Research Article
A New Method of Denoising of Vibration Signal and Its Application

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In order to improve the performance of the denoising method for vibration signals of rotating machinery, a new method of signal denoising based on the improved median filter and wavelet packet technology is proposed through analysing the characteristics of noise components and relevant denoising methods. Firstly, the window width of the median filter is calculated according to the sampling frequency so that the impulse noise and part of the white noise can be effectively filtered out. Secondly, an improved self-adaptive wavelet packet denoising technique is used to remove the residual white noise. Finally, useful vibration signals are obtained after the previous processing. Simulation signals and rotor experimental vibration signals were used to verify the performance of the method. Experiment results show that the method can not only effectively eliminate the mixed complex noises but also preserve the fault character details, which demonstrates that the proposed method outperforms the method based on the wavelet-domain median filter.

1. Introduction

It is the most direct and effective method in the fault diagnosis of rotating machinery to analyze the vibration signal and obtain the characteristic information of the running state of machinery [1, 2]. However, in the field measurement, due to the influence of electromagnetic interference and random noise of other equipment such as the motor and the data acquisition system, the final collected vibration signals are often polluted by different degrees of complex noise, and the useful signals carrying the characteristic information of mechanical operation state are submerged in the background noise. Therefore, how to separate the real mechanical vibration signal from the mixed signal is the primary task of fault diagnosis research. In recent years, many experts and scholars have carried out extensive research in this field [3–7].

After a long period of research, it is found that the vibration signal is a typical situation that is interfered by pulse noise and white noise [8, 9]. Because the pulse noise has the characteristics of large amplitude, short duration, and long time interval, the application of the median filter can effectively eliminate the pulse interference; while the spectrum width of the white noise is far greater than the bandwidth of the vibration signal of the rotor system, the use of wavelet denoising technology can filter out most of the Gaussian white noise. Therefore, a better denoising effect can be achieved by combining the median filter with wavelet denoising and setting it reasonably. However, at present, the signal denoising method based on the median filter does not adjust the filter window width adaptively according to the sampling frequency of the signal so that the processed signal will retain some noise or filter out some useful components [10]; the denoising method based on wavelet analysis, because it is unable to determine the optimal decomposition scale, threshold, and processing function, its results are greatly affected [11].

In order to improve the performance of the vibration signal denoising method, an improved signal denoising method is proposed in this paper. The method combines the median filter with window adaptive adjustment and wavelet packet denoising technology with adaptive adjustment of decomposition scale and threshold to filter out the impulse noise and white noise in the signal. Through the processing of the simulation signal and the vibration signal of the rotor

experimental platform, it can be seen that the impulse noise and the white noise in the noisy signal are obviously suppressed, and the signal-to-noise ratio is significantly improved, which proves the effectiveness and practicability of the method.

2. Denoising Theory

2.1. The Model of the Vibration Signal with Noise. The rotating machinery with the rotor-bearing system as its core component plays an important role in electric power, metallurgy, petrochemical, and other industries [8, 12]. However, in the production site of these industries, the environment is generally complex, and there is a common situation of multiunit joint operation. If the state monitoring and fault diagnosis of the key equipment are studied, the collected signals will contain a lot of noise. Therefore, the rotor vibration signal with noise can be expressed as

$$x(t) = s(t) + n(t). \tag{1}$$

In the formula, $s(t)$ is the vibration signal of the rotor; $n(t)$ is the environmental noise, one of which is mainly composed of impulse noise and white noise.

