Hindawi
Shock and Vibration
Volume 2020, Article ID 8816884, 7 pages
https://doi.org/10.1155/2020/8816884

Research Article

Study on the Measurement Method of the Crack Local Flexibility of the Beam Structure

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Received 2 July 2020; Revised 17 July 2020; Accepted 25 July 2020; Published 18 August 2020

Academic Editor: Jiawei Xiang

The crack which appears in the structure can be described by a local flexibility. With the occurrence and propagation of crack, the local flexibility will change. The change can effectively reflect the damage degree of the structure. In this paper, the measurement method of the crack local flexibility of the beam structure is presented. Firstly, a series of sample points are selected at the crack location and the possible value range of the crack local flexibility, and then these sample points are used as input parameters for the dynamic analysis of the beam structure. The vibration equation of beam structure is solved, and the frequency influence surface is drawn. In addition, the vibration signal of the beam is tested, and the first three order natural frequencies can be obtained. Thirdly, these frequencies measured are adopted to cut the natural frequency influence surfaces, and then the first three order natural frequency influence curves are drawn. The intersection points of these frequencies influence curves can indicate the crack local flexibility and the corresponding crack location. This method is suitable for measuring the local flexibility of crack with different shapes and types in the beam structure which have various cross sections.

1. Introduction

Structural damage is usually manifested firstly as the appearance of structural cracks and then expands under the long-term external load [1–3]. Crack propagation often leads to serious accidents. To ensure the safety and reliability of structures, researchers have developed many structural damage detection methods [4–7], such as acoustic emission, ray detection, and magnetic flux leakage detection. The crack which appears in the structure can be described by a local flexibility. With the occurrence and propagation of crack, the size of the flexibility will change. The crack local flexibility can effectively reflect the damage degree of the structure and has important theoretical and engineering significance in structural damage detection.

At present, the common crack models include the crack model based on the equivalent drop-section method [8], the crack model based on the principle of uniform crack beam [9, 10], and the crack model based on local flexibility [11–16]. The crack model based on local flexibility is established by the fracture mechanics theory and the introduced local flexibility. And it is used by many researchers. Zheng and Kessissoglou [11] used the Gaussian integral method to calculate the local flexibility coefficient caused by transverse cracks in a rectangular beam structure under pure bending moments. Chasalevris and Papadopoulos [12] studied the method of calculating the local flexibility of a circular cross-section axis with crack direction angle. Lee [13] established the finite element model of the multicrack cantilever beam structure based on the local flexibility method and discussed the identification method of the multicrack. Han and Chu [14] obtained the crack local flexibility when the elliptical crack circular axis was stretched and bent. Attar et al. [15] used the local flexibility method to analyze the free vibration characteristics of the cracked Timoshenko beam on an elastic foundation. Liu et al. [16] established a cantilever beam model with horizontally embedded cracks and analyzed the effect of crack local flexibility on the first three order natural frequencies and modes of the structure. Most of these studies have focused on the solution of crack local flexibility. However, due to the complexity of the structure, there is often a certain error between the calculated local flexibility of the crack and the actual value. Therefore, the study on the measurement method of the crack local flexibility is of great significance.

In this paper, the measurement method of the crack local flexibility is presented, which is suitable for the local flexibility measurement of crack with different types and shapes in the beam structure. The outline of this paper is as follows. Section 2 establishes and analyzes the dynamic model of cracked beam. In Section 3, by discussing the relationship between the natural frequency and the local flexibility of the crack, a method for measuring the crack local flexibility of the beam structure based on the natural frequency is proposed. In Section 4, an experiment is completed to verify the effectiveness of the method. The conclusion is summarized in Section 5.

