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Research Article

Behavior and Control of the Ballastless Track-Subgrade Vibration Induced by High-Speed Trains Moving on the Subgrade Bed with Mud Pumping

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This paper investigates behavior and control of the ballastless track-subgrade vibration induced by high-speed trains under mud pumping occurring in the subgrade bed. The characteristics of mud pumping occurring in the subgrade bed under the ballastless track structure are analyzed by visual observation and nondestructive testing. Then, based on the injection of the low-viscosity epoxy resin (LVER), the repair procedures for the mud pumping are proposed. A variety of on-site tests are performed on the ballastless track-subgrade with and without mud pumping and also after mud pumping reinforcement to analyze the vibration of the ballastless track-subgrade under the high-speed trains. The test results show that mud pumping can significantly increase the vertical vibration acceleration and displacement of the ballastless track structure and slightly decrease the vibration of subgrade surface. After mud pumping reinforcement, the abnormal vibration of the ballastless track-subgrade can be effectively controlled to make the vibration close to normal. In addition, the vibration ratio of the subgrade surface to the concrete base is proposed as a way to evaluate the effectiveness of the reinforcement of the mud pumping using the LVER, based on the vibration attenuation feature of the ballastless track-subgrade.

1. Introduction

High-speed railway has become one of the most efficient transport systems with the increasing operation speed of trains. Compared with air or road traffic, the high-speed railway has become more competitive for medium or short distances in terms of the effectiveness and economy of the passenger transportation systems and the energy efficiency and comfort associated with the rail transportation [1–4]. During the last two decades, the high-speed railway has been developed rapidly in a number of countries, especially in China. The total operating mileage of high-speed railway in mainland China reached 19,915 km by the end of 2015 [5] and exceeded 25,000 km by the end of February 2018, according to the Chinese government work report by Premier Li on March 5, 2018. In order to improve the dynamic performance and stability of the track and substructure, most of the high-speed railway lines around the world are paved with the ballastless track structure, which has the structural and operational advantages of higher longitudinal and lateral permanent stability, better smoothness, longer service life, and lower maintenance [6–8]. With the increase of the operating time of high-speed railway lines, some problems affecting the track irregularity inevitably occur in the ballastless track structure and substructure. These problems include the material and structure fatigue damage of the ballastless track structure [9–11] and the settlement and internal material failure of the substructure [12–15]. These problems can intensify the vibration of the train-track-substructure under high-speed trains, as well as the undesirable railway noise including rolling noise, impact noise, curve noise, mechanical noise, airborne noise, wheel/rail noise, and structure- and ground-borne noises [16]. Due to the diversity and variability of the subgrade materials and

structure, some issues that affect the track irregularity in the ballastless track structure on the subgrade sections are much more significant compared to the bridge and tunnel sections, such as mud pumping in the subgrade bed and the settlement of the subgrade caused by dynamic loading, water, and temperature [15, 17–20], as well as the foundation settlement under the subgrade [21–23].

In the past, subgrade settlement has been one of the fairly common problems in the substructure of high-speed railway. Track irregularity caused by subgrade settlement can intensify the vibration of the train-track-subgrade, leading to the reduction in the operation comfort and safety. Therefore, there are a number of literatures on analyzing the vibration of the train-track-subgrade generated by the track irregularity caused by subgrade settlement [24–27]. In order to reduce the subgrade settlement, some lightweight materials, such as lightweight foam concrete, are used to fill the subgrade under the ballastless track structure to decrease the subgrade self-weight [28, 29]. Besides, the contact loss between concrete base and subgrade caused by the differential settlement of the subgrade can deteriorate the mechanical properties of the ballastless track structure, causing the increase of the undesirable vibration of the train-track-subgrade. In order to analyze the vibration of the train-track-subgrade induced by the contact loss, some vehicle-track-subgrade-coupled system models based on the finite element method have been established, as well as the development experimental model tests [30–32]. The increase in vibration of the train-track-subgrade caused by the contact loss adversely affects the comfort and safety of high-speed rail travel and the lifetime of the ballastless track structure. Repair methods based on the uplift of the ballastless track structure combined with grouting technologies have been proposed to restore the contact between the layers and to reduce the vibration of the train-track-subgrade back to a normal condition [32, 33]. Furthermore, repeated mud pumping, which is caused by rainwater infiltration during high-speed railway operation, can squeeze out a large number of fine particles from the subgrade bed through cracks in the expansion joint of the concrete base, as shown in Figure 1, and thus lead to contact loss between the concrete base and the subgrade bed.

Previously, mud pumping occurring in the ballastless track-subgrade of high-speed railway was not so serious; however, in recent years, it has become more prevalent. Due to the fine particles in the subgrade bed squeezed out continuously during repeated mud pumping and the graded gravels of the subgrade bed affected by water, the supporting ability of the subgrade bed to the ballastless track structure is gradually deteriorated. The deterioration of the supporting ability of the subgrade bed does not directly affect the safety of the high-speed railway, but it can intensify the undesirable vibration of the train-track-subgrade system, decreasing the comfort of the passenger. Mud pumping in the ballastless track-subgrade has not attracted as much research attention as mud pumping in the ballast track bed and the contact layer between the subbase and pavement. In the fields of highway and ballast track, many researchers have performed some field and laboratory tests to analyze the development of the pore water pressure in the ballast track bed and the contact layer between the pavement and subbase and the effect of the excess pore water pressure on the migration of the fine particle [12, 16, 34–37]. According to the literature [38], mud pumping occurring in the ballastless track-subgrade and the ballast track bed can intensify the vibration and impact loads of the train-track-subgrade. Additionally, some corresponding remedial measures for mud pumping have been proposed [39–41]. However, there are only a few literatures on the characteristics and mechanism of mud pumping occurring in the ballastless track-subgrade and its effect on the vibration of the ballastless track-subgrade of high-speed railway. In order to analyze the characteristics of mud pumping in the ballastless track structure, some field investigations have been conducted by the authors on Chinese high-speed railway lines. It was found that there are a few cases of mud pumping in the ballastless track-subgrade during high-speed railway operation, as shown in Figure 1. The major characteristic of mud pumping in the ballastless track-subgrade is that a large number of fine particles are squeezed out of the subgrade bed. Mud pumping only occurs in the expansion joints of the concrete base of the ballastless track structure during the rainy season [42]. The authors established a full-scale model of the ballastless track-subgrade to analyze the vibration behavior of the model under normal and pumping conditions and after mud pumping reinforcement under cyclic dynamic loading and the reinforcement effect of mud pumping by using the LVER. This research results showed that mud pumping occurs in the subgrade bed under cyclic loading if the top layer of the subgrade bed is saturated with water, which causes contact loss between the concrete base and the subgrade bed, and increases the vibration of the concrete base of the ballastless track structure compared with the full-scale model under the normal condition. Mud pumping reinforced by using the LVER can restore the support ability of the subgrade bed for the concrete base and effectively control the abnormal vibration of the concrete base [43].

