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Research Article

Experimental Study of a Multipoint Random Dynamic Loading Identification Method Based on Weighted Average Technique

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Most of random dynamic loading identification research studies are about the original inverse pseudoexcitation method which does not fundamentally reduce the negative effect of ill-conditioned frequency response function matrix on accuracy of loading identification. This paper describes a new improved method based on weighted average technique to reduce peak errors between identified load spectrum and the actual load spectrum near some natural frequencies. Meanwhile, relative error of root mean square value between identified load and the actual load is reduced. The introduced selection method of threshold value is innovative which is the key of weighted average technique. This improved loading identification method is successfully applied to experiments of cantilever beam and thermal protection composite plate structure. Identification results prove that the proposed method is valid by good agreement between identified power spectrum density and the actual one. Moreover, this method has higher accuracy than inverse pseudoexcitation method in low-frequency band.

1. Introduction

Dynamic loading identification is the inverse problem in structural dynamics field. Great efforts have been made in dynamic loading identification as well as its engineering applications.

The development and application of dynamic loading identification in frequency domain preceded that in time domain. As pioneers, Bartlett and Flannelly [1] as early as 1979, by experimental tests, verified the accuracy of the dynamic loading identification method in the center of vibration wheel shaft of the dynamic model in American military UH-1H helicopter. Lee and Liu [2] combined the extended Kalman filter with intelligent regression least squares method to identify external dynamic loads acting on nonlinear tower structures. Samagassi et al. [3] reconstructed loads caused by multiple factors in linear elastic structures by the wavelet correlation vector method.

Because dynamic loading identification is an inverse problem in mathematics, many scholars [4–8] have done much work to overcome its ill-posedness in identification procedures. Callahan and Piergentili [9] identified dynamic loads with unknown positions by using frequency response function and singular value decomposition technique. Based on wavelet multiresolution analysis basis function expansion method, Qiao et al. [10] proposed a high precision time domain loading identification method to overcome the defect of ill-posed problem. Jacquelin et al. [11] analyzed a deconvolution loading identification technique in which the ill-posedness is solved by regularization. He et al. [12] combined interval extension with frequency response function- (FRF-) based least squares approach to identify load bounds for uncertain structures. Truncated total least squares (TTLS) method is applied to compute the perturbed part of the load.

Many scholars have made prominent contributions to the development and engineering application of random dynamic loading identification methods. Soize and Batou [13] used an uncertain computational model to identify stochastic loads acting on a nonlinear system for which a few experimental responses are usable. Ren et al. [14] combined maximum entropy regularization with creative conjugate gradient to identify random dynamic loading. Jia et al. [15] proposed a weighted regularization method based on orthogonal decomposition to alleviate the ill-posed problem in random dynamic loading identification. He et al. [16]

combined the modified regularization method with matrix perturbation method to identify random loads for stochastic structures. This method can solve ill-posed problems of traditional inverse pseudoexcitation method and reduce error propagation of the loading identification process. Jia et al. [17] discussed the error source and error range in random dynamic loading identification and then introduced the weighted total least square method to reduce the error expansion and accumulation in the stochastic dynamic loading identification model. Based on inverse pseudoexcitation method, Lin et al. [18] carried out some computer simulation studies on identification of random dynamic loads and then provided suggestions on selection and arrangement of response sensors. Guo and Li [19] drew support from acceleration responses and frequency response function matrix measured by experiments to identify random dynamic loads.

Researchers in References [20, 21] have discussed optimal arrangement of response sensors in dynamic loading identification processes through a large number of simulations and experiments. However, in actual engineering conditions, arrangement of response sensors is affected by structural design and external working environment [22, 23]. Sometimes, the installation positions of response sensors are limited or even fixed. Under these circumstances, it is necessary to optimize traditional methods to improve the accuracy of random dynamic loading identification and make these methods applicable to engineering application. Major contribution of this paper is to reduce peak errors between identified load spectrum and the actual load spectrum near some natural frequencies by introducing condition number weighted average technique to the original inverse pseudoexcitation method. Ultimately, the feasibility and effectiveness of the proposed method are demonstrated by random dynamic loading identification experiments of cantilever beam and thermal protection composite plate structures. The remainder of this paper is organized as follows. In Section 2, the model of improved random dynamic loading identification method based on condition number weighted average technique is introduced; in Sections 3 and 4, the improved method is applied to random dynamic loading identification of cantilever beam and thermal protection composite plate structures; finally, Section 5 gives some conclusions.

2. Model of Improved Random Loading Identification Based on Weighted Average Technique

The inverse pseudoexcitation method (IPEM) is applied to solve structural random dynamic loading identification problems. Due to considerable limitations in random dynamic loading identification procedure, some assumptions are given:

(1) Problems solved by the proposed method are limited to stationary random dynamic loading identification of linear systems

(2) Random responses for loading identification in this method are completely generated by stationary random loads to be identified

(3) The loading identification process is discrete including frequency response function matrices and response power spectrum density matrices

(4) Position of each random excitation to be identified is fixed, and its position remains unchanged in the whole process

In random vibration theory, according to transformation formula of power spectrum density matrices, the relation between response power spectrum density matrix S_{yy} and excitation power spectrum density matrix S_{FF} is represented as follows:

$$S_{yy} = H^* S_{FF} H^T, \tag{1}$$

where H^* is the conjugated matrix of frequency response function matrix H and H^T is the transpose of H.

