

Research Article

Frequency Response Function-Based Finite Element Model Updating Using Extreme Learning Machine Model

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A frequency response function- (FRF-) based surrogate model for finite element model updating (FEMU) is presented in this paper. Extreme learning machine (ELM) is introduced as the surrogate model of the finite element model (FEM) to construct the relationship between updating parameters and structural responses. To further improve the generalization ability, the input weights and biases of ELM are optimized by Lévy flight trajectory-based whale optimization algorithm (LWOA). Then, LWOA is also applied to obtain the best updating results, where the objective function is defined by the difference between analytical FRF data and experimental data. Finally, a plane truss is used to demonstrate the performance of the proposed method. The results show that, compared with second-order response surface (RS), radial basis function (RBF), traditional ELM, and other optimized ELM, a LWOA-ELM model has higher prediction accuracy. After updating, the FRF data and frequencies have a significant match to the experimental model. The proposed FEMU method is feasible.

1. Introduction

Due to the capacity for structural identification and health monitoring, finite element method has attracted much more attentions in the past few years. However, an initial finite element model (FEM) cannot reflect the actual structure precisely, because of simplifications and idealizations (ideal boundary conditions, material properties, etc.) while constructing the FEM [1–3]. In order to be used for further analysis, the initial FEM should be updated to minimize the error between the analytical responses and the experimental ones, which is called finite element model updating (FEMU) [4]. Several methods for FEMU have been developed. Among these methods, vibration-based techniques are widely used. This kind of technique can be classified into two main categories: the methods based on modal data and the ones based on frequency response function (FRF) data [5–9]. For the former, natural frequencies and mode shapes are introduced to define the objective function [10]. For the latter, FRF data provide enough response information directly without modal analysis, which can increase the

accuracy of model updating. The FRF-based methods have been gradually applied in FEMU [11, 12].

In addition, the model updating requires dozens of iterations to compute the analytical responses, which is time-consuming. The surrogate models, such as response surface (RS), Kriging model, radial basis function (RBF), neural network (NN), and support vector machine (SVM), are widely used to construct the relationship between the input parameters and output responses to replace the original FEM. Marwala [13] introduced multilayer perceptron (a neural network) as the surrogate model to update an H-type structure. Ren and Chen [14] presented a quadratic polynomial response surface-based method. A simply supported beam and a concrete bridge demonstrated the applicability and efficiency of the proposed method. Wang et al. [15] applied the Kriging model in a honeycomb Sandwich beam to reduce the solving time. Zhang and Hou [16] presented a SVM based response surface model for vehicle chassis components. With good generalization performance and fast learning speed, a learning method called extreme learning machine (ELM) for single-hidden layer feedforward

neural networks (SLFNs) has received many applications in data analysis, protein sequence classification, feature selection and clustering learning, etc. [17–19]. Although ELM does not need to adjust so many parameters during training process, generalization performance of ELM could be affected by the input weights and the biases. To improve the performance, several methods, such as genetic algorithm (GA) and particle swarm optimization (PSO), have been proposed to optimize the input weights and biases of ELM [20, 21].

Besides computational cost issue, the efficiency of the algorithms is another crucial step. Model updating is an optimization problem to minimize the difference between the analytical responses and experimental ones. Gradient-based techniques have a wide range of applications. However, the methods may be computationally expensive and are not feasible to achieve complicated engineering problems [22]. Intelligent algorithms (GA, PSO, etc.) can be used to reduce the difference between the FEM and the measured behavior [23]. Guo et al. [24] conducted the updating procedure by the hybrid genetic/pattern-search optimization algorithm. Boulkaibet et al. [25] adopted fish school search algorithm (FSS) and PSO to update two real structures. Whale optimization algorithm (WOA), which mimics the hunting behavior of humpback whales in nature, is a swarm-based optimization algorithm introduced by Mirjalili [26]. An improved whale optimization algorithm (WOA) based on Lévy flight (LWOA) is applied to infinite systems identification and classic engineering design problems [27, 28].

To the authors' best knowledge, ELM and LWOA have not been explored to solve the FEMU problem. To expand their applications and improve the efficiency of FEMU, in this paper, LWOA is firstly used to optimize input weights and the biases of ELM. Then, a LWOA-ELM based surrogate model is established by the updating parameters and corresponding FRF data to replace the initial FEM. The objective function is established using the residual between the analytical values and the experimental ones. Finally, the LWOA is employed to search the best updating parameters of FEMU.

