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Research Article

Research on the Multilayer Free Damping Structure Design

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The aim of this paper is to put forward a design model for multilayer free damping structures. It sets up a mathematical model and deduces the formula for its structural loss factor η and analyzes the change rules of η along with the change rate of the elastic modulus ratio q_1, the change rate of the loss factors of damping materials q_2, and the change rate of the layer thickness ratio q_3 under the condition with the layer thickness ratio $h_2 = 1, 3, 5, 10$ by software MATLAB. Based on three specific damping structures, the mathematical model is verified through ABAQUS. With the given structural loss factor ($\eta \geq 2$) and the layer number ($n = 3, 4, 5, 6$), 34 kinds of multilayer free damping structures are then presented. The study is meant to provide a more flexible and more diverse design solution for multilayer free damping structures.

1. Introduction

The damping structure is the structure with damping viscoelastic materials attached to mechanical parts to consume vibrational energy. When the machine vibrates, the damping layer bends and vibrates. Consequently, the tension stress and strain are generated within the damping material to dissipate the mechanical energy to achieve the vibration damping effect. Damping structures play a key role in vibration control, shock absorption, and noise reduction. They are commonly used in many domains such as aeronautics, aerospace, and mechanical and civil engineering. So many research works have been done on damping structures.

Moita et al. [1] developed a finite element model for vibration analysis of active-passive damped multilayer sandwich plates with a viscoelastic core sandwiched between elastic layers, including piezoelectric layers. Jin et al. [2] presented a unified formulation for vibration and damping analysis of a sandwich beam made up of laminated composite face sheets and a viscoelastic core with arbitrary lay-ups and general boundary conditions. Li and Narita [3] analyzed and did the optimal design for the damping loss factor of laminated plates under general edge conditions. Berthelot et al. [4] developed a synthesis of damping analysis of laminate materials and laminates with interleaved viscoelastic layers and sandwich materials. Araújo et al. [5] formulated a finite element model by using a mixed layer-wise approach for anisotropic laminated plates with a viscoelastic core. They considered two different deformation theories, and the optimal design and parameter estimation were also discussed. Akoussan et al. [6] proposed a high order continuous sensitivity analysis of the damping properties of viscoelastic composite plates according to their layer thicknesses. Yang et al. [7] presented an accurate solution approach based on the first-order shear deformation theory for the free vibration and damping analysis of thick sandwich cylindrical shells with a viscoelastic core under arbitrary boundary conditions. Chen et al. [8] presented an order-reduction-iteration approach for vibration analysis of viscoelastically damped sandwiches.

In light of earlier studies, it appears that great attention has been paid to investigations pertaining to the sandwich damping structure with viscoelastic materials, namely, the damping structure with the constraint layer. For that structure in vibration, shear deformation occurs between the constraint layer and the damping layer, and the vibration and deformation of the damping layer are limited and affected by the constraint layer, so the mechanical energy of vibration cannot be fully converted into the material internal energy, which influences the vibration damping effect, whereas the multilayer free damping structure in this paper is the structure composed of the base layer and the added multilayers of viscoelastic damping materials with different properties. During vibration, there is no shear deformation among those layers, and the vibration and deformation of each damping layer are not limited or affected by the upper layer, and then more mechanical energy of vibration can be converted into the material internal energy, which contributes to a better vibration damping effect. Moreover, for the multilayer free damping structure, we can choose different damping layer materials according to the characteristics of the base layer and adjust the thickness and the number of damping layers to achieve the desired vibration damping effect.

The following sections will introduce the mathematical model for the damping structure, the derivation of the structural loss factor η, the change rules of η along with the change rate of the elastic modulus ratio q_1, the change rate of the loss factors of damping materials q_2, and the change rate of the layer thickness ratio q_3 and the mathematical model verification. Finally, with the given structural loss factor ($\eta \geq 2$) and the layer number ($n = 3, 4, 5, 6$), 34 kinds of multilayer free damping structures are provided.

2. Multilayer Free Damping Structure Model

When the multilayer free damping beam is bending (as shown in Figure 1), the elongation ε of both the basic layer and the multilayer free damping layers along the longitudinal direction varies linearly with y.

$$\varepsilon = \frac{y\,d\theta}{dx} = \frac{y}{i\omega}\frac{\partial\omega}{\partial x}. \tag{1}$$

The normal tensile stress along the x direction is expressed as

$$\sigma_n = E_n\varepsilon_n. \tag{2}$$

$n = 1, 2, \ldots$ represents the basic layer and the multilayer free damping layers.

Plugging formula (1) into formula (2), the following is gained:

$$\sigma_n = \frac{E_n}{i\omega} y \frac{\partial\omega}{\partial x}. \tag{3}$$

In pure bending, the force acted on the cross section of the beam along the x direction (see Figure 2) is equal to zero.

$$\int_{-(H_1-\zeta)}^{\sum_{a=2}^{n} H_a+\zeta} \sigma\,dy = 0, \tag{4}$$

where H_1 stands for the thickness of the basic layer and H_a ($a = 2, \ldots, n$) is the thickness of the damping layers. ζ is the distance from the center line of the barycenter of the damping beam to the basic layer surface.

The curvature $(1/i\omega)\cdot(\partial\omega/\partial x)$ is a constant. If the above formula is equal to zero, then

$$\int_{-(H_1-\zeta)}^{\sum_{a=2}^{n} H_a+\zeta} Ey\,dy = 0. \tag{5}$$

Formula (5) can be expressed as

$$\sum_{k=2}^{n-1}\int_{\sum_{b=2}^{k} H_b+\zeta}^{\sum_{a=2}^{k+1} H_a+\zeta} \overline{E}_{k+1} y\,dy + \int_{\zeta}^{H_2+\zeta} \overline{E}_2 y\,dy + \int_{-(H_1-\zeta)}^{\zeta} E_1 y\,dy = 0. \tag{6}$$

By formula (6), ζ can be expressed as

$$\zeta = \frac{1}{2}\cdot\frac{E_1H_1^2 - \overline{E}_2H_2^2 - \sum_{k=2}^{n-1}\overline{E}_{k+1}H_{k+1}\left(2\sum_{a=2}^{k} H_a + H_{k+1}\right)}{E_1H_1 + \sum_{a=2}^{n}\overline{E}_aH_a}. \tag{7}$$

