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Research Article

Optimal Sensor Placement and Minimum Number Selection of Sensors for Health Monitoring of Transmission Towers

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Transmission towers are structurally complex, which makes it challenging to choose the right place and number of sensors for health monitoring. In this paper, optimal sensor placement of a cat-head-type transmission tower is conducted by using the Effective Independent Method (EIM) and a method is proposed for calculating the minimum number of sensors for structural health monitoring by combining EIM and Modal Assurance Criterion (MAC). The method for calculating the number of sensors prescribed in this paper derives a curve that shows the relationship between MAC value and the number of sensors. It is found that the MAC value decreases with increase in the number of sensors. When the number of sensors reaches a certain threshold, the curve tends to stabilize. Then, the number of sensors corresponding to the minimum MAC is proposed as the minimum number of sensors. Through calculation, the minimum number of sensors of the cat-head-type transmission tower is obtained. Also, the optimal sensor placement results show that the position of a large number of sensors includes the position of a smaller number of sensors.

1. Introduction

Transmission towers often operate in adverse and harsh environments with complex and varying loads and may experience failure, including collapse, from extreme conditions or vibration fatigue damage [1, 2]. To prevent such failures and ensure consistent, high-quality, and safe power provision to both the grid and dependent loads, health monitoring of transmission towers must be an integral consideration in the power ecosystem.

Structure health monitoring for transmission towers is increasingly attracting attention. In current literature [3–5], the prevailing method for structural health monitoring of transmission towers is based on Structural Dynamic Characteristics Identification. In this method, the health of the transmission tower can be monitored by measuring the vibration response of the tower structure and then identifying the parameters of its structure modal. It is important to point out here that the placement of vibration sensors directly affects the accuracy and completeness of the structural information of the tower structure. To maximize the value-added of sensors, sensor placement should strive to make transparent damage information, reduce monitoring costs, and improve detection efficiency [6].

There are many methods to choose from to optimize sensor placement, some of which include the Effective Independent Method (EIM), Energy Method, and Modal Assurance Criterion. The EIM [7] can be used to obtain the optimal placement scheme through removing the degrees of freedom with the greatest effect on the error of the estimation of the modal coordinate from the Fisher information matrix. This method was proposed by Kammer in 1991 and was originally applied in the selection of the best test points for experimental modal analysis. It was only later that it was extended to the field of structural health monitoring for optimizing sensor placements. The health monitoring applications [8–12] used EIM to optimize the sensor placement of the plates, dams, houses, and other structures and achieved good results. However, the EIM has some shortcomings such as information redundancy and avoidance of large energy measurement points. In response, some scholars [13–18] have proposed various methods to improve EIM. Since also sensors can more easily collect information of a measuring point with high energy output, some literatures reinforce the energy criterion when evaluating sensor placement. The Energy Method depends on the modal kinetic energy of the structure formed to optimize sensor placement [19–21]. The relationship between the Energy Method and EIM has also been studied [22]. The Modal Assurance Criterion is a method that can be used for the optimal sensor placement and evaluating the results of optimum sensor placement [23, 24].

There are only a few literatures that deal with the placement of sensors for tower structure. Yin et al. [5] present a methodology for identifying the "optimal" locations to install a given number of sensors on a structure and have brief inferences on using this methodology on transmission towers. Vergara et al. [25] compared the effects of a transmission tower and a transmission tower-line system and optimal sensor placement. Benny and Sai [26] evaluated a methodology for sensor placement in transmission line towers based on the concepts of entropy and model falsification. These researches on sensor arrangement mainly focus on optimizing the arrangement of sensors after giving the number of sensors, but there are few researches on the number of sensors that should be placed. The number of sensors is the prerequisite for optimal sensor placement, which in turn affects accuracy, sensitivity, and economy of acquiring the structural health information.

With these problems in mind, this paper proposes several sensor placement schemes of a cat-head-type transmission tower and a method for determining the number of sensors to be used for health monitoring. To solve this problem, the optimal sensor placement should first be performed by the EIM under different numbers of sensors. Then, the corresponding MAC matrices are calculated and the relationship of the maximum value of the nondiagonal elements of the MAC matrices with the sensor numbers is analyzed. These inputs are then used to fit the curve of the maximum nondiagonal element of the MAC matrix with the number of sensors. Finally, the point corresponding to the minimum value of the curve is taken as the best number of placed sensors.

2. Basic Theory of Optimal Sensor Placement for Transmission Tower

2.1. Effective Independence Method (EIM). The idea of the EIM is to rank each degree of freedom based on its contribution to the linear independence of the target modality (Fisher Information Matrix). The degrees of freedom with lesser contribution degree are then deleted and the degrees of freedom with greater contribution are retained. Thereby, the optimal sensor placement is realized.

The dynamic equation of a multi-degree-of-freedom linear time-invariant system is

$$\mathbf{M}\ddot{u}(t) + \mathbf{C}\dot{u}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{F}(t), \tag{1}$$

where **M** is the mass matrix, **C** is the damping matrix, **K** is the stiffness matrix, **u (t)** is the displacement response vector, and **F (t)** is the outer excitation vector. According to the modal analysis theory, the solution of equation (1) can be expressed as

$$\mathbf{u}(t) = \mathbf{\Phi}\mathbf{q}(t), \tag{2}$$

where the matrix $\mathbf{\Phi}$ is the modal shape matrix and **q (t)** is the modal coordinate vector. For m sensors ($m < s$) arranged at s positions, the degree of freedom of the system is s. If the influence of noise is considered, the output response of the sensor $\mathbf{U_s}$ can be expressed as

$$\mathbf{U_s} = \mathbf{\Phi_s}\mathbf{q} + \psi, \tag{3}$$

where the $s \times N$ matrix $\mathbf{\Phi_s}$ is the modal shape matrix, in which s is the number of degrees of freedom and N is the number of modal orders. ψ is a Gaussian white noise whose variance is σ^2; and **q** is a generalized modal coordinate vector.