In summary, there is still not enough research on the vibration of the ballastless track-subgrade induced by high-speed trains moving on the subgrade bed under normal condition, in a mud pumping state, and after mud pumping reinforcement. Besides, mud pumping occurring in the ballastless track-subgrade is usually reinforced using an injection method based on the LVER during Chinese high-speed railway operation. The composition and performance of the LVER, which has been widely applied in concrete gap repair, have been extensively studied [44–47]. The application of the LVER in China has led to some specifications being established for its composition and engineering performance [48–52]. Several years ago, this material was introduced to reinforce the contact loss between the concrete base and the subgrade bed caused by mud pumping. Meanwhile, based on these specifications [48–52], special provisional technical regulations for the application of the LVER in the reinforcement of mud pumping in the ballastless track-subgrade of high-speed railway were proposed by Chinese design and research institutions, including

(a) (b)

Figure 1: Mud pumping occurring in the ballastless track-subgrade during operation.

Southwest Jiaotong University, China Academy of Railway Sciences Corporation Limited, and China Railway Siyuan Survey and Design Group Co., Ltd. However, there is still no detailed research on the control method and the effect of the he injection of the LVER on the ballastless track-subgrade vibration induced by high-speed trains moving on the subgrade bed with mud pumping. Furthermore, there are some differences in the repair procedures between different operation management departments of the high-speed railways. After mud pumping has been reinforced by the injection of the LVER, the vibration acceleration and displacement of the track inspection car moving on the subgrade section with mud pumping are reduced to near normal conditions, according to test results from the track inspection car. But it is unknown what are the influences of mud pumping on the vibration distribution of the ballastless track-subgrade and the control vibration effect of the reinforcement procedures based on the injection of the LVER under the high-speed trains.

Therefore, in this paper, a variety of on-site tests for the ballastless track-subgrade with and without mud pumping were conducted to analyze the influence of mud pumping on the vibration of the ballastless track-subgrade, when the high-speed trains are moving at different speeds. Then combined with previous studies on the composition design and performance requirements of the LVER and the reinforcement procedures established based on the full-scale model [42, 43], special repair procedures are proposed to reinforce mud pumping in the ballastless track-subgrade. Further on-site tests were carried out to analyze the vibration of the ballastless track-subgrade after mud pumping reinforcement according to the repair procedures developed here.

2. On-Site Test Setup

2.1. General Situation of On-Site Test Point. The ballastless track-subgrade of this on-site test point is shown in Figure 2, which consists of the ballastless track structure and subgrade. In order to overcome expansion deformation of the concrete base of the ballastless track structure under different temperatures, the expansion joints of the concrete base along the longitudinal line are established at 20.108 m intervals. In order to prevent rainwater from infiltrating into the subgrade bed through the expansion joint, all the expansion joints have to be filled with waterproof material [53]. The ballastless track structure is placed on the subgrade bed consisting of the top and the bottom layers. The top and bottom layers of the subgrade bed are usually filled with graded crushed stone and the group A or B packing, respectively.

After the ballastless track-subgrade of a certain high-speed railway line has been in operation for several years, a few cases of mud pumping occurring in the ballastless track-subgrade gradually begin to emerge. In this research, a typical case of mud pumping in the ballastless track-subgrade is used as the on-site test point for this research. A large volume of slurry has been squeezed out of the subgrade bed at this on-site test point, as shown in Figure 1. Before the test began, the mud pumping area under the ballastless track structure must be detected by nondestructive means. Electromagnetic waves cannot pass and reflect through the ballastless track structure which contains several layers of reinforcing steel rebars [54, 55]; therefore, ground penetrating radar (GPR) is not suitable to detect mud pumping in the subgrade bed under the ballastless track structure, but it can accurately detect the subgrade bed under the waterproof seal layer without reinforcing steel rebars [56, 57]. According to this research based on frequency spectrum analysis [58], the contact loss between the ballastless track structure and the subgrade bed caused by mud pumping can be detected by the impact-echo method (IEM), which is an effective nondestructive testing technique widely used for detecting certain defects inside concrete elements or structures [59–61]. Therefore, the combination of GPR with IEM was used to detect the interlayer in the ballastless track-subgrade affected by mud pumping, as shown in Figure 3. The detection results from this test point showed that mud pumping only occurs within the width of the concrete base in the transverse direction, and the maximum distance of the interlayer affected by mud pumping in the longitudinal direction from the expansion joint of the concrete base is less than 1.0 m.

The high-speed train used in this research was the CRH2C type, which mainly consisted of eight carriages.

