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Review Article

Research Progress on High-Intermediate Frequency Extension Methods of SEA

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Statistical energy analysis (SEA) can accurately describe the average vibration characteristics through system energy flow and transmission feedback. It is a powerful tool to solve the problem of high-frequency acoustics-vibration. SEA is widely used in vehicles, ships, aviation, and other transportation engineering fields. However, the expansion of SEA, based on the assumption of modal equipartition and weak coupling, is limited to the intermediate frequency. Although the SEA basic theory can be extended by relaxing the hypothesis conditions or the analysis of the medium-frequency acoustics-vibration can be carried out using the finite element method (FEM) and SEA mixing method, there are still many challenges associated with these options. To improve the basic theory of SEA and knowledge of intermediate frequency extension methods, as well as attract the attention of domestic scholars, this paper describes classical SEA and intermediate frequency extension methods. First, coupling loss factor (CLF) error propagation and parameter acquisition in classical SEA are introduced, and the three relative error calculation methods of CLF are compared. Then, the method of obtaining parameters is described from three aspects of energy transfer, input load, and modal density. Second, SEA intermediate frequency extension technology (experimental statistical energy analysis (ESEA), finite element statistical energy analysis (FE-SEA), statistical modal energy distribution analysis (SMEDA), and waveguide analysis (WGA)) are introduced. Neutron structure assembly and modeling, interval and mixed interval analysis, interval variable and mixed interval variable response are also described, so as to justify the development of a hybrid, large-scale interval algorithm. Finally, the engineering application of the above method is introduced, the limitations and shortcomings of SEA and intermediate frequency extension methods are reviewed, and unsolved problems are further discussed.

1. Introduction

Statistical energy analysis (SEA) is a statistical method for studying vibration and acoustics from an energy perspective. It divides a system into weakly coupled substructures and describes the state of each subsystem with power and energy. SEA is currently the most widely used method to solve high-frequency acoustics-vibration problems [1] and to obtain the acoustics-vibration characteristics of the system through feedback of energy flow and transmission to the equilibrium equation [1]. Compared with the deterministic method, the size of the SEA model is reduced, which greatly improves computational efficiency. Generally, for a structure's acoustics system under high-frequency excitation, Monte Carlo simulation of the structure with the finite element method (FEM) and boundary element method (BEM) is inefficient due to the limitation of the theory and calculation period [2, 3]. SEA can accurately describe the average vibration characteristics of the system and is a powerful tool to solve the problem of high-frequency acoustics-vibration. SEA method is applied to solve high-frequency acoustics-vibration problems. The calculation accuracy depends on the accuracy of its parameters, namely, modal density, internal loss factor (ILF), coupling loss factor (CLF), input power, and system energy. Typical SEA subsystem models are shown in Figures 1 and 2. In the past few decades, the classical SEA theory has been expanded to address nonconservative coupled systems, strongly coupled systems, and indirectly coupled systems [4, 5]. In addition, the application scope of SEA has been extended in recent

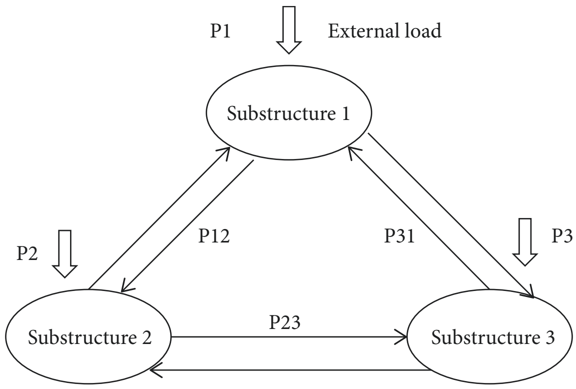

Figure 1: SEA model of three subsystems.

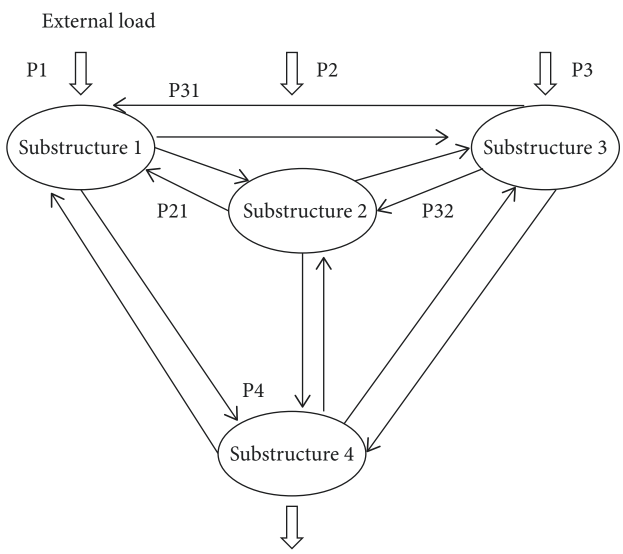

Figure 2: SEA model of four subsystems.

years due to further research in fields such as nonuniform mode energy statistical energy analysis and energy distribution [6, 7], low mode density and nonresonant response [8], SEA model variance prediction [9], SEA extension technology [10], finite element statistical energy analysis(FE-SEA) [11, 12], and transient statistical energy analysis.

Until now, SEA has been widely used in vehicles, ships, aviation, and other fields of transportation engineering [13, 14]. However, owing to the many assumptions involved in SEA, it is not suitable for acoustic array coupling problems in the intermediate frequency range. To solve the intermediate frequency problem, many scholars have carried out relevant research. Grice and Pinnington [15] combined FEM and the analytical impedance method to study the vibration of a beam element. Langley et al. [11, 16, 17] proposed an intermediate frequency solution combining FEM and SEA. In this hybrid finite element analysis framework, they built the deterministic components of the system and combined the advantages of FEM with SEA. This approach provided ideas for research to better solve the problem of medium and high-frequency acoustics-vibration. In this paper, the latest research on SEA and intermediate frequency extension is described.

2. Study of the Parameters of Classical SEA

2.1. Expression of Coupling Loss Factor (CLF). CLF is used to represent the amount of power flow or damping effect when one system is attached to another system. It is one of the important parameters that affect the calculation accuracy of the statistical energy model. CLF can be obtained by calculation, measurement, and other methods. The main calculation methods include infinite system impedance, analytic derivation method, power input method (PIM), empirical formula method, SEA numerical simulation method, and two-mode method [18]. The analytical calculation expression of CLF at point, line, and surface structural connections is given in Table 1:

2.2. CLF Parameter Acquisition Method. Power input method (PIM) [20–22] is one of the effective methods to determine the CLF. By measuring input power and average response energy of the system, as shown in Figures 3 and 4, it solves the energy balance equation to obtain the average loss factor of the frequency band. Studies of the application of PIM to obtain CLF have shown that Semprini and Barbaresi [23], Langhe and Sas [24], Gélat and Lalor [25], Bies and Hamid [20], and Fredö [26] used PIM to measure the vibration energy of plates from experiments in order to obtain CLF. Seçgin et al. [27] used PIM to conduct SEA on composite plates (I, L, and T connection) with three different connection modes and discuss the numerical solutions of different connection modes, as well as the accuracy of PIM. The theoretical assumption of this method is that, in a certain frequency band, the subsystem modes are sufficiently dense, the mode frequencies are uniformly distributed, and the measurement points can accurately reflect the time and space average characteristics. However, the development of this method is restricted in many cases, due to the existence of singular solutions in the inverse coupling matrix and the discrete mode frequency [14]. To overcome the disadvantages of PIM, some scholars have used mode data for CLF prediction. Mandale et al. [28] proposed a CLF estimation method, combining experimental test and mode analysis to verify the effect of different connection modes of vertical plates on CLF. Seçgin [18] proposed a modal-based method, determining SEA parameters to find CLF of a composite material connection plate. The feasibility of this method was verified by comparison. Totaro et al. [29–32] used mode data and FEM to evaluate CLF of an uncoupled subsystem. Simmons [31] estimated CLF of SEA subsystem mode parameters. They obtained the subsystem parameters through mode analysis and discuss the CLF solution of indirect contact within "double wall" substructures. Cotoni et al. [33] studied SEA parameter acquisition using FEM, component mode synthesis (CMS), and periodic structure theory. In terms of the research on transient random excitation CLF, Díaz-Cereceda et al. [34, 35] studied transient CLF, compared the transient and steady-state CLF of two subsystems with the same coupling vibration in the medium-strength coupling system, determined the undamped transient CLF by the numerical method, and gave the corresponding

Table 1: Expression of the coupling loss factor.

Structural connection	Analytical calculation expression of the coupling loss factor	
Point connection structure coupling loss factor	$\eta_{12} = (2\Omega_1\Omega_2)/(\pi\omega n_1(\omega)\lvert\Omega_3 + \Omega_4\rvert^2)$	
Line connection structure coupling loss factor	$\eta_{12} = ((LC_{\mathrm{w}})/(\pi\omega A_{\mathrm{p}}))\tau_{12}$	
Surface connection three-dimensional space coupling loss factor	$\eta_{12} = ((C_{\mathrm{s}}A_{\mathrm{c}})/(4\omega V_{\mathrm{s}}))\tau_{12}$	
Coupling loss factor between structure and sound field	Structure-sound field $\eta_{\mathrm{ps}} = (\rho_{\mathrm{s}}C_{\mathrm{s}}\sigma)/(\omega\rho_{\mathrm{p}})$	Sound field-structure $\eta_{\mathrm{sp}} = (\rho_{\mathrm{s}}C_{\mathrm{s}}\sigma n_{\mathrm{p}})/(\omega\rho_{\mathrm{p}}n_{\mathrm{s}})$

Ω_3 and Ω_4 are the input impedance of the structure point, and Ω_1 and Ω_2 are the real part of the input impedance of the structure point. L is the length of the line connection, C_{w} is the bending wave velocity, A_{p} is the area of plate 1, τ_{12} is the waveguide coefficient, and C_{s} is the sound propagation velocity. A_{c} is the area of the connection surface, V_{s} is the volume of space saved, ρ_{s} is the density of air, ρ_{p} is the density of the panel, σ is the acoustic radiation coefficient, n_{s} is the acoustic mode density, and n_{p} is the structural mode density. For simple-structure CLF, the finite/semiinfinite system impedance flow analysis method can be used [15, 19]. For complex-structure CLF, auxiliary technology based on the numerical or experimental process is required.

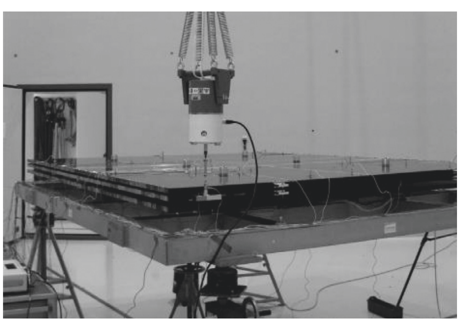

Figure 3: PIM method test input power.

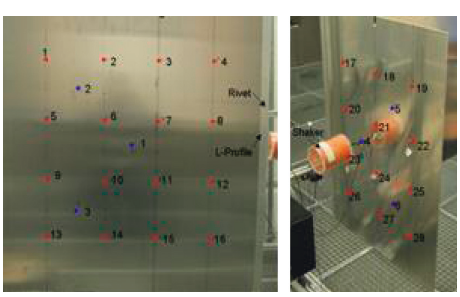

Figure 4: PIM method measurement point layout.

calculation formula. Wang [36] studied CLF of a linear vibration system and established a statistical energy analysis framework under parameter variation and random excitation. He studied the dimensionless force factors with weak, medium, and strong coupling and established the multi-objective parameter performance prediction and design optimization methods.

Although the above methods have made some progress in CLF parameter acquisition, CLF test accuracy has always been an unavoidable problem. Owing to structural limitations, it is difficult to install the input power device. The PIM input method must smoothly motivate subsystems one by one; moreover, the test is tedious, and the error is large. CLF calculation depends on the system energy matrix, and the energy is obtained by the finite average measurement points scattered throughout the subsystem, resulting in a large test error. To obtain CLF parameters more accurately, an improved input power method [37], dual formula method [38], spectral element method [39], matrix fitting method [40], and other theories have been developed in recent years.

2.3. Energy Transfer

2.3.1. Energy Path Planning. The energy transfer characteristics between subsystems of SEA have been the focus of classical statistical energy theory in recent years. Based on K shortest paths (KPS) and modified path analysis (MPS) algorithms, it has been applied in the research of statistical energy transfer characteristics. The energy transfer between subsystems is shown in Figures 5 and 6. Guasch and Aragonès [41] studied how to sort the shortest path planning among a multisource subsystem and derived the energy transmission path between substructures. They obtained the subsystem target of the SEA model in a systematic and efficient way and established the shortest path diagram of energy transfer of the SEA subsystem based on the graph theory. The MPS algorithm of improved path planning proposes the "shortest path" problem based on the KPS algorithm. MPS integrates "shortest loop" path planning and deviation path algorithm [42, 43], which can conduct in-depth research on the energy transfer characteristics of SEA. MPS can also obtain valuable information for noise control engineering [41]. Craik [44] introduced transmission path analysis in SEA, studied the energy driving mode from the source subsystem to the target subsystem, and classified the critical paths. Traditional path analysis primarily relies on experience and simple sorting, which has both a large error and high computational cost. To solve this problem, Guasch and Cortés [45] established the relationship between the SEA model and graph theory to decompose multipath complex sorting into multiple single-path problems. Aragonès and Guasch [46] proposed to solve the energy transfer problem by defining a random method. This method considers the mean value of energy loss factors, ignores variance, and transforms the random SEA energy transfer problem into an extended deterministic parameter SEA problem. Other energy transfer methods include using the standard error propagation formula to analyze the variability of energy transfer path and Monte Carlo simulation of the transfer path loss factor [47]. However, neither method forms a single sorted path list, which then cannot provide an energy transfer list. Guasch and Aragonès [48] solved the above problems. In the energy transfer path, not only are the energy mean value and variance considered, but the sorting of single paths can also be realized [48].

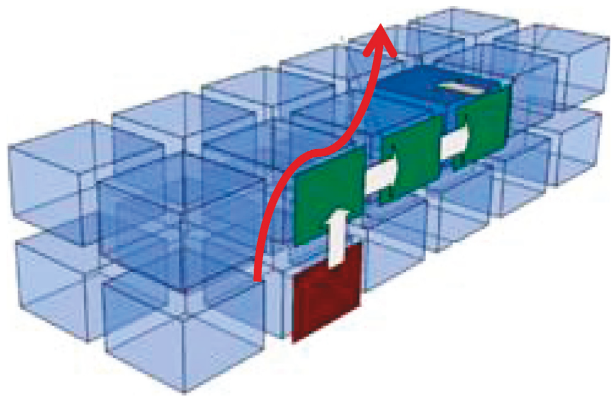

Figure 5: Primary energy transfer path between subsystems.