Response power spectrum density matrix S_{yy} is a Hermitian matrix. So, S_{yy} can be expressed as

$$S_{yy} = \sum_{j=1}^{r} \lambda_j \phi_j \phi_j^H, \tag{2}$$

where λ_j and ϕ_j is the jth characteristic pair, r is the order of S_{yy}, and ϕ_j^H is the conjugate transpose of ϕ_j.

Using the characteristics of each order to construct pseudoresponse $\tilde{y}_j$, the following equation is obtained:

$$\tilde{y}_j = \sqrt{\lambda_j}\phi_j e^{i\omega t}, \tag{3}$$

where ω is the angular frequency and $e^{i\omega t}$ is the unit harmonic excitation.

Then, S_{yy} is transformed as

$$S_{yy} = \sum_{j=1}^{r} \tilde{y}_j \tilde{y}_j^H, \tag{4}$$

where $\tilde{y}_j^H$ is the conjugate transpose of $\tilde{y}_j$.

The original IPEM cannot improve the ill-posed problem of the frequency response function matrix near natural frequencies of the structure. Therefore, the identification results of original IPEM have large fluctuations and errors compared with the actual load spectrum near the natural frequencies. In this section, this problem is improved obviously by introducing the weighted average method.

The constructed pseudoresponse $\tilde{y}_j$ is caused by pseudoexcitation $\tilde{f}_j$, and the mathematic relation between them is shown in the following equation:

$$\tilde{f}_j = H^{+*} \tilde{y}_j, \tag{5}$$

where H^{+*} is the conjugate inverse of frequency response function matrix H and the dimension of H is $m \times n$, $m \geq n$.

The frequency response function matrix H in (5) can be written into row vector form:

$$\tilde{f}_j = \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_m \end{bmatrix}^{+*} \begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \vdots \\ \tilde{y}_m \end{bmatrix}, \quad (j = 1, 2, \ldots, r), \tag{6}$$

where r is the rank of the response power spectrum matrix S_{yy}, $T_i\,(i = 1, 2, \ldots, m)$ is the row vector of H, and $\tilde{y}_1$, $\tilde{y}_2, \ldots, \tilde{y}_m$ are elements of pseudoresponse vector $\tilde{y}_j$.

The arbitrary n rows of frequency response function matrix H can compose square matrix $H_i\,(i = 1, 2, \ldots, k)$; meanwhile, the corresponding n pseudoresponse elements are chosen to compose new pseudoresponse vector $\widetilde{y}_j^i\,(i = 1, 2, \ldots, k)$, and the new pseudoexcitation $\widetilde{f}_j^i$ is shown in the following equation:

$$\widetilde{f}_j^i = H_i^{-1}\widetilde{y}_j^i. \tag{7}$$

The matrix condition number is a rough measurement scale to reflect the influence of coefficient matrix A and constant term b's errors on solution x in equation $Ax = b$. In this section, weighted average technique is used to improve the accuracy of loading identification. The condition number of a matrix is defined as

$$\operatorname{cond}(A) = \|A\|_p \left\|A^{-1}\right\|_p, \tag{8}$$

where $\|\cdot\|_p$ represents matrix norm. Since norms are equivalent, condition numbers defined by different norms are also equivalent.

The reciprocal of the condition number of the square matrix H_i is t_i:

$$t_i = \frac{1}{\operatorname{cond}(H_i)}. \tag{9}$$

In equation (9), the condition number of frequency response function matrix is calculated by 2-norm.

Threshold value t is defined as follows:

$$t = \frac{\sqrt{\sum_{i=1}^{k} t_i^2}}{k}, \tag{10}$$

where t is the $1/k$ of the arithmetic square root of t_i's quadratic sum.

Weight w_i is defined as follows:

$$w_i = \begin{cases} t_i, & t_i \ge t, \\ \dfrac{t_i}{2}, & t_i < t, \end{cases} \tag{11}$$

where $t_i < t$; in other words, the condition number of this square matrix H_i is big enough, set w_i equal to $t_i/2$ for reducing the impact of H_i in the inverse process.

Obtain pseudoexcitation $\widetilde{f}_j$ by the following formula:

$$\widetilde{f}_j = \frac{\sum_{i=1}^{k} w_i \widetilde{f}_j^i}{\sum_{i=1}^{k} w_i}, \tag{12}$$

hence, excitation power spectrum density matrix S_{FF} can be expressed by pseudoexcitation $\widetilde{f}_j$ as

$$S_{FF} = \sum_{j=1}^{m} \widetilde{f}_j \widetilde{f}_j^H = H^{+*} S_{yy} H^{+T}, \tag{13}$$

where H^{+T} is the transposition inverse of frequency response function matrix H. Flow chart of the above method is shown in Figure 1.

The medium- and high-frequency problems need to be solved with the statistical energy method. Therefore, this improved method is only used to solve problems of random dynamic loading identification in low frequency. The following experimental studies are all limited in the low-frequency band (under 200 Hz).

3. Experimental Validation of Cantilever Beam Structure

In order to verify effectiveness of the proposed improved method described in section 2, a validation experiment was performed for cantilever beam structure.