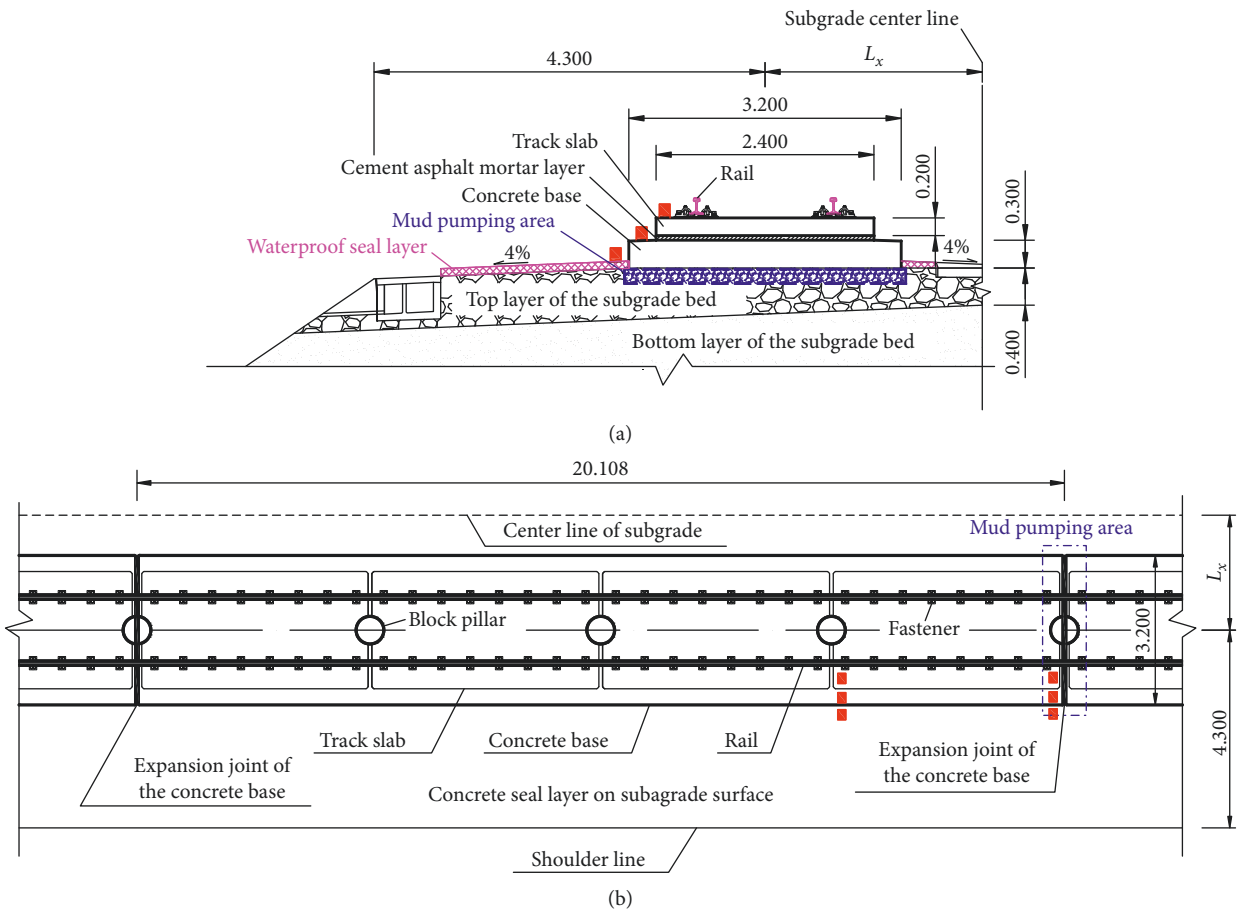

Figure 2: The structural characteristics of the ballastless track-subgrade and the layout of the test position (units in meter); ■, test position. (a) The cross-sectional profile of the ballastless track-subgrade. (b) The plane section of the ballastless track-subgrade.

(a) (b)

Figure 3: Detecting the interlayer of the ballastless track-subgrade affected by mud pumping. (a) Detecting for subgrade bed under the waterproof seal layer based on GPR. (b) Detecting for subgrade bed under the ballastless track structure based on IEM.

The key dimension parameters of the CRH2C train are shown in Figure 4. The wheelbase of the bogie, the center distance between adjacent bogies, and the center distance between both bogies of the same carriage are 2.5 m, 7.5 m, and 17.5 m, respectively. The maximum speed of the CRH2C train is 350 km/h; however, in general, the operating speed of the CRH2C train in China is between 200 km/h and 300 km/h. According to the structural vibration principle under an external load, the peak value of vibration acceleration wave should generally correspond to the time taken for the wheelset of the bogie to move through the testing cross section [62–64]. Therefore, the time interval between the first wheelset of the first bogie and the last wheelset of the last bogie of the same carriage moving through the test cross section can be obtained from the time corresponding to the first and the last peak acceleration of the vibration acceleration history curves. In this research, the ratio of the distance between the first wheelset of the first bogie and the

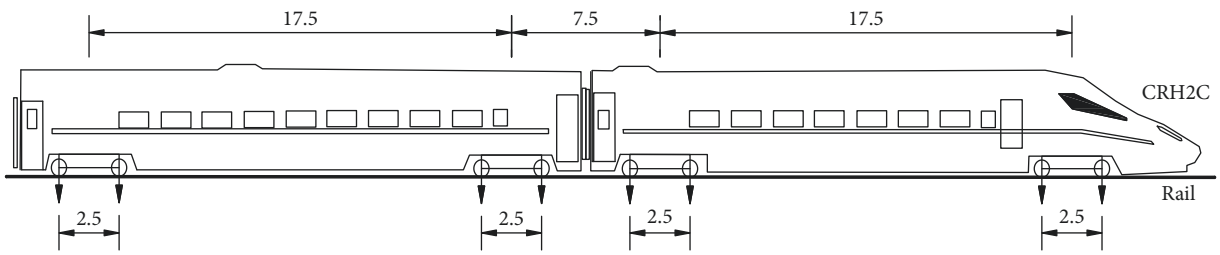

Figure 4: The key dimension parameters of the high-speed train moving on the on-site test section (units in meters).

last wheelset of the last bogie of the same carriage to the time interval is taken as the speed of the train moving through the test cross section.

2.2. Test Schemes. The maximum distance of the subgrade bed affected by mud pumping is less than 1.0 m from the expansion joint of the concrete base, and so only one test cross section in the mud pumping area is established near the end of the ballastless track structure. The other test cross section is installed near the other end of the same ballastless track structure under the normal condition, as shown in Figure 2(b). The aim is to compare the vibration of the ballastless track-subgrade when the subgrade bed is under normal and mud pumping conditions and after mud pumping reinforcement. The tested items include the vertical vibration acceleration and displacement of the track slab and concrete base of the ballastless track structure and the subgrade surface, as shown in Figure 2(a).

During high-speed railway operation, all the test sensors and their wires must be rigidly fastened to ensure none of the sensors can be destroyed by the vibration and wind pressure when the high-speed trains are moving on the test section at over 200 km/h or even 300 km/h. Because the sensors are very close to the rail, they may be sucked into the bogie and will destroy the bogie if they come away from the test position. The test sensors and their wires were fixed to the ballastless track-subgrade, as shown in Figure 5.

The test procedure consisted of two parts. One was to test the vibration (including vertical acceleration and displacement) of the ballastless track-subgrade under normal and mud pumping conditions under each high-speed moving train. Then after mud pumping reinforcement, the other was to test the vibration of the ballastless track-subgrade again, which is under the normal condition and after mud pumping reinforcement. During the train movement through the test section, the signals of all the sensors were collected by a dynamic acquisition system installed in a computer, which has amplification, filtering, and sampling functions for the voltage signals by the corresponding process software. Then, the voltage signals of all the sensors were converted into the vibration acceleration and displacement data.

2.3. Repair Method for Mud Pumping and Materials

2.3.1. Materials. After the first test of the ballastless track-subgrade had been completed, mud pumping in the ballastless track-subgrade must be immediately reinforced to restore the supporting ability of the subgrade bed for the ballastless track structure and return the vibration of the ballastless track-subgrade back to the normal condition. Nighttime possession of the high-speed railway in China usually lasts about 4 hours. Therefore, the repair procedures for mud pumping in the ballastless track-subgrade must be highly efficient. Besides, the cured materials injected into the mud pumping area under the ballastless track structure must have characteristics of appropriate flexibility, early strength, and low expansion rate. The cured materials of the LVER, which has been widely used in concrete gap repair [44–47], can meet the above requirements for repairing the ballastless track-subgrade with mud pumping. The mixed slurry of the LVER has advantages of low viscosity and high fluidity, and the cured materials have further advantages of early strength including compressive strength and tensile strength. These are the most important factors for reinforcement of mud pumping during operation.

