Research Article

Dispersion Compensation Method for Lamb Waves Based on Measured Wavenumber

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Received 5 September 2019; Revised 10 January 2020; Accepted 13 January 2020; Published 22 February 2020

Academic Editor: Jean-Mathieu Mencik

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In this study, a delta wavenumber dispersion compensation (ΔK -DC) method was developed and applied, not only with the theoretical wavenumber but also with the measured wavenumber. Dispersion compensation can be achieved by the following steps: relative wavenumber measurement, traveling distance estimation, phase compensation, and wave correction. The feasibility of ΔK -DC with the theoretical wavenumber and measured wavenumber was validated with a high-dispersive A_0 mode in a 2 mm steel plate experiment. The results showed that phase spectrum measurement was an effective method to construct the wavenumber curve, the propagation distances estimated by SAP² were very accurate, and the dispersive signals can be compensated perfectly by applying the phase compensation and wave correction methods for each wavepacket. The present results highlight the application of ΔK -DC on dispersion compensation without any material parameters of a waveguide.

1. Introduction

Ultrasonic-guided wave is an efficient approach to nondestructive testing (NDT) [1–4] and structural health monitoring (SHM) [5, 6] and material characterization applications [7]. Lamb waves are commonly used in plate structures because of the possibility of long traveling distance, large detecting area, and excellent inspecting sensitivities. Dispersion and multimode are fundamental properties of Lamb waves. Dispersion characteristics are that velocity depends on the product of excitation frequency and plate thickness and even relates to the layup orientation for anisotropic materials. Multimode property is that a finite waveguide can support infinite modes. A nonlinear relation between wavenumber and frequency results in wavepacket extension and waveform distortion when the Lamb wave propagates through a plate structure. The longer the propagation time is, the more serious the dispersion phenomenon is. In addition, considering the fact that the acquiring signal is the accumulation of multiple modes from the excitation point, scattered waves and new converted modes from the damage, and reflected waves from the

boundary, overlapping of these waveforms makes signals difficult to interpret accurate damage identification and high-resolution imaging hard to develop. Therefore, it is of great significance to explore the dispersion compensation method for developing efficient NDT methods.

More recently, there have been a number of techniques proposed for dispersion compensation or dispersion removal, such as the optimal design of the excitation waveform, signal domain transform, linearization mapping, warped frequency transform, and time reversal. Usually, a narrowband excitation signal can largely reduce the dispersion phenomenon in the frequency domain, but it makes signals more likely to overlap in the time domain. Signal domain transform is a widely used method to make waves free from dispersion and waveform deformation. Cai [8] and Wilcox [9] proposed a time-distance mapping method. Firstly, a dispersive Lamb wave is mapped from the time domain to the frequency domain using fast Fourier transform (FFT). Secondly, it is interpolated to the wavenumber domain based on the prior wavenumber knowledge, and finally, the signal is transformed to the distance domain using inverse fast Fourier transform (IFFT). Cai [10, 11]

developed a linear mapping technique. This method was realized based on the fact that the nonlinear relation between wavenumber and frequency leads to wave dispersion. Firstly, a linear wavenumber was constructed by expanding the nonlinear wavenumber in a Taylor series around the central frequency up to the first order. Then, in the wavenumber domain, the nonlinear wavenumber was mapped by the linear one to remove the dispersion effect. Backpropagation was also a signal processing tool of effective compensation, such as the time-reversal technique [12–15]. The dispersive signal was reversed in the time domain and reemitted from the sensor location to the actuator location. Fu [16] and Marchi [17, 18] presented the warped frequency transform (WFT), which maps signals from the frequency domain into the warped frequency domain by a warping function. Except for these commonly used compensation methods, Sicard [19] established a scalar diffraction theory, a numerical dispersion compensation technique with time recompression. Hua [20] performed the dispersion compensation by applying time reversed on the excitation signal with a given distance. Xu [21] developed a dispersion compensation method based on compressed sensing.