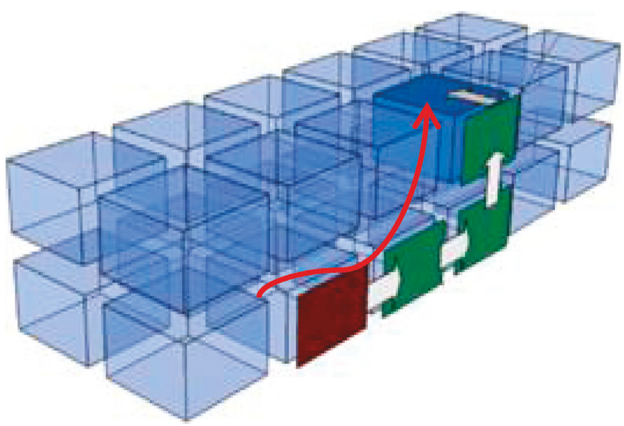

Figure 6: Nonprimary energy transfer paths between subsystems.

2.3.2. Energy Transfer Application. Culla et al. [47] discussed the benchmark structure composed of three aluminum plates, studied the parameterization method of the influence of ILF, CLF, and input power uncertainty on the energy propagation of the SEA model, and discussed the energy variability of the subsystem in the application of energy transfer. Büssow and Petersson [49] combined the energy uncertainty propagation theory and coupling loss factor sensitivity method and proposed an algorithm to calculate the factor of subsystem energy sensitivity to CLF. On this basis, the energy flow of various energy paths was analyzed. Culla et al. [50] used the resonant subsystem and velocity wave field to solve the energy flow, evaluated the confidence level of energy flow, and studied the properties of the confidence coefficient of energy flow.

2.4. Input Load Identification. Load identification is an inversion problem of structural dynamics analysis and is of great significance in reliability design, safety evaluation, and response prediction of engineering structures. At present, there are few studies on high-frequency identification of SEA [51]. In addition, existing recognition methods based on the mode method, FEM, and other methods have difficulty in accurately simulating the dynamic behavior of high frequency and cannot be widely applied to high-frequency loads. The main reason for this is that the dynamic characteristics of the high-frequency structure system are relatively complex, including the characteristics of high mode density, high degree of mode coupling, and sensitive boundary conditions. Secondly, the high-frequency noise in the measurement response may distort the original data and reduce the recognition accuracy. Based on the above problems, Xie et al. [51] proposed a high-frequency load identification method based on SEA theory, discussed load identification problems of various excitations, tested and simulated load identification of plate and shell assembly, and established a load power spectrum method. This method can accurately identify the input power of the load, predict the overall variation trend of the measured power spectrum, and is superior to the traditional load identification method. Chandra and Katipally [52, 53] studied the load identification of a flat plate structure based on the energy balance relationship between transient pulse excitation and a vibration system, and they further demonstrated the effectiveness of the energy analysis theory in the identification and analysis of transient vibration load. Mao et al. [54] studied the identification method of transient statistical impact load for a two-plate coupled structure and a three-plate coupled structure based on the energy analysis theory. From simulation and experimental study of the time-history function for the impact load spectrum of a given time-domain waveform, a new load identification parameter fitting method was proposed. This method can accurately identify the position and input energy of impact load. The impact loading time-history curve reconstructed by the fitting method is in good agreement with the actual impact loading time-history curve. The test process of multiexcitation transient impact load is shown in Figures 7 and 8. The identification method of transient SEA impact load mainly includes the following four steps:

(1) Establish the energy balance equation of the system based on the transient SEA theory, which includes determination of subsystem and model parameters.

(2) Determine the position and input energy of impact load according to the average kinetic energy response of the subsystem.

(3) Derive the amplitude spectrum of impact load from the determined input energy.

(4) Reconstruct the time-history curve of impact load according to the derived load amplitude spectrum.

2.5. Mode Density. Mode density calculation of composite materials has been widely studied by scholars at home and abroad for a long time. Its calculation theory includes the classical laminated theory (CLT), higher-order shear deformation theory considering the transverse shear effect (HOSDT) [55], block shear deformation theory [56, 57], and sandwich plate theory. Ordinary sandwich panel theory (OSPT) [58] is more accurate and efficient than CLT theory and HOSDT theory in predicting hierarchical models. Honeycomb sandwich plate is a special type of composite material, which has been widely used in spacecraft structures because of its high acoustic characteristics. In terms of the

Figure 7: External transient impact load test process.

Figure 8: Internal transient impact load test process.

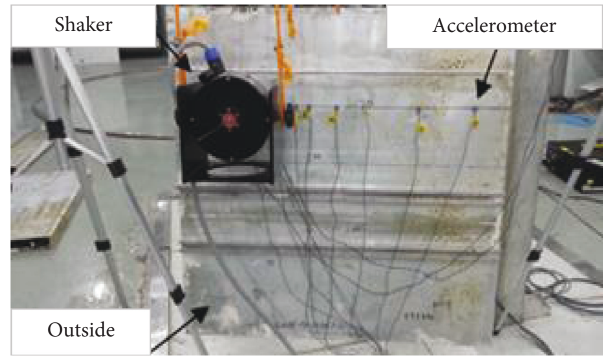

Figure 9: ESEA test process of excitation point A.

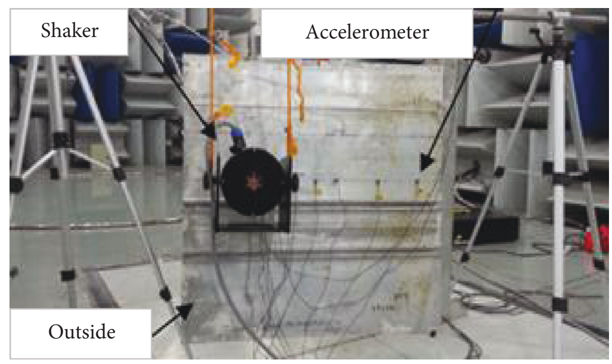

Figure 10: ESEA test process of incentive point B.

mode density research of the honeycomb plate, Zhou and Crocker [59] developed the mode density expression of the sandwich plate based on the sixth-order control equation and indicated that the flexural rigidity of the panel has a significant impact on the mode density of the high-frequency band. However, this expression only applies to sandwich plates with isotropic planes. To solve this problem, Renji et al. [60] studied the mode density of the "sandwich" honeycomb sandwich plate, derived the mode density expression of the orthogonal anisotropic honeycomb sandwich plate, and verified the correlation between mode density and shear force of the panel. They highlighted that ignoring transverse shear force or considering the panel as isotropic could lead to large errors in mode density estimation.