The estimated value of the generalized modal coordinate vector **q** is

$$\hat{q} = \left[\mathbf{\Phi_s^T}\mathbf{\Phi_s}\right]^{-1}\mathbf{\Phi_s}\mathbf{U_s}. \tag{4}$$

For a complex structure with s degrees of freedom, if m sensors need to be arranged for structure health monitoring to maximize the linear independence of the modal information, it is necessary to obtain the best estimate of the generalized coordinate **q**. The covariance matrix of the estimated deviation is

$$\begin{aligned}\mathbf{P} = \mathbf{E}\left[(\mathbf{q}-\hat{q})(\mathbf{q}-\hat{q})^{\mathbf{T}}\right] &= \left[\left(\frac{\partial \mathbf{\Phi_s q}}{\partial \mathbf{q}}\right)^T \left[\sigma^2\right]\left(\frac{\partial \mathbf{\Phi_s q}}{\partial \mathbf{q}}\right)\right]^{-1} \\ &= \left[\mathbf{\Phi_s^T}\left(\boldsymbol{\sigma}^2\right)^{-1}\mathbf{\Phi_s}\right]^{-1} = \mathbf{Q}^{-1},\end{aligned} \tag{5}$$

where **E** is the mathematical expectation and **Q** is the Fisher Information Matrix. To minimize the covariance matrix to get the best estimation, it is necessary to maximize the determinant of **Q**. To simplify the analysis, it is assumed that the noise is uncorrelated and has the same statistical properties in each sensor. Combined with equation (4), the Fisher Information Matrix **Q** can be expressed as

$$\mathbf{Q} = \mathbf{\Phi_s^T}\left(\boldsymbol{\sigma}^2\right)^{-1}\mathbf{\Phi_s} = \frac{1}{\sigma_0^2}\mathbf{\Phi_s^T}\mathbf{\Phi_s} = \frac{1}{\sigma_0^2}\mathbf{A}_0. \tag{6}$$

Obviously, the Fisher Information Matrix **Q** can be represented by the matrix $\mathbf{A_0}$. Then $\mathbf{A_0}$ can be represented in a matrix format:

$$\mathbf{A}_0=\mathbf{\Phi}_\mathbf{s}^\mathbf{T}\mathbf{\Phi}_\mathbf{s}=\begin{bmatrix}\varphi_{11} & \varphi_{21} & \cdots & \varphi_{s1}\\ \varphi_{12} & \varphi_{22} & \cdots & \varphi_{s2}\\ \vdots & \vdots & \ddots & \vdots\\ \varphi_{1N} & \varphi_{2N} & \cdots & \varphi_{sN}\end{bmatrix}\begin{bmatrix}\varphi_{11} & \varphi_{12} & \cdots & \varphi_{1N}\\ \varphi_{21} & \varphi_{22} & \cdots & \varphi_{2N}\\ \vdots & \vdots & \ddots & \vdots\\ \varphi_{s1} & \varphi_{s2} & \cdots & \varphi_{sN}\end{bmatrix}$$
$$=\left[\mathbf{\Phi}_1^\mathbf{T}\ \mathbf{\Phi}_2^\mathbf{T}\ \cdots\ \mathbf{\Phi}_\mathbf{s}^\mathbf{T}\right]\begin{bmatrix}\mathbf{\Phi}_1\\ \mathbf{\Phi}_2\\ \vdots\\ \mathbf{\Phi}_\mathbf{s}\end{bmatrix}=\sum_{i=1}^{s}\mathbf{\Phi}_\mathbf{s}^{i\mathbf{T}}\mathbf{\Phi}_\mathbf{s}^{i}. \tag{7}$$

where Φ_i^s is the ith row of the matrix $\mathbf{\Phi_s}$. $\mathbf{A_0}$ can be maximized by removing the degree of freedom that contributes less to the linear independence of target modes.

In order to reduce the degree of freedom $\mathbf{A_0}$, the contribution value of each degree of freedom of the target modes should be calculated. The determination of this value is given by the following procedure.

The characteristic equation of matrix $\mathbf{A_0}$ is

$$(\mathbf{A}_0-\lambda\mathbf{I})\mathbf{\Psi}=0, \tag{8}$$

where λ is the eigenvalue of $\mathbf{A_0}$ and $\mathbf{\Psi}$ is the eigenvector of $\mathbf{A_0}$.

It can be derived from equation (8) that

$$\mathbf{\Psi}^\mathbf{T}\mathbf{A}_0\mathbf{\Psi}=\boldsymbol{\lambda},\ \mathbf{\Psi}^\mathbf{T}\mathbf{\Psi}=\mathbf{I}. \tag{9}$$

with $\mathbf{F}$ set as a contribution matrix of each degree of freedom to target modes; and $\mathbf{F}$ can be expressed as

$$\mathbf{F}=[\mathbf{\Phi}_\mathbf{s}\mathbf{\Psi}]\otimes[\mathbf{\Phi}_\mathbf{s}\mathbf{\Psi}]\boldsymbol{\lambda}^{-1}. \tag{10}$$

where the symbol $\otimes$ represents a term-by-term matrix multiplication. Therefore, $\mathbf{F}$ is an $s\times N$ matrix. The jth item of the ith row of the matrix $\mathbf{F}$ represents the contribution of the ith degree of freedom to the jth eigenvalue. If each row of $\mathbf{F}$ is added, the column vector $\mathbf{E_D}$ can be obtained. The ith row of $\mathbf{E_D}$ represents the contribution of the ith degree of freedom to the linear independence of the entire mode shape matrix.

$$\mathbf{E_D}=\left[\sum_{j=1}^{N}F_{1j}\sum_{j=1}^{N}F_{2j}\cdot\sum_{j=1}^{N}F_{sj}\right]^T. \tag{11}$$

In addition, the contribution matrix of each degree of freedom to the linear independence of the target modes can also be represented by another matrix $\mathbf{E}$:

$$\mathbf{E}=\mathbf{\Phi}_\mathbf{s}\psi\boldsymbol{\lambda}^{-1}\left(\mathbf{\Phi}_\mathbf{s}\psi\right)^T=\mathbf{\Phi}_\mathbf{s}\mathbf{A}_0^{-1}\mathbf{\Phi}_s^T. \tag{12}$$

Using equation (7), the following equation can be noted:

$$\mathbf{E}=\mathbf{\Phi}_\mathbf{s}\mathbf{A}_0^{-1}\mathbf{\Phi}_s^T=\mathbf{\Phi}_s\left[\mathbf{\Phi}_s^T\mathbf{\Phi}_s\right]^{-1}\mathbf{\Phi}_s^T, \tag{13}$$

where E is an idempotent matrix and the ith element E_{ii} on the diagonal represents the contribution of the ith degree of freedom to the rank of the mode shape matrix $\mathbf{\Phi_s}$. Therefore, the elements on the diagonal of $\mathbf{E}$ represent the contribution of the linear independence of the target modes. After the matrix $\mathbf{E}$ is obtained, the degrees of freedom that have more contribution to the linear independence of the target modes are selected and the locations of the sensors can be obtained.