Together, most studies highlighted the needs for the prior knowledge on the dispersion characteristics (phase velocity, group velocity, or wavenumber) and the applications of FFT, IFFT, and interpolation operation. Time reversal does not need any prior dispersion knowledge nor material properties. The present study is to develop the ΔK -DC method, not only applied for the theoretical wavenumber but also applied for the measured wavenumber. When the theoretical wavenumbers are available, dispersion compensation can be approached by phase compensation and wave correction. When the structure properties are unknown, dispersion compensation can be achieved by the following steps: wavenumber measurement, traveling distance estimation, phase compensation, and wave correction. The phase spectrum measurement method is developed to measure the relative wavenumbers. The wavelet energy spectrum method is provided to extract the flight time of each wavepacket. The propagating distances are estimated by the group velocity and the flight times; finally, phase compensation and wave correction are executed to compensate dispersion separately. Stimulation and experimental setup are then performed to validate the performances of ΔK -DC for the theoretical wavenumber and the measured wavenumber.

2. Fundamental Investigation on ΔK -DC with Theoretical Wavenumber

A nonlinear relation between wavenumber and frequency results in dispersion effect, which can be minimized by compensating a nonlinear wavenumber to the corresponding linear one. The dispersive signals that are satisfactorily renovated to the waveform of the activation signal are the purpose of ΔK -DC.

2.1. The Basic Principle of ΔK -DC. Lamb waves can be modeled by imposing traction-free surface boundary conditions on the equations of motion, effectively describing the wave behavior. From Rose [22], the propagation of guided waves can be obtained from the dispersion characteristics of a plate structure and the spectrum of an excitation signal. Dispersive waves can be generated using the following equations:

$$Y(r, \omega) = Y(0, \omega)H(r, \omega), \quad (1a)$$

$$H(r, \omega) = A(r, \omega)e^{-iK(\omega)r}, \quad (1b)$$

where r is the propagation distance; ω is the angular frequency and ω_c is the angular frequency at the central frequency of the excitation signal; $Y(0, \omega)$ and $Y(r, \omega)$ are the spectrums of the excitation signal $y(0, t)$ and Lamb wave $y(r, t)$; and $H(r, \omega)$ is the transfer function, where $K(\omega)$ denotes the nonlinear wavenumber. The slightly varied amplitude $A(r, \omega)$ can be simplified as “1” to facilitate the following analysis. Based on the time-shifting properties of FFT, equation (1b) indicates that the complex phase shifts of $Y(0, \omega)$ in the frequency domain, $e^{-iK(\omega)r}$, are interpreted as a translation of $y(0, t)$ in the time domain, $K(\omega)r/\omega$.

According to the dispersion property and energy conservation, different frequency components in a wave packet will propagate at different velocities, which may result in the broadening of wavepackets and reduction of amplitude when they propagate through a structure. As discussed above, both dispersion phenomenon and waveform distortion of $y(r, t)$ can be achieved by linearizing the nonlinear wavenumber. The linear wavenumber, $K_{\text{lin}}(\omega)$, can be obtained by expanding $K(\omega)$ as the *Taylor* series around the central frequency up to the first order, as shown in equation (2). Equation (3) shows the 0-order coefficient is the wavenumber at ω_c and the 1-order coefficient is the reciprocal of group velocity at ω_c , where $c_p(\omega_c)$ and $c_g(\omega_c)$ express the phase velocity and the group velocity at ω_c , respectively. The delta wavenumber, $\Delta K(\omega)$, is calculated by equation (4).

$$K_{\text{lin}}(\omega) = K(\omega_c) + \left(\frac{dK(\omega)}{d\omega} \right) \Big|_{\omega=\omega_c} (\omega - \omega_c) = k_0 + k_1 (\omega - \omega_c), \quad (2)$$

$$\begin{aligned} k_0 &= \frac{\omega_c}{c_p(\omega_c)}, \\ k_1 &= \frac{1}{c_g(\omega_c)}, \end{aligned} \quad (3)$$

$$\Delta K(\omega) = K_{\text{lin}}(\omega) - K(\omega). \quad (4)$$

Substituting equations (2) and (3) into equation (1), the nondispersive signal, $Y'_{\text{lin}}(r, \omega)$, is obtained by using the linear wavenumber as

$$Y'_{\text{lin}}(r, \omega) = Y(0, \omega)e^{-iK_{\text{lin}}(\omega)r} = Y(0, \omega)e^{i\varphi_0 r}e^{-i(\omega r/c_g(\omega_c))}, \quad (5)$$

$$\varphi_0 = \omega_c \left(\frac{1}{c_g(\omega_c)} - \frac{1}{c_p(\omega_c)} \right). \quad (6)$$