After ζ is determined, the bending moment M can be given as

$$M = \int_{-(H_1-\zeta)}^{\sum_{a=2}^{n} H_a+\zeta} y\sigma\,dy. \tag{8}$$

Plugging formula (9) into (8), formula (10) can be gained:

$$M = \frac{\overline{B}}{i\omega}\frac{\partial\omega}{\partial x} \tag{9}$$

$$\begin{aligned}\overline{B} = \int_{-(H_1-\zeta)}^{\sum_{a=2}^{n} H_a+\zeta} Ey^2dy = \frac{E_1H_1^3}{3}\Bigg\{&\sum_{k=2}^{n-1}\overline{e}_{k+1}h_{k+1}\\ &\cdot\Bigg[3\sum_{a=2}^{k}h_a\left(h_a+h_{k+1}\right) + 6\sum_{a=2,b=2,a\neq b}^{k} h_ah_b + h_{k+1}^2\\ &+ 3\zeta\left(h_{k+1} + 2\sum_{a=2}^{k}h_a\right) + 3\zeta^2\Bigg]\\ &+\Bigg[\overline{e}_2\left(h_2^3 + 3h_2^2\frac{\zeta}{H_1} + 3h_2\frac{\zeta^2}{H_1^2}\right) + 1 - 3\frac{\zeta}{H_1}\\ &+ 3\frac{\zeta^2}{H_1^2}\Bigg]\Bigg\}.\end{aligned} \tag{10}$$

Among them, $\overline{e}_a = \overline{E}_a/E_1 = (E_a/E_1)(1+i\beta_a) = e_a(1+i\beta_a)$ and $h_a = (H_a/H_1)$ $(a = 2, \ldots, n)$.

Formula (7) can also be expressed as

$$\zeta = \frac{H_1}{2}\frac{1 - \overline{e}_2h_2^2 - \sum_{k=2}^{n-1}\overline{e}_{k+1}h_{k+1}\left(2\sum_{a=2}^{k}h_a + h_{k+1}\right)}{1 + \sum_{a=2}^{n}\overline{e}_ah_a}. \tag{11}$$

Plugging formula (11) into (10), the following can be given as

$$\begin{aligned}\overline{B} = \int_{-(H_1-\zeta)}^{\sum_{a=2}^{n} H_a+\zeta} Ey^2dy = B_1\Bigg\{&\sum_{k=2}^{n-1}\overline{e}_{k+1}h_{k+1}\\ &\cdot\Bigg[12\sum_{a=2}^{k}h_a\left(h_a+h_{k+1}\right) + 24\sum_{a=2,b=2,a\neq b}^{k} h_ah_b + 4h_{k+1}^2\end{aligned}$$

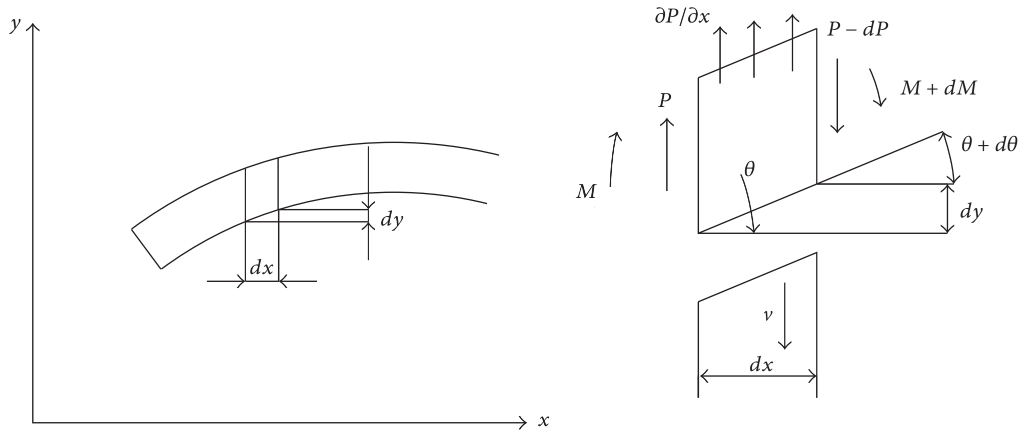


FIGURE 1: The vibration parameter description of the bending beam of the multilayer free damping structure. *Note.* ω is the angular velocity; v is the velocity of transverse vibration; θ is the displacement of the bending angle; P is the transverse force; m is the mass per unit length of beam; M is the bending moment; $\overline{B}$ is the composite bending stiffness.

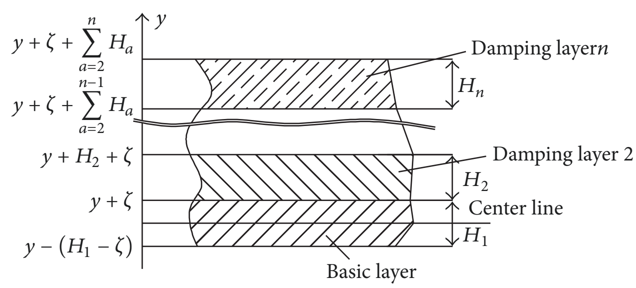


FIGURE 2: The sectional view of the multilayer free damping structure.

$$+ 6H_1 c \left(h_{k+1} + 2\sum_{a=2}^{k} h_a \right) + 3H_1^2 c^2 \Bigg] + \frac{1 + 2\overline{e}_2 \left(2h_2 + 3h_2^2 + 2h_2^3\right) + \overline{e}_2^2 h_2^4}{1 + \overline{e}_2 h_2} \Bigg\}. \quad (12)$$

Among them,

$$c = \frac{1 - \overline{e}_2 h_2^2 - \sum_{k=2}^{n-1} \overline{e}_{k+1} h_{k+1} \left(2\sum_{a=2}^{k} h_a + h_{k+1}\right)}{1 + \sum_{a=2}^{n} \overline{e}_a h_a}, \quad (13)$$

where $\overline{B}$ is the composite bending stiffness of the beam of the multilayer free damping structure and B_1 is the bending stiffness of the beam without damping layers.

