

Research Article

Reliability and Random Lifetime Models of Planetary Gear Systems

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Conventional reliability models of planetary gear systems are mainly static. In this paper, dynamic reliability models and random lifetime models of planetary gear systems are developed with dynamic working mechanism considered. The load parameters, the geometric parameters, and the material parameters are taken as the inputs of the reliability models and the random lifetime models. Moreover, failure dependence and dynamic random load redistributions are taken into account in the models. Monte Carlo simulations are carried out to validate the proposed models. The results show that the randomness of the load distribution is obvious in the system working process. Failure dependence has significant influences on system reliability. Moreover, the dispersion of external load has great impacts on the reliability, lifetime distribution, and redundancy of planetary gear systems.

1. Introduction

Planetary gear systems are widely used in technological systems, such as helicopters, cars, spacecraft, mining machineries, and so on, which have the advantages of compact structures, high transmission efficiency, and large transmission ratio. The load can be evenly distributed on planet gears to achieve the purpose of torque shunting. The normal operations of planetary gear systems are quite important for the safety and economy of technological systems. Hence, it is imperative to develop reliability models of planetary gear systems based on their practical working mechanism, environmental loads, and material properties.

Many efforts in reliability analysis of planetary gear systems have been made in the last few decades. Ye et al. developed a reliability-based optimization design method with stress and strength regarded as random variables. In their method, volume and efficiency were taken as the objective functions, while reliability and fatigue strength were set as boundary conditions. The reliability was calculated by the stress-strength interference (SSI) model [1–4]. Zhang

presented a reliability-based optimization design model, in which the design parameters were modeled as fuzzy variables and random variables [5]. Li et al. developed a reliability model of planetary gear systems in helicopters under the condition of partial loads with the effects of unequal load sharing considered [6]. Zhang introduced a method by combining floating-point encoding genetic algorithm with dynamic penalty function to improve the optimal design of planetary gear systems [7]. Wang developed a reliability model of planetary gear systems in wind power generators. In the literature, the SSI model was adopted with the contact force and the contact fatigue strength taken as the generalized stress and the generalized strength, respectively [8]. These innovative models provide a theoretical framework for reliability evaluation of planetary gear systems.

As a matter of fact, the operational process of a planetary gear system is dynamic and complex. At present, reliability models of planetary gear systems based on system working mechanisms are seldom reported. To analyze the reliability of planetary gear systems, the stress history and the strength degradation processes of the components in a system have to

be known. Currently, numerous dynamic models of planetary gear systems have been developed to analyze system motion characteristics. For instance, the spectral kurtosis technique was used by Barszcz and Randall to detect tooth cracks in a planetary gear system [9]. Kiracofe and Parker put forward compound analytical dynamic models for planetary gears [10]. Zhu et al. established dynamic models of planetary gear systems with both the flexibility of the pins and the gyroscopic effect taken into consideration [11]. These innovative models are quite effective for motion and dynamic characteristics analysis of planetary gear systems.

In general, many analytical dynamics models and dynamic simulation methods have been developed for dynamic analysis of planetary gears. Most of these models and methods are deterministic, which provide theoretical foundation for reliability analysis of planetary gears. However, they cannot be applied directly in reliability analysis. Many unique phenomena, such as failure dependence, stochastic strength degradation, and stochastic load redistribution, arise due to the emergence of dynamic random factors in the systems, which bring great difficulties in reliability modeling. Currently, reliability models of planetary gear systems are mainly static with components in a system regarded mutually statistically independent. The randomization of the data from deterministic dynamic models, associated with the mathematical treatments of these unique phenomena, has to be addressed when developing dynamic reliability models of planetary gear systems. This is the reason why dynamic reliability models of planetary gears considering working mechanism are seldom reported. In this paper, we concentrate on constructing dynamic reliability models of planetary gear systems with their working mechanism taken into account by employing the existing dynamic simulation methods.

In addition, conventional lifetime estimation of planetary gear systems has to rely on material fatigue tests under stress with constant amplitude. It is quite difficult to consider the comprehensive effects including the randomness of load, structural and material parameters as well as the failure dependence, and random load distribution in lifetime assessment. These problems will be addressed via the proposed dynamic reliability models.

The structure of this paper is organized as follows: In Section 2, the system logic structure and the problems in modeling are introduced. The reliability and random lifetime models are derived in Section 3. Numerical examples are given in Section 4 to demonstrate the proposed models and identify key factors which have great influences on reliability and lifetime distribution of planetary gear systems. Conclusions are summarized in Section 5.

2. System Logic Structure and Stochastic Dynamic Stress

2.1. Stochastic Dynamic Stress. The structure of a typical planetary gear system is shown in Figure 1. A planetary gear system is usually composed of a sun gear, a carrier, a ring gear, and some planet gears. To obtain the component reliability, the SSI model is adopted with the static root stress computed by [12]

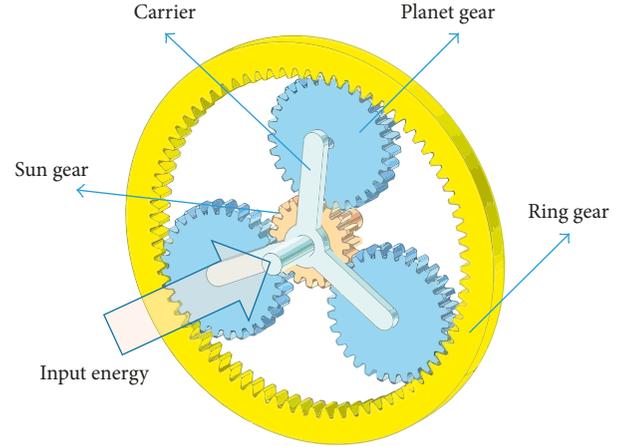


FIGURE 1: Structure of a planetary gear system.

$$s = \frac{2KT}{bd^2m} Y_1 Y_2 Y_3 Y_4, \quad (1)$$

where K , T , b , d , m , Y_1 , Y_2 , Y_3 , and Y_4 are load coefficient, torque on the gear, tooth width, diameter of dividing circle, normal modulus, tooth profile coefficient, stress correction coefficient, contact ratio factor, and helix angle coefficient, respectively. The equation is convenient in stress calculation. However, the working process of the planetary gear system is dynamic under fluctuant stress and the strength degrades under the dynamic stress. Therefore, the dynamic stress on each component should be calculated based the motion equations of the system and the material properties of the components.