For the finite element model, the process of using EIM for optimal sensor placement is shown in Figure 1. The mode shape matrix of the finite element modal can be extracted after modal analysis. The form of the mode shape matrix is arranged as shown in Figure 2. The mode shape matrix is an $n\times N$ matrix. The number of nodes is s. Thus, $n=3\times s$ if the mode shapes of X, Y, and Z directions are considered. During the process of OSP, the row in the mode shape matrix of the degree of freedom related to the minimum value of the diagonal in the $\mathbf{E}$ matrix will be removed at each time, which means that the position on this degree of freedom is not selected. After n-m times of iterations, m rows are left, which are the optimal sensor positions.

2.2. Evaluation Criterion. The Modal Assurance Criterion (MAC) [27] is an effective tool for evaluating the intersection angle of modal vectors. It can reflect the correlation of two space vectors. The calculation formula of MAC is

$$\mathrm{MAC}_{ij}=\left|\frac{\Phi_i\Phi_j}{\left(\Phi_i^T\Phi_i\right)\left(\Phi_j^T\Phi_j\right)}\right|, \tag{14}$$

where Φ_i is the ith modal vector and Φ_j is the jth modal vector and the values of MAC_{ij} fall between 0 and 1. The MAC_{ij} value will be 0 if the two vectors are orthogonal, while the value equals 1 if the two vectors are completely correlated. The smaller the value of MAC_{ij} is, the stronger the structural characteristics are reflected. It is generally believed that when MAC_{ij} is greater than 0.9, the two modal vectors are completely related and cannot be distinguished.

2.3. Method to Choose the Minimum Number of Sensors. From the concept of the EIM, if the target number of sensors is m $(m<n)$ and the degrees of freedom are n, then the dimension of modal matrix will change from $n\times N$ to $m\times N$ after EIM:

$$\begin{bmatrix}\varphi_{11} & \varphi_{12} & \cdots & \varphi_{1N}\\ \varphi_{21} & \varphi_{22} & \cdots & \varphi_{2N}\\ \vdots & \vdots & \ddots & \vdots\\ \varphi_{n1} & \varphi_{n2} & \cdots & \varphi_{nN}\end{bmatrix}\longrightarrow\begin{bmatrix}\varphi_{11} & \varphi_{12} & \cdots & \varphi_{1N}\\ \varphi_{21} & \varphi_{22} & \cdots & \varphi_{2N}\\ \vdots & \vdots & \ddots & \vdots\\ \varphi_{m1} & \varphi_{m2} & \cdots & \varphi_{mN}\end{bmatrix}. \tag{15}$$

The linear independence of the modal matrix after EIM will be worse than before, which means that the nondiagonal elements value of the MAC matrix gradually decreases. The maximum value of nondiagonal elements in the MAC matrix represents the worst correlation of two modal vectors. This paper therefore presents a method for calculating the minimum number of sensors for health monitoring of transmission tower structures. The steps are as follows:

(1) Use the EIM to obtain a series of placement schemes of differing number of sensors. k schemes can be obtained:

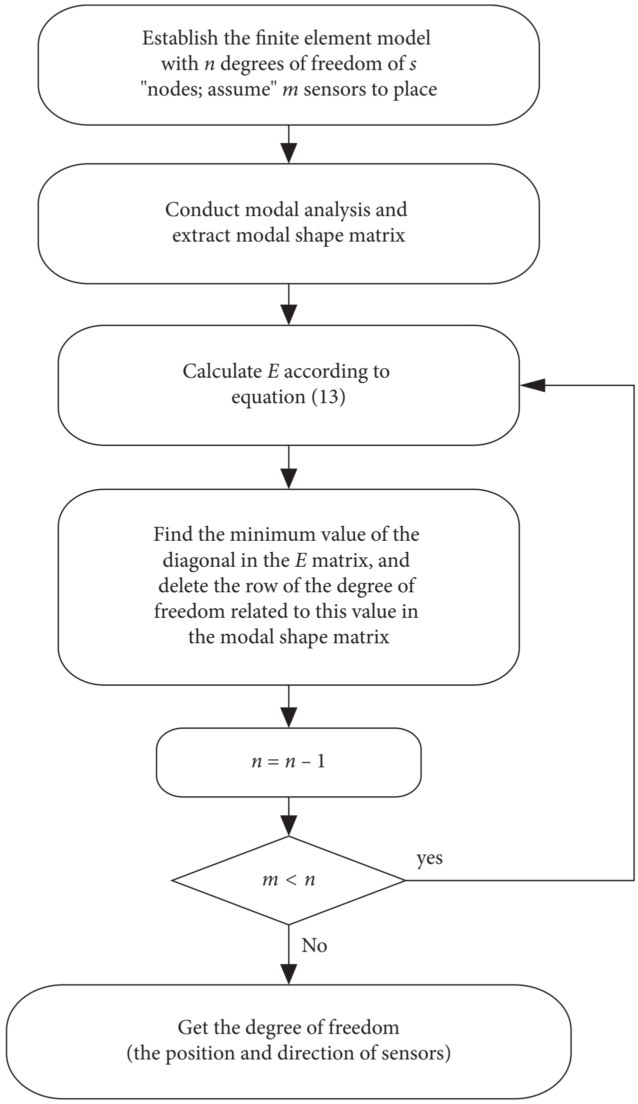


Figure 1: The process of EIM for optimal sensor placement.

$$X = \{m_1, m_2, m_3, \ldots m_k\}. \tag{16}$$

where m_1, m_2, m_3 $\ldots m_k$ represents the assumed number of sensors.