3. ESEA Method

3.1. ESEA Method Application. Classic SEA requires PIM to input power load into the structure to obtain system loss factor [20–22]. However, due to physical constraints, it is difficult to perform such a measurement. Ming [21] pointed out that the measurement error of input power is the main source of error of the SEA model. The purpose of experimental statistical energy analysis (ESEA) is to test the assembly structure to verify the CLF or to make theoretical estimates of it. When accurate theoretical estimation is not possible, ESEA obtains experimental data for the SEA model [61–63]. The test process is shown in Figures 9 and 10. This method does not need to input the measured power, but only needs to measure the average velocity square of the excitation point and response point (the square of the velocity transmission modulus) and calculate the space average. Guasch [64] proposed an ESEA method and established a statistical energy model composed of three subsystems by using the theory of direct and indirect propagation energy ratio and transfer rate. Bouhaj et al. [65] proposed a random energy method and studied the ESEA population mean problem by Monte Carlo simulation. They estimated the confidence interval of the normalized energy and compared the results with the energy matrix to verify the superiority of the method. Guasch and Magrans [66] proposed a new ESEA method in the framework of transmission path analysis. The energy contribution of the SEA model path group was determined by solving SEA algebraic matrix equations, and then the correlation between the ESEA model and TPA direct propagation path was verified.

3.2. ESEA Model Reduction. The ESEA method makes up for the deficiency of PIM in SEA by means of experimental measurement. However, ESEA requires all the plates to be measured by velocity admittance in order, which is difficult to test when encountering complex structures. To solve this problem, Manguan [67] proposed a matrix reduction technique based on ESEA. The reduction model can replace the complete model in the analysis, which has the advantage of retaining the original information. This method is based on the power reduction equation, and the energy is transmitted through the neglected subsystem. The reduction model can estimate the energy of the subsystem or the effective loss factor. The reduced ESEA loss matrix coefficients are based on the analyzed and neglected subsystem loss factors. The reduction technique has been shown to be effective in areas such as high-frequency acoustics-vibration. Because ESEA is relatively new, there are still many problems to be solved, for example, determining the impact of nonmeasurable quantities on the ESEA model, the wave transformation problem at the contact point (plane wave bending wave transformation) for subsystem connections, the transmission of bending waves between plates [68] in coupling loss factor measurements, and the influence of model reduction on the error of the calculation results in the subsystem.

4. FE-SEA Hybrid Method

In recent years, greater attention has been paid to the acoustics-vibration coupling problem of the intermediate frequency band, and the following three solutions have been gradually formed. The first is a type of the deterministic method which extends the application range of analytical frequency to intermediate frequency by improving the computational efficiency of the deterministic method. Research methods include modified integral formula, modified form function FEM method, multiscale FEM method, and high-order integral method. Although this method can improve the computational frequency, it ignores the uncertainty in the intermediate frequency problem. The second is a generalized SEA method. It allows SEA frequency to expand to the intermediate frequency band by relaxing the limitation of the basic hypothesis of statistical energy theory. Research methods include statistical modal energy analysis method and waveguide method. The third category is the FE-SEA method, which combines the finite element deterministic method and statistical energy nondeterministic method to solve the intermediate frequency band problem. Compared with other deterministic methods, hybrid FE-SEA method has certain technical advantages [11, 69].

4.1. Substructure Assembly and Modeling. As shown in Figures 11 and 12, FE-SEA combines the finite element method and statistical energy method through the principle of direct-hybrid field reciprocity [16, 70–73] and decomposes the complex structure into dynamic substructures of each assembly. The assembly principle is that FE components with the deterministic analysis method are defined as the main system, while SEA components with uncertain parameters are defined as subsystems. The main system and subsystems are connected by deterministic connection or deterministic components. The response of the assembly structure consists of two parts: one is provided by the degree of freedom of nodes from FE components, and the other is provided by the vibration energy of the SEA subsystem. In general, the uncertainty modeling method in the hybrid FE-SEA theory is nonparametric, which satisfies the hypothesis that the SEA subsystem is highly random and the characteristic frequency conforms to the statistical distribution [69, 71]. Nonparametric uncertainty modeling does not need to identify specific random physical parameters. It creates a set of uncertainties by introducing parameter uncertainties into FE components. This set is composed of two parts: one is a nonparametric SEA subsystem, and the other is a parameterized FE component [74]. In this paper, a combined method is proposed for uncertain modeling. This method combines the FE-SEA method with the Laplace method to evaluate the probability that the response variable of the system exceeds the limit value, and the effectiveness of this method is verified by two combination plates with uncertain characteristics.

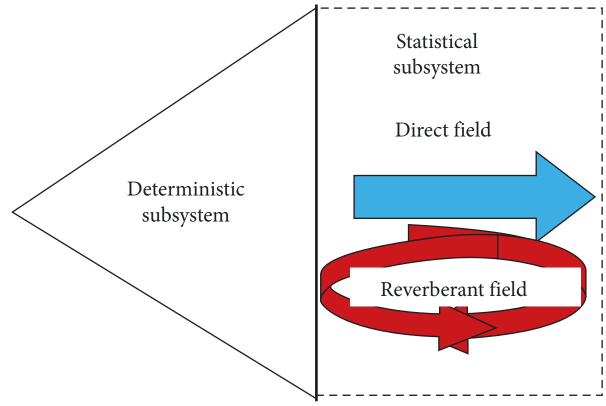

Figure 11: Direct-hybrid field reciprocal relationship.

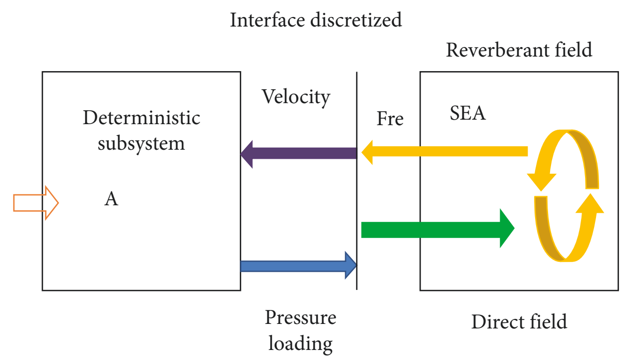

Figure 12: Direct-hybrid transfer relationship.

4.2. Parameter Uncertainty

4.2.1. Classification of Parameter Uncertainty. Parameter uncertainties in hybrid FE-SEA uncertainty problems are generally divided into two categories. The first category is probabilistic uncertainty, and the second category is nonprobabilistic uncertainty. Probabilistic uncertainty generally includes fuzzy set theory, stochastic theory, and other methods. The nonprobabilistic uncertainty includes the interval analysis method and hybrid parameter method. For the first class of the probabilistic uncertainty problem, the probability density function of uncertain parameters needs to be solved for fuzzy set theory and random theory [75, 76]. Unfortunately, in engineering practice, there are limitations of data sample space, data error, and other factors, which restrict the further development of this technology. However, for the second class of the probabilistic uncertainty problem, interval analysis does not require large sample data space and only requires less data and structural feature information to comprehensively consider the system uncertainty, so as to obtain an interval solution with real values.