The motion of the planetary gear systems is mainly analyzed via lumped parameter models as follows [13]:

$$M\ddot{X} + (C_1 + C_2 + 2wG)\dot{X} + KX = T, \quad (2)$$

where X , M , C_1 , C_2 , w , G , K , T represent the generalized coordinates of the system, the mass matrix, the support damping matrix, the meshing damping matrix, the angular velocity of the carrier, the gyro matrix, the system stiffness matrix including the support stiffness matrix, the meshing stiffness matrix and the centripetal stiffness matrix, and the external generalized load, respectively. Nevertheless, the stress on the components cannot be calculated directly in this way due to the lack of accurate geometric parameters of the components. Furthermore, despite the accuracy of the stress obtained via physical experiments, when considering the randomness of the load parameters and the material parameters, physical experiments are still infeasible for reliability estimation in practice. An alternative method to deal with this problem is to adopt the finite element method. In this paper, the stress is calculated by using the Adams software and the Ansys software [14]. The basic idea is to use the modal neutral file in the Ansys software to replace rigid bodies with flexible body in the Adams software [14]. Then the dynamic stress can be obtained via the simulations by using the Adams software. The motions of rigid bodies can be expressed as

$$\begin{aligned} M(q, t)\ddot{q} + \Phi \Xi_q^T(q, t)\lambda - Q(q, \dot{q}, t) = 0, \\ \Xi(q, t) = 0, \end{aligned} \quad (3)$$

where q is the generalized coordinate vector of the rigid bodies, λ is the lagrange multiplier, $Q(q, \dot{q}, t)$ is the generalized external forces on the rigid bodies, t is time, and Ξ is the constraint equations of the rigid bodies with respect to q . The motions of the flexible bodies can be calculated by [15]

$$M\ddot{q}_1 + \dot{M}\dot{q}_1 + Kq_1 + C\dot{q}_1 + \left[\frac{\partial \Omega}{\partial q_1} \right]^T q_1 - \frac{1}{2} \left[\frac{\partial M}{\partial q_1} \dot{q}_1 \right] = Q_1, \quad (4)$$

where C is the mass matrix, q_1 is the generalized coordinate vector of the flexible bodies, Ω is the constraint equation of the flexible bodies with respect to q_1 , and Q_1 is the generalized external forces on the flexible bodies. The friction coefficient is expressed by [16]

$$\zeta = \begin{cases} \frac{b_1 + b_2}{2} + \frac{1}{2} \left[(b_2 - b_1) \cos \left(\pi \frac{|v_1| - v_2}{v_3 - v_2} \right) \right], & v_2 \leq v_1 \leq v_3, \\ b_1, & |v_1| > v_2, \\ b_2 \sin \left(\pi \frac{|v_1|}{2v_2} \right), & v_1 < v_2, \end{cases} \quad (5)$$

where v_1 , v_2 , and v_3 are the relative velocity, the stick-slip conversion velocity, and the static-sliding conversion velocity, respectively. b_1 and b_2 are the sliding friction coefficient and the static friction coefficient, respectively. The dynamic stress obtained after the simulations, which contributes to the fatigue failure of the gears in the system, is deterministic. In order to acquire the statistical characteristics of stress in each time interval, the following sample matrix has to be determined via a large number of simulations or tests:

$$\begin{aligned} \Gamma = \begin{bmatrix} \psi_1^{(1)} & \psi_1^{(2)} & \dots & \psi_1^{(n)} \\ \psi_2^{(1)} & \psi_2^{(2)} & \dots & \psi_2^{(n)} \\ \dots & \dots & \dots & \dots \\ \psi_k^{(1)} & \psi_k^{(2)} & \dots & \psi_k^{(n)} \end{bmatrix}, \\ \psi_y^{(y_1)} = \left[\psi_y^{(y_1,1)}, \psi_y^{(y_1,2)}, \dots, \psi_y^{(y_1,n_1)} \right], \\ (y = 1, 2, \dots, k \cdot y_1 = 1, 2, \dots, n), \end{aligned} \quad (6)$$

where $\psi_y^{(y_1)}$ is the sample vector of the y th component in the y_1 th time interval and $\psi_y^{(y_1, y_2)}$ is the y_2 th stress sample of the y th component in the y_1 th time interval. Although the simulation method is much more efficient in sample acquisition than the method based on physical experiments, it is still quite time-consuming to obtain Γ considering the randomness of the input of the stress models, such as the input torque, input angular velocity, or the resistance torque on the output components. Nevertheless, provided that the

relationship between the inputs and the stress on each component is known, the process to gain Γ can be simplified. From the analysis above, it can be learnt that it is difficult to provide explicit mathematical equations to express the relationship. Thus, the response surface method [17] is used in this section to obtain Γ . The relationship between an input vector $\Theta = [\theta_1, \theta_2, \dots, \theta_{k_1}]$ ($\theta_{k_1} = t$) and the stress $s(t)$ on a component can be given by the following expression:

$$s(t) = \beta_0 + \sum_{i=1}^{k_1} \beta_i \theta_i + \sum_{i=1}^{k_1} \beta_{ii} \theta_i^2 + \sum_i \sum_j \beta_{ij} \theta_i \theta_j + \varepsilon. \quad (7)$$

Equation (7) is a generalized expression of response surface methods. The stress in this paper refers to the Mises equivalent stress. Equation (7) is used to establish the relationship between the input vector $\Theta = [\theta_1, \theta_2, \dots, \theta_{k_1}]$ and the output stress $s(t)$ on a gear in a specified time interval. $\beta = [\beta_0, \beta_1, \dots, \beta_{k_1}, \beta_{11}, \beta_{12}, \dots, \beta_{k_1 k_1}]$ is the pending vector, which is obtained via the input samples of Θ and the corresponding output samples of $s(t)$ by employing the dynamic simulations. ε is the higher-order omission item. The relationships between the input vector and the output stress on different gears have to be obtained separately by using Equation (7). After a limited number of simulations, this relationship can be determined through the regression method. Then, the stress distributions can be acquired directly via the distributions of the inputs as shown in Figure 2.

2.2. System Logic Structure. From the structures and the working mechanism of the planetary gear systems, it can be known that the failure of the sun gear, the carrier, or the ring gear could lead to the failure of the whole system. The remaining working duration of a planetary gear system after the failure of a planet gear depends on the material characteristics of the remaining planet gears, and the system can even work with less planet gears. In this case, the system can be viewed as a parallel system. However, the failure of a planetary gear system will speed up obviously after the failure of a planet gear due to the load redistribution. For the safety of the whole system, it is necessary to carry out maintenance or replacements after the partial failure of the system. Thus, the whole system is essentially a series-parallel system as shown in Figure 3.

