 Hz, increased by 72.3 Hz; the optimization effect is obvious. The optimization process is shown in Figure 12.

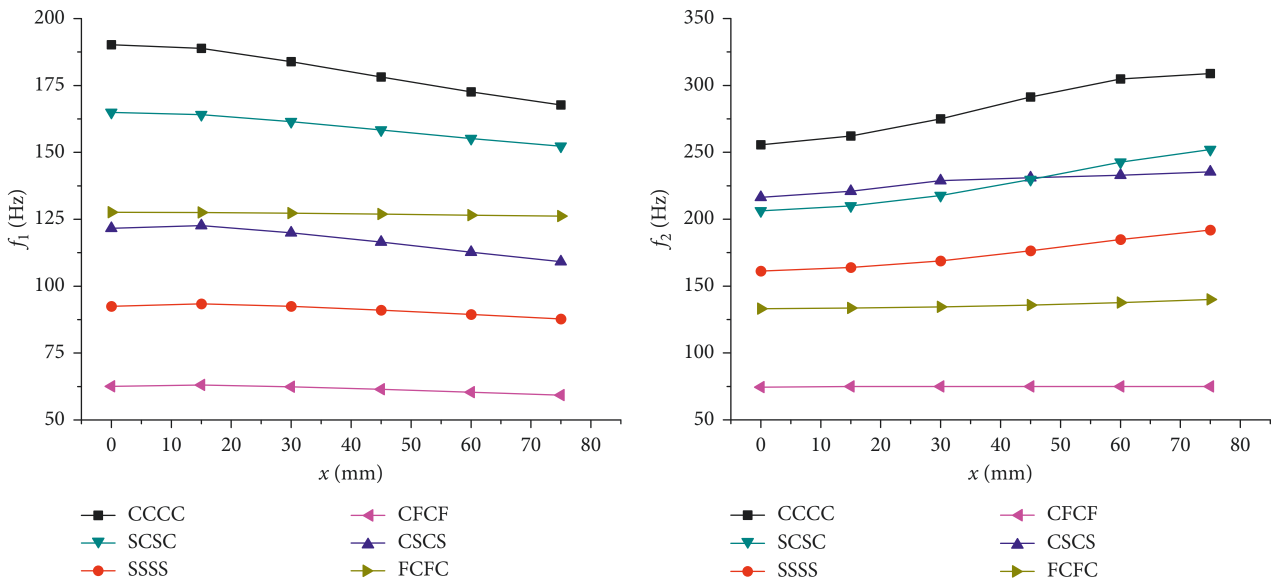
In addition, the difference of the first and second natural frequencies of the plate is also maximized (Model 3). Keeping all parameters same as Model 1 and Model 2, the difference between the two natural frequencies is 35 Hz. After optimization, the difference between the two natural frequencies reached 147.53 Hz, increased by 112. 53 Hz. Shapes of Model 3 before and after optimization are given in Figure 13.

If the cutout area of Model 1 is increased to $A_0 = 100^2\pi\,\text{mm}^2$, the other parameters keep unchanged (Model 4). A central flanging circular cutout is opened in the plate, with 100 mm radius and 6 mm flanging height. The second natural frequency is 267.2 Hz, which can reach 364.86 Hz after the optimization, and the optimization effect is quite obvious. From the optimization results of Model 2 and Model 4, it can be seen that the second-order natural frequency of the plate increases with the cutout area. At the same time, the optimization design of the cutout is more obvious. Figure 14 gives the shapes of Model 4 before and after optimization.

The optimal parameters of different models are given in Table 7.

5. Conclusions

The influence of a flanging ellipse cutout on the vibration performance of a rectangular plate is studied firstly in this paper. The results show that the cutout area, shape, position, rotation, flanging height, and the boundary conditions have great influence on vibration performance of the plate, the existing of the flanging increases the out-of-plane stiffness

