Research Article

Free Vibration of Rectangular Plates with Porosity Distributions under Complex Boundary Constraints

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Free vibration of rectangular plates with three kinds of porosity distributions and different boundary constraints has been performed by means of a semianalytical method. The distribution of porous varies along the thickness of the plate, in which the mechanical properties are defined by open-cell metal foam. Regardless of boundary conditions, displacement admissible functions are represented by combination of standard cosine Fourier series and auxiliary sine series. The kinetic energy and potential energy of plates are also expressed on the basis of first-order shear deformation theory (FSDT) and displacement admissible functions. Finally, the coefficients in the Fourier series which determine natural frequencies and modal shape are derived by means of the Rayleigh–Ritz method. Convergence and dependability of the current method are verified by comparing with the results of FEM and related literatures. In addition, some new results considering geometry parameters under classical and elastic boundary constraints are listed. The effects of geometry parameters, material parameters, and boundary constraints have been discussed in detail.

1. Introduction

As one of the most widely used structures, rectangular plates have been used in many branches of mechanical engineering. For the excellent mechanical properties, functionally graded materials (FGMs) have attracted researcher's attention in recent years. In addition, functionally graded (FG) porous material becomes a research hotspot as a new type of lightweight material because of the porous distribution through the thickness direction [1]. The aim of this paper is to establish a unified formulation to analyze the vibration behaviors of rectangular plates made of functionally graded (FG) porous materials under complex boundary constraints. The related literatures are reviewed as below.

Firstly, related literatures of FG structures are reviewed: Baferani et al. [2] studied vibration characteristics of functionally graded rectangular plates on the basis of classical plate theory (CPT). Zenkour [3] discussed the static response of the FG rectangular plate with simply supported boundary conditions by means of Navier solutions. Liu et al. [4] investigated the dynamic response of the FGM plate

using the element-free Galerkin method. Lanhe et al. [5] analyzed the dynamic behavior of FGM plates subjected to aero-thermo-mechanical loads by means of the differential quadrature method. Tornabene et al. [6–8] combined the generalized differential quadrature method and first-order shear deformation theory to investigate the free vibration behaviors of the moderately thick four-parameter functionally graded conical, cylindrical shells, and annular plates subjected to classical boundary conditions. Viola et al. [9] derived stress-strain recovery formulation to study the functionally graded spherical shells and panels under static loading. Li et al. [10, 11] used the energy method to investigate the vibration characteristics of composite cylindrical and spherical shells. The results in these references are compared with those obtained by published literatures and model experiment. To investigate the vibration characteristic of Levy-type thick functionally graded (FG) circular cylindrical shell, Hosseini-Hashemi et al. [12] extended the method of state space to establish the model. Qu et al. [13, 14] analyzed the vibration of FG doubly curved shells by means of the domain decomposition method. A new generic

exact solution was proposed by Fadaee et al. [15] to analyze the free vibration of FG doubly curved shallow shells. Li et al. [16–19] studied the free vibration of FG composite doubly curved shells by using a semianalytical method.

Moreover, many research studies have been conducted about structures made of porous materials: Fazzolari [20] studied the free vibration of three-dimensional functionally graded sandwich beams with two different types of porosity and arbitrary boundary conditions. Moreover, Ebrahimi and Zia [21] utilized Galerkin's method and the method of multiple scales to solve the nonlinear vibration of functionally graded porous beams. Chen et al. [22] implemented the vibration analysis of porous beams using Von Kármán's nonlinear strain-displacement relationships. Ebrahimi and Habibi [23] studied vibration behavior and deflection of rectangular plates made of functionally graded porous materials by means of the finite element method, where the material properties of functionally graded porous rectangular plates have been achieved by means of power-law form. Magnucka-Blandzi [24] derived the critical loads of metal foam circular plates by means of the Hamilton principle, where the influence of porosity on critical loads and comparison between homogeneous and porous plates was also conducted. A simple first-order shear deformation theory (FSDT) was applied to solve natural frequencies of FGP plates by Rezaei et al. [25] utilizing Hamilton's principle and variational method. Simultaneously, free vibration and buckling analyses of FGP nanocomposite plates reinforced by graphene were carried out by Yang et al. [26], where the Chebyshev–Ritz method was employed. On the basis of FSDT, Li et al. [27, 28] investigated the free vibration of functionally graded porous cylindrical shell, and the results are compared with those obtained by experiment. Wang and Wu also [29] considered two types of porosity distributions to study the free vibration of FGP cylindrical shell subject to immovable boundary restraints. Based on the theory of modified couple stress and first-order shear deformation, the vibration characteristics of functionally graded (FG) porous cylindrical microshell were investigated by Ghadiri and SafarPour [30]. Zhao et al. [31, 32] used the modified Fourier series approach to investigate the free vibration of FG porous doubly curved shells subject to general edge restraints based on FSDT.

From literatures mentioned above, it is easy to find that numerous research studies of structures with porosity distributions have been conducted in recent years. However, most research studies are limited to classical boundary conditions. Few research studies about vibration characteristics of structures with porosity distributions subjected to elastic boundary conditions have been analyzed. Therefore, it is necessary to propose a unified method to jointly analyze the vibration characteristics of porous rectangular plates with classical and elastic boundary conditions. In the current research, the displacement admissible function is expressed as modified Fourier series, which was proved reliable and efficient by previous studies [33–35]. Complex boundary conditions can be easily simulated by only varying the stiffness of the boundary springs. The convergence and exactness of the current method have been verified by comparing with the results of published literatures. In addition, the effects of geometry parameters, material parameters, and boundary conditions on dynamic behavior have also been analyzed.

2. Theoretical Formulations

2.1. Description of the Model. Figure 1(a) presents the outline and coordinates of porous rectangular plates, and the geometry parameters are, respectively, defined as follows: the length a , width b , and thickness of plate h . To illustrate the geometric relationship of the plates more comprehensively, a Cartesian coordinate system was fixed on the middle of the plate. The symbols of U , V , and W , respectively, denote displacements of porous rectangular plates in x , y , and z directions. Figure 1(b) denotes the boundary condition of plates, three groups of linear restrain springs (k_u , k_v , and k_w), and two groups of rotational springs (K_x and K_y) are assigned at the boundary of the plate to simulate complex boundary constraints. Meanwhile, partial view of plates with translational spring along u , v , and w directions and rotational spring is demonstrated in Figure 1(c). By setting proper spring stiffness values, various boundary constraints of the plate can be simulated.

Three types of porosity distributions are considered in current research. As displayed in Figure 2(c), in the case of uniform distribution, the pores are uniform over the whole plate. However, the size and density of pores varies along the thickness of plate in the case of Figure 2(a) (nonsymmetric distribution) and Figure 2(b) (symmetric distribution). For the porosity distributions displayed in Figures 2(a) and 2(b), E_2 , ρ_2 , and G_2 respectively, correspond to minimum Young's modulus, mass density, and shear modulus. Conversely, E_1 , ρ_1 , and G_1 , respectively, correspond to the minimum value. For example, E_2 and E_1 , respectively, correspond to values of Young's modulus on the bottom and top surfaces in the case of Figure 2(a).

As described in equations (1) to (3), the change of elastic modulus, shear modulus, and mass density in three different kinds of porosity distributions can be obtained.