In current reliability models or the models for reliability-based optimal design, the system reliability is always calculated according to classical reliability theory as follows:

$$R_{\text{sys}} = R_1 R_2 R_3 R_4, \quad (8)$$

where R_{sys} , R_1 , R_2 , R_3 and R_4 are the system reliability, the reliability of the ring gear, the reliability of the sun gear, the reliability of the parallel subsystem, and the reliability of the carrier, respectively. R_3 can be expressed by

$$R_3 = 1 - \prod_{i=1}^{K_1} (1 - R_{3i}), \quad (9)$$

where R_{3i} ($i = 1, 2, \dots, K_1$) is the reliability of the i th planet gear in the parallel subsystem with K_1 planet gears. From the derivation of Equation (9), it can be seen that the

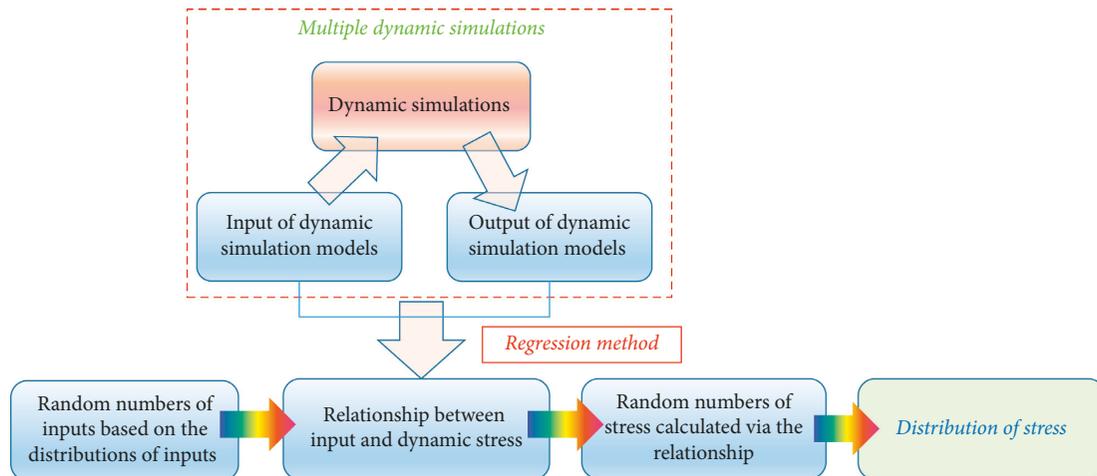


FIGURE 2: Method for distributions of stress.

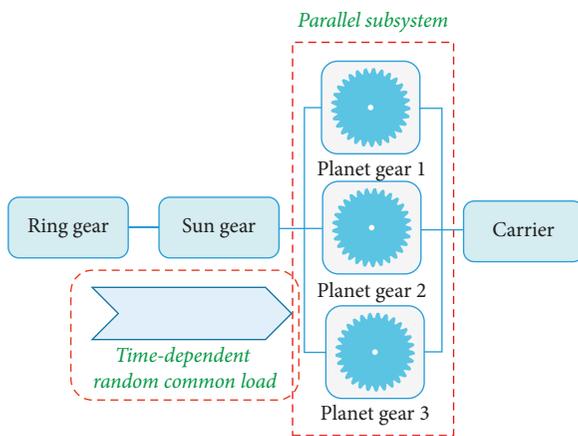


FIGURE 3: Reliability block diagram of a planetary gear system.

stress on each component in a system is mutually statistically correlative. Therefore, the components are statistically correlative, and the failure dependence should be taken into account in reliability modeling. As a matter of fact, failure dependence occurs under the condition of random common working environment, which significantly reduces the reliability of parallel systems. Under deterministic external load, failure dependence seldom takes place and the components seldom fail to work at the same time. However, the existence of random common load source greatly increases the possibility of simultaneous failure of components and reduces the effects of redundant designs, such as the well-known phenomenon of common cause failure in nuclear power plants. When failure dependence happens, the parallel system reliability cannot be calculated according to classical reliability theory. Therefore, failure dependence brings great difficulties in reliability estimation of planetary gear systems. In this case, the reliability calculated according to Equations (8) and (9) could lead to large computational error. Moreover, considering the strength degradation, the correlation between different components could show obvious dynamic characteristics. Meanwhile, this dynamic characteristic of failure dependence also significantly

increases the difficulty in random lifetime distribution modeling of planetary gear systems. It should be noted that failure dependence significantly affects the formulation of maintenance strategies and replacement strategies.

Besides, the components in parallel configuration in the subsystem share the same external load, and failure could take place on any component due to the randomness of the stress and the strength. Hence, a load redistribution could occur at any time and on any remaining components in the subsystem as shown in Figure 4. In Figure 4, t_1 and t_2 are random variables, and the load redistribution speeds up the strength degradation processes of the remaining components. The strength degradations of different components in the parallel subsystem are mutually dependent due to the stress dependence, and the load redistribution happens simultaneously on the remaining components. These problems in the subsystem have great influences on the failure dependence, dynamic reliability, and lifetime distribution of the whole system, which will be taken into consideration and analyzed in this paper.

3. Reliability Models and Lifetime Models

In practice, as an important component connecting the impeller and the generator in the wind generator, the gearbox always works with the highest failure rate in the wind generator under the harsh conditions of the external random wind load. The planetary gear is the most important part in the gearbox, which is crucial to the safe operation of the whole system. Besides, as important components, planetary gears are widely used in machine tools, automobiles, metallurgy, cranes, and aerospace products. Therefore, it is quite critical to investigate reliability models of planetary gears. Error, material parameters, and external load are key factors in the dynamics analysis of planetary gears. When considering the randomness of these factors, some new problems are encountered which bring large difficulties in reliability modeling of planetary gear systems. Current deterministic analytical dynamics models cannot be employed directly to solve these problems. For instance, the random external load causes

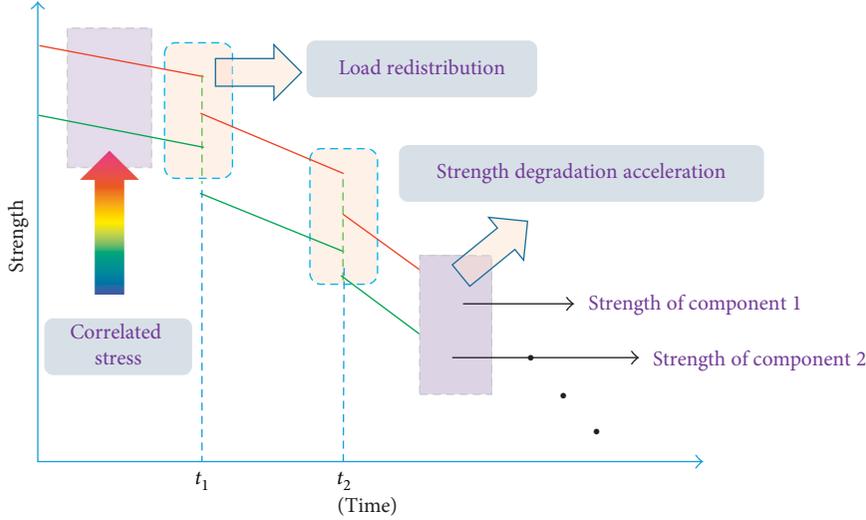


FIGURE 4: Strength degradation of components in a planetary gear system.

a random strength degradation. The random failure of the components results in the random load redistributions, which makes the mathematical expression of the stochastic strength degradation more difficult. Moreover, the dynamic failure dependence analysis of the components in a system, considering the random strength degradation and the random load redistribution, is seldom reported, which is quite important to the accurate reliability assessments of planetary gear systems. In this paper, dynamic reliability models of planetary gear systems are developed with the problems mentioned above taken into account.