3.1. Experimental Setup. Diagrammatic sketch of the experimental structure is shown in Figure 2. Setup of the cantilever beam experiment is shown in Figure 3. Four integrated circuit piezoelectric acceleration sensors and two force sensors are employed to measure vibratory signals for random dynamic loading identification proceeding. Experiment conditions are multi-input multi-output. The cantilever beam is divided into ten units, and serial number of each point is 1–10 from fixed end to free end. Sampling frequency of these vibration tests is 2048 Hz.

In the process of cantilever beam experiment, the low-pass filter noise reduction method has been adopted while M + P vibration control and data acquisition system are employed to collect information of frequency response functions and acceleration responses.

Geometry and material parameters of the cantilever beam are listed in Table 1.

3.2. Validation Procedure. Validating the proposed improved method described in Section 2 can be divided into three main steps:

(1) The first step is to obtain the frequency response functions between excitation points and response points. Every vibration exciter excites cantilever beam structure respectively and measures frequency response functions simultaneously. Distances from fixed end to each measuring point are listed in Table 2. The first three natural frequencies of the cantilever beam are extracted by a beforehand modal test and listed in Table 3.

(2) The second step is to obtain vibratory responses of the test structure. Two electromagnetic exciters excite the cantilever beam structure at the same time. And condition of the response-measured test is the same as condition in step (1). Moreover, positions of two exciters in these steps remain unchanged.

(3) The last step is to identify the random loads acting on the cantilever beam structure. Then compare relative error of the root mean square value between identified load and the actual one. According to random vibration theory, the root mean square value can be obtained by calculating the area under the power spectral curve line.

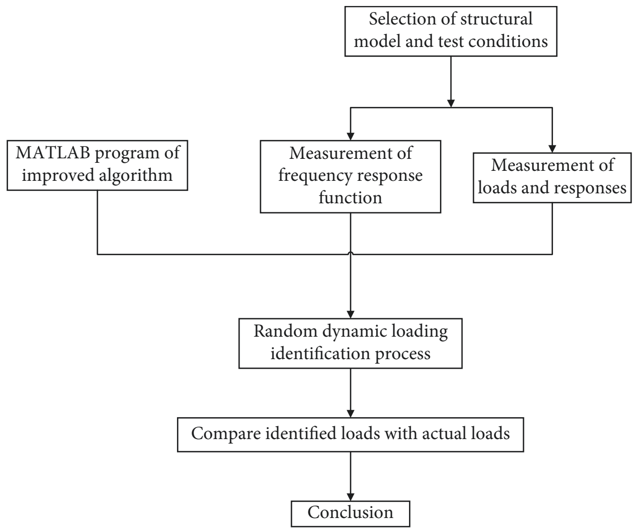

Figure 1: Flow chart of improved random dynamic loading identification method.

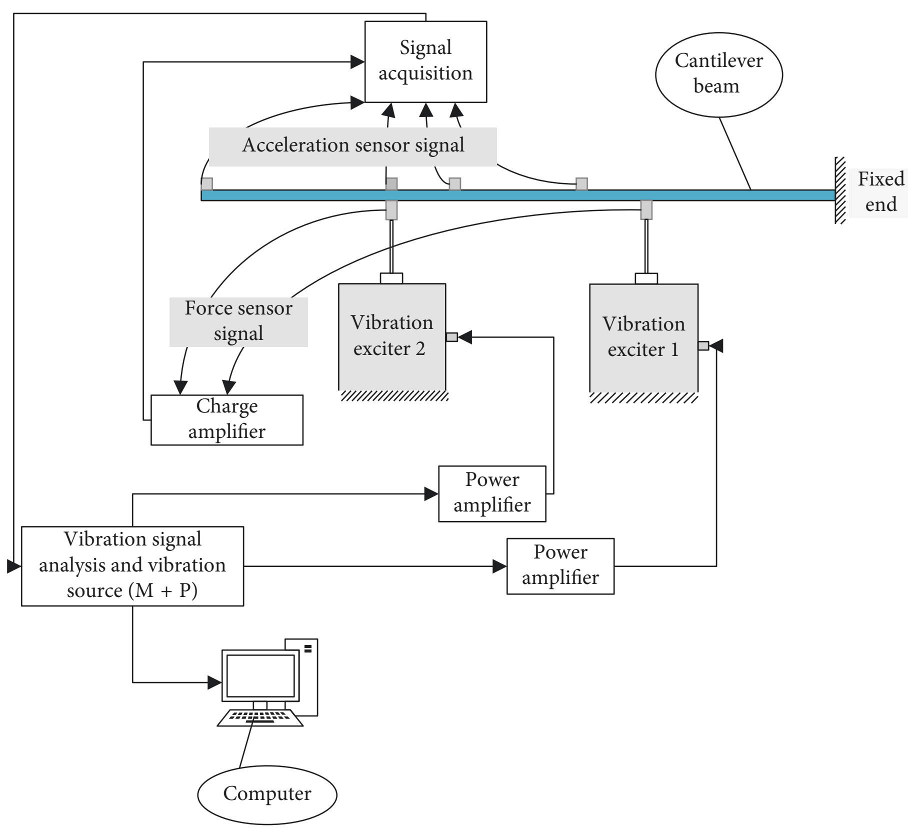

Figure 2: Diagrammatic sketch of cantilever beam test.