The rest of the paper is organized as follows. Section 2 presents the FRF and the objective function. The concepts of ELM, LWOA, and optimized ELM are provided in Section 3. In Section 4, an example is given to illustrate the FEMU based on the new method. The paper is concluded in Section 5.

2. Frequency Response Function and Objective Function

The motion equation of structure with structural damping can be written as

$$\mathbf{M}\ddot{x} + \mathbf{C}\dot{x} + \mathbf{K}x = \mathbf{F}, \quad (1)$$

where \mathbf{M} , \mathbf{C} , and \mathbf{K} are $n \times n$ matrices of mass, damping, and stiffness, respectively; x , \dot{x} , and \ddot{x} are $n \times 1$ vector of the

structural displacement, velocity, and acceleration; and \mathbf{F} is a $n \times 1$ vector of applied force.

Assuming a harmonic input, the corresponding analytical frequency response matrix is given by

$$\mathbf{H}(\omega) = (-\omega^2 \mathbf{M} + i\omega \mathbf{C} + \mathbf{K})^{-1}, \quad (2)$$

where ω is the frequency of excitation.

The acceleration FRF (AFRF) can be calculated by

$$\mathbf{H}(\omega) = \left(\mathbf{M} - i \frac{\mathbf{C}}{\omega} - \frac{\mathbf{K}}{\omega^2} \right)^{-1}. \quad (3)$$

FEMU utilizes information from the actual structural responses to update the parameters of the FEM. Model updating problem can be formulated in the following form:

$$\min F(x), \text{ s.t } x_{\text{lb}} < x_i < x_{\text{ub}}, \quad i = 1, 2, \dots, m, \quad (4)$$

where $F(x)$ is the objective function; x_i is the i th updating parameter; and x_{lb} and x_{ub} are the lower and upper bound of updating parameters, respectively.

Based on the AFRF data, the research sets the residual between the FEM data and experimental ones as the objective function. It can be described as

$$F = \sum_{i=1}^n |y_t(\omega_i) - y_a(\omega_i)|, \quad (5)$$

$$y(\omega_i) = \log_{10} \mathbf{H}(\omega_i), \quad (6)$$

where n is the number of selected frequency; ω_i is the selected frequency points; $y_t(\omega_i)$ is the experimental response at ω_i ; $y_a(\omega_i)$ indicates the corresponding response of the FEM. Considering the peak values at specified frequencies, which may overwhelm the influence of the smaller values, the logarithmic ($\log_{10} \mathbf{H}(\omega_i)$) is adopted.

3. Basic Theory

3.1. Extreme Learning Machine. Extreme learning machine (ELM) for single-hidden layer feedforward neural networks (SLFNs) proposed by Huang has been used extensively and successfully in different fields with fast learning speed and good generalization capability. Just like the traditional feedforward network algorithm, the ELM consists of three layers of neurons: an input layer, a hidden layer, and an output layer (as shown in Figure 1). \mathbf{w} is the input weight vector between the input nodes and the hidden nodes; β is the output weight vector between the hidden nodes and the output ones.

Suppose the N training dataset $(\mathbf{x}_i, \mathbf{t}_i)$, $\mathbf{x}_i = [x_{i1}, x_{i2}, \dots, x_{in}]^T \in \mathbf{R}^n$, $\mathbf{t}_i = [t_{i1}, t_{i2}, \dots, t_{im}]^T \in \mathbf{R}^m$ where \mathbf{x}_i is the input vector and \mathbf{t}_i is the output vector. The output is given by

$$\sum_{i=1}^L \beta_i g(\mathbf{w}_i \mathbf{x}_j + b_i) = \mathbf{t}_j, \quad j = 1, 2, \dots, N. \quad (7)$$

Equation (7) can be written as

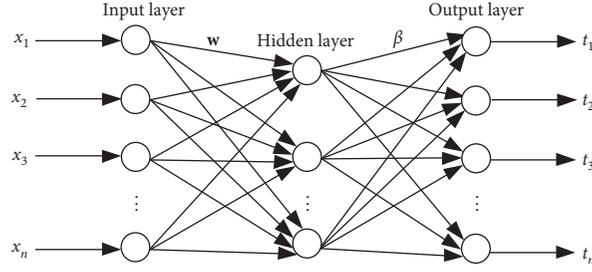


FIGURE 1: Extreme learning machine.