(2) Calculate the MAC matrix of each scheme separately, and extract the maximum nondiagonal element in the matrix:

$$\mathbf{Y} = \{\mathrm{MAC}_{\max}(m_1), \mathrm{MAC}_{\max}(m_2), \mathrm{MAC}_{\max}(m_3), \ldots, \mathrm{MAC}_{\max}(m_k)\}. \tag{17}$$

(3) Take the number of sensors in **X** as the variable **x** and the corresponding value in **Y** as the function value **y**, and use the sequence of the two variables to perform polynomial fitting to obtain the fitting curve:

$$y = f(x). \tag{18}$$

(4) Calculate the derivative of the fitting curve and set it to 0:

$$\frac{\mathrm{d}f(x)}{\mathrm{d}x} = 0, \tag{19}$$

$$
\begin{array}{cc}
 & \textit{(Modal order)} \\
 & \begin{array}{ccccc} 1 & 2 & 3 & \cdots & N \end{array} \\
\textit{(Sensor position)}\;
\begin{array}{c}
\text{Node 1-x} \\ \text{Node 2-x} \\ \vdots \\ \text{Node } s\text{-x} \\
\text{Node 1-y} \\ \text{Node 2-y} \\ \vdots \\ \text{Node } s\text{-y} \\
\text{Node 1-z} \\ \text{Node 2-z} \\ \vdots \\ \text{Node } s\text{-z}
\end{array}
&
\left[\begin{array}{ccccc}
x_1^1 & x_1^2 & x_1^3 & \cdots & x_1^N \\
x_2^1 & x_2^2 & x_2^3 & \cdots & x_2^N \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
x_s^1 & x_s^2 & x_s^3 & \cdots & x_s^N \\
\hdashline
y_1^1 & y_1^2 & y_1^3 & \cdots & y_1^N \\
y_2^1 & y_2^2 & y_2^3 & \cdots & y_2^N \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
y_s^1 & y_s^2 & y_s^3 & \cdots & y_s^N \\
\hdashline
z_1^1 & z_1^2 & z_1^3 & \cdots & z_1^N \\
x_2^1 & x_2^2 & x_2^3 & \cdots & z_2^N \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
z_s^1 & z_s^2 & z_s^3 & \cdots & z_s^N
\end{array}\right]
\end{array}
$$

Figure 2: The modal shape matrix of FE modal.

and, with this method, upon setting the slope of the curve to 0, the curve reaches the extreme point, and the value of x at this point can be interpreted as the minimum number of target sensors required.

3. Analysis Example

3.1. Finite Element Model. This paper presents a case study on a cat-head-type transmission tower in order to verify the optimal sensor placement method for structural health monitoring. In this example, it is assumed that the transmission tower bears the gravity of the overhead lines and the insulator strings and the horizontal force caused by the wind.

To facilitate the subsequent test verification, a 1 : 36 scale model of the cat-head-type transmission tower is analyzed in this paper. The finite element model of the transmission tower is shown in Figure 3. There are 194 nodes and 579 elements in the model. The element type of the finite element model is beam188 element, and the material parameters are set as follows: the elastic modulus is $7.5e7$, Poisson's ratio is 0.03, and the density is $2.7e{-}6$. The four nodes of the tower foot are fixed and the degrees of freedom of the remaining nodes are not constrained. The directions of three main axes are shown in Figure 3.

In this paper, the first 10 modes of the transmission tower are selected as the target modes. The Fisher Information Matrix 2-norm method is used to determine the target modes number. The detailed information of the method is introduced in literature [28]. The essence of the Fisher Information Matrix 2-norm method is to obtain the Fisher Information Matrix of the structure through the vibration mode information of each measuring point of the structure. The rate of change (ROC) of the 2-norm of the matrix on the i-th order and the $i+1$-th order is taken as the index to measure the influence of the number of modes, and the mode is selected by the change rate curve. The function of the ROC is

$$\mathrm{ROC} = \frac{\left\|Q_{i+1}\right\|_2 - \left\|Q_i\right\|_2}{\left\|Q_i\right\|_2}, \tag{20}$$

where Q is the 2-norm of Fisher Information Matrix. The ROC value changes with the increase of mode number i. When i reaches a certain value, the ROC value approaches zero, or i continues to increase but the ROC value has little change. Then, the whole modal information is concluded in the first i mode shapes.

The rate of change (ROC) of the Fisher Information Matrix 2-norm of the transmission tower is calculated through this method, as shown in Figure 4. It can be seen from the figure that the ROC will not change when the modal number is larger than 8. Thus, to ensure the integrity of the modal information, the first 10 modes are chosen as the target modes to conduct OSP. Through the modal analysis of the finite element model of the transmission tower, the first 10 natural frequencies of the transmission tower can be obtained, as shown in Table 1. The results show that the first 10 natural frequencies range between 13.259 and 117.067 Hz. The first 4 modal shapes are shown in Figure 5. It can be seen that the first modal shape is the torsional vibrations in the Z direction, and the second mode is the X direction bending vibrations. The third and fourth modes are the bending vibrations in the Y direction.

3.2. Optimal Sensor Placement. The optimal sensor placement is carried out by EIM and the results are shown in Table 2. The positions of the sensors are represented by "node number—direction." For example, "1701x" means that the sensor is arranged in the X direction of the node 1701. In this paper, the placement results of different target sensor numbers from 10 to 50 are calculated.

Table 2 lists the placement positions when the number of sensors is 50, 30, 20, and 10. As can be seen in Table 2, the placement directions of the sensors are all horizontal (X and Y), which indicates that the modal information in the X direction and Y direction contributes more to the Fisher Information Matrix. Also, it reflects that the main vibration directions of the transmission tower are in the X direction and Y direction.