Compared with the loss factor β_a of the viscoelastic damping materials of the damping layers, the loss factor β_1 of the basic layer is so small that it can be ignored. And also $\overline{B} = B(1 + i\eta)$, $\overline{e}_a = e_a(1 + i\beta_a)$, $a = 2, \ldots, n$, and η is the structural loss factor of the damping structure. So the following can be gained by plugging them into formula (12):

$$\frac{B}{B_1}(1 + i\eta) = \sum_{k=2}^{n-1} e_{k+1}(1 + i\beta_{k+1}) h_{k+1} \Bigg\{ 12\sum_{a=2}^{k} h_a (h_a + h_{k+1}) + 24 \sum_{a=2, b=2, a\neq b}^{k} h_a h_b + 4h_{k+1}^2$$
$$+ 6H_1 \left(h_{k+1} + 2\sum_{a=2}^{k} h_a \right) \frac{1 - e_2(1 + i\beta_2) h_2^2 - \sum_{k=2}^{n-1} e_{k+1}(1 + i\beta_{k+1}) h_{k+1} \left(2\sum_{a=2}^{k} h_a + h_{k+1}\right)}{1 + \sum_{a=2}^{n} e_a (1 + i\beta_a) h_a}$$
$$+ 3H_1^2 \left[\frac{1 - e_2(1 + i\beta_2) h_2^2 - \sum_{k=2}^{n-1} e_{k+1}(1 + i\beta_{k+1}) h_{k+1} \left(2\sum_{a=2}^{k} h_a + h_{k+1}\right)}{1 + \sum_{a=2}^{n} e_a (1 + i\beta_a) h_a} \right]^2 \Bigg\}$$

$$+\frac{1+2e_2(1+i\beta_2)(2h_2+3h_2^2+2h_2^3)+e_2^2(1+i\beta_2)^2h_2^4}{1+e_2(1+i\beta_2)h_2}. \tag{14}$$

Omitting $\beta_a\beta_b$, the real component can be expressed as

$$\begin{aligned}\frac{B}{B_1} = {} & \sum_{k=2}^{n-1} e_{k+1}h_{k+1}\left[12\sum_{a=2}^{k} h_a(h_a+h_{k+1})\right.\\ & + 24\sum_{a=2,b=2,a\neq b}^{k} h_a h_b + 4h_{k+1}^2\\ & \left.+\frac{3p_1H_1}{p_2^2}\left(2h_{k+1}+4\sum_{a=2}^{k}h_a+\frac{H_1p_1}{p_2^2}\right)\right]\\ & +\frac{1+2e_2(2h_2+3h_2^2+2h_2^3)+e_2^2h_2^4}{1+e_2h_2}.\end{aligned} \tag{15}$$

Among them, $p_1 = p_2[1-e_2h_2^2-\sum_{k=2}^{n-1}e_{k+1}h_{k+1}(2\sum_{a=2}^{k}h_a+h_{k+1})]$ and $p_2 = 1+\sum_{a=2}^{n}e_ah_a$.

Omitting $\beta_a\beta_b$, the imaginary component can be expressed as

$$\begin{aligned} i\frac{B}{B_1}\eta = {} & i\sum_{k=2}^{n-1} e_{k+1}h_{k+1}\beta_{k+1}\left[12\sum_{a=2}^{k}h_a(h_a+h_{k+1})\right.\\ & + 24\sum_{a=2,b=2,a\neq b}^{k} h_ah_b + 4h_{k+1}^2\\ & \left.+\frac{3p_1H_1}{p_2^2}\left(2h_{k+1}+4\sum_{a=2}^{k}h_a+\frac{H_1p_1}{p_2^2}\right)\right]\\ & -\frac{6H_1p_3}{p_2^2}\left[\sum_{k=2}^{n-1}e_{k+1}h_{k+1}\left(h_{k+1}+2\sum_{a=2}^{k}h_a\right)+\frac{H_1p_1}{p_2^2}\right]\\ & + i\beta_2e_2h_2\frac{3+6h_2+4h_2^2+2e_2h_2^3+e_2^2h_2^4}{(1+e_2h_2)^2}.\end{aligned} \tag{16}$$

Among them,

$$\begin{aligned} p_3 = {} & p_2\left[e_2\beta_2h_2^2\right.\\ & \left.+\sum_{k=2}^{n-1}e_{k+1}h_{k+1}\beta_{k+1}\left(2\sum_{a=2}^{k}h_a+h_{k+1}\right)\right]+\left(\sum_{a=2}^{n}e_a\beta_a\right)\\ & \cdot\left[1-e_2h_2^2-\sum_{k=2}^{n-1}e_{k+1}h_{k+1}\left(2\sum_{a=2}^{k}h_a+h_{k+1}\right)\right].\end{aligned} \tag{17}$$

Plugging formula (15) into (16) and omitting e_ae_b, the formula for η can be expressed as

$$\eta = \frac{6H_1(p_6+H_1p_4)+e_2h_2\beta_2(3+6h_2+4h_2^2)}{6H_1p_7+2+2e_2h_2(2+3h_2+2h_2^2)}. \tag{18}$$

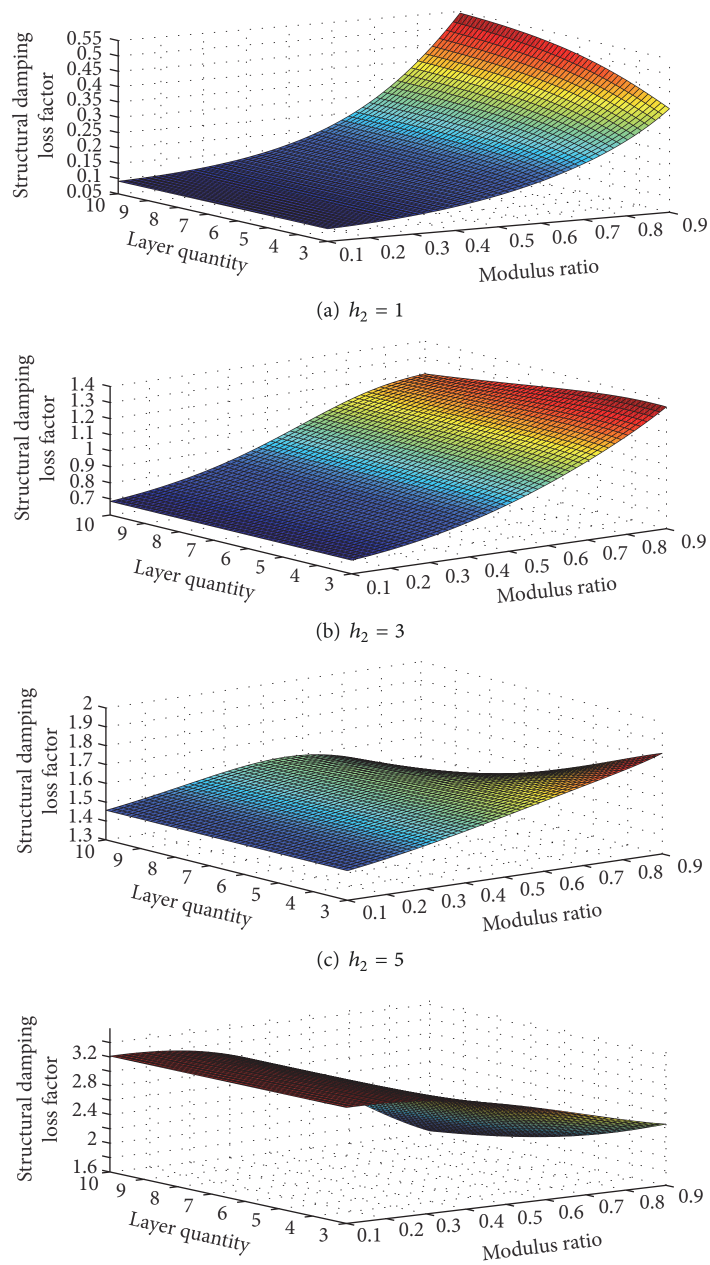
Here $a, b = 2, 3, \ldots, n$.