$$\mathbf{H}\beta = \mathbf{T}. \quad (8)$$

The hidden layer output matrix \mathbf{H} is given by

$$\mathbf{H} = \begin{pmatrix} g(\mathbf{w}_1 \mathbf{x}_1 + b_1) & \cdots & g(\mathbf{w}_L \mathbf{x}_1 + b_L) \\ \vdots & \ddots & \vdots \\ g(\mathbf{w}_1 \mathbf{x}_N + b_1) & \cdots & g(\mathbf{w}_L \mathbf{x}_N + b_L) \end{pmatrix}_{N \times L}, \quad (9)$$

$$\beta = \begin{pmatrix} \beta_1^T \\ \vdots \\ \beta_L^T \end{pmatrix}_{L \times m}, \quad (10)$$

$$\mathbf{T} = \begin{pmatrix} \mathbf{t}_1^T \\ \vdots \\ \mathbf{t}_N^T \end{pmatrix}_{N \times m},$$

where L denotes the number of hidden nodes; b_i is the bias; $g(\cdot)$ is the activation function. The minimum norm least squares of equation (8) can be calculated by

$$\hat{\beta} = \mathbf{H}^+ \mathbf{T}, \quad (11)$$

where \mathbf{H}^+ is the Moore–Penrose generalized inverse of \mathbf{H} .

Given the training samples, active function $g(\cdot)$, and hidden nodes number L , the specific process of ELM can be divided into three steps:

Step 1: randomly assign input weight vector \mathbf{w} and biases b

Step 2: calculate the hidden layer output matrix \mathbf{H}

Step 3: calculate the output weight $\hat{\beta}$

3.2. Lévy Flight-Based Whale Optimization Algorithm.

Whale optimization algorithm (WOA) is a new meta-heuristic algorithm, which simulates the social behavior of humpback whales. The basic WOA is divided into three operations: encircling, Bubble-net attacking behavior (exploitation phase), and search for prey (exploration period), where Bubble-net attacking consists of two approaches: shrinking encircling mechanism and spiral. Due to a lack of population diversity, WOA is easy to fall into local optimum. Lévy flight trajectory, which is the random walk step drawn from a Lévy distribution, is adapted to prevent WOA from local optimum and enhance the solution accuracy. The procedure of the LWOA is shown in Algorithm 1.

Firstly, humpback whales try to update their position towards the best search agent. The behavior is given as follows:

$$\mathbf{X}(t+1) = \mathbf{X}^*(t) - \mathbf{A} \cdot \mathbf{D}, \quad (12)$$

$$\mathbf{D} = |\mathbf{C} \cdot \mathbf{X}^*(t) - \mathbf{X}(t)|, \quad (13)$$

where $\mathbf{A} = 2a \cdot \mathbf{r} - a$; $a = 2 - 2(t/t_{\max})$; $\mathbf{C} = 2 \cdot \mathbf{r}$; \mathbf{r} is a random vector in $[0, 1]$; t is the current iteration; t_{\max} is the maximum iteration number; $\mathbf{X}(t)$ is the position vector; $\mathbf{X}^*(t)$ is the position vector of the best solution obtained so far.

Then, the humpback whales choose shrinking encircling mechanism or the spiral model with a probability of 50% to update the position, according to the following equation:

$$\mathbf{X}(t+1) = \begin{cases} \mathbf{X}^*(t) - \mathbf{A} \cdot \mathbf{D}, & \text{if } p < 0.5, \\ \mathbf{D}' \cdot e^{bl} \cdot \cos(2\pi l) + \mathbf{X}^*(t), & \text{if } p \geq 0.5, \end{cases} \quad (14)$$

where $\mathbf{D}' = |\mathbf{X}^*(t) - \mathbf{X}(t)|$ indicates the distance between the best solution obtained so far and the current position; b is a constant for defining the shape of the logarithmic spiral; l is a number in $[-1, 1]$; p is a random number in $[0, 1]$.

And then, humpback whales search randomly to update the position in the exploration phase. The mathematical model is as follows:

$$\mathbf{X}(t+1) = \mathbf{X}_r(t) - \mathbf{A} \cdot \mathbf{D}, \quad (15)$$

$$\mathbf{D} = |\mathbf{C} \cdot \mathbf{X}_r(t) - \mathbf{X}(t)|, \quad (16)$$

where $\mathbf{X}_r(t)$ is a random position vector chosen from the current population.

$|\mathbf{A}|$ is the parameter to choose mechanism to update position. If $|\mathbf{A}| \geq 1$, a random search agent is performed by equations (15) and (16). And if $|\mathbf{A}| < 1$, humpback whales update the position of the current search agent according to equations (12) and (13).