To show the positions of the sensors more clearly, the placement positions when the number of sensors is 20 and 10 are shown in Figure 6. The sensors are mainly located at the sharp corner and the two ends of the tower top. There are more sensors at the middle of the tower and fewer sensors at the tower leg. At the same time, some positions of the sensors are symmetrical at the middle and top of the tower and as the number of sensors is gradually reduced, some symmetric points are gradually deleted. In addition,

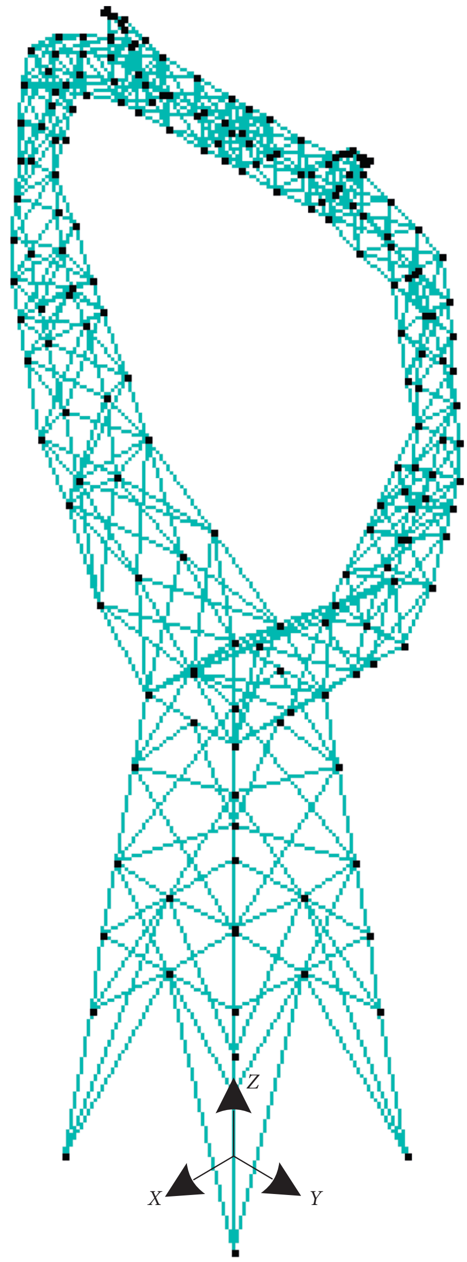


Figure 3: The finite element model of the transmission tower.

comparing the sensor arrangement with 10 and 20 sensors, it can be clearly seen that the positions of 20 sensors contain the positions of 10 sensors.

3.3. Effect Evaluation. The modal matrices calculated by the EIM are verified by the Modal Assurance Criterion. The MAC matrixes are shown in Figure 7. The horizontal axis is the modal order and the vertical axis is the MAC value. As can be seen in Figure 7, the value of the nondiagonal elements of the MAC increases as the number of sensor placements gradually reduces. When the numbers of sensors are 50, 30, 20, and 10, the maximum values of the nondiagonal elements of the MAC matrix are 0.2575, 0.3222, 0.3890, and 0.5989, and the average values are 0.0199, 0.0270, 0.0371, and 0.0608. This result indicates that when the number of sensors is large, the linear correlation between the various modes is small, and the modal information of the original structure can be well reflected. When the number of sensors is small, the correlation between some modes will be degraded, which has a negative impact on the acquisition of modal information.

3.4. Application Examples of Calculating the Minimum Number of Sensors. According to the EIM and the MAC, the maximum nondiagonal elements of the MAC matrix of the transmission tower are, respectively, calculated when the

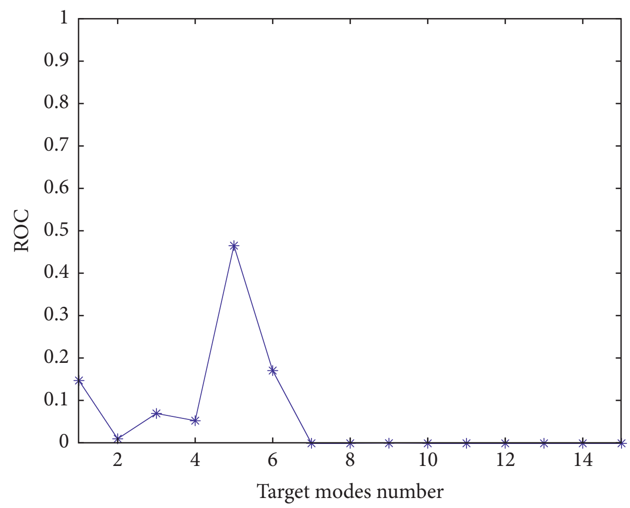


Figure 4: The rate of change of the Fisher Information Matrix 2-norm of the transmission tower.

Table 1: First 10 natural frequencies of the transmission tower.

Modal order	1	2	3	4	5	6	7	8	9	10
Natural frequency (Hz)	13.259	39.783	40.134	44.722	51.803	88.236	105.342	107.859	115.842	117.067