$$\begin{aligned} p_4 &= e_2\beta_2h_2^2+\sum_{a=2}^{n}e_a\beta_a+p_6.\\ p_5 &= h_{k+1}+2\sum_{a=2}^{k}h_a\\ p_6 &= \sum_{k=2}^{n-1}e_{k+1}h_{k+1}\beta_{k+1}p_5\\ p_7 &= \sum_{k=2}^{n-1}e_{k+1}h_{k+1}p_5\end{aligned} \tag{19}$$

3. Analysis of Influencing Factors of the Structural Loss Factor η

From formula (18), it is known that there are several factors to influence the structural loss factor η. They are the thickness of the basic layer H_1, the elastic modulus ratio of the damping material of each layer $e_a = E_a/E_1$, the layer thickness ratio of each damping layer $h_a = H_a/H_1$, the loss factor of the damping material of each layer β_a. In this damping structure, from the 2nd to the nth damping layer, the elastic moduli of the damping materials have the relation: $E_2 > E_3 > \cdots > E_n$, the layer thicknesses have the relation: $H_2 > H_3 > \cdots > H_n$, and the loss factors of the damping materials have the relation: $\beta_2 > \beta_3 > \cdots > \beta_n$, so there are the relations: $e_2 > e_3 > \cdots > e_n$ and $h_2 > h_3 > \cdots > h_n$. It is assumed that the elastic modulus ratio, the loss factor, and the layer thickness ratio of the damping layers have the relations: $e_n = q_1^{n-2}e_2$, $\beta_n = q_2^{n-2}\beta_2$, and $h_n = q_3^{n-2}h_2$. Those are then plugged into formula (19), and the following formulas are gained:

$$\begin{aligned} p_4 &= e_2\beta_2\frac{h_2^2+(q_1q_2)^2\left[1-(q_1q_2)^{n-2}\right]}{1-q_1q_2}+p_6\\ p_5 &= h_2\left[q_3^k+\frac{2q_3^2(1-q_3^{k-2})}{1-q_3}\right]\end{aligned}$$



(a) $h_2 = 1$

(b) $h_2 = 3$

(c) $h_2 = 5$

(d) $h_2 = 10$

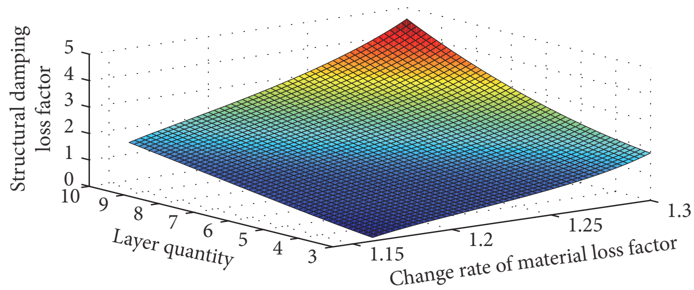
FIGURE 3: The change rule of η along with q_1.

$$p_6 = e_2\beta_2 \left(h_2 q_1 q_2 q_3^2\right)^2 \left\{ \frac{1-\left(q_1 q_2 q_3^2\right)^{n-2}}{1-q_1 q_2 q_3^2} + \frac{2}{1-q_3}\left[\frac{1-\left(q_1 q_2 q_3\right)^{n-2}}{1-q_1 q_2 q_3} - \frac{q_3\left(1-q_1^{n-2} q_2^{n-2} q_3^{2n-5}\right)}{1-q_1 q_2 q_3^2}\right]\right\}$$

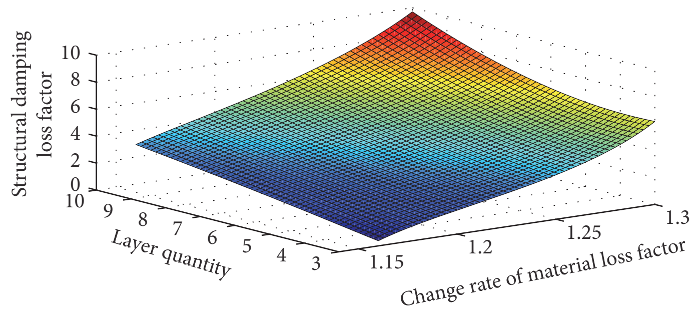
$$p_7 = e_2 \left(h_2 q_1 q_3^2\right)^2 \left\{ \frac{1-\left(q_1 q_3^2\right)^{n-2}}{1-q_1 q_3^2} + \frac{2}{1-q_3}\left[\frac{1-\left(q_1 q_3\right)^{n-2}}{1-q_1 q_3} - \frac{q_3\left(1-q_1^{n-2} q_3^{2n-5}\right)}{1-q_1 q_3^2}\right]\right\}. \tag{20}$$

Plugging formulas (20) into (18), the formula for the structural loss factor η can be worked out. Then it can be analyzed and calculated by software MATLAB.

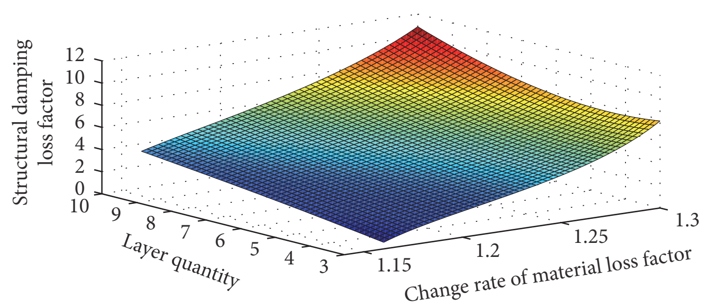
Figure 3 shows the change rule of the structural loss factor η along with the change rate of the elastic modulus q_1 under the condition with the layer thickness ratio $h_2 = 1, 3, 5, 10$ and the layer number $n = 3{\sim}10$. When $h_2 = 1$, η is increasing with