$y'_{\text{lin}}(r, t)$ is obtained by applying IFFT to $Y'_{\text{lin}}(r, \omega)$ as

$$y'_{\text{lin}}(r, t) = \text{IFFT}(Y'_{\text{lin}}(r, \omega)) = y\left(0, \frac{t-r}{c_g(\omega_c)}\right)e^{i\varphi_0 r}, \quad (7)$$

$$y_{\text{lin}}(r, t) = y\left(0, \frac{t-r}{c_g(\omega_c)}\right) = y'_{\text{lin}}(r, t)e^{-i\varphi_0 r}. \quad (8)$$

Equation (7) indicates $y'_{\text{lin}}(r, t)$ is an entire $r/c_g(\omega_c)$ delay of $y(0, t)$ in the time domain, with another phase factor φ_0 . $y_{\text{lin}}(r, t)$ is only an entire $r/c_g(\omega_c)$ delay of $y(0, t)$ in the time domain based on the fact that a constant time delay does not result in dispersion, as illustrated in equation (8). Usually, $c_g(\omega_c) \neq c_p(\omega_c)$, $\exp(-i\varphi_0) \neq 1$, so the waveform of $y'_{\text{lin}}(r, t)$ may differ from that of $y(r, t)$. By applying equations (1), (5), and (8), the relationship between $Y'_{\text{lin}}(r, \omega)$ and $Y(r, \omega)$ can be described in equation (9), with $Y'_{\text{lin}}(r, \omega)$ as the intermediate function. $y_{\text{lin}}(r, t)$ can be obtained by applying IFFT to $Y_{\text{lin}}(r, \omega)$, in equation (10).

$$Y_{\text{lin}}(r, \omega) = Y'_{\text{lin}}(r, \omega)e^{-i\varphi_0 r} = Y(r, \omega)e^{-i\Delta K(\omega)r}e^{-i\varphi_0 r}, \quad (9)$$

$$y_{\text{lin}}(r, t) = \text{IFFT}(Y_{\text{lin}}(r, \omega)). \quad (10)$$

To sum up, phase compensation and waveform correction are the specific steps for the compensation process, as indicated in equation (9).

2.2. Realization of ΔK -DC with Theoretical Wavenumber. The procedure of ΔK -DC with the known wavenumber and distance is given in detail, as shown in Figure 1. Compared with the traditional dispersion compensation algorithm, interpolation and transforming in the wavenumber domain are not required. The approach includes the following steps sequentially:

- Obtaining $K(\omega)$ by solving the dispersion equation, constructing $K_{\text{lin}}(\omega)$ with equation (2), and getting $\Delta K(\omega)$ with equation (4), respectively
- Applying N-point FFT to $y(r, t)$ for $Y(r, \omega)$
- $Y'_{\text{lin}}(r, \omega)$ is compensated by the phase-delay factor $e^{-i\Delta K(\omega)r}$ to $Y(r, \omega)$ with equation (9)
- Correcting the waveform with $e^{-i\varphi_0 r}$ to get $Y_{\text{lin}}(r, \omega)$
- Applying N-point IFFT to $Y_{\text{lin}}(r, \omega)$ for $y_{\text{lin}}(r, t)$

2.3. Simulation on ΔK -DC in a Steel Plate. To validate the feasibility of the ΔK -DC algorithm with the theoretical wavenumber, a single A_0 mode wave was activated in a homogeneous and isotropic steel plate (elastic modulus: 208.42 GPa, density: 7850 kg/m³, Poisson's ratio: 0.2959, and thickness: 2 mm). In a pitch-catch configuration, 3.5-cycle

FIGURE 1: Schematic of ΔK -DC for the theoretical wavenumber.

sinusoid tone bursts at a central frequency of 200 kHz modulated by a *Hanning* window were applied in turn to activate a single A_0 mode. And the sensor signal was received upon traveling 300 mm across the plate. The FEM simulation was accomplished using ABAQUS®/EXPLICIT, as shown in Figure 2(a). The sampling frequency was 10 MHz. Displacement gram in the thickness direction is illustrated in Figure 2(b). It can be seen that the wavepacket spreads out in space and time and amplitude reduces when Lamb waves propagate through the plate.