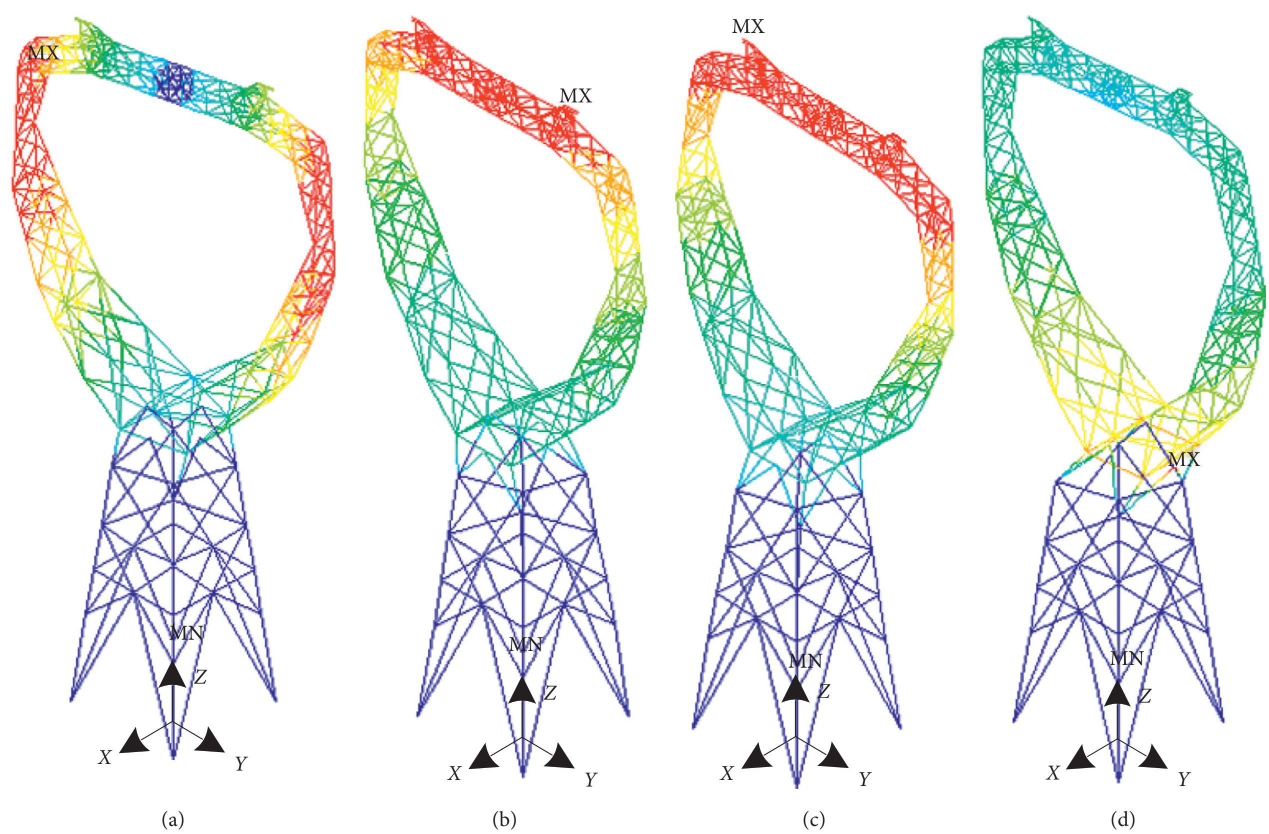
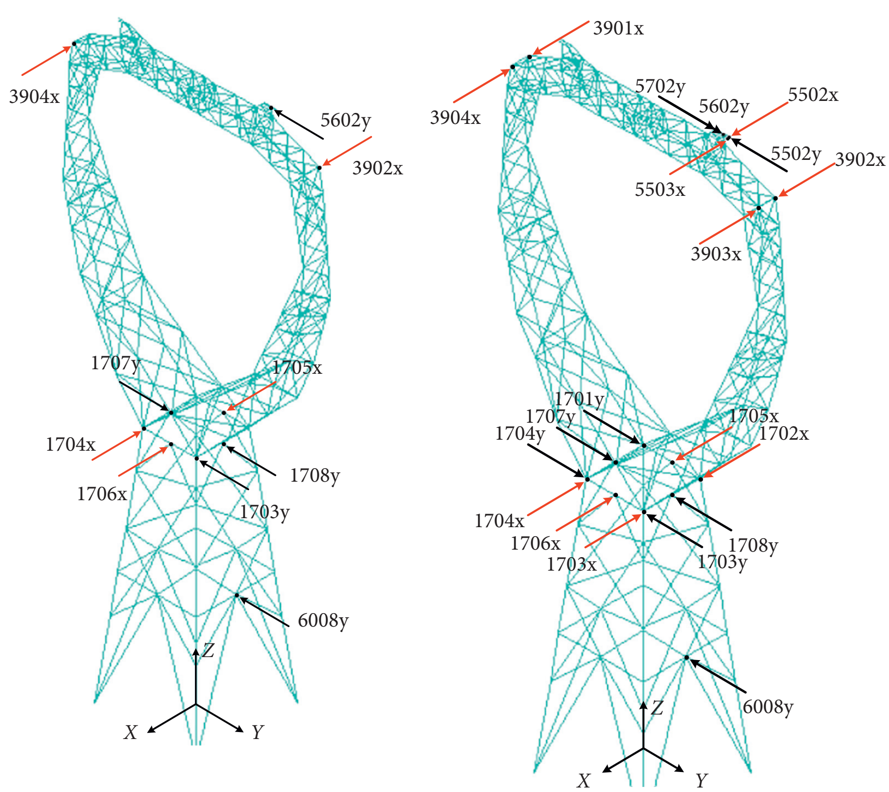


Figure 5: The modal shapes of the transmission tower. (a) The first-order modal shape. (b) The second-order modal shape. (c) The third-order modal shape. (d) The fourth-order modal shape.

Table 2: Sensor placements of different number of sensors.

Node numbering	Sensor position 50	30	20	10	Node numbering	Sensor position 50	30	20	10
1701*x*	●	●			5601*x*	●			
1702*x*	●	●	●		5604*x*	●			
1703*x*	●	●	●		6004*x*	●	●		
1704*x*	●	●	●	●	6006*x*	●	●		
1705*x*	●	●	●	●	1701*y*	●	●	●	
1706*x*	●	●	●	●	1702*y*	●	●		
1801*x*	●				1703*y*	●	●	●	●
1802*x*	●				1704*y*	●	●	●	
1803*x*	●				1707*y*	●	●	●	●
1804*x*	●				1708*y*	●	●	●	●
1901*x*	●				1802*y*	●			
1902*x*	●				1803*y*	●			
3404*x*	●	●			2001*y*	●			
3501*x*	●	●			2002*y*	●			
3502*x*	●				5501*y*	●			
3503*x*	●	●			5502*y*	●	●	●	
3901*x*	●	●	●		5503*y*	●			
3902*x*	●	●	●	●	5601*y*	●			
3903*x*	●	●	●		5602*y*	●	●	●	●
3904*x*	●	●	●	●	5603*y*	●	●		
4003*x*	●				5702*y*	●	●	●	
5501*x*	●				5703*y*	●	●		
5502*x*	●	●	●		5704*y*	●			
5503*x*	●	●	●		6006*y*	●	●		
5504*x*	●				6008*y*	●	●	●	●



(a) (b)

Figure 6: Locations of different numbers of sensors. (a) 10 sensors. (b) 20 sensors.