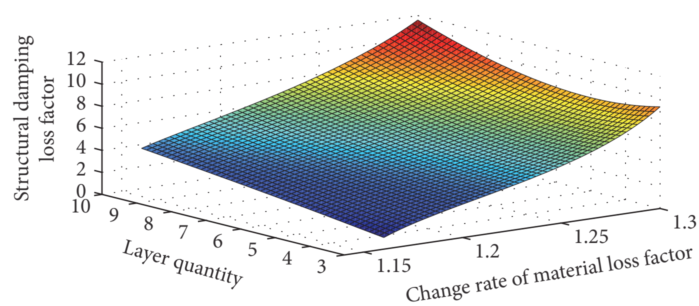
(a) $h_2 = 1$



(b) $h_2 = 3$



(c) $h_2 = 5$



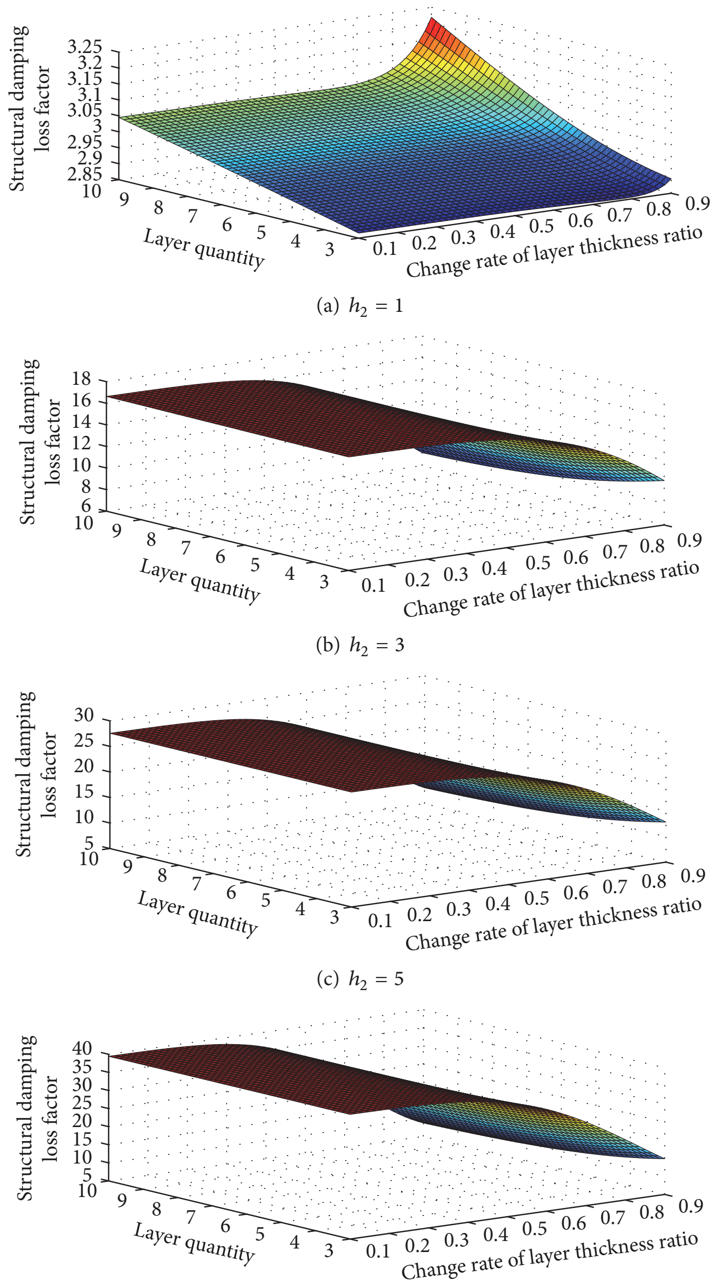
(d) $h_2 = 10$

Figure 4: The change rule of η along with q_2.

q_1, and the more the damping layers are, the more rapidly η increases. When $h_2 = 3$, η is increasing with q_1, and the more the damping layers, the more slowly η increases. When $h_2 = 5$, if $q_1 < 0.5$, η is increasing with q_1; if $q_1 > 0.5$ and $n \leq 5$, η is increasing with q_1, and the more the damping layers, the more slowly η increases; $n > 5$, η is decreasing with q_1, and the more the damping layers, the more rapidly η decreases. When $h_2 = 10$, η is decreasing with q_1, and the more the damping layers, the more rapidly η decreases.

Figure 4 shows the change rule of the structural loss factor η along with the change rate of the loss factors of damping materials q_2 under the same condition. When $h_2 = 1$, η is increasing with q_2, and the more the damping layers, the more rapidly η increases. When $h_2 = 3, 5, 10$, η is increasing with q_2, and, for those damping structures with $n \leq 5$, η increases obviously faster.

Figure 5 shows the change rule of the structural loss factor η along with the change rate of the layer thickness ratio q_3



(a) $h_2 = 1$

(b) $h_2 = 3$

(c) $h_2 = 5$

(d) $h_2 = 10$

FIGURE 5: The change rule of η along with q_3.

under the same condition. When $h_2 = 1$, if $0.1 < q_3 < 0.7$, η is decreasing slowly with q_3; if $0.7 < q_3 < 0.9$, η is increasing rapidly with q_3. When $h_2 = 3, 5, 10$, if $q_3 < 0.5$, η is decreasing slowly with q_3; if $q_3 > 0.5$, η is decreasing rapidly with q_3.

4. Mathematical Model Verification of the Multilayer Free Damping Structure

In Section 3, the change rules of η along with q_1, q_2, and q_3 are analyzed based on the mathematical model which has to be verified. Obviously, since there are a large number of design solutions, it is impossible to evaluate all of them. Therefore, this section will only evaluate three specific damping structures by ABAQUS to verify the model. For the three structures, the length of the basic layer is given as 50 cm, the width as 10 cm, and the thickness as 1 cm. Its material density is given as 7800 kg/m^3. The material parameter uses isotropic material with a Young's modulus of 210 GPa and a Poisson's ratio of 0.3. And they are all with the layer thickness ratio $h_2 = 5$ and the layer number $n = 5$ (as is shown in Figure 6).