2.2. The Method of the Median Filter. Median filter is a kind of nonlinear filter technology, which has good edge-preserving characteristics and the ability of suppressing impulse noise [8]. This method is essentially a window filter. The filtering operation is to scan the sample data by sliding a fixed length window and replace the data in the center of the current window with the median of the data in the window. With the end of the window moving, the filtering process of the whole sample signal will be completed. Since only one dimension signal is involved in vibration signal analysis in this paper, only one dimension discrete median filtering principle is discussed. Let the discrete sampling sequence of signal $x(t)$ be $x(n)$ ($n = 1, 2, 3, \ldots, n$); take the filtering window with the length of $L_d = 2d + 1$ (D is a positive integer) to conduct median filtering for this signal sequence. At the n th time, the data column in the window is represented as $W_d = \{x(n-d), \ldots, x(n), \ldots, x(n-d)\}$, and $n - d \geq 1$, $n + d \leq N$. The number of $2d + 1$ in W_d is arranged in the order of small to large, and then the intermediate value $y(n)$ is taken to replace the original $x(n)$, that is, the filtering task of a data point of the signal is completed. The mathematical expression of this process is

$$y(n) = \text{Med}[x(n-d), \ldots, x(n), \ldots, x(n-d)], \tag{2}$$

where $\text{Med}[\cdot]$ is the median of all numbers in the window.

The principle of median filter is simple, it is easy to realize its algorithm program by computer, and the impulse noise under half-window width can be eliminated basically. Therefore, as long as the appropriate window width is set, the median filter can effectively reduce the impulse noise in the vibration signal, but because of the characteristics of the filtering method itself, it cannot filter out the white noise.

2.3. Wavelet Packet Threshold Denoising Theory. Because wavelet analysis has good time-frequency analysis ability and multiresolution characteristics, it is especially suitable for processing nonstationary signals. Wavelet packet analysis is a more precise analysis method than wavelet analysis [13]. It decomposes the scale space and wavelet space at the same time, with higher time-frequency resolution. According to the characteristics of the decomposed signal, it selects the appropriate wavelet packet basis, especially suitable for processing complex signals, so wavelet packet is more widely used.

Wavelet packet threshold denoising method is developed on the basis of the wavelet threshold denoising method. Its implementation process can be divided into three steps [14]: (1) according to the set rules, multilayer wavelet packet decomposition is carried out to obtain all wavelet packet coefficients, and then the optimal wavelet packet basis is selected. (2) Set the appropriate threshold value, set the wavelet packet coefficients which are less than the threshold value to zero through the threshold function, and then keep the other wavelet packet coefficients unchanged or attenuate to a certain extent. (3) Reconstruct the signal with the processed wavelet packet coefficients. In this process, how to select wavelet packet basis, determine threshold, and threshold processing function are the three most difficult problems to solve and also three main factors affecting the denoising effect. Therefore, in-depth study of these three problems is very important to improve the denoising performance.

There are infinite orthonormal bases in the $L^2(R)$ space, which can be selected from wavelet base. For a specific problem $x(t) \in L^2(R)$, we need to define a cost function $M(x)$ to evaluate the most suitable wavelet packet basis. This function is usually defined as a real function about the sequence, which can reflect the concentration of wavelet coefficients and satisfy additivity. At present, the most common cost function is Shannon weaker information entropy of the sequence:

$$M(x) = -\sum_j p_j \log_2 p_j, \quad p_j = \frac{|x_j|^2}{\|x\|^2}, \tag{3}$$

where j is the decomposition scale. According to the size of $M(x)$ on different decomposition scales, the optimal wavelet packet basis can be determined.

Threshold denoising algorithm was proposed by Donoho [15, 16]. The algorithm determines the threshold value and threshold processing function according to the distribution of wavelet coefficients of the signal and noise on each decomposition scale. Its threshold value is selected as

$$T = \sigma\sqrt{2\ln N}, \tag{4}$$

where σ is the standard deviation of the noise and N is the length of the signal.

In the case of orthogonal wavelet packet decomposition, Donoho noise reduction theory gives two forms of hard threshold function and soft threshold function for threshold processing. The hard threshold method can be expressed as

$$\widehat{w}_{j,k} = \begin{cases} 0, & |w_{j,k}| < T, \\ w_{j,k}, & |w_{j,k}| \geq T. \end{cases} \qquad (5)$$

The soft threshold method can be expressed as

$$\widehat{w}_{j,k} = \begin{cases} 0, & |w_{j,k}| < T, \\ \text{sgn}(w_{j,k})(|w_{j,k}| - T), & |w_{j,k}| \geq T, \end{cases} \qquad (6)$$

where $w_{j,k}$ is the wavelet packet decomposition coefficient, $\widehat{w}_{j,k}$ is the wavelet packet subband coefficient estimated by thresholding; j is the scale index, k is the translation index, T is the threshold, and $\text{sgn}(\cdot)$ is the symbol function.