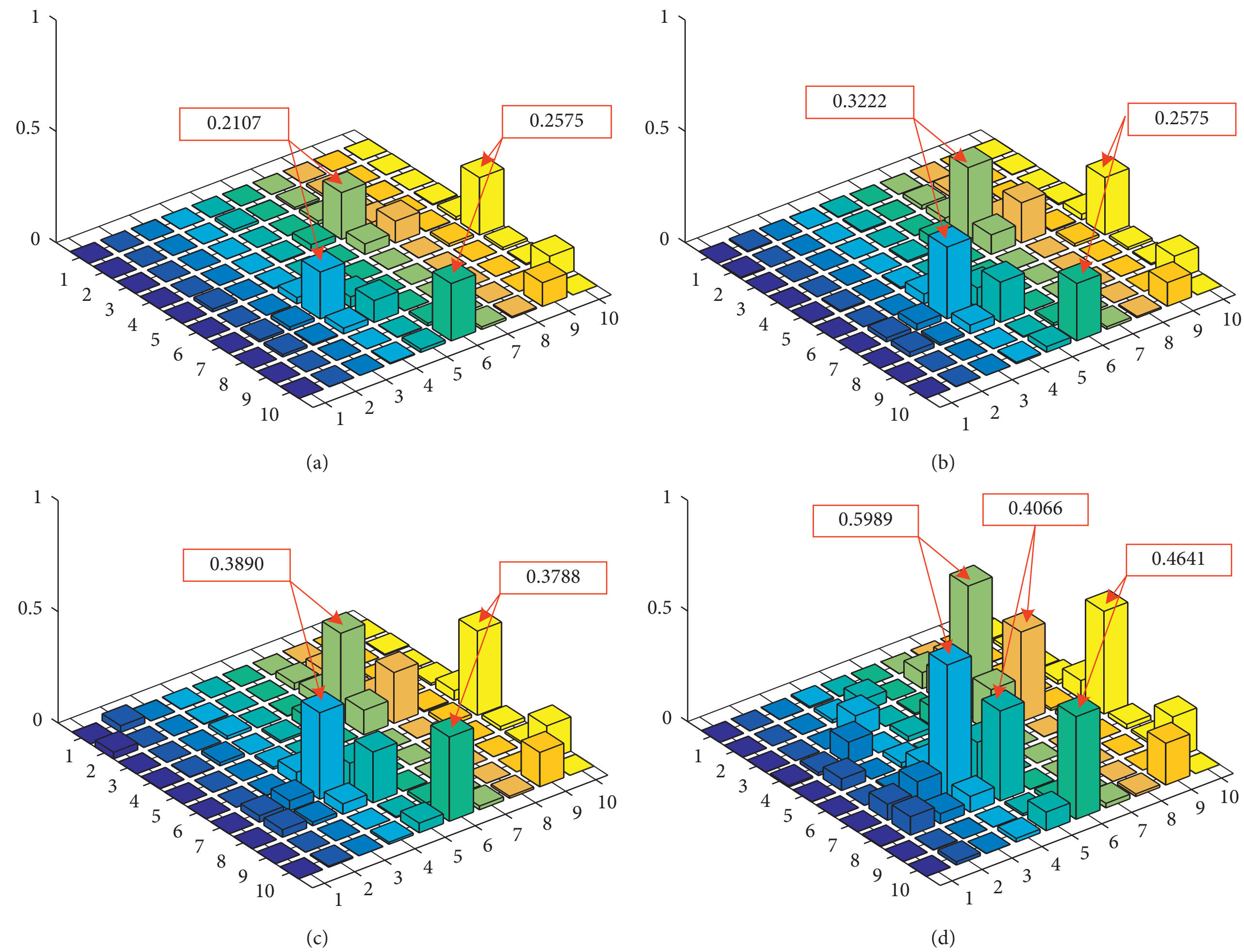


Figure 7: The MAC matrix corresponding to different sensor placement schemes. (a) 50 sensors. (b) 30 sensors. (c) 20 sensors. (d) 10 sensors.

numbers of sensors are 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, and 60. The polynomial fitting regression method can then be subsequently used to fit the curve of the number of sensors and the maximum nondiagonal elements of the MAC matrix. The curve is fitted using the curve fitting tool in MATLAB software. The curve fitting modal is a polynomial linear modal with 3 degrees, which is as follows:

$$f(x) = p1 \times x^3 + p2 \times x^2 + p3 \times x + p4. \tag{21}$$

By using the curve fitting tool, the parameters of the function and the goodness of fit are calculated, as shown in Table 3. The fitting curve is shown in Figure 8. The goodness of fit shows that R-squared is near 1, which means that the fitting modal is acceptable.

The scatter diagram and fitting curve are shown in Figure 9. It can be seen that the maximum values of the nondiagonal elements of the MAC matrix gradually decrease as the number of sensors increases. When this number of sensors exceeds 40, the maximum values of nondiagonal elements of the MAC matrices no longer change significantly. According to steps (3) and (4) mentioned above, an extreme point of the fitting curve can be obtained, and the number of sensors corresponding to the value of the extreme point is 46, which indicates that when the number of sensors exceeds 46, the maximum values of the nondiagonal elements of the MAC matrix will not change noticeably. In other words, the minimum number of sensors required to obtain the most comprehensive structural modal information is 46.

Figure 10 shows the sensor positions when the number of sensors is 46. The position of sensors is arranged mainly at the middle and the top of the tower. There are still some symmetric measurement points, similar to the position when the number of sensors is less than 46. In addition, as indicated by the circle in Figure 10, the position of the tower

Table 3: The parameters of the polynomial linear modal.