2. Dynamic Model of Cracked Beam

As shown in Figure 1, the total length of the beam is L, the crack is located at l, and the depth is a. The transverse free vibration equation of the crack-free beam is

$$q^2\frac{\partial^4 w(z,t)}{\partial z^4}+\frac{\partial^2 w(z,t)}{\partial t^2}=0, \tag{1}$$

where $q=\sqrt{\mathrm{EI}/\rho A}$; $w(z,t)$ represents the deflection of the beam; EI represents the bending stiffness of the beam; ρ represents the material density; and A represents the cross-sectional area of the beam structure.

The transverse free vibration equation of the cracked beam is

$$\begin{aligned} &q^2\frac{\partial^4 w_1(z,t)}{\partial z^4}+\frac{\partial^2 w_1(z,t)}{\partial t^2}=0, \quad 0\le z\le l,\\ &q^2\frac{\partial^4 w_2(z,t)}{\partial z^4}+\frac{\partial^2 w_2(z,t)}{\partial t^2}=0, \quad l\le z\le L. \end{aligned} \tag{2}$$

When $z\in[0,l]$, $w_1(z,t)$ represents the deflection of the beam on the left side of the crack. When $z\in[l,L]$, $w_2(z,t)$ represents the deflection of the beam on the right side of the crack.

The solution of the transverse free vibration equation of the cracked beam is

$$\begin{aligned} &w_1(z,t)=W_1(z)\sin(\omega t+\varphi), \quad 0\le z\le l,\\ &w_2(z,t)=W_2(z)\sin(\omega t+\varphi), \quad 0\le z\le L, \end{aligned} \tag{3}$$

where $W_1(z)$ is the natural mode function of the beam on the left side of the crack, $W_2(z)$ is the natural mode function of the beam on the right side of the crack, ω is the natural frequency of the structure, and φ is the initial phase.

The boundary condition of the crack is [17]

$$\begin{cases} W_1(z)=W_2(z),\\ \dfrac{\mathrm{d}^2W_1(z)}{\mathrm{d}z^2}=\dfrac{\mathrm{d}^2W_2(z)}{\mathrm{d}z^2},\\ \dfrac{\mathrm{d}^3W_1(z)}{\mathrm{d}z^3}=\dfrac{\mathrm{d}^3W_2(z)}{\mathrm{d}z^3}, \quad z=l,\\ \dfrac{\mathrm{d}W_1(z)}{\mathrm{d}z}+cEI\dfrac{\mathrm{d}^2W_1(z)}{\mathrm{d}z^2}-\dfrac{\mathrm{d}W_2(z)}{\mathrm{d}z}=0, \end{cases} \tag{4}$$

where c is the crack local flexibility.

Assuming that the beam structure with transverse cracks is subjected to bending moment M at both ends and the crack cross section is shown in Figures 1(b) and 1(c), the strain energy of the crack structure is [18]

$$U=\iint_{\Omega} J\mathrm{d}y\mathrm{d}x, \tag{5}$$

where J refers to the strain energy density function.

$$J=\frac{1-\nu^2}{E}K_I^2, \tag{6}$$

where ν is Poisson's ratio; E is Young's modulus of elasticity; and K_I is the I-mode crack stress intensity factor [19].

For the rectangular beam crack cross section,

$$K_I=\sigma\sqrt{\pi y}\,F_2\left(\frac{y}{h}\right), \tag{7}$$

where $\sigma=(Mh/2I_x)$; I_x is the moment of inertia of the rectangular beam section about the x axis, that is,

$$\begin{aligned} &I_x=\frac{bh^3}{12},\\ &F_2\left(\frac{y}{h}\right)=\sqrt{\left(\frac{2h}{\pi y}\right)\tan\left(\frac{\pi y}{2h}\right)}\times\frac{\left\{0.923+0.199\left[1-\sin(\pi y/2h)\right]^4\right\}}{\cos(\pi y/2h)}. \end{aligned} \tag{8}$$