Finally, the Lévy flight trajectory is employed to balance the exploitation and exploration of WOA. The mathematical model can be formulated by

$$\mathbf{X}(t+1) = \mathbf{X}(t) + \mu \text{sign} \left[\text{rand} - \frac{1}{2} \right] \oplus \text{Levy}, \quad (17)$$

where $\text{Levy} \sim \mu/|v|^{1/\beta}$; μ and v obey normal distribution, $\mu \sim N(0, \sigma_\mu^2)$ and $v \sim N(0, \sigma_v^2)$; $\sigma_\mu = [\Gamma(1 + \beta) \times \sin(\pi \times$

$\beta/2)/\Gamma((1 + \beta)/2) \times \beta \times 2^{(\beta-1)/2}]^{1/\beta}$ and $\sigma_v = 1$; \oplus means entry-wise multiplications; β is an index.

3.3. Optimized ELM. The prediction performance of ELM model can be assessed by root mean square error (RMSE) criterion and R^2 criterion:

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{j=1}^N (y_j - \hat{y}_j)^2}, \quad (18)$$

$$R^2 = 1 - \frac{\sum_{j=1}^N (y_j - \hat{y}_j)^2}{\sum_{j=1}^N (y_j - \bar{y})^2}, \quad (19)$$

where N is the number of observations; y_j is the analytical value calculated by FEM; \hat{y}_j is the response value predicted by ELM; \bar{y} indicates the average value of y . A value of R^2 close to 1 and RMSE close to 0 are always preferred, which reflect high accuracy of ELM.

In order to further improve accuracy of updating results, more reliable ELM models need to be created. The input weights and the biases are randomly generated in the ELM, which affect the prediction accuracy of the model. LWOA is utilized to optimize the input weights and biases of ELM. The minimized RMSE obtained by training samples is taken as the fitness function. It can be expressed as

$$\min \text{RMSE} = \sqrt{\frac{1}{N} \sum_{j=1}^N (y_j - \hat{y}_j(\mathbf{w}, b))^2}. \quad (20)$$

In the LWOA, the input weights and the biases are considered to be the humpback whales. The dimension of search space is $2 \times L + L$, where L denotes the number of hidden nodes. The optimal parameters \mathbf{w} and b corresponding to the minimum fitness value are obtained.

4. Model Updating Based on ELM and LWOA

4.1. Model Updating Procedure. FEMU is an inverse problem to update the parameters by minimizing the discrepancy between the measured responses and the analytical ones from FEM. In this study, LWOA-ELM model is constructed to replace the FEM, and LWOA is also applied to minimize the objective function and obtain the results. The flowchart of model updating using LWOA-ELM and LWOA is shown in Figure 2. The main steps can be illustrated as follows:

Step 1: create the FEM of the structure by the ANSYS software.

Step 2: select updating parameters (modulus of elasticity and material density are selected as the updating parameters in this paper).

Step 3: construct the LWOA-ELM model by training samples, and check whether the accuracy criterion is satisfied ($R^2 > 0.9$, $\text{RMSE} < 1.5 \times 10^{-4}$) by the model.

Step 4: search the global minimum solution of equation (5) by LWOA, and check whether the convergence criterion (maximum iterations) is satisfied.

Step 5: obtain the model updating results.

4.2. Numerical Study. A plane structure with modulus of elasticity of the element $E = 210$ GPa, material density $\rho = 7850$ kg/m³ is used to test the proposed method for model updating. The initial FEM is established using ANSYS software. Figure 3 shows the FEM of the plane truss. The structure has 26 nodes and 49 finite elements. Each node (except 1 and 25) includes two translations DOFs. In this paper, nodes 24 and 19 are selected as the exciting and measuring points, respectively.

Modulus of elasticity and material density of the FEM are selected as the updating parameters. To simulate initial FEM value, increase the modulus of elasticity by 10%, and decrease density by 10%. The parameters are shown in Table 1.

Latin hypercube sampling (LHS) [29] is used to get the design points of updating parameters. Modulus of elasticity and material density change within $\pm 20\%$ of the FEM values. 100 random samples are obtained for ELM. The corresponding response data are generated by equation (3). Thus, ELM model is established based on the input variables (updating parameters) and corresponding outputs (AFRF data). According to literature [17] and cross validation method, the activation function is sigmoidal function, and the number of hidden nodes is set to 80. For verification, ELM model is compared with frequently used surrogate models, RBF model and second-order RS model [14]. RBF neural network is a three-layer feedforward network with the hidden layer containing RBF units. RS is a mathematical and statistical technique and often applied in regression analysis. The models are constructed based on the same sample points. RBF is conducted in MATLAB neural network toolbox. The parameter spread is set to 25 by trial and error to keep a stable and minimum value of RMSE. Each algorithm is simulated 20 independent runs in MATLAB software. The results are presented as the average of 20 runs. Comparison of ELM with RS and RBF model is shown in Figure 4. It can be seen that the prediction accuracy of ELM is better than RS and RBF in Figure 4. RMSE of ELM is 0.0012 and R^2 is 1, RMSE of RBF is 0.0161 and R^2 is 0.9989, and RMSE of RS is 0.0314 and R^2 is 0.9963, which indicate that ELM model builds an accurate relationship between updating parameters and responses.