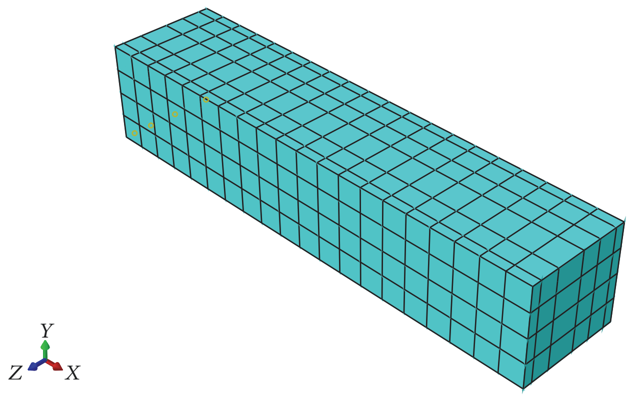
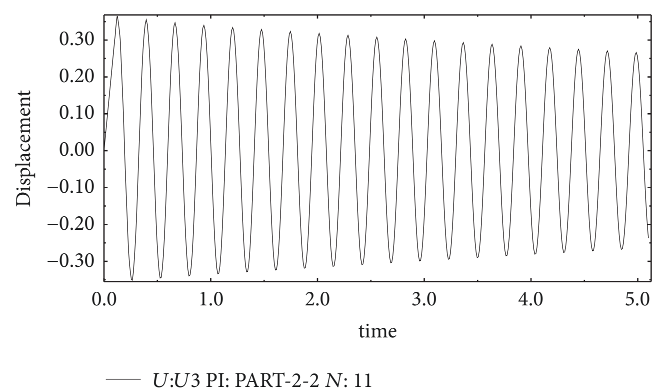
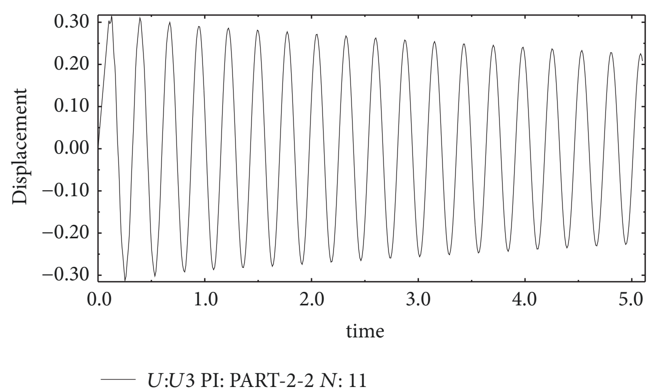


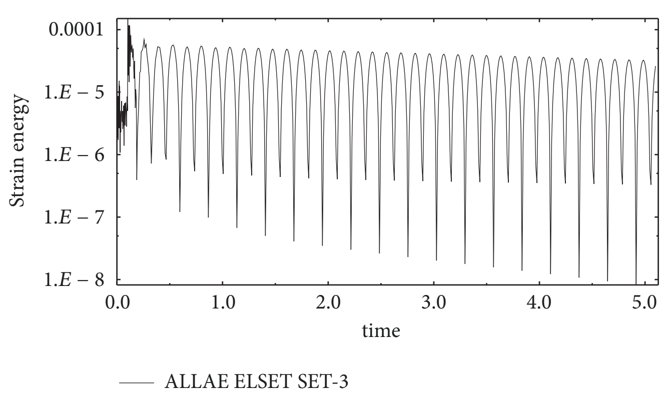
Figure 6: The damping structure.