In this research, the LVER of a particular brand, which is composed of two components and has been applied in actual engineering projects of mud pumping reinforcement, was used to reinforce mud pumping at this on-site test point. After the composition design of the LVER was completed, based on various tests according to the relevant specifications [48–52], the major performance parameters of the mixed slurry of the LVER and its cured material mechanical properties are shown in Tables 1 and 2.

The key indicators of the engineering performance requirements of the mixed slurry and the cured LVER, which are listed in Tables 1 and 2, were obtained from relevant provisional technical regulations for reinforcement of mud pumping in the ballastless track-subgrade proposed by the authors and China Railway Siyuan Survey and Design Group Co., Ltd. Based on a full-scale model for analyzing the reinforcement method of mud pumping in the ballastless track-subgrade [43] and some actual engineering projects, the key indicators of engineering performance requirements for the mixed slurry and the cured materials were verified and applied to actual engineering projects on high-speed railway. The test results show that the performance indicators of the mixed slurry and the cured LVER in this research can meet the requirements of the provisional technical regulations. In particular, the compressive strength and tensile strength of the cured LVER were not less than 15.0 MPa and 6.0 MPa after curing for 2 hours, respectively. This is very important for the LVER to be used as the material for mud pumping reinforcement of the ballastless track-subgrade under a limited nighttime possession of

(a) (b) (c) (d)

Figure 5: The test sensors and their wires installed on the ballastless track-subgrade.

Table 1: The major performance parameters of the mixed slurry of the LVER.

Test item	Required performance indicators	Detection value
Slurry density (kg/m^3)	>1000	1060
Viscosity (mPa)	≤100	76
Gelation time (min)	≤30	21

Table 2: The major mechanical properties of the cured LVER.

Test item	Required performance indicators	Detection value
Tensile strength (MPa)	≥6 (2 h)	8
	≥10 (24 h)	13
	≥15 (7 d)	18
Compressive strength (MPa)	≥15 (2 h)	19
	≥20 (24 h)	28
	≥30 (7 d)	39
Elongation at break (%)	≥2 (7 d)	4.6
Shrinkage ratio (%)	≤2 (7 d)	1.2
Adhesive strength (MPa)	≥3 (7 d)	5.3

high-speed railway. Therefore, the LVER of this particular brand can be used as the material for mud pumping reinforcement in this on-site test point.

2.3.2. Repair Procedures. Combined with previous research on the reinforcement procedures based on a full-scale model [43], the repair procedure for mud pumping in the subgrade bed during nighttime possession of high-speed railway was divided into four stages as follows:

(1) Identification of the area in the subgrade bed affected by mud pumping: during high-speed railway operation, the detection method for damage in the ballastless track-subgrade cannot be destructive to the ballastless track-subgrade. In this research, the combination of GPR and IEM was used to detect and identify the area in the subgrade bed affected by mud pumping, as shown in Figure 3. The test procedures of both nondestructive detection methods are carried out according to the respective operation manuals and are not detailed here.

(2) Location and construction of the injection holes and the exhaust and drain holes: after identification of the mud pumping area in the subgrade bed under the ballastless track structure, the injection holes and the exhaust and drain holes are located within the area affected by mud pumping, as shown in Figure 6. The diameter of all the injection holes in the concrete base and the exhaust and drain holes on subgrade surface was 10 mm, and that of the exhaust and drain holes at the center of the ballastless track structure is 22 mm. All the holes are drilled by a fixed small drilling equipment, and the drilling depth of all the holes was 10 cm below the bottom surface of the concrete base.

(3) Injection of the LVER into the mud pumping area: a grouting pipe was inserted into each of the injection holes to 10 cm below the bottom surface of the concrete base. The gap between the grouting pipe and the injection hole was sealed by quick-setting cement mortar, as shown in Figure 7(a). Then all the grouting pipes inserted into the injection holes were connected to the grouting equipment, which has an accurate pressure control function. Finally, the mixed slurry of the LVER was injected by the grouting equipment under a grouting pressure less than 0.2 MPa. The grouting process must be strictly followed from the distant hole to the nearest hole from the expansion joint of the concrete base. Besides, real-time monitoring was conducted to control the uplift displacement of the ballastless track structure at the same time. During injection of the mixed slurry, water in the mud pumping area is firstly squeezed out of the subgrade bed through the exhaust and drain holes close to the injected holes, as shown in Figure 7(b), and then, the mixed slurry of the LVER flows out from the exhaust and drain holes, as shown in Figure 7(c). The injection of the mixed slurry must immediately stop as soon as the uplifted displacement of the ballastless track structure reaches the limit value or a large volume of the mixed slurry flows out from the exhaust and drain hole close to the injection hole.

(4) Precise adjustment of the rail and cleaning up of the construction site: after all the holes have been injected completely, all the grouting pipes are cut off. All the exhaust and drain holes are then injected with quick-setting cement mortar. Finally, the quick-setting cement mortar and the cured materials of the LVER left on the surface of the ballastless

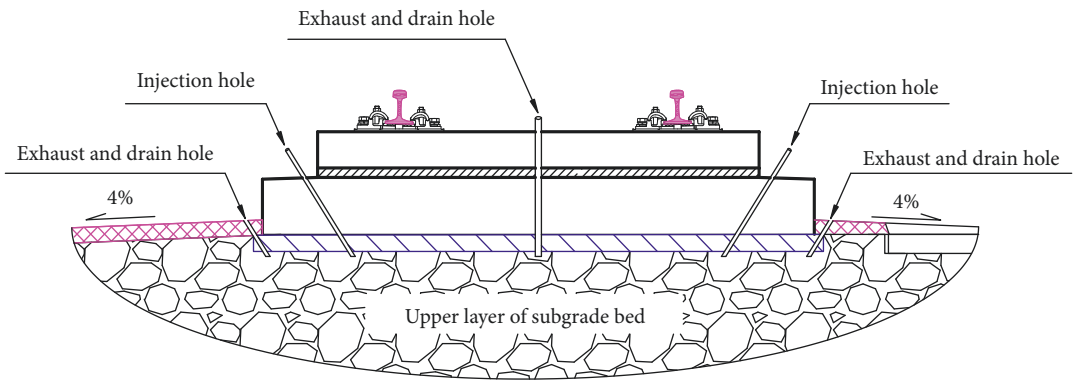

Figure 6: The layout of the injection and exhaust and drain holes in the ballastless track-subgrade.