To further improve the prediction ability of ELM model, the proposed LWOA-ELM model is constructed. Moreover, LWOA-ELM is compared with GA-ELM, PSO-ELM, and WOA-ELM. GA is a widely used optimization algorithm inspired from evolutionary process. PSO is a bio-inspired stochastic algorithm derived from biological entities. In order to make a fair comparison, the population size and the maximum iteration number are set to the same data. And other parameters are the best sets after trial and error. Finally, the population size is set to 20. The maximum iteration number is set to 100. Other parameters of these algorithms are listed in Table 2.

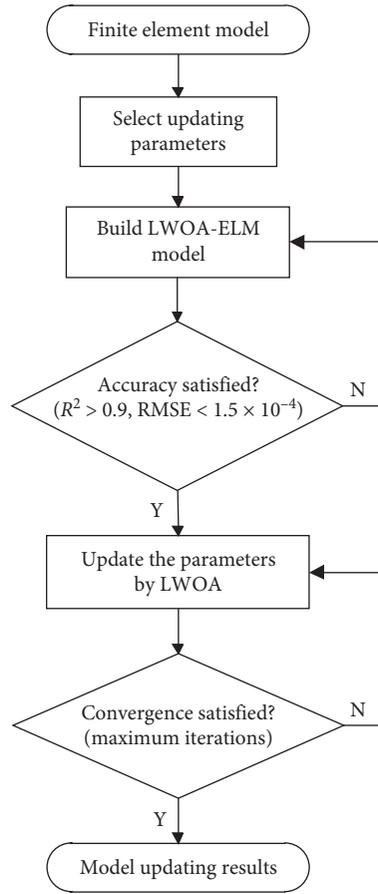


FIGURE 2: Flowchart of model updating.

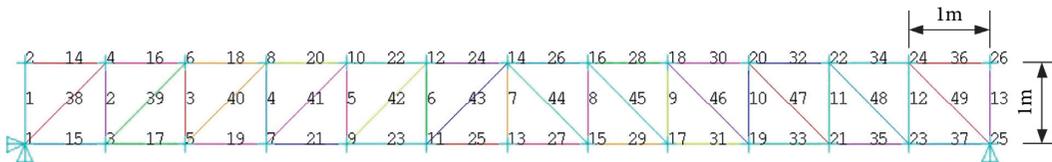


FIGURE 3: FEM of the plane truss.

The fitness convergence curves of GA-ELM, PSO-ELM, WOA-ELM, and LWOA-ELM are shown in Figure 5. From Figure 5, it is apparent that LWOA has a fast convergence than others. The smallest fitness value is LWOA. Figure 6 reflects RMSE of the four optimized ELM models.

As seen in Figure 6, the prediction accuracy of the four optimized ELM models is improved. LWOA-ELM is significantly better than GA-ELM, PSO-ELM, and WOA-ELM. The comparison results of the four algorithms are given in Table 3.

After building the LWOA-ELM model, LWOA is applied to find the best solution of the objective function of equation (5). At each time of iteration, the analytical values of FRF are predicted by LWOA-ELM instead of FEM. For the algorithm, the dimension is set as 2 (two updating parameters), and the maximum iteration number is set to 500. The population size is set to 50. The updated parameters

are listed in Table 4. The parameters are closer to experimental parameters, which prove the validity of this method.

Actually, the experimental responses usually contain noise. 5% random noise is added to experimental FRF. The $\log_{10}H(\omega_i)$ before and after updating compared with the experimental data is given in Figure 7. And Figure 8 shows the FRF of initial FEM, experimental, and updated model. As shown in Figures 7 and 8, the model has a good agreement with the experimental one. The results are given in Table 5.

From Tables 4 and 5, it can be observed that the noise affects the updating results. The maximum error is 0.86%. The error is still acceptable. To further illustrate the proposed method, the first five frequencies of the FEM, experimental, and updated model are shown in Table 6.