The above theory is the core content of the wavelet packet threshold denoising method, so improving formulas (4)–(6) to remove noise and retain useful signals is an important way to improve the denoising performance of this method, and it is also the research work to be carried out.

3. Improved Method of Vibration Signal Denoising

In order to reduce the impulse noise and white noise in the vibration signal at the same time, the denoising method which combines the median filter and wavelet threshold denoising has achieved a better effect [8, 17]. However, the median filter does not give a specific method of how to determine the window width, and wavelet denoising also has defects in the selection of threshold and threshold function, which make the denoised signal cannot achieve the best processing effect. In order to solve this problem, this paper proposes a new method combining the improved median filter and wavelet packet threshold denoising, which is used to eliminate the impulse noise and white noise in the rotor vibration signal.

3.1. Improved Method of the Median Filter. The key problem of the median filtering method is to determine the filter window width according to the signal characteristics. On the one hand, the window should not be too wide or the details of useful signals will be lost; on the other hand, the window should not be too narrow or too much impulse noise will remain. In order to filter out the impulse noise without losing useful signal, the window width should be 2 times of the pulse width. If the sampling interval of the vibration signal is T_s and the duration of the impulse noise is L_s, the reasonable window width L_d can be expressed as follows:

$$L_d = 2L_s F_s, \qquad (7)$$

where $F_s = 1/T$ is the sampling frequency. According to Li et al. [18], the impulse noise is mainly continuous $(5.36 \pm 2.48) \times 10^{-4}$ s. For the convenience of calculation and without loss of the useful signal, $L_s = 5.00 \times 10^{-4}$ s is taken. It can be seen from equation (7) that the window width is adaptively adjusted with F_s, which is more

conducive to eliminating the impulse noise and retaining useful signals.

3.2. Improved Method of Wavelet Packet Threshold Denoising. After the wavelet packet transform, the energy of the real signal is concentrated on the finite wavelet packet coefficients, and most of the wavelet packet coefficients are close to zero. However, after the white noise transform, it is still white noise, and the energy is evenly distributed on all the wavelet packet coefficients, and with the increase of the decomposition scale, the wavelet packet coefficients decrease rapidly. Therefore, the threshold method can reduce the noise pollution. However, equation (4) represents the unified threshold value, which is not effective in practical application and will produce overkill phenomenon. Based on this, this paper adopts a scale-based adaptive threshold, which can be expressed as [19]

$$T_{j,k} = \sigma_{j,k} \sqrt{2 \ln N}. \qquad (8)$$

The noise standard deviation $\sigma_{j,k}$ is estimated by the following empirical formula:

$$\sigma_{j,k} = \frac{1}{0.6745} \frac{1}{N} \sum_{k=1}^{N} |w_{j,k}|, \qquad (9)$$

where $w_{j,k}$ is the wavelet packet coefficient of the k-th subband of the j-th layer and N is the signal length.

For the threshold processing function, the hard threshold method keeps the coefficients larger than the threshold completely and sets the coefficients smaller than the threshold to zero. The soft threshold method sets all the coefficients smaller than the threshold value to zero and subtracts the threshold value from the coefficients larger than the threshold value to try to retain more signal components. In comparison, the soft threshold method has better denoising effect, but because the coefficients larger than the threshold are reduced to zero, the reconstructed signal characteristics will be weakened, and important feature information may be lost. Therefore, on the basis of formula (6), we improve it so that the coefficient shrinkage greater than the threshold value should not be too large, so we use the following formula to threshold:

$$\widehat{w}_{j,k} = \begin{cases} 0, & |w_{j,k}| < T, \\ \text{sgn}(w_{j,k})(|w_{j,k}|^3 - T^3)^{1/3}, & |w_{j,k}| \geq T. \end{cases} \qquad (10)$$

The presented threshold function is a compromise between the hard threshold method and the soft threshold method, which can overcome their shortcomings.