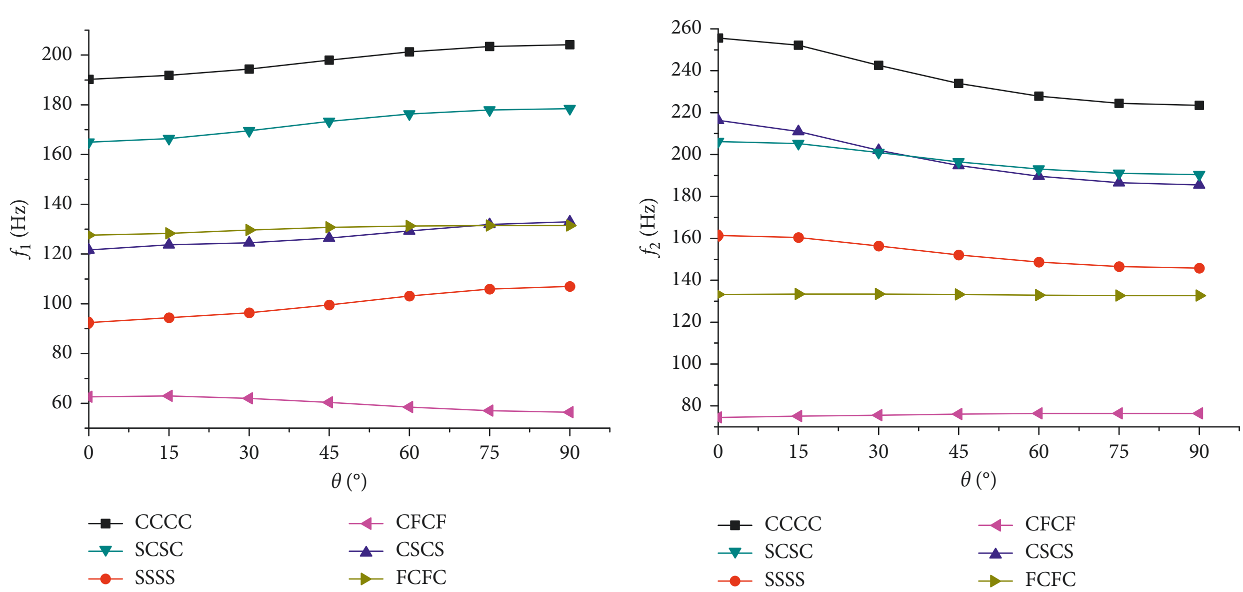


(a) (b)

Figure 7: Variations of the natural frequencies of a rectangular plate supposed differently with various x.

Table 5: Influence of cutout rotation on the first two frequencies of clamped-free plate.

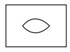
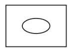
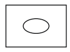
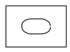
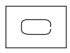
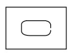
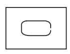
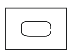
θ (°)	Natural frequency (Hz)	
	f_1	f_2
0	8.8815	27.399
15	8.876	27.609
30	8.7464	27.507
45	8.5695	27.466
60	8.3915	27.560
75	8.2598	27.738
90	8.2109	27.831



(a) (b)

Figure 8: Variations of the natural frequencies of a rectangular plate supposed differently with various θ.

Table 6: Influence of η on the first two frequencies of clamped-free plate.

η	Cutout shape	ϕ	Natural frequency (Hz) f_1	f_2
1.5		0.101	8.8301	27.842
2.0		0.116	8.8815	27.399
2.5		0.125	8.9092	27.134
3.0		0.131	8.9252	26.949
3.5		0.135	8.9357	26.815
4.0		0.137	8.9425	26.713
4.5		0.139	8.9474	26.634
5.0		0.141	8.9507	26.567