As presented in Table 6, the relative errors decrease from 10.55% to 0.038%. It is verified that the proposed method is

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Initialize relevant parameters
Generate a population  $\mathbf{X}$ , randomly. Calculate the fitness of each whale and get the best solution  $\mathbf{X}^*$ 
while ( $t < t_{\max}$ )
  for each search agent
    Update  $a$ ,  $A$ ,  $C$ ,  $l$  and  $p$ 
    if ( $p < 0.5$ )
      if ( $|A| < 1$ )
        Update the position of the current search agent by equation (12)
      else if ( $|A| \geq 1$ )
        Select a random search agent  $\mathbf{X}_r$ 
        Update the position of the current search agent by equation (15)
      end if
    else if ( $p \geq 0.5$ )
      Update the position of the current search agent by equation (14)
    end if
  end for
  for each search agent
    Update the position of the current search agent using Lévy flight by equation (17)
  end for
  Check if any search agent goes beyond the search and amend it
  Calculate the fitness of each whale
  Update  $\mathbf{X}^*$  if there is a better solution  $\mathbf{X}^*$ 
   $t = t + 1$ 
end while
Return  $\mathbf{X}^*$ 

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ALGORITHM 1: The Lévy flight based whale optimization algorithm (LWOA).

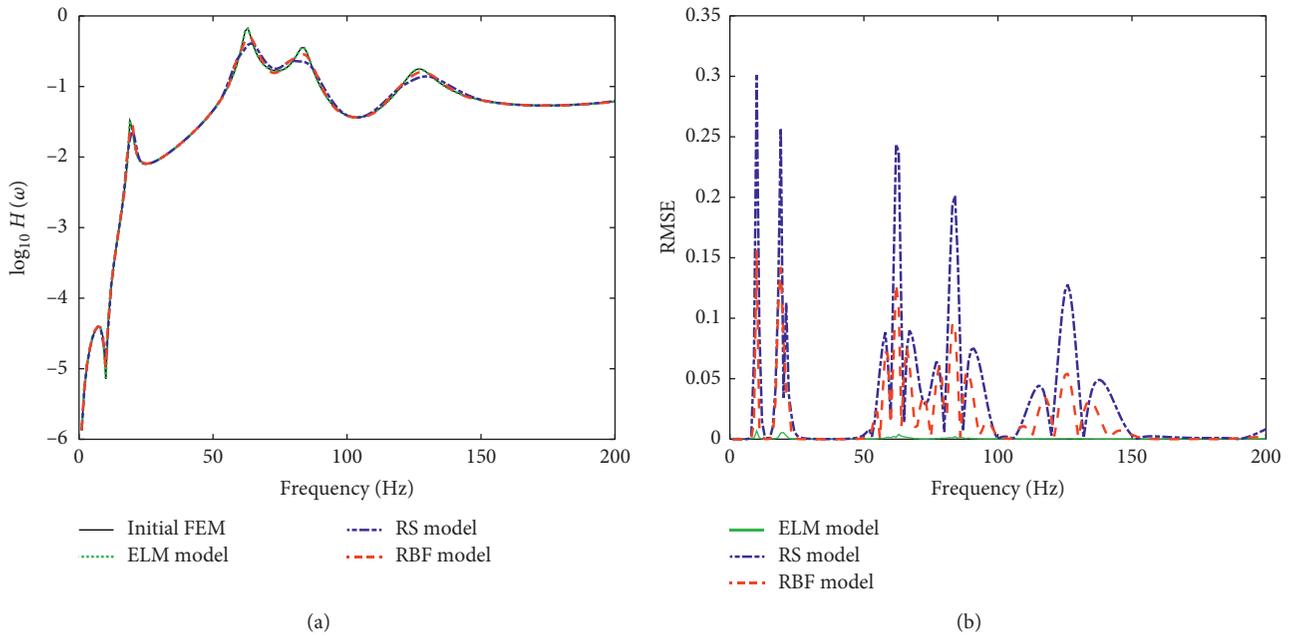


FIGURE 4: Comparison of ELM with RS and RBF model. (a) Initial FEM values and values predicted by ELM, RS, and RBF model. (b) RMSE of ELM, RS, and RBF model.

TABLE 1: Parameters of experimental model and FEM.