For the circular beam crack cross section,

$$K_I=\sigma\sqrt{\pi e}\,F_2\left(\frac{e}{h}\right), \tag{9}$$

where

$$\begin{aligned} &\sigma=\frac{4M}{\pi R^4}\sqrt{R^2-x^2},\\ &h=2\sqrt{R^2-x^2},\\ &e=\sqrt{R^2-x^2}-(R-a),\\ &F_2\left(\frac{e}{h}\right)=\sqrt{\left(\frac{2h}{\pi e}\right)\tan\left(\frac{\pi e}{2h}\right)}\times\frac{\left\{0.923+0.199\left[1-\sin(\pi e/2h)\right]^4\right\}}{\cos(\pi e/2h)}. \end{aligned} \tag{10}$$

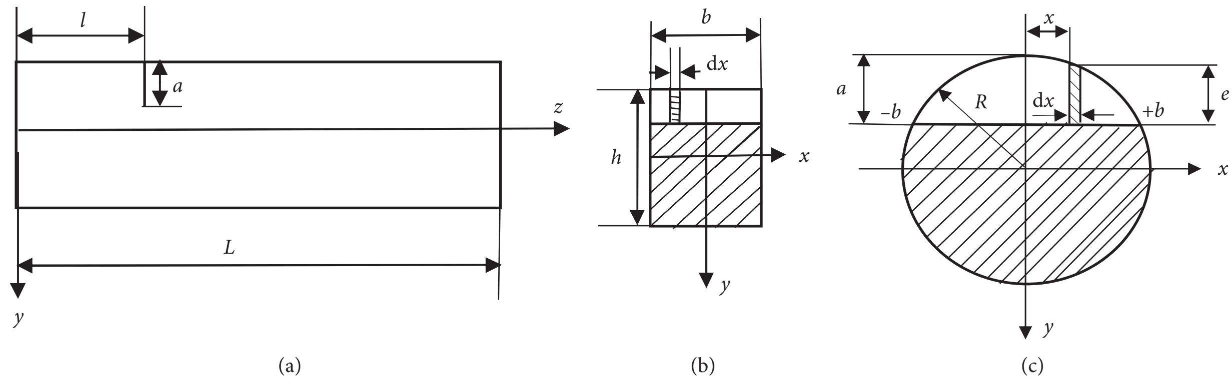

Figure 1: The model of the cracked beam. (a) Schematic diagram of cracked beam. (b) Rectangular beam crack cross section. (c) Circular beam crack cross section.

According to Karl's second theorem, the derivative of U for a certain load is equal to the corresponding displacement of the load, so

$$\delta = \frac{\partial U}{\partial M}, \tag{11}$$

where δ is the displacement corresponding to M.

The derivative of δ to M is the crack local flexibility c.

For the rectangular beam crack cross section, c is given as follows:

$$c = \frac{\partial^2 U}{\partial M^2} = \frac{72\pi}{E' b^2 h^4} \iint_{\Omega} y F_2^2 \frac{y}{h} \mathrm{d}y\, \mathrm{d}x. \tag{12}$$

For the circular beam crack cross section, c is given as follows:

$$c = \frac{\partial^2 U}{\partial M^2} = \frac{1-\nu^2}{E} \iint_{\Omega} \frac{32}{\pi^2 R^8} \left(R^2 - x^2\right) \pi e F_2^2 \left(\frac{e}{h}\right) \mathrm{d}y\, \mathrm{d}x. \tag{13}$$

3. Relationship between the Natural Frequency and the Crack Local Flexibility

As shown in Figure 2, in order to describe the change of the local flexibility of the structure caused by the crack, the crack is equivalent to a massless torsion wire spring, and the flexibility of the torsion wire spring can represent the crack local flexibility. According to the connection conditions at the crack, the deflections on both sides of the spring are equal, and the rotation angles can be connected through the stiffness matrix $\mathbf{K}_e$ of the crack element. When the finite element method is used to establish the model, the crack element mass matrix is zero, and the crack element stiffness matrix $\mathbf{K}_e$ can be obtained from the crack local flexibility [17]:

$$\mathbf{K}_e = \begin{bmatrix} \frac{1}{c}, & -\frac{1}{c}, \\ -\frac{1}{c}, & \frac{1}{c}. \end{bmatrix}. \tag{14}$$

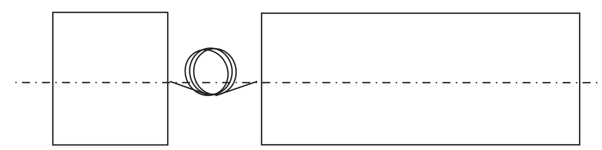

Figure 2: The model of the crack torsion line spring.