3.3. Results and Discussion. In comparative figures of identification results, "IPEM" represents the identification results of original inverse pseudoexcitation method. "Actual Load" represents excitation self-power spectrum density measured by force sensors. "CWAM" represents identification result of improved inverse pseudoexcitation method based on condition number weighted average technique.

The relative error is defined by the following equation:

$$\mathrm{RE} = \frac{\left|\mathrm{RMS}_{\mathrm{Identified}} - \mathrm{RMS}_{\mathrm{Actual}}\right|}{\mathrm{RMS}_{\mathrm{Actual}}} \times 100\%. \tag{14}$$

(1) Comparison of the third point's random excitation identification results is shown in Figure 4: relative errors of the root mean square value between identified load and the actual one in 5 Hz–180 Hz are listed in Table 4.

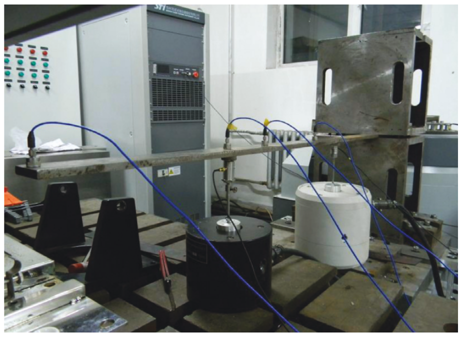

Figure 3: Photo of cantilever beam experiment setup.

Table 1: Geometry and material parameters of cantilever beam at reference temperature (RT: 25°C).

Length	Cross-sectional dimension	Density	Elastic modulus	Poisson ratio
0.9 m	0.05 m × 0.009 m	7.8×10^3 kg/m^3	200 GPa	0.3

Table 2: Distances between each measuring point and the fixed end.

Excitation 1	Excitation 2	Response 1	Response 2	Response 3	Response 4
0.27 m	0.63 m	0.36 m	0.54 m	0.63 m	0.9 m

Table 3: First three natural frequencies of the cantilever beam.

Fundamental frequency	Second natural frequency	Third natural frequency
9.7 Hz	48 Hz	115 Hz

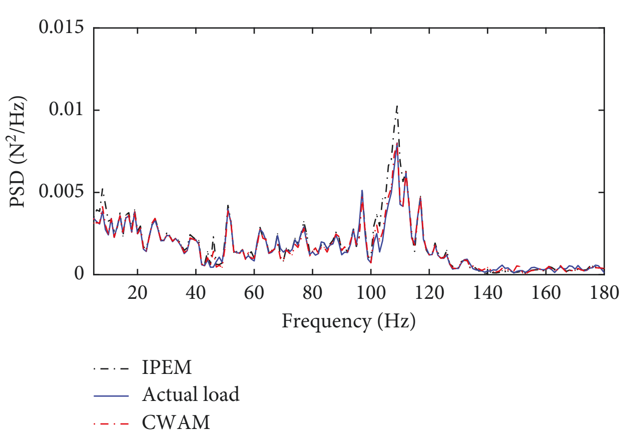

Figure 4: Comparison of the third point's random excitation identification results.

Table 4: The third point's excitation relative errors of root mean square value.

IPEM	11.07%
CWAM	6.27%

Relative errors of the root mean square value between identified load and the actual one in 5 Hz–180 Hz are listed in Table 4.

With 4.8% decrease of the relative error, compared with IPEM, CWAM significantly reduces peak errors between identified load spectrum and the actual load spectrum near 8 Hz, 45 Hz, and 115 Hz.

(2) Comparison of the seventh point's random excitation identification results is shown in Figure 5: relative errors of the root mean square value between identified load and the actual one in 5 Hz–180 Hz are listed in Table 5.

With 9.6% decrease of the relative error, compared with IPEM, CWAM significantly reduces peak errors between identified load spectrum and the actual load spectrum near 45 Hz, 100 Hz, and 115 Hz.

Through analysis of cantilever beam test results, CWAM significantly reduces peak errors between identified load spectrum and the actual load spectrum near some natural frequencies. The CWAM's relative error of the root mean square value between identified load and the actual one is less than IPEM's relative error in 5 Hz–180 Hz frequency band.

4. Experimental Validation of Thermal Protection Composite Plate Structure

In this section, a validation experiment was performed for thermal protection composite plate to verify the effectiveness of the proposed improved method described in Section 2. Moreover, extend the usability of this method for different structures.

4.1. Experimental Setup. Setup of the thermal protection composite plate experiment is shown in Figure 6. Diagrammatic sketch of the experimental structure is shown in Figure 7. Tests of thermal protection composite plate adopt multi-input multioutput experimental conditions. Sampling frequency is 1600 Hz. Test structure is thermal protection composite plate for the hypersonic vehicle. Thermal protection composite plate and electromagnetic exciters are fastened by bolts through the aluminum plate. In the processes of thermal protection composite plate experiment, the low-pass filter noise reduction method has been adopted while LMS vibration control and test system are employed to collect information of frequency response functions and acceleration responses.

Aluminum plate layer and composite layer are bonded together by glue. Delamination of the composite layer is shown in Figure 8.

Geometry and material parameters of aluminum plate and composite plate are listed in Tables 6–9.