Besides, analytical lifetime distribution models of planetary gear systems are further developed based on the proposed reliability models. The lifetime distribution models take the random load parameters, random geometric parameters, and random material parameters as the inputs. Furthermore, the lifetime distribution models consider the comprehensive effects of the random strength degradation, the random load distribution, and the dynamic failure dependence, which overcome the shortcomings of the lifetime assessment methods based on conventional fatigue tests and provide a theoretical framework for the random lifetime estimation of planetary gear systems. In practice, carriers are always designed with high strength and stiffness because of their important roles in energy transmission, and the system failure is mainly caused by other components in the systems. Therefore, in this section, we will concentrate on the behavior of the ring gear, the sun gear, and the parallel subsystem.

3.1. Reliability Models and Lifetime Distribution Models of the Subsystem. As mentioned above, the reliability analysis of the subsystem is the most complex compared with that of other components when considering the failure dependence and the random load redistributions. Thus, the dynamic reliability models of the subsystem will be established first. To consider the randomness of the input torque w_1 , input angular velocity w_2 , and the resistance torque w_3 in the j th

time interval, the stress associated its probability density function (PDF) on a planet gear is denoted by $T_j(w)$ and $f_{T_j(w)}(T_j(w))$ where $w = [w_1, w_2, w_3]$. Generally, the S-N curve model can be written as follows:

$$s^m N = C, \quad (10)$$

where m and C are material parameters and N is the total cycles to failure under the stress s . In the failure mode of fatigue, when considering the strength degradation path dependence (SDPD), the equivalent residual strength can be given by [18]:

$$r(n) = \begin{cases} r_0 \left(1 - \sum_{j=1}^{n-1} \frac{\int_{-\infty}^{\infty} T_j(w)^m f_{T_j(w)}(T_j(w)) dT_j(w)}{C} \right)^a, & n > 1, \\ r_0, & n = 1, \end{cases} \quad (11)$$

where r_0 is initial strength, m and C are material parameters for S-N curve, and $f_{T_j(w)}(T_j(w))$ is the PDF of the stress $T_j(w)$ in the i th time interval. For description convenience, the function Φ is defined as follows:

$$\Phi(z_1, z_2, T_j(w)) = \int_{z_2}^{z_1} f_{T_j(w)}(T_j(w)) dT_j(w). \quad (12)$$

The reliability of a component can be given by

$$R_{\text{com}}(n) = \prod_{h=1}^n \Phi(r(h), 0, T_h(w)). \quad (13)$$

For a parallel system composed of k components, in which the minimum initial strength is denoted by r_{\min} , with identical material parameters and stress, after n time intervals, the reliability of the parallel system in the condition that the k components still work normally can be calculated by

$$R_k(n) = \prod_{h=1}^n \Phi(r_{\min}(h), 0, T_{h+1}(w)). \quad (14)$$

For the k components with deterministic initial strength in ascending order denoted by r_{j2} ($j2 = 1, 2, \dots, k$), when considering the failure history of the components within the

n time intervals, the reliability of the parallel system in the condition that the $k2$ components still work normally can be calculated by

$$R_{k2}(n) = \sum_{h_1=1}^{p_1} \sum_{h_2=1}^{p_2} \dots \sum_{h_{(k-2)}=1}^{p_{(k-2)}} \left(\prod_{z3=1}^{k-2} W_{z3} \right) \prod_{hr=h_{(k-2)}+1}^n \Phi(r_{k-k2+1}(hr), 0, T_{hr}(w)),$$

$$(h_{z3-1} \leq h_{z3}, p_{z3-1} \leq p_{z3} \leq (n-1)),$$

$$W_1 = \begin{cases} \left[\prod_{d1=1}^{h_1-1} \Phi(r_1(d1), 0, T_{d1}(w)) \right] [1 - \Phi(r_2(h_1), r_1(h_1), T_{h_1}(w))], & h_1 \geq 2, \\ [1 - \Phi(r_2(h_1), r_1(h_1), T_{h_1}(w))], & h_1 = 1, \end{cases} \quad (15)$$

$$W_{k3} = \begin{cases} \left[\prod_{dk3=h_{k3-1}+1}^{h_{k3}-1} \Phi(r_{k3}(dk3), 0, T_{dk3}(w)) \right] [1 - \Phi(r_{k3+1}(h_{k3}), r_{k3}(h_{k3}), T_{h_{k3}}(w))], & h_{k3} \geq 2, \\ [1 - \Phi(r_{k3+1}(h_{k3}), r_{k3}(h_{k3}), T_{h_{k3}}(w))], & h_{k3} = 1, \end{cases} \quad (k3 \geq 2),$$

when $h_{x1} = h_{x2} = \dots = h_{xj}$,

$$\prod_{z3=x1}^{xj} W_{z3} = \begin{cases} \left[\prod_{dk3=h_{x1-1}+1}^{h_{x1}-1} \Phi(r_{x1}(dk3), 0, T_{dk3}(w)) \right] [1 - \Phi(r_{xj+1}(h_{x1}), r_{x1}(h_{x1}), T_{h_{x1}}(w))], & h_{x1} \geq 2, \\ [1 - \Phi(r_{xj+1}(h_{x1}), r_{x1}(h_{x1}), T_{h_{x1}}(w))], & h_{x1} = 1. \end{cases} \quad (16)$$

The reliability models above are derived under the assumption that the initial strength of each component is deterministic. When considering the randomness of these initial strength, provided that the PDFs of r_{j2} ($j2 = 1, 2, \dots, k$) are identical, which are denoted by $f_1(r_{j2})$ ($j2 = 1, 2, \dots, k$), the reliability above can be further modified as follows:

$$R_k(n) = k! \int_{-\infty}^{\infty} f_1(r_k) \int_{r_{-\infty}}^{r_k} f_1(r_{k-1}) \dots \int_{-\infty}^{r_2} f_1(r_1) \cdot \left[\prod_{h=0}^{n-1} \Phi(r_1(h), 0, T_{h+1}(w)) \right] dr_1 dr_2 \dots dr_k, \quad (17)$$

$$R_{k2}(n) = k! \int_{-\infty}^{\infty} f_1(r_k) \int_{r_{-\infty}}^{r_k} f_1(r_{k-1}) \dots \int_{-\infty}^{r_2} f_1(r_1) \cdot \left\{ \sum_{h_1=1}^{p_1} \sum_{h_2=1}^{p_2} \dots \sum_{h_{(k-2)}=1}^{p_{(k-2)}} \prod_{z3=1}^{k-2} W_{z3} \right. \\ \left. \times \left[\prod_{hr=h_{(k-2)}+1}^n \Phi(r_{k-k2+1}(hr), 0, T_{hr}(w)) \right] \right\} \cdot dr_1 dr_2 \dots dr_k, \quad (18)$$

The total reliability of the parallel system can be computed by

$$R_s(n) = \sum_{k3=1}^k R_{k3}(n). \quad (19)$$

In the dynamic reliability models of planetary gear systems derived above, all possible failure processes of the system are considered. Moreover, the possibility that a parallel system works with a specified number of components is calculated on the basis of the possibility that the parallel system works with more components, which simplifies the calculation and saves the computing cost. Besides, it should be noted that in Equation (18) that $R_{k3}(n)$ represents the reliability of the parallel system under the condition that the $k3$ components still work normally, rather than the reliability of Component $k3$.