The element stiffness matrix, mass matrix, and crack element stiffness matrix of the crack-free part of the cracked beam are assembled to obtain the total stiffness matrix $\mathbf{K}$ and total mass matrix $\mathbf{M}$ of the system. The vibration characteristic equation of the beam is as follows:

$$\left|\mathbf{K}(c,\beta) - \omega_i^2 \mathbf{M}\right| = 0, \tag{15}$$

where ω_i is the natural frequency of the structure, c is the local flexibility of the crack, and β is the relative position of the crack.

Equation (15) can also be expressed in the following form:

$$\omega_i = F(c,\beta), \quad i = 1,2,3. \tag{16}$$

The above formula can be regarded as solving the natural frequency ω_i of the structure through the crack parameters c and β when the functional relationship F is known.

After obtaining the natural frequency of the structure corresponding to the local flexibility and location of different cracks, a relationship between the local flexibility and location of cracks can be solved according to the natural frequency of the beam. This relationship can be expressed as follows:

$$(c,\beta) = F^{-1}(\omega_i), \quad i = 1,2,3. \tag{17}$$

Therefore, under the condition that the functional relationship $F^{-1}(\omega_i)$ is known, the crack position and crack local flexibility corresponding to any one natural frequency value can be solved. Since a natural frequency value corresponds to multiple crack locations and crack local flexibilities, in order to accurately determine the crack location and the crack local flexibility, multiple natural frequencies of vibration are often used. Because the first few natural

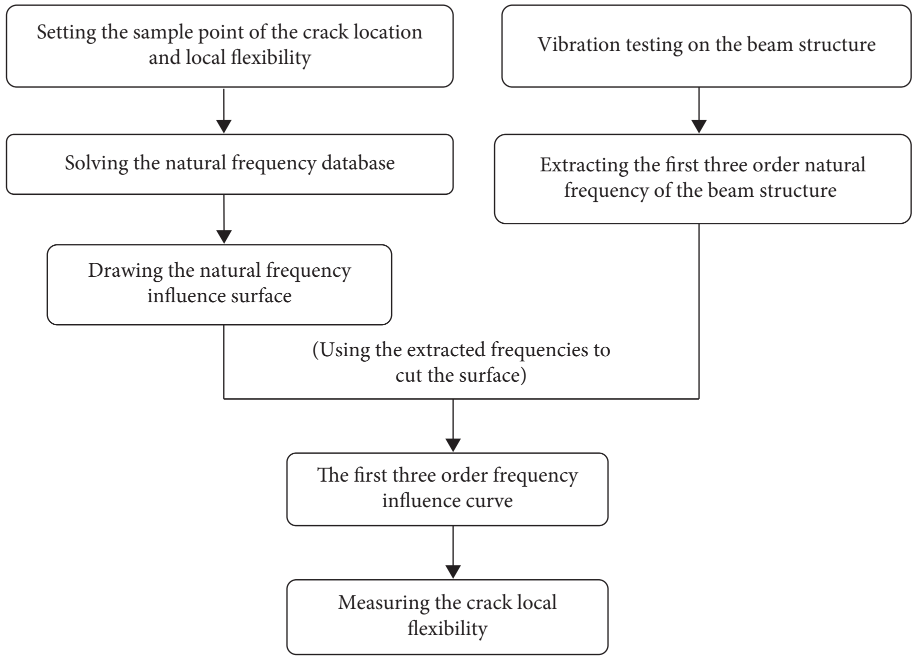

Figure 3: The flowchart of the method for measuring the crack local flexibility of the beam structure.

frequencies of the structure are easy to measure and have high accuracy in actual engineering, the first three order natural frequencies of the structure are usually selected.