Density of the fiber reinforced mullite matrix composite panels (face sheets) is 1580 kg/m^3.

4.2. Validation Procedure. The validation procedure can also be divided into three main steps:

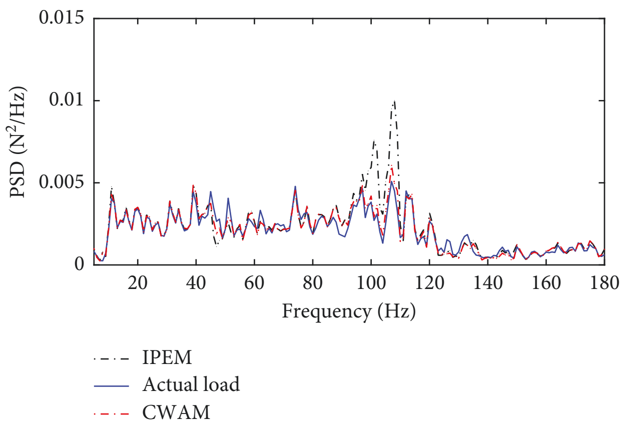

Figure 5: Comparison of the seventh point's random excitation identification results.

Table 5: The seventh point's excitation relative errors of root mean square value.

IPEM	12.04%
CWAM	2.44%

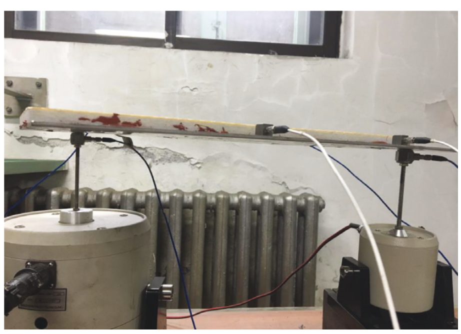

Figure 6: Photo of thermal protection composite plate experiment setup.

(1) The first step is to obtain the frequency response functions between excitation points and response points. Every vibration exciter excites thermal protection composite plate structure respectively and measures frequency response functions simultaneously. Sensor arrangement is shown in Figure 9, and their coordinates are listed in Table 10. Natural frequencies of the cantilever beam are extracted by a beforehand modal test and listed in Table 11.

(2) The second step is to obtain vibratory responses. Two electromagnetic exciters excite the thermal protection composite plate structure at the same time. And condition of response-measured test is the same as condition in step (1). Moreover, positions of two exciters in these steps remain unchanged.

(3) The last step is to identify random loads acting on the thermal protection composite plate structure. Then, compare relative error of root mean square value between identified load and the actual one. According to random vibration theory, the root mean square value can be obtained by calculating the area under the power spectral curve line.

4.3. Results and Discussion. In comparative figures of the following identification results, IPEM, actual load, and CWAM have the same meanings as those described in Section 3.3. The relative error is defined as Equation (14).

4.3.1. Excitation 1's Identification Result. Identification results of excitation 1 are shown in Figure 10.

Relative errors of the root mean square value between identified load and the actual one in 5 Hz–180 Hz are shown in Table 12. With 6.95% decrease of the relative error, compared with IPEM, CWAM significantly reduces peak errors between identified load spectrum and the actual load spectrum near 30 Hz and 110 Hz.

4.3.2. Excitation 2's Identification Result. Identification results of excitation 2 are shown in Figure 11.

Identification results of excitation 2 in 75 Hz–150 Hz are shown in Figure 12.

Excitation 2's relative errors of the root mean square value between identified load and the actual one in 5 Hz–180 Hz are shown in Table 13. With 4.9% decrease of the relative error, compared with IPEM, CWAM reduces peak errors between identified load spectrum and the actual load spectrum near 110 Hz.

Through analysis of thermal protection composite plate test results, CWAM reduces peak errors between identified load spectrum and the actual load spectrum near some natural frequencies. The CWAM's relative error of the root mean square value between identified load and the actual one is less than IPEM's relative error in 5 Hz–180 Hz regions.

5. Conclusions

A new improved random dynamic loading identification method has been proposed. Some conclusions can be summarized as follows:

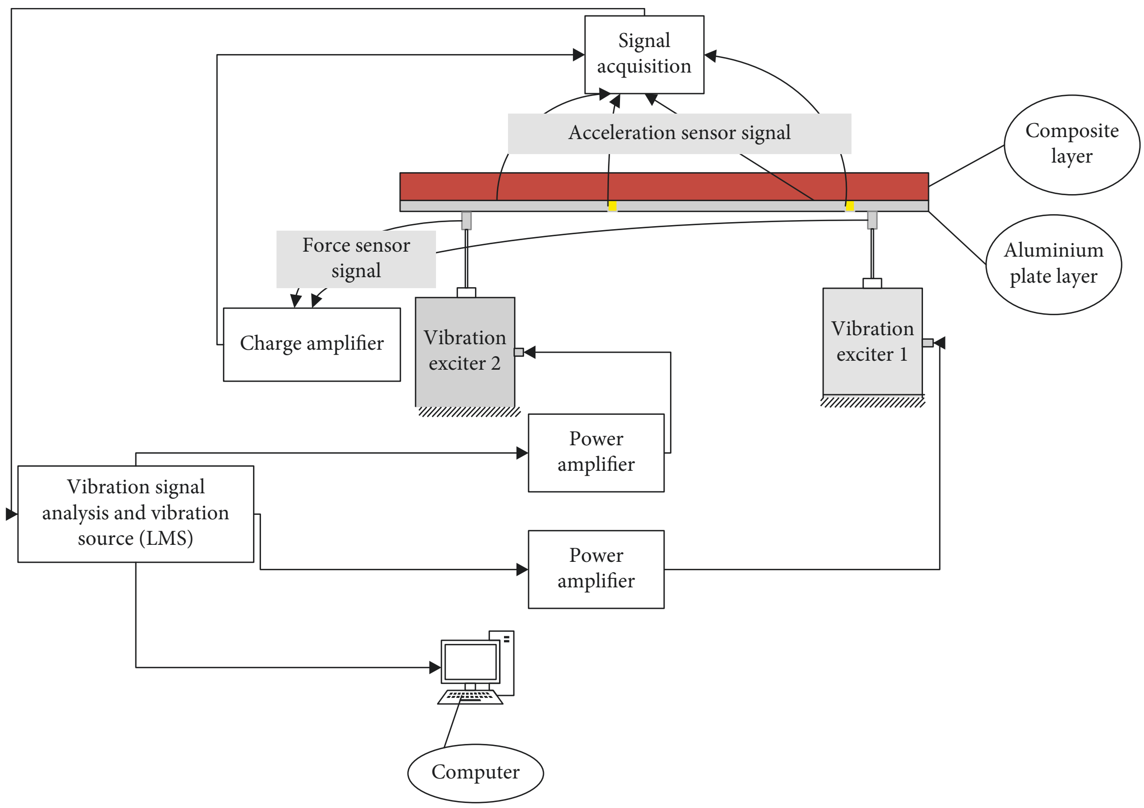

Figure 7: Connection of vibration exciter, force sensor, and composite plate.

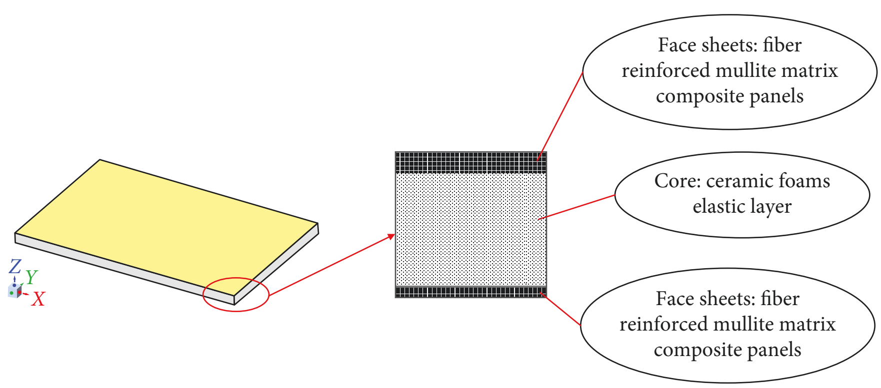

Figure 8: Delamination of the composite layer.

Table 6: Geometry and material parameters of the aluminum plate layer at reference temperature (25°C).

Length	Width	Thickness	Density	Elastic modulus	Poisson ratio
0.4 m	0.35 m	0.003 m	2.7×10^3 kg/m^3	70 GPa	0.33

Table 7: Geometry parameters of the composite plate.

Length	Width	Thickness
0.4 m	0.214 m	0.011 m

Table 8: Material parameters of face sheets at reference temperature (25°C).

E_X (GPa)	E_Y (GPa)	E_Z (GPa)	ν_{XY}	ν_{YZ}	ν_{ZX}	G_{XY} (GPa)	G_{YZ} (GPa)	G_{ZX} (GPa)
10.6	6.72	5.0	0.3	0.4	0.4	2.44	1.20	1.20

Table 9: Material parameters of core at reference temperature (25°C).