(a) (b) (c)

Figure 7: Mud pumping reinforcement process based on the injection of the LVER. (a) Grouting pipes. (b) Water squeezed out. (c) Mixed slurry flowing out.

track-subgrade must be removed by polishing. Then, the rails are precisely adjusted to ensure their smoothness. Finally, the construction site is cleaned up.

3. Test Results

3.1. Vertical Vibration Acceleration. Under the conditions of the ballastless track-subgrade with and without mud pumping, the vertical vibration acceleration history curves of the ballastless track-subgrade are shown in Figures 8 and 9, when the same high-speed train is moving through the test cross sections at a speed of 245 km/h.

The vibration acceleration response is one of the evaluation indicators for the vibration level of the structure. Under the same train moving through the test cross section at the same speed of 245 km/h, the peak values of the vertical vibration acceleration of the subgrade surface of the test cross sections with and without mud pumping in the subgrade bed are basically equal. However, the peak value of the vertical vibration acceleration of the ballastless track structure with mud pumping in the subgrade bed increases significantly, compared to the ballastless track structure without mud pumping or the subgrade bed under the normal condition. This shows that mud pumping can adversely intensify the vibration of the ballastless track structure above the mud pumping area.

After the mud pumping area had been reinforced according to the repair procedures in this research, the vertical vibration acceleration history curves of the ballastless track-subgrade are shown in Figure 10, when the same train is moving through the test sections at a speed of 283 km/h. Although the test train and its speed are different from the previous tests, the test results show that the reinforcement of the mud pumping according to the repair procedures can reduce the vibration of ballastless track structure.

In order to analyze the distributions of the vibration acceleration of the ballastless track-subgrade, several tests were conducted to get the vertical vibration acceleration history curves of the ballastless track-subgrade under different high-speed moving trains. In this research, the maximum amplitude of the vertical vibration acceleration of every test position under each wheelset for every train moving through the test cross section was averaged and taken as the vertical vibration acceleration of every test position under each train. Based on this method, the test results are listed in Tables 3 and 4. The distributions of the vertical vibration acceleration of the ballastless track-subgrade without mud pumping (or under the normal condition) and before and after mud pumping reinforcement under each high-speed train are shown in Figures 11 and 12.

Under the same high-speed train, the vertical vibration acceleration of the ballastless track structure under the subgrade bed with mud pumping is significantly greater than that under the subgrade bed without mud pumping. Especially, the increase in the vertical vibration acceleration of the concrete base under mud pumping occurring in the subgrade bed is much greater compared with the subgrade bed without mud pumping. Compared with the subgrade bed under the normal condition, the vertical vibration acceleration of the concrete base above the mud pumping area

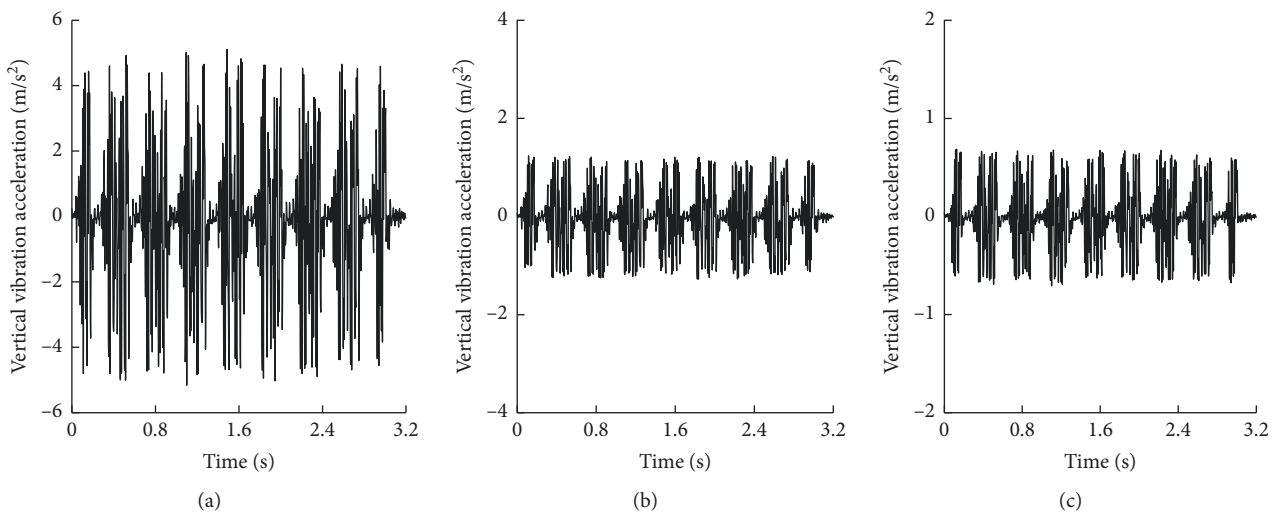

Figure 8: The vertical vibration acceleration history curves of the ballastless track-subgrade under the normal condition (v = 245 km/h). (a) The track slab. (b) The concrete base. (c) The subgrade surface.

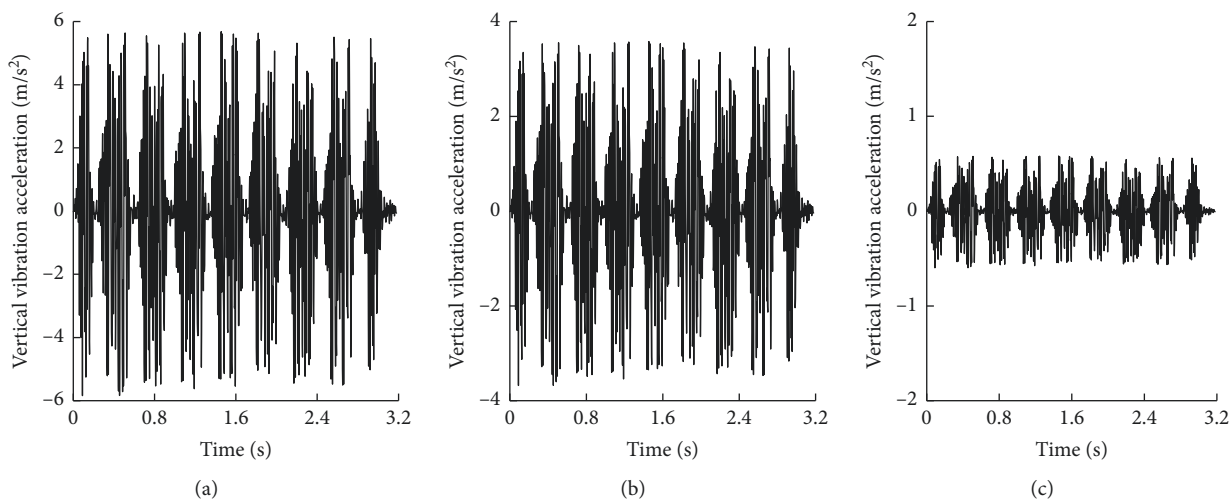

Figure 9: The vertical vibration acceleration history curves of the ballastless track-subgrade with mud pumping (v = 245 km/h). (a) The track slab. (b) The concrete base. (c) The subgrade surface.