According to equations ((15)–(17)), a method for measuring the crack local flexibility of the beam structure based on natural frequencies can be established. The flowchart of the method is shown in Figure 3. The steps are as follows:

(1) Select a series of sample points in the possible value range of the crack location and local flexibility.

(2) The sample points obtained in step (1) are used as input parameters of equation (15) to solve the natural frequency database corresponding to different crack positions and local flexibilities.

(3) According to the natural frequency database, the surface-fitting technique is employed to draw the natural frequencies influence surfaces.

(4) Perform vibration testing on the beam structure to obtain the first three order natural frequencies.

(5) Use the obtained first three order natural frequencies of the structure to cut the natural frequency influence surface and draw the first three order frequency influence curves corresponding to the crack local flexibility and the crack position. The intersection of the three influence curves (or the centroid of the triangle formed by the three intersections) can indicate the crack local flexibility and the corresponding crack location.

From the measurement process, it can be seen that the input parameters which are used to solve the natural frequency database are a series of sample points of the crack location and local flexibility. These input parameters have no relation with the type of crack, the shape of crack, and the calculation formula of the crack local flexibility. This method has the following characteristics:

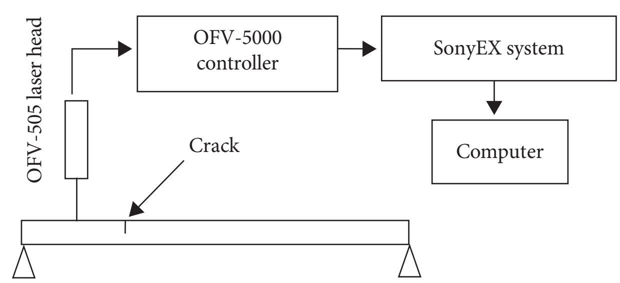

Figure 4: Measuring principle chart.

Table 1: The crack condition.

Condition	β	α	c (10^{-5} (rad/Nm^{-1}))
I	0.2	0.25	0.84
II	0.2	0.35	1.74
III	0.3	0.25	0.84
IV	0.3	0.35	1.74

(1) When solving the natural frequencies of the structure, it is not necessary to calculate the crack local flexibility. The crack local flexibility can be obtained by selecting a series of sample points within the possible value range of the local flexibility. Therefore, this method does not need to know the calculation formula of the crack local flexibility, and the formula is often difficult to be obtained.