It can be seen that the time duration rises from 17.5 μs of the excitation signal (shown in Figure 3(a)) to about 50 μs of the captured signal (shown in Figure 3(b)). The theoretical velocity dispersion curves can be obtained by solving the dispersion equation (shown in Figure 3(c)). The group velocity and the phase velocity at 200 kHz are 2794.4 m/s and 1724.8 m/s, respectively. Then, the nonlinear wavenumber $K(\omega)$, the linear wavenumber $K_{\text{lin}}(\omega)$, and the delta wavenumber $\Delta K(\omega)$ were successfully calculated with equations (2)–(4) (shown in Figure 3(d)). As illustrated in Figure 3(e), the phase compensation result of $y(r, t)$, $y'_{\text{lin}}(r, t)$, was still different from the waveform of $y(0, t)$ mainly because of the existence of φ_0 . Following this, it was necessary to correct the waveform as equation (10), and what stands out in Figure 3(f) was perfect compensation for the dispersive signal.

3. ΔK -DC with the Measured Wavenumber

When the material parameters are unavailable, such as *elastic modulus*, *density*, and *Poisson's ratio* for the isotropy structure and *stiffness matrix* and *layup* for anisotropic structures, the wavenumber could not be theoretically computed. In this event, dispersion compensation can be achieved by the following procedures: wavenumber measurement, traveling distance estimation, phase compensation, and wave correction. The feasibility of the scheme was verified in a 2 mm steel plate.

FIGURE 2: Simulation setup in a steel plate. (a) ABAQUS model. (b) Displacement gram in the thickness direction.

FIGURE 3: Procedure of ΔK -DC with the known wavenumber: (a) excitation signal in the time domain $y(0, t)$; (b) sensor signal in the time domain $y(r, t)$; (c) velocity dispersion curves $c_p(\omega)$ and $c_g(\omega)$; (d) wavenumber curves $K(\omega)$, $K_{lin}(\omega)$, and $\Delta K(\omega)$; (e) phase compensation result $y'_{lin}(r, t)$; (f) waveform correction result $y_{lin}(r, t)$.

3.1. Phase Spectrum Method of Wavenumber Measured. A phase spectrum measurement method is proposed to measure the wavenumber based on the phase difference between the excitation signal and the Lamb wave, expressed as equations (11)–(13). This process needs the spectrum of the excitation signal and Lamb waves but does not need any structure material parameters.

$$H(r, \omega) = \frac{Y(r, \omega)}{Y(0, \omega)} = e^{-i\Delta\Phi(\omega)} = e^{-iK(\omega)r}, \quad (11)$$

$$\Delta\Phi(\omega) = \text{unwrap}\left(\arctan\left[\frac{\text{Im}}{\text{Re}}\right]\right), \quad (12)$$

FIGURE 4: Procedure of SAP²: (a) excitation signal; (b) wavelet spectrum of (a); (c) SAP² of (a).

$$K(\omega) = \frac{\Delta\Phi(\omega)}{r}, \quad (13)$$

where $\Delta\Phi(\omega)$, the unwrapped phase of $H(r, \omega)$, can be calculated from the arctangent function in equation (12), where Re and Im represent the real part and the imaginary part of $H(r, \omega)$, respectively. Equation (13) presents that the dispersion number is the ratio of phase difference to propagation distance. Therefore, the propagation distance r for measuring the wavenumber should be accurately known, or the measured wavenumber is not accurate.