Type I nonsymmetric distribution:

$$\begin{cases} E(z) = E_1 \left[1 - e_0 \cos\left(\frac{\pi z}{2h} + \frac{\pi}{4}\right) \right], \\ G(z) = G_1 \left[1 - e_0 \cos\left(\frac{\pi z}{2h} + \frac{\pi}{4}\right) \right], \\ \rho(z) = \rho_1 \left[1 - e_m \cos\left(\frac{\pi z}{2h} + \frac{\pi}{4}\right) \right]. \end{cases} \quad (1)$$

Type II symmetric distribution:

$$\begin{cases} E(z) = E_1 \left[1 - e_0 \cos\left(\frac{\pi z}{h}\right) \right], \\ G(z) = G_1 \left[1 - e_0 \cos\left(\frac{\pi z}{h}\right) \right], \\ \rho(z) = \rho_1 \left[1 - e_m \cos\left(\frac{\pi z}{h}\right) \right]. \end{cases} \quad (2)$$

Type III uniform distribution:

FIGURE 1: Geometric characteristic symbols and coordinates of porous rectangular plates: (a) the outline and coordinates; (b) the edge restraints; (c) translational and rotational spring of differential elements.

FIGURE 2: Three typical kinds of porosity distributions: (a) type I nonsymmetric distribution; (b) type II symmetric distribution; (c) type III uniform distribution.

$$\begin{cases} E(z) = E_1 [1 - e_0 \beta], \\ G(z) = G_1 [1 - e_0 \beta], \\ \rho(z) = \rho_1 \sqrt{1 - e_0 \beta}. \end{cases} \quad (3)$$

In the equations above, e_0 and e_m correspond to the porosity coefficient and porosity density coefficient, respectively. The definition of e_0 and e_m are displayed in the following equations:

$$e_0 = 1 - \frac{E_2}{E_1} = 1 - \frac{G_2}{G_1}, \quad 0 < e_0 < 1, \quad (4)$$

$$e_m = 1 - \frac{\rho_2}{\rho_1}, \quad 0 < e_m < 1. \quad (5)$$

In addition, the correlation between porosity coefficient e_0 and porosity density coefficient e_m can be represented as follows [36, 37]:

$$e_m = 1 - \sqrt{1 - e_0}. \quad (6)$$

Specifically, the expression of β in uniform distribution can be rewritten as follows [22]:

$$\beta = \frac{1}{e_0} - \frac{1}{e_0} \left(\frac{2}{\pi} \sqrt{1 - e_0} - \frac{2}{\pi} + 1 \right)^2. \quad (7)$$

2.2. Energy Functional Expressions of Porous Rectangular Plates. On the basis of theory of first-order shear deformation theory (FSDT) [38–41], the relation between displacements of arbitrary point and corresponding point in the middle surface of the plate can be expressed as follows:

$$\begin{aligned} u(x, z, t) &= u_0(x, y, t) + z\theta(x, y, t), \\ v(x, y, z, t) &= v_0(x, y, t) + z\psi(x, y, t), \\ w(x, y, z, t) &= w_0(x, y, t), \end{aligned} \quad (8)$$

where u , v , and w denote the displacement components of arbitrary point; meanwhile, u_0 , v_0 , and w_0 , respectively, signify the displacement components of corresponding point in the middle surface of plate. θ and ψ , respectively, correspond to rotations of the normal about the middle surface about y and x directions. Similarly, the strain of arbitrary point and displacement components of corresponding point in the middle surface of the plate is expressed as follows:

$$\begin{aligned} \epsilon_{xx} &= \frac{\partial u_0}{\partial x} + z \frac{\partial \theta}{\partial x}, \\ \epsilon_{yy} &= \frac{\partial v_0}{\partial y} + z \frac{\partial \psi}{\partial y}, \\ \gamma_{xy} &= \left(\frac{\partial v_0}{\partial x} + \frac{\partial u_0}{\partial y} \right) + z \left(\frac{\partial \theta}{\partial x} + \frac{\partial \psi}{\partial y} \right), \end{aligned} \quad (9)$$

$$\gamma_{xz} = \frac{\partial w_0}{\partial x} + \theta,$$

$$\gamma_{yz} = \frac{\partial w_0}{\partial y} + \psi,$$

where ε_{xx} and ε_{yy} correspond to normal strains; in addition, γ_{xy} , γ_{xz} , and γ_{yz} signify the shear strains. The relation

between stress and strain of the plate can be expressed as follows based on general Hooke's law:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{11} & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}. \quad (10)$$

In equation (10), σ_{xx} and σ_{yy} denote normal stresses; meanwhile, τ_{xy} , τ_{xz} , and τ_{yz} signify shear stresses. The material stiffness coefficients Q_{mk} ($m, k = 1, 2, 4, 5, 6$) are expressed as follows:

$$\begin{aligned} Q_{11} &= \frac{E(z)}{1 - \nu(z)^2}, \\ Q_{12} &= \frac{\nu E(z)}{1 - \nu(z)^2}, \\ Q_{44} &= Q_{55} = Q_{66} = G(z). \end{aligned} \quad (11)$$

Integrating the stress along the thickness direction, the resultant force and moment expressions are as follows:

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ M_{xx} \\ M_{yy} \\ M_{xy} \\ Q_{xx} \\ Q_{yy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 & 0 & 0 \\ A_{21} & A_{22} & 0 & B_{12} & B_{22} & 0 & 0 & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} & 0 & 0 \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 & 0 & 0 \\ B_{21} & B_{22} & 0 & D_{12} & D_{22} & 0 & 0 & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \kappa A_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa A_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}. \quad (12)$$

In the matrix above, N_{ij} represent resultant forces; meanwhile, M_{ij} signify bending moments. In addition, Q_{ij} denote transverse shear forces. The value of the correction factor related to Q_{ij} is 5/6, which is represented by symbol κ .

In addition, the expressions of extensional stiffness A_{ij} , extensional-bending coupling stiffness B_{ij} , and bending stiffness D_{ij} are expressed as follows:

$$\begin{aligned} A_{ij} &= \int_{-h/2}^{h/2} Q_{ij}(z) dz, \\ B_{ij} &= \int_{-h/2}^{h/2} Q_{ij}(z) z dz, \\ D_{ij} &= \int_{-h/2}^{h/2} Q_{ij}(z) z^2 dz. \end{aligned} \quad (13)$$

The strain energy of the plate is expressed as follows:

$$\begin{aligned}
U_B &= \frac{1}{2} \iiint_V \{N_{xx}\epsilon_x^0 + N_{yy}\epsilon_y^0 + N_{xy}\gamma_{xy}^0 + M_{xx}\kappa_x + M_{yy}\kappa_y + M_{xy}\kappa_{xy} + Q_{xx}\gamma_{xz} + Q_{yy}\gamma_{yz}\} dV \\
&= \frac{1}{2} \int_0^a \int_0^b \left\{ A_{11} \left(\frac{\partial u_0}{\partial x} \right)^2 + A_{22} \left(\frac{\partial v_0}{\partial y} \right)^2 + 2A_{12} \left(\frac{\partial u_0}{\partial x} \right) \left(\frac{\partial v_0}{\partial y} \right) + A_{66} \left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \right)^2 + \kappa A_{55} \left(\frac{\partial w_0}{\partial x} + \theta \right)^2 \right. \\
&\quad + \kappa A_{44} \left(\frac{\partial w_0}{\partial y} + \psi \right)^2 + 2B_{11} \left(\frac{\partial u_0}{\partial x} \right) \left(\frac{\partial \theta}{\partial x} \right) + 2B_{22} \left(\frac{\partial v_0}{\partial y} \right) \left(\frac{\partial \psi}{\partial y} \right) + 2B_{12} \left(\frac{\partial u_0}{\partial x} \right) \left(\frac{\partial \psi}{\partial y} \right) + 2B_{12} \left(\frac{\partial v_0}{\partial y} \right) \left(\frac{\partial \theta}{\partial x} \right) \\
&\quad \left. + 2D_{12} \left(\frac{\partial \theta}{\partial x} \right) \left(\frac{\partial \psi}{\partial y} \right) + 2B_{66} \left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \right) \left(\frac{\partial \psi}{\partial x} + \frac{\partial \theta}{\partial y} \right) + D_{11} \left(\frac{\partial \theta}{\partial x} \right)^2 + D_{22} \left(\frac{\partial \psi}{\partial y} \right)^2 + 2D_{66} \left(\frac{\partial \psi}{\partial x} + \frac{\partial \theta}{\partial y} \right)^2 \right\} dx dy.
\end{aligned} \tag{14}$$