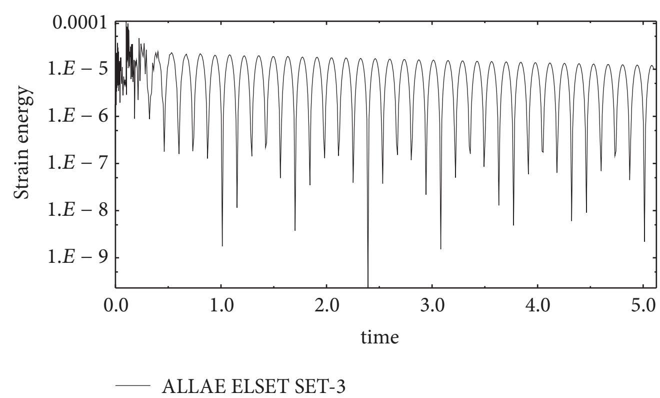
(a) $q_1 = 0.9$ amplitude of vibration



(b) $q_1 = 0.7$ amplitude of vibration



(c) $q_1 = 0.9$ strain energy



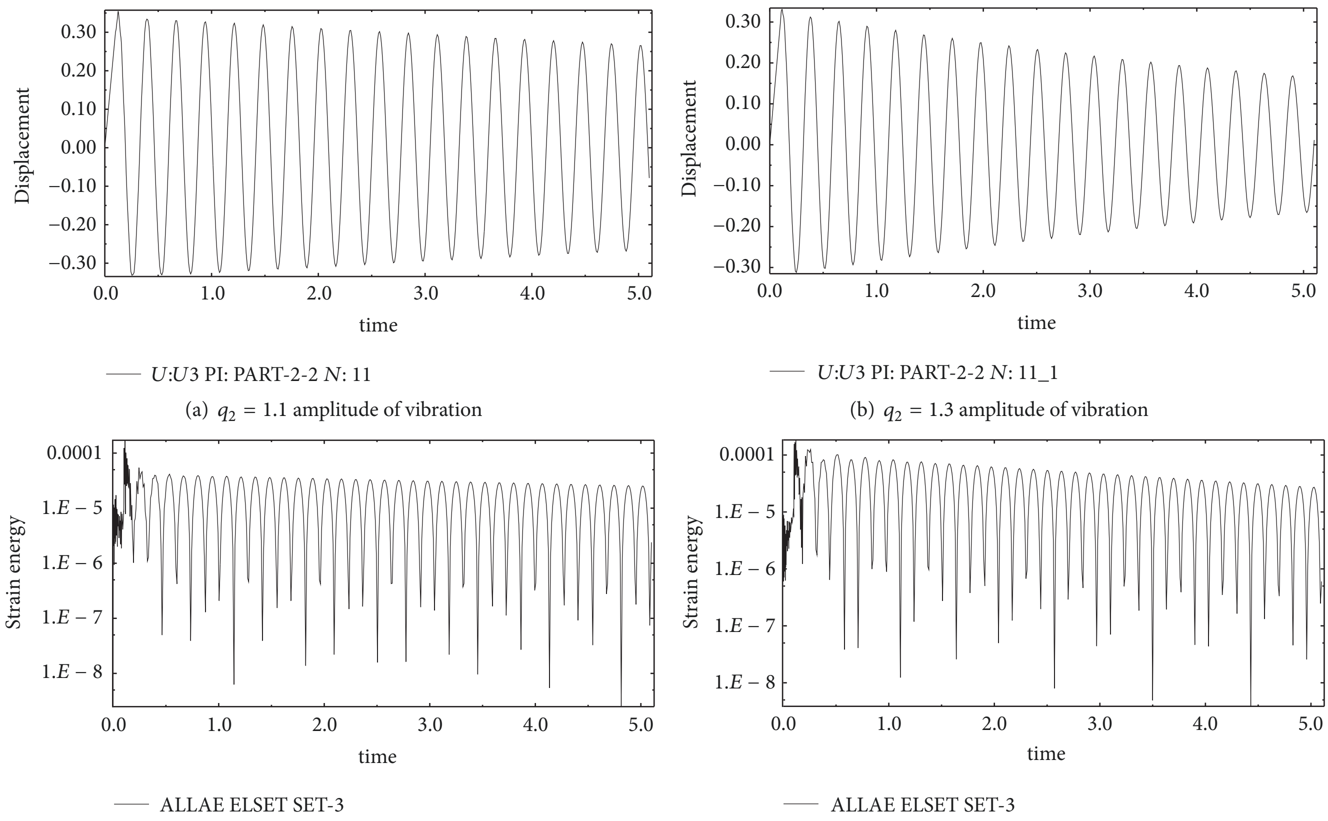
(d) $q_1 = 0.7$ strain energy

Figure 7: The amplitude and strain energy images of the first damping structure.

For the first damping structure, q_2 and q_3 are defined as fixed values and q_1 is defined as variables. For the second damping structure, q_1 and q_3 are defined as fixed values and q_2 is defined as variables. For the third damping structure, q_1 and q_2 are defined as fixed values and q_3 is defined as variables.

4.1. Finite Element Analysis for the First Damping Structure. The results of the finite element analysis of the first damping structure are shown in Figure 7. We set q_2 and q_3 as 1.1 and 0.9, respectively. If $q_1 = 0.9$, its amplitude of vibration is reduced by 27% and its strain energy is reduced by 28.6%. If $q_1 = 0.7$, its amplitude of vibration is reduced by 31.3% and its strain energy is reduced by 83.3%. The damping effect of the structure with $q_1 = 0.7$ is better than the one with $q_1 = 0.9$. That means the structural loss factor η of the structure with $q_1 = 0.7$ is greater than the one with $q_1 = 0.9$. These results are consistent with the previous results obtained from MATLAB in Figure 3(c).

4.2. Finite Element Analysis for the Second Damping Structure. The results of the finite element analysis of the second damping structure are shown in Figure 8. We set both q_1 and



(a) $q_2 = 1.1$ amplitude of vibration
(b) $q_2 = 1.3$ amplitude of vibration
(c) $q_2 = 1.1$ strain energy
(d) $q_2 = 1.3$ strain energy

Figure 8: The amplitude and strain energy images of the second damping structure.

q_3 as 0.9. If $q_2 = 1.1$, its amplitude of vibration is reduced by 25% and its strain energy is reduced by 20%. If $q_2 = 1.3$, its amplitude of vibration is reduced by 50% and its strain energy is also reduced by 50%. The damping effect of the structure with $q_2 = 1.3$ is better than the one with $q_2 = 1.1$. That means the structural loss factor η of the structure with $q_2 = 1.3$ is greater than the one with $q_2 = 1.1$. These results are consistent with the previous results obtained from MATLAB in Figure 4(c).

4.3. Finite Element Analysis for the Third Damping Structure. The results of the finite element analysis of the third damping structure are shown in Figure 9. We set q_1 and q_2 as 0.9 and 1.1, respectively. If $q_3 = 0.9$, its amplitude of vibration is reduced by 29.7% and its strain energy is reduced by 40%. If $q_3 = 0.7$, its amplitude of vibration is reduced by 35.5% and its strain energy is reduced by 50%. The damping effect of the structure with $q_3 = 0.7$ is better than the one with $q_3 = 0.9$. That means the structural loss factor η of the structure with $q_3 = 0.7$ is greater than the one with $q_3 = 0.9$. These results are consistent with the previous results obtained from MATLAB in Figure 5(c).

With the three damping structures, the mathematical model is validated by the comparisons between their MATLAB results and ABAQUS results, which are found to have a good agreement.

5. Application Selection of the Multilayer Free Damping Structure

Let us suppose that the structural loss factor η of the damping structure is equal to or greater than 2, the layer thickness ratio h_2 is 1, 3, 5, and 10, and the layer number n is 3, 4, 5, and 6. 34 kinds of design solutions are obtained by combining different damping materials of different thicknesses (as is shown in Table 1).

In Table 1, for example, with the layer $n = 3$ and the layer thickness ratio $h_2 = 10$, three sets of data ranges of q_1, q_2, and q_3 are gained. In Figure 3(d), when $q_2 = 1.1$ and $q_3 = 0.9$, q_1 should meet the condition $0.1 < q_1 < 0.9$. In Figure 4(d), when $q_1 = 0.9$ and $q_3 = 0.9$, q_2 should meet the condition $1.175 < q_2 < 1.3$. In Figure 5(d), when $q_1 = 0.9$ and $q_2 = 1.1$, q_3 should meet the condition $0.1 < q_3 < 0.9$. In line with the practical request, the relative parameters, such as the elastic modulus, the loss factors of damping materials, and the layer thickness, are further determined to achieve the desired vibration damping effect.

6. Conclusion

In this paper, a mathematical model for the multilayer free damping structure is presented and verified. And the relations of its structural loss factor η with the change rate of the

Table 1: Multilayer free damping structure parameters.

