Research Article

Dynamic Property Investigation of Sandwich Acoustic Black Hole Beam with Clamped-Free Boundary Condition

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The geometric parameters of the acoustic black hole (ABH) structure are changed in power exponent, and this feature can be used to control the flexural wave to achieve energy concentration, vibration attenuation, or noise reduction. However, in practice, the ABH structure often has a truncation due to the limitation of manufacturing, which will cause the reflection coefficient to increase significantly and seriously affect the ABH effect. In this paper, a semianalytical model of the sandwich-truncated ABH beam structure with aluminum in the middle layer and steel in the upper and lower layers is constructed based on the energy principle. The ABH effect of the sandwich beam under the clamped-free boundary condition is analyzed. Meanwhile, the effects of damping layer parameters, middle layer material, and thickness on the vibrational acceleration response of the ABH region and the uniform beam region of the sandwich beam are also studied. It is observed that, for the sandwich ABH beam structure, the influence of damping layer thickness on the acceleration response peak values of both the ABH region and the uniform region is very obvious in middle and high frequencies and the peaks at about 9 kHz are completely suppressed when the damping layer thickness reaches 3 mm. It also reveals that the use of aluminum as the middle layer material can bring a vibration attenuation at around 9 kHz both for the ABH region and the uniform beam region compared with using steel as the middle layer material. Experiments are carried out to verify the accuracy of simulation analysis.

1. Introduction

In the past two decades, the acoustic black hole (ABH) structure has attracted a lot of attention as a new method of vibration attenuation and noise reduction, and it can also bring positive effects to structural lightweighting [1–3]. Similar to the concept of the astronomical black hole, an acoustic black hole can reduce the velocity of sound wave propagation. Ideally, the sound wave velocity drops to zero to achieve zero reflection. Mironov found that, in the thin-plate wedge structure, if the thickness of the structure is reduced by a certain power function (the power exponent is not less than 2), the wave velocity of the flexural wave will decrease rapidly with the decrease of the thickness [4]. In an ideal case, the wave velocity can be reduced to zero to achieve zero reflection of the flexural wave and this structure is called the ABH structure [4]. By utilizing the above characteristics of the ABH structure, it is possible to design an ABH region

in beam or plate structures to achieve vibration attenuation [5], sound radiation reduction [6, 7], and energy harvesting [8, 9].

In the actual manufacturing, a truncation at the tip end of the ABH region is inevitable. Even a small thickness of truncation can significantly increase the reflection coefficient, seriously affecting the effectiveness of energy concentration and vibration attenuation [10]. In order to improve the energy concentration effect of the truncated ABH structure, some measures have been taken, such as the sticking damping material on the edge of the ABH area, which can significantly reduce the reflection coefficient [11, 12].

To analyze the performance of the ABH structure and the effect of damping layers or other attachments on the dynamic property of the entire system, we need to resort to some analytical or numerical approach. The analysis methods for ABH beam structures are geometrical acoustic

method, impedance method, semianalytical method, and finite element method (FEM). The geometrical acoustic method is based on the assumption that the thickness of the damping layer is much smaller than the thickness of the ABH structure. For ABH structures with a certain thickness of the damping layer, the calculation accuracy is greatly affected and the influence of variation of the thickness and length of the damping layer on the dynamic characteristics of the ABH structure could not be studied [13]. The structural impedance method considers a semi-infinite structure in modeling analysis, while the ABH structures existing in reality are often limited and have different boundary conditions [14]. Recently, Tang et al. established a semianalytical model of one-dimensional ABH structure based on the Lagrangian variational principle. The damping layers or other structures are attached to the whole system through energy terms, and the dynamic response of the ABH beam is studied by using rotational spring and translational spring to simulate different boundary conditions [15]. On the basis of the semianalytical method, Deng et al. used the Gaussian function to fit the displacement field of the ABH structure and established a semianalytical model with a passively constrained viscoelastic layer (PCVL) [16]. Wang et al. established a semianalytical model for power flow and structural intensity analysis of an ABH beam by using an improved Fourier series with auxiliary terms to construct the admissible function [17]. Besides the semianalytical method analysis, Ji et al. proposed an analytical expression of the wave reflection coefficient in a modified one-dimensional ABH and carried out a time-domain experiment based on a laser excitation technique to visualize the wave propagation [18]. On the contrary, the laminated beams or plates are an important type of structures in engineering with the advantages of lightweighting and high strength-mass ratio than the single-layered structure, and many researchers have carried out lots of studies on composite beams [19]. In the present work, we will adopt the semianalytical method to analyze a sandwich ABH beam. Different from the existing models, the object studied in this paper is a laminated sandwich ABH beam structure with an aluminum layer in the middle and steel in the upper and lower layers, so the segmentation and material variations for different layers are needed to be taken into consideration in the modeling procedure.