In the meantime, the dynamic energy of the plate is expressed as follows:

$$\begin{aligned}
T &= \frac{1}{2} \iint_S \left\{ I_0 \left[\left(\frac{\partial u_0}{\partial t} \right)^2 + \left(\frac{\partial v_0}{\partial t} \right)^2 + \left(\frac{\partial w_0}{\partial t} \right)^2 \right] \right. \\
&\quad \left. + 2I_1 \left(\frac{\partial u_0}{\partial t} \frac{\partial \theta}{\partial t} + \frac{\partial v_0}{\partial t} \frac{\partial \psi}{\partial t} \right) + I_2 \left[\left(\frac{\partial \theta}{\partial t} \right)^2 + \left(\frac{\partial \psi}{\partial t} \right)^2 \right] \right\} dS.
\end{aligned} \tag{15}$$

where I_0 , I_1 , and I_2 of equation (15) are signified as follows:

$$\begin{aligned}
I_0 &= \int_{-h/2}^{h/2} \rho(z) dz, \\
I_1 &= \int_{-h/2}^{h/2} \rho(z) z dz, \\
I_2 &= \int_{-h/2}^{h/2} \rho(z) z^2 dz.
\end{aligned} \tag{16}$$

In this paper, the boundary constraints of plate are modeled by using the penalty method, where penalty parameters are defined by stiffness values indicating translational and rotational spring. Complex boundary constraints of the plate can be generated by assigning proper values to penalty parameters. The potential energy stored in boundary springs can be written as follows:

$$\begin{aligned}
U_P &= \frac{1}{2} \int_{-h/2}^{h/2} \int_0^b \left\{ \left[k_{x0}^u u_0^2 + k_{x0}^v v_0^2 + k_{x0}^w w_0^2 + K_{xx0}^w \theta^2 + K_{xy0}^w \psi^2 \right]_{x=0} \right. \\
&\quad \left. \left[k_{xa}^u u_0^2 + k_{xa}^v v_0^2 + k_{xa}^w w_0^2 + K_{xxa}^w \theta^2 + K_{xya}^w \psi^2 \right]_{x=a} \right\} dy dz \\
&\quad + \frac{1}{2} \int_{-h/2}^{h/2} \int_0^a \left\{ \left[k_{y0}^u u_0^2 + k_{y0}^v v_0^2 + k_{y0}^w w_0^2 + K_{yy0}^w \theta^2 + K_{xy0}^w \psi^2 \right]_{y=0} \right. \\
&\quad \left. \left[k_{yb}^u u_0^2 + k_{yb}^v v_0^2 + k_{yb}^w w_0^2 + K_{yyb}^w \theta^2 + K_{xyb}^w \psi^2 \right]_{y=b} \right\} dx dz.
\end{aligned} \tag{17}$$

2.4. Admissible Displacement Function and Solution Procedures.

The selection of suitable displacement function plays an important role in guaranteeing the accuracy of solution. The displacement and rotation components of plate are generally expanded, regardless of boundary conditions, as combination of two-dimensional (2D) cosine Fourier coefficients with two sine supplementary functions. The displacement and rotation components of the middle surface are written as follows:

$$u_0(x, y, t) = \left(U(x, y) + \sum_{p=1}^{p=2} U_p^B(x, y) \right) A_{mn}, \tag{18}$$

$$v_0(x, y, t) = \left(V(x, y) + \sum_{p=1}^{p=2} V_p^B(x, y) \right) B_{mn}, \tag{19}$$

$$w_0(x, y, t) = \left(W(x, y) + \sum_{p=1}^{p=2} W_p^B(x, y) \right) C_{mn}, \tag{20}$$

$$\theta_0(x, y, t) = \left(\theta(x, y) + \sum_{p=1}^{p=2} \theta_p^B(x, y) \right) D_{mn}, \quad (21)$$

$$\psi_0(x, y, t) = \left(\psi(x, y) + \sum_{p=1}^{p=2} \psi_p^B(x, y) \right) E_{mn}, \quad (22)$$

where U , V , W , θ , and ψ represent the internal displacement component function of plate; meanwhile, U_p^B , V_p^B , W_p^B , θ_p^B , and ψ_p^B are the supplementary series of displacement components.

A_{mn} , B_{mn} , C_{mn} , D_{mn} , and E_{mn} , respectively, denote two-dimensional (2D) Fourier coefficients vector. The specific elements of these vectors are written as follows:

$$\begin{aligned} U = V = W = \theta = \psi &= \left\{ \begin{array}{l} \cos(\lambda_0 x) \cos(\lambda_0 y), \dots, \cos(\lambda_0 x) \cos(\lambda_n y), \dots, \cos(\lambda_0 x) \cos(\lambda_N y) \\ \cos(\lambda_m x) \cos(\lambda_n y), \dots, \cos(\lambda_m x) \cos(\lambda_N y), \dots, \cos(\lambda_M x) \cos(\lambda_N y) \end{array} \right\}, \\ U_1^B = V_1^B = W_1^B = \theta_1^B = \psi_1^B &= \left\{ \begin{array}{l} \cos(\lambda_0 x) \sin(\lambda_1 y), \dots, \cos(\lambda_m x) \sin(\lambda_1 y), \dots, \cos(\lambda_M x) \sin(\lambda_1 y) \\ \cos(\lambda_0 x) \sin(\lambda_2 y), \dots, \cos(\lambda_m x) \sin(\lambda_2 y), \dots, \cos(\lambda_M x) \sin(\lambda_2 y) \end{array} \right\}, \\ U_2^B = V_2^B = W_2^B = \theta_2^B = \psi_2^B &= \left\{ \begin{array}{l} \sin(\lambda_1 x) \cos(\lambda_0 y), \dots, \sin(\lambda_1 x) \cos(\lambda_n y), \dots, \sin(\lambda_1 x) \cos(\lambda_N y) \\ \sin(\lambda_2 x) \cos(\lambda_0 y), \dots, \sin(\lambda_2 x) \cos(\lambda_n y), \dots, \sin(\lambda_2 x) \cos(\lambda_N y) \end{array} \right\}, \\ A_{mn} &= \left\{ \begin{array}{l} A_{00}^u, \dots, A_{0n}^u, \dots, A_{0N}^u, \dots, A_{mn}^u, \dots, A_{mN}^u, \dots, A_{MN}^u \\ a_{01}, \dots, a_{m1}, \dots, a_{M1}, a_{02}, \dots, a_{m2}, \dots, a_{M2}, \\ b_{10}, \dots, b_{1n}, \dots, b_{1N}, b_{20}, \dots, b_{2n}, \dots, b_{2N} \end{array} \right\}^T, \\ B_{mn} &= \left\{ \begin{array}{l} B_{00}^u, \dots, B_{0n}^u, \dots, B_{0N}^u, \dots, B_{mn}^u, \dots, B_{mN}^u, \dots, B_{MN}^u \\ c_{01}, \dots, c_{m1}, \dots, c_{M1}, c_{02}, \dots, c_{m2}, \dots, c_{M2}, \\ d_{10}, \dots, d_{1n}, \dots, d_{1N}, d_{20}, \dots, d_{2n}, \dots, d_{2N} \end{array} \right\}^T, \\ C_{mn} &= \left\{ \begin{array}{l} C_{00}^w, \dots, C_{0n}^w, \dots, C_{0N}^w, \dots, C_{mn}^w, \dots, C_{mN}^w, \dots, C_{MN}^w \\ e_{01}, \dots, e_{m1}, \dots, e_{M1}, e_{02}, \dots, e_{m2}, \dots, e_{M2}, \\ f_{10}, \dots, f_{1n}, \dots, f_{1N}, f_{20}, \dots, f_{2n}, \dots, f_{2N} \end{array} \right\}^T, \\ D_{mn} &= \left\{ \begin{array}{l} D_{00}^\theta, \dots, D_{0n}^\theta, \dots, D_{0N}^\theta, \dots, D_{mn}^\theta, \dots, D_{mN}^\theta, \dots, D_{MN}^\theta \\ g_{01}, \dots, g_{m1}, \dots, g_{M1}, g_{02}, \dots, g_{m2}, \dots, g_{M2}, \\ h_{10}, \dots, h_{1n}, \dots, h_{1N}, h_{20}, \dots, h_{2n}, \dots, h_{2N} \end{array} \right\}^T, \\ E_{mn} &= \left\{ \begin{array}{l} E_{00}^\psi, \dots, E_{0n}^\psi, \dots, E_{0N}^\psi, \dots, E_{mn}^\psi, \dots, E_{mN}^\psi, \dots, E_{MN}^\psi \\ k_{01}, \dots, k_{m1}, \dots, k_{M1}, k_{02}, \dots, k_{m2}, \dots, k_{M2} \\ p_{10}, \dots, p_{1n}, \dots, p_{1N}, p_{20}, \dots, p_{2n}, \dots, p_{2N} \end{array} \right\}^T. \end{aligned} \quad (23)$$