Updating parameters	Experimental value	Initial FEM value
Modulus of elasticity (GPa)	210	231
Material density (kg/m^3)	7850	7065

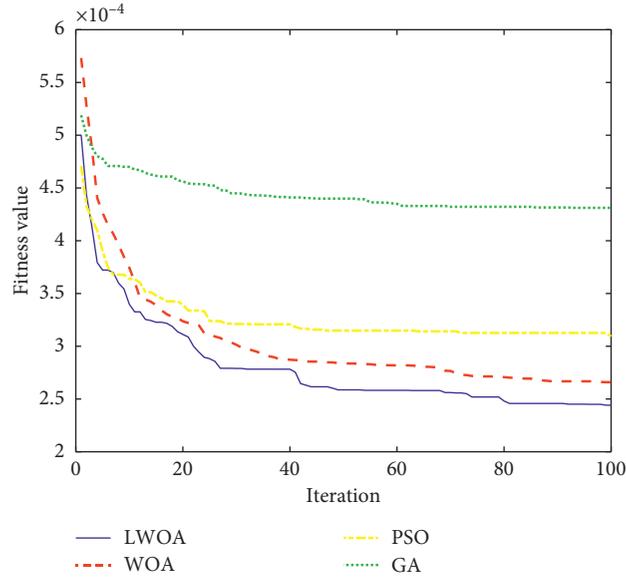


FIGURE 5: Convergence curves for LWOA, WOA, PSO, and GA.

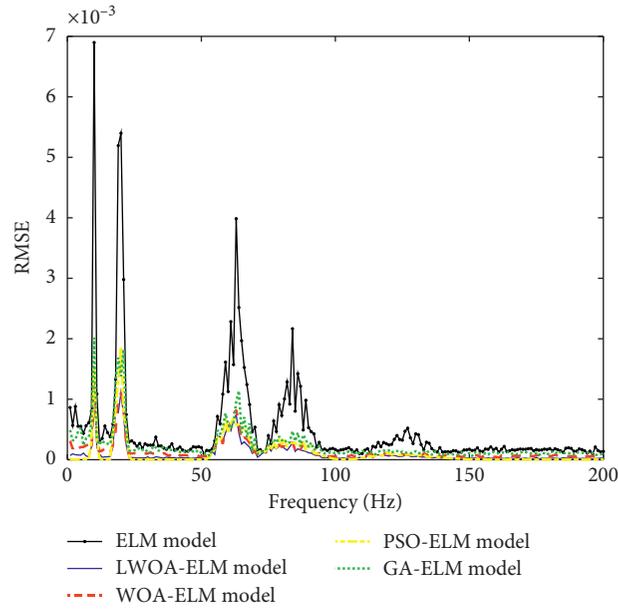


FIGURE 6: RMSE of the optimized ELM models.

TABLE 2: Parameters of algorithms.

Algorithm	Parameters	Values
GA	Crossover probability p_c	0.9
	Mutation probability p_m	0.1
PSO	Learning factors c_1	2
	Learning factors c_2	2

TABLE 3: Comparison results of the four algorithms.

Model	RMSE	R^2
GA-ELM	2.3252×10^{-4}	1
PSO-ELM	1.1490×10^{-4}	1
WOA-ELM	1.4250×10^{-4}	1
LWOA-ELM	1.0263×10^{-4}	1

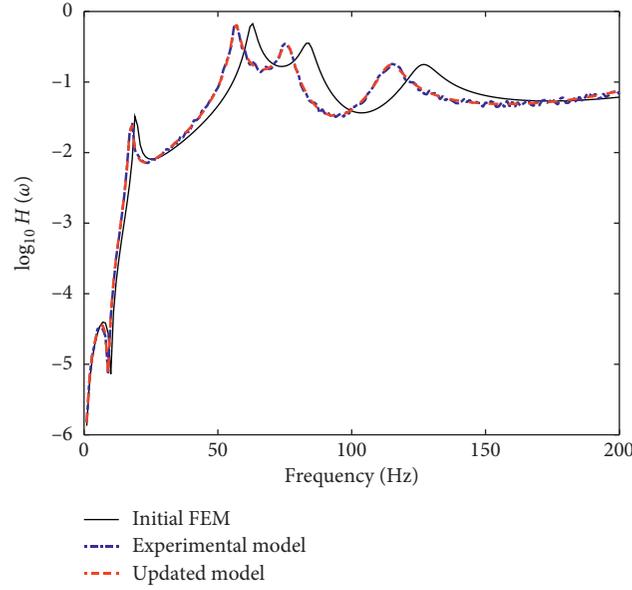


FIGURE 7: Result of FEMU.