In this paper, a sandwich ABH beam structure with truncation at the tip of the ABH region is proposed. Unlike previous one-dimensional ABH structures, the ABH beam investigated in this paper is a sandwich beam composed of three layers. The middle layer material of the proposed ABH beam is aluminum, and the upper and lower layers are steel. The sandwich ABH beam is an elongated load-bearing structure composed of three layers of materials bonded together, and its length is much larger than the width and thickness. The Euler–Bernoulli beam theory can analyze the dynamics of slender laminated beams accurately [19]. A semianalytical model of the ABH sandwich beam with truncation is established in this paper based on the Euler–Bernoulli laminated beam theory. The dynamic property of the ABH region and the uniform region of the sandwich ABH beam structure is analyzed. Then, the effects

of parameters of the damping layer and variations of the middle layer on the dynamic response of the proposed sandwich ABH beam are studied. Experimental measurement was also conducted to verify the accuracy of the numerical simulation.

2. Dynamic Model of a Sandwich ABH Beam

2.1. Modeling of a Sandwich ABH Beam. As shown in Figure 1, let us consider a center-symmetric sandwich beam with a truncated acoustic black hole region at the right end of the beam. Damping layers are attached to the ABH region, its thickness is $h_d(x)$, and its width is equal to the width of the bare beam. The width and height of the bare beam are much smaller than its length. The length of the sandwich ABH beam ranges from x_0 to x_{b3} , among which the length of the ABH region varies from x_0 to x_{b2} . The thickness of the uniform region of the sandwich beam is h_{b1} , h_{b2} , and h_{b3} . Since the beam in Figure 1 is a three-layered structure, the length of the ABH region is divided into two parts: (x_0, x_{b1}) and (x_{b1}, x_{b2}) . The thickness of the truncation is h_t , and the position of the point force excitation is x_f .

The thickness of the bare ABH beam is given by

$$h_b = \begin{cases} \varepsilon(x - x_0)^2 + \frac{h_t}{2}, & x_0 \leq x \leq x_{b2}, \\ h_{b1} + h_{b2} + h_{b3}, & x_{b2} \leq x \leq x_{b3}. \end{cases} \quad (1)$$

The entire system is symmetric about the x -axis. The ABH region of the sandwich beam is free, and the other end is supported by a rotating spring and a translational spring to achieve different boundary conditions. For example, when the stiffnesses of the rotating spring and the translational spring are set extremely high, the boundary condition of the sandwich ABH beam is clamped free. While fixing both the stiffnesses of the rotating spring and the translational spring zero, the boundary condition of the sandwich ABH beam is clamped-free. Besides, other boundary conditions can be obtained by changing the value of q and k .

In order to analyze the dynamic properties of the sandwich ABH beam, the following assumptions are made:

- (1) Shear deformation of the cross section perpendicular to the neutral axis is ignored
- (2) The movement of the beam only occurs in the x - z plane with small deflection
- (3) Only the normal stress of the damping layer is considered, regardless of the shear stress of the damping layer
- (4) Ignore the moment of inertia effect of the sandwich ABH beam

According to the Euler–Bernoulli beam theory, the displacement field of the ABH beam can be written as follows:

$$\{u, w\} = \left\{ -z \frac{\partial w}{\partial x}, w(x, t) \right\}, \quad (2)$$

FIGURE 1: An Euler-Bernoulli sandwich beam with truncated ABH.

where the vector $\{u, w\}$ represents the displacement of any point on the sandwich ABH beam, $w(x, t)$ represents the deflection curve of the beam at different times, and $\partial w / \partial x$ represents the deflection angle of the beam at position x . Based on the separation variable method, the deflection curve of the sandwich beam can be expressed as

$$w(x, t) = \sum_i P_i(t) \varphi_i(x) = \mathbf{P}^T \boldsymbol{\varphi} = \boldsymbol{\varphi}^T \mathbf{P}, \quad (3)$$

where $\varphi_i(x)$ is the admissible shape function solely depending on coordinate x and $P_i(t)$ is time dependent and corresponding to weight coefficients to be assigned to the shape function.

The kinetic energy of the system can be obtained by

$$T = T_{b1} + T_{b2} + T_{b3} + T_{\text{damp}}, \quad (4)$$

where T_{b1} , T_{b2} , and T_{b3} are the kinetic energies of the three layers of the sandwich ABH beam and T_{damp} is the kinetic energy of the damping layer attached on the beam. They can be computed as

$$\begin{aligned} T_{b1} &= \frac{1}{2} \int_{x_{b1}}^{x_{b3}} \rho_{b1} A_{b1} \left(\frac{\partial w}{\partial t} \right)^2 dx, \\ T_{b2} &= \frac{1}{2} \int_{x_0}^{x_{b3}} \rho_{b2} A_{b2} \left(\frac{\partial w}{\partial t} \right)^2 dx, \\ T_{b3} &= \frac{1}{2} \int_{x_{b1}}^{x_{b3}} \rho_{b3} A_{b3} \left(\frac{\partial w}{\partial t} \right)^2 dx, \\ T_{\text{damp}} &= \int_{x_{d1}}^{x_{d2}} \rho_{\text{damp}} A_{\text{damp}} \left(\frac{\partial w}{\partial t} \right)^2 dx, \end{aligned} \quad (5)$$