Density	Elastic modulus	Poisson ratio
320 kg/m^3	0.232 GPa	0.2

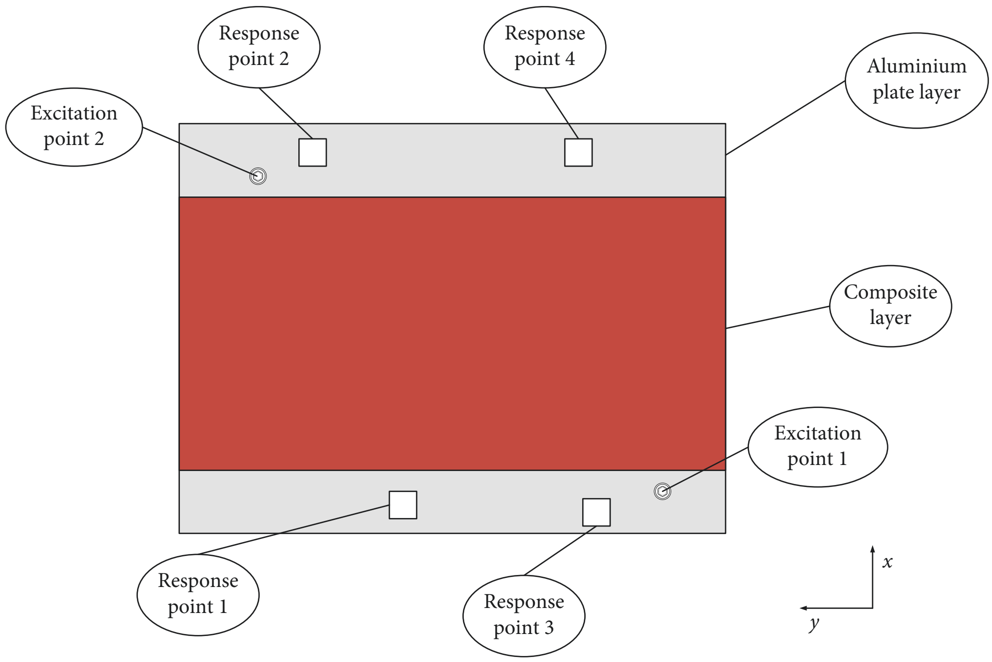

FIGURE 9: Sensor layout of excitation points and acceleration response points.

TABLE 10: Coordinates of excitation points and response points.

Excitation point (cm)	**1** (4.2, 4.6)	**2** (28.8, 33.8)	—	—
Response point (cm)	**1** (2.5, 24)	**2** (32, 30.9)	**3** (1.7, 8.9)	**4** (31.2, 9.7)

TABLE 11: Natural frequencies of the thermal protection composite plate.

Fundamental frequency	Second natural frequency
32 Hz	110 Hz

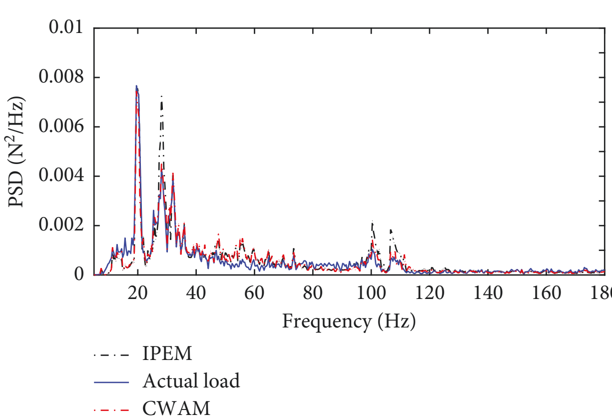

FIGURE 10: Identification results of excitation 1.

TABLE 12: Excitation 1's relative errors of the root mean square value.

IPEM	10.73%
CWAM	3.78%

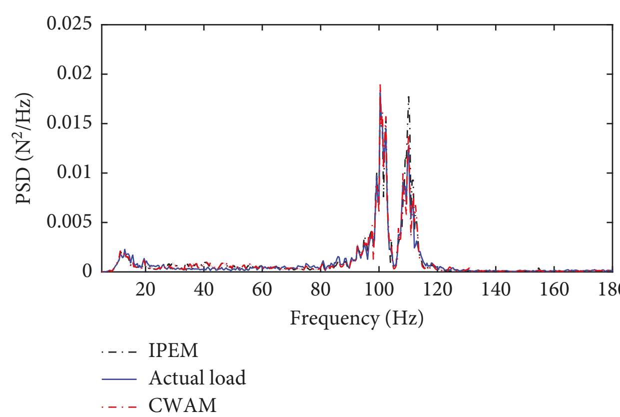

FIGURE 11: Identification results of excitation 2.

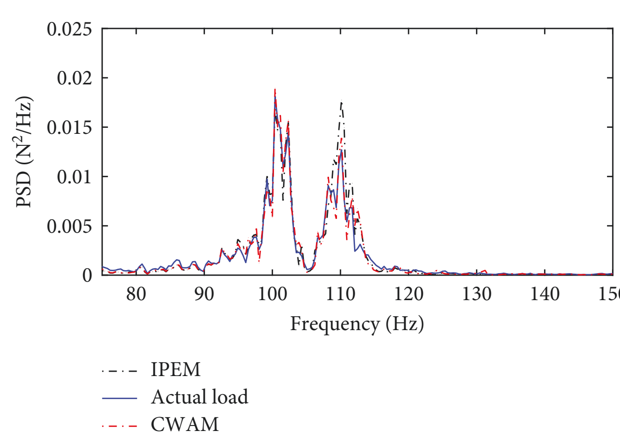

FIGURE 12: Identification results of excitation 2 in 75 Hz–150 Hz.

Table 13: Excitation 2's relative errors of the root mean square value.

IPEM	6.29%
CWAM	1.39%

(1) The accuracy of random dynamic loading identification method is improved through combining condition number weighted average technique with original inverse pseudoexcitation method. The influence of the matrix with large condition number is reduced by using weighted average technique.

(2) Loading identification experimental results for cantilever beam structure and thermal protection composite plate structure are all in good agreement with the actual self-power spectrum density curves, which proves the feasibility and effectiveness of the new improved method presented in this paper.

(3) The improved method's (CWAM) root mean square value relative error between identified load and the actual one is less than relative error of inverse pseudoexcitation method (IPEM) in the low-frequency band (under 200 Hz). Meanwhile, peak errors of load spectrum near some natural frequencies are reduced significantly.

(4) The proposed method in this paper can be further extended to identify multisource random dynamic loads, and the method can be optimized to improve the accuracy of multipoint random dynamic loading identification in future.