4.2.2. Interval Analysis. Interval analysis is an effective method for nonprobabilistic analysis of uncertain parameters. Commonly used methods include interval Gaussian elimination method, interval iteration method, nonprobability interval method, interval correspondence method, interval perturbation finite element method, and Legendre orthogonal polynomial method. [77–83]. In recent years, some new research methods for interval analysis have been developed. Wu et al. [84, 85] studied the dynamic response by using the Chebyshev polynomial interval method. The advantage of this method is that it does not need to determine the probability distribution of parameters but only needs to determine the parameter interval range and obtain the system response interval by solving the parameter equation. However, this method only applies to the case where the range of interval parameters is small, and it has a large solution error with a broad range of interval parameters. For larger parameter intervals, subinterval perturbation method can be used. The interval is divided into several subintervals to solve, and the subintervals of the response are found. Then, by assembling these subintervals, the global interval of the response can be obtained. Wang and Qiu [86] proposed an improved algorithm for interval parameter perturbation and applied this method to the prediction of external sound field noise. The improved interval parameter perturbation method preserved the higher-order terms to calculate the inverse of the interval matrix. They used the Taylor series correction algorithm to approximate interval matrix vector and compared the difference between Monte Carlo simulation and interval parameter perturbation examples. Zhang [87] proposed a finite element and distribution method for solving interval linear equations of the system. In this method, the interval values of nondeterministic parameters were separated, and each interval boundary was determined by calculating the extremum problem of the solution of the equation. Su et al. [88] proposed a method for inverse analysis of the unknown parameter interval, compared and studied the influence of different test accuracy and different target parameters on the results of inverse analysis, and deduced the convergence conditions for better understanding.

4.2.3. Hybrid Parameter Analysis. Interval parameter analysis is based on the nonprobabilistic uncertainty method, and probability distribution is based on the probabilistic uncertainty method. However, different types of parameter uncertainties may exist in actual engineering. In recent years, hybrid parameter uncertainty analysis has attracted increased attention. Gao et al. [89] proposed a random interval perturbation method for hybrid parameter analysis of uncertain structures and used this method to determine the random distribution and interval parameters. Yin et al. [90] introduced hybrid nonprobabilistic fuzzy and interval uncertainty into the composition of hybrid FE. Based on the combination of nonparametric hybrid fuzzy and interval parameter uncertainty, the hybrid model of nonparametric combination mode was established. They proposed a fuzzy interval finite element/statistical energy analysis (FIFE/SEA) framework to obtain the uncertain response of the combined system. In order to effectively deal with the uncertainty of hybrid parameters, Wang et al. [91] have introduced the first-order fuzzy interval perturbation method into the hybrid FE-SEA framework, proposed the first-order fuzzy interval perturbation FE/statistical energy analysis (FFIPFE/SEA), and verified the effectiveness of the method. Yin et al. [92] introduced interval parameter uncertainty and dynamic interval into a structural acoustic system based on hybrid FE-SEA framework, established a nonparametric equation and nonparametric hybrid model, and obtained interval parameter uncertainty parameters of the structural acoustic system. To further improve the computational efficiency, they put forward the second-order interval perturbation finite element/statistical energy analysis method (SIPFEM/SEA). In the SIPFEM/SEA method, the expectation of the second-order response quantity is obtained by the standard value of the interval parameter of the second-order Taylor series expansion. To perform the calculation efficiently, the nondiagonal elements of the Hessian matrix are ignored. By searching the target interval parameters to maximize or minimize the target function, the vibration energy boundary and response cross spectrum are obtained. However, this method is only suitable for narrow parameter interval analysis because it ignores the higher-order terms of Taylor series. Xia et al. [93] proposed a hybrid perturbation vertex method for uncertain structures and conducted acoustic analysis of hybrid random interval parameters. Chen et al. [94] developed a hybrid perturbation method for external sound field analysis with uncertain random interval parameters. Cicirello and Langley [95, 96] used the hybrid FE-SEA method and modal component synthesis (MCS) method to deal with the hybrid uncertainty of fuzzy random parameters and verified the effectiveness of the method. Literature [91, 97, 98] proposed a hybrid fuzzy random reliability analysis method based on transformation mode by studying the equivalent transformation between fuzzy variables and random transformation.

4.2.4. Fuzzy Parameter Analysis. Fuzzy set theory is an effective method to solve interval uncertainty problems. So far, there are two main methods for fuzzy analysis. The first one is based on the global variable optimization method, which has not only a high computational accuracy but also a large amount of computation. A large number of optimization problems make it difficult to be applied in engineering. The second method is based on interval algebra, in which fuzzy variables are considered as interval variables of each cutting level, and its uncertainty is predicted by using the classical interval algorithm. Interval fuzzy algebraic methods are mostly applied to the interaction between thin-walled structures and sound fields. The research results of interval parameter uncertainty based on fuzzy set theory are described next. Wang et al. [99] proposed the first-order subinterval perturbed finite element method and the improved subinterval perturbed finite element method. These methods are based on the finite element framework and uncertainty analysis theory and used the finite element method to solve the structural acoustic interval parameter problem with great ambiguity and uncertainty. The original fuzzy model is improved, the interval parameters are decomposed into several uncertain smaller intervals, and the subinterval matrix and vector are expanded by Taylor series. Finally, the response is reconstructed by using interval set operation and fuzzy decomposition theorem. Wang et al. [100] proposes a modified interval parameter perturbation finite element method (MIPPM), which uses the high-order term of Neumann series to approximate the inverse of the interval matrix. At the same time, the idea of reliability is introduced to establish the degree of satisfaction of the optimization model interval. Compared with the traditional method, this method can obtain more accurate interval response parameters. The numerical model of structure-cavity is shown in Figure 13 [100], which verifies the effectiveness of the algorithm. Xua et al. [101] proposed a nonintrusive, double polynomial chaos expansion and dimension-wise analysis (PCE-DW) method, which is used for quantitative statistical characteristics and interval parameter structure of the acoustic system response. Based on the sparse grid allocation strategy in the method of polynomial chaos expansion (PCE), program forecast was generated by the statistical characteristics; at the same time, the dimension-wise (DW) method was used to determine the system response interval expectation and standard variance. Finally, the validity of the algorithm is tested for two structure-acoustic systems with mixed uncertainties.

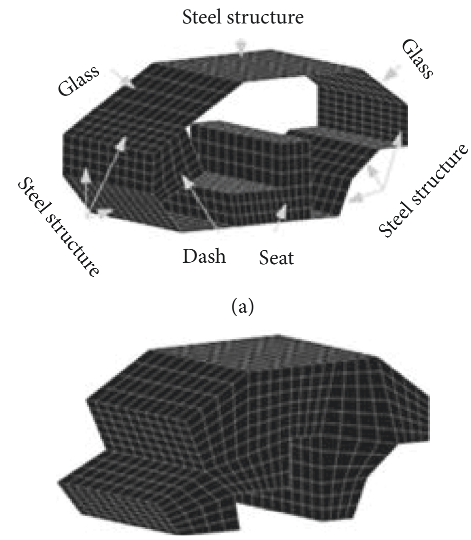

Figure 13: Structure-acoustic hybrid model validation example: (a) FEA model of the solid structure; (b) FEA model of the acoustic fluid.

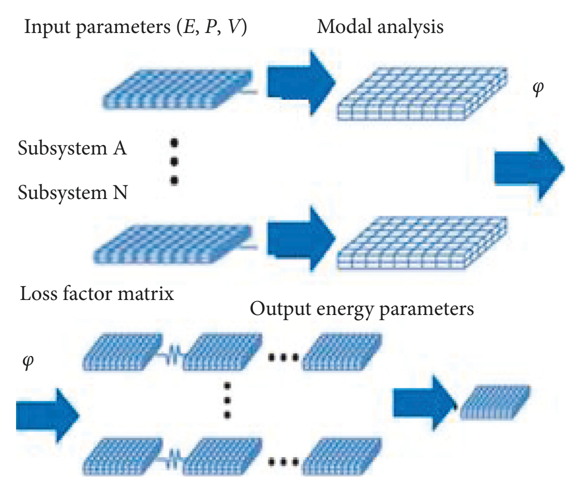

Figure 14: SMEDA method calculation flow.