where ρ_{b1} , ρ_{b2} , ρ_{b3} , and ρ_{damp} are the densities of the three layer materials and the damping layer material. A_{b1} , A_{b2} , A_{b3} , and A_{damp} are the cross-sectional areas of each layer of the sandwich ABH beam and the damping layer. The potential energy of the system can be found as

$$V = V_{b1} + V_{b2} + V_{b3} + V_{\text{damp}} + V_{\text{edge}}, \quad (6)$$

where V_{b1} , V_{b2} , and V_{b3} represent the potential energies of the three layers, V_{damp} represents the potential energy of the damping layer, and V_{edge} represents the potential energy of

the rotating spring and the translational spring. They are expressed as

$$V_{b1} = \frac{1}{2} \int_{x_{b1}}^{x_{b2}} E_{b1} I_{b1} \left(\frac{\partial^2 w}{\partial x^2} \right) dx, \quad (7)$$

$$V_{b2} = \frac{1}{2} \int_{x_{b0}}^{x_{b3}} E_{b2} I_{b2} \left(\frac{\partial^2 w}{\partial x^2} \right) dx, \quad (8)$$

$$V_{b3} = \frac{1}{2} \int_{x_{b1}}^{x_{b2}} E_{b3} I_{b3} \left(\frac{\partial^2 w}{\partial x^2} \right) dx, \quad (9)$$

$$V_{\text{damp}} = \frac{1}{2} \int_{x_{d1}}^{x_{d2}} E_{\text{damp}} I_{\text{damp}} \left(\frac{\partial^2 w}{\partial x^2} \right) dx, \quad (10)$$

$$V_{\text{edge}} = \frac{1}{2} k w(x_{b3}, t)^2 + \frac{1}{2} q \left(\frac{\partial w(x_{b3}, t)}{\partial x} \right)^2, \quad (11)$$

where E_{b1} , E_{b2} , and E_{b3} stand for Young's modulus of the three layers and E_{damp} stands for Young's modulus of the damping layer. I_{b1} , I_{b2} , and I_{b3} are the inertias of the three layers of the sandwich ABH beam at coordinate x . I_{damp} is the inertia of the damping layer at coordinate x . The damping layer is embedded as a part of the energy expression of the entire system, and it is fully coupled to the bare ABH beam. Considering the structural damping of the system, Young's modulus in equations (7)–(10) will be multiplied by $(1 + \eta_n)$, and η_n ($n = 1, 2, 3, \dots$) represents the damping loss factors of the three layers and the damping layer of the sandwich ABH beam.

The work done by the external force $f(t)$ acting on the ABH beam at x_f is given by

$$W = f(t) \cdot w(x_f, t). \quad (12)$$

The Lagrangian of the entire system can be expressed as

$$L = T + V - W. \quad (13)$$

According to the Hamiltonian equation, the following Lagrangian equation can be obtained after a series of transformation:

$$(\mathbf{K} - \omega^2 \mathbf{M}) \mathbf{A} = \mathbf{F}, \quad (14)$$

where $\mathbf{M}$ and $\mathbf{K}$ stand for the mass matrix and stiffness matrix of the whole system. Due to the consideration of the damping loss factors of the three layers and the damping layer, the stiffness matrix is complex. The response of the entire system under forced vibration can be obtained by solving equation (14). When $\mathbf{F}$ is set to zero, the natural frequencies and mode shapes of the sandwich ABH beam can be obtained.

The mass matrix of the entire system can be written as

$$\mathbf{M} = 2\mathbf{M}^{\text{steel}} + \mathbf{M}^{\text{AL}} + \mathbf{M}^{\text{damp}} = 2\mathbf{M}^{\text{steel_Uni}} + \mathbf{M}^{\text{AL_Uni}} + 2\mathbf{M}^{\text{steel_ABH}} + \mathbf{M}^{\text{AL_ABH}} + \mathbf{M}^{\text{damp}}. \quad (15)$$

For each part of the sandwich ABH beam, the mass matrices are expressed as

$$\mathbf{M}^{\text{damp}} = 2\rho_d \int_{x_{d1}}^{x_{d2}} \boldsymbol{\varphi} \boldsymbol{\varphi}^T h_d(x) dx, \quad (16)$$

$$\mathbf{M}^{\text{Uni}} = 2\rho h \int_{x_2}^{x_3} \boldsymbol{\varphi} \boldsymbol{\varphi}^T dx, \quad (17)$$

$$\mathbf{M}^{\text{ABH}} = 2\varepsilon \rho \int_{x_1}^{x_2} \boldsymbol{\varphi} \boldsymbol{\varphi}^T h_b(x) x^2 dx. \quad (18)$$

In (17) and (18), the upper and lower limits of the integral of the mass matrix are the starting and ending coordinates of the length of each layer along the x -axis. For example, when the equations are used to calculate the mass matrix of the steel layer, the upper and lower limits of the integral are (x_{b1}, x_{b2}) and (x_{b2}, x_{b3}) . When we use the equations to calculate the mass matrix of the aluminum layer, the upper and lower limits of the integral are (x_0, x_{b1}) and (x_{b1}, x_{b3}) .