where M and N , respectively, denote truncation number of internal and supplementary displacement component function. In addition, $\lambda_m = m\pi/a$ and $\lambda_n = n\pi/b$. By means of

combining equations (18)–(22) and equation (14), the strain energy can be rewritten as follows:

$$\begin{aligned}
U_B = \frac{1}{2} \int_0^a \int_0^b \left\{ A_{11} \left(\frac{\partial U}{\partial x} A_{mn} \right)^2 + A_{22} \left(\frac{\partial V}{\partial y} B_{mn} \right)^2 + 2A_{12} \left(\frac{\partial U}{\partial x} A_{mn} \right) \left(\frac{\partial V}{\partial y} B_{mn} \right) + A_{66} \left(\frac{\partial U}{\partial y} A_{mn} + \frac{\partial V}{\partial x} B_{mn} \right)^2 \right. \\
+ \kappa A_{55} \left(\frac{\partial W}{\partial x} C_{mn} + \theta A_{mn} \right)^2 + \kappa A_{44} \left(\frac{\partial W}{\partial y} C_{mn} + \psi B_{mn} \right)^2 + 2B_{11} \left(\frac{\partial U}{\partial x} A_{mn} \right) \left(\frac{\partial \theta}{\partial x} D_{mn} \right) + 2B_{22} \left(\frac{\partial V}{\partial y} B_{mn} \right) \left(\frac{\partial \Theta}{\partial y} E_{mn} \right) \\
+ 2B_{12} \left(\frac{\partial U}{\partial x} A_{mn} \right) \left(\frac{\partial \psi}{\partial y} E_{mn} \right) + 2B_{12} \left(\frac{\partial V}{\partial y} B_{mn} \right) \left(\frac{\partial \theta}{\partial x} D_{mn} \right) + 2D_{12} \left(\frac{\partial \theta}{\partial x} D_{mn} \right) \left(\frac{\partial \psi}{\partial y} E_{mn} \right) \\
\left. + 2B_{66} \left(\frac{\partial U}{\partial y} A_{mn} + \frac{\partial V}{\partial x} B_{mn} \right) \left(\frac{\partial \psi}{\partial x} E_{mn} + \frac{\partial \theta}{\partial y} D_{mn} \right) + D_{11} \left(\frac{\partial \theta}{\partial x} D_{mn} \right)^2 + D_{22} \left(\frac{\partial \psi}{\partial y} E_{mn} \right)^2 + 2D_{66} \left(\frac{\partial \psi}{\partial x} E_{mn} + \frac{\partial \theta}{\partial y} D_{mn} \right)^2 \right\} dx dy. \quad (24)
\end{aligned}$$

Similarly, the dynamic energy can also be rewritten as follows:

$$\begin{aligned}
T = \frac{1}{2} \int_0^a \int_0^b \left\{ I_0 \left[(UA_{mn})^2 + (VB_{mn})^2 + (WC_{mn})^2 \right] \right. \\
+ 2I_1 (UA_{mn} \theta D_{mn} + VB_{mn} \psi E_{mn}) \\
\left. + I_2 \left[(\theta D_{mn})^2 + (\psi E_{mn})^2 \right] \right\} dx dy. \quad (25)
\end{aligned}$$

Meanwhile, the energy stored in the boundary springs is expressed as follows:

$$\begin{aligned}
U_P = \frac{1}{2} \int_{-h/2}^{h/2} \int_0^b \left\{ \left[k_{x0}^u (UA_{mn})^2 + k_{x0}^v (VB_{mn})^2 + k_{x0}^w (WC_{mn})^2 + K_{xx0}^w (\theta D_{mn})^2 + K_{xy0}^w (\psi E_{mn})^2 \right]_{x=0} \right. \\
\left. + \left[k_{xa}^u (UA_{mn})^2 + k_{xa}^v (VB_{mn})^2 + k_{xa}^w (WC_{mn})^2 + K_{xxa}^w (\theta D_{mn})^2 + K_{xya}^w (\psi E_{mn})^2 \right]_{x=a} \right\} dy dz \\
+ \frac{1}{2} \int_{-h/2}^{h/2} \int_0^a \left\{ \left[k_{y0}^u (UA_{mn})^2 + k_{y0}^v (VB_{mn})^2 + k_{y0}^w (WC_{mn})^2 + K_{yy0}^w (\theta D_{mn})^2 + K_{xy0}^w (\psi E_{mn})^2 \right]_{y=0} \right. \\
\left. + \left[k_{yb}^u (UA_{mn})^2 + k_{yb}^v (VB_{mn})^2 + k_{yb}^w (WC_{mn})^2 + K_{yyb}^w (\theta D_{mn})^2 + K_{xyb}^w (\psi E_{mn})^2 \right]_{y=b} \right\} dx dz. \quad (26)
\end{aligned}$$

The Lagrangian energy function of the plate can be given as follows:

$$L = U_B + U_P - T. \quad (27)$$

The total expression of the Lagrangian energy function for the undetermined coefficients can be minimized by the minimum energy principle:

$$\frac{\partial L}{\partial \theta} = 0 \Rightarrow A_{mn}, B_{mn}, C_{mn}, D_{mn}, E_{mn}, F_{mn}. \quad (28)$$

By substituting equations (24)–(27) into equation (28) and performing the partial derivative of the Lagrangian energy function of plate to the unknown coefficient to 0, the following matrix can be obtained:

$$(K_S + K_P - \omega^2 M)G = 0. \quad (29)$$

where G is the vector consisting of unknown coefficients. K_S , K_P , and M , respectively, represent strain potential energy stiffness, elastic potential energy stiffness, and mass matrices

of the porous plate. The eigenvalues and the corresponding eigenvectors of the shells can be easily obtained by solving equation (29). The eigenvalues and the corresponding eigenvectors, respectively, correspond to modal frequencies and modal vectors (mode shapes).

3. Numerical Results and Discussion

In this section, some results and discussion are introduced to ensure the exactness and dependability of the current method when handling free vibration of rectangular plates with uniform thickness and porosity distributions. In addition, the solution may not be convergent when the penalty parameters are not defined as a suitable value [42, 43]. So it is very meaningful to do the convergence studies. On the basis of contents mentioned above, we elaborate the vibration characteristic of rectangular plates with different porosity distributions and complex edge constraints. Meanwhile, the influences of boundary restraints and geometry parameters on frequency parameters of plate are also discussed in detail.