Layers n	Layer thickness ratio h_2	Data range
3	1	$q_1 = 0.9, q_2 = 1.1, 0.1 < q_3 < 0.9$
	3	$q_1 = 0.9, q_2 = 1.1, 0.1 < q_3 < 0.9$
		$q_1 = 0.9, q_3 = 0.9, 1.215 < q_2 < 1.3$
	5	$q_1 = 0.9, q_2 = 1.1, 0.1 < q_3 < 0.9$
		$q_1 = 0.9, q_3 = 0.9, 1.19 < q_2 < 1.3$
	10	$q_1 = 0.9, q_3 = 0.9, 1.175 < q_2 < 1.3$
		$q_2 = 1.1, q_3 = 0.9, 0.1 < q_1 < 0.9$
		$q_1 = 0.9, q_2 = 1.1, 0.1 < q_3 < 0.9$
4	1	$q_1 = 0.9, q_2 = 1.1, 0.1 < q_3 < 0.9$
	3	$q_1 = 0.9, q_2 = 1.1, 0.1 < q_3 < 0.9$
		$q_1 = 0.9, q_3 = 0.9, 1.205 < q_2 < 1.3$
	5	$q_1 = 0.9, q_2 = 1.1, 0.1 < q_3 < 0.9$
		$q_1 = 0.9, q_3 = 0.9, 1.185 < q_2 < 1.3$
	10	$q_1 = 0.9, q_3 = 0.9, 1.17 < q_2 < 1.3$
		$q_2 = 1.1, q_3 = 0.9, 0.1 < q_1 < 0.9$
		$q_1 = 0.9, q_2 = 1.1, 0.1 < q_3 < 0.9$
5	1	$q_1 = 0.9, q_2 = 1.1, 0.1 < q_3 < 0.9$
		$q_1 = 0.9, q_3 = 0.9, 1.29 < q_2 < 1.3$
	3	$q_1 = 0.9, q_2 = 1.1, 0.1 < q_3 < 0.9$
		$q_1 = 0.9, q_3 = 0.9, 1.19 < q_2 < 1.3$
	5	$q_1 = 0.9, q_2 = 1.1, 0.1 < q_3 < 0.9$
		$q_1 = 0.9, q_3 = 0.9, 1.17 < q_2 < 1.3$
	10	$q_1 = 0.9, q_3 = 0.9, 1.16 < q_2 < 1.3$
		$q_2 = 1.1, q_3 = 0.9, 0.1 < q_1 < 0.9$
		$q_1 = 0.9, q_2 = 1.1, 0.1 < q_3 < 0.9$
6	1	$q_1 = 0.9, q_2 = 1.1, 0.1 < q_3 < 0.9$
		$q_1 = 0.9, q_3 = 0.9, 1.28 < q_2 < 1.3$
	3	$q_1 = 0.9, q_2 = 1.1, 0.1 < q_3 < 0.9$
		$q_1 = 0.9, q_3 = 0.9, 1.17 < q_2 < 1.3$
	5	$q_1 = 0.9, q_2 = 1.1, 0.1 < q_3 < 0.9$
		$q_1 = 0.9, q_3 = 0.9, 1.15 < q_2 < 1.3$
	10	$q_1 = 0.9, q_3 = 0.9, 1.155 < q_2 < 1.3$
		$q_2 = 1.1, q_3 = 0.9, 0.1 < q_1 < 0.9$
		$q_1 = 0.9, q_2 = 1.1, 0.1 < q_3 < 0.9$

elastic modulus ratio q_1, the change rate of the loss factors of damping materials q_2, and the change rate of the layer thickness ratio q_3 are studied by MATLAB. Then 34 design solutions of the damping structures are listed out for choice.

In practical application, only if some parameters (such as the structural loss factor, the base layer material property) are known can some other parameters (such as the damping material, the layer thickness, and the layer number) be set and then be compared. Finally, an optimal combination is selected to achieve the requested vibration damping effect.

As future work, it is necessary to improve the theory by taking into account the temperature and frequency which are the most important environmental factors affecting the dynamic properties of damping materials. Moreover, for optimization, an intensive study is needed into the relationships between the total layer thickness, the layer number, and the structural loss factor to obtain a more careful design solution.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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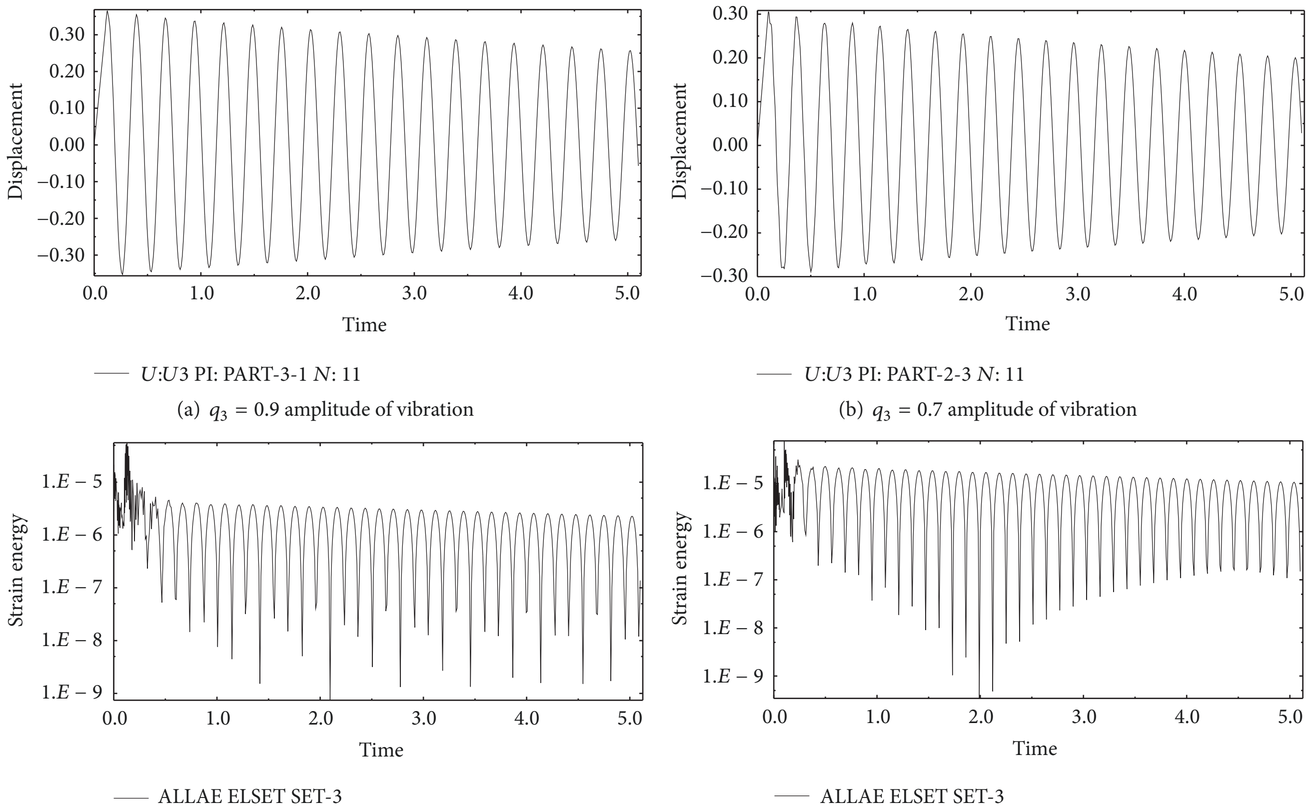

(a) $q_3 = 0.9$ amplitude of vibration

(b) $q_3 = 0.7$ amplitude of vibration

(c) $q_3 = 0.9$ strain energy

(d) $q_3 = 0.7$ strain energy

Figure 9: The amplitude and strain energy images of the third damping structure.

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