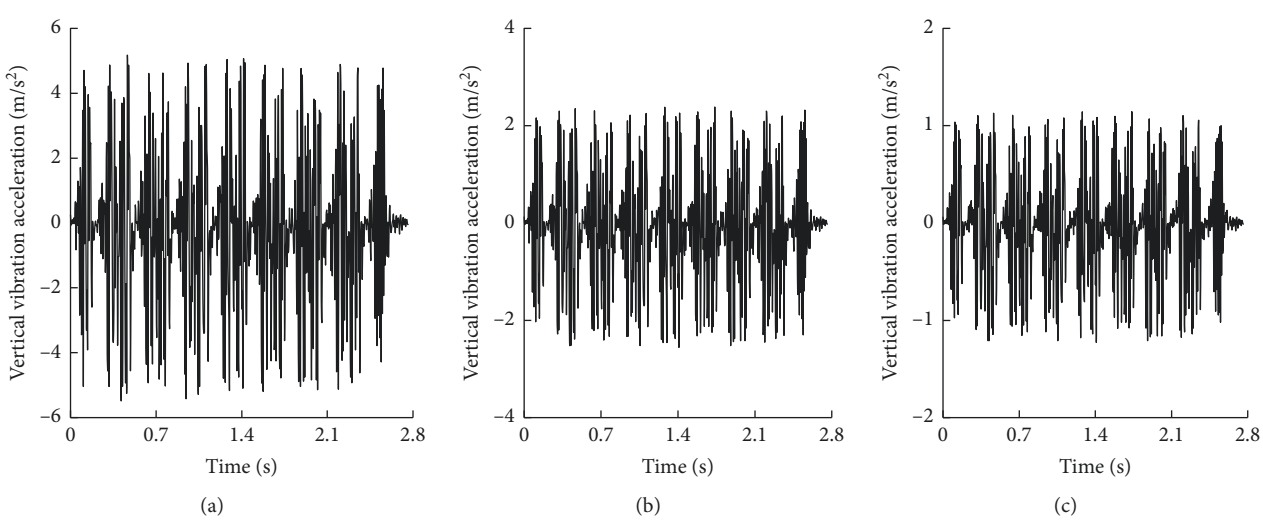

Figure 10: The vertical vibration acceleration history curves of the ballastless track-subgrade after mud pumping reinforcement (v = 283 km/h). (a) The track slab. (b) The concrete base. (c) The subgrade surface.

Table 3: The vertical vibration acceleration of the ballastless track-subgrade with and without mud pumping.

Testing content		Vertical vibration acceleration (m/s^2)					
		The ballastless track-subgrade without mud pumping			The ballastless track-subgrade with mud pumping		
Location (upper to lower)		Track slab	Concrete base	Subgrade surface	Track slab	Concrete base	Subgrade surface
Train moving speed (km/h)	245	4.63	1.21	0.65	5.51	3.46	0.56
	258	4.17	1.05	0.63	5.09	3.24	0.61
	276	4.25	1.16	0.72	5.24	3.53	0.59

Table 4: The vertical vibration acceleration of the ballastless track-subgrade without mud pumping and after mud pumping reinforcement.

Testing content		Vertical vibration acceleration (m/s^2)					
		The ballastless track-subgrade without mud pumping			The ballastless track-subgrade after mud pumping reinforcement		
Location (upper to lower)		Track slab	Concrete base	Subgrade surface	Track slab	Concrete base	Subgrade surface
Train moving speed (km/h)	250	4.38	1.17	0.87	4.52	1.65	0.85
	280	4.55	1.34	0.79	4.78	1.82	1.01
	283	4.86	1.39	0.98	5.05	2.38	1.14

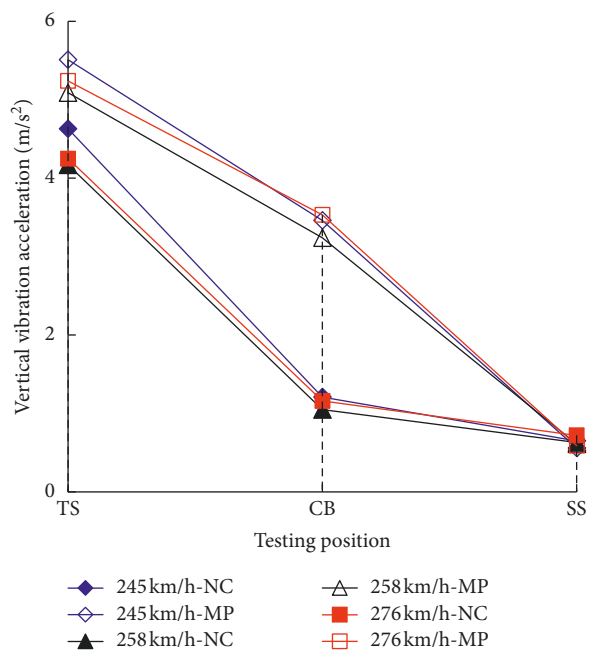

Figure 11: The distributions of the vertical vibration acceleration of the ballastless track-subgrade with and without mud pumping. NC, the subgrade bed under the normal condition (or without mud pumping); MP, the subgrade bed with mud pumping; TS, test position on the track slab; CB, test position on the concrete base; SS, test position on the subgrade surface.

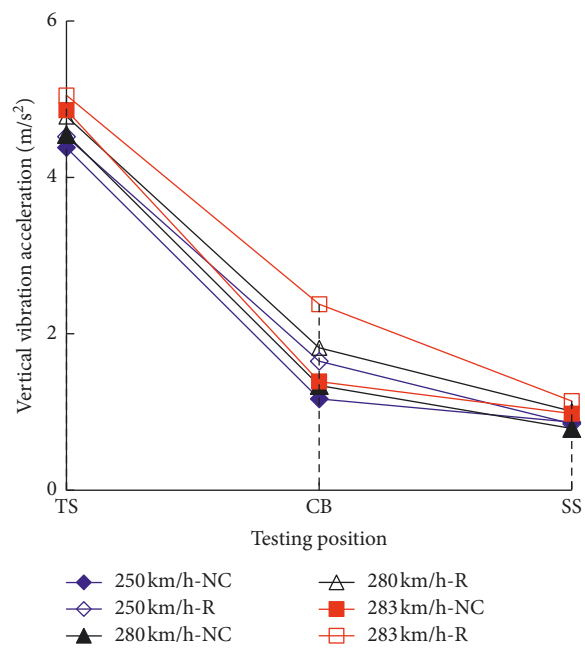

Figure 12: The distributions of the vertical vibration acceleration of the ballastless track-subgrade without mud pumping and after mud pumping reinforcement. *R* after mud pumping reinforcement.

increases by 1.86, 2.09, and 2.04 times, when trains are moving through the test sections at speeds of 245 km/h, 258 km/h, and 276 km/h, respectively. However, the vertical vibration acceleration of the subgrade surface under the subgrade bed with mud pumping reduces slightly. Therefore, mud pumping can significantly change the distributions of the vertical vibration acceleration of the ballastless track-subgrade and intensify the abnormal vibration of the ballastless track structure.