3.3. Improved Vibration Signal Denoising Method. Based on the above theoretical analysis, the specific implementation process of this denoising method can be designed. The whole algorithm steps are described as follows:

Step 1: according to the sampling frequency F_s of the signal, calculate the window width L_d of the median filter with equation (7).

Step 2: after the window width L_d is obtained, the noise signal is median filtered by formula (2)

Step 3: carry out wavelet packet decomposition for the median filtered signal, and use the cost function $M(x)$ expressed in formula (3) as the judgment basis to determine whether the decomposition continues or not, so as to determine the optimal decomposition scale and the optimal wavelet packet base

Step 4: use the improved threshold value and the expression of threshold function to process the coefficient $w_{j,k}$ of each wavelet packet and get the estimated new coefficient $\widehat{w}_{j,k}$

Step 5: reconstruct the signal by the new coefficient $w_{j,k}$ after the threshold shrinkage on each scale to get the denoising signal $\widehat{s}(n)$, which is an estimate of the real vibration signal $s(n)$

4. Simulation and Experimental Analysis

The improved denoising method is used to filter the simulation signal and the vibration signal of the rotor test-bed, so as to verify its effectiveness. In the process of wavelet packet decomposition, "db5" is chosen as the wavelet basis function. In order to further compare the signal quality before and after denoising, a quantitative performance evaluation index (SNR) is introduced to evaluate the denoising effect, which is defined as follows [20]:

$$\text{SNR} = 10 \lg \frac{\sum_{i=1}^{N} x^2(i)}{\sum_{i=1}^{N} n^2(i)}, \tag{11}$$

where $x(i)$ is the real vibration signal, $n(i)$ is the noise component added into the real signal, and N is the signal length.

4.1. Simulation Analysis. According to the vibration characteristics of the imbalance fault of the rotating machinery, the periodic vibration of the rotor with the rotation frequency $f_n = n/60$, 2 times and 3 times f_n, will be excited. If the noise component in the mixed vibration signal is $n(t)$, the simulation signal is constructed as follows:

$$x(t) = 3 \sin\left[2\pi f_n t\right] + 2 \sin\left[2\pi (2f_n)t\right] \\ + \sin\left[2\pi (3f_n)t\right] + n(t). \tag{12}$$

Take the rotor speed $n = 2800\,\text{r/min}\,(f_n = 100\,\text{Hz})$, sampling frequency $F_s = 5000\,\text{Hz}$, and sampling point 1024. The time-domain waveform of the real vibration signal of the rotor in the imbalance fault state is shown in Figure 1, and the signal state with the pulse noise and white noise is shown in Figure 2. The wavelet-domain median filtering method

and the method in this paper are, respectively, applied to denoise the noisy signal. The waveform of the denoised signal is shown in Figures 3 and 4. Table 1 lists the signal-to-noise ratio of the original signal and the noise reduced signal.

It can be seen from Figure 2 that the fault features of the mixed signal are almost completely submerged due to the large amount of noise, while the signals in Figures 3 and 4 after noise elimination clearly and accurately reflect the fault features of rotor imbalance, but compared with Figure 1, it can be found that Figure 4 is closer to the real fault vibration signal than Figure 3.

Therefore, compared with the wavelet-domain median filtering method, the method in this paper has better application effect in both eliminating the noise and protecting the details of the fault signal. The magnitude of the signal-to-noise ratio of the two noise reduction methods in Table 1 also fully proves this.