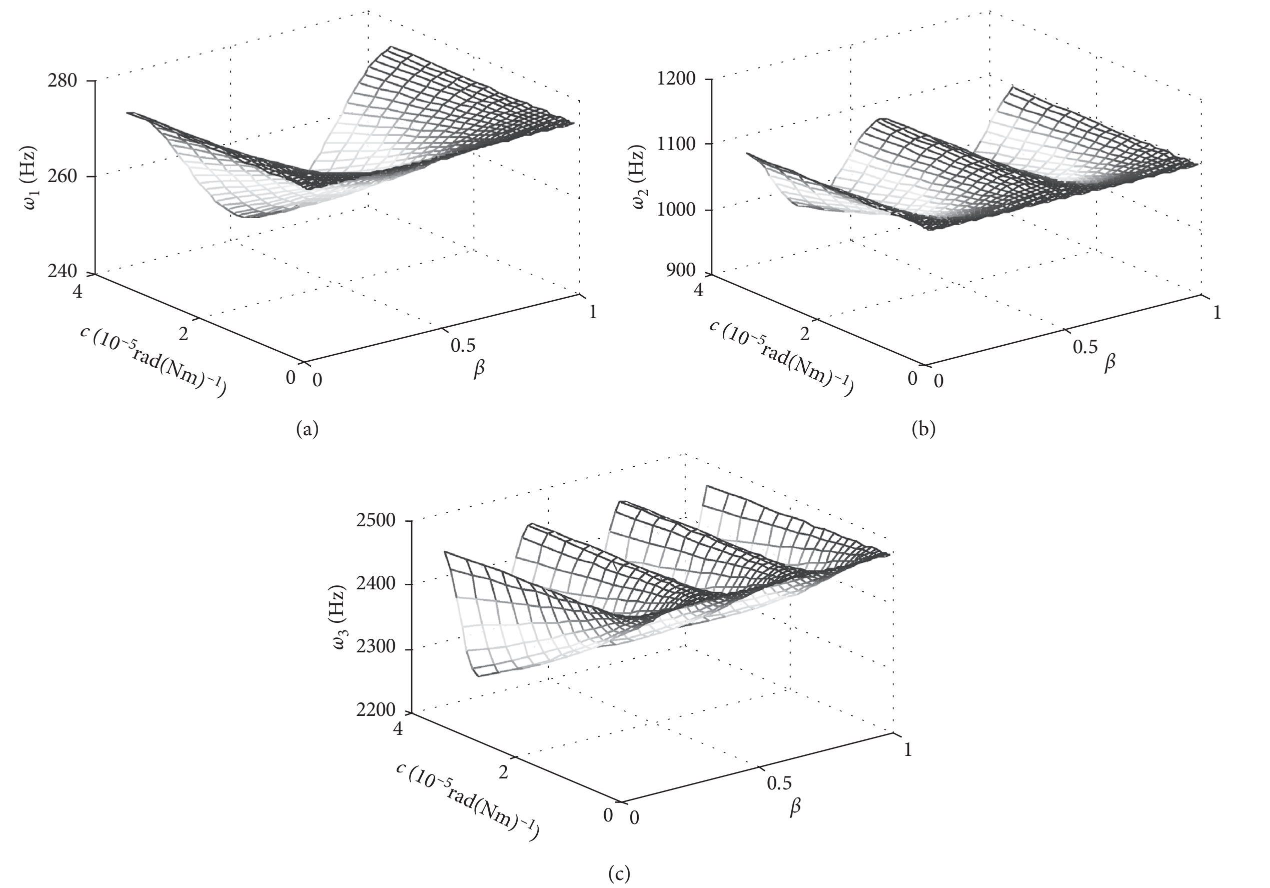

FIGURE 5: The first three order natural frequency influence surface. (a) The first order natural frequency affects the surface. (b) The second order natural frequency affects the surface. (c) The third order natural frequency affects the surface.

(2) It can be used to measure the crack local flexibility in beam structures with various cross sections, such as rectangular beam, circular beam, and hollow beam.

(3) It is suitable for measuring the local flexibility of crack with different shapes and types in the beam structure.

4. Experimental Study

In order to verify the effectiveness of the algorithm, the crack in the rectangular beam is taken as an example, and a test bed for measuring the local flexibility of crack is constructed. And the measurement principle is shown in Figure 4. The beam is simply supported at both ends, with length $L = 0.4$ m, width $b = 0.012$ m, and height $h = 0.02$ m. The crack is cut at location l, and the depth is a. The relative position is $\beta = l/L$, the relative depth is $\alpha = a/h$, Young's elasticity modulus $E = 1.7381 \times 10^{11}$ N/m^2, Poisson's ratio $\nu = 0.3$, and material density $\rho = 7348.9$ kg/m^3. The crack is processed by the wire-cutting machine. The cutting width is 0.02 mm. The crack type and shape are shown in Figure 1(b).

The crack conditions are shown in Table 1. c is the calculated local flexibility of the crack according to equation (12).