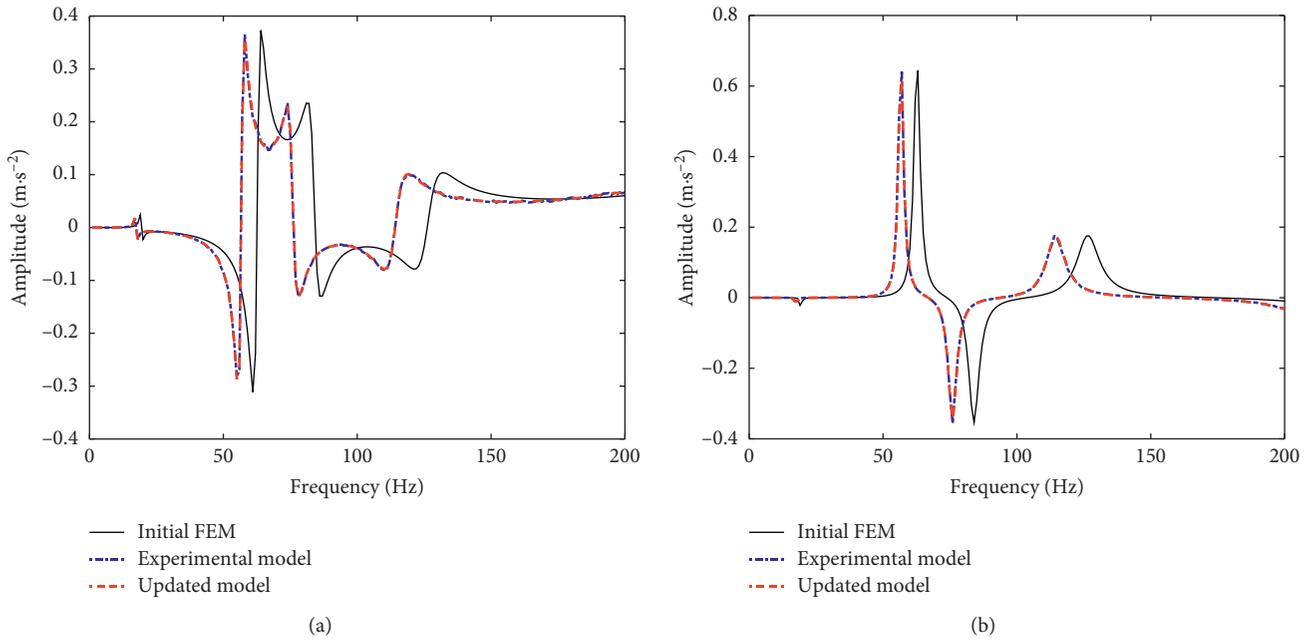


FIGURE 8: FRFs of initial FEM, experimental model, and updated model. (a) Values of the real part. (b) Values of the imaginary part.

TABLE 4: Values and errors of the updated parameters without noise.

Updating parameters	Experimental value	Initial value	Error (%)	ELM	Error (%)	LWOA-ELM	Error (%)
Modulus of elasticity (GPa)	210	231	10	210.0131	0.0063	210.0045	0.0021
Material density (kg/m ³)	7850	7065	-10	7851.2	0.0153	7850.4	0.0051

TABLE 5: Values and errors of the updated parameters with 5% noise.

Updating parameters	Experimental value	Initial value	Error (%)	LWOA-ELM	Error (%)
Modulus of elasticity (GPa)	210	231	10	211.8	0.86
Material density (kg/m ³)	7850	7065	-10	7911.2	0.78

TABLE 6: Natural frequencies of the FEM, experimental, and updated model.

Mode order	Initial FEM f_a (Hz)	Experimental model f_t (Hz)	Updated model f_u (Hz)	Error e_{at} (%)	Error e_{ut} (%)	Error reduction (%)
1	19.3290	17.4837	17.4904			
2	62.6328	56.6535	56.6753			
3	83.8904	75.8817	75.9109	10.55	0.038	99.63
4	126.4108	114.3429	114.3869			
5	195.6685	176.9864	177.0545			

Note: $e_{at} = |(f_a - f_t)/f_t| \times 100$; $e_{ut} = |(f_u - f_t)/f_t| \times 100$.

capable of updating the model, and the natural frequencies match the experimental model well.

5. Conclusion

A LWOA-ELM model based method for FEMU is proposed in this paper. LWOA is applied to optimize the input weights and biases of ELM. During the optimization process, LWOA-ELM model is introduced as the surrogate model to replace the initial FEM. The parameters are updated using LWOA, which obtain minimum of the objective function based on the AFRF data. The example results show that LWOA-ELM has better accuracy performance than that by the second-order RS, RBF, traditional ELM, GA-ELM, PSO-ELM, and WOA-ELM. With 5% noise, the errors of updated parameters are still less than 0.9%. The proposed method is suitable for FEMU. For future work, complex structure will be developed, and modal test will be performed.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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