4.2. Signal Denoising of Rotating Machinery. The laboratory rotor system can be used to simulate several typical faults of rotating machinery and collect vibration signals needed for fault diagnosis and research. Figure 5 shows the experimental double-span rotor system. The front and rear span rotors are supported by sliding bearings. The couplings between the two rotors and between the rotor and the motor are all flexibly connected. The eddy current sensor probes are arranged in a group of two perpendicular to each other, which are installed near the journal and around the disk with obvious vibration and easy to obtain signals. The single sensor at the end of the rotor is used to measure the real-time speed of the rotor. Figure 6 shows the time-frequency waveform of the original signal collected under the unbalanced fault condition of rotating speed $n = 3,000\,\text{r/min}$. It can be seen from Figure 6 that the real vibration signal of the rotor is seriously polluted by the noise, and the characteristics of the frequency domain are difficult to reflect the running state of the rotor. The wavelet-domain median filtering method and the method in this paper are used to process the rotor x-axis and y-axis sampling signals, respectively, and the denoising results are shown in Figures 7 and 8.

By comparing Figure 6 with Figures 7 and 8, it is found that, after denoising, the noise component in the original signal is obviously eliminated, the running state characteristics of the rotor are clearly visible, and the frequency-domain characteristics conform to the situation when the rotor is unbalanced. Compared with the median filtering method in the wavelet domain, the improved method has obvious advantages in the process of filtering impulse noise and white noise, and most of the noise in the signal is eliminated. This is mainly because this method can adaptively adjust a series of filtering parameters according to the sampling frequency and signal characteristics, so as to achieve better filtering effect; the time-frequency waveform of the signal in Figures 7 and 8 fully shows the effectiveness of the method.
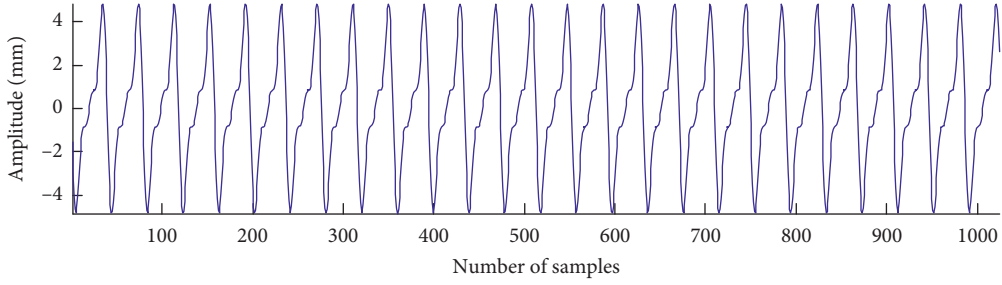
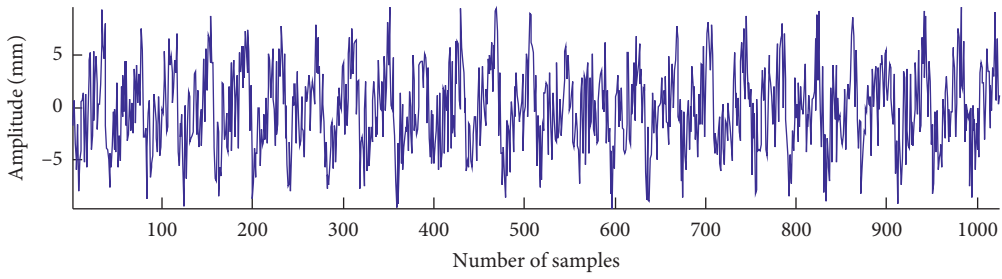FIGURE 1: The vibration signal of rotor imbalance.