Define the g th-order lifetime PDF of the parallel system as the PDF of the lifetime within which g components work normally. Then, the g th-order discrete lifetime PDF can be given by

$$\begin{aligned}
f_k(n) &= k! \int_{-\infty}^{\infty} f_1(r_k) \int_{r_{-\infty}}^{r_k} f_1(r_{k-1}) \cdots \int_{-\infty}^{r_2} f_1(r_1) \\
&\quad \cdot \left\{ \left[\prod_{h=0}^{n-1} \Phi(r_1(h), 0, T_{h+1}(w)) \right] \right. \\
&\quad \left. \times [1 - \Phi(r_1(n), 0, T_{n+1}(w))] \right\} dr_1 dr_2 \cdots dr_k, \\
f_{k2}(n) &= k! \int_{-\infty}^{\infty} f_1(r_k) \int_{r_{-\infty}}^{r_k} f_1(r_{k-1}) \cdots \int_{-\infty}^{r_2} f_1(r_1) \\
&\quad \cdot \left\{ \sum_{h_1=1}^{p_1} \sum_{h_2=1}^{p_2} \cdots \sum_{h_{(k-k2)}=1}^{p_{(k-k2)}} \prod_{z3=1}^{k-k2} W_{z3} \right. \\
&\quad \times \left[\prod_{hr=h_{(k-k2)}+1}^n \Phi(r_{k-k2+1}(hr), 0, T_{hr}(w)) \right] \\
&\quad - \sum_{h_{1a}=1}^{p_{1a}} \sum_{h_{2a}=1}^{p_{2a}} \cdots \sum_{h_{(k-k2)a}=1}^{p_{(k-k2)a}} \prod_{z3a=1}^{k-k2} W_{z3a} \\
&\quad \times \left[\prod_{hra=h_{(k-k2)a}+1}^{n+1} \Phi(r_{k-k2+1}(hra), 0, T_{hra}(w)) \right] \left. \right\} \\
&\quad \cdot dr_1 dr_2 \cdots dr_k, \\
&\quad (h_{z3a-1} \leq h_{z3a}, p_{z3a-1} \leq p_{z3a} \leq n),
\end{aligned} \tag{20}$$

3.2. Reliability Models and Lifetime Distribution Models of the Planetary Gear System.

In Section 3.1, the reliability models

where

$$\begin{aligned}
R_{Sk}(n) &= \int_{-\infty}^{\infty} f_{u_{10}}(u_{10}) \int_{-\infty}^{\infty} f_{u_{20}}(u_{20}) k! \int_{-\infty}^{\infty} f_1(r_k) \int_{r_{-\infty}}^{r_k} f_1(r_{k-1}) \cdots \int_{-\infty}^{r_2} f_1(r_1) \\
&\quad \times \left[\prod_{h=0}^{n-1} \Phi(\min(r_1(h), u_{11}(h), u_{22}(h)), 0, T_{h+1}(w)) \right] dr_1 dr_2 \cdots dr_k du_{10} du_{20}, \\
R_{Sk2}(n) &= \int_{-\infty}^{\infty} f_{u_{10}}(u_{10}) \int_{-\infty}^{\infty} f_{u_{20}}(u_{20}) k! \int_{-\infty}^{\infty} f_1(r_k) \int_{r_{-\infty}}^{r_k} f_1(r_{k-1}) \cdots \int_{-\infty}^{r_2} f_1(r_1) \left\{ \sum_{h_1=1}^{p_1} \sum_{h_2=1}^{p_2} \cdots \sum_{h_{(k-k2)}=1}^{p_{(k-k2)}} \prod_{z3=1}^{k-k2} W_{S_{z3}} \right. \\
&\quad \times \left[\prod_{hr=h_{(k-k2)}+1}^n \Phi(\min(r_{k-k2+1}(hr), u_{11}(hr), u_{22}(hr)), 0, T_{hr}(w)) \right] \left. \right\} dr_1 dr_2 \cdots dr_k du_{10} du_{20}, \\
W_{S1} &= \begin{cases} \left[\prod_{d1=1}^{h_1-1} \Phi(\min(r_1(d1), u_{11}(d1), u_{22}(d1)), 0, T_{d1}(w)) \right] \\ \quad \times [1 - \Phi(\min(r_2(h_1), u_{11}(h_1), u_{22}(h_1)), \min(r_1(h_1), u_{11}(h_1), u_{22}(h_1)), T_{h_1}(w)))] & h_1 \geq 2, \\ [1 - \Phi(\min(r_2(h_1), u_{11}(h_1), u_{22}(h_1)), \min(r_1(h_1), u_{11}(h_1), u_{22}(h_1)), T_{h_1}(w)))] & h_1 = 1, \end{cases} \\
W_{S_{k3}} &= \begin{cases} \left[\prod_{dk3=h_{k3-1}+1}^{h_{k3}-1} \Phi(\min(r_{k3}(dk3), u_{11}(dk3), u_{22}(dk3)), 0, T_{dk3}(w)) \right] \\ \quad \times [1 - \Phi(\min(r_{k3+1}(h_{k3}), u_{11}(h_{k3}), u_{22}(h_{k3})), \min(r_{k3}(h_{k3}), u_{11}(h_{k3}), u_{22}(h_{k3})), T_{h_{k3}}(w))] & h_{k3} \geq 2, \\ [1 - \Phi(\min(r_{k3+1}(h_{k3}), u_{11}(h_{k3}), u_{22}(h_{k3})), \min(r_{k3}(h_{k3}), u_{11}(h_{k3}), u_{22}(h_{k3})), T_{h_{k3}}(w))] & h_{k3} = 1, \end{cases} \\
&\quad (k3 \geq 2),
\end{aligned} \tag{24}$$

when $h_{x1} = h_{x2} = \cdots = h_{xj}$,

and the lifetime distribution models are developed, in which the failure dependence among each planet gear is taken into account. As stated in Section 2.2, the reliability of the ring gear $R_1(n)$ and the reliability of the sun gear $R_2(n)$ are also statistically dependent with $R_s(n)$. The residual strength of the ring gear and the residual strength of the sun gear are denoted by $u_1(n)$ and $u_2(n)$, respectively. From Equation (7), it can be known that the stress on the ring gear $s_1(j, w)$ and the stress on sun gear $s_2(j, w)$ can be mathematically expressed by