TABLE 1: Spring stiffness values correspond to different boundary conditions.

BC	$k_{u,x}$	$k_{v,x}$	$k_{w,x}$	$k_{w,xx}$	$k_{w,xy}$	$k_{u,y}$	$k_{v,y}$	$k_{w,y}$	$k_{w,yy}$	$k_{w,yx}$
FE	0	0	0	0	0	0	0	0	0	0
SSE	0	10^{15}	10^{15}	0	10^{15}	10^{15}	0	10^{15}	0	10^{15}
CE	10^{15}	10^{15}	10^{15}	10^{15}	10^{15}	10^{15}	10^{15}	10^{15}	10^{15}	10^{15}
E ¹ E	10^{10}	10^{15}	10^{15}	10^{15}	10^{15}	10^{10}	10^{15}	10^{15}	10^{15}	10^{15}
E ² E	10^{10}	10^{10}	10^{15}	10^{15}	10^{15}	10^{10}	10^{10}	10^{15}	10^{15}	10^{15}
E ³ E	10^{10}	10^{10}	10^{10}	10^{15}	10^{15}	10^{10}	10^{10}	10^{10}	10^{15}	10^{15}
E ⁴ E	10^{10}	10^{10}	10^{10}	10^{10}	10^{15}	10^{10}	10^{10}	10^{10}	10^{10}	10^{15}
E ⁵ E	10^{10}	10^{10}	10^{10}	10^{10}	10^{10}	10^{10}	10^{10}	10^{10}	10^{10}	10^{10}

TABLE 2: Constringency study of frequency parameters $\Omega = (\omega b^2/\pi^2)\sqrt{\rho h/D}$ with different truncated numbers of homogeneous rectangular plates.

Mode no.	$M * N * P$									Ref. [44]
	6 * 6 * 2	7 * 7 * 2	8 * 8 * 2	9 * 9 * 2	10 * 10 * 2	11 * 11 * 2	12 * 12 * 2	13 * 13 * 2	14 * 14 * 2	
1	5.3914	5.3912	5.3911	5.3911	5.3910	5.3910	5.3910	5.3910	5.3910	5.5132
2	6.7046	6.7040	6.7039	6.7037	6.7036	6.7036	6.7036	6.7036	6.7036	6.8531
3	8.4031	8.4027	8.4025	8.4024	8.4022	8.4022	8.4022	8.4022	8.4022	8.4309
4	8.7306	8.7304	8.7288	8.7287	8.7281	8.7281	8.7281	8.7281	8.7281	8.9311

TABLE 3: Comparison of frequency parameters $\Omega = (\omega b^2/\pi^2)\sqrt{\rho h/D}$ of homogeneous rectangular plates subjected to different boundary conditions.

Boundary condition	h/a	Method	Mode number					
			1	2	3	4	5	6
CCCC	0.3	Current	3.9716	4.9213	5.6007	6.3443	7.2322	7.4566
		Ref. [44]	4.0896	5.0651	5.6229	6.5367	7.4983	7.5321
	0.5	Current	2.5436	3.1425	3.3595	4.0441	4.3205	4.4677
		Ref. [44]	2.6471	3.2664	3.3723	4.2077	4.2887	4.5247
SSSS	0.3	Current	2.1738	3.1959	4.3472	4.3475	4.4351	4.8614
		Ref. [44]	2.1745	3.2406	4.3489	4.3489	4.5123	4.8623
	0.5	Current	1.3040	2.2821	2.6076	2.6080	2.9165	2.9322
		Ref. [44]	1.3047	2.3312	2.6094	2.6094	2.9174	2.9174

TABLE 4: Comparison of frequency parameters Ω of porous rectangular plates subjected to different boundary conditions.

h/a	Mode no.	SCSC		SFSF		SSSS	
		Present	Ref. [45]	Present	Ref. [45]	Present	Ref. [45]
0.1	1	0.0384	0.0384	0.0279	0.0279	0.0349	0.0348
	2	0.0650	0.0650	0.0339	0.0339	0.0551	0.0551
	3	0.1037	0.1037	0.0507	0.0507	0.0880	0.0880
	4	0.1150	0.1150	0.0777	0.0777	0.1136	0.1136
0.2	1	0.1434	0.1435	0.1069	0.1069	0.1323	0.1323
	2	0.2321	0.2326	0.1285	0.1287	0.2038	0.2039
	3	0.3531	0.3545	0.1871	0.1876	0.3137	0.3137
	4	0.3977	0.3980	0.2770	0.2781	0.3945	0.3946

To simplify the study, some symbols are employed to represent the boundary conditions of rectangular plates. For example, CE, SDE, SSE, FE, and EE, respectively, denote clamped edge, shear-diaphragm edge, shear-support edge, free edge, and elastic edge boundary conditions. In addition,

a simple letter string is employed to represent the boundary condition of plates. For example, CEFS denotes the plate subjected to clamped, elastic, free, and shear-support edges at $x = 0$, $y = 0$, $x = a$, and $y = b$, respectively. Unless otherwise stated, the material parameters of AI considered in current

FIGURE 3: The frequency parameters Ω of type I porous rectangular plate with different boundary spring stiffness values: (a) classical boundary conditions; (b) elastic boundary conditions.

research is defined as below: $E = 70$ GPa, $\rho = 2702$ kg/m³, and $\nu = 0.3$. Nondimensional frequency parameter is generally defined as $\Omega = \omega h \sqrt{\rho/E}$.

3.1. Convergence Studies. The first three frequency parameters of the clamped supported uniform plate are considered in Figure 3, in which the convergence of different spring stiffness values is investigated. The geometric dimensions are as follows: $a = 0.5$ m, $b = 1$ m, and $h = 0.3$ m. As displayed in Figure 3, when

the stiffness values are in the range of $10^{13} \sim 10^{20}$, the frequency parameters converge to a stable value. In other words, stiffness values in this range can guarantee the rigid boundary conditions. Meanwhile, when the stiffness values are less than 10^7 , the boundary conditions can be seen as free. Similarly, the stiffness values between 10^8 and 10^{12} can be regarded as elastic. In summary, the spring stiffness values of the different boundary conditions can be obtained as shown in Table 1.

In the admissible displacement function of plates with porosities, when the number of expansion terms of the Fourier

TABLE 5: Nondimensional frequency parameter Ω for porous rectangular plates subjected to different classical edge conditions ($e = 0.2$, $a/b = 0.5$, and $h/b = 0.2$).

Type	Mode no.	Boundary conditions					
		SCSC	SFSF	SSSS	CFCF	CSCS	CCCC
I	1	0.3768	0.3671	0.3768	0.5711	0.6033	0.6238
	2	0.4676	0.3769	0.4478	0.5885	0.7298	0.7756
	3	0.6867	0.4221	0.6470	0.6702	0.7536	0.9708
	4	0.7536	0.5815	0.7536	0.7195	0.9615	1.0097
	5	0.8942	0.6586	0.7536	0.8508	0.9674	1.1906
II	1	0.3768	0.3739	0.3768	0.5758	0.6085	0.6292
	2	0.4751	0.3769	0.4556	0.5935	0.7370	0.7823
	3	0.6951	0.4296	0.6566	0.6768	0.7536	0.9709
	4	0.7537	0.5908	0.7536	0.7196	0.9675	1.0178
	5	0.8943	0.6587	0.7537	0.8600	0.9711	1.1977
III	1	0.3763	0.3656	0.3763	0.5695	0.6016	0.6220
	2	0.4659	0.3763	0.4460	0.5869	0.7277	0.7735
	3	0.6845	0.4204	0.6447	0.6684	0.7525	0.9695
	4	0.7525	0.5794	0.7525	0.7186	0.9587	1.0071
	5	0.8930	0.6577	0.7525	0.8483	0.9661	1.1882

FIGURE 4: Nondimensional frequency parameter of type I porous rectangular plates with different boundary conditions: (a) classical boundary conditions; (b) elastic boundary conditions.

series is infinite, the corresponding results are equivalent to the closed analytical solutions. However, in actual operation, limited by hardware level and computational efficiency, only a limited number of the Fourier series expansion terms are used. Table 2 displays the nondimensional frequencies $\Omega = (\omega b^2/\pi^2)\sqrt{\rho h/D}$ of homogeneous rectangular plates with different truncated numbers. The aspect ratio of the plate in Table 2 is same with the plate of Figure 3, whereas the thickness is $h = 0.2$. As can be seen from Table 2, the convergence speed of the present method is very fast. When the truncated number

reaches $10 * 10 * 2$, the accuracy of the current method is relatively high. To ensure the convergence and efficiency simultaneously, the truncated number is chosen as $10 * 10 * 2$ in the following analysis.