5. SMEDA Method

5.1. SMEDA Method Application. Statistical modal energy distribution analysis (SMEDA) does not assume an even distribution of modal energy and is applicable to coupling of subsystems with low mode overlap, coupling of heterogeneous subsystems, local excitation, and coupled continuous elastic systems in general [6]. This method takes the two-mode formula [38] as the theoretical basis and the input parameters of subsystems as process variables to obtain the energy distribution of each subsystem. The calculation process is shown in Figure 14. The SMEDA equation depends only on the source characteristics (i.e., position, autospectral density, and mode). The information of each uncoupled subsystem (mode angular frequency, mode damping loss factor, and mode shape) can be obtained from the FEM of a subsystem with complex geometric or heterogeneous mechanical properties. Therefore, the SMEDA method can be regarded as an improvement over the traditional SEA method. The SMEDA model expands the SEA method to the intermediate frequency range by establishing the energy balance equation between different subsystems. It also takes the mode basis of the uncoupled subsystem estimated by FEM in the intermediate frequency domain as the input data. This method was originally developed by considering the constant mode damping factor of each subsystem, which could not describe the local distribution of dissipated materials. To overcome this problem, Maxit and Guyader [6, 102] proposed a statistical method of mode energy including dissipated materials. This method is based on the subsystem FEM and the homogenous material model of dissipation processing to process the structural plate cavity system. The Galerkin subsystem method is adopted to obtain the mode damping loss factor from the imaginary part of the mode projection by ignoring the crossover mode. The effectiveness of this method is verified by an experiment. Aragonès et al. [103] studied the energy transfer between subsystems by using SMEDA and identified and sorted the mode resonance and nonresonance paths of subsystems in the state of low mode overlap. Guyader et al. [104] applied the two-mode equation and SMEDA to study system uncertainty problems. In this method, coupling loss factor and damping loss factor are defined as uncertain parameters, and the system uncertainty is estimated by subsystem mode energy waveguide and total average energy. Maxit et al. [105] proposed a SMEDA expansion method considering the contribution of a nonresonant mode. Based on the mode reduction equation, this method studied the transfer loss between two cavities and proposed a new mode coupling scheme. Maxit et al. [105] studied the coupling and uncertainty of subsystems by using SMEDA. This method calculated coupling loss factor and internal loss factor from FEM results, estimated the overall statistical data of uncertain propagation with subsystem input parameters and coupling model natural frequency, and finally verified the effectiveness of this method through experiments and numerical calculation.

6. Waveguide Method

At present, the main solutions for the coupling problem of intermediate frequency acoustic vibration include the FE-SEA hybrid method, ESEA, SMEDA [6, 29], energy FEM [106–108], and waveguide analysis (WGA) [109]. WGA considers the angular power of the subsystem and the flow balance of the system, which can provide angular distribution of plate energy. WGA is more applicable when the size of the system in a certain direction is longer relative to the wave length. In terms of the application of waveguide theory, the WGA stiffness method was applied in literature [110–112] to decompose the wave components and study the forced vibration and energy distribution of a composite system of rectangular plates. In literature [113–116], wave FEM was used to analyze forced vibration of complex structures by using complex ray variational theory and Fourier series method, and the analysis efficiency and accuracy were verified. Zhou et al. [117] proposed Timoshenko beam time average power and energy function defined in the wave mode on the basis of the energy flow method and proved the incoherence between traveling waves and attenuated waves. Hopkins [117] considered the conversion of planar waves and bending waves between plates and determined the coupling loss factor of bending wave transfer between coupled plates by the experimental method. In [118–120], the dispersion relation of the waveguide structure with FEM was studied, but the stability was poor [121]. To solve this problem, the dual method [122, 123] was used to obtain the wave dispersion equation and analytical expression, as well as the simple boundary conditions of any combination. Dual analysis avoids boundary conditions and the poor accuracy and instability encountered in traditional wave propagation [124]. Langley and Cordioli [17, 125] applied the spectral finite element method (SFEM) to a truss structure in high-frequency acoustic analysis and, based on the structural element, extended SFEM to a 3D waveguide system. Although the WGA method has lower computational costs than the FEM method, the number of structural degrees of freedom has been greatly reduced which improves the computational efficiency and stability of the waveguide method. It can accurately describe the dynamics of simple structures such as truss structures and plate and shell structures, but there are still many problems in waveguide analysis of complex structures. WGA method is prone to the instable and poor accuracy in wave propagation. Although the dual method can solve similar problems, the accuracy of the calculation remains to be verified. In addition, the application of the dual method cannot be used to quickly modify the design of complex systems.

7. Conclusion and Reflection

Based on the above review, technical difficulties that may be encountered in the research of SEA and intermediate frequency extension methods are summarized:

7.1. Classic SEA and CLF. The classical SEA method has the following limitations:

(1) Assume that the mode is evenly divided, that is, the energy of the mode-free control frequency band.

(2) The ratio of coupling loss and the factor of low internal loss coefficient.

(3) The coupling between subsystems must be conservative and not applicable to the strongly coupled state.

(4) The excitation must be broadband, and the spatial distribution is irrelevant.

(5) The premise of the SEA model is weak coupling, which is controlled by the local mode among subsystems in the high-frequency band but is not applicable in the intermediate and low frequency band. Although the nonconservative and strongly coupled theory extends the basic SEA theory, the establishment of a balance equation with the general coupling mode and the solution of CLF are difficult and need to be further developed.

(6) In terms of CLF acquisition, it is difficult to arrange an excitation source device for actual measurement in the PIM method, and the experimental estimation of input power will generate a large error. Moreover, PIM cannot directly measure the energy of the system, and indirect testing will lead to cumulative errors. Test results depend on subsystem partition. Because there is no clear division criterion, the reproducibility of the analysis results is poor. The application of the principal component method and clustering method is the new trend of subsystem partition.

7.2. ESEA Method

(1) The problems existing in ESEA methods lie in uncertainties, such as subsystem partition uncertainty, no effective excitation and response sampling, difficulty in obtaining the quality of a heterogeneous subsystem, and digital uncertainty of matrix equation inversion. These factors may generate errors in the solution process.

(2) The ESEA method needs Monte Carlo simulation to study the population mean problem. Monte Carlo simulation depends on a certain probability model.

To obtain a small theoretical variance, a large sample interval is needed, which also easily leads to uncertainty problems. Other uncertainties include how to determine the impact of nonmeasurable quantities on the ESEA model. In a subsystem connection, the wave transformation uncertainty at the contact point and ESEA model reduction can increase the uncertainty of calculation results.

7.3. SMEDA and Waveguide Method

(1) Damping factors are usually implemented in the same way for both subsystem and coupling model layers, ignoring uncertainties and differences in damping factors in the coupling model layer.

(2) The WGA method is prone to instability in the wave propagation process, and the application of this method cannot make rapid design modifications for complex systems.

7.4. FE-SEA Method

(1) The deterministic part of the FE-SEA method includes FE components and requires the FEM method, so there is a risk of low interpolation accuracy and numerical error [125]. At the same time, because finer FEM is needed to obtain accurate calculation results, the calculation time cost will increase.

(2) FE-SEA interval analysis method does not need to determine the probability distribution of parameters but only needs to determine the parameter interval to obtain the system response. However, for large-scale interval parameters, the solution error is large, and the time cost is unacceptably high. The subinterval method is applied to solve the subinterval segmentation problem, but the precision and cumulative error of subinterval assembly cannot be effectively resolved.