$$s_1(j, w) = N_1(j)T_j(w), \tag{21}$$

$$s_2(j, w) = N_2(j)T_j(w),$$

where $N_1(j)$ and $N_2(j)$ can be obtained by Equation (7). The equivalent residual strength of the ring gear and the sun gear can be written as

$$u_{11}(j) = \frac{u_1(j)}{N_1(j)}, \tag{22}$$

$$u_{22}(j) = \frac{u_2(j)}{N_2(j)}.$$

The PDF of the initial strength of the ring gear and that of the sun gear are denoted by $f_{u_{10}}(u_{10})$ and $f_{u_{20}}(u_{20})$, respectively. Then, the system reliability can be calculated by

$$R_T(n) = \sum_{T1=1}^k R_{ST1}(n), \tag{23}$$

TABLE 1: Structural parameters and material parameters.

| Parameters | Value | Unit |
|---|------------------|---------------------------|
| Pressure angle | 20 | Degree |
| Helix angle | 0 | Degree |
| Tooth thickness | 3 | Mm |
| Density of gears | $7.8 \cdot 10^3$ | kg/m^3 |
| Number of teeth of sum gear | 36 | |
| Number of teeth of planet gear | 36 | |
| Number of planet gears | 3 | |
| Modulus | 2 | mm |
| Angular velocity of sun gear | 60 | Degree/second |
| Elastic modulus | $2 \cdot 10^5$ | MPa |
| Length of a time interval | 1 | H |
| m | 3 | |
| C | 10^8 | MPa^2 |
| Mean value of total resistance torque | 30 | $\text{N} \cdot \text{m}$ |
| Standard deviation of total resistance torque | 1.5 | $\text{N} \cdot \text{m}$ |
| Mean value of initial strength of a planet gear | 300 | MPa |
| Standard deviation of initial strength of a planet gear | 30 | MPa |
| Mean value of initial strength of sun gear | 250 | MPa |
| Standard deviation of initial strength of sun gear | 30 | MPa |
| Mean value of initial strength of ring gear | 250 | MPa |
| Standard deviation of initial strength of ring gear | 30 | MPa |

$$\prod_{z3=x1}^{xj} W_{z3} = \begin{cases} \left[\prod_{dk3=h_{x1-1}+1}^{h_{x1}-1} \Phi(\min(r_{x1}(dk3), u_{11}(dk3), u_{22}(dk3)), 0, T_{dk3}(w)) \right] \\ \times [1 - \Phi(\min(r_{xj+1}(h_{x1}), u_{11}(h_{x1}), u_{22}(h_{x1})), \min(r_{x1}(h_{x1}), u_{11}(h_{x1}), u_{22}(h_{x1})), T_{h_{x1}}(w))], & h_{x1} \geq 2, \\ [1 - \Phi(\min(r_{xj+1}(h_{x1}), u_{11}(h_{x1}), u_{22}(h_{x1})), \min(r_{x1}(h_{x1}), u_{11}(h_{x1}), u_{22}(h_{x1})), T_{h_{x1}}(w))], & h_{x1} = 1. \end{cases} \quad (25)$$

The lifetime PDF can be given by

$$f_S(n) = \sum_{k3=1}^k f_{sk3}(n), \quad (26)$$

where

$$f_{sk}(n) = \int_{-\infty}^{\infty} f_{u_{10}}(u_{10}) \int_{-\infty}^{\infty} f_{u_{20}}(u_{20}) k! \int_{-\infty}^{\infty} f_1(r_k) \int_{r_{-\infty}}^{r_k} f_1(r_{k-1}) \dots \int_{-\infty}^{r_2} f_1(r_1) \times \left\{ \left[\prod_{h=0}^{n-1} \Phi(\min(r_1(h), u_{11}(h), u_{22}(h)), 0, T_{h+1}(w)) \right] \right. \\ \left. \times [1 - \Phi(\min(r_1(n), u_{11}(n), u_{22}(n)), 0, T_{n+1}(w))] \right\} dr_1 dr_2 \dots dr_k du_{10} du_{20}. \quad (27)$$

For $k3 \neq k$

$$f_{sk3}(n) = \int_{-\infty}^{\infty} f_{u_{10}}(u_{10}) \int_{-\infty}^{\infty} f_{u_{20}}(u_{20}) k! \int_{-\infty}^{\infty} f_1(r_k) \int_{r_{-\infty}}^{r_k} f_1(r_{k-1}) \dots \int_{-\infty}^{r_2} f_1(r_1) \left\{ \sum_{h_1=1}^{p_1} \sum_{h_2=1}^{p_2} \dots \sum_{h_{(k-k3)}=1}^{p_{(k-k3)}} \prod_{z3=1}^{k-k3} W_{S_{z3}} \right. \\ \times \left[\prod_{hra=h_{(k-k3)a}+1}^n \Phi(\min(r_{k-k3+1}(hra), u_{11}(hra), u_{22}(hra)), 0, T_{hra}(w)) \right] - \sum_{h_{1a}=1}^{p_{1a}} \sum_{h_{2a}=1}^{p_{2a}} \dots \sum_{h_{(k-k3)a}=1}^{p_{(k-k3)a}} \prod_{z3a=1}^{k-k3} W_{S_{z3a}} \\ \left. \times \left[\prod_{hra=h_{(k-k3)a}+1}^{n+1} \Phi(\min(r_{k-k3+1}(hra), u_{11}(hra), u_{22}(hra)), 0, T_{hra}(w)) \right] \right\} dr_1 dr_2 \dots dr_k du_{10} du_{20}. \quad (28)$$