The stiffness matrix of the entire system is given by

$$\begin{aligned} \mathbf{K} &= 2\mathbf{K}^{\text{steel}} + \mathbf{K}^{\text{AL}} + \mathbf{K}^{\text{damp}} + \mathbf{K}^{\text{edge}} \\ &= 2\mathbf{K}^{\text{steel_Uni}} + \mathbf{K}^{\text{AL_Uni}} + 2\mathbf{K}^{\text{steel_ABH}} + \mathbf{K}^{\text{AL_ABH}} \\ &\quad + \mathbf{K}^{\text{damp}} + \mathbf{K}^{\text{edge}}. \end{aligned} \quad (19)$$

For each part of the sandwich ABH beam, the stiffness matrices are computed by

$$\mathbf{K}^{\text{Uni}} = \frac{2}{3} E h^3 \int_{x_2}^{x_3} \frac{\partial^2 \boldsymbol{\varphi}}{\partial x^2} \frac{\partial^2 \boldsymbol{\varphi}^T}{\partial x^2} dx, \quad (20)$$

$$\mathbf{K}^{\text{ABH}} = \frac{2}{3} E \varepsilon^3 \int_{x_1}^{x_2} \frac{\partial^2 \boldsymbol{\varphi}}{\partial x^2} \frac{\partial^2 \boldsymbol{\varphi}^T}{\partial x^2} x^6 dx, \quad (21)$$

$$\begin{aligned} \mathbf{K}^{\text{damp}} &= \frac{2}{3} E_d \int_{x_{d1}}^{x_{d2}} \frac{\partial^2 \boldsymbol{\varphi}}{\partial x^2} \frac{\partial^2 \boldsymbol{\varphi}^T}{\partial x^2} (3\varepsilon^2 x^4 h_d(x) \\ &\quad + 3\varepsilon x^2 h_d(x)^2 + h_d(x)^3) dx, \end{aligned} \quad (22)$$

$$\begin{aligned} \mathbf{K}^{\text{edge}} &= k \boldsymbol{\varphi}(x_{b3}) \boldsymbol{\varphi}^T(x_{b3}) \\ &\quad + q \frac{\partial \boldsymbol{\varphi}}{\partial x} \Big|_{x=x_{b3}} \frac{\partial \boldsymbol{\varphi}^T}{\partial x} \Big|_{x=x_{b3}}. \end{aligned} \quad (23)$$

Similar to the expression of the mass matrix, the upper and lower limits of the integral in equations (20) and (21) are (x_{b1}, x_{b2}) and (x_{b2}, x_{b3}) for the steel layer and (x_0, x_{b1}) and (x_{b1}, x_{b3}) for the aluminum layer.

Since the stiffness matrix $\mathbf{K}$ is complex, the eigenvalues of the proposed beam are given by

$$\omega^2 = \omega_n^2 (1 + i\eta), \quad (24)$$

where ω_n is the natural frequency and η is the loss factor of the sandwich ABH beam.

2.2. Shape Function Fitting the Displacement Field. To fit the displacement field of nonuniform beams or plates, the polynomial functions have been used to solve linear or nonlinear thickness variation of the nonuniform beams [20, 21] or plates [22, 23]. However, due to the thickness of the sandwich ABH beam dwindling to zero rapidly in the ABH region, the above functions are not able to accurately present the displacement of the sandwich ABH beam. The Mexican hat wavelets (MHWs) have been demonstrated suitable to analyze the displacement field of the ABH beam in [15]. The MHW function has the standard form after normalization [24]:

$$\varphi(x) = \frac{2}{\sqrt{3}} \pi^{-1/4} (1 - x^2) \exp\left(-\frac{x^2}{2}\right). \quad (25)$$

The MHW function is the second derivative of the Gaussian distribution function. It can be seen that, from equation (25), the MHW function is symmetric about the y -axis, with an amplitude of $2/\sqrt{3}\pi^{-1/4}$. The MHW function can be expanded as the following a set of functions after the wavelet transform:

$$\varphi_{j,k}(x) = \frac{2}{\sqrt{3}} \pi^{-1/4} 2^{j/2} \left[1 - (2^j x - k)^2 \right] \exp\left[-\frac{(2^j x - k)^2}{2}\right], \quad (26)$$

where the integer j is the scaling parameter determining the calculation accuracy of the present model and the integer k is the translation parameter which determines the dimensions of the mass matrix and the stiffness matrix. The values of the MHW under different scaling parameters j and translation parameters k are shown in Figure 2. It shows that, by changing the values of j and k , the MHW can be scaled and translated to fit the displacement field of different ABH beams.