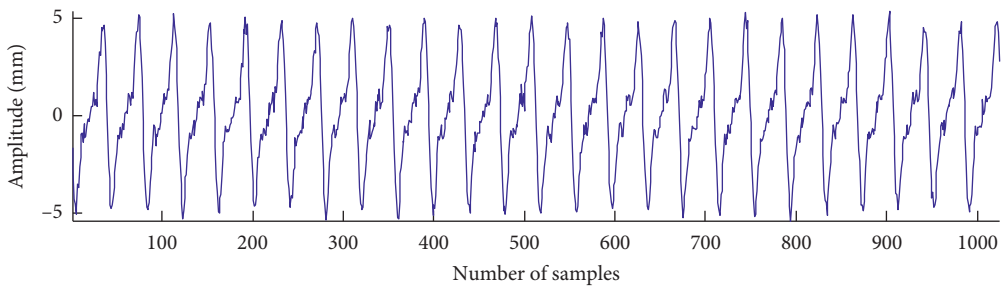FIGURE 2: The complex signal with heavy noise.

FIGURE 3: The signal processed by the wavelet and median filter.
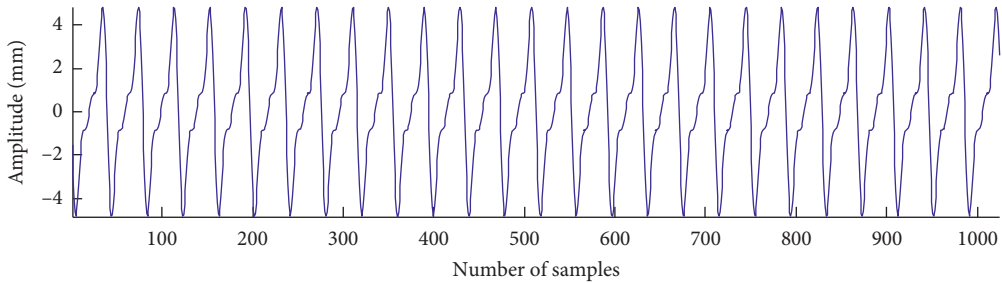

FIGURE 4: The signal processed by the improved method.

TABLE 1: The SNRs of the simulation signal before and after denoising (dB).

Sampling signal (including noise)	The signal after noise reduction	
	The wavelet and median filter	Improved method
0.8611	10.9968	13.8936
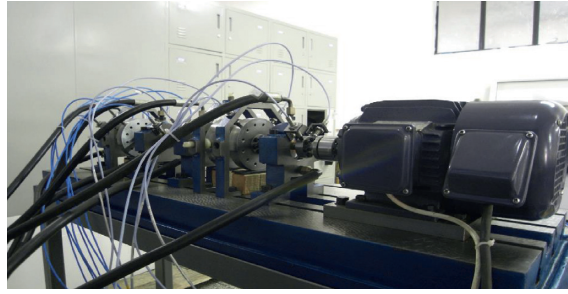
Figure 5: The two-span rotor-bearing rig.
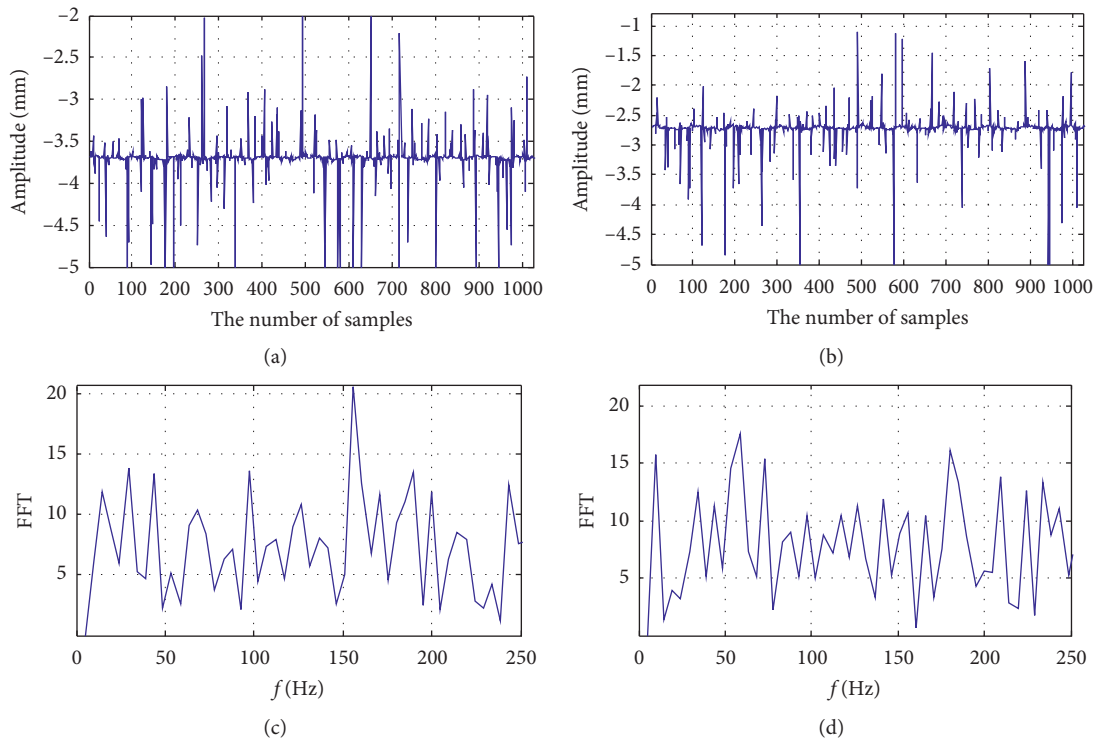