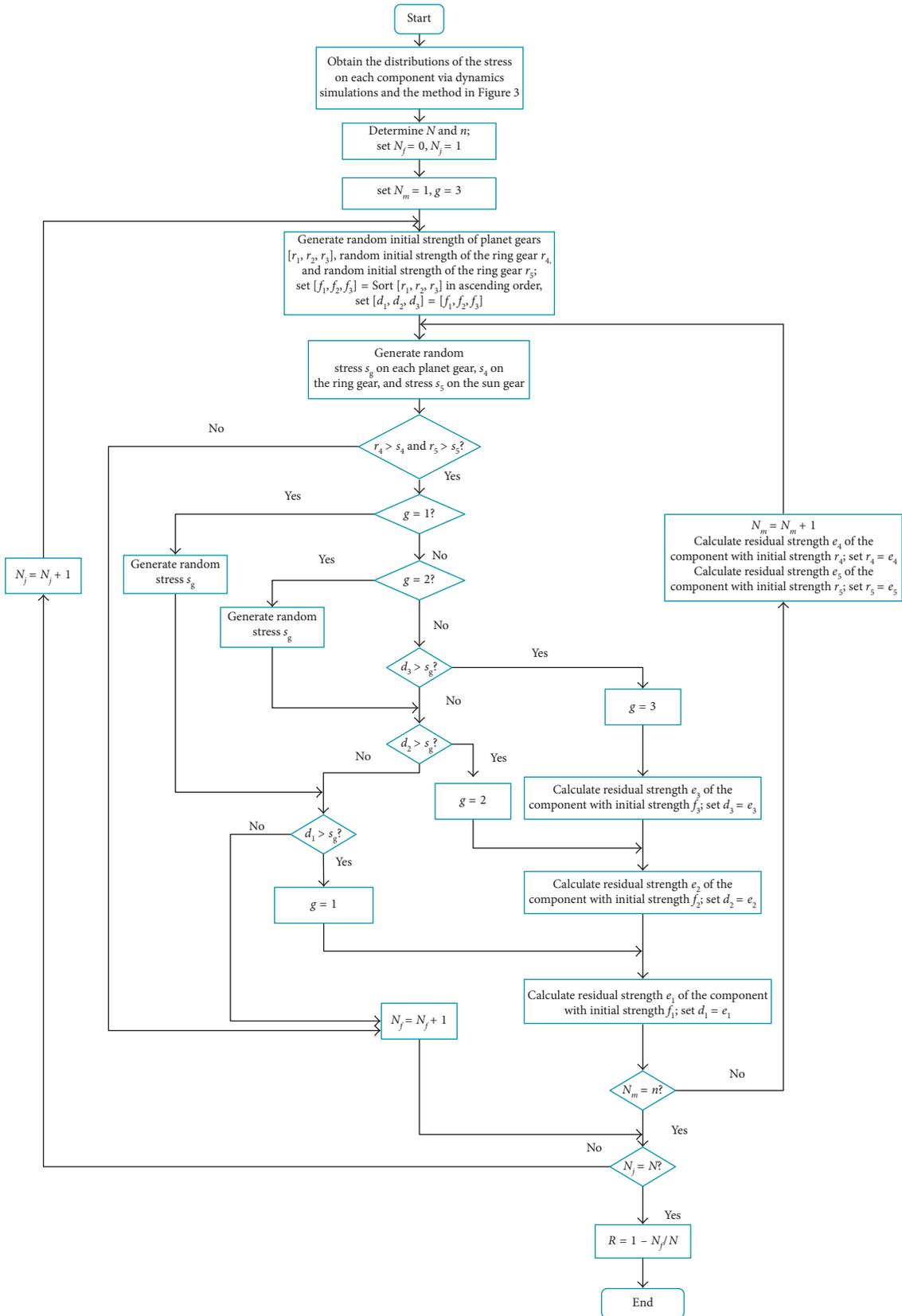


FIGURE 5: Flowchart for MCS of subsystem reliability.

4. Numerical Examples

Consider a planetary gear system with the structural and material parameters of its components listed in Table 1. In this numerical example, the sun gear operates at a constant angular velocity. The randomness of the output torque due to the uncertainty of the working environment is taken into account. The sum of the resistance torque on each planet gear is assumed to follow the normal distribution at each time interval. To validate the proposed models, Monte Carlo simulations (MCSs) are carried out in this section with the flowchart shown in Figure 5. The system reliability from the proposed models and the results from MCSs are shown in Figure 6. In addition, when the ring gear and the sun gear, whose reliability are calculated by Equation (12), are assumed to be independent to the subsystem, the system reliability from Equation (8) is plotted in Figure 6. In Table 1, pressure angle, helix angle, tooth thickness, density of gears, no. of teeth of sun gear, no. of teeth of planet gear, no. of planet gears, modulus, elastic modulus, angular velocity of sun gear, and length of a time interval are used for geometric modeling and material parameter input of planetary gear systems in the dynamic simulations. m and C are material parameters for S-N curve models. These two parameters and the parameter of initial strength are used to model the equivalent strength degradation paths of components. Moreover, the mean value and the standard deviation of a normal random variable are used to characterize the statistical distribution of the random variable. The stress is generated by the motion and mechanical parameters, such as the angular velocity of sun gear and the total resistance torque. The randomness of the stress comes from the randomness of these parameters. Hence, the stress distributions are obtained from the distribution of these parameters listed in Table 1 and dynamic simulations.

From Figure 6, it can be learnt that the reliability from the proposed models and that from MCSs show good agreement. The proposed analytical reliability models are effective for reliability evaluation of the planetary gear systems. The stress on each component is mutually statistically correlative because of the common external load. In the proposed models, the failure dependence between the planet gears and the failure dependence among the ring gear, the sun gear, and the parallel subsystem are taken into consideration. Besides, random load distributions are also considered in the models. The system reliability calculated under the assumption that the components are mutually independent is obviously lower than that calculated with failure dependence considered. Therefore, attention should be paid in the reliability assessment for mechanical systems which are mainly dependent system due to the working mechanism. At the early stage, the reliability is high, which seems almost invariable. Hence, a small change of reliability could lead to a sharp decrease in the failure rate. In the accidental failure period, on the one hand, the reliability decreases faster compared with that at the early stage. On the other hand, the reliability is obviously lower compared with that at the early stage. Thus, the failure rate shows a low descent speed in the accidental failure period as seen in the bathtub curve.

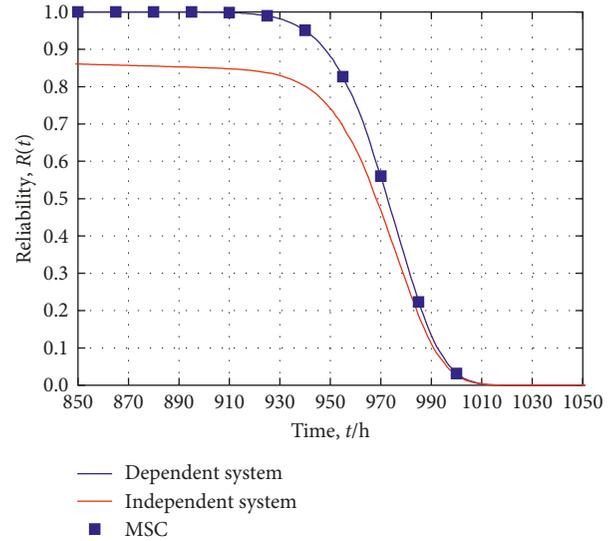


FIGURE 6: Reliability of independent system and dependent system and results from MSC.

To analyze the dispersion of the resistance torque on the system reliability, in the case where the standard deviations are $1.5 \text{ N} \cdot \text{m}$ and $0.75 \text{ N} \cdot \text{m}$, respectively, the system reliability is shown in Figure 7. In addition, the third-order lifetime PDF, the second-order lifetime PDF, and the first-order lifetime PDF of the parallel system in the case of different dispersions of the resistance torque are shown in Figures 8–10, respectively.