3.2. Wavelet Energy Spectrum for Traveling Distance Extraction. The aim of distance estimation is to determine whether the measured group velocity is accurate and use the estimated distances to compensate dispersion. Extracting the exact arrival time is a critical step to determine the propagation distance. The scale-averaged wavelet power (SAP²) based on CWT, which is defined in equations (14) and (15) [4, 23], is used to determine the exact energy peak of $y(r, t)$, where M is the largest scale and N is the sampling point of $y(r, t)$. The mean wave energy, E , over the entire time period is expressed in equation (15). Figure 4(b) displays SAP² of the excitation signal.

$$\text{SAP}^2(n) = \frac{1}{M} \sum_{i=1}^M |\text{CWT}(a_i, n)|^2, \quad i = 1, 2, \dots, M, \quad (14)$$

$$E = \frac{1}{N} \sum_{i=1}^M \text{SAP}^2(n), \quad n = 1, 2, \dots, N, \quad (15)$$

$$\text{time}_{\text{flight}} = \text{time}_{\text{arrival}}(y(r, t)) - \text{time}_{\text{arrival}}(y(0, t)), \quad (16)$$

$$r = c_g(\omega_c) \cdot \text{time}_{\text{flight}}. \quad (17)$$

According to Rose [22], wave propagation is energy transportation, and the group velocity is the velocity of energy transportation. The flight time of each wavepacket should be defined in terms of the time difference between the arrival time and the initial time moment, as expressed in equation (16). The arrival times of multiple wavepackets of $y(r, t)$ are determined at the moments that the energy reached

FIGURE 5: Schematic of ΔK -DC for the measured wavenumber.

its extremums in the time-scale domain. Looking at Figure 4(c), it is apparent that the initial time of the excitation signal is the 88th sampling point. Propagating distances are estimated by the group velocity at the central frequency and the flight time of each wavepacket, as expressed in equation (17).

3.3. Realization of ΔK -DC with Measured Wavenumber. As discussed above, the procedures of the ΔK -DC algorithm can be summarized, as displayed in Figure 5. The approach includes the following steps sequentially:

- (a) Applying FFT to the excitation signal and the direct wave signal to get $Y(0, \omega)$ and $Y(r, \omega)$
- (b) Obtaining the wavenumber, $K(\omega)$, by adopting the phase spectrum measurement method in equations (11)–(13)
- (c) Calculating the linear wavenumber, $K_{\text{lin}}(\omega)$, and the wavenumber increment, $\Delta K(\omega)$, in equations (2) and (4)

FIGURE 6: Sketch of the experiment setup with an actuator-sensor pair based on the Lamb wave.

- (d) Performing the wavelet spectrum measurement method to estimate the flight time and propagating distance in equations (14)–(17)
- (e) For each wavepacket, $Y'_{lin}(r, t)$ is compensated by the phase-delay factor $e^{-i\Delta K(\omega)r}$ to $Y(r, \omega)$ with equation (9)
- (f) For each wavepacket, correcting the waveform with $e^{-i\varphi_0 r}$ to get $Y_{lin}(r, \omega)$
- (g) Applying N-point IFFT to $Y_{lin}(r, f)$ to get $y_{lin}(r, t)$

3.4. Experiment Validation. To validate the ΔK -DC algorithm with the measured wavenumber, an experiment was conducted in an isotropic steel plate, with a size of $500 \times 500 \times 2$ mm. Two transducers (Fuji® AE204S) with a central frequency of 200 kHz were perfectly surface-bonded on the plate using coupling gel, as indicated in Figure 6, where the left transducer is a wave actuator to produce Lamb waves and the right is a sensor. A 3.5-cycle sinusoid tone burst at a central frequency of 200 kHz modulated by a *Hanning* window was generated by an arbitrary wave generator (Apobico® ADL5330). The excitation signals after being amplified to 50 V (peak to peak) (Rigol® DG4102) were applied to the actuator. The sensor signals were captured on the upper and lower surfaces upon traveling 100 mm by using a nonmetal ultrasonic detector (Sinotesting® TS-C6). The sampling frequency was 10 MHz. The NFFT point was set to 10,000. The resolution was 1 kHz. The antisymmetric mode can be obtained by subtraction of the captured signals, as highlighted in Figure 7. There are two wavepackets: one for the direct A_0 mode and another for the reflected A_0 wave from the boundary.