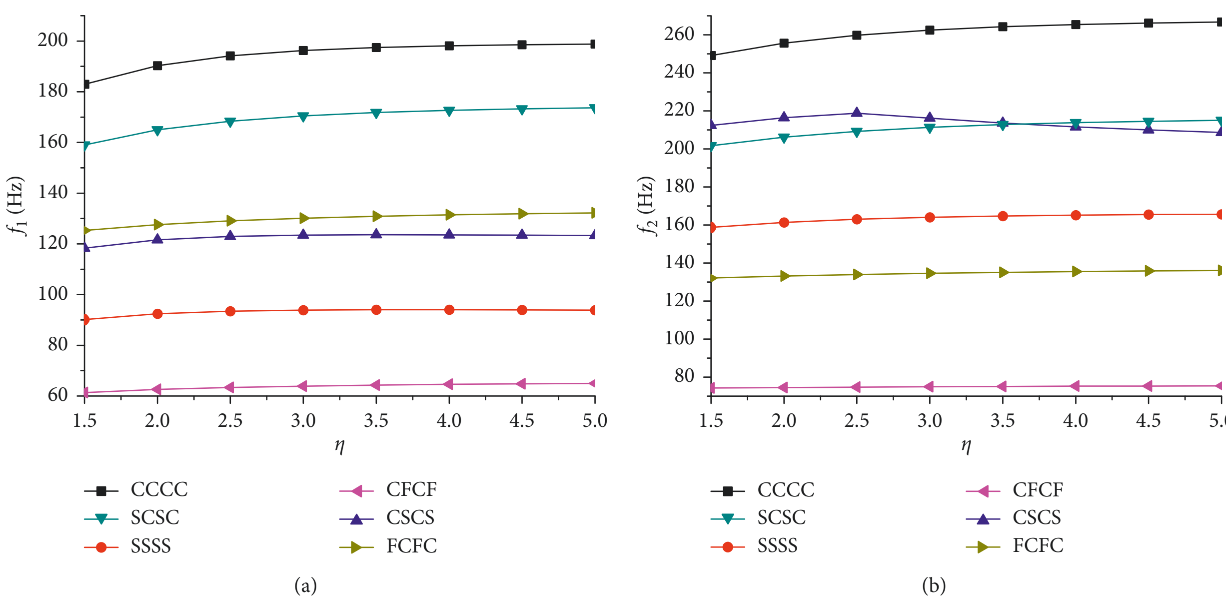


Figure 9: Variations of the natural frequencies of a rectangular plate supposed differently with various η.

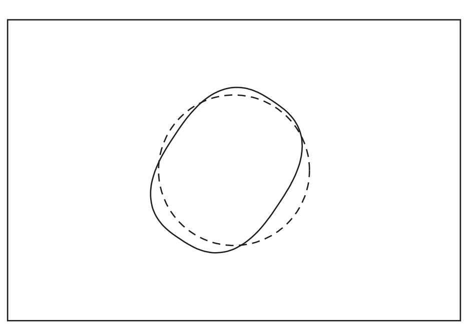


Figure 10: Shapes of Model 1 before and after optimization.

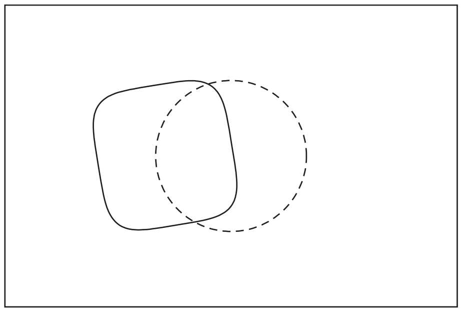


Figure 11: Shapes of Model 2 before and after optimization.

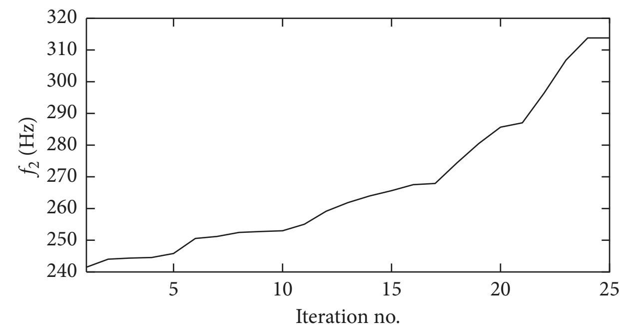


Figure 12: Optimization process of Model 2.

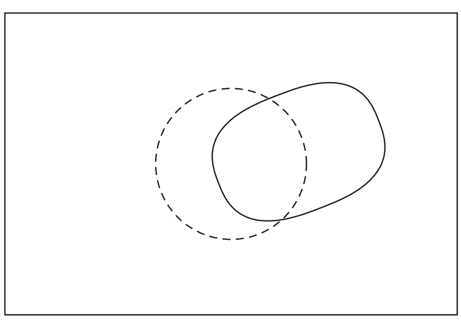


Figure 13: Shapes of Model 3 before and after optimization.

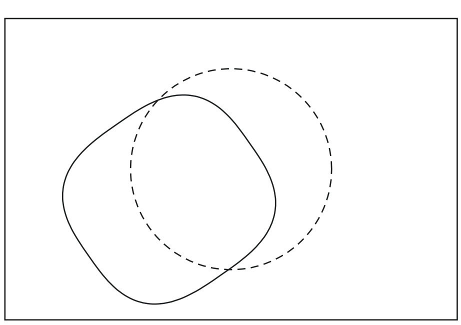


Figure 14: Shapes of Model 4 before and after optimization.

of the plate. Then, the frequency of the plate is optimized by using Multi-island Genetic Algorithm in ISIGHT software, and the natural frequency of the plate has been improved obviously for different models, and increasing

Table 7: Optimal results of a rectangular plate with flanging cutout.

Design parameters	Model 1	Model 2	Model 3	Model 4
A_0 (mm^2)	$75^2\pi$	$75^2\pi$	$75^2\pi$	$100^2\pi$
a (mm)	82.75	67.15	85.48	97.73
b (mm)	61.95	70.28	59.93	91.17
h (mm)	8.28	5.89	9.57	7.98
η	2.69	4.33	2.70	2.97
x_0 (mm)	−4.05	−64.69	67.00	−67.82
y_0 (mm)	6.40	11.32	−12.89	10.56
θ (°)	56.48	9.34	21.08	34.94

of cutout area can also improve the effect of optimal design.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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