(3) For mixed interval analysis, interval perturbation method based on Taylor series expansion has been widely used. The approximate results of the expansion of first-order Taylor series by linear function may make the relative error of nonlinear function larger. The approximation accuracy of the nonlinear function can be improved by using the second-order Taylor series expansion. However, because standard values of interval parameters of the second-order Taylor series expansion are used, the Hessian matrix nondiagonal elements and higher-order terms of the Taylor series are ignored. Therefore, these values are only applicable to the analysis of narrow parameter intervals, and the analysis of large interval parameters will have a large error.

With the above technical difficulties, further scientific research and technical requirements can be considered as follows:

(1) In the field of high-frequency SEA, the test errors and correction algorithms of CLF, especially the calculation of CLF of complex systems, are the focus of further research.

(2) *High-frequency input load identification.* High-frequency load identification has the characteristics of high mode density, high degree of mode coupling, sensitive boundary conditions, etc. The load power spectrum method and load spectrum noise reduction algorithm can provide possible ideas for load identification.

(3) *Intermediate frequency SEA analysis.* The development of a hybrid interval algorithm suitable for larger intervals is the development trend of the FE-SEA method. The improvement of calculation accuracy and efficiency of large intervals is also the focus of future development of this technology.

Appendix

A. Example of Error Propagation for Coupling Loss Factors of Two Substructures

The calculation formula for the CLF system of a typical two-substructure system is as follows:

$$\begin{Bmatrix} P_{1,\text{in}} \\ P_{2,\text{in}} \end{Bmatrix} = \omega \begin{bmatrix} \eta_{11} + \eta_{12} & -\eta_{21} \\ -\eta_{12} & \eta_{21} + \eta_{22} \end{bmatrix} \begin{Bmatrix} \langle E_1 \rangle \\ \langle E_2 \rangle \end{Bmatrix}, \quad \text{(A.1)}$$

where η_{ii} and E_i, respectively, represent the internal loss factor and average energy of subsystem i, η_{ij} represents the coupling loss factor $(i \neq j)$ from subsystem i to subsystem $j\,(i \neq j)$, and $P_{1,\text{in}}$ represents the input power of subsystem i. These parameters satisfy the consistency relation:

$$\eta_{ij} n_i = \eta_{ji} n_j, \quad \text{(A.2)}$$

where n_i and n_j represent the mode density per Hz of subsystem i and subsystem j, respectively. The basic property of the SEA energy equation is the power and energy exchange between subsystems. The energy exchange formula can be obtained by motivating subsystem i and subsystem j, respectively:

$$P_{12} = \omega \left(\eta_{12} \langle E_1 \rangle - \eta_{21} \langle E_2 \rangle \right), \quad \text{(A.3)}$$

where P_{ij} is the power transferred from subsystem i to subsystem j. CLF is calculated by numerical simulation of the same vibration sound problem and equation (A.1) to obtain the average energy. There are three different methods to calculate the coupling loss factor. Two of them are based on the SEA equation, and the other one is based on power conversion and connection. The first expression is obtained from the power balance of equation (A.1):

$$\eta_{12} = \frac{P_{1,in}/\omega - \eta_{11} \langle E_1 \rangle}{\langle E_1 \rangle - (n_1/n_2) \langle E_2 \rangle}. \quad \text{(A.4)}$$

The second expression is obtained from equation (A.2):

$$\eta_{12} = \frac{\eta_{22} \langle E_2 \rangle}{\left(\langle E_1 \rangle - (n_1/n_2) \langle E_2 \rangle \right)}. \quad \text{(A.5)}$$

The third expression is obtained from equation (A.3):

$$\eta_{12} = \frac{P_{12}}{\omega\left(\langle E_1\rangle - \left(n_1/n_2\right)\langle E_2\rangle\right)}. \tag{A.6}$$

In order to check the reliability of these calculations, error analysis is required, and the relative error calculation formula is

$$r_L = \frac{L-\overline{L}}{L}, \tag{A.7}$$

where r_L represents the system relative error, $\overline{L}$ is the estimated value, and L is the accurate value. Classical SEA error analysis requires the following assumptions:

(1) $\langle E_1\rangle \geq \langle E_2\rangle$, which satisfies the weak coupling hypothesis of classical SEA theory.

(2) In order to simplify error calculation, it is assumed that the two subsystems have the same attributes, that is, $\eta_{11} = \eta_{22}$, $n_1 = n_2$.

(3) Set the internal loss factor as a constant value. Reduce the influence of the internal loss factor error on the overall error. The error expression of the first expression is

$$\begin{aligned} r_{\eta_{12}} &\approx \left(r_{P_{1,\text{in}}} - r_1\right)\frac{\left(P_{1,\text{in}}\right)}{\omega\langle E_1\rangle\left(\left(P_{1,\text{in}}/\left(\omega\langle E_1\rangle\right)\right) - \eta_{11}\right)} \\ &= \left(r_{P_{1,\text{in}}} - r_1\right)\left(1 + \frac{\langle E_1\rangle\eta_{11}}{\left(P_{1,\text{in}}/\omega\right) - \eta_{11}\langle E_1\rangle}\right) \\ &= \left(r_{P_{1,\text{in}}} - r_1\right)\left(1 + \frac{\langle E_1\rangle}{\langle E_2\rangle}\right) \\ &= \left(1 + \frac{\langle E_1\rangle}{\langle E_2\rangle}\right)r_{P_{1,\text{in}}} - \left(1 + \frac{\langle E_1\rangle}{\langle E_2\rangle}\right)r_1, \end{aligned} \tag{A.8}$$

where $r_{P_{1,\text{in}}}$ and r_1, respectively, represent the calculation error of $P_{1,\text{in}}$ and $\langle E_1\rangle$. Similarly, the error formulas of expressions (A.2) and (A.3) can be derived:

$$\begin{aligned} r_{\eta_{12}} &= \frac{r_2 - r_1}{1 - r_1} \approx r_2 - r_1, \\ r_{\eta_{12}} &\approx r_{p_{12}} - r_1, \end{aligned} \tag{A.9}$$

where r_2 is the calculation error of $\langle E_2\rangle$ and $r_{p_{12}}$ is the calculation error of p_{12}. The above error calculation can correct the SEA model effectively.