According to steps (1) ~ (3) of crack local flexibility measurement method in Section 3, the first three order natural frequency influence surfaces are drawn, as shown in Figure 5. When using equation (15) to solve the natural frequency database corresponding to different crack locations and local flexibilities, the numerical model needs to be revised because of inconsistency between the numerical model and the actual structure, such as damping and support conditions. Considering the influence of the error between the numerical model and the actual structure on different models of the structure, a model correction coefficient $k_i, i = 1, 2, 3$, is used to modify the first three models of the numerical model. Equations (16) and (17) establish the relationship between the natural frequency of the structure and the local flexibility and location of the crack, and the corrected result is as follows:

$$\begin{aligned} \omega_i' &= k_i F(c, \beta), \quad i = 1, 2, 3, \\ (c, \beta) &= (k_i F)^{-1} (\omega_i'), \quad i = 1, 2, 3, \end{aligned} \tag{18}$$

where ω_i' is the ith natural frequency calculated by the correction model and k_i is the ith modal correction coefficient, which can be determined by comparing the measured value of the natural frequency with the calculated value using the numerical model.

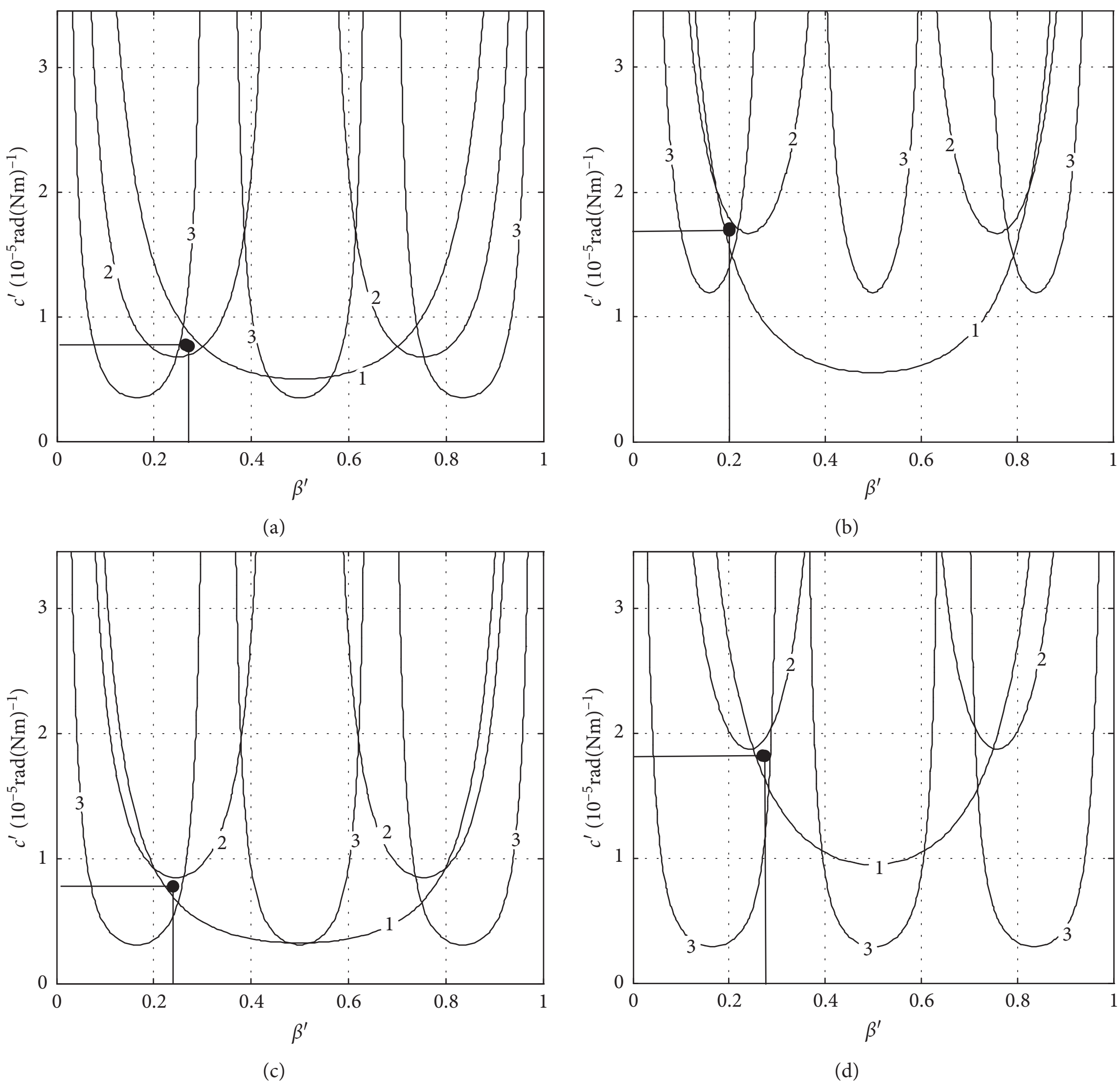

Figure 6: Measurement results of the local flexibility of crack. (a) Condition I (b) Condition II. (c) Condition III. (d) Condition IV. 1: the first order natural frequency influence curve. 2: the second order natural frequency influence curve. 3: the third order natural frequency influence curve.

Vibration testing of beam structure is performed, and the first three order natural frequencies are obtained. The measured first three order natural frequencies are adopted to cut the natural frequency influence surfaces, and the first three order natural frequency influence curves are obtained. The crack local flexibility and the corresponding crack location can be indicated by the intersection points of these frequencies influence curves. The measurement results are shown in Figure 6 and Table 2. ε_β and ε_c are the measurement errors of crack position and local flexibility.

$$\varepsilon_\beta = \left|\beta' - \beta\right| \times 100\%,$$
$$\varepsilon_c = \frac{\left|c' - c\right|}{c} \times 100\%, \qquad (19)$$