(a)

(b)

(c)

(d)

Figure 6: Time-domain and frequency-domain waveforms of rotor vibration signals.

(a)

(b)

Figure 7: Continued.
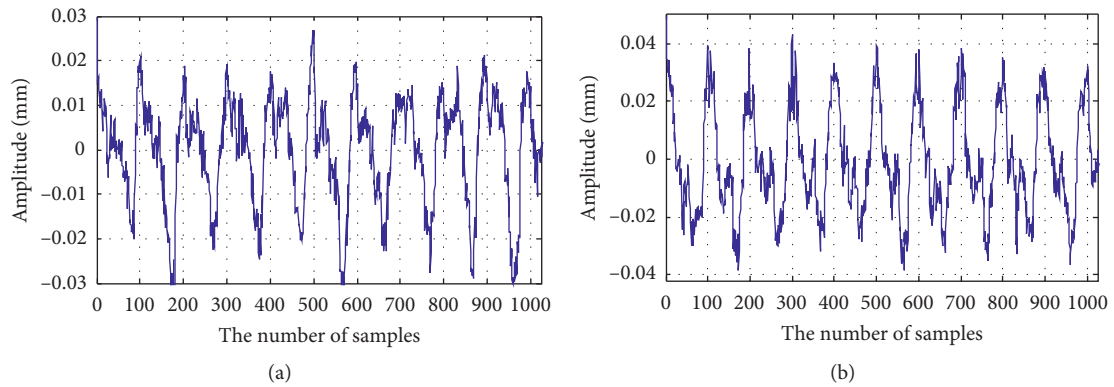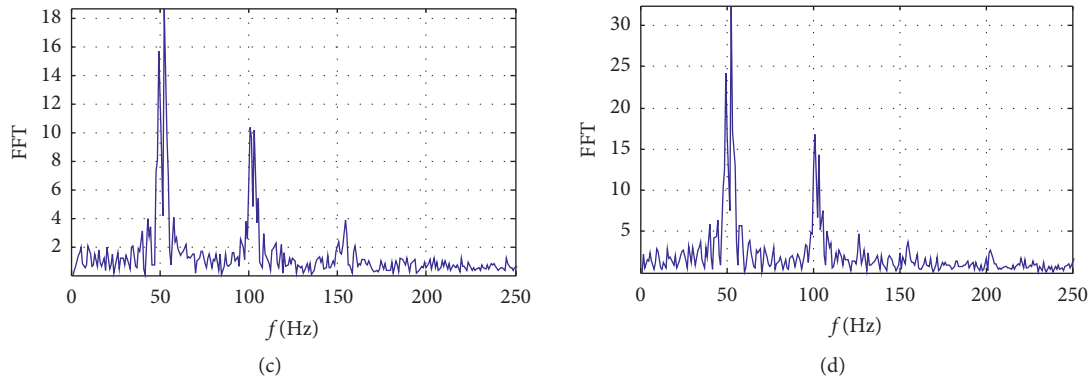
(c)