In order to further confirm the accuracy and effectiveness of current method, the comparison results are listed in Tables 3 and 4. The research object in Table 3 is a homogeneous rectangular plate with an aspect ratio of 0.5. Meanwhile, type I porous rectangular plates are considered in Table 4. The aspect ratios and porosity coefficient in Table 4 are,

TABLE 6: Fundamental frequencies for porous rectangular plates subjected to different elastic edge conditions with different porosity coefficients ($e=0.2$, $a/b=0.5$, and $h/b=0.2$).

Type	Mode no.	Boundary conditions				
		$E^1E^1E^1E^1$	$E^2E^2E^2E^2$	$E^3E^3E^3E^3$	$E^4E^4E^4E^4$	$E^5E^5E^5E^5$
I	1	0.3998	0.3829	0.3546	0.3539	0.3534
	2	0.5972	0.3912	0.3828	0.3828	0.3828
	3	0.6238	0.5052	0.3912	0.3912	0.3912
	4	0.7756	0.6237	0.4614	0.4611	0.4603
	5	0.8752	0.7059	0.5052	0.5052	0.5052
II	1	0.3998	0.3829	0.3556	0.3549	0.3544
	2	0.5973	0.3912	0.3829	0.3829	0.3829
	3	0.6292	0.5052	0.3912	0.3912	0.3912
	4	0.7823	0.6292	0.4633	0.4631	0.4623
	5	0.8753	0.7059	0.5052	0.5052	0.5052
III	1	0.3997	0.3828	0.3542	0.3536	0.3531
	2	0.5969	0.3911	0.3828	0.3828	0.3828
	3	0.6220	0.5050	0.3911	0.3911	0.3911
	4	0.7735	0.6220	0.4608	0.4606	0.4598
	5	0.8740	0.7055	0.5050	0.5050	0.5050

TABLE 7: Nondimensional frequency parameter Ω of porous rectangular plates with different aspect ratios (CCCC) ($e=0.2$ and $h/b=0.2$).

Type	Mode no.	Aspect ratio a/b						
		0.5	1.0	1.5	2.0	2.5	3.0	3.5
I	1	0.6238	0.3112	0.2455	0.2258	0.2181	0.2144	0.2124
	2	0.7756	0.5429	0.3558	0.2841	0.2522	0.2362	0.2274
	3	0.9708	0.5429	0.5002	0.3811	0.3130	0.2763	0.2551
	4	1.0097	0.7207	0.5186	0.4855	0.3971	0.3346	0.2967
	5	1.1906	0.7207	0.5559	0.4867	0.4493	0.4081	0.3511
II	1	0.6292	0.3160	0.2496	0.2296	0.2218	0.2180	0.2160
	2	0.7823	0.5493	0.3612	0.2887	0.2564	0.2402	0.2312
	3	0.9709	0.5493	0.5063	0.3869	0.3181	0.2809	0.2594
	4	1.0178	0.7208	0.5254	0.4855	0.4032	0.3400	0.3016
	5	1.1977	0.7208	0.5559	0.4926	0.4493	0.4144	0.3568
III	1	0.6220	0.3101	0.2446	0.2249	0.2172	0.2136	0.2115
	2	0.7735	0.5412	0.3545	0.2830	0.2512	0.2353	0.2265
	3	0.9695	0.5412	0.4986	0.3796	0.3118	0.2752	0.2541
	4	1.0071	0.7197	0.5169	0.4848	0.3956	0.3333	0.2955
	5	1.1882	0.7197	0.5551	0.4851	0.4487	0.4066	0.3498

FIGURE 5: Nondimensional frequency parameter of type I porous rectangular plates with different aspect ratios (a/b).

TABLE 8: Nondimensional frequency parameter Ω for porous rectangular plates with different thickness ratios h/b ($e = 0.2$ and $a/b = 0.5$).

Type	Mode no.	Thickness ratio						
		0.02	0.04	0.06	0.08	0.1	0.15	0.2
I	1	0.0114	0.0440	0.0936	0.1556	0.2258	0.4200	0.6238
	2	0.0147	0.0564	0.1192	0.1968	0.2841	0.5241	0.7756
	3	0.0207	0.0783	0.1634	0.2666	0.3811	0.6900	0.9708
	4	0.0291	0.1085	0.2203	0.3494	0.4855	0.7282	1.0097
	5	0.0293	0.1087	0.2232	0.3588	0.4867	0.8393	1.1906
II	1	0.0117	0.0450	0.0956	0.1586	0.2296	0.4250	0.6292
	2	0.0151	0.0578	0.1217	0.2005	0.2887	0.5303	0.7823
	3	0.0212	0.0801	0.1667	0.2714	0.3869	0.6975	0.9709
	4	0.0298	0.1109	0.2243	0.3546	0.4855	0.7282	1.0178
	5	0.0300	0.1111	0.2275	0.3647	0.4926	0.8459	1.1977
III	1	0.0113	0.0437	0.0932	0.1549	0.2249	0.4186	0.6220
	2	0.0147	0.0561	0.1186	0.1959	0.2830	0.5225	0.7735
	3	0.0206	0.0779	0.1626	0.2655	0.3796	0.6879	0.9695
	4	0.0289	0.1080	0.2194	0.3482	0.4848	0.7272	1.0071
	5	0.0291	0.1082	0.2223	0.3574	0.4851	0.8372	1.1882

TABLE 9: Nondimensional frequency parameter Ω of porous rectangular plates with different porosity coefficients ($h/b = 0.2$ and $a/b = 0.5$).

Type	Mode no.	Porosity coefficients						
		0.1	0.2	0.3	0.4	0.5	0.6	0.7
I	1	0.6342	0.6238	0.6128	0.6011	0.5886	0.5751	0.5603
	2	0.7886	0.7756	0.7619	0.7473	0.7316	0.7146	0.6959
	3	0.9876	0.9708	0.9535	0.9357	0.9176	0.8995	0.8822
	4	1.0266	1.0097	0.9918	0.9728	0.9524	0.9305	0.9065
	5	1.2109	1.1906	1.1691	1.1462	1.1217	1.0954	1.0671
II	1	0.6367	0.6292	0.6216	0.6140	0.6067	0.5999	0.5943
	2	0.7917	0.7823	0.7729	0.7635	0.7544	0.7459	0.7389
	3	0.9876	0.9709	0.9539	0.9365	0.9189	0.9017	0.8858
	4	1.0304	1.0178	1.0051	0.9924	0.9800	0.9683	0.9586
	5	1.2141	1.1977	1.1810	1.1642	1.1476	1.1318	1.1180
III	1	0.6334	0.6220	0.6097	0.5961	0.5809	0.5637	0.5435
	2	0.7877	0.7735	0.7581	0.7412	0.7224	0.7010	0.6758
	3	0.9873	0.9695	0.9502	0.9291	0.9055	0.8786	0.8472
	4	1.0256	1.0071	0.9870	0.9650	0.9405	0.9127	0.8799
	5	1.2099	1.1882	1.1645	1.1386	1.1097	1.0768	1.0382

respectively, set as 0.5 and 0.1. The comparison results of Tables 3 and 4 demonstrate the high accuracy and reliability of the current method.