The key to fitting the shape function of the semianalytical model using MHW is to determine the value ranges of j and k . Meanwhile, the mass matrix $\mathbf{M}$ and the stiffness matrix $\mathbf{K}$ should be eliminated when singularity occurs during calculation. When the length domain of the sandwich ABH beam is (x_0, x_{b3}) , the value of k should be

$$\begin{aligned} k &= [-4 + m - j + l(x_0 2^j), u(x_{b3} 2^j) + 4 - m + j], \\ j &= 0, 1, 2, \dots, m, \quad m \leq 5, \end{aligned} \quad (27)$$

FIGURE 2: MHW values under different scaling and translation parameters.

where $l(x)$ presents the greatest integer that is less than or equal to x and $u(x)$ denotes the least integer that is greater than or equal to x . m is the power exponent of the ABH region of the sandwich beam. For example, when the length domain of the sandwich beam is $(-6, 6)$ and $m = 2$, $j = 0$, the value range of k is $(-8, 8)$. It can be known from equation (27) that the longer the length of the sandwich ABH beam, the larger the range of k , and the higher the power exponent of the ABH region, the larger the value of j . For any given sandwich ABH beam, the displacement field can be accurately fitted by adjusting the values of j and k to fulfill the calculation accuracy requirements.

3. Numerical Simulation Results and Discussion

3.1. FEM Validation of the Proposed Model. The geometric parameters and material parameters of the sandwich ABH beam are shown in Table 1. The boundary condition of the sandwich ABH beam is clamped free. There is a truncation at the end of the ABH region, with a thickness of 2 mm. In order to verify the accuracy of the proposed semianalytical model of the sandwich ABH beam, the natural frequencies of the sandwich ABH beam is calculated, respectively, by the finite element method (FEM) and the semianalytical model. The model used in FEM validation is shown in Figure 3. In order to ensure the accuracy of the FEM calculation results, the mesh size of the ABH region is 1 mm, while the mesh size of the remaining part is 4 mm.

To verify the computational accuracy of the established model, the first 30 natural frequencies of the sandwich ABH beam under the clamped-free boundary condition without considering damping layers are calculated, as shown in Table 2. It can be seen that the first 30 natural frequencies obtained by the presented model and FEM are approximately equal with an error less than 2 percent. The modal shapes of the bare ABH beam calculated with the present

TABLE 1: Geometrical and material parameters of the sandwich ABH beam.

Geometrical parameters	Material parameters
$\varepsilon = 0.006$	$\rho_{b1} = 7800 \text{ kg/m}^3$, $\rho_{b2} = 2700 \text{ kg/m}^3$, and $\rho_{b3} = 7800 \text{ kg/m}^3$
$m = 2$	$E_{b1} = 210 \text{ GPa}$, $E_{b2} = 69 \text{ GPa}$, and $E_{b3} = 210 \text{ GPa}$
$h_{b1} = 0.5 \text{ cm}$, $h_{b2} = 1 \text{ cm}$, and $h_{b3} = 0.5 \text{ cm}$	$\eta_{b1} = 0.005$, $\eta_{b2} = 0.005$, and $\eta_{b3} = 0.005$
$x_0 = 5 \text{ cm}$	$\rho_d = 1200 \text{ kg/m}^3$
$x_{d1} = 6 \text{ cm}$ and $x_{d2} = 8 \text{ cm}$	$E_d = 12 \text{ MPa}$
$x_{b1} = 7.58 \text{ cm}$, $x_{b2} = 8.87 \text{ cm}$, and $x_{b3} = 45 \text{ cm}$	$\eta_d = 0.5$
$h_d = 0.2 \text{ cm}$ and $h_t = 0.2 \text{ cm}$

FIGURE 3: Model used in FEM validation.

model and FEM method are shown in Figure 4. In Figure 4, the starting point of the x -axis is the right end of the bare ABH beam and it can be seen that the modal shapes computed by the present model also agree well with those

TABLE 2: Comparison of the first 30 natural frequencies obtained by the present model and FEM.

Natural frequency	Present model	FEM	Error (%)
ω_1	121.65	121.74	-0.070
ω_2	142.21	142.56	-0.247
ω_3	751.74	748.92	0.376
ω_4	869.24	865.82	0.392
ω_5	1765.8	1767.2	-0.078
ω_6	2050.4	2044.6	0.286
ω_7	2331.9	2321.5	0.447
ω_8	3493.5	3469.3	0.408
ω_9	3895.3	3875.3	0.515
ω_{10}	4298.9	4301.5	-0.060
...	...	...	...
ω_{21}	13285	13216	0.521
ω_{22}	14816	14723	0.634
ω_{23}	16024	15922	0.640
ω_{24}	16016	15929	0.544
ω_{25}	17386	17295	0.528
ω_{26}	17936	17837	0.554
ω_{27}	19141	19019	0.639
ω_{28}	19131	19352	-1.143
ω_{29}	21105	21023	0.390
ω_{30}	21224	21118	0.501

calculated by the FEM method. It is proved that the presented model has good precision for solving the dynamic response of the sandwich ABH beam.