To begin the ΔK -DC process, the first step is to measure the wavenumber, $K(\omega)$, using the phase spectrum measurement method. Figures 8(a) and 8(b) display the spectrum of the excitation signal and the direct A_0 wave. After the phase is unwrapped, the phase difference spectrum, $\Delta\Phi(\omega)$, can be calculated based on equation (12), as shown in Figure 8(c). Then, the wavenumber is computed based on equation (13); the linear wavenumber is constructed based

FIGURE 7: Raw signal with direct and reflected A_0 waves, $y(r, t)$.

on equation (2); the delta wavenumber is obtained based on equation (4); Figure 8(d) presents the above wavenumber relation. At frequency 200 kHz, the measured group velocity was 2647.9 m/s vs. theoretical value of 2794.4 m/s and the measured phase velocity was 1895.7 m/s vs. theoretical value of 1724.8 m/s.

The wavelet energy spectrum method was used to estimate the traveling distance of each wavepacket. Looking at Figure 8(e), it is apparent that the arrival time of the direct A_0 mode and the reflected A_0 mode is the 465th and 1981st sampling points. And the arrival time of the excitation signal was the 88th sampling point (shown in Figure 4(c)). Based on equations (14)–(16), the estimated distances of the direct A_0 mode and the reflected A_0 mode were 99.8 mm and 501.2 mm, which were in good agreement with the experimental expectations, 100 mm and 500 mm.

Then, phase compensation was executed for the direct A_0 wave with 99.8 mm and reflected A_0 wave with 501.2 mm. Phase compensation is illustrated in Figures 8(f) and 8(g). It can be seen that the waveforms were compressed in the time domain but still different from the excitation. Lastly, phase compensation was applied. From Figure 8(h), we can see that, under the circumstances of multiple-wavepacket arrival, the ΔK -DC method can get good compensation to separately compensate each wavepacket using different traveling distances.

FIGURE 8: Continued.

FIGURE 8: Procedure of ΔK -DC with the unknown wavenumber: (a) spectrum of the excitation signal $Y(0, \omega)$; (b) spectrum of the direct A_0 wave $Y_{A_0}(r, \omega)$; (c) phase spectrums $\Phi(0, \omega)$, $\Phi(r, \omega)$, and $\Delta\Phi(\omega)$; (d) wavenumber curves $K(\omega)$, $K_{lin}(\omega)$, and $\Delta K(\omega)$; (e) SAP^2 of the raw signal in Figure 7; (f) phase compensation result for the direct A_0 wave $Y'_{lin}(r, t)$ (direct A_0 wave); (g) phase compensation result for the reflected A_0 wave $Y'_{lin}(r, t)$ (reflected A_0 wave); (h) ΔK -DC result after correction $Y_{lin}(r, t)$.

4. Conclusion

This project was undertaken to propose a new ΔK -DC method for dispersion compensation of Lamb waves, which is applicable for waveguides with known or unknown material parameters. The applicability of ΔK -DC with the theoretical wavenumber and measured wavenumber was validated with a high-dispersive A_0 mode in a 2 mm steel plate experiment. The results showed that ΔK -DC can get good compensation.

With regard to the ΔK -DC method, some limitations need to be acknowledged. Firstly, the propagation distance should be accurately known to measure the wavenumber. Secondly, when multiple modes propagate in a waveguide, it is necessary to identify the mode, measure the dispersion curve, and then apply ΔK -DC to each mode separately. Further investigation and experimentation of ΔK -DC into anisotropy composite media are strongly recommended for developing NDT techniques, such as damage location, damage quantification, and high resolution of damage imaging.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

Our heartfelt thanks are due to Prof. Han and Dr. Qin for their academic supervision and personal support. This work was supported by the Emergency Management Project of Natural Science Foundation of China (Grant no. 61842103), Youth Science Foundation (Grant no. 11604304), and Natural Science Foundation of Shanxi, China (Grant no. 201801D121156).

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