Function parameter	$p1$	$p2$	$p3$	$p4$
Value	$-5.447e-06$	0.000805	−0.03967	0.9126
Goodness of fit	SSE	R-squared	Adjusted R-squared	RMSE
Value	0.0009511	0.9923	0.9891	0.01166

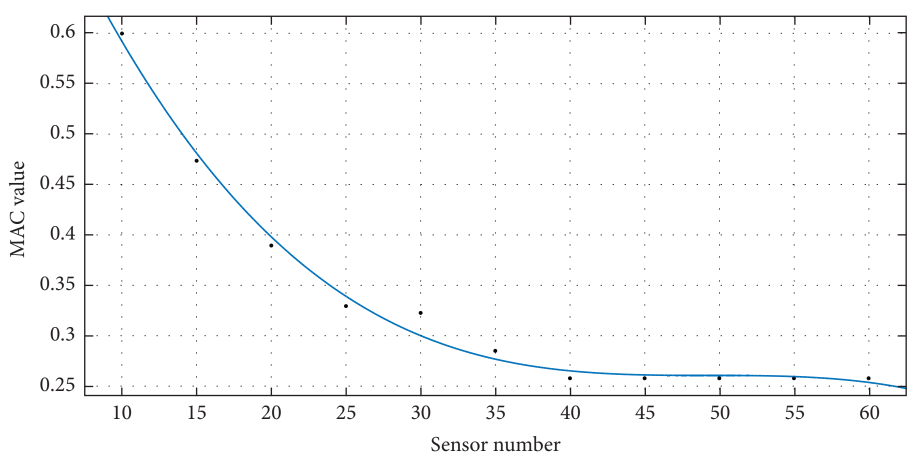


Figure 8: The curve fitting modal of the MAC value using the curve fitting tool in MATLAB.

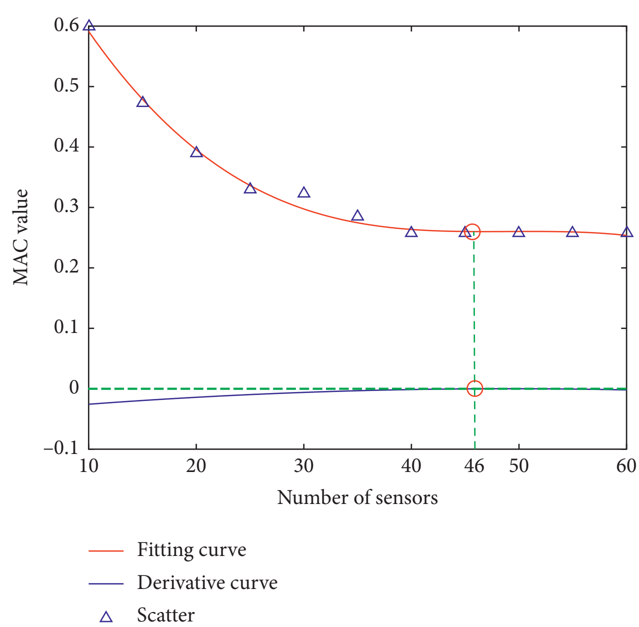


Figure 9: Scatter diagram and the fitting curve when the MAC value changes with the number of sensors.

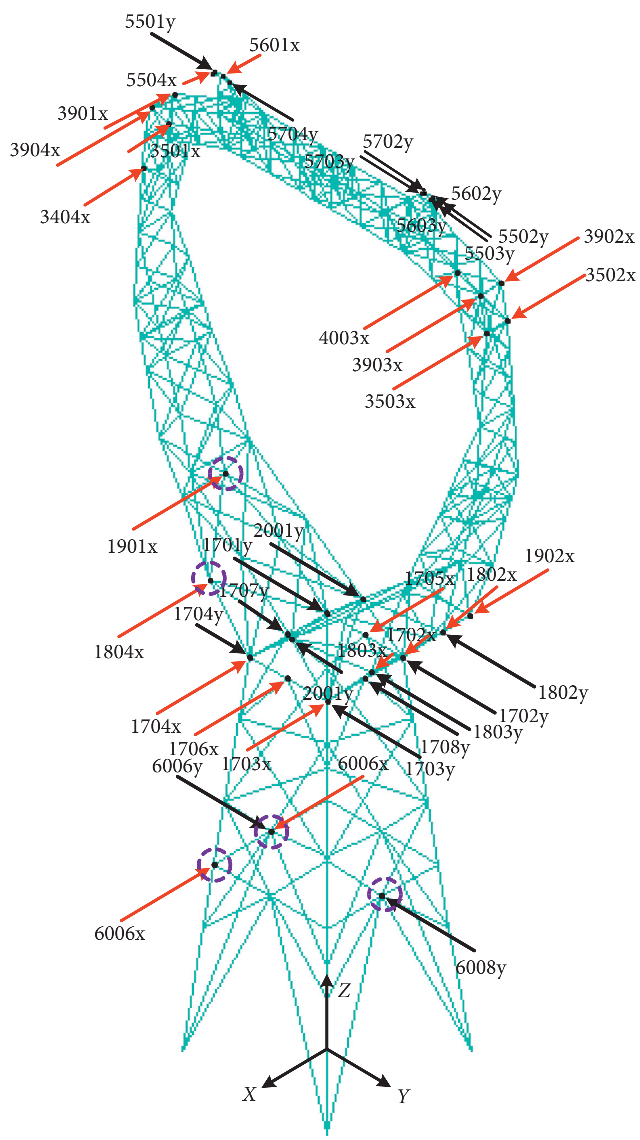


Figure 10: The sensor placement when the number of sensors is 46.

leg and the middle of the tower is increased by some points compared with the position when the number of sensors is less than 46.

4. Conclusion

(1) The sensor placement of the transmission tower model is optimized by the EIM. The position and direction of the sensors are obtained when the numbers of target sensors are, respectively, 60, 55, 50, 45, 40, 35, 30, 25, 20, 15, and 10. The results show that when the number of target sensors is large, the position of sensors is concentrated and symmetrically arranged. In addition, the position of the large number of sensors includes the position of a smaller number of sensors.

(2) The MAC matrix values are calculated when the numbers of sensors are different. The results indicate that as the number of sensors decreases, the MAC value gradually increases. The average value of nondiagonal elements is below 0.1, which shows that the optimal sensor placement can guarantee the acquisition of the modal information.

(3) A method for choosing the number of sensors is proposed based on the EIM and MAC. A cat-head-type transmission tower modal is used to verify this method. The curve by which MAC values change with the number of sensors is obtained. The result shows that the minimum number of sensors required to obtain the most comprehensive structural modal information is 46.

Data Availability

The data supporting the results of this study can be obtained upon request to the corresponding author.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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