The increasing abnormal vibration acceleration of the ballastless track structure under a high-speed train has an adverse effect on the life span of the ballastless track structure and the comfort of the high-speed railway. Therefore, the maintenance department of the high-speed railway is trying to explore effective reinforcement methods for mud pumping in the subgrade bed under the ballastless track structure. Based on the above repair procedures using the LVER, it was found that the vertical vibration acceleration of the ballastless track-subgrade could be reduced to near normal conditions, as shown in Figure 12. In other words, after mud pumping in the subgrade bed is reinforced

according to the repair procedures reported here, the vertical vibration acceleration of the ballastless track-subgrade can be controlled to be approximately equal to that of the ballastless track-subgrade under the normal condition.

3.2. Vertical Vibration Displacement. The same processing method as mentioned above was adopted to process the test data of the vertical vibration displacement. The vertical vibration displacements of every test position under mud pumping and after mud pumping under each train in this research were obtained and are listed in Tables 5 and 6. The distributions of the vertical vibration displacement of the ballastless track-subgrade without mud pumping (or under the normal condition) and before and after mud pumping reinforcement under each train are shown Figures 13 and 14.

Figure 13 shows that the vertical vibration displacement of the ballastless track structure under mud pumping in the subgrade bed increases significantly compared with the subgrade bed without mud pumping, when the same high-speed train is moving through the test sections at the same speed. Compared with the subgrade bed without mud pumping, the vertical vibration displacement of the concrete base close to the expansion joint under the subgrade bed with mud pumping increases by 1.53, 1.23, and 1.06 times under the trains moving through the test sections at speeds of 245 km/h, 258 km/h, and 276 km/h, respectively. However, the vertical vibration displacement of the subgrade surface decreases slightly. The reason for the differences in the variation of the vertical vibration displacement under the subgrade bed with and without mud pumping is that a large number of the fine particles in the subgrade bed are squeezed out by repeated mud pumping, causing a reduction in the supporting ability and stiffness of the subgrade bed affected by the infiltration of rainwater.

After the supporting ability and stiffness of the subgrade bed have been reduced by mud pumping, the vibration displacement of the ballastless track structure increases under the same high-speed moving train, and the vibration acceleration of the ballastless track structure should decrease. However, both vertical vibration displacement and acceleration of the ballastless track structure above the subgrade bed with mud pumping increase. This is because the high stiffness of the ballastless track structure plays an increasingly important role on the vibration of the ballastless track structure under high-speed moving train, while mud pumping in the subgrade bed deteriorates gradually, causing a reduction in the supporting ability and stiffness of the subgrade bed in the mud pumping area, even forming a gap between the subgrade bed and the ballastless track structure, as shown in Figure 15.

Under the high-speed train, the increase of the vertical vibration displacement of the ballastless track structure has an adverse effect on the comfort and safety of the high-speed railway. Therefore, once mud pumping occurring in the subgrade bed under the ballastless track structure has been identified, it should be immediately reinforced during operation. After mud pumping reinforcement according to the repair procedures reported in this research, the vertical vibration displacement distribution of the ballastless track-subgrade under each high-speed moving train was recorded is shown in Figure 14. After mud pumping reinforcement, the vertical vibration displacement of the ballastless track structure decreases significantly and is approximately equal to that of the ballastless track structure on the subgrade bed without mud pumping, when the same train is moving through the test sections at the same speed. This shows that the subgrade bed, where the mud pumping has been reinforced according to the repair procedures in this research, can be restored to the supporting ability for the ballastless track structure.

3.3. Vibration Ratio between Layers for Evaluating the Reinforcement Effect. The vibration of the ballastless track-subgrade under high-speed moving trains decreases gradually from the ballastless track structure to the subgrade surface, regardless of whether the subgrade bed is with and without mud pumping. Compared with the subgrade bed without mud pumping, the vibration reduction between the ballastless track structure and the subgrade surface increases abnormally when mud pumping occurs in the subgrade bed. Especially, the vibration attenuation gradient between the concrete base and the subgrade surface is the largest. Based on this characteristic, the vibration ratio of the subgrade surface to the concrete base is proposed as a way for evaluating the vibration of the ballastless track-subgrade with mud pumping and the reinforcement effect in the subgrade bed. In these on-site tests, when the subgrade bed under the ballastless track structure is normal, with mud pumping and after mud pumping reinforcement, the vertical vibration acceleration ratios of the subgrade surface to the concrete base are in ranges of 1.34–1.86, 5.31–6.18, and 1.80–2.09, respectively, and the vertical vibration displacement ratios are in ranges of 1.33–2.43, 3.67–5.43, and 2.22–3.00, respectively. Obviously, the repair method in this research for mud pumping can decrease the abnormal vibration of the ballastless track structure and make the vibration of the ballastless track-subgrade close to normal. This shows that the mud pumping area between the concrete base and the subgrade bed should be effectively filled with the cured materials of the LVER, and the supporting ability of the subgrade bed to the ballastless track structure has been restored.

Therefore, the repair method based on the injection of the LVER for mud pumping in the ballastless track-subgrade is an effective control method to reduce the ballastless track-subgrade vibration generated by mud pumping. What is more, in addition to the nondestructive detection methods employed, the vibration ratio of the subgrade surface to the concrete base in this research can be used as another method to evaluate the reinforcement effect.

4. Conclusions

In this research, a typical case of mud pumping occurring in the ballastless track-subgrade and its characteristics are analyzed by visual observation and nondestructive testing, and then in order to investigate and control the abnormal

Table 5: The vertical vibration displacement of the ballastless track-subgrade with mud pumping.