B. Interval Variables with Parameter Uncertainty

In the hybrid FE-SEA model, the interval parameter of the FE component can be expressed by vector and its expression is

$$a = \left(a_i\right)\left(i = 1, 2, \ldots, n\right), \tag{B.1}$$

where i represents the number of components; for a given fixed value a, the average energy E of the subsystem and the crosspower spectrum function S_{qq} of the main system can be expressed as follows:

$$E = E(a), \tag{B.2}$$

$$S_{qq} = S_{qq}(a). \tag{B.3}$$

If the parameter set a is considered as an interval vector, it can be expressed as

$$\begin{aligned} a^I &= \left[a^l, a^u\right] = \left(a_i^I\right), \\ a_i^I &= \left[a_i^l, a_i^u\right], \end{aligned} \tag{B.4}$$

where $a \in a^I$ and $a_i \in a_i^I$ $(i = 1, 2, \ldots, n)$ in which a^l and a^u, respectively, represent the upper and lower limits of interval set a, and a_i^l and a_i^u, respectively, represent the upper and lower boundary of interval parameter a_i. Since the input parameter is an interval variable, the response variables of equations (B.2) and (B.3) can be expressed as

$$E^I = \left[E^l, E^u\right] = E\left(a^I\right), \tag{B.5}$$

$$S_{qq}^I = \left[S_{qq}^l, S_{qq}^u\right] = S_{qq}\left(a^I\right), \tag{B.6}$$

where E^l and E^u and S_{qq}^l and S_{qq}^u satisfy the following relation:

$$\begin{aligned} E^l &= \min\left\{E \middle| E = E(a), a \in a^I\right\}, \\ E^u &= \max\left\{E \middle| E = E(a), a \in a^I\right\}, \\ S_{qq}^l &= \min\left\{S_{qq} \middle| S_{qq} = S_{qq}(a), a \in a^I\right\}, \\ S_{qq}^u &= \max\left\{S_{qq} \middle| S_{qq} = S_{qq}(a), a \in a^I\right\}. \end{aligned} \tag{B.7}$$

Based on the principle of interval transformation, equation (A.10) can be expressed as

$$\begin{aligned} a^I &= \left[a^l, a^u\right] = \left[a^m - \Delta a, a^m + \Delta a\right] = a^m + \Delta a^I = a^m + \Delta a e^I, \\ a_i^l &= \left[a_i^l, a_i^u\right] = \left[a_i^m - \Delta a_i, a_i^m + \Delta a_i\right] = a_i^m + \Delta a_i^l \\ &= a_i^m + \Delta a_i e^I, \quad (i = 1, 2, \ldots, n), \end{aligned} \tag{B.8}$$

where $a^m = (a_i^m) = ((a^l + a^u)/2), a_i^m = ((a_i^l + a_i^u)/2)$, $\Delta a = (\Delta a_i) = ((a^u - a^l)/2), \Delta a_i = ((a_i^u - a_i^l)/2)$, $\Delta a^I = [-\Delta a, \Delta a]$ $\Delta_i^I = [-\Delta a_i, \Delta a_i]$, and $e^I = [-1, 1]$; at the mean variable a^m, the interval variables E^I and S_{qq}^I are expanded by using the second-order Taylor series, and then equations (B.2) and (B.3) can be expressed as

$$\begin{aligned} E^I = E\left(a^I\right) &= E\left(a^m\right) + \sum_{i=1}^n \frac{\partial E\left(a^m\right)}{\partial a_i}\Delta a_i^I \\ &+ \frac{1}{2}\sum_{i=1}^n\sum_{j=1}^n \frac{\partial^2 E\left(a^m\right)}{\partial a_i \partial a_j}\Delta a_i^I \Delta a_j^I, \end{aligned} \tag{B.9}$$

$$\begin{aligned} S_{qq}^I = S_{qq}\left(a^I\right) &= S_{qq}\left(a^m\right) + \sum_{i=1}^n \frac{\partial S_{qq}\left(a^m\right)}{\partial a_i}\Delta a_i^I \\ &+ \frac{1}{2}\sum_{i=1}^n\sum_{j=1}^n \frac{\partial^2 S_{qq}\left(a^m\right)}{\partial a_i \partial a_j}\Delta a_i^I \Delta a_j^I, \end{aligned} \tag{B.10}$$

where $E\left(a^m\right)$ and $S_{qq}\left(a^m\right)$ are the value of interval variable E^I and S_{qq}^I at mean variable a^m. Similarly, according to equation (B.9) and (B.10), the expression of the p element at the interval response vector S_{qq}^I can be obtained:

$$S_{qq,p}^I = S_{qq,p}\left(a^I\right) = S_{qq,p}\left(a^m\right) + \sum_{i=1}^{n} \frac{\partial S_{qq,p}\left(a^m\right)}{\partial a_i} \Delta a_i^I + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial^2 S_{qq,p}\left(a^m\right)}{\partial a_i \partial a_j} \Delta a_i^I \Delta a_j^I. \quad \text{(B.11)}$$

If equations (B.9) and (B.11) are expressed as the gradient vector and HESSIAN matrix, the equation become

$$\begin{aligned} E^I &= E\left(a^I\right) = E\left(a^m\right) + \left(E'\left(a^m\right)\right)^T \Delta a^I + \frac{1}{2}\left(\Delta a^I\right)^T E''\left(a^m\right) \Delta a^I, \\ S_{qq,p}^I &= S_{qq,p}\left(a^I\right) = S_{qq,p}\left(a^m\right) + \left(S_{qq,p}'\left(a^m\right)\right)^T \Delta a^I + \frac{1}{2}\left(\Delta a^I\right)^T S_{qq,p}''\left(a^m\right) \Delta a^I, \end{aligned} \quad \text{(B.12)}$$

where $E'\left(a^m\right)$ and $S_{qq,p}'\left(a^m\right)$ are the gradient vector, $E''\left(a^m\right)$ and $S_{qq.p}''\left(a^m\right)$ are the Hessian matrix at mean variable a^m. In the above interval solution method, when the number of interval parameters is large, the calculation of the Hessian matrix will be very heavy. In addition, it is difficult to solve hybrid partial derivatives in the Hessian matrix.

C. Hybrid Interval Variables with Parameter Uncertainty

If the objective information of uncertain parameters is not enough to construct the probability model, then fuzzy set theory can be used as the nonprobability method to describe uncertain parameters. The membership function of the fuzzy uncertainty vector $a^F = \left(a_1^F, a_2^F, \ldots, a_1^F\right)$ is expressed as

$$L_i = f_i\left(a_i^F\right), \quad a_i^F \in U_i, L_i \in [0,1], 1, 2, \ldots, n, \quad \text{(C.1)}$$

where L_i is the membership function value and U_i is the domain of vector a^F. If the subjective expert opinion cannot construct the membership function, the interval model is the lower bound and upper bound of uncertainty, and the uncertain parameters are described by the nonprobability method. Interval uncertainty is represented by the interval vector $p^I = \left(P_1^I, P_2^I, \ldots, P_I^I\right)$, and then interval uncertainty can be expressed as

$$\begin{aligned} P^I &= \left[\underline{P}, \overline{P}\right] = P^m + \Delta P^I, \\ \Delta P^I &= \left[-\Delta P, \Delta P\right], \\ p^m &= \frac{\underline{P} + \overline{P}}{2}, \\ \Delta P &= -\frac{\underline{P} - \overline{P}}{2}, \end{aligned} \quad \text{(C.2)}$$

where $\underline{P}$ and $\overline{P}$, respectively, represent the upper limit and lower limit of the interval vector, p^m represents the mean of the interval vector, ΔP^I represents the interval vector error, and ΔP represents the error interval range. In engineering practice, both of these situations may exist simultaneously. Uncertainty analysis under hybrid fuzzy and interval input parameters is required. Considering the addition function G of a fuzzy variable a^F and an interval variable P^I, the function G can be expressed as

$$G\left(a^F, P^I\right) = a^F + P^I, \quad \text{(C.3)}$$

where $P^I = \left[\underline{P}, \overline{P}\right]$, If formula (B.5) is converted by the interval algorithm, then

$$G\left(a^F, P^I\right) = \left[a^F + \underline{P}, a^F + \overline{P}\right], \quad \text{(C.4)}$$

where $a^F + \underline{P}$ is the lower limit of the hybrid interval and $a^F + \overline{P}$ is the upper limit of the hybrid interval.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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