where β' is the measured value of crack relative position and c' is the measured value of crack local flexibility.

It can be seen from Figure 6 and Table 2 that the relative position error of the crack does not exceed 7.0%, and the error of the local flexibility of the crack does not exceed 8.2%. So, this method can effectively measure the local flexibility of the crack and its corresponding crack position. The experimental results verify the feasibility and correctness of the method. In the experiment, the calculated value of the crack local flexibility is used to estimate the measurement accuracy. The stress intensity factor of the crack in the rectangular section is used to solve the calculated value of the crack local flexibility. When the measurement accuracy is estimated in the measurement of the local flexibility of other types of cracks, the stress intensity factor of the corresponding type of crack needs to be obtained to calculate the crack local flexibility.

Table 2: Measurement results of the local flexibility of crack.

Condition	ω_1 (Hz)	ω_2 (Hz)	ω_3 (Hz)	β'	c' (10^{-5} (rad/Nm^{-1}))	Relative error %	
						ε_β	ε_c
I	271.0	1077.9	2452.1	0.27	0.77	7.0	8.2
II	270.5	1046.0	2389.6	0.19	1.76	1.0	1.1
III	272.6	1072.1	2455.0	0.24	0.79	6.0	6.0
IV	267.0	1040.0	2456.5	0.27	1.85	3.0	6.3

5. Conclusion

The crack local flexibility is an important parameter to describe the damage degree of the beam structure. This paper discusses a method of measuring local flexibility of crack based on natural frequency by combining structural dynamics analysis and vibration testing. This method is easy to operate and can be used to measure the local flexibility of crack with different shapes and types in the beam structure which have various cross sections. It has an important reference value for the dynamic analysis and damage identification of cracked beam structure.

Data Availability

The raw/processed data required to reproduce these findings cannot be shared at this time due to technical or time limitations.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The authors acknowledge the financial support from National Natural Science Foundation of China (nos. 51075314 and 51175399) and Natural Science Foundation of Shaanxi Province (no. 2014JM7269).

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