Testing content		Vertical vibration displacement (mm)					
		The ballastless track-subgrade without mud pumping			The ballastless track-subgrade with mud pumping		
Location (upper to lower)		Track slab	Concrete base	Subgrade surface	Track slab	Concrete base	Subgrade surface
Train moving speed (km/h)	245	0.32	0.15	0.11	0.54	0.38	0.07
	258	0.29	0.13	0.09	0.47	0.29	0.06
	276	0.31	0.16	0.12	0.51	0.33	0.09

Table 6: The vertical vibration displacement of the ballastless track-subgrade after mud pumping has been reinforced.

Testing content		Vertical vibration displacement (mm)					
		The ballastless track-subgrade without mud pumping			The ballastless track-subgrade after mud pumping reinforcement		
Location (upper to lower)		Track slab	Concrete base	Subgrade surface	Track slab	Concrete base	Subgrade surface
Train moving speed (km/h)	250	0.29	0.13	0.06	0.31	0.18	0.06
	280	0.34	0.17	0.07	0.37	0.20	0.09
	283	0.39	0.21	0.08	0.42	0.27	0.10

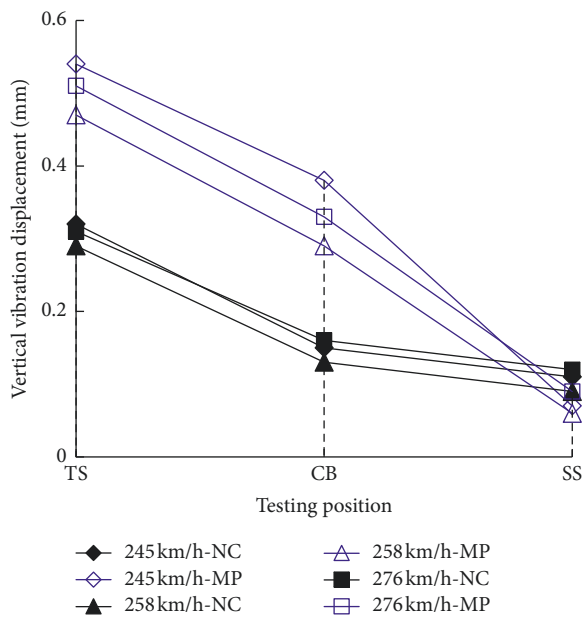

Figure 13: The distribution of the vertical vibration displacement of the ballastless track-subgrade with and without mud pumping.

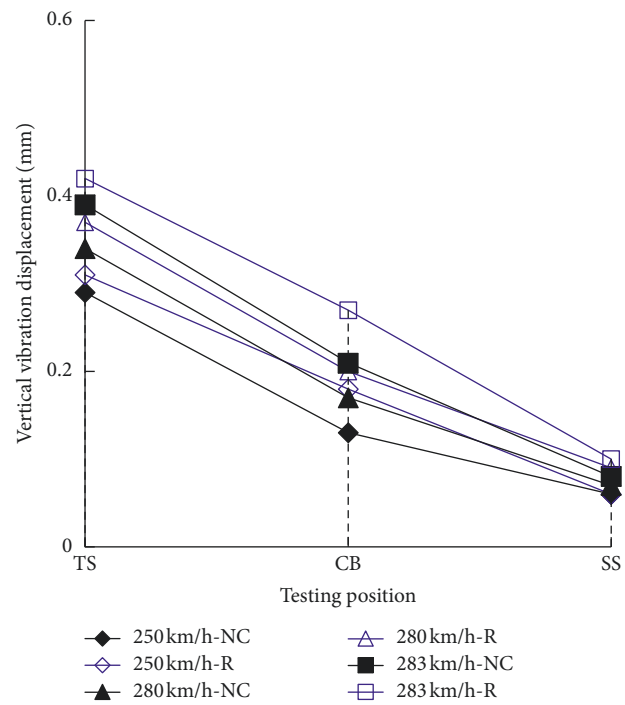

Figure 14: The distribution of the vertical vibration displacement of the ballastless track-subgrade without mud pumping and after mud pumping reinforcement.

vibration of the ballastless track-subgrade generated by mud pumping under the high-speed moving trains, the repair procedures for mud pumping are established, and a variety of on-site tests are conducted. The main conclusions can be drawn as follows:

(1) Mud pumping only occurs in the subgrade bed near the expansion joint of the concrete base. A large number of fine particles in the subgrade bed are squeezed out by repeated mud pumping. Based on the detection of GPR and IEM, it was found that the subgrade bed under the ballastless track structure affected by mud pumping is less than 1.0 m in the longitudinal direction away from the expansion joint of the concrete base.

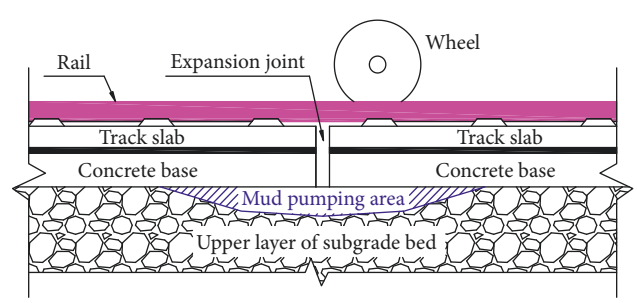

Figure 15: The longitudinal profile of the ballastless track-subgrade with mud pumping.

(2) Under the same high-speed train moving through the ballastless track-subgrade at the same speed, the vertical vibration acceleration and displacement of the ballastless track structure upon the subgrade bed with mud pumping can be significantly intensified, compared with the subgrade bed without mud pumping. The vertical vibration acceleration and displacement of the concrete base increase by 1.86–2.09 times and 1.06–1.53 times, respectively.

(3) After mud pumping in the ballastless track structure is reinforced according to the repair procedures established in this research, the abnormal vibration including acceleration and displacement of the ballastless track structure under the high-speed moving train can be reduced to near normal levels. This shows that the supporting ability of the subgrade bed to the ballastless track structure has been restored due to the mud pumping area between the concrete base and the subgrade bed filled by the cured LVER.

(4) Based on the vibration attenuation characteristics of the ballastless track-subgrade under the high-speed train, the vibration ratio of the subgrade surface to the concrete base is proposed as a way for evaluating the vibration of the ballastless track-subgrade generated by mud pumping and the reinforcement effect of mud pumping in the ballastless track-subgrade.

In this research, based on the vibration attenuation features of the ballastless track-subgrade, the vibration ratio of the subgrade surface to the concrete base can be used as an important indicator for analyzing the abnormal vibration of the ballastless track-subgrade induced by mud pumping and for evaluating the reinforcement effect of mud pumping. But there is still one unresolved issue that must be addressed before the vibration ratio can be practically used as the evaluation indicator for detecting the contact loss between the concrete base and the subgrade bed caused by mud pumping during high-speed railway operation. The unresolved issue is that the specific vibration ratio coefficient corresponds to the contact loss level between the concrete base and the subgrade bed caused by mud pumping.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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