From Figure 7, it can be seen that although the mean value of the resistance torque keeps constant, the system reliability is significantly affected by the dispersion of the resistance torque. In general, large dispersion of the resistance torque results in low system reliability, because large dispersion increases the possibility of large stress which always causes the common cause failure and reduces the effects of redundant design. Hence, in the design of planetary gear systems, besides the deterministic design stress, the practical stress dispersion caused by the uncertainty of external loads, dimensional error, and material parameters should also be paid enough attentions.

In addition, from Figures 8–10, it can be learnt that random load redistributions exist in the operational duration of the planetary gear systems, which is seldom reported in current reliability models of planetary gear systems. Due to the randomness of the external load, the failure of each component is stochastic, which results in the randomness of the load redistribution. In general, the decrease in the dispersion of external load causes the improvement of the mean value of the system lifetime and reduces the dispersion of the system lifetime in each order, which is beneficial for the usage of the planetary gear systems. In addition, with the increase of the number of the redundant components, the mean value of the system lifetime is raised and the dispersion of the system lifetime is decreased. Furthermore, these effects are more obvious with the increase of the number of the redundant components in a system. Hence, the proposed

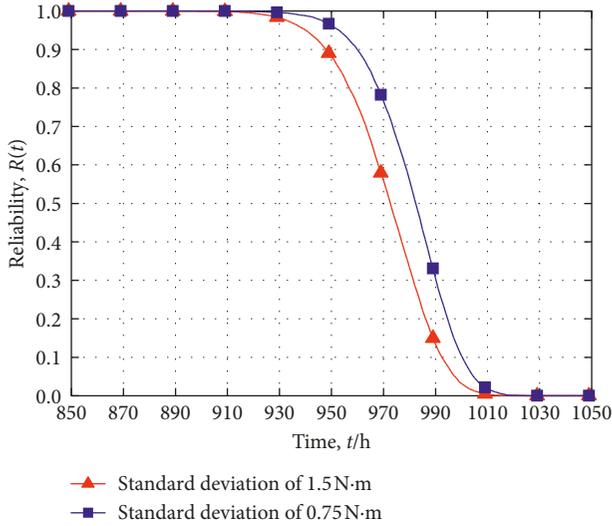


FIGURE 7: System reliability with different resistance torque dispersions.

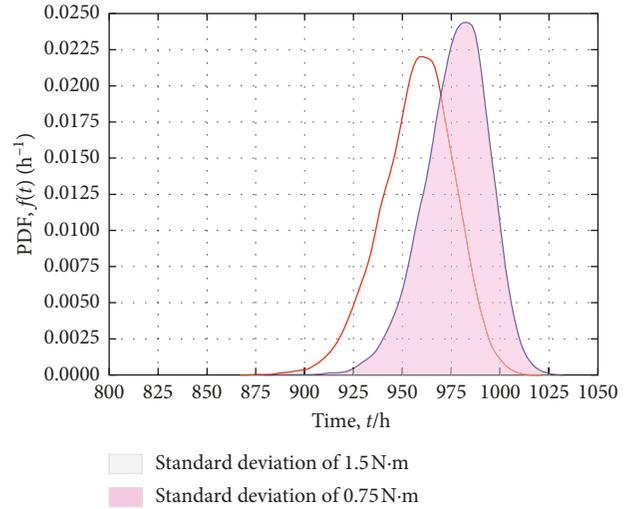


FIGURE 9: The second-order lifetime PDF with different resistance torque dispersions.

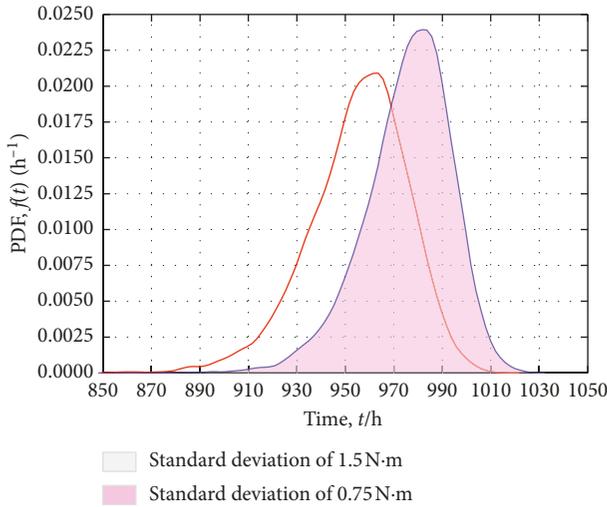


FIGURE 8: The third-order lifetime PDF with different resistance torque dispersions.

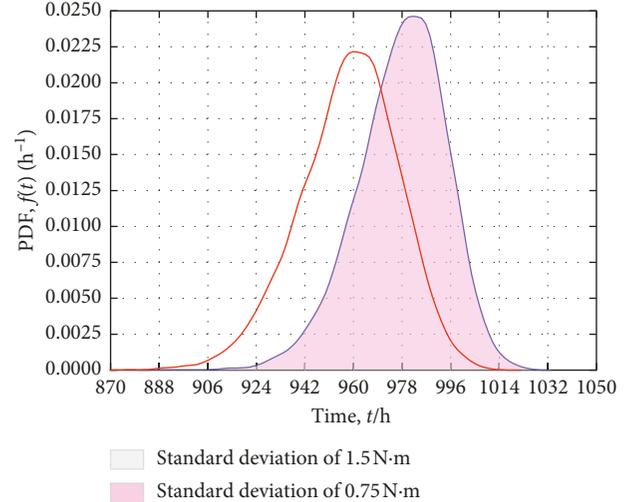


FIGURE 10: The first-order lifetime PDF with different resistance torque dispersions.

models are helpful for the design and the draw-up of maintenance strategies of the planetary gear systems.

5. Conclusions

Reliability and random lifetime models of planetary gear systems are developed in this paper. Conventional reliability models of planetary gear systems are mainly static models without detailed information about the stress and strength in the models. In this paper, the dynamic working mechanism is considered when establishing the dynamic system reliability models. Besides, conventional lifetime models are constructed based on fatigue test under constant stress, which cannot be used in the situation of dynamic random stress. In this paper, the lifetime distribution models of planetary gear systems are derived based on the proposed dynamic reliability models with the load parameters, the

geometric parameters, and the material parameters taken as the inputs. Furthermore, failure dependence of components in a planetary gear system and the random dynamic load redistributions are taken into account in the reliability models and the lifetime distribution models. MCSs are performed in this paper to validate the proposed models. The results in numerical examples show that the randomness of the load distribution is obvious in the system working process. Failure dependence has significant influences on system reliability. Moreover, the dispersion of external load has great impacts on the reliability, the lifetime distributions, and the redundancy of the planetary gear systems.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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