3.2. Free Vibration of Porous Rectangular Plates. The convergence, accuracy, and reliability of the current method have been verified in the previous study. More results of three types of porous rectangular plates subjected to various edge conditions are presented in this section. In addition, the effects of boundary conditions and geometry parameters are also discussed in detail.

The first five frequency parameters of three kinds of porous rectangular plates subjected to classical boundary conditions are displayed in Table 5. The aspect and thickness ratio are same with Table 2; meanwhile, the porosity coefficient $e = 0.2$. At the same time, the first frequency parameters of type I porous plate with different thicknesses and classical boundary conditions are presented in Figure 4(a).

We can discover that the frequency parameters of CCCC boundary conditions are highest in all classical conditions. With the weakening of boundary conditions, the frequency parameters also decrease.

However, the boundary conditions of structures are often not ideal classical boundaries in the practical engineering applications and variety of possible boundary cases can be encountered. One advantage of the current method is various types of elastic restrains at edges can be performed by adopting artificial spring technology. The same plates of Table 5 with elastic boundary conditions are studied in Table 6. Meanwhile, first frequency parameters of type I porous plates with different thickness ratios and elastic boundary conditions are presented in Figure 4(b). It can be concluded from Table 6 that the frequencies decreased when the boundary conditions changed from $E^1E^1E^1E^1$ to $E^5E^5E^5E^5$. However, the difference between $E^3E^3E^3E^3$ and $E^5E^5E^5E^5$ is so tiny that we can hardly find it in Figure 4(b). In addition, we can conclude from Figure 4 that, with the

FIGURE 6: Nondimensional frequency parameter of CCCC porous rectangular plates with different porosity coefficients.

increase of thickness ratios, the frequencies increase regardless of boundary conditions.

The frequency parameters of porous plates with different aspect ratios subjected to CCCC boundary conditions are displayed in Table 7. The thickness ratio and porosity coefficient are same with the plate of Table 5. Meanwhile, the relation between frequency parameter and aspect ratios of type I porous rectangular plate is also displayed in Figure 5. It is notable that, with the increase of aspect ratio, the frequencies decrease, especially when the aspect ratio is smaller than 2.

Table 8 shows the frequency parameters of porous plates with different thickness ratios under CCCC boundary conditions. The aspect ratio and porosity coefficient are same with the plate of Table 5. The relation discovered in Figure 4

appears again in Table 8. When the thickness ratio is set as 0.2, the plate of Table 8 with different porosity coefficients is chosen as research object in Table 9.

Figures 6 and 7 shows the variations of nondimensional frequency parameter Ω of SSSS and CCCC porous plates with respect to different porosity volume indexes, respectively. The geometric constants of the above porous plate are same as Figure 4. From Figures 6 and 7, we find different types of porosity distribution have remarkable effect on vibration characteristics of porous rectangular plates. For the type III porosity distribution function, the frequency parameter decreases with the increase of the porosity coefficients. However, the relation between porosity coefficients and type II porosity distribution becomes different when the mode number varies.

FIGURE 7: Nondimensional frequency parameter of SSSS porous rectangular plates with different porosity coefficients.

In addition, it can be seen that boundary conditions also have an effect on vibration characteristics of porous rectangular plates with different types of porosity distribution.

4. Conclusions

The paper provides a generalized method to investigate the free vibration of porous rectangular plates subjected to different edge conditions. Three types of porosity distributions are investigated on the basis of first-order shear deformation theory. The boundary conditions at the edge of the plate were simulated by the penalty method. To test the convergence, the effect of penalty boundary springs and truncated numbers is examined. The reliability and accuracy of the current method were verified by comparing with published literatures.

Numerical results of porous rectangular plates with classical and elastic boundary conditions are displayed. The results demonstrate that, with the increase of thickness ratios, the frequencies also increase regardless of boundary conditions. Meanwhile, the relation between porosity coefficients and frequencies varies with boundary conditions.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

- [1] T. T. Banh and D. Lee, "Topology optimization of multi-directional variable thickness thin plate with multiple materials," *Structural and Multidisciplinary Optimization*, vol. 59, no. 5, pp. 1503–1520, 2019.
- [2] A. H. Baferani, A. R. Saidi, and E. Jomehzadeh, "An exact solution for free vibration of thin functionally graded rectangular plates," *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, vol. 225, no. 3, pp. 526–536, 2011.
- [3] A. M. Zenkour, "Generalized shear deformation theory for bending analysis of functionally graded plates," *Applied Mathematical Modelling*, vol. 30, no. 1, pp. 67–84, 2006.
- [4] D. Y. Liu, C. Y. Wang, and W. Q. Chen, "Free vibration of FGM plates with in-plane material inhomogeneity," *Composite Structures*, vol. 92, no. 5, pp. 1047–1051, 2010.
- [5] W. Lanhe, W. Hongjun, and W. Daobin, "Dynamic stability analysis of FGM plates by the moving least squares differential quadrature method," *Composite Structures*, vol. 77, no. 3, pp. 383–394, 2007.
- [6] F. Tornabene, "Free vibration analysis of functionally graded conical, cylindrical shell and annular plate structures with a four-parameter power-law distribution," *Computer Methods in Applied Mechanics and Engineering*, vol. 198, no. 37–40, pp. 2911–2935, 2009.
- [7] F. Tornabene and E. Viola, "Free vibrations of four-parameter functionally graded parabolic panels and shells of revolution," *European Journal of Mechanics—A/Solids*, vol. 28, no. 5, pp. 991–1013, 2009.
- [8] F. Tornabene and E. Viola, "Free vibration analysis of functionally graded panels and shells of revolution," *Mechanica*, vol. 44, no. 3, pp. 255–281, 2009.
- [9] E. Viola, L. Rossetti, N. Fantuzzi, and F. Tornabene, "Generalized stress-strain recovery formulation applied to functionally graded spherical shells and panels under static loading," *Composite Structures*, vol. 156, pp. 145–164, 2016.
- [10] F. Pang, H. Li, H. Chen, and Y. Shan, "Free vibration analysis of combined composite laminated cylindrical and spherical shells with arbitrary boundary conditions," *Mechanics of Advanced Materials and Structures*, vol. 26, pp. 1–18, 2019.
- [11] H. Li, F. Pang, X. Miao, S. Gao, and F. Liu, "A semi analytical method for free vibration analysis of composite laminated cylindrical and spherical shells with complex boundary conditions," *Thin-Walled Structures*, vol. 136, pp. 200–220, 2019.
- [12] S. Hosseini-Hashemi, M. R. Ilkhani, and M. Fadaee, "Identification of the validity range of Donnell and Sanders shell theories using an exact vibration analysis of functionally graded thick cylindrical shell panel," *Acta Mechanica*, vol. 223, no. 5, pp. 1101–1118, 2012.
- [13] Y. Qu, X. Long, G. Yuan, and G. Meng, "A unified formulation for vibration analysis of functionally graded shells of revolution with arbitrary boundary conditions," *Composites Part B: Engineering*, vol. 50, pp. 381–402, 2013.
- [14] Y. Qu and G. Meng, "Prediction of acoustic radiation from functionally graded shells of revolution in light and heavy fluids," *Journal of Sound and Vibration*, vol. 376, pp. 112–130, 2016.
- [15] M. Fadaee, M. R. Ilkhani, and S. Hosseini-Hashemi, "A new generic exact solution for free vibration of functionally graded moderately thick doubly curved shallow shell panel," *Journal of Vibration and Control*, vol. 22, no. 15, pp. 3355–3367, 2016.
- [16] H. Li, F. Pang, X. Wang, Y. Du, and H. Chen, "Free vibration analysis for composite laminated doubly-curved shells of revolution by a semi analytical method," *Composite Structures*, vol. 201, pp. 86–111, 2018.
- [17] H. Li, F. Pang, Y. Li, and C. Gao, "Application of first-order shear deformation theory for the vibration analysis of functionally graded doubly-curved shells of revolution," *Composite Structures*, vol. 212, pp. 22–42, 2019.
- [18] F. Pang, H. Li, F. Jing, and Y. Du, "Application of first-order shear deformation theory on vibration analysis of stepped functionally graded paraboloidal shell with general edge constraints," *Materials*, vol. 12, no. 1, p. 69, 2019.
- [19] H. Li, F. Pang, Q. Gong, and Y. Teng, "Free vibration analysis of axisymmetric functionally graded doubly-curved shells with un-uniform thickness distribution based on ritz method," *Composite Structures*, vol. 225, Article ID 111145, 2019.
- [20] F. A. Fazzolari, "Generalized exponential, polynomial and trigonometric theories for vibration and stability analysis of porous FG sandwich beams resting on elastic foundations," *Composites Part B: Engineering*, vol. 136, pp. 254–271, 2018.
- [21] F. Ebrahimi and M. Zia, "Large amplitude nonlinear vibration analysis of functionally graded Timoshenko beams with porosities," *Acta Astronautica*, vol. 116, pp. 117–125, 2015.
- [22] D. Chen, S. Kitipornchai, and J. Yang, "Nonlinear free vibration of shear deformable sandwich beam with a functionally graded porous core," *Thin-Walled Structures*, vol. 107, pp. 39–48, 2016.
- [23] F. Ebrahimi and S. Habibi, "Deflection and vibration analysis of higher-order shear deformable compositionally graded porous plate," *Steel and Composite Structures*, vol. 20, no. 1, pp. 205–225, 2016.
- [24] E. Magnucka-Blandzi, "Dynamic stability of a metal foam circular plate," *Journal of Theoretical and Applied Mechanics*, vol. 47, no. 2, pp. 421–433, 2009.
- [25] A. S. Rezaei, A. R. Saidi, M. Abrishamdar, and M. H. P. Mohammadi, "Natural frequencies of functionally graded plates with porosities via a simple four variable plate theory: an analytical approach," *Thin-Walled Structures*, vol. 120, pp. 366–377, 2017.
- [26] J. Yang, D. Chen, and S. Kitipornchai, "Buckling and free vibration analyses of functionally graded graphene reinforced porous nanocomposite plates based on Chebyshev-Ritz method," *Composite Structures*, vol. 193, pp. 281–294, 2018.
- [27] H. Li, F. Pang, H. Chen, and Y. Du, "Vibration analysis of functionally graded porous cylindrical shell with arbitrary boundary restraints by using a semi analytical method," *Composites Part B: Engineering*, vol. 164, pp. 249–264, 2019.
- [28] H. Li, F. Pang, Y. Ren, X. Miao, and K. Ye, "Free vibration characteristics of functionally graded porous spherical shell with general boundary conditions by using first-order shear