3.2. ABH Effect of the Sandwich Beam. According to the geometric and material parameters in Table 1, the acceleration responses of the ABH region (measuring point $x_m = 8.87$ cm) and the uniform beam region (measuring point $x_m = 10.87$ cm) of the sandwich ABH beam without damping layers are calculated. They are compared with the acceleration responses of the sandwich uniform beam at the same measuring points, which are shown in Figure 5. A harmonic driving force of 10 N is applied at $x_f = 25$ cm, and the reference acceleration is 1 m/s^2 . It is shown that, for a sandwich ABH beam structure with a truncation and no damping layers, the acceleration peak values and corresponding frequencies of the ABH region are very similar to that of the sandwich uniform beam at low frequencies and the peak values are significantly greater than the sandwich uniform beam in 2 kHz to 8 kHz. It should be noted that when the location of the driving point is different, the vibration response of the structure also changes which is mainly caused by the modal participation coefficient changing of some modes [25]. So there are only six peaks below 10 kHz in the calculation when the driving point is located at $x_f = 25$ cm. The above phenomenon in the ABH region indicates that there is significant energy concentration in this area, which makes the vibration acceleration of the ABH region markedly increased. Meanwhile, as the calculation frequency increases from 0 to 10 kHz, the gap between two corresponding frequencies of the bare ABH beam and the uniform beam peaks also increases. For the uniform beam region of the sandwich ABH beam, the peak

values of the acceleration are attenuated at middle frequency and high frequency, but it still increases at about 2 kHz.

To further investigate the ABH effect, the flexural wave focalization effect of the bare ABH beam is studied in the time domain with the commercial software COMSOL Multiphysics. The position and amplitude of the driving force is the same as above, and the frequency of the driving force is 100 kHz. The displacement of the bare ABH beam along the z -axis is shown in Figure 6. In Figure 6, the direction of the view is a top view and the ABH regions are located at the right end of the beam. Figure 6(a) is the displacement of the bare ABH beam when the action time is $1.1e - 4$ s, and at this time, we can see that the flexural wave is propagating from the position of the excitation point to both ends of the ABH beam. Figure 6(b) is the displacement of the bare ABH beam when the action time is $2.8e - 4$ s, and at this time, the flexural wave has been focalized in the ABH region. The wavelength of the flexural wave is compressed in this region, forming a high energy region in the acoustic black hole.

To conclude above analysis, for a sandwich-truncated ABH beam structure without damping layers, the ABH region produces a certain energy concentration effect in middle and high frequencies, but due to the existence of the truncation, the acceleration attenuation of the uniform beam region is not obvious and even get increased at some frequencies. In the actual manufacturing, it is impossible to achieve an ABH area without truncation. So that other measures should be taken to improve the vibration reduction effect of the sandwich ABH beam in application.

3.3. Effect of the Damping Layer. To evaluate the vibration reduction performance of the sandwich ABH beam, the influence of the damping layer parameters on the dynamic responses of the ABH region and uniform region of the sandwich ABH beam will be analyzed. The damping layers coupled with the sandwich ABH beam have a thickness of 2 mm and a length of 20 cm (along the x -axis direction) with a damping loss factor of 0.5. The magnitude and position of the harmonic force and measuring points keep unchanged. It is observed from Figure 7 that even if the thickness of the damping layers is only 2 mm, they can still have a very significant vibration reduction effect on the sandwich ABH beam, especially in the middle and high frequencies. For the ABH region, the attenuation of the acceleration peak values at about 9 kHz reaches 18.8 dB and, in 2 kHz–4 kHz domain, it also reaches more than 7 dB. Besides, for the uniform beam region, the attenuation is 12.5 dB at about 4 kHz and it reaches more than 7 dB in other corresponding frequencies.

To summarize, in order to achieve an obvious vibration reduction effect, it is important to attach damping layers on the ABH region of the sandwich ABH beam with truncation even if the length and thickness of the damping layer are much smaller than the sandwich ABH beam.

Figure 8 shows the effect of the damping loss factor of the damping layer on the vibrational acceleration of the ABH region and the uniform beam region. As observed in Figure 8, the larger the damping loss factor of the damping

FIGURE 4: Modal shapes of the bare ABH beam calculated with the present model and FEM method: (a) first mode; (b) sixth mode; (c) twelfth mode; (d) twenty-ninth mode.

FIGURE 5: Acceleration responses of the bare ABH beam and the uniform beam: (a) ABH region ($x_m = 8.87$ cm); (b) uniform beam region ($x_m = 10.87$ cm).