Data Availability

All data included in this study are available upon request by contact with the corresponding author.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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References

[1] F. D. Bartlett and W. G. Flannelly, "Model verification of force determination for measuring vibratory loads," *Journal of the American Helicopter Society*, vol. 24, no. 2, pp. 10–18, 1979.

[2] M. H. Lee and Y. W. Liu, "Input load identification of nonlinear tower structural system using intelligent inverse estimation algorithm," *Procedia Engineering*, vol. 79, pp. 540–549, 2014.

[3] S. Samagassi, A. Khamlichi, A. Driouach, and E. Jacquelin, "Reconstruction of multiple impact forces by wavelet relevance vector machine approach," *Journal of Sound and Vibration*, vol. 359, pp. 56–67, 2015.

[4] P. C. Hansen and D. P. O'Leary, "The use of the L-curve in the regularization of discrete ill-posed problems," *SIAM Journal on Scientific Computing*, vol. 14, no. 6, pp. 1487–1503, 1993.

[5] F. Presezniak, P. A. G. Zavala, G. Steenackers et al., "Acoustic source identification using a generalized weighted inverse beamforming technique," *Mechanical Systems and Signal Processing*, vol. 32, pp. 349–358, 2012.

[6] T. Reginska, "A regularization parameter in discrete ill-posed problems," *SIAM Journal on Scientific Computing*, vol. 17, no. 3, pp. 740–749, 1996.

[7] H. lee and Y. S. park, "Error analysis of indirect force determination and a regularisation method to reduce force determination error," *Mechanical Systems and Signal Processing*, vol. 9, no. 6, pp. 615–633, 1995.

[8] M. E. Hochstenbach, L. Reichel, and G. Rodriguez, "Regularization parameter determination for discrete ill-posed problems," *Journal of Computational and Applied Mathematics*, vol. 273, no. 273, pp. 132–149, 2015.

[9] J. O. Callahan and F. Piergentili, "Force estimation using operational data," in *Proceedings of 14th International Modal Analysis Conference*, p. 1586, Dearborn, MI, USA, February1996.

[10] B. Qiao, X. Zhang, X. Luo, and X. Chen, "A force identification method using cubic B-spline scaling functions," *Journal of Sound and Vibration*, vol. 337, pp. 28–44, 2015.

[11] E. Jacquelin, A. Bennani, and P. Hamelin, "Force reconstruction: analysis and regularization of a deconvolution problem," *Journal of Sound and Vibration*, vol. 265, no. 1, pp. 81–107, 2003.

[12] Z. C. He, X. Y. Lin, and E. Li, "A novel method for load bounds identification for uncertain structures in frequency domain," *International Journal of Computational Methods*, vol. 15, no. 6, article 1850051, 2018.

[13] C. Soize and A. Batou, "Identification of stochastic loads applied to a nonlinear dynamical system using an uncertain computational model," *Mathematical Problems in Engineering*, vol. 2008, Article ID 181548, 17 pages, 2008.

[14] C. P. Ren, N. J. Wang, and C. S. Liu, "Identification of random dynamic force using an improved maximum entropy regularization combined with a novel conjugate gradient," *Mathematical Problems in Engineering*, vol. 2017, Article ID 9125734, 14 pages, 2017.

[15] Y. Jia, Z. Yang, and Q. Song, "Experimental study of random dynamic loads identification based on weighted regularization method," *Journal of Sound and Vibration*, vol. 342, pp. 113–123, 2015.

[16] Z. C. He, Z. Zhang, and E. Li, "Multi-source random excitation identification for stochastic structures based on matrix perturbation and modified regularization method," *Mechanical Systems and Signal Processing*, vol. 119, pp. 266–292, 2019.

[17] Y. Jia, Z. Yang, N. Guo, and L. Wang, "Random dynamic load identification based on error analysis and weighted total least squares method," *Journal of Sound and Vibration*, vol. 358, pp. 111–123, 2015.

[18] J. H. Lin, X. L. Guo, H. Zhi, W. P. Howson, and F. W. Williams, "Computer simulation of structural random loading identification," *Computers and Structures*, vol. 79, no. 4, pp. 375–387, 2001.

[19] X. L. Guo and D. S. Li, "Experiment study of structural random loading identification by the inverse pseudo excitation method," *Structural Engineering and Mechanics*, vol. 18, no. 6, pp. 791–806, 2004.

[20] T. Wang, Z. Wan, X. Wang, and Y. Hu, “A novel state space method for force identification based on the Galerkin weak formulation,” *Computers and Structures*, vol. 157, pp. 132–141, 2015.

[21] T. Lai, T. H. Yi, and H. N. Li, “Parametric study on sequential deconvolution for force identification,” *Journal of Sound and Vibration*, vol. 377, pp. 76–89, 2016.

[22] M. Ge, H. R. Hardy Jr., and H. Wang, “A retrievable sensor installation technique for acquiring high frequency signals,” *Journal of Rock Mechanics and Geotechnical Engineering*, vol. 4, no. 2, pp. 127–140, 2012.

[23] R. Z. M. da Silveira, L. M. Campeiro, and F. G. Baptista, “Analysis of sensor installation methods in impedance-based SHM applications,” *Procedia Engineering*, vol. 168, pp. 1751–1754, 2016.