- deformation theory," *Thin-Walled Structures*, vol. 144, Article ID 106331, 2019.
- [29] Y. Wang and D. Wu, "Free vibration of functionally graded porous cylindrical shell using a sinusoidal shear deformation theory," *Aerospace Science and Technology*, vol. 66, pp. 83–91, 2017.
 - [30] M. Ghadiri and H. SafarPour, "Free vibration analysis of size-dependent functionally graded porous cylindrical microshells in thermal environment," *Journal of Thermal Stresses*, vol. 40, no. 1, pp. 55–71, 2017.
 - [31] J. Zhao, F. Xie, A. Wang, C. Shuai, J. Tang, and Q. Wang, "A unified solution for the vibration analysis of functionally graded porous (FGP) shallow shells with general boundary conditions," *Composites Part B: Engineering*, vol. 156, pp. 406–424, 2019.
 - [32] J. Zhao, F. Xie, A. Wang, C. Shuai, J. Tang, and Q. Wang, "Vibration behavior of the functionally graded porous (FGP) doubly-curved panels and shells of revolution by using a semi-analytical method," *Composites Part B: Engineering*, vol. 157, pp. 219–238, 2019.
 - [33] W. L. Li, "Free vibrations of beams with general boundary conditions," *Journal of Sound and Vibration*, vol. 237, no. 4, pp. 709–725, 2000.
 - [34] W. L. Li, "Vibration analysis of rectangular plates with general elastic boundary supports," *Journal of Sound and Vibration*, vol. 273, no. 3, pp. 619–635, 2004.
 - [35] W. L. Li, X. Zhang, J. Du, and Z. Liu, "An exact series solution for the transverse vibration of rectangular plates with general elastic boundary supports," *Journal of Sound and Vibration*, vol. 321, no. 1-2, pp. 254–269, 2009.
 - [36] M. R. Barati, "Investigating dynamic response of porous inhomogeneous nanobeams on hybrid Kerr foundation under hygro-thermal loading," *Applied Physics A*, vol. 123, no. 5, p. 332, 2017.
 - [37] M. R. Barati and A. M. Zenkour, "Investigating post-buckling of geometrically imperfect metal foam nanobeams with symmetric and asymmetric porosity distributions," *Composite Structures*, vol. 182, pp. 91–98, 2017.
 - [38] H. Li, F. Pang, X. Miao, and Y. Li, "Jacobi-Ritz method for free vibration analysis of uniform and stepped circular cylindrical shells with arbitrary boundary conditions: a unified formulation," *Computers & Mathematics with Applications*, vol. 77, no. 2, pp. 427–440, 2019.
 - [39] F. Pang, H. Li, J. Cui, Y. Du, and C. Gao, "Application of flügge thin shell theory to the solution of free vibration behaviors for spherical-cylindrical-spherical shell: a unified formulation," *European Journal of Mechanics—A/Solids*, vol. 74, pp. 381–393, 2019.
 - [40] H. Li, F. Pang, X. Miao, Y. Du, and H. Tian, "A semi-analytical method for vibration analysis of stepped doubly-curved shells of revolution with arbitrary boundary conditions," *Thin-Walled Structures*, vol. 129, pp. 125–144, 2018.
 - [41] F. Pang, C. Gao, J. Cui, Y. Ren, H. Li, and H. Wang, "A semianalytical approach for free vibration characteristics of functionally graded spherical shell based on first-order shear deformation theory," *Shock and Vibration*, vol. 2019, Article ID 7352901, 18 pages, 2019.
 - [42] L. E. Monterrubio, "Free vibration of shallow shells using the Rayleigh-Ritz method and penalty parameters," *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, vol. 223, no. 10, pp. 2263–2272, 2009.
 - [43] D. Sármany, F. Izsák, and J. J. W. van der Vegt, "Optimal penalty parameters for symmetric discontinuous Galerkin discretisations of the time-harmonic Maxwell equations," *Journal of Scientific Computing*, vol. 44, no. 3, pp. 219–254, 2010.
 - [44] K. M. Liew, K. C. Hung, and M. K. Lim, "A continuum three-dimensional vibration analysis of thick rectangular plates," *International Journal of Solids and Structures*, vol. 30, no. 24, pp. 3357–3379, 1993.
 - [45] A. S. Rezaei and A. R. Saidi, "Exact solution for free vibration of thick rectangular plates made of porous materials," *Composite Structures*, vol. 134, pp. 1051–1060, 2015.