FIGURE 6: Displacement of bare ABH beam at different times: (a) $t = 1.1e-4$ s; (b) $t = 2.8e-4$ s.

FIGURE 7: Effects of the damping layers on the acceleration responses: (a) ABH region ($x_m = 8.87$ cm); (b) uniform beam region ($x_m = 10.87$ cm).

FIGURE 8: Effects of the damping loss factor of the damping layers on acceleration responses: (a) ABH region ($x_m = 8.87$ cm); (b) uniform beam region ($x_m = 10.87$ cm).

layer, the greater the vibration acceleration attenuation of the sandwich ABH beam. In the ABH region, for every 0.1 increase in the damping loss factor of the damping layer, the vibration acceleration of the sandwich ABH beam can achieve about 1.2 dB attenuation, among which the maximum value is 1.3 dB. Furthermore, the attenuation of the acceleration peak values corresponding to the high frequencies is slightly larger than the attenuation of the acceleration peak values corresponding to the low frequencies. For the uniform beam region, the attenuation trend of the vibration acceleration peak values corresponding to each frequency is similar to that of the ABH region as the damping loss factor of the damping layer increases. However, the influence of the damping loss factor variation on the uniform beam region is greater than that on the ABH region. The maximum peak values attenuation of the uniform beam region is 1.7 dB at about 9 kHz.

The influence of the thickness of the damping layer on the vibration acceleration response of the sandwich ABH beam structure is also studied. The results are shown in Figure 9, and it reveals that a significant vibration acceleration attenuation could be reached with every 1 mm increase in the thickness of the damping layer both for the ABH region and the uniform region of the sandwich ABH beam. The higher the corresponding frequencies, the greater the peak values attenuation. When the thickness of the damping layer reaches 3 mm, the acceleration attenuations generated of the ABH region and the uniform beam region are 30.1 dB and 21.4 dB at about 9 kHz compared with the thickness of the damping layer being 2 mm. With this parameter, it has the effect of completely suppressing the peaks of acceleration response at this corresponding frequency.

The mode shapes of ABH beams with different damping layer thickness at around 9 kHz are presented in Figure 10. It can be observed from Figure 10 that, for the three ABH beams with different damping layer thicknesses varying from 1 mm to 3 mm, the mode shape of each ABH beam exists at 9150 Hz. The modal loss factor is also computed to reveal the mechanism of vibration peak attenuation which is as shown in Figure 11. In Figure 11, we can see that the modal loss factor gets increased with the increasing of the damping layer thickness in a certain frequency domain and it can be concluded that the mechanism of the peaks attenuation in Figure 9 is the increase of the modal loss factor of the system. At 9150 Hz, the damping loss factors of the three damping ABH beams with damping layer thickness varying from 1 mm to 3 mm are $6.78e-4$, $2.99e-2$, and $3.87e-1$, respectively, and the huge difference in modal loss factors at 9150 Hz is the reason of the disappearance of the peak of the damping ABH beam with damping layer of 3 mm thickness which is shown in Figure 9.

To summarize, it can be seen that the influence of the thickness variation of the damping layers on the acceleration response of the sandwich ABH beam is much greater than the variation of the damping loss factor. Therefore, in the application of the sandwich ABH beam, the selection of a damping layer of appropriate thickness can significantly

improve the vibration attenuation effect of the entire system.

3.4. Effect of the Middle Layer. Unlike the traditional single-layer ABH beam structures, the sandwich ABH beam with aluminum in the middle layer and steel in the upper and lower layers is studied in this paper. The effect of aluminum or steel as the middle layer material on vibration acceleration response of the sandwich ABH beam is shown in Figure 12. It is observed that the effect of the middle layer material on both the peak values and the corresponding frequencies is not obvious in the low-frequency domain. However, the frequencies corresponding to the peak values become larger as the material of the middle layer is replaced by aluminum from steel in middle- and high-frequency domains. For the ABH region, the peak value of vibration acceleration of the ABH beam with aluminum in the middle layer is 5.4 dB lower than that of steel in the middle layer when the frequency is around 9 kHz. However, it is 8.9 dB larger than that of steel in the middle layer when the frequency is at about 6 kHz. For the uniform beam region, the peak value of vibration acceleration of the ABH beam with aluminum in the middle layer is 4.7 dB lower than that of steel in the middle layer when the frequency is around 9 kHz.

The influence of the thickness variation of the aluminum layer on the vibration acceleration of the sandwich ABH beam structure will be further investigated. The thickness of the middle aluminum layer is 4 mm, 6 mm, and 8 mm, respectively. The thickness of the steel layers changes with the thickness of the middle aluminum layer, so as to ensure that the total thickness of the sandwich ABH beam remains 20 mm. Figure 13 shows the acceleration responses of the ABH region and the uniform beam region of the sandwich ABH beam structure when the thickness of the aluminum layer is changing from 4 mm to 8 mm. As can be seen, the larger the thickness of the aluminum layer, the larger the frequencies corresponding to the peaks of the acceleration in middle- and high-frequency domains. At low frequency, the peak values and the frequencies corresponding to the peak values are not affected obviously by the thickness variation of the aluminum layer. But at the frequency of around 9 kHz, the larger the thickness of the aluminum layer, the smaller the peak values of both the ABH region and the uniform beam region.