(d)

FIGURE 7: Time-domain and frequency-domain waveforms of the signals after denoising using MF-WD.
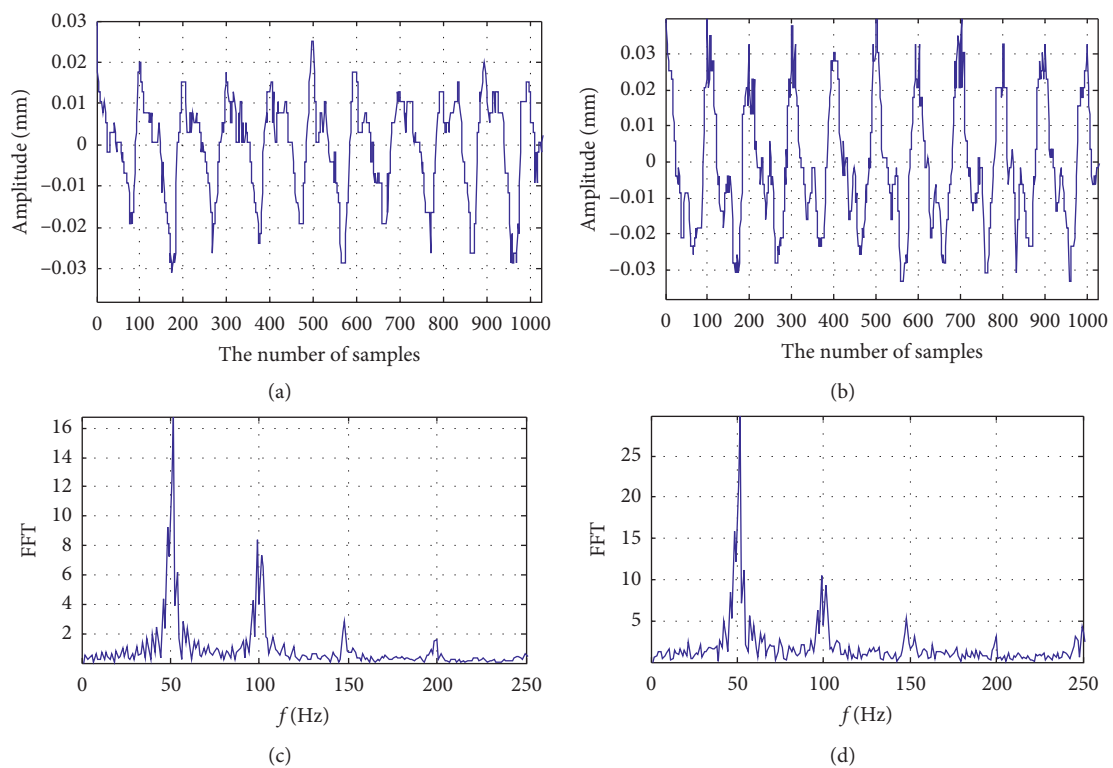

(a)

(b)

(c)

(d)

FIGURE 8: Time-domain and frequency-domain waveforms of the signals after denoising using the proposed method.

5. Conclusions

The rotor vibration signal is interfered by the pulse noise and white noise, which will seriously affect its analysis and processing effect. The improved median filtering method and the wavelet packet threshold denoising method can be used to eliminate two kinds of common noises. Simulation and experimental research show that the improved noise reduction method of the rotor vibration signal can effectively eliminate the interference of the mixed complex noise on the vibration signal while retaining the details of the fault signal. By analysing and comparing the time-frequency waveform and signal-to-noise ratio of the denoised signal, it is further proved that the denoising method proposed in this paper is superior to the general wavelet-domain median filtering denoising method.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

Acknowledgments

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