4. Experimental Validations

An experiment system was conducted to validate the accuracy of the numerical simulation. The geometric parameters of the sandwich ABH beam are the same as in Table 1. The non-ABH end of the sandwich beam is clamped on the fixture, and the length of the clamped part is 4 cm so that the entire beam achieves the clamped-free boundary condition. The force rod of the shaker (Modal shop K2007E01) is attracted to the action point of the sandwich ABH beam by a magnetic seat. The location of the harmonic driving force of 10 N is $x_f = 25$ cm, and acceleration sensors

FIGURE 9: Effects of the damping layer thickness on acceleration responses: (a) ABH region ($x_m = 8.87$ cm); (b) uniform beam region ($x_m = 10.87$ cm).

FIGURE 10: Mode shapes of ABH beams with different damping layer thickness at around 9 kHz: (a) $h = 1$ mm, frequency = 9141.59 Hz; (b) $h = 2$ mm, frequency = 9158.9 Hz; (c) $h = 3$ mm, frequency = 9158.22 Hz.

are placed in the ABH region and the uniform beam region to measure the acceleration response. The $m+p$ data acquisition system (VibPlot-8) generates a sinusoidal signal

from 20 Hz to 12000 Hz to feed the shaker and collects the signals of two acceleration sensors. Photograph and schematic diagram of the experimental system are shown in

FIGURE 11: Modal loss factor of ABH beams with different damping layer thickness: (a) calculation frequency from 0 to 10 kHz; (b) calculation frequency from 8500 to 9500 Hz.

FIGURE 12: Effects of the middle layer material on acceleration responses: (a) ABH region ($x_m = 8.87$ cm); (b) uniform beam region ($x_m = 10.87$ cm).

Figure 14. The sandwich ABH beam structure was manufactured, with the middle layer material being aluminum and the upper and lower layers being steel. The aluminum material is 2A12 aluminum alloy, and the steel material is 45 steel. The three layers of the sandwich ABH beam are bonded by thin metal glue.

The acceleration response of the ABH region of the sandwich ABH beam structure was measured and compared with the numerical simulation results, as shown in Figure 15. It can be seen from Figure 15 that the numerical simulation results in 0 to 8 kHz domain agree well with the experimental results, in terms of both the acceleration peak values and corresponding frequencies. At 9 kHz, an increasing error compared with the measured peak values and corresponding frequency of the acceleration is mainly

because the shear and torsional effects are ignored in the Euler–Bernoulli beam theory, resulting in an error in calculation of the vibration of the sandwich ABH beam in the high-frequency domain.

5. Conclusions

In this paper, a sandwich acoustic black hole beam has been proposed and a semianalytical model of the sandwich ABH beam with truncation has been established. The displacement field of the proposed ABH beam is fitted by the Mexican wavelet function, and the accuracy of the model of the sandwich ABH beam has been verified by the FEM method.

We have investigated the vibrational acceleration responses of the ABH region and the uniform region of the

FIGURE 13: Effect of the aluminum layer thickness on acceleration responses: (a) ABH region ($x_m = 8.87$ cm); (b) uniform beam region ($x_m = 10.87$ cm).

FIGURE 14: Experimental set-up: (a) photograph of the experiment system; (b) schematic of the experiment system.

sandwich ABH beam with the middle layer being aluminum and the upper and lower layers being steel. Simulations reveal that, for the bare ABH beam, there is a certain energy concentration effect in the ABH region in middle and high frequencies. The damping layers attached on the sandwich ABH beam can significantly improve the vibration suppression effect of both the ABH region and the uniform beam region. In the high-frequency domain of 9 kHz, the

ABH region and the uniform beam region have attenuation effects of 18.8 dB and 10.9 dB when the damping layers are applied, and the attenuation effect being more than 7 dB in middle- and low-frequency domains. It has been shown that, for every 0.1 increase in the damping loss factor of the damping layers, the acceleration peak values of the ABH region and the uniform beam region can be attenuated by about 1 dB. Numerical results also indicate that the influence

FIGURE 15: Comparison between the numerical simulation and the experimental results.

of the damping layer thickness on the dynamic response of the sandwich ABH beam is very significant. When the thickness of the damping layer is increased to 3 mm, the high-frequency acceleration peaks at 9 kHz can be completely suppressed. It has been concluded that the material and thickness of the middle layer of the sandwich ABH beam can bring a vibration attenuation at around 9 KHz both for the ABH region and the uniform beam region in the present model.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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